UNIT-I

Gyroscopic Couple and Static & Dynamic Force Analysis

1.0 INTRODUCTION

'Gyre' is a Greek word, meaning 'circular motion. A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.

A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.1. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance to this motion is called gyroscopic effect.

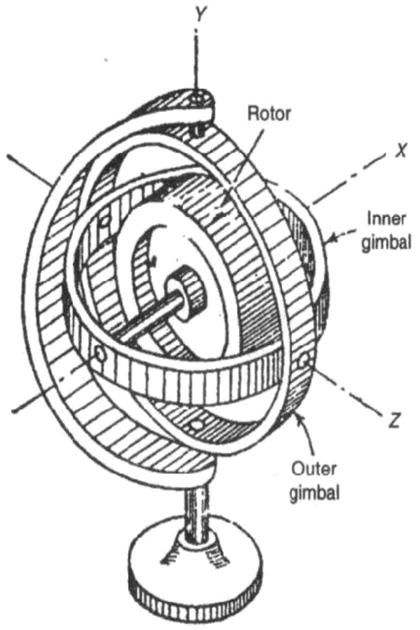


Fig.1: Gyroscope Mechanism

1.1 ANGULAR MOTION

A rigid body, (Fig.2) spinning at a constant angular velocity ω rad/s about a spin axis through the mass centre. The angular momentum 'H' of the spinning **vector** whose magnitude ω'. I represents the mass amount of inertia of the rotor about the axis of spin.

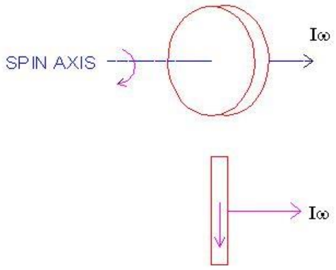
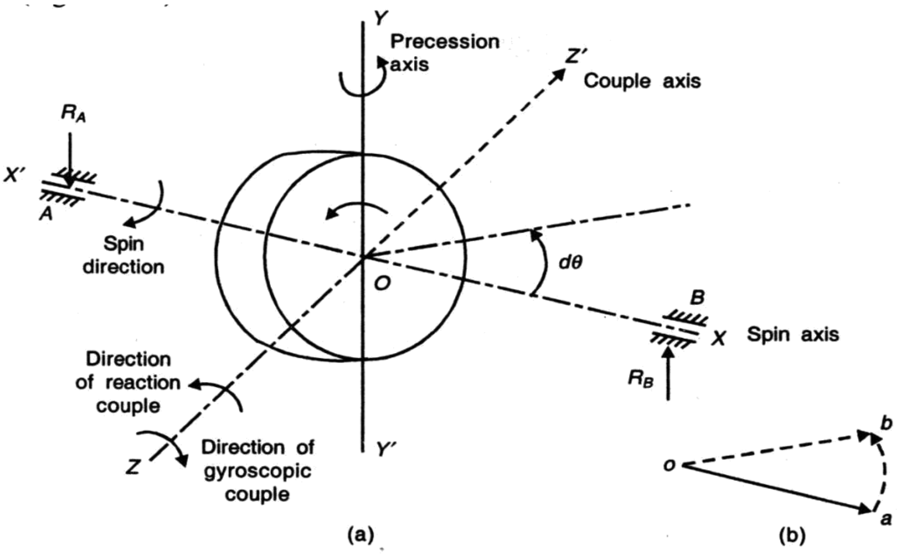


Fig.2: spinning body

The direction of the angular momentum can be found from the right hand screw rule or the right hand thumb rule. Accordingly, if the fingers of the right hand are bent in the direction of rotation of rotor, then the thumb indicates the direction of momentum.

1.2 GYROSCOPIC COUPLE

Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis (Fig.3).



The angular momentum of the rotating mass is given by,

$$H = mk^2\ \omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle δθabout Y-axis in the plane *XOZ*, then the angular momentum varies from *H* to *H* + δ*H*, where δ*H* is the change in the angular momentum, represented by vector ab [Figure 15.2(b)]. For the small value of angle of rotation 5^0, we can write

$$ab = oa \times \delta\theta$$

$$\delta H = H \times \delta\theta$$

$$= I\omega\delta\theta$$

However, the rate of change of angular momentum is:

$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\,\delta\theta}{\delta t}\right)$$

$$= I\omega\frac{d\theta}{dt}$$

$$\mathbf{C = I\omega\omega_p}$$

Where C = gyroscopic couple (N-m)

ω= angular velocity of rotary body (rad/s)

ω_p= angular velocity of precession (rad/s)

1.3 Direction of Spin vector, Precession vector and Couple/Torque vector with forced precession

To determine the direction of spin, precession and torque/couple vector, right hand screw rule or right hand rule is used. The fingers represent the rotation of the disc and the thumb shows the direction of the spin, precession and torque vector (Fig.4).

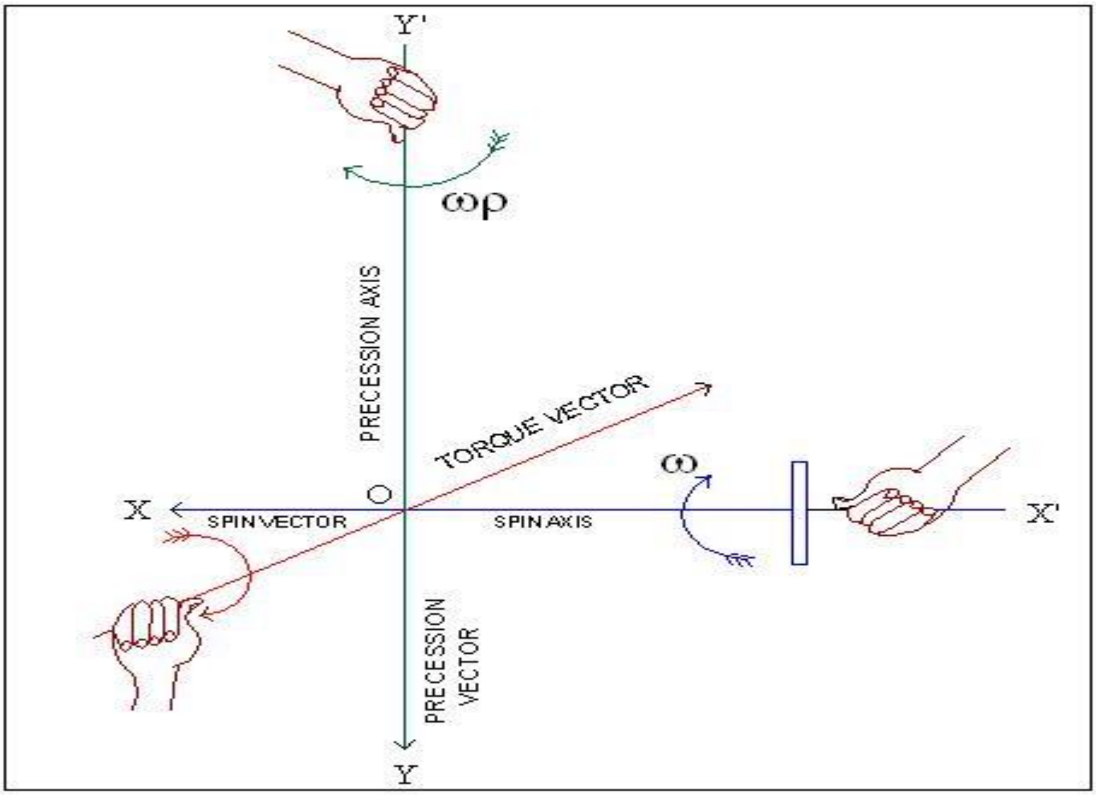


Fig.4. Direction of Spin vector, Precession vector and Couple/Torque vector

The method of determining the direction of couple/torque vector is as follows.

Case (i):

Consider a rotor rotating in anticlockwise direction when seen from the right (Fig.5 and Fig. 6), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector, the following procedure is used

- Turn the spin vector through 90^0 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction

