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1 Measuring Instruments

1.1 Introduction

The measurement of a given quantity is the result of comparison between the quantity to be measured and a definite standard. The instruments which are used for such measurements are called **measuring instruments**. The three basic quantities in the electrical measurement are current, voltage and power. The measurement of these quantities is important as it is used for obtaining measurement of some other quantity or used to test the performance of some electronic circuits or components etc.

The **necessary requirements** for any measuring instruments are :

1) With the introduction of the instrument in the circuit, the circuit conditions should not be altered. Thus the quantity to be measured should not get affected due to the instrument used.
2) The power consumed by the instruments for their operation should be as small as possible.

The instrument which measures the current flowing in the circuit is called **ammeter** while the instrument which measures the voltage across any two points of a circuit is called **voltmeter**. But there is no fundamental difference in the operating principle of analog voltmeter and ammeter. The action of almost all the analog ammeters and voltmeters depends on the deflecting torque produced by an electric current. In ammeters such a torque is proportional to the current to be measured. In voltmeters this torque is decided by a current which is proportional to the voltage to be measured. Thus all the analog ammeters and voltmeters are basically current measuring devices. The instruments which are used to measure the power are called **power meters or wattmeters.**

1.2 Classification of Measuring Instruments

Electrical measuring instruments are mainly classified as:

a) Indicating instruments b) Recording instruments c) Integrating instruments

a) Indicating instruments : These instruments make use of a dial and pointer for showing or indicating magnitude of unknown quantity. The examples are ammeters, voltmeter etc.

b) Recording instruments : These instruments give a continuous record of the given electrical quantity which is being measured over a specific period.

The examples are various types of recorders. In such recording instruments, the readings are recorded by drawing the graph. The pointer of such instruments is provided with a marker i.e. pen or pencil, which moves on graph paper as per the reading. The X-Y plotter is the best example of such an instrument.

c) Integrating instruments : These instruments measure the total quantity of electricity delivered over period of time. For example a household energymeter registers number of revolutions made by the disc to give the total energy delivered, with the help of counting mechanism consisting of dials and pointers.

1.3 Essential Requirements of an Instrument

In case of measuring instruments, the effect of unknown quantity is converted into a mechanical force which is transmitted to the pointer which moves over a calibrated scale. The moving system of such instrument is mounted on a pivoted spindle. For satisfactory operation of any **indicating instrument,** following systems must be present in an instrument.

1) Deflecting system producing deflecting torque T_d

2) Controlling system producing controlling torque T_c

3) Damping system producing damping torque.

Let us see the various ways in which these torques are obtained in an indicating instrument.

1.4 Deflecting System

In most of the indicating instruments the mechanical force proportional to the quantity to be measured is generated. This force or torque deflects the pointer. The system which produces such a deflecting torque is called **deflecting system** and the torque is denoted as T_d The deflecting torque overcomes,

1) The inertia of the moving system

2) The controlling torque provided by controlling system

3) The damping torque provided by damping system.

The deflecting system uses one of the following effects produced by current or voltage, to produce deflecting torque.

1) Magnetic Effect : When a current carrying conductor is placed in uniform magnetic field, it experiences a force which causes to move it. This effect is mostly used in many instruments like moving iron attraction and repulsion type, permanent magnet moving coil instruments etc.

2) Thermal Effect : The current to be measured is passed through a small element which heats it to cause rise in temperature which is converted to an e.m.f. by a thermocouple attached to it.

When two dissimilar metals are connected end to end to form a closed loop and the two junctions formed are maintained at different temperatures, then e.m.f. is induced which causes the flow of current through the closed circuit which is called a **thermocouple.**

3) Electrostatic Effects : When two plates are charged, there is a force exerted between them, which moves one of the plates. This effect is used in electrostatic instruments which are normally voltmeters.

4) Induction Effects : When a non-magnetic conducting disc is placed in a magnetic field produced by electromagnets which are excited by alternating currents, an e.m.f. is induced in it.

If a closed path is provided, there is a flow of current in the disc. The interaction between induced currents and the alternating magnetic fields exerts a force on the disc which causes to move it. This interaction is called an **induction effect**. This principle is mainly used in energymeters.

5) Hall Effect : If a bar of semiconducting material is placed in uniform magnetic field and if the bar carries current, then an e.m.f. is produced between two edges of conductor. The magnitude of this e.m.f. depends on flux density of magnetic field, current passing through the conducting bar and hall effect co-efficient which is constant for a given semiconductor. This effect is mainly used in flux-meters.

Thus the deflecting system provides the deflecting torque or operating torque for movement of pointer from its zero position. It acts as the prime mover for the deflection of pointer.

1.5 Controlling System

This system should provide a force so that current or any other electrical quantity will produce deflection of the pointer proportional to its magnitude. The important functions of this system are,

1) It produces a force equal and opposite to the deflecting force in order to make the deflection of pointer at a definite magnitude. If this system is absent, then the pointer will swing beyond its final steady position for the given magnitude and deflection will become indefinite.
2) It brings the moving system back to zero position when the force which causes the movement of the moving system is removed. It will never come back to its zero position in the absence of controlling system.

Controlling torque is generally provided by springs. Sometimes gravity control is also used.

1.5.1 Gravity Control

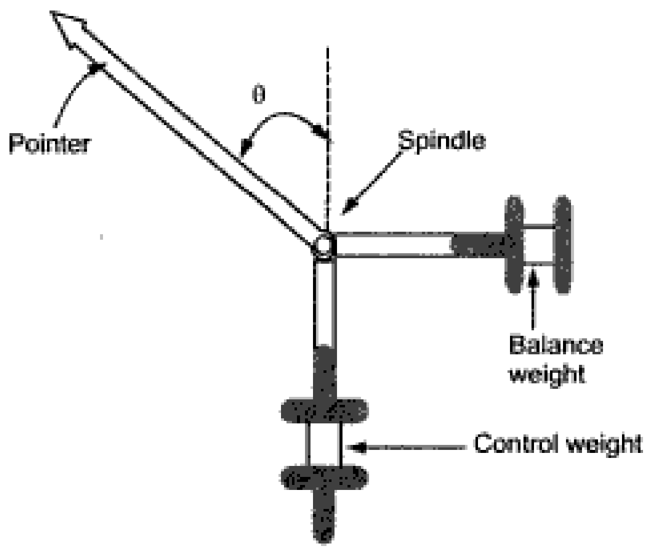


Fig. 1.1 Gravity control

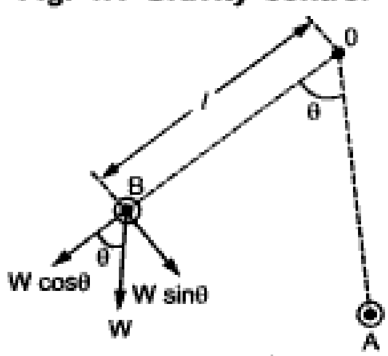


Fig. 1.2

This type of control consists of a small weight attached to the moving system whose position is adjustable. This weight produces a controlling torque due to gravity. This weight is called **control weight**.

The Fig. 1.1 shows the gravity control system. At the zero position of the pointer, the controlling torque is zero. This position is shown as position A of the weight in the Fig. 1.2. If the system deflects, the weight position also changes, as shown in the Fig. 1.2.

The system deflects through an angle θ. The control weight acts at a distance l from the center. The component W sin θ of this weight tries to restore the pointer back to the zero position. This is nothing but the controlling torque T_c.

Thus,

controlling torque $\boxed{T_c = W \sin\theta \times l}$

$= K \sin\theta$

here $K = Wl$

$=$ gravity constant

Now generally all meters are current sensing meters where,

Deflecting torque $\boxed{T_d = K_t I}$

where $K_t =$ another constant.

In equilibrium position, $\boxed{T_d = T_c}$

$\therefore \quad K_t I = K \sin\theta$

$\therefore \quad \boxed{I \propto \sin\theta}$

Thus the deflection is proportional to current i.e. quantity to be measured.

Key Point: *But as it is a function of sin θ, the **scale for the instrument using gravity control is not uniform.***

Its **advantages** are :

1) Its performance is not time dependent.

2) It is simple and cheap.

3) The controlling torque can be varied by adjusting the position of the control weight.

4) Its performance is not temperature dependent.

Its **disadvantages** are :

1) The scale is nonuniform causing problems to record accurate readings.

2) The system must be used in vertical position only and must be properly levelled. Otherwise it may cause serious errors in the measurement.

3) As delicate and proper levelling required, in general it is not used for indicating instruments and portable instruments.

1.5.2 Spring Control

Two hair springs are attached to the moving system which exerts controlling torque. To employ spring control to an instrument, following requirements are essential.

1) The spring should be non-magnetic.

2) The spring should be free from mechanical stress.

3) The spring should have a small resistance, sufficient cross-sectional area.

4) It should have low resistance temperature co-efficient.

The arrangement of the springs is shown in the Fig. 1.3.

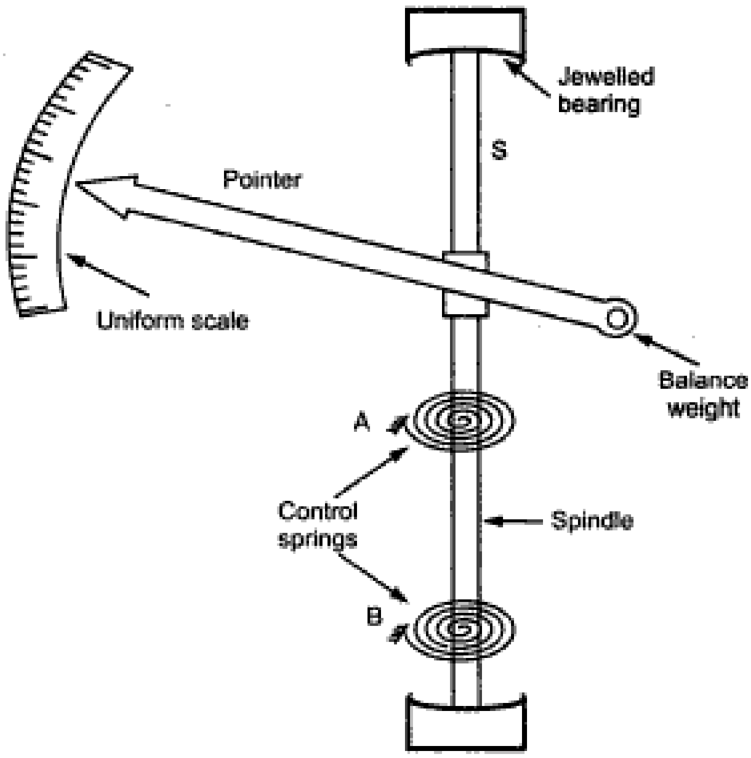


Fig. 1.3 Spring control

The springs are made up of non-magnetic materials like silicon bronze, hard rolled silver or copper, platinum silver and german silver. For most of the instruments, phosphor bronze spiral springs are provided. Flat spiral springs are used in almost all indicating instruments.

The inner end of the spring is attached to the spindle while the outer end is attached to a lever or arm which is actuated by a set of screw mounted at the front of the instrument. So zero setting can be easily done. The controlling torque provided by the instrument is directly proportional to the angular deflection of the pointer.

The controlling torque produced by spiral spring is given by,

$$T_c = \frac{E\,b\,t^3}{12\,L}\,\theta = K_s\theta$$

where E = Young's modulus of spring material in N/m^2

t = thickness in metres

b = depth in metres

L = length in metres

K_s = spring constant = $\frac{Ebt^3}{12L}$

$$\therefore \quad T_c \propto \theta$$

Now deflecting torque is proportional to current.

$$\therefore \quad T_d \propto I$$

At equilibrium, $T_d = T_c$

$$\therefore \quad I \propto \theta$$

Key Point: *Thus the deflection is proportional to the current. Hence the* ***scale of the instrument using spring control is uniform.***

When the current is removed, due to spring force the pointer comes back to initial position. The spring control is very popular and is used in almost all indicating instruments.

1.5.3 Comparison of Controlling Systems

	Gravity Control	Spring Control
1.	Adjustable small weight is used which produces the controlling torque.	Two hair springs are used which exert controlling torque.
2.	Controlling torque can be varied.	Controlling torque is fixed.
3.	The performance is not temperature dependent.	The performance is temperature dependent.
4.	The scale is nonuniform.	The scale is uniform.
5.	The controlling torque is proportional to sin θ.	The controlling torque is proportional to θ.

6.	The readings can not be taken accurately.	The readings can be taken very accurately.
7.	The system must be used in vertical position only.	The system need not be necessarily in vertical position.
8.	Proper levelling is required as gravity control.	The levelling is not required.
9.	Simple, cheap but delicate.	Simple, rigid but costlier compared to gravity control.
10.	Rarely used for indicating and portable instruments.	Very popularly used in most of the instruments.

Table 1.1

1.6 Damping System

The deflecting torque provides some deflection and controlling torque acts in the opposite direction to that of deflecting torque. So before coming to the rest, pointer always oscillates due to inertia, about the equilibrium position.Unless pointer rests, final reading can not be obtained. So to bring the pointer to rest within short time, damping system is required. The system should provide a damping torque only when the moving system is in motion. Damping torque is proportional to velocity of the moving system but it does not depend on operating current. It must not affect controlling torque or increase the friction.

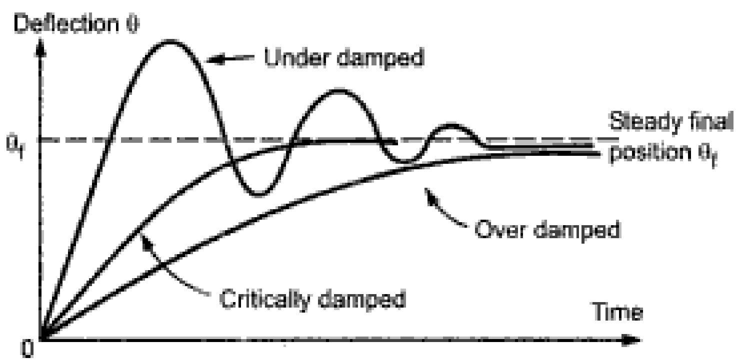


Fig. 1.4

The quickness with which the moving system settles to the final steady position depends on relative damping. If the moving system reaches to its final position rapidly but smoothly without oscillations, **the instrument is said to be critically damped.** If the instrument is **under damped,** the moving system will oscillate about the final steady position with a decreasing amplitude and will take sometime to come to rest. While the instrument is said to be **over damped** if the moving system moves slowly to its final steady position. In over damped case the response of the system is very slow and sluggish. In practice slightly under damped systems are preferred. The time response of damping system for various types of damping conditions is shown in the Fig. 1.4.

The following methods are used to produce damping torque.

1) Air friction damping 2) Fluid friction damping 3) Eddy current damping.

1.6.1 Air Friction Damping

This arrangement consists of a light aluminium piston which is attached to the moving system, as shown in the Fig. 1.5.

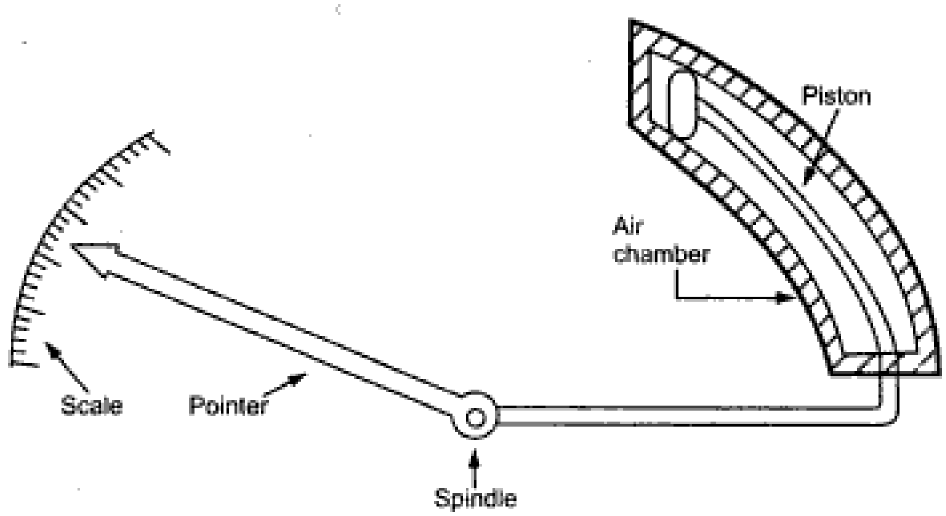


Fig. 1.5 Air friction damping

The piston moves in a fixed air chamber. It is close to one end. The clearance between piston and wall chambers is uniform and small. The piston reciprocates in the chamber when there are oscillations. When piston moves into the chamber, air inside is compressed and pressure of air developed due to friction opposes the motion of pointer. There is also opposition to motion of moving system when piston moves out of the chamber. Thus the oscillations and the overshoot gets reduced due to to and fro motion of the piston in the chamber, providing necessary damping torque. This helps in settling down the pointer to its final steady position very quickly.

1.6.2 Fluid Friction Damping

Fluid friction damping may be used in some instruments. The method is similar to air friction damping, only air is replaced by working fluid. The friction between the disc and fluid is used for opposing motion. Damping force due to fluid is greater than that of air due to more viscosity. The disc is also called vane.

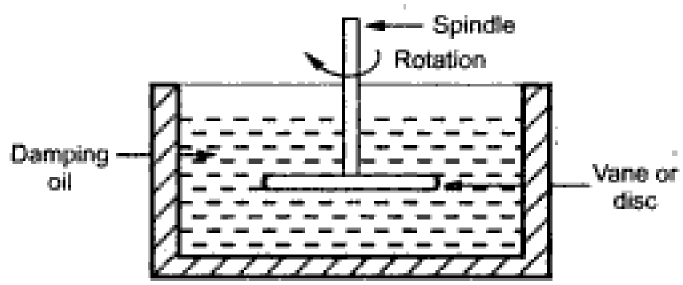


Fig. 1.6 Fluid friction damping

The arrangement is shown in the Fig. 1.6. It consists of a vane attached to the spindle which is completely dipped in the oil. The frictional force between oil and the vane is used to produce the damping torque, which opposes the oscillating behaviour of the pointer.

The advantages of this method are :

1) Due to more viscosity of fluid, more damping is provided.

2) The oil can also be used for insulation purposes.

3) Due to up thrust of oil, the load on the bearings is reduced, thus reducing the frictional errors.

The disadvantages of this method are :

1) This can be only used for the instruments which are in vertical position.

2) Due to oil leakage, the instruments can not be kept clean.

1.6.3 Eddy Current Damping

This is the most effective way of providing damping. It is based on the **Faraday's law** and **Lenz's law**. When a conductor moves in a magnetic field cutting the flux, e.m.f. gets induced in it. And direction of this e.m.f. is so as to oppose the cause producing it.

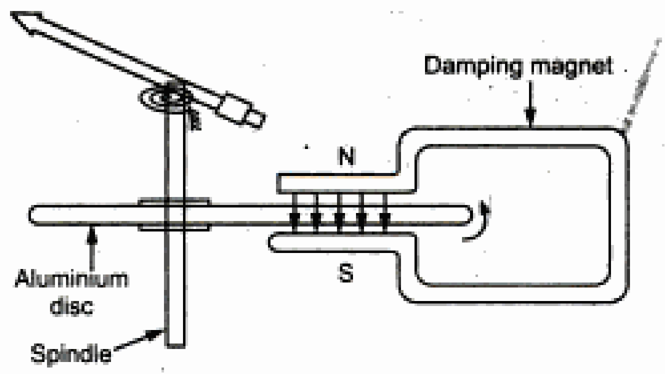


Fig. 1.7 Eddy current damping

In this method, an aluminium disc is connected to the spindle. The arrangement of disc is such that when it rotates, it cuts the magnetic flux lines of a permanent magnet. The arrangement is shown in the Fig. 1.7.

When the pointer oscillates, aluminium disc rotates under the influence of magnetic field of damping magnet. So disc cuts the flux which causes an induced e.m.f. in the disc. The disc is a closed path hence induced e.m.f.circulates current through the disc called **eddy current**. The direction of such eddy current is so as oppose the cause producing it. The cause is relative motion between disc and field. Thus it produces an opposing torque so as to reduce the oscillations of pointer. This brings pointer to rest quickly. This is most effective and efficient method of damping.

1.7 D'Arsonval Galvanometer

The use of D'Arsonval galvanometer is very common in variety of measuring instruments. The galvanometer is basically used in an instrument for detecting the presence of small voltages or currents in a circuit or to indicate zero current in applications like bridge circuits. Thus galvanometer has to be very much sensitive.

1.7.1 Construction

The construction of D'Arsonval galvanometer is shown in the Fig. 1.8.

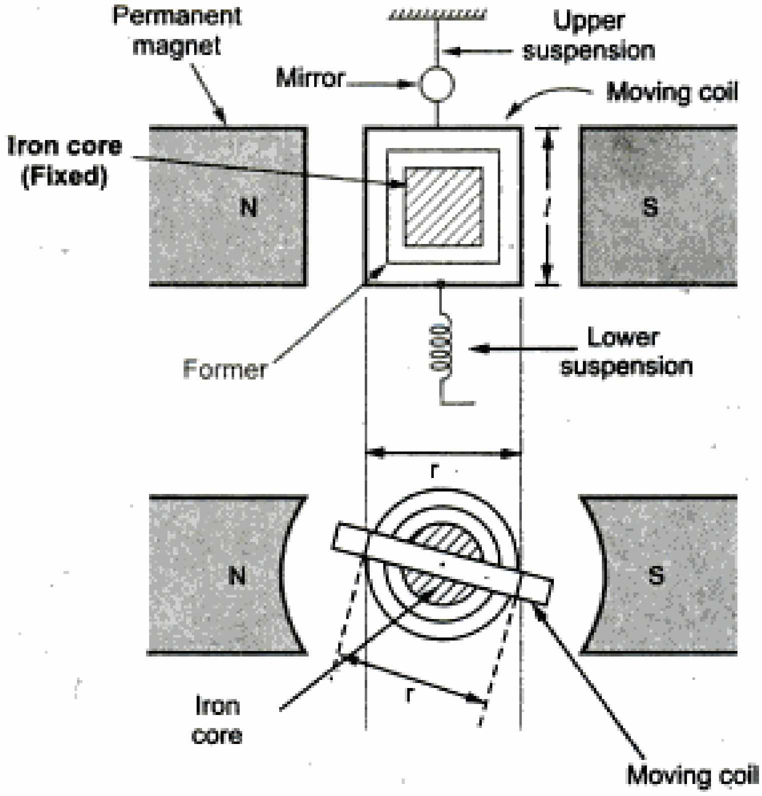


Fig. 1.8 Construction of D'Arsonval galvanometer

It consists of the following parts,

1. Moving coil : The moving coil is rectangular or circular in cross-section, carrying number of turns of fine wire. It carries the current proportional to the quantity to be measured. It is suspended in the air gap between the poles of a permanent magnet and iron core. It is free to turn about its vertical axis. The pole faces are of particular shape such that the magnetic field is radial.

2. Iron core : It is spherical if coil is circular and cylindrical if coil is rectangular. It is basically used to provide low reluctance path to the magnetic flux and to produce strong magnetic field. This ensures higher deflecting torque and better sensitivity of the galvanometer. The air gap is about 1/16 inches i.e. about 1.5 mm. If small moment of inertia is necessary, the iron core can be omitted but it decreases the sensitivity.

3. Suspension : The suspension is a single fine strip of phosphor-bronze and serves as one lead of the coil. The other lead takes the form of a loosely coiled spiral of fine wire leading downwards from the bottom of the coil. This is lower suspension. This type of galvanometer requires a perfect levelling so that the suspension coils remain straight and in central position without rubbing the poles or iron core. In galvanometers which do not require the perfect levelling, taut suspensions with straight flat strips are used, which are kept under tension from both sides.

4. Damping : The damping is eddy current damping. The eddy currents developed in the metal former on which coil is mounted, are responsible to produce damping torque. For effective damping a low resistance is connected across the galvanometer terminals. By adjusting the value of this resistance damping can be changed and critical damping can be achieved.

5. Indication : The suspension carries a small mirror upon which a beam of light is cast through a glass window in the outer brass case surrounding the instrument. The beam of light is reflected on the scale. The scale is **usually 1m** away from the mirror.

6. Zero adjustment : A torsion head is provided for the adjustment of the coil position and zero setting.

1.7.2 Torque Equation

The various parameters involved in torque equation are,

l = Length of coil measured along vertical axis in m.

r = Width of coil in m.

N = Number of turns of coil.

B = Flux density in air gap in Wb/m^2 or Tesla.

i = Current through coil in A.

K = Spring constant or restoring constant in Nm/rad.

α = Angle between plane of coil and direction of magnetic field.

A = Area of coil in $m^2 = l \times r$.

θ_f = Final steady state deflection of coil in rad.

$$F = \text{Force on each side of a coil} = N\,B\,i\,l \sin\alpha \text{ N} \quad \text{... (1)}$$

$$T_d = \text{Deflecting torque} = F \times d = N\,B\,i\,l \sin\alpha\, r \quad \text{... (2)}$$

$$\therefore \quad T_d = N\,B\,i\,A \sin\alpha$$

As the field is radial in nature, $\alpha = 90°$ hence $\sin\alpha = 1$.

$$\therefore \quad T_d = N\,B\,i\,A = Gi = G\,i \text{ Nm} \quad \text{... (3)}$$

where $G = N\,B\,A$ = Galvanometer constant

The restoring torque provided by the spring is directly proportional to the final deflection of the coil.

$$\therefore \quad T_c = K\theta_f \qquad \text{... (4)}$$

For final steady state position of coil,

$$T_d = T_c$$

$$\therefore \quad Gi = K\theta_f$$

$$\therefore \quad \boxed{\theta_f = \frac{Gi}{K}} \qquad \text{... (5)}$$

The scale is calibrated in mm. The scale is at a distance of 1 m from the mirror as shown in the Fig. 1.9.

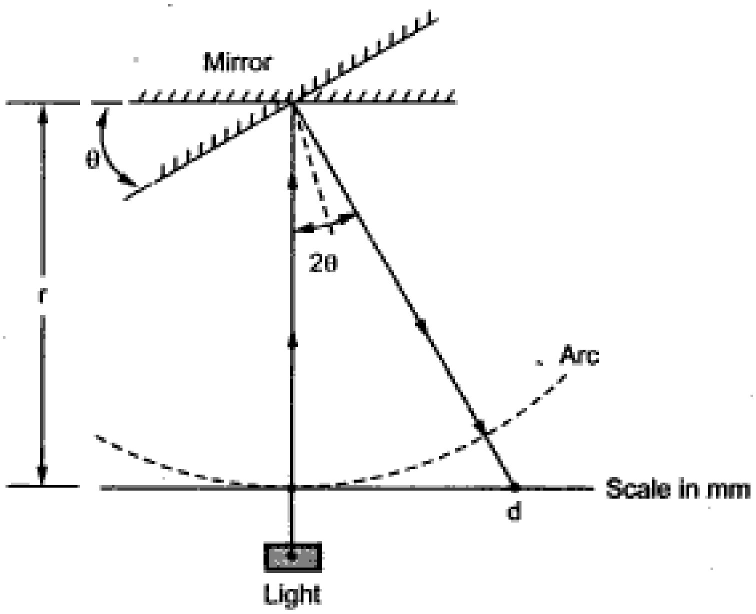


Fig. 1.9 Measurement of deflection in mm

For small deflection, the radius of arc and angle of turning, decide the deflection. The angle through which the beam gets reflected is $2\theta_f$ if mirror is turned through θ_f.

$$\therefore \quad \text{d in mm} = 2\theta_f \times r$$

$$\therefore \quad \boxed{d = \frac{2\,G\,i\,r}{K}\ \text{mm}} \qquad \text{... (6)}$$

Key Point : *Usually r = 1 m = 1000 mm for the galvanometer.*

1.7.3 Intrinsic Constants of Galvanometer

The various intrinsic constants of galvanometer are,

1. Displacement constant (G) : The constant G defined in torque equation of a galvanometer is called displacement constant.

$$\boxed{G = NBA = NBl \times r \ \ Nm/A} \quad \text{... (7)}$$

2. Constant of inertia (J) : The inertia of the system opposes the motion. Thus inertia produces a retarding torque given by,

$$\boxed{T_i = J\frac{d^2\theta}{dt^2}} \quad \text{... (8)}$$

where J = Constant of inertia about axis of rotation in kg-m²

$\frac{d^2\theta}{dt^2}$ = Angular acceleration

θ = Deflection at any time t

3. Damping constant : Another torque retarding the motion is friction in air and elastic hysteresis in the suspension. It is assumed to be proportional to the angular velocity of the moving system.

$$\therefore \quad \boxed{T_D = D\frac{d\theta}{dt}} \quad \text{... (9)}$$

where D = Damping constant in Nm/rad s^{-1}

4. Control constant : The elasticity of the suspension is proportional to the displacement which produces controlling torque. This is required to bring the moving system back to the original position.

$$\boxed{T_c = K\theta} \quad \text{... (10)}$$

where K = control constant or restoring constant in Nm/rad

This constant is also called stiffness constant.

1.7.4 Dynamic Behaviour of Galvanometer

The dynamic behaviour of galvanometer is analysed through its equation of motion.

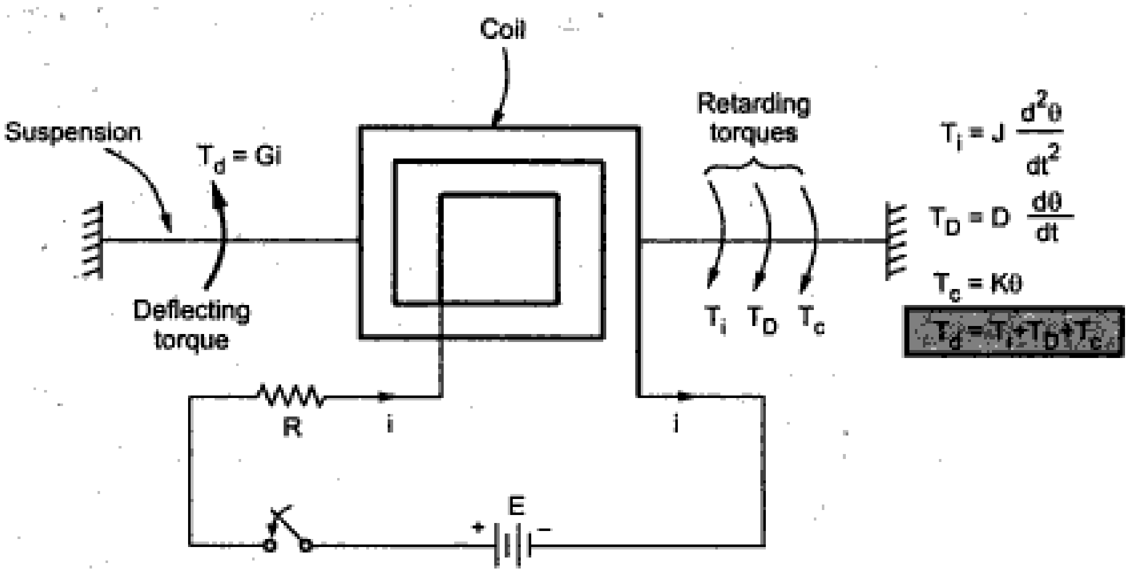


Fig. 1.10 Torques acting in galvanometer motion

Thus $J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta = G\,i$ ... (11)

This is second order differential equation governing the galvanometer motion. The solution of this equation has two parts, 1) Complementary function, 2) Particular integral.

The **complementary function** (C.F.) represents the **transient** behaviour while **particular integral** (P.I.) represents the **steady state** condition i.e. final deflection of the moving system.

The behaviour of system before it achieves the steady state is transient behaviour. When transient behaviour dies out, the system achieves final steady state position.

The auxiliary equation of above differential equation is obtained as,

$$Jm^2 + Dm + K = 0 \qquad \text{... (12)}$$

The roots of this equation are,

$$m_1 = \frac{-D + \sqrt{D^2 - 4JK}}{2J}, \quad m_2 = \frac{-D - \sqrt{D^2 - 4JK}}{2J}$$

Hence the solution has two exponential terms of powers m_1 and m_2

$$\therefore \quad \theta = A\,e^{m_1 t} + B\,e^{m_2 t} \quad \text{(C.F.)} \qquad \text{... (13)}$$

where A, B = constants

Now when steady state is reached then $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ are zero as moving system attains a final steady state position of θ_f. Using in equation (11) we get P.I. as,

$$K\theta_f = G\,i \quad \text{i.e.} \quad \theta_f = \frac{G\,i}{K} \qquad \text{... as derived earlier}$$

Thus the complete solution is the addition of C.F. and P.I.

$$\therefore \quad \theta = \underbrace{A\,e^{m_1 t} + B\,e^{m_2 t}}_{\text{Transient term}} + \underbrace{\theta_f}_{\text{Steady state}} \qquad \text{... (14)}$$

Now the transient terms may be purely exponential or oscillatory which depends on the nature of roots m_1 and m_2. This defines the various damping conditions of the system.

1.7.4.1 Underdamped Motion

Both m_1 and m_2 are complex conjugates of each other having negative real part.

$$\therefore \quad D^2 - 4JK < 0 \qquad \text{... Underdamped}$$

Key Point: *The transient behaviour is damped oscillations i.e. oscillations of decreasing amplitude. After sometime amplitude becomes zero and system achieves steady state.*

The roots are imaginary and complex conjugates of each other represented as,

$$m_1, m_2 = \frac{-D \pm \sqrt{(-1)^2\left[4JK - D^2\right]}}{2J} \quad \text{but} \quad \sqrt{-1} = j$$

$$\therefore \quad m_1, m_2 = -\frac{D}{2J} \pm j\sqrt{4JK - D^2} = -\alpha \pm j\omega_d \qquad \text{... (15)}$$

where $\alpha = \frac{D}{2J}$ and $\omega_d = \frac{\sqrt{4JK - D^2}}{2J}$

Thus the solution becomes,

$$\theta = A\,e^{(-\alpha + j\omega_d)t} + B\,e^{(-\alpha - j\omega_d t)} + \theta_f \qquad \text{...(16)}$$

$$\therefore \quad \theta = e^{-\alpha t}\left[A\,e^{+j\omega_d t} + B\,e^{-j\omega_d t}\right] + \theta_f$$

But, $e^{+j\theta} = \cos\theta + j\sin\theta$, $e^{-j\theta} = \cos\theta - j\sin\theta$ ... trigonometry

$$\therefore \quad \theta = e^{-\alpha t}\left[A(\cos\omega_d t + j\sin\omega_d t) + B(\cos\omega_d t - j\sin\omega_d t)\right] + \theta_f$$

$$\therefore \quad \theta = e^{-\alpha t}\left[(A+B)\cos\omega_d t + j(A-B)\sin\omega_d t\right] \quad \dots (17)$$

$$\therefore \quad \theta = e^{-\alpha t}\left[P\cos\omega_d t + Q\sin\omega_d t\right] \quad \dots (18)$$

where $\boxed{P = A + B}$ and $\boxed{Q = j(A - B)}$

Let $$\theta = F e^{-\alpha t}\sin(\omega_d t + \alpha) \quad \dots (19)$$

$$\therefore \quad \theta = F e^{-\alpha t}\left[\sin\omega_d t\cos\alpha + \cos\omega_d t\sin\alpha\right] \quad \dots (20)$$

Comparing (18) and (20),

$$P = \sin\alpha \quad \text{and} \quad Q = \cos\alpha$$

Hence $\boxed{\alpha = \tan^{-1}\frac{P}{Q}}$ and $\boxed{F = \sqrt{P^2 + Q^2}}$

Thus the final equation for θ for underdamped case is,

$$\boxed{\theta = F e^{-\frac{D}{2J}t}\sin(\omega_d t + \alpha) + \theta_f} \quad \dots (21)$$

where $\boxed{\omega_d = \frac{\sqrt{4JK - D^2}}{2J} = \text{damped frequency of oscillations in rad/s}}$

The nature of such oscillations is shown in the Fig. 1.11.

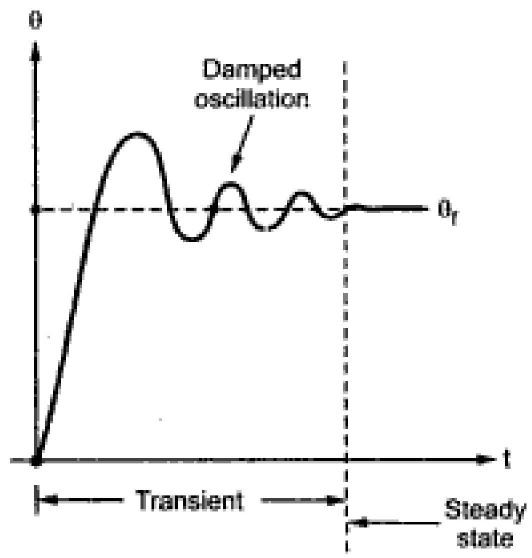


Fig. 1.11 Underdamped motion of galvanometer

The constants F and α are to be obtained from initial conditions.

Obtaining F and α :

To obtain F and α i.e. P and Q use initial condition i.e. at t = 0 and θ = 0 in the equation (21),

$$\therefore \quad 0 = F\sin(\alpha) + \theta_f$$

$$\therefore \quad \sin\alpha = -\frac{\theta_f}{F} = P \quad \text{... (22)}$$

Differentiating equation (21) with time,

$$\frac{d\theta}{dt} = F\left(-\frac{D}{2J}\right)e^{\frac{-D}{2J}t}\sin(\omega_d t + \alpha) + F\,e^{-\frac{D}{2J}t}\,\omega_d\cos(\omega_d t + \alpha)$$

But at t = 0, $\frac{d\theta}{dt} = 0$

$$\therefore \quad 0 = -\left(\frac{D}{2J}\right)(F)\sin\alpha + F\omega_d\cos\alpha \quad \text{... (23)}$$

$$\therefore \quad \tan\alpha = \omega_d \times \frac{2J}{D} = \frac{\sqrt{4JK - D^2}}{2J} \times \frac{2J}{D}$$

$$\therefore \quad \boxed{\tan\alpha = \frac{\sqrt{4JK - D^2}}{D}} \quad \text{... (24)}$$

Using (22) in (23),

$$0 = -\left(\frac{D}{2J}\right)(F)\left(-\frac{\theta_f}{F}\right) + F\omega_d\cos\alpha$$

$$\therefore \quad \cos\alpha = -\frac{D\theta_f}{2\,J\,F\omega_d} = Q \quad \text{... (25)}$$

But $\sin^2\alpha + \cos^2\alpha = 1$

$$\therefore \left(-\frac{\theta_f}{F}\right)^2 + \left(-\frac{D\theta_f}{2\,J\,F\omega_d}\right)^2 = 1$$

$$\therefore \quad \boxed{F = -\theta_f\sqrt{\frac{4\,J^2\omega_d^2 + D^2}{4\,J^2\omega_d^2}}} \quad \text{... (26)}$$

Using $\omega_d = \frac{\sqrt{4JK - D^2}}{2J}$ in equation (26),

$$\therefore \quad F = -\theta_f \left[\frac{2\sqrt{JK}}{\sqrt{4JK - D^2}}\right] \qquad \text{... (27)}$$

Using (24) and (26) in equation (21),

$$\theta = -\theta_f \left[\frac{2\sqrt{JK}}{\sqrt{4JK - D^2}}\right] e^{-\frac{D}{2J}t} \sin(\omega_d t + \alpha) + \theta_f$$

$$\therefore \quad \theta = \theta_f \left[1 - \frac{2\sqrt{JK}}{\sqrt{4JK - D^2}} \, e^{-\frac{D}{2J}t} \sin(\omega_d t + \alpha)\right] \qquad \text{... (28)}$$

The angular frequency is ω_d hence,

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi} \frac{\sqrt{4\,JK - D^2}}{2J} \text{ Hz} \qquad \text{... (29)}$$

$$\therefore \quad T_d = \text{time period} = \frac{1}{f_d} = 2\pi \left[\frac{2J}{\sqrt{4JK - D^2}}\right] \qquad \text{... (30)}$$

1.7.4.2 Undamped Motion

The motion existing when damping is made zero i.e. D = 0 is called undamped motion of the system. The roots m_1 and m_2 are purely imaginary with zero real part for this case.

> **Key Point:** *The oscillations with zero damping are natural oscillations without opposition, having highest frequency. This frequency of undamped oscillations is called natural frequency of oscillations denoted as ω_n.*

$$\therefore \quad \omega_n = \sqrt{\frac{K}{J}} \text{ rad/s} \qquad \text{(Putting D = 0 in } \omega_d\text{)} \qquad \text{... (31)}$$

$$\alpha = \tan^{-1}\left[\frac{\sqrt{4JK - D^2}}{D}\right] = \tan^{-1} \infty = 90^\circ \qquad \text{... D = 0}$$

Thus the solution of undamped motion is,

$$\theta = \theta_f \left[1 - \frac{2\sqrt{JK}}{2\sqrt{JK}} \, e^0 \sin(\omega_n t + 90^\circ)\right]$$

$$\therefore \quad \boxed{\theta = \theta_f [1 - \cos\omega_n t]} \quad \text{... (32)}$$

These oscillations are shown in the Fig. 1.12.

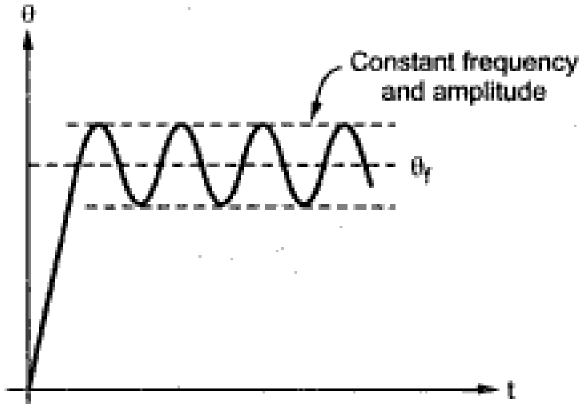


Fig. 1.12 Undamped motion of galvanometer

These are the oscillations with constant frequency and amplitude about the final position θ_f. Such oscillations are called **sustained oscillations.**

1.7.4.3 Critically Damped Motion

For critically damping, the roots m_1 and m_2 are equal, real and negative. Thus $D^2 - 4\,JK = 0$ i.e. $D^2 = 4\,JK$ and $D = 2\sqrt{JK}$ for critical damping.

$$\therefore \quad \boxed{m_1 = m_2 = \frac{-D}{2J}} \quad \text{and} \quad \boxed{D = 2\sqrt{JK}} \quad \text{... Critically damped}$$

Key Point: *For this case, the transient response is not oscillatory but purely exponential such that pointer attains the steady position θ_f very quickly.*

As the response is not oscillatory, equation (28) **is not applicable** for this case.

The solution for the critically damped case is,

$$\theta = \theta_f + e^{-\frac{D}{2J}t}[A + B\,t] \quad \text{... (33)}$$

Using $t = 0,\ \theta = 0,$

$$0 = \theta_f + A \quad \text{... (34)}$$

Differentiating (33) and using $t = 0,\ \frac{d\theta}{dt} = 0,$

$$\frac{d\theta}{dt} = -\frac{D}{2J}\,e^{-\frac{D}{2J}t}[A + B\,t] + e^{-\frac{D}{2J}t}\,B$$

$$\therefore \quad 0 = -\frac{D}{2J}A + B \qquad \text{... (35)}$$

$$\therefore \quad A = -\theta_f \quad \text{and} \quad B = -\frac{D\theta_f}{2J}$$

$$\therefore \quad \boxed{\theta = \theta_f\left[1 - e^{-\frac{D}{2J}t}\left(1 + \frac{D}{2J}t\right)\right]} \qquad \text{... (36)}$$

The value of damping constant for the critical damping is denoted by D_c and $D_c = 2\sqrt{JK}$.

Thus, $$\frac{D}{2J} = \frac{2\sqrt{JK}}{2J} = \sqrt{\frac{K}{J}} = \omega_n$$

Using in equation (36),

$$\boxed{\theta = \theta_f\left[1 - e^{-\omega_n t}(1 + \omega_n t)\right]} \qquad \text{... (37)}$$

1.7.4.4 Overdamped Motion

The amount of damping is mathematically measured by defining a ratio of actual damping D and critical damping D_c. This is called a **damping ratio** and denoted by a greek letter ξ. THis is also called **relative damping.**

$$\therefore \quad \boxed{\xi = \frac{D}{D_c} = \frac{D}{2\sqrt{JK}} = \text{Damping ratio}} \qquad \text{... (38)}$$

Thus when the actual damping is more than the damping for critical case, the motion is called overdamped and the roots m_1 and m_2 are real, **unequal** and negative.

> **Key Point:** *For overdamped case, the transient motion is purely exponential and nonoscillatory. The pointer attains final position θ_f exponentially, taking more time than that of critical damping.*

More the value of damping, the pointer response is slow and sluggish, taking more time to attain the final position.

> **Key Point:** *For critical damping $\xi = 1$ while for overdamped case the damping ratio* $\xi > 1$.

The solution for overdamped motion interms of ξ is given as,

$$\theta = \theta_f \left[1 + \frac{\xi + \sqrt{\xi^2 - 1}}{2\sqrt{\xi^2 - 1}} e^{-\omega_n t\left(\xi - \sqrt{\xi^2 - 1}\right)} - \frac{\xi - \sqrt{\xi^2 + 1}}{2\sqrt{\xi^2 - 1}} e^{-\omega_n t\left(\xi + \sqrt{\xi^2 - 1}\right)} \right] \quad \text{... (39)}$$

The pointer motion for critically damped and the overdamped case is shown in the Fig. 1.13.

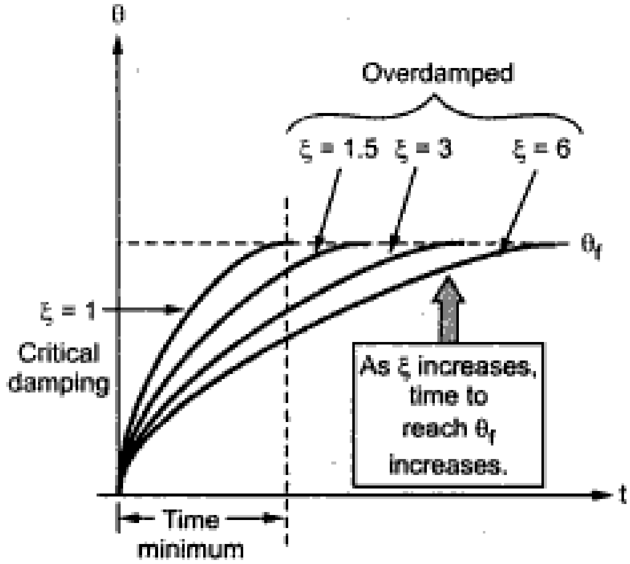
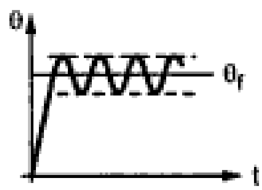
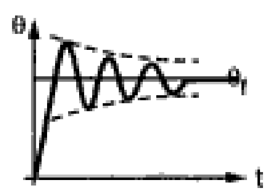


Fig. 1.13 Critically and overdamped motion

Key Point: *As this motion is slow, practically overdamping is avoided in the instruments.*

The critical damping is preferred practically for the instruments.

Motion	Range of ξ	Nature of roots m_1 and m_2	Response
Undamped	$\xi = 0$	Purely complex with zero real part.	 Pure oscillations
Underdamped	$0 < \xi < 1$	Comple/ conjugates with negative real part.	 Damped oscillations

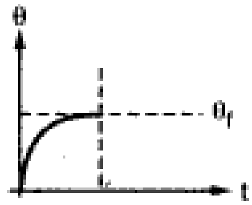
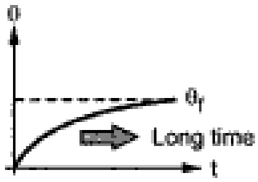
Critically damped	$\xi = 1$	Real, equal, negative.	 Purely exponential and quick
Overdamped	$\xi > 1$	Real, unequal, negative.	 Purely exponential but very slow

Table 1.2 : Types of galvanometer motion

1.7.5 Logarithmic Decrement

Consider the underdamped galvanometer motion as shown in the Fig. 1.14.

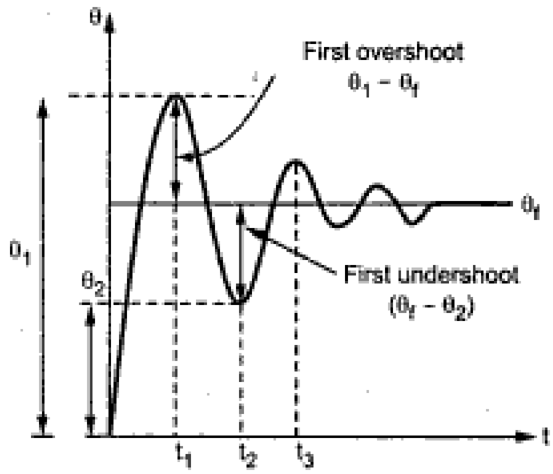


Fig. 1.14 Overshoot and undershoot in motion

The amount by which the pointer exceeds its final position θ_f, during first attempt is called the **first overshoot.** This is maximum in all the overshoots existing in the transient period.

$$\therefore \qquad \theta_1 = \text{Maximum deflection at } t = t_1$$

$$\boxed{\text{First overshoot} = \theta_1 - \theta_f} \qquad \text{... (40)}$$

Thus at $t = t_1$, the deflection is maximum equal to θ_1. According to maxima theorem, the time t_1 at which deflection is maximum, must satisfy $d\theta / dt\big|_{t=t_1} = 0$.

Rearrange equation (28) interms of ξ as,

$$\theta = \theta_f \left[1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin(\omega_d t + \alpha)\right] \quad \text{... (41)}$$

where $\omega_d = \omega_n \sqrt{1-\xi^2}$, $\alpha = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$

At $t = t_1$, $\frac{d\theta}{dt} = 0$

i.e. $\frac{d\theta}{dt} = \frac{-\theta_f}{\sqrt{1-\xi^2}}\left\{(-\xi\omega_n) e^{-\xi\omega_n t} \sin(\omega_d t + \alpha) + e^{-\xi\omega_n t}\omega_d \cos(\omega_d t + \alpha)\right\} = 0$

As ξ, ω_d and ω_n are not zero, the above equation gets satisfied by,

$(-\xi\omega_n)\sin(\omega_d t + \alpha) + \omega_d \cos(\omega_d t + \alpha) = 0$

$$\therefore \tan(\omega_d t + \alpha) = \frac{\omega_d}{\xi\omega_n} = \frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n} = \frac{\sqrt{1-\xi^2}}{\xi} = \tan\alpha$$

This equation is satisfied when $\omega_d t = n\pi$ because $\tan(n\pi + \theta) = \tan\theta$.

$$\therefore \quad t = \frac{n\pi}{\omega_d}$$

For first overshoot $n = 1$, $t = t_1$

$$\therefore \quad t_1 = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \quad \text{... (42)}$$

Putting in equation (41),

$$\therefore \quad \theta_1 = \theta_f\left[1 + e^{-\pi\xi/\sqrt{1-\xi^2}}\right] \quad \text{... (43)}$$

Note that $\sin(\omega_d t_1 + \alpha)\Big|_{t_1 = \frac{\pi}{\omega_d}} = +\sin\alpha = \sqrt{1-\xi^2}$

$$\therefore \quad \text{First overshoot} = \theta_1 - \theta_f = \theta_f\left(e^{-\pi\xi/\sqrt{1-\xi^2}}\right) \quad \text{... (44)}$$

At $n = 2$, $\quad t_2 = \frac{2\pi}{\omega_d}$

$$\therefore \quad \theta_2 = \theta_f \left[1 - e^{-2\pi\xi/\sqrt{1-\xi^2}}\right]$$

$$\therefore \quad \boxed{\text{First undershoot} = \theta_f - \theta_2 = \theta_f \left(e^{-2\pi\xi/\sqrt{1-\xi^2}}\right)} \quad \text{... (45)}$$

Taking ratio of first over and undershoots,

$$\frac{\theta_1 - \theta_f}{\theta_f - \theta_2} = \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{e^{-2\pi\xi/\sqrt{1-\xi^2}}} = e^{+\pi\xi/\sqrt{1-\xi^2}} \quad \text{... (46)}$$

$$\therefore \quad ln\left[\frac{\theta_1 - \theta_f}{\theta_f - \theta_2}\right] = \frac{\pi\xi}{\sqrt{1-\xi^2}} \quad \text{... (47)}$$

The natural logarithm of the ratio of two successive swings is called logarithmic decrement and denoted by λ.

$$\therefore \quad \boxed{\lambda = ln\left[\frac{\theta_1 - \theta_f}{\theta_f - \theta_2}\right] = \frac{\pi\xi}{\sqrt{1-\xi^2}}} \quad \text{... (48)}$$

Now $$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

while $$T_o = \frac{2\pi}{\omega_n}$$

$$\therefore \quad \frac{T_o}{T_d} = \sqrt{1-\xi^2} \quad \text{... (49)}$$

$$\therefore \quad \boxed{\lambda = \pi\xi\left(\frac{T_d}{T_o}\right)} \quad \text{... (50)}$$

where T_o = Time period corresponding to ω_n.

Thus equation (28) can be expressed as,

$$\theta = \theta_f \left[1 - \frac{T_d}{T_o} e^{-2\pi\xi t/T_o} \sin\left(\frac{2\pi t}{T_d} + \sin^{-1}\frac{T_o}{T_d}\right)\right] \quad \text{... (51)}$$

$$\therefore \quad \theta = \theta_f \left[1 - \frac{\omega_n}{\omega_d} e^{-\omega_d \lambda t/\pi} \sin\left(\omega_d t + \sin^{-1}\frac{\pi\xi}{\lambda}\right)\right] \quad \text{... (52)}$$

1.7.6 Settling Time

The time required by the pointer to achieve the steady state when the complete transient behaviour dies out is called **settling time** denoted as T_s.

Practically as the exponential term is present in the equation of motion, ξ is kept between 0.6 to 1 and motion is underdamped hence oscillatory. While a particular band is defined about θ_f denoted by $\pm \Delta$. When transient oscillations decrease and enter into $\theta_f \pm \Delta$ zone and remain thereafter within this interval then it is said that the pointer has achieved the steady state. This is shown in the Fig. 1.15.

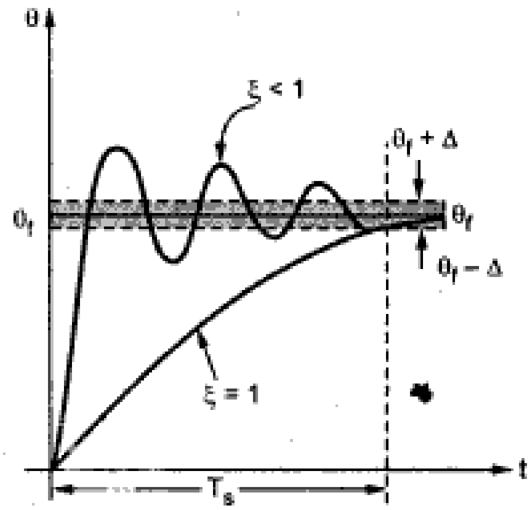


Fig. 1.15 Settling time

For ± 2% band defined from accuracy point of view,

$$T_s = \frac{4}{\xi \omega_n} \quad \text{... (53)}$$

This is possible for ξ between 0.6 to 1.0 hence practically ξ is designed between this range.

1.7.7 Effect of External Resistance on Damping

The damping means opposition to the motion by dissipating the energy of rotation. In galvanometer the damping is provided by two types,

1) Mechanical damping : This is due to the friction present in the mechanical motion of the pointer. This is not very significant. The damping torque produced due to such mechanical effects is given by,

$$T_m = D_m \frac{d\theta}{dt} \quad \text{... (54)}$$

where D_m = Mechanical damping constant

2) Electromagnetic damping : This is effective damping than the mechanical damping. It is produced due to induced effects when coil moves in a magnetic field. Thus when coil moves in a magnetic field,

i) The eddy currents are induced in the metal former.

ii) The e.m.f. is induced in coil which circulates current through coil.

These two effects cause damping called electromagnetic damping.

Let R = Resistance of galvanometer circuit = $R_g + R_x$

where R_g = Resistance of galvanometer coil

R_x = External resistance connected for damping

when coil rotates, e.m.f is induced in it which is given by,

$$e = 2N \times B\,l\,v \quad \text{... (55)}$$

where $v = \frac{r}{2}\omega = \frac{r}{2}\frac{d\theta}{dt}$ = linear velocity

$$\therefore \quad e = 2\,N\,B\,l\frac{r}{2}\frac{d\theta}{dt} = N\,B\,A\frac{d\theta}{dt} \quad \text{... } l \times r = \text{Area A}$$

But $G = N\,B\,A$

$$\therefore \quad e = G\frac{d\theta}{dt} \quad \text{... (56)}$$

$$\therefore \quad i = \frac{e}{R} = \frac{G}{R}\frac{d\theta}{dt} \quad \text{... (57)}$$

The torque produced due to this current flowing through the coil is,

$$T_{coil} = N \times B\,i\,l \times r = N\,B\,A\,i = G\,i$$

$$\therefore \quad T_{coil} = G \times \frac{G}{R}\frac{d\theta}{dt} = \frac{G^2}{R}\frac{d\theta}{dt} = D_{coil}\frac{d\theta}{dt} \quad \text{... (58)}$$

where D_{coil} = Damping constant of coil circuit

$$\therefore \quad \boxed{D_{coil} = \frac{G^2}{R}} \quad \text{... (59)}$$

Now let us find damping due to the metal former. It consists of one strip i.e. N = 1.

$$\therefore \quad T_f = B\,i\,l \times r = B\,A\,i \quad \text{... (60)}$$

Now $BA = \frac{G}{N}$ and $i = \frac{G}{NR_f}\frac{d\theta}{dt}$ ... as N = 1 for former

where R_f = Resistance of former

$$\therefore \quad T_f = \frac{G}{N} \times \frac{G}{NR_f} \frac{d\theta}{dt} = \frac{G^2}{N^2 R_f} \frac{d\theta}{dt} = D_{former} \frac{d\theta}{dt} \quad \text{... (61)}$$

where D_{former} = Damping constant of former

$$\therefore \quad \boxed{D_{former} = \frac{G^2}{N^2 R_f}} \quad \text{... (62)}$$

Total electromagnetic damping $= (D_{coil} + D_{former}) \dfrac{d\theta}{dt}$

$$\therefore \quad T_e = \left[\frac{G^2}{R} + \frac{G^2}{N^2 R_f}\right] \frac{d\theta}{dt}$$

$$\therefore \quad T_e = D_e \frac{d\theta}{dt} \quad \text{... (63)}$$

where $D_e = \dfrac{G^2}{R} + \dfrac{G^2}{N^2 R_f}$ = damping constant due to electromagnetic damping ... (64)

The total damping due to both effects is,

$$\boxed{T_D = T_m + T_e = [D_m + D_e] \frac{d\theta}{dt} = D \frac{d\theta}{dt}} \quad \text{.... (65)}$$

where $D = D_m + D_e$

1.7.7.1 Critical Resistance for Damping

The mechanical damping is very small and can be neglected.

$$\therefore \quad D = D_e = \frac{G^2}{R} + \frac{G^2}{N^2 R_f}$$

Practically the damping due to metal former is also very small.

$$\therefore \quad D = \frac{G^2}{R}$$

For the critical damping, $\xi = 1$, $R = R_c$ and $D = D_c = 2\sqrt{Jk}$

$$\therefore \quad 2\sqrt{JK} = \frac{G^2}{R_c}$$

$$\therefore \quad R_c = \frac{G^2}{2\sqrt{J\,K}} \quad \text{... (66)}$$

$$\therefore \quad R_x = R_c - R_g = \frac{G^2}{2\sqrt{JK}} - R_g \quad \text{... (67)}$$

This is the value of external resistance required to adjust damping to the critical damping. It is called external critical damping resistance (ECDR).

1.8 Sensitivity of Galvanometer

The sensitivity of galvanometer can be defined with respect to current, voltage or resistance of galvanometer. Thus there are three sensitivities associated with a galvanometer.

1.8.1 Current Sensitivity

It is defined as the deflection obtained per unit current.

$$S_i = \frac{\theta_f}{i} = \frac{G\,i}{K\,i} = \frac{G}{K} \text{ rad/A} \quad \text{... (1)}$$

Practically the values of current and deflection are very small. Hence the current sensitivity is expressed in mm/μA.

$$\therefore \quad S_i = \frac{d}{i} \text{ but } d = 2\theta_f\, r = \frac{2\,G\,i}{K}\, r$$

$$\therefore \quad S_i = \frac{2\,G\,r}{K} \text{ m/A}$$

As usually r = 1m = 1000 mm,

$$S_i = \frac{2000\,G}{K \times 10^6} = \frac{G}{500\,K} \text{ mm/μA} \quad \text{... (2)}$$

1.8.2 Voltage Sensitivity

It is defined as the deflection obtained in scale divisions per unit voltage impressed on the galvanometer.

$$S_v = \frac{d}{i\,R_g} \text{ m/V} \quad \text{... (3)}$$

But $$d = \frac{2Gir}{K} = \frac{2000\ Gi}{K} \text{ in mm for } r = 1 \text{ m}$$

$$\therefore \quad S_v = \frac{2000\ G\,i}{i\,K\,R_g \times 10^6} \text{ mm}/\mu\text{V}$$

$$\therefore \quad \boxed{S_v = \frac{G}{500\ K\,R_g} \text{ mm}/\mu\text{V}} \quad \text{... (4)}$$

1.8.3 Megohm Sensitivity

It is the resistance of the circuit in megaohm so that the deflection is one scale division when one volt is impressed on the galvanometer.

$$\therefore \quad \boxed{S_0 = \frac{d}{i \times 10^{-6}}\ \text{M}\Omega/\text{scale division}} \quad \text{... (5)}$$

But $$d = \frac{2000\ G\,i}{K} \text{ mm for } r = 1 \text{ m},$$

$$\therefore \quad \boxed{S_0 = \frac{G}{500\ K}\ \text{M}\Omega/\text{mm}} \quad \text{... (6)}$$

For high sensitivity, G must be large and K should be small. As G = NBA, to get high sensitivity, coil must be having more N, more cross-section area and must be placed in high flux density magnetic field. Hence the coil has large number of turns of fine wire as area of cross-section cannot be increased beyond limits. The K can be decreased by using small stiffness constant springs.

Example 1.1 : *A D'Arsonval galvanometer has the following data :*

flux density = 8×10^{-3} Wb/m^2, number of turns = 300, length of coil = 15 mm, width of coil = 30 mm, spring constant = 2.5×10^{-9} Nm/rad ; moment of inertia = 10×10^{-9} kg – m^2, damping constant = 2×10^{-9} Nm/rad - s^{-1}, resistance of the coil = 80 Ω. Calculate,

i) deflection of galvanometer for a curent of $1\,\mu A$.

ii) current sensitivity if the scale is kept 1 m away from the mirror.

(JNTU, Nov.-04, Set-1, Nov.-03, Set-4)

Solution : $B = 8\times10^{-3}$ Wb/m², N = 300, l = 15 mm, r = 30 mm

$K = 2.5\times10^{-9}$ Nm/rad, $J = 10\times10^{-9}$ Kg – m², $D = 2\times10^{-9}$ Nm/rad s⁻¹, $R_g = 80\ \Omega$

$$A = l\times r = 15\times30 = 450\ \text{mm}^2 = 450\times10^{-6}\ \text{m}^2$$

$$G = NBA = 300\times8\times10^{-3}\times450\times10^{-6} = 1.08\times10^{-3}\ \text{Nm/A}$$

$$i = 1\ \mu A$$

i) $$\theta_f = \frac{G\,i}{K} = \frac{1.08\times10^{-3}\times1\times10^{-6}}{2.5\times10^{-9}}$$

$$= \mathbf{0.432\ rad.}$$

∴ $$d \text{ in mm} = 2\theta_f\ r \qquad \ldots r = 1\text{ m} = 1000\text{ mm}$$

∴ $$d = 2\times0.432\times1000$$

$$= \mathbf{864\ mm}$$

ii) $$S_i = \frac{\theta_f}{i} = \frac{d}{i}$$

$$= \mathbf{864\ mm/\mu A}$$

Example 1.2 : *A galvanometer has the following parameters :*

$B = 10\times10^{-3}\ Wb/m^2$, N = 200 turns, length of coil = 16 mm,

$K = 12\times10^{-9}\ Nm/rad$, $J = 50\times10^{-9}\ Kg - m^2$, $D = 5\times10^{-9}\ Nm/rad\ s^{-1}$,

Resistance of the coil = 120 Ω. Calculate,

i) deflection of the galvanometer in radians and in mm when a current of 1 μA flows through it, the scale being 1 m away

ii) the current sensitivity *iii) the voltage sensitivity*

iv) the megaohm sensitivity *v) the frequency of damped oscillation*

vi) the period of free oscillations *vii) the first maximum deflection*

viii) the relative damping *ix) the logarithmic decrement*

Sketch the typical curve of the motion of the galvanometer for the above data.

(JNTU,Nov.-04, Set-2, Nov.-03, Set-3)

Solution : Assuming square coil, $A = l^2 = 16 \times 16 = 256 \text{ mm}^2$

$$\therefore \quad G = NBA = 200 \times 10 \times 10^{-3} \times 256 \times 10^{-6} = 5.12 \times 10^{-4} \text{ Nm/A}$$

i) $$\theta_f = \frac{G\,i}{K} = \frac{5.12 \times 10^{-4} \times 1 \times 10^{-6}}{12 \times 10^{-9}}$$

$$= \mathbf{0.04266 \text{ rad}}$$

$$\therefore \quad d = 2\theta_f \times r = 2 \times 0.04266 \times 1000 \qquad \text{... } r = 1 \text{ m} = 1000 \text{ mm}$$

$$\therefore \quad d = \mathbf{85.3333 \text{ mm}}$$

ii) $$S_i = \frac{d}{i} = \mathbf{85.3333 \text{ mm}/\mu A}$$

iii) $$S_v = \frac{d}{i\,R_g}$$

$$= \frac{85.333}{1 \times 120}$$

$$= \mathbf{0.711 \text{ mm}/\mu V}$$

iv) $$S_0 = \frac{d}{i \times 10^6} = \frac{85.333}{1 \times 10^{-6} \times 10^6}$$

$$= \mathbf{85.333 \text{ M}\Omega/\text{mm}}$$

v) $$\omega_d = \frac{\sqrt{4JK - D^2}}{2J}$$

$$= \frac{\sqrt{4 \times 50 \times 10^{-9} \times 12 \times 10^{-9} - (5 \times 10^{-9})^2}}{2 \times 50 \times 10^{-9}}$$

$$= 0.4873 \text{ rad/sec}$$

$$\therefore \quad f_d = \frac{\omega_d}{2\pi} = \frac{0.4873}{2\pi}$$

$$= \mathbf{0.0775 \text{ Hz}}$$

vi) $T_o = \frac{2\pi}{\omega_n}$ and $\omega_n = \sqrt{\frac{K}{J}}$

$\therefore$ $T_o = \frac{2\pi}{\sqrt{\frac{K}{J}}} = \frac{2\pi}{\sqrt{\frac{12\times10^{-9}}{50\times10^{-9}}}}$

$= \mathbf{12.825\ sec.}$

vii) $\theta_1 = \theta_f\left[1+e^{-\pi\xi/\sqrt{1-\xi^2}}\right]$

where $\xi = \frac{D}{D_c} = \frac{D}{2\sqrt{JK}} = \frac{5\times10^{-9}}{2\sqrt{50\times10^{-9}\times12\times10^{-9}}} = 0.102$

$\therefore$ $\theta_1 = 0.04266\left[1+e^{-\pi\times0.102/\sqrt{1-(0.102)^2}}\right]$

$= 0.07358\ \text{rad} = 2\times1000\times0.07358\ \text{mm}$

$= \mathbf{147.164\ \ mm}$ ... First maximum deflection

viii) ξ = relative damping = **0.102**

ix) $\lambda = \frac{\pi\xi}{\sqrt{1-\xi^2}} = \frac{\pi\times0.102}{\sqrt{1-(0.102)^2}}$

$= \mathbf{0.3221}$

The typical curve of motion is shown in the Fig. 1.16.

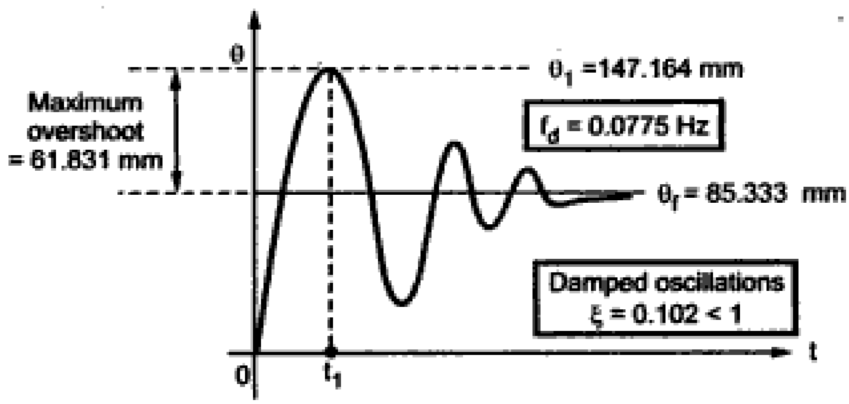


Fig. 1.16

1.9 Permanent Magnet Moving Coil Instruments (PMMC)

The permanent magnet moving coil instruments are most accurate type for d.c. measurements.The action of these instruments is based on the motoring principle. When a current carrying coil is placed in the magnetic field produced by permanent magnet, the coil experiences a force and moves. As the coil is moving and the magnet is permanent, the instrument is called permanent magnet moving coil instrument. This basic principle is called **D'Arsonval principle.** The amount of force experienced by the coil is proportional to the current passing through the coil.

The PMMC instrument is shown in the Fig. 1.17.

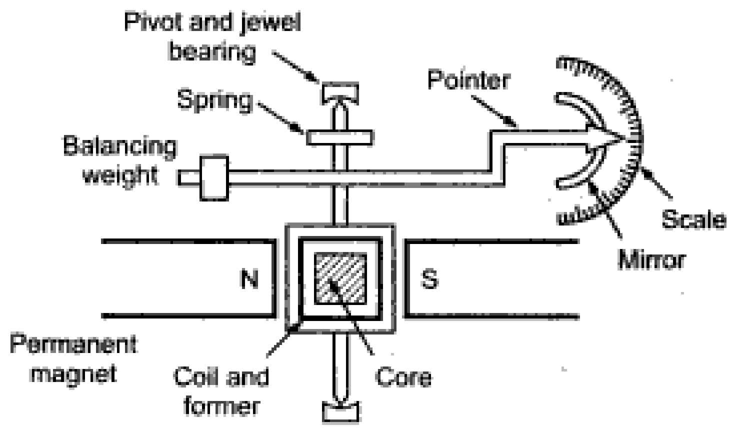


Fig. 1.17 Construction of PMMC instrument

The moving coil is either rectangular or circular in shape. It has number of turns of fine wire. The coil is suspended so that it is free to turn about its vertical axis. The coil is placed in uniform, horizontal and radial magnetic field of a permanent magnet in the shape of a horse-shoe. The iron core is spherical if coil is circular and is cylindrical if the coil is rectangular. Due to iron core, the deflecting torque increases, increasing the sensitivity of the instrument.

The controlling torque is provided by two phosphor bronze hair springs.

The damping torque is provided by eddy current damping. It is obtained by movement of the aluminium former, moving in the magnetic field of the permanent magnet.

The pointer is carried by the spindle and it moves over a graduated scale. The pointer has light weight so that it can deflect rapidly. The mirror is placed below the pointer to get the accurate reading by removing the parallax. The weight of the instrument is normally counter balanced by the weights situated diametrically opposite and rigidly connected to it. The scale markings of the basic d.c. PMMC instruments are usually linearly spaced as the deflecting torque and hence the pointer deflection are directly proportional to the current passing through the coil.

The top view of PMMC instrument is shown in the Fig. 1.18.

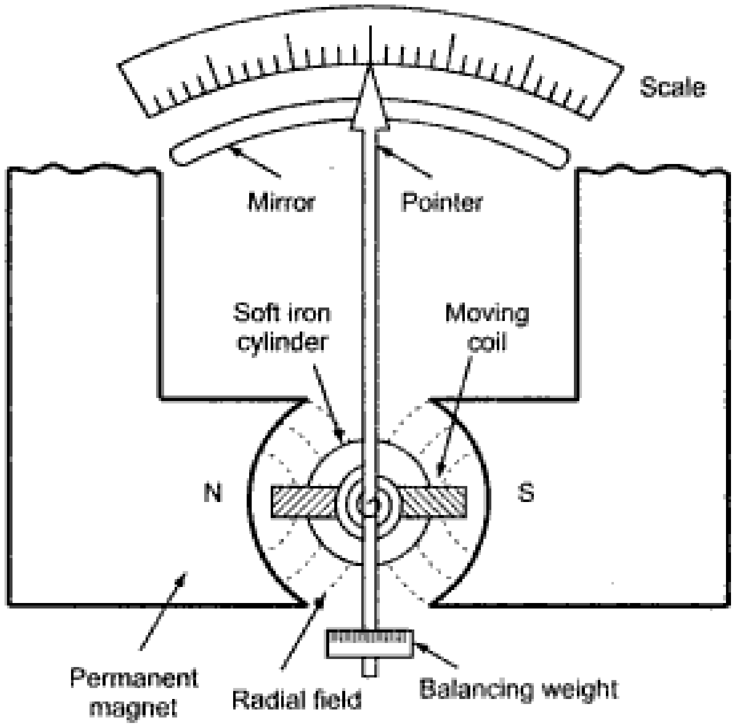


Fig. 1.18 PMMC instrument

In a practical PMMC instrument, a Y shaped member is attached to the fixed end of the front control spring. An eccentric pin through the instrument case engages the Y shaped member so that the zero position of the pointer can be adjusted from outside.

1.9.1 Torque Equation

The equation for the developed torque can be obtained from the basic law of the electromagnetic torque. The deflecting toque is given by,

$$\boxed{T_d = NBAI}$$

where T_d = deflecting torque in N-m

B = flux density in air gap, Wb/m^2

N = number of turns of the coil

A = effective coil area m^2

I = Current in the moving coil, amperes

$\therefore \quad T_d = GI$

where $G = NBA$ = constant

The controlling torque is provided by the springs and is proportional to the angular deflection of the pointer.

$$T_c = K\theta$$

where T_c = controlling torque

K = spring constant, Nm/rad or Nm/deg

θ = angular deflection

For the final steady state position,

$$T_d = T_c$$

$$\therefore \quad GI = K\theta$$

$$\therefore \quad \theta = \left(\frac{G}{K}\right) I$$

or

$$\boxed{I = \left(\frac{K}{G}\right)\theta}$$

Key Point : *Thus the deflection is directly proportional to the current passing through the coil.*

The pointer deflection can therefore be used to measure current.

As the direction of the current through the coil changes, the direction of the deflection of the pointer also changes. Hence such instruments are well suited for the d.c. measurements.

In the micro ammeters and milliammeters upto about 20 mA, the entire current to be measured is passed through the coil. The springs carry current to the coil. Thus the current carrying capacity of the springs, limits the current which can be safely carried. For higher currents, the moving coil is shunted by sufficient resistance. While the voltmeters having high ranges use a moving coil together with sufficient series resistance, to limit the instrument current. Most d.c. voltmeters are designed to produce full scale deflection with a current of 20, 10, 5 or 1 mA.

The power requirement of PMMC instrument is very small, typically of the order of 25 μW to 200 μW. Accuracy is generally of the order of 2 to 5% of the full scale reading.

➡ **Example 1.3 :** *A PMMC instrument has a coil of dimensions 10 mm × 8 mm. The flux density in the air gap is 0.15 Wb/m². If the coil is wound for 100 turns, carrying a current of 5 mA then calculate the deflecting torque. Calculate the deflection if the spring constant is* 0.2×10^{-6} *Nm/degree.*

Solution : The deflecting torque is given by,

$$T_d = NBAI = 100 \times 0.15 \times (A) \times 5 \times 10^{-3} \text{ Nm}$$

Now $A = \text{area} = 10 \times 8 = 80 \text{ mm}^2 = 80 \times 10^{-6} \text{m}^2$

$\therefore \quad T_d = 100 \times 0.15 \times 80 \times 10^{-6} \times 5 \times 10^{-3} = 6 \times 10^{-6} \text{ Nm}$

Now $T_d = T_c = K\theta$

$\therefore \quad 6 \times 10^{-6} = 0.2 \times 10^{-6} \times \theta$

$\therefore \quad \theta = \dfrac{6 \times 10^{-6}}{0.2 \times 10^{-6}} = \textbf{30 degrees}$

1.9.2 Advantages

The various advantages of PMMC instruments are,

1) It has uniform scale.
2) With a powerful magnet, its torque to weight ratio is very high. So operating current is small.
3) The sensitivity is high.
4) The eddy currents induced in the metallic former over which coil is wound, provide effective damping.
5) It consumes low power, of the order of 25 W to 200 μW.
6) It has high accuracy.
7) Instrument is free from hysteresis error.
8) Extension of instrument range is possible.
9) Not affected by external magnetic fields called stray magnetic fields.

1.9.3 Disadvantages

The various disadvantages of PMMC instruments are,

1) Suitable for d.c. measurements only.
2) Ageing of permanent magnet and the control springs introduces the errors.
3) The cost is high due to delicate construction and accurate machining.
4) The friction due to jewel-pivot suspension.

1.9.4 Taut Band Instrument

The friction due to jewel-pivot suspension can be eliminated by using taut band movement. The working principle of taut band instrument is same based on D'Arsonval's principle. The main difference is the method of mounting the coil.

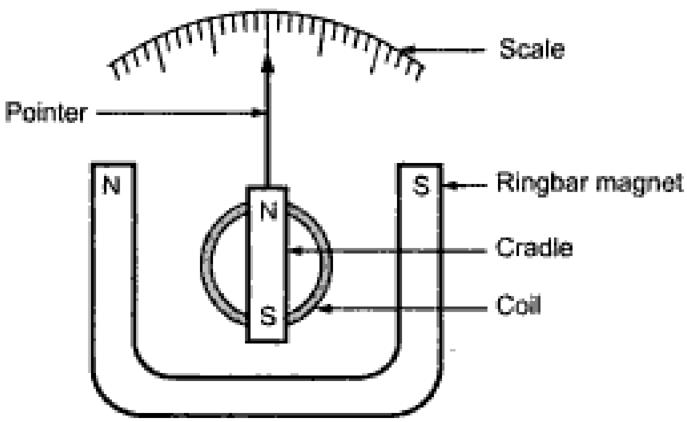


Fig. 1.19 Taut band instrument

In the taut band instrument the movable coil is suspended by means of two torsion ribbons. The ribbons are placed under sufficient tension to eliminate any sag. This tension is provided by the tension string. The coil is mounted in a cradle and surrounded by ring bar magnet. The construction is shown in the Fig 1.19.

The taut band instrument can be used in any position while jewel-pivot instrument should be used vertically. The sensitivity of the taut band instruments is higher than jewel-pivot instruments. The taut band instruments are relatively insensitive to shocks and temperature and are capable of withstanding overloads.

1.9.5 Temperature Compensation

The basic PMMC instrument is sensitive to the temperature.The magnetic field strength and spring tension decrease with increase in temperature. The coil resistance increases with increase in the temperature. Thus pointer reads low for a given current. The meter tends to read low by approximately 0.2 % per ° C rise in the temperature. Hence the temperature compensation is provided by appropriate use of series and shunt resistances of copper and manganin.

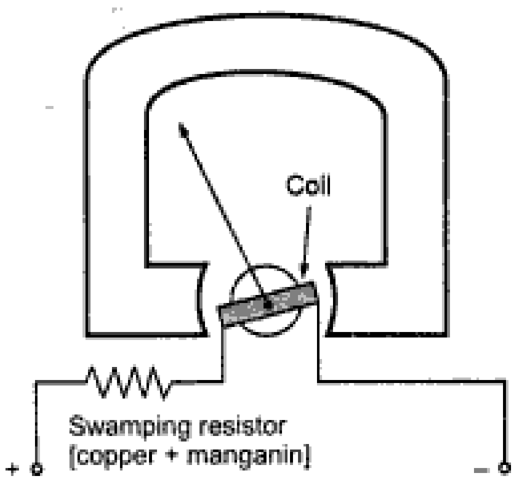


Fig. 1.20 Simple temperature compensation

The simple compensation circuit uses a resistance in series with the movable coil, as shown in the Fig. 1.20. The resistor is called a **swamping resistor**. It is made up of manganin having practically zero temperature coefficient, combined with copper in the ratio of 20/1 or 30/1.

The resultant resistance of coil and the swamping resistor increases slightly as temperature increases, just enough to compensate the change in springs and magnet due to temperature. Thus the effect of temperature is compensated.

More complicated but complete cancellation of temperature effects can be obtained by using the swamping resistors in series and parallel combination as shown in the Fig. 1.21.

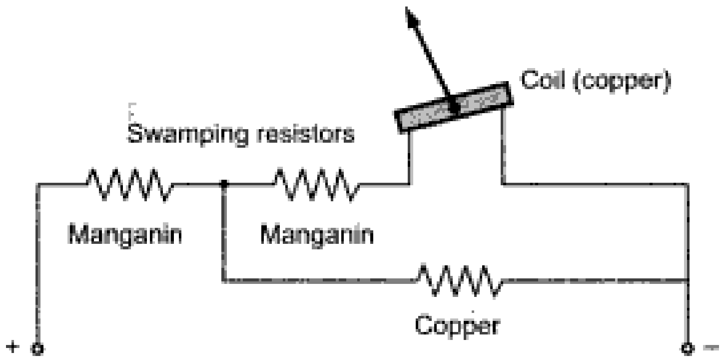


Fig. 1.21 Improved temperature compensation

In this circuit, by correct proportioning of copper and manganin parts, complete cancellation of the temperature effects can be achieved.

1.9.6 Errors in PMMC Instrument

The basic sources of errors in PMMC instruments are friction, temperature and aging of various parts. To reduce the frictional errors ratio of torque to weight is made very high.

The most serious errors are produced by the heat generated or by changes in the temperature. This changes the resistance of the working coil, causing large errors. In case of voltmeters, a large series resistance of very low temperature coefficient is used. This reduces the temperature errors.

The aging of permanent magnet and control springs also cause errors. The weakening of magnet and springs cause opposite errors. The weakening of magnet cause less deflection while weakening of the control springs cause large deflection, for a particular value of current. The proper use of material and preageing during manufacturing can reduce the errors due to weakening of the control springs.

1.10 Moving Iron Instruments

The moving iron instruments are classified as :

i) Moving iron attraction type instruments and

ii) Moving iron repulsion type instruments

1.10.1 Moving Iron Attraction Type Instruments

The basic working principle of these instruments is very simple that a soft iron piece if brought near the magnet gets attracted by the magnet.

The construction of the attraction type instrument is shown in the Fig. 1.22.

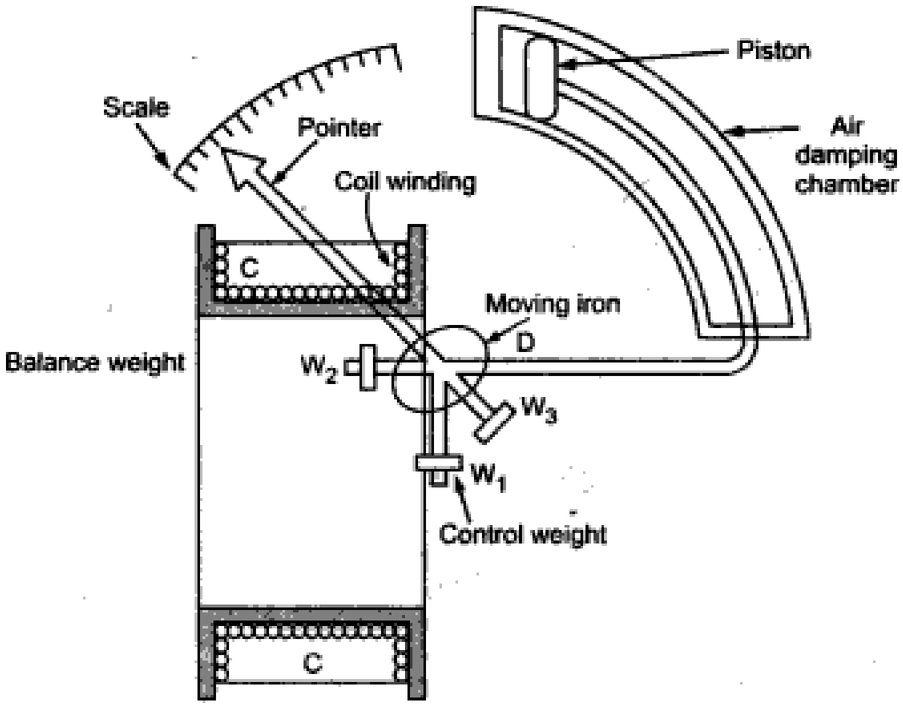


Fig. 1.22 Moving iron attraction type instrument

It consists of a fixed coil C and moving iron piece D. The coil is flat and has a narrow slot like opening. The moving iron is a flat disc which is eccentrically mounted on the spindle. The spindle is supported between the jewel bearings. The spindle carries a pointer which moves over a graduated scale. The number of turns of the fixed coil are dependent on the range of the instrument.For passing large current through the coil only few turns are required.

The controlling torque is provided by the springs but gravity control may also be used for vertically mounted panel type instruments.

The damping torque is provided by the air friction. A light aluminium piston is attached to the moving system. It moves in a fixed chamber. The chamber is closed at one end. It can also be provided with the help of vane attached to the moving system.

The operating magnetic field in moving iron instruments is very weak. Hence eddy current damping is not used since it requires a permanent magnet which would affect or distort the operating field.

1.10.2 Moving Iron Repulsion Type Instrument

These instruments have two vanes inside the coil, the one is fixed and other is movable. When the current flows in the coil, both the vanes are magnetised with like polarities induced on the same side. Hence due to the repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of the moving vane. The repulsion type instruments are the most commonly used instruments.

The two different designs of repulsion type instruments are :

i) Radial vane type and

ii) Co-axial vane type

1.10.2.1 Radial Vane Repulsion Type Instrument

The Fig. 1.23 shows the radial vane repulsion type instrument. Out of the other moving iron mechanisms, this is the most sensitive and has most linear scale.

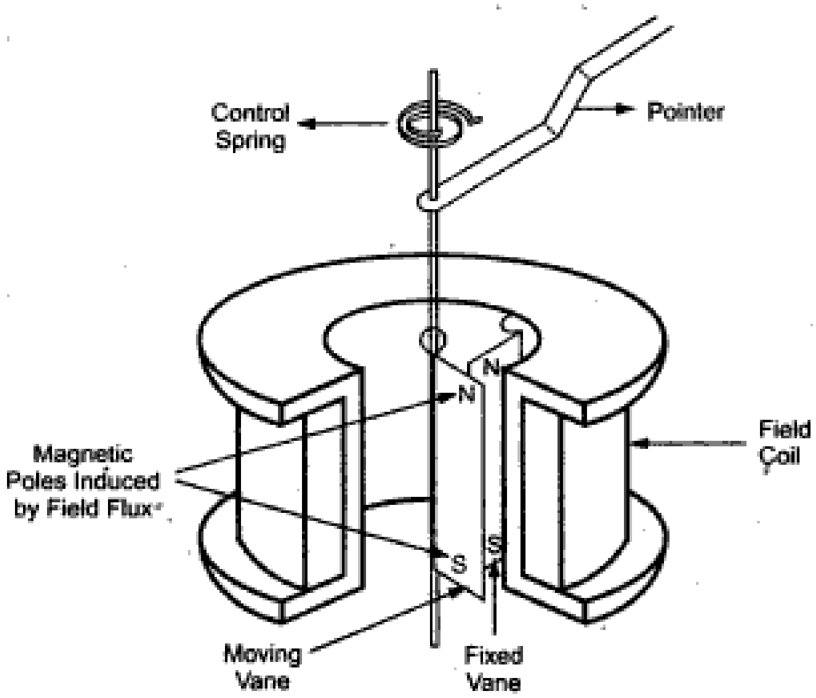


Fig. 1.23 Radial vane repulsion type instrument

The two vanes are radial strips of iron. The fixed vane is attached to the coil. The movable vane is attached to the spindle and suspended in the induction field of the coil. The needle of the instrument is attached to this vane.

Eventhough the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane. Hence the deflection of the pointer is always in the same direction.The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to read amperes or volts. The calibration is accurate only for the frequency for which it is designed because the impedance is different for different frequencies.

1.10.2.2 Concentric Vane Repulsion Type Instrument

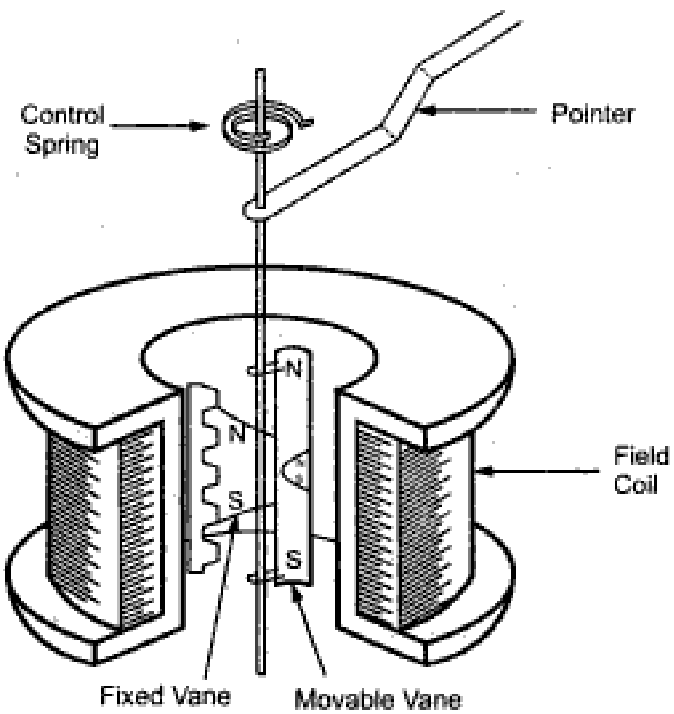


Fig. 1.24 Concentric vane repulsion type instrument

Fig. 1.24 shows the concentric vane repulsion type instrument. The instrument has two concentric vanes. One is attached to the coil frame rigidly while the other can rotate coaxially inside the stationary vane.

Both the vanes are magnetised to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force. As the movable vane is attached to the pivoted shaft, the repulsion results in a rotation of the shaft. The pointer deflection is proportional to the current in the coil. The concentric vane type instrument is moderately sensitive and the deflection is proportional to the square of the current through coil. Thus the instrument is said to have square law response. Thus the scale of the instrument is non-uniform in nature. Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction. Hence moving iron instruments can be used for both a.c. and d.c. measurements. Due to square law response, the scale of the moving iron instrument is non-uniform.

1.10.3 Torque Equation of Moving Iron Instruments

Consider a small increment in current supplied to the coil of the instrument.Due to this current let $d\theta$ be the deflection under the deflecting torque T_d. Due to such deflection, some mechanical work will be done.

$$\therefore \quad \boxed{\text{Mechanical work} = T_d \, d\theta}$$

There will be a change in the energy stored in the magnetic field due to the change in inductance. This is because the vane tries to occupy the position of minimum reluctance hence the force is always in such a direction so as to increase the inductance of coil. The inductance is inversely proportional to the reluctance of the magnetic circuit of coil.

Let
I = initial current
L = instrument inductance
θ = deflection
dI = increase in current
$d\theta$ = change in deflection
dL = change in inductance

In order to effect an increment dI in the current, there must be an increase in the applied voltage given by,

$$e = \frac{d(LI)}{dt}$$

$$= I\frac{dL}{dt} + L\frac{dI}{dt} \qquad \text{as both I and L are changing.}$$

The electrical energy supplied is given by,

$$eIdt = \left(I\frac{dL}{dt} + L\frac{dI}{dt}\right)Idt$$

$$= I^2\, dL + IL\, dI$$

The stored energy increases from $\frac{1}{2}LI^2$ to $\frac{1}{2}(L+dL)(I+dI)^2$

Hence the change in the stored energy is given by,

$$= \frac{1}{2}(L+dL)(I+dI)^2 - \frac{1}{2}LI^2$$

Neglecting higher order terms, this becomes, $IL\, dI + \frac{1}{2}I^2\, dL$

The energy supplied is nothing but increase in stored energy plus the energy required for mechanical work done.

$$\therefore \quad I^2 dL + IL\, dI = IL\, dI + \frac{1}{2}I^2\, dL + T_d \cdot d\theta$$

$$\therefore \qquad T_d \cdot d\theta = \frac{1}{2} I^2 \, dL$$

$$\therefore \qquad \boxed{T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}}$$

While the controlling torque is given by,

$$T_c = K\,\theta$$

where K = spring constant

$$\therefore \qquad K\,\theta = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad \text{under equilibrium}$$

$$\therefore \qquad \theta = \frac{1}{2} \frac{I^2}{K} \frac{dL}{d\theta}$$

Thus the deflection is proportional to the square of the current through the coil. And the instrument gives square law response.

1.10.4 Advantages

The various advantages of moving iron instruments are,

1) The instruments can be used for both a.c. and d.c. measurements.
2) As the torque to weight ratio is high, errors due to the friction are very less.
3) A single type of moving element can cover the wide range hence these instruments are cheaper than other types of instruments.
4) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable.
5) These are capable of giving good accuracy. Modern moving iron instruments have a d.c. error of 2% or less.
6) These can withstand large loads and are not damaged even under severe overload conditions.
7) The range of instruments can be extended.

1.10.5 Disadvantages

The various disadvantages of moving iron instruments are,

1) The scale of the moving iron instruments is not uniform and is cramped at the lower end. Hence accurate readings are not possible at this end.
2) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.

3) The increase in temperature increases the resistance of coil, decreases stiffness of the springs, decreases the permeability and hence affect the reading severely.

4) Due to the non linearity of B-H curve, the deflecting torque is not exactly proportional to the square of the current.

5) There is a difference between a.c. and d.c. calibrations on account of the effect of inductance of the meter. Hence these meters must always be calibrated at the frequency at which they are to be used. The usual commercial moving iron instrument may be used within its specified accuracy from 25 to 125 Hz frequency range.

6) Power consumption is on higher side.

1.10.6 Errors in Moving Iron Instruments

The various errors in the moving iron instruments are,

1) **Hysteresis error :** Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetise quickly or to work with lower flux densities.

2) **Temperature error :** The temperature error arises due to the effect of temperature on the temperature coefficient of the spring. This error is of the order of 0.02% per °C change in temperature. Errors can cause due to self heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence manganin is generally used for the series resistances.

3) **Stray magnetic field error :** The operating magnetic field in case of moving iron instruments is very low. Hence effect of external i.e. stray magnetic field can cause error. This effect depends on the direction of the stray magnetic field with respect to the operating field of the instrument.

4) **Frequency error :** These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This causes errors in the instrument.

5) **Eddy current error :** When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instrument. The eddy current affects the instrument current causing the change in the deflecting torque. This produces the error in the meter reading. As eddy currents are frequency dependent, frequency changes cause eddy current error.

Example 1.4 : *The inductance of a moving iron instrument is given by*

$$L = (12 + 6\theta - \theta^2)\ \mu H$$

where θ is the deflection in radians from zero position. The spring constant is 12×10^{-6} *Nm/radians. Calculate the deflection for a current of 8 A.*

Solution : The rate of change of inductance with deflection is,

$$\frac{dL}{d\theta} = \frac{d}{d\theta}(12 + 6\theta - \theta^2)$$

$$= 6 - 2\theta\ \mu H/\text{radians} = (6 - 2\theta) \times 10^{-6}\ H/\text{radians}$$

From the torque equation,

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta}$$

$$\therefore \quad \theta = \frac{1}{2} \times \frac{(8)^2}{12 \times 10^{-6}} \times [6 - 2\theta] \times 10^{-6}$$

$$\therefore \quad 0.375\,\theta = 6 - 2\,\theta$$

$$\therefore \quad \theta = 2.526 \text{ radians} = \mathbf{144.74°}$$

1.11 Basic D.C. Ammeter

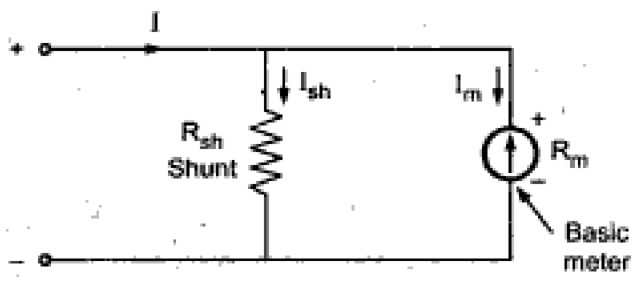


Fig. 1.25 Basic d.c. ammeter

The basic d.c. ammeter is nothing but a D'Arsonval galvanometer. The coil winding of a basic movement is very small and light and hence it can carry very small currents. So as mentioned earlier, for large currents, the major part of current is required to be bypassed using a resistance called **shunt**. It is shown in the Fig. 1.25.

The shunt resistance can be calculated as :

Let R_m = internal resistance of coil

R_{sh} = shunt resistance

I_m = full scale deflection current

I_{sh} = shunt current

I = total current

Now $\quad I = I_{sh} + I_m$

As the two resistances R_{sh} and R_m are in parallel, the voltage drop across them is same.

$$\therefore \quad I_{sh} R_{sh} = I_m R_m$$

$$\therefore \quad R_{sh} = \frac{I_m R_m}{I_{sh}}$$

but $\quad I_{sh} = I - I_m$

$$\therefore \quad R_{sh} = \frac{I_m R_m}{(I - I_m)}$$

$$\therefore \quad \boxed{R_{sh} = \frac{R_m}{m-1}} \quad \text{where} \quad m = \frac{I}{I_m}$$

The m is called **multiplying power** of the shunt and defined as the ratio of total current to the current through the coil. It can be expressed as,

$$\boxed{m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}}$$

The shunt resistance may consist of a constant temperature resistance wire within the case of the meter or it may be external shunt having low resistance.

Thus to increase the range of ammeter 'm' times, the shunt resistance required is 1/(m–1) times the basic meter resistance. This is nothing but **extension of ranges of an ammeter.**

➡ **Example 1.5 :** *A 2 mA meter with an internal resistance of 100 Ω is to be converted to 0-150 mA ammeter.Calculate the value of the shunt resistance required.*

Solution : Given values are,

$$R_m = 100\ \Omega,\ I_m = 2\text{ mA},\ I = 150\text{ mA}$$

$$R_{sh} = \frac{I_m R_m}{I - I_m}$$

$$\therefore \quad R_{sh} = \frac{2\times10^{-3}\times100}{[150\times10^{-3} - 2\times10^{-3}]}$$

$$= \mathbf{1.351\ \Omega}$$

➡ **Example 1.6 :** *A moving coil ammeter has fixed shunt of 0.01 Ω. With a coil resistance of 750 Ω and a voltage drop of 400 mV across it, the full scale deflection is obtained.*

a) Calculate the current through shunt.

b) Calculate the resistance of meter to give full scale deflection if the shunted current is 50 A.

Solution : a) The drop across the shunt is same as drop across the coil.

$$\therefore \quad I_{sh} R_{sh} = 400 \text{ mV}$$

$$\therefore \quad I_{sh} = \frac{400 \times 10^{-3}}{0.01} = \mathbf{40 \ A}$$

b) The voltage across shunt for shunted current of 50 A is,

$$V_{sh} = I_{sh} R_{sh} = 50 \times 0.01$$

$$= 0.5 \text{ V}$$

For this voltage the meter should give full scale deflection. In first case, the current through meter for full scale deflection was,

$$I_m = \frac{400 \text{ mV}}{R_m} = \frac{400 \times 10^{-3}}{750}$$

$$= 5.33 \times 10^{-4} \text{ A}$$

The same I_m must flow for new voltage across the meter of 0.5 V

$$\therefore \quad I_m R'_m = 0.5$$

$$\therefore \quad 5.33 \times 10^{-4} R'_m = 0.5$$

$$\therefore \quad R'_m = \mathbf{937.5 \ \Omega}$$

This is the resistance of the meter required for 50 A shunted current to give full scale deflection.

1.12 Multirange Ammeters

The range of the basic d.c. ammeter can be extended by using number of shunts and a selector switch. Such a meter is called **multirange ammeter** and is shown in the Fig. 1.26.

R_1, R_2, R_3 and R_4 are four shunts. When connected in parallel with the meter, they can give four different ranges I_1, I_2, I_3 and I_4. The selector switch S is multiposition switch, having low contact resistance and high current carrying capacity. The make before break type switch is used for the range changing.

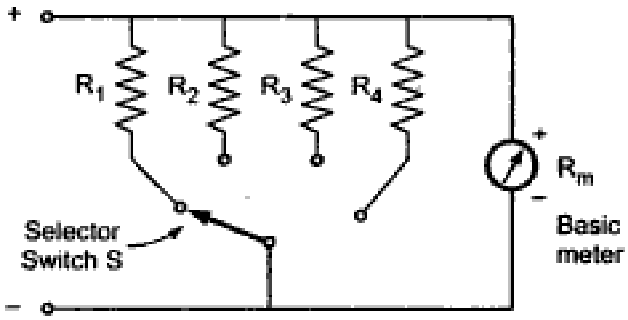


Fig. 1.26 Multirange ammeter

If the ordinary switch is used, while range changing the switch remains open and full current passes through the meter. The meter may get damaged due to such high current. So make before break switch is used. The design of such switch is so that it makes contact with next terminal before completely breaking the contact with the previous terminal.

The multirange ammeters are used for the ranges upto 50 A. While using the multirange ammeter, highest range should be used first and thus the current range should be decreased till good upscale reading is obtained. All the shunts are very precise resistances and hence cost of such multirange ammeter is high.

The mathematical analysis of basic d.c. ammeter is equally applicable to such multirange ammeter. Thus,

$$R_1 = R_m / m_1 - 1$$

$$R_2 = R_m / m_2 - 1 \quad \text{and so on,}$$

where m_1, m_2, m_3........ are the shunt multiplying powers for the currents I_1, I_2, I_3.......

Example 1.7 : *Design a multirange d.c. milliammeter with a basic meter having a resistance 75 Ω and full scale deflection for the current of 2 mA. The required ranges are 0 - 10 mA, 0 - 50 mA and 0 - 100 mA.*

Solution : The first range is 0 - 10 mA,

$$I_1 = 10 \text{ mA}$$

While $I_m = 2 \text{ mA}$ and $R_m = 75\ \Omega$

$$R_1 = \frac{I_m R_m}{(I_1 - I_m)} = \frac{2 \times 75}{(10-2)}$$

$$= \mathbf{18.75\ \Omega}$$

The second range is 0 - 50 mA,

$$\therefore \quad I_2 = 50 \text{ mA}$$

$$R_2 = \frac{I_m R_m}{(I_2 - I_m)} = \frac{2 \times 75}{(50-2)}$$

$$= \mathbf{3.125\ \Omega}$$

The third range is 0 - 100 mA,

$$\therefore \quad I_3 = 100 \text{ mA}$$

$$R_3 = \frac{I_m R_m}{(I_3 - I_m)} = \frac{2 \times 75}{(100-2)} = 1.53\ \Omega$$

The designed multirange ammeter with a selector switch is shown in the Fig. 1.27.

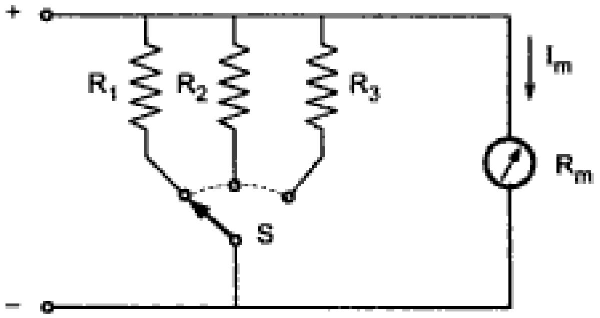


Fig. 1.27

1.13 The Aryton Shunt or Universal Shunt

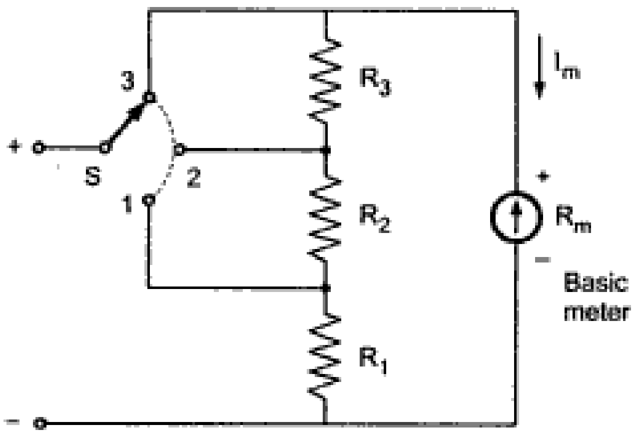


Fig. 1.28 Ammeter with Aryton shunt

We have seen that in multirange ammeter, a make before break switch is must. The aryton shunt or the universal shunt eliminates the possibility of having a meter without a shunt. The meter with the aryton shunt is shown in the Fig. 1.28.

The selector switch S, selects the appropriate shunt required to change the range of the meter. When the position of the switch is '1' then the resistance R_1 is in parallel with the series combination of R_2, R_3 and R_m. Hence current through the shunt is more than the current through the meter, thus protecting the basic meter. When the switch is in the position '2', then the series resistance of R_1 and R_2 is in parallel with the series combination of R_3 and R_m. The current through the meter is more than through the shunt in this position. In the position '3', the resistances R_1, R_2 and R_3 are in series and acts as the shunt. In this position, the maximum current flows through the meter. This increases the sensitivity of the meter.

The voltage drop across the two parallel branches is always equal.

Thus, $I_{sh} R_{sh} = I_m R_m$

But in position 1, R_1 is in parallel with $R_2 + R_3 + R_m$

$$\therefore \quad I_1 [R_1] = I_m [R_2 + R_3 + R_m] \quad \text{... (1)}$$

where I_1 is the first range required.

In position 2, $R_1 + R_2$ is in parallel with $R_3 + R_m$.

$$\therefore \quad I_2 (R_1 + R_2) = I_m (R_3 + R_m) \quad \text{... (2)}$$

where I_2 is the second range required.

In position 3, $R_1 + R_2 + R_3$ is in parallel with R_m.

$$\therefore I_3 (R_1 + R_2 + R_3) = I_m R_m \quad \text{... (3)}$$

where I_3 is the third range required.

The current range I_3 is the minimum while I_1 is the maximum range possible. Solving the equations (1), (2) and (3) the required Aryton shunt can be designed.

➡ **Example 1.8 :** *Design an Aryton shunt to provide an ammeter with the current ranges 1 A, 5 A and 10 A. A basic meter resistance is 50 Ω and fullscale deflection current is 1 mA.*

Solution : The required meter with Aryton shunt is shown in the Fig. 1.29.

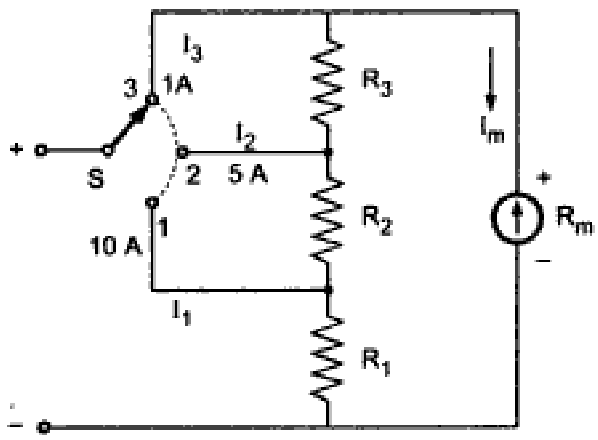


Fig. 1.29

In position 1, R_1 is shunt with $R_2 + R_3 + R_m$,

$$\therefore \quad I_1 R_1 = I_m (R_2 + R_3 + R_m)$$

where $I_1 = 10$ A, $I_m = 1$ mA and $R_m = 50\ \Omega$

$$\therefore \quad 10 R_1 = 1 \times 10^{-3} (R_2 + R_3 + 50)$$

$$\therefore \quad R_1 = 10^{-4} (R_2 + R_3 + 50) \quad \text{... (1)}$$

In position 2, $R_1 + R_2$ is shunt with $R_3 + R_m$,

$\therefore \quad I_2(R_1 + R_2) = I_m(R_3 + R_m)$

where $\quad I_2 = 5\text{ A}$

$\therefore \quad 5(R_1 + R_2) = 1\times10^{-3}(R_3 + 50)$

$\therefore \quad R_1 + R_2 = 2\times10^{-4}(R_3 + 50)$ ... (2)

In position 3, $R_1 + R_2 + R_3$ is shunt with R_m,

$\therefore \; I_3(R_1 + R_2 + R_3) = I_m R_m$

where $\quad I_3 = 1\text{A}$

$\therefore \quad R_1 + R_2 + R_3 = 1\times10^{-3} \times 50$

$\therefore \quad R_1 + R_2 + R_3 = 0.05$ ... (3)

From (3), $\quad R_1 + R_2 = 0.05 - R_3$

Substituting in (2) we get, $0.05 - R_3 = 2\times10^{-4}(R_3 + 50)$

$\therefore \quad 0.05 - R_3 = 2\times10^{-4}R_3 + 0.01$

$\therefore \quad 1.0002\, R_3 = 0.04$

$\therefore \quad R_3 = \mathbf{0.0399\ \Omega}$

$\therefore \quad R_1 + R_2 = 0.05 - 0.0399 = 0.01$

$\therefore \quad R_2 = 0.01 - R_1$

Substituting in (1), $R_1 = 10^{-4}[\,0.01 - R_1 + 0.0399 + 50]$

$\therefore \quad R_1 = 10^{-6} - 10^{-4}R_1 + 5\times10^{-3}$

$\therefore \quad 1.0001R_1 = 5.00139\times10^{-3}$

$\therefore \quad R_1 = \mathbf{0.005\ \Omega}$

$\therefore \quad R_2 = \mathbf{0.005\ \Omega}$

Thus the various sections of the Aryton shunt are 0.005 Ω, 0.005 Ω and 0.0399 Ω.

1.13.1 Precautions to be taken while using an Ammeter

The following precautions must be taken while using an ammeter :

1) As the ammeter resistance is very low, it should never be connected across any source of e.m.f. Always connect an ammeter in series with the load.
2) The polarities must be observed correctly. The opposite polarities deflect the pointer in opposite direction against the mechanical stop and this may damage the pointer.

3) While using multirange ammeter, first use the highest current range and then decrease the current range until sufficient deflection is obtained. So to increase the accuracy, finally select the range which will give the reading near full scale deflection.

1.14 Requirements of a Shunt

1) The temperature coefficient of shunt and the meter should be low and should be as equal as possible.
2) The shunt resistances should be stable and constant with time.
3) The shunt resistances should not carry currents which will cause excessive temperature rise.
4) The type of material used to join the shunts should have low thermo dielectric voltage drop i.e. the soldering of joints should not cause a voltage drop.
5) Due to the soldering, the values of resistance should not be change.
6) The resistances should have low thermal electromotive force with copper.

The manganin is usually used for the shunts of d.c. instruments while the constantan is useful for the shunts of a.c. instruments.

1.15 Basic D.C. Voltmeter

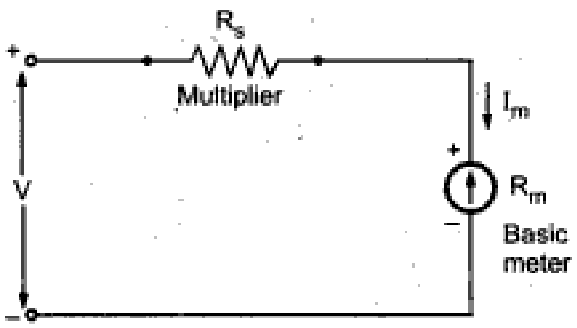


Fig. 1.30 Basic d.c. voltmeter

The basic d.c. voltmeter is nothing but a PMMC D'Arsonoval galvanometer. The resistance is required to be connected in series with the basic meter to use it as a voltmeter. This series resistance is called a **multiplier**. The main function of the multiplier is to limit the current through the basic meter so that the meter current does not exceed the full scale deflection value. The voltmeter measures the voltage across the two points of a circuit or a voltage across a circuit component. The basic d.c. voltmeter is shown in the Fig. 1.30.

The voltmeter must be connected across the two points or a component, to measure the potential difference, with the proper polarity.

The multiplier resistance can be calculated as :

Let R_m = internal resistance of coil i.e. meter

R_s = series multiplier resistance

I_m = full scale deflection current

V = full range voltage to be measured

From Fig. 1.30, $\therefore \quad V = I_m (R_m + R_s)$

$\therefore \quad V = I_m R_m + I_m R_s$

$\therefore \quad I_m R_s = V - I_m R_m$

$$\therefore \quad \boxed{R_s = \frac{V}{I_m} - R_m}$$

The **multiplying factor** for multiplier is the ratio of full range voltage to be measured and the drop across the basic meter.

Let $\quad v$ = drop across the basic meter = $I_m R_m$

$\therefore \quad m$ = multiplying factor = $\frac{V}{v}$

$$= \frac{I_m (R_m + R_s)}{I_m R_m}$$

$$\therefore \quad \boxed{m = 1 + \frac{R_s}{R_m}}$$

Hence multiplier resistance can also be expressed as,

$$R_s = (m - 1) R_m$$

Thus to increase the range of voltmeter 'm' times, the series resistance required is (m–1) times the basic meter resistance. This is nothing but **extension of ranges of a voltmeter.**

➡ **Example 1.9 :** *A moving coil instrument gives a full scale deflection with a current of 40 μA, while the internal resistance of the meter is 500 Ω. It is to be used as a voltmeter to measure a voltage range of 0 – 10 V. Calculate the multiplier resistance needed.*

Solution : Given values are, $R_m = 500\ \Omega$, $I_m = 40\ \mu A$ and $V = 10\ V$

Now $\quad R_s = \frac{V}{I_m} - R_m = \frac{10}{40 \times 10^{-6}} - 500 = \mathbf{249.5\ k\Omega}$

This is the required multiplier resistance.

Example 1.10 : *A moving coil instrument gives a full scale deflection for a current of 20 mA with a potential difference of 200 mV across it. Calculate : i) Shunt required to use it as an ammeter to get a range of 0 - 200 A ii) Multiplier required to use it as a voltmeter of range 0 - 500 V*

Solution : The meter current $I_m = 20$ mA

Drop across meter , $V_m = 200$ mV

Now $V = I_m R_m$

$\therefore \quad 200 \text{ mV} = (20 \text{ mA}) R_m$

$\therefore \quad R_m = 10\ \Omega$

i) For using it as an ammeter, I = 200 A

$$R_{sh} = \frac{I_m R_m}{I - I_m} = \frac{20 \times 10^{-3} \times 10}{200 - 20 \times 10^{-3}}$$

$$= \mathbf{0.001\ \Omega}$$

This is the required shunt.

ii) For using it as a voltmeter, $V = 500$ V

$$\therefore \quad R_s = \frac{V}{I_m} - R_m$$

$$= \frac{500}{20 \times 10^{-3}} - 10 = \mathbf{24.99\ k\Omega}$$

This is the required multiplier.

1.16 Multirange Voltmeters

The range of the basic d.c. voltmeter can be extended by using number of multipliers and a selector switch. Such a meter is called **multirange voltmeter** and is shown in the Fig. 1.31.

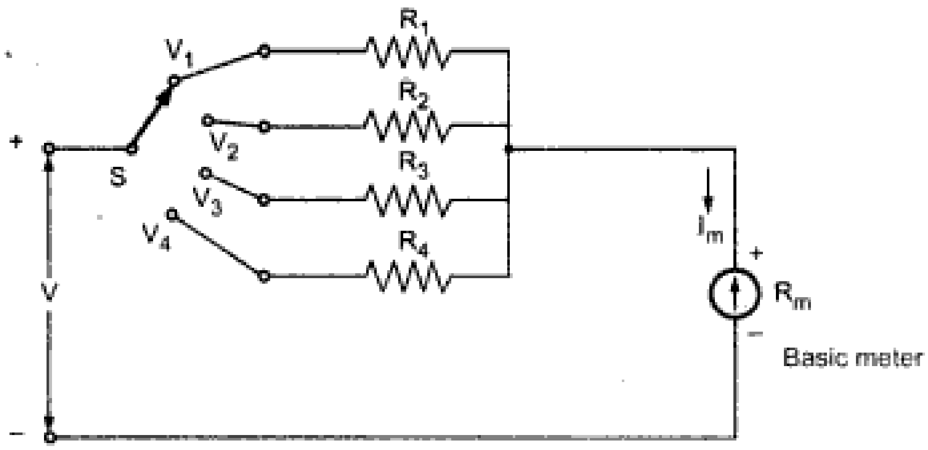


Fig. 1.31 Multirange voltmeter

The R_1, R_2, R_3 and R_4 are the four series multipliers. When connected in series with the meter, they can give four different voltage ranges as V_1, V_2, V_3 and V_4. The selector switch S is multiposition switch by which the required multiplier can be selected in the circuit.

The mathematical analysis of basic d.c. voltmeter is equally applicable for such multirange voltmeter. Thus,

$$R_1 = \frac{V_1}{I_m} - R_m \qquad R_2 = \frac{V_2}{I_m} - R_m \qquad \text{and so on.}$$

1.16.1 Practical Multirange Voltmeter

More practical arrangement of multiplier resistances is shown in the Fig. 1.32.

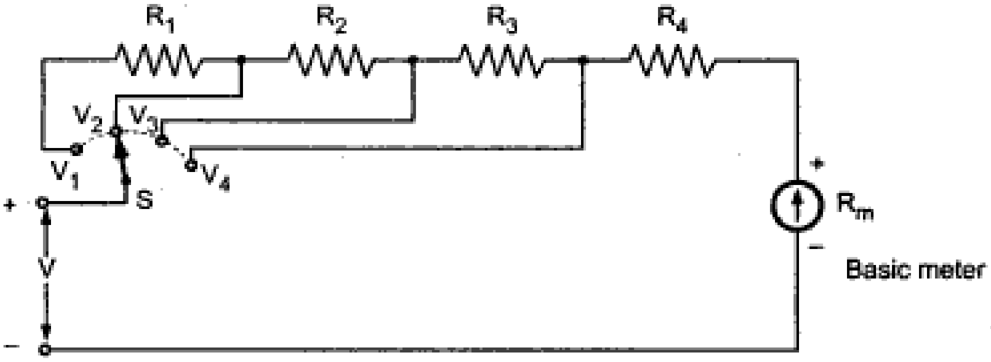


Fig. 1.32

In this arrangement, the multipliers are connected in a series string. The connections are brought out from the junctions of the resistances. The selector switch is used to select the required voltage range.

When the switch S is at position V_1, $R_1 + R_2 + R_3 + R_4$ acts as a multiplier resistance. While when the switch S is at position V_4 then the resistance R_4 only acts as multiplier resistance. The V_4 is the lowest voltage range while V_1 is the maximum voltage range.

The multiplier resistances can be calculated as :

In position V_4, the multiplier is R_4 only. The total resistance of the circuit is say R_T.

$$R_T = \frac{V_4}{I_m}$$

$$\therefore \quad \boxed{R_4 = R_T - R_m} \qquad \text{... (1)}$$

In position V_3, the multiplier is $R_3 + R_4$

$$\therefore \quad R_T = V_3 / I_m$$

$$\therefore \quad R_3 + R_4 = R_T - R_m$$

$$\therefore \quad \boxed{R_3 = R_T - (R_m + R_4)} \quad \text{... (2)}$$

In position V_2, the multiplier is $R_2 + R_3 + R_4$

$$\therefore \quad R_T = V_2 / I_m$$

$$\therefore \quad (R_2 + R_3 + R_4) = R_T - R_m$$

$$\therefore \quad \boxed{R_2 = R_T - (R_m + R_3 + R_4)} \quad \text{... (3)}$$

In position V_1, the multiplier is $R_1 + R_2 + R_3 + R_4$

$$\therefore \quad R_T = \frac{V_1}{I_m}$$

$$\therefore \quad R_1 + R_2 + R_3 + R_4 = R_T - R_m$$

$$\therefore \quad \boxed{R_1 = R_T - (R_m + R_2 + R_3 + R_4)} \quad \text{... (4)}$$

Using the equations (1), (2), (3) and (4) multipliers can be designed. The advantage of this arrangement is that the multiplier except R_4 have standard resistance values and can be obtained commercially in precision tolerances. The first resistance i.e. R_4 only is the resistance having special value and must be manufactured specially to meet the circuit requirements.

Example 1.11 : *A basic D'Arsonoval movement with an internal resistance of 50 Ω and a full scale deflection current of 2 mA is to be used as a multirange voltmeter. Design the series string of multipliers to obtain the voltage ranges of 0 - 10 V, 0 - 50 V, 0 - 100 V, 0 - 500 V.*

Solution : The arrangement is shown in the Fig. 1.33.

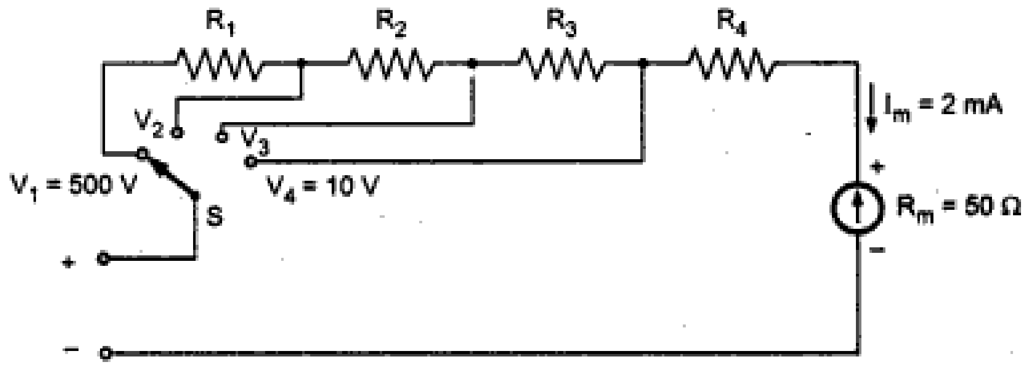


Fig. 1.33

For a meter, $R_m = 50\,\Omega$ and $I_m = 2$ mA

For position $V_4 = 10$ V, Series multiplier is R_4.

$$\therefore \quad R_4 = \frac{V_4}{I_m} - R_m = \frac{10}{2\times10^{-3}} - 50 \qquad \text{.... as } R_s = \frac{V}{I_m} - R_m$$

$$= \mathbf{4.95\ k\Omega}$$

For position $V_3 = 50$ V, series multiplier is $(R_4 + R_3)$.

$$\therefore \quad (R_4 + R_3) = \frac{V_3}{I_m} - R_m$$

$$R_3 = \frac{50}{2\times10^{-3}} - 50 - 4950$$

$$= \mathbf{20\ k\Omega}$$

For position $V_2 = 100$ V, series multiplier is $(R_2 + R_3 + R_4)$.

$$\therefore \quad (R_2 + R_3 + R_4) = \frac{V_2}{I_m} - R_m$$

$$\therefore \quad R_2 = \frac{100}{2\times10^{-3}} - 50 - 4950 - 20000 = \mathbf{25\ k\Omega}$$

For position $V_1 = 500$ V, multiplier is $(R_1 + R_2 + R_3 + R_4)$.

$$\therefore \quad (R_1 + R_2 + R_3 + R_4) = \frac{V_1}{I_m} - R_m$$

$$\therefore \quad R_1 = \frac{500}{2\times10^{-3}} - 50 - 4950 - 25000 - 20000$$

$$\therefore \quad R_1 = \mathbf{200\ k\Omega}$$

Thus R_1, R_2, R_3 and R_4 forms a series string of multipliers.

1.17 Sensitivity of Voltmeters

In a multirange voltmeter, the ratio of the total resistance R_T to the voltage range remains same. This ratio is nothing but the reciprocal of the full scale deflection current of the meter i.e. $1/I_m$. This value is called **sensitivity** of the voltmeter.

Thus the sensitivity of the voltmeter is defined as,

$$S = \frac{1}{\text{Full scale deflection current}}$$

$$\therefore \quad \boxed{S = \frac{1}{I_m}\ \Omega/\text{V or k}\Omega/\text{V}}$$

Key Point : *The sensitivity range is specified on the meter dial and it indicates the resistance of the meter for a one volt range.*

The internal resistance of the voltmeter is not the same in each of its ranges. The higher is the range of the voltmeter, greater is its internal resistance. Internal resistance of a voltmeter can be obtained from its sensitivity as,

Internal resistance of voltmeter = Maximum voltage (range) × Sensitivity in Ω/V

The sensitivity is useful in calculating the resistance of a multiplier in d.c voltmeter.

Consider the practical multirange voltmeter circuit shown in the Fig. 1.34.

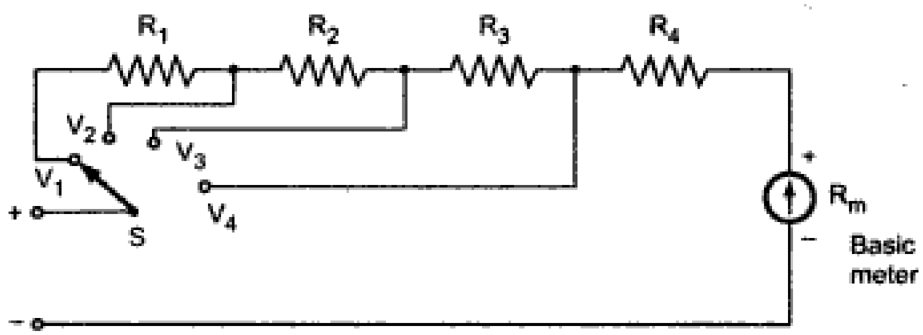


Fig. 1.34

Let S = sensitivity rating in Ω/V

R_m = internal resistance of basic meter or coil

Then the multiplier resistance can be obtained as,

$$R_1 = S\,V_1 - (R_m + R_2 + R_3 + R_4)$$

$$R_2 = S\,V_2 - (R_m + R_3 + R_4)$$

$$R_3 = S\,V_3 - (R_m + R_4)$$

$$R_4 = S\,V_4 - R_m$$

where V_1, V_2, V_3 and V_4 are the required voltage ranges.

Key Point : *This method is called the* ***sensitivity method*** *of calculating the multiplier resistances.*

Example 1.12 : *Solve the Example 1.11, by the sensitivity method.*

Solution : The basic meter has $R_m = 50\ \Omega$ and $I_m = 2$ mA

Now $$S = \frac{1}{I_m} = \frac{1}{2\times10^{-3}} = 500\ \Omega/V$$

while $V_1 = 500$ V, $V_2 = 100$ V, $V_3 = 50$ V, $V_4 = 10$ V

$\therefore$ $$R_4 = S\,V_4 - R_m = 500 \times 10 - 50$$

$$= \mathbf{4.95\ k\Omega}$$

then $R_3 = SV_3 - (R_m + R_4)$

$= 500 \times 50 - (50 + 4.95 \times 10^3) = \mathbf{20\ k\Omega}$

then $R_2 = SV_2 - (R_m + R_3 + R_4)$

$= 500 \times 100 - (50 + 20 \times 10^3 + 4.95 \times 10^3)$

$= \mathbf{25\ k\Omega}$

and $R_1 = SV_1 - (R_m + R_2 + R_3 + R_4)$

$= 500 \times 500 - (50 + 25 \times 10^3 + 20 \times 10^3 + 4.95 \times 10^3)$

$= \mathbf{200\ k\Omega}$

These are the same values as obtained before in Example 1.11.

Example 1.13 : *Calculate the value of the multiplier resistance on the 500 V range of a d.c. voltmeter, that uses 50 μA meter movement with an internal resistance of 200 Ω.*

Solution : The sensitivity of the meter is,

$$S = \frac{1}{I_m} = \frac{1}{50 \times 10^{-6}} = 20000 \quad \Omega/V$$

Now $R_s = SV - R_m$

where V = voltage range

$\therefore$ $R_s = 20000 \times 500 - 200 = \mathbf{9.99\ M\Omega}$

Example 1.14 : *The meter A has a range of 0 - 100 V and multiplier resistance of 25 kΩ. The meter B has a range 0 - 1000 V and a multiplier resistance of 150 kΩ. Both meters have basic meter resistance of 1 kΩ. Which meter is more sensitive ?*

Solution : For meter A, $R_s = 25\ k\Omega$, $R_m = 1\ k\Omega$, $V = 100\ V$

Now $R_s = SV - R_m$

$\therefore$ $25 \times 10^3 = S \times 100 - 1 \times 10^3$

$\therefore$ $S = 260\ \Omega/V$

For meter B, $R_s = 150\ k\Omega$, $R_m = 1\ k\Omega$, $V = 1000\ V$

$R_s = SV - R_m$

$\therefore$ $150 \times 10^3 = S \times 1000 - 1 \times 10^3$

$\therefore$ $S = 151\ \Omega/V$

The meter A is more sensitive than meter B.

1.17.1 Loading Effect

While selecting a meter for a particular measurement, the sensitivity rating is very important. A low sensitive meter may give the accurate reading in low resistance circuit but will produce totally inaccurate reading in high resistance circuit.

The voltmeter is always connected across the two points between which the potential difference is to be measured. If it is connected across a low resistance then as voltmeter resistance is high, most of the current will pass through a low resistance and will produce the voltage drop which will be nothing but the true reading. But if the voltmeter is connected across the high resistance then due to two high resistances in parallel, the current will divide almost equally through the two paths. Thus the meter will record the voltage drop across the high resistance which will be much lower than the true reading.

Thus the low sensitivity instrument when used in high resistance circuit gives a lower reading than the true reading. This is called **loading effect** of the voltmeters. It is mainly caused due to low sensitivity instruments.

➡ **Example 1.15 :** *The Fig. 1.35 shows a simple series circuit of R_1 and R_2 connected to a 250 V d.c. source. If the voltage across R_2 is to be measured by the voltmeters having*

i) a sensitivity of 500 Ω/V ii) a sensitivity of 10,000 Ω/V

Find which voltmeter will read more accurately. Both the meters are used on the 150 V range.

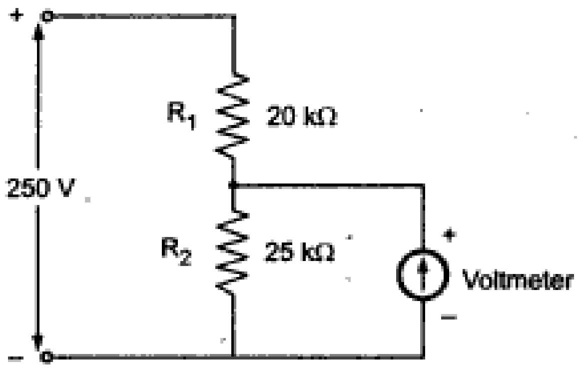


Fig. 1.35

Solution : By the voltage divider rule, the voltage across R_2 is,

$$V = \frac{250}{(20+25)} \times 25$$

$$= 138.88 \text{ V}$$

This is the true voltage across R_2.

Case i) $S = 500\ \Omega/V$

The voltmeter resistance will be,

$$R_V = S \times V = 500 \times 150$$

$$= 75\ k\Omega$$

$$\therefore \quad R_{eq} = R_2 \parallel R_V$$

$$= \frac{25 \times 75}{(25+75)}$$

$$= 18.75\ k\Omega$$

Hence the voltage across R_{eq} is,

$$V = \frac{R_{eq}}{(R_{eq}+R_1)} \times 250$$

$$= \frac{18.75}{(18.75+20)} \times 250$$

$$= 120.96\ V$$

Thus first voltmeter will read **120.96 V**.

Case ii) $S = 10{,}000\ \Omega/V$

The voltmeter resistance will be

$$R_V = S\ V$$

$$= 10000 \times 150 \qquad \text{as voltage range 150 V}$$

$$= 1.5\ M\Omega$$

$$\therefore \quad R_{eq} = R_2 \parallel R_V$$

$$= \frac{25 \times 1.5 \times 10^6 \times 10^3}{(25 \times 10^3 + 1.5 \times 10^6)} = 24.59\ k\Omega$$

Hence the voltage across R_{eq} is,

$$V = \frac{R_{eq}}{(R_{eq}+R_1)} \times 250 = \frac{24.59}{(24.59+20)} \times 250$$

$$= 137.86\ V$$

Thus the second voltmeter reads more accurately.

Key Point : *Thus the high sensitivity voltmeter gives more accurate reading, though the voltage range for both the meters is same.*

Example 1.16 : *Two different voltmeters are used to measure the voltage across R_b shown in the Fig. 1.36.*

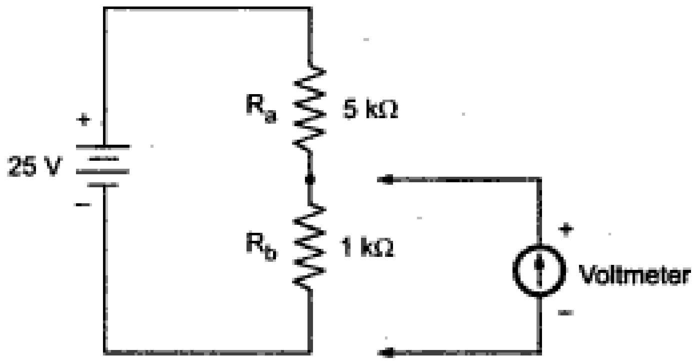


Fig. 1.36

The two meters used are as follows :

i) Meter with sensitivity 1 kΩ / V and range 5 V.

ii) Meter with sensitivity 20 kΩ / V and range 5 V.

Calculate : a) True voltage across R_b

b) Reading on voltmeter 1

c) Reading on voltmeter 2

d) % error in the two voltmeters

e) % accuracy of the two voltmeters

Solution : a) By voltage divider rule, the true voltage across R_b without any meter is,

$$V = \frac{R_b}{R_a + R_b} \times 25$$

$$= \frac{1}{(1+5)} \times 25$$

$$= \mathbf{4.167\ V}$$

b) Consider first voltmeter with S = 1 kΩ/V

$$\therefore \quad R_V = S \times V_{range} = 1 \times 5$$

$$= \mathbf{5\ k\Omega}$$

Thus circuit becomes,

$$R_{eq} = R_b \,||\, R_V = \frac{1 \times 5}{1+5}$$

$$= 0.833\ k\Omega$$

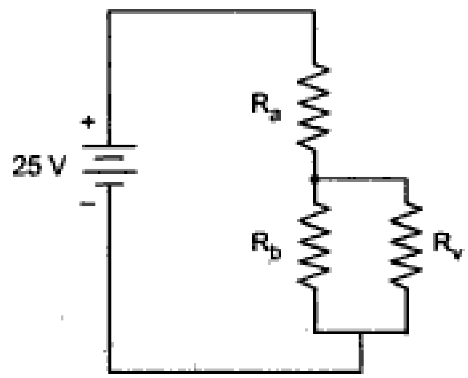


Fig. 1.37

Hence the voltmeter reading is,

$$V_1 = \frac{R_{eq}}{(R_{eq}+R_a)} \times 25 = \frac{0.833}{(5+0.833)} \times 25$$

$$= \mathbf{3.571\ V}$$

c) Consider second voltmeter with S = 20 kΩ/V

$$\therefore \quad R_V = S \times V_{range} = 20 \times 5$$

$$= 100\ k\Omega$$

The circuit becomes as shown in the Fig. 1.37 (a) earlier, now with R_V = 100 kΩ.

$$\therefore \quad R_{eq} = R_b \,||\, R_V = \frac{100 \times 1}{(100+1)}$$

$$= 0.99\ k\Omega$$

Hence the voltage reading is,

$$V_2 = \frac{R_{eq}}{(R_a+R_{eq})} \times 25$$

$$= \frac{0.99}{(0.99+5)} \times 25$$

$$= \mathbf{4.132\ V}$$

d) The percentage error can be calculated as :

$$\%\text{ error in voltmeter 1} = \frac{\text{true value} - \text{measured value}}{\text{true value}} \times 100$$

$$= \frac{4.167 - 3.571}{4.167} \times 100$$

$$= \mathbf{14.3\%}$$

$$\% \text{ error in voltmeter 2} = \frac{4.167 - 4.132}{4.167} \times 100$$

$$= \mathbf{0.84\%}$$

e) The percentage accuracy can be obtained as :

% A for voltmeter 1= 100 – % error = 100 – 14.3 = **85.7%**

% A for voltmeter 2= 100 – 0.84 = **99.16%**

Thus voltmeter 2 is 99.16% accurate while voltmeter 1 is 85.7% accurate.

1.17.2 Precautions to be taken while using a Voltmeter

The following precautions must be taken while using a voltmeter :

1) The voltmeter resistance is very high and it should always be connected across the circuit or component whose voltage is to be measured.
2) The polarities must be observed correctly. The wrong polarities deflect the pointer in the opposite direction against the mechanical stop and this may damage the pointer.
3) While using the multirange voltmeter, first use the highest range and then decrease the voltage range until the sufficient deflection is obtained.
4) Take care of the loading effect. The effect can be minimised by using high sensitivity voltmeters.

1.17.3 Requirements of a Multiplier

1) Their resistance should not change with time.
2) The change in their resistance with temperature should be small.
3) They should be non-inductively wound for a.c. meters.

Commonly used resistive materials for construction of multiplier are manganin and constantan.

1.18 Ammeter and Voltmeter

The meters which are connected in series with the circuit whose current is to be measured are called **ammeters**. The power loss in ammeter is I^2R_a where R_a is ammeter resistance. To have low power loss, ammeter resistance must be very low.

The meters which are connected in parallel with the circuit whose voltage is to be measured are called **voltmeters.** The power loss in voltmeters is V^2/R_V where R_V is voltmeter resistance. To have low power loss, voltmeter resistance must be very high.

The construction and working principle of both the meters is same. Both are basically current sensing devices but they have following differences :

Sr. No.	Ammeter	Voltmeter
1.	It is a current measuring device which measures current through circuit.	It is a voltage measuring device which measures potential difference between the two points of a circuit.
2.	Always connected in series with circuit.	Always connected in parallel with the circuit.
3.	The resistance is very very small.	The resistance is very very high.
4.	Deflecting torque is produced by current to be measured directly.	Deflecting torque is produced by a current which is proportional to the voltage to be measured.

Table 1.3

1.19 Electrostatic Instruments

Basically electrostatic instruments are all voltmeters. Practically such instruments may be used for measurement of current and power but both the types of measurements require measurement of voltage across a known impedance. The main advantage of such instruments is the measurement of high voltages in both a.c. and d.c. circuits without any errors due to eddy current losses and hysteresis.

1.19.1 Principle of Operation

The operation of all the electrostatic instruments is based on the principle that there exists a force between the two plates with opposite charges. This force can be obtained using the principle that the mechanical work done is equal to the stored energy if there is a relative motion of plates.

Consider two plates A and B where plate A is fixed while B is movable. Two plates are oppositely charged and plate B is restrained by a spring connected to fixed point. Let the force of attraction between the two plates be F newton. Let the capacitance between the two plates be C farad.

The energy stored E is the given by,

$$E = \frac{1}{2} C V^2 \text{ J} \qquad \dots (1)$$

When applied voltage increases by dV, the current flowing through capacitance also changes and it is given by,

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV)$$

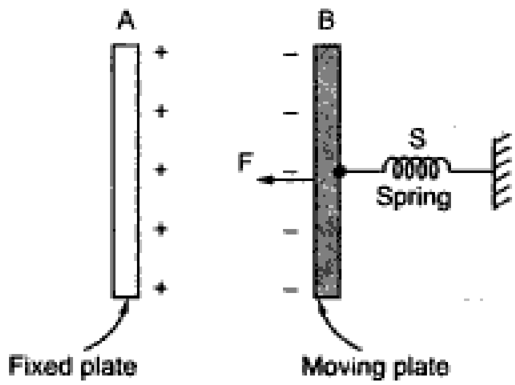


Fig. 1.38 Principle of electrostatic voltmeter

$$\therefore \quad i = C\frac{dV}{dt} + V\frac{dC}{dt} \qquad \text{... (2)}$$

The input energy is given by,

$$V\,i\,dt = CV\,dV + V^2\,dC \qquad \text{... (3)}$$

Also due to change in applied voltage by value dV, the capacitance increases by dC. Because plate B moves towards a fixed plate A which decreases the distance of separation between two plates increasing net capacitance.

Thus the new energy stored is given by,

$$E' = \frac{1}{2}(C+dC)(V+dV)^2 \qquad \text{... (4)}$$

The change in stored energy is given by,

$$E' - E = \frac{1}{2}(C+dC)(V+dV)^2 - \frac{1}{2}CV^2$$

$$= \frac{1}{2}(C+dC)(V^2+2V\,dV+dV^2) - \frac{1}{2}CV^2$$

$$= \frac{1}{2}CV^2 + CV\,dV + \frac{1}{2}C\,dV^2 + \frac{1}{2}V^2\,dC + 2\,VdV\,dC$$

$$+\frac{1}{2}dC\,dV^2 - \frac{1}{2}CV^2$$

Neglecting higher order terms of small quantities such as dC and dV, we can write,

$$E' - E = \frac{1}{2}V^2\,dC + CV\,dV \qquad \text{... (5)}$$

From the principle of the conservation of energy, we can write,

input energy = increment in stored energy + mechanical work done.

$$\therefore \quad CV\,dV + V^2\,dC = \left(\frac{1}{2}V^2\,dC + CV\,dV\right) + (F\,dx)$$

$$\therefore \quad F\,dx = \frac{1}{2}V^2\,dC$$

$$\therefore \quad \boxed{F = \frac{1}{2}V^2\frac{dC}{dx}} \quad \ldots (6)$$

From above expression it is clear that the force of attraction is directly proportional to the square of the applied voltage V.

The above theory can be extended to the rotational motion, with the angular deflection θ in place of the linear displacement x.

$$\therefore \quad T_d = \frac{1}{2}V^2\frac{dC}{d\theta} \quad \ldots (7)$$

If the meter uses the spring control with torsional spring constant K then,

$$T_c = K\theta \quad \ldots (8)$$

But $\quad T_d = T_c$ for steady position

$$\therefore \quad \frac{1}{2}V^2\frac{dC}{d\theta} = K\theta$$

$$\therefore \quad \boxed{\theta = \frac{1}{2K}V^2\frac{dC}{d\theta}} \quad \ldots (9)$$

1. Such an instrument can be used for a.c. and d.c. measurements as the deflection is proportional to the square of the voltage to be measured.
2. It shows square law response hence the scale is nonuniform which is compressed at the lower end.

1.20 Types of Electrostatic Voltmeters

Following are the two types of electrostatic voltmeter.

i) **Quadrant type electrostatic voltmeter** which is used to measure voltages upto 10 kV to 20 kV.

ii) **Attracted disc type electrostatic voltmeter** which is used to measure voltages above 20 kV.

1.20.1 Quadrant Type Electrostatic Voltmeter

The instrument consists of four fixed metal double quadrants arranged such that there is a small air gap between the quadrants and the total assembly forms shallow circular box. Inside this box a double sectored needle is suspended by means of a phosphor bronze thread. The needle is suspended such that it is placed equidistant from above and below quadrant plates as shown in the Fig. 1.39.

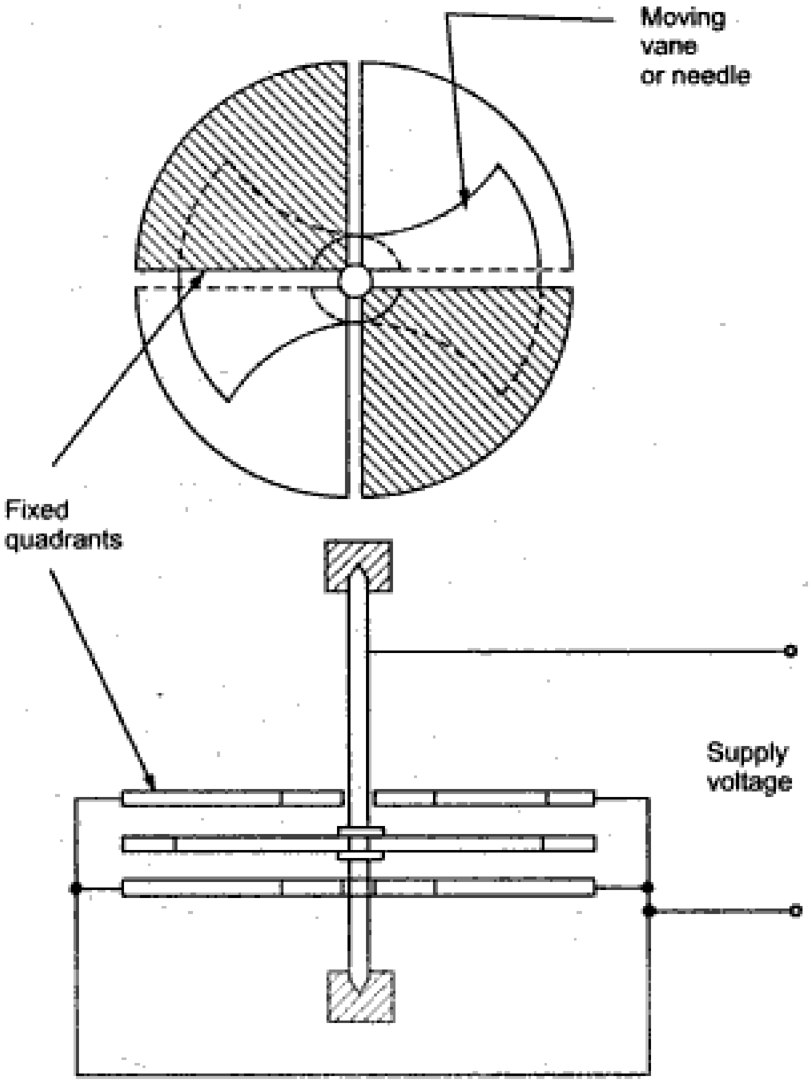


Fig. 1.39 Quadrant electrometer

As shown in the above Fig. 1.39 the fixed quadrants are connected together. The voltage to be measured either a.c. or d.c. is connected between the fixed quadrants and the moving needle. This needle rotates due to the electrostatic force set up due to the charge accumulation on the quadrant plates. Then the suspension exerts a controlling torque and the needle settles at the position where both the torques, controlling and deflection, are equal.

There are two types of the electrical connections in the quadrant electrometer,

i) Heterostatic connection

ii) Idiostatic connection

1.20.2 Heterostatic Connection

In this type of connection, a high voltage battery is used to charge the needle to a voltage considerably higher than the voltage to be measured. The connection diagram is as shown in the Fig. 1.40.

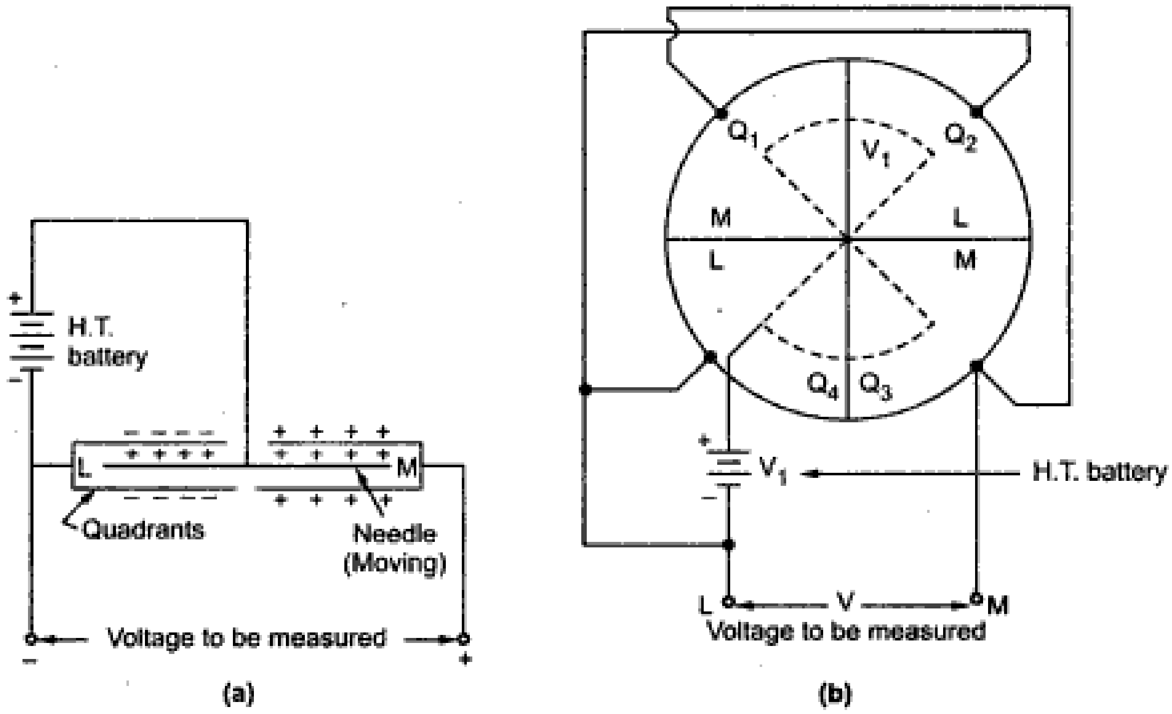


Fig. 1.40 Heterostatic connection

In this connection, the quadrants are connected together in diagonally opposite pairs. The moving vane i.e. needle is positively charged due to battery. The deflecting force due to top and bottom quadrants on movable needle cancels each other on both sides. The only deflecting force responsible is force of attraction between left quadrant and right moving sector and force of repulsion between right quadrant and left moving sector.

1.20.2.1 Theory of Heterostatic Connection

To obtain the torque equation for the heterostatic connection, consider only one half portion of the needle with two quadrants adjacent to it. This is shown in the Fig. 1.41.

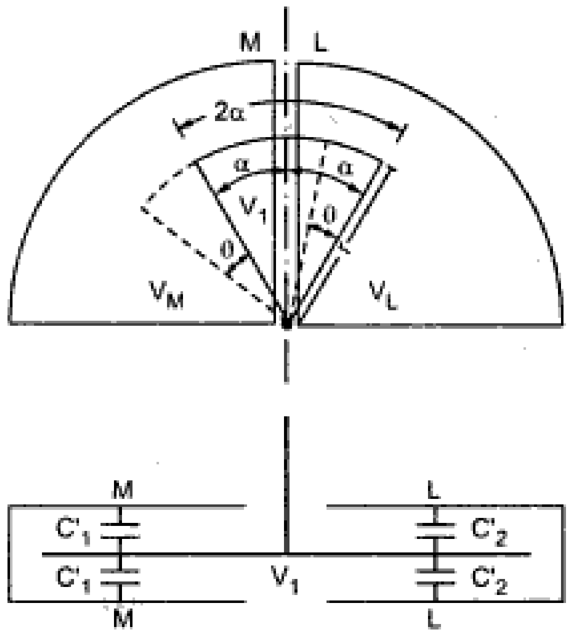


Fig. 1.41 Position of needle with some deflection

The needle is considered as a sector of circle with radius r. Now this arrangement of two quadrants with needle exactly in between, resembles the two capacitors placed side by side. At equilibrium position, as needle is placed symmetrically, the capacitances C_1 and C_2 are equal. But when needle rotates, the value of one capacitor becomes greater than other.

Let V_1 = Potential of needle, V = Voltage being measured

V_L = Potential of quadrant L, V_M = Potential of quadrant M

Let the needle deflects in anticlockwise direction through an angle θ.

Let C_1 = Capacitance of left hand capacitor

C_2 = Capacitance of right hand capacitor

d = Distance of needle from either top or bottom plates of quadrants

$$\therefore \quad C_1' = \frac{\varepsilon A}{d} \qquad \ldots (1)$$

$$A = \text{Area of vane} = \frac{r \times (\alpha + \theta) \times r}{2} = \frac{1}{2} r^2 (\alpha + \theta)$$

Note that needle spans through an angle (α +θ) under quadrant M and half as there are two faces of vanes.

$$\therefore \quad C_1' = \frac{\varepsilon r^2(\alpha + \theta)}{2d} \quad \text{... (2)}$$

The two capacitances C_1' are in parallel hence,

$$C_1 = 2\,C_1' = \frac{\varepsilon r^2(\alpha + \theta)}{d} \quad \text{... (3)}$$

And

$$C_2 = 2\,C_2' = \frac{\varepsilon r^2(\alpha - \theta)}{d} \quad \text{... (4)}$$

The energy stored in $C_1 = \frac{1}{2} C_1 (V_1 - V_L)^2$

The energy stored in $C_2 = \frac{1}{2} C_2 (V_1 - V_M)^2$

The total energy stored is,

$$W = \frac{1}{2} C_1 (V_1 - V_L)^2 + \frac{1}{2} C_2 (V_1 - V_M)^2 \quad \text{... (5)}$$

Let T_θ be the torque in the position θ then for an infinitesimal change $d\theta$ of the needle, the work done in moving system is $T_\theta\, d\theta$. This work done is equal to the increase in the stored energy dW.

$$\therefore \quad T_\theta d\theta = dW$$

$$\therefore \quad T_\theta = \frac{dW}{d\theta} \quad \text{... (6)}$$

$$\therefore \quad T_\theta = \frac{d}{d\theta}\left[\frac{1}{2} C_1 (V_1 - V_L)^2 + \frac{1}{2} C_2 (V_1 - V_M)^2\right]$$

$$\therefore \quad T_\theta = \frac{1}{2}(V_1 - V_L)^2 \frac{dC_1}{d\theta} + \frac{1}{2}(V_1 - V_M)^2 \frac{dC_2}{d\theta} \quad \text{... (7)}$$

Now

$$\frac{dC_1}{d\theta} = \frac{d}{d\theta}\left[\frac{\varepsilon r^2(\alpha + \theta)}{d}\right] = \frac{\varepsilon r^2}{d}$$

$$\frac{dC_2}{d\theta} = \frac{d}{d\theta}\left[\frac{\varepsilon r^2(\alpha - \theta)}{d}\right] = -\frac{\varepsilon r^2}{d}$$

Using in the equation (7),

$$T_\theta = \frac{1}{2}(V_1 - V_L)^2 \times \frac{\varepsilon r^2}{d} - \frac{1}{2}(V_1 - V_M)^2 \frac{\varepsilon r^2}{d}$$

$$\therefore \quad T_\theta = \frac{\varepsilon r^2}{2d}\left[(V_1 - V_L)^2 - (V_1 - V_M)^2\right] \quad \text{... (8)}$$

But the medium is air hence $\varepsilon = \varepsilon_0$,

$$\therefore \quad T_\theta = \frac{\varepsilon_0 r^2}{2d} \{(V_M - V_L)[2V_1 - (V_L + V_M)]\} \quad \ldots (9)$$

The above expression is obtained considering only two quadrants and half needle hence for all four quadrants the deflecting torque will be **doubled**.

$$\therefore \quad \boxed{T_\theta = \frac{\varepsilon_0 r^2}{d} \{(V_M - V_L)[2V_1 - (V_L + V_M)]\}} \quad \ldots (10)$$

Key Point : *The torque is positive only when* $2V_1 > (V_L + V_M)$*. Now V is the potential to be measured and is equal to* $V_M - V_L$.

$$\therefore \quad T_\theta = \frac{\varepsilon_0 r^2}{2d} \{V[2V_1 - (V_L + V_M)]\} \quad \ldots (11)$$

If potential of needle V_1 is very large compared to voltage to be measured then,

$$T_\theta = \frac{\varepsilon_0 r^2}{d} \times V \times 2V_1$$

$$\therefore \quad \boxed{T_\theta \; \alpha \; 2\,V\,V_1} \quad \ldots \text{For heterostatic with V, large}$$

Key Point : *Thus in heterostatic connection, the* ***uniform scale*** *is obtained.*

1.20.3 Idiostatic Connection

This is connection generally used in commercial instruments. In this type of connection, needle is connected to any one of the pairs of quadrant as shown in the Fig. 1.42, directly without external voltage.

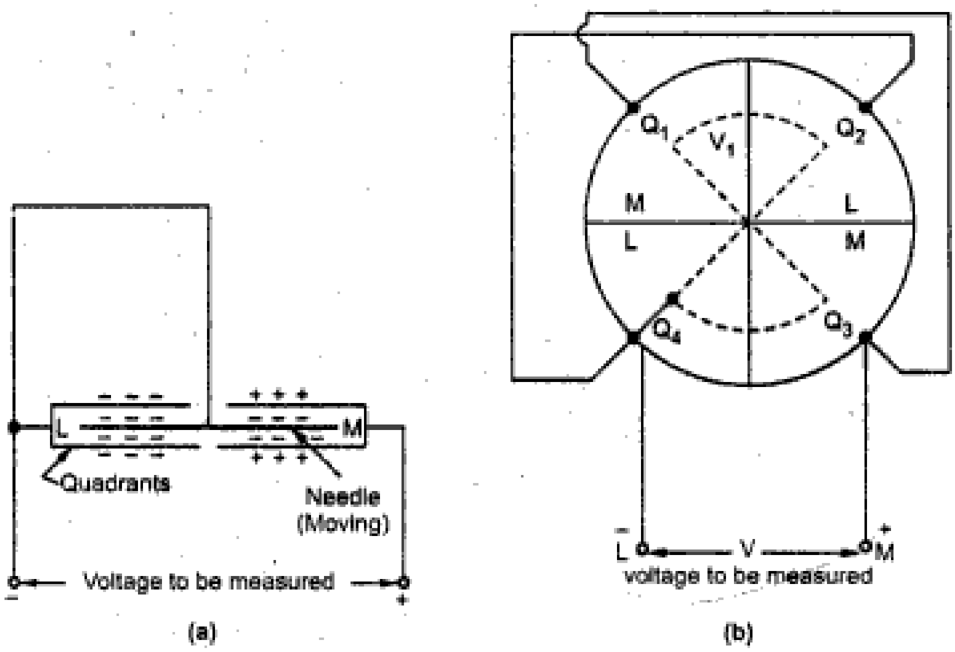


Fig. 1.42 Idiostatic connection

The moving needle is negatively charged, the left hand quadrant is negatively charged and the right hand quadrant is positively charged. The force of attraction on needle due to top and bottom parts of right hand quadrant cancel each other. So there is no motion of needle due to right hand quadrant. Similarly the force of repulsion on needle due to top and bottom parts of left hand quadrant also cancel each other.

Thus the right hand positively charged quadrant attracts the part of the needle near to left hand quadrant while the left hand negatively charged quadrant repels the part of the needle to right hand quadrant. This rotats the needle and hence the pointer.

1.20.3.1 Theory of Idiostatic Connection

For idiostatic connection, the external voltage applied to the needle $V_1 = 0$ V. And the potential of quadrant is nothing but the voltage V_1 which is applied to the needle directly.

$$\therefore \quad V_1 = V_L$$

While the voltage to be measured is $V = V_M - V_L$.

Thus using these values in the expression of T_θ,

$$T_\theta = \frac{\varepsilon_0 r^2}{d}\{(V)[2V_L - V_L - V_M]\} = \frac{\varepsilon_0 r^2}{d}\{(V)(-V)\}$$

$$\therefore \quad \boxed{T_\theta = \frac{\varepsilon_0 r^2}{d} V^2} \quad \dots (12)$$

The negative sign is neglected as it indicates the direction of rotation oppdsite to that which has been assumed.

$$\therefore \quad \boxed{T_\theta \propto V^2} \quad \dots \text{For idiostatic}$$

Key Point : *As torque is proportional to square of the applied voltage, the scale is* ***nonuniform*** *for idiostatic connection. Note that as deflecting torque is proportional to the square of the voltage to be measured, the* ***idiostatic connection*** *is used for* ***a.c. measurements.***

Example 1.17 : *An electrostatic voltmeter is controlled by a spring with a constant 4×10^{-6} Nm/rad and has a full scale deflection of 90° when voltage of 1500 V is applied to it. The capacitance at zero voltage is 10 pF. Find its capacitance when the pointer indicates 1500 V.*

Solution : $K = 4\times10^{-6}$ Nm/rad, $V = 1500$ V, $C_0 = 10$ pF, $\theta' = 90° = \frac{\pi}{2}$ rad

$$\theta = \frac{1}{2K} V^2 \frac{dC}{d\theta}$$

$$\therefore \quad \frac{\pi}{2} = \frac{1}{2\times4\times10^{-6}}\times(1500)^2\times\frac{dC}{d\theta}$$

$$\therefore \quad \frac{dC}{d\theta} = 5.585 \times 10^{-12} \text{ F/rad}$$

$$\therefore \quad \frac{C' - C_0}{\theta' - \theta_0} = 5.585 \times 10^{-12}$$

$$\therefore \quad C' - C_0 = (5.585 \times 10^{-12}) \left(\frac{\pi}{2} - 0\right) = 8.7728 \times 10^{-12}$$

$$\therefore \quad C' = C_0 + 8.7728 \times 10^{-12} = \mathbf{18.7728 \text{ pF}}$$

1.20.4 Kelvin Multicellular Voltmeter

It is one of the most important commercial form of an electrostatic voltmeter. It is basically a quadrant electrometer with large number of needles and only one quadrant. Basically it is used for a voltage range of 100 to 1000 volts. By modifying basic voltmeter it is possible to measure voltages of the range of 40V only.

Thus to obtain a very high force for very small voltages, a large number of cells are used in the instruments hence it is called **multicellular**. The kelvin multicellular voltmeter is as shown in the Fig. 1.43.

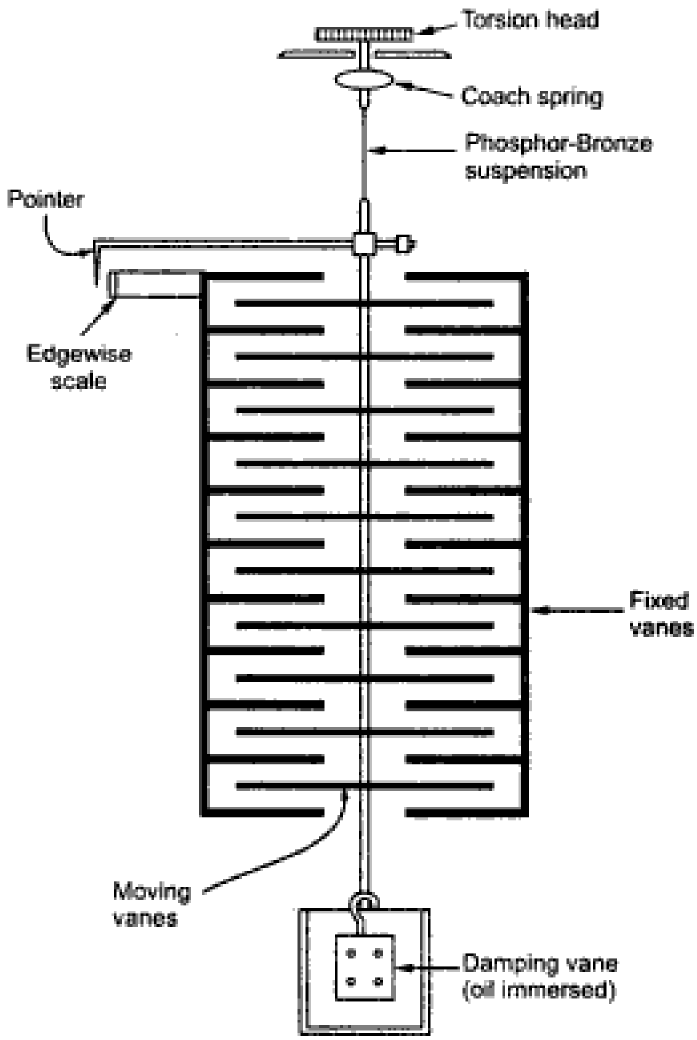


Fig. 1.43 Kelvin multicellular voltmeter

The moving mechanism is suspended with the help of Phospor-Bronze suspension. To protect the suspension against the vibration, the coach spring is used. For the zero adjustment, warm wheel actuated torsion head is used. The pointer and scale are of edgewise type alongwith oil immersed damping vane.

It is very essential to use a safety collar above the pointer to avoid movements of the system caused due to short circuit of quadrants. Two guard plates are fitted inside the case of the instrument. These are electrically connected to the moving mechanism and metal case of the instrument.

In such instruments, number of cells used increases as the voltage to be measured decreases. For example to measure voltage of the order of 150 V, 10 to 15 cells are required while for the measurement of 3000 V one cell may be sufficient.

The deflection torque for n cells is given by

$$T = n \times \text{torque of 1 cell}$$

$$\therefore \quad \boxed{T = n \frac{\varepsilon}{d} r^2 V^2}$$

1.21 Attracted Disc Electrostatic Voltmeter

The attracted disc type instruments are generally used for the measurement of voltages above 20 kV. The system consists of two plates such that one plate can move freely while other is fixed. Both the plates are perfectly insulated from each other. The voltage to be measured is applied across the plates as a supply voltage as shown in the Fig. 1.44. Due to the supply voltage, electrostatic field gets produced which developes a force of attraction between the two plates. Due to the force of attraction, the movable plate gets deflected. In this mechanism the controlling torque is provided by a spring.

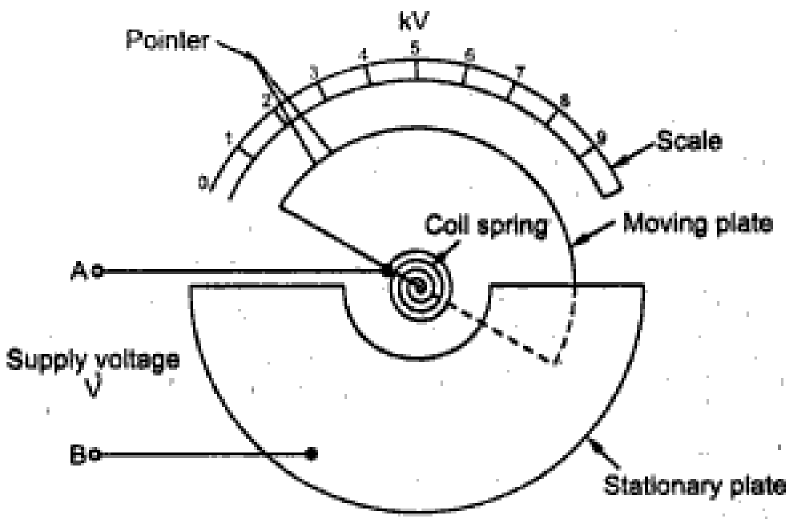


Fig. 1.44 Attracted disc type electrostatic instrument

1.21.1 Kelvin Absolute Electrometer

This instrument is of attracted disc type electrostatic voltmeter. The basic structure of this instrument is as shown in the Fig. 1.45.

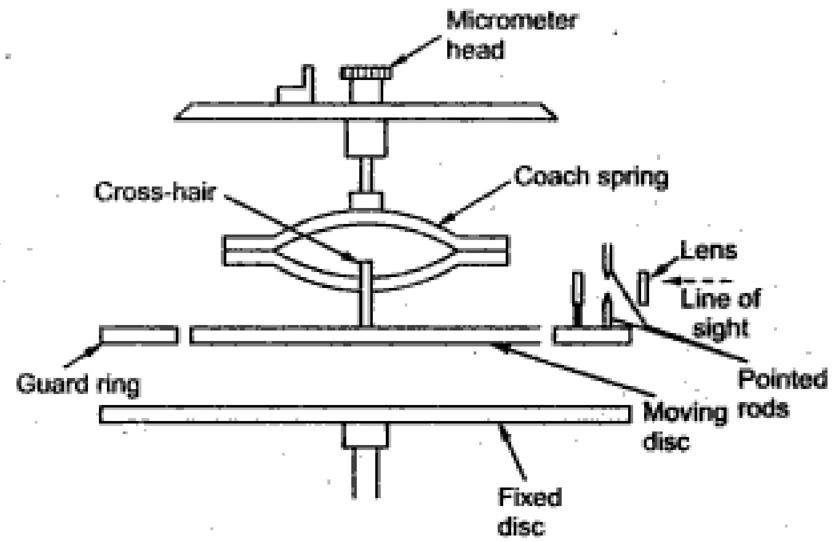


Fig. 1.45 Kelvin absolute electrometer

The moving disc is suspended exactly above the centre of fixed disc from a micrometer head with the help of a spring. The moving disc is surrounded by a ring called **guard ring**, keeping some air gap between moving disc and ring. This guard ring is very useful in reducing fringing effects. It is connected electrically with the moving disc.

With the help of a device consisting lenses and finely pointed rod it is possible to determine zero setting of a disc consisting a fine cross-hair.

In actual practice, the potential to be measured is supplied to the two disc. Due to the force of attraction, the moving disc is attracted downwards. With the help of micrometer head, the moving plate is brought back to zero position setting again. This movement required is observed accurately.

For accurate measurement, it is very important to calibrate the instrument properly. During the calibration, first instrument is short circuited. Then the moving plate is set to its zero position. Then known weights are added to the disc. Then the calibration is done by observing the movements required to bring the moving disc back to the original zero position. Thus instrument measures the force of attraction produced by the potential difference between the two discs. Then the potential difference can be expressed interms of this force.

1.21.2 Theory of Attracted Disc Type Voltmeter

As we have seen already that the force of attraction F between the two parallel plates with potential difference V between them is given by,

$$F = \frac{1}{2} V^2 \frac{dC}{dx} \text{ N} \quad \text{... (1)}$$

Let A be the area of each plate and $\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of the medium. Let d be the distance between the two plates.

Then the capacitance C is given by,

$$C = \frac{A\varepsilon}{d} \text{ farads} \quad \text{... (2)}$$

Differentiating equation (2) with respect to x,

$$\frac{dC}{dx} = -\frac{A\varepsilon}{d^2} \quad \text{... (3)}$$

The negative sign indicates that with decrease in distance of seperation d, capacitance C increases. So neglecting negative sign, the force of attraction can be rewritten as,

$$\boxed{F = \frac{1}{2} V^2 \left(\frac{A\varepsilon}{d^2}\right) \text{ N}} \quad \text{... (4)}$$

The potential difference V between the two plates is given by,

$$\boxed{V = \sqrt{\frac{2Fd^2}{A\varepsilon}} \text{ volts}} \quad \text{... (5)}$$

Thus this instrument gives an absolute determination of voltage as it is given interms of force and linear dimensions. The deflecting force is adequate only when the voltage to be measured is high. For avoiding the errors due to corona effect, special construction is necessary to ensure good insulation. The superior dielectric strength of a high vacuum is used in modern instruments which enables to get more force for a given voltage with very small clearance between the plates.

1.22 Advantages and Disadvantages of Electrostatic Instruments

The various **advantages** of electrostatic instruments are,

1) They give correct measurement in both a.c. as well as d.c. circuits.

2) They are most useful in high voltage measurements.

3) There are no frequency and waveform variations as the deflection is proportional to the square of voltage.

4) As there is no iron part in the working system of these instruments, they are free from errors due to eddy currents and hysteresis.

5) The power loss in these instruments is very small.

6) Costly resistance wires are not used in this instruments.

7) The operating current is very small hence does not affect the performance of other circuits connected to the same supply.

The various **disadvantages** of electrostatic instruments are,

1) They are not suitable for low voltage measurements.

2) These are large in size, bulky and not very robust.

3) These are expensive instruments.

4) The scale is not uniform.

1.23 Extension of Range of Electrostatic Instruments

The range of various instruments can be extended using multipliers. Similarly the range of electrostatic instruments is also extended using multipliers. The multipliers used for electrostatic instruments are of two types,

1. Resistance potential divider 2. Capacitance multipliers

1.23.1 Resistance Potential Divider

The use of resistance potential divider for extending the range of an electrostatic instrument is shown in the Fig. 1.46.

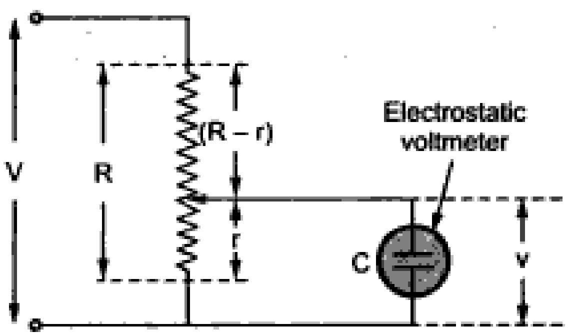


Fig. 1.46 Use of resistance potential divider

R = Total resistance of potential divider

V = Voltage to be measured

r = The resistance whose voltage drop is applied to an electrostatic voltmeter

v = The voltage across an electrostatic voltmeter

C = The capacitance of an electrostatic voltmeter

The resistance r and capacitance C forms a parallel circuit and the equivalent impedance is,

$$Z = r \parallel -j X_C \qquad \text{where } X_C = \frac{1}{\omega C}$$

$$= \frac{r \times \left(\frac{-j}{\omega C}\right)}{r - \frac{j}{\omega C}} = \frac{-j r}{r\omega C - j} = \frac{-j r \times j}{[r\omega C - j] \times j}$$

$$\therefore \quad Z = \frac{r}{1 + j r\omega C} \qquad \ldots(1)$$

Thus the equivalent impedance across the voltage V is,

$$Z_T = R - r + Z = R - r + \frac{r}{1 + j r\omega C} = \frac{(R-r)(1 + j r\omega C) + r}{(1 + j r\omega C)}$$

$$\therefore \quad Z_T = \frac{R + j\omega r C (R - r)}{(1 + j r\omega C)} \qquad \ldots(2)$$

The factor by which voltage is changed due to potential divider is called its **multiplying power** and given by,

$$m = \frac{V}{v} = \frac{Z_T}{Z} = \frac{\frac{R + j\omega r C (R-r)}{(1 + j r\omega C)}}{\frac{r}{1 + j r\omega C}}$$

$$\therefore \quad m = \frac{R + j\omega r C (R - r)}{r} = \frac{R}{r} + j\omega C(R - r) \qquad \ldots(3)$$

The numerical value of multiplying power m is,

$$\boxed{m = \sqrt{\left(\frac{R}{r}\right)^2 + \omega^2 C^2 (R - r)^2}} \qquad \ldots(4)$$

If ω, C and r are very small then $\omega^2 C^2 r^2 \leq 1$ and can be neglected,

$$\therefore \qquad m = \frac{R}{r}\sqrt{1+\frac{\omega^2 C^2 r^2 (R-r)^2}{R^2}} \approx \frac{R}{r} \qquad \ldots (5)$$

1. At high voltages, the cost of this method is high.
2. At high voltages, the power loss and wastage is excessive.
3. At high voltages, the accuracy is very less due to stray capacitance effects.
4. When used for a.c. measurements, it should be wound noninductively and capacitor leakage resistance must be high.

Thus this method is not suitable for high voltages but used for d.c. measurements as capacitance potential divider can not be used for d.c. circuits.

1.23.2 Capacitance Multipliers

The capacitance multiplier method is nothing but the use of capacitance potential divider. There are two methods of connecting capacitor for potential division.

Method 1 : In first method, a single capacitor is connected in series with the voltmeter and the voltage to be measured is applied across the combination as shown in the Fig. 1.47.

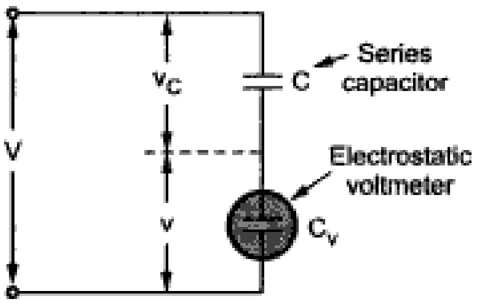


Fig. 1.47 Capacitor multiplier-method

Let $\quad C$ = Series capacitor

C_v = Capacitor of voltmeter

v = Voltage across voltmeter

V = Voltage to be measured

The total capacitance across the supply is,

$$C_t = \frac{C \times C_v}{C + C_v} \qquad \ldots \text{C and } C_v \text{ in series}$$

The total impedance across the supply is,

$$Z_t = \frac{1}{j\omega C_t} = \frac{(C+C_v)}{j\omega C C_v} \quad \dots (1)$$

The impedance of voltmeter is,

$$Z = \frac{1}{j\omega C_v} \quad \dots (2)$$

Thus the multiplying power of the multiplier is,

$$m = \frac{V}{v} = \frac{Z_t}{Z} = \frac{\frac{(C+C_v)}{j\omega C C_v}}{\frac{1}{j\omega C_v}}$$

$$\therefore \quad \boxed{m = \frac{C+C_v}{C} = 1+\frac{C_v}{C}} \quad \dots (3)$$

1. To have high value of multiplying power, the voltmeter capacitor must be high.
2. The voltmeter capacitor varies with the deflection of the moving needle hence the voltmeter must be calibrated alongwith the series multiplier capacitor.

Method 2 : In many practical cases a set of capacitors connected in series across the voltage to be measured is used. The voltmeter is connected across one of the suitable capacitors as shown in the Fig. 1.48.

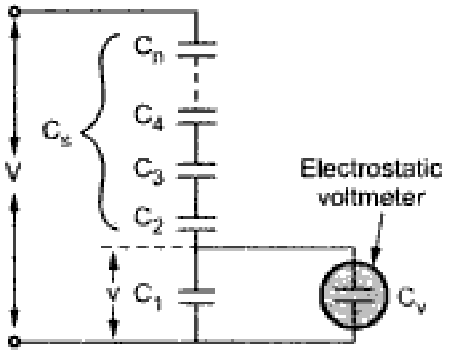


Fig. 1.48 Capacitor multiplier-method 2

The capacitors C_1 and C_v are in parallel hence their resultant is $C_1 + C_v$. While C_2, C_3 ... C_n are in series and their equivalent is C_s where,

$$\frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2}+\dots+\frac{1}{C_n}$$

Thus C_s and $(C_1 + C_v)$ are in series hence the resultant capacitor across the voltage V is,

$$C_t = \frac{C_s (C_1 + C_v)}{C_s + C_1 + C_v} \quad \text{... (4)}$$

$$\therefore \quad Z_t = \frac{1}{j\omega C_t} = \frac{C_s + C_1 + C_v}{j\omega C_s (C_1 + C_v)} \quad \text{... (5)}$$

While across the voltage v the capacitor is $C_1 + C_v$ hence the impedance is,

$$Z = \frac{1}{j\omega (C_1 + C_v)} \quad \text{... (6)}$$

Thus the multiplying power is,

$$m = \frac{Z_t}{Z} = \frac{V}{v} = \frac{\dfrac{C_s + C_1 + C_v}{j\omega C_s (C_1 + C_v)}}{\dfrac{1}{j\omega(C_1 + C_v)}}$$

$$\therefore \quad \boxed{m = 1 + \left[\frac{C_1 + C_v}{C_s}\right]}$$

If C_1 is large with respect to C_v, then there is no appreciable change in the multiplying power alongwith the deflection of the pointer.

Example 1.18 : *An absolute electrometer has a movable circular vane having 75 mm diameter. At the time of measurement, the plates are 5 mm apart and the force of attraction is 3×10^{-3} N. Find the voltage across the plates if the air is used as a dielectric.*

Solution : d = 5 mm, r = radius of vane = $\frac{75}{2}$ = 37.5 mm, F = 3×10^{-3} N

$$A = \pi r^2 = \pi \times (37.5 \times 10^{-3})^2 = 4.4178 \times 10^{-3} \text{ m}^2$$

$$V = \sqrt{\frac{2 F d^2}{A \varepsilon}} \quad \text{where } \varepsilon = \varepsilon_0 = 8.854 \times 10^{-12} \text{ as air}$$

$$\therefore \quad V = \sqrt{\frac{2 \times 3 \times 10^{-3} \times (5 \times 10^{-3})^2}{4.4178 \times 10^{-3} \times 8.854 \times 10^{-12}}} = \mathbf{1958.27\ V}$$

Example 1.19 : *The movable vane of a quadrant electrometer is moved through 30 scale divisions when connected idiostatically to a potential difference of 70 V. When connected heterostatically with the quadrants connected to small voltage V and needle connected to 1000 V, the deflection is 12 scale divisions. Find the value of V.*

Solution : For idiostatic connection,

$$\theta_i \propto T_0 \alpha V^2 \qquad \text{where } \theta_i = 30, V = 70 \text{ V}$$

For heterostatic connection,

$$\theta_h \propto T_0 \alpha 2V_1V \qquad \text{where } \theta_h = 12, V_1 = 1000 \text{ V}$$

$$\therefore \quad \frac{\theta_i}{\theta_h} = \frac{V^2}{2V_1V}$$

$$\therefore \quad \frac{30}{12} = \frac{(70)^2}{2\times 1000\times V}$$

$$\therefore \quad V = \mathbf{1.02\ V}$$

Example 1.20 : *An electrostatic voltmeter has two parallel plates, one moving and other fixed. The force exerted on moving plate is 4×10^{-3} N when 20 kV voltage is applied between the plates. Find the change in capacitance for a movement of 1 mm of the movable plate. The radius of movable plate is 50 mm and air is used as a dielectric.*

Solution : $F = 4\times10^{-3}$ N, V = 20 kV, r = 50 mm

$$\therefore \quad A = \pi r^2 = \pi\times(50\times10^{-3})^2 = 7.8539\times10^{-3} \text{ m}^2$$

$$V = \sqrt{\frac{2Fd^2}{A\varepsilon_0}}$$

$$\therefore \quad 20\times10^3 = \sqrt{\frac{2\times4\times10^{-3}\times d^2}{7.8539\times10^{-3}\times8.854\times10^{-12}}}$$

$$\therefore \quad d = 0.05896 \text{ m} = 58.96 \text{ mm}$$

The distance between plates is 58.96 mm

Let d_1 = 58.96 mm

d_2 = New distance after movement of 1 mm

$$\therefore \quad d_2 = 58.96 - 1 = 57.96 \text{ mm}$$

Now $$C = \frac{\varepsilon_0 A}{d}$$

$$\therefore \quad dC = \text{Change in capacitance} = C_2 - C_1$$

$$= \varepsilon_0 A\left[\frac{1}{d_2} - \frac{1}{d_1}\right]$$

$$= 8.854\times10^{-12}\times7.8539\times10^{-3}\left[\frac{1}{57.96\times10^{-3}} - \frac{1}{58.96\times10^{-3}}\right]$$

$$\therefore \quad dC = \mathbf{2.0348\times10^{-14}\ F} \qquad \text{... Change in capacitance}$$

Example 1.21 : *The capacitance of 0 – 1000 V electrostatic voltmeter increases uniformly from 36 to 42 pF from zero to full scale deflection. It is required to extend the range of voltmeter to 10000 V by using an external series capacitor. Calculate the value of the series capacitor.*

Solution : V = 10000 V, v = 1000 V

$$\therefore \quad m = \frac{V}{v} = \frac{10000}{1000} = 10$$

For full scale, $C_v = 42$ pF

Now $m = 1 + \frac{C_v}{C}$

$$\therefore \quad 10 = 1 + \frac{42}{C}$$

$$\therefore \quad C = \mathbf{4.667\ pF} \quad \text{... Series capacitor required}$$

Examples with Solutions

Example 1.22 : *A galvanometer gives a deflection of 200 mm on a linear scale distant 2 m for a steady current of 1 μA. The period of undamped oscillations is 3.1415 sec and moment of inertia 2×10^{-6} kg – m². Calculate the resistance required to obtain critical damping. Assume damping due to other effects to be negligible.*

Solution : r = 2 m = 2000 mm, d = 200 mm, T_o = 3.1415 sec, $J = 2 \times 10^{-6}$ kg – m²,

$$d = 2r\,\theta_f$$

$$\therefore \quad \theta_f = \frac{d}{2r} = \frac{200}{2 \times 2000} = 0.05 \text{ rad}$$

But $\theta_f = \frac{G\,i}{K}$

$$\therefore \quad 0.05 = \frac{G \times 1 \times 10^{-6}}{K}$$

$$\therefore \quad G = 50 \times 10^{3}\ K \quad \text{... (1)}$$

$$T_o = 2\pi\sqrt{\frac{J}{K}} = 3.1415$$

$$\therefore \quad 3.1415 = 2\pi\sqrt{\frac{2 \times 10^{-6}}{K}}$$

$$\therefore \quad K = 8 \times 10^{-6} \text{ Nm/rad}$$

$\therefore \quad G = 0.4 \text{ Nm/A}$

For critical damping,

$$R_c = \frac{G^2}{2\sqrt{JK}} = \frac{(0.4)^2}{2\sqrt{2\times10^{-6}\times8\times10^{-6}}} = \mathbf{20\ k\Omega}$$

Example 1.23 : *The deflection of a galvanometer is zero with no current and 70 mm with a steady current of 6.2 μA. Its first maximum deflection, after application of a step voltage is 128 mm. The maximum deflection in the next cycle is 90 mm. Determine : i) current sensitivity ii) logarithmic decrement and iii) relative damping ξ.*

Solution : i) $$S_i = \frac{\theta_f}{i} = \frac{70}{6.2} = \mathbf{11.2903\ mm/\mu A}$$

ii) $$\theta_1 = \theta_f\left[1+e^{-\pi\xi/\sqrt{1-\xi^2}}\right] = \text{first maximum deflection}$$

and $$\theta_3 = \theta_f\left[1+e^{-3\pi\xi/\sqrt{1-\xi^2}}\right] = \text{second maximum deflection}$$

$$\therefore \quad \theta_1-\theta_f = \theta_f\ e^{-\pi\xi/\sqrt{1-\xi^2}}$$

and $$\theta_3-\theta_f = \theta_f\ e^{-3\pi\xi/\sqrt{1-\xi^2}}$$

$$\therefore \quad \frac{\theta_1-\theta_f}{\theta_3-\theta_f} = \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{e^{-3\pi\xi/\sqrt{1-\xi^2}}} = e^{+2\pi\xi/\sqrt{1-\xi^2}}$$

But $$\lambda = \frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$$\therefore \quad \frac{\theta_1-\theta_f}{\theta_3-\theta_f} = e^{2\lambda}$$

i.e. $$\frac{128-70}{90-70} = e^{2\lambda}$$

$$\therefore \quad \frac{58}{20} = e^{2\lambda} \quad \text{i.e.} \quad ln\left(\frac{58}{20}\right) = 2\lambda$$

$$\therefore \quad \lambda = \mathbf{0.5323}$$

iii) $$\lambda = \frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$$\therefore \quad 0.5323 = \frac{\pi\xi}{\sqrt{1-\xi^2}} \quad \text{i.e.} \quad \left(\frac{0.5323}{\pi}\right)^2 = \frac{\xi^2}{(1-\xi^2)}$$

$$\therefore \quad 0.02871 = \frac{\xi^2}{1-\xi^2}$$

$$\therefore \quad \xi^2 = 0.02791$$

$$\therefore \quad \xi = \mathbf{0.167}$$

Example 1.24 : *The inductance of a moving iron instrument is given by $L = (10 + 5\theta - \theta^2)\ \mu H$ where θ is the deflection in radians from zero position. The spring constant is 12×10^{-6} Nm/rad. Estimate the deflection for a current of 5 A.*

(JNTU, May-04, Set - 4)

Solution : The rate of change of inductance is,

$$\frac{dL}{d\theta} = (5-2\theta)\ \mu H/rad = (5-2\theta) \times 10^{-6}\ H/rad$$

$$\theta = \frac{1}{2}\frac{I^2}{K}\frac{dL}{d\theta} = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} \times (5-2\theta) \times 10^{-6}$$

$$\therefore \quad \theta = 1.04166\ (5-2\theta)$$

$$\therefore \quad \theta = 1.6891\ \text{rad} = \mathbf{96.782^\circ}$$

Example 1.25 : *The deflecting torque of an ammeter varies as the square of the current passing through it. If a current of 5 A produces a deflection of 90°, what will be the deflection for a current of 10 A when the instrument is, i) spring controlled ii) gravity controlled.*

(JNTU, May-04, Set - 1)

Solution : The deflection torque varies as square of the current.

$$\therefore \quad T_d = K_d\ I^2$$

i) Spring controlled

$$T_c = K\theta$$

$$\therefore \quad T_c = T_d \quad \text{i.e. } K\theta = K_d\ I^2$$

$$\therefore \quad \theta = \frac{K_d}{K}\ I^2 = K_1 I^2$$

$$\therefore \quad 90^\circ = K_1 \times (5)^2 \quad \text{i.e.} \quad K_1 = 3.6$$

$$\therefore \quad \theta = K_1\ I^2 = 3.6 \times (10)^2 = \mathbf{360^\circ} \qquad \text{... for } I = 10\ A$$

(ii) Gravity controlled

$$T_c = K_g\ \sin\theta$$

$\therefore \quad T_c = T_d \quad$ i.e. $\quad K_g \sin\theta = K_d \; I^2$

$$\therefore \quad \sin\theta = \frac{K_d}{K_g} I^2 = K_2 I^2$$

$$\therefore \quad \sin(90°) = K_2 \times (5)^2 \quad \text{i.e.} \quad K_2 = \frac{1}{25}$$

$$\therefore \quad \sin\theta = K_2 I^2 = \frac{1}{25} \times (10)^2 = 4 \qquad \text{... for } I = 10 \text{ A}$$

$$\therefore \quad \theta = \sin^{-1} 4$$

But this is mathematically undefined. Thus for I = 10 A, with gravity control, the instrument cannot achieve steady state and may get damaged.

Example 1.26 : *A moving coil instrument whose resistance is 25 Ω gives a full scale deflection with a current of 1 mA. This instrument is to be used with a manganin shunt to extend its range to 100 mA. Calculate the error caused by a 10 °C rise in temperature when,*

i) Copper moving coil is connected directly across the manganin shunt.

ii) A 75 Ω manganin resistance is used in series with the instrument moving coil. The temperature coefficient of copper is 0.004/°C and that of manganin is 0.00015/°C.

(JNTU, May-05, Set - 4)

Solution : The arrangement is shown in the Fig. 1.49.

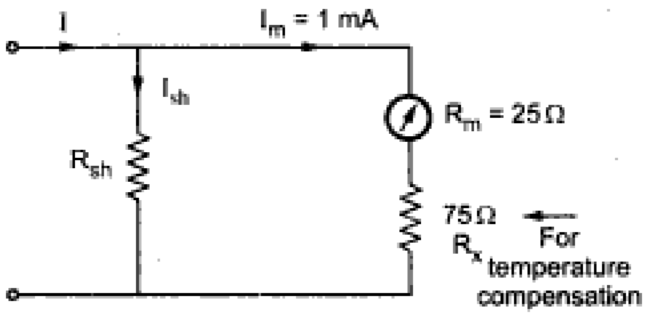


Fig. 1.49

i) Initially copper coil is directly across the shunt **without** 75 Ω resistance in series.

$$I = 100 \text{ mA}, \; I_m = 1 \text{ mA}$$

$$\therefore \quad m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$$\therefore \quad \frac{100}{1} = 1+\frac{25}{R_{sh}}$$

$$\therefore \quad R_{sh} = \mathbf{0.2525\ \Omega}$$

when temperature increases by 10 °C i.e. $\Delta t = 10$ °C

$$\boxed{R_2 = R_1[1+\alpha_1 \Delta t]}$$ where α_1 = resistance temperature coefficient at t_1 °C

$$\therefore \quad R'_m = R_m[1+\alpha_c \Delta t] = 25[1+0.004\times 10] = 26\ \Omega$$

$$R'_{sh} = R_{sh}[1+\alpha_m \Delta t] = 0.2525[1+0.00015\times 10]$$

$$= 0.25287\ \Omega$$

when I = 100 mA then,

$$I'_m = \frac{R'_{sh}}{R'_{sh}+R'_m}\times I = \frac{0.25287}{0.25287+26}\times 100$$

$$= 0.9632\ \text{mA}$$

But I_m = 1 mA required for full scale deflection.

$$\therefore \quad \%\ \text{error} = \frac{(I'_m - I_m)}{I_m}\times 100 = \frac{(0.9632-1)}{1}\times 100$$

$$= \mathbf{-3.679\ \%}$$

ii) Now $R_x = 75\ \Omega$ manganin resistance is connected in series with the meter.

$$\therefore \quad R_{total} = R_m + R_x = 25+75 = 100\ \Omega$$

$$\therefore \quad m = 100 = 1+\frac{R_{total}}{R_{sh}}$$

$$\therefore \quad R_{sh} = \frac{100}{99} = \mathbf{1.01\ \Omega}$$

After, 10 °C rise in temperature,

$$R'_{total} = R'_m + R'_x = 26+R_x[1+\alpha_m \Delta t]$$

$$= 26+75[1+0.00015\times 10] = 101.1125\ \Omega$$

$$R'_{sh} = R_{sh}[1+\alpha_m \Delta t] = 1.01[1+0.00015\times 10] = 1.01151\ \Omega$$

When I = 100 mA then,

$$I'_m = \frac{R'_{sh}}{R'_{sh} + R'_{total}} \times I = \frac{1.01151}{1.01151 + 101.1125} \times 100$$

$$= 0.9904 \quad A$$

But $\quad I_m = 1$ mA required for full scale deflection

$$\therefore \quad \% \text{ error} = \frac{I'_m - I_m}{I_m} \times 100 = \frac{0.9904 - 1}{1} \times 100 = \mathbf{-0.96} \ \%$$

Thus the use of manganin resistance in series with the coil provides the temperature compensation, reducing the error. Such a resistance is called **swamping resistor.**

Example 1.27 : *The spring constant of 3000 V electrostatic voltmeter is 7.06×10^{-6} Nm/rad. The full scale deflection of the instrument is 80°. Assuming the rate of change of capacitance with the angular deflection to be constant over the operating range, calculate the total change of capacitance from zero to full scale.*

(JNTU, May-05, Set - 1)

Solution : $K = 7.06 \times 10^{-6}$ Nm/rad, V = 3000 V, $\theta = 80° = \left(\frac{80 \times \pi}{180}\right)$ rad

$$\theta = \frac{1}{2K} V^2 \frac{dC}{d\theta}$$

$$\therefore \quad \frac{80 \times \pi}{180} = \frac{1}{2 \times 7.06 \times 10^{-6}} \times (3000)^2 \times \frac{dC}{d\theta}$$

$$\therefore \quad \frac{dC}{d\theta} = \frac{2 \times 7.06 \times 10^{-6} \times 80 \times \pi}{180 \times (3000)^2} = 2.19 \text{ pF/rad}$$

$$\therefore \text{ dC for zero to full scale} = \frac{dC}{d\theta} \times (d\theta)$$

$$= 2.19 \times \left(\frac{80 \times \pi}{180}\right) = \mathbf{3.0586 \ pF}$$

Example 1.28 : *An electrostatic voltmeter is constructed with six parallel, semicircular fixed plates equispaced at 4 mm intervals and five interleaved semicircular movable plates that move in plane midway between the fixed plates in air. The instrument is spring controlled. If the radius of the movable plates is 40 mm, calculate the spring constant if 10 kV corresponds to full scale deflection of 100°. Neglect edge effects and plate thickness. The permittivity of air is 8.85×10^{-12} F/m.*

(JNTU, May-05, Set - 3)

Solution : Let the deflection is θ radians. At this time the plates overlap as shown in Fig. 1.50 (a) and form a capacitor as shown in the Fig. 1.50 (b).

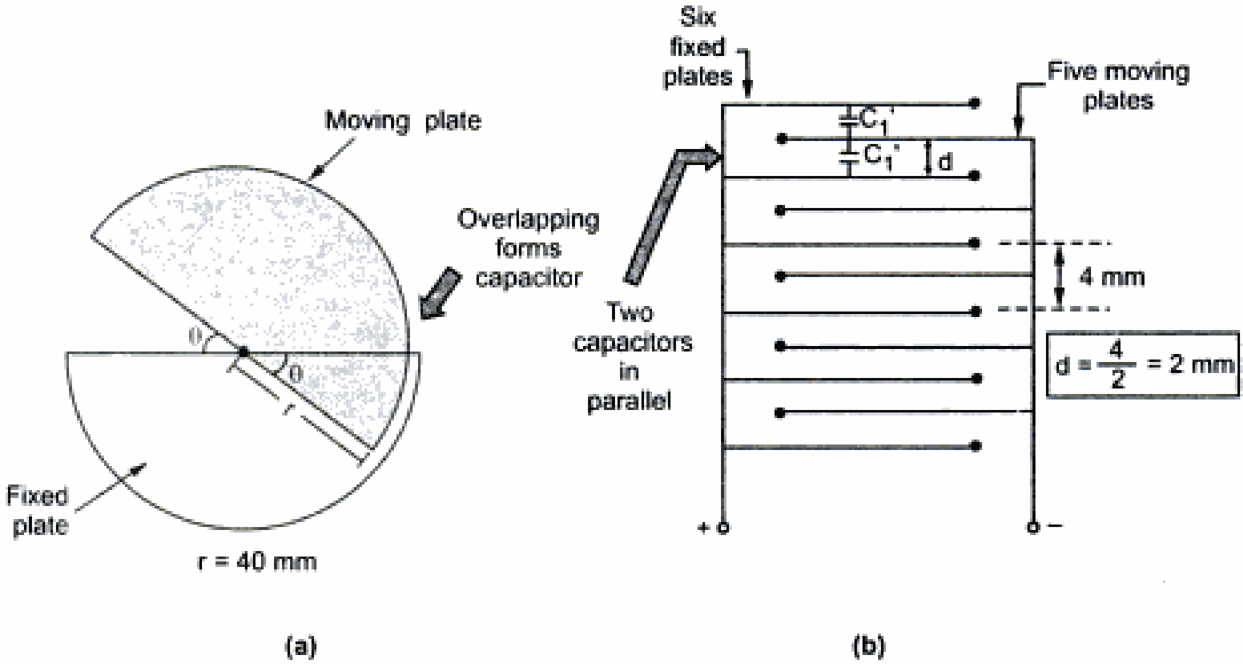


Fig. 1.50

When moving plate moves, it forms two capacitors in parallel with respect to upper and lower fixed plates as shown in the Fig. 1.50 (b).

Now
$$C_1' = \frac{\varepsilon_0 \ A}{d}$$

$$A = \frac{r \times r\theta}{2} = \frac{r^2\theta}{2}$$

$$C = 2\ C_1' = \frac{2 \times \varepsilon_0 \times r^2\ \theta}{d \times 2} = \frac{\varepsilon_0\ r^2\ \theta}{d}$$

$$\therefore \quad \frac{dC}{d\theta} = \frac{\varepsilon_0\ r^2}{d}$$

The torque exerted by each unit is,

$$T_d\ /\text{unit} = \frac{1}{2}\ V^2\ \frac{dC}{d\theta}$$

There are 5 such units,

$$\therefore \quad T_d = \frac{5}{2} V^2 \frac{dC}{d\theta} = \frac{5}{2} V^2 \frac{\varepsilon_0 r^2}{d}$$

while $T_c = K\theta$ ... spring constant

But $T_c = T_d$

$$\therefore \quad K\theta = \frac{5}{2} V^2 \frac{\varepsilon_0 r^2}{d}$$

Now $V = 10$ kV, $\theta = 100° = \frac{100 \times \pi}{180}$ rad, $\varepsilon_0 = 8.85 \times 10^{-12}$

$$\therefore \quad K \times \frac{100\pi}{180} = \frac{5}{2} \times \frac{(10 \times 10^3)^2 \times 8.85 \times 10^{-12} \times (40 \times 10^{-3})^2}{2 \times 10^{-3}}$$

$$\therefore \quad K = \mathbf{1.014 \times 10^{-3}\ Nm/rad = 17.708 \times 10^{-6}\ Nm/degree}$$

Review Questions

1. *Explain the construction of D'Arsonval galvanometer.*
2. *Derive the torque equation of D'Arsonval galvanometer.*
3. *Define the intrinsic constants of D'Arsonval galvanometer.*
4. *Derive the dynamic behaviour of D'Arsonval galvanometer.*
5. *Explain the following motions in D'Arsonval galvanometer, i) underdamped ii) overdamped iii) Critically damped.*
6. *What is relative damping ? Explain its effect on galvanometer motion.*
7. *Explain logarithmic decrement for a galvanometer.*
8. *Derive the expression for the first overshoot in a galvanometer motion.*
9. *Explain the effect of external resistance on damping of galvanometer.*
10. *Derive the value of external resistance for critical damping.*
11. *Which are the damping effects present in galvanometer ? Which is significant ?*
12. *Define the sensitivity of galvanometer in 3 ways.*
13. *Describe the construction and working of PMMC instrument.*
14. *Derive the equation for deflection in spring controlled PMMC instrument.*
15. *How is the current range of a PMMC instrument extended with the help of shunts ?*
16. *Describe the working of universal shunt used for multirange ammeters. Derive expressions for the resistances of different sections.*
17. *Describe how a potential divider arrangement is used for multipliers used for multirange voltmeters. Derive expressions for resistance of different sections.*
18. *State the advantages, disadvantages and errors in PMMC instruments.*

19. *Describe the general requirements of shunts for ammeters and multipliers.*
20. *Explain the working of attraction type and repulsion type moving iron instruments with neat diagrams.*
21. *Derive the torque equation for moving iron instruments.*
22. *Describe the constructional details and working of the electrodynamometer type instrument.*
23. *A moving coil instrument has following data : number of turns = 100, width of coil = 20mm, depth of coil = 30 mm, flux density in air gap 0.1 Wb/m^2. Calculate the deflecting torque when carrying a current of 10 mA. Also calculate the deflection if spring constant is 2×10^{-6} Nm/degrees.*

    [Ans : 60×10^{-6} Nm, 30°]
24. *Design a multirange ammeter with the ranges of 1A, 5 A, 25 A and 125 A employing individual shunts in each case. A D'Arsonval movement with an internal resistance of 730 Ω and full scale current of 5 mA is available.* **[Ans. : 3.67 Ω, 0.73 Ω, 0.146 Ω, 0.0292 Ω]**
25. *A basic D'Arsonval movement with full scale reading of 50 μA and an internal resistance of 1800 Ω is available. It is to be converted into 0-1 V, 0-5 V, 0-25 V and 0-125 V multirange voltmeter using individual multipliers. Calculate the values of the individual multipliers.*

    [Ans. : 18.2 kΩ, 98.2 kΩ, 498.2 kΩ, 2498.2 kΩ]
26. *Write a short note on Taut Band instrument.*
27. *Explain the temperature compensation in PMMC instruments.*
28. *State the advantages and disadvantages of moving iron instrument.*
29. *What is sensitivity of voltmeters ? Explain.*
30. *What is a loading effect ? Explain with the suitable example.*
31. *State the basic requirement of any measuring instrument. How the various measuring instruments are classified ?*
32. *Which torques are necessary for the successful operation if any indicating instrument ? Explain briefly how these torques are produced in various instruments.*
33. *Which are the various effects with which deflecting torque is produced ?*
34. *Differentiate between spring control and gravity control methods used to produce the controlling torque.*
35. *Explain the various methods of providing damping torque in an indicating instrument.*
36. *Why scale of moving iron instruments is non-uniform while that of PMMC instruments is uniform.*
37. *How the range of d.c. ammeter and d.c. voltmeter can be extended ? Derive the expressions to calculate shunt resistance and multiplier resistance.*
38. *Explain principle of electrostatic instruments.*
39. *Explain briefly quadrant type electrometer.*
40. *Explain attracted-disc type electrometer with neat diagrams.*

41. *Write a note on*

    i) Kelvin multicellular voltmeter

    ii) Kelvin absolute electrometer.

42. *List advantages and disadvantages of electrostatic instruments.*

43. *Explain the extension of range of electrostatic instruments.*

❑❑❑

2 Instrument Transformers

2.1 Introduction

In heavy currents and high voltage a.c. circuits, the measurement can not be done by using the method of extension of ranges of low range meters by providing suitable shunts. In such conditions, specially constructed accurate ratio transformers called **instrument transformers**. These can be used, irrespective of the voltage and current ratings of the a.c. circuits. These transformers not only extend the range of the low range instruments but also isolate them from high current and high voltage a.c. circuits. This makes their handling very safe. These are generally classified as (i) current transformers and (ii) potential transformers.

2.2 Current Transformers (C.T.)

The large alternating currents which can not be sensed or passed through normal ammeters and current coils of wattmeters, energymeters can easily be measured by use of current transformers along with normal low range instruments.

A transformer is a device which consists of two windings called primary and secondary. It transfers energy from one side to another with suitable change in the level of current or voltage. A current transformer basically has a primary coil of one or more turns of heavy cross-sectional area. In some, the bar carrying high current may act as a primary. This is connected in series with the line carrying high current.

The secondary of the transformer is made up of a large number of turns of fine wire having small cross-sectional area. This is usually rated for 5 A. This is connected to the coil of normal range ammeter. Symbolic representation of a current transformer is as shown in the Fig. 2.1.

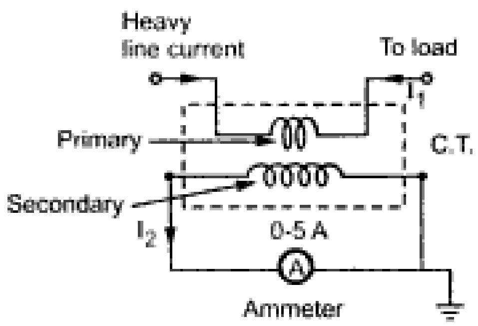


Fig. 2.1 Current transformer

2.2.1 Working Principle

These transformers are basically step up transformers i.e. stepping up a voltage from primary to secondary. Thus the current reduces from primary to secondary. So from current point of view, these are step down transformers, stepping down the current value considerably from primary to secondary.

Let N_1 = Number of turns of primary

N_2 = Number of turns of secondary

I_1 = Primary current

I_2 = Secondary current

For a transformer,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

As N_2 is very high compared to N_1, the ratio I_1 to I_2 is also very high for current transformers. Such a current ratio is indicated for representing the range of current transformer. For example, consider a 500 : 5 range then it indicates that C.T. Steps down the current from primary to secondary by a ratio 500 to 5.

$$\therefore \quad \frac{I_1}{I_2} = \frac{500}{2}$$

Knowing this current ratio and the meter reading on the secondary, the actual high line current flowing through the primary can be obtained.

➡ **Example 2.1 :** *A 250 : 5, current transformer is used along with an ammeter. If ammeter reading is 2.7 A, estimate the line current.*

Solution : $$\frac{I_1}{I_2} = \frac{250}{5}$$

But as ammeter is in secondary, $I_2 = 2.7$ A

$$\therefore \quad \frac{I_1}{2.7} = \frac{250}{5}$$

$$\therefore \quad I_1 = 135 \text{ A}$$

So line current is **135 A.**

2.3 Construction of Current Transformers

There are two types of constructions used for the current transformers which are,

1. Wound type 2. Bar type

2.3.1 Wound Type Current Transformer

In wound type construction, the primary is wound for more than one full turn, on the core. The construction is shown in the Fig. 2.2.

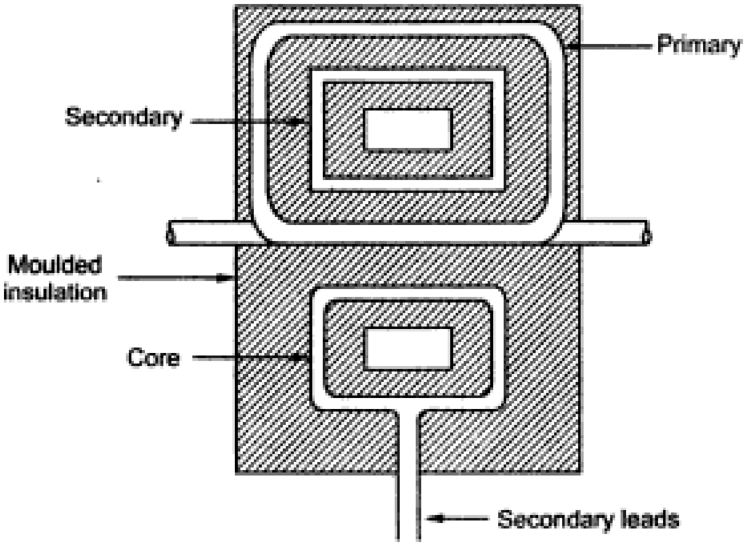


Fig. 2.2 Wound type current transformer

In a low voltage wound type current transformer, the secondary winding is wound on a bakelite former. The heavy primary winding is directly wound on the top of the secondary winding with a suitable insulation in between the two. Otherwise the primary is wound completely separately and then taped with suitable insulating material and assembled with the secondary on the core.

The current transformers can be ring type or window type. Some commonly used shapes for the stampings of window type current transformers are shown in the Fig. 2.3.

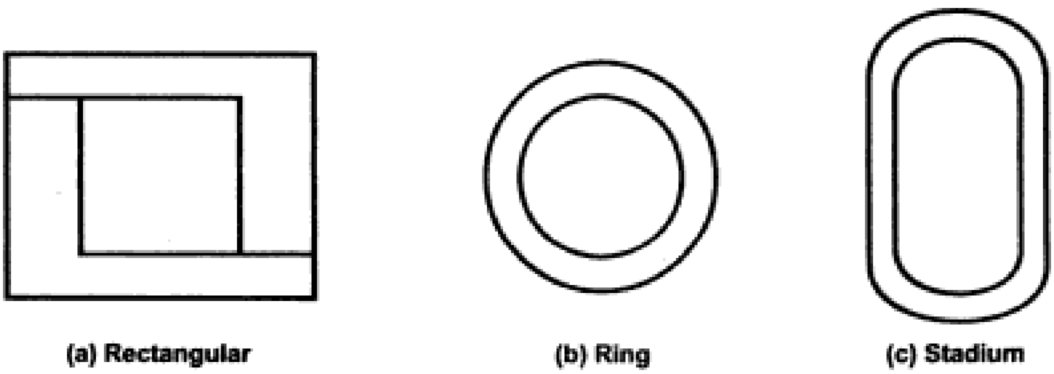


Fig. 2.3 Stampings for current transformers

The core material for wound type is nickel-iron alloy or an oriented electrical steel. Before installing the secondary winding on core it is insulated with the help of end collars and circumferential wraps of pressboards. Such pressboards provide additional insulation and protection to the winding from damage due to the sharp corners.

2.3.2 Bar Type Current Transformer

In this type of current transformer, the primary winding is nothing but a bar of suitable size. The construction is shown in the Fig. 2.4.

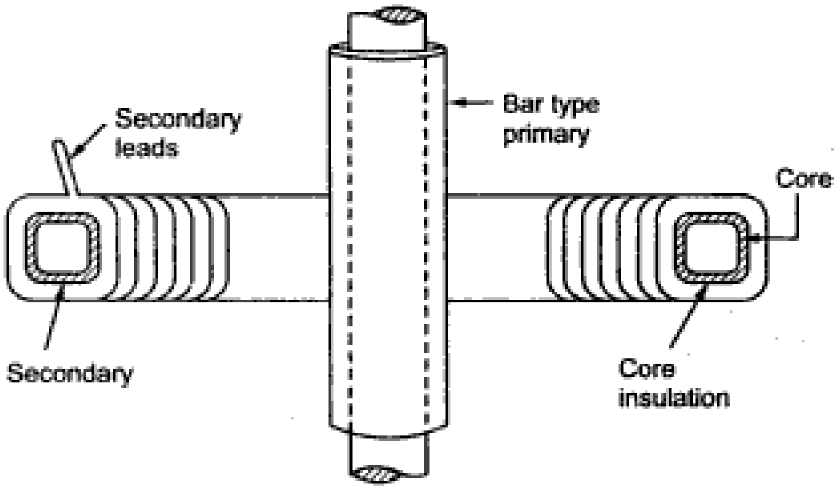


Fig. 2.4 Bar type current transformer

The insulation on the bar type primary is bakelized paper tube or a resin directly moulded on the bar. Such bar type primary is the integral part of the current transformer. The core and the secondary windings are same in bar type transformer.

The stampings used for the laminations in current transformers must have high cross-sectional area than the ordinary transformers. Due to this, the reluctance of the interleaved corners remains as low as possible. Hence the corresponding magnetizing current is also small. The windings are placed very close to each other so as to reduce the leakage reactance. To avoid the corona effect, in bar type transformer, the external diameter of the tube is kept large.

The windings are so designed that without damage, they can withstand short circuit forces which may be caused due to short circuit in the circuit in which the current transformer is inserted.

For small line voltages, the tape and varnish are used for insulation. For line voltages above 7 kV the oil immersed or compound filled current transformers are used.

2.4 Why Secondary of C.T. Should not be Open ?

It is very important that the secondary of C.T. should not be kept open. Either it should be shorted or must be connected in series with a low resistance coil such as current coils of wattmeter, coil of ammeter etc. If it is left open, then current through secondary becomes zero hence the ampere turns produced by secondary which generally oppose primary ampere turns becomes zero. As there is no counter m.m.f., unopposed primary m.m.f. (ampere turns) produce high flux in the core. This produce excessive core losses, heating the core beyond limits. Similarly heavy e.m.f.s will be induced on the primary and secondary side. This may damage the insulation of the winding. This is danger from the operator point of view as well. It is usual to ground the C.T. on the secondary side to avoid a danger of shock to the operator.

Hence never open the secondary winding circuit of a current transformer while its primary winding is energised.

Thus most of the current transformers have a short circuit link or a switch at secondary terminals. When the primary is to be energised, the short circuit link must be closed so that there is no danger of open circuit secondary.

2.5 Potential Transformers (P.T.)

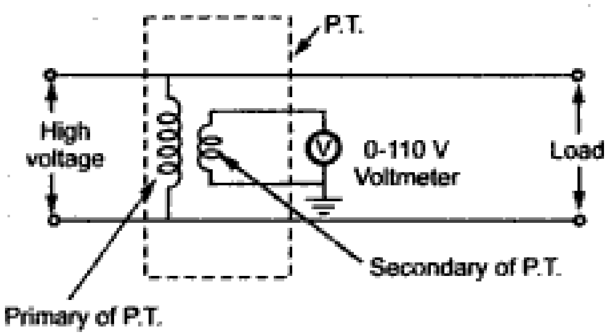


Fig. 2.5 Potential transformer

The basic principle of these transformers is same as current transformers. The high alternating voltage are reduced in a fixed proportion for the measurement purpose with the help of potential transformers. The construction of these transformers is similar to the normal transformer. These are extremely accurate ratio step down transformers. The windings are low power rating windings. Primary winding consists of large number of turns while secondary has less number of turns and usually rated for 110 V, irrespective of the primary voltage rating. The primary is connected across the high voltage line while secondary is connected to the low range voltmeter coil. One end of the secondary is always grounded for safety purpose. The connections are shown in the Fig. 2.5.

As a normal transformer, its ratio can be specified as,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

So if voltage ratio of P.T. is known and the voltmeter reading is known then the high voltage to be measured, can be determined.

➡ **Example 2.2 :** *A 11000 : 110, potential transformer is used along with a voltmeter reading 87.5 V. Estimate the value of line voltage.*

Solution : For a P.T.

$$\frac{V_1}{V_2} = \frac{11000}{110}$$

and $\qquad V_2 = 87.5 \text{ V}$

$$\therefore \quad \frac{V_1}{87.5} = \frac{11000}{110}$$

$$\therefore \quad V_1 = 8750 \text{ V}$$

This is the value of high voltage to be measured.

2.5.1 Construction

The potential transformer use larger core and conductor sizes compared to conventional power transformer. In potential transformer, economy of material is not an important consideration at the time of design. The accuracy is an important consideration.

The shell type or core type construction is preferred for potential transformer. The shell type is used for low voltage while core type for high voltage transformers. At the time of assembly special core is required to reduce the effect of air gap at the joints.

The coaxial primary and secondary windings are used, to reduce the leakage reactance. The secondary winding which is a low voltage winding is always next to the core. The primary winding is a single coil in low voltage transformers. For high voltages, insulation is the main problem. Hence in high voltage potential transformers, primary is divided into number of small sections of short coils to reduce the need of insulation between coil layers.

The cotton tape and varnished cambric are used as the insulations for windings. Hard fiber separators are used in between the coils. The oil immersed potential transformers are used for the voltage levels above 7 kV.

For oil filled potential transformers, oil filled bushings are used. Two bushings are required when no side of the line is at earth potential.

The overall construction of single phase, two winding potential transformer is shown in the Fig. 2.6.

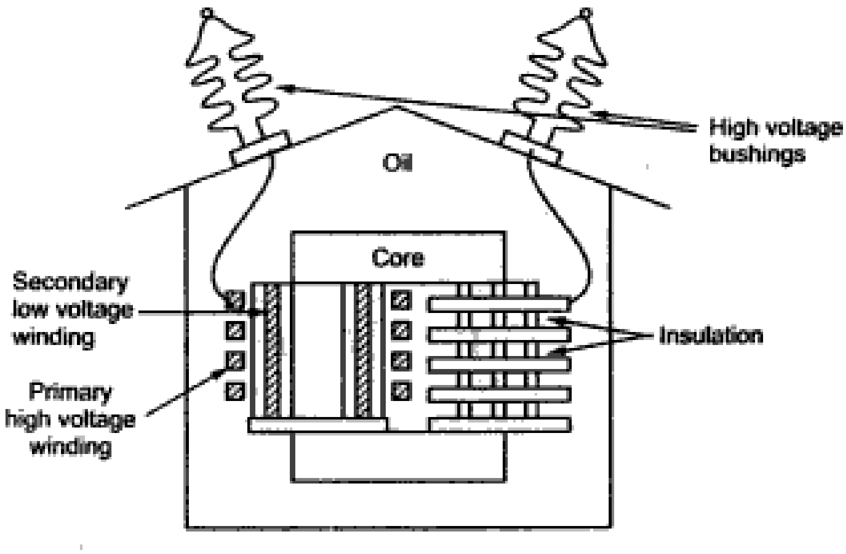


Fig. 2.6 Single phase potential transformer

2.6 Comparison of C.T. and P.T.

The comparison of C.T. and P.T. is given in the following table,

Sr. No.	Current Transformer	Potential Transformer
1.	It can be treated as series transformer under virtual short circuit conditions.	It can be treated as parallel transformer under open circuit secondary.
2.	Secondary must be always shorted.	Secondary is nearly under open circuit conditions.
3.	A small voltage exists across its terminals as connected in series.	Full line voltage appears across its terminals.
4.	The winding carries full line current.	The winding is impressed with full line voltage.
5.	The primary current and excitation varies over a wide range.	The line voltage is almost constant hence exciting current and flux density varies over a limited range.
6.	The primary current is independent of the secondary circuit conditions.	The primary current depends on the secondary circuit conditions.
7.	Needs only one bushing as the two ends of primary winding are brought out through the same insulator. Hence there is saving in cost.	Two bushings are required when neither side of the line is at ground potential.

Table 2.1

2.7 Ratios of Instrument Transformers

The various ratios defined for the instrument transformers are,

1. Actual ratio [R]

The actual transformation ratio is defined as the ratio of the magnitude of actual primary phasor to the corresponding magnitude of actual secondary phasor.

$$\therefore \quad R = \frac{\text{Magnitude of actual primary current}}{\text{Magnitude of actual secondary current}} \quad \text{... For C.T.}$$

$$R = \frac{\text{Magnitude of actual primary voltage}}{\text{Magnitude of actual secondary voltage}} \quad \text{... For P.T.}$$

The actual ratio is also called **transformation ratio.**

2. Nominal ratio [K_n]

The nominal ratio is defined as the ratio of rated primary quantity to the rated secondary quantity, either current or voltage.

$$\therefore \quad K_n = \frac{\text{Rated primary current}}{\text{Rated secondary current}} \quad \text{... For C.T.}$$

$$K_n = \frac{\text{Rated primary voltage}}{\text{Rated secondary voltage}} \quad \text{... For P.T.}$$

3. Turns ratio [n]

$$\therefore \quad n = \frac{\text{Number of turns of secondary winding}}{\text{Number of turns of primary winding}} \quad \text{... For C.T.}$$

$$n = \frac{\text{Number of turns of primary winding}}{\text{Number of turns of secondary winding}} \quad \text{... For P.T.}$$

2.7.1 Ratio Correction Factor (RCF)

It is the ratio of transformation i.e. actual ratio to the nominal ratio.

$$\therefore \quad \text{RCF} = \frac{R}{K_n}$$

i.e.

$$R = \text{RCF} \times K_n$$

The ratio which is indicated on the name plate of a transformer is always its nominal ratio.

2.8 Burden of an Instrument Transformer

The nominal ratio of an instrument transformer, does not remain constant in practice as the load on the secondary changes. It changes because of effect of secondary current, power factor and magnetising as well as core loss components of current and this causes errors in the measurements. For the particular class of transformers the specific loading at rated secondary winding voltage is specified such that the errors do not exceed the limits. Such a permissible load is called **burden** of an instrument transformer.

Thus the permissible load across the secondary winding expressed in **volt-amperes** at the rated secondary winding voltage or current, such that errors do not exceed the limits is called burden of an instrument transformer.

$$\text{Total secondary winding burden} = \frac{(\text{Secondary winding induced voltage})^2}{\text{Total impedance of secondary circuit including load and winding}}$$

$$= \left(\begin{array}{l}\text{Secondary winding} \\ \text{current}\end{array}\right)^2 \times \left[\begin{array}{l}\text{Total impedance of secondary circuit} \\ \text{including load and winding}\end{array}\right]$$

If only the impedance of the load is considered then **burden due to only load** can be obtained.

$$\text{Secondary winding burden due to load} = \frac{(\text{Secondary winding induced voltage})^2}{\text{Impedance of the load on secondary}}$$

$$= \begin{pmatrix}\text{Secondary winding} \\ \text{current}\end{pmatrix}^2 \times [\text{Impedance of the load on secondary}]$$

2.9 Theory of Current Transformers

Consider the equivalent circuit of a current transformer as shown in the Fig. 2.7 along with the load.

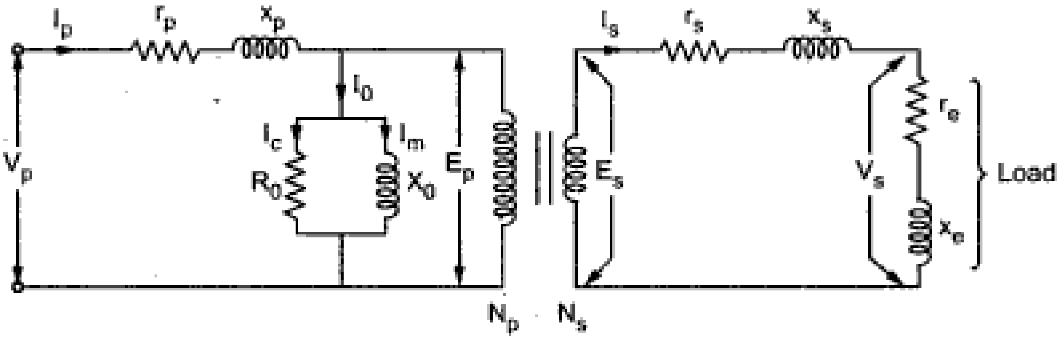


Fig. 2.7 Equivalent circuit of current transformer

The various symbols are,

n = Turns ratio $= \dfrac{\text{Secondary turns}}{\text{Primary turns}} = \dfrac{N_s}{N_p}$

r_p = Resistance of primary winding

x_p = Reactance of primary winding

r_s = Resistance of secondary winding

x_s = Reactance of secondary winding

r_e = Resistance of external burden i.e. load on secondary

x_e = Reactance of external burden i.e. load on secondary

E_p = Primary induced voltage

E_s = Secondary induced voltage

V_s = Secondary terminal voltage

I_p = Primary current

I_s = Secondary current

I_0 = No load current or exciting current

I_c = Core loss component of I_0 i.e. $I_0 \cos\phi_0$

I_m = Magnetising component of I_0 i.e. $I_0 \sin\phi_0$

ϕ = Working flux of transformer

δ = **Angle between E_s and I_s**

= Phase angle of total impedance of secondary including burden

$$= \tan^{-1}\left(\frac{x_s + x_e}{r_s + r_e}\right)$$

θ = Phase angle of transformer

Δ = Phase angle of load or burden i.e. $r_e + j\,x_e$

$$= \tan^{-1}\frac{x_e}{r_e}$$

α = **Angle between I_0 and working flux ϕ.**

2.9.1 Derivation of Actual Ratio

Consider the phasor diagram of the transformer with a lagging p.f. load, as shown in the Fig. 2.8.

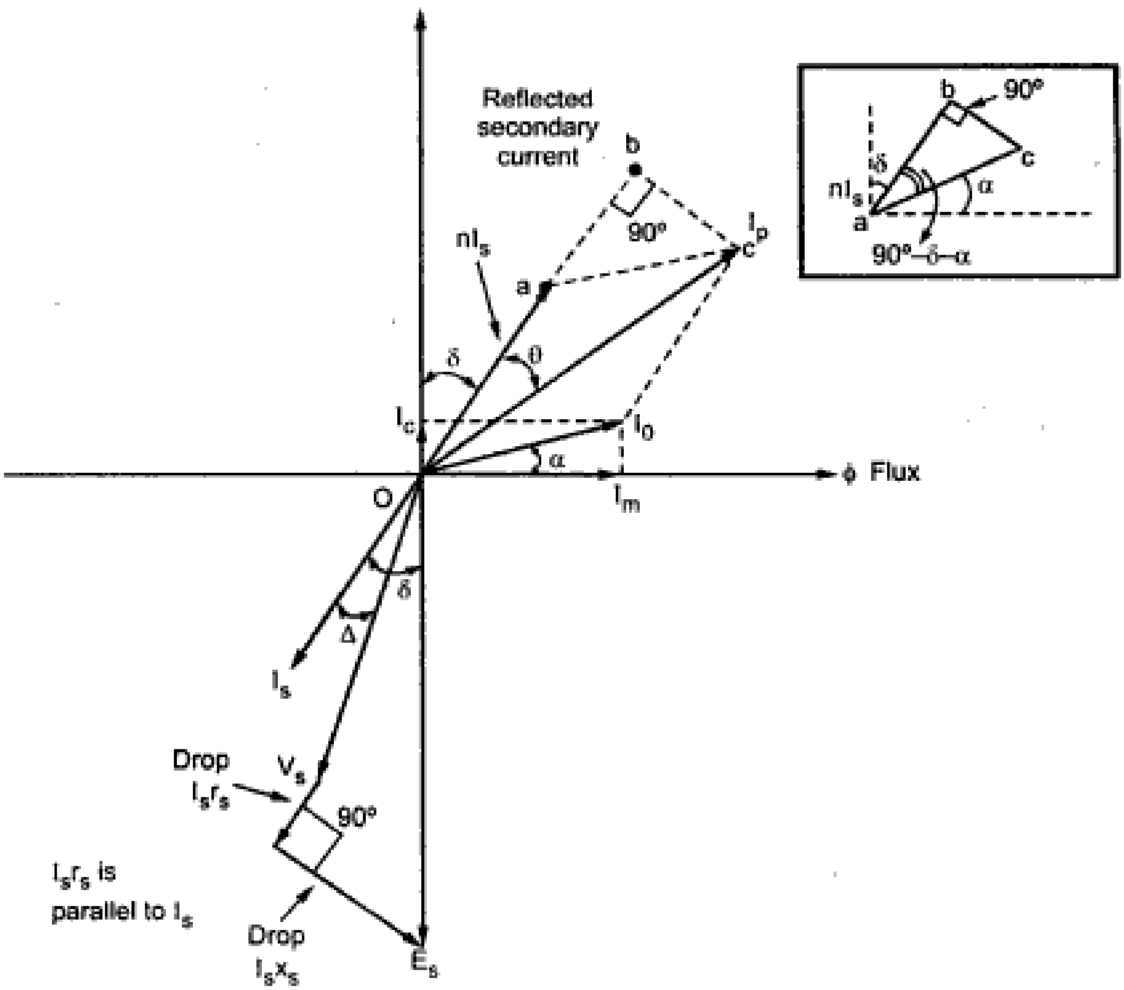


Fig. 2.8 Phasor diagram of current transformer

Consider $\angle$ bac as shown in the small section which is,

$$\angle bac = 90°-\delta-\alpha, \quad ac = I_0, \quad Oa = n\,I_s, \quad Oc = I_p$$

$$\therefore \quad bc = ac\sin(90°-\delta-\alpha) = I_0\sin[90°-(\delta+\alpha)] = I_0\cos(\delta+\alpha)$$

$$\therefore \quad ab = ac\cos(90°-\delta-\alpha) = I_0\cos[90°-(\alpha+\delta)] = I_0\sin(\delta+\alpha)$$

From right angle triangle Obc,

$$(Oc)^2 = (Ob)^2+(bc)^2 = (Oa+ab)^2+(bc)^2$$

$$= [n\,I_s+I_0\sin(\delta+\alpha)]^2+[I_0\cos(\delta+\alpha)]^2$$

$$= n^2\,I_s^2+2n\,I_s\,I_0\sin(\delta+\alpha)+I_0^2\sin^2(\delta+\alpha)+I_0^2\cos^2(\delta+\alpha)$$

i.e.
$$I_p = \sqrt{n^2\,I_s^2+2n\,I_s\,I_0\sin(\delta+\alpha)+I_0^2} \qquad \text{... (1)}$$

$$\therefore \quad \text{Actual ratio} = R = \frac{I_p}{I_s} \qquad \text{... As per definition}$$

$$\therefore \quad R = \frac{\sqrt{n^2\,I_s^2+2n\,I_s\,I_0\sin(\delta+\alpha)+I_0^2}}{I_s} \qquad \text{... (2)}$$

Practically for properly designed transformer $I_0 << n\,I_s$,

$$\therefore \quad R = \frac{\sqrt{n^2\,I_s^2+2n\,I_s\,I_0\sin(\delta+\alpha)+I_0^2\sin^2(\delta+\alpha)}}{I_s}$$

... Adjusting I_0^2 as $I_0^2\sin^2(\delta+\alpha)$

$$\therefore \quad R = \frac{n\,I_s+I_0\sin(\delta+\alpha)}{I_s} = n+\frac{I_0}{I_s}\sin(\delta+\alpha) \qquad \text{... (3)}$$

This is approximate value of actual ratio but practically very close to actual result. The equation (3) can be further expanded as,

$$R = n+\frac{I_0}{I_s}[\sin\delta\cos\alpha+\cos\delta\sin\alpha]$$

But $I_0\cos\alpha = I_m$ and $I_0\sin\alpha = I_c$

$$\therefore \quad R = n+\frac{I_m}{I_s}\sin\delta+\frac{I_c}{I_s}\cos\delta$$

$$\therefore \quad \boxed{R = n+\frac{I_m\sin\delta+I_c\cos\delta}{I_s}} \qquad \text{... (4)}$$

Note that δ is **positive** for **lagging** p.f. load while **negative for leading** p.f. load.

2.9.2 Derivation of Phase Angle (θ) of Transformer

The phase angle θ is defined as the angle between reversed secondary current phasor i.e. reflected secondary current phasor and the primary current.

Sign convention : θ is **positive** if reflected secondary current **leads** primary current. θ is **negative** if secondary current **lags** primary current.

$$\therefore \quad \theta = n I_s \wedge I_p$$

From the phasor diagram,

$$\tan\theta = \frac{bc}{Ob} = \frac{bc}{Oa + ab} = \frac{I_0 \cos(\delta+\alpha)}{n I_s + I_0 \sin(\delta+\alpha)}$$

Now $\tan\theta \approx \theta$ as θ is very small.

$$\therefore \quad \theta = \frac{I_0 \cos(\delta+\alpha)}{n I_s + I_0 \sin(\delta+\alpha)} \quad \textbf{radians} \qquad \text{... (5)}$$

But $I_0 << n I_s$ hence neglecting from denominator,

$$\theta = \frac{I_0 \cos(\delta+\alpha)}{n I_s} = \frac{I_0 [\cos\delta \cos\alpha - \sin\delta \sin\alpha]}{n I_s}$$

$$\therefore \quad \theta = \frac{I_m \cos\delta - I_c \sin\delta}{n I_s} \quad \text{radians} \qquad \text{... (6)}$$

Converting to degrees,

$$\boxed{\theta = \frac{180^\circ}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n I_s}\right] \quad \text{degrees}} \qquad \text{... (7)}$$

2.9.3 Errors in Current Transformer

For an instrument transformers, it is necessary that the transformation ratio must be exactly equal to turns ratio and phase of the secondary terms (voltage and current) must be displaced by exactly 180° from that of the primary terms (voltage and current). Two types of errors affect these characteristics of an instrument transformer which are,

1. Ratio error
2. Phase angle error

2.9.3.1 Ratio Error

In practice it is said that current transformation ratio I_2 / I_1 is equal to the turns ratio N_1 / N_2. But actually it is not so. The current ratio is not equal to turns ratio because of magnetizing and core loss components of the exciting current. It also gets affected due to the secondary current and its power factor. The load current is not a constant fraction of the primary current. Similarly in case of potential transformers, the voltage ratio V_2 / V_1 is also not exactly equal to N_2 / N_1 due to the factors mentioned above. Thus the transformation ratio is not constant but depends on the load current, power factor of load and exciting current of the transformer. Due to this fact, large error is introduced in the measurements done by the instrument transformers. Such an error is called **ratio error**.

The ratio error is defined as,

$$\% \text{ Ratio error} = \frac{\text{nominal ratio} - \text{actual ratio}}{\text{actual ratio}} \times 100$$

$$\% \text{ Ratio error} = \frac{K_n - R}{R} \times 100 \quad \text{... (8)}$$

2.9.3.2 Phase Angle Error

In the power measurements, it is must that the phase of secondary current is to be displaced by exactly 180° from that of primary current for C.T. While the phase of secondary voltage is to be displaced by exactly 180° from that of primary voltage, for P.T. but actually it is not so. The error introduced due to this fact is called **phase angle error**. It denoted by angle θ by which the phase difference between primary and secondary is different from 180°.

The phase angle error is given by,

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right] \text{ degrees} \quad \text{... (9)}$$

Approximate results : In practice, the loads are inductive and δ is positive and very small.

$\therefore$ $\sin\delta \approx 0$ and $\cos\delta = 1$ hence the equations (4) and (7) becomes,

$$R = n + \frac{I_c}{I_s} \quad \text{... (10 (a))}$$

and

$$\theta = \left(\frac{180}{\pi}\right)\left(\frac{I_m}{n\, I_s}\right) \text{ degrees} \quad \text{... (11 (a))}$$

Interms of I_p, these can be written using $n = \frac{I_p}{I_s}$.

$\therefore$

$$R = n + \frac{n\, I_c}{I_p} \quad \text{... (10 (b))}$$

and

$$\theta = \left(\frac{180}{\pi}\right)\left(\frac{I_m}{I_p}\right) \text{ degrees} \quad \text{... (11 (b))}$$

2.9.4 Characteristics of Current Transformers

Let us study the effect of various parameters on the characteristics of current transformers,

1. Effect of power factor of secondary circuit

The p.f. of the secondary circuit depends on the p.f. of the burden on secondary. This directly affects the two errors of the transformer.

a. Ratio error : For all inductive loads, δ is positive hence $\sin(\delta+\alpha)$ is positive hence actual ratio R is **always greater** than the turns ratio n. (Refer equation (3)). For capacitive burdens, δ is negative and R is less than the turns ratio.

b. Phase angle error : When load is inductive and δ is small positive then θ is positive. But as δ approaches 90° as load becomes highly inductive then θ becomes negative. For capacitive load, δ is negative and θ is always positive.

2. Effect of change in I_p

The I_p and I_s are directly related thus as I_p changes, I_s also changes. For low values of I_p, I_0 is dominating thus I_m and I_c are also dominating parts of I_p. Thus errors are higher. As I_p increases, part of I_0 becomes insignificant from I_p hence I_c and I_m also are less significant compared to I_p i.e. I_s. Thus errors are less.

3. Effect of change in burden on secondary

The secondary winding circuit burden increases means volt-ampere rating increases. Due to increased secondary current, secondary flux increases which induces more voltage on secondary. Thus both I_m and I_c increases to keep flux constant. Due to this, errors also increase. Thus more secondary burden means more errors.

4. Effect of change in frequency

If frequency is increased at constant voltage then the flux and flux density decreases. Thus there is reduction in I_m and I_c and hence errors also get reduced.

➡ **Example 2.3 :** *The no load current components of a current transformer are, magnetizing component = 102 A core loss component = 38 A. The current transformation ratio is 1000 / 5 A. Calculate the approximate ratio error at full load.*

Solution : I_m = 120 A, I_c = 38 A K_n = nominal ratio = $\frac{1000}{5} = 200$

At full load, I_s = 5 A

$$R = n + \frac{I_c}{I_s} \quad \cdots \text{Using approximate results}$$

Now $n = \text{turns ratio} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = \frac{1000}{5} = 200$

$\therefore \quad R = 200 + \frac{38}{5} = 207.6$

$\therefore \quad \%\text{ ratio error} = \frac{K_n - R}{R} \times 100 = \frac{200 - 207.6}{207.6} \times 100 = \mathbf{-3.66\%}$

Example 2.4 : *A current transformer has a single turn primary and 400 secondary turns. The magnetizing current is 90 A while core loss current is 40 A. Secondary circuit phase angle is 28°. Calculate the actual primary current and ratio error when secondary carries 5 A current.*

Solution : $I_m = 90$ A, $I_c = 40$ A, $\delta = 28°$, $I_s = 5$ A.

$$n = \frac{N_s}{N_p} = \frac{400}{1} = 400$$

$$K_n = \frac{I_p}{I_s} = \frac{N_s}{N_p} = 400$$

$$R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s}$$

$$= 400 + \frac{90 \sin 28° + 40 \cos 28°}{5} = 415.514$$

$$I_p = \text{actual primary current} = R\, I_s$$

$$= 415.514 \times 5 = \mathbf{2077.5703\ A}$$

$\therefore \quad \%\text{ ratio error} = \frac{K_n - R}{R} \times 100 = \frac{400 - 415.514}{415.514} \times 100 = \mathbf{-3.733\%}$

Example 2.5 : *A current transformer has turns ratio 1:399 and is rated as 2000/5 A. The core loss component is 3 A and magnetizing component is 8 A, under full load conditions. Find the phase angle and ratio errors under full load condition if secondary circuit power factor is 0.8 leading.*

Solution : $I_c = 3$ A, $I_m = 8$ A, $\cos\delta = 0.8$ leading

$\therefore \quad \delta = -36.8698°$, negative as leading

$$n = \frac{N_s}{N_p} = \frac{399}{1}$$

$$K_n = \frac{I_p}{I_s} = \frac{2000}{5} = 400$$

$$R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s} \quad \text{where} \quad I_s = 5\text{ A (full load)}$$

$$= 399 + \frac{8\sin(-36.869°) + 3 \times 0.8}{5} = 398.52$$

$$\therefore \quad \%\text{ ratio error} = \frac{K_n - R}{R} \times 100 = 0.3713\%$$

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{nI_s}\right] \text{degrees}$$

$$= \frac{180}{\pi}\left[\frac{8 \times 0.8 - 3 \times \sin(-36.8698°)}{399 \times 5}\right]$$

$$= 0.2355° = 14.13'$$

2.10 Theory of Potential Transformers

The loading of potential transformer is very small in practice hence exciting current I_0 is of the order of I_s i.e. secondary winding current. While in a normal power transformer I_0 is very small compared to I_s.

The equivalent circuit of potential transformer is shown in the Fig. 2.9.

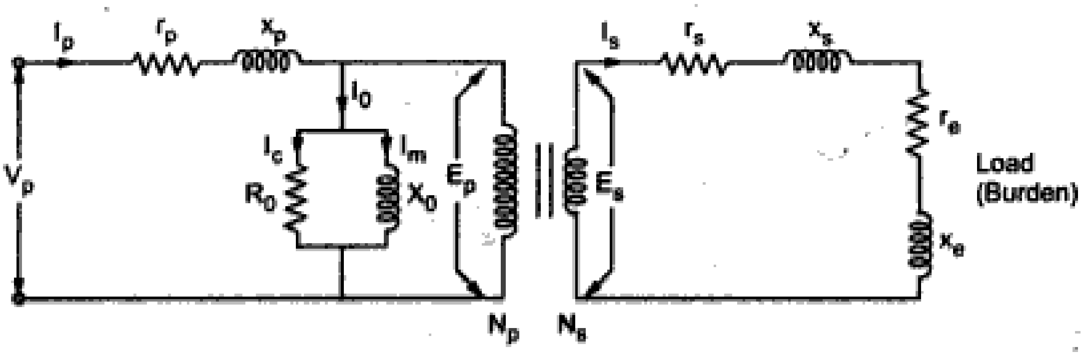


Fig. 2.9 Equivalent circuit of potential transformer

The various symbols are,

ϕ = Working flux

N_p = Primary turns

N_s = Secondary turns

I_p = Primary current

I_s = Secondary current

I_m = Magnetising component of I_0

I_c = Core less component of I_0

I_0 = No load current i.e. exciting current

r_s, x_s = Resistance and reactance of secondary winding

r_p, x_p = Resistance and reactance of primary winding

r_e, x_e = Resistance and reactance of burden

E_p = Primary induced voltage

E_s = Secondary induced voltage

Δ = Phase angle of secondary load current = $\tan^{-1} \frac{x_e}{r_e}$

V_p = Primary applied voltage

V_s = Secondary terminal voltage

For P.T.

$$n = \frac{N_p}{N_s} = \frac{E_p}{E_s}$$

The phasor diagram is shown in the Fig. 2.10.