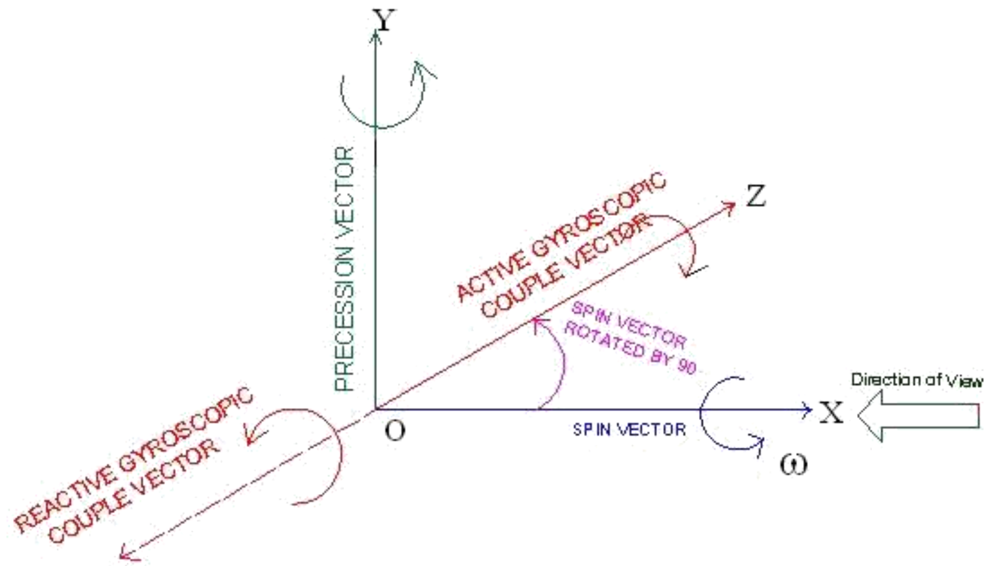


Fig. 5 Direction of active and reactive gyroscopic couple/torque vector

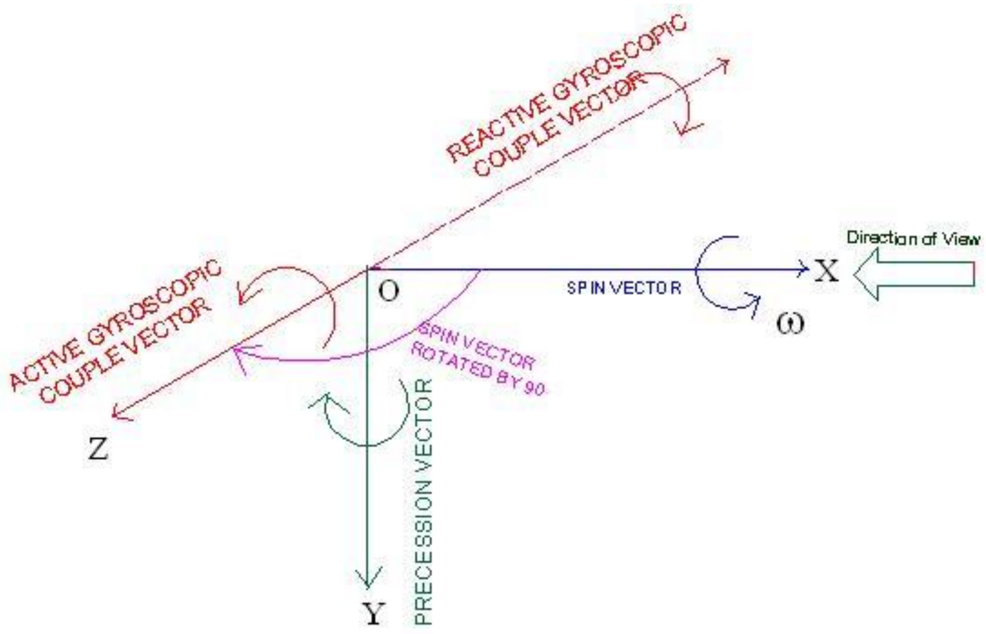


Fig. 6 Direction of active and reactive gyroscopic couple/torque vector

Case (ii):

Consider a rotor rotating in clockwise direction when seen from the right (Fig.7 and Fig. 8), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector,

- Turn the spin vector through 90^0 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction.

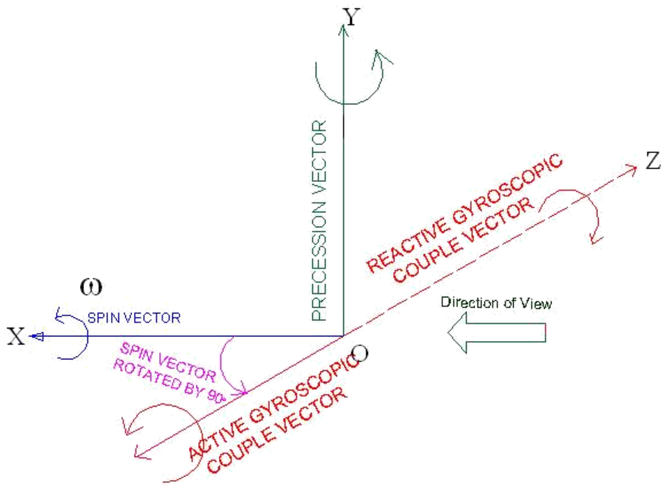


Fig. 7 Direction of active and reactive gyroscopic couple/torque vector

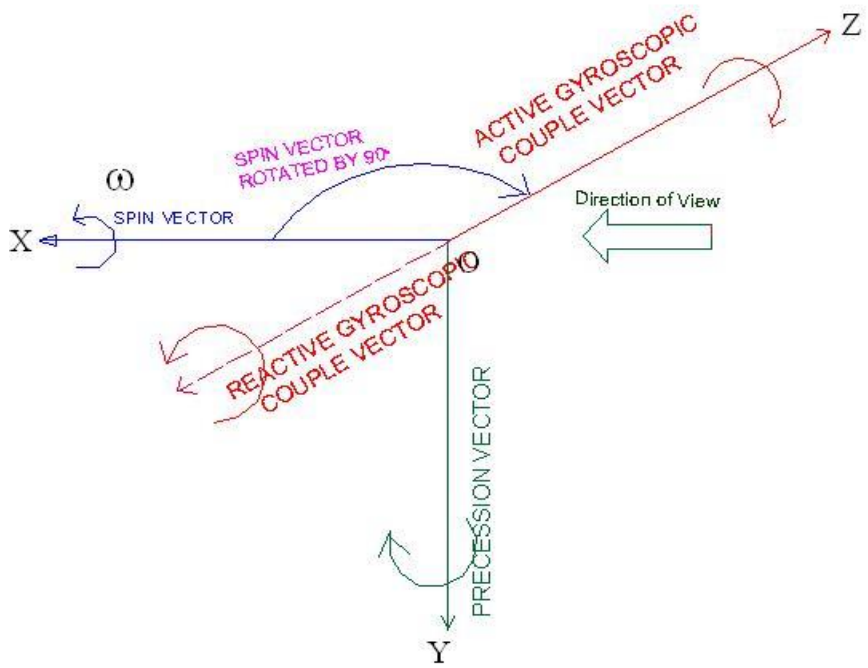


Fig. 8 Direction of active and reactive gyroscopic couple/torque vector

The resisting couple/ reactive couple will act in the direction opposite to that of the gyroscopic couple. This means that, whenever the axis of spin changes its direction, a **gyroscopic couple** is applied to it through the bearing which supports the spinning axis.

Please note that, for analyzing the gyroscopic effect of the body, always reactive gyroscopic couple is considered.

Problem 1

A disc of 5 kg mass with radius of gyration 70 mm is mounted at span on a horizontal shaft spins at 720 rpm in clockwise direction when viewed from the right hand bearing. If the shaft precesses about the vertical axis at 30 rpm in clockwise direction when viewed from the top, determine the reactions at each bearing due to mass of the disc and gyroscopic effect.

Solution Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 720}{60}$$

$$= 75.4 \text{ rad/s}$$

Angular velocity of precession: $\omega_p = \dfrac{2\pi N_p}{60}$

$$= \frac{2\pi \times 30}{60} = 3.14 \text{ rad/s}$$

Moment of inertia: $I = mk^2$

$$= 5 \times 0.07^2 = 0.0245 \text{ kg m}^2$$

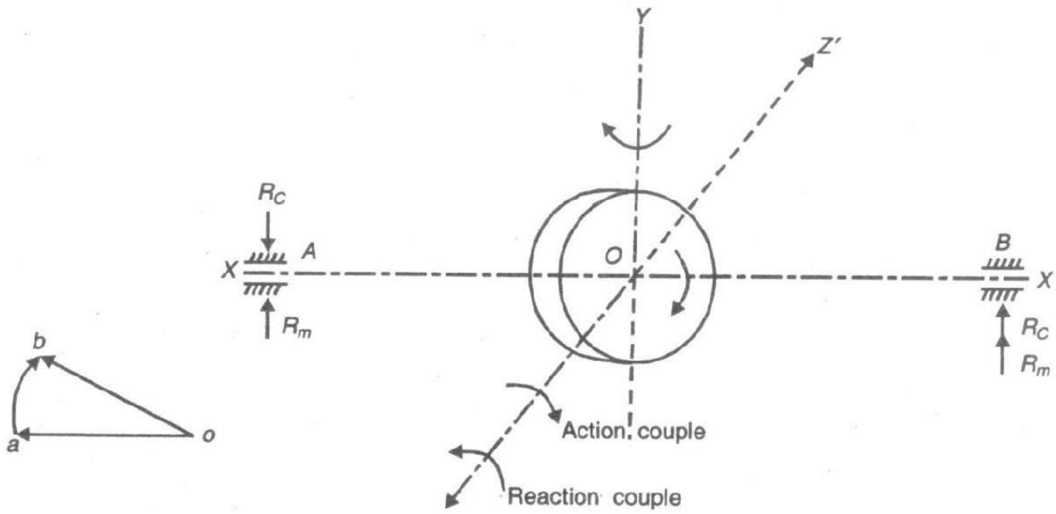


FIG. 9a

Gyroscopic couple: $C = I\,\omega\omega_p$

$$= 0.0245 \times 75.4 \times 3.14$$

$$= 5.8 \text{ Nm}$$

This couple induces reaction R_c at the bearing support.

$$R_c \times \frac{120}{1000} = 5.8$$

or $R_c = 48.3$ N

Reaction on the bearings due to weight of the disc, R_m = mg/2 = 5x9.81 /2 = 24.53 N

The angular momentum vector and induced reactive gyroscopic couple acting in anticlockwise direction as shown in fig.

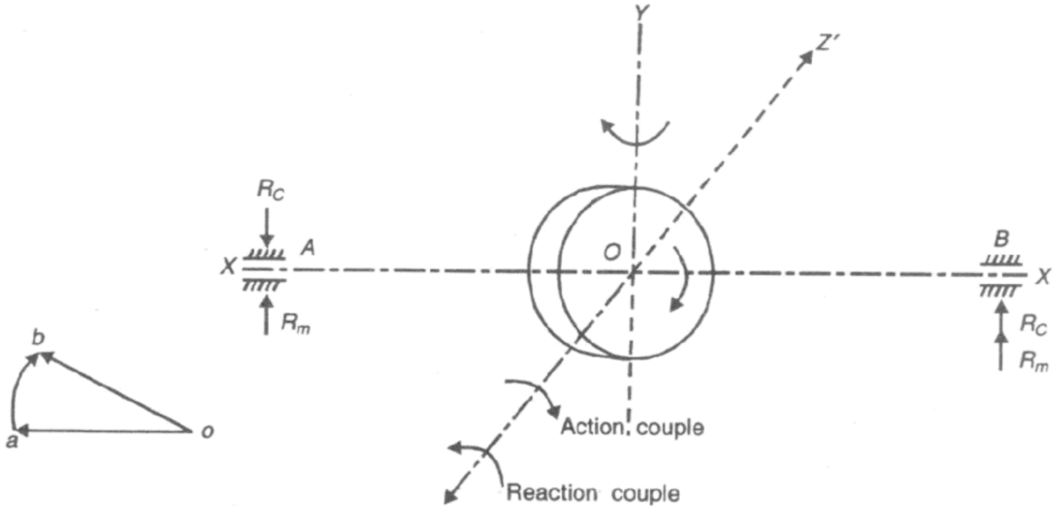


FIG.9b

Gyroscopic couple: $C = I\,\omega\omega_p$

$$= 0.0245 \times 75.4 \times 3.14$$

$$= 5.8 \text{ Nm}$$

This couple induces reaction R_c at the bearing support.

$$R_c \times \frac{120}{1000} = 5.8$$

or $R_c = 48.3$ N

The reaction R_c acts in upward direction at right hand bearing and in downward d at left hand bearing.

The reaction due to weight of the disc acts in upward direction. Therefore,

Reaction at bearing A: $R_A = R_c - R_m$

$$= 48.43 - 24.53$$

$$= \mathbf{23.9\ N(\downarrow)}$$

Reaction at bearing B: $R_B = R_c + R_m$

$$= 48.43 + 24.53$$

$$= \mathbf{72.96\ N(\uparrow)}$$

1.4 GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion:

(i) Steering—The turning of ship in a curve while moving forward

(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis.

(iii)Rolling—Sideway motion of the ship about longitudinal axis

For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

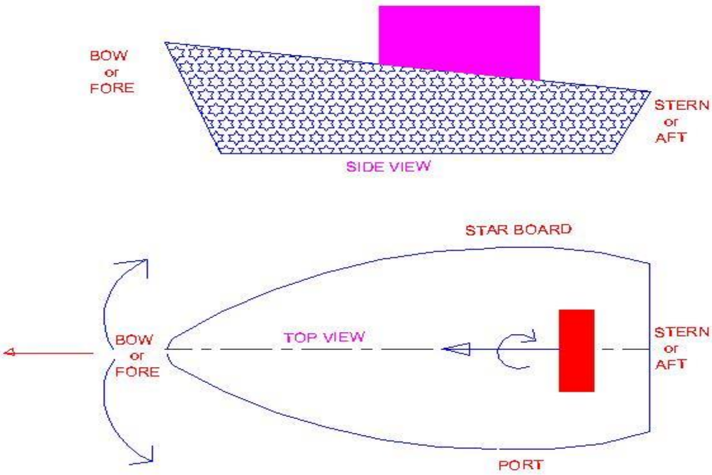
1.4.1 Ship Terminology

(i) Bow –It is the fore end of ship

(ii) Stern –It is the rear end of ship

(iii) Starboard –It is the right hand side of the ship looking in the direction of motion

(iv) Port –It is the left hand side of the ship looking in the direction of motion

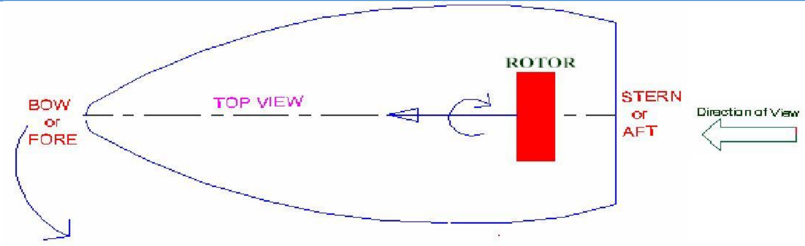


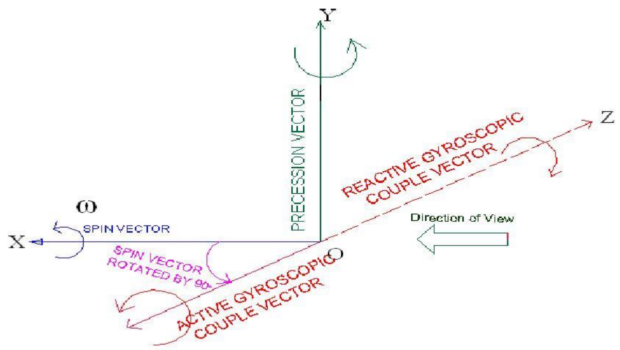
Consider a gyro-rotor mounted on the ship along longitudinal axis (X-axis) as shown in Fig. and rotate in clockwise direction when viewed from rear end of the ship. The angular speed of the rotor is rad/s. The direction of angular momentum vector *oa*, based on direction of rotation of rotor, is decided using right hand thumb rule as discussed earlier. The gyroscopic effect during the three types of motion of ship is discussed.

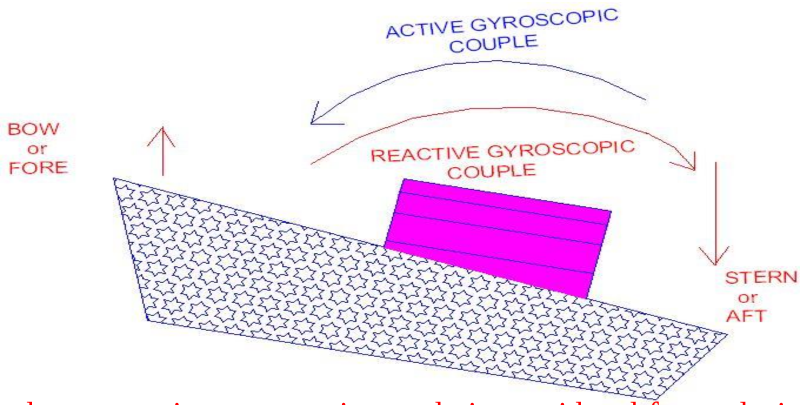
1.4.2 Gyroscopic effect on Steering of ship

(i) Left turn with clockwise rotor

When ship takes a left turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way.