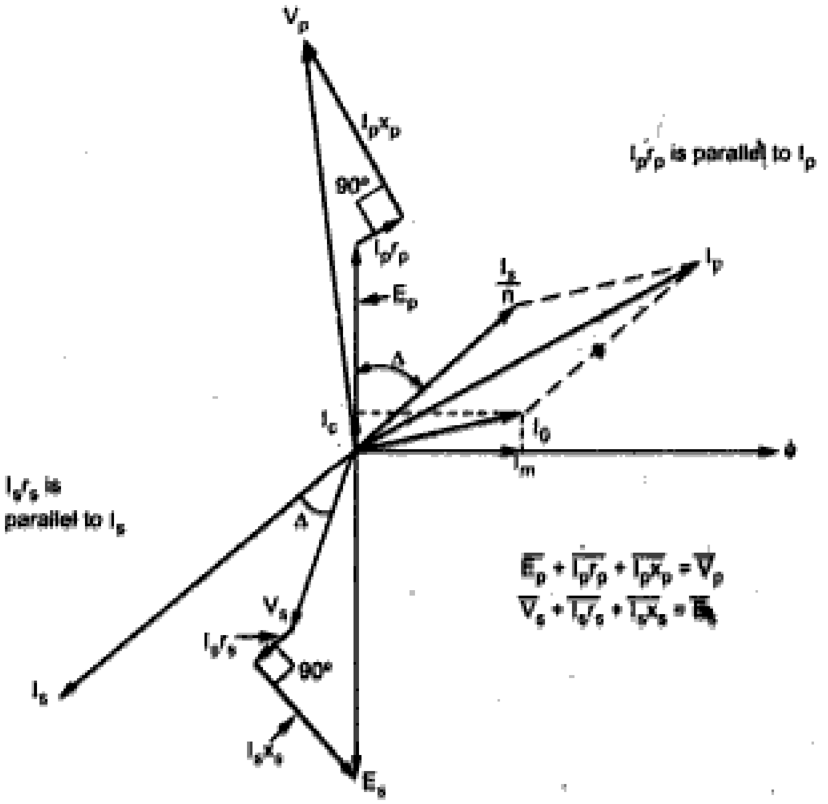
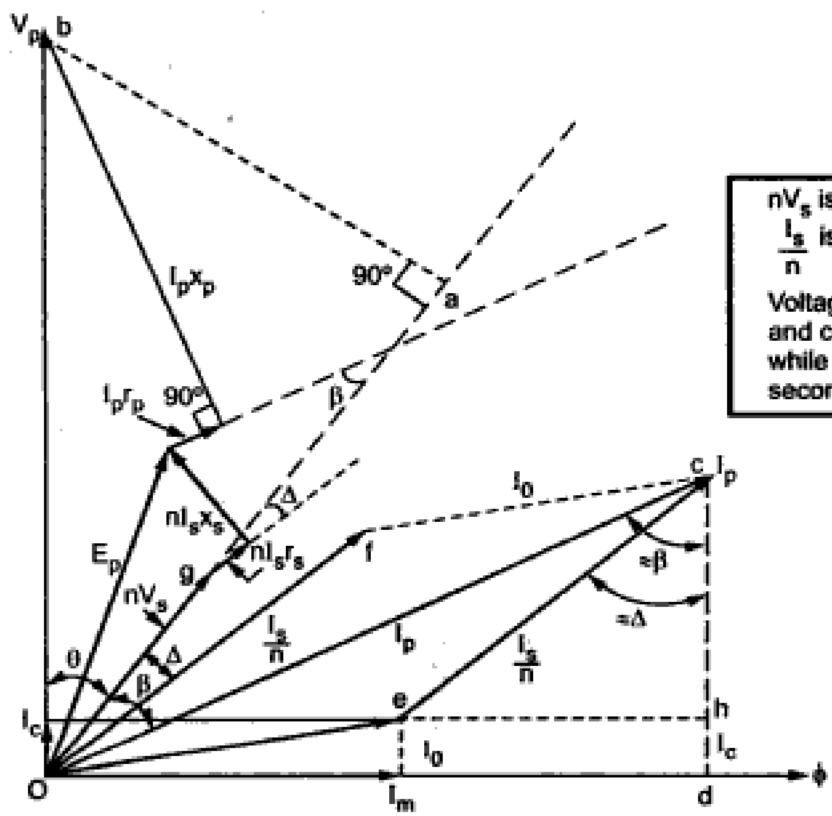


Fig. 2.10 Phasor diagram of a potential transformer

2.10.1 Derivation of Actual Ratio

Consider the phasor diagram with all the quantities referred to the primary side.

In the phasor diagram,



nV_s is V_s reversed
$\frac{I_s}{n}$ is I_s reversed.
Voltages are multiplied by n and currents divided by n while transferring from secondary to primary.

Fig. 2.11 Phasor diagram referred to primary

$Oe = I_0$, $Oc = I_p$, $Of = \frac{I_s}{n}$, $Og = n V_s$, $Ob = V_p$

θ = Phase angle of transformer = V_p ^ V_s reversed

Δ = Phase angle of secondary load = $n V_s \wedge \frac{I_s}{n}$

β = Phase angle between I_p and V_s reversed

Oa is n V_s extended and ba is perpendicular drawn from b on n V_s extended.

$$\therefore \quad Oa = Ob \cos\theta = V_p \cos\theta \quad \text{... (1)}$$

Now Oa is made up of various components.

$$Oa = n V_s + n I_s r_s \cos\Delta + n I_s x_s \sin\Delta + I_p r_p \cos\beta + I_p x_p \sin\beta \quad \text{... (2)}$$

Equating (1) and (2)

$$\therefore \quad V_p \cos\theta = n V_s + n I_s (r_s \cos\Delta + x_s \sin\Delta) + I_p (r_p \cos\beta + x_p \sin\beta) \quad \text{... (3)}$$

For potential transformer the load is nothing but a voltmeter hence θ is very small hence both V_p as well as $n V_s$ can be assumed to be perpendicular to ϕ.

Thus approximately, $\angle Ocd \approx \beta$ and $\angle ecd \approx \Delta$

$$\therefore \quad cd = I_p \cos\beta = ch + hd = \frac{I_s}{n}\cos\Delta + I_c \quad \text{... (4)}$$

$$\text{and} \quad I_p \sin\beta = I_m + \frac{I_s}{n}\sin\Delta \quad \text{... (5)}$$

As θ is very very small, $\cos\theta \approx 1$

$$\therefore \quad V_p \cos\theta = V_p \quad \text{... (6)}$$

Using in (3), all the results of (4), (5) and (6)

$$V_p = n V_s + n I_s (r_s \cos\Delta + x_s \sin\Delta) + r_p \left[\frac{I_s}{n}\cos\Delta + I_c\right] + x_p\left[I_m + \frac{I_s}{n}\sin\Delta\right]$$

$$= n V_s + I_s \cos\Delta\left(n r_s + \frac{r_p}{n}\right) + I_s \sin\Delta\left(n x_s + \frac{x_p}{n}\right) + I_c r_p + I_m x_p$$

$$\therefore \quad V_p = n V_s + \frac{I_s}{n}\cos\Delta(n^2 r_s + r_p) + \frac{I_s}{n}\sin\Delta(n^2 x_s + x_p) + I_c r_p + I_m x_p \quad \text{... (7)}$$

Now $n^2 r_s + r_p = R_{1e}$ = equivalent resistance referred to primary

$n^2 x_s + x_p = X_{1e}$ = equivalent reactance referred to primary

$$\therefore \quad V_p = n V_s + \frac{I_s}{n}[R_{1e}\cos\Delta + X_{1e}\sin\Delta] + I_c r_p + I_m x_p \quad \text{... (8)}$$

Thus the actual ratio is,

$$R = \frac{V_p}{V_s} = n + \frac{\frac{I_s}{n}[R_{1e}\cos\Delta + X_{1e}\sin\Delta] + I_c r_p + I_m x_p}{V_s} \quad \text{... (9)}$$

The result also can be derived interms of parameters referred to secondary. The equation of V_p can be written as,

$$V_p = n V_s + \frac{n^2}{n} I_s \cos\Delta\left(r_s + \frac{r_p}{n^2}\right) + \frac{n^2}{n} I_s \sin\Delta\left(x_s + \frac{x_p}{n^2}\right) + I_c r_p + I_m x_p$$

But $r_s + \frac{r_p}{n^2} = R_{2e}$ = equivalent resistance referred to secondary

$x_s + \frac{x_p}{n^2} = X_{2e}$ = equivalent reactance referred to secondary

$$\therefore \quad V_p = n V_s + n I_s [R_{2e}\cos\Delta + X_{2e}\sin\Delta] + I_c r_p + I_m x_p \quad \text{... (10)}$$

$$\therefore \quad \boxed{R = \frac{V_p}{V_s} = n + \frac{n\, I_s [R_{2e} \cos\Delta + X_{2e} \sin\Delta] + I_c\, r_p + I_m\, x_p}{V_s}} \quad \text{... (11)}$$

2.10.2 Derivation of Phase Angle θ

From the phasor diagram shown in the Fig. 2.11.

$$\tan\theta = \frac{ab}{Oa} = \frac{I_p\, x_p \cos\beta - I_p\, r_p \sin\beta + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s + n\, I_s\, r_s \cos\Delta + n\, I_s\, x_s \sin\Delta + I_p\, r_p \cos\beta + I_p\, x_p \sin\beta}$$

In the expression of Oa, the terms other than nV_s are very small and can be neglected.

Similarly as θ is very small, $\tan\theta \approx \theta$.

$$\therefore \quad \theta = \frac{I_p\, x_p \cos\beta - I_p\, r_p \sin\beta + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s}$$

$$= \frac{x_p \left[\frac{I_s}{n} \cos\Delta + I_c\right] - r_p \left[I_m + \frac{I_s}{n} \sin\Delta\right] + n\, I_s\, x_s \cos\Delta - n\, I_s\, r_s \sin\Delta}{n\, V_s}$$

$$= \frac{I_s \cos\Delta \left(\frac{x_p}{n} + n\, x_s\right) - I_s \sin\Delta \left(\frac{r_p}{n} + n\, r_s\right) + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

$$= \frac{\frac{I_s \cos\Delta}{n}\left(x_p + n^2\, x_s\right) - \frac{I_s \sin\Delta}{n}\left(r_p + n^2 r_s\right) + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

Now $r_p + n^2\, r_s = R_{1e}$ and $x_p + n^2\, x_s = X_{1e}$

$$\therefore \quad \theta = \frac{\frac{I_s}{n} \cos\Delta\, X_{1e} - \frac{I_s}{n} \sin\Delta\, R_{1e} + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

$$\therefore \quad \theta = \frac{\frac{I_s}{n}\left(X_{1e} \cos\Delta - R_{1e} \sin\Delta\right) + I_c\, x_p - I_m\, r_p}{n\, V_s} \text{ radians} \quad \text{... (12)}$$

Interms of quantities referred to secondary,

$$R_{2e} = \frac{R_{1e}}{n^2} \quad \text{and} \quad X_{2e} = \frac{X_{1e}}{n^2}$$

$$\therefore \quad \theta = \frac{n\, I_s \left(X_{2e} \cos\Delta - R_{2e} \sin\Delta\right) + I_c\, x_p - I_m\, r_p}{n\, V_s}$$

$$\therefore \quad \boxed{\theta = \frac{I_s}{V_s}\left(X_{2e} \cos\Delta - R_{2e} \sin\Delta\right) + \frac{I_c\, x_p - I_m\, r_p}{n\, V_s} \text{ radians}} \quad \text{... (13)}$$

It can be noted that the phase angle θ is treated **positive** when V_s reversed i.e. **n V_s leads the primary winding voltage V_p.** The θ is treated **negative when nV_s lags the primary winding voltage V_p.**

Once R and θ are obtained then the errors in potential transformers are,

$$\% \text{ ratio error } = \frac{K_n - R}{R} \times 100$$

and

$$\text{Phase angle error } = \theta \text{ radians}$$

2.10.3 Characteristics of Potential Transformers

Let us study the effect of various parameters on the characteristics of potential transformers.

1. Effect of power factor of secondary circuit

The p.f. of secondary circuit depends on the p.f. of the secondary burden. As p.f. decreases, angle Δ increases. This shifts I_p towards I_0, as seen from the phasor diagram. Thus V_p and V_s become almost in phase with E_p and E_s respectively. Thus V_p becomes more compared to E_p but as V_p is constant as supply voltage, the result is the reduction in E_p. Similarly the voltage V_s also reduces compared to E_s. Hence from equation (11) it is clear that, **R increases as V_s decreases due to decrease in the p.f. of secondary circuit.** While V_s advances in phase and V_p retards hence negative phase angle reduces as lagging p.f. of secondary decreases.

2. Effect of change in burden on secondary

The change in burden on secondary changes the secondary current and secondary VA. As the secondary current increases, primary current increases. This increases the various drops. For constant V_p, V_s decreases. This increases the transformation ratio R. Thus **ratio error increases as the secondary current i.e. burden increases.**

Due to increased voltage drop, V_p advances while V_s retards in phase. Thus the phase angle between V_p and V_s increases and becomes more negative as the secondary current increases.

3. Effect of change in frequency

If frequency is increased at constant voltage then the flux and flux density decreases. This reduces the values of I_m, I_c and hence I_0. This tends to reduce the errors. But increase in frequency increases the leakage reactance increasing the voltage drops. This increases the ratio and ratio error. Thus effects are opposite to each other and the resultant effect depends on relative values of I_0 and the leakage reactance.

While the phase angle increases due to increase in leakage reactance as well as decrease in I_0.

4. Effect of primary voltage

The primary voltage is practically constant and there is hardly any significant change in it hence there is no significant change in the errors.

Example 2.6 : *A potential transformer has a ratio 1000/100 V and has following parameters :*

Primary resistance = 96 Ω, Secondary resistance = 0.88 Ω

Primary reactance = 67.2 Ω, Total equivalent reactance = 115 Ω.

No load current is 0.03 A at 0.4 power factor lagging.

Calculate, i) Phase angle error at no load.

ii) Burden in VA at unity p.f. at which the phase angle will be zero.

Solution : $r_p = 96\ \Omega$, $r_s = 0.88\ \Omega$, $x_p = 67.2\ \Omega$, $X_{1e} = 115\ \Omega$.

Now $$n = \frac{E_p}{E_s} = \frac{1000}{100} = 10$$

i) $$\theta = \frac{\frac{I_s}{n}(X_{1e}\cos\Delta - R_{1e}\sin\Delta) + I_c\,x_p - I_m\,r_p}{n\,V_s}$$

On no load, $I_s = 0$

$$\therefore \quad \theta = \frac{I_c\,x_p - I_m\,r_p}{n\,V_s}$$

$$\cos\phi_0 = 0.4, \quad I_0 = 0.03\ \text{A}$$

$$I_c = I_0\cos\phi_0 = 0.012\ \text{A}$$

$$I_m = I_0\sin\phi_0 = 0.02749\ \text{A}$$

$$\therefore \quad \theta = \frac{0.012\times 67.2 - 0.02749\times 96}{10\times 100}\ \text{rad}$$

$$= -1.8326\times 10^{-3}\ \text{radians} = -0.105° = \mathbf{-6.3'}$$

ii) At unity p.f. $\cos\Delta = 1$, $\sin\Delta = 0$

$$\therefore \quad \theta = \frac{\frac{I_s}{n}X_{1e}\cos\Delta + I_c\,x_p - I_m\,r_p}{n\,V_s}$$

$$\therefore \quad 0 = \frac{\frac{I_s}{10}\times 115 + 0.012\times 67.2 - 0.02749\times 96}{10\times 100}$$

$$\therefore \quad I_s = 0.1593\ \text{A}$$

$$\therefore \quad \text{Load in } V_A = V_s\,I_s = 100\times 0.1593 = \mathbf{15.93\ VA}$$

2.11 Reduction of Errors in Instrument Transformers

The ratio and phase angle errors can be minimized by using following methods :

1. Reducing the core loss and magnetising components of I_0

The equations of ratio and phase angle errors show that the errors depend on the core loss and magnetising components of no load current I_0. The following precautions are taken to reduce I_c and I_m,

1. Choosing low reductance core.
2. Using materials of high permeability.
3. Providing smaller magnetic paths to the flux.
4. Using large cross-section of the core.
5. Keeping flux density in the core to low value.
6. Taking suitable precautions while designing the assembly and interleaving the core.

2. Reduction of resistance and leakage reactance

The errors depend on the voltage drops which are in turn depend on the values of resistance and leakage reactance.

The resistance can be reduced by increasing cross-section of conductors and decreasing the length of the mean turn.

The leakage reactances depend on the leakage fluxes. Thus mutual coupling between the windings must be high hence windings must be as close as possible. Similarly keeping flux density as high as practicable, the number of turns required are less hence leakage reactances get reduced.

3. Providing turns compensation

For the potential transformer,

$$R = n + \frac{\frac{I_s}{n}\left[R_{1e}\cos\Delta + X_{1e}\sin\Delta\right] + I_c\, r_p + I_m\, x_p}{V_s}$$

At no load, secondary current $I_s = 0$ hence,

$$R = n + \frac{I_c\, r_p + I_m\, x_p}{V_s}$$

Thus on no load, actual ratio exceeds by $\left(I_c\, r_p + I_m\, x_p\right) / V_s$. The remedy for this is to reduce the primary turns or increasing the number of secondary turns. Thus for one particular value of load, the actual ratio can be made equal to nominal ratio. This reduces the ratio error over the entire range of the burden. This is called **turns compensation**. However, the turns compensation does not affect the phase angle θ.

For the current transformer the approximate expression for R is,

$$R = n + \frac{I_c}{I_s}$$

Thus actual ratio becomes more than the nominal ratio. So turns ratio is reduced by reducing the secondary winding turns. This makes actual transformation ratio equal to the nominal ratio. This is **turns compensation** for the current transformer.

2.12 Advantages and Disadvantages of Instrument Transformers

The advantages of instrument transformers can be listed as,

1. The normal range voltmeter and ammeter can be used along with these transformers to measure high voltage and currents.
2. The rating of low range meter can be fixed irrespective of the value of high voltage or current to be measured.
3. These transformers isolate the measurement from high voltage and current circuits. This ensures safety of the operator and makes the handling of the equipments very easy and safe.
4. These can be used for operating many types of protecting devices such as relays or pilot lights.
5. Several instruments can be fed economically by single transformer.

Disadvantage :

The only disadvantage of these instrument transformers is that they can be used only for a.c. circuits and not for d.c. circuits.

➡ **Example 2.7 :** *A current transformer has a single turn primary and 400 turns secondary. The secondary is supplying a pure resistive load of 2 Ω at 5 A. The magnetising m.m.f. required to set up the flux in the core is 100 AT. The frequency is 50 Hz. While core has cross-sectional area of 8 cm². Calculate the ratio and phase angle of the current transformer. Also obtain the maximum flux density in the core. Neglect iron losses and copper losses.*

Solution : For any transformer,

$$\bar{I}_0 = \text{No load current} = \bar{I}_c + \bar{I}_m$$

where I_c = Core loss component i.e. active component of I_0

I_m = Magnetising i.e. reactive component of I_0

In this problem, core loss i.e. iron loss is to be neglected.

$\therefore \quad I_c = 0$

While $\quad I_m = \dfrac{\text{Magnetising ampere turns (m.m.f.)}}{\text{Primary turns}}$

$N_p = 1$ and $N_s = 400$ hence $n = \dfrac{N_s}{N_p} = 400$

$\therefore \quad I_m = \dfrac{\text{AT for magnetisation}}{N_p} = \dfrac{100}{1} = 100 \text{ A}$

$\therefore \quad I_0 = 100 \text{ A}$

Now $\quad I_s = 5$ A and $R_s = 2\ \Omega$ purely resistive.

$I_2' =$ Reversed secondary current $= n\, I_s = 2000$ A

As load is purely resistive, $\delta = 0°$ and E_s and I_s are in phase as shown in the Fig. 2.12.

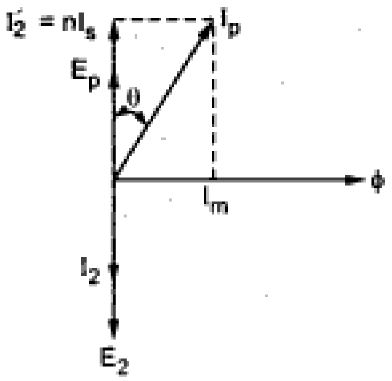


Fig. 2.12

$$I_p = \sqrt{(n\, I_s)^2 + I_m^2}$$

$$= \sqrt{(2000)^2 + (100)^2}$$

$$= 2002.498 \text{ A}$$

$$\text{Actual ratio} = \frac{\text{Actual } I_p}{\text{Actual } I_s}$$

$$\therefore \quad R = \frac{2002.498}{5} = 400.499$$

While $\quad \tan\theta = \dfrac{I_m}{n\, I_s} = \dfrac{100}{2000} = 0.05$

$\therefore \quad \theta = 2.86° = 2°\ 51.6'$

For transformer, $E_s = 4.44\, f \phi_m N_s$ and $E_s = I_s R_s$

$\therefore \quad (5 \times 2) = 4.44 \times 50 \times \phi_m \times 400$

$\therefore \quad \phi_m = 1.126 \times 10^{-4}$ Wb

$\therefore \quad B_m = \dfrac{\phi_m}{A} = \dfrac{1.126 \times 10^{-4}}{8 \times 10^{-4}}$ ... A = Area of cross-section

$= \mathbf{0.1407\ Wb/m^2}$

Example 2.8 : *A particular bar type current transformer has 300 secondary turns. The secondary winding carriers a burden of ammeter having resistance of 1 Ω and inductive reactance of 0.53 Ω while the secondary winding resistance is 0.25 Ω and reactance 0.35 Ω. The magnetising m.m.f. required is 85 A while the current component for core losses is 50 A. Find,*

i) the primary current when secondary carries 5A

ii) the ratio error

iii) the reduction in the number of turns of secondary to obtain zero ratio error.

Solution : $r_e = 1\ \Omega,\ x_e = 0.53\ \Omega,\ r_s = 0.25\ \Omega,\ x_s = 0.35\ \Omega$

$$\therefore \quad Z_s = (r_s + r_e) + j(x_s + x_e) = 1.25 + j\,0.88\ \Omega$$

$$= 1.5286\ \angle 35.1455°\ \Omega$$

$$\therefore \quad \delta = \tan^{-1}\frac{0.88}{1.25} = 35.1455°$$

Now $\quad N_p = 1$ as bar type, $N_s = 300$

$$\therefore \quad K_n = n = \frac{N_s}{N_p} = 300 \quad \text{(for C.T.)}$$

$$I_m = \frac{\text{Magnetising AT}}{N_p} = \frac{85}{1} = 85\ \text{A}$$

While $\quad I_c = 50$ A (Given)

Now $\quad R = n + \dfrac{I_m \sin\delta + I_c \cos\delta}{I_s}$ and $I_s = 5$ A

$$= 300 + \frac{(85 \times \sin 35.1455) + (50 \times \cos 35.1455)}{5} = 317.963$$

i) $\quad I_p = R \times I_s = 317.963 \times 5 =$ **1589.8152 A**

ii) $\quad \%\ \text{ratio error} = \dfrac{K_n - R}{R} \times 100 = \dfrac{300 - 317.963}{317.963} \times 100 =$ **− 5.65%**

iii) For zero ratio error,

$$K_n = R = 300$$

Let $\quad n' =$ New ratio of the turns

$$\therefore \quad 300 = n' + \frac{I_m \sin\delta + I_c \cos\delta}{I_s}$$

$$\therefore \quad n' = 300 - 17.963 = 282.037$$

$\therefore \quad n' = \frac{N'_s}{N_p} = 282.037$

$\therefore \quad N'_s = \mathbf{282.037}$

Thus from 300, the secondary turns must be reduced to 282.037 i.e. by approximately 18, to have zero ratio error.

Example 2.9 : *A potential transformer has a ratio 2000/100 V and has following parameters :*

Primary resistance = 105 Ω, *Secondary resistance = 0.7 Ω*

Primary reactance = 75.2 Ω, *Secondary reactance = 0.087 Ω*

No load current is 0.03 A at 0.36 p.f. lagging. Find

i) Phase angle error on no load

ii) Phase angle error on a load of 5 A at 0.92 lag p.f.

iii) Burden in VA at unity p.f. to have zero phase angle.

Solution : $r_p = 105\ \Omega,\ r_s = 0.7\ \Omega,\ x_p = 75.2\ \Omega,\ x_s = 0.087\ \Omega$

$$n = \frac{E_p}{E_s} = \frac{2000}{100} = 20 \quad \text{(for P.T.)}$$

$I_0 = 0.03$ A, $\cos\phi_0 = 0.36$, $\sin\phi_0 = 0.9329$

$$\therefore \quad I_c = I_0 \cos\phi_0 = 0.03 \times 0.36 = 0.0108 \text{ A}$$

$$I_m = I_0 \sin\phi_0 = 0.03 \times 0.9329 = 0.02798 \text{ A}$$

i) On no load, $I_s = 0$

$$\therefore \quad \theta = \frac{I_c x_p - I_m r_p}{n V_s} = \frac{0.0108 \times 75.2 - 0.02798 \times 105}{20 \times 100}$$

$$= -1.062 \times 10^{-3} \text{ radians} = -0.0608° = \mathbf{-3.65'}$$

ii) $I_s = 5$ A, $\cos\Delta = 0.92$, $\sin\Delta = 0.3919$

$$\therefore \quad \theta = \frac{\frac{I_s}{n}[X_{1e}\cos\Delta - R_{1e}\sin\Delta] + I_c x_p - I_m r_p}{n V_s} \text{ radians}$$

$$X_{1e} = x_p + n^2 x_s = 75.2 + (20)^2 \times 0.087 = 110\ \Omega$$

$$R_{1e} = r_p + n^2 r_s = 105 + (20)^2 \times 0.7 = 385\ \Omega$$

$$\therefore \quad \theta = \frac{\frac{5}{20}[110 \times 0.92 - 385 \times 0.3919] + 0.0108 \times 75.2 - 0.02798 \times 105}{20 \times 100}$$

$$= -7.273 \times 10^{-3} \text{ radians} = -0.4167° = \mathbf{-25'}$$

iii) At unity p.f., $\cos\Delta = 1$, $\sin\Delta = 0$, $\theta = 0$ required.

$$\therefore \quad 0 = \frac{\frac{I_s}{20}[110 - 0] + 0.0108 \times 75.2 - 0.02798 \times 105}{20 \times 100}$$

$$\therefore \quad I_s = 0.3864 \text{ A}$$

$\therefore$ Burden in $V_A = V_s I_s = 100 \times 0.3864 =$ **38.64 VA**

➡ **Example 2.10 :** *A ring core current transformer with a nominal ratio of 500/5 and a bar primary has a secondary resistance of 0.5 Ω and negligible secondary reactance. The resultant of magnetising and iron loss components of the primary current associated with a full load secondary current of 5 A in a non-inductive burden of 1 Ω is 3 A at a 0.04 p.f. Calculate the true ratio and the phase angle error of transformer on full load. Calculate the total flux in the core assuming a frequency of 50 Hz.*

Solution : $K_n = \frac{500}{5} = 100$, $N_p = 1$, $I_s = 5$ A, $r_e = 1\ \Omega$

$r_s = 0.5\ \Omega$, $x_s = 0\ \Omega$, $I_0 = 3$ A, $\cos\phi_0 = 0.4$, $\sin\phi_0 = 0.9165$

$$\therefore \quad I_c = I_0 \cos\phi_0 = 1.2 \text{ A}$$

$$I_m = I_0 \sin\phi_0 = 2.7495 \text{ A}$$

$$Z_s = r_s + r_e + j\,0 = 1.5\ \Omega, \quad \delta = 0°$$

$$\therefore \quad \alpha = 90 - \cos^{-1} 0.4 = 23.5782°$$

$$n = K_n = 100$$

$$R = n + \frac{I_0}{I_s} \sin(\delta + \alpha)$$

$$= 100 + \frac{3}{5} \sin(23.5782°) = \mathbf{100.24}$$

$$\text{While } \theta = \frac{I_0 \cos(\delta + \alpha)}{n I_s + I_0 \sin(\delta + \alpha)} \text{ radians}$$

$$= \frac{3\cos(23.5782°)}{100 \times 5 + 3\sin(23.5782°)} = 5.485 \times 10^{-3} \text{ radians}$$

$$= \frac{180°}{\pi} \times 5.485 \times 10^{-3} \text{ degrees} = 0.314°$$

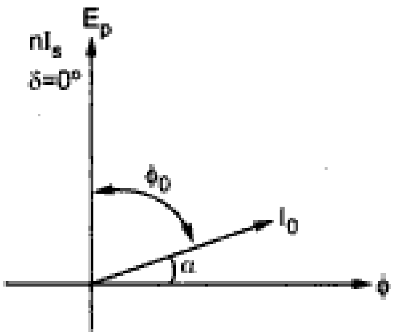


Fig. 2.13

$$E_s = I_s \times Z_s = 5 \times 1.5 = 7.5 \text{ V}$$

and $$E_s = 4.44 f\phi_m N_s \text{ and } N_s = n \times N_p = 100$$

$$\therefore \quad 7.5 = 4.44 \times 50 \times \phi_m \times 100$$

$$\therefore \quad \phi_m = \mathbf{337.83 \times 10^{-6} \text{ Wb}}$$

Example 2.11 : *A current transformer with 5 primary turns has a secondary burden consisting of a resistance of 0.16 Ω and an inductive reactance of 0.12 Ω, when the primary current is 200 A, the magnetising current is 1.5 A and the iron loss current is 0.4 A. Find the number of secondary turns needed to make the current ratio 100.1 and the phase angle.* **(JNTU, May-2005, Set-3)**

Solution : $$Z_s = r_s + j\, x_s = 0.16 + j\, 0.12\, \Omega = 0.2 \angle 36.86° \Omega$$

$$\therefore \quad \delta = 36.86°, \ \sin\delta = 0.6, \ \cos\delta = 0.8$$

$$I_m = 1.5 \text{ A}, \ I_c = 0.4 \text{ A}, \ I_p = 200 \text{ A}, \ N_p = 5$$

$$\frac{I_p}{I_s} = 100.1 = R$$

$$\therefore \quad I_s = \frac{I_p}{100.1} = 1.998$$

Now $$R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s}$$

$$\therefore \quad 100.1 = n + \frac{1.5 \times 0.6 + 0.4 \times 0.8}{1.998}$$

$$\therefore \quad n = 99.4893$$

$$n = \frac{N_s}{N_p} \quad \text{for C.T.}$$

$$\therefore \quad N_s = n \times N_p = 99.4893 \times 5 = 497.44$$

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right]$$

$$= \frac{180}{\pi}\left[\frac{1.5\times 0.8 - 0.4\times 0.6}{99.4893\times 1.998}\right] = \mathbf{0.276°}$$

Example 2.12 : *A single phase potential transformer has a turns ratio of 3810/63. The nominal secondary voltage is 63 V and the total equivalent resistance and leakage reactance referred to the secondary side are 2 Ω and 1 Ω respectively. Calculate the ratio and phase angle errors when the transformer is supplying a burden of 100 + j 200 Ω.*

Solution : $R_{2e} = 2\ \Omega, \quad X_{2e} = 1\ \Omega,\ V_s = 63\ V$

$$n = \frac{3810}{63} = 60.4761$$

Neglecting no load component of current,

$$R = n + \frac{nI_s[R_{2e}\cos\Delta + X_{2e}\sin\Delta]}{V_s}$$

$$\text{Burden} = r_e + jx_e = 100 + j\,200\ \Omega$$

$$\therefore \quad \Delta = \tan^{-1}\frac{x_e}{r_e} = 63.434948°$$

$$\therefore \quad R = 60.4761 + \frac{60.4761\,[2\times\cos\Delta + \sin\Delta]}{\dfrac{V_s}{I_s}}$$

Now $$\frac{V_s}{I_s} = Z_s = \sqrt{100^2 + 200^2} = 223.6067\ \Omega$$

$$\therefore \quad R = 60.9599 \text{ and } K_n = \text{Nominal ratio} = n = 60.4761$$

$$\therefore \quad \%\ \text{ratio error} = \frac{K_n - R}{R}\times 100 = -0.799\ \% \approx \mathbf{-0.8\ \%}$$

$$\theta = \frac{I_s}{V_s}[X_{2e}\cos\Delta - R_{2e}\sin\Delta] \text{ in radians}$$

$$= \frac{1}{223.6067}[\cos 63.43 - 2\sin 63.43]$$

$$= -5.9\times 10^{-3}\ \text{rad} = \frac{-5.9\times 10^{-3}\times 180°}{\pi}\ \text{degrees}$$

$$= \mathbf{-0.338°}$$ ... Phase angle error

2.13 Difference between Instrument and Power Transformers

No.	Power Transformer	Instrument Transformer
1)	Mainly used to change the voltage levels in a power system.	Mainly used to extend the ranges of the instruments while measuring parameters like voltage, current, power, energy etc.
2)	They are required to transform huge amount of power to the load.	They are required to transform very small power as their loads are generally delicate moving elements of the instruments.
3)	They can be used to step up or step down the voltage.	They are basically step down transformers and used alongwith devices such as protective relays, indicators etc.
4)	The exciting current is a small fraction of the secondary winding load current.	As the load itself is small, the exciting current is of the order of the secondary winding load current.
5)	The cost is main consideration in the design while efficiency and regulation are the second considerations.	Accuracy is the main consideration while designing to keep ratio and phase angle errors to minimum. Cost is the second consideration.
6)	As they handle large power, the heat dissipation is the major consideration and cooling arrangement is necessary.	The power output is very small as loads are light hence heating is not severe.
7)	The limitation on the load is due to temperature rise.	The accuracy is the main load limitation factor and not the temperature rise.
8)	Examples are distribution transformers, transformers used for transmission.	Examples are current and potential transformers.

2.14 Power Factor Meters

The power in single phase a.c. circuit is given by

$$P = V I \cos\phi$$

where $\cos\phi$ = Power factor of the circuit

Thus by using precise voltmeter, ammeter and wattmeter in the circuit, the readings of V, I and P can be obtained. Then power factor can be calculated as,

$$\boxed{\cos\phi = \frac{P}{VI}}$$

But this method is not accurate. The errors in all the meters together cause the error in power factor calculation. Similarly the method is not suitable for the circuits whose power factor is varying according to circuit and load conditions. Hence it is necessary to have a meter which can directly indicate the power factor of the circuit. Such a meter which indicates the instantaneous power factor of the circuit is called **power factor meter.**

Basic construction of power factor meter is similar to a wattmeter. It has two circuits, current circuit and a voltage circuit. The current circuit carries current or fraction of current in the circuit whose power factor is to be measured. The voltage coil is split into two parallel paths, one inductive and one non-inductive. The currents in the two paths are proportional to the voltage of the circuit. Thus the deflection depends upon the phase difference between the main current through current circuit and the currents in the two branches of the voltage circuit i.e. power factor of the circuit.

There are two types of power factor meters,

1) Electrodynamometer type 2) Moving iron type

Let us discuss, these types of power factor meters.

2.15 Single Phase Electrodynamometer Type Power Factor Meter

The construction of electrodynamometer type power factor meter is similar to the construction of electrodynamometer type wattmeter. The basic construction of electrodynamometer type power factor meter is shown in the Fig. 2.14 (a).

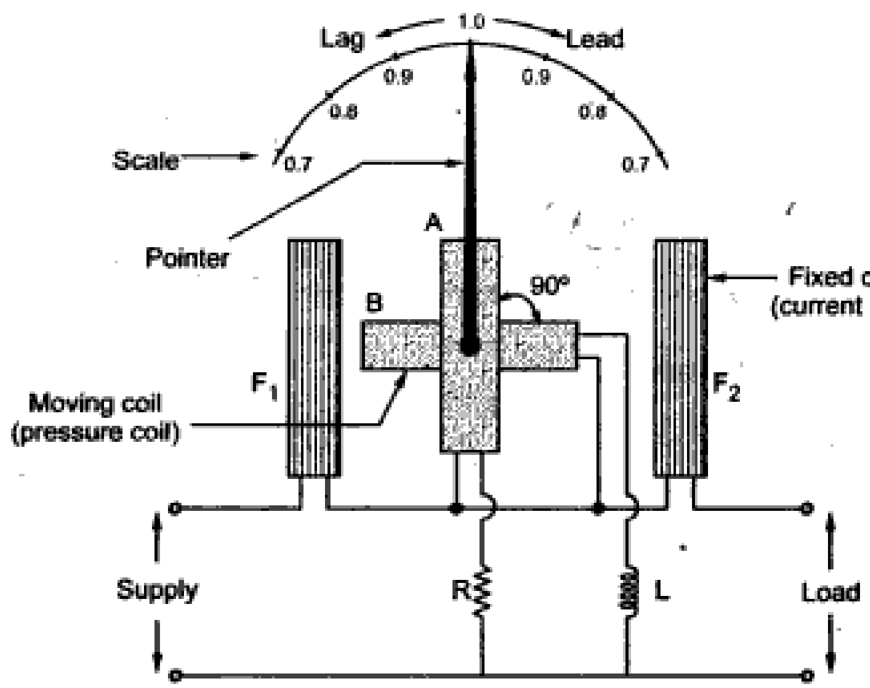


Fig. 2.14 (a) Single phase electrodynamometer type power factor meter

The F_1-F_2 are the two fixed coils which are connected in series. The A-B are the two moving coils which are rigidly connected to each other so that their axes are at 90° to each other. The moving coils A-B move together and carry the pointer which indicates the power factor of the circuit.

The fixed coils F_1-F_2 carry the main current in the circuit. If the current is large, the fraction of the current is passed through the fixed coils. Thus the magnetic field produced by the fixed coils is proportional to the main current.

The moving coils A-B are identical. These are connected in parallel across the supply voltage and hence called pressure coils or voltage coils. The currents through coils A and B are proportional to the supply voltage. The coil A has non-inductive resistance R in series with it while the coil B has an inductance L in series with it. The values of R and L are so adjusted that the coils A and B carry equal currents at normal frequency. So at normal frequency $R = \omega L$. The current through coil A is in phase with the supply voltage while the current through coil B lags the supply voltage by nearly 90° due to highly inductive nature of the circuit. Due to L, current through coil B is frequency dependent while current through coil A is frequency independent.

The currents in the coils A and B are equal and produce the magnetic fields of equal strength, which have phase difference of 90° between them. The coils are also mutually perpendicular to each other.

The controlling torque is absent. The contacts to the moving coils are made with the help of extremely fine ligaments which give no controlling effect on the moving system.

2.15.1 Working of Meter

Consider the position of the moving system as shown in the Fig. 2.14 (b).

Assume that the current through coil B lags the voltage exactly by 90°.

Also assume that the field produced by the fixed coils is uniform and in the direction X-X as shown in the Fig. 2.14 (b) .

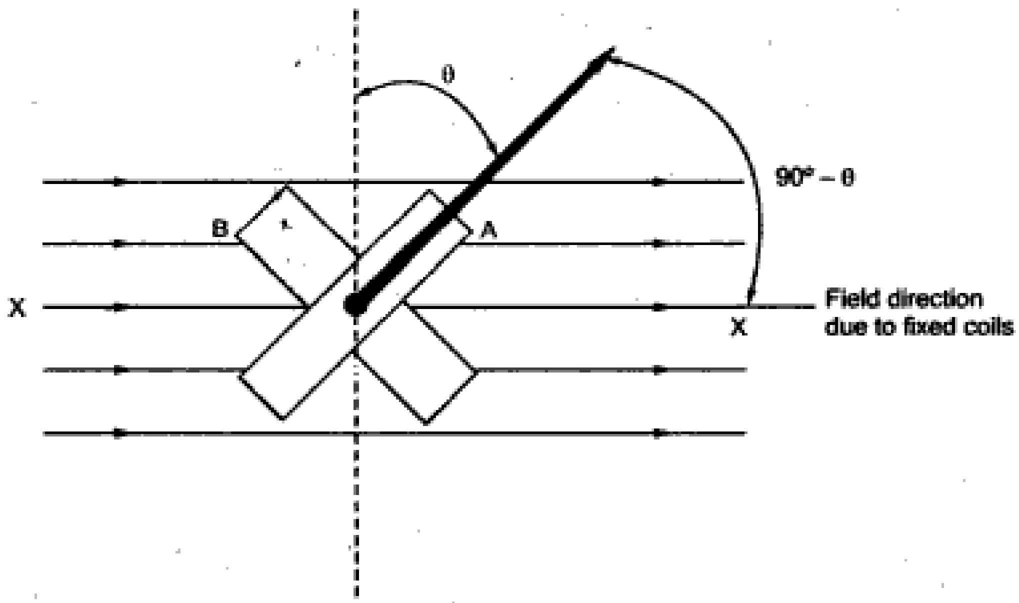


Fig. 2.14 (b)

Due to interaction of the fields produced by the currents through various coils, both the coils A and B experience a torque. The windings are arranged in such a manner that the torques experienced by coil A and B are opposite to each other. Hence the pointer attains an equilibrium position when these two torques are equal.

The torque on each coil, for a given coil current will be maximum when the coil is parallel to the field produced by $F_1 - F_2$ i.e. direction X-X.

Let ϕ = Power factor angle

θ = Angle of deflection

The θ is measured from the vertical axis, in the equilibrium position.

Similar to a dynamometer type wattmeter, torque on coil A is given by,

$$T_A = K\,VI \cos\phi \cos(90°-\theta) \quad \text{... (1)}$$

where K = Constant

The equation is similar to the torque equation of a dynamometer type instrument. The current through coil A is in phase with system voltage V and it moves in a magnetic field which is proportional to system current I and $dM/d\theta$ which is generally constant for radial field is not constant for parallel field and is proportional to $\cos(90° - \theta)$.

Similarly current in coil B lags the supply voltage by 90° and it moves in same field. Hence the torque on B is proportional to $\cos(90° - \phi)$ i.e. $\sin\phi$ and $\cos\theta$.

$$\therefore \quad T_B = K\,VI \sin\phi \cos\theta \quad \text{... (2)}$$

In equilibrium position, $T_A = T_B$

$$\therefore \cos\phi \cos(90°-\theta) = \sin\phi \cos\theta$$

$$\therefore \quad \sin\theta = \tan\phi \cos\theta$$

$$\therefore \quad \tan\theta = \tan\phi$$

$$\therefore \quad \boxed{\theta = \phi}$$

Thus the angular position taken up by the moving coils is equal to the system power factor angle. The scale of the instrument can then be calibrated interms of power factor values.

The operation of the instrument is dependent on the specific supply frequency. If the frequency is different or it contains harmonics then inductance of choke coil changes, due to which there will be serious errors in the instruments reading. Thus the operation of the meter is not dependent on the values of current and voltage but dependent on the frequency and the waveform.

2.16 Moving Iron Power Factor Meter

The advantages of moving iron power factor meter over the dynamometer type are,

1. The working forces in moving iron are larger.
2. All coils in moving iron are fixed so no ligaments are required.
3. A scale extends over 360°.

But due to the losses in the iron parts, the accuracy of moving iron power factor meters is much less than electrodynamometer type.

There are two types of moving iron power factor meters,

1. Rotating field type 2. Alternating field type

2.16.1 Rotating Field Type Moving Iron Power Factor Meter

It consists of three fixed coils whose axes are displaced from each other by 120°. The coils are supplied from a three phase supply through current transformers (C.T.) The Fig. 2.15 shows the construction of rotating field type moving iron power factor meter. The coils F_1 , F_2 and F_3 are the fixed coils. The coil F_1 is supplied from phase R, coil F_2 from phase Y and coil F_3 from phase B. The coil Q is placed at the centre of the three fixed coils and is connected across any two lines of the supply through a series resistance.

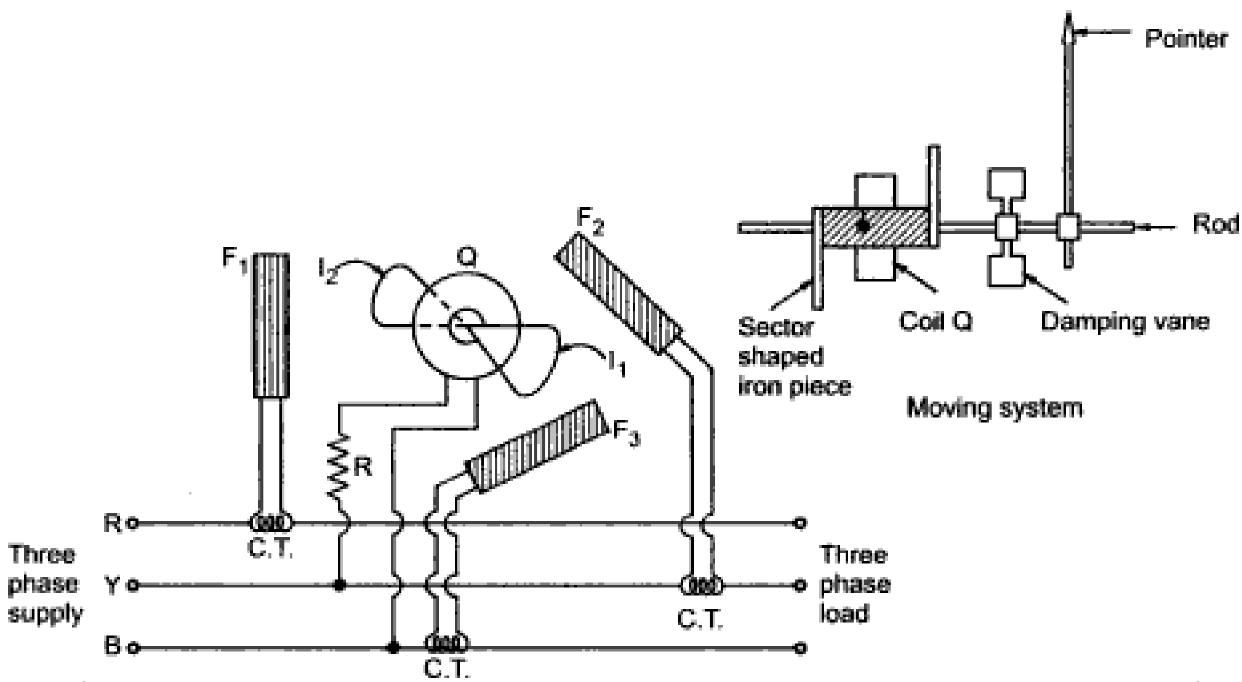


Fig. 2.15 Rotating field type moving iron power factor meter

Inside coil Q, there is a short pivoted iron rod. The rod carries two sector shaped vanes I_1 and I_2, at its ends. The same rod carries damping vanes and a pointer. The control springs are absent. The moving system is shown seperately in the Fig. 2.15.

The coil Q and the iron system produce an alternating flux which interacts with the flux produced by the coils F_1, F_2 and F_3. Due to resistance R, the current in coil Q is in phase with the supply voltage. So the deflection of the moving system is approximately equal to the power factor angle of the three phase circuit. The flux produced by the coils F_1, F_2 and F_3 is rotating magnetic flux which creats an induction motor action. It tries to keep moving system continuously rotating. But it sets moving system in a definite position due to use of high resistivity iron parts. Such high resistivity parts reduce the induced currents and stops the continuous rotation.

The meter can be used for balanced loads. It is also called Westinghouse power factor meter. It is calibrated at the normal supply frequency and can cause serious errors if used at any other frequency.

2.16.2 Alternating Field Type Moving Iron Power Factor Meter

This instrument consists of three moving irons and vanes, which are fixed to the common spindle. The spindle carries the damping vanes and the pointer. The moving iron vanes are sector shaped similar to those used in the rotating field type meter. The arcs of these sectors have an angle of 120° with respect to each other. These iron sectors are separated from each other on the spindle by the non-magnetic pieces denoted as S. The Fig. 2.16 shows the construction of this instrument where Q_1, Q_2 and Q_3 are the iron sectors. These iron sectors are magnetised by the coils P_1, P_2 and P_3. These are voltage coils.

Please refer Fig. 2.16 on next page.

These coils are connected across the three phases. Thus the currents through them are proportional to the phase voltages of the three phase system.

The current coil is divided into two equal parts F_1 and F_2, parallel to each other. The current coil carries one of the three line currents. One part F_1 of the current coil is on one side of the moving system and other F_2 on other side.

When connected in the circuit, the moving system moves and attains such a position in which mean torque on one of the iron pieces gets neutralized by the torques produced by the other two iron pieces. In this position, the deflection of the pointer is equal to phase angle between the currents and voltages of the three phase system. The instrument is used for the balanced loads but can be modified for unbalanced loads. The voltage coils are at different levels hence the resultant flux is not rotating but alternating.

This instrument is also called Nalder-Lipman power factor meter.

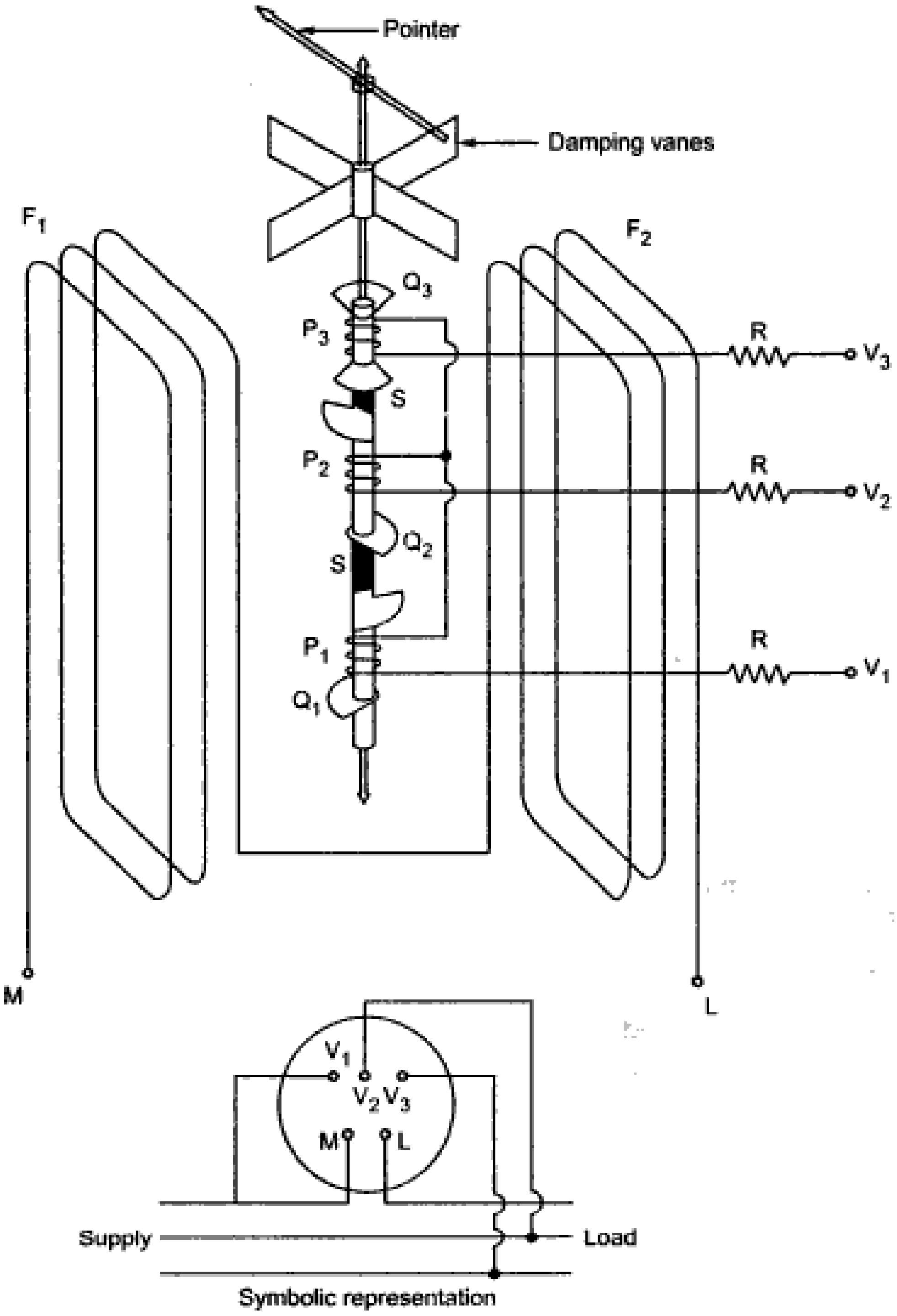


Fig. 2.16 Alternating field type moving iron power factor meter

2.17 Frequency Meters

The meters which are used in the circuit to indicate the frequency of the supply are called frequency meters.

The frequency meters are classified based on the principle of operation as,

1. Mechanical resonance type frequency meter
2. Electrical resonance type frequency meter
3. Weston type frequency meter

The mechanical resonance type frequency meter is called vibrating reed type frequency meter. The electrical resonance type frequency meter is called ferro-dynamic frequency meter. Let us discuss these frequency meters.

2.17.1 Vibrating Reed Type Frequency Meter

This meter works on the principle of mechanical resonance. The meter consists of number of thin steel strips called reeds. The bottom of the reed is rigidly fixed to an electromagnet. The upper part of the reed is free and bent at right angles. This upper part is called a flag. An electromagnet has a laminated iron core, which carries an excitation coil having large number of turns. This coil is connected across the voltage whose frequency is to be measured. The flags are painted white to have good visibility on the black background. The basic construction of this type of meter and the construction of reed is shown in the Fig. 2.17.

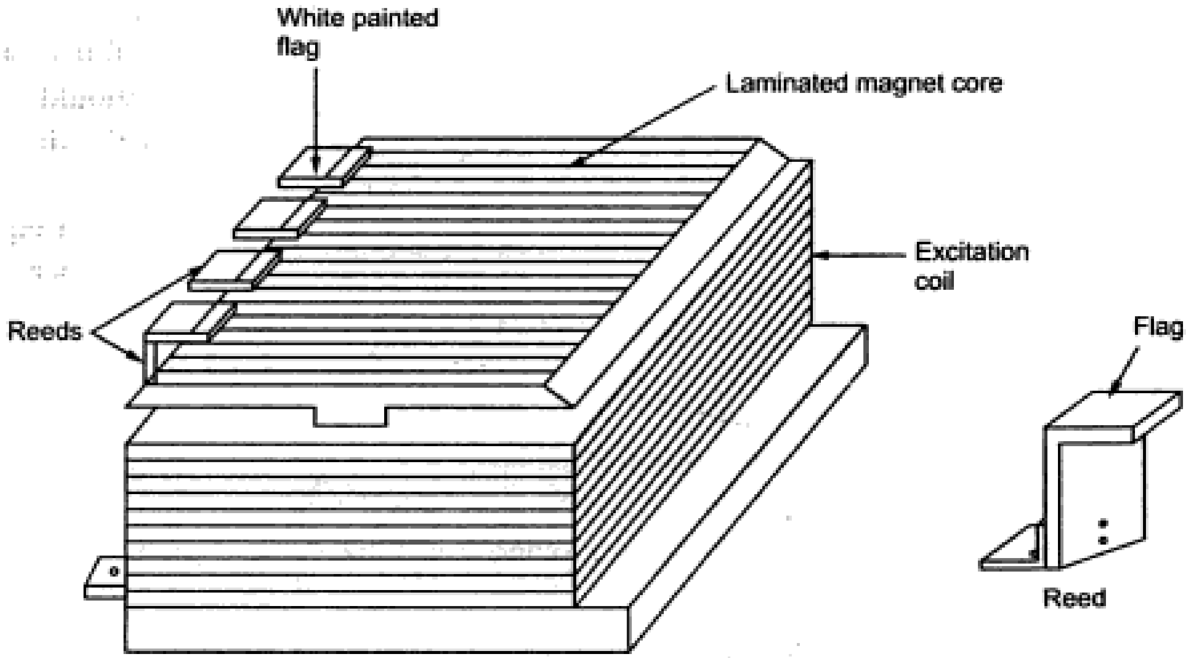


Fig. 2.17 Vibrating reed type frequency meter

The reeds are manufactured such that their weights and dimensions are different. Hence their natural frequencies of vibration are different. The reeds are arranged in the ascending order of their natural frequencies and the natural frequencies are generally differ by half cycle. So natural frequency of first reed may be 48 Hz, next may be 48.5 Hz, next may be 49 Hz and so on.

When meter is connected in the system, the coil carries current i which alternates at the supply frequency. This produces an alternating flux. This produces a force of attraction on the reeds which is proportional to **square of the current** i^2 and hence all the reeds vibrate with a force which varies at twice the supply frequency. But the reed whose natural frequency is twice the frequency of supply voltage will be in resonance and will vibrate most. The tuning in such meters is so precise that for a 1 to 2% change in the frequency away from resonating frequency, the amplitude of vibration decreases drastically and becomes negligible. Thus when a reed corresponding to 50 Hz is vibrating with maximum amplitude, other reeds vibrate but with negligible amplitudes which can not be noticed. This is shown in the Fig. 2.18.

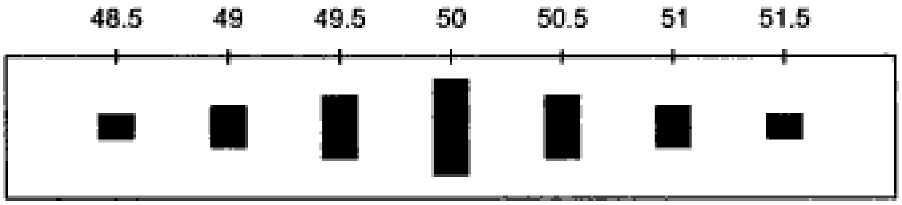


Fig. 2.18 Vibrating reeds

The advantages of this frequency meter are that the reading is not affected by the changes in the waveform of the supply voltage and simple mechanism. But if supply voltage is low, the vibrations may not be noticed. So supply voltage should not be low for the effective operation. One more limitation of the meter is that the difference in the frequencies of the adjacent reeds is 0.5 only. So reading corresponding to less than half the frequency difference can not be obtained. So precise frequency measurement is not possible. The accuracy of the meter depends on the proper tuning of the reeds.

2.17.2 Electrical Resonance Type Frequency Meter

The Fig. 2.19 shows the construction of the electrical resonance type frequency meter.

It consists of a laminated iron core. On one end of the core a fixed coil is wound which is called magnetizing coil. This coil is connected across the supply whose frequency is to be measured. This coil carries current which has same frequency as that of supply. On the same core, a moving coil is pivoted which carries a pointer. A capacitor C is connected across the terminals of the fixed coil.

Let $\quad I$ = Current through magnetizing coil

ϕ = Flux in the iron core

The flux ϕ is assumed to be in phase with the current I.

This flux induces the voltage in the moving coil which always lags flux ϕ by 90°.

Let $\quad i$ = Current through moving coil

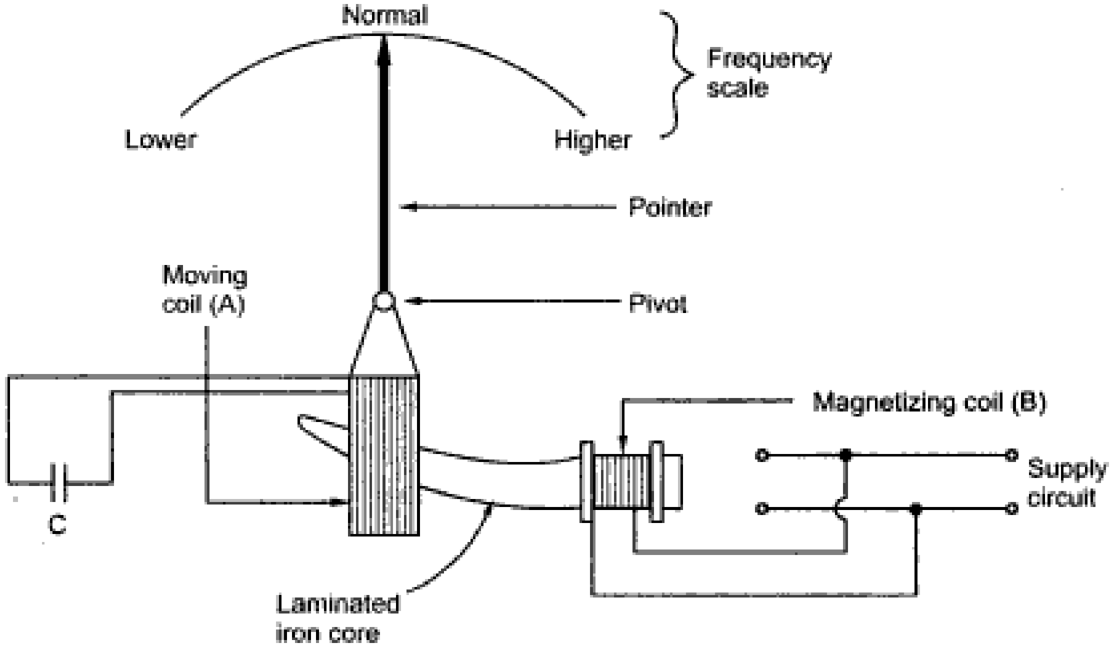


Fig. 2.19 Electrical resonance type frequency meter

The phase of the current i depends on the inductance of the moving coil and the capacitor C.

Consider the different cases and the corresponding phasor diagrams to understand the working of the meter, as shown in the Fig. 2.20 (a), (b) and (c).

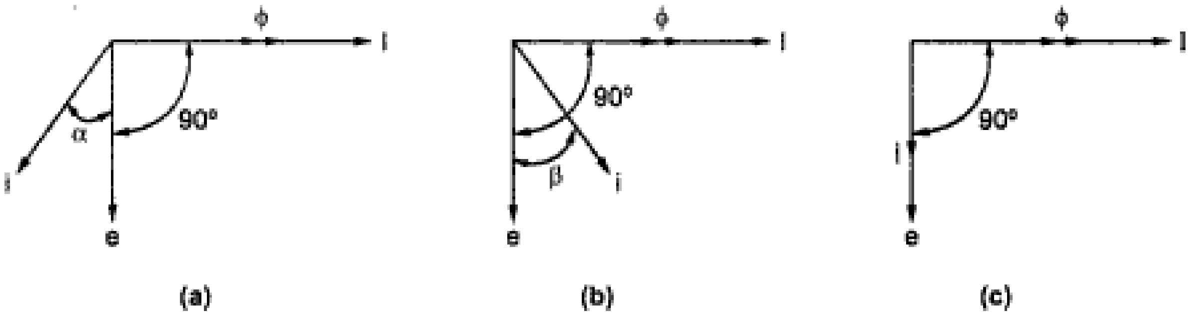


Fig. 2.20

In Fig. 2.20 (a) the circuit of moving coil A is assumed to be inductive, hence current i lags the induced voltage e by angle α. Hence the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90+\alpha) \qquad \text{... (1)}$$

In Fig. 2.20 (b) the circuit of moving coil A is assumed to be largely capacitive, hence current i leads the induced voltage e by angle β. Hence the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90-\beta) \qquad \text{... (2)}$$

This torque is in opposite direction to the torque produced in case of inductive nature of the moving coil circuit.

The Fig. 2.20 (c) shows the resonance condition where the inductive reactance is equal to the capacitive reactance. So current i is in phase with e and the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90^\circ) = 0$$

Hence under resonance condition, torque acting on the moving coil is zero.

Now the capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is constant for a given frequency. But the inductive reactance $X_L = \omega L$ not only depends on the frequency but also depends on the position of the moving coil on the core. Nearer the moving coil to the magnetizing coil, higher is its inductance. Thus for a given frequency, moving coil moves in such a way to achieve a position where $X_L = X_C$ and electrical resonance is achieved. At this position, torque on the moving coil is zero and the pointer indicates the corresponding frequency. The design of the instrument is such that for a normal frequency, the coil takes a mean position. The capacitor C is chosen such that electrical resonance takes place at this mean position and pointer indicates the normal frequency.

If frequency is higher than the normal value, then $X_C = \frac{1}{2\pi f C}$ decreases. Hence $X_L = 2\pi f L$ must decrease in order to achieve resonance. So moving coil moves away from the magnetizing coil on the core and pointer moves to the right of the mean position, indicating higher frequency.

If frequency is lower than normal value, $X_C = \frac{1}{2\pi f C}$ increases. So to achieve $X_L = X_C$, the moving coil moves towards the magnetizing coil where inductance increases. Thus pointer moves to the left of the mean position, indicating the lower frequency.

An important advantage of the instrument is that the great sensitivity is achieved as the inductance of the moving coil changes slowly with variation of its position on the core. This meter is also called **ferro-dynamic frequency meter.**

2.17.3 Weston Frequency Meter

This is moving iron type instrument. It works on the changes in current distribution between two parallel circuits, one of which is inductive and other non-inductive, when the frequency changes. This is due to the fact that the impedance of the inductive circuit changes with the change in the frequency. ($X_L = 2\pi f L$)

The Fig. 2.21 shows the constructional details of the Weston frequency meter.

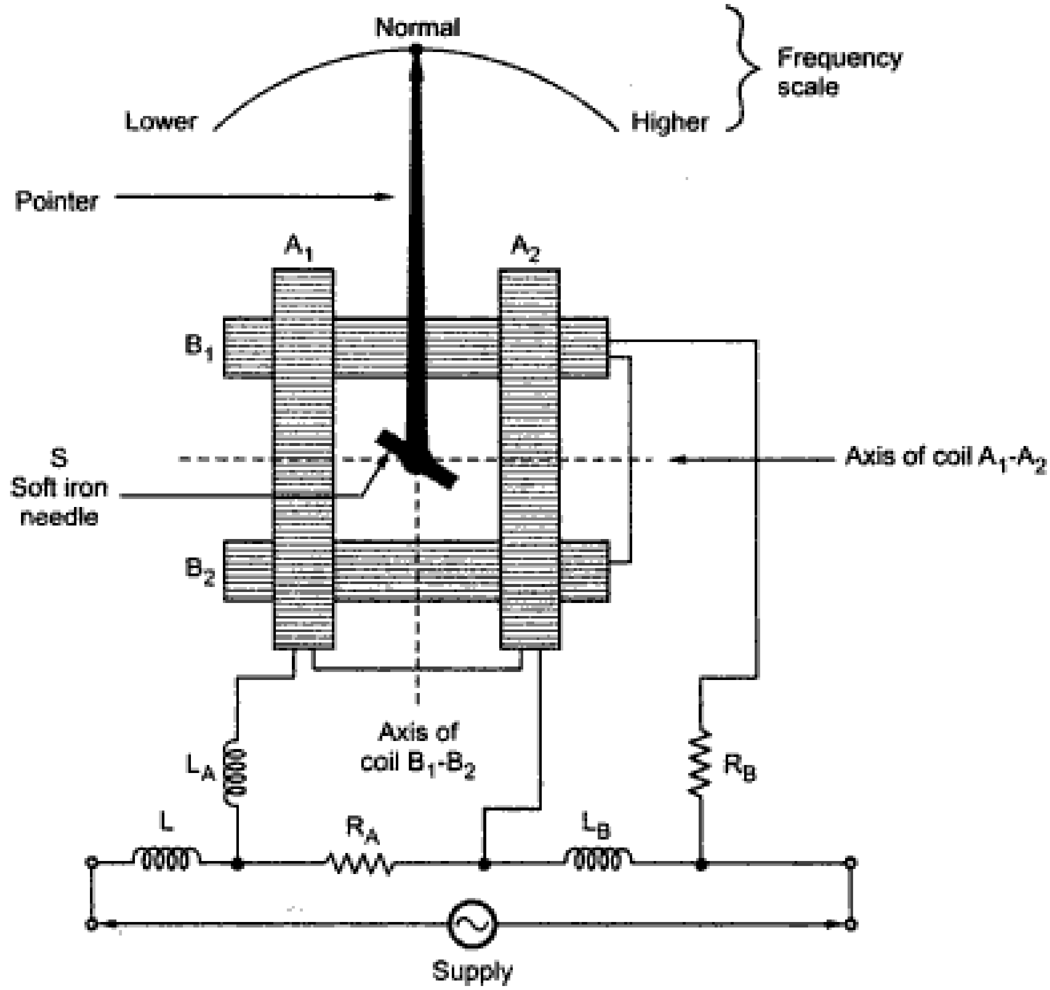


Fig. 2.21 Weston frequency meter

It consists of two fixed coils, each divided in two parts A_1-A_2 and B_1-B_2. The axes of the two coils are mutually perpendicular to each other. At the centre of the axes, a soft iron needle is pivoted which is thin and long. The needle carries a pointer and damping vanes. There is no controlling device to produce controlling torque.

The coil A is connected in series with an inductor L_A across a non-inductive resistance R_A. The coil B is connected in series with a non-inductive resistance R_B across an inductance L_B. The resistance R_A and L_B are in series with another inductor L and the combination is across the supply voltage. The main purpose of inductor L is for damping out the harmonics in the waveform of the current. This eliminates the errors caused due to the harmonics.

When the meter is connected across the supply, both the coils carry currents. The two magnetic fields produced by the two currents are at right angles to each other. These fields act upon the soft iron needle, causing its deflection. So position of needle and hence the pointer depends on the currents through the coils A and B.

In practice, the values of R_A , R_B, L_A and L_B are so choosen that the equal currents flow through the coils and needle takes the mean position, which indicates the normal frequency.

If the frequency increases above the normal value, then reactances L_A and L_B increase while non-inductive resistances R_A and R_B remain same. So impedance of the coil A increases. Hence the current through coil A is reduced. While voltage drop across R_A remains same. While the current through coil B increases due to its parallel combination with coil A. This makes the magnetic field produced by coil B more stronger. So the needle moves in such a way that it lies more nearly parallel to the axis of the coil B. So needle tries to become vertical and pointer deflects to the right indicating higher frequency. When the frequency decreases than the normal value, the opposite action takes place and pointer deflects to the left.

2.18 Phase Sequence Indicators

For determining the phase sequence of a three phase supply, phase sequence indicators are used. There are two types of phase sequence indicators,

1. Rotating type 2. Static type

2.18.1 Rotating Type Phase Sequence Indicator

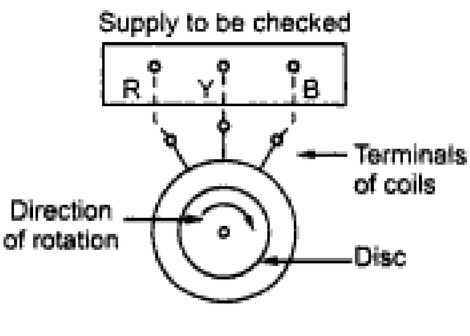


Fig. 2.22 Rotating type phase sequence indicator

This type of indicator works on the principle of induction. This working principle is the same based on which three phase induction motor works.

It consists of three stationary coils, separated from each other by 120° in space. The three ends of these coils, after connecting them in star are brought out for the connection purpose. The three phase supply whose phase sequence is to be observed is given to the three ends of these star connected coils. A disc is mounted on the top of the coils as shown in the Fig. 2.22.

When a star connected coils are excited by a three phase supply, then each coil produces an alternating flux. So all three fluxes are separated from each other by 120° in space. The resultant flux due to the interaction of these three fluxes is of rotating type called rotating magnetic field whose axis rotates in space with a certain speed.

This rotating flux pass over the disc and though the disc is stationary, there is cutting of flux as flux is rotating. Hence e.m.f. gets induced in this disc which circulates eddy currents through disc. These eddy currents produce a flux, which interacts with the rotating flux to produce a torque and the disc starts rotating. The direction of rotation is dependent on the direction of rotation of rotating field which in turn depends on the phase sequence. A particular direction is marked on the disc for a phase sequence of R-Y-B. If the supply connected has a phase sequence R-Y-B, disc rotates in the direction indicated on it. If disc rotates in opposite direction to that indicated on the disc, it confirms that the phase sequence of supply used is opposite to that marked on the terminals.

2.18.2 Static Type Phase Sequence Indicator

In static type of indicator few arrangements of lamps with inductor or capacitor are used. From the proportional darkness and brightness of the lamps used, phase sequence is identified.

Consider an arrangement where two lamps and one inductor are used as shown in the Fig. 2.23. Two lamps form the load in two lines R and Y while the inductor forms a load in line B.

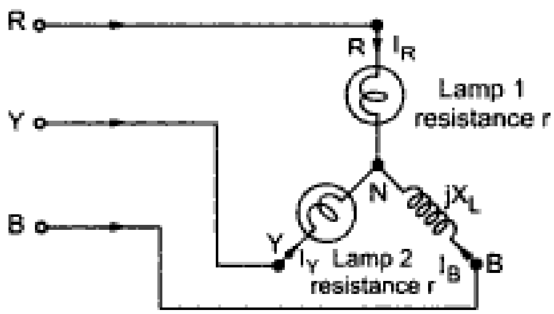


Fig. 2.23

The arrangement is star connected across the supply. Let us assume phase sequence to be R-Y-B.

For star connection, $V_L = \sqrt{3}V_{ph}$

and $\overline{V}_{RY} = \overline{V}_R - \overline{V}_Y$, $\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$ and $\overline{V}_{BR} = \overline{V}_B - \overline{V}_R$

where $\overline{V}_R$, $\overline{V}_Y$ and $\overline{V}_B$ are phase voltages across two lamps and inductor.

$\therefore \quad V_R = I_R \times r, \quad V_Y = I_Y \times r \quad$ and $\quad V_B = jX_L I_B$

$\therefore \quad \overline{V_{RY}} = \overline{V_R} - \overline{V_Y} = \overline{I_R}\,r - \overline{I_Y}\,r$

$\therefore \quad \overline{V_{RY}} + \overline{I_Y}\,r - \overline{I_R}\,r = 0 \qquad \ldots (1)$

and $\overline{V_{YB}} = \overline{V_Y} - \overline{V_B} = \overline{I_Y r} - j\overline{X_L I_B}$

$$\therefore \overline{V_{YB}} + j\overline{X_L I_B} - \overline{I_Y r} = 0 \quad \text{... (2)}$$

and $\overline{I_R} + \overline{I_Y} + \overline{I_B} = 0$ ... (3)

From (1), $$I_R = \frac{V_{RY} + I_Y r}{r} = \frac{V_{RY}}{r} + I_Y \quad \text{... (4)}$$

From (2), $$rI_Y = V_{YB} + jX_L I_B \quad \text{... (5)}$$

Now $I_B = -I_R - I_Y$

$$\therefore \quad rI_Y = V_{YB} + jX_L(-I_R - I_Y)$$

Using (4), $$rI_Y = V_{YB} + jX_L\left[-\frac{V_{RY}}{r} - I_Y - I_Y\right]$$

$$\therefore \quad rI_Y = V_{YB} - j\frac{X_L}{r}V_{RY} - 2jX_L I_Y$$

$$\therefore \quad I_Y = \frac{V_{YB} - \frac{jX_L}{r}V_{RY}}{(r + 2jX_L)} = \frac{rV_{YB} - jX_L V_{RY}}{r(r + 2jX_L)} \quad \text{... (6)}$$

$$\therefore \quad I_R = \frac{V_{RY}}{r} + \frac{V_{YB} - \frac{jX_L}{r}V_{RY}}{(r + 2jX_L)}$$

$$= \frac{(r + 2jX_L)V_{RY} + V_{YB}r - jX_L V_{RY}}{r(r + 2jX_L)}$$

$$\therefore \quad I_R = \frac{[rV_{RY} + 2jX_L V_{RY}] + [V_{YB}r - jX_L V_{RY}]}{r(r + 2jX_L)} \quad \text{... (7)}$$

Dividing (7) by (6),

$$\frac{I_R}{I_Y} = \left\{\frac{[rV_{RY} + 2jX_L V_{RY}] + [V_{YB}r - jX_L V_{RY}]}{r(r + 2jX_L)}\right\} \times \left\{\frac{r(r + 2jX_L)}{rV_{YB} - jX_L V_{RY}}\right\}$$

$$= 1 + \frac{rV_{RY} + 2jX_L V_{RY}}{rV_{YB} - jX_L V_{RY}} = 1 + \frac{rV_{RY}\left[1 + j\frac{2X_L}{r}\right]}{rV_{RY}\left[\frac{V_{YB}}{V_{RY}} - j\frac{X_L}{r}\right]}$$

$$\therefore \quad \boxed{\frac{I_R}{I_Y} = 1 + \frac{1 + j\frac{2X_L}{r}}{\left[\frac{V_{YB}}{V_{RY}}\right] - j\frac{X_L}{r}}} \quad \text{... (8)}$$

If supply is balanced and V_{RY} is taken as reference then,

$$V_{RY} = V\angle 0^\circ = V(1 + j0)\ V$$

$$V_{YB} = V\angle -120^\circ = V(1\angle -120^\circ) = V\ [\ -0.5 - j\ 0.866\ ]$$

$$V_{BR} = V\angle -240^\circ = V(1\angle -240^\circ) = V\ [-\ 0.5 + j\ 0.866\ ]$$

$$\therefore \quad \frac{V_{YB}}{V_{RY}} = \frac{V\angle -120^\circ}{V\angle 0^\circ} = 1\angle -120^\circ = -\ 0.5 - j\ 0.866 \quad \text{... (9)}$$

Substituting in (8),

$$\frac{I_R}{I_Y} = 1 + \frac{1 + \frac{j2X_L}{r}}{-0.5 - j0.866 - \frac{jX_L}{r}}$$

Now let $X_L = r$ at the supply frequency then,

$$\frac{I_R}{I_Y} = 1 + \frac{(1 + j2)}{-0.5 - j0.866 - j} = 1 + \frac{2.23\angle 63.43^\circ}{1.93\angle -105^\circ}$$

$$= 1 + 1.155\angle 168.43^\circ = 1 + \{-1.1315 + j0.231\}$$

$$= -\ 0.1315 + j\ 0.231 = 0.266\angle 120^\circ$$

Hence I_R is lesser than I_Y and the drop $I_R r$ across Lamp 1 is just 26.66 % of that across Lamp 2 which is $I_Y r$. Hence **for the phase sequence R-Y-B, Lamp 1 glows dimmer while Lamp 2 glows brighter.**

While for other phase sequence i.e. R-B-Y, Lamp 1 glows brighter while Lamp 2 dimmer. From this condition phase sequence can be identified.

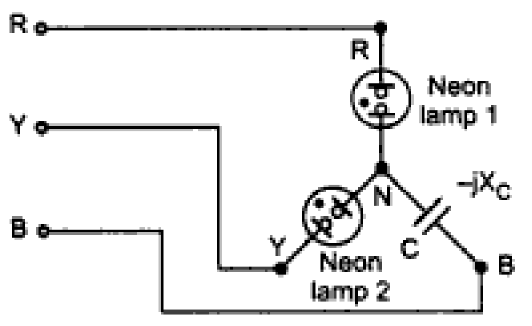


Fig. 2.24

Instead of inductor, a capacitor of value $X_C = X_L$ can be used and Neon lamps can be used. Neon lamps have property that if voltage is less than breakdown voltage of lamp then it does not glow at all. By this, confusion of dimmer and brighter can be avoided. The arrangement is shown in the Fig. 2.24.

It can be proved that due to capacitor C, for the phase sequence of R-Y-B, the ratio I_R to I_Y is 3.66 and the Lamp 1 glows bright while Lamp 2 does not glow at all. While for the phase sequence of R-B-Y, Lamp 1 will remain dark and Lamp 2 glows bright. From these conditions phase sequence of the supply can be identified.

2.19 Synchroscopes

The process of switching of an alternator with a common busbar without any interruption is called **synchronization.** The machine which is to be synchronised with the busbar is called **incoming machine.** To have effective synchronization certain conditions are to be fulfilled,

1. The terminal voltage of incoming machine must be same as that of busbar.
2. The frequency of incoming machine must be same as that of busbar.
3. The phase of the voltage of incoming machine must be same as the phase of the busbar voltage.

Practically a voltmeter and synchronizing lamps are used to decide the instant of switching such that all the above conditions are satisfied. But this method is based on personal judgement hence not accurate. The accurate device which is used to determine the difference in frequency and phase of voltages of incoming machine and busbar is called **synchroscope.** It consists of a rotating pointer which indicates the exact moment of closing the synchronizing switch. The rotation of the pointer is proportional to the difference in frequencies and phases of the voltages of incoming machine and busbar. It rotates in clockwise or anticlockwise direction depending whether the incoming machine is faster or slower than busbar. The synchroscope dial is shown in the Fig. 2.25. When pointer stops rotating it indicates that frequencies are same. When pointer stops in vertical position, it indicates that two voltages are in phase. Thus the correct instant of synchronizing can be accurately decided.

There are two types of synchroscopes,

1. Electrodynamometer or Weston type synchroscope.
2. Moving iron type synchroscope.

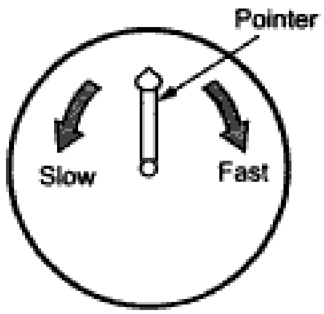


Fig. 2.25 Dial of synchroscope

2.19.1 Electrodynamometer or Weston Type Synchroscope

Practically to achieve the correct instant of synchronization, the static and dynamic methods are combined together. The static method includes the use of lamp while the dynamic method includes the use of electrodynamometer type synchroscope. The arrangement is shown in the Fig. 2.26.

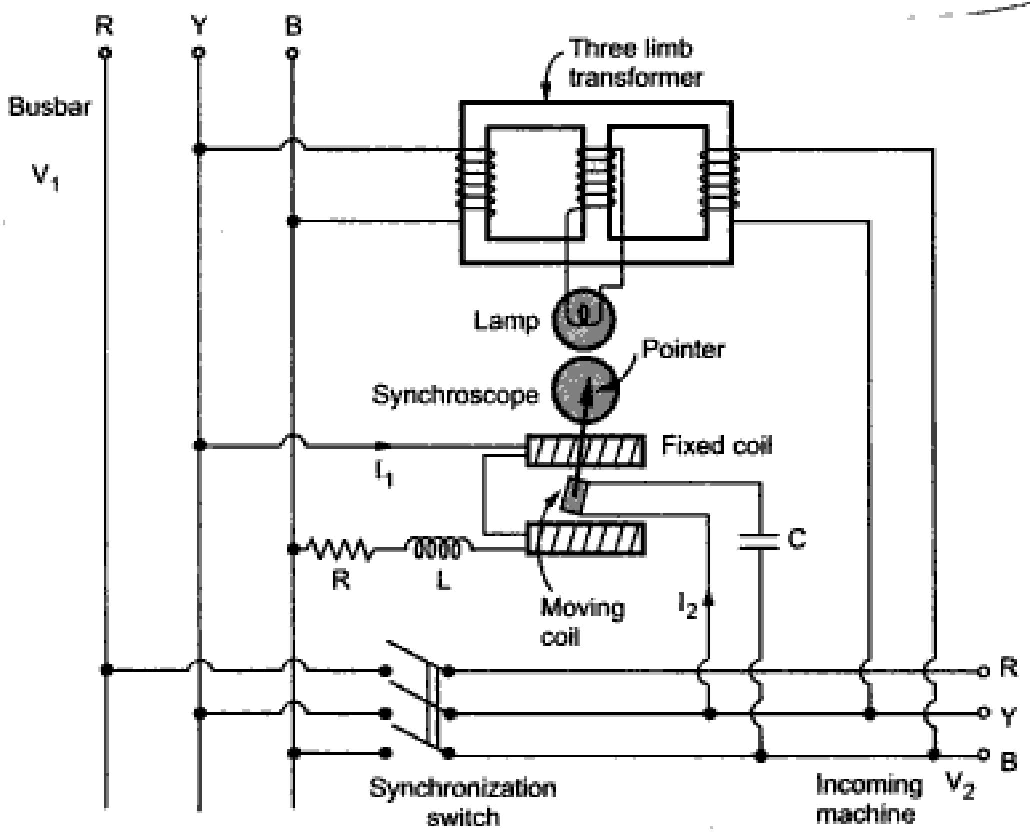


Fig. 2.26 Electrodynamometer type synchroscope

Static Part : It consists of three limbed transformer. One of the outer limbs is excited by busbar voltage V_1 while other outer limb is excited by the incoming machine voltage V_2. The central limb carries the lamp. The fluxes produced by the two outer limbs are forced through the central limb. The phasor sum of these two fluxes is the net flux in the central limb. This is responsible to induced an e.m.f. in the central limb which operates the lamp.

The outer limb windings are so arranged that if the two voltages V_1 and V_2 are in phase, two fluxes in the central limb help each other and maximum e.m.f. gets induced in the central limb. This makes lamp glow with maximum brightness. If the two voltages V_1 and V_2 are 180° out of phase, two fluxes in the central limb oppose each ohter and resultant flux in the central limb is zero. Thus no e.m.f. is induced in it and lamp remains dark.

If the frequency of V_2 is different than the frequency of V_1 then lamp flickers i.e. lamp becomes alternately dark and bright. The flickering frequency is equal to difference in the frequencies of V_1 and V_2. Thus when the lamp is flickering with very slow rate and when the lamp is maximum bright then the synchronizing switch must be closed. But this part does not detect whether incoming machine is faster or slower. This can be known using the synchroscope.

Dynamic Part : This consists of an electrodynamometer type synchroscope. It consists of fixed coil divided into two parts while the moving coil consists of a pointer. The fixed coil is connected to busbar with a resistor and inductor in series with it. The moving coil is connected to the terminals of incoming machine with a capacitor in series. The inductor and capacitor are used in fixed coil and moving coil circuit respectively because when the two voltages are in phase then due to L and C, the two currents are in exact quadrature (90°) to each other. Thus no torque will act on the pointer. The control springs are arranged such that under this condition, pointer remains in vertical position. This is shown in the Fig. 2.27. The current I_1 lags V_1 while I_2 leads V_2 such that I_1 and I_2 are in quadrature. If V_1 and V_2 are 180° out of phase, still the currents I_1 and I_2 will be in quadrature and pointer will remain stationary. But in this situation, the lamp will be dark. Practically it is very difficult to achieve exact stationary position of the pointer. It oscillates about its vertical position on the scale.

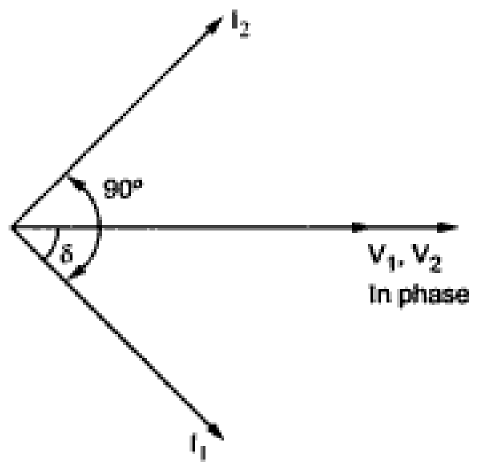


Fig. 2.27 (a) Phasor diagram when V_1, V_2 are in phase

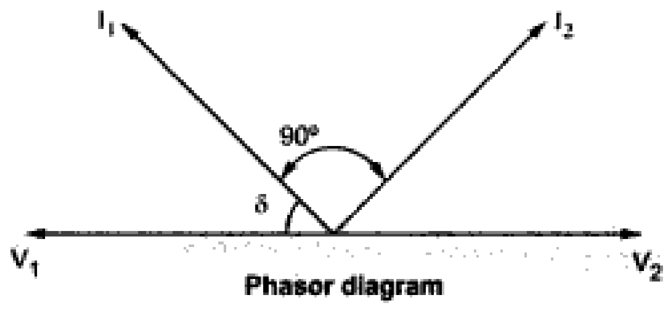


Fig. 2.27 (b) Phasor diagram when V_1, V_2 are in antiphase

Thus when pointer is oscillating very slowly about its central position and the lamp is maximum bright, then the synchronizing switch is operated. The movement of pointer in clockwise or anticlockwise direction tells whether the incoming machine is faster or slower.

2.19.2 Moving Iron Synchroscope

The construction of moving iron synchroscope is similar to the Nalder-Lipman power factor meter. It consists of two moving irons and vanes which are mounted on the common spindle. The moving iron vanes are sector shaped, separated by a brass piece. The vanes are connected to iron cylinders C_1 and C_2. The axes of moving iron vanes are 180° out of phase with respect to each other. The cylinders carry two pressure coils P_1 and P_2. The coils P_1 and P_2 are excited by incoming machine voltage. One coil is connected to incoming machine through resistance R while the other coil through inductor L. The fixed coil is divided into two parts A and A′ and excited from the busbar voltage. The control springs are not used. The fixed coil is designed to carry small current.

The construction of moving iron synchroscope is shown in the Fig. 2.28.

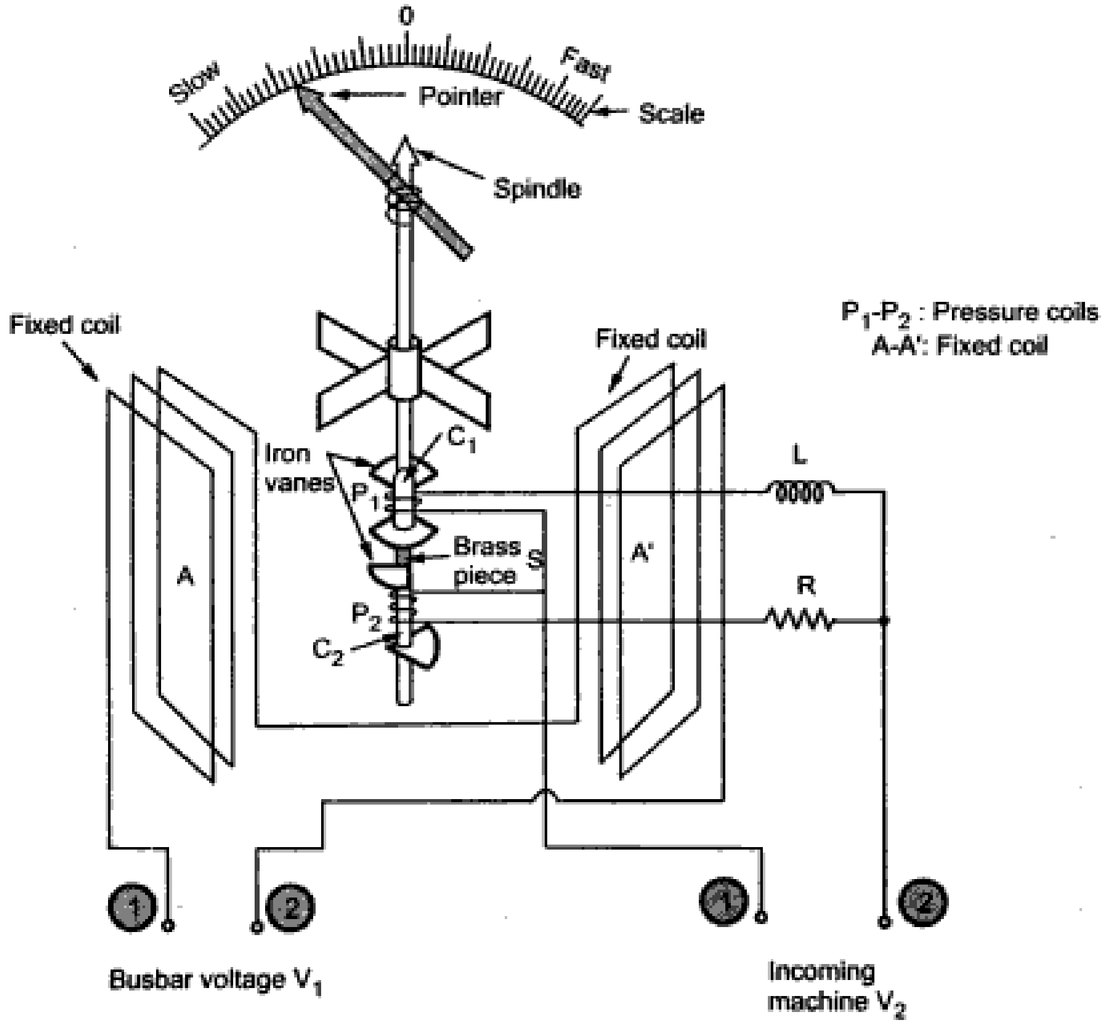


Fig. 2.28 Moving iron synchroscope

Working : When the frequency of V_2 is same as that of V_1 then the pointer is stationary. Its deflection is proportional to the phase difference between the two voltages V_1 and V_2. If the frequencies are different, then pointer rotates continuously. The speed of rotation is proportional to the difference in two frequencies. The direction of rotation indicates whether incoming machine is fast or slow.

The R and L connected in the pressure coils P_1 and P_2 ensure that there is phase difference of 90° between the currents of two pressure coils.

Let V_1 = Busbar voltage

V_2 = Voltage of incoming machine

The torque produced by the pressure coils P_1 and P_2 when the frequencies of two voltages is same then,

$$T_1 = K V_1 V_2 \sin\theta \cos(\pm \alpha)$$

$$T_2 = K V_1 V_2 \sin(90° - \theta) \cos(90° \pm \alpha)$$

i.e. $$T_2 = K V_1 V_2 \cos\theta \sin(\pm\alpha)$$

where θ = Deflection of pointer from vertical position

α = Phase difference between V_1 and V_2

If pointer has to achieve stationary vertical position, $T_1 = T_2$ as they act in opposite direction.

$$\therefore \quad \sin\theta \cos(\pm \alpha) = \cos\theta \sin(\pm \alpha)$$

$$\therefore \quad \boxed{\theta = \pm \alpha}$$

Thus the deflection of pointer is proportional to the phase difference between V_1 and V_2 and it is stationary.

If the frequencies of two voltages differ then,

$$T_1 = K V_1 V_2 \sin\theta \cos(\pm 2\pi f' t \pm \alpha)$$

$$T_2 = K V_1 V_2 \sin(90° - \theta) \cos[90° - (\pm 2\pi f' t \pm \alpha)]$$

i.e. $$T_2 = K V_1 V_2 \cos\theta \sin[\pm 2\pi f' t \pm \alpha]$$

In equilibrium condition, $T_1 = T_2$ and

$$\sin\theta \cos(\pm 2\pi f' t \pm \alpha) = \cos\theta \sin[\pm 2\pi f' t \pm \alpha]$$

$$\boxed{\theta = \pm 2\pi f' t \pm \alpha}$$

Thus the pointer rotates with a speed proportional to the difference in the two frequencies. The direction of rotation depends on whether f' is positive or negative i.e. whether frequency of incoming machine is higher or lower than that of busbar voltage.

Key Point : *The correct instant of switching is when pointer is stationary and is in vertical position.*

This type of synchroscope is very common in use because of its **advantages** such as,

1. Operation is simple.
2. The 360° scale is available.
3. The correct instant of switching can be decided.
4. Very cheap hence economical.

Examples with Solutions

➠ **Example 2.13 :** *The resistance and reactance of the secondary of a 500/5 A current transformer are 0.02 Ω and 0.03 Ω respectively and the transformer characteristics are given by,*

E.M.F. on secondary V	0.3	0.5	1.0	1.5	2.0
Magnetising current A	0.9	1.5	2.5	3.2	4.0
Coreloss component A	0.5	1.3	2.7	3.9	4.6

An ammeter, a wattmeter current coil and an induction relay are connected in series with the secondary winding. Their resistances are 0.08 Ω, 0.1 Ω and 0.14 Ω and their reactances are 0.09 Ω, 0.07 Ω and 0.08 Ω respectively. If the secondary current is 4 A, calculate ratio and phase angle error a) When all meter are in the circuit. b) Only wattmeter is in the circuit. Calculate the load VA in each case.

Solution : a) All meters are in the secondary winding

$$r_s = 0.2\,\Omega, \quad x_s = 0.033\,\Omega$$

$\therefore Z_s = (r_s + \text{resistances of all meters}) + j\,(x_s + \text{reactances of all meters})$

$$= (0.02 + 0.08 + 0.1 + 0.14) + j(0.09 + 0.07 + 0.08 + 0.03)$$

$$= 0.34 + j\,0.27 = 0.4341\ \angle 38.453°\,\Omega$$

$$\therefore \quad \delta = 38.453°$$

$$E_s = I_s\,Z_s = 4 \times 0.4341 = 1.737\ \text{V}$$

To obtain corresponding I_c and I_m, draw the graphs from the given observation table, as shown in the Fig. 2.29.

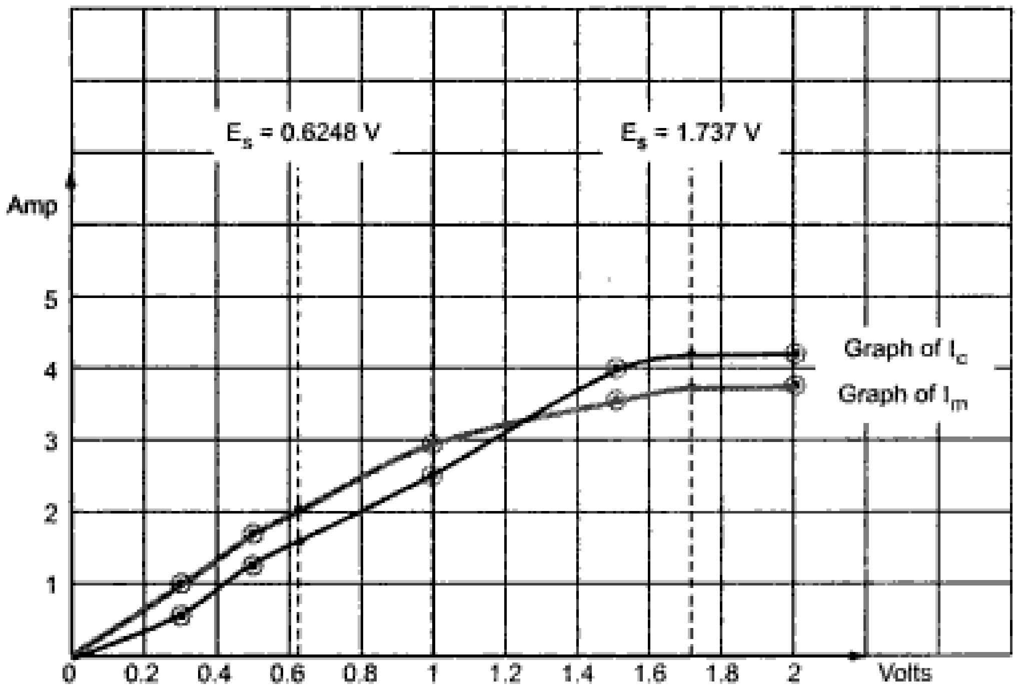


Fig. 2.29

From the graph, for E_s = 1.737 V, I_m = 3.7 A, I_c = 4.3 A

$$\therefore \quad R = n + \frac{I_c \cos\delta + I_m \sin\delta}{I_s} \quad \text{and} \quad n = \frac{I_p}{I_s} = \frac{500}{5} = 100$$

$$= 100 + \frac{4.3 \times \cos(38.453^\circ) + 3.7 \times \sin(38.453^\circ)}{4} = 101.417$$

$$\therefore \quad \%\text{ ratio error} = \frac{K_n - R}{R} \times 100 \quad \text{where} \quad K_n = n = 100$$

$$= \frac{100 - 101.417}{101.417} \times 100 = \mathbf{-1.397\%}$$

$$\theta = \frac{180^\circ}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right] \text{ degrees}$$

$$= \frac{180^\circ}{\pi}\left[\frac{3.7 \cos(38.453^\circ) - 4.3 \sin(38.453^\circ)}{100 \times 4}\right] = 0.032^\circ = \mathbf{1.91'}$$

$$\text{Load VA} = E_s\, I_s = 1.737 \times 4 = \mathbf{6.95\ VA}$$

b) Only wattmeter is in the secondary winding

$$\therefore \quad Z_s = (r_s + 0.1) + j(x_s + 0.07) = 0.12 + j\,0.1$$

$$= 0.1562 \angle 39.805° \ \Omega$$

$$\therefore \quad \delta = 39.805°$$

$$\therefore \quad E_s = I_s Z_s = 4 \times 0.1562 = 0.6248 \text{ V}$$

From the graph, $I_c = 1.6$ A and $I_m = 1.9$ A

Solving for R, ratio error and θ again,

$$R = 100.6113$$

$$\% \text{ ratio error} = -0.607\%$$

$$\theta = 0.062° = 3.73'$$

$$\text{and} \quad \text{Load VA} = E_s I_s = 0.6248 \times 4 = \mathbf{2.5 \ VA}$$

Example 2.14 : *A current transformer of turns ratio 1 : 199 is rated as 1000/5 A, 25 VA. The core loss is 0.1 W and magnetising current is 7.2 A, under rated conditions. Determine the phase angle and ratio errors for the rated burden and rated secondary current at 0.8 p.f. i) lagging and ii) leading. Neglect winding resistance and reactance.*

Solution :

$$n = \frac{N_s}{N_p} = \frac{199}{1} = 199, \quad K_n = \frac{I_p}{I_s} = \frac{1000}{5} = 200$$

$$I_m = 7.2 \text{ A}, \quad \text{Iron loss} = 0.1 \text{ W}$$

The VA rating is same on both sides.

$$\therefore \quad E_p I_p = E_s I_s = \text{VA}$$

$$\therefore \quad 25 = E_s \times 5 = E_p \times 1000$$

$$\therefore \quad E_s = 5 \text{ V} \quad \text{and} \quad E_p = 0.025 \text{ V}$$

$$I_c = \frac{\text{Iron loss}}{E_p} = \frac{0.1}{0.025} = 4 \text{ A}$$

Case i) $\cos\delta = 0.8$ lagging, $\delta = 36.86°$, positive for lagging

$$\therefore \quad R = n + \frac{I_c \cos\delta + I_m \sin\delta}{I_s} = 199 + \frac{4 \times 0.8 + 7.2 \times 0.6}{5}$$

$$= 200.504$$

$$\therefore \quad \% \text{ ratio error} = \frac{K_n - R}{R} \times 100 = \mathbf{-0.251\%}$$

$$\therefore \quad \theta = \frac{180°}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right] = +0.193° = +\mathbf{11.6'}$$

Case ii) $\cos\delta = 0.8$ leading, $\delta = -36.86°$, negative for leading

$$R = 199 + \frac{4 \times 0.8 + 7.2 \times (-0.6)}{5} = 198.776$$

$$\therefore \quad \%\text{ ratio error} = \frac{200 - 198.776}{198.776} \times 100 = \mathbf{+\ 0.615\%}$$

$$\therefore \quad \theta = \frac{180°}{\pi}\left[\frac{7.2 \times 0.8 - 4 \times (-0.6)}{199 \times 5}\right] = 0.469° = \mathbf{28.18'}$$

Example 2.15 : *At its rated load of 25 VA, a 100/5 current transformer has an iron loss of 0.2 W and a magnetizing current of 1.5 A. Calculate its ratio error and phase angle when supplying rated output to a meter having a ratio of resistance to reactance of 5.*

Solution : The nominal ratio

$$K_n = \frac{100}{5} = 20$$

$n = K_n = 20 =$ turn ratio ... No other data given.

Secondary burden = VA on secondary = 25

VA on primary = VA on secondary = 25

$\therefore \quad E_p I_p = 25, \quad I_p = 100\text{ A}, \quad I_s = 5\text{ A}$

$$\therefore \quad E_p = \frac{25}{100} = 0.25\text{ V}$$

$$\text{Core loss component} = I_c = \frac{\text{iron loss}}{E_p} = \frac{0.2}{0.25} = 0.8\text{ A}$$

Magnetising component $= I_m = 1.5$ A

Now $\dfrac{R_2}{X_2} = 5$

$$\therefore \quad \delta = \tan^{-1}\frac{X_2}{R_2} = \tan^{-1}\frac{1}{5} = 11.3099°$$

$\therefore \quad \cos\delta = 0.98058, \quad \sin\delta = 0.19611$

i) Actual ratio $R = n + \frac{I_c \cos\delta + I_m \sin\delta}{I_s} = 20 + \frac{0.8 \times 0.98058 + 1.5 \times 0.19611}{5}$

$= 20.2157$

$\therefore$ Ratio error $= \frac{K_n - R}{R} \times 100 = \frac{20 - 20.2157}{20.2157} \times 100 = \mathbf{-1.067\ \%}$

ii) Phase angle $\theta = \frac{180°}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right]$ degrees

$= \frac{180}{\pi}\left[\frac{1.5 \times 0.98058 - 0.8 \times 0.19611}{20 \times 5}\right] = \mathbf{0.7528°}$

Example 2.16 : *A power primary C.T. has 300 secondary turns. The total resistance and reactance for the secondary circuit are* 1.5 Ω *and* 1.0 Ω *respectively. When 5 A flows through the secondary winding, the magnetizing m.m.f. is 100 AT and the iron loss component is 40 A. Determine the ratio and phase angle errors of the C.T. at this load.* **(JNTU, Nov.-2004, Set-4)**

Solution : $N_p = 1 =$ Primary turns, $N_s = 300$

$$n = \frac{N_s}{N_p} = 300$$

$$R_2 = 1.5\ \Omega \quad \text{and} \quad X_2 = 1.0\ \Omega$$

$$\therefore \quad Z_2 = \sqrt{R_2^2 + X_2^2} = 1.8027\ \Omega$$

$$\delta = \tan^{-1}\frac{X_2}{R_2} = \tan^{-1}\frac{1}{1.5} = 33.69°$$

$$\therefore \quad \cos\delta = 0.832 \quad \text{and} \quad \sin\delta = 0.5546$$

$$E_s = I_s Z_2 = 5 \times 1.8027 = 9.0135\ \text{V}$$

Now $\frac{E_p}{E_s} = \frac{1}{n}$ i.e. $E_p = \frac{E_s}{n} = \frac{9.0135}{300} = 0.03$ V

$I_c = 40$ A = Iron loss component

$$I_m = \frac{\text{Magnetizing m.m.f.}}{\text{Primary winding turns}} = \frac{100}{1} = 100\ \text{A}$$

$$\therefore \quad R = n + \frac{I_m \sin\delta + I_c \cos\delta}{I_s} = 300 + \frac{[100 \times 0.5546 + 40 \times 0.832]}{5}$$

$= 317.748$

$$\therefore \quad \text{\% ratio error} = \frac{K_n - R}{R} \times 100 \qquad \ldots K_n = n = 300$$

$$= \frac{300 - 317.748}{317.748} \times 100 = \mathbf{-5.585\ \%}$$

$$\theta = \frac{180}{\pi}\left[\frac{I_m \cos\delta - I_c \sin\delta}{n\, I_s}\right] = \frac{180}{\pi}\left[\frac{100 \times 0.832 - 40 \times 0.5546}{300 \times 5}\right]$$

$$= \mathbf{2.33\ degrees}$$

Example 2.17 : *A 800/5 A, 50 Hz current transformer with a single turn primary has a secondary burden comprising a non reactive resistance of 4 Ω. The secondary winding of 160 turns has a resistance of 0.2 Ω. At the rated secondary current, calculate : i) flux in the core, ii) the actual ratio of primary to secondary current iii) the phase angle between the primary and secondary currents. No load primary current of 6 A lags by 30° to the reversed secondary voltage.* **(JNTU, Nov.-2003, Set-2)**

Solution : $K_n = 800/5, \; I_s = 5 \text{ A (rated)}, \; I_0 = 6 \text{ A}$

$$n = K_n = \frac{800}{5} = 160, \; N_s = 160$$

As burden is purely resistive, $\delta = 0$

Secondary impedance = 4 + 0.2 = 4.2 Ω = Z_2

$$E_s = Z_2 \times I_s = 4.2 \times 5 = 21 \text{ V}$$

But $E_s = 4.44\ f\, \phi_m\ N_s$

$$\therefore \quad 21 = 4.44 \times 50 \times \phi_m \times 160$$

i) $\phi_m = \mathbf{0.5912\ mWb}$ ... Flux in the core

ii) α = Angle between working flux and ϕ_0

$= 90° - 30° = 60°$ ... Refer Fig. 2.30

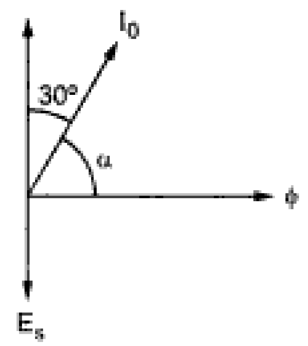


Fig. 2.30

$$R = n + \frac{I_0}{I_s} \sin(\delta + \alpha) = 160 + \frac{6}{5} \sin(60°)$$

$$= \mathbf{161.0392}$$

iii)

$$\theta = \frac{180}{\pi} \left[\frac{I_0 \cos(\delta + \alpha)}{n\, I_s + I_0 \sin(\delta + \alpha)} \right] \text{ degrees}$$

$$= \frac{180}{\pi} \left[\frac{6 \times \cos(60°)}{160 \times 5 + 6 \sin(60°)} \right] = \mathbf{0.2134° = 12'48''}$$

➡ **Example 2.18 :** *A potential transformer has a primary resistance of 300 Ω, a primary reactance of 600 Ω, a secondary resistance of 0.75 Ω and a secondary reactance of 1.5 Ω. The primary to secondary turns ratio is 20 : 1, the primary voltage is 2000 V. Neglect the magnetizing and core loss current. Determine the voltage ratio correction factor, ratio error and the phase angle error when the burden on the secondary of the transformer is, a) 50 VA at 0.6 p.f. lagging b) 50 VA at unity p.f. and c) 25 VA at 0.6 p.f. leading.* **(JNTU, May-2004, Set - 3)**

Solution : $r_p = 300\ \Omega$, $x_p = 600\ \Omega$, $x_s = 1.5\ \Omega$, $r_s = 0.75\ \Omega$

$$n = \frac{N_p}{N_s} = 20, \quad E_p = 2000 \text{ V}$$

$$n = \frac{E_p}{E_s} \quad \text{i.e. } E_s = \frac{E_p}{n} = \frac{2000}{20} = 100 \text{ V}$$

a) Burden 50 VA at 0.6 p.f. lagging

$$\therefore \quad E_s I_s = 50 \qquad \ldots V_s = E_s = 100 \text{ V}$$

$$\therefore \quad I_s = \frac{50}{E_s} = \frac{50}{100} = 0.5 \text{ A}$$

$$\Delta = \cos^{-1}(0.6) = 53.13°$$

$$\therefore \quad R = \frac{V_p}{V_s} = n + \frac{\left(\frac{I_s}{n}\right)[R_{1e} \cos\Delta + X_{1e} \sin\Delta]}{V_s} \qquad \ldots I_c = I_m = 0$$

$$R_{1e} = n^2 r_s + r_p = (20)^2 \times 0.75 + 300 = 600\ \Omega$$

$$X_{1e} = n^2 x_s + x_p = (20)^2 \times 1.5 + 600 = 1200\ \Omega$$

$$\therefore \quad R = 20 + \frac{\left(\frac{0.5}{20}\right)[600 \times 0.6 + 1200 \times 0.8]}{100}$$

$$= \mathbf{20.33} \qquad \ldots \text{Actual ratio}$$

$$\text{RCF} = \frac{R}{K_n} = \frac{20.33}{20} = 1.0165 \qquad \ldots K_n = n = 20$$

$$\% \text{ ratio error} = \frac{K_n - R}{R} \times 100 \qquad \ldots K_n = n = 20$$

$$= \frac{20 - 20.33}{20.33} \times 100 = -\mathbf{1.6232\ \%}$$

$$\theta = \frac{\frac{I_s}{n}(X_{1e}\cos\Delta - R_{1e}\sin\Delta)}{n\,V_s} \qquad \ldots I_c = I_m = 0A$$

$$= \frac{\frac{0.5}{20}[1200 \times 0.6 - 600 \times 0.8]}{20 \times 100} = \mathbf{0.003\ rad = 0.172° = 10.3'}$$

b) Burden 50 VA at unity p.f.

$\therefore I_s = 0.5$ A and $\Delta = \cos^{-1} 1 = 0°$, $\sin \Delta = 0$

Repeat the above calculations with new value of Δ.

$\therefore$ R = **20.15**, RCF = **1.0075**, % of ratio error = **– 0.744 %**, θ = **0.015 rad = 51.33′**

c) Burden 2 VA at 0.6 p.f. leading

$$E_s I_s = 25$$

$$\therefore \quad I_s = \frac{25}{E_s} = \frac{25}{100} = 0.25 \text{ A}$$

$$\Delta = \cos^{-1} 0.6 = 53.13° \text{ but leading}$$

$$\therefore \quad R = \frac{V_p}{V_s} = n + \frac{\left(\frac{I_s}{n}\right)[R_{1e}\cos\Delta - X_{1e}\sin\Delta]}{V_s}$$

$$= 20 + \frac{\left(\frac{0.25}{20}\right)[600 \times 0.6 - 1200 \times 0.8]}{100} = \mathbf{19.925}$$

$$\therefore \quad \text{RCF} = \frac{R}{K_n} = \frac{19.925}{20} = \mathbf{0.99625}$$

$$\therefore \quad \% \text{ ratio error} = \frac{K_n - R}{R} \times 100 = \mathbf{0.3764\ \%}$$

$$\theta = \frac{\frac{I_s}{n}(X_{1e}\cos\Delta + R_{1e}\sin\Delta)}{n\,V_s} = \frac{\frac{0.25}{20}[1200 \times 0.6 + 600 \times 0.8]}{20 \times 100}$$

$$= \mathbf{0.0075\ rad = 0.4297° = 25.46'}$$

Review Questions

1. *Draw neat connection diagram for measuring high voltage and high current with the help of P.T. and C.T. What purpose do they serve ?*
2. *What is ratio error? State its expression.*
3. *How to obtain actual transformation ratio? On which factor it depends ?*
4. *What is phase angle error and on which factors it depends ?*
5. *Phase angle error has no effect while only current or voltage measurement. State true or false with reason.*
6. *Write precise an approximate expression to calculate phase angle error.*
7. *Write a technical note on the errors in the instrument transformers.*
8. *How to minimize the ratio error and phase angle errors in the instrument transformers.*
9. *Explain the construction of : i) Current transformer ii) Potential transformer*
10. *A current transformer has bar primary and 200 secondary turns. The secondary current is 5 A to a purely resistive load of 1 Ω. Magnetising m.m.f. required is 80 AT. The frequency is 50 Hz while area of cross-section of core is 10 cm. Calculate the actual ratio, phase angle and maximum flux density in the core. Neglect core and copper losses.* **(Ans. : 2000.64, 4.573°, 0.112 Wb/m²)**
11. *The exciting current of a current transformer of ratio 1000/5 A, when operating at full primary current and with a secondary burden of non-inductive resistance of 1 is 1 A at 0.4 p.f. Calculate :*
    i) phase displacement between primary and secondary currents
    ii) the ratio error on full load. **(Ans. : 0.0525°, – 0.04 %)**
12. *A current transformer with single turn primary has 300 secondary turns and R = 1.5 Ω and X = 1 Ω. When secondary carries 5 A current, magnetising m.m.f. of 100 A and iron loss of 1.2 W, calculate ratio and phase angle errors.*
    (Hint : I_c *= Iron loss /* V_1*,* $V_2 = I_2 \times Z_2$*)* **(Ans. : – 5.54 %, 2.34°)**
13. *Show a neat connections diagram of a 3 phase energymeter used for measurement of energy, incorporating C.T. and P.T. Explain why C.T. and P.T. are used.*
14. *Write a note on : i) Power factor meter ii) Frequency meter.*
15. *Explain the construction and working principle of*
    i. Weston frequency meter ii. Electrodynamometer type power factor meter.
16. *Explain the working of rotating type phase sequence indicator.*
17. *Explain the working of static type phase sequence indicator.*
18. *Write a note on phase sequence indicators.*
19. *Why Neon lamps are more suitable in static type phase sequence indicators?*
20. *What is synchroscope ? Where it is used ?*
21. *Explain the construction and working of electrodynamometer type synchroscope.*
22. *Explain the construction and working of moving iron type synchroscope.*

❑❑❑

3

Measurement of Power

3.1 Introduction to Power Measurement

In a d.c. circuit if V_L is the voltage supplied to load and I_L is the load current then the d.c. load power is given by the product of the load supply voltage V_L and the load current I_L. Thus employing voltmeter and ammeter, power can be measured.

$$P_{dc} = V_L \, I_L \text{ watts} \quad \dots (1)$$

If R_L is the resistance of the load then,

$$R_L = \frac{V_L}{I_L}$$

$$\therefore \quad \boxed{P_{dc} = V_L \, I_L = \frac{V_L^2}{R_L} = I_L^2 \, R_L \quad \text{watts}} \quad \dots (2)$$

3.1.1 Necessity of Wattmeter

Consider the circuit using voltmeter and ammeter for the measurement of power, as shown in the Fig. 3.1.

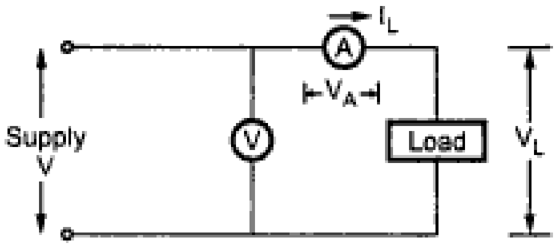


Fig. 3.1 Power measurement

The ammeter measures the load current I_L and there is voltage drop $V_A = I_L R_A$ across the ammeter where R_A is the ammeter resistance.

$$\therefore \quad V_L = V - V_A$$

$$\therefore \quad P_{dc} = V_L I_L = (V - V_A) I_L$$

$$\therefore \quad P_{dc} = V I_L - V_A I_L$$

where $V I_L$ = Power measured by the meters

P_{dc} = Power consumed by load.

$V_A I_L$ = Power consumed by ammeter.

$$\therefore \begin{bmatrix}\text{Power measured} \\ \text{by meters}\end{bmatrix} = \begin{bmatrix}\text{Power consumed} \\ \text{by load}\end{bmatrix} + \begin{bmatrix}\text{Power loss in the} \\ \text{instrument(ammeter)}\end{bmatrix}$$

Hence the product of ammeter and voltmeter does not give correct power consumed by the load.

If the voltmeter is shifted across the load to measure the load voltage, the circuit becomes as shown in the Fig. 3.2.

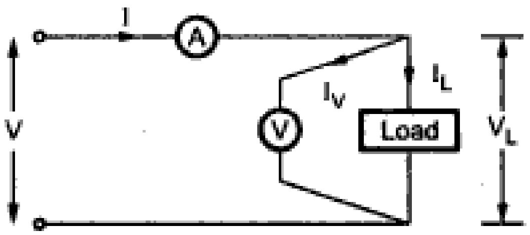


Fig. 3.2 Power measurement

Now as voltmeter is across the load, it measures V_L correctly but ammeter measures current I which is sum of I_L and I_V.

$$\therefore \quad I = I_L + I_V$$

$$\therefore \quad P_{dc} = V_L I_L = V_L (I - I_V) = I V_L - V_L I_V$$

where $I V_L$ = Power measured by meters

P_{dc} = Power consumed by load

$V_L I_V$ = Power consumed by voltmeter

$$\therefore \begin{bmatrix}\text{Power measured}\\ \text{by meters}\end{bmatrix} = \begin{bmatrix}\text{Power consumed}\\ \text{by load}\end{bmatrix} + \begin{bmatrix}\text{Power loss in the}\\ \text{instrument (voltmeter)}\end{bmatrix}$$

Thus by any method, the power measured is higher than the power actually consumed by the load. The power loss in the instrument near the load causes the error.

Key Point : *To avoid such errors in power measurement a device called **wattmeter** is used, which gives direct reading of power.*

3.1.2 A. C. Power

In an a.c. circuit, the instantaneous power fluctuates with time and is the product of the instantaneous values of voltage and current in the circuit.

Thus, $v = V_m \sin \omega t, \; i = I_m \sin(\omega t \pm \phi)$

$$p = v\,i = V_m I_m \sin\omega t \sin(\omega t \pm \phi) = \text{instantaneous power}$$

Then the average power consumption is given by,

$$P_{ac} = \frac{1}{2\pi}\int_0^{2\pi} p\, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin(\omega t)\sin(\omega t \pm \phi)\, d(\omega t)$$

This gives the average power consumption in a.c. circuit as,

$$\therefore \quad \boxed{P_{ac} = V\, I \cos\phi \quad \text{watts}} \qquad \text{... (3)}$$

where V = r.m.s. value of the voltage $= \frac{V_m}{\sqrt{2}}$

I = r.m.s. value of the current $= \frac{I_m}{\sqrt{2}}$

$\cos\phi$ = Power factor of circuit

ϕ = Power factor angle $= \hat{V I}$

The above expression shows that only ammeter and voltmeter is not sufficient to measure power. The measurement of $\cos\phi$ is also required. Therefore in a.c. circuits also, the **wattmeter** is used which senses the angle between voltage and current and directly gives the power consumption of the circuit in watts considering the effect of $\cos\phi$ into account.

Let us study the electrodynamometer principle as the wattmeters used are generally of electrodynamometer type.

3.2 Electrodynamometer Type Instruments

The electrodynamometer type instrument is a **transfer instrument**. A transfer instrument is one which is calibrated with a d.c. source and used without any modifications for a.c. measurements. Such a transfer instrument has same accuracy for a.c. and d.c. measurements. The electrodynamometer type instruments are often used in accurate a.c. voltmeters and ammeters, not only at the powerline frequency but also in the lower audiofrequency range. With some little modifications, it can be used as a wattmeter for the power measurements.

Why PMMC instruments can not be used for a.c. measurements ?

The PMMC instrument cannot be used on a.c. currents or voltages. If a.c. supply is given to these instruments, an alternating torque will be developed. Due to moment of inertia of the moving system, the pointer will not follow the rapidly changing alternating torque and will fail to show any reading. In order that the instrument should be able to read a.c. quantities, the magnetic field in the air gap must change along with the change in current. This principle is used in the electrodynamometer type instrument. Instead of a permanent magnet, the electrodynamometer type instrument uses the current under measurement to produce the necessary field flux.

3.2.1 Construction

The Fig. 3.3 shows the construction of the electrodynamometer type instrument.

The various parts of the electrodynamometer type instrument are :

Fixed Coils : The necessary field required for the operation of the instrument is produced by the fixed coils. A uniform field is obtained near the center of coil due to division of coil in two sections. These coils are air cored. Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire. The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

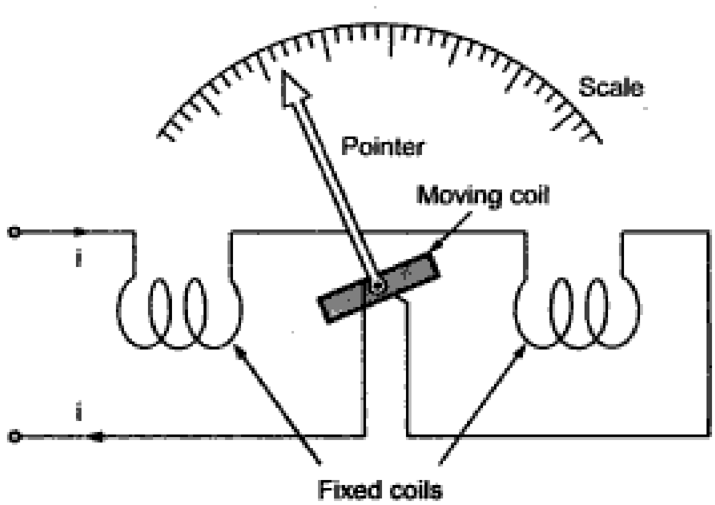


Fig. 3.3 Electrodynamometer type instrument

Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

Moving coil : The moving coil is wound either as a self-sustaining coil or else on a non-metallic former. If metallic former is used, then it would induce eddy currents in it. The construction of moving coil is made light as well as rigid. It is air cored.