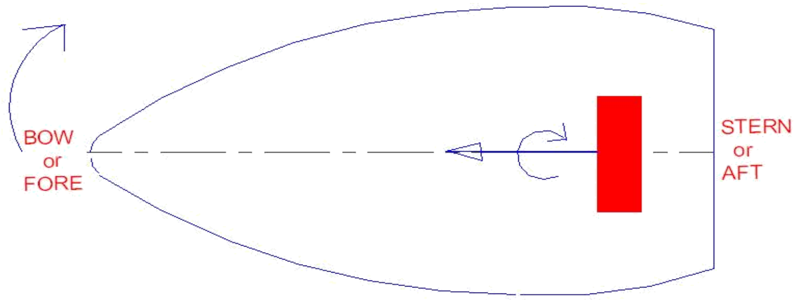


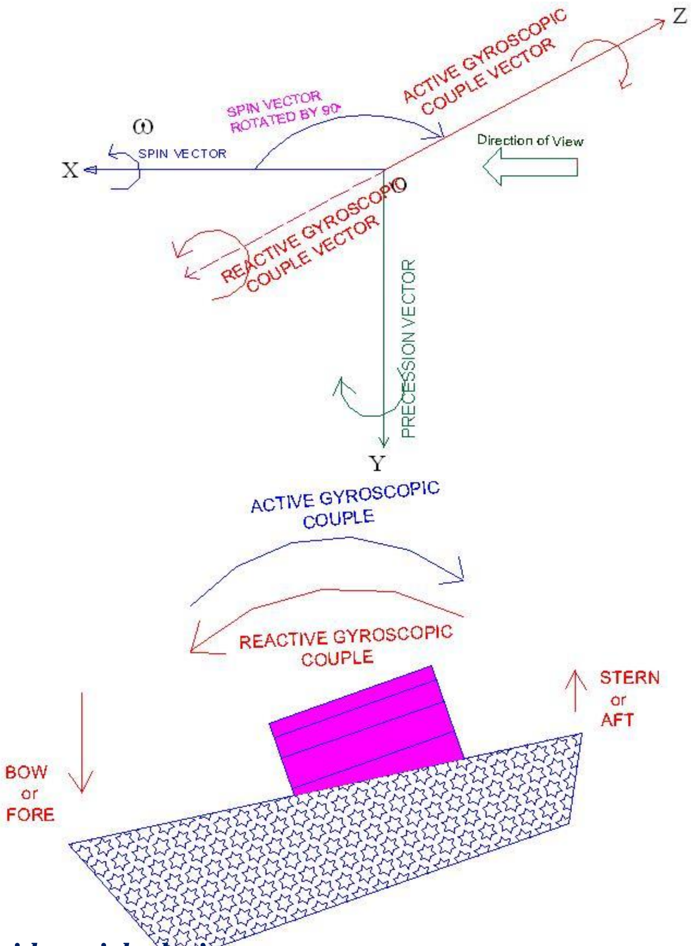


Note that, always reactive gyroscopic couple is considered for analysis. From the above analysis (Fig.12), the couple acts over the ship between stern and bow. This reaction couple tends to raise the front end (bow) and lower the rear end (stern) of the ship.

(ii) Right turn with clockwise rotor

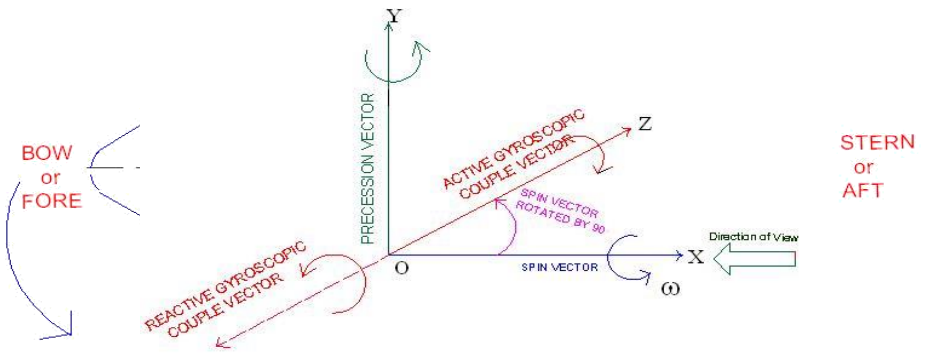
When ship takes a right turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple acts on the ship is analyzed (Fig). Again, the couple acts in vertical plane, means between stern and bow. Now the reaction couple tends to lower the bow of the ship and raise the stern.

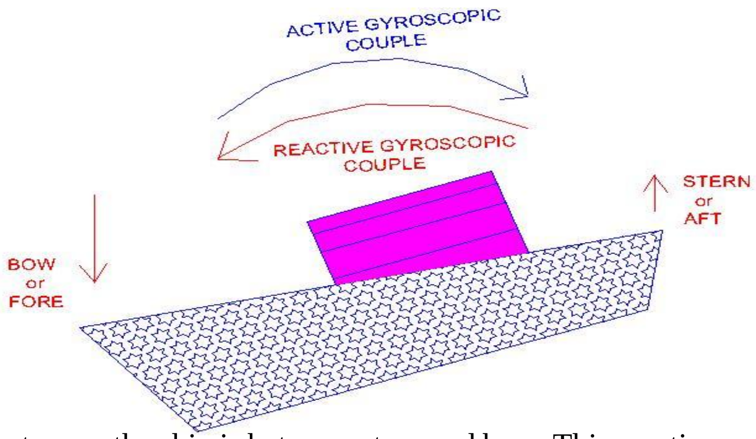




(iii) Left turn with anticlockwise rotor

When ship takes a left turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way (Fig.).





The couple acts over the ship is between stern and bow. This reaction couple tends to press or dip the front end (bow) and raise the rear end (stern) of the ship.

(iv) Right turn with anticlockwise rotor

When ship takes a right turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is according to Fig 20. Now, the reaction couple tends to raise the bow of the ship and dip the stern.

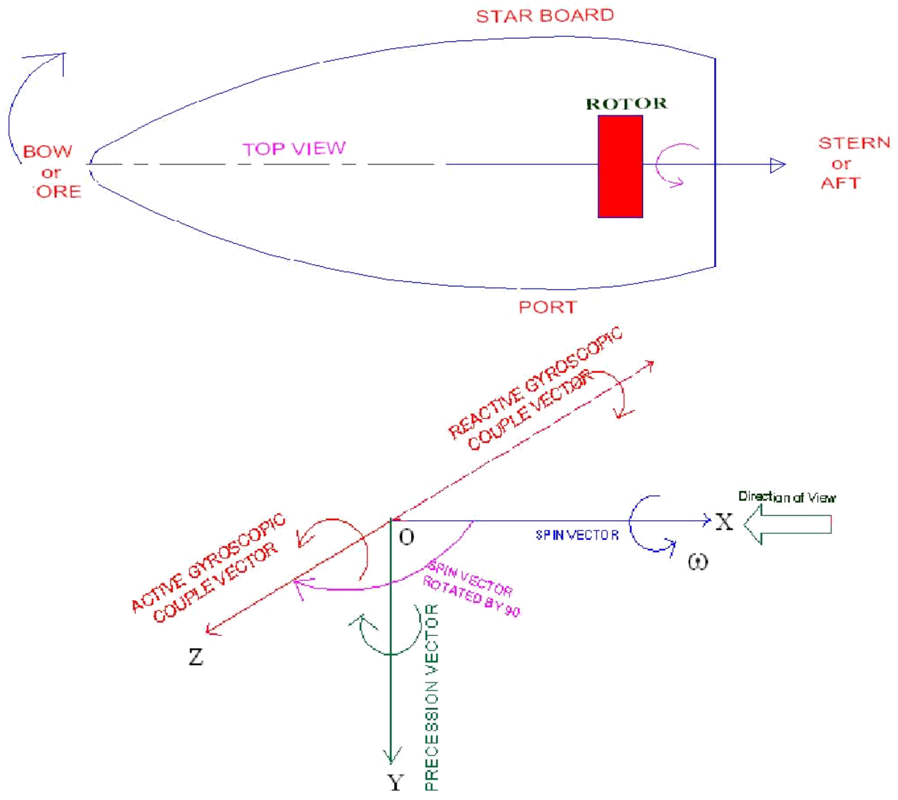


Fig. 21

1.4.3 Gyroscopic effect on Pitching of ship

The pitching motion of a ship generally occurs due to waves which can be approximated as sine wave. During pitching, the ship moves up and down from the horizontal position in vertical plane (Fig.22. & Fig. 23)

Fig.22 Pitching action of ship

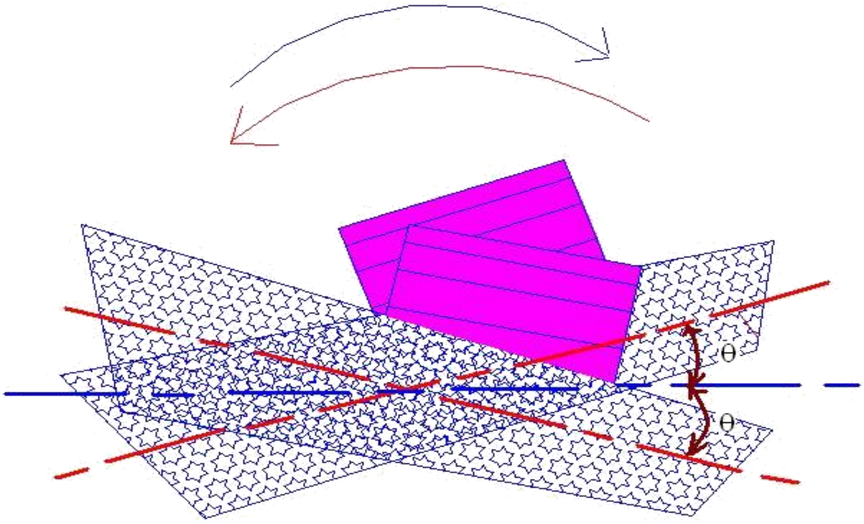


Fig.23 Pitching action of ship

Let θ = angular displacement of spin axis from its mean equilibrium position

A = amplitude of swing

(= angle in degree $\times \dfrac{2\pi}{360°}$)

and ω_0 = angular velocity of simple hormonic motion $\left(= \dfrac{2\pi}{\text{time period}}\right)$

The angular motion of the rotor is given as

$$\theta = A \sin \omega_0 t$$

Angular velocity of precess: $$\omega_p = \frac{d\theta}{dt}$$

$$= \frac{d}{dt}(A \sin \omega_0 t)$$

or $$\omega_p = A\omega_0 \cos \omega_0 t$$

The angular velocity of precess will be maximum when $\cos \omega_0 t = 1$

or $$\omega_{p\max} = A\omega_0$$

$$= A \times \frac{2\pi}{t}$$

Thus the gyroscopic couple: $$C = I\,\omega\,\omega_p$$

Consider a rotor mounted along the longitudinal axis and rotates in clockwise direction when seen from the rear end of the ship. The direction of momentum for this condition is shown by vector *ox* (Fig.24). When the ship moves up the horizontal position in vertical plane by an angle $\delta\theta$from the axis of spin, the rotor axis (X-axis) processes about Z-axis in XY-plane and for this case Z-axis becomes precession axis. The gyroscopic couple acts in anticlockwise direction about Y-axis and the reaction couple acts in opposite direction, i.e. in clockwise direction, which tends to move towards **right side** (Fig.25). However, when the ship pitches down the axis of spin, the direction of reaction couple is reversed and the ship turns towards **left side** (Fig. 26).