Controlling : The controlling torque is provided by springs. These springs act as leads to the moving coil.

Moving system : The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case a high accuracy is desired.

Damping : The damping torque is provided by air friction, by a pair of aluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would be distorted by eddy current damping, it is not employed.

Shielding : The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So shielding is done to protect it from stray magnetic fields. It is done by enclosing in a casing of high permeability alloy.

Cases and Scales : Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid. The case is supported by adjustable levelling screws.

A spirit level may be provided to ensure proper levelling.

For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit.

The electrodynamometer instruments can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

For using electrodynamometer instrument as a wattmeter to measure the power, the fixed coils acts as a current coil and must be connected in series with the load. The moving coil acts as a voltage coil or pressure coil and must be connected across the supply terminals. The wattmeter indicates the supply power. When current passes through the fixed and moving coils, both coils produce the magnetic fields. The field produced by fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

3.2.2 Torque Equation

Let i_1 = Instantaneous value of current in fixed coil

i_2 = Instantaneous value of current in moving coil

L_1 = Self inductance of fixed coils

L_2 = Self inductance of moving coil

M = Mutual inductance between fixed and moving coils

The electrodynamometer instrument can be represented by an equivalent circuit as shown in the Fig. 3.4.

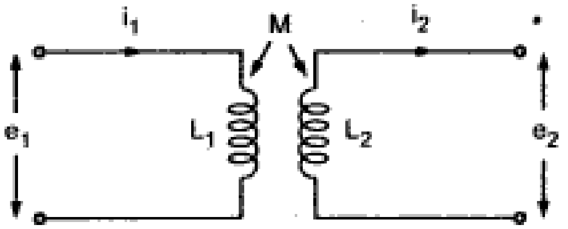


Fig. 3.4

The flux linkages of coil 1 are,

$$\phi_1 = L_1 i_1 + M i_2$$

The flux linkages of coil 2 are,

$$\phi_2 = L_2 i_2 + M i_1$$

Now $e_1 = \frac{d\phi_1}{dt}$

and $e_2 = \frac{d\phi_2}{dt}$

$$\begin{aligned} \text{Electrical input energy} &= e_1 i_1 dt + e_2 i_2\, dt \\ &= i_1\, d\,\phi_1 + i_2\, d\,\phi_2 \\ &= i_1\, d\,(L_1 i_1 + M i_2) + i_2\, d\,(L_2 i_2 + M i_1) \\ &= i_1 L_1\, d\,i_1 + i_1^2\, d\,L_1 + i_1\, i_2\, d\,M + i_1\, M\, d\,i_2 + \\ &\quad i_2 L_2 d\,i_2 + i_2^2\, d\,L_2 + i_1\, i_2 d\,M + i_2\, M\, d\,i_1 \end{aligned} \quad \text{... (1)}$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by,

$$\text{Energy stored} = \frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M$$

$$\begin{aligned} \text{Change in stored energy} &= d\left[\frac{1}{2} L_1\, i_1^2 + \frac{1}{2} L_2\, i_2^2 + i_1\, i_2\, M\right] \\ &= i_1\, L_1\, d\,i_1 + \frac{1}{2}\, i_1^2\, d\,L_1 + i_2\, L_2\, d\,i_2 + \frac{1}{2}\, i_2^2\, d\,L_2 + \\ &\quad i_1\, M\, d\,i_2 + i_2\, M\, d\,i_1 + i_1\, i_2\, d\,M \end{aligned} \quad \text{... (2)}$$

From the principle of conservation of energy,

Energy input = Energy stored + Mechanical energy

$\therefore$ Mechanical energy = Energy input – Energy stored

Substracting (2) from (1),

$$\text{Mechanical energy} = \frac{1}{2} i_1^2 \, dL_1 + \frac{1}{2} i_2^2 \, dL_2 + i_1 \, i_2 \, dM$$

The self inductances L_1 and L_2 are constants and hence dL_1 and dL_2 are zero.

$$\therefore \quad \text{Mechanical energy} = i_1 \, i_2 \, dM$$

If T_i is the instantaneous deflecting torque and $d\theta$ is the change in the deflection then

$$\text{Mechanical energy} = \text{Mechanical work done}$$

$$= T_i \, d\theta$$

$$\therefore \quad i_1 \, i_2 \, dM = T_i \, d\theta$$

$$\therefore \quad T_i = i_1 \, i_2 \frac{dM}{d\theta}$$

This is the expression for the instantaneous deflecting torque. Let us see its operation on a.c. and d.c.

D.C. operation : For d.c. currents of I_1 and I_2,

$$T_d = I_1 \, I_2 \frac{dM}{d\theta}$$

The controlling torque is provided by springs hence

$$T_c = K\theta$$

$$\text{In steady state,} \quad T_d = T_c$$

$$\therefore \quad I_1 \, I_2 \frac{dM}{d\theta} = K\theta$$

$$\therefore \quad \theta = \frac{I_1 \, I_2}{K} \cdot \frac{dM}{d\theta}$$

Thus the deflection is proportional to the product of the two currents and the rate of change of mutual inductance.

A.C. operation : In a.c. operation, the total deflecting torque over a cycle must be obtained by integrating T_i over one period.

Average deflecting torque over one cycle is,

$$T_d = \frac{1}{T}\int_0^T T_i \, dt$$

$$T = \text{Time period of one cycle}$$

$$\therefore \quad T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T i_1 \, i_2 \, dt$$

Now if the two currents are sinusoidal and displaced by a phase angle ϕ then

$$i_1 = I_{m1} \sin \omega t$$

and $$i_2 = I_{m2} \sin (\omega t - \phi)$$

$$\therefore \quad T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_{m1} \sin \omega t \cdot I_{m2} \sin (\omega t - \phi) d(\omega t)$$

$$= \left(\frac{I_{m1} \, I_{m2}}{2} \right) \cos\phi \frac{dM}{d\theta}$$

$$= I_1 \, I_2 \cos\phi \frac{dM}{d\theta}$$

where I_1, I_2 are the r.m.s. values of the two currents as,

$$I_1 = \frac{I_{m1}}{\sqrt{2}} \quad \text{and} \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

As $$T_c = K \theta$$

Hence in steady state, $T_c = T_d$

$$\therefore \quad I_1 \, I_2 \cos\phi \frac{dM}{d\theta} = K \theta$$

$$\therefore \quad \boxed{\theta = \frac{I_1 \, I_2}{K} \cos\phi \frac{dM}{d\theta}}$$

Thus the deflection is decided by the product of r.m.s. values of two currents, cosine of the phase angle (power factor) and rate of change of mutual inductance.

For d.c. use, the deflection is proportional to square of current and the scale is non-uniform and crowded at the ends. For a.c. use the instantaneous torque is proportional to the square of the instantaneous current. The i^2 is positive and as current varies, the deflecting torque also varies.

But moving system, due to intertia cannot follow rapid variations and thus finally meter shows the average torque.

Thus the deflection is the function of the mean of the squared current. The scale is thus calibrated in terms of the square root of the average current squared i.e. r.m.s. value of the a.c. quantity to be measured.

If an electrodynamometer instrument is calibrated with a d.c. current of 1A and pointer indicates 1 A d.c. on scale then on a.c., the pointer will deflect upto the same mark but 1A in this case will indicate r.m.s. value.

Thus as it is a transfer instrument, there is direct connection between a.c. and d.c. Hence the instrument is often used as a **calibration instrument.**

The instrument can be used as an ammeter to measure currents upto 20 A while using as a voltmeter it can have low sensitivity of about 10 to 30 Ω/V.

The Fig. 3.5 (a), (b) and (c) shows the connections of the electrodynamometer instrument as ammeter, voltmeter and the wattmeter.

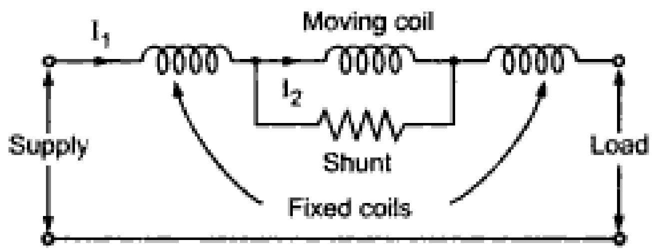


Fig. 3.5 (a) Electrodynamometer ammeter upto 100 mA

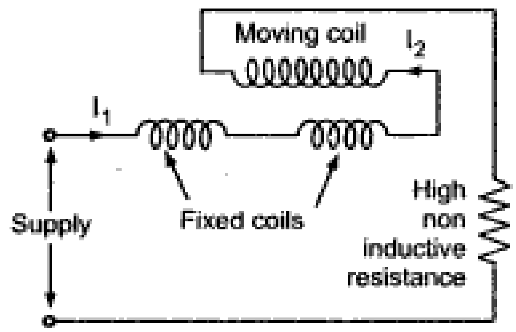


Fig. 3.5 (b) Electrodynamometer voltmeter

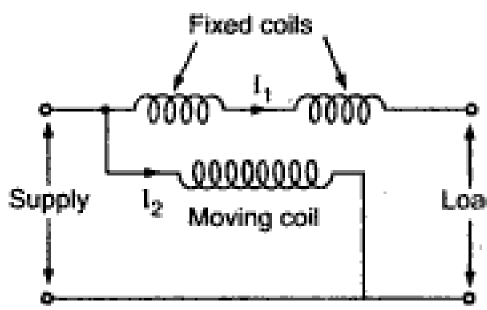


Fig. 3.5 (c) Electrodynamometer wattmeter

3.2.3 Advantages of Electrodynamic Instruments

1) As the coils are air cored, these instruments are free from hysteresis and eddy current losses.
2) They have a precision grade accuracy.
3) These instruments can be used on both a.c. and d.c. They are also used as a transfer instruments.
4) Electrodynamometer voltmeters are very useful where accurate rms values of voltage, irrespective of waveforms, are required.
5) Free from hysteresis errors.
6) Low power consumption.
7) Light in weight.

3.2.4 Disadvantages of Electrodynamic Instruments

1) These instruments have a low sensitivity due to a low torque to weight ratio. Also it introduces increased frictional losses. To get accurate results, these errors must be minimized.
2) They are more expensive than other type of instruments.
3) These instruments are sensitive to overloads and mechanical impacts. Therefore care must be taken while handling them.
4) They have a non-uniform scale.
5) The operating current of these instruments is large due to the fact that they have weak magnetic field.

3.2.5 Errors in Electrodynamometer Instruments

The various errors in electrodynamometer instruments are,

1. **Torque to weight ratio :** To have reasonable deflecting torque, m.m.f. of the moving coil must be large enough. Thus m.m.f. = NI hence current through moving coil should be high or number of turns should be large. The current can not be made very high because it may cause excessive heating of springs. Large number of turns hence is the only option but it increases weight of the coil. This makes the system heavy reducing torque to weight ratio. This can cause frictional errors in the reading.

2. **Frequency errors :** The changes in the frequency causes to change self inductances of moving coil and fixed coil. This causes the error in the reading. The frequency error can be reduced by having equal time constants for both fixed and moving coil circuits.

3. **Eddy current errors :** In metal parts of the instrument the eddy currents get produced. The eddy currents interact with the instrument current, to cause change in

the deflecting torque, to cause error. Hence metal parts should be kept as minimum as possible. Also the resistivity of the metal parts used must be high, to reduce the eddy currents.

4. Stray magnetic field error : Similar to moving iron instruments the operating field in electrodynamometer instrument is very weak. Hence external magnetic field can interact with the operating field to cause change in the deflection, causing the error. To reduce the effect of stray magnetic field, the shields must be used for the instruments.

5. Temperature error : The temperature errors are caused due to the self heating of the coil, which causes change in the resistance of the coil. Thus temperature compensating resistors can be used in the precise instrument to eliminate the temperature errors.

3.2.6 Comparison of Various Types of Instruments

The comparison of PMMC, moving iron and electrodynamometer type instruments is summarized in the Table 3.1.

Meter Type	Control	Damping	Suitability	Application
PMMC	Spring	Eddy current	D.C.	Widely used for d.c. current and voltage measurements in low and medium impedance circuits.
Moving Iron	Spring or Gravity	Air friction	D.C. and A.C.	Used for rough indication of currents and voltages. Widely used for the indicator type instruments on panels.
Electrodynamometer	Spring	Air friction	D.C. and A.C.	Used mainly as wattmeter. Also may be used as ammeter or voltmeter. Widely used as a calibration instrument and as a transfer instrument.

Table 3.1

3.3 Single Phase Dynamometer Wattmeter

An electrodynamometer type wattmeter is used to measure power. It has two coils, fixed coil which is **current coil** and moving coil which is **pressure coil** or **voltage coil.** The current coil carries the current of the circuit while pressure coil carries current proportional to the voltage in the circuit. This is achieved by connecting a series resistance in voltage circuit.

The connections of an electrodynamometer wattmeter in the circuit are shown in the Fig. 3.6.

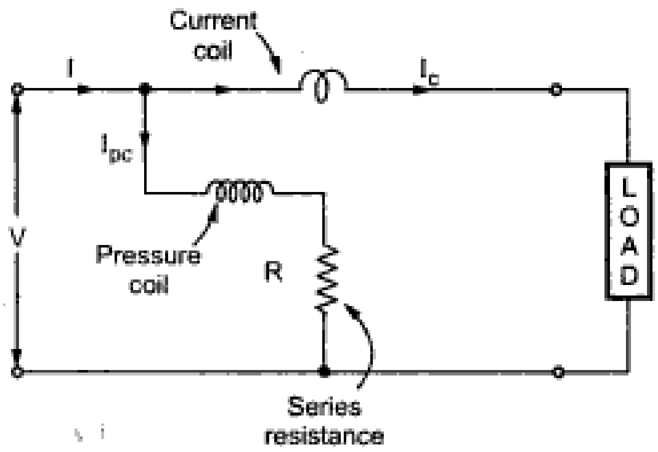


Fig. 3.6 Electrodynamometer wattmeter

I_c = Current through current coil

I_{pc} = Current through pressure coil

R = Series resistance

V = R.M.S. value of supply voltage

I = R.M.S. value of current

3.3.1 Torque Equation

According to theory of electrodynamic instruments,

$$T_i = i_1 i_2 \frac{dM}{d\theta} \qquad \dots (1)$$

Let v = Instantaneous voltage = $V_m \sin \omega t = \sqrt{2} V \sin \omega t$ ... (2)

Due to high series resistance, pressure coil is treated to be purely resistive.

Key Point: *The current I_{pc} is in phase with V as pressure coil is purely resistive.*

i_{pc} = Instantaneous value = $\frac{v}{R_p}$ where $R_p = r_{pc} + R$

$$\therefore \quad i_{pc} = \frac{\sqrt{2}\,V}{R_p} \sin \omega t = \sqrt{2}\, I_{pc} \sin \omega t \qquad \dots (3)$$

If current coil current lags the voltage by angle ϕ then its instantaneous value is,

$$i_c = \sqrt{2} I_c \sin(\omega t - \phi) \qquad \dots (4)$$

Now $i_1 = i_c$ and $i_2 = i_{pc}$ hence,

$$T_i = [\sqrt{2} I_{pc} \sin \omega t][\sqrt{2} I_c \sin(\omega t - \phi)] \frac{dM}{d\theta}$$

$$= 2 I_c I_{pc} \sin(\omega t) \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$\therefore \quad T_i = I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} \quad \ldots (5)$$

Key Point : *Thus instantaneous torque has a component of power which varies as twice the frequency of current and voltage.*

$$\therefore \quad T_d = \text{Average deflecting torque} = \frac{1}{T}\int_0^T T_i \, d(\omega t)$$

$$= \frac{1}{T}\int_0^T I_c I_{pc} [\cos\phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} d(\omega t)$$

$$\therefore \quad T_d = I_c I_{pc} \cos\phi \frac{dM}{d\theta} \quad \ldots (6)$$

where $I_{pc} = \dfrac{V}{R_p}$

For a spring controlled wattmeter.

$$T_c = K\theta \quad \ldots (7)$$

But $T_d = T_c$

$$\therefore \quad I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K\theta$$

$$\therefore \quad \boxed{\theta = \frac{1}{K} I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K_1 I_c I_{pc} \cos\phi} \quad \ldots (8)$$

where $K_1 = \dfrac{1}{K}\dfrac{dM}{d\theta}$

$$\therefore \quad \boxed{\theta = K_1 I_c \frac{V}{R_p} \cos\phi = K_2 P} \quad \ldots (8)$$

where $K_2 = \dfrac{K_1}{R_p}$ and $P = V I_c \cos\phi = \text{Power}$

$$\therefore \quad \boxed{\theta \propto P} \quad \ldots (10)$$

Key Point : *Thus the wattmeter deflection when calibrated gives the power consumption of the circuit.*

3.3.2 Reading on Wattmeter

The Fig. 3.7 shows symbolic representation of wattmeter.

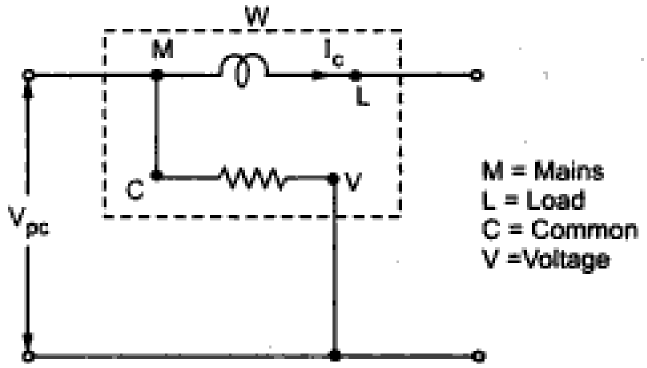


Fig. 3.7 Symbolic representation of wattmeter

Thus if, I_c = Current through current coil

V_{pc} = Voltage across pressure coil

Then wattmeter reading is,

$$W = V_{pc} I_c \cos (V_{pc} \wedge I_{pc})$$

Key Point: *The angle between V_{pc} and I_{pc} may or may not be power factor angle ϕ. It depends on the wattmeter connection in the circuit.*

Thus if wattmeter is connected in a three phase circuit such that $I_c = I_{ph}$ and $V_{pc} = V_{ph}$ then only $V_{pc} \wedge I_{pc} = \phi$.

Key Point : *In a three phase circuit, angle between V_{pc} and I_{pc} is to be obtained from the corresponding phasor diagram.*

3.3.3 Shape of Scale of Dynamometer Wattmeter

The deflection is given by

$$\theta = \frac{V}{R_p} I_c \cos\phi \frac{1}{K} \frac{dM}{d\theta} = K_2 P \frac{dM}{d\theta}$$

where $K_2 = \frac{1}{K R_p}$ and $P = V I_c \cos\phi$ = Power measured

Thus the deflection is directly proportional to power being measured and the scale is uniform over the range in which $(dM / d\theta)$ remains constant.

Practically the wattmeters are designed such that $dM/d\theta$ remains almost constant over a range of 40° to 50° on either side of zero mutual inductance position. The M varies linearily in this zone with respect to θ. Thus if zero mutual inductance position is kept in the middle of the scale then M varies linearily for the deflections upto 80° to 100° and thus scale is uniform over the range of 80° to 100°. Practically this covers the entire scale range. The shape of scale and variation in mutual inductance is shown in the Fig. 3.8.

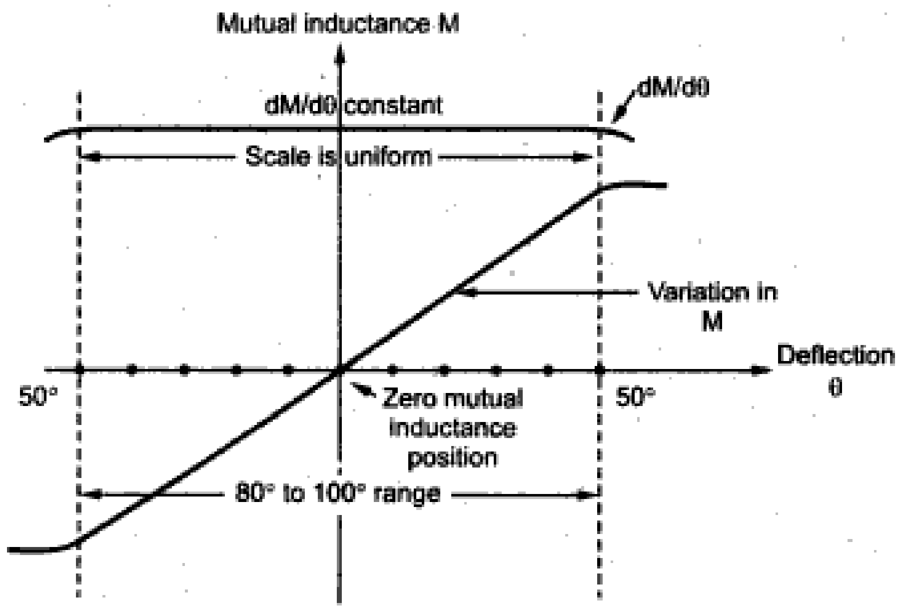


Fig. 3.8 Shape of scale of dynamometer wattmeter

3.4 Errors in Wattmeter

Practically there are errors in dynamometer wattmeter due to pressure coil parameters such as inductance, capacitance and due to method of connections. Some corrections are to be applied to compensate for these errors.

3.4.1 Error Due to Pressure Coil Inductance

In case of ideal wattmeter, the current in the pressure coil is in phase with the applied voltage because the pressure coil is assumed to be purely resistive without any reactance. But if it is having inductance the current in the pressure coil lags behind the supply voltage by some angle. Because of this, an error is introduced in the measurement of true power by the wattmeter. Some correction factor must be applied to get exact reading from wattmeter. This can be derived as given below.

Let r_p = Resistance of pressure coil

L = Inductance of pressure coil

R = Resistance in series with pressure coil

R_p = Total resistance of pressure coil circuit = $r_p + R$

V = Voltage supplied to pressure coil

I = Current in current coil circuit

I_p = Current in pressure coil circuit

Z_p = Impedance of pressure coil circuit = $(r_p + R) + j\omega L$

$$= \sqrt{(r_p + R)^2 + (\omega L)^2}$$

Let the current in the pressure coil circuit is lagging behind the supply voltage by an angle β.

$$\therefore \quad \tan\beta = \frac{\omega L}{R_p} = \frac{\omega L}{r_p + R}$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{\omega L}{r_p + R}\right)$$

If we assume that the load power factor to be lagging then the corresponding phasor diagram is as shown in the Fig. 3.9.

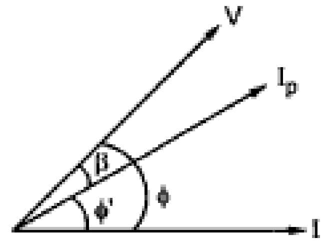


Fig. 3.9

From the Fig. 3.9 we have,

$$\phi = \phi' + \beta$$

$$\therefore \quad \phi' = \phi - \beta$$

Now, Actual wattmeter reading

$$= \frac{I_1 I_2}{K} \cos\phi \, \frac{dM}{d\theta}$$

Here $I_1 = I, \quad I_2 = I_p = \frac{V}{Z_p}, \quad \phi = \phi'$

$$\text{Actual wattmeter reading} = \frac{I_p . I}{K} \cos\phi' \frac{dM}{d\theta}$$

$$= \left(\frac{V}{Z_p K}\right) I \cos(\phi - \beta) \frac{dM}{d\theta}$$

Now, $R_p = Z_p \cos\beta$

$\therefore$ $Z_p = \dfrac{R_p}{\cos\beta}$

$$\text{Actual wattmeter reading} = \frac{V\,I \cos(\phi - \beta)}{\left[\dfrac{R_p}{\cos\beta}\right] \cdot K} \frac{dM}{d\theta} = \frac{V\,I}{KR_p} \cos\beta \cos(\phi - \beta) \frac{dM}{d\theta}$$

In the absence of inductance $Z_p = R_p$ and $\beta = 0$ and the wattmeter gives true power and it reads correctly at all power factors and frequencies.

$$\therefore \quad \boxed{\text{True power} = \frac{I_p \cdot I}{K} \cos\phi \frac{dM}{d\theta} = \frac{V\,I \cos\phi}{K\,R_p} \frac{dM}{d\theta}}$$

If we take ratio of true power and actual wattmeter reading then,

$$\frac{\text{True power}}{\text{Actual wattmeter reading}} = \frac{\left[\dfrac{VI \cos\phi}{K\,R_p}\right] \dfrac{dM}{d\theta}}{\left[\dfrac{VI\cos\beta \cos(\phi - \beta)}{K\,R_p}\right] \dfrac{dM}{d\theta}} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)}$$

$$\therefore \quad \boxed{\text{True power} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)} \times \text{Actual wattmeter reading}}$$

Thus for lagging loads the correction factor is given by,

$$\boxed{\text{Correction factor} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)}} \quad \text{... For lagging p.f.}$$

It can be seen from the phasor diagram that for lagging power factor condition the wattmeter reads high since the inductance of pressure coil will try to bring the current in pressure coil nearly in phase with current in current coil than would be the case if this inductance were zero. Thus error will be involved in wattmeter reading which would be serious at low power factor condition if proper precaution is not taken.

In case of leading power factor condition, the effect of pressure coil inductance is to increase the phase angle between load current and pressure coil current and wattmeter reads low. The corresponding phasor diagram is shown in the Fig. 3.10.

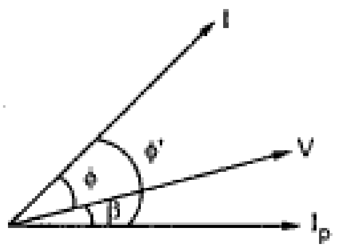


Fig. 3.10

For leading power factor, $\boxed{\text{Correction factor} = \dfrac{\cos\phi}{\cos\beta \cdot \cos(\phi+\beta)}}$ **... For leading p.f.**

Now we will find the error which is produced because of inductance of pressure coil at **lagging power factor** condition.

$$\text{Actual wattmeter reading} - \text{True power} = \text{Actual wattmeter reading} - \left[\frac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)}\right] \text{Actual wattmeter reading}$$

$$= \left[1 - \frac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)}\right] \times \text{Acutal wattmeter reading}$$

As β is small, $\cos\beta$ will be nearly equal to unity.

$$\therefore \quad \text{Error} = \left[1 - \frac{\cos\phi}{\cos(\phi-\beta)}\right] \times \text{Actual wattmeter reading}$$

$$= \left[1 - \frac{\cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right] \times \text{Actual wattmeter reading}$$

$$= \left[\frac{\cos\phi\cos\beta + \sin\phi\sin\beta - \cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right] \times \text{Actual wattmeter reading}$$

$$= \left[\frac{\cos\phi + \sin\phi\sin\beta - \cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right] \times \text{Actual wattmeter reading}$$

$$= \left[\frac{\sin\phi\sin\beta}{\cos\phi + \sin\phi\sin\beta}\right] \times \text{Actual wattmeter reading}$$

$$\therefore \quad \boxed{\text{Error} = \left[\frac{\sin\beta}{\cot\phi + \sin\beta}\right] \times \text{Actual wattmeter reading}}$$

Now let us find the percentage error.

Consider the relation $\dfrac{\text{True power}}{\text{Actual wattmeter reading}} = \dfrac{\cos\phi}{\cos\beta \cdot \cos(\phi-\beta)}$

$$= \frac{\cos\phi}{\cos\beta[\cos\phi\cos\beta + \sin\phi\sin\beta]} = \frac{\cos\phi}{\cos\beta\cos\phi\cos\beta[1+\tan\phi\tan\beta]}$$

$$= \frac{(1/\cos^2\beta)}{1+\tan\phi\tan\beta} = \frac{\sec^2\beta}{1+\tan\phi\cdot\tan\beta} = \frac{1+\tan^2\beta}{1+\tan\phi\tan\beta}$$

Now as β is small therefore $\tan^2\beta << 1$,

$$\frac{\text{True power}}{\text{Actual wattmeter reading}} = \frac{1}{1+\tan\phi \cdot \tan\beta}$$

$$\therefore \quad \boxed{\text{Actual wattmeter reading} = [1+\tan\phi \cdot \tan\beta]\times \text{True power}}$$

$$\therefore \quad \text{Error} = \text{Actual wattmeter reading} - \text{True power}$$

$$= [1+\tan\phi \cdot \tan\beta]\times \text{True power} - \text{True power}$$

$$= [\tan\phi \cdot \tan\beta]\times \text{True power}$$

$$\%\ \text{Error} = \frac{\text{Actual wattmeter reading} - \text{True power}}{\text{True power}}\times 100$$

$$\boxed{\%\ \text{Error} = [\tan\phi \cdot \tan\beta]\times 100}$$

But true power = VI cos ϕ. Hence the error is given as,

$$\boxed{\text{Error} = [\tan\phi \cdot \tan\beta]\times \text{VI}\cos\phi = \text{VI}\sin\phi \cdot \tan\beta}$$

From the above equation it is clear that the error is considerable at low power factors.

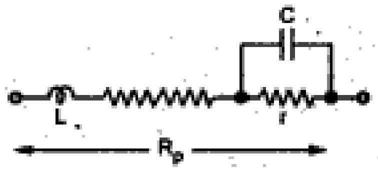


Fig. 3.11

Compensation of error :

The error due to pressure coil inductance can be compensated as shown in the Fig. 3.11.

A capacitor C is connected in parallel with a portion of series resistance (multiplier resistance) as shown in the Fig. 3.11.

The total impedance of the circuit shown in above figure is given by,

$$Z_p = (R_p - r) + j\omega L + \frac{(r)(-j/\omega C)}{r - \frac{j}{\omega C}}$$

$$= (R_p - r) + j\omega L + \frac{(r)(-j)(\omega Cr + j)}{(\omega Cr - j)(\omega Cr + j)}$$

$$\therefore \quad Z_p = (R_p - r) + j\omega L + \frac{r - j\omega Cr^2}{1+\omega^2 C^2 r^2}$$

With proper selection of circuit constants at power frequencies $\omega^2 C^2 r^2 << 1$

$$\therefore \quad Z_p \approx (R_p - r) + j\omega L + r - j\omega Cr^2 \approx R_p + j(\omega L - \omega Cr^2)$$

$$\approx R_p + j\omega(L - Cr^2)$$

If $L = Cr^2$ then $Z_p \approx R_p$ and $\beta = 0°$ which is desired.

Thus the error caused by pressure coil inductance is almost completely eliminated. This type of compensation is slightly affected by change in frequency and can be used for frequencies where $\omega^2 C^2 r^2 << 1$. It can be applied to frequencies upto 10 kHz.

3.4.2 Error Due to Pressure Coil Capacitance

In addition to inductance of pressure coil there may be capacitance due to interturn capacitance of the series resistance. The effect of capacitance is exactly opposite to that of inductance. Thus the wattmeter reads low on lagging power factors.

The phase angle between pressure coil current and applied voltage depends upon the reactance of the pressure coil circuit. The inductive reactance is normally greater than capacitive reactance, thus the phase angle increases with increase in frequency.

If the capacitive reactance of the pressure coil circuit is equal to its inductive reactance, there will be no error due to these effects since the two errors will neutralize each other.

3.4.3 Error Due to Method of Connection

There are two ways of connecting wattmeter in a given circuit. These are respectively shown in the Fig. 3.12 (a) and (b).

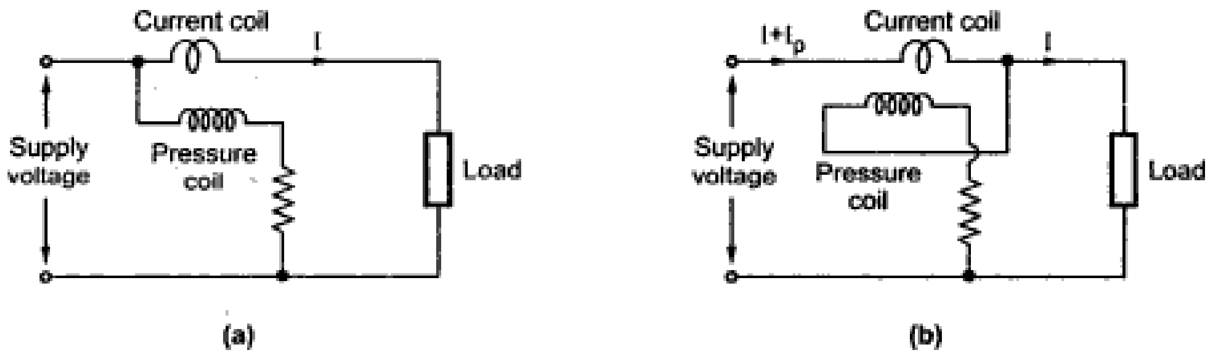


Fig. 3.12

Because of the power loss in the current and pressure coils, error is introduced in the measurement of power.

In connection shown in the Fig. 3.12 (a), pressure coil is connected on the supply side and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across current coil. Thus wattmeter measures power loss in its current coil in addition to power consumed by load.

Power indicated by wattmeter = Power consumed by load + Power loss in current coil

$\therefore$ Power indicated by wattmeter = Power consumed by load + $I^2 R_c$

If wattmeter connections are as shown in the Fig. 3.12 (b) the current coil is on supply side and hence it carries pressure coil current plus the load current. Thus wattmeter reads in addition to power consumed in load, the power loss in pressure coil.

$$\text{Power indicated by wattmeter} = \text{Power consumed by load} + \text{Power loss in pressure coil circuit}$$

$$= \text{Power consumed by load} + V^2 / R_p$$

With small load current, the voltage drop in current coil is small so connections in Fig. 3.12 (a) introduces small error. Alternatively if load current is large, the pressure coil current is very small as compared with load current. Hence power loss in pressure coil circuit is small as compared with power consumed by load. Thus connection shown in Fig. 3.12 (a) is preferable for small currents while for large currents the connections shown in Fig. 3.12 (b) are preferable.

But if load current is high and the power factor is small, connection shown in Fig. 3.12 (b) results in large error as the total power measured is small. In this case a compensating coil may be used for compensation of error which is explained further in low power factor wattmeters.

3.4.4 Eddy Current Errors

The current coil produces an alternating magnetic field because of which eddy currents are induced in the solid metal parts and within the thickness of the conductors. These currents produce field and tries to reduce flux produced by current coil. The phase angle between fluxes in current coil and potential coil is increased which decreases the deflecting torque produced by the instrument for lagging power factors.

The wattmeter reads low for lagging power factors and reads high for leading power factors. To minimize this error, solid metal parts are avoided as far as possible. Standard conductors are used for the current coil if it carries large current. Thus in practice a special type of wattmeter called low power factor wattmeter (LPF) is used.

➡ **Example 3.1 :** *A wattmeter has a current coil of 0.03 Ω resistance and a pressure coil of 6000 Ω resistance. Calculate the percentage error if the wattmeter is so connected that*

i) The current coil is on the load side ii) The pressure coil is on the load side.

a) If the load takes 20 A at a voltage of 220 V and 0.6 power factor in each case.

b) What load current would give equal errors with the two connections ?

Solution : Power consumed by load,

$$P_T = VI\cos\phi$$

$$= 220 \times 20 \times 0.6$$

$$\therefore \quad P_T = 2640 \text{ watt}$$

Now given that $R_c = 0.03\ \Omega$, $\quad R_p = 6000\ \Omega$

i) Consider that current coil is on the load side

Power indicated by wattmeter,

$$P_W = \text{Power consumed by load + Power loss in current coil}$$

$$= P_T + I^2 R_c$$

$$= 2640 + (20)^2 (0.03)$$

$$= 2652 \text{ watt}$$

$$\% \text{ Error} = \frac{P_W - P_T}{P_T} \times 100 = \frac{2652 - 2640}{2650} \times 100$$

$$\% \text{ Error} = \mathbf{0.45\ \%}$$

ii) Consider the pressure coil is on the load side

Power indicated by wattmeter,

$$P_W = \text{Power consumed by load + Power loss in pressure coil}$$

$$= P_T + V^2 / R_p$$

$$= 2640 + \frac{(220)^2}{6000}$$

$$= 2648.06 \text{ watt}$$

$$\% \text{ Error} = \frac{P_W - P_T}{P_T} \times 100 = \frac{2648.06 - 2640}{2640} \times 100$$

$$\therefore \quad \% \text{ Error} = \mathbf{0.3055\ \%}$$

iii) Power consumed by load remains same. Thus to get equal error with two connections power indicated in both cases by wattmeter must be same.

∴ Power indicated by wattmeter with current coil on load side = Power indicated by wattmeter with pressure coil on load side

$$P_T + I^2 R_c = P_T + V^2 / R_p$$

$$I^2 R_c = \frac{V^2}{R_p}$$

$$\therefore \quad I = \frac{1}{\sqrt{R_p \cdot R_c}} \cdot V = \frac{1}{\sqrt{(6000)(0.03)}} (220)$$

$$\therefore \quad I = \mathbf{16.39\ A}$$

At this current equal errors are obtained.

Example 3.2 : *An electrodynamometer wattmeter is used for measurement of power in a single phase circuit. The load voltage is 100 V and the load current is 10 A at a power factor of 0.2. The wattmeter voltage circuit has a resistance of 3000 Ω and an inductance of 30 mH. Estimate the percentage error in the wattmeter reading when pressure coil is connected i) on the supply side and ii) on the load side. The current coil has a resistance of 0.1 Ω and negligible inductance. The frequency is 50 Hz.*

Solution : Power consumed by load, $P_T = VI\cos\phi = 100 \times 10 \times 0.2 = 200\ W$

$$\cos\phi = 0.2, \qquad \phi = 78.46^\circ$$

$$R_p = 3000\ \Omega, \qquad L = 30\ mH$$

$$X_L = 2\pi fL = (2\pi)(50)(30 \times 10^{-3}) = 9.42\ \Omega$$

$$\beta = \tan^{-1}\frac{X_L}{R_p} = \tan^{-1}\frac{9.42}{3000} = 0.00313\ rad$$

Consider pressure coil is connected on load side.

Using the expression for effect of inductance,

$$\text{Actual Wattmeter Reading} = [1 + \tan\phi \cdot \tan\beta]\ \text{True power}$$

$$= [1 + \tan 78.46 \cdot \tan(0.00313\ rad)]\,200$$

$$= [1 + (4.897)(3.13 \times 10^{-3})]\,200$$

$$= 203.06\ W \quad \text{... Use proper mode of calculator}$$

$$\text{Power loss in pressure coil} = \frac{V^2}{R_p} = \frac{(100)^2}{3000} = 3.33\ W$$

$$\text{Total Wattmeter Reading} = 203.06 + 3.33 = 206.39\ W$$

$$\%\ \text{Error} = \frac{P_W - P_T}{P_T} \times 100$$

$$= \frac{206.39 - 200}{200} \times 100$$

$$= \mathbf{3.196\ \%}$$

Now consider that pressure coil is on supply side

$$\text{Total power} = \text{True power} + I^2 R_c = 200 + (10)^2(0.1)$$

$$= 200 + 10 = 210\ W$$

$$\text{Impedance of load} = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

$$\text{Resistance of load, } R_L = Z\cos\phi = (10)\ (0.2) = 2\ \Omega$$

$$\text{Resistance of load, } X_L = Z\sin\phi = (10)\ (0.979) = (9.797)\ \Omega$$

The current coil acts as a load.

$$\text{Total resistance} = 2 + 0.1 = 2.1\ \Omega$$

$$\therefore \quad \text{Total resistance} = 9.797\ \Omega$$

$$\text{Total impedance of current coil} = \sqrt{(2.1)^2 + (9.797)^2} = 10.01\ \Omega$$

$$\text{Total p.f. of load} = \frac{R_T}{Z_T} = \frac{2.1}{10.01} = 0.2097$$

$$\phi = 77.89$$

$$\tan\phi = 4.662$$

$$\text{Reading of wattmeter, } P_W = (1 + \tan\phi \cdot \tan\beta)\ \text{power}$$

$$= \left[1 + (4.662)\left(3.13 \times 10^{-3}\right)\right] 210$$

$$= 213.06\ \text{W}$$

$$\%\ \text{Error} = \frac{P_W - P_T}{P_T} \times 100$$

$$= \frac{213.06 - 200}{200} \times 100$$

$$\therefore \quad \%\ \text{Error} = \mathbf{6.53\ \%}$$

3.5 Low Power Factor Electrodynamic Type Wattmeter

If any circuit is operating at low power factor then power in that circuit is difficult to measure with ordinary electrodynamometer wattmeters. The reading of the wattmeter is inaccurate on account of following reasons,

1. The deflecting torque on the moving system is small as the power factor is low even though the current and pressure coils are fully excited.
2. The inductance of pressure coil introduces considerable error at low power factors.

In order to get accurate reading from the wattmeter when it is measuring low power, extra adjustments are required to be made so that there will be compensation of the errors.

When power to be measured is low then the current in the circuit is high as the power factor is low. Thus in this case pressure coil can not be connected to supply side as otherwise large error will be produced because of large current flowing in current coil and corresponding power loss in current coil circuit is measured by wattmeter.

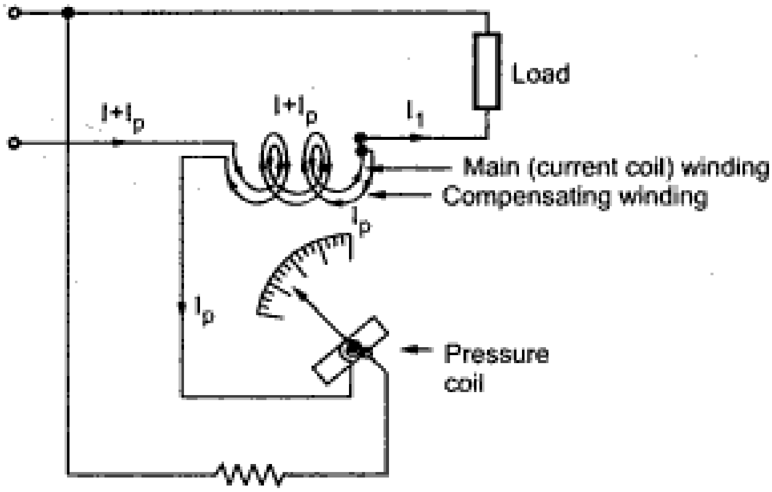


Fig. 3.13

If pressure coil is connected to load side, power consumed by pressure coil is measured by wattmeter which is appreciable in comparison with power to be measured which is small. Hence it is necessary to compensate for pressure coil current in low power factor wattmeter. The compensated wattmeter is shown in Fig. 3.13.

As shown in the Fig. 3.13 the compensating coil is connected in series with the potential coil and is made as identical and coincident with current coil as possible. The current coil carries current $I+I_p$ and produces its own field proportional to this current. The compensating coil carries current I_p and produces field proportional to this current. This field acts in opposite direction to the field produced by current coil.

Thus the resultant field is due to current I only. Hence error due to pressure coil current is neutralized.

Thus at no load condition, the wattmeter should not deflect as the resultant current coil field is zero.

In case of low power factor wattmeter, the pressure coil circuit is designed for low resistance to increase the current flowing through it so as to have increased torque. In low power factor wattmeter the value of pressure coil current is 10 times the current in case of high power factor wattmeters.

We have already seen in previous section that the pressure coil inductance introduces error whose magnitude is given by $VI \sin\phi \tan\beta$. If power factor is low then ϕ is large and hence $\sin\phi$ is large. Thus the error introduced in the measurement is appreciable which must be compensated. It is compensated by connecting a capacitor across a part of series resistance in the pressure coil circuit which is shown in the Fig. 3.14.

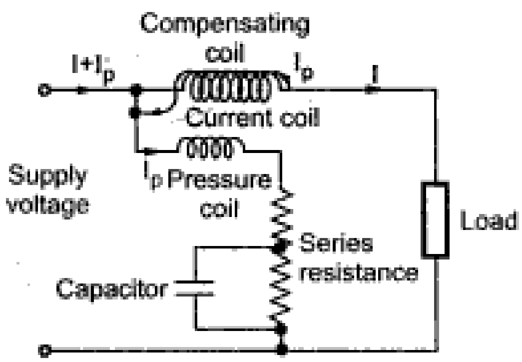


Fig. 3.14 Low power factor wattmeter

3.6 Power in Three Phase System

In a three phase system, the load can be star connected or delta connected having impedance per phase as,

$$Z_{ph} = R_{ph} + j\,X_{ph}\ \ \Omega/ph = |Z|\ \angle\phi\ \ \Omega/ph$$

The reactance X_{ph} may be positive or negative depending on inductive or capacitive load. It is positive for inductive and negative for capacitive load.

$$\phi = \text{Angle between } V_{ph} \text{ and } I_{ph}$$

For star connected load	For delta connected load
$V_L = \sqrt{3}V_{ph}$	$I_L = \sqrt{3}I_{ph}$
$I_L = I_{ph}$	$V_L = V_{ph}$
V_R , V_Y , V_B = V_{ph}	I_{RY}, I_{YB}, I_{BR} = I_{ph}
V_{RY} , V_{YB}, V_{BR} = V_L	I_R, I_Y , I_B = I_L
$\bar{V}_{RY} = \bar{V}_R - \bar{V}_Y$	$\bar{I}_{RY} - \bar{I}_{BR} = \bar{I}_R$
$\bar{V}_{YB} = \bar{V}_Y - \bar{V}_B$	$\bar{I}_{YB} - \bar{I}_{RY} = \bar{I}_Y$
$\bar{V}_{BR} = \bar{V}_B - \bar{V}_R$	$\bar{I}_{BR} - \bar{I}_{YB} = \bar{I}_B$

For any load, star or delta connected, the three phase total power is given by,

$$P = \sqrt{3}\,V_L\ I_L\ \cos\phi = 3\,[V_{ph}\ I_{ph}\ \cos\phi]$$

The wattmeters must be properly connected in a three phase system to measure the total power.

3.7 Examples of Wattmeter Connections and Corresponding Readings

Case i) : Consider delta connected inductive load and wattmeter connected as shown to measure power.

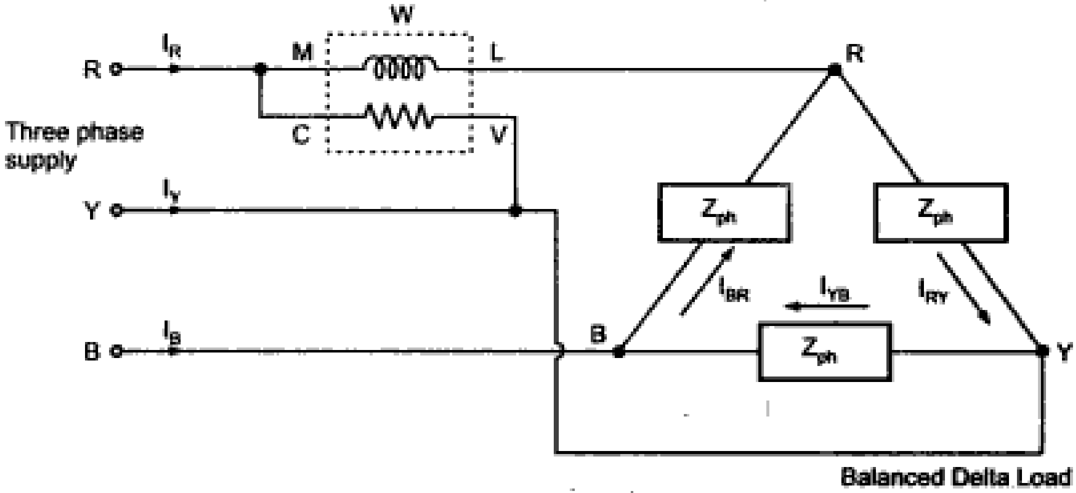


Fig. 3.15

Find the reading on it. Will it measure phase power ? Let us find it out.

For wattmeter : $I_c = I_R = I_L$ = Line current

while $V_{pc} = V_{RY} = V_L = V_{ph}$ as delta load

$$\therefore \quad W = V_{pc} I_c \cos\left(V_{pc} \,{}^\wedge\, I_c\right)$$

$$= V_{RY} . I_R . \cos\left(V_{RY} \,{}^\wedge\, I_R\right)$$

Now $I_R = I_L$ and $V_{RY} = V_L = V_{ph}$.

Hence angle between V_{RY} and I_R is not 'ϕ'.

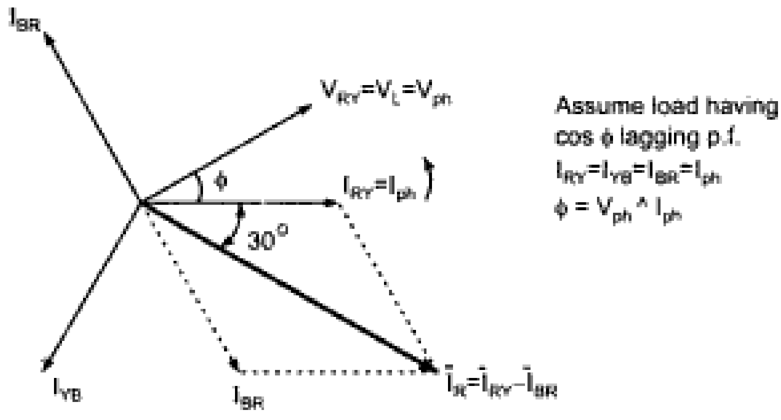


Fig. 3.16

To find $V_{RY} \,{}^\wedge\, I_R$ let us draw phasor diagram as shown in the Fig. 3.16.

For delta connected load, $\overline{I_R} = \overline{I_{RY}} - \overline{I_{BR}}$

Phase current I_{RY} lags phase voltage V_{RY} assuming that load p.f. is cos ϕ lagging.

From phasor diagram it is clear that,

$$V_{RY} \,{}^\wedge\, I_R = (30 + \phi)^\circ$$

$$\therefore \quad W = V_{RY} \times I_R \times \cos(30 + \phi)$$

$$\therefore \quad \boxed{W = V_L I_L \cos\,(30 + \phi)\ W}$$

This is not a phase power reading.

Case ii) : Now let us shift the same wattmeter in such a way that it has to read phase power $V_{ph}\ I_{ph} \cos\phi$. For this $I_c = I_{ph} = I_{RY}$ or I_{YB} or I_{BR} and $V_{pc} = V_{ph} = V_L = V_{RY}$ or V_{YB} or V_{BR}. Accordingly wattmeter coils must be connected such that $I_c = I_{ph}$ and $V_{pc} = V_{ph}$, as $I_{ph} \,{}^\wedge\, V_{ph} = \phi$ and then it will read $V_{ph}\ I_{ph} \cos\phi$ which is phase power. The connections can be shown as in the Fig. 3.17.

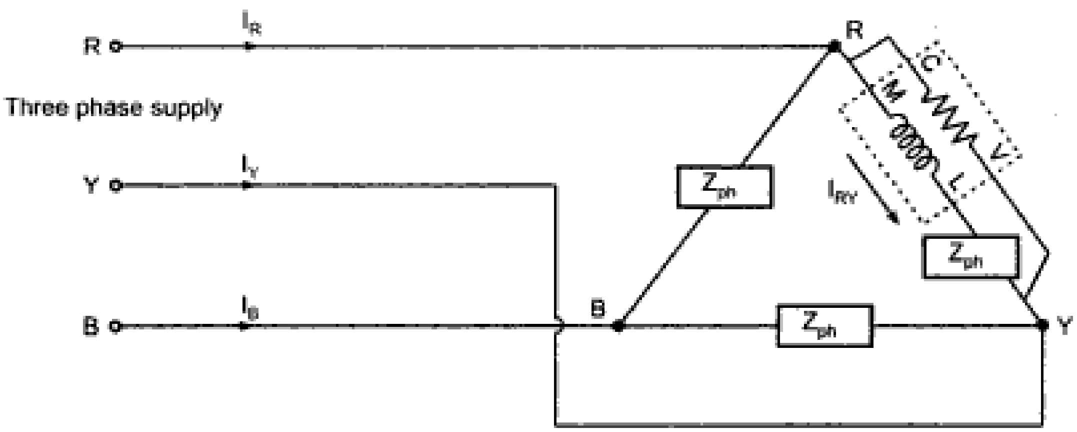


Fig. 3.17

For wattmeter : $I_c = I_{ph} = I_{RY}$ and $V_{pc} = V_{RY} = V_{ph}$

$\therefore$ Wattmeter will read, $W = I_c V_{pc} \cos(I_c \wedge V_{pc}) = I_{RY} V_{RY} \cos(I_{RY} \wedge V_{RY})$

$$\therefore \quad \boxed{W = I_{ph} V_{ph} \cos\phi = \textbf{phase power}}$$

Key Point : *Wattmeter will not always measure phase power. Its reading depends on the angle between current sensed by its current coil and voltage sensed by its pressure coil. This depends on its connections in the circuit.*

Three wattmeters measuring power in three phases may be connected to measure power. But connecting wattmeter to measure phase power is not always possible if neutral point of star connected load is not available out side. Similarly in delta connected load to measure phase current it is necessary to open delta load to insert current coil of the wattmeter is discussed above, which is not practicable. The best method of measuring power whether load is star or delta connected, balanced or unbalanced, neutral is available or not is, using only two wattmeters. The method is called **Two Watttmeter Method.**

Let us discuss first the single wattmeter method and three wattmeter method alongwith the corresponding limitaions before discussing the two wattmeter method.

3.8 Single or One Wattmeter Method

This can be only used for balanced three phase load. When the load is balanced, total power can be calculated as,

$$\boxed{P = 3 V_{ph} I_{ph} \cos\phi = 3 \times \text{(wattmeter reading)}}$$

Hence one wattmeter is to be used to measure single phase power and then reading is to be multiplied by three.

Key Point : *Wattmeter must be connected in such a way that its current coil must carry* I_{ph} *and its voltage coil must be across* V_{ph}.

3.8.1 Star Connected Load

This can be achieved by connecting wattmeter as shown in the Fig. 3.18.

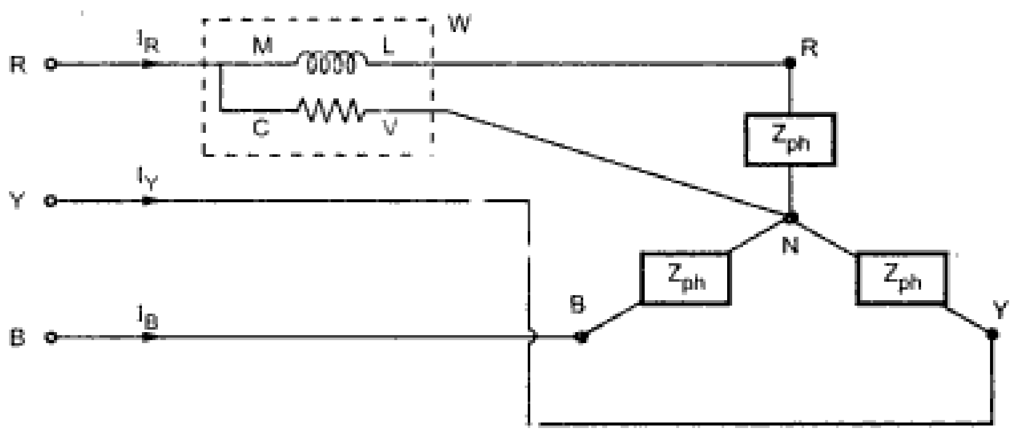


Fig. 3.18 Wattmeter measuring single phase power

Now $I_c = I_R = I_L = I_{ph}$ as load is star connected

But voltage coil must be connected so as to measure V_{ph} i.e. as shown in the Fig. 3.18 i.e. voltage coil across one line terminal and other to a neutral point.

$$V_{pc} = V_{RN} = V_{ph}$$

$$\therefore \quad W = V_{pc} I_c \cos (V_{pc} \wedge I_c)$$

$$\boxed{W = V_{RN} I_R \cos (V_{RN} \wedge I_R) = V_{ph} I_{ph} \cos\phi}$$

$$\therefore \text{ Total power, } P = 3 V_{ph} I_{ph} \cos\phi$$

$$= 3 W \text{ watts}$$

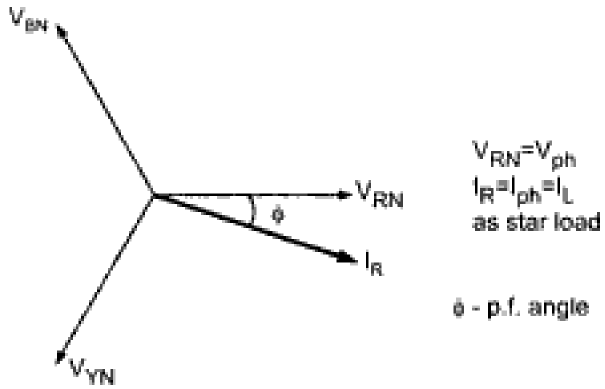


Fig. 3.19

Let Z_{ph} is having power factor as $\cos\phi$ lagging. Then phasor diagram can be drawn as in Fig. 3.19.

For Z_{ph} having $\cos\phi$ leading p.f., only change is I_{ph} will lead V_{ph} by ϕ degrees while for Z_{ph} having $\cos\phi = 1$ i.e. unity p.f. (Resistive load), V_{ph} and I_{ph} will be in phase with each other ($\phi = 0°$).

3.8.2 Delta Connected Load

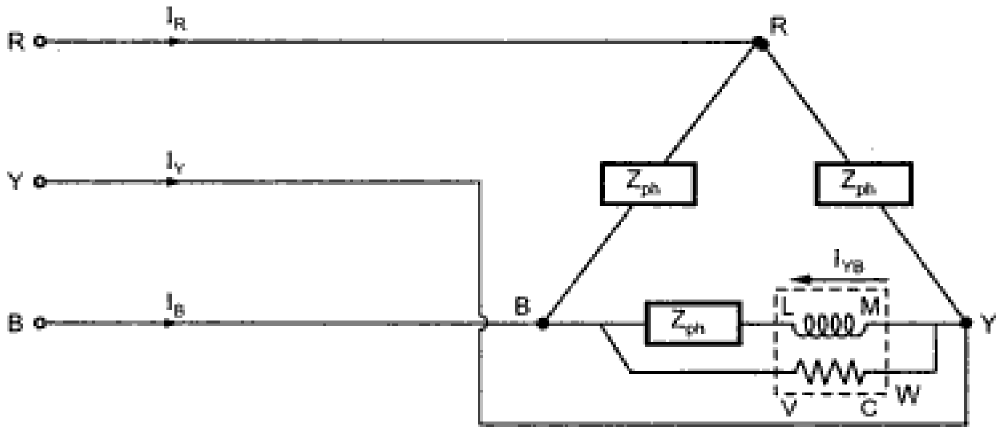


Fig. 3.20 Wattmeter measuring single phase power

In this connection, $I_c = I_{YB} = I_{ph}$

While $V_{pc} = V_{YB} = V_{ph}$

Here as current coil must read I_{ph}, it is necessary to open delta and insert current coil in it.

$$W = V_{pc} I_c \cdot \cos (V_{pc} \wedge I_c)$$

$$\therefore \quad \boxed{W = V_{YB} \cdot I_{YB} \cdot \cos(V_{YB} \wedge I_{YB}) = V_{ph} \, I_{ph} \, \cos\phi}$$

$\therefore$ Total power = 3 W watts

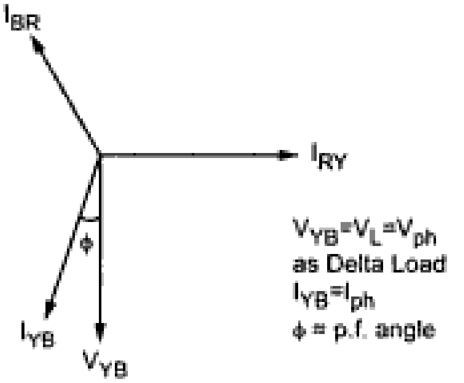


Fig. 3.21

If load is having cos ϕ lagging power factor, phasor diagram can be shown as in Fig. 3.21.

But this method has certain disadvantages.

3.8.3 Disadvantages

1) Applicable only for balanced loads.
2) For using this method, for star connected loads, neutral point must be available otherwise voltage coil can not be connected so as to measure phase voltage.

   Similarly for delta connected load, it must be possible to open the closed delta so as to insert current coil to measure phase current. **This may not be possible in practice.**

Key Point : *It can be noticed that for Star or Delta connected load, wattmeter may be connected so as to read any combination of phase voltage and phase current and not necessarily only the combinations shown in the Fig. 3.18 and 3.20.*

3.9 Three Wattmeter Method

If the load is unbalanced then we can use three wattmeters which will measure power consumed in each phase and then all the three readings can be added to get the total power. If load is unbalanced, power consumed in each phase will be different and hence three wattmeters are necessary. The connections of the wattmeter are exactly similar to the connection of one wattmeter in the method discussed above.

The connections for Star connected load can be shown as in the Fig. 3.22

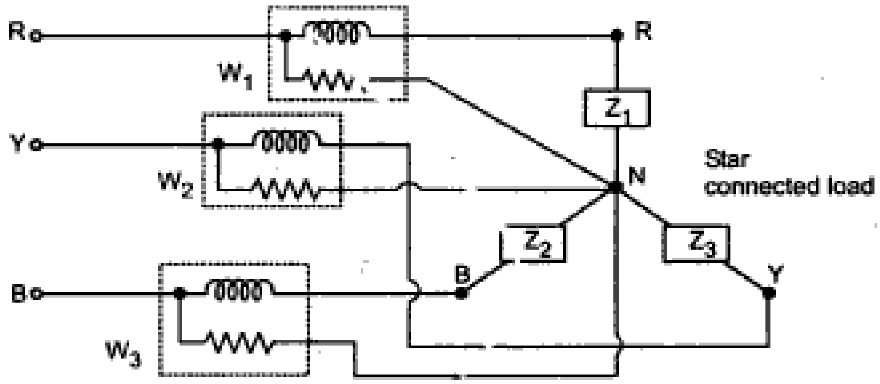


Fig. 3.22 Star connected load

If load is balanced, $W_1 = W_2 = W_3$

If load is unbalanced, $W_1 \neq W_2 \neq W_3$

$$\text{Total power} = W_1 + W_2 + W_3 \qquad \text{(for unbalanced load)}$$

$$= 3\,W_1 \text{ or } 3\,W_2 \text{ or } 3\,W_3 \qquad \text{(for balanced load)}$$

The connections for Delta connected load can be shown as in the Fig. 3.23.

If load is balanced, $W_1 = W_2 = W_3$

and total power is, $P = W_1 + W_2 + W_3 = 3W_1$ or $3W_2$ or $3W_3$

In this method, the power can be measured for unbalanced load but the disadvantages of requirement of neutral point for star load and arrangement for insertion of current coil in closed delta, still continues.

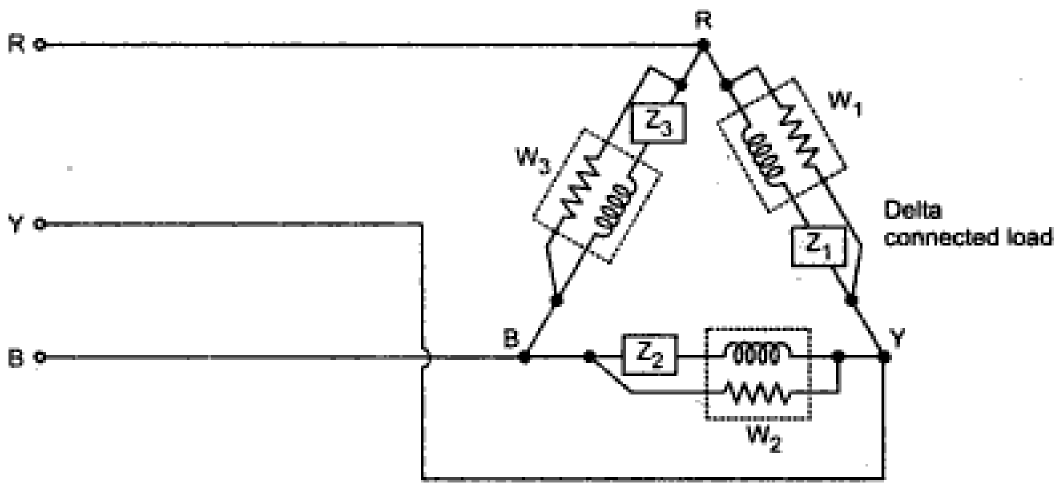


Fig. 3.23 Delta connected load

Key Point : *Thus practically single wattmeter as well as three wattmeter methods are rarely used for the industrial loads, due to their limitations.*

3.10 Blondel's Theorem

The Blondel's theorem is about number of wattmeters required in a polyphase system for the measurement of total power. It states that,

If a network is supplied using n conductors, the total power is the sum of the readings of n wattmeters so arranged that a current coil of each wattmeter is in each line and the corresponding pressure coil is connected between that line and a common point. If the common point is located on one of the lines then the power may be measured by n – 1 wattmeters.

Thus in a three phase system if common point for connecting pressure coils is located on one of the lines then only 2 wattmeters are sufficient for measuring the power. This is called two wattmeter method.

3.11 Two Wattmeter Method

Method of Connection :

The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeter is connected between its own current coil terminal and the line without a current coil. For example, the current coils are inserted in the lines R and Y then the pressure coils are connected between R-B for one wattmeter and Y-B for other wattmeter, as shown in the Fig. 3.24.

The connections are same for star or delta connected load. It can be shown that when two wattmeters are connected in this way, the algebraic sum of the two wattmeter readings gives the total power dissipated in the three phase circuit.

> If W_1 and W_2 are the two wattmeter readings then total power is,
>
> $$W = W_1 + W_2 = \text{Three phase power}$$

3.11.1 Proof of Two Wattmeter Method

Consider star connected load and two wattmeters connected as shown in the Fig.3.24.

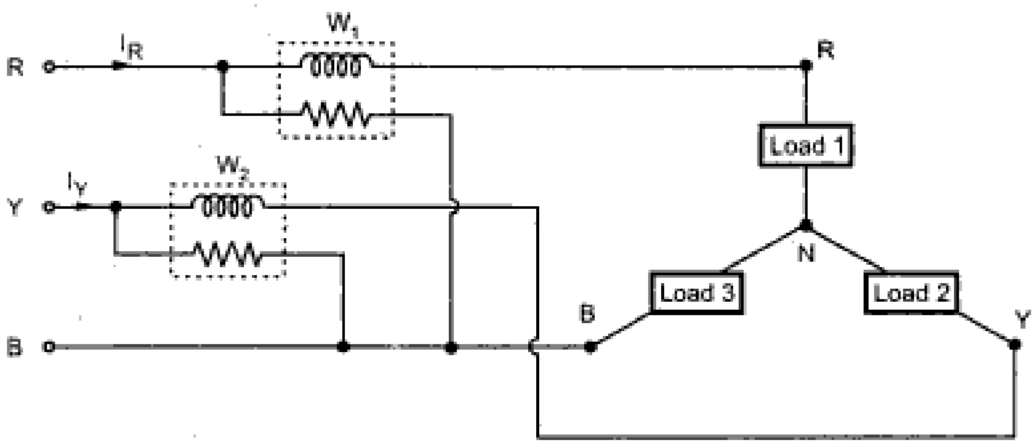


Fig. 3.24 Two wattmeter method for star connected load

a) For unbalanced load :

Let us consider the instantaneous values of different current and voltages.

Note that the r.m.s. values are indicated in capital letters like I_R, V_{RY} etc. while instantaneous values are indicated in small letters like i_r , v_{ry} etc.

It is important to note that wattmeter gives average value which is $W = I_c \times V_{pc} \times \cos\left(I_c \hat{} V_{pc}\right)$ if I_c and V_{pc} are r.m.s. values of current though current coil and voltage across voltage coil, respectively. And instantaneously wattmeter reads just the product of instantaneous values of current through current coil and voltage across pressure coil.

$$W_{instantaneous} = i_c \times v_{pc}$$

According to the connections shown in the Fig. 3.24 instantaneously W_1 and W_2 will show following readings.

$$W_1 = i_r \times v_{rb} \quad \text{and} \quad W_2 = i_y \times v_{yb}$$

Now $\quad v_{rb} = v_r - v_b$ and $v_{yb} = v_y - v_b$

where v_r, v_y and v_b are instantaneous values of phase voltages V_R, V_Y and V_B respectively.

∴ Substituting in W_1 and W_2,

$$W_1 = i_r \times (v_r - v_b) \text{ and } W_2 = i_y \times (v_y - v_b)$$

$$\therefore \quad W_1 + W_2 = i_r(v_r - v_b) + i_y(v_y - v_b)$$

$$= i_r v_r - i_r v_b + i_y v_y - i_y v_b$$

$$= i_r v_r + i_y v_y - (i_r + i_y) v_b$$

Now according to Kirchhoff's current law to neutral point.

$$i_r + i_y + i_b = 0$$

$$\therefore \quad i_r + i_y = -i_b$$

Substituting above, $W_1 + W_2 = i_r v_r + i_y v_y + i_b v_b$

As v_r, v_y and v_b are instantaneous values of phase voltages and i_r, i_y, i_b are instantaneous values of line currents which are same as phase currents as load is star connected,

$$\boxed{W_1 + W_2 = P_R + P_y + P_B}$$

where P_R, P_y and P_B are instantaneous values of powers consumed by each phase of the load at the instant considered regardless of power factor. Hence at any instant, addition of two wattmeter readings always gives instantaneous total power consumed by a three phase load.

The result can be easily proved for delta connected load. Hence by two wattmeter method we can measure total power though load is star or delta connected, balanced or unbalanced.

Note that instantaneously power is always fluctuating and wattmeter pointer also tries to show this fluctuating power. But due to inertia of the moving system, pointer can not respond to these fluctuations. And hence wattmeter reads average value of the power and hence $W_1 + W_2$ gives the average value of the total power consumed by 3 phase load.

b) For balanced load : Let us consider the r.m.s. values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.

$$W_1 = I_R \times V_{RB} \times \cos\left(I_R \wedge V_{RB}\right)$$

$$W_2 = I_Y \times V_{YB} \times \cos\left(I_Y \wedge V_{YB}\right)$$

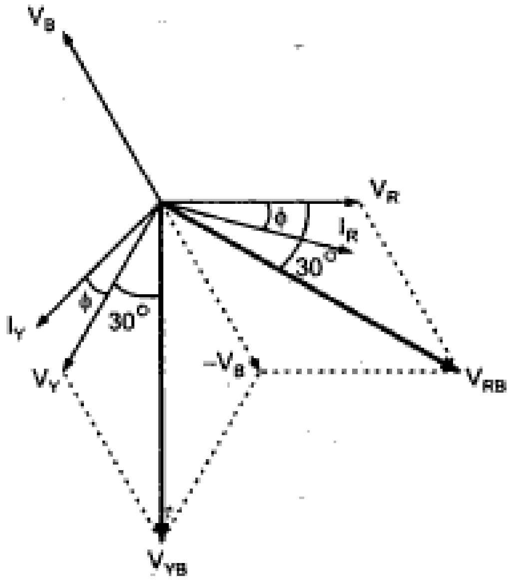


Fig. 3.25

To find angle between (I_R and V_{RB}) and (I_Y and V_{YB}) let us draw phasor diagram. (Assuming load p.f. be cos ϕ lagging)

$$\overline{V}_{RB} = \overline{V}_R - \overline{V}_B$$

and $$\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$$

$$V_R \wedge I_R = \phi \quad \text{and} \quad V_Y \wedge I_Y = \phi$$

$$V_R = V_Y = V_B = V_{ph}$$

and $$V_{RB} = V_{YB} = V_L$$

$$I_R = I_Y = I_L = I_{ph}$$

From Fig. 3.25, $I_R \wedge V_{RB} = 30 - \phi$

and $I_Y \wedge V_{YB} = 30 + \phi$

$\therefore$ $$W_1 = I_R V_{RB} \cos(30 - \phi)$$

$\therefore$ $$\boxed{W_1 = V_L I_L \cos(30 - \phi)}$$

and $$W_2 = I_Y V_{YB} \cos(30 + \phi)$$

$\therefore$ $$\boxed{W_2 = V_L I_L \cos(30 + \phi)}$$

$\therefore$ $$W_1 + W_2 = V_L I_L \left[\cos(30 - \phi) + \cos(30 + \phi)\right]$$

$$= V_L I_L [\cos 30 \cos \phi + \sin 30 \sin \phi + \cos 30 \cos \phi - \sin 30 \sin \phi]$$

$$= 2\ V_L I_L\ \cos 30 \cos \phi\ = 2 V_L I_L \frac{\sqrt{3}}{2} \cos\phi$$

$$\therefore \quad W_1 + W_2 = \sqrt{3}\, V_L\ I_L\ \cos\phi \ = \text{Total 3 phase power}$$

Consider delta connected balanced load, as shown in the Fig. 3.26.

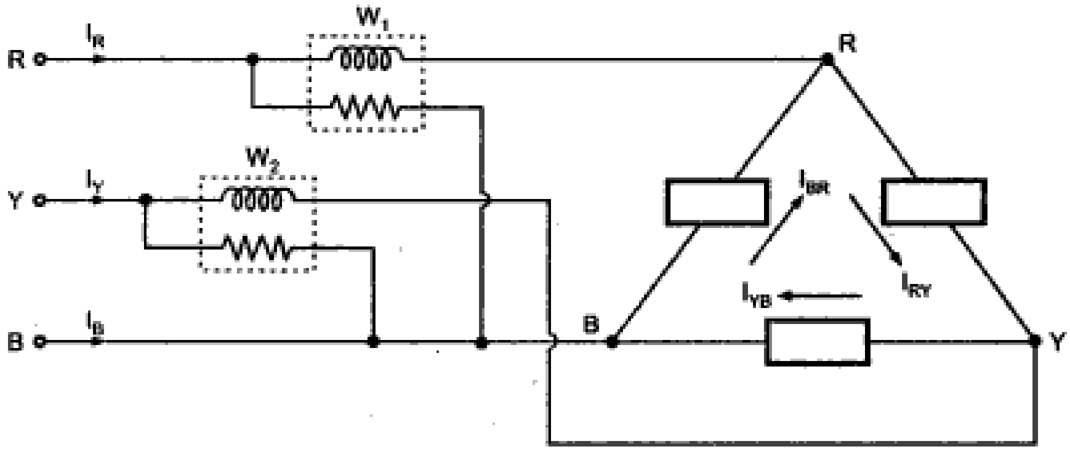


Fig. 3.26 Two wattmeter method for delta connected load

For W_1, $\quad I_c = I_R$ and $V_{pc} = V_{RB}$

and W_2, $\quad I_c = I_Y$ and $V_{pc} = V_{YB}$

$$\therefore \quad W_1 = I_R V_{RB} \cos\left(I_R \wedge V_{RB}\right)$$

$$W_2 = I_Y V_{YB} \cos\left(I_Y \wedge V_{YB}\right)$$

To find $I_R \wedge V_{RB}$ and $I_Y \wedge V_{YB}$ let us draw phasor diagram. Assume load having cos ϕ lagging p.f.

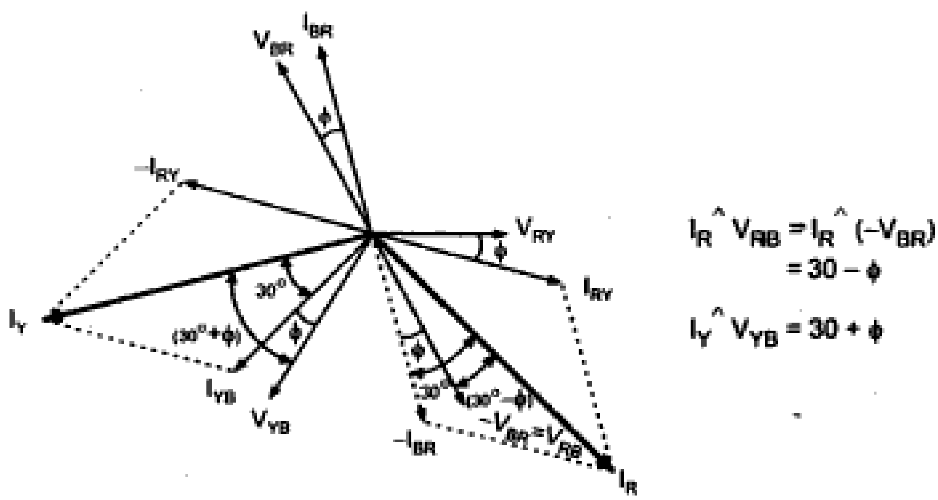


Fig. 3.27 Delta connected load, lagging p.f.

$$V_{RB} = V_{YB} = V_{phase} = V_{line}$$

and $$I_R = I_Y = I_{line}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} \quad \text{and} \quad \bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$$

$\therefore$ $$W_1 = I_R V_{RB} \cos(30-\phi) = I_L V_L \cos(30-\phi)$$

$$W_2 = I_Y V_{YB} \cos(30+\phi) = I_L V_L \cos(30+\phi)$$

$\therefore$ $$W_1 + W_2 = V_L I_L [\cos(30-\phi) + \cos(30+\phi)] = \sqrt{3} V_L I_L \cos\phi$$

$$= \text{Total power consumed by three phase load.}$$

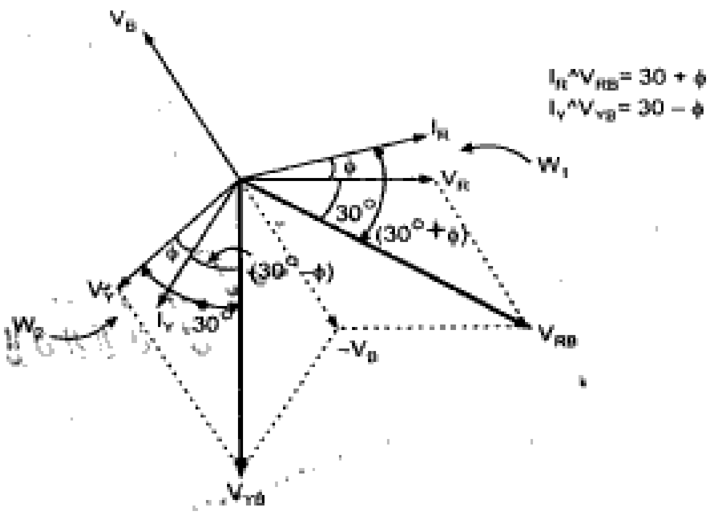


Fig. 3.28 Star connectd leading p.f

Key Point : *It can be observed that whether load is star of delta, the expressions for* W_1 *and* W_2 *remain same.*

For load having leading power factor, I_{ph} will lead V_{ph} by angle ϕ and it can be observed from phasor diagram that W_1 reading and W_2 reading will get interchanged as $30-\phi$ will become $30+\phi$ and viceversa.

Refer Fig. 3.28 for leading power factor phasor diagram.

I_R leads V_R by ϕ , I_Y leads V_Y by ϕ ,

$$W_1 = I_R V_{RB} \cos(I_R \wedge V_{RB})$$

$\therefore$ $$\boxed{W_1 = V_L I_L \cos(30+\phi)}$$

While, $$W_2 = I_Y V_{YB} \cos(I_Y \wedge V_{YB})$$

$\therefore$ $$\boxed{W_2 = V_L I_L \cos(30-\phi)}$$

Reading of W_1 and W_2 are interchanged compared to lagging power factor load.

But $$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

$\therefore$ $$\boxed{W_1 + W_2 = \text{Total 3 phase power}}$$

For **star or delta lagging p.f. load,** $W_1 = V_L I_L \cos(30-\phi)$

and $W_2 = V_L I_L \cos(30+\phi)$

For **star or delta leading p.f. load,** $W_1 = V_L I_L \cos(30+\phi)$

and $W_2 = V_L I_L \cos(30-\phi)$

For **star or delta unity p.f. load,** $\cos\phi = 1$ and $\phi = 0°$

$$W_1 = W_2 = V_L I_L \cos 30°$$

➡ **Example 3.3 :** *A 3-phase 10 kVA load has a p.f. of 0.342. The power is measured by two wattmeter method. Find the reading of each wattmeter when i) p.f. is leading and ii) p.f. is lagging.*

Solution : The given values are, 10 kVA, $\cos\phi = 0.342$

We know that volt-ampere input for a three phase circuit is given by,

$$S = \sqrt{3}\, V_L\, I_L$$

$$\therefore \quad 10 \times 10^3 = \sqrt{3}\, V_L\, I_L$$

$$\therefore \quad V_L\, I_L = 5773.5027 \quad \text{VA}$$

i) p.f. is leading $\cos\phi = 0.342, \quad \phi = \cos^{-1}(0.342) = 70°$

$$W_1 = V_L I_L \cos(30+\phi) = 5773.5027 \cos(30+70)$$

$$= \mathbf{-1002.5582 \quad W}$$

$$W_2 = V_L I_L \cos(30-\phi) = 5773.5027 \cos(30-70)$$

$$= \mathbf{4422.7596 \quad W}$$

ii) p.f. is lagging $W_1 = V_L I_L \cos(30-\phi) = \mathbf{4422.7596 \quad W}$

$$W_2 = V_L I_L \cos(30+\phi) = \mathbf{-1002.5582 \quad W}$$

Key Point : *It can be observed that when p.f. is changed from lagging to leading the readings of W_1 and W_2 get interchanged.*

3.12 Power Factor Calculation by Two Wattmeter Method

In case of balanced load, the p.f. can be calculated from W_1 and W_2 readings.

For balanced, lagging p.f. load, $W_1 = V_L I_L \cos(30-\phi)$

$$W_2 = V_L I_L \cos(30+\phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \qquad \text{...(1)}$$

$$W_1 - W_2 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi]$$

$$= V_L I_L [2 \sin 30 \sin\phi] = V_L I_L \left[2 \times \frac{1}{2} \times \sin\phi\right]$$

$$\therefore \quad W_1 - W_2 = V_L I_L \sin(\phi) \quad \ldots(2)$$

Taking ratio of (1) and (2),

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\therefore \quad \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

$$\therefore \quad \phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]$$

$$\therefore \quad \boxed{\text{p.f. } \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}}$$

For **leading p.f.** we get **tan ϕ negative**. But cosine of negative angle is positive.

Key Point : *So cos ϕ is always positive but its nature must be determined by observing sign of tan ϕ.*

3.13 Effect of P.F. on Wattmeter Readings

For a lagging p.f.

$$W_1 = V_L I_L \cos(30-\phi)$$

$$W_2 = V_L I_L \cos(30+\phi)$$

Consider different cases,

Case i) $\cos\phi = 0 \quad \phi = 90°$

$$\therefore \quad W_1 = V_L I_L \cos(30-90) = +\frac{1}{2} V_L I_L$$

$$W_2 = V_L I_L \cos(30+90) = -\frac{1}{2} V_L I_L$$

i.e. $W_1 + W_2 = 0$

$$|W_1| = |W_2| \quad \text{but} \quad W_2 = -W_1$$

Note : Wattmeter can not show negative reading as it has only positive scale. Indication of negative reading is that pointer tries to deflect in negative direction i.e. to the left of zero. In such case, reading can be converted to positive by interchanging either pressure coil connections i.e. ($C \leftrightarrow V$) or by interchanging current coil connections ($M \leftrightarrow L$). Remember that interchanging connections of both the coils will have no effect on the wattmeter reading.

Key Point : *Such a reading obtained by interchanging connections of either of the two coils will be positive on wattmeter but must be taken as negative for calculations.*

In the case discussed above W_1 will show positive reading with normal connections while W_2 will try to deflect in negative direction and hence W_2 reading must be obtained by reversing connections of either of the two coils and must be taken as negative.

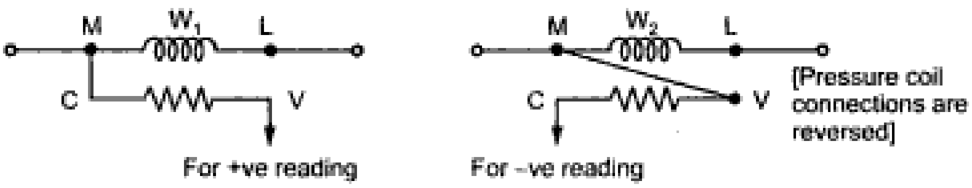


Fig. 3.29 Negative reading on wattmeter

So on wattmeter $W_1 = W_2$ but W_2 must be taken as negative as this reading will be obtained by reversing connections of any one coil.

Case ii) $\cos\phi = 0.5, \ \phi = 60°$

$$\therefore \quad W_1 = V_L I_L \cos(30-60) = V_L I_L \cos 30$$

$$= \text{Positive}$$

$$W_2 = V_L I_L \cos(30+60) = 0$$

$$\therefore \quad W_1 + W_2 = W_1 = \text{Total power.}$$

One wattmeter shows zero reading for $\cos\phi = 0.5$. For all power factors between 0 to 0.5 W_2 shows negative and W_1 shows positive, **for lagging p.f.**

Case iii) $\cos\phi = 1, \ \phi = 0°$

$$W_1 = V_L I_L \cos(30+0) = V_L I_L \cos 30 = +\text{ve}$$

$$W_2 = V_L I_L \cos(30-0) = V_L I_L \cos 30 = +\text{ve}$$

$\therefore$ Both W_1 and W_2 are equal and positive. For all power factors between 0.5 to 1 both wattmeter gives +ve reading.

In short, the result can be summarised as,

Range of p.f.	Range of ' ϕ '	W_1 sign	W_2 sign	Remark
$\cos\phi = 0$	$\phi = 90°$	positive	negative	$\lvert W_1 \rvert = \lvert W_2 \rvert$
$0 \angle \cos\phi \angle 0.5$	$90° \angle \phi \angle 60°$	positive	negative	
$\cos\phi = 0.5$	$\phi = 60°$	positive	0	
$0.5 \angle \cos\phi \angle 1$	$60° \angle \phi \angle 0°$	positive	positive	
$\cos\phi = 1$	$\phi = 0°$	positive	positive	$W_1 = W_2$

Table 3.2

Key Point : *The Table 3.2 is applicable for lagging power factors but same table is applicable for leading power factors by interchanging columns of W_1 and W_2.*

The curve of p.f. against K is shown in the Fig. 3.30 where,

$$K = \frac{\text{Smaller reading}}{\text{Larger reading}}$$

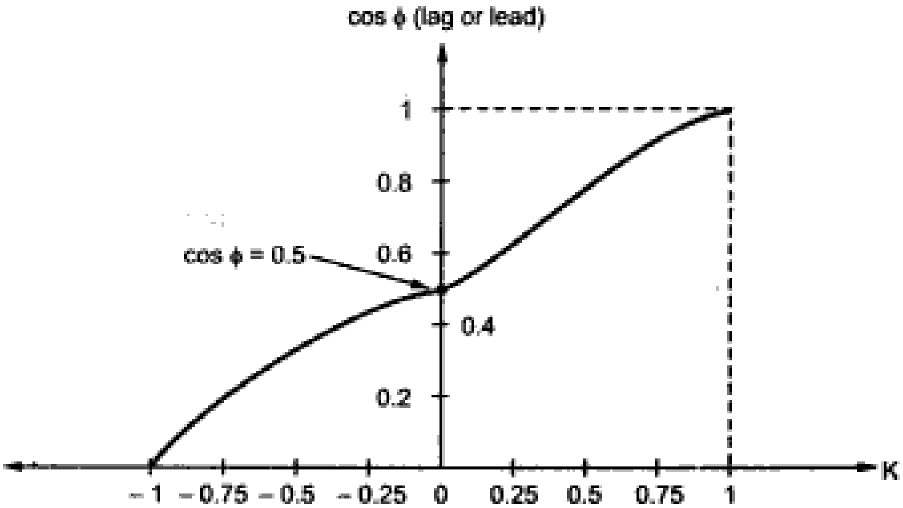


Fig. 3.30 Effect of p.f. on wattmeter readings

3.14 Reactive Volt-Amperes by Two Wattmeter Method

We have seen that,

$$W_1 - W_2 = V_L I_L \sin\phi$$

The total reactive volt-amperes for a 3 phase circuit is given by,

$$Q = \sqrt{3}\, V_L I_L \sin\phi = \sqrt{3}\,(W_1 - W_2) \quad \text{VAR}$$

Thus reactive volt-amperes of a 3 phase circuit can be obtained by multiplying the difference of two wattmeter readings by $\sqrt{3}$.

So,

$$S = \text{Apparent power } \sqrt{3}\, V_L\, I_L \qquad \text{VA or kVA}$$

$$P = \text{Active power} = \sqrt{3}\, V_L\, I_L \cos\phi = W_1 + W_2 \qquad \text{W or kW}$$

$$Q = \text{Reactive power} = \sqrt{3}\, V_L\, I_L \sin\phi = \sqrt{3}\,(W_1 - W_2) \quad \text{VAR or kVAR}$$

Example 3.4 : *Two wattmeters connected to measure the input to a balanced 3 ϕ circuit indicate 2000 W and 500 W respectively. Find the power factor of the circuit :-*

i) When both readings are positive.

ii) When the latter is obtained after reversing the connection to the current coil of one instrument.

Solution : Case i) Both readings positive

$$W_1 = 2000 \text{ W} \quad \text{and} \quad W_2 = 500 \text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-500)}{(2000+500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(1.0392)\} = \cos(46.102^\circ)$$

$$= \mathbf{0.6933 \quad lagging}$$

Case ii) W_2 obtained by reversing the connections hence negative

$$\therefore \quad W_1 = 2000 \text{ W} \quad \text{and} \quad W_2 = -500 \text{ W}$$

$$\therefore \quad \cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(2000-(-500))}{(2000-500)}\right]\right\}$$

$$= \cos\{\tan^{-1}(2.8867)\} = \cos(70.89^\circ)$$

$$= \mathbf{0.3273 \quad lagging}$$

3.15 Advantages of Two Wattmeter Method

The various advantages of two wattmeter method are,

1. The method is applicable for balanced as well as unbalanced loads.
2. Neutral point for star connected load is not necessary to connect the wattmeters.
3. The delta connected load, need not be opened for connecting the wattmeters.
4. Only two wattmeters are sufficient to measure total 3 phase power.
5. If the load is balanced not only the power but power factor also can be determined.
6. Total reactive volt amperes can be obtained using two wattmeter readings for balanced loads.

3.16 Disadvantages of Two Wattmeter Method

The few disadvantages of this method are,

1. Not applicable for three phase, 4 wire system
2. The signs of W_1 and W_2 must be identified and noted down correctly otherwise it may lead to the wrong results.

3.17 Modified Version of 2 Wattmeter Method as One Wattmeter Method

This method can be used only for balanced loads. In this method, only one wattmeter is used. Its current coil is introduced in any one line while its voltage coil is connected to other two lines, one after the other in sequence, with the help of two way switch.

The sum of the two readings gives us total active power in three phase circuit.

Its connection can be shown as in the Fig. 3.31.

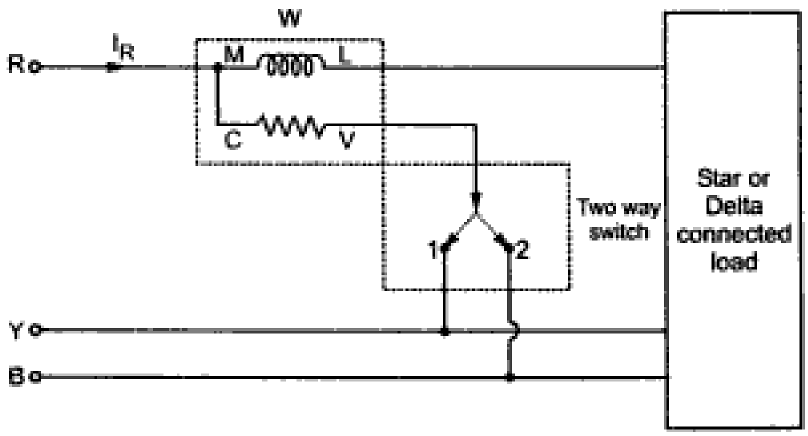


Fig. 3.31 Modified 2 wattmater method as 1 wattmeter method

In the Fig. 3.31 current coil is introduced in line R and hence it carries current I_R.

While one end of pressure coil is connected to 'R' only and second end gets connected to 'Y' when switch is in position 1 and gets connected to 'B' when switch is in position 2.

Assume Star Connected Load : Let us find the two readings on that wattmeter. Assume load having p.f. cos ϕ lagging.

In switch position 1 : $I_c = I_R$

$$V_{pc} = V_{RY}$$

$$\therefore \quad W_1 = I_R V_{RY} \cos(I_R \wedge V_{RY})$$

In switch position 2 : $I_c = I_R$

$$V_{pc} = V_{RB}$$

$$\therefore \quad W_2 = I_R V_{RB} \cos(I_R \wedge V_{RB})$$

Let us find $I_R \wedge V_{RY}$ and $I_R \wedge V_{RB}$ from phasor diagram as shown in the Fig. 3.32.

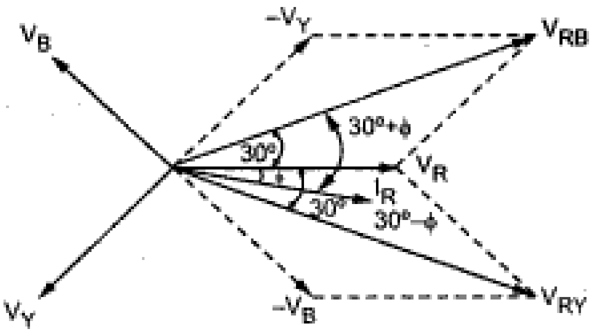


Fig. 3.32

$$\overline{V}_{RB} = \overline{V}_R - \overline{V}_B$$

$$\overline{V}_{RY} = \overline{V}_R - \overline{V}_Y$$

I_R lags V_R by angle 'ϕ'.

$$\therefore \quad I_R \wedge V_{RY} = (30 - \phi)$$

$$I_R \wedge V_{RB} = (30 + \phi)$$

$\therefore$ In position 1, $W_1 = I_R V_{RY} \cos(30 - \phi) = I_L V_L \cos(30 - \phi)$

and In position 2, $W_2 = I_R V_{RB} \cos(30 + \phi) = I_L V_L \cos(30 + \phi)$

$$\therefore \quad \boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi = \text{Total power.}}$$

Key Point : *Any unbalanced condition of the load may create large errors in this method hence not preferred over 2 wattmeter method.*

3.18 One Wattmeter Method for Reactive Voltamperes Measurement

This can be used for balanced load. In this method, current coil of wattmeter is connected in any one line and pressure coil is connected across remaining two lines. The connection is as shown in the Fig. 3.33.

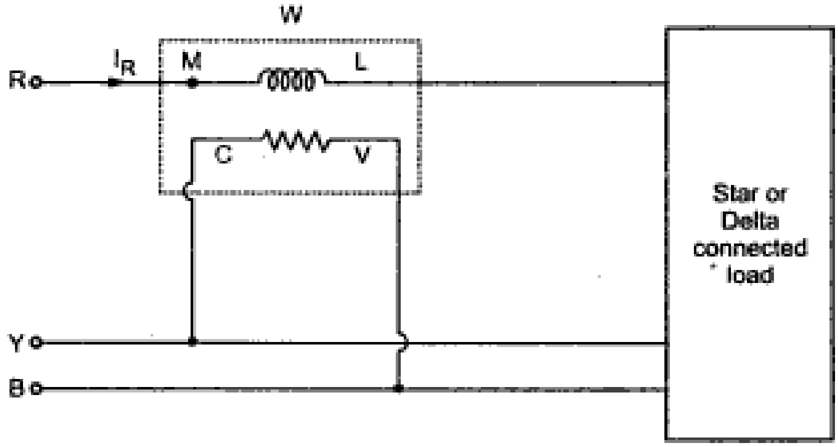


Fig. 3.33 Reactive voltamperes measurement

$$W = I_c V_{pc} \cos(I_C \wedge V_{PC})$$

$$= I_R V_{YB} \cos(I_R \wedge V_{YB})$$

To find $I_R \wedge V_{YB}$, assume load to be star connected having $\cos\phi$ lagging p.f.

$\therefore$ Phasor diagram is as in the Fig. 3.34.

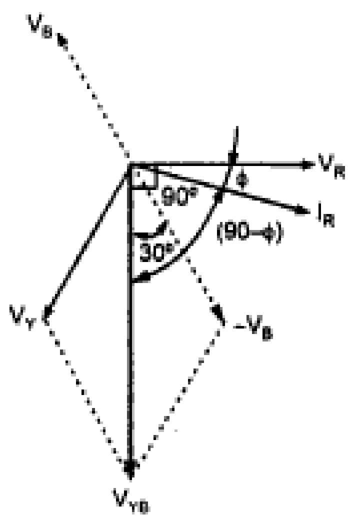


Fig. 3.34

$$\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$$

$$I_R \wedge V_{YB} = 90 - \phi$$

$$\therefore \quad W = I_R V_{YB} \cos(90 - \phi)$$

$$= I_R V_{YB} \sin\phi = I_L V_L \sin\phi$$

Thus in this method the wattmeter reading is,

$$\boxed{W = V_L I_L \sin\phi}$$

But total reactive volt amperes are $\sqrt{3}\, V_L I_L \sin\phi$.

Key Point : *This reading must be multiplied by $\sqrt{3}$ to get total reactive volt amperes.*

$$\therefore \quad \boxed{\sqrt{3}\, W = \text{Total reactive volt amperes}}$$

Example 3.5 : *In a particular test the two wattmeter readings are 4 kW and 1 kW. Calculate the power and power factor if*

i) Both meters read direct ii) One meter connections reversed.

Solution : i) Both meters read direct

$$\therefore \quad W_1 = +4 \text{ kW} \qquad W_2 = +1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = \mathbf{5\ kW}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 3}{5}\right]\right\}$$

$$\therefore \quad \cos\phi = \cos(46.102°) = \mathbf{0.6933\ lagging}$$

ii) When one meter reversed

$$W_1 = +4 \text{ kW} \qquad W_2 = -1 \text{ kW}$$

$$\therefore \quad P = W_1 + W_2 = 4 - 1 = \mathbf{3\ kW}$$

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}[4-(-1)]}{[4-1]}\right]\right\} = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\times 5}{3}\right]\right\}$$

$$= \cos(70.8933°) = \mathbf{0.3273\ lagging}$$

Example 3.6 : *A three phase, 400 V load has power factor of 0.6 lagging. The two wattmeters read a total input power of 20 kW. Find the reading of each wattmeter.*

Solution : $W_1 + W_2 = 20$ kW = P, $V_L = 400$ V, $\cos\phi = 0.6$

Now $\quad P = \sqrt{3}\, V_L\, I_L\, \cos\phi$

$\therefore \quad 20\times10^3 = \sqrt{3}\times400\times I_L \times 0.6$

$\therefore \quad I_L = 48.1125 \text{ A}$

$\phi = \cos^{-1} 0.6 = 53.13°$

$\therefore \quad W_1 = V_L\, I_L\, \cos(30-\phi) = 400\times48.1125\times\cos(30°-53.13°)$

$= 17698 \text{ W}$

$= \mathbf{17.698\ kW}$

And $\quad W_2 = V_L\, I_L\, \cos(30+\phi) = 400\times48.1125\times\cos(30°+53.13°)$

$= 2302 \text{ W}$

$= \mathbf{2.302\ kW}$

Example 3.7 : *The power input in a three phase three wire delta connected balanced load is measured by the two wattmeter method. The reading of wattmeter A is 5000 W and wattmeter B is – 1000 W (with reversal of connection).*

i) Find the power factor of circuit.

ii) If the voltage of the circuit is 440 V, 50 Hz , what is the value of capactiance connected in delta at the source to cause the whole of the power measured by the wattmeter A only ?

Solution : $W_1 = +5000$ W, $\quad W_2 = -1000$ W

i)
$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

$$= \cos\left\{\tan^{-1}\left(\frac{\sqrt{3}\,[5000-(-1000)]}{[5000+(-1000)]}\right)\right\}$$

$$= \cos\left\{\tan^{-1}\left(\frac{\sqrt{3}\times6000}{4000}\right)\right\} = \cos\ (68.948)$$

$$= \mathbf{0.3592\ lag}$$

ii) Now capacitance is connected at the source which does not affect the active power consumption.

$\therefore \quad W = 5000 - 1000 = 4000 \text{ W}$

Before connecting capacitor

$$W = \sqrt{3}\, V_L\, I_L\, \cos\phi$$

$$\therefore \quad I_L = \frac{4000}{\sqrt{3}\times 440\times 0.3592} = 14.612\ A$$

For delta, $V_{ph} = V_L = 440\ V$ and $I_L = \sqrt{3} I_{ph}$ i.e. $I_{ph} = 8.4362\ A$

$$\therefore \quad |Z_{ph}| = \frac{V_{ph}}{I_{ph}} = \frac{440}{8.4362} = 52.1561\ \Omega\ ,\ \phi = 68.948^\circ$$

$$\therefore \quad R_{ph} = Z_{ph} \cos\phi = 18.7352\ \Omega$$

$$X_{ph} = Z_{ph} \sin\phi = 48.6749\ \Omega \text{ (inductive)}$$

After connecting capacitor

$$W_A = W = 4000\ W,\quad W_B = 0\ W$$

But $W_B = V_L\, I_L\, \cos(30+\phi) = 0$

i.e. $30 + \phi = 90^\circ$ i.e. $\phi = 60^\circ$

$$\therefore \quad \cos\phi = \cos(60^\circ) = 0.5 \text{ lagging}$$

Due to pure capacitor, R_{ph} remains same.

$$\therefore \quad R_{ph} = 18.7352\ \Omega$$

Now $\tan\phi = \dfrac{X'_{ph}}{R_{ph}}$ where X'_{ph} = New reactance

$$\therefore \quad \tan(60^\circ) = \frac{X'_{ph}}{18.7352}$$

$$\therefore \quad X'_{ph} = 32.4503\ \Omega$$

$$\therefore \quad X_{cph} = X_{ph} - X'_{ph} = 48.6749 - 32.4503 = 16.2246\ \Omega$$

But $X_{cph} = \dfrac{1}{2\pi f C}$

$$\therefore \quad C = \frac{1}{2\pi\times 50\times 16.2246}$$

$$= 196.1896\ \mu F$$

3.19 Extension of Range of Wattmeter using Instrument Transformers

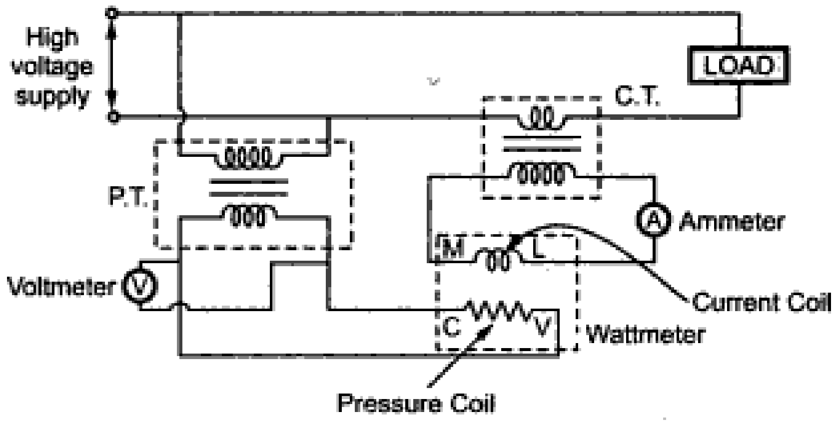


Fig. 3.35 Power measurement using C.T. and P.T.

For very high voltage circuits, the high rating wattmeters are not available to measure the power. The range of wattmeter can be extended using instrument transformers, in such high voltage circuits. The connections are shown in the Fig. 3.35.

The primary winding of C.T. is connected in series with the load and secondary is connected in series with an ammeter and the current coil of a wattmeter.

The primary winding of P.T. is connected across the supply and secondary is connected across voltmeter and the pressure coil of the wattmeter. One secondary terminal of each transformer and the casings are grounded.

Now both C.T. and P.T. have errors like ratio error and phase angle error. For precise measurements, these errors must be considered. If not considered, these errors may cause inaccurate measurements. The correction must be applied to such errors to get the accurate results.

3.19.1 Phasor Diagrams and Correction Factors

Consider the various parameters as,

V = Voltage across the load

I = Load current

ϕ = Phase angle between V and I

V_s = Voltage across secondary of P.T.

= Wattmeter pressure coil voltage

I_s = Current in secondary of C.T.

= Wattmeter current coil current

I_p = Current in pressure coil of wattmeter

α = Phase angle between currents in current coil and pressure coil of wattmeter.

δ = Phase angle of P.T.

θ = Phase angle of C.T.

β = Angle by which I_p logs V_s due to inductance of pressure coil.

The phasor diagrams for lagging and leading P.f. loads are shown in the Fig. 3.36 (a) and (b) respectively.

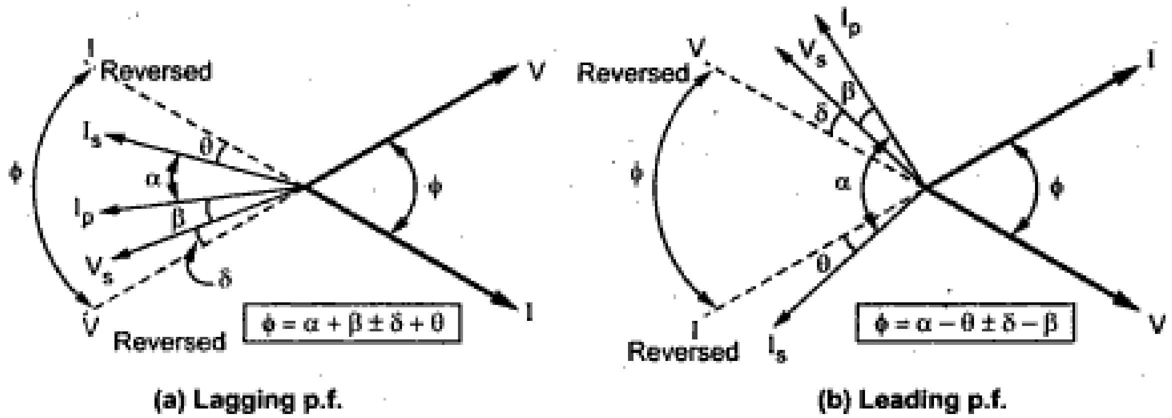


Fig. 3.36

Lagging power factor : For lagging p.f., θ is positive i.e. I_s leads reversed I while phase of P.T. may be positive or negative i.e. δ can be positive or negative. For phasor diagram shown in Fig. 3.36 (a) δ is negative i.e. V_s lags reversed V hence,

$\phi = \alpha + \beta + \delta + \theta$	...	δ negative	Lagging p.f.
$\phi = \alpha + \beta - \delta + \theta$	...	δ positive	

Leading power factor : For leading p.f. the ϕ is given by

$\phi = \alpha - \beta - \delta - \theta$	...	δ negative	Leading p.f.
$\phi = \alpha - \beta + \delta - \theta$	...	δ positive	

Correction Factor : The correction factor, neglecting transformation ratio errors is,

$$K = \frac{\cos \phi}{\cos \beta \ \cos \alpha}$$

where $\alpha = \phi - \beta - \delta - \theta$ ... lagging p.f. (with δ negative)

and $\alpha = \phi + \beta + \delta + \theta$ ... leading p.f. (with δ negative)

Key Point: *Change the sign of δ in the expression of α if δ is positive.*

$$\therefore \quad \text{True Power} = K \times \begin{matrix}\text{actual ratio}\\ \text{of C.T.}\end{matrix} \times \begin{matrix}\text{actual ratio}\\ \text{of P.T.}\end{matrix} \times \begin{matrix}\text{wattmeter}\\ \text{reading}\end{matrix}$$

$$\text{True Power} = K \times \begin{bmatrix}\text{R.C.F}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}\text{R.C.F.}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{nominal ratio}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}\text{nominal ratio}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{wattmeter}\\ \text{reading}\end{bmatrix}$$

This concept of using C.T. and P.T. for single phase power measurement can be extended for three phase power measurement. The connections are basically similar to the two wattmeter method as shown in the Fig. 3.37.

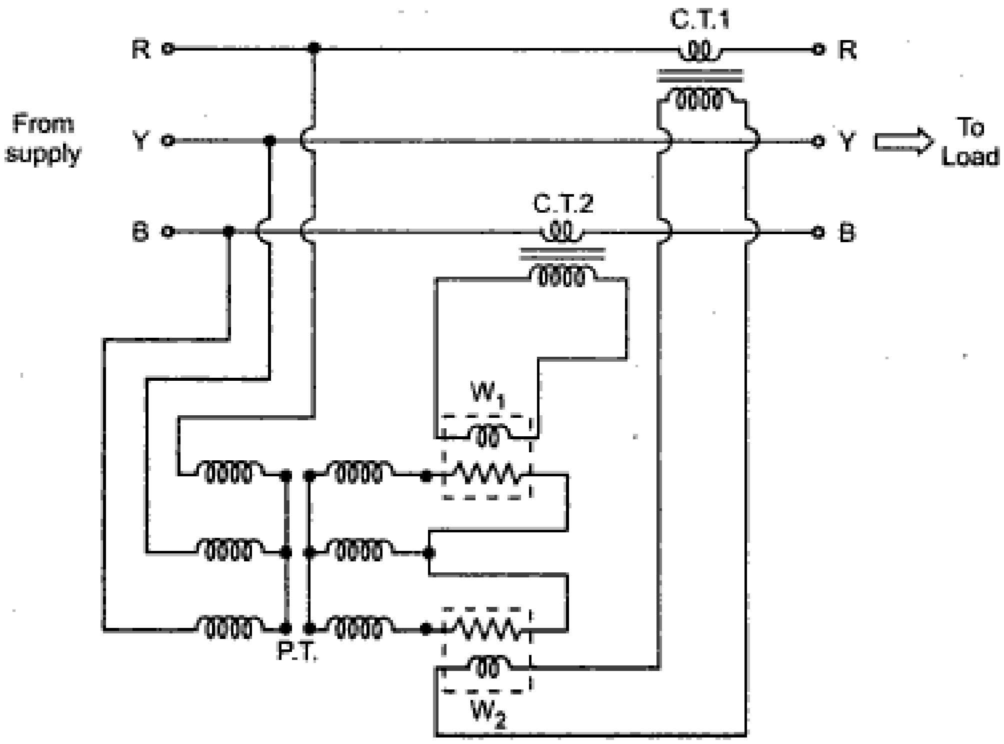


Fig. 3.37 Three phase power measurement using C.T. and P.T.

Example 3.8 : *A reading of 400 W is indicated on a 100 V / 5 A wattmeter used in connection with voltage and current transformers of nominal ratio 100 / 1 and 20 / 1 respectively. If the wattmeter pressure coil has a resistance of 400 Ω and an inductance of 20 mH and the ratio errors and the phase differences of the voltage and current transformers are + 1% and 50 min and – 0.5% and 100 min respectively. Compute the true value of the power measured. The load phase angle is 60° lagging and the frequency is 50 Hz.* **[JNTU, Nov.-2003, Set-3]**

Solution : $\delta = +50' = +0.833°$, $\theta = 100' = +1.667°$, K_n (P.T.) = 100, K_n (C.T.) = 20

$$X_{Lp} = 2\pi f L_p = 2\pi \times 50 \times 20 \times 10^{-3} = 6.2831\,\Omega$$

$$\therefore \quad \beta = \tan^{-1}\left[\frac{X_{Lp}}{R_P}\right] = \tan^{-1}\left[\frac{6.2831}{400}\right] = 0.9°$$

ϕ = 60° lagging, wattmeter reading = 400 W.

The phasor diagram is shown in the Fig. 3.38.

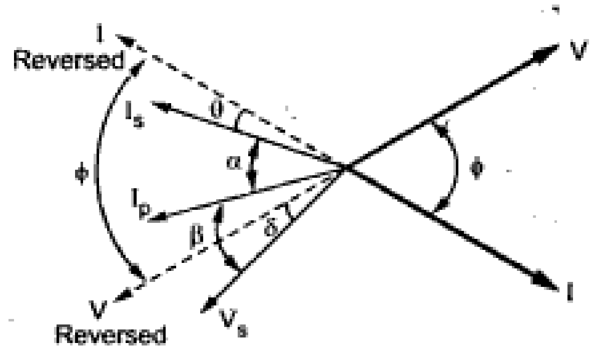


Fig. 3.38

Note : δ is positive hence V_s leads V reversed by δ.

$$\phi = \theta + \alpha + \beta - \delta$$

$$60° = 1.667° + \alpha + 0.9° - 0.833°$$

$$\therefore \quad \alpha = 58.266°$$

$$K = \frac{\cos\phi}{\cos\beta\ \cos\alpha} = \frac{\cos 60°}{\cos(0.9°)\cos(58.266°)} = 0.9507$$

$$\%\ \text{ratio error} = \frac{K_n - R}{R} \times 100$$

$$\therefore \quad 1 = \frac{100 - R}{R} \times 100 \text{ i.e. } R = 99.0099$$

$$\therefore \quad \text{R.C.F. of P.T.} = \frac{R}{K_n} = \frac{99.0099}{100} = \mathbf{0.990099}$$

$$\text{\% ratio error} = \frac{K_n - R}{R} \times 100$$

$$\therefore \quad -0.5 = \frac{20 - R}{R} \times 100 \text{ i.e. } R = 20.1005$$

$$\therefore \quad \text{R.C.F. of C.T.} = \frac{R}{K_n} = \frac{20.1005}{20} = \mathbf{1.005}$$

$$\therefore \quad \begin{bmatrix}\text{True}\\ \text{power}\end{bmatrix} = K \times \begin{bmatrix}\text{R.C.F}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{R.C.F.}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of}\\ \text{C.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of}\\ \text{P.T.}\end{bmatrix} \times \begin{bmatrix}\text{wattmeter}\\ \text{reading}\end{bmatrix}$$

$$= 0.9507 \times 0.990099 \times 1.005 \times 100 \times 20 \times 400$$

$$= \mathbf{756794.8439 \text{ W}}$$

Example 3.9 : *A potential transformer with a nominal ratio of 2000/100 V, Ratio correction factor of 0.995 and a phase angle of – 22′ is used with a current transformer of nominal ratio 100/5 A, ratio correction factor of 1.005 and a phase angle error of 10′, to measure the power (I_s leads I_p) to a single phase inductive load. The meters connected to these instrument transformers read correct readings of 102 volts, 4 amperes and 375 watts. Determine the true values of voltage, current and power supplied to the load.*

[JNTU, May-2004, Set-4]

Solution : $\delta = -22' = -\left(\frac{22}{60}\right)^{\circ} = -0.367^{\circ}$, $\theta = 10' = \left(\frac{10}{60}\right)^{\circ} = 0.167^{\circ}$

$\beta = 0^{\circ}$ as pressure coil inductance is negligible and not given.

The phasor diagram is shown in the Fig. 3.39.

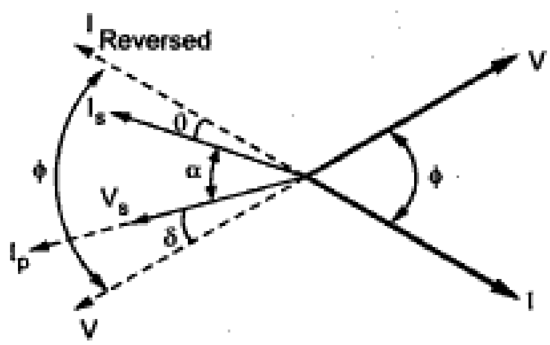


Fig. 3.39

As $\beta = 0$, V_s and I_p are in phase.

$$\therefore \quad W = V_s \, I_s \cos(V_s \,\hat{}\, I_s)$$

$$375 = 102 \times 4 \times \cos\alpha$$

$$\therefore \quad \alpha = \cos^{-1}\left(\frac{375}{102 \times 4}\right) = 23.2025°$$

$$\therefore \quad \phi = \alpha + \delta + \theta = 23.2025 + 0.367 + 0.167$$

$$= 23.7365°$$

Note that when δ is negative, V_s lags V reversed and use positive δ in the expression of ϕ.

$$\therefore \quad K = \frac{\cos\phi}{\cos\beta \cos\alpha} = \frac{\cos(23.7365°)}{\cos 0° \cos 23.2025°} = 0.9959$$

$$\therefore \quad \text{True power} = K \times \begin{bmatrix}\text{R.C.F} \\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{R.C.F.} \\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of} \\ \text{P.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of} \\ \text{C.T.}\end{bmatrix} \times \begin{bmatrix}\text{wattmeter} \\ \text{reading}\end{bmatrix}$$

$$= 0.9959 \times 0.995 \times 1.005 \times \frac{2000}{100} \times \frac{100}{5} \times 375$$

$$= \mathbf{149381.2654 \ W}$$

For P.T., $R = K_n \times \text{R.C.F.} = \frac{2000}{100} \times 0.995 = 19.9$

and $R = \frac{V_P}{V_s}$ i.e. $V_P = R \times V_s = 19.9 \times 102$

$\therefore \quad V_P = \mathbf{2029.8 \ V}$ ... True voltage

For C.T., $R = K_n \times \text{R.C.F.} = \frac{100}{5} \times 1.005 = 20.1$

and $R = \frac{I_P}{I_s}$ i.e. $I_P = R \times I_s = 20.1 \times 4$

i.e. $I_P = \mathbf{80.4 \ A}$ ... True current

3.20 Three Phase Wattmeter

Similar to single phase dynamometer wattmeter, a three phase dynamometer wattmeter is available. It consists of two sets of fixed and moving coils, mounted together in one case. The moving coils are placed on the same spindle. The Fig.3.40. shows the construction of a three phase wattmeter.

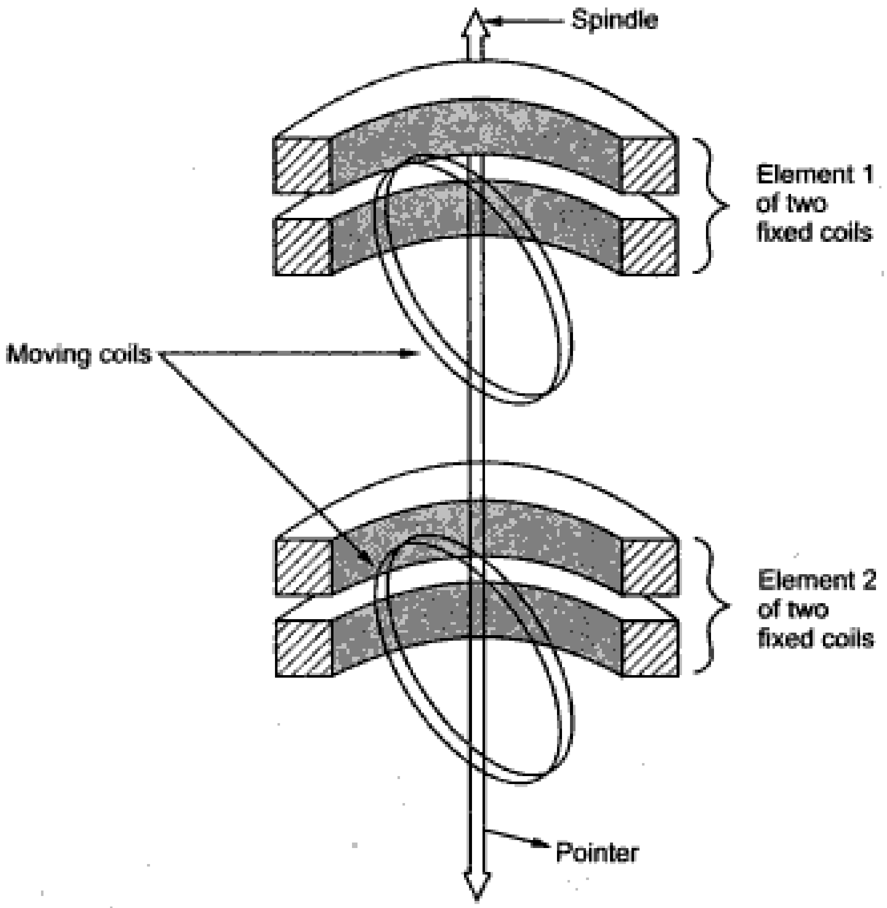


Fig. 3.40 Three phase two element wattmeter

Due to two sets of current and pressure coils, its connections are similar to the connections of two single phase wattmeters to measure three phase power. Each element experiences a torque which is proportional to the power measured by that element. The net torque on the moving system is the sum of the deflecting torques produced on each of the two elements.

$\therefore \quad T_{d1} \propto W_1 \quad \text{and} \quad T_{d2} \propto W_2$

$\therefore \quad T_d \propto (T_{d1} + T_{d2}) \propto (W_1 + W_2) \propto W$

where T_d = Total deflecting torque

W_1 = Power measured by element 1

W_2 = Power measured by element 2

W = Total power

Thus the total deflecting torque is proportional to the total power.

As the coils are mounted very near each other, errors due to mutual interference are possible. To eliminate such errors, the laminated iron shield is placed in between the two elements.

The compensation for mutual interference can be obtained by using the resistances as shown in the Fig. 3.41.

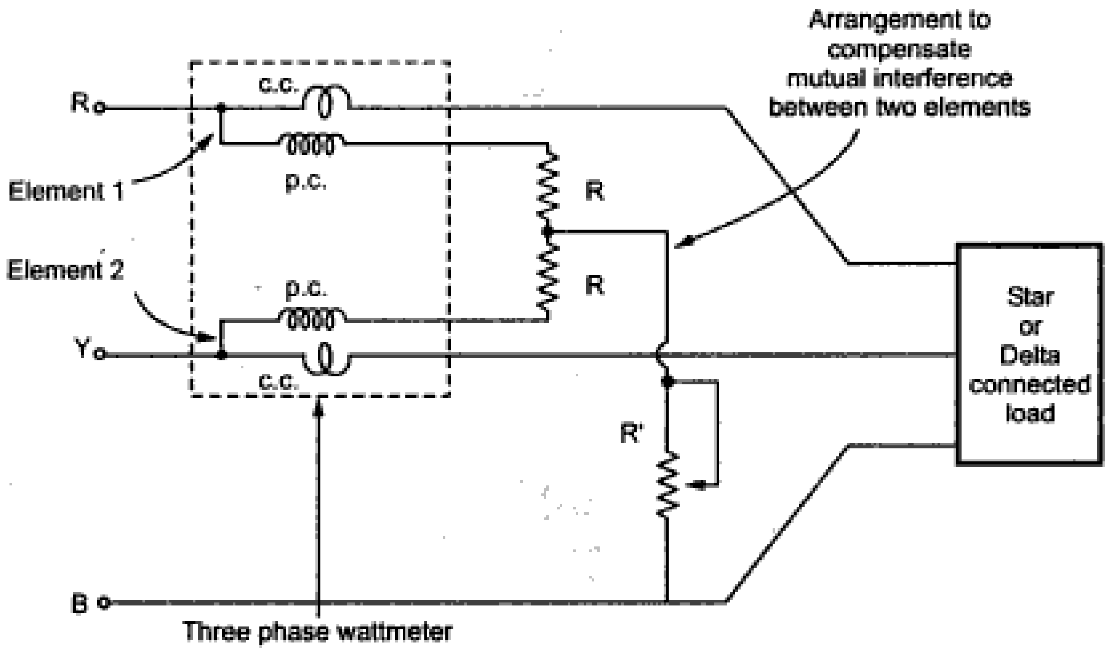


Fig. 3.41 Connections of three phase wattmeter

The value of R can be adjusted using R' to compensate the errors due to mutual effects between the two elements.

Examples with Solutions

➠ **Example 3.10 :** *A 500 V, 20 A dynamometer instrument is used as a wattmeter. Its current coil has 0.1 Ω resistance and pressure coil has 25 kΩ resistance with 0.1 H inductance. The meter was calibrated on d.c. supply. What is the error in the instrument if it is used to measure the power in a circuit with supply voltage of 500 V, load current of 24 A at 0.2 p.f. Assume that pressure coil is connected across load.*

[JNTU, May-2004, Set-1]

Solution : $R_c = 0.1\ \Omega$, $R_p = 25\ k\Omega$, $L_p = 0.1\ H$

$$\therefore \quad X_{Lp} = 2\pi f L_p = 2\pi \times 50 \times 0.1 = 31.4159\ \Omega$$

$$\therefore \quad \beta = \tan^{-1}\frac{X_{Lp}}{R_p} = \tan^{-1}\frac{31.4159}{25\times10^3} = 0.072° = 0.001256 \text{ rad}$$

$$\cos\phi = 0.2 \quad \text{i.e.} \quad \phi = 78.463°$$

The pressure coil is connected on the load side,

$$\begin{bmatrix}\text{Actual wattmeter}\\ \text{reading}\end{bmatrix} = [1 + \tan\phi \tan\beta]\times \text{True power}$$

$$\text{True power } P_T = VI\cos\phi = 500\times 24\times 0.2 = 2400 \text{ W}$$

$$\therefore \begin{bmatrix}\text{Actual wattmeter}\\ \text{reading}\end{bmatrix} = [1 + \tan(78.463^\circ)\tan(0.072^\circ)]\times 2400$$

$$= 2414.7749 \text{ W}$$

$$\therefore \quad \% \text{ error} = \frac{\text{Measured} - \text{True}}{\text{True}}\times 100 = \frac{2414.7749 - 2400}{2400}\times 100$$

$$= \mathbf{0.6156\ \%}$$

Example 3.11 : *A certain circuit takes 10 A at 200 V and the power absorbed is 1000 W. If the current coil of the wattmeter has a resistance of 0.15 Ω and its pressure coil has a resistance of 5000 Ω and inductance of 0.3 H, find*
i) the error due to resistance for each of the two possible methods of connection.
ii) the error due to the inductance if the frequency of 50 Hz.
iii) the total error in each case. **[JNTU, May-2004, Set-2]**

Solution : $R_c = 0.15\ \Omega$, $R_p = 5000\ \Omega$, $L_p = 0.3$ H, I = 10 A, V = 200 V

i) Error due to method of connection

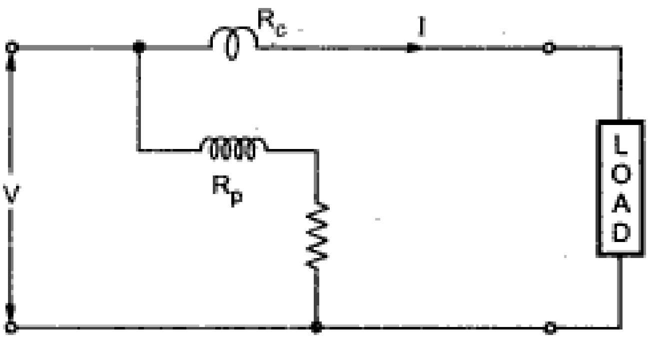


Fig. 3.42

[Power indicated by wattmeter] = [Power consumed by load] + $I^2 R_c$

$$= 1000 + (10)^2 \times 0.15 = 1015 \text{ W}$$

$$\therefore \quad \text{Error} = \mathbf{15\ W}$$

$$\therefore \quad \% \text{ error} = \frac{\text{Measured} - \text{True}}{\text{True}}\times 100 = \frac{15}{1000}\times 100$$

$$= \mathbf{1.5\ \%}$$

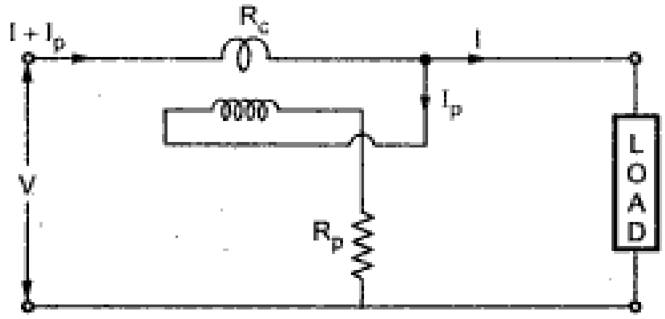


Fig. 3.43

$$[\text{Power indicated by wattmeter}] = [\text{Power consumed by load}] + \frac{V^2}{R_p}$$

$$= 1000 + \frac{(200)^2}{5000} = 1008 \text{ W}$$

$\therefore$ Error = **8 W**

$$\therefore \quad \% \text{ error} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100 = \frac{8}{1000} \times 100 = \mathbf{0.8\ \%}$$

ii) $X_{Lp} = 2\pi f L_p = 2\pi \times 50 \times 0.3 = 94.2477\ \Omega$

$$\therefore \quad \beta = \tan^{-1}\frac{X_{Lp}}{R_p} = \tan^{-1}\left[\frac{94.2477}{5000}\right] = 1.0798°$$

$\therefore$ [Actual wattmeter reading] = $[1 + \tan\phi \tan\beta] \times$ True power

True power = $VI \cos\phi$

$$\therefore \quad \cos\phi = \frac{1000}{10 \times 200} = 0.5 \quad \text{i.e.} \quad \phi = 60°$$

$\therefore$ [Actual wattmeter reading] = $[1 + \tan 60° \tan 1.0798°] \times 1000$ = 1032.6462 W

$\therefore$ Error = 1032.6462 − 1000 = **32.6462 W**

$$\therefore \quad \% \text{ error} = \frac{32.6462}{1000} \times 100 = \mathbf{3.2646\%}$$

iii) Total error

$$\begin{bmatrix}\text{Total error when} \\ \text{pressure coil on} \\ \text{supply side}\end{bmatrix} = 15 + 32.6462 = \mathbf{47.6462\ W}$$

$$= \mathbf{4.7646\%}$$

$$\begin{bmatrix} \text{Total error when} \\ \text{pressure coil on} \\ \text{load side} \end{bmatrix} = 8 + 32.6462 = \mathbf{40.6462\ W}$$

$$= \mathbf{4.06462\%}$$

Example 3.12 : *The power in a single phase high voltage circuit is measured by using instrument transformers with voltmeter, ammeter and wattmeter. Observed readings on the instruments (assuming no errors) are 115 V, 4.5 A and 200 W. Characteristics of the transformers are,*

P.T. : Nominal ratio : 11500/115 V, ratio correction factor 0.995, phase angle −25′

C.T. : Nominal ratio : 25/5 A, ratio correction factor 0.997, phase angle +15′.

Neglecting the voltage phase angle in the voltmeter, calculate the true power.

[JNTU, Nov.-2003, Set-1]

Solution : $\delta = -25' = -\left(\frac{25}{60}\right)^0 = -0.416°, \quad \theta = +15' = \left(\frac{15}{60}\right)^0 = 0.25°, \quad \beta = 0°.$

The phasor diagram is shown in the Fig. 3.44.

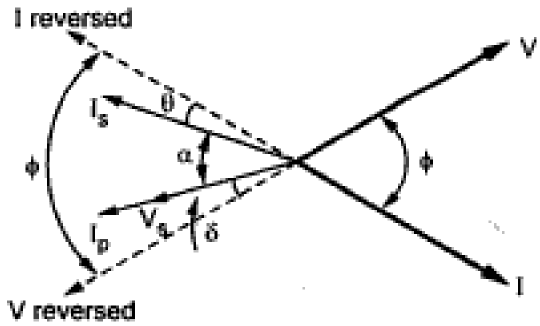


Fig. 3.44

V_s and I_p in phase as $\beta = 0°$.

$$\phi = \alpha + \delta + \theta$$

and $$W = V_s I_s \cos(V_s \wedge I_s)$$

$$200 = 115 \times 4.5 \times \cos(\alpha)$$

$$\therefore \quad \alpha = \cos^{-1}[0.3864] = 67.264°$$

$$\therefore \quad \phi = 67.264° + 0.416° + 0.25° = 67.93°$$

$$\therefore \quad K = \frac{\cos\phi}{\cos\beta \cos\alpha} = \frac{\cos(67.93°)}{\cos(0°) \times \cos(67.264°)} = 0.9722$$

$$\therefore \quad \begin{bmatrix}\text{True}\\ \text{power}\end{bmatrix} = K \times \begin{bmatrix}\text{R.C.F.}\\ \text{of P.T.}\end{bmatrix} \times \begin{bmatrix}\text{R.C.F.}\\ \text{of C.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of}\\ \text{P.T.}\end{bmatrix} \times \begin{bmatrix}K_n \text{ of}\\ \text{C.T.}\end{bmatrix} \times \begin{bmatrix}\text{wattmeter}\\ \text{reading}\end{bmatrix}$$

$$= 0.9722 \times 0.995 \times 0.997 \times \frac{11500}{115} \times \frac{25}{5} \times 200$$

$$= \mathbf{96443.6983\ W}$$

Example 3.13 : *The pressure coil of an electrodynamometer wattmeter has a resistance of 6600 Ω. When the voltage applied to the pressure coil is 120 V and a current of 20 A flows in the series coil, the deflection is 160°. What additional resistance must be connected in the pressure coil circuit to make the meter constant equal to 20 W per degree ?* **[JNTU, May-2005, Set-2]**

Solution : $R_p = 6600\ \Omega$, $V = 120$ V, $I = 20$ A, $\theta = 160°$

$$\theta = \frac{V}{R_p} \times I_c \cos\phi \times \frac{1}{K} \times \frac{dM}{d\theta}$$

where K = Spring constant

Assuming $\frac{dM}{d\theta}$ constant throughout the operation,

$$\theta = \frac{K'}{R_p} \times P \quad \text{where} \quad K' = \frac{1}{K}\frac{dM}{d\theta} \quad \text{and} \quad P = VI\cos\phi$$

$$\therefore \quad 160° = \frac{K'}{6600} \times [120 \times 20 \times 1] \qquad \text{... } \cos\phi = 1$$

$$\therefore \quad K' = 440 \text{ degrees } \Omega / W$$

If Meter constant = 20 W/degree then $\theta' = \frac{P}{\text{Meter constant}}$

$$\therefore \quad \theta' = \frac{2400}{20} = 120°$$

At this time, $R'_p = R_p + R_x$

$$\therefore \quad 120° = \frac{440}{R'_p} \times 2400$$

$$\therefore \quad R'_p = 8800\ \Omega = 6600 + R_x$$

$$\therefore \quad R_x = \mathbf{2200\ \Omega} \qquad \text{... Additional resistance required}$$

Review Questions

1. *What is power ? Why wattmeter is essential ?*
2. *Explain the principle of operation of electrodynamometer type instrument.*
3. *Explain the construction of electrodynamometer type instrument.*
4. *How dynamometer type instrument is used as an ammeter, voltmeter and wattmeter ?*
5. *Explain the theory of dynamometer type wattmeter.*
6. *Write about the shape of scale of dynamometer type wattmeter.*
7. *How the range of the dynamometer type wattmeter can be extended using instrument transformers?*
8. *Which errors are possible to occur in wattmeter and how are they compensated ?*
9. *What is Blondel's theorem ?*
10. *Explain one wattmeter method of measurement of power in a three phase system with balanced load.*
11. *Explain the three wattmeter method of measurement of power in a three phase system with balanced load.*
12. *With the help of connection diagram and phasor diagram, show that two wattmeters are sufficient to measure active power in a three phase three wire system with*

    i) balanced star connected load

    ii) balanced delta connected load.
13. *Explain how the power factor can be calculated by two wattmeter method.*
14. *Explain the effect of power factor on two wattmeter readings in a two wattmeter method.*
15. *Two wattmeter method is used to measure power in a 3-phase balanced load. Find the power factor if*

    i) the two readings are equal and have the same sign,

    ii) the two readings are equal and have opposite sign,

    iii) the reading of one wattmeter is zero,

    iv) the reading of one wattmeter is half of the other wattmeter.

    (Ans. : (i) = 1, (ii) = 0, (iii) = 0.5, (iv) = 0.8660)
16. *Find the power and power factor of the balanced circuit in which the wattmeter readings are 5 kW and 0.5 kW, the latter being obtain after the reversal of the current coil terminal of the wattmeter.*

    (Ans. : Power = 4.5 kW, Power factor = 0.4271 lagging)
17. *How reactive volt amperes are measured by 2 wattmeter method ?*
18. *State the advantages of 2 wattmeter method.*
19. *State the disadvantages of 2 wattmeter method.*
20. *Explain the modification in a two wattmeter method to use it as a single wattmeter method with a tap key. Draw the relevant circuit and phasor diagrams.*
21. *Explain the connection of a single wattmeter to measure reactive volt amperes of a three phase system. Draw the phasor diagram.*

22. *A 10 kW, three-phase induction motor having full-load efficiency of 85% is connected to a 400 volt supply. The motor when running on full-load draws a current of 20 amp from supply. If two wattmeters are connected to measure the total power input to the motor, determine the reading on each wattmeter. Also find out total reactive power of the motor in terms of wattmeter readings.*
23. *A three-phase, 440 V, 50 Hz induction motor takes a line current of 30 A and delivers 10 kW output power under full-load. Assuming its efficiency as 90%.*

    Calculate :

    i) p.f. of the motor,

    ii) readings on the two wattmeters connected to measure the input power to the motor,

    iii) total reactive power.
24. *Two wattmeter method is used to measure power, consumed by delta connected load. Each branch of load having impedance of $20 \angle 60° \Omega$. Supply voltage is 400 V. Calculate the total power and readings on individual wattmeters.*
25. *Power input measurement to a synchronous motor is done using a two wattmeter method. Each of the wattmeters reads 40 kW at certain operating condition. If the power factor is changed to 0.8 lead now, what would be the new wattmeter readings ?* **[Ans. : 22.6 kW, 57.4 kW]**
26. *Power input to a 3-phase, 415 volts, 50 Hz induction motor is measured using two wattmeters. It was observed that one wattmeter reads 7.5 kW while the other reads 2.5 kW after reversing the connections of pressure coil. Calculate :*

    i) Total power ii) Power factor iii) Line current.

    Also identify which wattmeter was reading 2.5 kW if it is given that the two wattmeters were connected in lines R and Y.
27. *Write short note on measurement of reactive power in a three phase balanced circuit using single wattmeter.*
28. *Write a note on three phase wattmeter.*
29. *Explain the features of low power factor wattmeter. Prove that the error due inductance of pressure coil in the wattmeter is $[\tan\phi \tan\beta]$ times actual power.*
30. *Explain the error in wattmeter reading due to method of connection.*
31. *Derive the expression for the true power when the range of wattmeter is extended using C.T. and P.T.*
32. *Write a note on low power factor wattmeter.*

□□□

4 Measurement of Energy

4.1 Introduction

The energy is defined as the power delivered over a time interval.

$$\text{energy} = \text{power} \times \text{time}$$

The electrical energy is defined as the work done over a time interval t and mathematically expressed as,

$$E = \int_0^t \text{power}(dt) = \int_0^t vi\,dt \quad \text{... (1)}$$

where v = voltage in volts and i = current in amperes

The energy is measured in joules (J) or watt-sec (W-s). Thus energy of one joule means the power of 1 watt over a time interval of 1 second.

An electrical energy can also be expressed in the unit watt-hour (Wh) or kilowatt-hour (kWh). Thus one kilowatt-hour energy means the expenditure of 1 kW power over a time interval of 1 hour. The domestic electric energy expenditure is measured in kWh and 1 kWh is called 1 unit of energy.

4.2 Single Phase Energymeter

Induction type instruments are most commonly used as energy meters. Energy meter is an integrating instrument which measures quantity of electricity. Induction type of energy meters are universally used for domestic and industrial applications. These meters record the energy in kilowatt-hours (kWh).

The Fig. 4.1 (See Fig. 4.1 on next page) shows the induction type single phase energymeter.

It works on the principle of induction i.e. on the production of eddy currents in the moving system by the alternating fluxes. These eddy currents induced in the moving system interact with each other to produce a driving torque due to which disc rotates to record the energy.

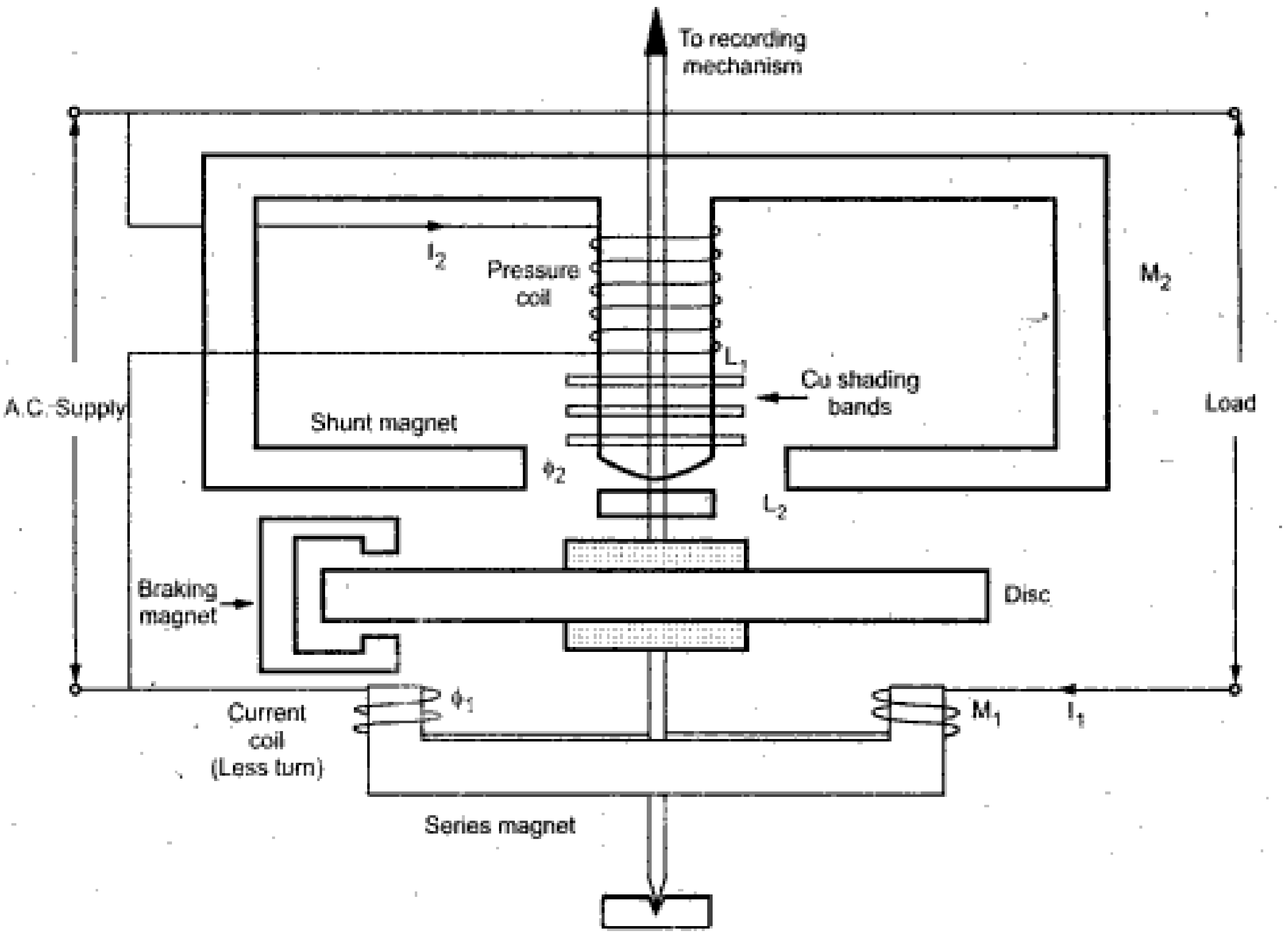


Fig. 4.1 Induction type single phase energymeter

In the energy meter there is no controlling torque and thus due to driving torque only, a continuous rotation of the disc is produced. To have constant speed of rotation braking magnet is provided.

4.2.1 Construction

There are four main parts of operating mechanism,

1) Driving system 2) Moving system 3) Braking system 4) Registering system.

1) Driving system : It consists of two electromagnets whose core is made up of silicon steel laminations. The coil of one of the electromagnets, called **current coil, is** excited by load current which produces flux further. The coil of another electromagnet is connected across the supply and it carries current proportional to supply voltage. This coil is called **pressure coil.** These two electromagnets are called **series** and **shunt** magnets respectively.

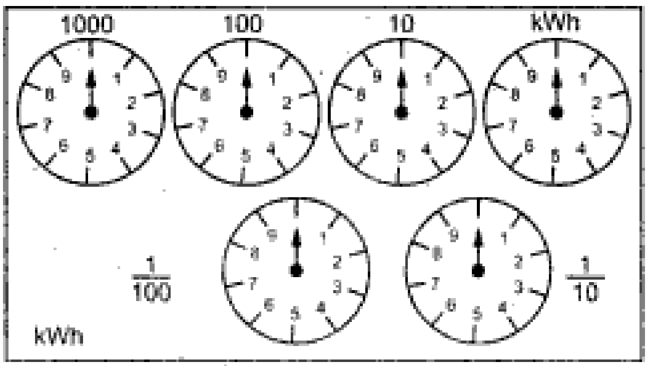
The flux produced by shunt magnet is brought in exact quadrature with supply voltage with the help of copper shading bands whose position is adjustable.

2) Moving system : Light aluminium disc mounted in a light alloy shaft is the main part of moving system. This disc is positioned in between series and shunt magnets. It is supported between jewel bearings. The moving system runs on hardened steel pivot. A pinion engages the shaft with the counting mechanism. There are no springs and no controlling torque.

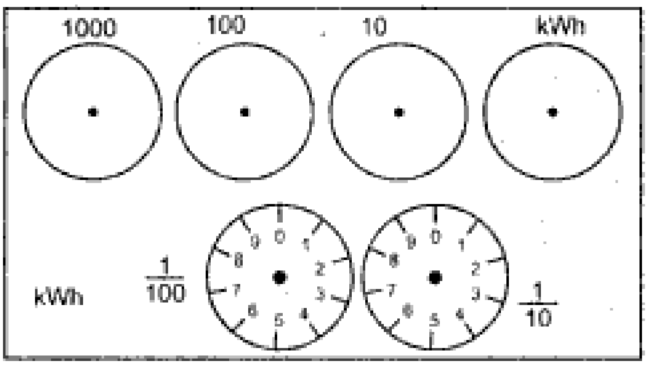
3) Braking system : A permanent magnet is placed near the aluminium disc for braking mechanism. This magnet reproduced its own field. The disc moves in the field of this magnet and a braking torque is obtained. The position of this magnet is adjustable and hence braking torque is adjusted by shifting this magnet to different radial positions. This magnet is called **Braking magnet.**

4) Registering mechanism : It records continuously a number which is proportional to the revolutions made by the aluminium disc. By a suitable system, a train of reduction gears, the pinion on the shaft drives a series of pointers. These pointers rotate on round dials which are equally marked with equal divisions.

Practically the pointer type registering mechanism is used. The pointer indicates one kWh when the disc completes certain number of revolutions. The second dial represents 10 kWh, third 100 kWh while on the other sides, dials measuring 1/100 and 1/10 kWh are also provided. The Fig. 4.2 (a) shows the pointer type register while the Fig. 4.2 (b) shows the cyclometer type register. In some meters the cyclometer type registering mechanism is used.



(a) Pointer register



(b) Cyclometer register

Fig. 4.2 Registering mechanisms used in induction energy meter

4.3 Theory of Single Phase Induction Type Energymeter

Since the pressure coil is carried by shunt magnet M_2 which is connected across the supply , it carries current proportional to the voltage. Series magnet M_1 carries current coil which carries the load current. Both these coils produce alternating fluxes ϕ_{sh} and ϕ_{se} respectively. These fluxes are proportional to currents in their coils. Parts of each of these fluxes link with the disc and induces e.m.f. in it. Due to these e.m.f.s eddy currents are induceed in the disc. The eddy current induced by the electromagnet M_2 react with magnetic field produced by M_1. Also eddy currents induced by electromagnet M_1 react with magnetic field produced by M_2. Thus each portion of the disc experiences a mechanical force and due to motor action, disc rotates. The speed of disc is controlled by the C shaped magnet called braking magnet. When disc rotates in the air gap, eddy currents are induced in disc which oppose the cause producing them i.e. relative motion of disc with respect to magnet. Hence braking torque T_b is generated. This is proportional to speed N of disc. By adjusting position of this magnet, desired speed of disc is obtained. Spindle is connected to recording mechanism through gears which record the energy supplied.

A simple functional diagram of driving mechanism is shown in the Fig. 4.3.

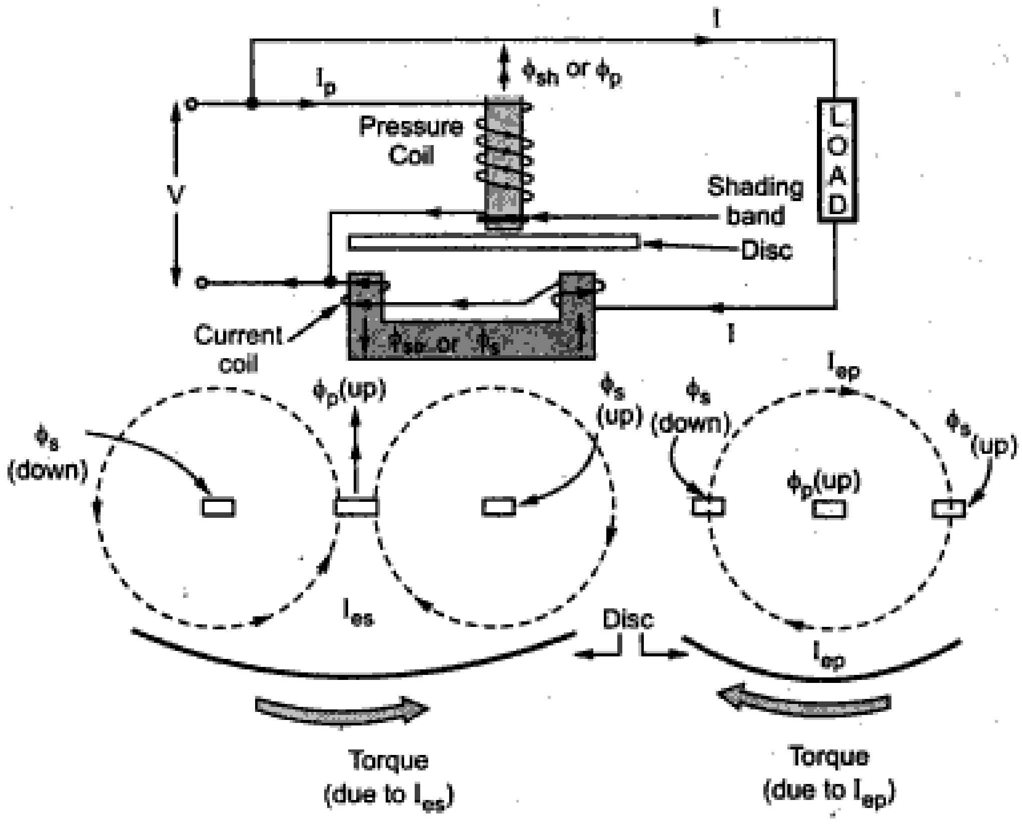


Fig. 4.3 Functional diagram of induction type energymeter

The current I_p produces the total flux ϕ_{pt} which has two components ϕ_g and ϕ_p. The major portion is ϕ_g which flows through the side gaps as the reluctance of this part is very small. While ϕ_p flows across the air gap and across the disc and is responsible to produce the eddy e.m.f. E_{ep} in the disc which produces the eddy current I_{ep} in the disc. The ϕ_p is small and is in phase with I_p. It is proportional to I_p and hence to supply voltage V as it produces I_p through pressure coil. ϕ_p lags the supply voltage V by an angle slightly less than 90°.

The current coil carries the load current I and produces the flux ϕ_s. This is proportional to I and in phase with it. This flux is responsible to induce eddy e.m.f. E_{es} in the disc which produces the eddy current I_{es} in the disc. This interacts with the flux ϕ_p to produce the torque while the eddy current I_{ep} interacts with ϕ_s to produce the torque. These two torques are opposite in direction and the net torque produced is the difference between these two torques.

4.3.1 Torque Equation

The phasor diagram is shown in the Fig. 4.4.

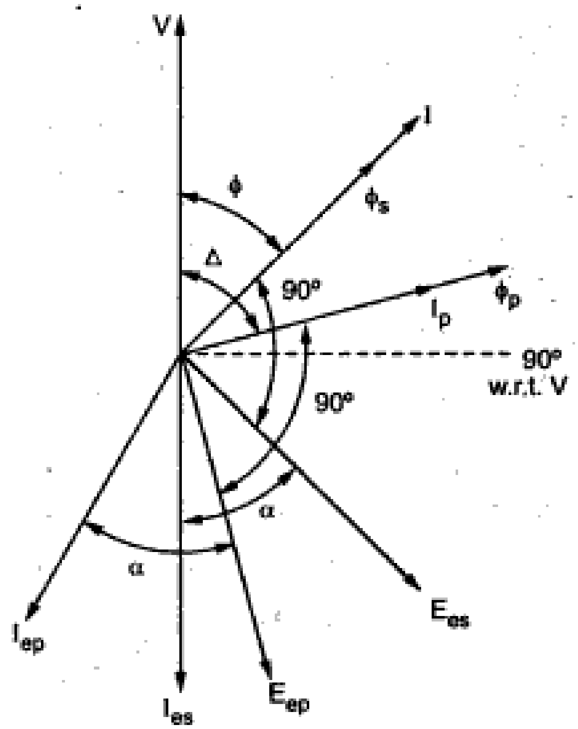


Fig. 4.4 Phasor diagram of single phase induction type energymeter

Let,

V = supply voltage

I = load current

= current coil current

I_p = pressure coil current

Δ = phase angle between V and I_p

$\Delta \approx 90°$

E_{ep} = eddy e.m.f. induced due to ϕ_p

E_{es} = eddy e.m.f. induced due to ϕ_s

α = phase angle of eddy currents

I_{ep} = eddy current due to E_{ep}

I_{es} = eddy current due to E_{es}

The current I_p lags V by Δ and Δ is made 90° using copper shading bands. The current I lags V by ϕ which depends on the load. The flux ϕ_s and I are in phase. The E_{ep} lags ϕ_p by 90° while E_{es} lags ϕ_s by 90°. The eddy currents I_{es} and I_{ep} lags E_{es} and E_{ep} respectively by angle α.

The interaction between ϕ_p and I_{es} produces torque T_1.

While the interaction between ϕ_s and I_{ep} produces torque T_2.

$$\therefore\ T_1 \propto \phi_p\, I_{es} \cos(\phi_p \wedge I_{es}) \quad \text{and} \quad T_2 \propto \phi_s\, I_{ep} \cos(\phi_s \wedge I_{ep})$$

$$\phi_p \wedge I_{es} = \alpha + \phi \quad \text{and} \quad \phi_s \wedge I_{ep} = 180 - \phi + \alpha \qquad \ldots \Delta = 90°$$

$$\therefore\ T_d \propto T_1 - T_2 \ \alpha\ \{[\phi_p\, I_{es} \cos(\alpha + \phi)] - [\phi_s\, I_{ep} \cos(180 - \phi + \alpha)]\}$$

$$\text{Now}\quad \phi_p \propto V, \quad \phi_s \propto I, \quad I_{es} \propto \phi_s \propto I, \quad I_{ep} \propto \phi_p \propto V$$

$$\therefore\ T_d \propto V\,I[\cos(\alpha + \phi) - \cos(180 - \phi + \alpha)]$$

$$\propto VI[(\cos\alpha\, \cos\phi - \sin\alpha\, \sin\phi) - (\cos(180 - \phi)\cos\alpha - \sin(180 - \phi)\sin\alpha)]$$

$$\propto VI[\cos\alpha\, \cos\phi - \sin\alpha\, \sin\phi - \cos(180 - \phi)\cos\alpha + \sin(180 - \phi)\sin\alpha]$$

$$\propto 2\,VI\cos\alpha\, \cos\phi \qquad \ldots \cos(180 - \phi) = -\cos\phi,\ \sin(180 - \phi) = \sin\phi$$

$$\therefore \quad \boxed{T_d = K_3\, VI\cos\phi} \qquad (\alpha \text{ is constant}) \qquad \ldots (1)$$

Key Point: *Thus the deflecting torque is proportional to the true power in the circuit.*

If Δ is considered,

$$\boxed{T_d \propto VI\sin(\Delta - \phi)} \qquad \ldots (2)$$

But practically Δ is achieved to be exactly 90° with the help of copper shading bands so that T_d is proportional to power in the circuit.

The braking torque is due to eddy currents induced in the aluminium disc. The magnitude of eddy currents is proportional to the speed N of the disc. Hence the braking torque T_b is also proportional to the speed N.

$$\therefore \quad \boxed{T_b \propto N} \quad \text{i.e.} \quad \boxed{T_b = K_4 N} \qquad \text{... (3)}$$

For the steady speed of rotation, $T_d = T_b$.

$$\therefore \quad K_3 \, VI \cos\phi = K_4 N$$

$$\therefore \quad \boxed{N = K \, VI \cos\phi = K[\text{power}]} \qquad \text{... (4)}$$

$$\text{Total number of revolutions} = \int_0^t N \, dt = \int_0^t K \, (\text{power}) \, dt$$

$$\therefore \quad \boxed{\text{Total number of revolutions} = K \int_0^t P \, dt = K \times \text{energy}} \qquad \text{... (5)}$$

Thus the number of revolutions of the disc in a given time is the energy consumption by the circuit in that time.

$$\boxed{K = \text{Meter constant} = \frac{N}{\text{energy}} = \frac{\text{Number of revolutions}}{\text{kWh}}} \qquad \text{... (6)}$$

Thus the number of revolutions of the disc per kWh of energy consumption is called the **meter constant**.

4.4 Errors and Compensations

There are various errors present in the single phase induction type energymeter. The driving system can cause the errors due to inaccurate phase angles, abnormal frequencies, effect of temperature on the resistance and unsymmetrical magnetic circuit. The braking system also can cause error due to change in the strength of the braking magnet, change in resistance of the disc, abnormal friction of moving disc etc. To get accurate reading, these errors are required to be compensated. Hence some adjustments are provided in the energymeter to minimize these errors.

4.4.1 Lag Adjustment or Power Factor Adjustment

It is absolutely necessary that meter should measure correctly for all power factor conditions of the loads. This is possible when the flux produced due to current in the pressure coil lags the applied voltage by 90°. But the iron loss and resistance of winding do not allow the flux to lag by exact 90° with respect to the voltage.

The arrangement used to adjust this angle to be 90° is called lag adjustment. A magnetic shunt circuit is introduced in the device which allows the main portion of the shunt magnet flux to bypass the gap in which the disc rotates. It is possible to produce an m.m.f. in the proper phase relation to the shunt magnet flux to bring ϕ_{sh} i.e. ϕ_p in exact quadrature with the voltage. This is shown in the Fig. 4.5. The lag coil is used in addition to shunt coil. The lag coil is few turns of fairly thick wire placed around the central limb of the shunt magnet. The part of shunt flux i.e. ϕ_p links with the lag coil to induce an e.m.f. in it. This produces the lag coil current I_L. This current produces a m.m.f. AT_L in phase with I_L. Thus now the phase of ϕ_p is decided by the combined m.m.f. of lag coil and shunt coil. This can be adjusted by adjusting the resistance connected across the lag coil. When resistance increases, current and m.m.f. of lag coil decreases which decreases the value of the lag angle of coil hence ϕ_{sh} lags behind the voltage by exactly 90°.

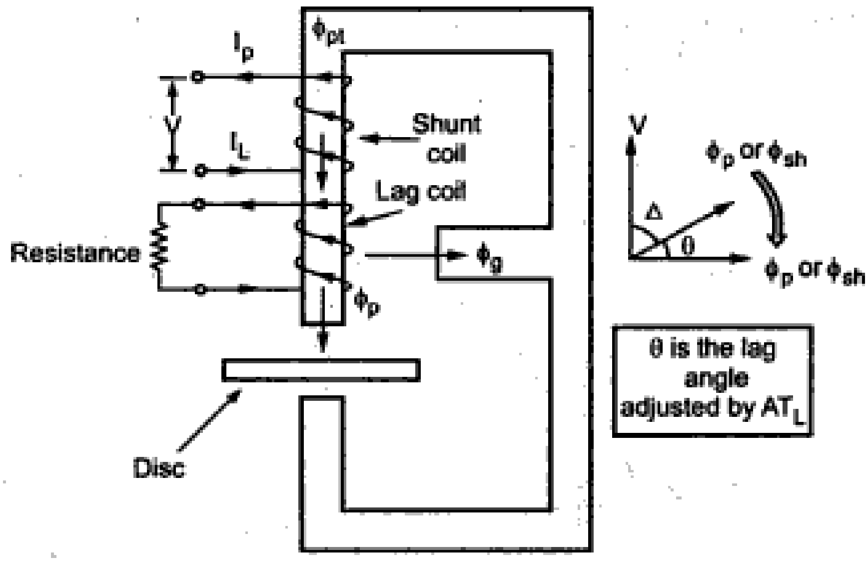


Fig. 4.5 Lag adjustment

Instead of lag coil and resistance, many time copper shading bands are placed on the central limb of the shunt magnet. These bands are adjustable. By moving these bands along the axis of the central limb, the lag adjustment can be achieved. When bands are moved upwards, the e.m.f. induced in them increases increasing the m.m.f. produced, hence lag angle increases. When bands are moved down, the m.m.f. produced by the bands decreases which decreases the lag angle. Thus the ϕ_{sh} can be brought in exact quadrature with the voltage V.

This adjustment is also called **power factor adjustment, quadrature adjustment** or **inductive load adjustment.**

4.4.2 Light Load Adjustment or Friction Adjustment

Inspite of proper design of the bearings and registering mechanism, there is bound to exist some friction. Due to this, speed of the meter gets affected which cause the error in the measurement of the energy.

To compensate for this, a metallic loop or strip is provided between central limb of shunt magnet and the disc. Due to this strip an additional troque independent of load is produced which acts on the disc in the direction of rotation. This compensates for the friction and meter can be made to read accurately. This is shown as L_2 in the Fig. 4.6.

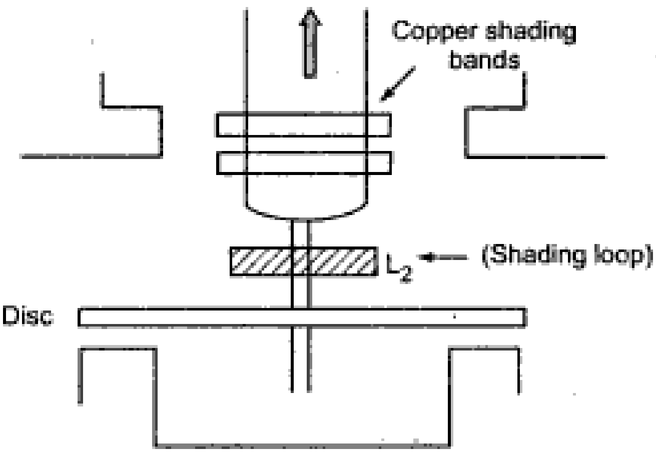


Fig. 4.6 Shading loop for friction adjustment

The shading loop L_2 is also called light load plate.

The interaction between the portions of the flux which are shaded and unshaded by this loop and the currents they induce in the disc generates a small driving torque whose value can be adjusted by lateral movement of the loop L_2. This additional driving torque overcomes the frictional error. This torque is practically independent of the load and depends on line voltage hence remains constant. The friction error is dominant at rated voltage and very low current i.e. at light loads. The shading loop can be moved laterally to adjust the speed to provide necessary compensation.

4.4.3 Creeping Adjustment

In some meters, the disc rotates slowly and continuously when there is no load. The rotation of disc without any current through current coil and only due to excitation of press ıre coil is called **creeping.** This is due to friction overcompensation. The torque produce‹ due to light load adjustment may keep disc rotating. To prevent creeping, two holes are drilled in the disc, 180° opposite to each other. When the hole comes under the shunt magnet pole, it gets acted upon by a torque opposite to its rotation. This is shown in the Fig. 4.7.

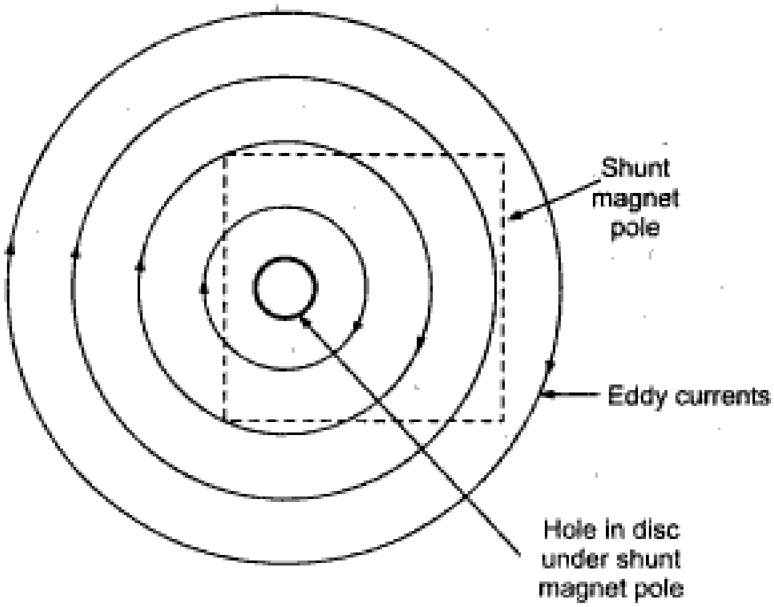


Fig. 4.7 Creeping adjustment

When a hole comes under the shunt magnet, the circular eddy current paths in the disc get distorted. This distortion is responsible to produce torque in opposite direction to the rotation of the disc. This stops creeping. The torque is not very large so as to cause errors under normal operating conditions.

In some cases, a small piece of iron is attached to the edge of the disc. The force of attraction of the braking magnet on the iron piece is responsible to prevent rotation of disc on no loads.

4.4.4 Overload Compensation

When the disc rotates in the field of series magnetic field under load conditions, it cuts the series flux and dynamically, e.m.f. is induced in the disc. This produces eddy currents in the disc which interacts with series magnet flux to produce braking torque. This is proportional to square of the current. This is called self braking torque and at large loads its value is very high, causing serious errors in the measurement. To minimize this braking torque, the full load speed of the disc is kept very low about 40 r.p.m. The current coil series flux is kept minimum compared to shunt magnet flux.

Practically an overload compensating device in the form of a saturable magnetic shunt for the series magnet core is used. This is shown in the Fig. 4.8.

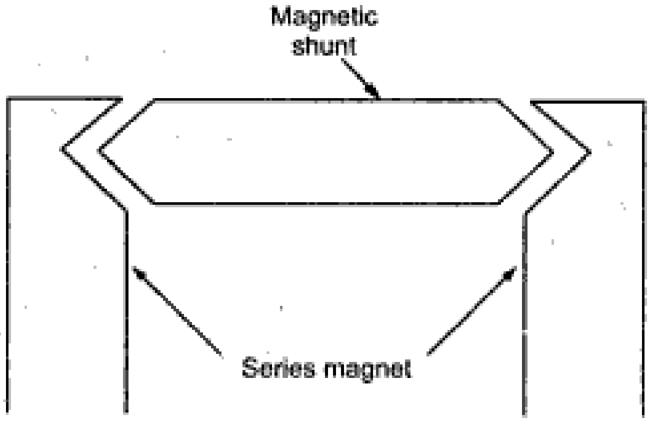


Fig. 4.8 Overload compensation

At high loads, magnetic shunt saturates and diverts some of the series magnetic flux. This compensates for the self braking torque.

4.4.5 Voltage Compensation

When supply voltage varies, the energymeter can cause errors. This is because of two reasons,

i) Nonlinear magnetic characteristics of shunt magnet core.

ii) The braking torque which is proportional to square of the supply voltage.

The voltage compensation is provided by the saturable magnetic shunt which diverts a large proportion of the flux into the active path when the supply voltage increases. The compensation can be provided by increasing the side limb reluctance, by providing holes in the side limbs.

4.4.6 Temperature Compensation

As temperature increases, the resistance of the copper and aluminium parts increases. This has following effects,

i) Small reduction in shunt magnet flux.

ii) Reduction in angle of lag between V and ϕ_p.

iii) Reduction in torque produced by all shading bands.

iv) Increase in eddy current resistance path.

v) Decrease in angle of lag α of the eddy currents.

These various effects neutralize each other and hence errors due to temperature are not serious. But at low lagging power factor loads, such effects may cause serious errors. These effects are compensated by providing a temperature shunt on the brake magnet. Special magnetic materials such as Mutemp is used for the shunt whose permeability decreases considerably as temperature increases. This provides temperature compensation and does not allow the disc to rotate faster as temperature increases.

4.4.7 Main Speed Adjustment

The measurement of energy is dependent on the speed of the rotating disc. For accurate measurement, speed of the disc must be also proportionate. The speed of the meter can be adjusted by means of changing the effective radius of the braking magnet. Moving the braking magnet in the direction of the spindle, decreases the value of the effective radius, decreasing the braking torque. This increases the speed of the meter. While the movement of the braking magnet in the outward direction i.e. away from the centre of the disc, increases the radius, decreasing the speed of the disc. The fine adjustments of the speed can be achieved by providing an additional flux divertor.

4.5 Advantages of Induction Type Energymeter

The various advantages of induction type energymeters are,

1. Its construction is simple and strong.
2. It is cheap in cost.
3. It has high torque to weight ratio, so frictional errors are less and we can get accurate reading.
4. It has more accuracy.
5. It requires less maintenance.
6. Its range can be extended with the help of instrument transformers.

4.6 Disadvantages of Induction Type Energymeter

1. The main disadvantage is that it can be used only for a.c. circuits.
2. The creeping can cause errors.
3. Lack of symmetry in magnetic circuit may cause errors.

4.7 Three Phase Energymeter

In a three phase, four wire system, the measurement of energy is to be carried out by a three phase energy meter. For three phase, three wire system, the energy measurement can be carried out by two element energy meter, the connections of which are similar to the connections of two wattmeters for power measurement in a three phase, three wire system. So these meters are classified as i) three element energymeter and ii) two element energymeter.

4.7.1 Three Element Energymeter

This meter consists of three elements. The construction of an individual element is similar to that of a single phase energymeter. The pressure coils are denoted as P_1, P_2 and P_3. The current coils are denoted as C_1, C_2 and C_3. All the elements are mounted in a vertical line in common case and have a common spindle, gearing and registering mechanism. The coils are connected in such a manner that the net torque produced is sum of the torques due to all the three elements. These are employed for three phase, four wire system where fourth wire is a neutral wire.

The current coils are connected in series with the lines while pressure coils are connected across a line and a neutral. Fig. 4.9 shows a three phase energymeter.

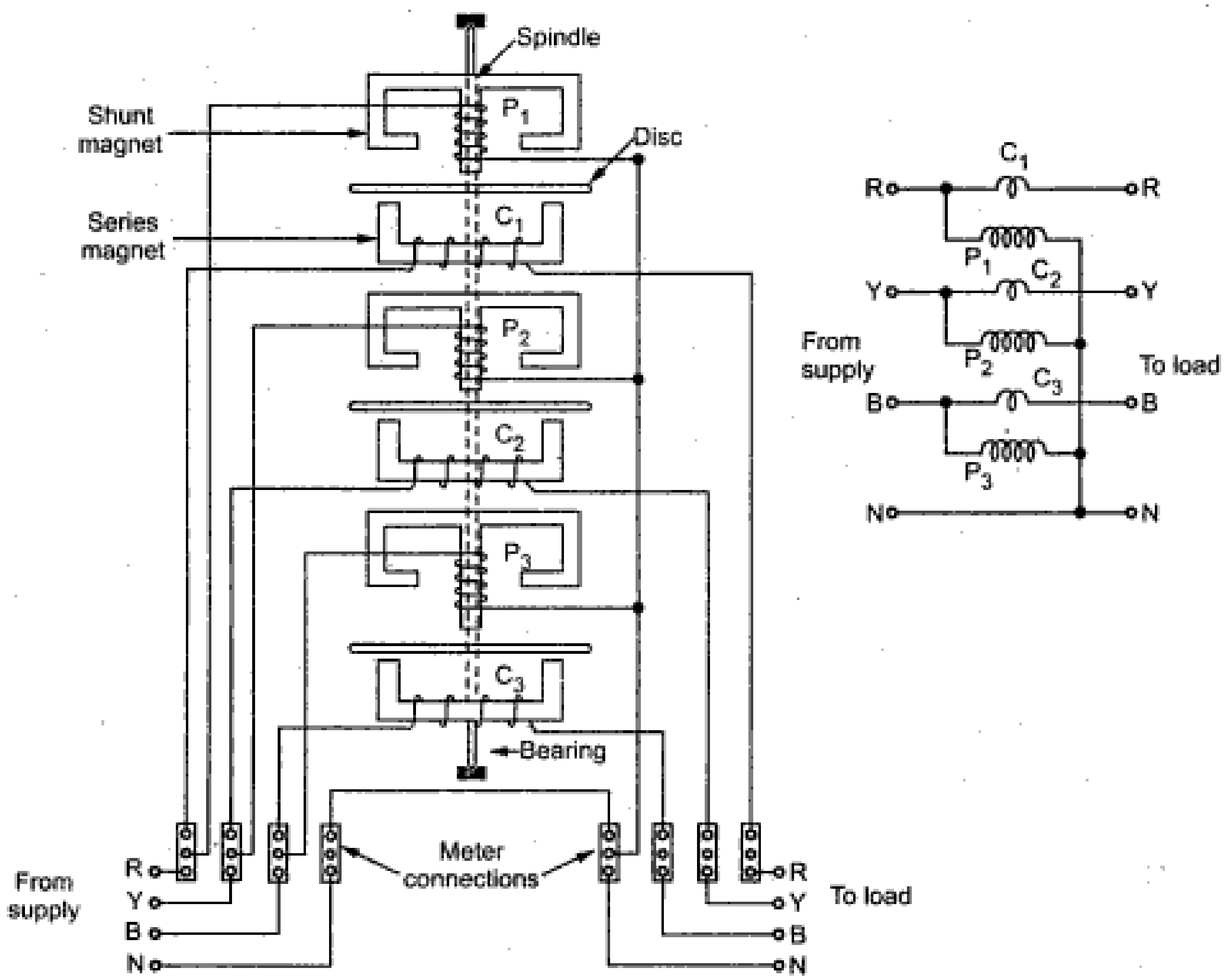


Fig. 4.9 Three element energymeter

One unit of three element, three phase element is always cheaper than three units of single phase energymeter. But due to interaction between eddy currents produced by one element with the flux produced by another element, there may be errors in the measurement by three phase energymeter. Such errors may be reduced by suitable adjustments.

4.7.2 Two Element Energymeter

The Fig. 4.10 shows a two element energymeter and a simplified connection diagram.

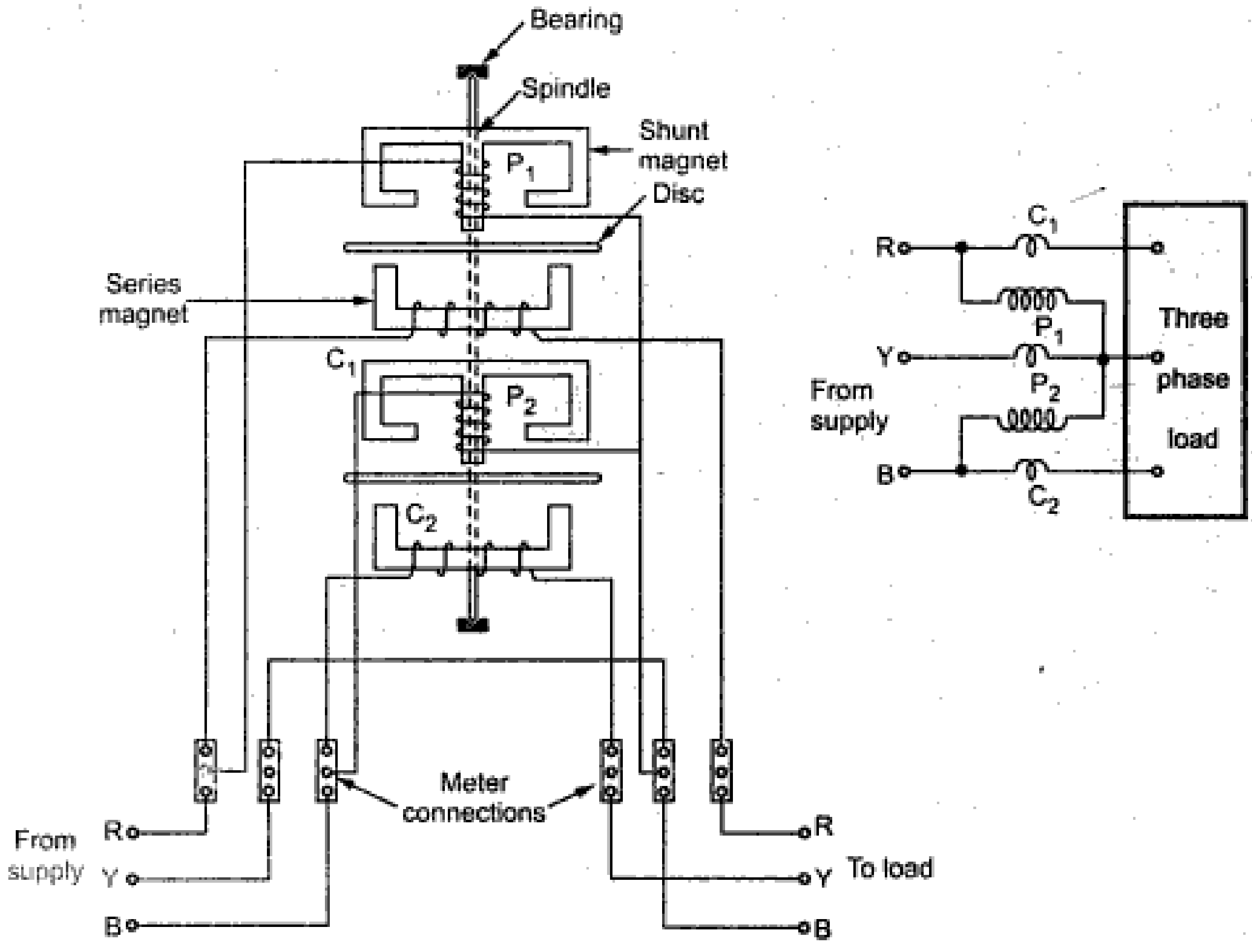


Fig. 4.10 Two element energymeter

This energymeter is used for three phase, three wire systems. The meter is provided with two discs each for an element. The shunt magnet is carrying pressure coil while a series magnet carries a current coil. The pressure coils are connected in parallel and the current coils in series. The connections are similar to the connections of two wattmeters for power measurement in three phase, three wire system. Torque is produced in same manner as in a single phase energymeter, in each element. The total torque on the registering mechanism connected to moving system, is sum of the torques of the individual elements.

4.8 Calibration of an Energymeter

Calibrating the energymeter means to find out the error in the measurement of energy by energymeter.

Every energymeter has its own characteristic constant specified by the manufacturer which relates the energy measured in joules and the number of revolutions of the disc. For example say 'x' revolutions corresponds to the measurement of 'y' joules. But practically the value of 'x' is very large and can not be measured in the laboratory. Hence using this constant, energy recorded for certain less number of revolutions say 5, is calculated in the laboratory for the calibration purpose. This energy is denoted as E_r. Thus E_r can be caculated from 'x' as,

$$E_r = \frac{5\,x}{y} \quad \text{joules}$$

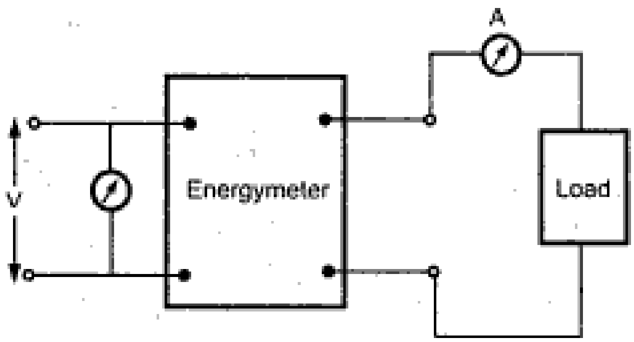


Fig. 4.11

To have zero error, the actual energy consumed by the load for the time corresponding to the 5 revolutions must be same as E_r. This energy is called actual energy consumed or the true energy denoted as E_t. Experimental set up used in the laboratory to obtain the value of E_t is shown in the Fig. 4.11.

For various loads, the time required to complete the 5 revolutions of disc is measured with the help of stop watch. The voltage and current readings are observed on the ammeter and voltmeter connected in the circuit. The readings can be tabulated as :

Sr. No.	Voltage (V)	Current (A)	Time for revolutions	True energy E_t = VI t J
1				
2				
3				

Now E_r is fixed for the 5 revolutions, while E_t is obtained practically. Hence error for each load condition can be obtained as,

$$\%\ \text{error} = \frac{E_r - E_t}{E_t} \times 100$$

The graph of % error against the load current I can be obtained, which is called **calibration curve** for the energymeter. When there is no load, I = 0 and hence true energy E_t is also zero. While E_r is also zero. Hence the error is also zero. Thus calibration curve passes through origin. The errors can be positive or negative. Such a curve is shown in the Fig. 4.12.

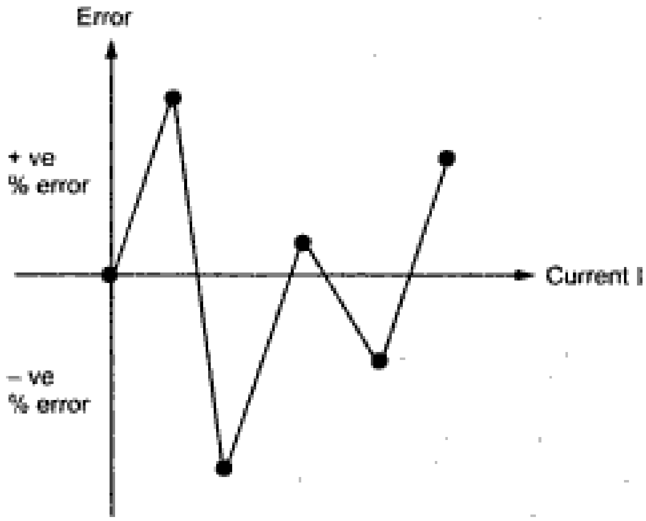


Fig. 4.12

Once the calibration curve is obtained, by observation of the curve, in which range of the load current error is severe, can be easily predicted. And if error is not within the permissible limits then by using the various adjustments discussed earlier, the error can be minimised.

➠ **Example 4.1 :** *An energymeter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 20 A at 230 V at 0.8 p.f. for an hour. If it actually makes 360 revolutions, find the percentage error.* **[JNTU, May-2005, Set-1, Set-3, Nov.-2003, Set-2]**

Solution : $I = 20$ A, $V = 230$ V, $\cos\phi = 0.8$, $t = 1$ hour $= 3600$ sec

$$K = \text{meter constant} = 100 \text{ rev/kWh}$$

$$E_t = \text{true energy} = VI\cos\phi \times t = 230 \times 20 \times 0.8 \times 3600$$

$$= 13.248 \times 10^6 \text{ J i.e. watt-sec} \qquad \text{... } 1 \text{ W} = 1 \text{ J/s}$$

$$= \frac{13.248 \times 10^6}{3600 \times 10^3} \text{ kWh} = 3.68 \text{ kWh}$$

Number of revolutions = 360

$$\therefore \qquad E_r = \frac{360}{100} = 3.6 \text{ kWh}$$

$$\therefore \qquad \% \text{ error} = \frac{E_r - E_t}{E_t} \times 100 = \frac{3.6 - 3.68}{3.68} \times 100$$

$$= -2.174\%$$

Negative sign indicates that E_r is less than E_t and the meter is **slow.**

4.9 Use of C.T. and P.T. in Energy Measurement

The single phase energymeter connections are exactly similar to the connections of a wattmeter along with C.T. and P.T. for power measurement as shown in Fig. 3.60. The pressure coil of wattmeter is replaced by pressure coil of energymeter and current coil of wattmeter is replaced by current coil of energymeter.

The Fig. 4.13 shows a two element energymeter with C.T. and P.T.

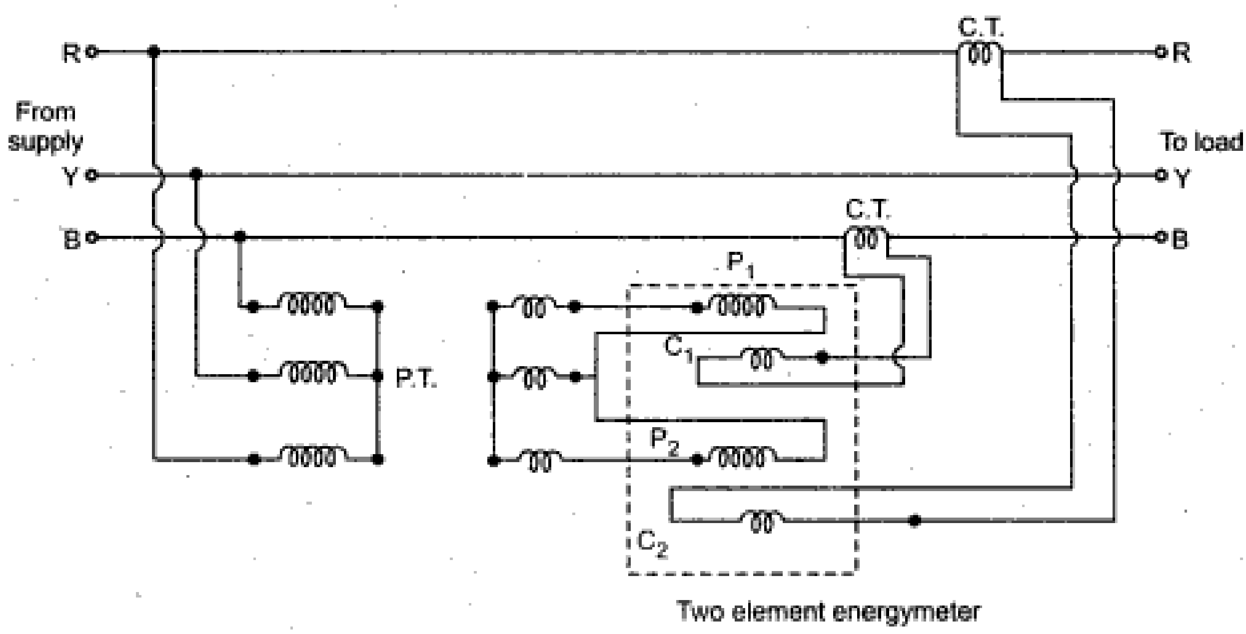


Fig. 4.13 Two element energymeter with C.T. and P.T.

The connections with three element, three phase energymeter are shown in the Fig. 4.14.

But three phase energymeter connections along with C.T. and P.T. are little bit different than single phase but the basic principle of extending the ranges by using C.T. and P.T. remains the same.

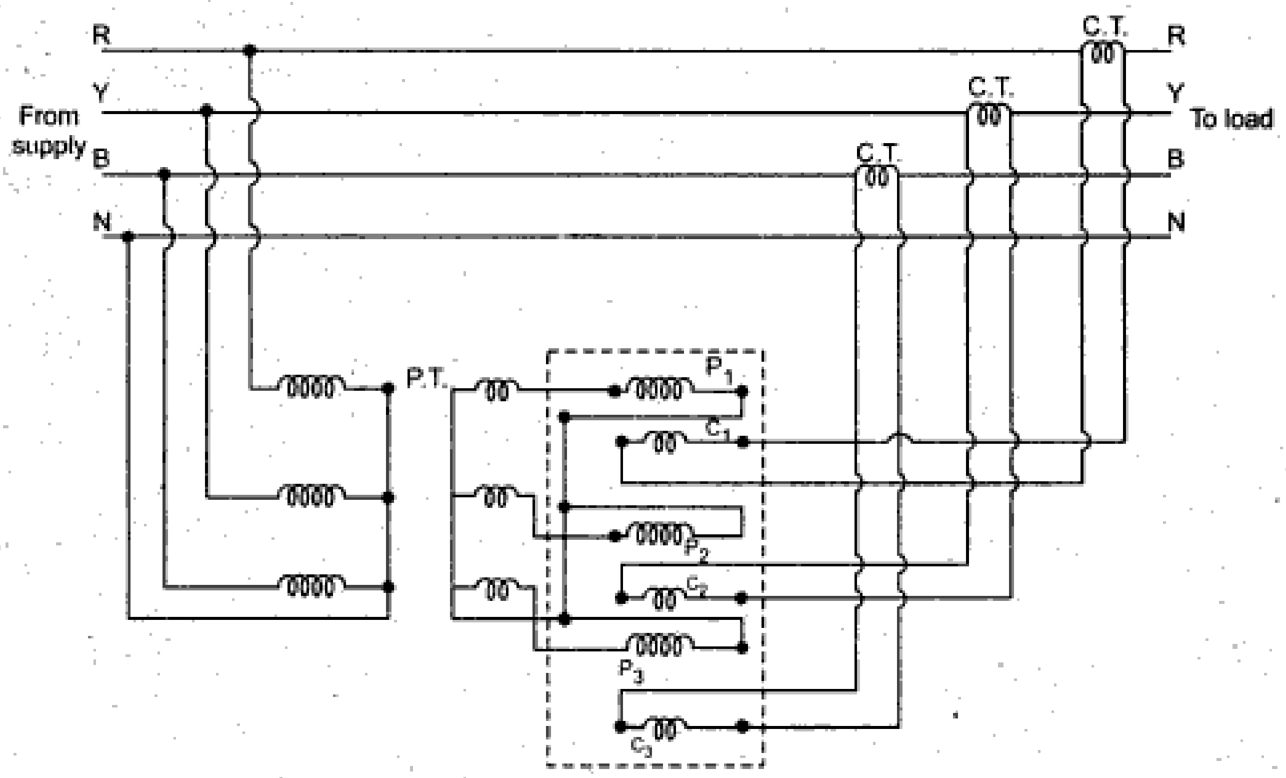


Fig. 4.14 Three element energymeter with C.T. and P.T.

4.10 Introduction to Tariff

A large number of consumers are fed by the power stations in order to meet their energy requirements. The demand of power is not same but it goes on varying from time to time. This results in variable load on the power station. The electrical supply company sells this energy to the consumers at the reasonable rate. Thus the rate at which energy is sold i.e. tariff is an important factor while studying the economic aspects of electric supply. The rules or rates which are framed for supply of electricity to various consumers is nothing but tariff. It can alternatively defined as the rate at which electrical energy is supplied to the consumers.

The tariff should be framed in such a way that it should recover the total cost of producing electrical energy and also it should provide marginal profit on the capital investment.

Following points must be considered.

i) Whether the consumer is able to pay it or not.

ii) It should be simple in calculations.

iii) Services rendered.

iv) The annual cost of production i.e. total of running and fixed charges.

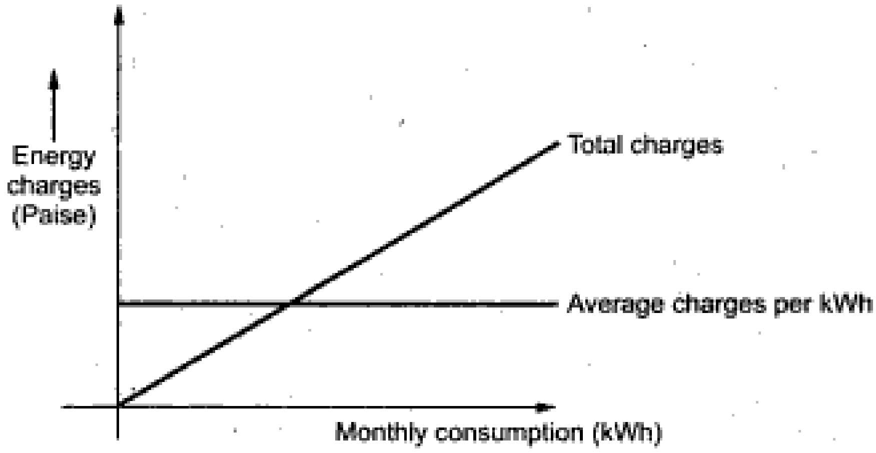


Fig. 4.15

In this type of tariff, there is no differentiation between various types of consumers. All the consumers are sharing equal burden of capital investment. The price charged per unit is constant i.e. independent of number of units consumed.

Apart from the simplicity following are the limitations of this type of tariff.

i) The cost per unit delivered is high.

ii) There is no discrimination between different types of consumers viz domestic and industrial consumers. The maximum demand on the station is mostly decided by these industrial or bulk consumers so the domestic consumers unnecessarily have to pay higher.

In order to give proper justice to different types of consumers, following differences must be observed carefully,

i) Bulk consumer and domestic consumer :

Based on the number of units consumed these are the two types of consumers. Because of cheapness in generating large number of units and supplying it, the bulk consumer may pay at lower cost than domestic consumer but the bulk consumer may have high maximum demand which may increase the capital cost of station so proper compromise must be made between the two facts.

ii) Time characteristics of energy consumption :

There are certain consumers who use electrical energy during the time when the station is supplying peak load i.e. such consumers try to improve the load factor and diversity of the station and hence the economy of the station. The tariff should take into consideration this fact.

iii) Difference in consumers p.f. :

The consumers whose appliances are running at very low p.f. try to shatter the economy of the station and such consumers must be penalise while fixing the tariff.

iv) Difference in system of supply :

The tariff for the consumers who are using HT supply should be smaller compared to consumers using LT supply.

The above considerations lead to give the best possible tariff but it may be complicated for ordinary consumer to follow so following points are applied to simple tariff to modify it

i) Depending on the quantity of energy consumed, a discount must be offered.

ii) Special types of consumers with special types of loads should be offered special tariffs.

iii) A special discount must be offered to encourage the consumers having their demand during off peak hours.

4.12.2 Flat Rate Tariff

When various types of consumers are charged at different rates then the tariff is called flat rate tariff. In this type, the consumers are grouped into various classes and each class is charged at different rate. The rate for various types of consumers is obtained by taking into consideration its load and diversity factor.

Such type of tariff may be fair to different types of consumers. It has also advantage that this type of tariff is simple and easily understood by the consumers. The disadvantages of this type of tariff are as follows

i) It varies depending on the way in which supply is used. Seperate meters are used for various types of loads. This makes the tariff expensive and complicated.

ii) A particular class of consumers is charged at the same rate irrespective of the magnitude of energy consumed but a big consumer should be charged at a lower rate.

iii) It is difficult to derive the load and diversity factor used in fixing tariff.

4.12.3 Block Rate Tariff

In this type of tariff, a given block of energy is charged at a specified rate and the succeeding blocks of energy are charged at progressively reduced rate. The main consideration here is that as the number of units generated increase, the cost of generation per unit decreases. Therefore the consumer having large demand in terms of number of units have to pay less as compared to the consumers with lower demand. The energy consumption is divided into blocks and each block is charged with fixed rate.

The advantage with this type of tariff is that the user gets incentive to consume more energy which increase load factor of the system and cost of generation is reduced. The disadvantage is that it does not take into consideration consumer's demand.

It is applicable to majority of residential and small commercial consumers.

4.12.4 Two Part Tariff

In two part tariff, the rate of electricity is based on the maximum demand of the consumer and the units consumed.

The total charge is split into two components the fixed charges and running charges. The fixed charges depend on the maximum demand of the consumer while the running charges depend upon the number of units consumed by the consumer.

$$\text{Total charge} = \text{Rs}\,(y \times kW + z \times kWh)$$

where y = charge per kW of maximum demand

z = charge per kWh of maximum demand

The industrial consumers with appreciable maximum demand is charged with this type of tariff.

Advantages of this tariff are,

i) It is simple and can be easily understood by the consumers.

ii) The fixed charges which depend on the maximum demand but independent of units consumed are recovered.

Disadvantages :

i) Irrespective of whether the consumer has consumed the energy or not, he has to pay the fixed charges.

ii) Error may occur in calculating maximum demand of the consumer.

4.12.5 Three Part Tariff

In this type of tariff the total charge to be made from the consumer is split into three parts namely fixed, semi-fixed and running charges.

$$\text{Total charge} = \text{Rs}\left(x + y \times kW + z \times kWh\right)$$

where x = Fixed charge which has to be paid which covers the interests depreciations and labour cost of collecting revenues.

y = Charge per kW of maximum demand

z = Charge per kWh of energy consumed

In the two part tariff if we add the fixed charges then it gives three part tariff which is applicable to big consumers.

4.12.6 Maximum Demand Tariff

It is similar to a two part tariff except with the difference that in this type of tariff, the maximum demand of the consumer is actually measured by a maximum demand indicator installed at consumers premises. In two part tariff the maximum demand is assessed nearly on the basis of the rateable value. This type of tariff is applicable to big consumers but not suitable for a residential consumer as a separate maximum demand indicator is required.

4.12.7 Power Factor Tariff

By taking into consideration the p.f. of the consumer's load if the tariff is fixed then it is called power factor tariff.

The power factor is vital in case of ac systems. The low p.f. leads to large kVA rating of equipment required, greater conductor size required, larges losses and poor voltage regulation. Thus penalty is to be taken from the consumers having their equipments running at very low factor. Various types of power factor tariffs are as follows.

i) Sliding scale or average p.f. tariff :

In this case average p.f. say e.g. 0.8 is taken as reference. If the p.f. of the consumer falls below this reference factor then suitable additional charges are taken from that consumer on the contarary, if the p.f. is above the reference, special discount is offered to the consumer.

ii) kWh and kVAR tariff :

In this type of tariff both active (kWh) and reactive power (kVAR) are charged separately. If p.f. is low then it takes higher kVAR and correspondingly it is required to pay higher charges. For low kVARh of a consumer, it is required to pay less. Thus the consumer will try to improve power factor.

iii) kVA maximum demand tariff :

It is improved form of two part tariff. The fixed charges are made on the basis of maximum demand in kVA and not in kW.

As kVA is inversely proportional to power factor therefore a consumer having low p.f. has to pay more fixed charges. This forces the consumers to operate their equipments at improved power factor.

Key Point : *The block rate tariff is used for LT consumers while three part tariff, maximum demand tariff and power factor tariff are used for HT consumers.*

4.13 Merz Price Maximum Demand Indicator

The meters used to record the maximum power consumed by the consumer during a particular period are called **maximum demand indicators.**

The merz price maximum demand indicator is available as a unit together with an energymeter. The energymeter records total energy consumption while the maximum demand indicator indicates the maximum value of the average power over equal intervals of time.

4.13.1 Construction

It consists of a special disc mechanism which drives the pointer through gearing arrangement, which is coupled to the energymeter spindle. The dial system is coupled to the energymeter spindle for a fixed interval of time. Generally this time interval is of 30 minutes duration. After the end of this interval, reset device resets the driving mechanism bringing to zero position. But the pointer is held by special friction device which indicates the energy consumed during that interval of time.

This pointer position remains fixed unless and until in next intervals of time, the energy consumed exceeds the one indicated by pointer. Thus pointer then indicates new energy consumption which is more than the previous. Thus inall, the pointer indicates the maximum demand expressed in energy consumed per half hour, for any given period of time.

The Fig. 4.16 shows the merz price maximum demand indicator.

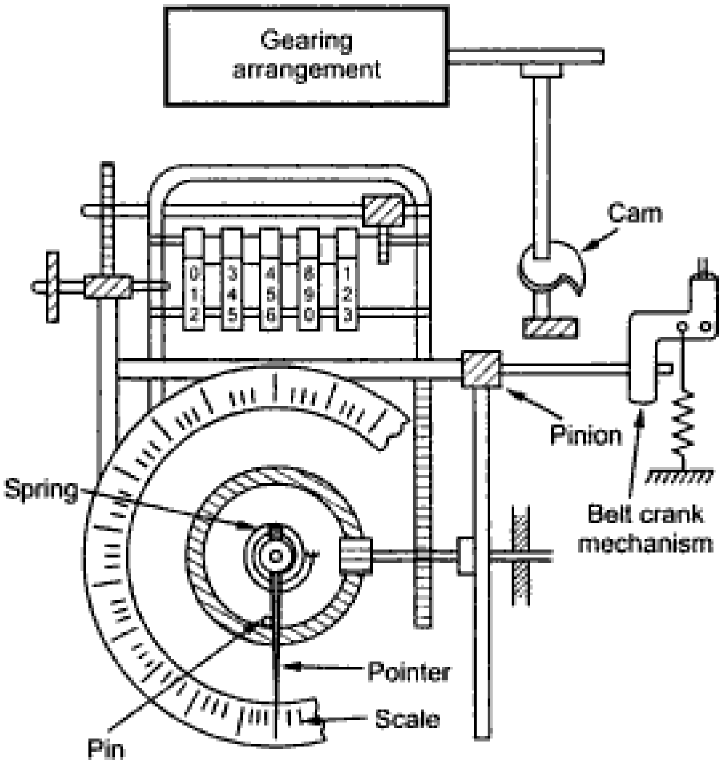


Fig. 4.16 Merz price maximum demand indicator

The pin provided drives the pointer forward for the set period of time interval. When the period ends, the cam controlled by a timing gear momentarily disengages the pinion with the help of bell crank mechanism. Under the spring force, the driving mechanism and pin comes to zero position. The pointer remains at its position indicating the energy consumed during the past interval of time.

In some cases reset of the driving mechanism is achieved using a switch actuated by small synchronous motor of the electric clock type.

The average maximum demand can be obtained as,

$$\begin{bmatrix}\text{Average maximum} \\ \text{demand in kW}\end{bmatrix} = \frac{\text{Maximum energy recorded over a time interval in kWh}}{\text{time interval in hours}}$$

4.13.2 Advantages

The basic advantages of this type of meter are,

1. Accurate maximum demand is measured.
2. The scale is uniform.

4.13.3 Disadvantages

The various disadvantages of this type of meter are,

1. Very costly due to complicated gearing mechanism.
2. If the maximum demand occurs in a time interval and continues over only a small part of the next interval then it can not be measured accurately.

4.14 Phantom Loading

The phantom loading is also called **ficticious loading**. It is the method of testing an energy meter. When the capacity of the meter to be tested is very high then tremendous loss of power occurs due to ordinary loading. Hence high capacity ficticious loads are used to test such meters, to avoid wastage of power.

In this method, pressure coil is excited by a normal supply voltage while the current coil is excited by a small battery voltage connected across it. As impedance of current coil circuit is small, small voltage is enough to circulate the rated current through the current coil. Then the total power supplied for the test is sum of power supplied to small pressure coil current at normal voltage and due to rated current at very low voltage. Thus the overall power loss during the test is very small.

The arrangement for the phantom loading is shown in the Fig. 4.17

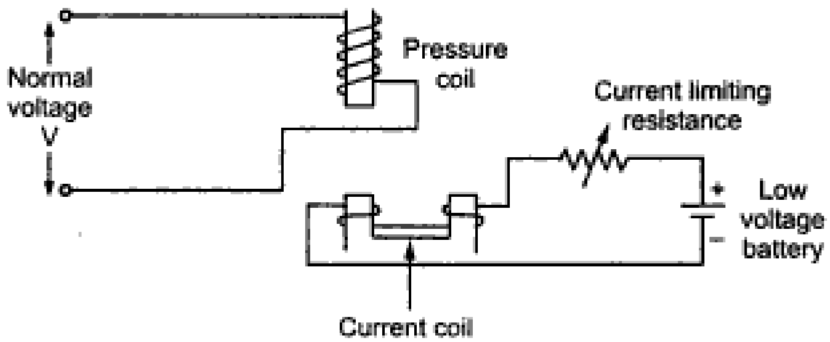


Fig. 4.17 Phantom loading of energymeter

➡ **Example 4.2 :** *A 220 V, 5 A d.c. energymeter is tested at its marked ratings. The resistance of the pressure circuit is 8800 Ω and that of current coil is 0.1 Ω. Calculate the power consumed when testing the meter with phantom loading with current circuit excited by a 6 V battery.* **[JNTU, May-2004, Set-2]**

Solution : When the loading is direct, the arrangement is shown in the Fig. 4.18 (a).

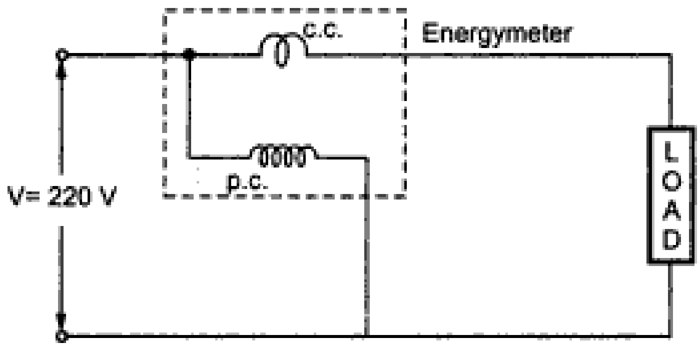


Fig. 4.18 (a)

$R_p = 8800\ \Omega,\quad R_c = 0.1\ \Omega$

$\therefore \quad P_p$ = Power consumed in pressure coil

$$= \frac{V^2}{R_p} = \frac{(220)^2}{8800} = 5.5\ \text{W}$$

P_c = Power consumed in current coil

$$= V \times I_c = 220 \times 5 = 1100\ \text{W}$$

$\therefore \quad P_t = P_p + P_c =$ **1105.5 W** ... Direct loading

Under the phantom loading, the arrangement is shown in the Fig. 4.18 (b).

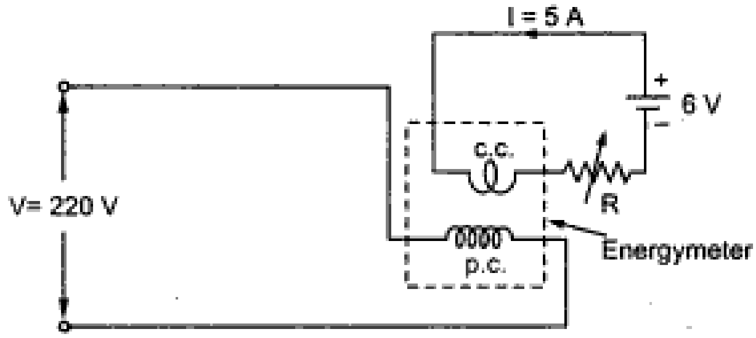


Fig. 4.18 (b)

The power consumed in pressure coil remains same as before.

$\therefore \quad P_p = 5.5 \text{ W}$

Now the load is not present but 5 A is adjusted using battery of 6 V against current coil resistance $R_c = 0.1\Omega$.

$\therefore \quad P_c = VI = 6 \times 5 = 30 \text{ W}$

$\therefore \quad P_t = P_p + P_c = 5.5 + 30 = \mathbf{35.5 \text{ W}}$ ... Phantom loading

Thus the power consumption due to the phantom loading is considerably less than the direct loading of the energymeter.

4.15 Testing of Meter using Rotating Substandard Meter

This test is conducted for short period of time hence called **short period test**. A rotating substandard meter is used alongwith the meter under test. The current coils of two meters are connected in series while pressure coils in parallel. The two meters are started and stopped simultaneously for short period of time.

When the predetermined load is adjusted, then the meter under test is allowed to make certain number of revolutions. At the same time, the number of revolutions made by rotating substandard meter, in the same time are observed.

If the constants of meters are same then error can be directly obtained. But if meter constants are different then error is required to be calculated.

Let K_x = Meter constant in number of revolutions per kWh for meter under test

K_s = Meter constant in number of revolutions per kWh for substandard meter

N_x = Number of revolutions made by meter under test

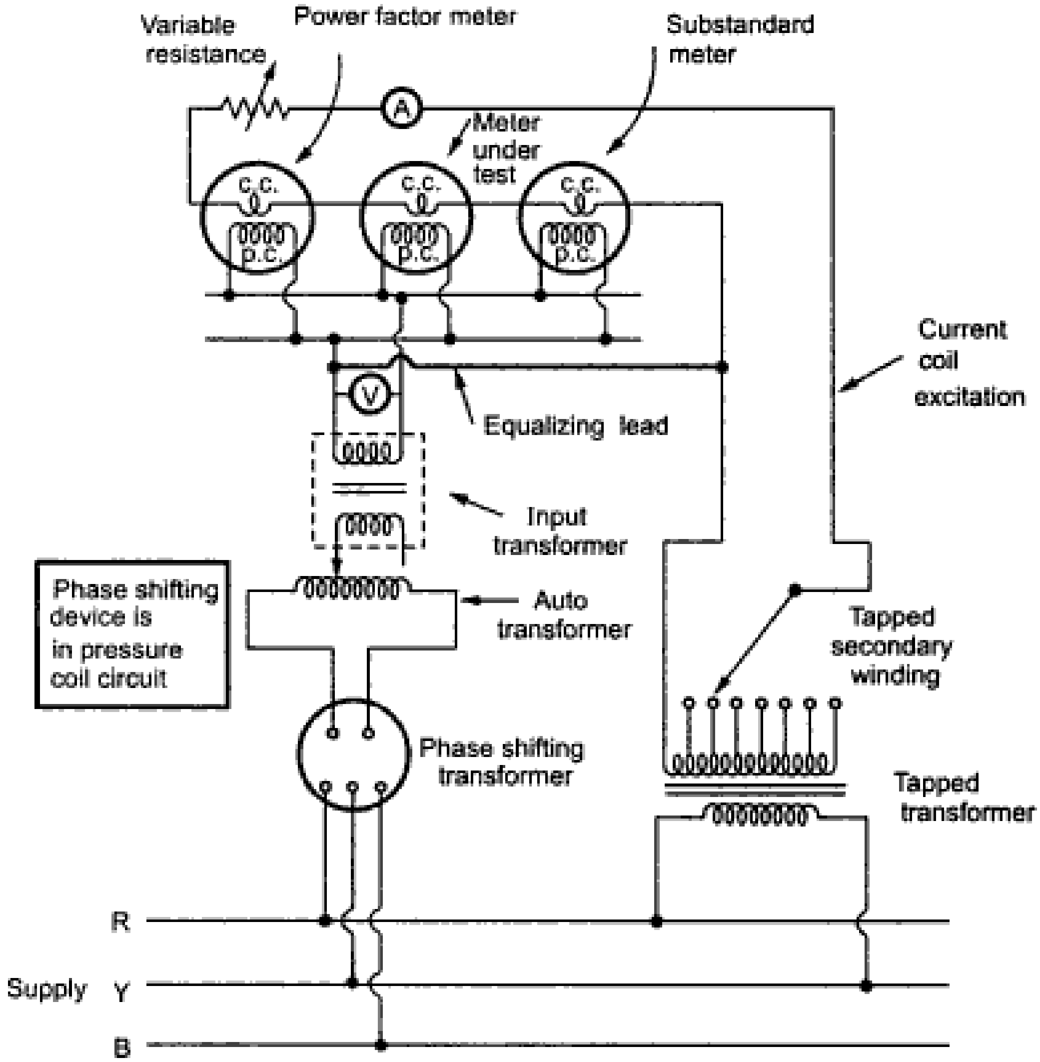


Fig. 4.19 Phantom loading of a.c. meter using rotating substandard meter

The alternator with movable stator is in current coil circuit. The angle through which the stator is moved gives the phase angle between voltage and current. The frequency of both supplies is same as alternators run at same speed. The arrangement is shown in the Fig. 4.20.

Please refer Fig. 4.20 on next page.

The equalizing lead is provided so as to maintain the potential difference between current and pressure coil of the meters to be zero. The method is also called **phantom loading with R.S.S. meter.**

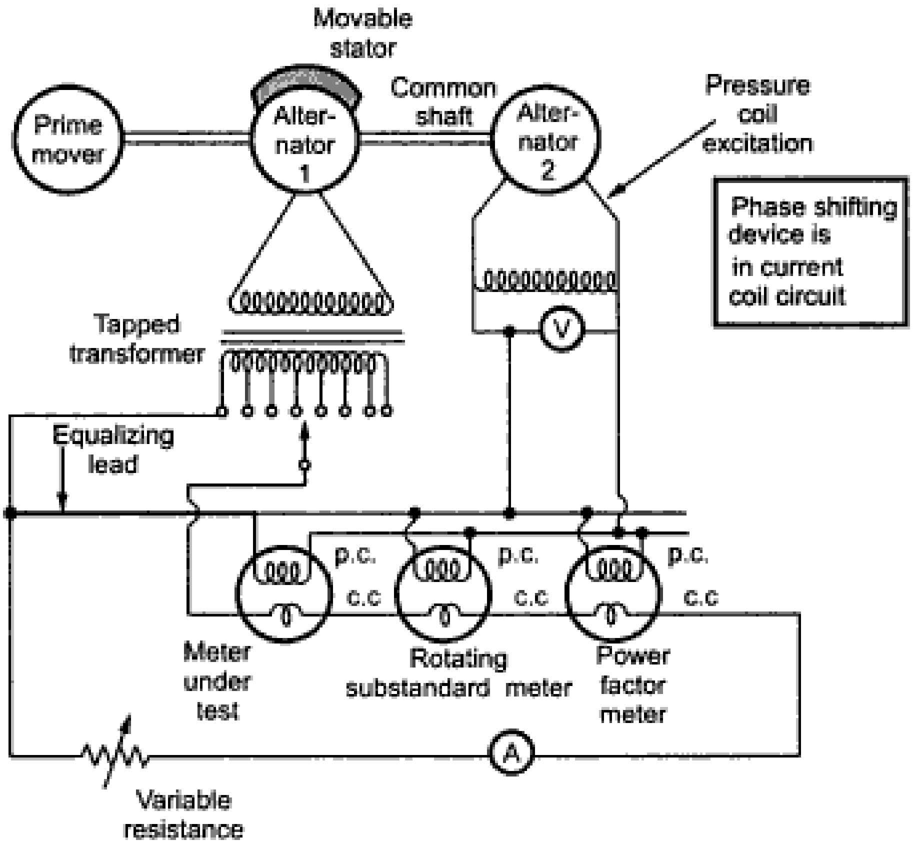


Fig. 4.20 Phantom loading of a.c. meter using rotating substandard meter

4.17 Trivector Meter

The meter which measures the kVAh and kVA of the maximum demand simultaneously is called a trivector meter.

It consists of a kWh meter and a reactive kVARh meter together. A special summator arrangement is used in between them. Both the meters drive the summater via a complicated gearing arrangement such that the summator records the kVAh accurately at all the power factors.

$$\text{kVAh} = \sqrt{(\text{kWh})^2 + (\text{kVARh})^2}$$

It uses five different gearing systems,

1. Watt-hour meter driving alone at normal speed, unity power factor condition.
2. Watt-hour meter speed slightly reduced and reactive meter speed is reduced considerably. This represents phase angle of 22.5° and power factor of 0.925.
3. Both speeds reduced by the same factor and corresponds to power factor of 0.707, phase angle 45°.
4. Watt-hour meter speed is considerably reduced and speed of reactive meter is slightly reduced. This corresponds to phase angle 67.5° and power factor 0.38.
5. Reactive meter driving alone at normal speed representing zero power factor.

The ratchet coupling is linked to the main common register shaft to which final drive from each gear system is connected. Thus the shaft is always driven by the direct drive which has the maximum speed. At that time all other four slower shafts are idle on ratchets.

As the power factor changes, other gear drive system drives the shaft at higher speed and drive shifts to different ratchet. For a given V-I product, the speed of the kWh meter varies as cosϕ where ϕ varies between 0° to 90° lagging. The variation of percentage speed against the phase angle curves are provided for the trivector meter as shown in the Fig. 4.21.

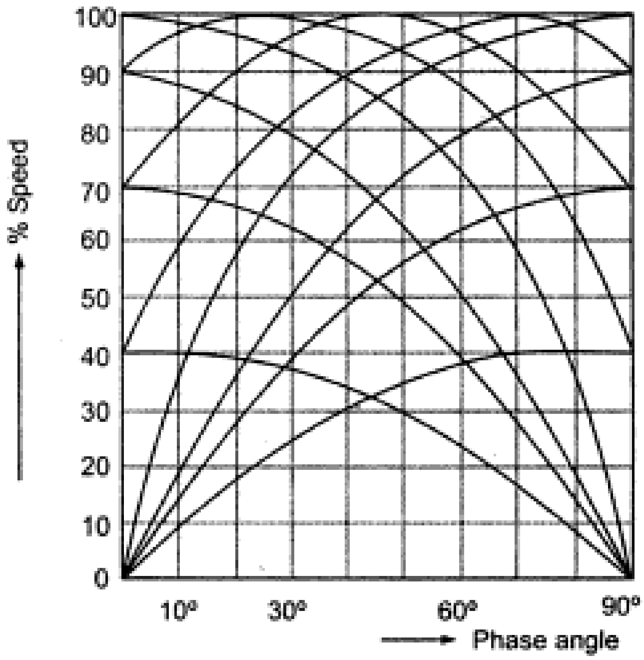


Fig. 4.21 Speed-phase angle curves for trivector meter

Examples with Solutions

➡ **Example 4.3 :** *A correctly adjusted 240 V, induction watt-hour meter has meter constant of 600 revolutions per kWh. Determine the speed of the disc for a current of 10 A, at a power factor of 0.8 lagging. If the lag adjustment is altered so that the phase angle between flux and applied voltage is 86°, calculate the error introduced at 1) unity p.f., 2) 0.5 p.f. lagging.* **[JNTU, May-2004, Set-1, Nov.-2003, Set-1]**

Solution : The energy consumed in one minute is,

$$E = VI\cos\phi \times t = 240 \times 10 \times 0.8 \times \frac{1}{60} = Wh = 32\ Wh = 0.032\ kWh$$

Revolutions in one minute = $E \times K = 0.032 \times 600 = 19.2$

∴ Speed of disc = **19.2 r.p.m.**

When lag adjustment is altered,

$$\text{Speed } N = KVI \sin(\Delta - \phi)$$

$$\Delta = 90° \quad \text{for correct lag adjustment}$$

$\therefore \quad N = KVI \cos\phi$ for correct lag adjustment

Given, $\quad \Delta = 86°$

i) Unity p.f., $\quad \phi = 0°$

$$\therefore \quad \% \text{ error} = \frac{\sin(86°-0°)-\cos(0°)}{\cos(0°)} \times 100 = \mathbf{-0.2436\%}$$

ii) 0.5 p.f. lagging, $\phi = 60°$

$$\therefore \quad \% \text{ error} = \frac{\sin(86°-60°)-\cos(60°)}{\cos(60°)} \times 100 = \mathbf{-12.326\%}$$

Example 4.4 : *The meter constant of a 230 V, 10 A watt-hour meter is 1800 revolutions per kWh. The meter is tested at half load and rated voltage and unity power factor. The meter is found to make 80 revolutions in 138 seconds. Determine the meter error at half load.* **[JNTU, May-2004, Set-3]**

Solution : K = 1800 rev/kWh, V = 230 V, I = 10 A, $\cos\phi = 1$, Half load

$$I_{HL} = \frac{10}{2} = 5 \text{ A.,} \quad t = 138 \text{ sec}$$

$$\therefore \quad E_t = V I_{HL} \cos\phi \times t = 230 \times 5 \times 1 \times 138 = 158700 \text{ J i.e. watt.sec.}$$

$$= \frac{158700}{3600 \times 10^3} \text{ kWh} = 0.04408 \text{ kWh}$$

$$E_r = \frac{\text{No. of revolutions}}{K} = \frac{80}{1800} = 0.04444 \text{ kWh}$$

$$\therefore \quad \% \text{ error} = \frac{E_r - E_t}{E_t} \times 100 = \frac{0.04444 - 0.04408}{0.04408} \times 100 = \mathbf{0.817\%}$$

As E_r is more than E_t, **meter is fast.**

Example 4.5 : *An energymeter is designed to make 100 revolutions of disc for one unit of energy. Calculate the number of revolutions made by it when connected to load carrying 40 A at 230 V and 0.4 p.f. for an hour. If it actually makes 360 revolutions, find the percentage error.* **[JNTU, May-2004, Set-4]**

Solution : K = 100 rev/kWh, I = 40 A, V = 230 V, $\cos\phi = 0.4$, t = 1 hour

$E_t = VI\cos\phi \times t = 40 \times 230 \times 0.4 \times 1 = 3680 \text{ Wh} = 3.68 \text{ kWh}$

$\therefore$ No. of revolutions $= E_t \times K = 3.68 \times 100 =$ **368** revolutions

$$E_r = \frac{\text{No. of revolutions(actual)}}{K} = \frac{360}{100} = 3.6 \text{ kWh}$$

$\therefore$ Speed of disc = **60.04 r.p.m. = 1 r.p.s.**

At half load, I = 5/2 = 2.5 A, t = 59.5 sec

$$\therefore \quad E_t = VI\cos\phi \times t = 220 \times 2.5 \times 1 \times 59.5 \text{ J i.e. watt-sec}$$

$$= 0.00909027 \text{ kWh}$$

$$N = 30 \text{ revolutions}$$

$$\therefore \quad E_r = \frac{N}{K} = \frac{30}{3275} = 0.0091603 \text{ kWh}$$

$$\therefore \quad \% \text{ error} = \frac{E_r - E_t}{E_t} \times 100 = \mathbf{0.77\%}$$

As E_r is more than E_t, **meter is fast.**

Review Questions

1. *Explain the construction of induction type single phase energymeter.*
2. *Explain the working of induction type single phase energymeter.*
3. *Derive the torque equation for induction type single phase energymeter.*
4. *Which are the possible errors in induction type single phase energymeter ?*
5. *What is lag adjustment used in induction type single phase energymeter ?*
6. *Explain the friction adjustment in induction type single phase energymeter.*
7. *How creeping adjustment is provided in induction type single phase energymeter ?*
8. *Explain overload, voltage and temperature compensation used in energymeters.*
9. *State the advantages and disadvantages of induction type energymeter.*
10. *Explain the calibration of single phase energymeter.*
11. *Explain the three element and two element three phase induction type energymeters.*
12. *How C.T and P.T. can be used to extend the range of energymeter ?*
13. *What is tarrif ? Explain the various types of tarrif used.*
14. *Draw and explain the working of Merz price maximum demand indicator.*
15. *What is phantom loading ? When it is used ?*
16. *Explain the testing of meter using rotating substandard meter.*
17. *How the phantom loading test is carried out with rotating substandard meter using phase shifting device ?*
18. *Write a note on trivector meter.*

❑❑❑

5 Potentiometers

5.1 Introduction

A **potentiometer** is an instrument used to measure an unknown e.m.f. which is compared with known e.m.f. Thus it is a device used for measurement of unknown e.m.f. by comparison. The unknown e.m.f. is compared with a known e.m.f. which is obtained from a **standard cell** or any **reference voltage source**. The main advantage of the comparison method of the e.m.f. measurement is that the potentiometer is capable of providing high degree of accuracy as the measurement result is not dependent on the actual deflection of the pointer. Instead of that the accuracy is dependent on the accuracy with which the reference voltage is known.

Basically a potentiometer uses **balance** or **null condition** during the measurement of unknown e.m.f. Actually no current flows in the circuit of the unknown e.m.f. during measurement. Thus no power is consumed in such circuit.

A basic application of the potentiometer is to measure unknown e.m.f. or voltage. It can also be used to determine current. The unknown current can be obtained by using potentiometer by measuring the voltage drop across the standard resistor due to the unknown current.

In the field of the electrical measurements, a potentiometer is most widely used as standard for the calibration of the voltmeters, ammeters and wattmeters. The modern potentiometers are available with high precision and wide range because of the availability of very precise accurate standard e.m.f. in the form of standard cell.

5.2 Principle of Potentiometer

The potentiometer works on the principle of opposing the unknown e.m.f. by a known e.m.f. with the negative terminals of both the e.m.f.s connected together, while the positive terminals connected together through a galvanometer as shown in the Fig. 5.1.

When the e.m.f.s are of same values, there is no deflection on galvanometer. Thus to measure the unknown e.m.f. by using above method, the known e.m.f. used must be variable. Another important requirement is that known e.m.f. should be varied to give a larger number of known values but it is practically very difficult.

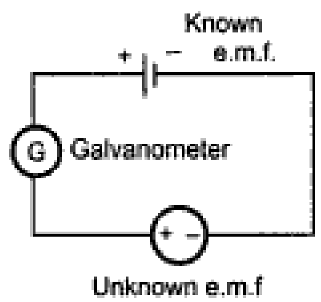


Fig. 5.1 Representation of method of balancing e.m.f.

Hence alternatively, the unknown e.m.f. is connected in parallel with and in opposition to a voltage drop measured across the resistor is shown in the Fig. 5.2.

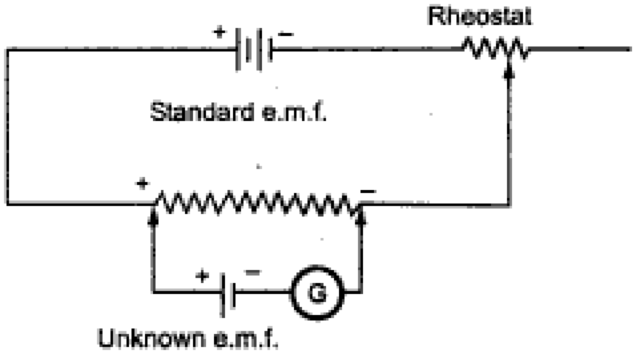


Fig. 5.2 Alternative method of balancing e.m.f.

The main advantage of this method is that the current in the resistor can be varied easily to obtain any desired voltage with very fine adjustment. The voltage drop across resistor can be determined by calibrating the resistor with standard cell.

The potentiometers are classified as d.c. potentiometers and a.c. potentiometers. There are various forms of the d.c. potentiometers used widely practically. The basic, simplest type of the d.c. potentiometer is the slide wire potentiometer. Let us study the slide wire potentiometer in detail.

5.3 Slide Wire D.C. Potentiometer

The slide wire d.c. potentiometer is the basic and simplest type of the d.c. potentiometer as shown in the Fig. 5.3.

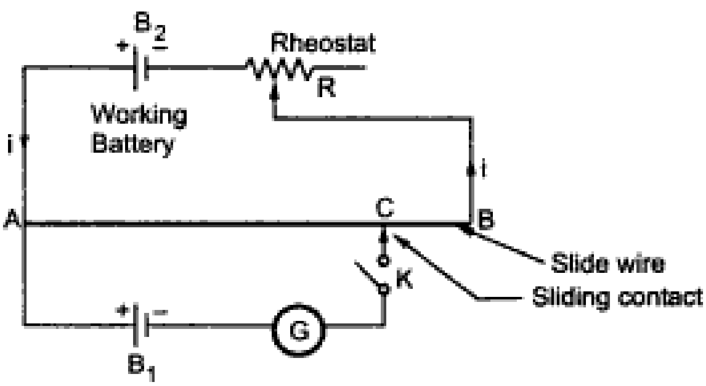


Fig. 5.3 Basic slide wire potentiometer

A basic potentiometer circuit consists of a slide wire AB of uniform cross section and unit length. Generally slide wire is made up of manganin. Let r be the resistance per unit length of slide wire. The battery B_2 supplies a current through the slide wire which is limited with the help of regulating resistance i.e. rheostat. The battery B_1, whose e.m.f. is to be measured is connected in series with a galvanometer G and switch K.

When the switch K is opened, the current through slide wire is i. If the sliding contact is at position C, let the length AC be l units, then the voltage drop across AC is given by i r l.

Consider that switch K is closed which puts the battery B_1 in the circuit. The battery B_1 whose e.m.f. is to be measured is connected such that the voltage drop along the slide wire and e.m.f. of B_1 oppose each other. The deflection in the galvanometer G depends on the magnitudes of voltage drop across the slide wire portion AC and e.m.f. of B_1. If the voltage drop across length l of the slide wire is greater than e.m.f. of battery B_1, then the current will flow in the direction A to C through the galvanometer. Similarly if the e.m.f. of battery B_1 is greater than the voltage drop across the length l of the slide wire, then the current will flow in the direction C to A through the galvanometer. The most important condition exhibiting the basic principle of the potentiometer is that no current flows through the galvanometer when the two e.m.f.s are equal.

Generally a scale is provided along with the slide wire which enables to measure the length of portion AC.

Hence to measure e.m.f. of a battery, first adjust a current through slide wire with switch K open. Then insert battery whose e.m.f. is to be measured. By closing switch K, adjust sliding contact such that the galvanometer shows zero deflection. Measure the length of the portion of the slide wire with the help of scale provided. Then the unknown e.m.f. E of battery is given by,

$$E = i\,(r\,l) \quad \text{... (5.1)}$$

where r is the resistance per unit length, i is the working current adjusted using rheostat R.

If e.m.f.s of the two batteries B_1 and B_2 are to be compared then insert the first battery B_1 in series with the galvanometer and then adjust the sliding contact such that no current flows through the galvanometer. Measure the length of the slide wire portion say l_1. Repeat the same procedure with battery B_2 in the circuit. Let the length measured be l_2. Let e.m.f. of batteries B_1 and B_2 be E_1 and E_2 respectively, then we can write,

$$E_1 = i\,(r\,l_1) \quad \text{... (5.2)}$$

$$E_2 = i\,(r\,l_2) \quad \text{... (5.3)}$$

Thus, we can write,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \quad \text{... (5.4)}$$

From above equation it is clear that the ratio of two lengths gives the ratio of the two e.m.f.s.

If one of the batteries used is a standard cell, say battery B_2 of known voltage, the e.m.f. of battery B_1 is given by,

$$E_1 = E_2\left(\frac{l_1}{l_2}\right) \quad \text{... (5.5)}$$

While using basic slide wire potentiometer following precautions must be taken.

1. The supply battery B_2 should be of high capacity so that a constant current flows through the slide wire throughout the measurement.
2. A small resistance should be used in series with the galvanometer to protect it during the initial adjustments of contact C. It also takes care that no appreciable current is taken from the standard cell.
3. The accuracy of the measurement depends on how accurately ratio (l_1 / l_2) is determined. Hence in such elementary potentiometers, the length of the slide wire used should be large enough so that percentage error in measurement reduces. Now a days in the potentiometers used for precise measurement, by connecting a number of resistance coils in series with short slide wire the effect of larger length can be obtained.

5.3.1 Standardisation of Potentiometer

Standardisation of a potentiometer is a process of adjusting the working current supplied by the supply battery such that the voltage drop across a portion of sliding wire matches with the standard reference source.

The practical set up for standardising a d.c. potentiometer is as shown in the Fig. 5.4.

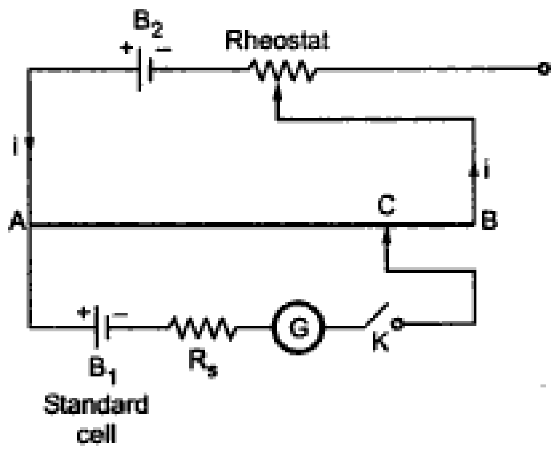


Fig. 5.4 Practical set up for d.c. potentiometer standardisation

A battery of sufficient capacity i.e. B_2 is connected in series with a rheostat R_h which regulates the working or standard current flowing through the slide wire.

A standard cell B_1 usually a Weston standard cell of e.m.f. 1.0186 volts is connected to galvanometer and a switch K through a series resistance R_s. By properly adjusting R_s full sensetivity of the galvanometer can be obtained.

A slide wire with total length of 200 cm and resistance of 200 Ω is connected which is indicated by points A and B.

During standardisation process, switch K is closed and the sliding contact is placed at the mark of 101.86 cm along the slide wire as indicated by point C in the Fig. 5.4. Thus we can observe some deflection in the galvanometer. Now by adjusting the value of rheostat R_h we can get null deflection in the galvanometer.

Under the condition of null deflection, the voltage drop along 101.86 cm portion of the slide wire equals the e.m.f. of standard Weston cell. This is nothing but the standardisation of a potentiometer and once the potentiometer is standardised, the rheostat is not disturbed. In other words, the working or standard current is kept constant.

After standardising a potentiometer, it is used as direct reading potentiometer as the voltage along the slide wire at any point is proportional to the length of the slide wire where the point is obtained by moving sliding contact along the wire to get null deflection in the galvanometer for any battery whose e.m.f. is to be measured.

5.4 Crompton's D.C. Potentiometer

The basic slide wire potentiometer considered in the previous section is not the practical form of the d.c. potentiometer. The main drawback of the basic slide wire is that the length illustrated can not be read with great precision. Not only this, but the long slide wire is very awkward as the size of the instrument increase due to it. Hence in practical laboratory form of a d.c. potentiometer, the size is reduced as instead of long slide wire calibrated dialed resistors or small circular wires of one or more turns are used.

A basic slide wire potentiometer was first modified by the scientist R.E. Crompton to a general form known as **Crompton's D.C. potentiometer** as shown in the Fig. 5.5.

This potentiometer consists of a graduated slide wire AC which is connected in series with large number of coils. The coils are selected such that a resistance of each coil is equal to the resistance of the slide wire. Instead of one sliding contact used in the basic slide wire potentiometer, two sliding contacts P_1 and P_2 are used here. The first sliding contact slides over the slide wire while the second one P_2 over the studs of the resistance coils.

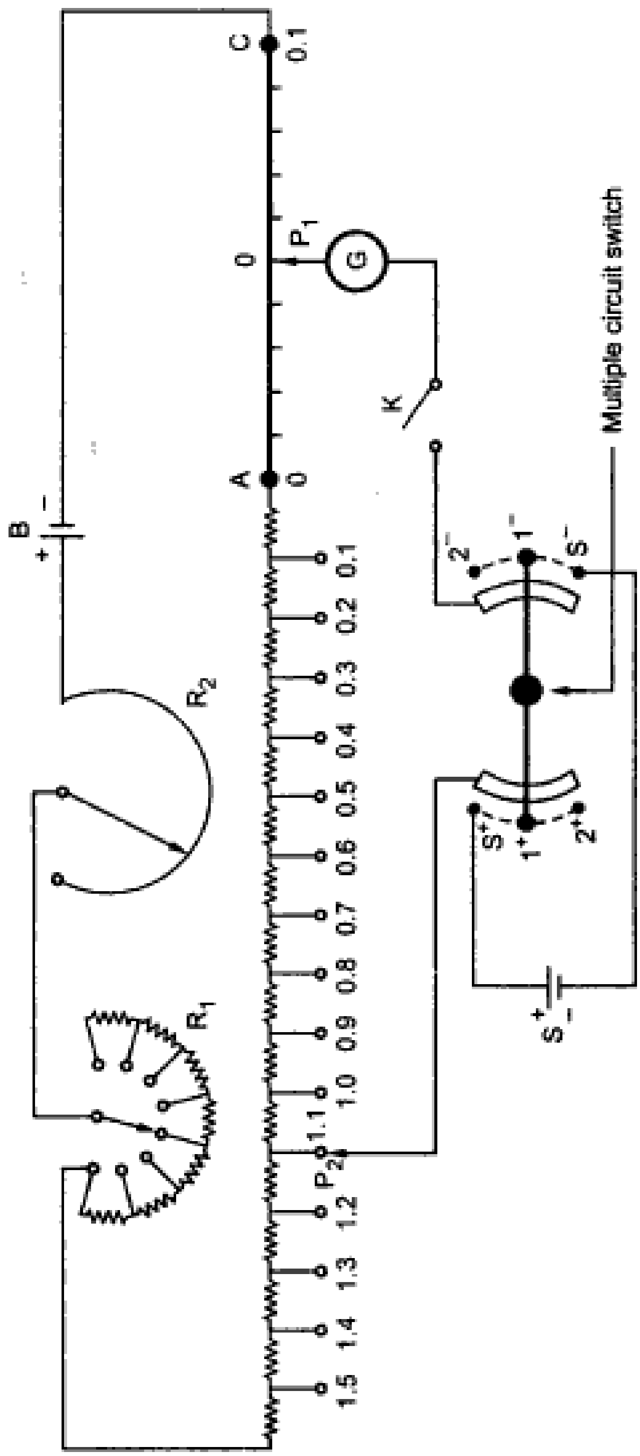


Fig. 5.5 Crompton's D.C. potentiometer

The battery B is of sufficient capacity to provide working current and it is connected in series with regulating resistance. The regulating resistance is realized by series connection of two variable resistance R_1 and R_2 where R_1 consists of number of resistance coils while R_2 in the form of slide wire.

A galvanometer G and switch K are connected in series between points P_1 and P_2 through multiple circuit switch or change over switch. The main advantage of the multiple circuit switch is that we can connect a standard cell S (such as Weston cell) between terminals S^+ and S^- or the batteries whose e.m.f.s are to be measured between the terminals 1^+ and 1^- or 2^+ and 2^-. The terminals are marked with positive (+) and negative (–) signs so as to avoid damage to the potentiometer due to the wrong polarity of battery connected.

First the potentiometer is standardised by using a standard cell S, say Weston cell with e.m.f. of 1.0186 V as shown in the Fig. 5.5. The multiple circuit switch is thrown to the terminals S^+ and S^-. The sliding contacts P_1 and P_2 are set at 0.0186 along the slide wire and 1.0 on the stud respectively. The switch K is closed and the null deflection in the galvanometer is obtained by varying R_1 and R_2. Here R_1 is used for coarse adjustment while R_2 is used for fine adjustment. Once the potentiometer is standardised, resistances R_1 and R_2 are left undisturbed. Now the potentiometer is direct reading potentiometer.

Now connect a battery whose e.m.f. is to be measured between terminals 1^+ and 1^- with the polarities indicated by the terminals and change over the multiple circuit switch to the terminals 1^+ and 1^-. By adjusting P_1 and P_2, the potentiometer can be again balanced which gives the reading of e.m.f. of the battery under measurement directly. For example as shown in the Fig. 5.5. if P_1 is at 0.05 and P_2 is at 1.1 then the e.m.f. of the battery is 1.15 V.

The main drawback of this instrument is that it is necessary to restandardise the potentiometer if it takes considerable time for the potentiometer balance adjustment during the measurement. It is advisable to allow the current to flow through the slide wire before the measurement to get stable balance of potentiometer in quick time.

5.5 Duo-Range Potentiometer

By keeping direct reading feature, the basic potentiometer may be modified to add a second range with usually a second factor such as 0.01. Such a potentiometer is called **duo-range potentiometer**.

The duo-range potentiometer is the modified version of Crompton's d.c. potentiometer in which additional range selector switch is used as shown in the Fig. 5.6.

The working battery B of sufficient capacity is connected in series with two variable resistances R_1 and R_2 which are used for regulating the current through potentiometer during standardisation.

It is very important to design a circuit in such a way that the instrument should enable the user to change the measuring range without standardising the instrument again and again either by adjusting R_1 and R_2 or by changing the voltage of working battery B. In other words, once the instrument is standardised for X1 range, it should not be needed to standardise again for X0.1 range. This condition can be fulfilled only if the voltage V remains same for both the positions of range selector switch S. And this is possible only if the total battery current I_T is same for both the ranges.

The operation of duo-range potentiometer can be explained in simple form as below. First consider that the range selector switch S is at position M i.e. on range of X1. Then the total measuring resistance R_M gets shunted by series combination of range resistors R_3 and R_4 as shown in the Fig. 5.7 (a). The current through R_M is I_M while total current is I_T. Now consider that the range selector switch S is moved to position N i.e. on range of X0.1. Then the range resistor R_3 shunts the series combination of total measuring resistance R_M and range resistor R_4 as shown in the Fig. 5.7 (b). Now the current flowing through the branch consisting R_M and R_4 is I'_M which is (0.1) I_M and still total current is I_T.

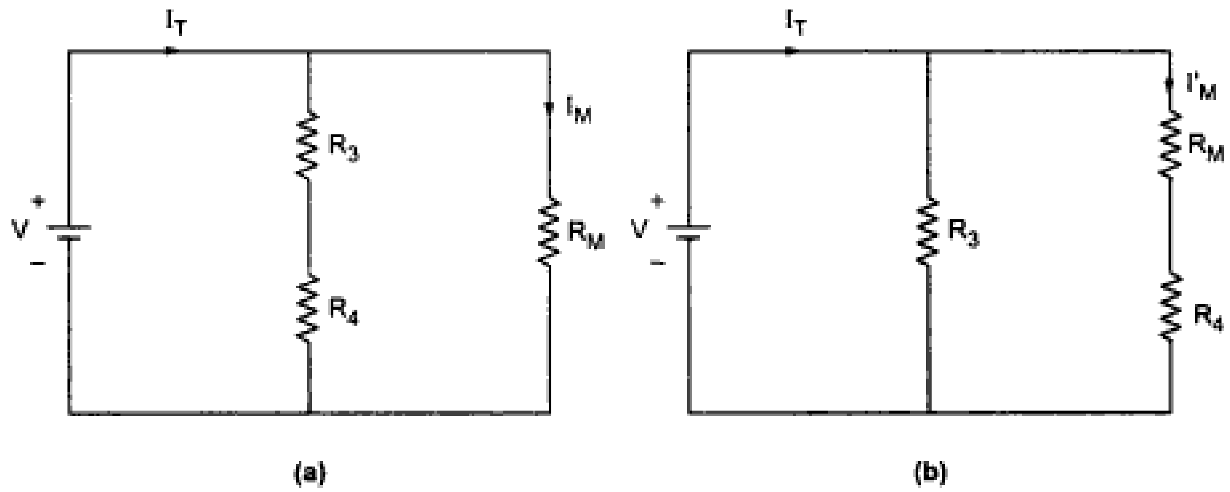


Fig. 5.7 Simplified circuits of duo-range potentiometer for different range selections

In order to have the same current I_T for both ranges, the condition is,

$$(R_3 + R_4) \parallel R_M = R_3 \parallel (R_M + R_4)$$

$$\therefore \quad \frac{(R_3 + R_4) R_M}{R_3 + R_4 + R_M} = \frac{R_3 (R_M + R_4)}{R_3 + R_4 + R_M}$$

$$\therefore \quad R_3 R_M + R_4 R_M = R_3 R_M + R_3 R_4$$

$$R_M = R_3 \qquad \text{... (2)}$$

Above equation (2) indicates that range resistance R_3 must be selected same as total measuring resistance R_M so as to keep total current, supplied by battery B, same for both the ranges.

Now the second condition states that the current I'_M when switch S is at position N must be equal to 0.1 I_M where I_M is the current through R_M when switch S is at position M.

$$\therefore \quad I'_M = 0.1 \, I_M$$

$$\therefore \quad \frac{V}{R_M + R_4} = 0.1 \left[\frac{V}{R_M} \right]$$

$$\therefore \quad R_M = 0.1 \, (R_M + R_4)$$

$$\therefore \quad R_4 = 9 \, R_M = 9 \, R_3 \qquad \text{... (3)}$$

Thus by properly designing values of R_3 and R_4 we can achieve high resolution in measurement using duo-range potentiometers.

5.5.1 Advantages of Duo-Range Potentiometer

Following are the advantages of the duo-range potentiometer.

1) Due to the dual range, the precision of reading is increased by one decimal point.
2) Due to the inherent accuracy of dial resistors as compared to that of slide wire, the accuracy of reading is increased.

5.6 Vernier Potentiometer

By using basic simple potentiometers, the precision of 100 μV for readings upto 1.6 V can be obtained. Using such instruments it is very difficult to get accurate readings mainly due to the non uniformity of slide wire and maintaining good potential contacts. Some of the practical applications demand highly precised and accurate measurements. The limitations due to the slide wire in basic potentiometers are eliminated in the vernier potentiometers. The instrument with normal range of 1.6 V with 10 μV precision and lower range of 0.16 V with 100 μV precision is as shown in the Fig. 5.8.