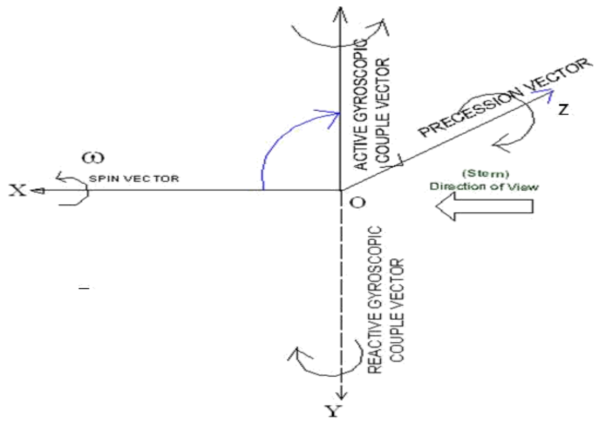


Fig. 24

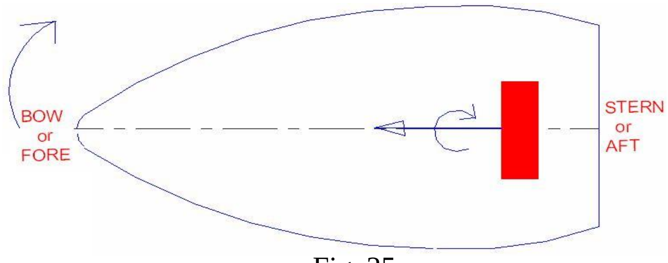


Fig. 25

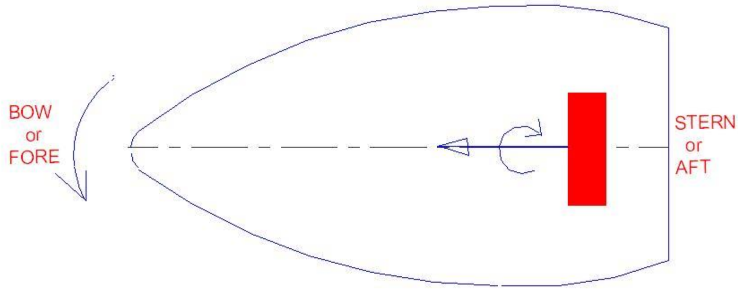


Fig. 26

Similarly, for the anticlockwise direction of the rotor viewed from the rear end (Stern) of the ship, the analysis may be done.

1.4.4 Gyroscopic effect on Rolling of ship.

The axis of the rotor of a ship is mounted along the longitudinal axis of ship and therefore, there is **no** precession of this axis. Thus, no effect of gyroscopic couple on the ship frame is formed when the ship rolls.

Fig.27

Problem 2

A turbine rotor of a ship has a mass of 3500 kg and rotates at a speed of 2000 rpm. The rotor has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from the stern (rear) end. Determine the magnitude of gyroscopic couple and its direction for the following conditions

(i) When the ship runs at a speed of 12 knots and steers to the left in a curve of 70 m radius

(ii) When the ship pitches 6° above and 6° below the horizontal position and the bow (Front) end is lowered. The pitching motion is simple harmonic with periodic time 30 sec.

(iii) When the ship rolls and at a certain instant, it has an angular velocity of 0.05 rad/s clockwise when viewed from the stern

Also find the maximum angular acceleration during pitching.

Solution Given, 1 knot = 1.86 kmph, the linear velocity of the ship:

$$V = 1.86 \times 12 = 22.32 \text{ kmph}$$

$$= \frac{22.32 \times 1000}{3600} = 6.2 \text{ m/s}$$

Angular velocity of the rotor:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60}$$

$$= 209.44 \text{ rad/s}$$

Precession velocity: $\omega_p = \frac{V}{R} = \frac{6.2}{70} = 0.08857$ rad/s

Moment of inertia: $I = mk^2 = 3500 \times 0.5^2 = 875$ kg m^2

Gyroscopic couple: $C = I\omega\omega_p$

$$= 875 \times 209.44 \times 0.08857$$

$$= \mathbf{16231.34 \text{ Nm}}$$

When ship steers to the left, the reaction gyroscopic couple action is in anticlockwise direction and the bow of the ship is raised and stern is lowered, as shown in Fig.28.

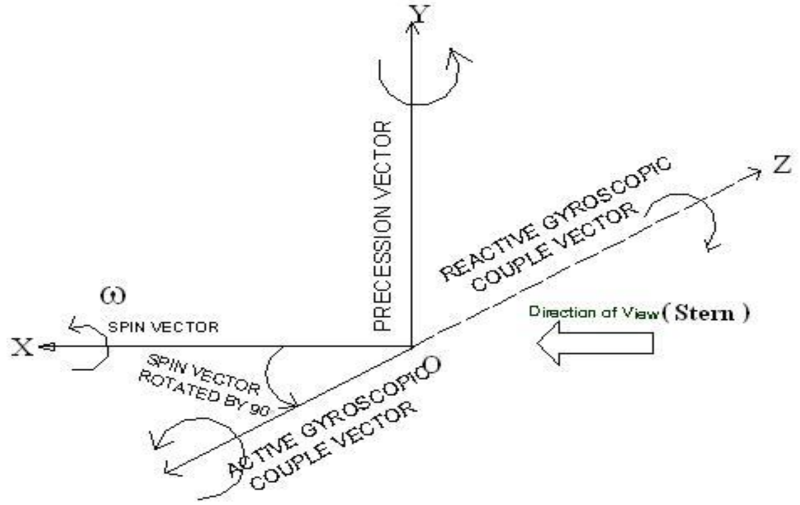


Fig.28

(ii) Amplitude of swing: $A = \dfrac{6° \times 2\pi}{360°} = 0.1047$ rad

Angular displacement: $\theta = A \sin \omega_0 t$

Angular velocity of precession: $\omega_p = \dfrac{d\theta}{dt} = A\omega_0 \cos \omega_0 t$

Maximum angular velocity of precession:

$$\omega_{pmax} = \omega_0 A$$

where $\omega_0 = \dfrac{2\pi}{\text{time period of oscillation}} = \dfrac{2\pi}{30}$

$= 0.2094$ rad/s

$$\omega_{pmax} = 0.2094 \times 0.1047 = 0.022 \text{ rad/s}$$

Maximum couple for pitching:

$C_{max} = I\omega\omega_{pmax}$
$= 875 \times 209.44 \times 0.022$
$= 4031.72$ Nm

The effect of gyroscopic couple due to pitching is shown in Fig.29. the reactive gyroscopic couple will act in anticlockwise direction seen from top and it will turn ship towards the left side.

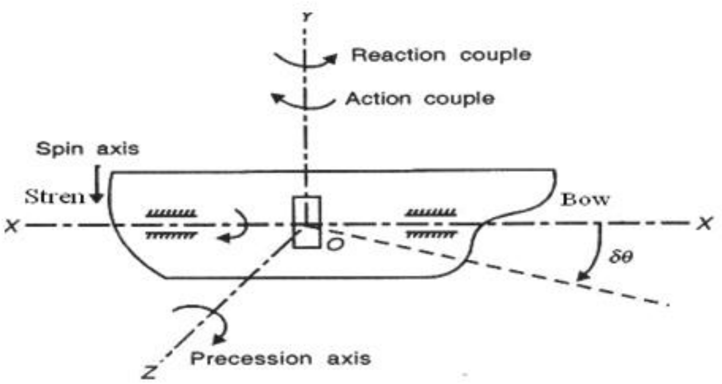


Fig.29

iii) Angular velocity of precession while the ship rolls is:

$$\omega p = 0.05 \text{ rad/s}$$

and gyroscopic couple : C $= I\omega\omega p$

$= 875 \times 209.44 \times 0.05$

$= 9163$ Nm

Since the ship rolls in the same plane as the plane of spin, there **is no gyroscopic effect**.