The main difference in the simple potentiometer and vernier potentiometer is that it uses three measuring dials. The slide wire is not used in this type of the potentiometer. The main dial i.e. the first dial measures upto 1.5 V on (X1) range in steps of 0.1 volts. The second dial reads upto 0.1 V in steps of 0.001 volts on (X0.001) range. It consists of 102 studs. The third dial again with 102 studs measures from – 0.00001 to + 0.001 volts on (X0.00001) range. This third dial provides true zero and negative setting. The function of range selector switch S and range resistances R_1 and R_2 is already explained in previous section. The resistances of the second dial

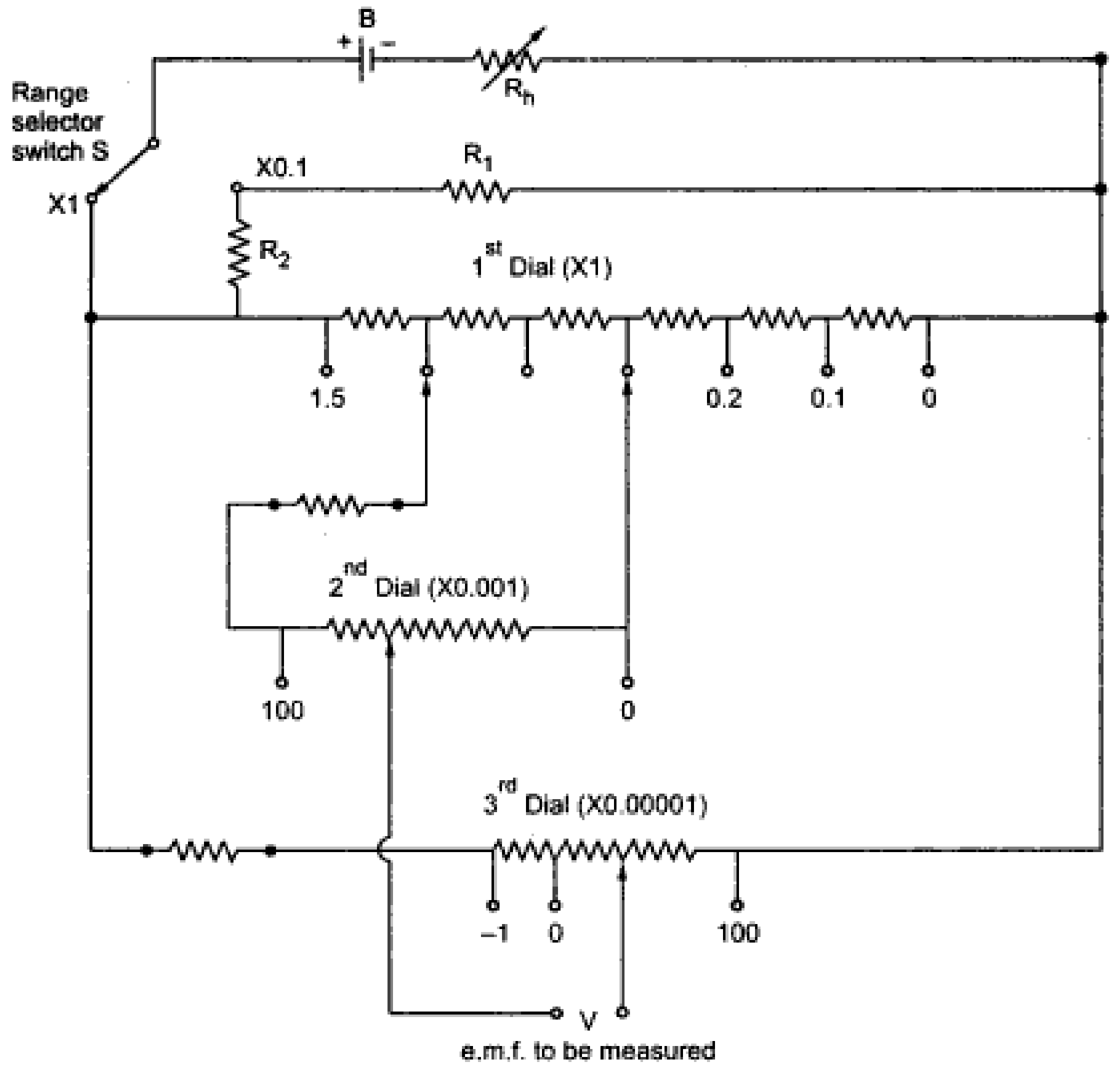


Fig. 5.8 Vernier potentiometer

shunt two of the coils of the main dial. The moving arm of the second dial carries two contacts which are placed two studs away from each other.

The vernier potentiometer reads voltages of 10 μV on X1 range while 1 μV on X0.1 range. To have voltage reading of 0.1 μV one more range of X0.01 may be provided. But it is not possible to read such small voltages as stray thermal and contact potentials in potentiometer, galvanometer and measuring circuits are uncontrollable. These potentials are of the order of one to several microvolts. These potentials can be reduced by properly selecting metals for resistors, terminals and connecting leads. Sometimes thermal shields are also used to enhance process of reduction of potentials.

5.7 Simple Potentiometer with Independent Calibrating Circuit

In general, the calibration of the potentiometer is done by using standard cell dial circuit. It provides a **standard cell balance resistance** to be used for a particular standard cell used. Many times it is necessary to check the standard cell balance during the measurement also. So under this condition, the operator has to check the standard cell balance using separate standard cell dial circuit, without disturbing settings of the potentiometer. But practically the potentiometer settings has to be readjusted after every standard cell balance check. This disadvantage is overcome by using simple potentiometer with independent calibrating circuit incorporated in it. Now a days, all the modern potentiometers incorporate separate standard cell dial circuit as shown in the Fig. 5.9.

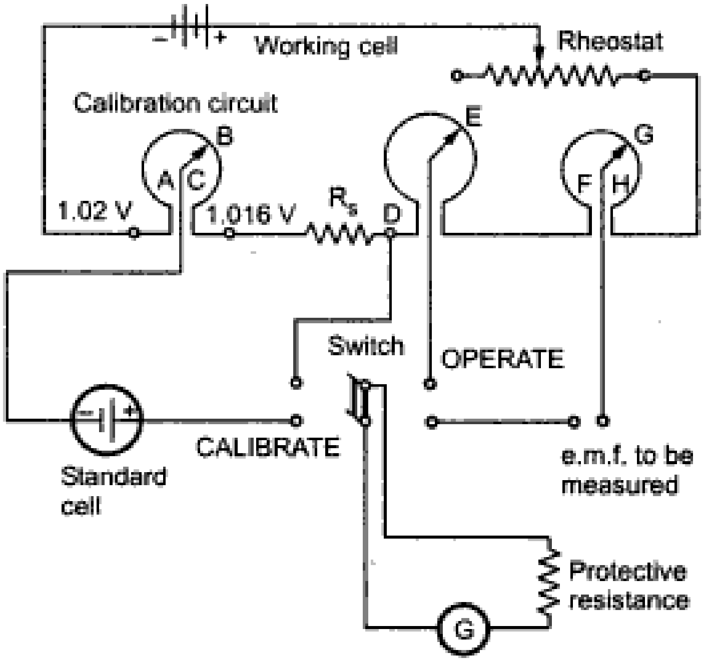


Fig. 5.9 Potentiometer with independent calibration circuit

In the potentiometer shown in the Fig. 5.9, the calibration circuit A-D can be set for any range of standard cell e.m.f from 1.016 V to 1.02 V. The resistance R_s provides 1.016 V while the standard slide wire AC provides 0.04 V. These change in the e.m.f. of standard cell due to the changes in temperature are also accounted for the operation using this calibration circuit. The slide wire of the standard cell dial measures the e.m.f of the standard cell which is also connected to the potentiometer through the switch. First the switch is put at **CALIBRATE** position and the rheostat is adjusted such that the current through galvanometer G is zero. Thus with this adjustment the working current is fixed at proper value. Then the switch is put at **OPERATE** position. The unknown e.m.f is then measured by using measuring circuit consisting measuring circuit dial and slide wire.

The main advantage of this type of the potentiometer is that during measurement also the calibration circuit is checked for the set value of the working current by simply throwing switch to **CALIBRATE** position, without disturbing the settings of potentiometer. This facilitates to increase speed of measurement by reducing the time that is required for the calibration using separate standard cell dial.

5.8 Potentiometer with True Zero

In the basic potentiometer it is very difficult to obtain true zero as the two contacts can not coincide correctly. To overcome this drawback, the circuit is modified as shown in the Fig. 5.10.

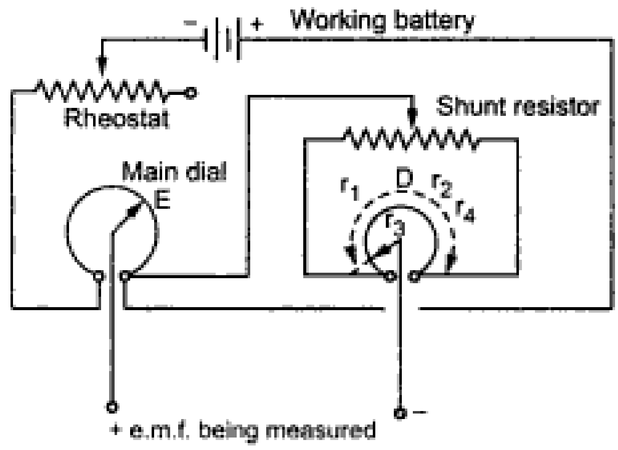


Fig. 5.10 Modified potentiometer with true zero

The slide wire BC is shunted by shunt resistor which is tapped at D. This serves as 0 on main dial. When the contact is in a position, then $\frac{R_1}{R_3} = \frac{R_2}{R_4}$ is zero. If the slider can travel lower than zero position providing negative reading. The slider movement above zero gives positive reading. The typical range of slider wire is from − 0.005 V to + 0.1.5 V.

5.9 Brook's Deflectional Potentiometer

It is observed that using conventional potentiometers, the measurement of continuously changing voltage is very difficult. Even voltage is changing slowly, it is very difficult to manipulate the changes of the dials. Hence for the measurement of the continuously changing voltage deflectional potentiometers are more effective.

The brook's deflectional potentiometer is as shown in the Fig. 5.11.

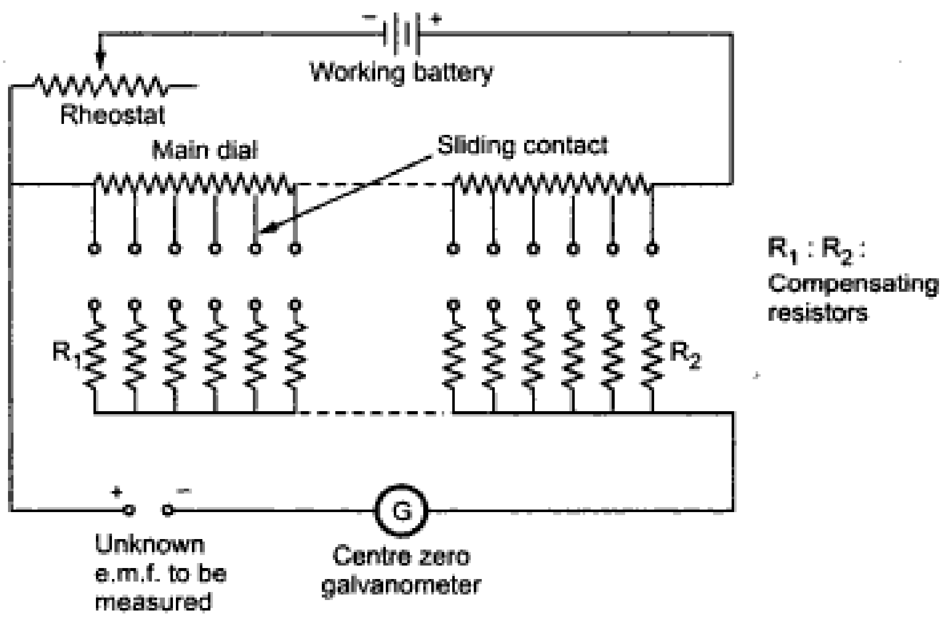


Fig. 5.11 Brook's deflectional potentiometer

This type of potentiometer incorporates only one or two main dials only. The main dials of the deflectional potentiometers consist decade resistance boxes. To indicate deflection, a centre zero type galvanometer is used. The galvanometer circuit includes the compensating resistors R_1 and R_2 which have values so adjusted that they indicate the resistance of the potentiometer for the unknown e.m.f. to the measured. Irrespective of the position of the sliding contacts, the resistance of the compensating resistors will remain same. That means the current through the galvanometer is directly proportional to the current due to out of balance. Thus whatever may be the setting of main dial, the galvanometer can be calibrated so as to give the current for out of balance e.m.f directly. Then the value of unknown e.m.f is the sum of the galvanometer reading and the main dial reading.

5.10 Volt-Ratio Box

As per the discussion of d.c. potentiometers in earlier sections, the maximum voltage measured is less than 2 V. In practical applications if the voltage to be measured is greater than 2 V, then along with the potentiometer a **volt-box** or **ratio-box** or **volt-ratio box** is used. It is based on the concept of potential divider. It consists of a high resistance having number of tappings with properly adjusted resistances between various pairs of tappings as shown in the Fig. 5.12.

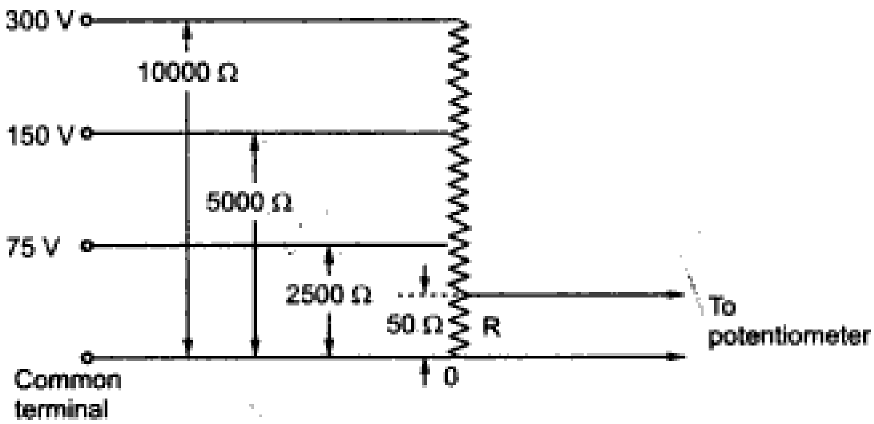


Fig. 5.12 Volt-ratio box

Consider that the voltage of the order of 150 V is to be measured. Then this voltage is connected between the terminals 150 V and common terminal. The leads to the potentiometer are taken from two tapping points say 50 Ω as shown in the Fig. 5.12. Now if the voltage on the potentiometer is 1.3 V then the actual voltage to be measured is given by

$$V_{unknown} = 1.3\left(\frac{5000}{50}\right) = 130 \text{ V}$$

Thus by using simple concept of potential divider we can measure unknown voltage.

5.11 Applications of D.C. Potentiometers

The main application of d.c. potentiometer is measurement of voltage. But it may be also used for measurement of resistance, current and power. The d.c. potentiometer is also useful in the calibration of voltmeters, ammeters, wattmeters etc. Let us take a review of few applications of d.c. potentiometers.

5.11.1 Calibration of Voltmeter

The practical set up for calibration of voltmeter is as shown in the Fig. 5.13 (a) & (b).

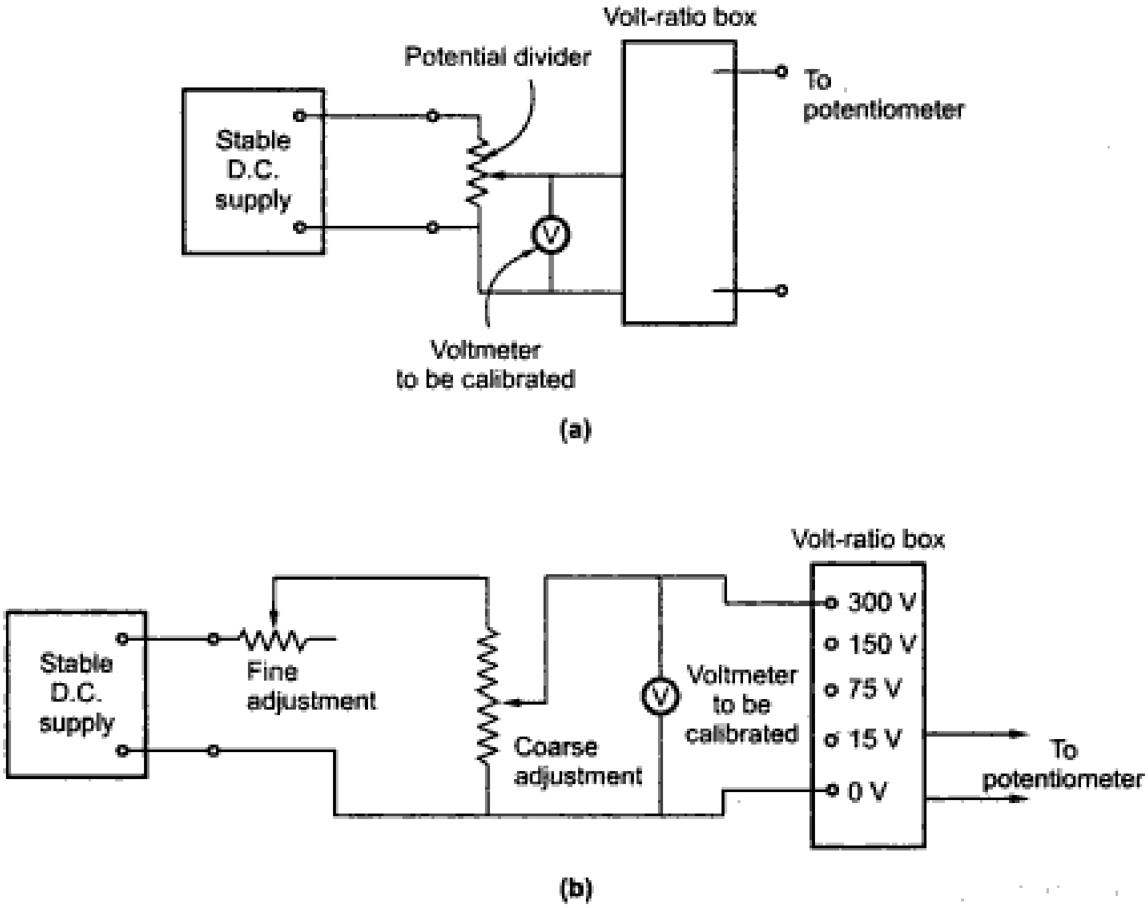


Fig. 5.13 Calibration of voltmeter

In both the circuits the concept used is of potential divider. The main important requirement of these circuits is the use of very stabilized D.C. supply. The only difference is that in the second circuit two rheostats used which are useful in coarse and fine adjustment. With these adjustments it is possible to adjust a voltage such that the pointer of voltmeter exactly coincides the major division. The voltage across voltmeter is stepped down using volt-ratio box. For high accuracy it is advisable to measure voltages near the maximum range of potentiometer.

$$\% \text{ error} = \frac{I_{ind} - I_{act}}{I_{ind}} \times 100$$

where I_{ind} = current indicated by ammeter and

$$I_{act} = I_s = \frac{V_s}{R_s}$$

The calibration curve of ammeter can be obtained by plotting % error against the reading of ammeter i.e. I_{ind}.

5.11.3 Calibration of Wattmeter

By using d.c. potentiometer it is possible to calibrate wattmeter too. The practical setup is as shown in the Fig. 5.15.

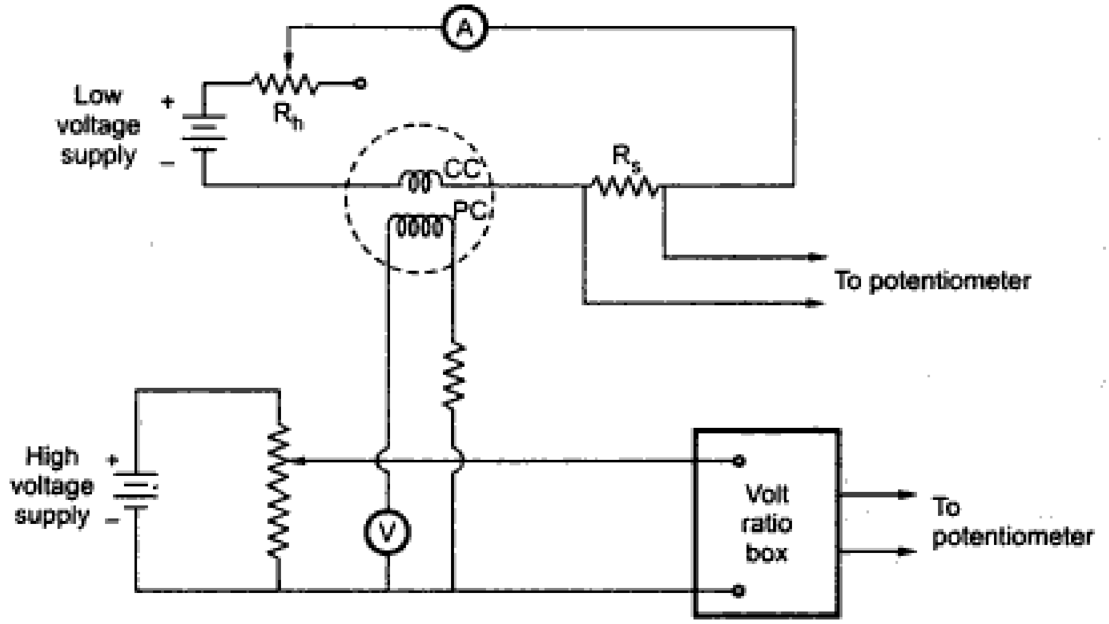


Fig. 5.15 Calibration of wattmeter

A low voltage supply supplies a current to current coil (CC) of wattmeter. This current is adjusted by using a rheostat R_h in series with low voltage supply. The potential divider circuit is supplied by a high voltage supply. The voltage is stepped down by volt-ratio box and the tappings are adjusted accordingly. A voltmeter measures voltage V and ammeter measures current I which gives true power as,

$$W_{ind} = V\ I$$

This value can be compared with a value indicated by wattmeter. If two values are not matching, a positive or negative error is indicated which is given by,

$$\% \text{ error} = \frac{W_{ind} - W_{act}}{W_{ind}} \times 100$$

5.11.4 Measurement of Resistance

The set up for measurement of resistance is as shown in the Fig. 5.16.

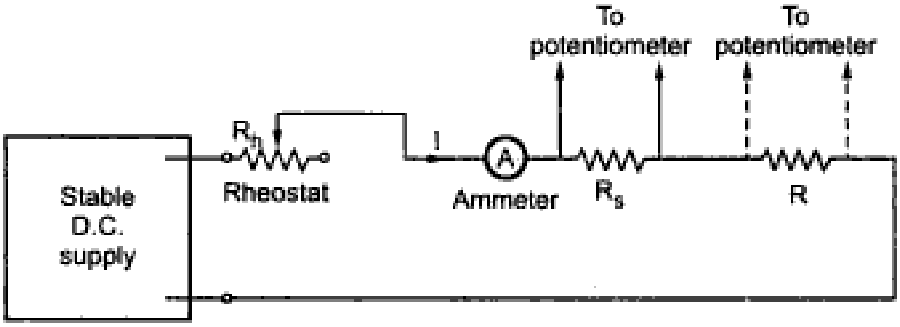


Fig. 5.16 Measurement of unknown resistance

A resistor whose resistance is to be measured is connected in series with a standard resistor of resistance R_s. The current through the circuit is supplied by a stable D.C. supply and it is controlled by a rheostat R_h. The current is adjusted such that the drop across each resistors is of the order of 1 V. Due to the current I, voltages are developed across R_s and R. Both are then measured by using a d.c. potentiometer.

Let the voltage across standard resistance be V_{RS}. Then, we can write,

$$V_{RS} = I\ R_s \quad \text{... (1)}$$

Let the voltage across unknown resistance be V_R. Then, we can write,

$$V_R = I\ R \quad \text{... (2)}$$

Dividing (2) by (1),

$$\frac{V_R}{V_{RS}} = \frac{R}{R_s}$$

Hence the unknown resistance is given by,

$$\boxed{R_s = R\left(\frac{V_R}{V_{RS}}\right)} \quad \text{... (3)}$$

The basic requirement of above measurement method is that the current flowing through the circuit should remain same during measurement of voltages across R and R_s. This need can be fulfilled by using a stabilized D.C. supply at the input.

5.11.5 Measurement of Power

The circuit diagram for measurement of power is as shown in the Fig. 5.17.

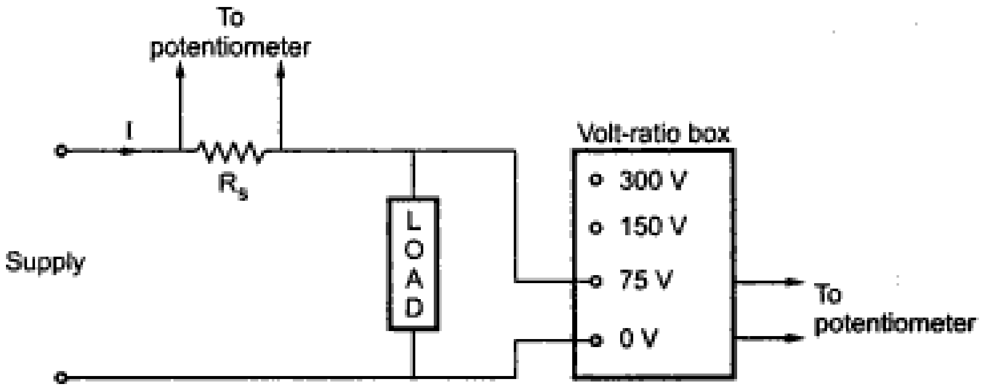


Fig. 5.17 Measurement of power

The power across load can be calculated as,

$$P = I V$$

In above circuit, voltage across standard resistance R_s is measured using potentiometer. Let it be V_{RS}. Then current flowing through it is given by,

$$I = \frac{V_{RS}}{R_s} \quad \text{... (4)}$$

For the measurement of voltage V across load, volt-ratio box is connected across load. The output of volt-ratio box is then connected to potentiometer. Let it be V. Then the voltage across load is given by

$$V = k\ V_R \quad \text{... (5)}$$

where k is the multiplying factor of volt-ratio box and V_R is actual reading of potentiometer when it is connected across volt-ratio box. Thus from equation (4) and (5), the power is given by,

$$P = V\,I = (k\,V_R)\left(\frac{V_{RS}}{R_s}\right)$$

$$\boxed{P = k\left(\frac{V_R\ V_{RS}}{R_s}\right)} \quad \text{... (6)}$$

5.12 Self Balancing Potentiometer

The self balancing potentiometers have the ability of automatic self balancing. As their action is automatic, the attention of the operator is not required constantly. Another advantage of the potentiometer is that it plots a curve of the quantity being measured. These potentiometers are panel mountable and can be used as monitoring displays for the quantities being measured.

In general, in normal potentiometers, any unbalance e.m.f produces a deflection through the galvanometer. But in self balancing potentiometers first the e.m.f. being measured is balanced against known e.m.f. In self balancing potentiometer, the unbalanced e.m.f. is applied to the input of an amplifier which amplifies signal and then this amplified signal is given to the input of the motor. This drives a motor and thus the motor moves a sliding contact to balance the potentiometer automatically.

But the main disadvantage of this system is that working with the d.c. signals. Basically the unbalance e.m.f. is of d.c. type. This d.c. signal is applied to the d.c. amplifier. But output of the d.c. amplifier is not stable and it may suffer due to the drift in the signal level. Hence the self balancing potentiometer with d.c. amplifier is not used commercially in the industrial applications.

To overcome this limitation, between an amplifier and the potentiometer, a converter is used. The main component in this section is reed switch which is excited by an a.c. current. When the reed switch operates, the switch changes the condition at primary of the transformer by reversing the direction of the current in the primary winding. For every vibration of the reed switch, this process takes place. As a effect of the reversing current in the primary current, the e.m.f. induced in the secondary of the transformer is a.c. So the output of the transformer is given to the a.c amplifier. Thus using reed switch a d.c. e.m.f. is converted to an a.c. e.m.f. So the amplified a.c. output is proportional to the unbalance d.c. e.m.f. inputted to the reed converter.

The output of the amplifier is then connected to the control winding of the two phase induction motor; while the other winding is connected to the supply voltage. The phase difference of 90° is maintained between the output of the converter and the supply line voltage. This is achieved by using a simple capacitor at the converter side. Because of this, the amplifier output voltage either leads or lags the a.c. supply voltage by 90°. Depending upon the phase of the output of the amplifier, the direction of the motor is decided. But the phase of the e.m.f. obtained as output of the amplifier depends on the polarity of the unbalance d.c. e.m.f. input to the converter. If the measured e.m.f. is greater than the balancing voltage of the potentiometer, then motor rotates in one direction. Similarly if the measured e.m.f. is less than the balancing voltage of the potentiometer, then motor rotates in other direction because the polarity of unbalance e.m.f. reverses and thus the output of the amplifier gets shifted by 180°.

The potentiometer balancing action can be realized by connecting shaft of the motor to the slide wire of the potentiometer mechanically.

Ultimately if the e.m.f. being measured and the potentiometer voltage both are equal, then the output of the amplifier becomes zero since unbalance condition is not present. Thus irrespective of polarity and the magnitude of the amplifier output, the motor rotates due to the output of the amplifier and achieves the sliding contact to reach to the balance condition.

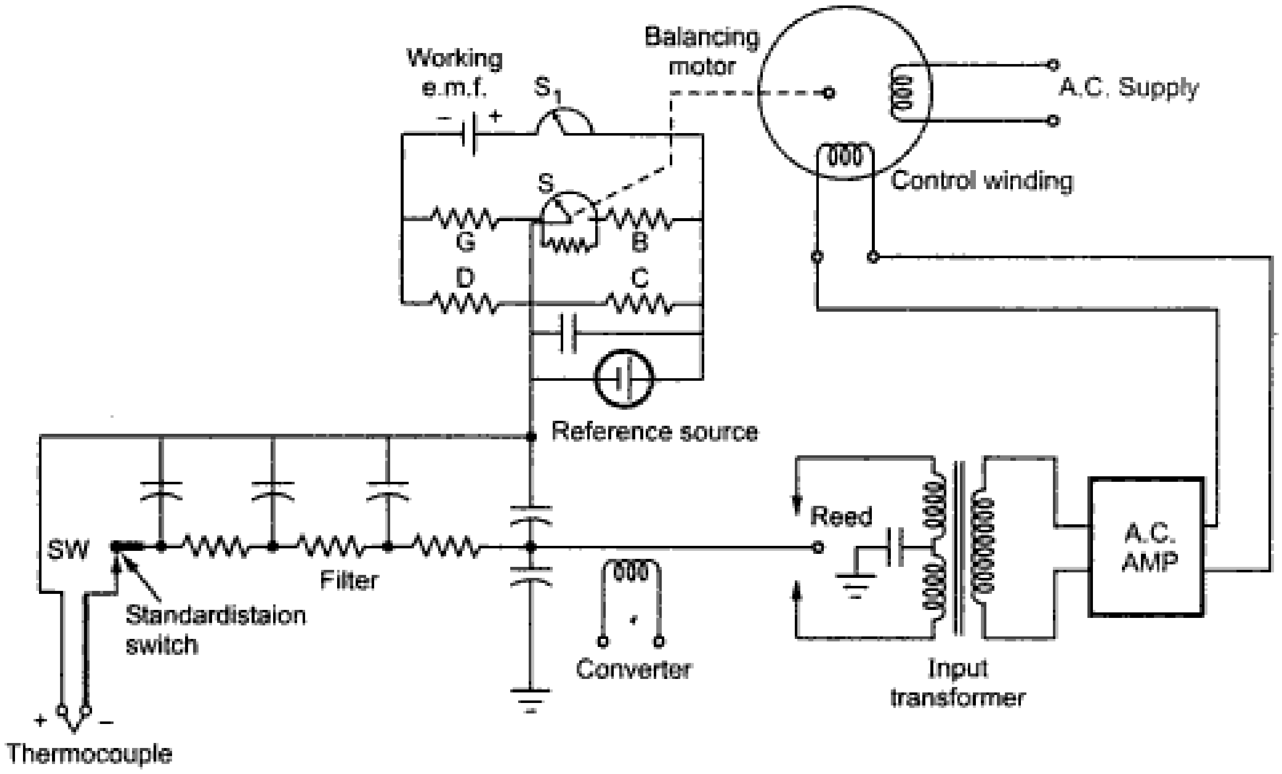


Fig. 5.18 Self balancing potentiometer

In above circuit, the temperature is recorded. The thermocouple is used to measure the temperature difference. But directly this signal can not be applied to the amplifier, in spite the e.m.f produced by the variation in temperature at not and cold junctions of the thermocouple is compensated by an electrical circuit.

The resistor named D in the circuit is made up of nickel copper alloy compensates for the change in the temperature at reference junction. The resistor G is used balance voltage drop across D. The significance of this resistor is that zero suppression is also possible. The resistor and slide wire S together measures the signal while the calibration of the circuit with reference voltage is carried out with the help of resistor B. To adjust working current, rheostat S_1 is used.

The set-up consists filter section which is used to remove unwanted stray a.c. signals.

The motor which maintains the balance position of the potentiometer is coupled with a pen mechanism. So whenever the variable being measured varies, the pen mechanism is actuated, thus the pen moves over a chart and the parameter is recorded on the strip chart.

5.13 A.C. Potentiometers

The basic principle of operation of d.c. potentiometer and an a.c. potentiometer is exactly same. But in a d.c. potentiometer, the balance between voltage drop across the slide wire and the magnitude of unknown voltage is obtained. While in an a.c. potentiometer, the two voltages should be balanced in magnitude as well as phase.

Broadly a.c. potentiometers are classified on the basis of method of measurement of unknown voltages. There are two types of a.c. potentiometers as,

i) **Polar type a.c. potentiometer** in which the magnitude and phase angle of unknown voltage are measured on different scales directly. The phase angle is measured with respect to some reference phaser. As the voltage measured is represented in polar form as $V \angle \theta°$, the a.c. potentiometer is called polar type a.c. potentiometer.

ii) **Co-ordinate type a.c. potentiometer** in which the two components of an unknown voltage are measured on two different scales. One of the components measured is **inphase** component while remaining is **quadrature** component. Both the components are 90° out of phase with each other. If the inphase component and quadrate components are represented by V_A and V_B respectively then the magnitude and phase angle of an unknown voltage can be represented as given below.

$$V = \sqrt{V_A^2 + V_B^2} \quad \text{and}$$

$$\theta = \tan^{-1}\left(\frac{V_B}{V_A}\right)$$

For the measurement of all angles upto 360°, the provision is made in such potentiometers to read positive as well as negative values of V_A and V_B. As the two components of the unknown voltage represent rectangular form of voltage, the potentiometer is called co-ordinate type a.c. potentiometer.

5.13.1 Basic Requirements of A.C. Potentiometers

1. In a.c. potentiometers, the basic requirement is that, at all the instants of time both the voltages being compared must be equal with respect to magnitude and phase both. Hence the current in the potentiometer circuit must have same phase and frequency as compared to the voltage being measured. Hence in a.c. potentiometers, the current in the potentiometer circuit is derived from the voltage being measured.

2. While comparing two voltages in a.c. potentiometers it is necessary to measure the potentiometer voltage accurately as a.c. reference is not available in the circuit.

3. The a.c. source must supply exactly sinusoidal signal. It is necessary in a.c. potentiometers as the detectors used in it are vibration galvanometers. Being tuned circuits, these galvanometers may respond hormonics present in one or both the voltages. Due to this, achieving balance point is a very difficult task.

The variable capacitor and resistor shown in the circuit diagram are so adjusted that exact quadrature component between the two stator winding currents is obtained.

An electrodynamometer type amme.er is used to measure a.c. as well as d.c. currents during the standardisation of an a.c. potentiometer.

5.14.1 Standardisation of Drysdale-Tinsley A.C. Potentiometer

In standardisation of a.c. potentiometer, both d.c. as well as a.c. standardisations are done. The d.c. standardisation is done first by replacing vibration galvanometer by D'Arsonval galvanometer. A standard cell such as Weston cell is used for d.c. standardisation. Then by adjusting sliding contacts null deflection in galvanometer is achieved. The reading of a precision ammeter included in battery supply is noted. During a.c. standardisation again vibration galvanometer is used. The ammeter is still included in the supply circuit but now this circuit is without standard cell. By properly adjusting resistance in the circuit, the r.m.s. value of current in slide wire is made same as that of d.c. current noted in d.c. standardisation.

5.14.2 Measurement of Unknown e.m.f.

The circuit diagram for measurement of unknown e.m.f. using an a.c. potentiometer is as shown in the Fig. 5.20.

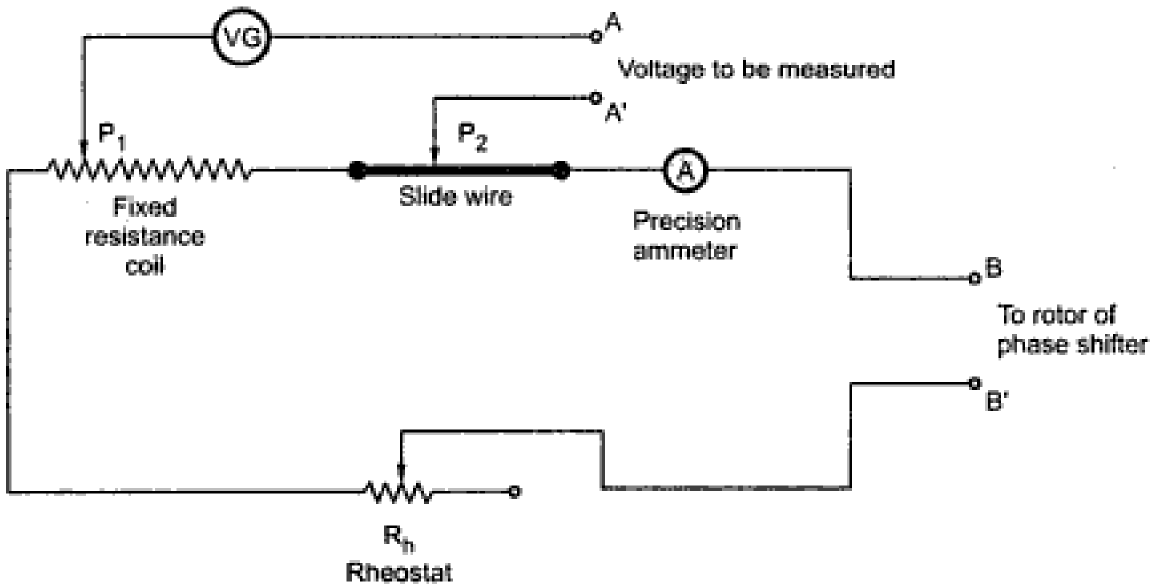


Fig. 5.20 Measurement of unknown e.m.f.

An e.m.f. to be measured is connected across terminals A-A'. The sliding contacts P_1 and P_2 and the position of rotor in phase shifter are adjusted simultaneously till the balance is obtained as indicated by the null deflection of vibration galvanometer.

At balance, the magnitude of the unknown e.m.f. is obtained from P_1 and P_2. And the phase angle is obtained from the scale reading which is mounted on the top of the instrument. Thus the unknown e.m.f. can be expressed in polar form as $E\angle\theta°$.

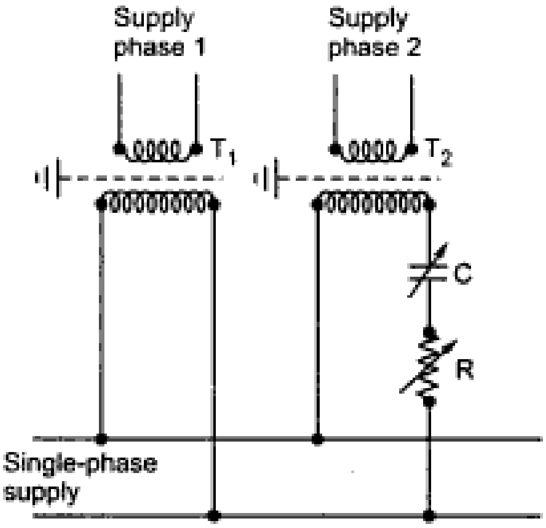


Fig. 5.22 Phase splitting circuit

5.15.1 Standardisation of Gall-Tinsley A.C. Potentiometer

First of all d.c. standardisation of potentiometer is done by using standard cell and D'Arsonval type galvanometer. Then without disturbing this setting, a.c. standardisation is done by adjusting slide wire current to give zero deflection. Then previous galvanometer is replaced by vibration galvanometer and also direct current supply is replaced by a.c. supply. Then the rheostat is adjusted till the current in the quadrature potentiometer wire is same as that in the in-phase potentiometer magnitudewise. Also these two currents must be exactly in quadrature.

5.15.2 Measurement of Unknown e.m.f.

The e.m.f. to be measured is connected across the terminals A-A' using selector switch S_3. The sliding contacts of both the potentiometers are adjusted till the null deflection is obtained in the vibration galvanometer.

Under the balance condition, the in-phase component of the unknown e.m.f. is obtained from in-phase potentiometer while the quadrature component of the unknown e.m.f. is obtained from quadrature potentiometer. If needed the polarity of the test voltage may be reversed by using sign changing switches S_1 and S_2 to balance the potentiometers.

If V_A and V_B are the two potentiometer readings, the magnitude and phase angle of unknown e.m.f. are given by,

$$V = \sqrt{V_A^2 + V_B^2} \quad \text{and}$$

$$\theta = \tan^{-1}\left(\frac{V_B}{V_A}\right)$$

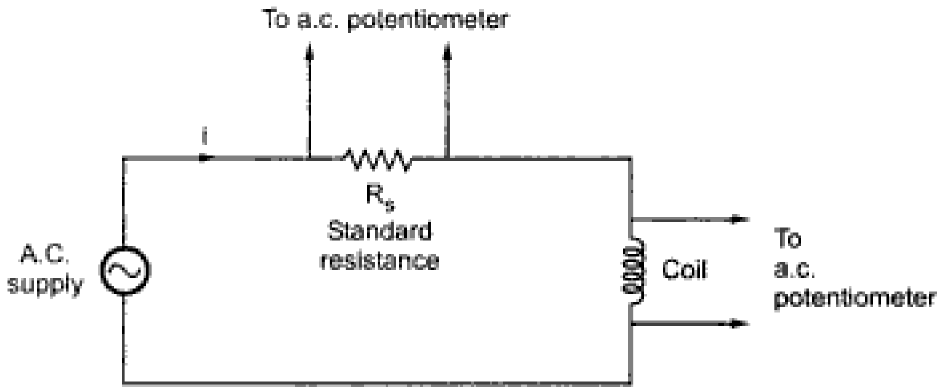


Fig. 5.23 Measurement of self reactance of coil

Voltage across coil = $v_c = V_c \angle\theta_c$ in polar form ... (1)

Voltage across $R_s = v_s = V_s\angle\theta_s$ ... (2)

The current through coil can be calculated as,

$$i = \frac{v_s}{R_s} = \frac{V_s\angle\theta_s}{R_s} \quad \text{... (3)}$$

The impedance of coil can be calculated as,

$$Z = \frac{v_c}{i} = \frac{V_c\angle\theta_c}{\left[\frac{V_s}{R_s}\angle\theta_s\right]} = \frac{R_s V_c}{V_s}\angle\theta_c - \theta_s \quad \text{... (4)}$$

We can write this impedance Z in rectangular form in real part and imaginary part as resistance and reactance.

The resistive part of impedance is given by,

$$R = Z\cos(\theta_c - \theta_s) = \frac{R_s V_c}{V_s}\cos(\theta_c - \theta_s) \quad \text{... (5)}$$

The reactive part of impedance is given by,

$$X = Z\sin(\theta_c - \theta_s) = \frac{R_s V_c}{V_s}\sin(\theta_c - \theta_s) \quad \text{... (6)}$$

Thus equation (6) represents reactance of the coil.

Examples with Solutions

Example 5.1 : *A current in the circuit is measured using a simple slide wire. It is observed that voltage drop across 0.1 Ω standard resistor is balanced at a length of 75 cm. Find the magnitude of the current if the standard cell is of e.m.f. 1.45 V balanced at length of 50 cm.*

Solution : From the data about the length on the slide wire balancing the standard cell, we can write,

$$\text{Voltage drop per unit length} = \frac{\text{e.m.f. of standard cell}}{\text{length of slide wire at the balanced condition}}$$

$$\therefore \quad \text{Voltage drop per unit length} = \frac{1.45}{50} = 0.029 \text{ V/cm}$$

Now voltage drop due to unknown current is balanced at the length of slide wire equal to 75 cm. Hence the corresponding voltage across standard resistor is given by,

$$\text{Voltage across standard resistor} = 75 \text{ (Voltage drop per unit length)}$$

$$= 75 \ (0.029)$$

$$= 2.175 \text{ V}$$

Hence unknown current can be obtained as,

$$I = \frac{\text{Voltage drop across 0.1 standard resistor}}{\text{Standard resistor}}$$

$$\therefore \quad I = \frac{2.175}{0.1} = \mathbf{21.75 \text{ A}}$$

Example 5.2 : *Design a volt-ratio box with a resistance of 20 Ω/V and ranges 3 V, 10 V, 30 V, 100 V. The volt-ratio box is to be used with a potentiometer having a measuring range of 1.6 V.*

Solution : Let the output voltage of potentiometer be 1.5 V. The resistance of volt-ratio box per volt is 20 Ω/V.

∴ Total output resistance R = (20) (1.5) = 30 Ω

For volt-ratio box,

$$\text{Actual voltage measured } (V_A) = \begin{bmatrix}\text{Measured voltage} \\ \text{on potentiometer}(V_m)\end{bmatrix}\begin{bmatrix}\dfrac{\text{Resistance of tapping}}{\text{Output resistance of potentiometer}}\end{bmatrix}$$

Example 5.3 : *A Crompton's potentiometer consists of a resistance dial having 15 steps of 10 Ω each and a series connected slide-wire of 10 Ω divided into 100 divisions. If the working current of the potentiometer is 10 mA and each division of slide wire can be read accurately upto $\frac{1}{5}^{th}$ of its span, calculate the resolution of the potentiometer in volts.*

Solution : The total resistance of Crompton's potentiometer is the combination of the resistance of dial and the resistance of slide wire. Hence we can write,

$$R_T = R_{Dial} + R_{Slide\,wire}$$

But the resistance of dial is given by,

$$R_{Dial} = \text{(No. of steps) (Resistance of each step)} = (15)\,(10)$$

$$= 150\ \Omega$$

Hence total resistance of the potentiometer is given by,

$$R_T = R_{Dial} + R_{Slide\,wire} = 150 + 10 = 160\ \Omega$$

The working current of potentiometer is $I = 10\text{ mA} = 10 \times 10^{-3}$ A.

Then the voltage range of the potentiometer is given by

$$V = (R_{Slide\,wire})\,(I) = (10)\,(10 \times 10^{-3}) = 0.1\text{ V}$$

Now the slide wire has 100 divisions, thus each division represents a voltage of $\frac{0.1}{100} = 0.001$ V.

Each division of the slide wire can be read accurately upto $\frac{1}{5}^{th}$ of its span, then the resolution of the Crompton's potentiometer is given by,

$$\text{Resolution} = (0.001)\left(\frac{1}{5}\right) = \mathbf{0.0002\ V}$$

Example 5.4 : *The voltage-ratio box shown in the Fig. 5.25 is designed in such a way that when 200 V is applied to the input terminals, the output voltage of 4 V is available at the output terminals of the box. Additionally the total resistance at input terminals must be equal to 1 MΩ. Determine voltage box ratio and values of R_1 and R_2.*

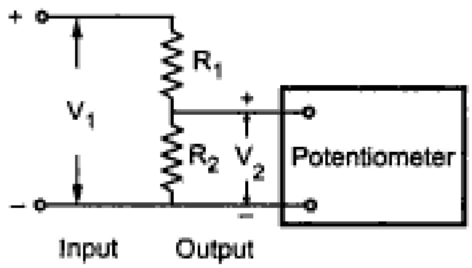


Fig. 5.25

Solution : The voltage box ratio can be obtained as follows.

$$\text{Ratio} = \frac{\text{Input voltage (i.e. unknown voltage)}}{\text{Output voltage}} = \frac{200}{4} = \frac{50}{1}$$

Thus **the voltage ratio box has ratio 50 : 1.**

Now by potential (or voltage) divider rule,

$$V_2 = V_1 \left[\frac{R_2}{R_1 + R_2}\right]$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{R_1 + R_2}{R_2} = \frac{50}{1}$$

Simplifying, we get,

$$R_1 + R_2 = 50\ R_2$$

$$\therefore \quad R_1 = 49\ R_2 \qquad \text{... (i)}$$

But the condition is that total resistance at the input terminals (i.e. $R_1 + R_2$) must be 1 MΩ.

$$\therefore \quad R_1 + R_2 = 1 \times 10^6 \qquad \text{... (ii)}$$

Solving equations (i) and (ii) we get,

$$R_1 = \mathbf{20\ k\Omega}$$

$$R_2 = \mathbf{980\ k\Omega}$$

Example 5.5 : *In the low resistance measurement technique using potentiometer following readings were observed.*

i) Voltage drop across low resistance under test = 0.97825 V

ii) Voltage drop across a 0.1 Ω standard resistance in series with unknown resistance = 1.02575 V

The resistance of the standard resistor at the temperature of test is 1.00024 Ω. By setting the potential dial to zero and breaking current passing through unknown resistance the thermal e.m.f. of latter produces a galvanometer deflection equivalent to 19 μV, in the direction same as that produced by an increase of the potentiometer reading during the voltage measurement. Calculate unknown resistance.

Solution : The drop across low resistance under test = 0.97825 V

The thermal e.m.f. with unknown resistance = 19 μV = 0.000019 V.

Hence actual voltage drop across unknown resistance is given by,

$$V_R = 0.97825 - 0.000019 = 0.978231 \text{ V}$$

Thus the unknown resistance is given by,

$$R = \frac{V_R}{V_s} \times R_s$$

where R_s = Standard resistor = 1.00024

V_s = Voltage across R_S = 1.02575

$$\therefore \quad R = \text{Unknown resistor} = \frac{0.978231}{1.02575} \times 0.100024$$

$$= \mathbf{0.095399\ \Omega}$$

Example 5.6 : *Measurement for the determination of impedance of a coil are made on a co-ordinate type potentiometer. The results are as follows :*

Voltage across 1 Ω standard resistance in series with the coil is + 0.952 V on in-phase dial and – 0.34 V on quadrature dial.

Voltage across 10 : 1 potential divider connected to a terminal of coil is + 1.35 V on in-phase dial and + 1.28 V on quadrature dial. Calculate resistance and reactance of the coil.

Solution : Current through the coil $= I = \frac{0.952 - j\,0.34}{1} = (0.952 - j\,0.34)$ A

Similarly voltage across coil $= V = 10\,(1.35 + j\,1.28) = (13.5 + j\,12.8)$ V

$\therefore$ Impedance of the coil $= Z = \frac{V}{I} = \frac{13.5 + j\,12.8}{0.952 - j\,0.34}$

$$\therefore \quad Z = \frac{18.387\ \angle 44.118°}{1.018\ \angle -19.653°}$$

$$\therefore \quad Z = 18.062\ \angle 63.77°\ \Omega$$

This impedance can be represented in rectangular form as

$$Z = 7.983 + j\,16.2\ \Omega = R + j\,X$$

Hence, the resistance of the coil R = **7.983 Ω ≈ 8 Ω**

and the reactance of the coil X = **16.2 Ω**

Example 5.7 : *Power is measured with an a.c. potentiometer. The voltage across a 0.1 Ω standard resistance connected in series with load is $(0.35 - j\,0.1)$ V. The voltage across 300 : 1 potential divider connected to supply is $(0.8 + j\,0.15)$ V. Determine power consumed by load and power factor.*

Solution : Current through load $= I = \frac{(0.35 - j\,0.1)}{0.1} = (3.5 - j\,1)$ A

$$= 3.64 \angle -15.94° \text{ A}$$

Voltage across load $= V = 300(0.8 + j\,0.15) = 300(0.8139 \angle 10.62°)$

$\therefore \quad V = 244.182 \angle 10.62°$ V

Phase angle of load $= 10.62° - (-15.94°) = 26.56° = \phi$

Power factor of load $= \cos\phi = \cos(26.56)° =$ **0.8944**

Power consumed by load $= |V|\ |I| \cos\phi$

$= (244.182)\,(3.64)\,(0.8944)$

= 794.96 W

Example 5.8 : *A non-reactive resistor of 1000 Ω is connected in series with a coil and a capacitor at 50 Hz. If the voltages across R, coil and C are (0.6 − j 0.24) V, (0.6 + j 0.4) V and (− 0.1 − j 0.4) V respectively. Calculate power dissipated and energy stored in each component.*

Solution : The circuit elements in series are as given below.

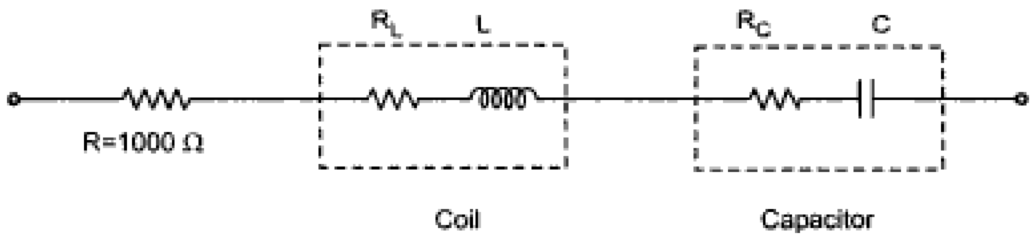


Fig. 5.26

Voltage across 1000 Ω resistor $= V_R = (0.6 - j\,0.24)$ V

Hence current in the resistor and hence through series connection is given by,

$$I = \frac{V_R}{1000} = \frac{0.6 - j\,0.24}{1000} = \frac{0.6462 \angle -21.8°}{1000} = 0.6462 \angle -21.8° \text{ mA}$$

The impedance of the coil is given by,

$$Z_{coil} = \frac{V_{coil}}{I} = \frac{0.6 + j\,0.4}{0.6462 \times 10^{-3} \angle -21.8} = \frac{0.7211 \angle 33.69°}{0.6462 \times 10^{-3} \angle -21.8°}$$

$\therefore \quad Z_{coil} = 1035.7655 \angle 55.49°\ \Omega = (586.81 + j\,853.49)\ \Omega$

But $\quad Z_{coil} = R_L + j\,X_L = R_L + j\,(2\pi f L)$

Hence comparing expressions for Z_{coil}, we can write,

$$\text{Resistance of the coil} = R_L = 586.81\ \Omega$$

$$\text{Reactance of the coil} = X_L = 853.49\ \Omega$$

Hence inductance of coil can be obtained as,

$$X_L = 2\pi fL = 853.49$$

$$\therefore \quad L = \frac{853.49}{2\pi f} = \frac{853.49}{2\times\pi\times 50} = 2.7167\ \text{H}$$

Similarly the impedance of the capacitor is given by,

$$Z_C = \frac{V_C}{I} = \frac{-0.1 - j\,0.4}{0.6462\times 10^{-3}\ \angle -21.8^\circ} = \frac{0.4123\ \angle -104.03}{0.6462\times 10^{-3}\ \angle -21.8^\circ}$$

$$\therefore \quad Z_C = 638.0377\ \angle -82.23^\circ\ \Omega = (86.26 - j\,632.17)\ \Omega$$

But $$Z_C = R_C - j X_C = R_C - j\left(\frac{1}{2\pi fC}\right)$$

Hence comparing expressions for Z_C, we can write,

$$\text{Resistance of the capacitor} = R_C = 86.26\Omega$$

$$\text{Reactance of the capacitor} = X_C = 632.17\ \Omega$$

Hence capacitance is given by,

$$X_C = \frac{1}{2\pi fC} = 632.17$$

$$\therefore \quad C = \frac{1}{2\times\pi\times 50\times 632.17} = 5.035\ \mu\text{F}$$

The power dissipated,

i) Across non-reactive resistor $= P_1 = (|I|^2)\cdot 1000 = (0.6462\times 10^{-3})^2(1000)$

$$\therefore P_1 = \mathbf{0.4175\ W}$$

ii) Across coil $= P_2 = (|I|^2)\ R_L = (0.6462\times 10^{-3})^2(586.81)$

$$\therefore P_2 = \mathbf{0.245\ mW}$$

iii) Across capacitor $= P_3 = (|I|^2)\ R_C = (0.6462\times 10^{-3})^2(86.26)$

$$\therefore P_3 = \mathbf{0.036\ mW}$$

The mean energy stored,

i) By Coil $= \frac{1}{2} L|I|^2 = \frac{1}{2} \times 2.7167 \times (0.6462 \times 10^{-3})^2 =$ **0.567 μJ**

ii) By Capacitor $= \frac{1}{2} C|V_C|^2 = \frac{1}{2} \times 5.035 \times 10^{-6} \times (0.4123)^2 =$ **0.427 μJ**

Examaple 5.9 : *A potentiometer that is accurate to ± 0.0001 volts (standard deviation) is used to measure current through a standard resistance of 0.1 ± 0.1 Ω % (standard deviation). The voltage across the resistance is measured to be 0.2514 volts. What is the current and to what accuracy it has been determined.* **[Nov.-2004, Set-2]**

Solution : i) Current through standard resistor R_s is given by,

$$I = \frac{\text{Voltage drop across standard resistor}}{\text{Standard resistor}} = \frac{V_s}{R_s}$$

$$\therefore \quad I = \frac{0.2514}{0.1} = \mathbf{2.514\ A}$$

2) The fractional standard deviation of current is,

$$\frac{\delta I}{I} = \sqrt{\left(\frac{\delta V_s}{V_s}\right)^2 + \left(\frac{\delta R_s}{R_s}\right)^2}$$

$$\therefore \quad \frac{\delta I}{I} = \sqrt{\left(\frac{0.0001}{0.2514}\right)^2 + \left(\frac{0.1}{100}\right)^2}$$

$$\therefore \quad \frac{\delta I}{I} = 0.001076$$

Hence percentage accuracy of measurement of current $= \frac{\delta I}{I} 100$

$= (0.001076)\ (100)$

$=$ **0.1076%**

Example 5.10 : *During the measurement of a low resistance using a potentiometer the following readings were obtained. Voltage drop across the low resistance under test = 0.4221 V.*

Voltage across the 0.1 Ω standard resistance = 1.0235 V

Calculate the value of unknown resistance, current through it and power lost in it.

[May-2004, Set-4]

ii) The total resistance of the circuit (R_T) is the addition of two resistances; namely resistance of slide wire and resistance of rheostat i.e. $R_T = R_{slide\ wire} + R_h$.

But $$R_T = \frac{\text{Battery Voltage}}{\text{Working Current}} = \frac{3}{5 \times 10^{-3}} = 600\ \Omega$$

Thus the resistance of the series rheostat is given by,

$$R_h = R_T - R_{slide\ wire} = 600 - 400 = \mathbf{200\ \Omega}$$

iii) The range of total voltage measurement is given by,

$$\text{Range of Measurement} = (\text{Working current})(R_{slide\ wire}) = (5 \times 10^{-3})(400)$$
$$= \mathbf{2\ V}$$

iv) Length of 200 cm i.e. 2000 mm represents 2 V, hence 1 mm represents $\left(\frac{2}{2000}\right) = 1$ mV. But the instrument can read upto $\frac{1}{5}^{\text{th}}$ of 1 mm. Hence the resolution of the instrument is,

$$\text{Resolution} = \frac{1}{5}(1\ \text{mV}) = \mathbf{0.2\ mV}$$

Example 5.12 : *A slide wire potentiometer has a battery of 4 V and negligible internal resistance. The resistance of slide wire is 100 Ω and its length is 200 cm. A standard cell of 1.018 V is used for standardizing the potentiometer and the rheostat is adjusted so that balance is obtained when the sliding contact is at 101.8 cm. Find the working current of slide wire and the rheostat setting. If the slide wire has division marked in mm and each division can be interpolated to one-fifth, calculate the resolution of the instrument.* **[Nov.-2003, Set-2, May-2005, Set-1]**

Solution : i) The slide wire potentiometer is standardized with an e.m.f. of 1.018 V with the sliding contact at 101.8 cm. This indicates that the length 101.8 cm represents voltage 1.018 V.

Thus resistance of 101.8 cm length wire $= \left[\frac{101.8}{200}\right] \times 100 = 50.9\ \Omega$

Hence working current is given by,

$$I = \frac{1.018}{50.9} = \mathbf{20\ mA}$$

ii) The total resistance of the circuit (R_T) is the addition of two resistances; namely resistance of slide wire and resistance of rheostat. i.e.

$$R_T = R_{slide\ wire} + R_h$$

$\therefore \quad Z = R + jX = 8.2406 + j\,16.4561$

Comparing, we get,

Resistance of the coil = **8.2406 Ω**

Reactance of the coil = **16.4561 Ω**

Example 5.15 : *In the measurement of power by a polar potentiometer, the following readings were obtained.*

i) Voltage across a 0.2 Ω standard resistance in series with load = 1.46 ∠ 32° V.

ii) Voltage across a 200 : 1 potential divider across the line = 1.37 ∠ 56° V.

Estimate the current, voltage, power and power factor of the load. **[May-2005, Set-1]**

Solution : Standard resistance and load are in series. Thus we get current through load = I = Current through standard resistance

$$= \frac{V}{R_s} = \frac{1.46 \angle 32^\circ}{0.2} = \mathbf{7.3 \angle 32^\circ \; A}$$

Thus the magnitude of the load current is,

$$|I| = 7.3 \text{ A}$$

Voltage across load = $V_{Load} = 200\ (1.37 \angle 56^\circ) = \mathbf{274 \angle 56^\circ \; V}$

Angle between voltage and current is,

$$\phi = 56 - 32 = 24^\circ$$

Hence power factor is given by,

$$\text{P.F.} = \cos\phi = \cos 24 = \mathbf{0.91334}$$

The power consumed by load is given by,

$$P_{Load} = |V|\,|I| \cos\phi$$

$$= (274)\ (7.3)\ (0.91354)$$

$$= \mathbf{1.8272 \text{ kW}}$$

Review Questions

1. *Explain the principle of basic potentiometer with neat diagram.*
2. *What is standardisation of potentiometer ? Why it is necessary ?*
3. *With neat diagram explain Crompton's d.c. potentiometer.*
4. *Explain Duo-range potentiometer with suitable circuit diagram. Write advantages of the same.*
5. *Explain the scheme used for obtaining Duo-range in Duo-range potentiometer.*

6. *Write a note on vernier potentiometer.*
7. *What is volt-ratio box ? Explain how volt-ratio box works*
8. *Write applications of d.c. potentiometers.*
9. *What is a.c. potentiometer ? Give classification of a.c. potentiometer.*
10. *List the basic requirements of a.c. potentiometer.*
11. *Explain briefly the connection diagram of Drysdale phase shifter.*
12. *Explain how Drysdale-Tinsley a.c. potentiometer is standardised.*
13. *How is the unknown e.m.f. measured using Drysdale-Tinsley a.c. potentiometer ?*
14. *Explain briefly co-ordinate type a.c. potentiometer.*
15. *Write a note on applications of a.c. potentiometer.*
16. *Explain self balancing potentiometer with the help of circuit diagram.*
17. *Explain true zero in potentiometer.*
18. *Write a note on simple potentiometer with calibrating circuit.*

6

Resistance Measurements

6.1 Introduction

The measurement of resistance is as important as the measurement of any other electrical parameter. From the point of view of measurement, the basic knowledge of resistance measurement is necessary to understand the working of other instruments used for the measurement of other electrical quantities. Basically resistances are classified as low resistances, medium resistances and high resistances. This classification is based on the values of the resistances. But practically this classification truly indicates different techniques or methods applied for the measurement of resistances such as low resistance measurements, medium resistance measurements and so on.

6.2 Classification of Resistances

Let us study first, the basic methods used to measure resistance. From the measurement point of view the resistances are classified as :

i) **Low resistances** : All the resistances of the order of 1 Ω or less are classified as low resistances.

ii) **Medium resistances** : From 1 Ω onwards upto 0.1 MΩ the resistances are classified as medium resistances.

iii) **High resistances** : Resistances of the order of 0.1 MΩ and higher are classified as high resistances. The classification of resistances given above is not rigid but forms the basis for the methods used for the measurement of the resistances of different classes.

6.3 Voltmeter-Ammeter Method

The resistance is nothing but the ratio of voltage and current. Thus measurement of voltage and current separately is sufficient for the resistance measurement. Thus voltmeter-ammeter method is based on the principle of measuring voltage and current separately. This is very simple method as the instruments required to measure voltage and current are readily available in the laboratory.

Mathematically the resistance value is given by,

$$R_x = \frac{V}{I}$$

where R_x = resistance value to be measured

V = voltmeter reading

I = ammeter reading

The two types of connections are used in this method. The connections are shown in the Fig. 6.1(a) and (b).

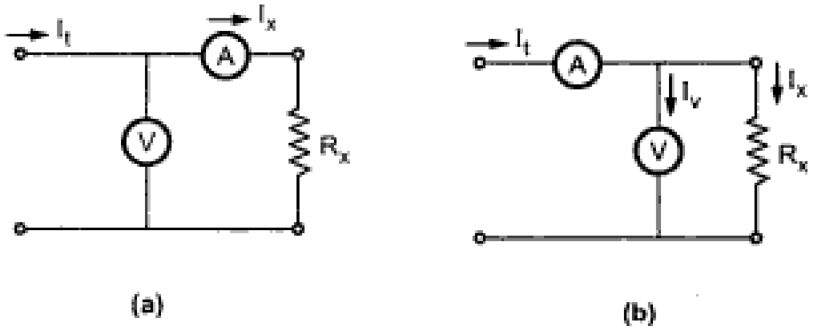


Fig. 6.1 Voltmeter-ammeter method

The **ratio of voltmeter reading and ammeter reading gives the value of unknown resistance** R_x correctly, if the ammeter resistance is zero and the voltmeter resistance is infinite. Practically this is impossible and hence both the circuits shown, give the inaccurate value of the resistance.

In the Fig. 6.1 (a), the ammeter carries the current which is the true current passing through the resistance i.e. I_x. But the voltmeter reading is not the true voltage across the resistance. It is the sum of the voltages across the resistance and across the ammeter.

$$\therefore \quad V = V_R + V_a$$

$$= I_x R_x + I_x R_a$$

where R_a = ammeter resistance

$$\therefore \quad R_m = \frac{V}{I} = \frac{I_x [R_x + R_a]}{I_x}$$

where R_m = measured value of resistance

$$\therefore \quad R_m = R_x + R_a$$

$$\therefore \quad R_x = R_m - R_a$$

Thus the actual value of resistance is same as the measured value if the ammeter resistance is zero.

Actually the measured value is higher than the actual value of the resistance.

But practically if the value of resistance is large compared to ammeter resistance ($R_x >> R_a$) then the effect of ammeter resistance becomes negligible. Hence the circuit is used to measure the resistances in medium resistance range and is not suitable for the low resistance measurement.

In the Fig. 6.1 (b), the voltmeter reading is the true voltage across the resistance to be measured but the ammeter carries the total current which is sum of the current through resistance and the current through voltmeter.

$$\therefore \quad I_t = I_x + I_v$$

$$= \frac{V}{R_x} + \frac{V}{R_v}$$

where R_v = voltmeter resistance

$$\therefore \quad R_m = \frac{V}{I} = \frac{V}{\left[\frac{V}{R_x} + \frac{V}{R_v}\right]}$$

$$\therefore \quad R_m = \frac{R_x R_v}{R_x + R_v}$$

$$\therefore \quad R_x + R_v = \frac{R_x R_v}{R_m}$$

$$\therefore \quad R_x \left[1 - \frac{R_v}{R_m}\right] = -R_v$$

$$\therefore \quad R_x = \frac{-R_v R_m}{R_m - R_v} = \frac{R_m}{1 - \frac{R_m}{R_v}} \qquad \text{adjusting –ve sign}$$

Thus the actual value of resistance $\mathbf{R_x}$ is same as the measured value $\mathbf{R_m}$ if $\mathbf{R_m/R_v}$ is zero i.e. the voltmeter resistance is infinite.

Thus the method gives the measured value which is smaller than the actual value.

But practically if the value of resistance to be measured is very small as compared to the voltmeter resistance ($R_x << R_v$) then the effect of R_m/R_v becomes negligibly small and method gives true value of resistance. Hence this circuit is used for the low resistance measurement.

The accuracy of voltmeter-ammeter method depends on the accuracy of the two meters. Similarly the reading is not available directly and required to be calculated from the readings. The division point between the two circuits is at the resistance value for which the relative errors due to both the methods are equal. This is possible for the resistance value given by,

$$R_x = \sqrt{R_a R_v}$$

For the resistance values greater than this value, the circuit in Fig. 6.1 (a) should be used while for the resistance values less than this, the circuit in the Fig. 6.1 (b) should be used.

To avoid this difficulty, a practical circuit shown in the Fig. 6.2 can be used.

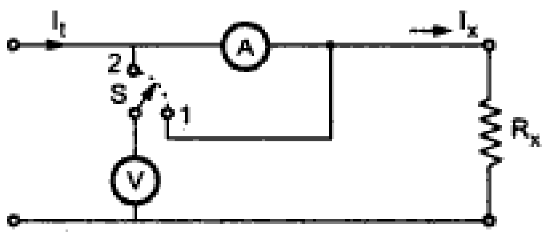


Fig. 6.2

While using this circuit, the switch S is used which is to be kept in position 1 and observe the ammeter reading. Change the switch to position 2. If the ammeter reading does not change, restore the switch in position 1. This indicates the value of resistance to be measured is low.

If the ammeter reading decreases after throwing the switch to position 2, keep it in the position 2 itself to take the readings. This indicates that the resistance to be measured is a high value resistance.

Thus the accurate value of resistance can be obtained.

6.4 Series Type Ohmmeter

Instead of measuring both voltage and current, many instruments keep one of the two quantities constant. Thus the measurement of other quantity is nothing but proportional to the value of the resistance. If current is kept constant, a voltmeter reading across the resistance is directly proportional to the value of the resistance. This is the principle of the **ohmmeter**. Similarly if the voltage is kept constant, an ammeter in series will have deflection proportional to the conductance but the meter can be calibrated in terms of the resistance.

The values of R_1 and R_2 can be determined from the value of R_h which is half scale deflection resistance.

$$R_h = R_1 + [R_2 \| R_m]$$

$$= R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

For half scale deflection, the battery current is,

$$I_h = \frac{V}{2R_h}$$

This is because the total resistance presented to the battery is then $2R_h$, as $R_x = R_h$.

To produce full scale deflection current, battery current must be doubled.

$$\therefore \quad I_t = 2I_h = \frac{2V}{2R_h} = \frac{V}{R_h}$$

When terminals A-B are shorted,

$$I_{fsd} = \text{meter current}$$

$$I_2 = \text{current through } R_2$$

$$\therefore \quad I_2 = I_t - I_{fsd}$$

Now voltage across shunt R_2 is the voltage across the meter.

$$\therefore \quad V_{sh} = V_m$$

$$\therefore \quad I_2 R_2 = I_{fsd} R_m$$

$$R_2 = \frac{I_{fsd} R_m}{I_2}$$

$$= \frac{I_{fsd} R_m}{I_t - I_{fsd}}$$

But $$I_t = \frac{V}{R_h}$$

$$\therefore \quad R_2 = \frac{I_{fsd} R_m}{\frac{V}{R_h} - I_{fsd}}$$

$$\therefore \quad \boxed{R_2 = \frac{I_{fsd} R_m R_h}{V - I_{fsd} R_h}} \qquad \dots (1)$$

Now $$R_h = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

$$\therefore \quad R_1 = R_h - \frac{R_2 R_m}{R_2 + R_m}$$

Substituting value of R_2 we get,

$$R_1 = R_h - \frac{\dfrac{I_{fsd} R_m R_h R_m}{V - I_{fsd} R_h}}{\dfrac{I_{fsd} R_m R_h}{V - I_{fsd} R_h} + R_m}$$

$$\therefore \quad \boxed{R_1 = R_h - \frac{I_{fsd} R_m R_h}{V}} \qquad \text{... (2)}$$

From the equations (1) and (2), the resistances R_1 and R_2 can be determined.

The series ohmmeter is a popular instrument and is used extensively.

Example 6.1 : *A 50 Ω basic movement requiring a full scale current of 1 mA is to be used as an ohmmeter. The internal battery voltage is 3 V. A half scale deflection marking desired is 1000 Ω. Calculate*

i) values of R_1 *and* R_2

ii) maximum value of R_2 *to compensate for a 5 % drop in battery voltage.*

Solution : The given values are, $R_h = 1000\ \Omega$, $R_m = 50\ \Omega$, $V = 3$ V, $I_{fsd} = 1$ mA

i) Now $$R_1 = R_h - \frac{I_{fsd} R_m R_h}{V}$$

$$= 1000 - \frac{1 \times 10^{-3} \times 50 \times 1000}{3}$$

$$= \mathbf{983.33\ \Omega}$$

and $$R_2 = \frac{I_{fsd} R_m R_h}{V - I_{fsd} R_h} = \frac{1 \times 10^{-3} \times 50 \times 1000}{3 - 1 \times 10^{-3} \times 1000} = \mathbf{25\ \Omega}$$

ii) Due to 5% drop in battery voltage, the voltage becomes,

$$V = 3 - 0.05 \times 3$$

$$= \mathbf{2.85\ V}$$

Hence the corresponding value of R_2 is,

$$R_2 = \frac{I_{fsd} R_m R_h}{V - I_{fsd} R_h} = \frac{1 \times 10^{-3} \times 50 \times 1000}{2.85 - 1 \times 10^{-3} \times 1000}$$

$$= \mathbf{27.027\ \Omega}$$

The Fig. 6.7 shows the basic Wheatstone bridge circuit.

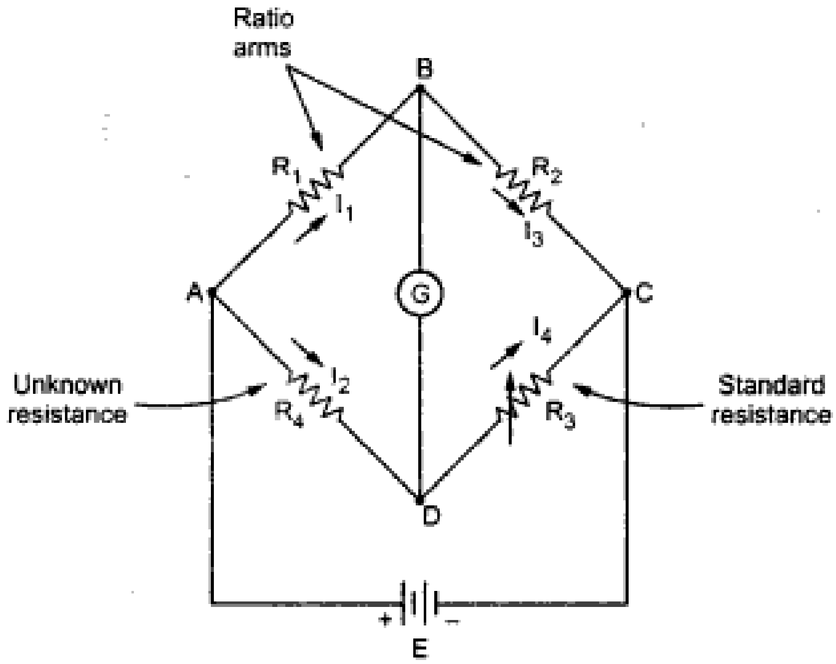


Fig. 6.7 Wheatstone bridge

The arms consisting the resistances R_1 and R_2 are called **ratio arms.** The arm consisting the standard known resistance R_3 is called **standard arm.** The resistance R_4 is the **unknown resistance** to be measured. The battery is connected between A and C while galvanometer is connected between B and D.

6.6.1 Balance Condition

When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection. Thus bridge works on the **principle of null deflection** or **null indication.**

To have zero current through galvanometer, the points B and D must be at the same potential. Thus potential across arm AB must be same as the potential across arm AD.

Thus $$I_1 R_1 = I_2 R_4 \quad \text{... (1)}$$

As galvanometer current is zero,

$$I_1 = I_3 \quad \text{and} \quad I_2 = I_4 \quad \text{... (2)}$$

Considering the battery path under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1 + R_2} \quad \text{... (3)}$$

and $$I_2 = I_4 = \frac{E}{R_3 + R_4} \quad \text{... (4)}$$

Using (3) and (4) in (1),

$$\frac{E}{R_1 + R_2} \times R_1 = \frac{E}{R_3 + R_4} \times R_4$$

$$\therefore \quad R_1 (R_3 + R_4) = R_4 (R_1 + R_2)$$

$$\therefore \quad R_1 R_3 + R_1 R_4 = R_1 R_4 + R_2 R_4$$

$$\therefore \quad \boxed{R_4 = R_3 \frac{R_1}{R_2}} \quad \text{... (5)}$$

This is required balance condition of Wheatstone bridge.

The following points can be observed,

1. It depends on the ratio of R_1 and R_2 hence these arms are called **ratio arms.**
2. As it works on null indication, the results are not dependent on the calibration and characteristics of galvanometer.
3. The standard resistance R_3 can be varied to obtain the required balance.

6.6.2 Industrial Form of Wheatstone Bridge

In an industrial or laboratory form of Wheatstone bridge, the resistances R_1, R_2 and R_3 are mounted in a box. The values are selected by the dial switches. The ratio R_1 / R_2 is adjusted as per the requirement using **ratio selector switch.** The resistances R_1 and R_2 generally consist of four resistors each of 10, 100, 1000 and 10,000 Ω respectively. The resistance R_3 can be adjusted using 4 dial or 5 dial decade arrangement and thus its value can be adjusted from 0 Ω to 10,000 Ω or 10,0000 Ω depending upon 4 or 5 dial decades available. The battery is connected across the battery terminals while the unknown resistance is connected across $X_1 - X_2$ terminals as shown in the Fig. 6.8. The galvanometer is connected across the terminals marked G. The Fig. 6.8 shows this commercial form of the Wheatstone bridge.

Please refer Fig. 6.8 on next page.

While using the bridge, connect R_4, battery and the galvanometer. Select the ratio R_1 / R_2 using ratio selector switch. Adjust R_3 till the galvanometer deflection is zero. Then R_4 is given by $(R_1 / R_2) R_3$. Repeat the set for various values of the ratio R_1 / R_2 and find the average to minimise the errors.

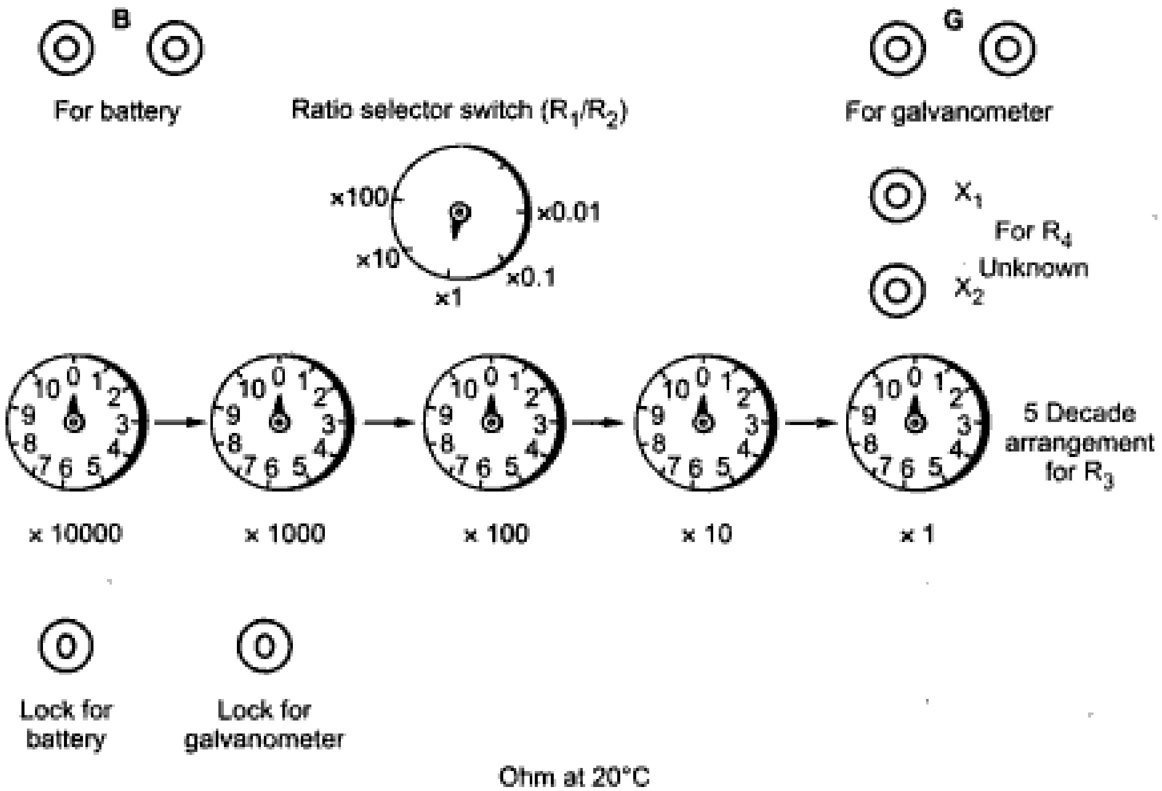


Fig. 6.8 Commercial form of Wheatstone bridge

6.7 Sensitivity of Wheatstone Bridge

When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection. The amount of deflection depends on the **sensitivity** of the galvanometer. This sensitivity can be expressed as amount of deflection per unit current.

$$\text{Sensitivity S} = \frac{\text{deflection D}}{\text{current I}}$$

As the current is in microampere and deflection can be measured in mm, radians or degrees, the sensitivity is expressed as mm/μA, radians/μA or degrees/μA. More is the sensitivity of a galvanometer, more is its deflection for the same amount of current.

Another way of representing the galvanometer sensitivity is the amount of deflection per unit voltage across the galvanometer. This is called **voltage sensitivity** of the galvanometer. Mathematically it is denoted as,

$$S_V = \frac{\theta}{e}$$

where e = voltage across galvanometer

θ = deflection of galvanometer

It is measured in degrees per volts or radians per volts.

While the **bridge sensitivity** is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance. It is denoted as S_B.

$$\therefore \quad S_B = \frac{\theta}{\Delta R / R}$$

where $\Delta R/R$ = unit fractional change in unknown resistance.

6.8 Wheatstone Bridge Under Small Unbalance

The bridge sensitivity can be calculated by solving the bridge for small unbalance.

At balance condition, $R_4 = R_3 \frac{R_1}{R_2}$

i.e. $\frac{R_4}{R_3} = \frac{R_1}{R_2}$

Let the resistance R_4 is changed by ΔR creating the unbalance. Due to this, the e.m.f. appears across the galvanometer. To obtain this e.m.f., let us use Thevenin's method. Remove the branch of galvanometer and obtain the voltage across the open circuit teminals.

$$E_{AB} = I_1 R_1 \quad \text{... (1)}$$

$$I_1 = \frac{E}{R_1 + R_2} \quad \text{... (2)}$$

$$E_{AD} = I_2 (R_4 + \Delta R) \quad \text{... (3)}$$

$$I_2 = \frac{E}{R_3 + R_4 + \Delta R} \quad \text{... (4)}$$

$$V_{BD} = V_{TH} = E_{AD} - E_{AB} \quad \text{... (5)}$$

$$\therefore \quad V_{TH} = \frac{E(R_4 + \Delta R)}{R_3 + R_4 + \Delta R} - \frac{E}{R_1 + R_2} R_1$$

$$\therefore \quad V_{TH} = E \left\{ \frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_1}{R_1 + R_2} \right\} \quad \text{... (6)}$$

As $\frac{R_4}{R_3} = \frac{R_1}{R_2}$ then $\frac{R_1}{R_1+R_2} = \frac{R_4}{R_4+R_3}$

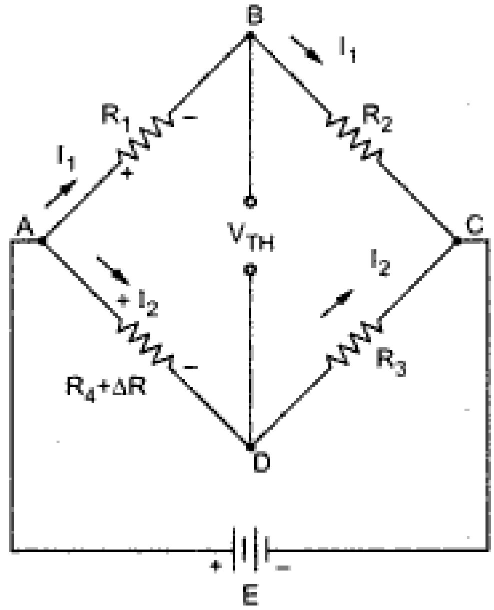


Fig. 6.9 Bridge under unbalance

Using above relation in equation (6),

$$V_{TH} = E\left\{\frac{R_4+\Delta R}{R_3+R_4+\Delta R} - \frac{R_4}{R_3+R_4}\right\}$$

$$= E\left\{\frac{R_3R_4 + R_3\Delta R + R_4^2 + R_4\Delta R - R_3R_4 - R_4^2 - R_4\Delta R}{(R_3+R_4)(R_3+R_4+\Delta R)}\right\}$$

$$= \frac{E R_3 \Delta R}{(R_3+R_4)^2 + (R_3+R_4)\Delta R}$$

But as ΔR is very small, $(R_3+R_4)\Delta R << (R_3+R_4)^2$

$$\therefore \quad \boxed{V_{TH} = V_g = \frac{E R_3 \Delta R}{(R_3+R_4)^2}} \quad ... (7)$$

Now $S_B = \frac{\theta}{\Delta R / R}$ = bridge sensitivity

and $\Delta R/R = \Delta R / R_4$ as there is change in R_4.

From the galvanometer sensitivity S_V,

$\theta = S_V \times e$ where e = voltage across galvanometer = V_g

Using θ in the expression of S_B,

Thus Thevenin's equivalent is as shown in the Fig. 6.11.

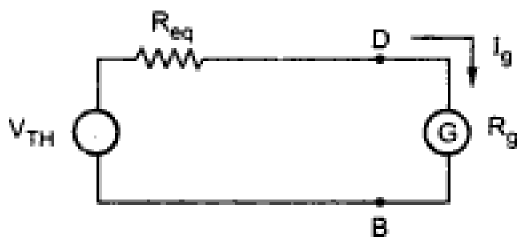


Fig. 6.11

Let R_g = Galvanometer resistance

I_g = Galvanometer current

$$\therefore \quad \boxed{I_g = \frac{V_{TH}}{R_{eq} + R_g}} \qquad \text{... (11)}$$

where V_{TH} = Thevenin's voltage

6.8.2 Galvanometer Current Under Unbalanced Condition

Let the resistance R_4 is changed by ΔR which has caused the unbalance in the bridge.

As derived earlier,

$$V_{TH} = \frac{E\, R_3\, \Delta R}{(R_3 + R_4)^2} \qquad \text{... (12)}$$

and now

$$R_{eq} = (R_1 \| R_2) + (R_3 \| R_4 + \Delta R)$$

$$= \frac{R_3 (R_4 + \Delta R)}{R_3 + R_4 + \Delta R} + \frac{R_1 R_2}{R_1 + R_2}$$

Neglecting ΔR compared to R_3 and R_4,

$$R_{eq} = \frac{R_3 R_4}{R_3 + R_4} + \frac{R_1 R_2}{R_1 + R_2} \qquad \text{... (13)}$$

$$\therefore \quad I_g = \frac{V_{TH}}{R_{eq} + R_g} \qquad \text{... (14)}$$

For **bridge with equal arms** $R_1 = R_2 = R_3 = R_4 = R$ then

$$V_{TH} = \frac{E\, R\, \Delta R}{4\, R^2} = \frac{E\, \Delta R}{4\, R}$$

and

$$R_{eq} = \frac{R^2}{2\, R} + \frac{R^2}{2R} = R$$

$$\therefore \quad \boxed{I_g = \frac{\frac{E\, \Delta R}{4R}}{R + R_g} = \frac{E(\Delta R / 4R)}{R + R_g}} \qquad \text{... (15)}$$

6.8.3 S_B Interms of Current Sensitivity of Galvanometer

The deflection of galvanometer for a small change in unknown resistance R_4 is,

$$\theta = S_V\ e = S_V\ V_g = \frac{S_V\ E\, R_3\ \Delta R}{(R_3 + R_4)^2}$$

While $$S_V = \frac{S_i}{R_{eq} + R_g}$$

where S_i = Current sensitivity of galvanometer

$$\therefore \quad \theta = \frac{S_i\ E\, R_3\ \Delta R}{(R_{eq} + R_g)(R_3 + R_4)^2}$$

and $$S_B = \frac{\theta}{\Delta R / R_4} = \frac{S_i\, E R_3 R_4}{(R_{eq} + R_g)(R_3 + R_4)^2} \quad \text{... (16)}$$

where $R_{eq} = (R_1 || R_2) + (R_3 || R_4 + \Delta R)$

This is the bridge sensitivity interms of current sensitivity of the galvanometer.

Key Point: *Practically R_{eq} can be assumed to be R_{TH} as ΔR is small compared to actual values of the resistances.*

$$\therefore \quad R_{eq} \approx R_{TH} = [R_1 || R_2] + [R_3 || R_4]$$

6.9 Measurement Errors

The Wheatstone bridge is used to measure the resistances in the range 1 Ω to few megaohms. But certain errors occur during the measurement using the Wheatstone bridge. These errors are as follows :

i) The main error is because of limiting errors of the three known resistances. Hence very precise resistances are required having tolerance of 1% or even 0.1%.

ii) The insufficient sensitivity of the null detector may cause the error.

iii) **Heating effect :** When the current passes through the resistances, due to the heating effect (I^2R) the temperature increases. Hence the values of the resistances of the bridge arms change due to the heating effect. The excessively high current may cause the permanent change in the resistance values. This may cause the serious error in the measurement. To avoid this, power dissipation in the arms must be calculated well in advance and currents must be limited to a safe value.

It is used by the telephone companies to locate the cable faults. The faults may be of the type line to line short or line to ground short.

Example 6.2 : *The Wheatstone bridge is shown in the Fig. 6.12. Calculate the value of unknown resistance, assuming the bridge to be in balanced condition.*

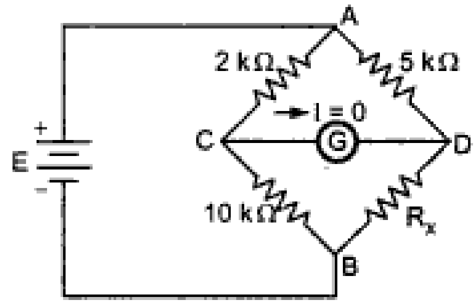


Fig. 6.12

Solution : As per the bridge shown in the Fig. 6.7 earlier,

R_1 = 10 kΩ, R_2 = 2 kΩ, R_3 = 5 kΩ and $R_4 = R_x$

Under balanced condition,

$$R_4 = R_x = \frac{R_1}{R_2} R_3 = \frac{10}{2} \times 5 = 5 \times 5 = \mathbf{25\ k\Omega}$$

Thus unknown resistance is **25 kΩ.**

Example 6.3 : *Calculate the current through the galvanometer for the bridge shown in the Fig. 6.13.*

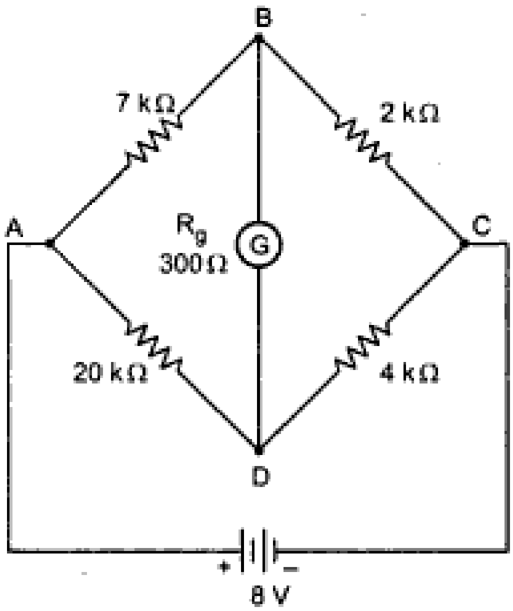


Fig. 6.13

Solution : From the Fig. 6.13.

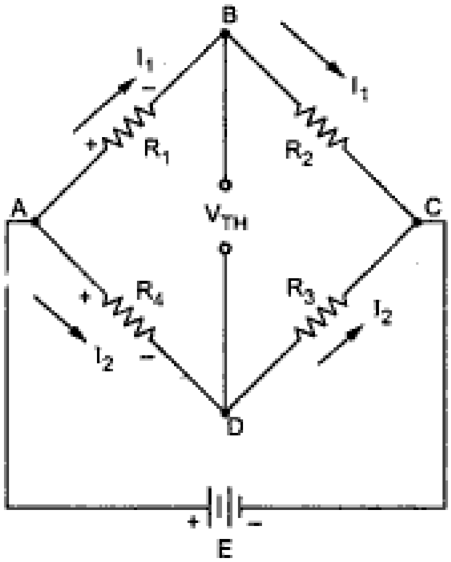


Fig. 6.14

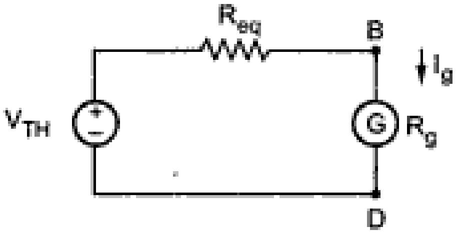


Fig. 6.15

$R_1 = 7\ k\Omega,\quad R_2 = 2\ k\Omega,$

$R_3 = 4\ k\Omega,\quad R_4 = 20\ k\Omega,\quad E = 8\ V.$

Use Thevenin's equivalent for I_g.

$$V_{TH} = V_{BD} = V_{AD} - V_{AB}$$

$$= I_2 R_4 - I_1 R_1$$

$$= \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$= 8\left\{\frac{20}{20+4} - \frac{7}{7+2}\right\}$$

$$= 0.444\ V$$

Thus B is positive w.r.t D.

Now $R_{eq} = [R_1 \| R_2] + [R_3 \| R_4]$ ... with E shorted

$$= \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 4.888\ k\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{0.444}{4.888 \times 10^3 + 300}$$

$$= \mathbf{85.62\ \mu A}$$

This is the current through the galvanometer.

6.12 Carey-Foster Slide Wire Bridge

A **Carey-Foster slide wire bridge** is the elaborated form of the Wheatstone bridge. This type of the bridge is most extensively used for the comparison of the two nearly equal resistances. The circuit arrangement for the Carey-Foster bridge is as shown in the Fig. 6.16.

Please refer Fig. 6.16 on next page.

The arms consisting resistances P and Q are nominal equal ratio arms. The resistance R is the resistance under test; while S is the standard resistance. A slide wire of length L is introduced between the resistances R and S as shown in the Fig. 6.16.

Initially a balanced condition is obtained by adjusting the sliding contact on the slide wire at a distance l_1 from the left hand side of the slide wire. After this the positions of resistances R and S are interchanged and a new balance point is obtained. Let the distance from the left hand of the slide wire be l_2.

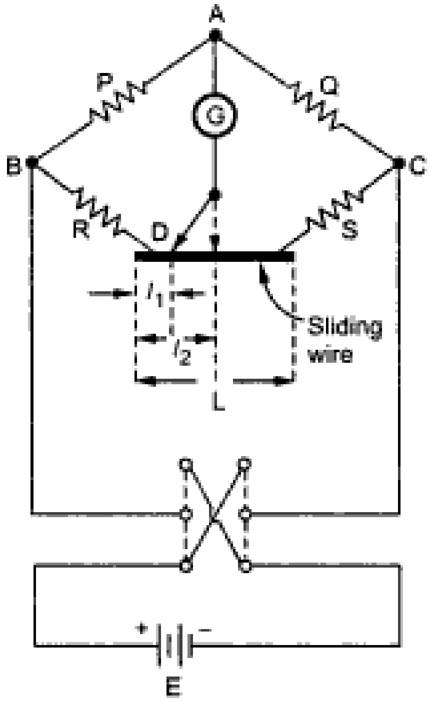


Fig. 6.16 Carey-Foster slide wire bridge

The first balance condition is given by,

$$\boxed{\frac{P}{Q} = \frac{R + l_1 r}{S + (l - l_1)\, r}} \quad \text{... (1)}$$

where r = resistance of slide wire per unit length of the slide wire.

Similarly the second balance condition is given by,

$$\boxed{\frac{P}{Q} = \frac{S + l_2 r}{R + (l - l_2)\, r}} \quad \text{... (2)}$$

Comparing equations (1) and (2), we can write,

$$\frac{R + l_1 r}{S + (l - l_1)\, r} = \frac{S + l_2 r}{R + (l - l_2)\, r}$$

Adding 1 on both the sides of above equations, we get,

$$\frac{R + l_1 r}{S + (l - l_1)\, r} + 1 = \frac{S + l_2 r}{R + (l - l_2)\, r} + 1$$

$$\therefore \quad \frac{R + l_1 r + S + lr - l_1 r}{S + (l - l_1)\, r} = \frac{S + l_2 r + R + lr - l_2 r}{R + (l - l_2)\, r}$$

$$\therefore \quad \frac{R + S + lr}{S + (l - l_1)\, r} = \frac{S + R + lr}{R + (l - l_2)\, r}$$

$$\therefore \qquad R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \qquad \dots (7)$$

Now $\qquad R_{cb} + R_{ab} = R_y$

$$\therefore \qquad R_{cb} = R_y - R_{ab} \qquad \dots (8)$$

Substituting (7) into (8),

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2} = R_y \left[1 - \frac{R_2}{R_1 + R_2}\right]$$

$$\therefore \qquad R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \qquad \dots (9)$$

Substituting these values of R_{cb} and R_{ab} in the equation (4) we get,

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$\therefore \qquad R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$$\therefore \qquad \boxed{R_x = \frac{R_1 R_3}{R_2}} \qquad \dots (10)$$

Thus equation (10) represents standard bridge balance equation for the Wheatstone bridge. Thus the effect of the connecting lead resistance is completely eliminated by connecting the galvanometer to an intermediate position 'b'.

This principle forms the basis of the construction of Kelvin's Double Bridge which is popularly called Kelvin Bridge.

6.14 Kelvin's Double Bridge Method for Low Resistance Measurement

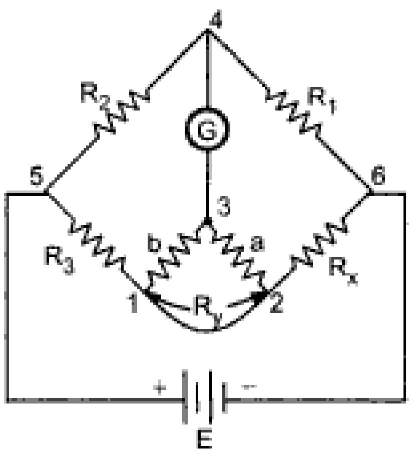


Fig. 6.18 Kelvin's double bridge

This bridge consists of another set of ratio arms hence called **double bridge**. The Fig. 6.18 shows the circuit diagram of Kelvin's Double Bridge.

The second set of ratio arms is the resistances 'a' and 'b'. With the help of these resistances the galvanometer is connected to point '3'. The **galvanometer gives null indication** when the potential of the terminal '3' is same as the potential of the terminal '4'.

$$\therefore \quad E_{513} = I\,R_3 + I\,\frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]$$

$$\therefore \quad E_{513} = I\left[R_3 + \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]\right] \quad \ldots (7)$$

Now $E_{45} = E_{513}$ ... for balancing

$$\therefore \quad \frac{I R_2}{R_1+R_2}\left[R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}\right] = I\left[R_3+\frac{b}{a+b}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2}\left[R_3+\frac{b}{a+b}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore \quad R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = \left[1+\frac{R_1}{R_2}\right]\left[R_3+\frac{b\,R_y}{R_y+a+b}\right]$$

$$\therefore \quad R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = R_3+\frac{R_1R_3}{R_2}+\frac{b\,R_y}{R_y+a+b}+\frac{R_1 b\,R_y}{R_2(R_y+a+b)}$$

$$\therefore \quad R_x = \frac{R_1R_3}{R_2}+\frac{b\,R_y}{R_y+a+b}+\frac{R_1 b\,R_y}{R_2(R_y+a+b)}-\frac{(a+b)R_y}{(R_y+a+b)}$$

$$\therefore \quad R_x = \frac{R_1R_3}{R_2}+\frac{b\,R_1\,R_y}{R_2(R_y+a+b)}-\frac{aR_y}{(a+b+R_y)}$$

$$\therefore \quad \boxed{R_x = \frac{R_1R_3}{R_2}+\frac{b\,R_y}{(R_y+a+b)}\left[\frac{R_1}{R_2}-\frac{a}{b}\right]} \quad \ldots (8)$$

But $\frac{a}{b} = \frac{R_1}{R_2}$ thus $\frac{R_1}{R_2}-\frac{a}{b}=0$

$$\therefore \quad \boxed{R_x = \frac{R_1R_3}{R_2}} \quad \ldots (9)$$

This is the standard equation of the bridge balance. The resistances a, b and R_y are not present in this equation. **Thus the effect of lead and contact resistances is completely eliminated.**

Key Point : *The important condition for this bridge balance condition is that the ratio of the resistances of ratio arms must be same as the ratio of the resistances of the second ratio arms.*

In a typical Kelvin's double bridge, the range of a resistance covered is 1 Ω to 10 μΩ with an accuracy of ± 0.05 % to ± 0.2 %.

The ratio of R_1 and R_2 is selected in such a way that the larger part of the variable standard resistance is used and hence R_x is determined to the largest possible number of significant figures. This increases the measurement accuracy.

Example 6.4 : *In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridge uses,*

Standard resistance = 100.03 μΩ

Inner ratio arms = 100.31 Ω and 200 Ω

Outer ratio arms = 100.24 Ω and 200 Ω

The resistance of the connecting leads from standard to unknown resistance is 700 μΩ. Calculate the unknown resistance under this condition.

Solution : From the given data,

$$R_3 = 100.03\ \mu\Omega,\ R_2 = 100.24\ \Omega,\ R_1 = 200\ \Omega$$

$$b = 100.31\ \Omega,\ a = 200\ \Omega,\ R_y = 700\ \mu\Omega$$

Thus unknown resistance is,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{b R_y}{[R_y + a + b]}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

... Refer equation (8) of section 6.14.

$$= \frac{200 \times 100.03 \times 10^{-6}}{100.24} + \frac{100.31 \times 700 \times 10^{-6}}{[700 \times 10^{-6} + 200 + 100.31]}\left\{\frac{200}{100.24} - \frac{200}{100.31}\right\}$$

$$= 1.9958 \times 10^{-4} + (2.3381 \times 10^{-4})(1.3923 \times 10^{-3})$$

$$= 1.999 \times 10^{-4}\ \Omega = \mathbf{199.905\ \mu\Omega}$$

6.15 Measurement of High Resistance

The measurement of high resistance of the order of hundreds and thousands of megaohms is often required in electrical equipments. The examples of such resistances are :

i) Insulation resistance of components like machines, cables etc.

ii) Leakage resistance of capacitors

iii) Resistance of high circuit elements like vacuum tube circuits.

But there are certain difficulties in measurement of such high resistances. Because of very high resistance, very small currents flow through the measuring circuits, which is very difficult to sense. The various other difficulties are :

i) Presence of **leakage currents** : The leakage currents are produced and are of comparable magnitude to the current being measured. Such currents cause errors. These currents depend on humidity and hence are unpredictable. Hence leakage currents mus. be eliminated from the measurement.

ii) The stray charges may appear due to electrostatic effect. Such charges and alternating fields can also cause serious measurement errors.

iii) One point of the circuit may be connected to earth for accuracy in measurements.

iv) When the voltage is applied to the insulation resistance, it takes some time for charging and absorbing currents. The measurement should be delayed till these currents vanish completely. In some cases, this may take very long time hence the testing conditions include the time between the application of voltage and the observation of the reading. This time must be specified for the accuracy.

v) Very high voltage is required in order to raise the current magnitudes. The galvanometer should be very sensitive and proper steps must be taken to prevent the damage of galvanometer due to high voltages.

6.15.1 Use of Guard Circuits

Some form of guard circuits are generally used to eliminate errors due to leakage currents. The Fig. 6.21 shows the basic principle of guard circuit.

The high resistance mounted on a piece of insulating material is measured by ammeter - voltmeter method. In Fig. 6.21 (a), it can be seen that the microammeter carries the current which is a sum of leakage current I_L and the resistance current I_R. Hence the reading obtained by such measurement will not be a true reading due to the error caused by leakage current.

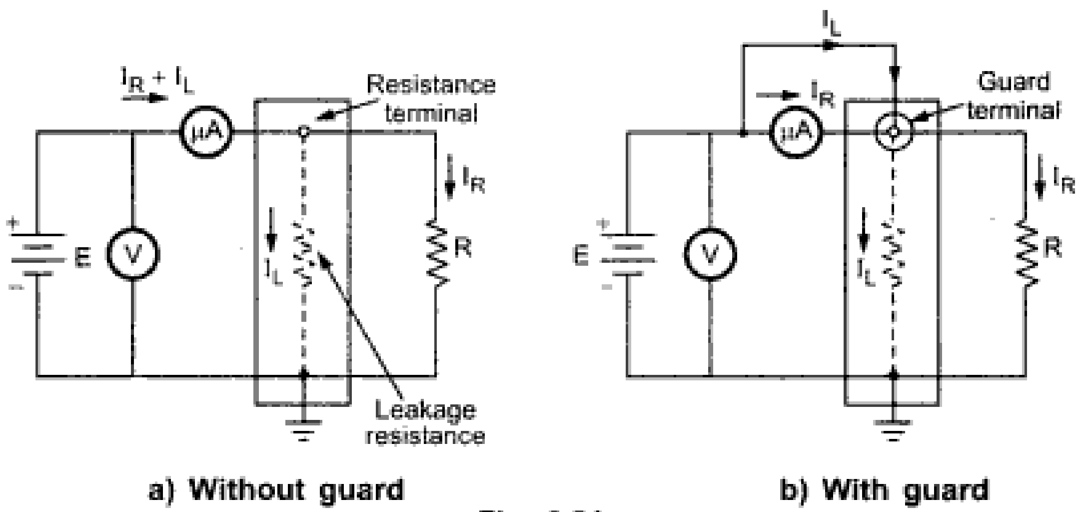


Fig. 6.21

For this, a guard terminal is added to the resistance terminal block, as shown in the Fig. 6.21 (b). This terminal surrounds the resistance entirely and is connected to the battery side of the microammeter. The leakage current I_L has now a separate path and it bypasses the microammeter. The current through microammeter is I_R only and hence the high resistance can be determined correctly. The guard and resistance terminal are at the same potential hence no current can flow in between them.

The same principle is used to guard the Wheatstone bridge. This is shown in the Fig. 6.22.

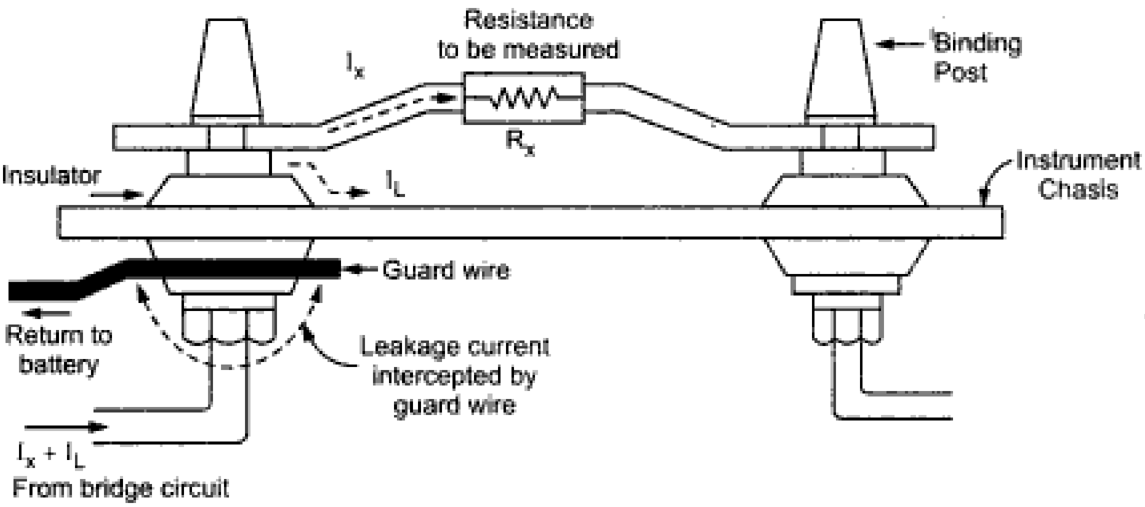


Fig. 6.22 Simple guard circuit

Without guard, the leakage current I_L along the insulated surface adds to the current I_x causing the error. A guard wire shown thick, completely surrounds the surface of the insulated post and obstructs the leakage current and returns it back towards battery. In schematic diagram, the guard around the R_x binding post is shown by small circle around the terminal. It does not touch to any part of the bridge and is directly connected to positive of battery. The Wheatstone bridge with guard terminal is shown in the Fig. 6.23.

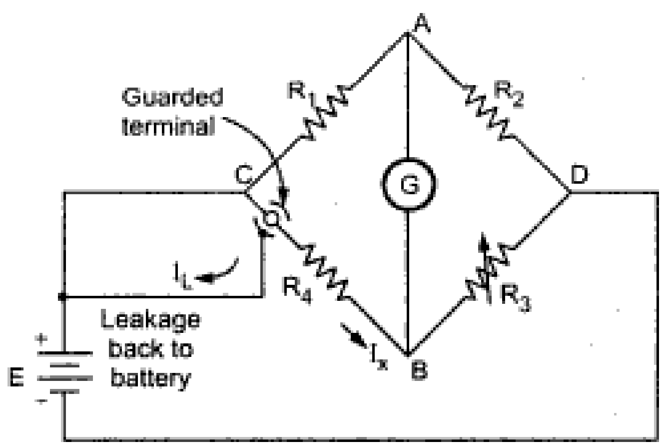


Fig. 6.23 Guarded Wheatstone bridge

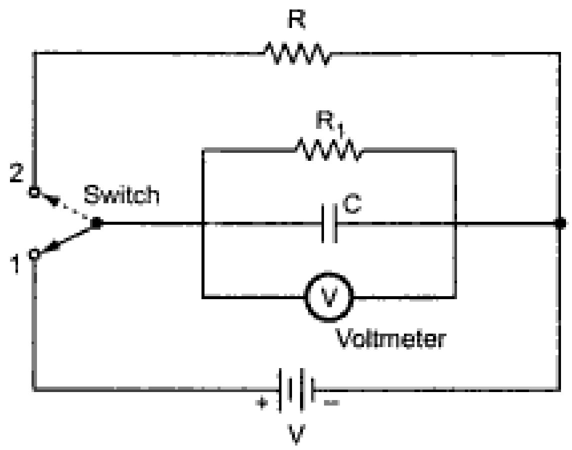


Fig. 6.28 Measurement of high resistance using loss of charge method include leakage resistance of C

The circuit consists high insulation resistance R to be measured alongwith capacitor of known value C shunted with electrostatic voltmeter and leakage resistance R_1.

Initially, capacitor C is charged to suitable voltage say V_1 by moving switch to position 1. Then switch is moved to position 2. The capacitor starts discharging through parallel combination of R and R_1. At certain instant t voltage across capacitor C i.e. V_2 is measured using electrostatic voltmeter. Thus in time t, voltage across capacitor drops down from V_1 to V_2. Let the equivalent resistance through which C discharges be denoted by R' where $R \parallel R_2$.

The expression for current any instant t is given by,

$$i = -\frac{dq}{dt} = -C\frac{dV}{dt} \quad \text{... (2)}$$

But $$i = \frac{\text{Potential drop across R'}}{R'} = \frac{V}{R'} \quad \text{... (3)}$$

Comparing equations (2) and (3), we can write,

$$\frac{V}{R'} = -C\frac{dV}{dt} \quad \text{or} \quad \frac{dV}{V} = -\frac{dt}{RC}$$

Integrating both sides,

$$[ln\ V]_{V_1}^{V_2} = \left[\frac{-t}{R'C}\right]_0^t$$

i.e. $$ln\frac{V_2}{V_1} = -\frac{t}{R'C}$$

$\therefore$ $$V_2 = V_1 e^{-\frac{t}{R'C}} \quad \text{... (4)}$$

Thus if time t is known, the resistance R' can be obtaining by measuring voltages V_1 and V_2. The same test is repeated with unknown resistance R removed. Then C discharges through only R_1. Then expression is given by,

$$\boxed{V_2 = V_1 \, e^{-\frac{t}{R_1 C}}} \quad \text{... (5)}$$

Thus the value of the leakage resistance of the capacitor can also be found out using this method. Note that in this method, leakage resistance of the voltmeter can be neglected if its value of low. If it is high, then it must be considered alongwith R_1.

6.16.3 Megohm Bridge

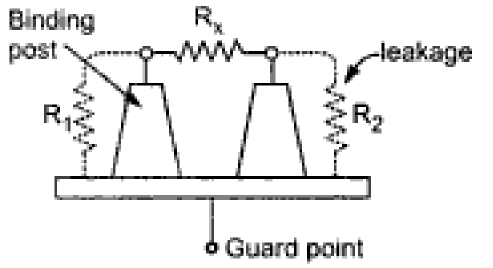


Fig. 6.29 Three terminal resistance

To avoid the leakage current external to the bridge, the junction of ratio arms R_A and R_B is brought out as a separate guard terminal on the front panel of the instrument. This can be used to connect **three terminal resistance**. The three terminal resistance is shown in the Fig. 6.29.

The high resistance is connected between two binding posts which are fixed to metal plate. The two main terminals of the resistor are connected to the R_x terminals in the bridge. The third terminal is the common point of resistances R_1 and R_2, which represents the leakage paths, from the main terminal along the insulating post of the metal plate. The guard is connected to the guard terminal on the front panel of the bridge as shown in the Fig. 6.30.

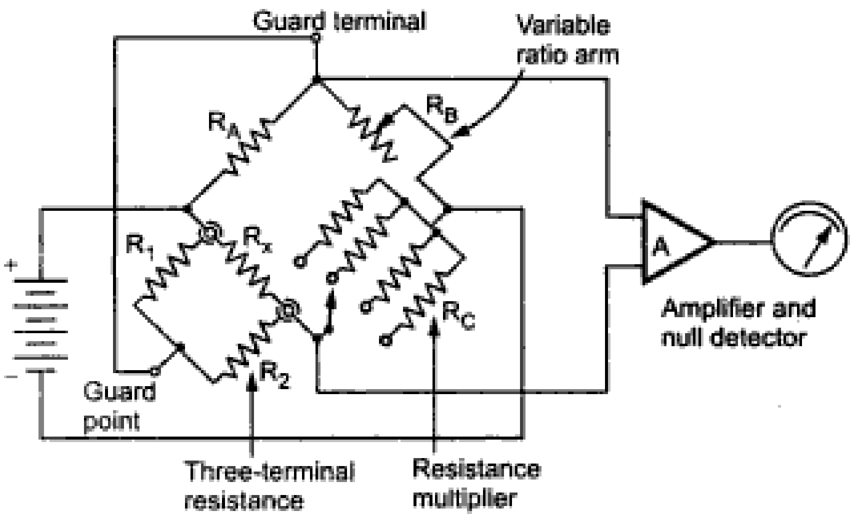


Fig. 6.30 Guarded bridge with three terminal resistor

This connection puts R_1 in parallel with ratio arm resistance R_A but since R_1 is very much larger than R_A, its shunting effect is negligible. Similarly R_2 in parallel with galvanometer has no effect as R_2 is much higher than galvanometer. It slightly reduces the sensitivity of the galvanometer. Thus the effect of external leakage path can be removed by using the guard circuit on the three terminal resistance.

If guard circuit is absent, leakage resistances R_1 and R_2 would be placed directly across R_x and giving large error in the measurement.

6.16.4 Megger

Resistances of the order of 0.1 MΩ and upwards are classified as high resistances.These high resistances are measured by portable, instrument known as **megger**. It is also used for testing the insulation resistance of cables.

6.16.4.1 Principle of Operation

It is based on the principle of electromagnetic induction. The Fig 6.31 shows the construction of megger.

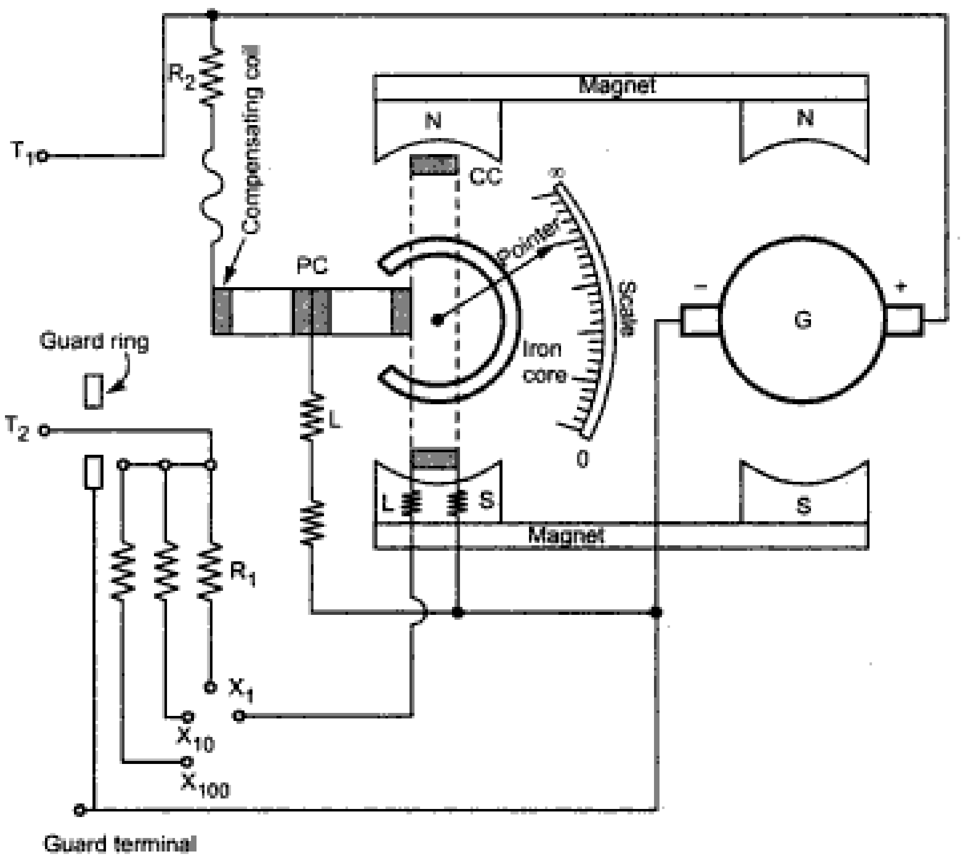


Fig. 6.31 Megger

When a current carrying conductor is placed in a uniform magnetic field it experiences a mechanical force whose magnitude depends upon the strength of current and magnetic field. While its direction depends on the direction of current and magnetic field.

6.16.4.2 Construction

It consists of a permanent magnet which provides the field for both the generator G and ohmmeter. The moving element of the ohmmeter consist of three coil viz. current or deflection coil, pressure or control coil and compensating coil. These coils are mounted on a central shaft which are free to rotate over a stationary C-shaped iron core.

The coils are connected to the circuit through flexible leads called ligaments which do not produce a restoring torque on the moving element, consequently the moving element takes up any position over the scale when the generator handle is stationary.

The current coil is connected in series with resistance R_1 between one generator terminal and the test terminal T_2. The series resistance R_1 protects the current coil in the event of the test terminals getting short circuited and also controls the range of the instrument. The pressure coil, in series with a compensating coil and protection resistance R_2 is connected across the generator terminals. The compensating coil is included in the circuit to ensure better scale proportions. The scale is calibrated reversely means the normal position of pointer indicates infinity while full scale deflection indicates zero resistance.

6.16.4.3 Working

When the current flows from the generator, through the pressure coil, the coil tends to set itself at right angles to the field of the permanent magnet.

When the test terminals are open, corresponding to infinite resistance, no current flows through deflection coil. Thus the pressure coil governs the motion of the moving element making it move to its extreme anticlockwise position. The pointer comes to rest at the infinity end of the scale.

When the test terminals are short circuited i.e. corresponding to zero resistance, the current from the generator flowing through the current coil is large enough to produce sufficient torque to overcome the counter-clockwise torque of the pressure coil. Due to this, pointer moves over a scale showing zero resistance.

When the high resistance to be tested is connected between terminals T_1 and T_2 the opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale. The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.

The guard ring is provided to eliminate the error due to leakage current. The supply to the meter is usually given by a hand-driven permanent magnet d.c. generator sometimes motor-driven generator may also be used.

6.16.4.4 Applications

The megger can be used to determine whether there is sufficiently high resistance between the conducting part of a circuit and the ground. This resistance is called insulation resistance.

The megger can also be used to test continuity between any two points. When connected to the two points, if pointer shows full deflection then there is an electrical continuity between them.

6.17 Bridge in Controlled Circuits

The unbalanced bridge causes a potential difference across the galvanometer. This potential causes current to flow through the galvanometer. If the bridge is used in control circuit, the galvanometer is not used and the potential developed due to unbalanced conditions across the two terminals is used to drive some other control circuit. This potential is called **output voltage** or **error voltage**.

While using bridge in such circuit, one arm of the bridge consists of a resistance which is sensitive to the parameter under control. The various parameters are strain, pressure, temperature, light etc, while parameter sensitive elements are strain gauges, temperature sensitive resistors (thermistors), photoresistors etc. Under normal value of the parameter, the sensitive resistance value R_V is such that R_2/R_V is same as R_1/R_3 and the bridge is perfectly balanced. The error signal is zero.

But when the physical parameter changes then R_V changes. This causes unbalance in the bridge which produces the error voltage. This voltage is used to drive some circuitry, which restores the condition of the parameter, thus bringing R_V back to its normal value, maintaining the bridge balance. The entire control action depends on the change in R_V and the production of the error voltage. Hence Wheatstone bridge used in such circuits is called **error detector.** The error voltage is generally very small and needs to amplify before using it for any control purposes.

The Wheatstone bridge used as an error detector is shown in the Fig. 6.32.

Please refer Fig. 6.32 on next page.

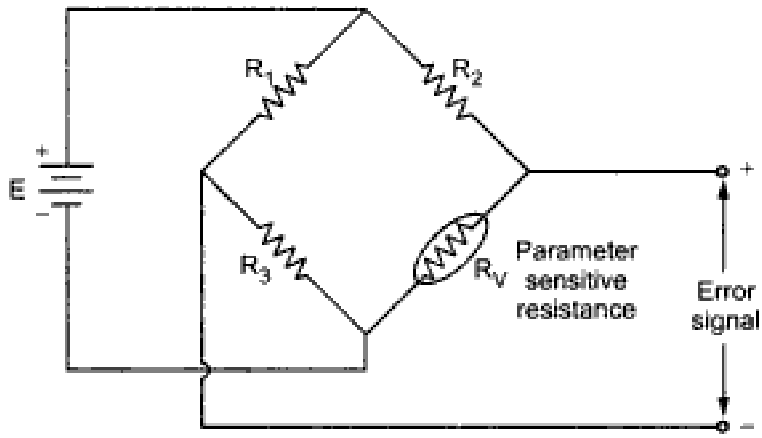


Fig. 6.32 Wheatstone bridge as error detector

Examples with Solutions

Example 6.5 : *The wheatstone bridge is shown in the Fig. 6.33. The galvanometer has a current sensitivity of 12 mm/μA. The internal resistance of galvanometer is 200 Ω. Calculate the deflection of the galvanometer caused due to 5 Ω unbalance in the arm BD.*

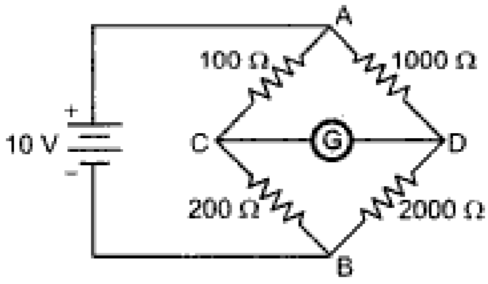


Fig. 6.33

Solution : From the given bridge,

$$R_1 = 100\ \Omega \quad R_2 = 1000\ \Omega$$

$$R_3 = 200\ \Omega \quad R_4 = 2000\ \Omega$$

Now $$R_1 R_4 = 100 \times 2000 = 200000$$

$$R_2 R_3 = 200 \times 1000 = 200000$$

For $R_4 = 2000\ \Omega$, the bridge is balanced. But there is unbalance of 5 Ω in the resistance of arm BD i.e. R_4.

$$R_4 = 2000 + 5$$

$$= 2005\ \Omega$$

Due to this imbalance current will flow through the galvanometer.

By Thevenin's equivalent,

$$V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 10\left[\frac{200}{100+200} - \frac{2005}{1000+2005}\right]$$

$$= 10\ [0.6667 - 0.6672]$$

$$= \mathbf{-\ 5.213\ mV}$$

The negative sign indicates that D is more positive than C.

$$R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

$$= \frac{100\times200}{(100+200)} + \frac{1000\times2005}{(1000+2005)}$$

$$= \mathbf{733.888\ \Omega}$$

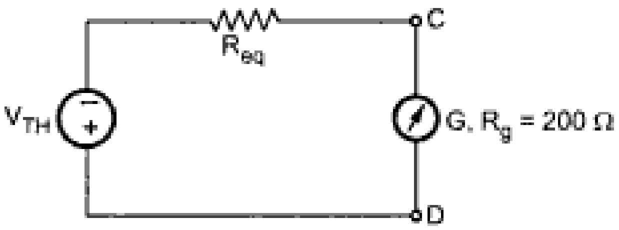


Fig. 6.33 (a)

Hence Thevenin's equivalent is,

$$\therefore \quad I_g = \frac{V_{TH}}{R_{eq}+R_g}$$

$$= \frac{5.213\times10^{-3}}{733.888+200}$$

$$= \mathbf{5.582\ \mu A}$$

Now deflection of galvanometer is proportional to its sensitivity.

$$S = \frac{D}{I}$$

$$\therefore \quad D = S\times I = 12\ mm/\mu A \times 5.582\ \mu A$$

$$= \mathbf{66.98\ mm}$$

Example 6.6 : *The four arms of the Wheatstone bridge have the following resistances, AB = 1000 Ω, BC = 1000 Ω, CD = 120 Ω, DA = 120 Ω. The bridge is used for strain measurement and supplied from 5 V ideal battery. The galvanometer has sensitivity of 1 mm/μA with internal resistance of 200 Ω. Determine the deflection of the galvanometer if arm DA increases to 121 Ω and arm CD decreases to 119 Ω.*

Solution : The bridge given is shown in the Fig. 6.34.

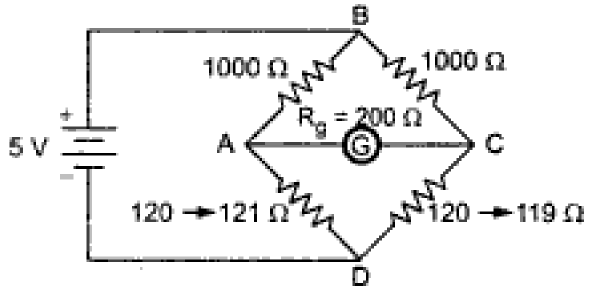


Fig. 6.34

Now $R_1 = 1000\ \Omega \quad R_2 = 1000\ \Omega$

$R_3 = 121\ \Omega \quad R_4 = 119\ \Omega$

Let us calculate Thevenin's equivalent due to change in R_3 and R_4.

$$V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 5\left[\frac{121}{1000+121} - \frac{119}{1000+119}\right]$$

$$= 5\ [0.1079 - 0.1063]$$

$$= \mathbf{7.975\ mV}$$

$$R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

$$= \frac{121\times1000}{121+1000} + \frac{119\times1000}{119+1000}$$

$$= 107.9393 + 106.3449$$

$$= \mathbf{214.2842\ \Omega}$$

Thevenin's equivalent circuit is,

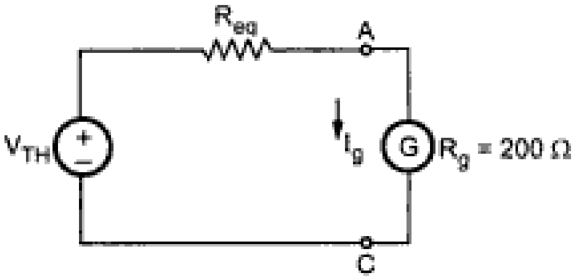


Fig. 6.34 (a)

$$I_g = \frac{V_{TH}}{R_{eq}+R_g}$$

$$= \frac{7.975\times10^{-3}}{214.2842+200}$$

$$= \mathbf{19.24\ \mu A}$$

Now the deflection of the galvanometer is proportional to its sensitivity.

$$S = \frac{D}{I}$$

$$\therefore \quad D = S \times I = 1 \text{ mm}/\mu A \times 19.24 \mu A$$

$$= \mathbf{19.24 \ mm}$$

This is the deflection of the galvanometer.

Example 6.7 : *Using the approximation of slightly unbalanced bridge, calculate the current through the galvanometer having internal resistance of 125 Ω, for the bridge shown in the Fig. 6.35.*

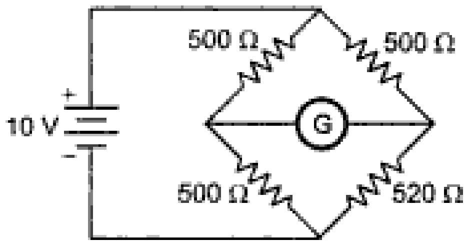


Fig. 6.35

Solution : For the bridge shown,

$$R = 500 \ \Omega \quad \text{and} \quad \Delta r = 20 \ \Omega$$

Using approximate result,

$$V_{TH} = \frac{E \Delta r}{4 R} = \frac{10 \times 20}{4 \times 500}$$

$$= \mathbf{0.1 \ V}$$

while $R_{eq} = R$

$$= \mathbf{500 \ \Omega}$$

$$R_g = 125 \ \Omega \quad \text{given}$$

$$\therefore \quad I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{0.1}{500 + 125}$$

$$= \mathbf{160 \ \mu A}$$

Example 6.8 : *In the Fig. 6.36, the Kelvin's double bridge is shown. The ratio of* R_a *to* R_b *is 1200 Ω while* R_1 *is 10 Ω and* $R_1 = 0.5\ R_2$*. Calculate the value of unknown resistance* R_x.

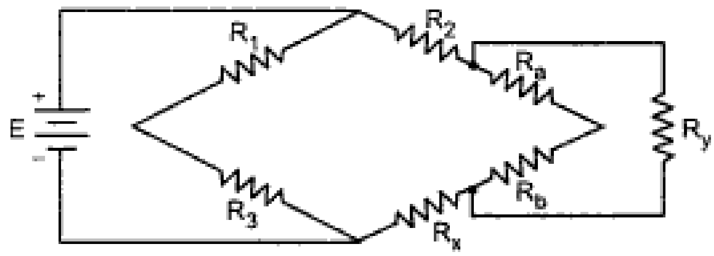


Fig. 6.36

Solution : From the Fig. 6.36 and the balance condition we can write,

$$\frac{R_x}{R_2} = \frac{R_3}{R_1}$$

For Kelvin's double bridge,

$$\frac{R_3}{R_1} = \text{ratio of resistances of ratio arms}$$

and $$\frac{R_b}{R_a} = \text{ratio of resistances of second ratio arms}$$

$$\therefore \quad \frac{R_3}{R_1} = \frac{R_b}{R_a}$$

$$\therefore \quad \frac{R_x}{R_2} = \frac{R_b}{R_a} = \frac{1}{1200}$$

Now $$R_1 = 10\ \Omega$$

$$R_1 = 0.5R_2$$

$$\therefore \quad R_2 = \frac{R_1}{0.5} = \frac{10}{0.5}$$

$$= \mathbf{20\ \Omega}$$

$$\therefore \quad \frac{R_x}{20} = \frac{1}{1200}$$

$$\therefore \quad R_x = \frac{20}{1200} = \mathbf{0.0167\ \Omega}$$

This is the value of unknown resistance R_x given by Kelvin's double bridge.

Example 6.9 : *The temperature dependent resistor is used in one arm of a wheatstone bridge. The other resistances are*

R_1=10 kΩ , R_2=10 kΩ and R_3 = 10 kΩ, E = 10 V

Calculate the temperature at which the bridge is balanced. Also calculate the error voltage at 60°C. The variation of resistance against temperature is shown in the Fig.6.37.

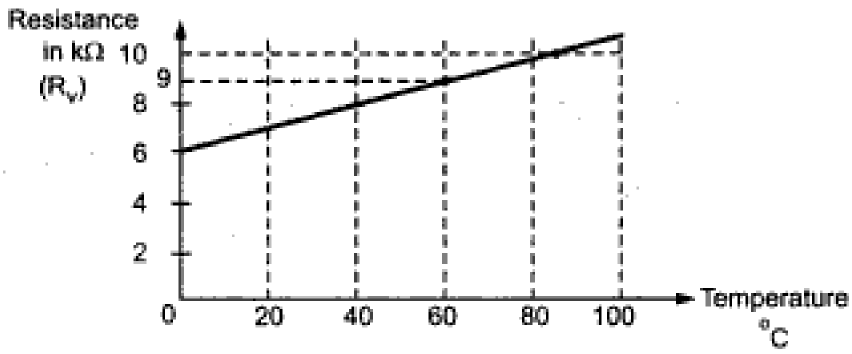


Fig. 6.37

Solution : For bridge balance,

$$\frac{R_2}{R_V} = \frac{R_1}{R_3}$$

$$\therefore \quad R_V = \frac{R_2 R_3}{R_1} = \frac{10 \times 10}{10}$$

$$= \mathbf{10\ k\Omega}$$

The R_V is 10 kΩ when temperature is 80 °C, from the Fig. 6.37. Thus bridge is balanced at 80 °C.

At 60 °C, the value of R_V is 9 kΩ from the Fig. 6.37. Thus the error voltage can be determined by Thevenin's equivalent voltage.

$$\therefore \quad e = E\left[\frac{R_3}{R_1 + R_3} - \frac{R_V}{R_2 + R_V}\right]$$

$$= 10\left[\frac{10}{10+10} - \frac{9}{10+9}\right]$$

$$= 10\ [0.5 - 0.4736] = 10\ [0.0264]$$

$$= \mathbf{0.264\ V}$$

The error voltage can also be determined by approximation of slightly unbalanced bridge.

$$\Delta r = 10 - 9 = 1 \text{ k}\Omega$$

$$\therefore \quad e = \frac{E\,\Delta r}{4\,R} = \frac{10 \times 1}{4 \times 10}$$

$$= \mathbf{0.25\ V}$$

Example 6.10 : *The Wheatstone bridge is used for the strain measurement. Determine the supply voltage required to produce 10 mV error output voltage when one arm of the bridge, sensitive to stress changes to 121 ohms due to stress while all other arms remain at 120 ohms.*

Solution : The bridge is shown in the Fig. 6.38.

Now under balanced condition $R_V = 120\ \Omega$.

The error voltage i.e. Thevenin's voltage is given as 10 mV.

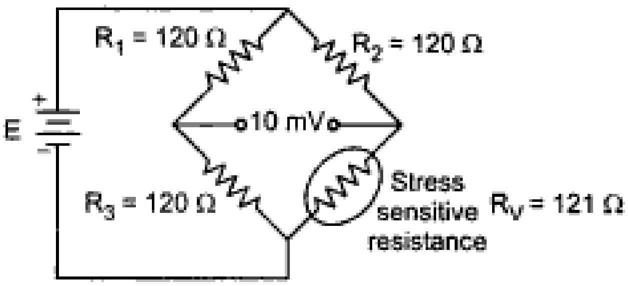


Fig. 6.38

$$\therefore \quad E_{TH} = E\left[\frac{R_3}{R_1 + R_3} - \frac{R_V}{R_2 + R_V}\right]$$

$$\therefore \quad 10 \times 10^{-3} = E\left[\frac{120}{120+120} - \frac{121}{120+121}\right]$$

$$\therefore \quad 10 \times 10^{-3} = E\,[0.5 - 0.5020]$$

$$\therefore \quad E = \frac{10 \times 10^{-3}}{-2.0746 \times 10^{-3}}$$

$$= \mathbf{-\,4.82\ V}$$

The negative sign indicates reversal of polarity. Thus the supply voltage required is 4.82 V.

For this problem, approximation of slightly unbalanced bridge also can be used.

$$E_{TH} = \frac{E\,\Delta r}{4\,R}$$

Now $\quad \Delta r = 121 - 120$

$$= 1\ \Omega$$

$$\therefore \quad 10\times10^{-3} = \frac{E\times 1}{4\times 120}$$

$$\therefore \quad E = 4\times120\times10\times10^{-3}$$

$$= \mathbf{4.8\ V}$$

Example 6.11 : *A Wheatstone's bridge circuit consists of ratio arms as 1000 Ω and 100 Ω. The adjustable arm is adjusted to its maximum of 4 kΩ. The supply voltage of 10 V is used for the bridge.*

i) Draw the circuit diagram.

ii) Determine maximum unknown resistance which can be measured.

iii) If galvanometer internal resistance is 80 Ω and its sensitivity is 70 mm/μA, find the unbalance in bridge required to cause the deflection of 3 mm if the unknown resistance equal to its maximum value is used in the circuit. Neglect internal resistance of the battery.

Solution : i) The circuit diagram is shown in the Fig. 6.39.

ii) At bridge balance ;

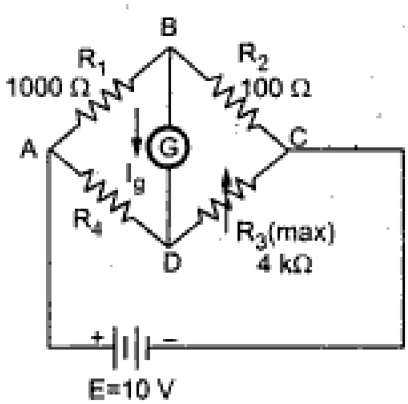


Fig. 6.39

$$R_1\,R_3 = R_2\,R_4$$

$$\therefore \quad R_4 = \frac{R_1\,R_3}{R_2} = \frac{1000\times4\times10^3}{100}$$

$$= \mathbf{40\ k\Omega}$$

iii) $R_{TH} = [R_1 \parallel R_2] + [R_3 \parallel R_4]$

$$= \left[\frac{1000\times100}{1000+100}\right] + \left[\frac{40\times10^3\times4\times10^3}{40\times10^3+4\times10^3}\right]$$

$$= 90.9090 + 3.6363\times10^3 = 3.7272\ k\Omega$$

S_i = Current sensitivity = 70 mm/μA ... Given

Now $$\theta = \frac{S_i\,E R_3\,\Delta R}{(R_{TH}+R_g)\,(R_3+R_4)^2}$$ ... Assuming $R_{eq} = R_{TH}$

$$\therefore \quad 3 = \frac{[70\times10^6]\times10\times4\times10^3\times\Delta R}{[3.7272\times10^3+80][4\times10^3+40\times10^3]^2}$$ ... $S_i = 70\times10^6$ mm/A

$$\therefore \quad \Delta R = \frac{3\times3807.2\times1.936\times10^9}{70\times10^6\times10\times4\times10^3} = \mathbf{7.8972\ \Omega}$$

This much unbalance is necessary to cause the deflection of 3 mm.

➠ **Example 6.12 :** *In a Wheatstone bridge, the resistance in each arm is 150 Ω. The maximum allowable power dissipation in each arm is 0.4 W. If the battery voltage is 25 V and internal resistance is 1 Ω then obtain the series resistance required between battery and bridge so as to keep current in each arm upto the permissible value.*

Solution : The bridge is shown in the Fig. 6.40. As the bridge is balanced, the galvanometer current is zero and the current in each arm is same as I amperes. So total current drawn from battery is '2I' amperes.

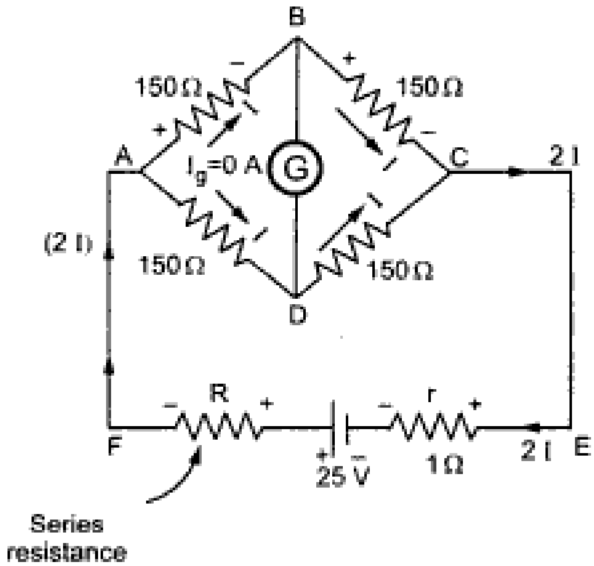


Fig. 6.40

$$P = \text{Power dissipation in each arm}$$

$$= I^2 R_{arm} = 0.4 \text{ W} \quad \text{... Given}$$

$$\therefore \quad I^2 = \frac{0.4}{150} = 2.666 \times 10^{-3}$$

$$\therefore \quad I = \mathbf{0.05163 \text{ A}}$$

This is permissible value of current through each arm.

Applying KVL to the loop ABCEFA,

$$-I \times 150 - I \times 150 - 2\,Ir + 25 - 2\,I\,R = 0$$

$$\text{Using } I = 0.05163 \text{ A}$$

$$\therefore \; -7.7445 - 7.7445 - 0.10326 + 25 - 0.10326\,R = 0$$

$$\therefore \quad R = \mathbf{91.1073\ \Omega}$$

This is the required series resistance.

a) At balance, $R = \frac{P}{Q} \cdot S = \frac{1000}{1000}(0.001) = \mathbf{0.001\ \Omega}$

b) Current under balance condition,

$$I = \frac{V}{R_b + R + S} = \frac{100}{5 + 0.001 + 0.001} = \mathbf{19.99\ A}$$

c) The value of R is changed by 0.1 %.

$\therefore$ New value of $R = 0.001 \times 0.1 = 0.0001\ \Omega$

$$V_{ac} = \left[\frac{R + r + S}{R_b + R + r + S}\right] V$$

Neglecting r,

$$V_{ac} = \frac{R + S}{R_b + R + S} V = \frac{0.0001 + 0.001}{5 + 0.0001 + 0.001} \times 100 = 29.995\ mV$$

$$V_{ab} = \frac{P}{P + Q} V_{ac} = \frac{1000}{1000 + 1000}(29.995 \times 10^{-3}) = 14.9978\ mV$$

$$V_{amd} = \left[\frac{R + \frac{P_r}{p + q + r}}{R + S + \frac{(p + q)\,r}{p + q + r}}\right] V_{ab}$$

Neglecting r,

$$V_{amd} = \frac{R}{R + S} V_{ab} = \frac{0.0001}{0.0001 + 0.001}(14.9978 \times 10^{-3})$$

$$= 1.3634\ mV$$

Hence output voltage is given as,

$$V_{out} = V_{ab} - V_{amd} = 14.9978\ mV - 1.3634\ mV$$

$$= \mathbf{0.01362\ V}$$

Example 6.16 : *A highly sensitive galvanometer can detect a current as low as 0.1 nA. This galvanometer is used in Wheatstone bridge as a detector. The resistance of galvanometer is negligible. Each arm of the bridge has a resistance of 1 kΩ. The input voltage applied to the bridge is 20 V. Calculate the smallest change in the resistance, which can be detected.* **[JNTU, Nov.-2003, Set-2]**

Solution : For the bridge,

$$R = 1000\ \Omega \quad E = 20\ V$$

The current which can be detected by the galvanometer is 0.1 nA.

$$\therefore \quad I_g = 0.1 \text{ nA} = 0.1 \times 10^{-9} \text{ A}$$

For small change in the resistance Δr , the Thevenin's approximate voltage is

$$V_{TH} = \frac{E\,\Delta r}{4\,R}$$

while $R_{eq} = R$

$$\therefore \quad I_g = \frac{V_{TH}}{R_{eq}} \quad \text{as } R_g = 0\,\Omega$$

$$\therefore \quad 0.1 \times 10^{-9} = \frac{E\,\Delta r}{4\,R \times R}$$

$$\therefore \quad 0.1 \times 10^{-9} = \frac{20 \times \Delta r}{4 \times 1000 \times 1000}$$

$$\therefore \quad \Delta r = \frac{4 \times 10^{6} \times 0.1 \times 10^{-9}}{20}$$

$$= 20\ \mu\Omega$$

Thus the smallest change in the resistance which can be detected is 200 μΩ.

Review Questions

1. *Explain the basic voltmeter-ammeter method used for the resistance measurement.*
2. *Explain the operation of*
   i) Series type ohmmeter
   ii) Shunt type ohmmeter
3. *State the advantages of using the bridge circuits for the measurement.*
4. *Explain the working of Wheatstone bridge and derive its balance condition.*
5. *What is the sensitivity of the Wheatstone bridge ?*
6. *Derive the expression for the current through the galvanometer in case of unbalanced Wheatstone bridge.*
7. *What are the sources of errors in case of the Wheatstone bridge ?*
8. *Explain the applications and the limitations of the Wheatstone bridge.*
9. *Why Kelvin's bridge is preferred ? Derive the bridge balance equation for the Kelvin's double bridge.*
10. *What are the difficulties in measurement of high resistance ? Explain the use of guard circuits.*
11. *What is three terminal resistance ? Explain its use.*
12. *How the bridge is used in the measurement of the parameters like stress, strain, temperature etc. ?*

13. *Explain loss of charge method for high resistance measurement.*
14. *Explain construction and working of megger. Write application of megger.*
15. *A bridge is shown in the Fig. 6.44. Calculate the current through galvanometer.*

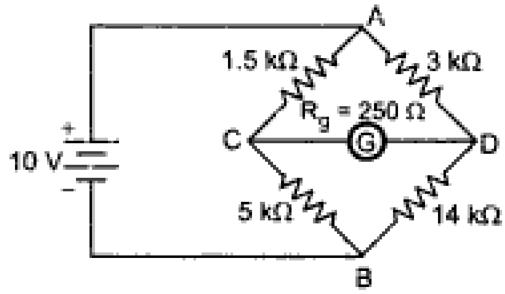


Fig. 6.44 **(Ans. : 138.76 μA)**

16. *A galvanometer is connected as shown in the Fig. 6.45. The galvanometer sensitivity is 8 mm/μA and internal resistance of 150 Ω. Calculate the deflection of galvanometer caused by 10 Ω unbalance in the arm BC.* **(Ans. : 3.5 mm)**

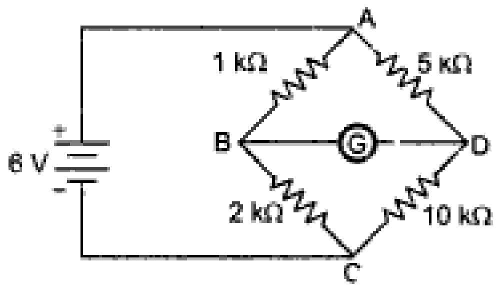


Fig. 6.45

17. *Given a centre zero 200-0-200 μA movement having an internal resistance of 125 Ω. Calculate the current through the galvanometer given in the Fig. 6.46 by approximate method.* **(Ans. : 151.5 μA)**

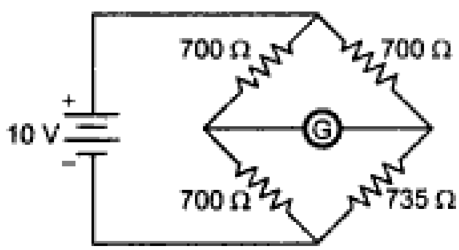


Fig. 6.46

❑❑❑

7

A.C. Bridges

7.1 Introduction

The bridges are used for not only the measurement of resistances but also used for the measurement of various component values like capacitance, inductance etc.

A **bridge circuit** in its simplest form consists of a network of four resistance arms forming a closed circuit. A source of current is applied to two opposite junctions. The current detector is connected to other two junctions.

The bridge circuits use the **comparison measurement** methods and operate on **null-indication principle**. The bridge circuit compares the value of an unknown component with that of an accurately known standard component. Thus the accuracy depends on the bridge components and not on the null indicator. Hence high degree of accuracy can be obtained.

In a bridge circuit, when no current flows through the null detector which is generally galvanometer, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called **balancing condition** or **balancing equation**. This equation gives us the value of the unknown component.

7.1.1 Advantages of Bridge Circuit

The various **advantages** of the bridge circuit are,

1) The balance equation is independent of the magnitude of the input voltage or its source impedance. These quantities do not appear in the balance equation expression.

2) The measurement accuracy is high as the measurement is done by comparing the unknown value with the standard value.

3) The accuracy is independent of the characteristics of a null detector and is dependent on the component values.

4) The balance equation is independent of the sensitivity of the null detector, the impedance of the detector or any impedance shunting the detector.

5) The balance condition remains unchanged if the source and detector are interchanged.

6) The bridge circuit can be used in the control circuits. When used in such control applications, one arm of the bridge contains a resistive element that is sensitive to the physical parameter like pressure, temperature etc. which is to be controlled.

7.2 Types of Bridges

The two types of bridges are,

1) D.C. bridges and 2) A.C. bridges

The **d.c. bridges** are used to measure the **resistances** while the **a.c. bridges** are used to measure the **impedances** consisting capacitances and inductances. The d.c. bridges use the d.c. voltage as the excitation voltage while the a.c. bridges use the alternating voltage as the excitation voltage.

The two types of d.c. bridges are,

1. Wheatstone bridge 2. Kelvin bridge

The various types of a.c. bridges are,

1. Capacitance comparison bridge
2. Inductance comparison bridge
3. Maxwell's bridge
4. Hay's bridge
5. Anderson bridge
6. Schering bridge
7. Wien bridge

Let us now discuss the various types of the bridges in detail.

7.3 A.C. Bridges

An a.c. bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance. The source is an a.c. supply which supplies a.c. voltage at the required frequency. For high frequencies, the electronic oscillators are used as the source. The balance detectors commonly used for a.c. bridges are head phones, tunable amplifier circuits or vibration galvanometers. The headphones are used as detectors at the frequencies of 250 Hz to 3 to 4 kHz. While working with single frequency a tuned detector is the most sensitive detector. The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz but are commonly used below 200 Hz. Tunable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.

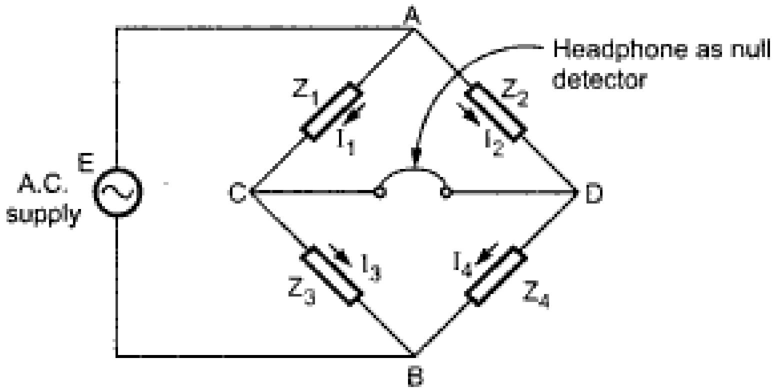


Fig. 7.1 A.C. Wheatstone bridge

The simple a.c. bridge is the outcome of the Wheatstone bridge. The impedances at audio and radio frequency range can be easily determined by such simple a.c. Wheatstone bridge. It is shown in the Fig. 7.1.

This is similar to d.c. Wheatstone bridge. The bridge arms are impedances. The bridge is excited by a.c. supply and pair of headphones is used as a null detector. The null response is obtained by varying one of the bridge arms.

7.3.1 Sources and Detectors

For bridge measurements at very low frequencies, the power line itself may act as a source of supply to the bridge circuit. For bridge measurements at higher frequencies electronic oscillators are used as a source of supply to the bridge circuit. These electronic oscillators are used as a source of supply universely because,

i) The output waveform is very close to sine wave.

ii) The output frequency is very stable.

iii) The output frequency is easily determinable with accuracy and also it is easily adjustable.

iv) The output power is sufficient to drive the bridge circuits.

A typical oscillator has a frequency range of 40 Hz to 125 kHz with power output of 7 W.

For the a.c. bridges commonly used detectors are as follows.

i) **Headphones :** These are the most commonly used as detectors in a.c. bridges. The frequency range over which headphones can be used as detector effectively is 250 Hz upto 3 to 4 kHz.

ii) **Vibration galvanometers :** For low audio frequency ranges and power ranges, these detectors are extremely effective. Even though these type of detectors are manufactured to work at various frequency ranges starting from 5 Hz to 1000 Hz. These detectors can be effectively used below 200 Hz with greater sensitivity than the headphones.

iii) **Tuneable amplifier detectors :** The transistor amplifier can be tuned electrically to any desired frequency and then it can be made to respond to a narrow bandwidth at a bridge frequency. The output of such amplifier is connected to the indicating instruments. The frequency range for these detectors is 10 Hz to 100 kHz.

7.3.2 Bridge Balance Equation

For bridge balance, the potential of point C must be same as the potential of point D. These potentials must be equal interms of amplitude as well as phase.

Thus the drop from A to C must be equal to drop across A to D, in both magnitude and phase for the bridge balance.

$$\therefore \quad \bar{E}_{AC} = \bar{E}_{AD} \qquad \dots (1)$$

The vector notation indicates, both amplitude and phase to be considered.

$$\therefore \quad \overline{I_1 Z_1} = \overline{I_2 Z_2} \qquad \dots (2)$$

When the bridge is balanced, no current flows through the headphones.

$$\therefore \quad \bar{I}_3 = \bar{I}_1 \quad \text{and} \quad \bar{I}_4 = \bar{I}_2$$

Now

$$\bar{I}_1 = \frac{\bar{E}}{\bar{Z}_1 + \bar{Z}_3} \qquad \dots (3)$$

and

$$\bar{I}_2 = \frac{\bar{E}}{\bar{Z}_2 + \bar{Z}_4} \qquad \dots (4)$$

Substituting (3) and (4) into (2) we get,

$$\frac{\bar{E} \cdot \bar{Z}_1}{\bar{Z}_1 + \bar{Z}_3} = \frac{\bar{E} \cdot \bar{Z}_2}{\bar{Z}_2 + \bar{Z}_4}$$

$$\therefore \quad \overline{Z_1 Z_2} + \overline{Z_1 Z_4} = \overline{Z_1 Z_2} + \overline{Z_2 Z_3}$$

$$\therefore \quad \overline{Z_1 Z_4} = \overline{Z_2 Z_3} \qquad \dots (5)$$

The equation (5) is the **balancing equation** in the impedance form.

In the **admittance** form the condition can be expressed as,

$$\overline{Y_1 Y_4} = \overline{Y_2 Y_3} \qquad \dots (6)$$

The admittance is the reciprocal of the impedance.

Now in the polar form the impedances are expressed as,

$$\overline{Z_1} = Z_1 \angle \theta_1$$

$$\overline{Z_2} = Z_2 \angle \theta_2$$

$$\overline{Z_3} = Z_3 \angle \theta_3$$

$$\overline{Z_4} = Z_4 \angle \theta_4$$

where Z_1, Z_2, Z_3, Z_4 are the magnitudes and θ_1, θ_2, θ_3 and θ_4 are the phase angles.

Note that the product of the impedances must be carried out in **polar form** where magnitudes get multiplied and phase angles get added.

Substituting in equation (3) we get,

$$Z_1 \angle \theta_1 \times Z_4 \angle \theta_4 = Z_2 \angle \theta_2 \times Z_3 \angle \theta_3$$

$$\therefore \quad Z_1 Z_4 \angle \theta_1 + \theta_4 = Z_2 Z_3 \angle \theta_2 + \theta_3 \qquad \ldots (7)$$

The equation (7) gives the two conditions to be satisfied for the bridge balance.

Equating magnitudes of both sides we get the magnitude condition as,

$$\boxed{Z_1 Z_4 = Z_2 Z_3} \qquad \ldots (8)$$

Equating phase angles we get,

$$\boxed{\theta_1 + \theta_4 = \theta_2 + \theta_3} \qquad \ldots (9)$$

Key Point : *Thus the products of the magnitudes of the opposite arms must be equal while sum of the phase angles of the opposite arms must be equal.*

Thus the bridge must be balanced for both the conditions magnitude as well as phase. The phase angles depend on the components of the individual impedances.

Key Point : *The phase angles are positive for the inductive impedances and negative for the capacitive impedances.*

For inductive branch,	$Z_L = R + jX_L = \lvert Z_L \rvert \angle +\theta$
For capacitive branch,	$Z_C = R - jX_C = \lvert Z_C \rvert \angle -\theta$
where	$X_L = 2\pi f L \Omega$ and $X_C = \dfrac{1}{2\pi f C} \Omega$

Example 7.1 : *The impedances of the basic a.c. bridge are,*

$Z_1 = 50\ \Omega \angle 80°$, $\quad Z_2 = 250\ \Omega \angle 0°$, $\quad Z_3 = 200\ \Omega \angle 30°$

Calculate the constants of the unknown impedance.

Solution : The bridge balance equation is,

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{magnitude condition}$$

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The equivalent series circuit is, shown in the Fig. 7.4.

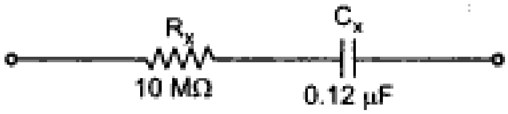


Fig. 7.4

7.5 Inductance Comparison Bridge

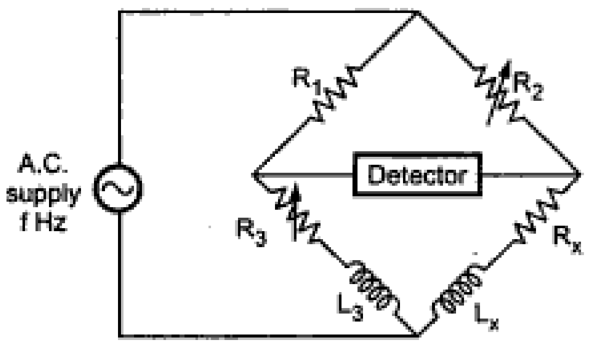


Fig. 7.5 Inductance comparison bridge

By this bridge, unknown inductance L_x and its internal resistance R_x can be determined. The schematic diagram of bridge is shown in the Fig. 7.5.

The bridge consists of pure resistances R_1 and R_2 in the ratio arms. The third arm consists of a standard inductor L_3 and the variable resistance R_3 while the fourth arm consists of the unknown inductor L_x with its internal resistance R_x

Here $Z_1 = R_1$ and $Z_2 = R_2$

$$Z_3 = R_3 + j X_{L_3}$$

$$= R_3 + j(\omega L_3)\ \Omega$$

$$Z_4 = R_x + j X_{L_x}$$

$$= R_x + j(\omega L_x)\ \Omega$$

From balance condition,

$$Z_1 \bar{Z}_4 = Z_2 \bar{Z}_3$$

$$\therefore \quad R_1 [R_x + j\omega L_x] = R_2 [R_3 + j\omega L_3]$$

$$\therefore \quad R_1 R_x + j\omega R_1 L_x = R_2 R_3 + j\omega R_2 L_3$$

Equating real parts,

$$R_1 R_x = R_2 R_3$$

$$\therefore \quad \boxed{R_x = \frac{R_2 R_3}{R_1}} \quad \ldots (1)$$

Equating imaginary parts,

$$\omega R_1 L_x = \omega R_2 L_3$$

$$\therefore \quad \boxed{L_x = \frac{R_2 L_3}{R_1}} \quad \ldots (2)$$

By using equations (1) and (2) , unknown elements can be determined.

In this bridge, R_2 is selected as inductive balance control and R_3 as resistance balance control. The balance is obtained by alternately varying L_3 or R_3.

Example 7.4 : *An inductance comparison bridge is used to measure the inductive impedance at a frequency of 1.5 kHz. The bridge constants at bridge balance are,*

$L_3 = 8$ mH, $R_1 = 1$ kΩ , $R_2 = 25$ kΩ, $R_3 = 50$ kΩ

Find the equivalent series circuit of unknown impedance.

Solution : From bridge balance equation of inductance comparison bridge,

$$R_x = \frac{R_2 R_3}{R_1}$$

$$= \frac{25 \times 10^3 \times 50 \times 10^3}{1 \times 10^3}$$

$$= 1.25 \text{ M}\Omega$$

and

$$L_x = \frac{R_2 L_3}{R_1}$$

$$= \frac{25 \times 10^3 \times 8 \times 10^{-3}}{1 \times 10^3}$$

$$= 200 \text{ mH}$$

The equivalent series circuit is,

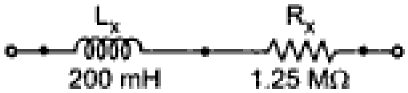


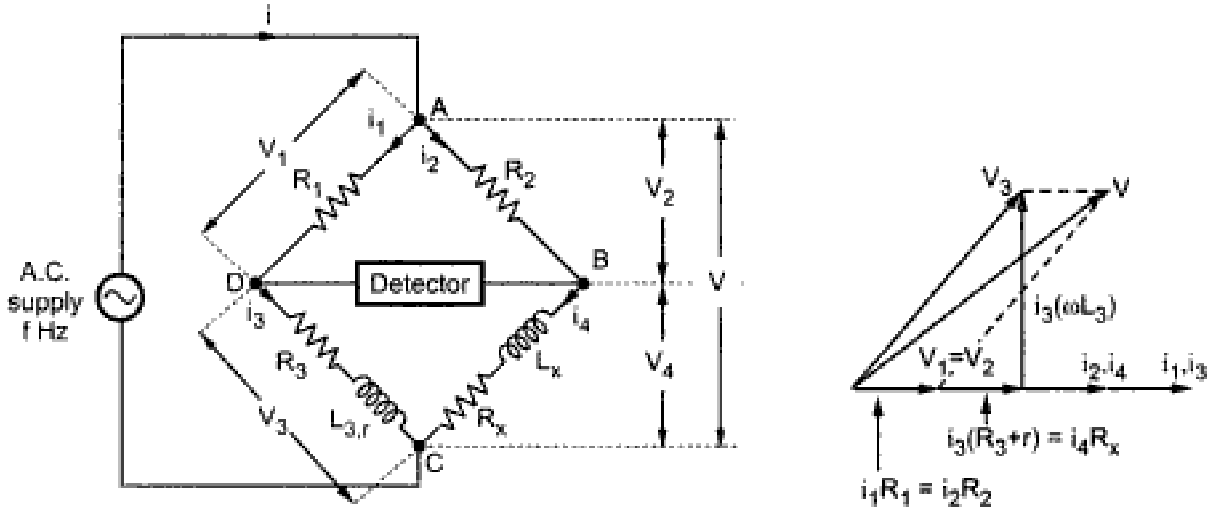
Fig. 7.6

7.6 Maxwell's Bridge

Maxwell's bridge can be used to measure inductance by comparison either with a variable standard self inductance or with a standard variable capacitance. These two measurements can be done by using the Maxwell's bridge in two different forms.

7.6.1 Maxwell's Inductance Bridge

Using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in Fig. 7.7 (a).



(a) Circuit diagram (b) Phasor diagram

Fig. 7.7 Maxwell's inductance bridge

Consider Maxwell's inductance bridge as shown in the Fig 7.7 (a). Two branches consist of non-inductive resistances R_1 and R_2. One of the arms consists variable inductance with series resistance r. The remaining arm consists unknown inductance L_x.

At balance, we get condition as

$$\frac{R_1}{[(R_3+r)+j\omega L_3]} = \frac{R_2}{R_x+j\omega L_x} \quad \text{... (1)}$$

$$\therefore \quad R_1[R_x+j\omega L_x] = R_2[(R_3+r)+j\omega L_3]$$

$$\therefore \quad R_1R_x+j\omega R_1L_x = R_2(R_3+r)+j\omega R_2L_3$$

Equating imaginary terms, we can write

$$R_1 L_x = R_2 L_3$$

$$\therefore \quad \boxed{L_x = \frac{R_3}{R_1} L_3} \quad \text{... (2)}$$

Equating real terms, we can write,

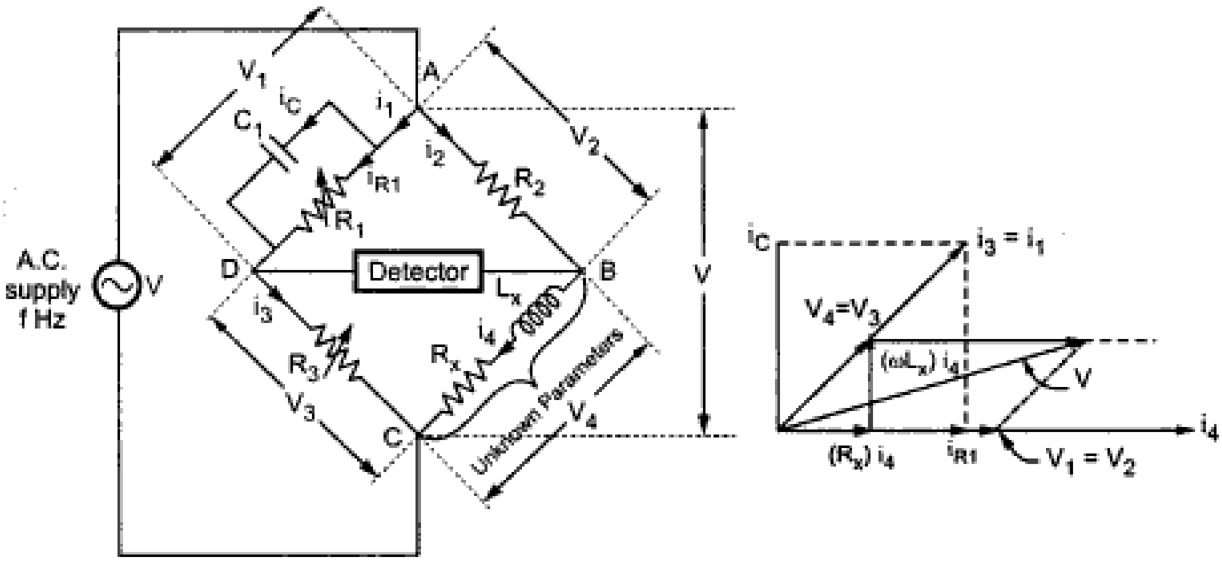
$$R_1 R_x = R_2(R_3+r)$$

$$\therefore \quad \boxed{R_x = \frac{R_2}{R_1}(R_3+r)} \quad \text{... (3)}$$

Under the balanced condition, the vector diagram for Maxwell's inductance bridge is as shown in the Fig. 7.7 (b).

7.6.2 Maxwell's Inductance Capacitance Bridge

Using this bridge, we can measure inductance by comparing with a variable standard capacitor. The bridge circuit diagram is as shown in the Fig. 7.8 (a).



(a) Circuit diagram (b) Phasor diagram

Fig. 7.8 Maxwell's inductance capacitance bridge

One of the ratio arms consists of resistance and capacitance in parallel. Hence it is simple to write the bridge equations in the admittance form.

The general bridge balance equation is,

$$\overline{Z_1}\,\overline{Z_x} = \overline{Z_2}\,\overline{Z_3}$$

$$\therefore \quad \overline{Z_x} = \frac{\overline{Z_2}\,\overline{Z_3}}{\overline{Z_1}} = \overline{Z_2}\,\overline{Z_3}\,\overline{Y_1} \qquad \ldots (4)$$

where $\overline{Y_1} = \frac{1}{\overline{Z_1}}$ i.e. R_1 in parallel with C_1

$$\overline{Z_2} = R_2$$

$$\overline{Z_3} = R_3$$

$$\overline{Z_x} = R_x + j\,\omega L_x$$, as L_x in series with R_x

Now $$\overline{Y_1} = \frac{1}{R_1} + j\,\omega C_1 \qquad \ldots (5)$$

as $$\overline{Z_1} = R_1 \,\|\, j\left(\frac{1}{\omega C_1}\right)$$ as $\frac{1}{j} = -j$

Solution : The bridge is shown in the Fig. 7.9.

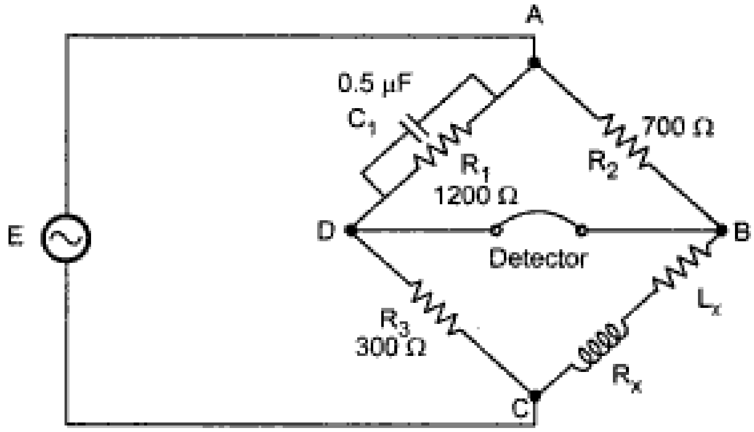


Fig. 7.9

From the bridge,

$$C_1 = 0.5\ \mu F, \quad R_1 = 1200\ \Omega$$

$$R_2 = 700\ \Omega, \quad R_3 = 300\ \Omega$$

From bridge balance equation,

$$R_x = \frac{R_2\ R_3}{R_1} = \frac{700 \times 300}{1200}$$

$$= \mathbf{175\ \Omega}$$

And $$L_x = R_2\ R_3\ C_1 = 700 \times 300 \times 0.5 \times 10^{-6}$$

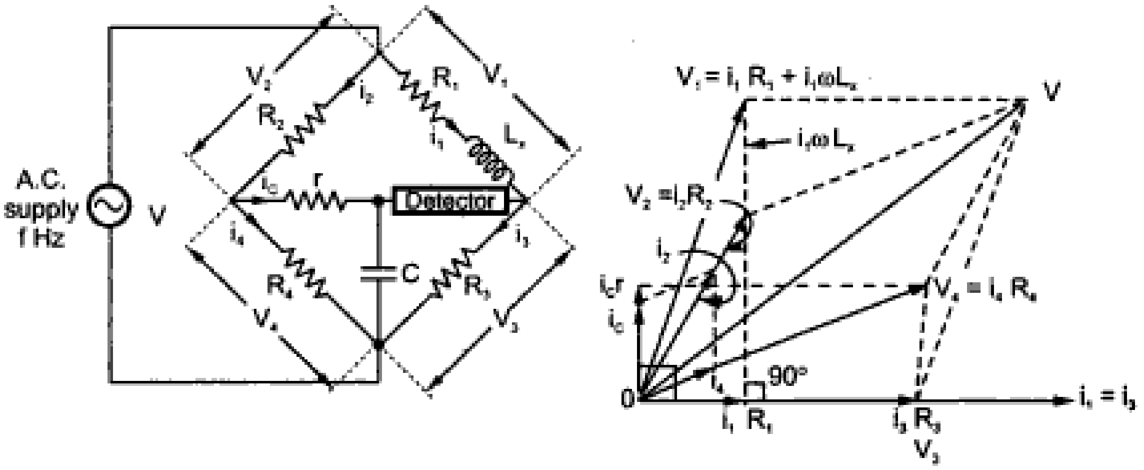
$$= \mathbf{105\ mH}$$

Key Point : *If the branches are not given in standard form as they are assumed for deriving bridge balance equation, derive the bridge balance equation again from the basic condition* $\overline{Z_1}\ \overline{Z_4} = \overline{Z_3}\ \overline{Z_2}$.

7.7 Anderson Bridge

It is another important a.c. bridge used for the measurement of self inductance interms of a standard capacitor. Actually this bridge is nothing but modified Maxwell's bridge in which also the value of self inductance is obtained by comparing it with a standard capacitor. This bridge is basically used for the precise measurement of inductance over a wide range of value. The Anderson bridge is as shown in the Fig. 7.10 (a).

One arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x. This resistance R_1 includes resistance of the inductor. C is the standard capacitor with r, R_2, R_3 and R_4 are non-inductive known resistances.



(a) Circuit diagram (b) Phasor diagram

Fig. 7.10

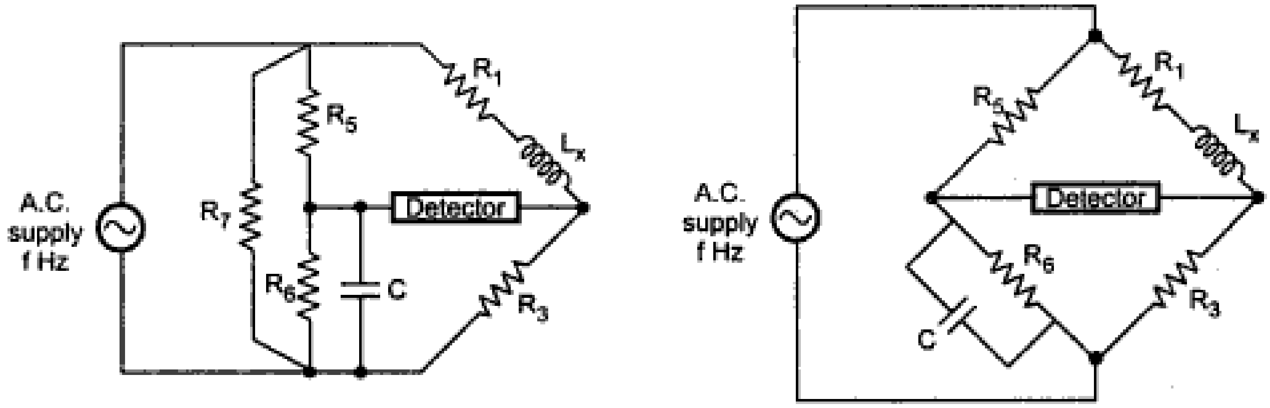
The bridge balance equations are,

$i_1 = i_3,\ i_2 = i_4 + i_C,\ V_2 = i_2 R_2,\ V_3 = i_3 R_3$

$$V_1 = V_2 + i_C r \text{ and } V_4 = V_3 + i_C r,\ V_1 = i_1 R_1 + i_1 \omega L_1,\ V_4 = i_4 R_4$$

$$V = \overline{V}_2 + \overline{V}_4 = \overline{V}_1 + \overline{V}_3$$

To find balance equations transforming a star formed by R_2, R_4 and r into its equivalent delta as shown in the Fig. 7.11 (a) and (b).



(a) (b)

Fig. 7.11 Transformed Anderson bridge

The elements in equivalent delta are given by,

$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$

Now R_7 shunts the source, hence it does not affect the balance condition. Thus by neglecting R_7 and rearranging a network as shown in the Fig. 7.11 (b), we get a Maxwell inductance bridge.

Thus, balance equations are given by,

$$L_x = C R_3 R_5 \text{ and}$$

$$R_1 = R_3 \frac{R_5}{R_6}$$

Substituting values of R_5 and R_6, we can write,

$$L_x = \frac{C R_3}{R_4}[R_2 r + R_4 r + R_2 R_4] \quad \text{and}$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

If the capacitor used is not perfect, the value of inductance remains unchanged, but the value of R_1 changes.

This method can also be used to measure the capacitance of the capacitor C if a calibrated self inductance is available.

7.7.1 Advantages of Anderson Bridge

The **advantages** of Anderson's bridge are,

1. Can be used for accurate measurement of capacitance interms of inductance.
2. Other bridges require variable capacitor but a fixed capacitor can be used for Anderson's bridge.
3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q.

7.7.2 Disadvantages of Anderson Bridge

The disadvantages of Anderson's bridge are,

1. It is more complicated than other bridges.
2. Uses more number of components.
3. Balance equations are also complicated to derive.
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

Example 7.6 : *An Anderson a.c. bridge is as follows :*

Arm AB : Unknown inductance R_x and L_x ,

Arm BC : Non-reactive resistance $R_2 = 1000\ \Omega$

Arm CD : Non-reactive resistance $R_4 = 1000\ \Omega$

Arm DA : Non-reactive resistance $R_3 = 500\ \Omega$

Arm DE : Resistance $r = 100\ \Omega$

Arm EB : Detector and a.c. supply between AC

Arm EC : Capacitor $C = 3\ \mu F$

State the expressions for L_x and R_x and find the values of them for given values of elements.

Solution : The bridge is shown in the Fig. 7.12.

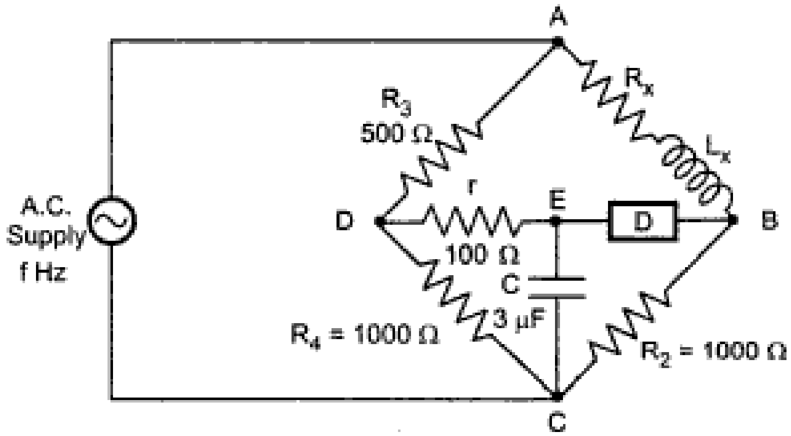


Fig. 7.12

It can be noticed that the names of resistances in branches AD and BC are reversed compared to what is assumed in the derivation earlier.

Hence change R_2 and R_3 in the equations. And R_1 is denoted as R_x.

$$R_x = \frac{R_2\ R_3}{R_4} = \frac{1000 \times 500}{1000} = \mathbf{500\ \Omega}$$

and

$$L_x = \frac{C\ R_2}{R_4}\left[R_3\ r + R_4\ r + R_3\ R_4\right]$$

$$= \frac{3 \times 10^{-6} \times 1000}{1000}[500 \times 100 + 1000 \times 100 + 500 \times 1000]$$

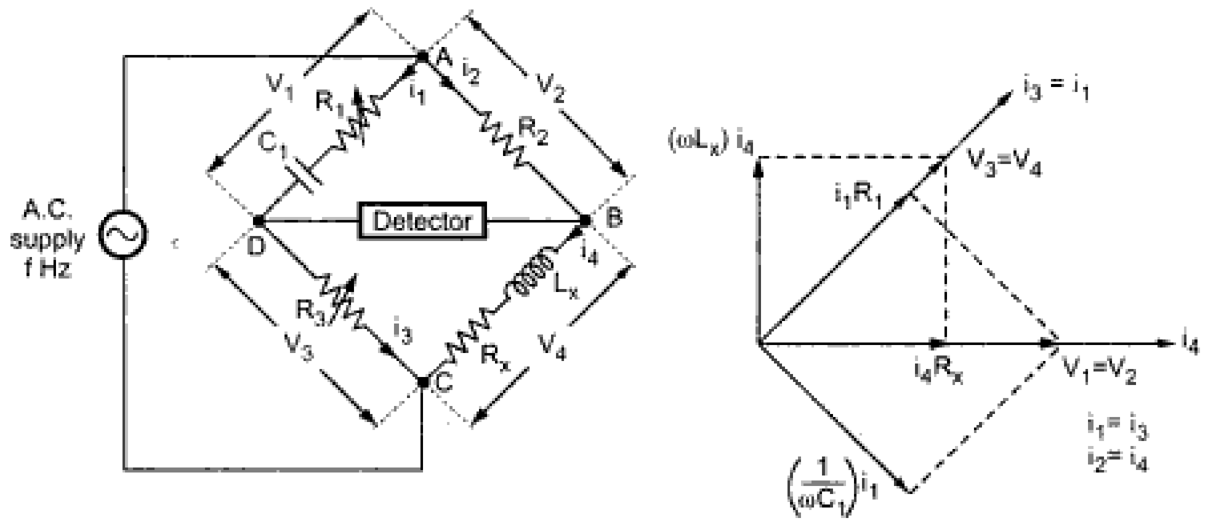
$$= \mathbf{1.95\ H}$$

Key Point : *In examination, derive the balanced equation again and then use the given values.*

7.8 Hay's Bridge

The limitation of Maxwell's bridge is that it cannot be used for high Q values. The Hay's bridge is suitable for the coils having high Q values.

The difference in Maxwell's bridge and Hay's bridge is that the Hay's bridge consists of resistance R_1 in series with the standard capacitor C_1 in one of the ratio arms. Hence for larger phase angles R_1 needed is very low, which is practicable. Hence bridge can be used for the coils with high Q values. The Hay's bridge is shown in the Fig. 7.13 (a). Under balanced condition, the phasor diagram is as shown in the Fig. 7.13 (b).



(a) Circuit diagram (b) Phasor diagram

Fig. 7.13 Hay's bridge

The various constants of the bridge are :

$$Z_1 = R_1 - j X_{C_1} = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2 \quad \text{and} \quad Z_3 = R_3$$

$$Z_4 = Z_x = R_x + j(\omega L_x)$$

At the balance condition,

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

$$\therefore \quad \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right]\left[R_x + j(\omega L_x)\right] = R_2 R_3$$

$$\therefore \quad R_1 R_x - j\left(\frac{R_x}{\omega C_1}\right) + j\omega R_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

$$\therefore \quad \left[R_x R_1 + \frac{L_x}{C_1}\right] + j\left[\omega R_1 L_x - \frac{R_x}{\omega C_1}\right] = R_2 R_3 \quad \ldots (1)$$

Equating the real parts of both sides,

$$R_x R_1 + \frac{L_x}{C_1} = R_2 R_3 \quad \ldots (2)$$

Equating the imaginary parts of both sides of (1),

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0 \quad \ldots (3)$$

To obtain R_x and L_x, solve equations (2) and (3) simultaneously.

From (3),

$$\omega R_1 L_x = \frac{R_x}{\omega C_1}$$

$$\therefore \quad L_x = \frac{R_x}{\omega^2 R_1 C_1} \quad \ldots (4)$$

Substituting in (2),

$$R_x R_1 + \frac{R_x}{\omega^2 R_1 C_1^2} = R_2 R_3$$

$$\therefore \quad R_x \left[R_1 + \frac{1}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

$$\therefore \quad R_x \left[\frac{1+\omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

$$\therefore \quad \boxed{R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 C_1^2}} \quad \ldots (5)$$

Substituting (5) in (4) we get,

$$L_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{(1+\omega^2 R_1^2 C_1^2)\, \omega^2 R_1 C_1}$$

$$\therefore \quad \boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2}} \quad \ldots (6)$$

Using equations (5) and (6) the unknown resistance and the inducıance can be calculated. The expressions are frequency (ω) sensitive and hence source frequency must be accurately known. The inductance balance equation depends on the losses of the inductor.

From the phase angle balance equation, the opposite sets of phase angles must be equal. As Z_2 and Z_3 are purely resistive, the inductive phase angle must be equal to the capacitive phase angle.

The inductive and capacitive phase angles can be determined from the impedance triangles shown in the Fig. 7.14.

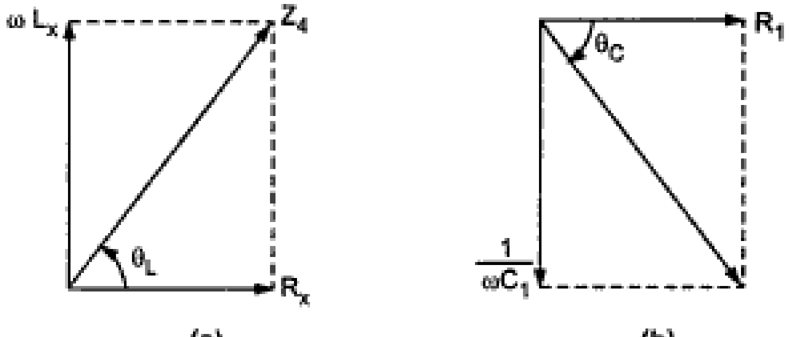


Fig. 7.14

$$\tan\theta_L = \frac{X_{L_x}}{R_x} = \frac{\omega L_x}{R_x}$$

$$= Q \qquad \ldots (7)$$

while $$\tan\theta_C = \frac{X_{C_1}}{R_1}$$

$$= \frac{1}{\omega C_1 R_1} \qquad \ldots (8)$$

The two angles must be equal at bridge balance.

$$\therefore \quad \boxed{Q = \frac{1}{\omega C_1 R_1}} \qquad \ldots (9)$$

Substituting in the equation (6) we get,

$$\boxed{L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}} \qquad \ldots (10)$$

For a large value of Q, $1/Q^2$ becomes small and can be neglected.

$$\therefore \quad \boxed{L_x = R_2 R_3 C_1} \qquad \ldots \text{for high Q}$$

This is same as Maxwell's bridge equation.

The commercial Hay's bridge measure the inductances in the range 1μH - 100 H with ± 2% error.

7.8.1 Advantages of Hay's Bridge

The **advantages** of Hay's bridge are,

i) It is best suitable for the measurement of inductance with high Q, typically greater than 10.

ii) It gives very simple expression for Q factor interms of elements in the bridge.

iii) It requires very low value resistor R_1 to measure high Q inductance.

7.8.2 Disadvantage of Hay's Bridge

It is only suitable for measurement of high Q inductance. Consider expression for unknown inductance.

$$L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}$$

For high Q inductances, $(1 / Q^2)$ term can be neglected. But for low Q measurements, $(1 / Q^2)$ term is significant, hence cannot be neglected. Hence Hay's bridge is not suitable for the measurement of low Q inductances. In such cases, Maxwell's bridge is preferred.

Example 7.7 : *Calculate the unknown inductance and resistance measured by Hay's bridge. The bridge elements at the balancing condition are*

R_1 = 5.1 kΩ, C_1 = 2 μF , R_2 = 7.9 kΩ , R_3 = 790 Ω

The supply angular frequency is 1000 rad/sec.

Solution : From the Hay's bridge balance equations,

$$R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 {C_1}^2}$$

$$= \frac{(1000)^2 \times 5.1\times 10^3 \times (2\times 10^{-6})^2 \times 7.9\times 10^3 \times 790}{1+(1000)^2 (5.1\times 10^3)^2 (2\times 10^{-6})^2}$$

$$= \frac{127316.4}{1+104.04}$$

$$= \mathbf{1.212\ k\Omega}$$

and

$$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 {C_1}^2}$$

$$= \frac{7.9\times 10^3 \times 790\times 2\times 10^{-6}}{1+104.04}$$

Rationalising, $Z_1 = R_3\left(1+\omega^2 R_4^2 C_4^2\right)\left\{\frac{-\frac{j}{\omega C_2}\left(R_4 + j\omega R_4^2 C_4\right)}{R_4^2 + \omega^2 R_4^4 C_4^2}\right\}$

$$\therefore \quad R_x - j\frac{1}{\omega C_x} = \frac{R_3\left(1+\omega^2 R_4^2 C_4^2\right)}{R_4^2\left(1+\omega^2 R_4^2 C_4^2\right)}\left\{\frac{R_4^2 C_4}{C_2} - \frac{j R_4}{\omega C_2}\right\}$$

Equating real and imaginary parts,

$$R_x = \frac{R_3}{R_4^2} \times \frac{R_4^2 C_4}{C_2} = \frac{R_3 C_4}{C_2} \quad \text{... (1)}$$

$$-j\frac{1}{\omega C_x} = -j\frac{R_3}{R_4^2} \times \frac{R_4}{\omega C_2} = -j\left[\frac{1}{\frac{R_4}{R_3}\omega C_2}\right]$$

$$\therefore \quad \omega C_x = \frac{R_4}{R_3}\omega C_2$$

$$\therefore \quad C_x = \frac{R_4}{R_3} C_2 \quad \text{... (2)}$$

The equations (1) and (2) gives the required values of C_x and R_x.

7.9.1 Power Factor and Loss Angle

i) **Power factor (p.f.) :** The power factor of the series RC combination is defined as the cosine of the phase angle of the circuit. Thus,

$$\text{p.f.} = \cos\phi_x = \frac{R_x}{Z_x}$$

For phase angles very close to 90°, the reactance is almost equal to the impedance,

$$\therefore \quad \text{p.f.} = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)}$$

$$\text{p. f.} = \omega R_x C_x$$

ii) Loss angle (δ) : For a series combination of R_x and C_x, the angle between the voltage across the series combination and voltage across the capacitor C_x is called **loss angle** δ. This is shown in the Fig. 7.16 (a).

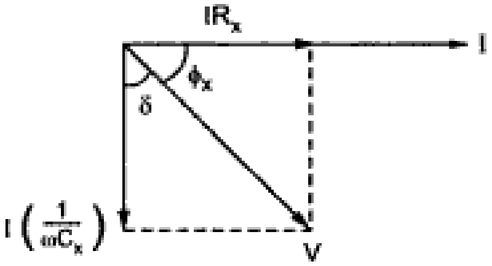


Fig. 7.16 (a)

$$\text{Now } \tan\delta = \frac{I R_x}{I\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$$

$$\therefore \quad \tan\delta = \omega\left(\frac{R_3 C_4}{C_2}\right)\left(\frac{R_4}{R_3} C_2\right) = \omega R_4 C_4$$

Thus loss angle can be measured, knowing the values of ω, R_4 and C_4.

iii) Dissipation factor (D) : For R_x - C_x series circuit, it is cotangent of the phase angle ϕ_x.

$$\therefore \quad D = \cot\phi_x = \frac{1}{\tan\phi_x} = \frac{1}{\left[\frac{I\left(\frac{1}{\omega C_x}\right)}{I R_x}\right]} = \omega R_x C_x = \omega R_4 C_4$$

> **Key Point :** *The quality factor Q = X/R = 1/ω CR hence* ***dissipation factor is reciprocal of quality factor Q*** *and gives the information about quality of the capacitor.*

Thus if the resistance R_4 is fixed, then the dial of capacitor C_4 can be directly calibrated to give dissipation factor D i.e. quality of the capacitor. As the term ω is present in the equation, the calibration of C_4 dial holds good for only one particular frequency. The different frequency can be used but a correction should be made to multiply the C_4 dial reading by the ratio of the two frequencies.

Similarly if the resistance ratio is maintained at fixed value, the dial of C_3 can be graduated in terms of direct readings of C_x.

Commercial Schering bridge measures the capacitors from 100 pF - 1 μF, with ± 2% accuracy.

> **Key Point :** *The bridge is widely used for testing small capacitors at low voltages with very high precision.*

The phasor diagram is shown in the Fig. 7.16 (b) at the balance condition.

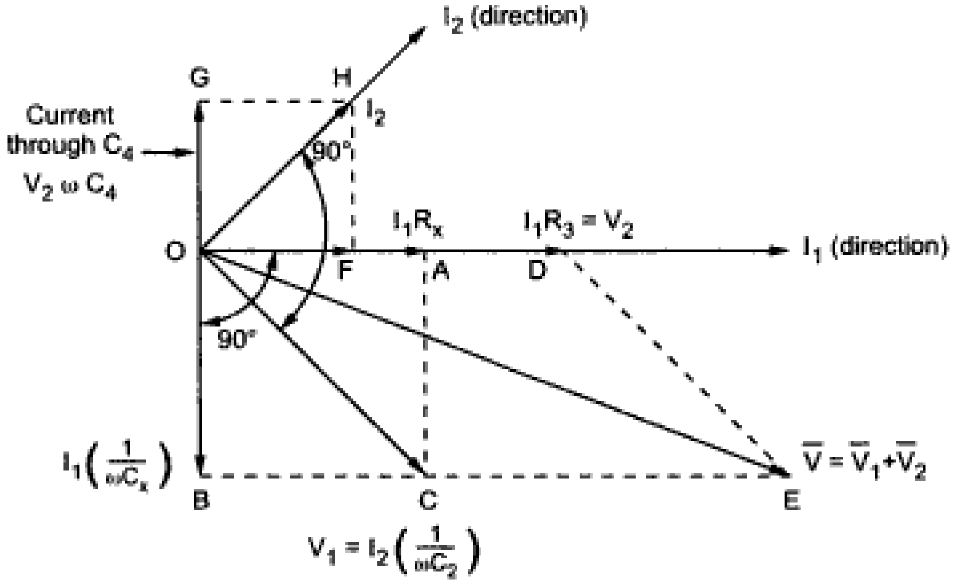


Fig. 7.16 (b)

I_1 is chosen reference. Now V_1 is drop across R_x and C_x. Thus $OA = I_1 R_x$ and $OB = I_1\left(\frac{1}{\omega C_x}\right)$ and I_1 leads capacitor drop by 90°. Thus $OC = V_1$ is $\overline{OA} + \overline{OB}$. Now $V_1 = I_2\left(\frac{1}{\omega C_2}\right)$ i.e. drop across C_2. So current I_2 leads V_2 by 90°. Then $V_2 = I_1 R_3$ which is OD in phase with I_1. And $\overline{V_1} + \overline{V_2} = \overline{V}$ is supply voltage OE. V_2 is also drop across R_4 and C_4. Let OF is current through R_4 thus $OF = V_2 / R_4$ in phase with V_2 while OG is current through C_4 which is $V_2 \omega C_4$ and is leading V_2 by 90°. The $OH = I_2$ which is vector sum of OF and OG.

Example 7.8 : *The Schering bridge has the following constants :*

Arm AB - capacitor of 1 μF in parallel with 1.2 kΩ resistance
Arm AD - resistance of 4.7 kΩ
Arm BC - capacitor of 1 μF
Arm CD - unknown capacitor C_x and R_x.
The frequency of supply is 0.5 kHz. Calculate the unknown capacitance and its dissipation factor.

Solution : From the given information,

$$R_1 = 1.2\ \text{k}\Omega \qquad C_1 = 1\ \mu\text{F}$$

$$R_2 = 4.7\ \text{k}\Omega \qquad C_3 = 1\ \mu\text{F}$$

From the balance equations,

$$R_x = \frac{R_2 C_1}{C_3} = \frac{4.7\times10^3 \times 1\times10^{-6}}{1\times10^{-6}}$$

$$= \mathbf{4.7\ k\Omega}$$

$$C_x = \frac{R_1 C_3}{R_2} = \frac{1.2\times10^3 \times 1\times10^{-6}}{4.7\times10^3}$$

$$= \mathbf{0.255\ \mu F}$$

The dissipation factor,

$$D = \omega C_x R_x = 2\pi f C_x R_x$$

$$= 2\pi \times 0.5 \times 10^3 \times 0.255 \times 10^{-6} \times 4.7 \times 10^3 = \mathbf{3.765}$$

7.10 High Voltage Schering Bridge

When the Schering bridge is used for the measurement of small capacitances at low voltages, then it suffers from errors. To avoid this, high voltage Schering bridge is used. The bridge is shown in the Fig. 7.17.

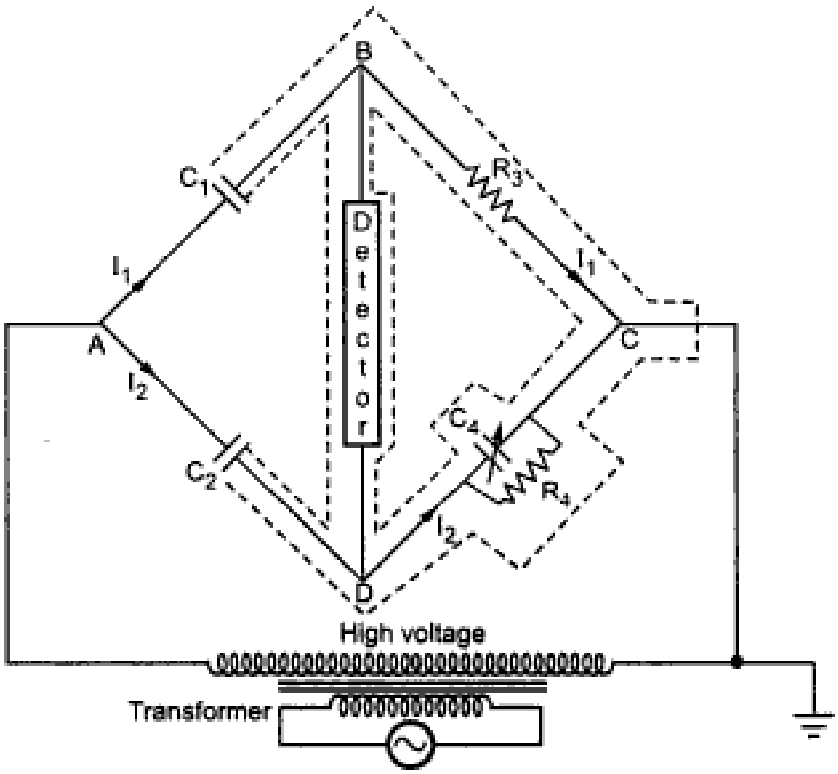


Fig. 7.17 High voltage Schering bridge

The high voltage is obtained from step up transformer at 50 Hz. The detector used is vibration galvanometer.

and $$Z_4 = R_4$$

The balance condition is,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$\therefore \quad \overline{Z_2} = \frac{\overline{Z_1 Z_4}}{\overline{Z_3}} = Z_1 \overline{Z_4 Y_3}$$

$$\therefore \quad R_2 = \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

$$\therefore \quad R_2 = R_4 \left[\frac{R_1}{R_3} + j\omega R_1 C_3 - j\frac{1}{\omega C_1 R_3} + \frac{C_3}{C_1}\right]$$

$$\therefore \quad R_2 = R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1}\right] + jR_4 \left[\omega R_1 C_3 - \frac{1}{\omega C_1 R_3}\right]$$

Equating real parts of both sides,

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1}$$

$$\therefore \quad \boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}} \quad \ldots (1)$$

Equating imaginary parts of both sides,

$$\omega R_1 C_3 - \frac{1}{\omega C_1 R_3} = 0$$

$$\therefore \quad \omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{R_1 C_1 R_3 C_3}} \quad \ldots (2)$$

$$\therefore \quad \boxed{f = \frac{1}{2\pi\sqrt{R_1 C_1 R_3 C_3}}} \quad \ldots (3)$$

The equation (1) gives the resistance ratio while the equation (3) gives the frequency of applied voltage.

Generally in Wien bridge, the selection of the components is such that

$$R_1 = R_3 = R$$

and $$C_1 = C_3 = C$$

Comparing real terms, we get,

$$R_1 = R_3 \frac{C_4}{C_2} \quad \text{... (2)}$$

Similarly comparing imaginary terms, we get,

$$L_1 = R_2 R_3 C_4 \quad \text{... (3)}$$

7.13.1 Advantages of Owen's Bridge

The **advantages** of the Owen's bridge are,

1) From the equation of balance conditions, it is clear that both conditions are independent of each other even if R_2 and C_2 are variables. Moreover these two elements are connected in the same arm in series. Hence it is very easy to achieve balance condition properly.
2) The balance equations are of very simple form.
3) The balance equations are independent of the frequency term.
4) It is possible to use this bridge over a wide range of inductances values.

7.13.2 Disadvantages of Owen's Bridge

The **disadvantages** of Owen's bridge are,

1) With the inductances of large Q-factor values, the value of C_2 becomes very large.
2) For measurement of wide range of inductances interms of capacitance, the capacitor C_2 is required to be variable. This increases the cost of the bridge.

7.13.3 Application of Owen's Bridge

The main application of the Owen's bridge is to measure incremental inductance. To measure incremental inductance, it is necessary to feed coil from a.c. as well as d.c. voltage supply. Thus it is necessary to modify original Owen's bridge such that both a.c. and d.c. sources are included in it. The modified Owen's bridge used for the measurement of the incremental inductance is as shown in the Fig. 7.22.

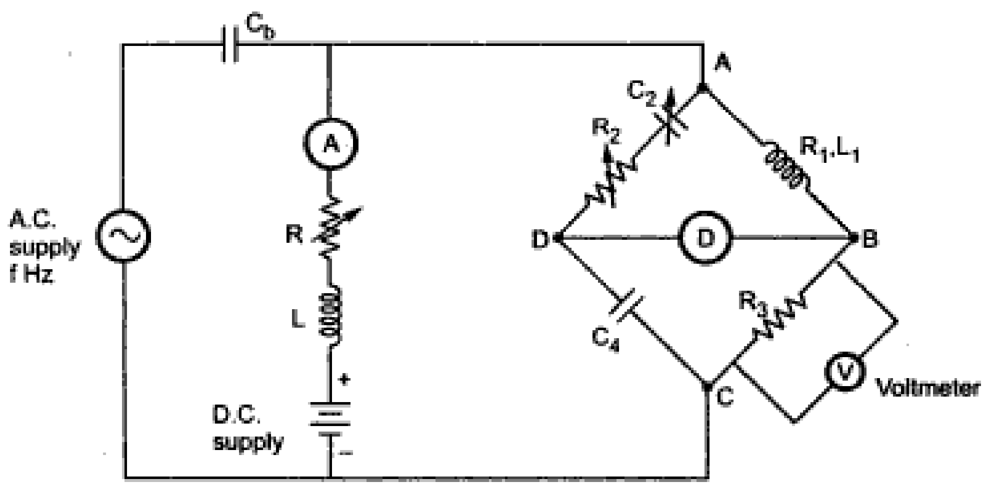


Fig. 7.22 Owen's bridge for measurement of an incremental inductance

In the bridge circuit, the coil L_1 is fed with a.c. source as well as d.c. source. Both the sources are connected in parallel. The capacitor C_b is the blocking capacitor which is used to prevent d.c. current from flowing into a.c. sources. Similarly a large value inductance L is used in series with the d.c. supply. It stops the a.c. current from flowing into the d.c. supply. As this bridge consists both the types of sources, it is necessary that no d.c. current should affect any a.c. balance conditions. In actual bridge circuit, no d.c. current can flow through detector as the bridge consists capacitors C_2 and C_4.

To measure the d.c. current, a moving coil type ammeter is used. While the component of the a.c. current can be obtained from the reading on the voltmeter. The voltmeter used is a special type of voltmeter known as **Valve voltmeter** which is insensitive to the d.c. voltage. The current obtained from this reading is nothing but the current through R_3 as well as current through coil L_1 at balance.

At balance the incremental inductance as per previous derivation is given by,

$$L_1 = R_2 R_3 C_4$$

But $$L_1 = N^2 \Big/ \left(\frac{l}{\mu A}\right) \qquad \text{... (4)}$$

Incremental permeability $$\mu = \frac{L_1 \, l}{N^2 A} \qquad \text{... (5)}$$

where l = length of flux path

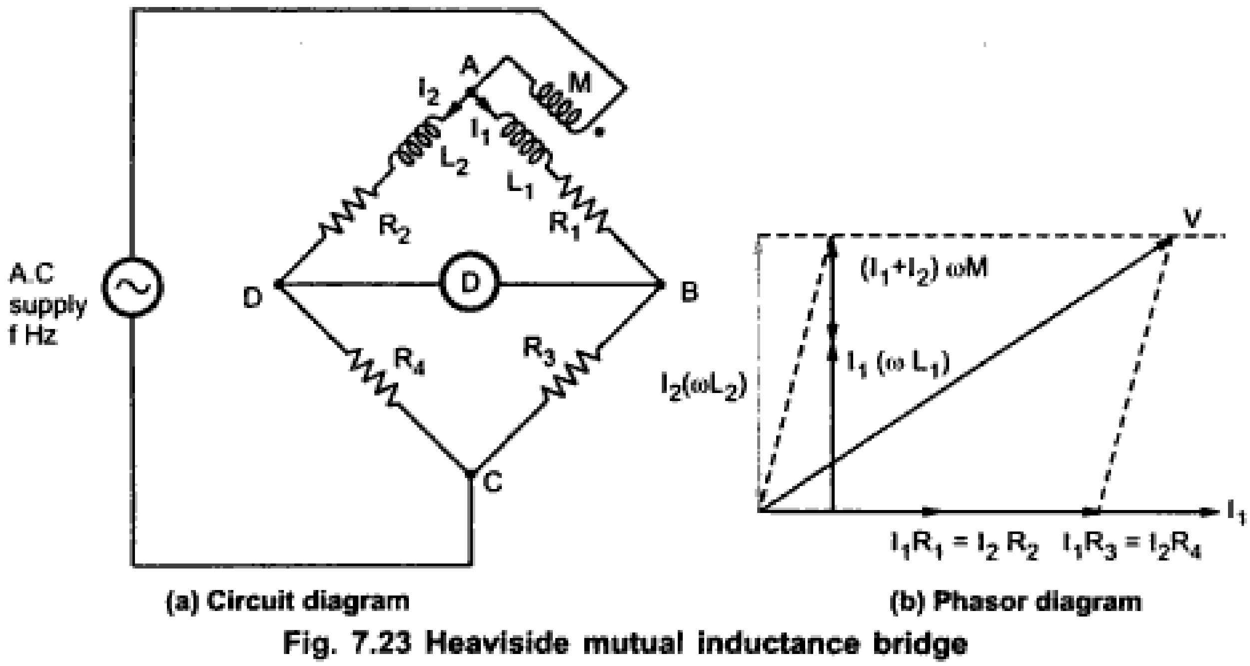
A = area of flux path

L_1 = incremental inductance

7.14 Heaviside Mutual Inductance Bridge

As the name suggests, Heaviside mutual inductance bridge measures mutual inductance interms of self inductances.

Let M be the unknown mutual inductance between two coils of self inductances of L_1 and L_2. The bridge consists coils L_1, L_2 and non-inductive resistance R_1, R_2, R_3 and R_4 as shown in the Fig. 7.23.



(a) Circuit diagram (b) Phasor diagram

Fig. 7.23 Heaviside mutual inductance bridge

The balance condition can be written as follows,

Voltage drop across branch BC = Voltage drop across branch DC

$$\therefore \quad I_1 R_3 = I_2 R_4 \quad \text{... (1)}$$

Also another balance condition is given by,

Voltage drop between A-B-C nodes = Voltage drop between A-D-C nodes

$$\therefore \quad (I_1 + I_2)(j\omega M) + I_1 (R_1 + j\omega L_1 + R_3) = I_2 (R_2 + j\omega L_2 + R_4) \quad \text{... (2)}$$

Substituting value of $I_1 = \frac{R_4}{R_3} I_2$ from equation (1) in equation (2), we get,

$$I_2 \left(\frac{R_4}{R_3} + 1\right)(j\omega M) + \frac{R_4}{R_3}(R_1 + j\omega L_1 + R_3) I_2 = I_2 (R_2 + j\omega L_2 + R_4)$$

$$\therefore \quad j\omega M\left(\frac{R_4}{R_3} + 1\right) + \frac{R_4}{R_3}(R_1 + R_3) + \frac{R_4}{R_3} j\omega L_1 = (R_2 + R_4) + j\omega L_2$$

$$\therefore \quad \frac{R_1 R_4}{R_3} + R_4 + j\omega \left[L_1\left(\frac{R_4}{R_3}\right) + \left(\frac{R_4}{R_3} + 1\right) M\right] = (R_2 + R_4) + j\omega L_2 \quad \text{... (3)}$$

Equating real terms, we get,

$$\frac{R_1 R_4}{R_3} + R_4 = R_2 + R_4$$

$$\therefore \quad R_1 = \frac{R_3}{R_4} R_2 \quad \text{... (4)}$$

Equating imaginary terms, we get,

$$\left(\frac{R_4}{R_3}\right) L_1 + \left(\frac{R_4}{R_3} + 1\right) M = L_2$$

$$\therefore \quad M\left[\frac{R_4 + R_3}{R_3}\right] = L_2 - \frac{R_4}{R_3} L_1$$

$$\therefore \quad M\left[\frac{R_4 + R_3}{R_3}\right] = \frac{L_2 R_3 - R_4 L_1}{R_3}$$

$$\therefore \quad M = \frac{L_2 R_3 - L_1 R_4}{R_3 + R_4} \quad \text{... (5)}$$

From equations (6) and (7), the value of unknown inductance L_2 is given by,

$$L_2 = (M_1 - M_2)\left(1 + \frac{R_4}{R_3}\right) \quad \text{... (8)}$$

Another condition for R_2 can be derived as follows.

Consider equation (4). With switch S_1 open, we can write,

$$(R_1 + r_1) = \frac{R_3}{R_4}(R_2 + R) \quad \text{... (9)}$$

Similarly with switch S_1 closed, we can write,

$$(R_1 + r_2) = \frac{R_3}{R_4} + (R_2) \quad \text{... (10)}$$

Hence, solving equations (9) and (10), we get,

$$R_2 = (r_1 - r_2)\frac{R_4}{R_3} \quad \text{... (11)}$$

When we use equal ratio arms i.e. $R_3 = R_4$, then

$$L_2 = 2(M_1 - M_2) \quad \text{... (8 - a)}$$

$$R_2 = (r_1 - r_2) \quad \text{... (11 - a)}$$

7.15 Shielding and Grounding of Bridges

The various bridges discussed uptil now consist of the lumped impedances, which do not interact in any way.

In practice, the stray capacitances between the various bridge elements and ground, and between the bridge arms themselves exist. These stray capacitances shunt the bridge arms and cause considerable error in the measurement.

The stray capacitances are uncertain in magnitude. They often vary with the adjustment of bridge arms and position of operator.

The **shielding and grounding** of bridge is one way of reducing the effect of stray capacitances. But this technique does not eliminate the stray capacitances but makes them constant in value and hence they can be compensated.

One very effective and popular method of eliminating the stray capacitances and the capacitances between the bridge arms is using a ground connection called **Wagner Ground Connection.** This connection is shown in the Fig. 7.25.

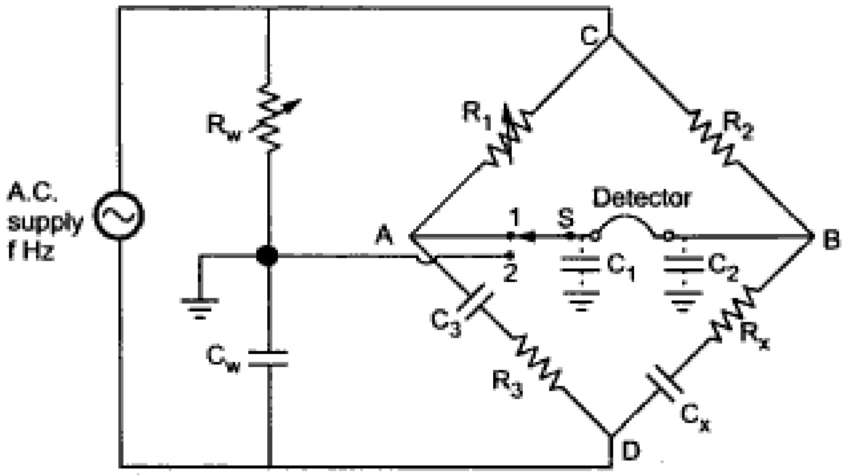


Fig. 7.25 Wagner ground connection

The circuit is a capacitance bridge where C_1 and C_2 represent the stray capacitances. The Wagner's connection is the use of separate arm consisting of the resistance R_w and the capacitance C_w across the terminals C and D, forming a potential divider. This arm is also called **guarding arm**. The procedure for the adjustment is as follows :

The switch S is connected in series with the detector. The switch is connected to position 1 and R_1 is then adjusted to get null response i.e. minimum sound in headphones.

The switch is then thrown to position 2, which connects the detector to the Wagner ground point. The resistance R_w of the Wagner connection is now adjusted to get the minimum sound.

The switch is again thrown back to position 1. There is some imbalance present now. The resistances R_1 and R_3 are then adjusted to get minimum sound.

This procedure is repeated till a null is obtained on both the switch positions 1 and 2. This null obtained at both the positions indicates that the points 1 and 2 are at the same potential. But the point 2 is at the ground potential due to Wagner ground connection. Hence points 1 and 2 are at the ground potential. Thus the stray capacitances C_1 and C_2 are effectively short circuited. Thus they have no effect on the normal bridge balancing.

There are capacitances existing from points C and D to ground but the addition of the Wagner ground connection eliminates them as current through these capacitances will enter through Wagner ground connection.

Key Point : *As the addition of Wagner ground connection does not affect the balance conditions, the procedure for the measurement remains unchanged.*

Examples with Solutions

➡ **Example 7.10 :** *The basic a.c. bridge consists of the following constants :*
Arm AB : R = 400 Ω
Arm BC : R = 150 Ω in series with C = 0.2 μF
Arm CD : unknown
Arm DA : R = 100 Ω in series with L = 10 mH.
The source oscillator frequency is 1 kHz. Determine the constants of the arm CD.

Solution : The bridge is shown in the Fig. 7.26.

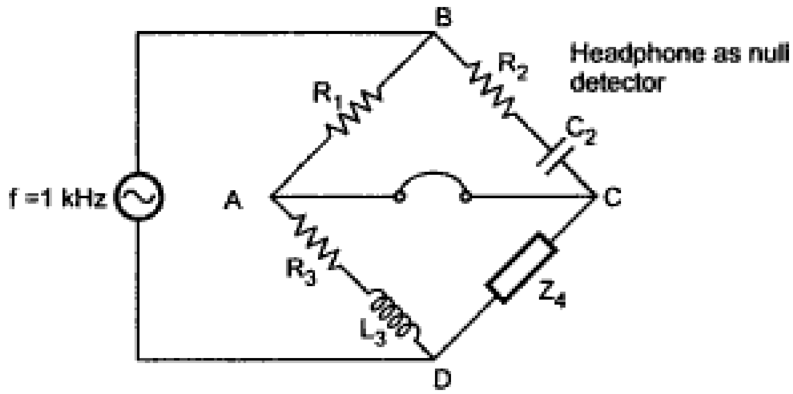


Fig. 7.26

Now $\bar{Z}_1 = 400 + j\,0\,\Omega = 400\angle 0°\ \Omega$

$$\bar{Z}_2 = R_2 - j\,X_{C_2} = 150 - j\left(\frac{1}{2\pi f C_2}\right)\ \Omega$$

$$= 150 - j\left(\frac{1}{2\pi\times1\times10^3\times0.2\times10^{-6}}\right)\ \Omega$$

$$= 150 - j\,795.8\ \Omega$$

$$= 809.78\angle -79.32°\ \Omega \qquad \text{using r} \rightarrow \text{p conversion}$$

$$\bar{Z}_3 = R_3 + j\,X_{L_3} = 100 + j\,(2\pi f L_3)$$

$$= 100 + j\left(2\pi\times1\times10^3\times10\times10^{-3}\right)$$

$$= 100 + j\,62.83\ \Omega$$

$$= 118.1\angle +32.14°\ \Omega \qquad \text{using r} \rightarrow \text{p conversion}$$

$$Z_1 Z_4 = Z_2 Z_3 \qquad \text{magnitude}$$

$$\therefore \quad 400\,Z_4 = 809.78\times118.1$$

$$\therefore \quad Z_4 = 239.087\ \Omega$$

and $\theta_1 + \theta_4 = \theta_2 + \theta_3$ phase angle

$\therefore \quad 0° + \theta_4 = -79.32° + 32.14°$

$\therefore \quad \theta_4 = -47.18°$

$\therefore \quad \bar{Z}_4 = 239.087 \angle -47.18° \, \Omega$

$= 162.5 - j\, 175.36 \, \Omega$

Thus it is a series combination of resistance and capacitance.

$R_4 = 162.5 \, \Omega$

and $X_{C_4} = 175.36 \, \Omega$

$$\therefore \quad 175.36 = \frac{1}{2 \pi f C_4}$$

$$\therefore \quad C_4 = \frac{1}{2 \pi \times 1 \times 10^3 \times 175.36} = \mathbf{0.9076 \, \mu F}$$

Example 7.11 : *A capacitance comparison bridge is used to measure the capacitive impedance at a frequency of 3 kHz. The bridge constants at bridge balance are,*

$C_3 = 10 \, \mu F$

$R_1 = 1.2 \, k\Omega$

$R_2 = 100 \, k\Omega$

$R_3 = 120 \, k\Omega$

Find the equivalent series circuit of the unknown impedance.

Solution : From the bridge balance equations,

$$R_x = \frac{R_2 R_3}{R_1} = \frac{100 \times 10^3 \times 120 \times 10^3}{1.2 \times 10^3}$$

$= 10 \, M\Omega$

while $C_x = \dfrac{R_1 C_3}{R_2}$

$$= \frac{1.2 \times 10^3 \times 10 \times 10^{-6}}{100 \times 10^3} = 0.12 \, \mu F$$

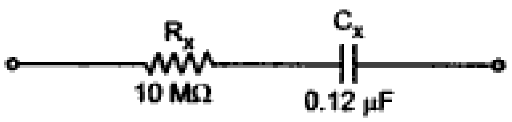


Fig. 7.27

The equivalent series circuit is, shown in the Fig. 7.27.

Example 7.12 : *The bridge is balanced at 1000 Hz. It has following components :*

Arm AB = 0.2 μF pure capacitance
Arm BC = 500 Ω pure resistance
Arm DA = 300 Ω parallel with 0.1 μF
Find the constants of arm CD, considering it as a series circuit.

Solution : The bridge is shown in the Fig. 7.28.

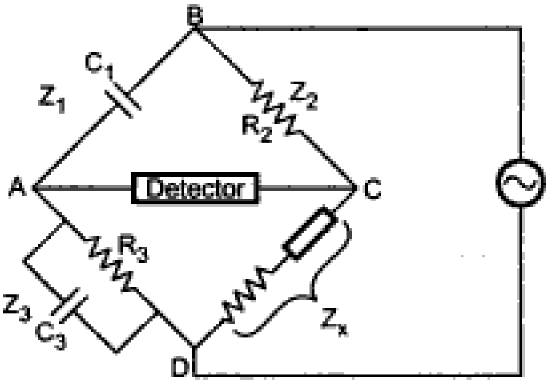


Fig. 7.28

$$Z_1 = 0 - j\,X_{C1} = 0 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2$$

$$\therefore \quad Z_3 = R_3 \,||\, X_{C3}$$

$$\therefore \quad Y_3 = \frac{1}{R_3} + j\left(\frac{1}{X_{C3}}\right) = \frac{1}{R_3} + j\,(\omega C_3)$$

and $\quad Z_4 = Z_X$

The basic balance equation is,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{Z_2}{Z_1 Y_3}$$

$$= \frac{R_2}{-\frac{j}{\omega C_1}\left[\frac{1}{R_3} + j\omega C_3\right]}$$

Now, $$-\frac{1}{j} = j$$

$$\therefore \quad Z_4 = \frac{j\omega C_1 R_2}{\frac{1}{R_3} + j\omega C_3}$$

$$= \frac{0 + j2\pi \times 1000 \times 0.2 \times 10^{-6} \times 500}{\frac{1}{300} + j2\pi \times 1000 \times 0.1 \times 10^{-6}}$$

$$= \frac{0 + j0.6283}{3.33 \times 10^{-3} + j6.283 \times 10^{-4}}$$

$$= \frac{0.6283 \angle 90°}{3.3888 \times 10^{-3} \angle +10.68°}$$

$$= 185.407 \angle +79.32°$$

$$= 34.36 + j\,182.19\ \Omega$$

$$\therefore \quad Z_x = 34.36 + j\,182.19\ \Omega$$

$$\therefore \quad R_x = 34.36\ \Omega$$

and $$X_L = 182.19\ \Omega = 2\pi f L_x$$

$$\therefore \quad L_x = \frac{182.19}{2\pi \times 1000} = 0.029\ \text{H} = \mathbf{29\ mH}$$

Example 7.13 : *An a.c. bridge is balanced at 2 kHz with the following components in each arm.*

Arm AB = 10 kΩ
Arm BC = 100 μF in series with 100 kΩ
Arm AD = 50 kΩ

Find the unknown impedance R ± jX in the arm DC, if the detector is between BD.

Solution : The bridge is shown in the Fig. 7.29.

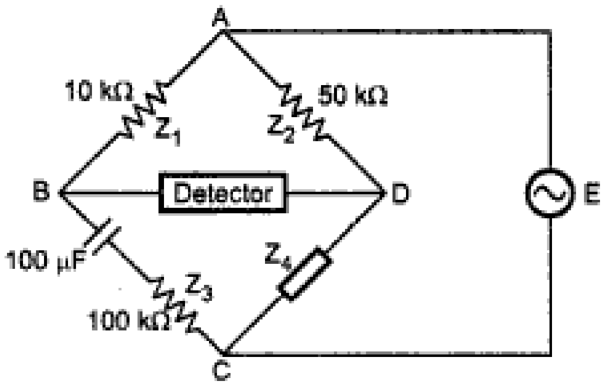


Fig. 7.29

$$Z_1 = 10\ k\Omega$$

$$Z_2 = 50\ k\Omega$$

$$Z_3 = 100 \times 10^3 - j\left(\frac{1}{\omega C_3}\right)$$

$$= 100\times10^3 - j\left(\frac{1}{2\pi\times2\times10^3\times100\times10^{-6}}\right)$$

$$= 100\times10^3 - j0.7957\ \Omega$$

$$Z_4 = Z_X$$

From the basic balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad Z_4 = \frac{Z_2 Z_3}{Z_1}$$

$$= \frac{50\times10^3\ [100\times10^3 - j\,0.7957]}{10\times10^3}$$

$$= 5\ [100\times10^3 - j\,0.7957]$$

$$= 500\times10^3 - j3.9788\ \ \Omega$$

$$= R_x - j\,X_C$$

$$\therefore \quad R_x = 500\times10^3\ \Omega = \mathbf{500\ k\Omega}$$

and $$X_C = \mathbf{3.9788\ \Omega}$$

$$= \frac{1}{2\pi f\,C_x}$$

$$\therefore \quad C_x = \frac{1}{2\pi\times2\times10^3\times3.9788} = \mathbf{20\ \mu F}$$

Example 7.14 : *In a bridge shown in the Fig. 7.30, find the constants of Z_x, considering as series circuit.*

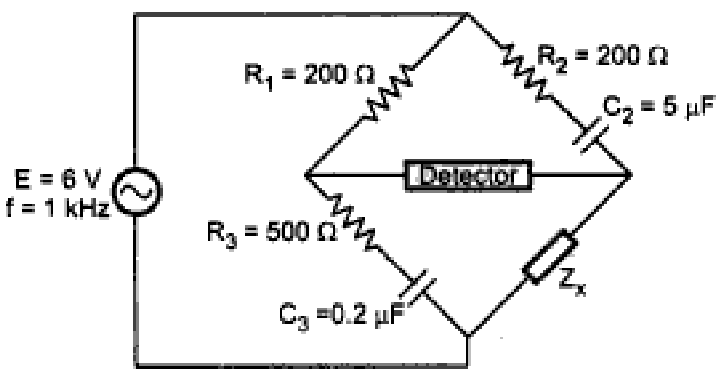


Fig. 7.30

$$\therefore \quad C_x = \frac{1}{2\pi \times 1 \times 10^3 \times 147.5791}$$

$$= \mathbf{1.08\ \mu F}$$

Example 7.16 : *Given the Maxwell bridge as shown in the Fig. 7.32, find the equivalent series resistance and inductance of* R_x *and* L_x *at balance.*

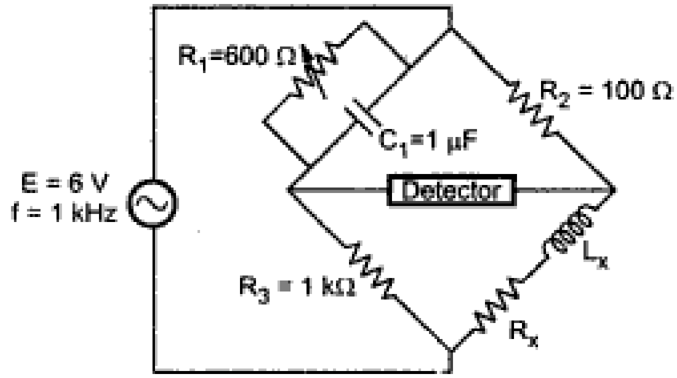


Fig. 7.32

Solution : $Z_1 = R_1 \parallel X_{C1}$

$$\therefore \quad Y_1 = \frac{1}{R_1} + j\,\omega C_1$$

$$= \frac{1}{600} + j\left(2\pi \times 1000 \times 1 \times 10^{-6}\right)$$

$$= 1.66 \times 10^{-3} + j\,6.283 \times 10^{-3}$$

$$Z_2 = 100\ \Omega$$

$$Z_3 = 1000\ \Omega$$

$$Z_4 = R_x + j\,X_L = Z_x$$

From the basic balance equation,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad Z_4 = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

$$= 100 \times 1000 \times [1.66 \times 10^{-3} + j\ 6.283 \times 10^{-3}]$$

$$= 166 + j\ 628.3\ \Omega$$

$$= R_x + j\,X_L\ \Omega$$

$$\therefore \quad R_x = \mathbf{166\ \Omega}$$

$$\therefore \quad X_L = 628.3 = 2\pi f L_x$$

$$\therefore \quad L_x = \frac{628.3}{2\pi \times 1000}$$

$$= \mathbf{0.099\ H}$$

Example 7.17 : *An a.c. bridge circuit for measurement of effective inductance and capacitance of an iron cored coil is as follows : Arm AB : the unknown impedance, Arm BC : a pure resistance of 10 Ω, Arm CB : a loss free capacitance of 1 μF and Arm AD : a capacitance of 0.135 μF in series with 842 Ω resistance. Obtain the balance equations of the bridge and determine the unknown parameters in the arm AB.*

Solution : From the given information the bridge is as shown in the Fig. 7.33.

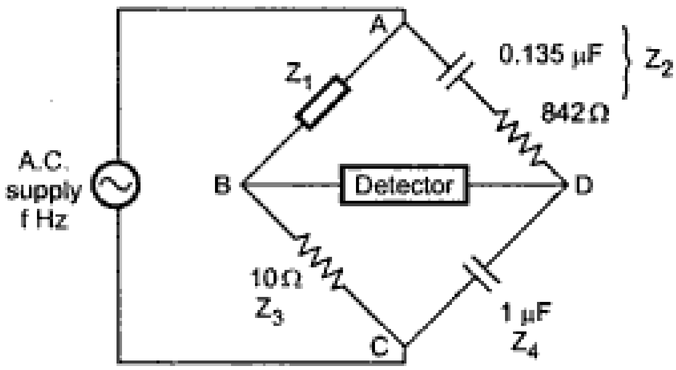


Fig. 7.33

The general balance equation is,

$$Z_1 Z_4 = Z_2 Z_3$$

$$\therefore \quad \boxed{Z_1 = \frac{Z_2 Z_3}{Z_4}}$$

Let the frequency is ω rad/sec.

$$\therefore \quad Z_2 = \left[842 - j\frac{1}{\omega\, 0.135 \times 10^{-6}}\right]$$

$$= 842 - j\frac{7.4074 \times 10^6}{\omega}\ \Omega$$

$$Z_3 = 10 + j\,0 = 10\ \angle 0^\circ\ \Omega$$

$$Z_4 = 0 - j\,\frac{1}{\omega \times 1 \times 10^{-6}} = -j\frac{10^6}{\omega} = \frac{10^6}{\omega}\ \angle -90^\circ\ \Omega$$

$$Z_1 = \frac{\left[842 - j\frac{7.4074\times10^6}{\omega}\right][10]}{\frac{10^6}{\omega}\angle -90^\circ}$$

$$= \left[8420 - j\frac{7.4076\times10^7}{\omega}\right]\frac{1}{10^6}\angle +90^\circ$$

$$= \left[8420 - j\,\frac{7.4076\times10^7}{\omega}\right]\left[+j\,\frac{\omega}{10^6}\right]$$

$$= j\omega\frac{8420}{10^6} + \frac{7.4076\times10^7}{10^6}$$

$$= 74.076 + j\omega\, 8420\times10^{-6}$$

Comparing Z_1 with $R_1 + j\omega L_1$

∴ R_1 = **74.076 Ω** and L_1 = 8420 μH = **8.42 mH.**

These are the unknown parameters of arm AB.

Example 7.18 : *The arms of an a.c. Maxwell's Bridge are arranged as follows. AB and BC are non-reactive resistor of 100 Ω each. DA is a standard variable inductor L_1 of resistance 32.7 Ω and CD comprises a standard variable resistance R in series with a coil of unknown impedance. Balance was obtained with L_1 = 47.8 mH and R = 1.36 Ω. Find the resistance and inductance of coil.*

Solution : The Maxwell's bridge is as shown in the Fig. 7.34.

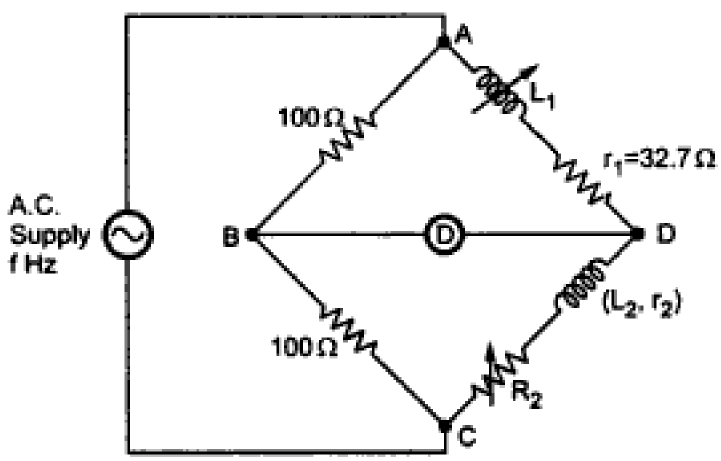


Fig. 7.34 Maxwell's bridge

The balance is obtained when L_2 = 47.8 mH and R_2 = 1.36 Ω.

At balance,

$$100(r_1 + j\omega L_1) = 100[(R_2 + r_2) + j\omega L_2]$$

Equating real and imaginary terms, we get,

$L_1 = L_2$ and $R_2 + r_2 = r_1$ or $r_2 = r_1 - R_1$

∴ Inductance of coil in branch CD is $L_2 = L_1 =$ **4.7 mH**

Resistance of coil in branch CD is given by

$$r_2 = r_1 - R_1 = 32.7 - 1.36$$

$$= \mathbf{31.34\ \Omega.}$$

➡ **Example 7.19 :** *In a Maxwell's inductance comparison bridge arm ab consists of a coil with inductance L_1 and resistance r_1 in series with a non-inductive resistance R. The arm bc and ad are each of non-inductive resistances of 100 Ω . Arm ad consists of standard variable inductor L_2 of resistance 32.7 Ω. Balance is obtained when L_2 = 47.8 mH and R = 1.36 Ω. Find resistance and inductance of the coil in arm ab.*

[JNTU, May-2004, Set-2]

Solution : A Maxwell's inductance comparison bridge is as shown in the Fig. 7.35.

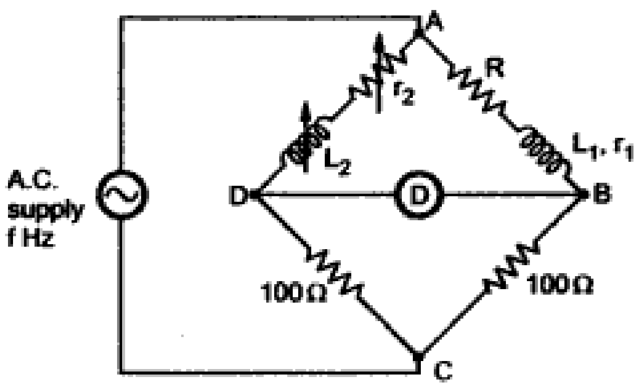


Fig. 7.35

Under balance condition, we can write,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \quad \text{... (i)}$$

But impedance of branch ab is given by,

$$Z_1 = [(R + r_1) + j\omega L_1] \quad \text{... (ii)}$$

Similarly the impedance of branch bc and cd are given as,

$$Z_3 = 100\ \Omega \quad \text{and} \quad Z_4 = 100\ \Omega \quad \text{... (iii)}$$

The impedance of branch ad is given by,

$$Z_2 = [32.7 + j\omega L_2] \quad \text{... (iv)}$$

Putting values in equation (i), we get,

$$[(R + r_1) + j\omega L_1][100] = [32.7 + j\omega L_2][100]$$

i.e. $$(R + r_1) + j\omega L_1 = 32.7 + j\omega L_2 \quad \text{... (v)}$$

Under balanced condition, $L_2 = 47.8$ mH and $R = 1.36\ \Omega$.

Hence putting values in equation (v), we get,

$$(1.36 + r_1) + j\omega L_1 = 32.7 + j\omega\left(47.8 \times 10^{-3}\right)$$

Equating real terms, we get,

$$1.36 + r_1 = 32.7$$

$$\therefore \quad r_1 = \mathbf{31.34\ \Omega}$$

Equating imaginary terms, we get

$$\omega L_1 = \omega\left(47.8 \times 10^{-3}\right)$$

i.e. $$L_1 = \mathbf{47.8 \times 10^{-3}\ H = 47.8\ mH}$$

Thus the values of resistance and inductance of the coil in arm ab are 31.34 Ω and 47.8 mH respectively.

➡ **Example 7.20 :** *The four impedances of an a.c. bridge are*

$Z_{AB} = 400\angle 50°\ \Omega$, $Z_{AD} = 200\angle 40°\ \Omega$,
$Z_{BC} = 800\angle -50°\ \Omega$, $Z_{CD} = 400\angle 20°\ \Omega$

Find out whether the bridge is balanced under these conditions are not.

[JNTU, Nov.-2004, Set-2]

Solution : For an a.c. bridge, the balance conditions are given by,

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{... condition of balance for magnitudes}$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \quad \text{... condition of balance for phases.}$$

Consider the basic a.c. bridge with four impedances as shown in the Fig. 7.36.

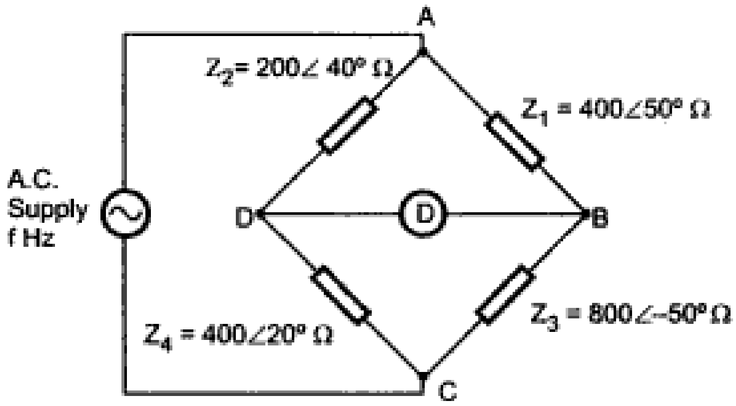


Fig. 7.36

Applying the condition of balance for the magnitudes, we get,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \quad \text{... (a)}$$

i.e. $(400)\ (400) = (200)\ (800)$

i.e. $160000 = 160000$

That means condition of balance for magnitudes is satisfied.

Applying the condition of balance for phases, we get

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \quad \text{... (b)}$$

$$\text{L.H.S.} = \angle\theta_1 + \angle\theta_4 = [50°+20°] = 70°$$

$$\text{R.H.S.} = \angle\theta_2 + \angle\theta_3 = [40°-50°] = -10°$$

As the values on L.H.S. and R.H.S. of equation (b) are not equal, **the condition of balance for phases is not satisfied.**

Thus for above given conditions, the bridge is in unbalanced condition because eventhough condition of balance for magnitudes is satisfied ; condition of balance for phases is not satisfied.

➡ **Example 7.21 :** *A four arm a.c. bridge a-b-c-d has following impedances.*

Arm ab : $Z_1 = 200\angle 60°\ \Omega$, *Arm ad :* $Z_2 = 400\angle -60°\ \Omega$

Arm bc : $Z_3 = 300\angle 0°\ \Omega$, *Arm cd :* $Z_4 = 600\angle 30°\ \Omega$

Determine whether it is possible to balance the bridge under above conditions.

[JNTU, Nov.-2003, Set-3]

Solution : Consider the basic a.c. bridge with four arms as shown in the Fig. 7.37.

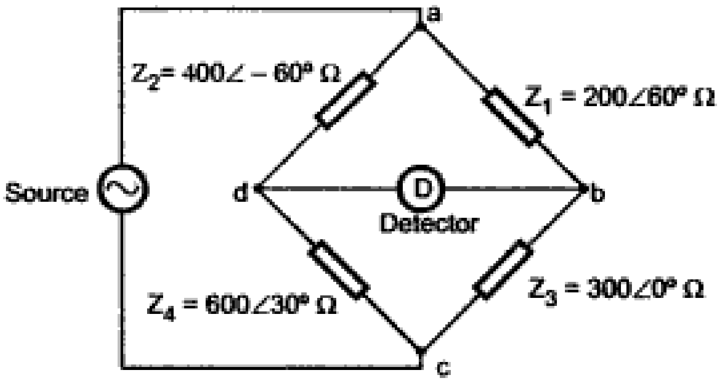


Fig. 7.37

For an a.c. bridge shown, the balance conditions for the magnitudes and phases are given as follows.

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \quad \text{... condition of balance for magnitudes}$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \quad \text{... condition of balance for phases}$$

Applying condition of balance for magnitudes, we get,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \quad \text{... (a)}$$

$$\text{L.H.S.} = Z_1 \cdot Z_4 = (200)\ (600) = 120000$$

$$\text{R.H.S.} = Z_2 \cdot Z_3 = (400)\ (300) = 120000$$

As values of L.H.S. and R.H.S. of equation (a) are equal, **the condition of balance for magnitudes is satisfied.**

Applying condition of balance for phases, we get,

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3 \quad \text{... (b)}$$

$$\text{L.H.S.} = \angle\theta_1 + \angle\theta_4 = [60° + 30°] = 90°$$

$$\text{R.H.S.} = \angle\theta_2 + \angle\theta_3 = [-60° + 0°] = -60°$$

As values of L.H.S. and R.H.S. of equation (b) are not equal, **the condition of balance for phases is not satisfied.**

Thus for above given conditions, the bridge is in unbalanced condition, because eventhough the condition of balance for the magnitudes is satisfied, the condition of balance for phases is not satisfied.

$\therefore \qquad Z_2 = 294.81 \angle -10.67° \ \Omega$

For balance, the condition is given by,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

Hence under balance condition impedance of branch CD i.e. $Z_4 = Z_x$ is given by,

$$Z_4 = \frac{Z_2 \cdot Z_3}{Z_1}$$

Substituting values of Z_1, Z_2 and Z_3, we get,

$$Z_4 = \frac{(294.81 \angle -10.67°)\ (500)}{795.77 \angle -90°}$$

$\therefore \qquad Z_4 = 185.2356 \angle 79.33° \ \Omega$

Thus positive angle of impedance Z_4 indicates it is an inductive impedance of series R-L circuit. Thus we can write,

$$Z_4 = R_4 + j X_{L_4} = (34.2967 + j\,182.03)\ \Omega$$

Thus inductive reactance can be written as,

$$X_{L_4} = \omega L_4 = 2 \pi f L_4 = 182.03$$

$$2 \times \pi \times 1 \times 10^3 \times L_4 = 182.03$$

$\therefore \qquad L_4 = \mathbf{28.97\ mH}$

And $\qquad R_4 = \mathbf{34.2967\ \Omega}$

Thus branch CD is a series R-L circuit consisting $R_4 = 34.2967\ \Omega$ and $L_4 = 28.97$ mH.

Example 7.23 : *The arms of five node bridge are as follows :*

Arm ab : an unknown impedance (R_1, L_1) in series with a non-variable resistor r_1.

Arm bc : a non-inductive resistor $R_3 = 100\ \Omega$

Arm cd : a non-inductive resistor $R_4 = 200\ \Omega$

Arm da : a non-inductive resistor $R_2 = 250\ \Omega$

Arm de : a variable non-inductive resistor r.

Arm ec : a lossless capacitor $C = 1\ \mu F$, .

An a.c. supply is connected between a and c. Detector is between b and e. Calculate the resistance R_1 and inductance L_1 when under balance condition $r_1 = 43.1\ \Omega$ and $r = 229.7\ \Omega$. **[JNTU, Nov.-2003, Set-1]**

Solution : The a.c. bridge is as shown in the Fig. 7.39.

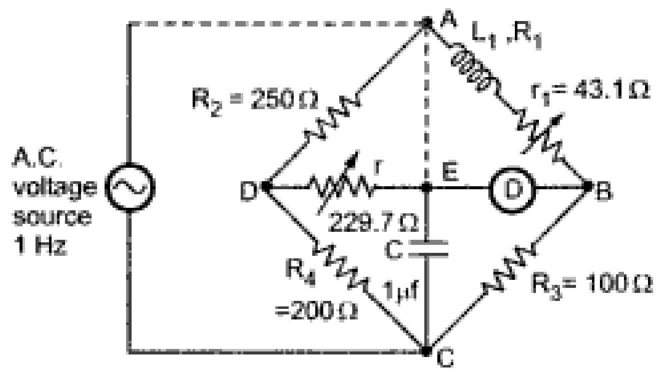


Fig. 7.39

From circuit arrangement, it is Anderson's bridge. The value of unknown resistor is given by,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1 = \frac{(250)(100)}{200} - 43.1 = \mathbf{81.9\ \Omega}$$

Unknown inductance is given by,

$$L_1 = \frac{CR_3}{R_4}\left[(R_2 + R_4)\, r + R_2 R_4\right]$$

$$= \frac{1 \times 10^{-6} \times 100}{200}\left[(250 + 200)\,(229.7) + (250)\,(200)\right]$$

$$= \mathbf{0.0766\ H} = \mathbf{76.6825\ mH}$$

➠ **Example 7.24 :** *The four arms of the bridge are as follows :*

Arm ab : An imperfect capacitor C_1 with an equivalent series resistance of r_1

Arm bc : A non-inductive resistance R_3

Arm cd : A non-inductive resistance R_4

Arm da : An imperfect capacitor C_2 with an equivalent resistance of r_2 in series with resistance R_2.

A supply at 450 Hz is connected between terminals a and c and the detector is connected between b and d. At the balance condition :

$R_2 = 4.8\ \Omega$, $R_3 = 200\ \Omega$, $R_4 = 2850\ \Omega$, and $C_2 = 0.5\ \mu F$, $r_2 = 0.4\ \Omega$

Calculate values of C_1 and r_1 and also of the dissipating factor for the capacitor.