Angular velocity of precess during pitching is:

$$\omega_p = \frac{d\theta}{dt} = A\omega_0 \cos \omega_0 t$$

Therefore, angular acceleration:

$$\alpha = \frac{d^2\theta}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

Maximum angular acceleration:

$$\alpha_{max} = -A\omega_0^2$$

$$= 0.1047 \times 0.2094^2$$

$$= 0.00459 \text{ rad/s}^2$$

Problem 3

A ship is propelled by a rotor of mass of 2000 kg rotates at a speed of 2400 rpm. The radius of gyration of rotor is 0.4 m and spins clockwise direction when viewed from bow (front) end. Find the gyroscopic couple and its effect when;

(i) the ship takes left turn at a radius of 350 m with a speed of 35 kmph

(ii) the ship pitches with the bow rising at an angular velocity of 1 rad/s

(iii)the ship rolls at an angular velocity of 0.15 rad/s

Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2400}{60} = 251.33 \text{ rad/s}$$

Linear velocity: $V = 35 \text{ kmph} = \frac{35 \times 1000}{3600} = 9.72 \text{ m/s}$

Moment of inertia: $I = mk^2 = 2000 \times 0.4^2 = 320 \text{ kg m}^2$

Steering towards left

Angular velocity of precession: $\omega_p = \frac{V}{R} = \frac{9.72}{350} = 0.0278 \text{ rad/s}$

Gyroscopic couple: $C = I\omega\omega_p$

$= 320 \times 251.33 \times 0.0278$

$= 2235.8 \text{ Nm}$

The reaction gyroscopic couple will act in anticlockwise and will tend to lower the bow as shown in Figure 30.

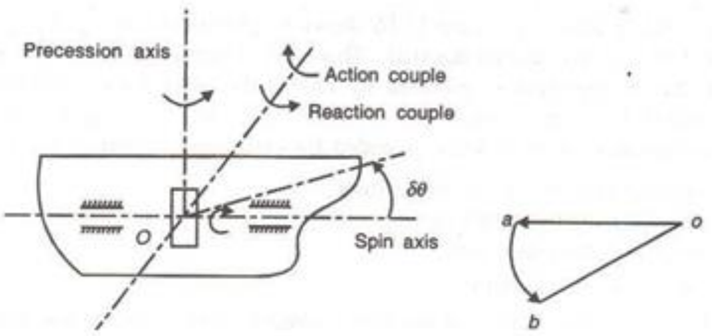


Fig.30

Pitching. Angular velocity of precession during pitching ω_p = 1.0 rad/s

Gyroscopic couple: C = 320 x 251.33 x 1.0

= 80425.6 **Nm Ans.**

The reaction gyroscopic couple acting in anticlockwise direction will tend to turn the bow towards the Right side as shown in Figure 31.

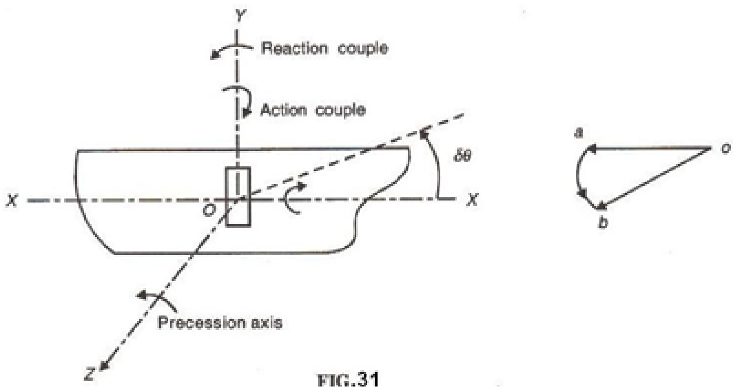


FIG.31

Rolling, Gyroscopic couple: C= I6XQp

= 320 x 251.33 x 0.15 = 12063.84 Nm

During rolling, the ship rolls in the same plane as the plane of spin and there will be no gyroscopic effect.

1.5 Gyroscopic Effect on Aeroplane

Aero planes are subjected to gyroscopic effect when it taking off, landing and negotiating left or right turn in the air.

Let

ω = Angular velocity of the engine rotating parts in rad/s,
m = Mass of the engine and propeller in kg,
rW = Radius of gyration in m,
I = Mass moment of inertia of engine and propeller in kg m^2,
V = Linear velocity of the aeroplane in m/s,
R = Radius of curvature in m,
ω_p =Angular velocity of precession = $\frac{V}{R}$ rad/s

∴ Gyroscopic couple acting on the aero plane = $C = I\,\omega\omega_p$

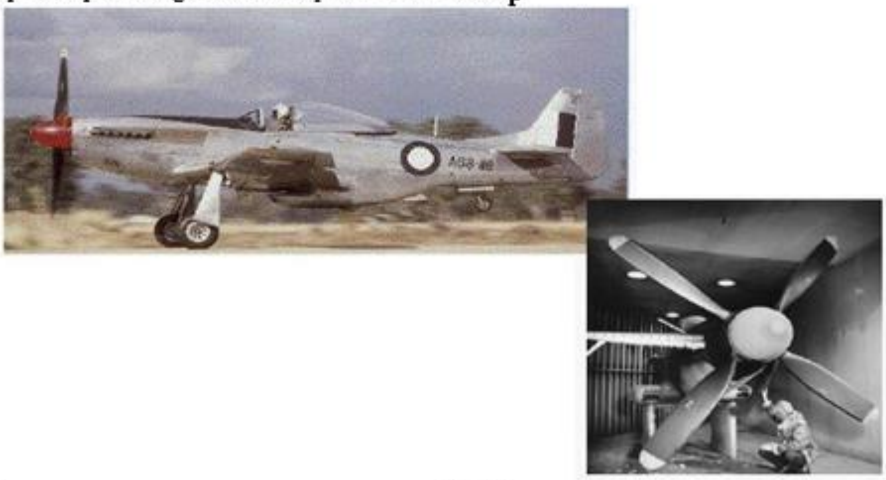


Fig.32

Let us analyze the effect of gyroscopic couple acting on the body of the aero plane for various conditions.

Case (i): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT

Fig.33

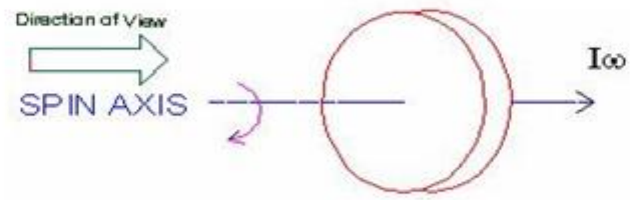


Fig.34

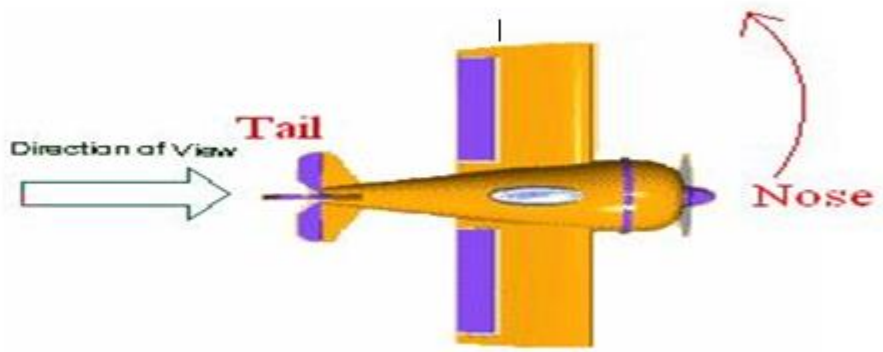


Fig.35

Fig.36

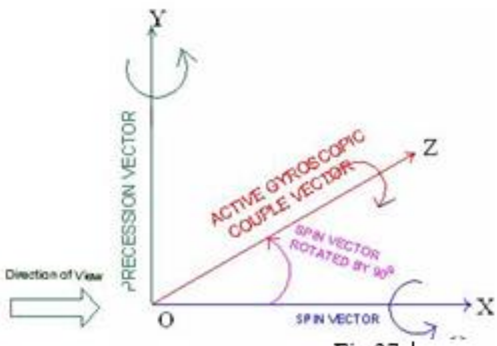


Fig.37

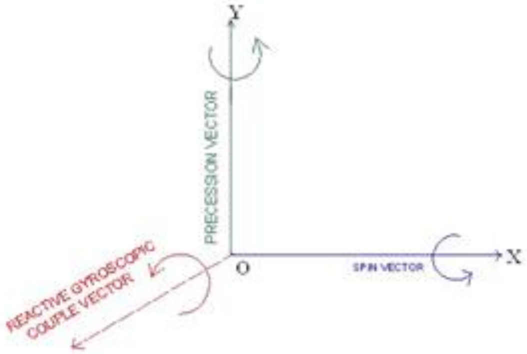


Fig.38

According to the analysis, the reactive gyroscopic couple tends to dip the tail and raise the nose of aeroplane.

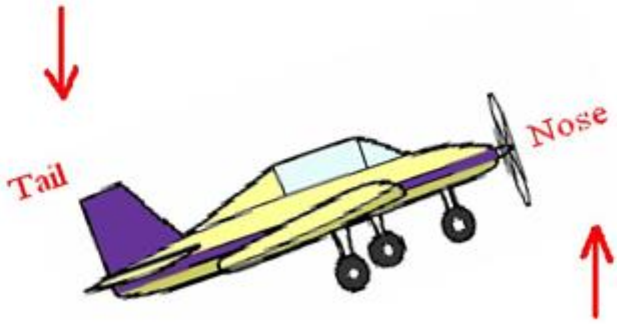
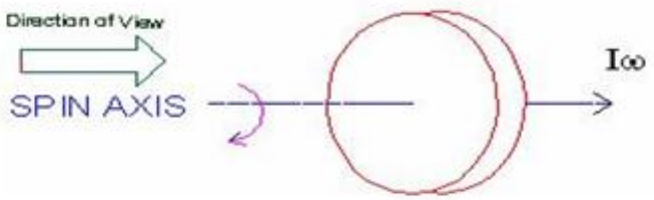


Fig.39

Case (ii): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT

Fig.40



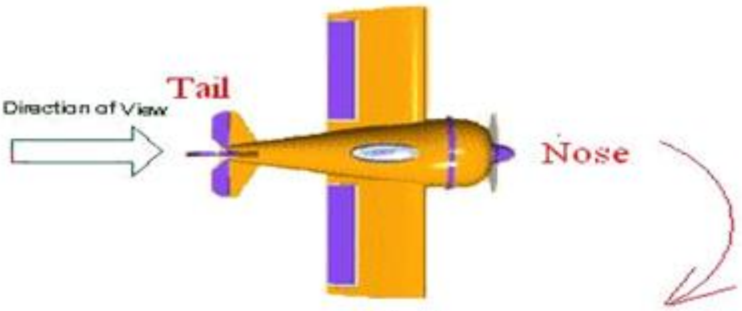


Fig.42

Fig.43

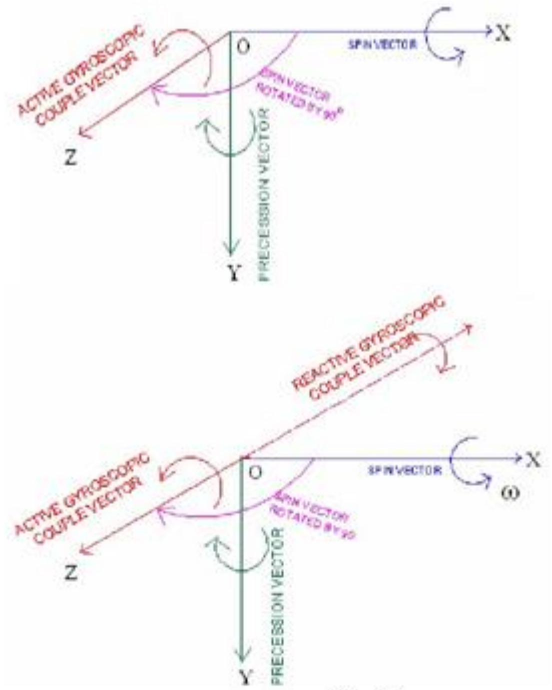


Fig. 44

According to the analysis, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.

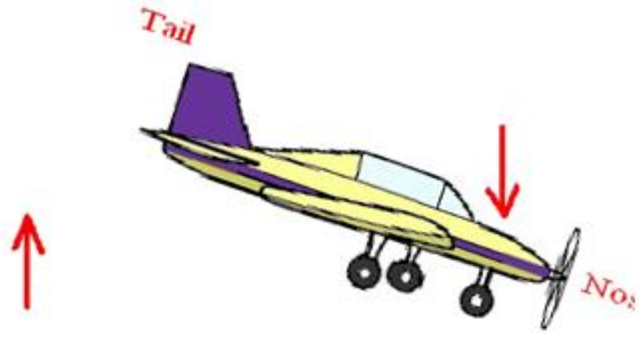


Fig.45

Case (iii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT

Fig.46

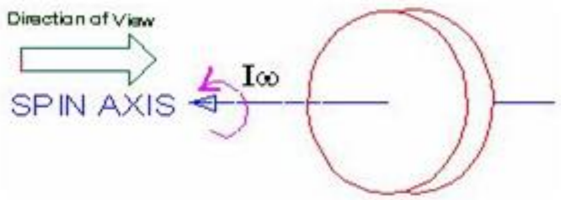


Fig.47

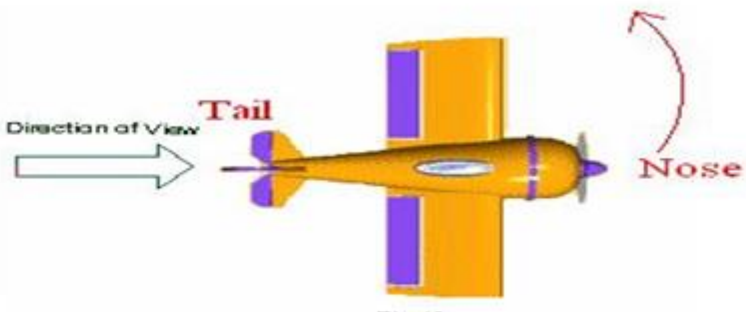


Fig.48

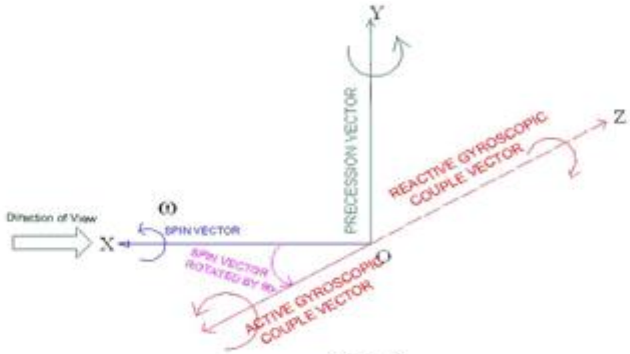
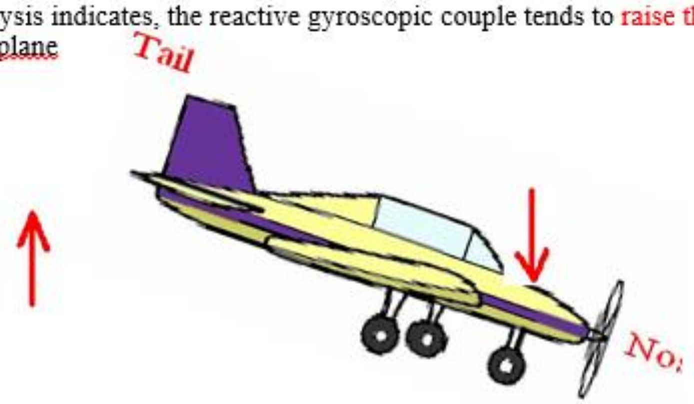


Fig.49

The analysis indicates, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane



Case (IV): **PROPELLER direction when seen from rear end and Aero plane turns towards RIGHT**

Fig.51

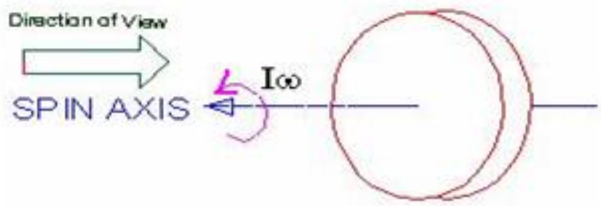


Fig.52

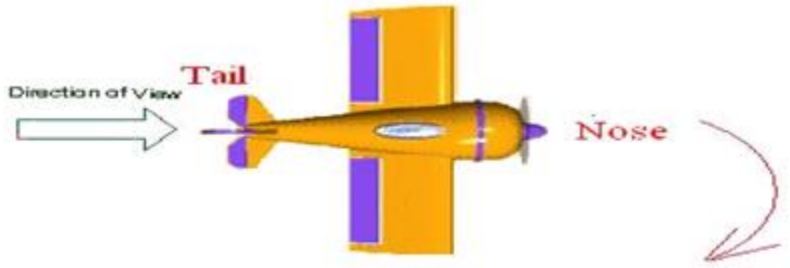


Fig.53

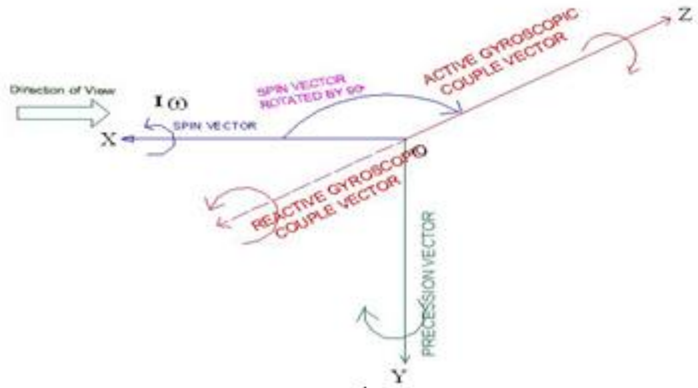


Fig.54

The analysis and dip the nose of aeroplane.