[JNTU, Nov.-2003, Set-2, May-2005, Set-4]

Solution : The bridge is as shown in the Fig. 7.40

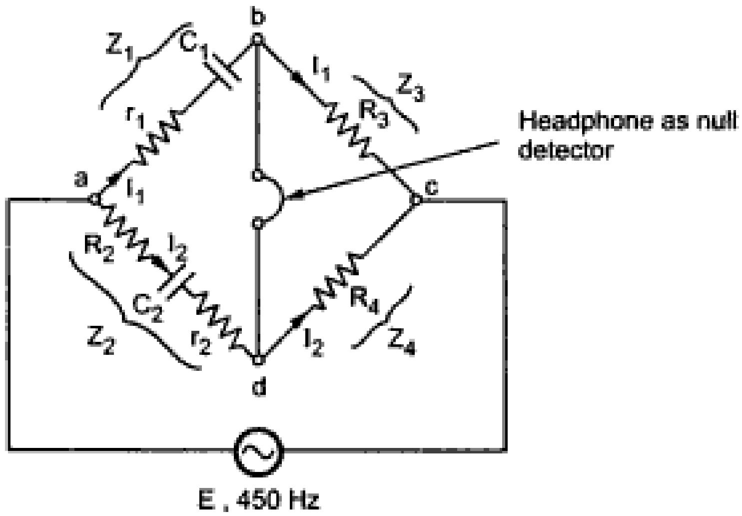


Fig. 7.40

$$Z_1 = r_1 - j\frac{1}{\omega C_1}\,\Omega$$

$$Z_2 = (R_2 + r_2) - j\frac{1}{\omega C_2} = (4.8 + 0.4) - j\,\frac{1}{2\pi \times 450 \times 0.5 \times 10^{-6}}$$

$$= 5.2 - j\,707.3553\,\Omega$$

$$= 707.3744 \angle -89.5788^\circ\,\Omega$$

$$Z_3 = 200 + j\,0\,\Omega = 200 \angle 0^\circ\,\Omega$$

$$Z_4 = 2850 + j\,0\,\Omega = 2850 \angle 0^\circ\,\Omega$$

At a bridge balance, no current flows through the detector.

$$\therefore \quad I_1 = \frac{E}{Z_1 + Z_3} \quad \text{and} \quad I_2 = \frac{E}{Z_2 + Z_4}$$

Now $\quad I_1 Z_1 = I_2 Z_2 \quad$ for null deflection of detector

$$\therefore \quad \frac{E\,Z_1}{Z_1 + Z_3} = \frac{E\,Z_2}{Z_2 + Z_4}$$

$$\therefore \quad Z_1 Z_4 = Z_2 Z_3 \qquad \text{... Balance equation}$$

$$\therefore \quad 2850\left[r_1 - j\frac{1}{\omega C_1}\right] = 200 \angle 0^\circ \times 707.3744 \angle -89.5788^\circ$$

$$\therefore \quad r_1 - j\frac{1}{\omega C_1} = 49.6403 \angle -89.5788^\circ = 0.3649 - j\,49.6389\,\Omega$$

Comparing both sides,

$$r_1 = \mathbf{0.3649\ \Omega} \qquad \text{and} \qquad \frac{1}{\omega C_1} = 49.6389$$

$$\therefore \qquad C_1 = \frac{1}{2\pi \times 450 \times 49.6389} = \mathbf{7.125\ \mu F}$$

$$\text{Dissipating factor} = \omega r_1 C_1 = 2\pi \times 450 \times 0.3649 \times 7.125 \times 10^{-6}$$

$$= \mathbf{0.007351}$$

Example 7.25 : *An a.c. bridge circuit is used to measure the properties of a sample sheet steel at 2 kHz. At balance arm ab is test specimen. Arm bc is 100 Ω . Arm cd is 0.1 μF capacitor and branch da is 834 Ω is series with 0.12 μF capacitor. Calculate the effective impedance of the specimen under test conditions.* **[JNTU, May-2004, Set-3]**

Solution : From given data, the bridge is as shown in the Fig. 7.41.

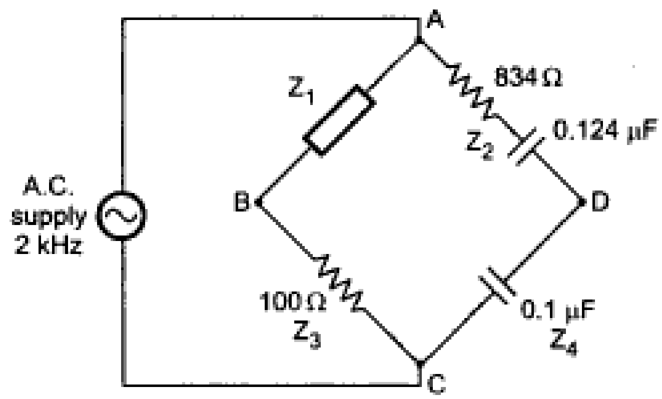


Fig. 7.41

The balance condition is

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$\therefore \qquad Z_1 = \frac{Z_2 \cdot Z_3}{Z_4}$$

Calculating Z_2 separately,

$$Z_2 = 834 + \frac{1}{j(2\times\pi\times 2\times 10^3 \times 0.124\times 10^{-6})}$$

$$= (834 - j\ 641.7538)\ \Omega = 1052.33 \angle -37.57°\ \Omega$$

Similarly, $$Z_4 = -j\frac{1}{(2\times\pi\times 2\times 10^3 \times 0.1\times 10^{-6})}$$

$$= -j\ 795.7747\ \Omega = 795.7747 \angle -90°\ \Omega$$

and $$Z_2 = R_2 - j\frac{1}{\omega C_2} = R_2 + \frac{1}{j\omega C_2} = \frac{1 + j\omega C_2 R_2}{j\omega C_2}$$

Hence we can write,

$$\left(\frac{R_1}{1 + j\omega C_1 R_1}\right)(R_4) = \left(\frac{1 + j\omega C_2 R_2}{j\omega C_2}\right)(R_x + j\omega L_x)$$

Simplifying we get,

$$R_4 = \omega^2 R_1 C_2 \,(R_1 R_4 C_1 - L_x) \qquad \text{... (1)}$$

and $$R_4 = \frac{R_1 (R_x C_2 - R_4 C_1)}{R_2 C_2} \qquad \text{... (2)}$$

Using equations, we can write,

$$R_4 = \frac{R_1 (R_x C_2 - R_4 C_1)}{R_2 C_2}$$

$$\therefore \quad R_2 R_4 C_2 = R_1 R_x C_2 - R_1 R_4 C_1$$

$$\therefore \quad R_x = \frac{R_4 (R_1 C_1 + R_2 C_2)}{R_1 C_2}$$

$$\therefore \quad R_x = \frac{10 \times 10^3 \left[50 \times 10^3 \times 0.003 \times 10^{-6} + 20 \times 10^3 \times 150 \times 10^{-12}\right]}{20 \times 10^3 \times 0.003 \times 10^{-6}}$$

$$\therefore \quad R_x = \mathbf{25.5\ \Omega}$$

Using equation (1), we can write,

$$R_4 = \omega^2 R_1 C_2 (R_1 R_4 C_1 - L_x)$$

$$\therefore \quad \frac{R_4}{\omega^2 R_1 C_2} = R_1 R_4 C_1 - L_x$$

$$\therefore \quad L_x = R_1 R_4 C_1 - \frac{R_4}{\omega^2 R_1 C_2} = R_4 \left[R_2 C_1 - \frac{1}{\omega^2 R_1 C_2}\right]$$

$$L_x = 10 \times 10^3 \left[50 \times 10^3 \times 150 \times 10^{-12} - \frac{1}{(10^6)^2 \times 20 \times 10^3 \times 0.003 \times 10^{-6}}\right]$$

$$\therefore \quad L = \mathbf{74.998\ mH}$$

Example 7.27 : *The four arms of the Maxwell's capacitance bridge at balance are :*

Arm ab : unknown inductance L_1 having an inherent resistance R_1

Arm bc : A non-inductive resistance of 1000 Ω,

Amr cd : A capacitor of 0.5 μF in parallel with a resistance of 1000 Ω .

Arm da : A resistance of 1000 Ω

Determine the values of R_1 and L_1. Draw the phasor diagram of the bridge.

[JNTU, Nov.-2003, Set-4]

Solution : From the given information, the Maxwell's capacitance bridge is as shown in the Fig. 7.43.

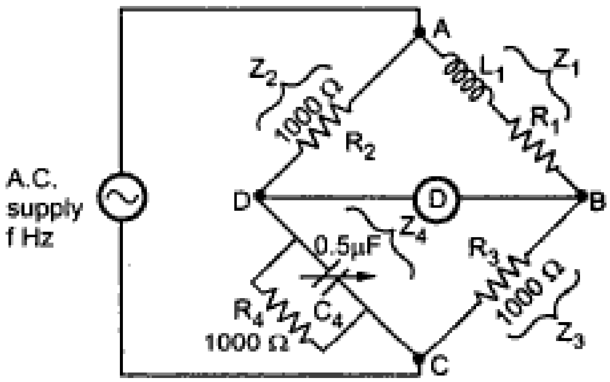


Fig. 7.43

The equation for balance is,

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

i.e.

$$[R_1 + j\omega L_1]\left[\frac{R_4}{1+j\omega C_4 R_4}\right] = (R_2)(R_3)$$

$\therefore$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 R_4 C_4$$

Equating real terms, we get,

$$R_1 R_4 = R_2 R_3$$

$\therefore$

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{(1000)(1000)}{1000} = \mathbf{1000\ \Omega}$$

Equating imaginary terms, we get

$$\omega L_1 R_4 = \omega R_2 R_3 R_4 C_4$$

$\therefore$

$$L_1 = R_2 R_3 C_4 = (1000)(1000)(0.5 \times 10^{-6}) = \mathbf{0.5\ H}$$

Example 7.28 : *In a heavyside Campbell bridge used for the measurement of a self inductance L_x with the equal ratio i.e. $R_3 = R_4$, the following results were obtained. With switch open $M = 15.8$ mH, $r = 25.7\ \Omega$ and with switch closed $M = 0.2$ mH and $r = 1.2\ \Omega$. Find the resistance and self inductance of the coil.* **[JNTU, May-2004, Set-4]**

Solution : The self inductance of the coil is given by,

$$L_2 = 2(M_1 - M_2)$$

$$= 2\left(15.8 \times 10^{-3} - 0.2 \times 10^{-3}\right)$$

$$= 2\left(15.6 \times 10^{-3}\right)$$

$$= \mathbf{31.2 \times 10^{-3}\ H = 31.2\ mH}$$

The resistance of the coil is given by,

$$R_2 = r_1 - r_2 = 25.7 - 1.2 = \mathbf{24.5\ \Omega}$$

Example 7.29 : *A sheet of bakellite 4.5 mm thick is tested at 50 Hz between electrodes 0.12 m in diameter. The Schering bridge employs a standard air capacitor C_2 of 106 pF capacitance, non-reactive resistance R_4 of $\left(\frac{1000}{\pi}\right)\Omega$ in parallel with variable capacitor C_4 and a non-reactive variable resistance R_3. A balance is obtained with $C_4 = 0.5\ \mu F$ and $R_3 = 260\ \Omega$.*

Calculate capacitance, power factor and relative permittivity. **[JNTU, May-2004, Set-4]**

Solution : For Schering bridge the equations at balance are,

$$r_1 = \frac{C_4}{C_2} R_3 \quad \text{and} \quad C_1 = \frac{R_4}{R_3} C_2$$

$$\therefore \quad r_1 = \frac{C_4}{C_2} R_3 = \frac{0.5 \times 10^{-6} \times 260}{106 \times 10^{-12}} = \mathbf{1.23 \times 10^6\ \Omega}$$

$$C_1 = \frac{R_4}{R_3} C_2 = \frac{(1000/\pi)}{260} \times 106 \times 10^{-12} = \mathbf{1.30\ pF}$$

The power factor of the sheet $= \omega C_1 r_1 = 2 \times \pi \times 50 \times 130 \times 10^{-12} \times 1.23 \times 10^6$

$$= \mathbf{0.05}$$

The capacitance $C_1 = \varepsilon_r \varepsilon_0 \dfrac{A}{d}$

Hence relative permittivity is given by,

$$\varepsilon_r = \frac{C_1 d}{\varepsilon_0 A} = \frac{130 \times 10^{-12} \times 4.5 \times 10^{-3}}{8.854 \times 10^{-12} \times \left(\frac{\pi}{2} \times 0.12\right)^2} = 5.9$$

Example 7.30 : *A condensor brushing forms arm ab of a Schering bridge and a standard capacitor of 500 pF and negligible loss forms arm ad. Arm bc consists a non-inductive resistance of 300 Ω . When the bridge is balanced, arm cd has resistance of 72.6 Ω in parallel with a capacitance of 0.148 μF. The supply frequency is 50 Hz. Calculate the capacitance and dielectric loss angle of capacitor.* **[JNTU, Nov.-2004, Set-3]**

Solution : The bridge can be drawn as shown in the Fig. 7.44.

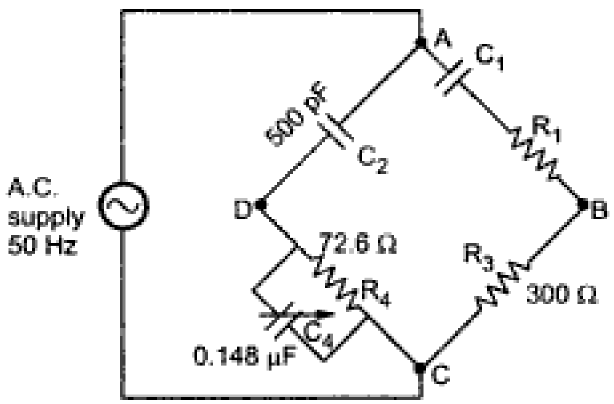


Fig. 7.44

The unknown capacitance in the Schering bridge is given by,

$$C_1 = C_2 \frac{R_4}{R_3} = 0.5 \times 10^{-9} \times \frac{72.6}{300} = \mathbf{0.121\ \mu F}$$

The dielectric loss angle of capacitor is,

$$\delta = \tan^{-1}(\omega C_4 R_4)$$

$$= \tan^{-1}\left(2 \times \pi \times 50 \times 0.148 \times 10^{-6} \times 72.6\right)$$

$$= \mathbf{89.98°}$$

Review Questions

1. *Derive the bridge balance equation for the basic a.c. bridge.*
2. *What is the capacitance comparison bridge ? Derive its balance equation.*
3. *Why the inductance comparison bridge is used ? Derive its balance equation.*
4. *How Schering bridge is used for the measurement of unknown capacitor ? Derive its balance equation. State its advantages.*
5. *How the quality of the capacitor can be obtained using Schering bridge ?*
6. *Write a note on Wagner ground connection.*
7. *Write a note on high voltage Schering bridge.*
8. *Draw the Anderson bridge and explain its working.*
9. *Which measurements can be carried out by Maxwell bridge ? Derive the balance equation and expressions for the unknown components.*
10. *List the advantages of using standard capacitor in Maxwell bridge.*
11. *State the advantages and limitations of the Maxwell bridge.*
12. *What is Hay's bridge ? Derive its balance equation. When it is preferred over Maxwell bridge ?*
13. *Compare Maxwell bridge with Hay's bridge.*
14. *What is Wien's bridge ? Derive the expression for the frequency.*
15. *An a.c. bridge has the following constants*

    Arm AB - Capacitor of 0.5 μF in parallel with 1 kΩ resistance.

    Arm AD - resistance of 2 kΩ

    Arm BC - capacitor of 0.5 μF

    Arm CD - unknown C_x and R_x in series

    Frequency - 1 kHz

    Determine the unknown capacitance and dissipation factor. **[Ans. : 2 kΩ, 0.25 μF, 3.1416]**

❑❑❑

8

Magnetic Measurements

8.1 Introduction

The measurements of various properties of a magnetic material are called magnetic measurements. The magnetic materials play a very important role in the operation of electrical machines hence measurement of various characteristics of a magnetic materials is important from the point of view of designing and manufacturing of electrical machines.

The magnetic measurements include,

1. Measurement of flux density B in a specimen of ferro-magnetic material.
2. Measurement of magnetising force H, producing the flux density B, in air.
3. Determination of B-H curve and the hysteresis loop.
4. Determination of eddy current and the hysteresis losses.
5. Testing of permanent magnets.

For such magnetic measurements following tests are performed :

1. D.C. Tests : These are used to determine B-H curve and hysteresis loop of ferro-magnetic materials. The direct current is used to have variable m.m.f. and fluxmeter is used to measure the flux density. A ballistic galvanometer can be used to measure flux density. Such tests are also called ballistic tests.

2. A.C. Tests : When a ferro-magnetic material is subjected to a cycle of magnetisation and demagnetisation then the eddy current and hysteresis losses occur. Hence alternating current is used to determine iron losses, having provision of a variable frequency and form factor. Such tests are carried out at power, audio or radio frequencies.

3. Steady State Tests : The flux in the air gap plays an important role in the operation of various electrical equipments. Such a flux is measured using steady state tests. Such tests give steady state value of the flux in the air gap of a magnetic material.

The results of magnetic measurements are not very accurate because of following reasons :

1. The magnetic materials are nonhomogeneous.
2. The condition at the time of calculations are different than the conditions existing at the time of testing of magnetic material.
3. Various groups of test specimens have no uniformity.

8.2 Ballistic Galvanometer

The ballistic galvanometer and the fluxmeter are the necessary instruments for various types of magnetic measurements. The ballistic galvanometer is used to measure a **quantity of electricity** passed through it. The ballistic galvanometer has a **search coil** connected to its terminals. When there is change in the flux linked with the search coil, an e.m.f. is induced in the search coil. The electricity proportional to this e.m.f is measured by the ballistic galvanometer.

The ballistic galvanometer is usually of D'Arsonval type. The electricity passing through the ballistic galvanometer is in the form of transitory current which is instantaneous in nature. It exists only till the change of flux is associated with the search coil. Thus the ballistic galvanometer gives a 'throw' proportional to the quantity of current instantaneously passing through it. It does not give steady deflection. The scale of the ballistic galvanometer is calibrated in such a way that from its throw, the quantity of electricity and the change in the flux producing it, can be measured.

The throw of the ballistic galvanometer is proportional to the electricity only if the discharge of electricity through it gets completed before any appreciable change in position of the moving system. Hence the moving system of the galvanometer is made to have large moment of inertia and due to this, it keeps on vibrating for long time for about 10-15 seconds. The large moment of inertia is achieved by the addition of weights to the moving system. Thus the oscillations of the galvanometer moving system are damped with large time period and small damping ratio. The small damping ensures that the first deflection (throw) is high and large in amplitude. The throw or deflection is measured at the extreme point of the first throw using a lamp and scale.

The dead beat galvanometer is best. It has to have electromagnetic damping such that it can be determined from the constants of the instrument. Air damping should not be present as it is indeterminate. To bring the moving system back rapidly, a key is provided with which galvanometer terminals are short circuited.

The special construction of the ballistic galvanometer also includes,

1. The moving coil is free from magnetic material.

2. The terminals, coil and all the connections are made up of copper to avoid thermoelectric e.m.f.s. at the junctions.
3. The suspension strip is selected carefully and mounted precisely.
4. In precision instrument, the suspension is non-conducting and current is led in and out of the coil with the help of delicate spirals of very thin strip of copper.

8.2.1 Theory of Ballistic Galvanometer

The quantity of electricity through the galvanometer is instantaneous and for very short period of time. While discharging it gives energy to the moving system which is dissipated gradually in friction and damping.

Thus let, θ = deflection in radians

b = damping constant

a = moment of inertia

c = controlling constant

During the actual motion, electricity is not present once gets discharged hence the equation of motion becomes,

$$a\frac{d^2\theta}{dt^2}+b\frac{d\theta}{dt}+c\theta = 0 \quad \text{... (1)}$$

The solution of this differential equation is,

$$\theta = A\,e^{m_1 t}+B\,e^{m_2 t} \quad \text{... (2)}$$

where A, B = constants

m_1, m_2 = roots of quadratic = $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

As damping is very small, b is very small hence (b^2-4ac) term is negative and the **roots are imaginary in nature.**

The solution of second order differential equation having complex conjugate roots is damped sinusoidal oscillations given by,

$$\theta = e^{-\frac{bt}{2a}}\,b\sin\left(\frac{\sqrt{4ac-b^2}}{2a}\,t+\alpha\right)$$

But as b is very small hence $4ac-b^2 \approx 4ac$

$$\therefore \quad \theta = e^{\frac{-bt}{2a}}\,b\sin\left(\sqrt{\frac{c}{a}}\,t+\alpha\right) \quad \text{... (3)}$$

From the initial conditions when t = 0, θ = 0 hence α = 0,

$$\therefore \qquad \theta = e^{-\frac{bt}{2a}} b \sin\left(\sqrt{\frac{c}{a}}\, t\right) \qquad \text{... (4)}$$

During the discharge of the electricity through the instrument for the short period of time τ, the deflecting torque proportional to instantaneous current i exists given by Gi where G is displacement constant.

Hence the torque produced by Gi is represented as,

$$Gi = a\frac{d^2\theta}{dt^2}$$

$$\therefore \qquad \int_0^{\tau} Gi\, dt = \int_0^{\tau} a\frac{d^2\theta}{dt^2}\, dt = a\frac{d\theta}{dt} \qquad \text{... (5)}$$

But $\int_0^{\tau} i\, dt$ represents the charge Q coulombs.

$$\therefore \qquad QG = a\frac{d\theta}{dt} \qquad \text{... (6)}$$

where $\frac{d\theta}{dt}$ is the angular velocity of the system at the end of the period τ i.e. at the beginning of the first deflection.

$$\therefore \qquad \boxed{\frac{d\theta}{dt} = \frac{G}{a} Q} \qquad \text{... (7)}$$

Now differentiating equation (4),

$$\frac{d\theta}{dt} = -\frac{b}{2a} e^{-\frac{bt}{2a}} b \sin\left(\sqrt{\frac{c}{a}}\, t\right) + e^{-\frac{bt}{2a}} b \cos\left(\sqrt{\frac{c}{a}}\, t\right)\sqrt{\frac{c}{a}} \qquad \text{... (8)}$$

As the period of discharge is small, at the end of discharge, t = 0. Using above,

$$\therefore \qquad \boxed{\frac{d\theta}{dt} = b\sqrt{\frac{c}{a}}} \qquad \text{... (9)}$$

From the equations (7) and (9),

$$b\sqrt{\frac{c}{a}} = \frac{G}{a} Q$$

$$\therefore \quad \boxed{b = \frac{G}{a}\sqrt{\frac{a}{c}}\,Q} \quad \text{... (10)}$$

Substituting value of b in equation (4),

$$\boxed{\theta = e^{-\frac{bt}{2a}}\left[\frac{G}{a}\sqrt{\frac{a}{c}}\,Q\right]\sin\left(\sqrt{\frac{c}{a}}\,t\right)} \quad \text{... (11)}$$

Key Point : *This shows that the deflection of ballistic galvanometer is proportional to the charge Q and it is oscillatory in nature.*

The waveform of θ is as shown in the Fig. 8.1 with the angular frequency ω.

$$\omega = \sqrt{\frac{c}{a}} = 2\pi f$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$$

$$\therefore \quad \boxed{T' = \frac{1}{f} = 2\pi\sqrt{\frac{a}{c}} = \text{Time period of oscillations}} \quad \text{... (12)}$$

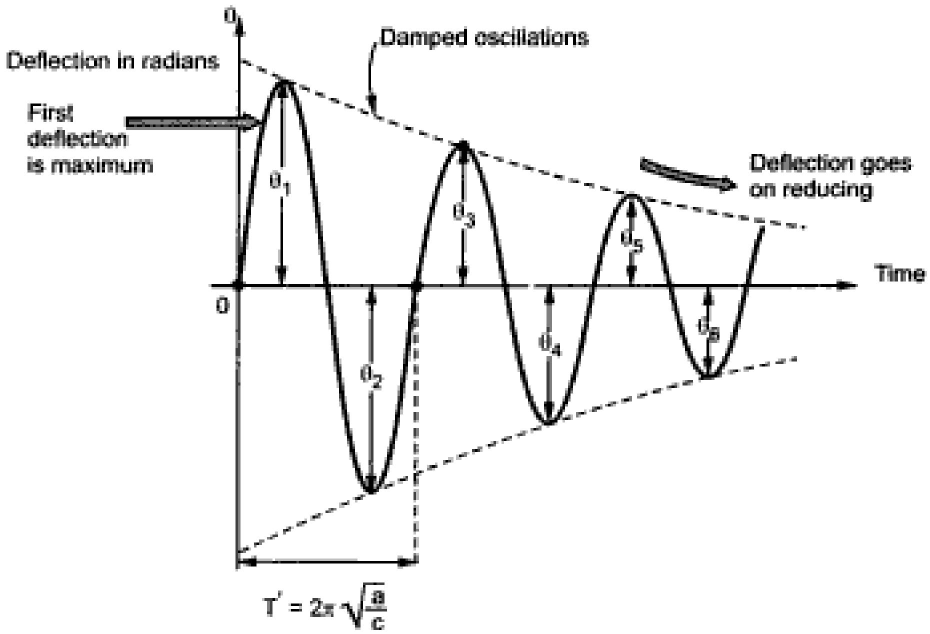


Fig. 8.1 Waveform of θ

The graph shows successive decreasing maxima at the instants $\frac{T'}{4}, \frac{3T'}{4}, \frac{5T'}{4}$ ... etc.

Using in θ,

$$\therefore \quad \theta_1 = \frac{G}{a}\,Q\sqrt{\frac{a}{c}}\,e^{-\frac{\pi}{4}\frac{b}{\sqrt{ac}}} \quad \text{... } t = \frac{T'}{4} = \frac{2\pi}{4}\sqrt{\frac{a}{c}}$$

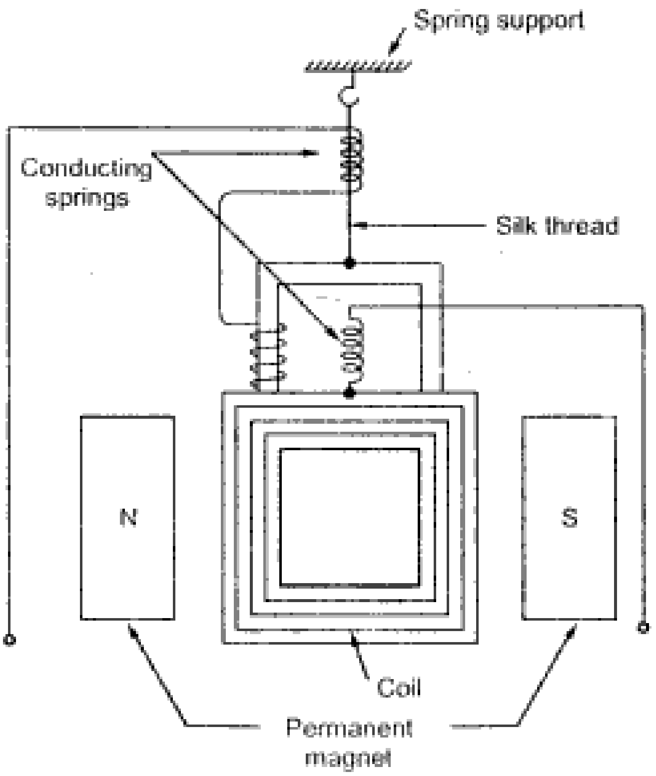


Fig. 8.2 Fluxmeter

The pointer is fitted to the moving system of the fluxmeter and the scale is calibrated interms of flux turns. Such a fluxmeter is designed by Grassot and hence it is called Grassot fluxmeter. The spirals of silver springs keep the controlling torque to minimum.

As controlling torque is minimum, pointer takes time to come back to the zero position. But readings may be taken by observing the difference in deflections at the beginning and end of the change in flux, without waiting for pointer to restore its zero position.

The resistance R_S of the search coil connected to the flux meter must be small. The inductance of search coil may be large.

Let ϕ_1 and ϕ_2 are the interlinking fluxes at the beginning and the end of the change in flux to be measured respectively. The corresponding deflections are θ_1 and θ_2 respectively. Then,

$$d\theta = \text{Change in deflection} = \theta_2 - \theta_1$$

$$d\phi = \text{Change in flux} = \phi_2 - \phi_1$$

and it can be proved that,

$$G d\theta = N\, d\phi$$

where G = Constant of flux meter called displacement constant

N = Number of turns on search coil

$$\therefore \quad \boxed{d\theta = \frac{N}{G} d\phi}$$

In modern fluxmeters, the coil is supported with pivots and mounted in jewelled bearings. The current is passed to the coil through fine ligaments.

The fluxmeter has an advantage over ballistic galvanometer as the deflection is same irrespective of the time taken for the corresponding change in flux, interlinking with the search coil. Another advantage of fluxmeter is it is portable.

The differences between flux meter and ballistic galvanometer are,

Sr. No.	Fluxmeter	Ballistic Galvanometer
1.	Controlling torque is very small.	Controlling torque is high.
2.	Heavy electromagnetic damping.	Electromagnetic damping is not heavy.
3.	Less sensitive.	More sensitive.
4	Less accurate	More accurate.
5.	The deflection is independent of the time taken by the flux changes.	The deflection is dependent on the time taken by the flux changes.

Table 8.1

8.4 Measurement of Flux Density (B)

Let us study the measurement of flux density in a ring specimen. A coil with sufficient number of turns is wound on a ring specimen. This coil is called search coil or a B coil. This coil is then connected to a fluxmeter or a ballistic galvanometer, as shown in the Fig. 8.3.

Please refer Fig. 8.3 on next page.

The magnetising winding carries a current I which produces the flux to be measured. The current I in the magnetising coil is reversed using a reversing switch. Thus flux linkages associated with the search coil also change inducing e.m.f. in it. This e.m.f. drives a current through a ballistic galvanometer, causing the corresponding deflection.

8.4.1 Theory of Flux Density Measurement

Let ϕ = Flux linking with search coil

N = Number of turns of search coil

R = Resistance of ballistic galvanometer circuit

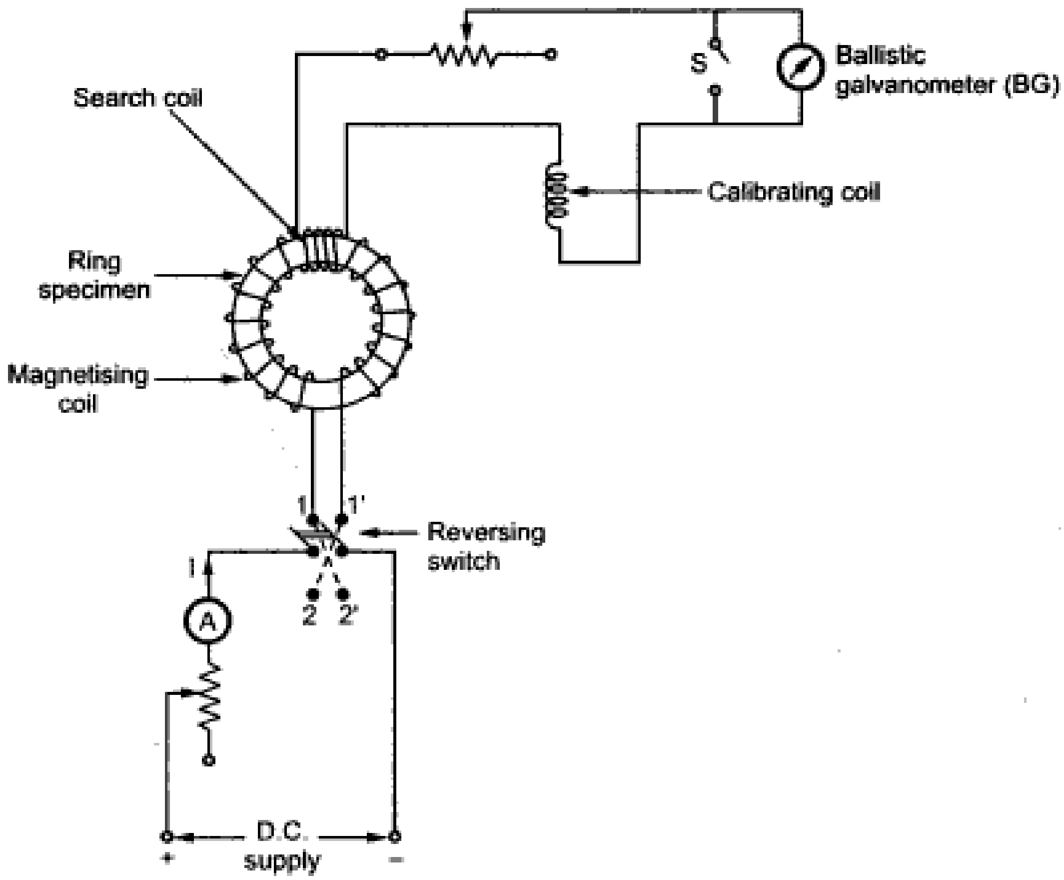


Fig. 8.3 Measurement of flux density

t = Time required to reverse current I i.e. time required to reverse flux ϕ.

The average e.m.f. induced in the search coil is given by,

$$e = N \frac{d\phi}{dt} \text{ volts}$$

Initial flux = ϕ, After reversal = $-\phi$

$$\therefore \quad d\phi = \phi - (-\phi) = 2\phi$$

$$dt = t$$

$$\therefore \quad e = \frac{N2\phi}{t} \text{ V}$$

Thus the average current through ballistic galvanometer is,

$$i = \frac{e}{R} = \frac{2N\phi}{Rt} \text{ A}$$

$$\therefore \quad B_i = B' - \mu_0 H \left[\frac{A_c}{A} - 1\right]$$

This is called correction of air flux.

8.5 Measurement of Magnetising Force H

The magnetising force H is also measured by using a search coil and a ballistic galvanometer. The arrangement is shown in the Fig. 8.4.

In such method H can not be obtained directly but is to be calculated by measuring flux density by the method of current reversal, as described earlier.

The position of search coil shown in the Fig. 8.4 is for the measurement of flux density B_a in air, by reversing current I with the help of reversing switch. The search coil used for such measurement is called H coil. Once B_0 is measured then H can be obtained as,

$$H = \frac{B_0}{\mu_0} \ A/m$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m

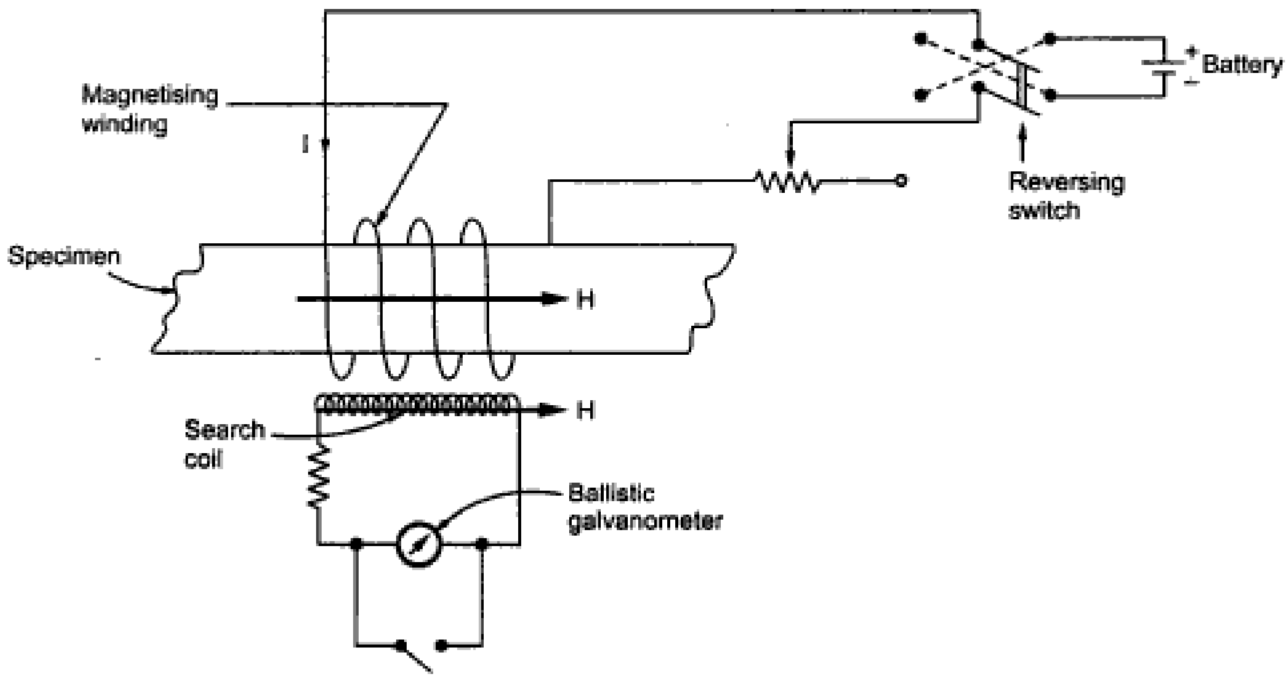


Fig. 8.4 Measurement of H

Thus search coil is placed in the air gap itself if H in the air gap is to be determined. While if magnetising force N within the ferro-megnetic material specimen is to be obtained then H is measured on the surface of the specimen as the tangential components of field are of equal in magnitude for both the sides of the interface.

Such a potentiometer is used for the measurement of the magnetic potential drop across a given part of a magnetic circuit such as a joint, the measurement of magnetic leakage or measurement of m.m.f. around a closed path. The important advantage of the device is that the results are same whether the strip on which the helix is wound is straight or of other shape.

As the method does not give uniform values of H over the entire length AB and hence method is rarely used in practice for direct measurement of H. The potentiometer is also called **Chattock** magnetic potentiometer.

8.7 Measurement of Leakage Factor

In the electrical machines and other devices, the flux crossing the air gap is called useful flux while the flux in the actual pole is called total flux. The useful flux is less than the total flux due to leakage flux. The leakage is specified by a factor called leakage factor which is the ratio of the total flux to the useful flux denoted as λ.

$$\lambda = \frac{\text{Total flux}}{\text{Useful flux}}$$

So to measure leakage factor, the flux crossing the air gap and reaching the armature from pole must be measured alongwith the measurement of flux in the pole bodies. This is possible by the flux meter. The ballistic galvanometer is not suitable for such measurements due to high inductance of the field winding.

The arrangement for such measurement is shown in the Fig. 8.6.

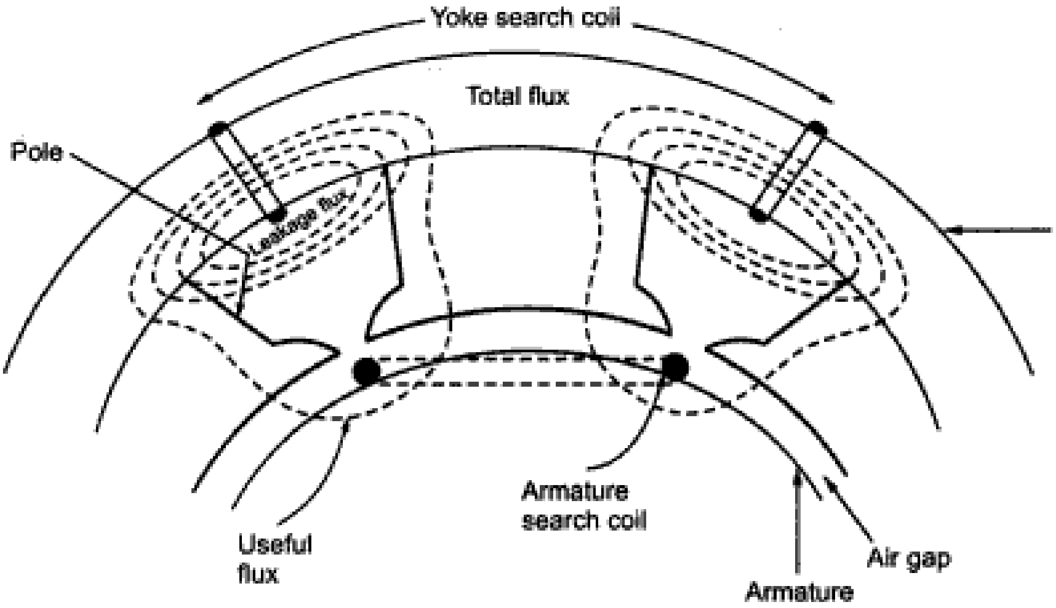


Fig. 8.6 Measurement of leakage factor

The yoke of the machine carries the total flux produced by the field winding would on the poles. Two search coils are wound on the yoke of the machine, one each on the either side of the pole as shown in the Fig. 8.6. Though the yoke carries total flux, there exists half of the total flux on either side of the poles. Hence to measure the total flux the two search coils are connected in series. The coils are connected to the fluxmeter which gives the total flux of the machine.

The flux which reaches to the armature through air gap is called useful flux. A search coil is placed on the stationary armature such that it is in contact with the entire useful flux. It is then connected to the fluxmeter which gives the reading of the useful flux.

The ratio of the two readings thus obtained, is the leakage factor of the machine. The search coils with only one turn are preferred in such measurements so that flux meter directly gives the reading of the required flux.

8.8 Determination of B-H Curve

There are two methods by which B-H curve can be obtained for the magnetic material specimen,

1. Method of reversals 2. Step by step method

8.8.1 Method of Reversals

A ring specimen with known dimensions is taken for the test. A thin tape is wound on the ring. The search coil insulated by the paraffined wax is wound over the tape. Another layer of tap is wound on the search coil. Then the magnetising winding is wound uniformly on the specimen. The overall circuit used is as shown in the Fig. 8.7. (See Fig. 8.7 on next page).

The complete specimen is demagnetised before the test. Using the variable resistance, the magnetising current is adjusted to its lower value, at the beginning of the test. The ballistic galvanometer switch K is closed and reversing switch S is thrown backward and forward for about twenty times. This brings the iron specimen into a 'reproducible cyclic magnetic state'. The galvanometer key K is now opened and the flux in the specimen corresponding to this value of H is measured from the deflection of the ballistic galvanometer, by reversing the switch S. The change in flux, measured by the galvanometer, when the reversing switch S is quickly reversed, will be twice the flux in the specimen, corresponding to the value of H applied. This value of H can be obtained as,

$$\boxed{H = \frac{NI_1}{l}}$$

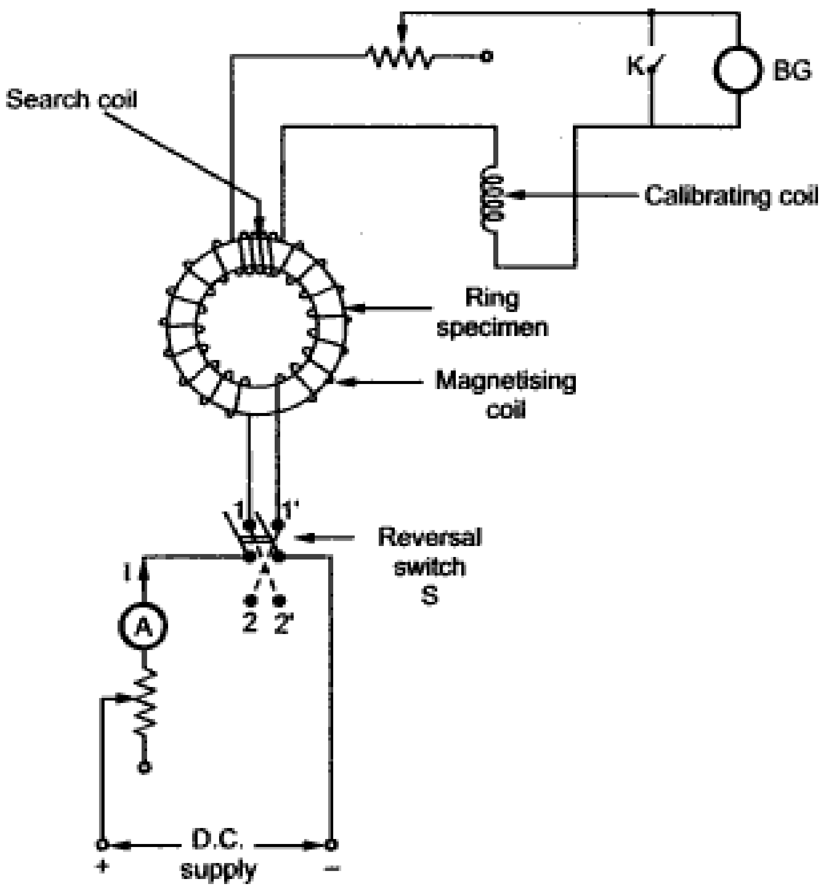


Fig. 8.7 Method of reversals

where N = Number of turns on the magnetising winding

I_1 = Corresponding mangetising current

l = Mean circumference length of specimen in m

While the flux density B is obtained by dividing the flux measured by the area of cross-section of the specimen.

The procedure is repeated for the different values of H by increasing H upto the maximum testing point value. The graph of B against H gives the required B – H curve for the specimen.

8.8.2 Step by Step Method

In this method reversal of magnetising current is not used. The magnetising current in the winding is supplied through a potential divider as shown in the Fig. 8.8.

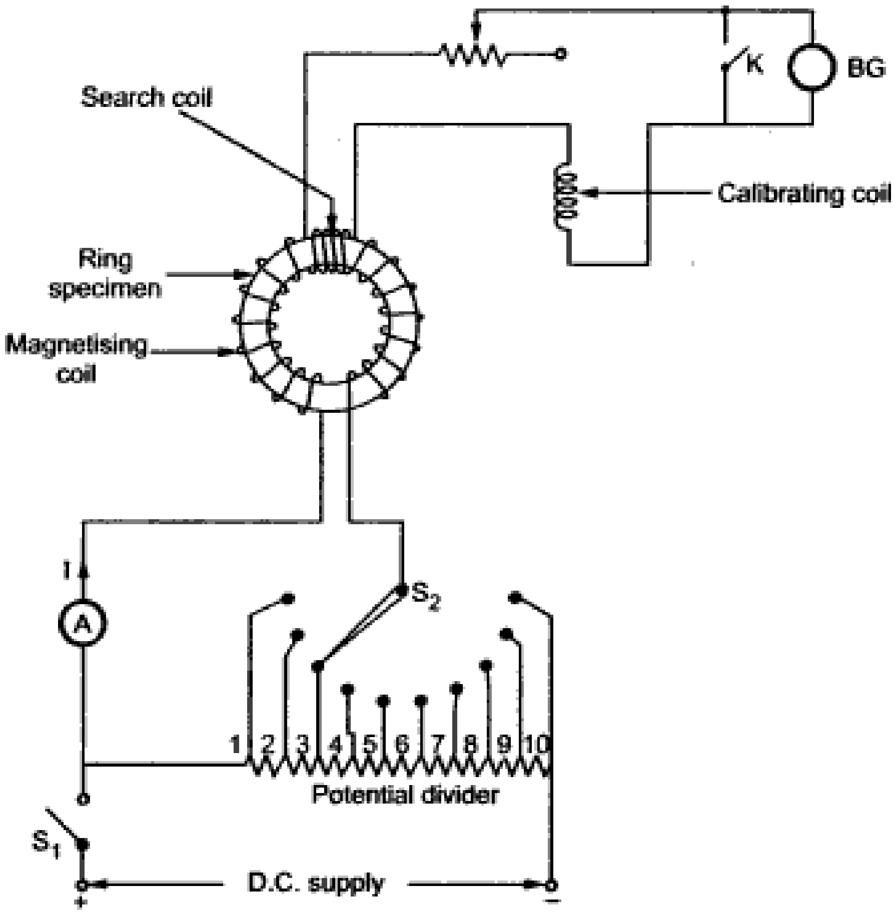


Fig. 8.8 Step by step method

The potential divider has number of tappings. The tappings are arranged in such a way that the magnetising force H increases in suitable number of steps, upto the required maximum value.

The specimen is completely demagnetised before starting the test. The switch S_1 is closed with switch S_2 at tapping 1. Due to this there will be some change in the flux hence there will be increase in the flux density from 0 to B_1. This value can be obtained by observing the deflection of the ballistic galvanometer. Recording the value of corresponding magnetising current the corresponding value of the magnetising force H_1 can be obtained. The switch S_2 is instantaneously changed to tapping 2 which increases the magnetizing force to H_2. Due to this there is increase in flux and hence flux density by the amount ΔB. This can be determined from the galvanometer throw. Hence B_2 at H_2 can be obtained as $B_1 + \Delta B$. The procedure is repeated for various tappings till maximum value of H is achieved. The graph of B against H is then plotted which is nothing but the B – H curve for the specimen under test. This is shown in the Fig. 8.9.

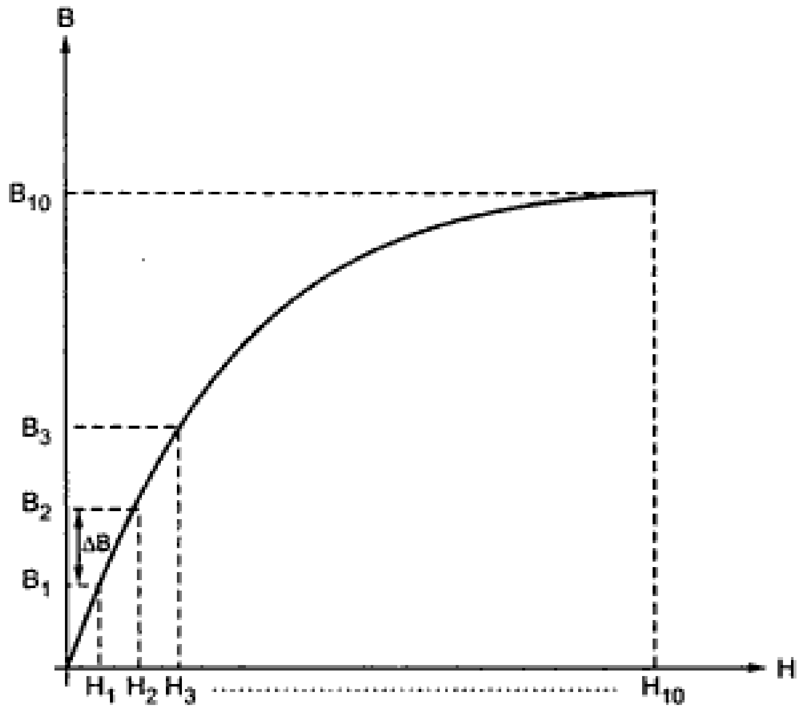


Fig. 8.9 B-H curve from step by step method

Example 8.2 : *A moving coil ballistic galvanometer of 200 Ω resistance gives a throw of 70 divisions when the flux through a search coil to which it is connected is reversed. Find the flux density given that the galvanometer constant is 100 μC per division and the search coil has 1200 turns, a mean area of 60 cm² and a resistance of 15 Ω.*

Solution : K = 100 μC/division, θ′ = 70, N = 1200, A = 60 cm²

$$R = \text{Total resistance} = 200 + 15 = 215\ \Omega$$

$$B = \frac{RK\theta'}{2\,NA} = \frac{215 \times 100 \times 10^{-6} \times 70}{2 \times 1200 \times 60 \times 10^{-4}}$$

$$= \mathbf{0.1045\ Wb/m^2}$$

Example 8.3 : *A ring specimen has a mean length of 1m with a cross-sectional area of 300 mm². It is wound with the magnetising winding having 120 turns. The search coil has 180 turns and is connected to a ballistic galvanometer having constant of 1.5 μC per scale division. The total resistance of the galvanometer circuit is 1500 Ω. On reversing a current of 10 A in the magnetising winding the galvanometer shows a deflection of 75 scale divisions. Calculate the flux density in the specimen and value of relative permeability at this flux density.*

Solution : Total m.m.f. of coil = $N_1 I_1$ = 120 × 10 = 1200 AT

$$\therefore \qquad H = \frac{NI}{l} = \frac{1200}{1} = 1200 \text{ AT/m}$$

Charge through galvanometer is,

$$Q = K\theta' = 1.5 \times 10^{-6} \times 75 = 1.125 \times 10^{-4} \text{ C}$$

Now ϕ is the flux through the ring.

Flux linkages of search coil,

$$\psi = N_2 \phi = 180 \phi$$

Change in flux linkages due to reversal of current is,

$$\Delta\psi = 2(180\phi) = 360\phi$$

$$e = \frac{\Delta\psi}{\Delta t} = \frac{360\phi}{\Delta t} \quad \text{where } \Delta t = \text{time of reversal}$$

$$\therefore \qquad i = \frac{e}{R} = \frac{360\phi}{1500\,\Delta t} = \frac{0.24\phi}{\Delta t} \text{ A}$$

Charge through search coil is,

$$Q = i\,\Delta t = 0.24\phi \quad \text{C}$$

Equating the charges,

$$0.24\phi = 1.125 \times 10^{-4}$$

$$\therefore \qquad \phi = 4.6875 \times 10^{-4} \text{ Wb}$$

$$\therefore \qquad B = \frac{\phi}{A} = \frac{4.6875 \times 10^{-4}}{300 \times 10^{-6}} = \mathbf{1.5625\ Wb/m^2}$$

and $\qquad B = \mu H$

$$\therefore \qquad \mu = \mu_0 \mu_r$$

$$\therefore \qquad \mu_r = \frac{\mu}{\mu_0} = \frac{1.3021 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{1036.165 \approx 1037}$$

➡ **Example 8.4 :** *A ballistic galvanometer has a circuit resistance of 4000 Ω and a constant of 0.1 μC per scale division is connected one after the other in two parts of d.c. machine.*

1. With a coil of 2 turns wound round the field coil of a d.c. machine, producing 110 divisions reading on galvanometer.

2. with armature surface with 3 turns, measuring flux entering the armature, producing 140 divisions reading on galvanometer.

The readings are obtained by breaking the normal field current. Calculate the flux per pole and leakage coefficient.

Solution : Let ϕ be flux linking with a search coil of N turns.

$\therefore$ flux linkages $\psi = N\phi$

The field current is broken so flux becomes zero in time Δt.

$$\therefore \quad \Delta\psi = N\phi \text{ in time } \Delta t$$

$$\therefore \quad e = \frac{\Delta\psi}{\Delta t} = \frac{N\phi}{\Delta t}$$

$$\therefore \quad i = \frac{e}{R} = \frac{N\phi}{R\Delta t}$$

$$\therefore \quad Q = i \times \Delta t = \frac{N\phi}{R}$$

But for a galvanometer charge is given by,

$$Q = K\theta'$$

$$\therefore \quad K\theta' = \frac{N\phi}{R}$$

$$\therefore \quad \phi = \frac{K\theta' R}{N}$$

i) For field coil i.e. pole flux, N = 2, $\theta' = 110$

$$\therefore \quad \phi_t = \frac{0.1 \times 10^{-6} \times 110 \times 4000}{2} = \mathbf{0.022\ Wb}$$

ii) For armature flux, N = 3, $\theta' = 140$

$$\therefore \quad \phi_a = \frac{0.1 \times 10^{-6} \times 140 \times 4000}{3} = \mathbf{0.0186\ Wb}$$

Thus flux per pole is 0.022 Wb i.e. total flux and useful flux is 0.0186 Wb.

$$\therefore \quad \lambda = \text{Leakage factor} = \frac{\phi_t}{\phi_a} = \frac{0.022}{0.0186} = \mathbf{1.1828}$$

8.9 Magnetic Testing Under A.C. Conditions

Whenever a piece of magnetic material is subjected to alternating current, it goes under a cycle of magnetisation and demagnetisation. There exists a hysteresis, due to which power loss occurs in the form of hysteresis loss and eddy current loss. This loss is called **iron loss**. The knowledge of iron loss in ferro-magnetic materials plays an important role for the designers. The iron loss depends on,

i) Frequency of alternating field to which it is subjected.

ii) Maximum value of flux density B_m.

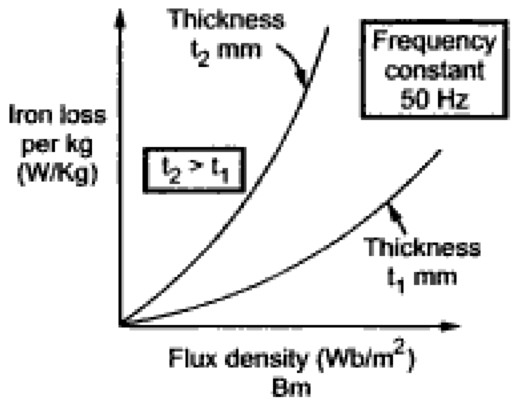


Fig. 8.10 Typical iron loss curves

In practice, for various materials, the curves are obtained at typical frequency giving the variation between iron loss per kg against the maximum flux density. Such curves are called **iron loss curves**. The curves help the designers to select the proper material for the proper application. The Fig. 8.10 shows typical iron loss curves at a frequency of 50 Hz.

The total iron loss has two components,

1) Hysteresis loss and 2) Eddy current loss.

8.9.1 Hysteresis Loss

For a given volume and thickness of laminations, these losses depend on the operating frequency and maximum flux density in the core. **Basically hysteresis loss per unit volume is the area of the hysteresis loop of that material.**

Practically Steinmetz has given the formula for the hysteresis loss per unit volume as,

$$P_h = \eta f B_m^k \quad \text{watts/m}^3$$

where η = Hysteresis coefficient

f = Frequency

B_m = Maximum flux density

K = Steinmetz coefficient varies between 1.6 to 2

Practically K is taken as 1.67.

For a given specimen of thickness t and certain volume, it is given by,

$$P_h = K_h f B_m^{1.67} \quad W \qquad \text{... (1)}$$

where K_h = Hysteresis loss constant

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➡ **Example 8.6 :** *A specimen is tested having total weight of 15 kg for the iron loss. It is found that for a given value of flux density, the iron loss is 19.4 W at 40 Hz and 31 W at 60 Hz. Calculate the eddy current and hysteresis loss in watts/kg at 50 Hz at the same value of flux density and total iron loss at 50 Hz.*

Solution : It is known that $P_h \propto f$ and $P_e \propto f^2$ for given B_m

$$\therefore \qquad P_i = P_h + P_e = A\,f + B\,f^2$$

$$\therefore \qquad 19.4 = A(40) + B(40)^2 \quad \text{and} \quad 31 = A(60) + B(60)^2$$

Solving, $\quad A = 0.4216 \qquad B = 0.0015833$

At 50 Hz, $\quad P_i = 0.4216 \times 50 + 0.0015833 \times (50)^2 = \mathbf{25.03825\ W}$

... Total P_i at 50 Hz

$$P_h = \frac{Af}{\text{mass}} = \frac{0.4216 \times 50}{15} = \mathbf{1.4053\ W/kg}$$

$$P_e = \frac{B\,f^2}{\text{mass}} = \frac{0.0015833 \times 50^2}{15} = \mathbf{0.2638\ W/kg}$$

➡ **Example 8.7 :** *In an iron loss test on a 10 kg specimen of steel laminations, the peak flux density and the form factor are maintained constant and following results are obtained.*

Frequency Hz	*25*	*40*	*50*	*60*	*80*
Total iron loss W	*19*	*37*	*52*	*70*	*108*

Calculate the eddy current and hysteresis loss per kg at 50 Hz.

Solution : The total iron loss is,

$$P_i = P_h + P_e = A\,f + B\,f^2$$

$$\therefore \qquad \frac{P_i}{f} = A + B\,f$$

From the given table,

f	25	40	50	60	80
P_i / f	0.76	0.925	1.04	1.1666	1.35

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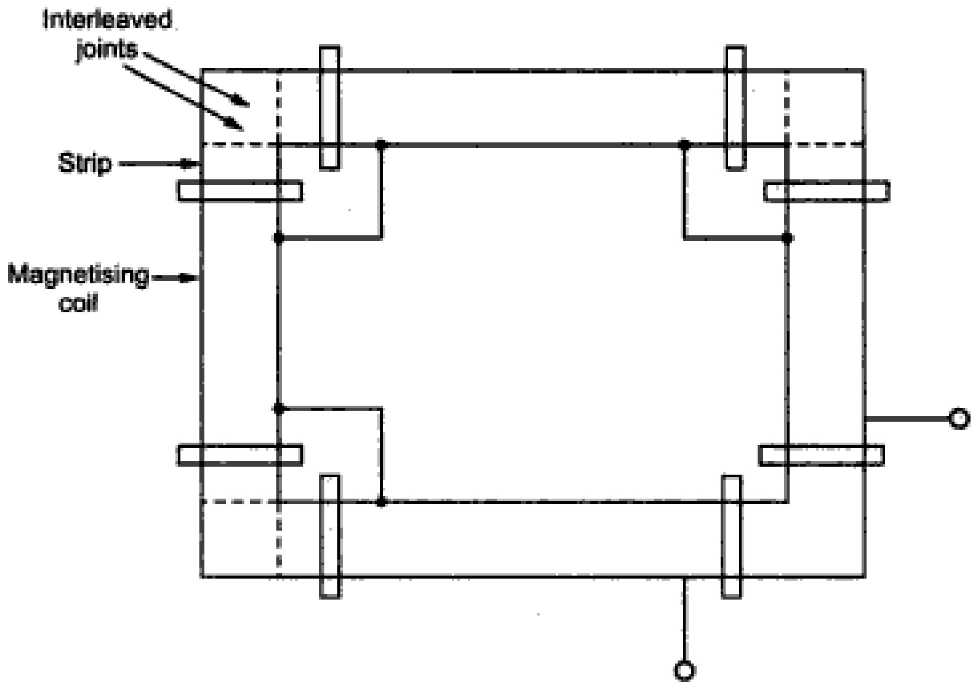

Fig. 8.14 Epstein square

2. Lioyed Fisher square :

In this square, the strips are built up into four stacks. Each stack consists of two types of strips, one cut in the direction of rolling and other cut perpendicular to the direction of rolling. The stacks are inserted inside four magnetising coils having large cross-sectional area. All these coils are connected in series to form a **primary winding.**

Below each magnetising coil, there are two similar single layer coils are placed. These are **secondary coils.** The four coils are connected in series and such two groups exist. Thus two seperate secondary windings exist.

The strips are projected beyond the coil and arranged in such a way that each is perpendicular to the plane of the square. The right angled corner pieces are used at the corners to form the corner joints. The corner pieces and strips are overlapped due to which the cross-section of the iron is doubled at the corners. **Hence a correction is required to be applied for the measured value of iron loss.**

The Fig. 8.15 shows an arrangement at Lioyed-Fisher square.

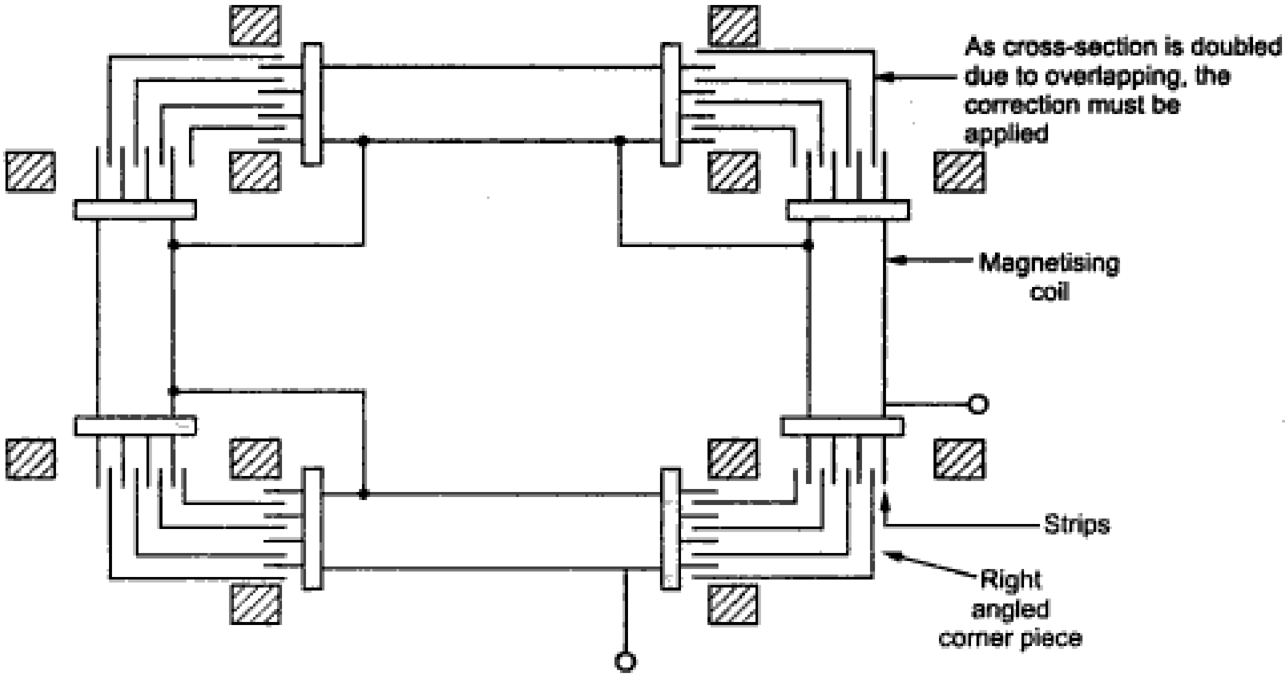

Fig. 8.15 Lioyed Fisher square

	Epstein square	Lioyed-Fisher square
1.	The value of flux density is not same in corners and in square sides. The correction for this is difficult to apply. Hence results not accurate.	The correction is required for doubling of cross-section at the corners. This is easy to apply. Once applied, the results are highly accurate and reliable.
2.	The performance is poor for anistropic materials as the direction of flux at the corners is partially perpendicular to the flux path in other parts of strip.	Suitable for anistropic material as the corner pieces eliminates the difficulty of direction of flux existing in Epstein square.
3.	Not preferred due to its inaccuracy.	Very commonly used due to its accuracy.
4.	The strips are in the plane of the square.	The strips are perpendicular to the plane of the square.

Table 8.2

The correction is required for this value of flux density as S_2 encloses the flux in air gap between specimen and the coil, in addition to the flux in the specimen.

$$\therefore \quad B_m = B'_m - \mu_0 H_m \left[\frac{A_c}{A_s} - 1\right] \quad \text{... (1)}$$

where B_m = Maximum flux density required

A_c = Cross-section of coil in m^2, A_s = Cross-section and specimen in m^2

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Dividing the two equations,

$$\frac{R_s + j\omega L_s}{R_3} = \frac{R_2 + r + j\omega L_2}{R_4}$$

Equating real and imaginary parts,

$$\boxed{R_s = \frac{R_3}{R_4}(R_2 + r)} \quad \text{and} \quad \boxed{L_s = \frac{R_3}{R_4} L_2} \qquad \text{... (3)}$$

Now R_s = Effective resistance of a-b including winding resistance R_w and the resistance representing core losses R_c

$$\therefore \quad I_1^2 R_s = \text{Iron loss + Copper loss in winding} = P_i + I_1^2 R_w$$

$$\therefore \quad P_i = I_1^2 [R_s - R_w] \qquad \text{... (4)}$$

Now $I = I_1 + I_2$

From (1), $I_1 R_3 = (I - I_1) R_4$

$$\therefore \quad I_1 = I\left[\frac{R_4}{R_3 + R_4}\right] \qquad \text{... (5)}$$

$$\therefore \quad \boxed{P_i = I^2 \left[\frac{R_4}{R_3 + R_4}\right]^2 [R_s - R_w]} \qquad \text{... (6)}$$

The current I is measured on ammeter and R_s, R_w can be measured. Thus iron loss can be obtained.

The L_s obtained by equation (3) can be expressed as,

$$L_s = \frac{N^2}{S} \quad \text{where} \quad S = \text{reluctance} = \frac{l_s}{\mu_0 \mu_r A_s}$$

N = Number of turns of magnetising coil

l_s = Length of mean flux path in specimen

A_s = Cross-sectional area of specimen

μ_r = A.C. relative permeability of the specimen

$$\therefore \quad L_s = \frac{N^2}{\dfrac{l_s}{\mu_0 \mu_r A_s}}$$

$$\therefore \quad \boxed{\mu_r = \frac{l_s L_s}{N^2 \mu_0 A_s} = \frac{R_3 l_s L_2}{N^2 R_4 \mu_0 A_s}} \qquad \text{... (7)}$$

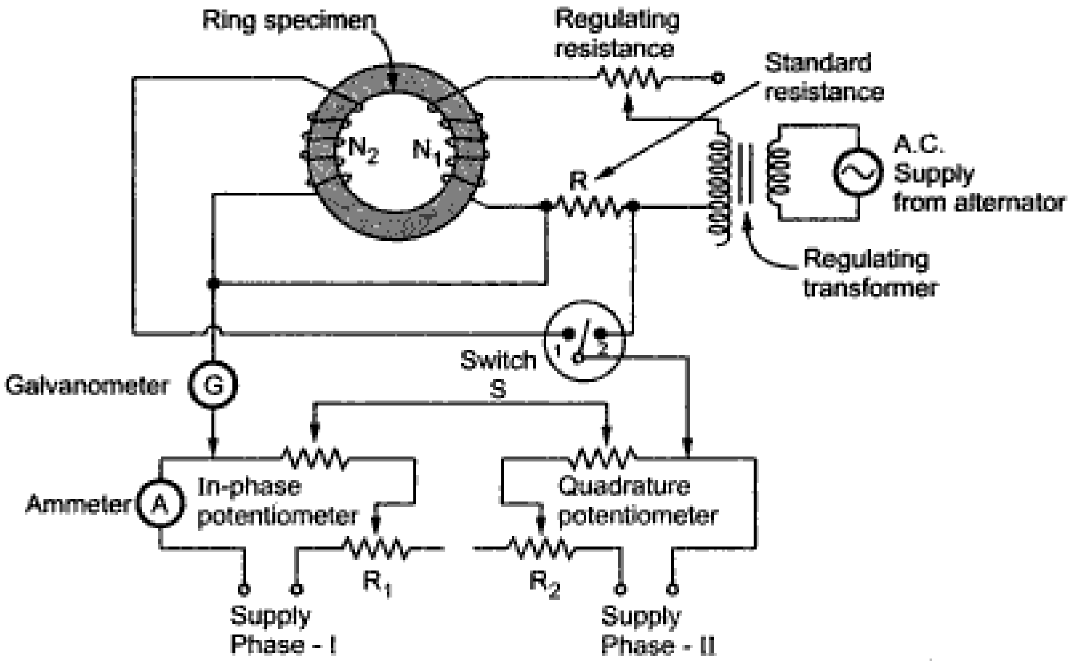

Fig. 8.20 Measurement of iron loss using a.c. potentiometer method

When supply is given to the primary, the voltage E_2 is induced in the secondary. According to transformer equation, it is given by,

$$E_2 = 4.44\, \phi_m\, f N_2 = 4.44\, B_m\, A f N_2$$

$$\therefore \quad B_m = \frac{E_2}{4.44\, A f N_2} \quad \text{... (1)}$$

For sinusoidal waveform $K_f = 1.11$.

The E_2 can be measured as,

i) Put the switch S to position 1.

ii) Set the quadrature potentiometer at zero.

iii) Adjust the in-phase potentiometer till galvanometer G shows zero deflection.

iv) The setting of in-phase potentiometer for balance gives the value of E_2.

Note that the potentiometer must be standardized first.

Then put the switch to position 2 and adjust both the potentiometers till galvanometer shows zero deflection.

The total current has two parts as core loss component I_c and magnetising component I_m.

The in-phase potentiometer reading gives the drop $I_c R$ while the quadrature potentiomenter reading gives the drop $I_m R$.

$$\therefore \qquad I_c = \frac{\text{Reading of in - phase potentiometer}}{R}$$

and
$$I_m = \frac{\text{Reading of quadrature potentiometer}}{R}$$

$$\therefore \qquad \boxed{P_i = \text{Iron loss} = I_c E_2 \left(\frac{N_1}{N_2}\right)}$$

The method is very effective as both core loss and magnetising components of currents are obtained separately.

8.14 Oscillographic Method

The magnetic measurements can be done using cathode ray oscilloscope (C.R.O.). The Fig. 8.21 shows the arrangement of C.R.O. for such measurements.

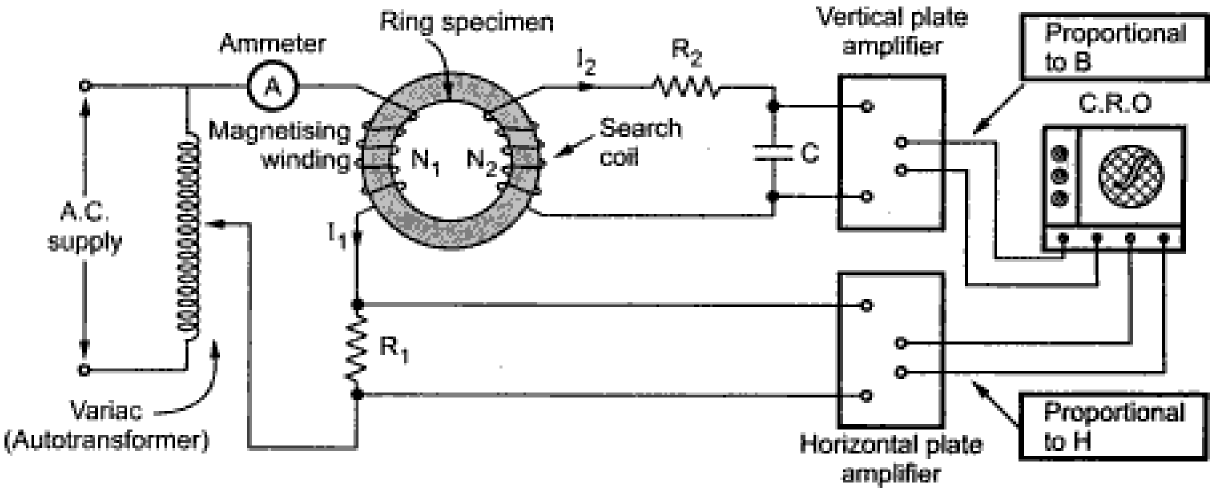

Fig. 8.21 Testing of magnetic material using oscillographic method

The ring specimen has two windings, magnetising winding having N_1 turns and search coil having N_2 turns. In series with magnetising winding, an ammeter and non-inductive resistance R_1 are connected. The magnetising circuit is excited with a supply from an autotransformer. The magnetising current I_1 flows through the circuit, causing a drop $I_1 R_1$ across R_1.

$$\therefore \qquad \boxed{V_1 = I_1 R_1 \propto \text{Magnetic field in test specimen}}$$

The voltage V_1 which is directly proportional to the magnetic field in the test specimen is connected to x-plate amplifier representing horizontal axis on C.R.O.

Examples with Solutions

Example 8.9 : *A fluxmeter is connected to a search coil of 100 turns and the mean area of the coil is 5 cm^2. The search coil is placed at the centre of a standard solenoid 1 m long uniformly wound with 800 turns. When a current of 5 A is reversed, a deflection of 10 scale divisions is obtained with the fluxmeter. Calculate the calibration constant of the instrument in Wb-turns per division.*

Solution : The magnetic field strength inside the solenoid is,

$$H = \frac{NI}{l} \qquad \text{where } N = 800,\ I = 5\text{ A},\ l = 1\text{ m}$$

$$\therefore \quad H = \frac{800 \times 5}{1} = 4000 \text{ AT/m}$$

The flux passing through the search coil is,

$$\phi = B A = \mu_0 \mu_r H A \qquad \text{... A = area} = 5 \times 10^{-4} \text{ m}^2$$

$$\mu_r = 1 \text{ for air cored solenoid}$$

$$\therefore \quad \phi = 4\pi \times 10^{-7} \times 1 \times 4000 \times 5 \times 10^{-4} = 2.5132 \times 10^{-6} \text{ Wb}$$

$$\therefore \quad \text{Wb - turns} = \phi \times \text{turns of search coil} = 2.5132 \times 10^{-6} \times 100$$

$$= 2.5132 \times 10^{-4} \text{ Wb - turns}$$

When the current is reversed, the new current is – 5 A. Hence change in current is 5 – (– 5) = 10 A i.e. twice the original value. Hence the change in weber-turns is also twice the original value when current is reversed.

$$\therefore \text{Change in Wb - turns} = 2 \times 2.5132 \times 10^{-4} = 5.0265 \times 10^{-4} \text{ Wb - turns}$$

$$\therefore \text{Calibration constant} = \frac{\text{Change in Wb - turns}}{\text{Corresponding deflection}} = \frac{5.0265 \times 10^{-4}}{10}$$

$$= \mathbf{5.0265 \times 10^{-5}} \textbf{ Wb - turns/division}$$

Example 8.10 : *In loss tests on a sample of iron laminations the following results were recorded,*

a) At 60 Hz, 250 V, total iron loss = 200 W

b) At 40 Hz, 100 V, total iron loss = 40 W.

Calculate the eddy current and hysteresis loss for each test. The steinmetz index is 1.6.

(JNTU, May-05 Set-1, Set-3, Nov-04 Set-4)

Solution : For a constant form factor,

$$P_e = K_e B_m^2 f^2 \text{ and } P_h = K_h f B_m^{1.6}$$

$$\therefore \quad P_i = P_e + P_h = K_e B_m^2 f^2 + K_h f B_m^{1.6}$$

As voltage is changed in the above two tests, B_m is not same for both.

$$\therefore \quad (P_i)_1 = K_e\, B_{m1}^2\, f_1^2 + K_h\, f_1\, B_{m1}^{1.6} \quad \text{... (1)}$$

$$\text{and} \quad (P_i)_2 = K_e\, B_{m2}^2\, f_2^2 + K_h\, f_2\, B_{m2}^{1.6} \quad \text{... (2)}$$

Key Point : *The maximum flux density varies directly with voltage and inversely with frequency.*

$$\therefore \quad B_m \propto \frac{V}{f}$$

$$\therefore \quad B_{m1}\, f_1 \propto V_1 \text{ and } B_{m2}\, f_2 \propto V_2 \quad \text{... (3)}$$

$$\frac{B_{m1}\, f_1}{B_{m2}\, f_2} = \frac{V_1}{V_2}$$

$$\text{Now} \quad (P_i)_1 = 200\text{ W},\ f_1 = 60\text{ Hz},\ V_1 = 250\text{ V}$$

$$(P_i)_2 = 40\text{ W},\ f_2 = 40\text{ Hz},\ V_2 = 100\text{ V}$$

$$\therefore \quad \frac{B_{m1} \times 60}{B_{m2} \times 40} = \frac{250}{100}$$

$$\therefore \quad B_{m1} = 1.667\ B_{m2} \quad \text{... (4)}$$

Using (4) in (1) and (2),

$$200 = K_e\ B_{m1}^2\ (60)^2 + K_h\ B_{m1}^{1.6} \times 60 \quad \text{... (5)}$$

$$40 = K_e\left(\frac{B_{m1}}{1.6667}\right)^2 (40)^2 + K_h\left(\frac{B_{m1}}{1.6667}\right)^{1.6} \times 40 \quad \text{... (6)}$$

$$\text{Let} \quad K_e\ B_{m1}^2 = A \text{ and } K_h\ (B_{m1})^{1.6} = B \quad \text{... (7)}$$

$$\therefore \quad 200 = 3600\text{ A} + 60\text{ B and } 40 = 575.9769\text{ A} + 17.6639\text{ B}$$

Solving above simultaneously,

$$\therefore \quad A = 0.03902 \text{ and } B = 0.99218$$

$$\text{i.e.} \quad K_e\ B_{m1}^2 = 0.03902 \text{ and } K_h\ (B_{m1})^{1.6} = 0.99218$$

At 250 V_1 60 Hz :

$$P_e = K_e\ B_{m1}^2\ f_1^2 = 0.03902 \times (60)^2 = \mathbf{140.472\ W}$$

$$P_h = K_h\ B_{m1}^{1.6}\ f_1 = 0.99218 \times 60 = \mathbf{59.528\ W}$$

At 100 V, 40 Hz :

$$P_e = K_e\left(\frac{B_{m1}}{1.6667}\right)^2 (40)^2 = \frac{0.03902}{(1.6667)^2} \times 40^2 = \mathbf{22.466\ W}$$

$$P_h = K_h\left(\frac{B_{m1}}{1.667}\right)^{1.6} \times 40 = \frac{0.99218}{(1.667)^{1.6}} \times 40 = \mathbf{17.534\ W}$$

➠ Example 8.11 : *In a test on a specimen of total weight 13 kg, the measured values of iron loss at a given value of flux density were 17.2 W at 40 Hz and 28.9 W at 60 Hz. Estimate the values of hysteresis and eddy current losses at 50 Hz for the same value of peak flux density.* **(JNTU, May-2004, Set-3)**

Solution :

$$P_e = K_e f^2 B_m^2 \text{ W/kg}$$

$$P_h = K_h f B_m^k \text{ W/kg}$$

As B_m is constant,

$$P_e = K_1 f^2 \text{ and } P_h = K_2 f$$

$$\therefore \quad P_i = P_e + P_h = K_1 f^2 + K_2 f$$

$$\therefore \quad \frac{17.2}{13} = K_1 (40)^2 + K_2 (40) \qquad \text{... (1)}$$

$$\therefore \quad \frac{28.9}{13} = K_1 (60)^2 + K_2 (60) \qquad \text{... (2)}$$

Solving above equations simultaneously,

$$K_1 = 1.9871 \times 10^{-4}, \quad K_2 = 0.02512$$

Thus at 50 Hz,

$$P_e = K_1 \times (50)^2 = \mathbf{0.4967 \text{ W/kg}}$$

$$P_h = K_2 \times (50) = \mathbf{1.256 \text{ W/kg}}$$

➠ Example 8.12 : *The mutual inductance between magnetising winding and a search coil wound on a specimen is 9 mH. The search coil has 20 turns. The area of cross-section of specimen is 5000 mm². The reversal of current of 3 A in magnetising winding produces the throw of 60 galvanometer divisions. Calculate the value of flux density in the specimen if the reversal of current in the magnetising winding produces a galvanometer deflection of 36 divisions.* **(JNTU, May-2004, Set-1)**

Solution : Let

$$\psi = \text{flux linkages} = \text{flux} \times \text{turns} \quad \text{Wb-turns}$$

$$M = \frac{N\phi}{I} = \frac{\psi}{I}$$

$$\therefore \quad \psi = M I$$

When current of 3A is reversed i.e. becomes – 3 A.

$$\Delta\psi = M \Delta I = 9 \times 10^{-3} \times [6] = 54 \text{ mWb - turns}$$

$$K = \text{galvanometer constant} = \frac{\Delta\psi}{\theta}$$

$$= \frac{54 \times 10^{-3}}{60} = 9 \times 10^{-4} \ \frac{\text{Wb - turns}}{\text{division}}$$

For : θ = 36 divisions,

$$\Delta\psi = K\theta = 9 \times 10^{-4} \times 36 = 0.0324 \text{ Wb-turns}$$

Now $\Delta I = 2\,I$ ... as current is reversed

$\therefore$ $\Delta\psi = M\Delta I = 2\,M\,I$

$\therefore$ $M\,I = \frac{\Delta\psi}{2} = \frac{0.0324}{2} = 0.0162$ Wb-turns

But $MI = N\phi = 0.0162$

$\therefore$ $\phi = \frac{0.0162}{N} = \frac{0.0162}{20} = 8.1 \times 10^{-4}$ Wb ... N = 20 turns of coil

$\therefore$ $B = \frac{\phi}{A} = \frac{8.1 \times 10^{-4}}{5000 \times 10^{-6}} = \mathbf{0.162\ Wb/m^2}$

Example 8.13 : *An iron ring has a mean diameter of 0.15 m and a cross-sectional area of 34.5 mm². It is wound with a magnetising winding of 330 turns and a secondary winding of 220 turns. On reversing a current of 12 A in a magnetising winding, a ballistic galvanometer gives a throw of 272 divisions. With a Hibbert's magnetic standard with 10 turns and flux of 0.00025 Wb, gives a reading of 102 scale divisions. Other conditions remain same. Find the relative permeability of the specimen.*

(JNTU, May-2004, Set-4)

Solution : In a Hibbert's magnetic standard, the flux slides through the gap of permanent magnet.

The change in flux linkages of the coil when it goes down the Hibbert's standard is,

$$\Delta\psi = \text{Flux} \times \text{number of turns of Hibbert's coil}$$

$$= 0.00025 \times 10 = 0.0025 \text{ Wb-turns}$$

$$\theta = \text{Throw of galvanometer} = 102 \text{ divisions}$$

$\therefore$ $K = \text{Galvanometer constant} = \frac{\Delta\psi}{\theta}$

$$= \frac{0.0025}{102} = 2.4509 \times 10^{-5} \ \frac{\text{Wb - turns}}{\text{division}}$$

When connected to search coil, θ = 272 divisions.

Electrical Measurements

✎ **Chapterwise University Questions with Answers**

1

Measuring Instruments

Q.1 *How are measuring instruments classified ? Also explain the basic issues concerned with the measurement of electrical quantities.* **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer section 1.2.

Q.2 *What are the requirements of an electrical indicating instrument? Discuss.* **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer section 1.3.

Q.3 *Classify the electrical measuring instruments based on how the deflecting torque is produced.* **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer section 1.2.

Q.4 *Explain deflecting system, controlling system and damping system with reference to an electrical indicating instrument.* **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer sections 1.4, 1.5 and 1.6.

Q.5 *Explain different types of deflecting systems which are operating in an indicating instrument.* **[Nov.-2003, Set-3, 3 Marks]**

Ans. : Refer section 1.4.

Q.6 *Discuss the role of controlling systems in an indicating instrument.* **[Nov.-2003, Set-3, 3 Marks]**

Ans. : Refer section 1.5.

Q.7 *How are controlling torques produced ? Explain with neat sketches the different methods of producing controlling torque indicating their relative merits and demerits.* **[Nov.-2003, Set-3, 4 Marks]**

Ans. : Refer sections 1.5.1, 1.5.2 and 1.5.3.

Q.8 *Discuss why indicating instruments with gravity control have a non-uniform and with spring control have a uniform scale? Explain.* **[Nov.-2003, Set-4, 5 Marks]**

Ans. : Refer section 1.5.

Q.23 *Explain with a neat sketch, the working of a PMMC instrument.*
[Nov.-2004, Set-3, 5 Marks]

Ans. : Refer section 1.9.

Q.24 *What are the errors in a moving coil instrument ? How these errors are compensated ?*
[Nov.-2004, Set-3, 5 Marks]

Ans. : Refer sections 1.9.6 and 1.9.5.

Q.25 *How moving iron instruments are classified ? Explain with the neat sketches the working of an attraction type moving iron instrument.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer section 1.10.1.

Q.26 *What are the different types of instruments that are used as ammeters and voltmeters? What are the errors that occur in ammeters and voltmeters?*
[May-2005, Set-1, Set-2, 5 Marks]

Ans. : Refer sections 1.9, 1.9.6, 1.10 and 1.10.6.

Q.27 *Describe how can we obtain different voltage ranges by using a multirange d.c. voltmeter. Discuss about sensitivity and loading effects of PMMC voltmeters.*
[May-2005, Set-1, Set-2, 5 Marks]

Ans. : Refer sections 1.16 and 1.17.

Q.28 *Describe the working of a quadrant electrometer. Derive the deflection in the case of*

i) Heterostatic connection and

ii) Idiostatic connection. If the instrument is spring controlled, which of these instruments can be used for measurement of low voltages? **[May-2005, Set-1, 5 Marks]**

Ans. : Refer section 1.20.

Q.29 *The spring constant of 3000 V electrostatic voltmeter is 7.06×10^{-6} Nm/rad. The full scale deflection of the instrument is 80°. Assuming the rate of change of capacitance with angular deflection to be constant over the operating range. Calculate the total change of capacitance from zero to full scale.* **[May-2005, Set-1, 5 Marks]**

Ans. : Refer example 1.27.

Q.30 *Derive the expression for deflection of a rotary type electrostatic instrument using spring control. Comment upon the scale of the instrument.* **[May-2005, Set-3, 5 Marks]**

Ans. : Refer section 1.20.

Q.31 *An electrostatic voltmeter is constructed with six parallel, semicircular fixed plates equispaced at Ymm intervals and five interleaved semicircular movable plates that move in planer midway between the fixed plates in air. The instrument is spring controlled. If the radius of movable plates 40 mm, calculate the spring constant if 10 kV corresponds to full scale deflection of 1000. Neglect edge effects and plate thickness. The permittivity of air is 8.85×10^{-12} F/M.* **[May-2005, Set-3, 5 Marks]**

Ans. : Refer example 1.28.

2 Instrument Transformers

Q.1 *Describe the various advantages and disadvantages of instrument transformer for extension of range of a.c. instrument.* **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer section 2.12.

Q.2 *Write short notes on the following :*

i) Single phase power factor meter.

ii) Ratiometer type frequency meter. **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer sections 2.15 and 2.17.

Q.3 *"Never open the secondary circuit of a current transformer while its primary is energized". Justify.* **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer section 2.14.

Q.4 *A 800/5 A, 50 Hz current transformer with a single turn primary has a secondary burden comprising a non reactive resistance of 4 Ω. The secondary winding of 160 turns has a resistance of 0.2 Ω. At rated secondary current, calculate*

i) Flux in the core.

ii) The actual ratio of primary to secondary current.

iii) The phase angle between the primary and secondary currents. No load primary current of 6 A lags by 30° the reversed secondary voltage.

[Nov.-2003, Set-2, 5 Marks]

Ans. : Refer example 2.17.

Q.5 *Write short note on synchroscope.* **[Nov.-2003, Set-2, Set-3, 5 Marks]**

Ans. : Refer section 2.19.

Q.6 *With a suitable diagram, explain the working of a 3 phase power factor meter.*

[Nov.-2003, Set-2, 5 Marks]

Ans. : Refer section 2.16.2.

Q.7 *With suitable diagram explain the working of elecrodynamometer type frequency meter.*

[Nov.-2003, Set-3, 5 Marks]

Ans. : Refer section 2.17.2.

Q.8 *Write short notes on following :*

i) Resonance type frequency meter.

ii) Advantages and disadvantages of moving iron power factor meter.

[Nov.-2003, Set-4, 5 Marks]

Ans. : Refer sections 2.17.2 and 2.16.

Q.9 *List out the differences between instrument transformers and power transformers.*

[May-2004, Set-3, 5 Marks]

Ans. : Refer section 2.13.

Q.10 *A potential transformer has a primary resistance of 300 Ω, a primary reactance of 600 Ω, a secondary resistance of 0.75 Ω and a secondary reactance of 1.5 Ω. The primary to secondary turns ratio is 20 : 1, the primary voltage is 2000 V. Neglecting the magnetizing and core loss current. Determine the voltage ratio correction factor, the ratio error and phase angle error when the burden on the secondary of the transformer is :*

a) 50 VA at 0.6 power factor lagging.

b) 50 VA at unity power factor.

c) 25 VA at 0.6 power leading. **[May-2004, Set-3, 5 Marks]**

Ans. : Refer example 2.18.

Q.11 *Draw and explain the equivalent circuit and phasor diagram of a potential transformer.*

[May-2004, Set-3, 5 Marks]

Ans. : Refer section 2.10.

Q.12 *Explain with a neat sketch, the working of weston synchroscope.*

[May-2004, Set-4, 5 Marks]

Ans. : Refer section 2.19.1.

Q.13 *What are the different types of frequency meters available and hence explain with a neat sketch the construction and working principle of reed frequency meter.*

[May-2004, Set-4, 5 Marks]

Ans. : Refer section 2.17.1.

Q.14 *With a neat sketch explain the working principle of a single phase dynamometer type power factor meter.* **[Nov.-2004, Set-3, 5 Marks]**

Ans. : Refer section 2.15.

Q.15 *Describe with a diagram the construction and working of synchroscope.*

[Nov.-2004, Set-1, 5 Marks]

Ans. : Refer section 2.19.

Q.16 *Describe with a diagram the construction and operation of a vibrating reeds type frequency meter.* **[Nov.-2004, Set-1, 5 Marks]**

Ans. : Refer section 2.17.1.

Q.17 *Explain the construction and working of moving iron type power factor meters.*

[Nov.-2004, Set-2, 5 Marks]

Ans. : Refer section 2.16.

Q.18 *Describe the construction and working of weston type synchroscope.*

[Nov.-2004, Set-2, 5 Marks]

Ans. : Refer section 2.19.1.

Q.19 *Explain the principle of working of an electrical resonance type frequency meter.*

[Nov.-2004, Set-3, 5 Marks]

Ans. : Refer section 2.17.2.

Q.20 *Explain the effects of secondary burden on the ratio and phase errors of a current transformer.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer section 2.9.3.

Q.21 *A power primary C.T. has 300 secondary turns. The total resistance and reactance for the secondary circuit are 1.5 Ω and 1.0Ω. respectively. When 5 A flows through the secondary winding, the magnetizing m.m.f. is 100 AT and the iron loss component is 40 A. Determine the ratio and phase angle errors of the C.T. at this load.*

[Nov.-2004, Set-4, 5 Marks]

Ans. : Refer example 2.16.

Q.22 *Explain with a diagram the construction and working of a Nalder - Lipmen type 3 phase power factor meter.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer section 2.16.2.

Q.23 *Describe with a diagram the construction and working of a electrodynamic type synchroscope.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer section 2.19.1.

Q.24 *Explain weston type synchroscope.* **[May-2005, Set-4, 5 Marks]**

Ans. : Refer section 2.19.1.

Q.25 *Explain the constructional features used in potential transformers to reduce the ratio and phase angle errors.* **[May-2005, Set-2, 5 Marks]**

Ans. : Refer section 2.11.

Q.26 *Explain the characteristics of potential transformers in detail.* **[May-2005, Set-2, 5 Marks]**

Ans. : Refer section 2.10.

Q.27 *With neat sketch, explain how high currents and voltages can be measured with the help of instrument transformers. Describe the advantanges of instruments transformers for extension of range of current and voltage on high voltage a.c. systems.*

[May-2005, Set-3, 5 Marks]

Ans. : Refer sections 2.2, 2.5 and 2.12.

Q.28 *A current transformer with 5 primary turns has a secondary burden consisting of a resistance of 0.16 and an inductive resistance of 1.12. When the primary current is 200 A, the magnetizing current is 1.5 A and the iron loss current is 0.4 A. Determine the expressions used, the number of secondary turns needed to make the current ratio 100:1 and also the phase angle under these conditions.* **[May-2005, Set-3, 5 Marks]**

Ans. : Refer example 2.11.

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3 Measurement of Power

Q.1 *The power in a single phase high voltage circuit is measured by using instrument transformers with voltmeter, ammeter and wattmeter. Observed readings of the instruments (assuming no errors) are 115 V, 4.5 A and 200 W. Characteristics of the transformers are :*

PT : Nominal ratio : 11500/115 V, ratio correction factor 0.995, phase angle – 25°.

CT : Nominal ratio 25/5 A, ratio correction factor 0.997, phase angle +15°.

Neglecting the voltage phase angle in the voltmeter, calculate the true power.

[Nov.-2003, Set-1, 5 Marks]

Ans. : Refer Example 3.12.

Q.2 *Explain, how power in high voltage circuits is measured using instrument transformers.*

[Nov.-2003, Set-3, 5 Marks]

Ans. : Refer section 3.19.

Q.3 *A reading of 400 W is indicated on a 100 V/ 5 A wattmeter used in conjuction with voltage and current transformers of nominal ratio 100/1 and 20/1 respectively. If the wattmeter pressure coil has a resistance of 400 Ω and an inductance of 20 mH and the ratio errors and the phase differences of the voltage and current transformers are +1 percent and 50 min and – 0.5 percent and 100 min respectively. Compute the true value of the power measured. The load phase angle is 60° lagging and the frequency is 50 Hz.* **[Nov.-2003, Set-3, 5 Marks]**

Ans. : Refer example 3.8.

Q.4 *Obtain the expression for power, interms of correction factor, wattmeter reading, actual ratio of P.T. and C.T., in case of power measurement along with instrument transformers.*

[Nov.-2003, Set-4, 5 Marks]

Ans. : Refer section 3.19.

Q.5 *In a 11 kV, 100 A, 3 phase (balanced) supply, voltmeter of 0-110 V, Ammeter 0-25 A and wattmeter with pressure coil rating of 110 V and current coil rating of 5 A are to be used for measuring the voltages, currents and power. Draw wiring diagram using necessary instrument transformers. What type of errors are expected in such measurements? And how to minimize these errors?* **[Nov.-2003, Set-4, 5 Marks]**

Ans. : Refer section 3.19.

Q.6 *Explain the construction and theory of operation of a single phase elecrodynamometer type wattmeter.* **[May-2004, Set-2, 5 Marks]**

Ans. : Refer sections 3.2.1 and 3.3.1.

Q.7 *A certain circuit takes 10 A at 200 V and the power absorbed is 1000 W. If the wattmeter's current coil has a resistance of 0.15 Ω and its pressure coil a resistance of 5000 Ω and an inductance of 0.3 H, find*

i) The error due to the resistance for each of the two possible methods of connection.

ii) The error due to the inductance if the frequency is 50 Hz.

iii) Total error in each case. **[May-2004, Set-2, 5 Marks]**

Ans. : Refer example 3.11.

Q.8 *A potential transformer with a nominal ratio of 2000 / 100 V, an RCF of 0.995 and a phase angle* $(V_s \text{ lags } V_p)$ *of −22′ is used with a current transformer with a nominal ratio of 100/5 A, an RCF of 1.005 and a phase angle error* $(I_s \text{ leads } I_p)$ *of 10′ to measure the power in a single phase inductive load. The meters connected to these transformers give correct readings of 102 V, 4 A and 375 watts. Determine the true values of the voltage, current and power supplied to the load.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer Example 3.9.

Q.9 *Explain with the aid of a phasor diagram the error caused by the inductance of the pressure coil of a dynamometer wattmeter. Indicate the dependance of the error on load power factor and supply frequency.* **[May-2004, Set-1, 5 Marks]**

Ans. : Refer section 3.4.1.

Q.10 *A 500 V, 20 A dynamometer instrument is used as a wattmeter. Its current coil has 0.1 W resistance and pressure coil has 25 kW resistance and 0.1 H inductance. The meter was calibrated on d.c supply. What is the error in the instrument if it is used to measure the power in a circuit with supply voltage 500 V, load current 24 A at 0.2 p.f. ? Assume that the pressure coil is connected across the load.* **[May-2004, Set-1, 5 Marks]**

Ans. : Refer example 3.10.

Q.11 *Explain clearly how range of wattmeter can be extended using instrument transformer.*

[Nov.-2004, Set-1, 5 Marks]

Ans. : Refer section 3.19.

Q.12 *Explain the working of a 3 phase dynamometer wattmeter. Describe how mutual effects between the two elements of the wattmeter are eliminated.* **[Nov.-2004, Set 2, 10 Marks]**

Ans. : Refer section 3.20.

Q.13 *Prove that the true power =* $\cos\phi/[\cos\phi * \cos(\phi-\beta)]$ **actual wattmeter reading for electrodynamometer type of wattmeter where* $\cos\phi$ *= power factor of the circuit,* $\beta = \tan^{-1} wL/R$. **[Nov.-2004, Set-3, 5 Marks]**

Ans. : Refer section 3.4.

Q.14 *Describe the method of measurement of reactive power in three phase circuits using single dynamometer type wattmeter.* **[Nov.-2004, Set-1, 5 Marks]**

Ans. : Refer section 3.18.

Q.15 *Explain with a neat circuit diagram how reactive power can be measured in balanced 3 phase system using single wattmeter ?* **[Nov.-2004, Set-3, 5 Marks]**

Ans. : Refer section 3.18.

Q.16 *Explain the following errors for electrodynamometer wattmeters.*

i) Mutual inductance effects. ii) Errors due to connections.

iii) Eddy currents. iv) Stray Magnetic fields.

v) Vibration of moving system. vi) Temperature errors. **[May-2005, Set-1, 10 Marks]**

Ans. : i) Due to mutual inductance the phase angle β increases by Δβ given by,

$$\beta' = \beta + \Delta\beta = \tan^{-1}\frac{\omega L}{R_p} + \tan^{-1}\frac{\omega M}{R_p}$$

For (ii), (iii) refer section 3.4.

iv) As the operating field is weak, external magnetic fields affect the reading of wattmeters.

v) Due to alternating supply, torque is pulsating at double the supply frequency. If this matches with the frequency of vibration or moving system then resonance occurs and starts vibrating with that frequency. Such vibrations can cause errors.

vi) As temperature increases, resistance of pressure and current coil increases. This reduces the operating field. This affects deflecting torque. The stiffness of spring also decreases, reducing controlling torque.

Q.17 *Describe the construction and working of electrodynamometer wattmeter. Derive the expression for torque when the instrument is used on a.c.* **[May-2005, Set-2, 5 Marks]**

Ans. : Refer section 3.3.

Q.18 *The pressure coil of an electrodynamometer wattmeter has a resistance of 6600. When the voltage applied to the pressure coil is 120 V and a current of 20 A flows in the series coil, the deflection is 1600. What additional resistance must be connected in the pressure coil circuit to make the meter constant equal to 20 W per degree ?* **[May-2005, Set-2, 5 Marks]**

Ans. : Refer example 3.13.

Q.19 *Explain the errors caused due to pressure coil inductance and pressure coil capacitance in electrodynamometer wattmeter.* **[May-2005, Set-3, Set-4, 5 Marks]**

Ans. : Refer section 3.4.

Q.20 *Discuss the shape of scale of electrodynamometer wattmeters with the help of a neat sketch.* **[May-2005, Set-3, Set-4, 5 Marks]**

Ans. : Refer section 3.3.3.

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Q.7 *A 220, 5 A, d.c. energymeter is tested at its marked ratings. The resistance of the pressure circuit is 8800 Ω and that of current coil is 0.1 Ω. Calculate the power consumed when testing the meter with phantom loading with current circuit excited by a 6 V battery.* **[May-2004, Set-2, 5 Marks]**

Ans. : Refer example 4.2.

Q.8 *Explain the testing of energymeter using R.S.S. meter.* **[May-2004, Set-3, 5 Marks]**

Ans. : Refer section 4.15.

Q.9 *The meter constant of a 230 V, 10 A, watt-hour meter is 1800 revolutions per kWh. The meter is tested at half load and rated voltage and unity power factor. The meter is found to make 80 revolutions in 138 second. Determine the meter error at half load.* **[May-2004, Set-3, 5 Marks]**

Ans. : Refer example 4.4.

Q.10 *What are the various adjustments that are required to be made in a single phase energymeter, so that it reads correctly ? Explain each of them to the extent required. Also give all the details about an induction type single phase energymeter.* **[Nov.-2004, Set-3, 10 Marks]**

Ans. : Refer section 4.4.

Q.11 *Explain the various types of errors occurring in an energymeter and also the method of compensation to overcome these errors.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer section 4.4.

Q.12 *Explain with a neat sketch the construction and working principle of a single phase energymeter and derive an expression to show that number of revolutions/sec. is proportional to power.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer sections 4.2.1 and 4.3.

Q.13 *Draw a neat sketch showing the construction of a single phase induction type energymeter. Give the theory and operation of the instrument.* **[May-2005, Set-1, Set-3, 5 Marks]**

Ans. : Refer sections 4.2.1 and 4.3.

Q.14 *An energymeter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it when connected to a load carrying 20 A at 230 V at 0.8 p.f. for an hour. If it actually makes 360 revolutions, find the percentage error.* **[May-2005, Set-1, Set-3, 5 Marks]**

Ans. : Refer example 4.1.

Q.15 *Draw a neat circuit diagram of a single phase watt-hour meter and explain its working. What are the various sources of errors and how they are compensated ?* **[May-2005, Set-2, Set-4, 5 Marks]**

Ans. : Refer sections 4.2.1 and 4.4.

Q.16 *A large consumer has a kVA demand and a KVAh tariff measured by "sine" and "cosine" watt-hour type meters each equipped with a Merz price demand indicator. The tariff is Rs. 40 per month per kVA of demand plus 30 paise per kVAh. Determine the*

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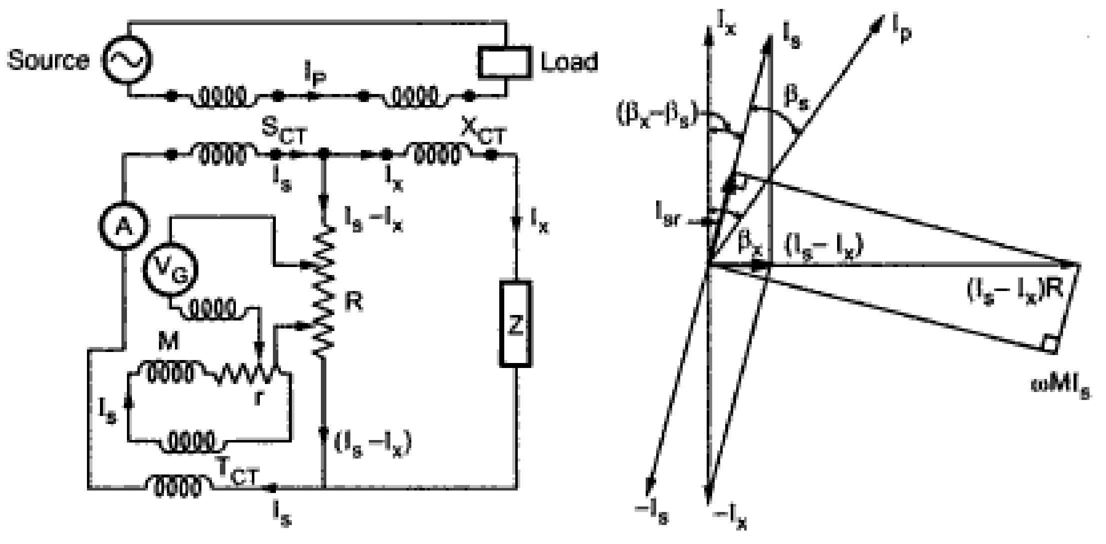

(a) Circuit diagram **(b) Phasor diagram**

Fig. 1

T_{CT} is the isolating current transformer which is used along with ammeter A and suitable burden Z. The value of non-inductive resistance R is decided by required sensitivity and range. The variable mutual inductor M is used in series with slide wire potentiometer r.

To eliminate inductive interference in measuring circuit first of all the leads of resistance R are disconnected and short circuited. The values of r and M are made zero and full rated primary current is allowed to flow. Now the leads are connected and the measuring circuit is so adjusted that there is no deflection on galvanometer. Thus inductive interference is eliminated.

The phasor difference of the two currents through the non-inductive resistance R and the potential difference across R is balanced by the voltage in the secondary of M and voltage drop across part of slide wire r. As the T_{CT} is of unity ratio, the current through primary is same as secondary current of S_{CT}. The vibration galvanometer gives the indication of balance through the adjustment of M and r.

Q.7 *Explain how potentiometer is employed in measuring resistance power and calibration of wattmeter.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer sections 5.11.4, 5.11.5 and 5.11.3.

Q.8 *During the measurement of a low resistance using a potentiometer the following readings were obtained. Voltage drop across the low resistance under test = 0.4221 V voltage drop across the 0.1 Ω standard resistance = 1.0235 V. Calculate the value of unknown resistance, current and power lost in it.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer example 5.10.

Q.18 *In the measurement of power by a polar potentiometer, the following readings were obtained : Voltage across a 0.2 Ω standard resistance in series with the load = 1.46 ∠ 32° V. Voltage across a 200:1 potential divider across the line = 1.37 ∠56° V. Estimate the current, voltage, power and power factor of the load.*

[May-2005, Set-1, 5 Marks]

Ans. : Refer example 5.15.

Q.19 *Describe the construction, principle of operation of a Duo-range potentiometer by drawing its circuit diagram. Also explain its advantages.* **[May-2005, Set-2, 10 Marks]**

Ans. : Refer section 5.5.

Q.20 *Describe the construction and working of a polar type potentiometer. How is it standardized ? What are the functions of the transfer instrument and the phase shifting transformer ?* **[May-2005, Set-2, Set-3, 10 Marks]**

Ans. : Refer section 5.14.

Q.21 *Describe the working and construction of a potentiometer with the help of a diagram.*

[May-2005, Set-3, 5 Marks]

Ans. : Refer section 5.2.

Q.22 *A basic slide wire potentiometer has a working battery voltage of 3.0 V with negligible internal resistance. The resistance of slide wire is 400 and its length is 200 cm. A 200 cm scale is placed along the slide wire. The slide wire has 1 mm scale divisions and it is possible to read up to 1/5 of a division. The instrument is standardized with 1.018 V standard cell with sliding contact at the 101.8 cm mark on scale. Calculate*

i) Working current.

ii) The resistance of series rheostat.

iii) The measurement range.

iv) The resolution of instrument. **[May-2005, Set-3, 5 Marks]**

Ans. : Refer example 5.11.

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6

Resistance Measurements

Q.1 *Draw the circuit of Kelvin double bridge for measurement of low resistances. Derive the condition for balance.* **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer section 6.14.

Q.2 *A highly sensitive galvanometer can detect a current as low as 0.1 nA. This galvanometer is used in a Wheatstone bridge as a detector. The resistance of galvanometer is negligible. Each arm of the bridge has a resistance of 1 kΩ. The input voltage applied to the bridge is 20 V. Calculate the smallest change in the resistance, which can be detected.* **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer Example 6.16.

Q.3 *A Kelvin double bridge has each of the ratio arms $P = Q = p = q = 1000\ \Omega$. The e.m.f. of the battery is 100 V and a resistance of 5 Ω is included in the battery circuit. The galvanometer has a resistance of 500 Ω and the resistance of the link connecting the unknown resistance to the standard resistance may be neglected. The bridge is balanced when the standard resistance $S = 0.001\ \Omega$.*

i) Determine the value of unknown resistance.

ii) Determine the current (approximate value) through the unknown resistance R at balance.

iii) Determine the deflection of the galvanometer when the unknown resistance, R is changed by 0.1 percent from its value at balance. The galvanometer has a sensitivity of 200 mm/ μA. **[Nov.-2003, Set-3, 10 Marks]**

Ans. : Refer Example 6.15.

Q.4 *A modified Wheatstone bridge network is constituted as follows.*

AB is resistance P in parallel with resistance p ; BC is resistance Q in parallel with a resistance q ; CD and DA are resistance R and S respectively. The nominal values of P, Q and S are each 10 Ω. With resistance R in circuit, balance is obtained with $p = 30{,}000\ \Omega$ and $q = 25{,}000\ \Omega$. With R replaced by a standard resistance of 10 Ω, balance is obtained when $p = 15{,}000\ \Omega$ and $q = 40{,}000\ \Omega$. Calculate the value of R. **[Nov.-2003, Nov.-2004, Set-4, 10 Marks]**

Ans. : Refer Example 6.14

Q.5 *Describe any one method of measuring a very high value of resistance.* **[May-2004, Set-2, 5 Marks]**

Ans. : Refer section 6.16.

Q.6 *How do you classify the resistances from the point of view of measurements ?* **[May-2004, Set-3, 5 Marks]**

Ans. : Refer section 6.2.

Q.7 *Describe the method of measurement of medium resistances by Wheatstone bridge method derive the conditions for balance.* **[May-2004, Set-3, Set-4, 5 Marks]**

Ans. : Refer section 6.6.

Q.8 *What are the various factors to be considered in the precision measurement of medium resistances using Wheatstone bridge ? Explain.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer section 6.9.

Q.9 *A Kelvin double bridge is balanced with the following constants : Outer ratio arm 100 Ω and 1000 Ω ; Inner ratio arms, 99.92 Ω and 1000.6 Ω ; Resistance of link 0.1 Ω ; Standard resistance 0.00377 Ω. Calculate the value of unknown resistance.* **[Nov.-2004, Set-1, 5 Marks]**

Ans. : Refer Example 6.13.

Q.10 *What are the different problems associated with measurement of low resistances ? Explain.* **[Nov.-2004, Set-2, 5 Marks]**

Ans. : Refer section 6.14.

Q.11 *How these problems are eliminated by using Kelvin's double bridge ? Explain.* **[Nov.-2004, Set-2, 5 Marks]**

Ans. : Refer section 6.14.1.

Q.12 *Explain the working of a Megger with relevant equations.* **[Nov.-2004, Set-3, 5 Marks]**

Ans. : Refer section 6.16.4.

Q.13 *Explain the working of shunt type Ohm-meter with a neat diagram.* **[Nov.-2004, Set-3, 5 Marks]**

Ans. : Refer section 6.5.

Q.14 *What are the various limitations of Wheatstone bridge for measurement of high and low resistances ?* **[May-2005, Set-3, 3 Marks]**

Ans. : Refer section 6.9.

Q.15 *Derive an expression for current through the galvanometer connected in Wheatstone bridge for a small unbalance.* **[May-2005, Set-3, 3 Marks]**

Ans. : Refer section 6.8.2.

Q.16 *Describe the substitution method of measurement of medium resistances. List the factors on which are accuracy of the method depends upon.* **[May-2005, Set-3, 4 Marks]**

Ans. : Refer sections 6.6, 6.8 and 6.10.

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7

A.C. Bridges

Q.1 *The arms of a five node bridge are as follows :*

arm ab : an unknown impedance (R_1, L_1) in series with a non-variable inductive resistor r_1,

arm bc : a non-inductive resistor $R_3 = 100\ \Omega$, arm cd : a non-inductive resistor $R_4 = 200\ \Omega$

arm da : a non-inductive resistor $R_2 = 250\ \Omega$, arm de : a variable non-inductive resistor r,

arm ec : a lossless capacitor $C = 1\ \mu F$, and arm be : a detector. An A.C. supply is connected between a and c.

Derive the expressions for balance condition.

Calculate the resistance and inductance R_1, L_1 when under balance conditions $r_1 = 43.1\ \Omega$ and $r = 229.7\ \Omega$. **[Nov.-2003, Set-1, 10 Marks]**

Ans. : Refer Example 7.23.

Q.2 *The four arms of a bridge are :*

arm ab : an imperfect capacitor C_1 with an equivalent series resistance of r_1.

arm bc : a non-inductive resistance R_3, arm cd : a non-inductive resistance R_4.

arm da : an imperfect capacitor C_2 with an equivalent resistance of r_2 in series with a resistance R_2. A supply of 450 Hz is given between terminal a and c and the detector is connected between b and d. At balance : $R_2 = 4.8\ \Omega$, $R_2 = 200\ \Omega$, $R_4 = 2850\ \Omega$ and $C_2 = 0.5\mu F$ and $r_2 = 0.4\ \Omega$.

Calculate the value C_1 and r_1 and also of the dissipating factor for this capacitor.
[Nov.-2003, Set-2, 10 Marks ; May-2005, Set-4, 5 Marks]

Ans. : Refer Example 7.24.

Q.3 *A four arm A.C. bridge a b c d has the following impedances.*

Arm a b : $Z_1 = 200 \angle 60° \Omega$

Arm a d : $Z_2 = 400 \angle -60° \Omega$

Arm b c : $Z_3 = 300 \angle 0° \Omega$

Arm c d : $Z_4 = 600 \angle 30° \Omega$

Ans. : Refer Example 7.29.

Q.10 *In a Heavy side Campbell bridge as shown for measurement of a self inductance L_2 with the equal ratio i.e., $R_3 = R_4$ the following results were obtained. With switch open $M = 15.8$ mH, $r = 25.7\ \Omega$ and with switch close $M = 0.2$ mH and $r = 1.2\ \Omega$. Find the resistance and self inductance of the coil.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer Example 7.28.

Q.11 *Deduce the conditions for balancing of bridges in A.C. bridges.* **[Nov.-2004, Set-1, 5 Marks]**

Ans. : Refer section 7.3.2.

Q.12 *The four impedances of an A.C. bridge are $Z_{AB} = 400\angle 50°\Omega$, $Z_{AD} = 200\angle 40°\Omega$, $Z_{BC} = 800\angle -50°\Omega$, $Z_{CD} = 400\angle 20°\Omega$. Find out whether the bridge is balanced under these conditions or not.* **[Nov.-2004, Set-2, 5 Marks]**

Ans. : Refer Example 7.20.

Q.13 *Mention the two conditions for balancing of A.C. bridges.* **[Nov.-2004, Set-2, 5 Marks]**

Ans. : Refer section 7.3.2.

Q.14 *A condenser bushing forms arm ab of a Schering bridge and a standard capacitor of 500 pF capacitance and negligible loss, forms arm ad. Arm bc consists of a non-inductive resistance of 300 Ω. When the bridge is balanced, arm cd has a resistance of 72.6 Ω in parallel with a capacitance of 0.148 μF. The supply frequency is 50 Hz. Calculate the capacitance and dielectric loss angle of capacitor.* **[Nov.-2004, Set-3, 10 Marks]**

Ans. : Refer Example 7.30.

Q.15 *Explain the difference between balance conditions for D.C. and A.C. bridges.* **[Nov.-2004, Set-4, 5 Marks]**

Ans. : Refer sections 7.2 and 7.3.

Q.16 *An A.C. bridge circuit working at 1000 Hz, have its arms as follows. Arm AB is 0.2 microfarad capacitance. Arm BC is a 500 Ohms resistance ; arm CD contains an unknown impedance and arm DA has a 300 ohms resistance in parallel with a 0.1 microfarad capacitor. Find the R and L or C constants of arm CD considering it as a series circuit.* **[Nov.-2004, Set-4; May-2005, Set-1; 5 Marks]**

Ans. : Refer Example 7.22.

Q.17 *Derive the equations for balance in the case of Maxwells inductance capacitance bridge. Give its advantages. Draw the phasor diagram for balanced conditions.* **[May-2005, Set-1, 5 Marks]**

Ans. : Refer sections 7.6.2 and 7.6.3

Q.18 *Explain what is meant by sliding balance. How is this condition avoided by choosing variables for manipulation of balance i.e., why variables are so chosen that the two equations for balance are independent of each other ?* **[May-2005, Set-2, 5 Marks]**

Ans. : Refer section 7.3.2.

Q.19 *Why is it preferable in bridge circuits, that the equations for balance are independent of frequency ? Explain.* **[May-2005, Set-2, 5 Marks]**

8

Magnetic Measurements

Q.1 *With a suitable diagram explain the working of ballistic galvanometer.* **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer section 8.2.

Q.2 *Show that by using fluxmeter the leakage factor can be measured in the specimen.* **[Nov.-2003, Set-1, 5 Marks]**

Ans. : Refer section 8.7.

Q.3 *Write short note on the following :*

Measurement of permeability. **[Nov.-2003, Set-2, 5 Marks]**

Ans. : Refer section 8.12.

Q.4 *Write short note on fluxmeter.* **[Nov.-2003, Set-3, 5 Marks]**

Ans. : Refer section 8.3.

Q.5 *Explain with suitable diagram the working of ballistic galvanometer in the magnetic measurement. Show that the instrument is proportional to the total charge.* **[Nov.-2003, Set-4, 10 Marks]**

Ans. : Refer section 8.2.

Q.6 *Describe the method of obtaining hysteresis loop of a ring specimen using fluxmeter.* **[May-2004, Set-1, 5 Marks]**

Ans. : Refer section 8.8.

Q.7 *The mutual inductance between magnetising winding and a search coil wound on a specimen in 9 mH. The search coil has 20 turns and the specimen has a cross-sectional area of 5000 sq. mm reversal of current of 3 A in magnetising winding produces galvanometer through of 60 divisions. Calculate the value of flux density in the specimen, if the reversal of current in the magnetising winding produces a galvanometer deflection of 36 divisions.* **[May-2004, Set-1, 5 Marks]**

Ans. : Refer example 8.12.

Q.8 *Describe the method of reversal used for the determination of B-H loop of a sample material. State the advantages of this method over step by step method. How do you calculate hysteresis loss in this specimen ?*

[May-2004, Set-2; Nov.-2004, Set-3, 10 Marks]

Ans. : Refer section 8.8.1.

Q.9 *Describe with a diagram method of getting the relative permeability of the bar specimen using a fluxmeter.* **[May-2004, Set-3, 5 Marks]**

Ans. : Refer sections 8.4 and 8.5.

Once B and H are known, $B = \mu_0 \mu_r H$ hence μ_r can be obtained.

Q.10 *In a test on a specimen of total weight 13 kg the measured values of iron loss at a given value of flux density were 17.2 watts at 40 Hz and 28.9 watts at 60 W. Estimate the values of hysteresis and eddy current losses at 50 Hz for the same value of peak flux density.* **[May-2004, Set-3, 5 Marks]**

Ans. : Refer example 8.11.

Q.11 *Describe the step by step method of getting B-H curve of a ring specimen using ballistic galvanometer.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer section 8.8.2.

Q.12 *An iron ring has a mean diameter of 0.15 m and a cross-sectional area of 34.5 sq.mm. It is wound with a magnetising winding of 330 turns and a secondary winding of 220 turns. On reversing a current of 12 A in the magnetising winding a ballistic galvanometer gives a throw of 272 scale division. With a HMS with 10 turns and flux of 0.00025 Wb, gives a reading of 102 scale division. Other conditions remain the same. Find the relative permeability of the specimen.* **[May-2004, Set-4, 5 Marks]**

Ans. : Refer example 8.13.

Q.13 *Describe the construction and working of a moving coil ballistic galvanometer.*

[Nov.-2004, Set-1, 5 Marks]

Ans. : Refer section 8.2.

Q.14 *Describe the method of experimental measurement of flux density in a specimen of magnetic material using ballistic galvanometer.* **[Nov.-2004, Set-1, 5 Marks]**

Ans. : Refer section 8.4.

Q.15 *Describe the method of measurement of iron loss using Lioyed-Fisher square.*

[Nov.-2004, Set-2, 5 Marks]

Ans. : Refer section 8.10.

Q.16 *How the value of A.C. permeability of magnetic material is determined by using Maxwell's bridge.* **[Nov.-2004, Set-2, 5 Marks]**

Ans. : Refer section 8.12.1.

Q.17 *Explain how B-H curve for the ring specimen by using potentiometer is obtained.*

[Nov.-2004, Set-4, 5 Marks]

Ans. : Refer section 8.6.

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