Fig.55

Case (v): PROPELLER rotates in CLOCKWISE direction when seen from rear upwards

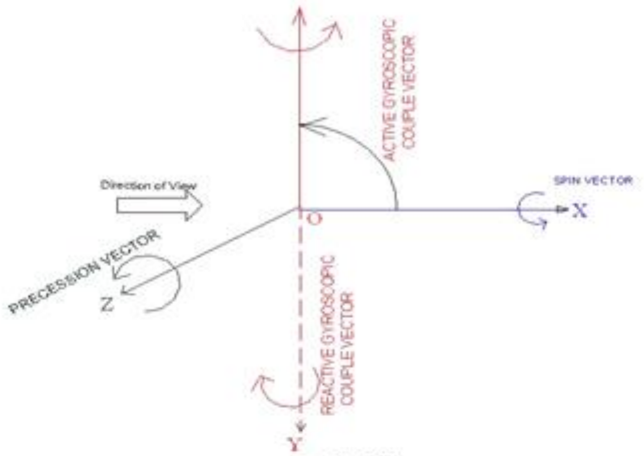
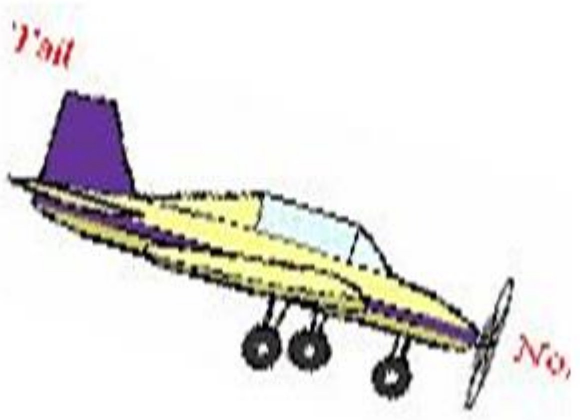


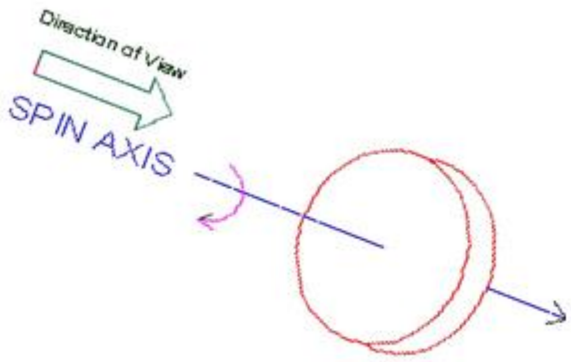
Fig.57

The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right

Fig.58

Case (vi): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards





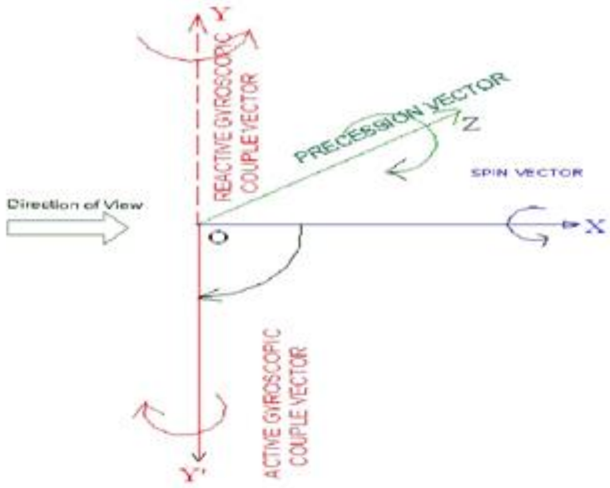


Fig. 61

The reactive gyroscopic couple tends to turn the nose of aeroplane toward left

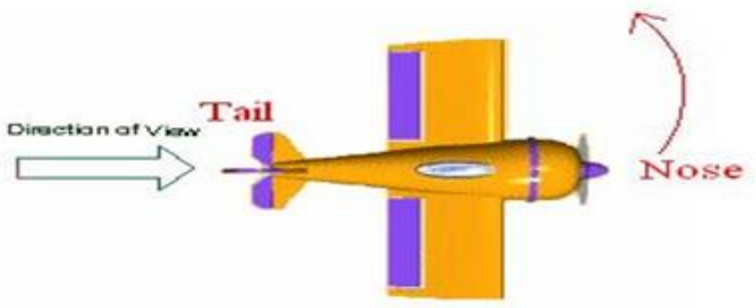


Fig.62

Case (vii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards

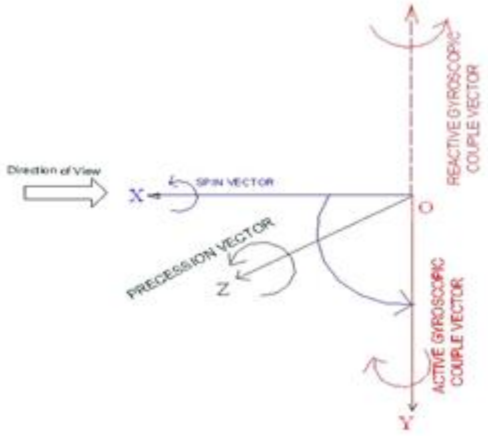


Fig.63

The reactive gyroscopic couple tends to turn the nose of aeroplane toward left

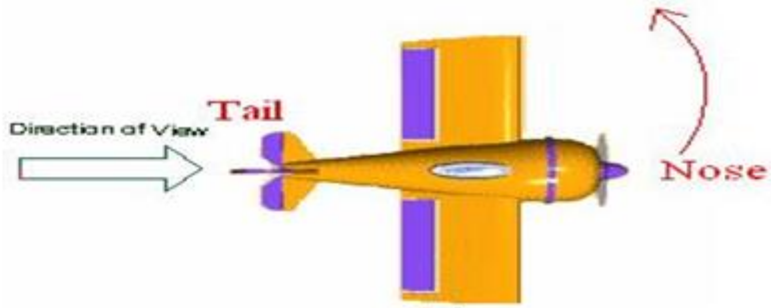


Fig.64

Case (viii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards

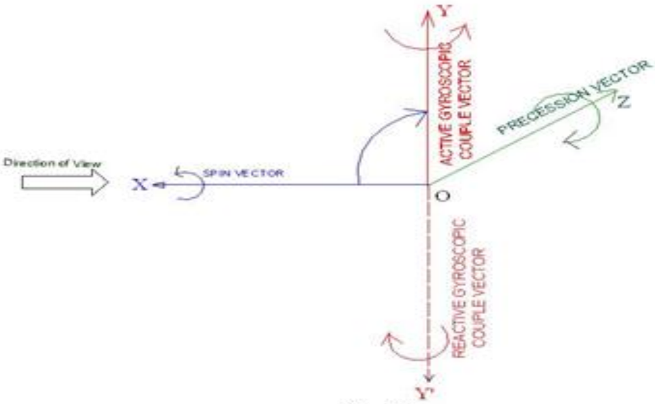


Fig.65

The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right

Problem 4

An aeroplane flying at a speed of 300 kmph takes right turn with a radius of 50 m. The mass of engine and propeller is 500 kg and radius of gyration is 400 mm. If the engine runs at 1800 rpm in clockwise direction when viewed from tail end, determine the gyroscopic couple and state its effect on the aeroplane. What will be the effect if the aeroplane turns to its left instead of right?

Solution Angular velocity of aeroplane engine:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.49 \text{ rad/s}$$

Angular velocity of precession: $\omega_p = \frac{V}{R}$

or $$\omega_p = \frac{300 \times 1000}{3600} \times \frac{1}{50}$$

$$= 1.67 \text{ rad/s}$$

Moment of inertia: $I = mk^2 = 500 \times 0.4^2$

$$= 80 \text{ kg m}^2$$

Gyroscopic couple: $c = I\omega\omega_p$

$$= 80 \times 188.49 \times 1.67$$

$$= \mathbf{25182.26 \text{ Nm}}$$ **Ans.**

1.6 Stability of Automotive Vehicle

A vehicle running on the road is said to be stable when no wheel is supposed to leave the road surface. In other words, the resultant reactions by the road surface on wheels should act in upward direction. For a moving vehicle, one of the reaction is due to gyroscopic couple produced by the rotating wheels and rotating parts of the engine. Let us discuss stability of two and four wheeled vehicles when negotiating a curve/turn.

1.6.1 Stability of Two Wheeler negotiating a turn

Fig.71

Fig. 71 shows a two wheeler vehicle taking **left turn** over a curved path. The vehicle is inclined to the vertical for equilibrium by an angle known as angle of heel.

Let

m = *Mass of the vehicle and its* rider in kg,

W = *Weight of the vehicle and* its rider in newtons = $m.g$,

h = *Height of the Centre of* gravity of the vehicle and rider,

r_W = *Radius of the wheels,*

R = *Radius of track or curvature,*

I_W = *Mass moment of inertia of each wheel,*

I_E = *Mass moment of inertia of the rotating parts of the engine,*

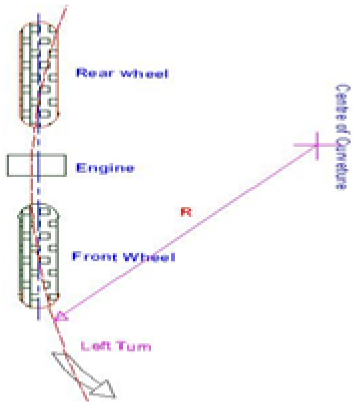
ω_W = Angular velocity of the wheels,

ω_E = Angular velocity of the engine rotating parts,

G = *Gear ratio* = ω_E / ω_W,

v = *Linear velocity* $_W \times r_W$, *of the vehicle* = ω

θ = Angle of heel. It is inclination of the



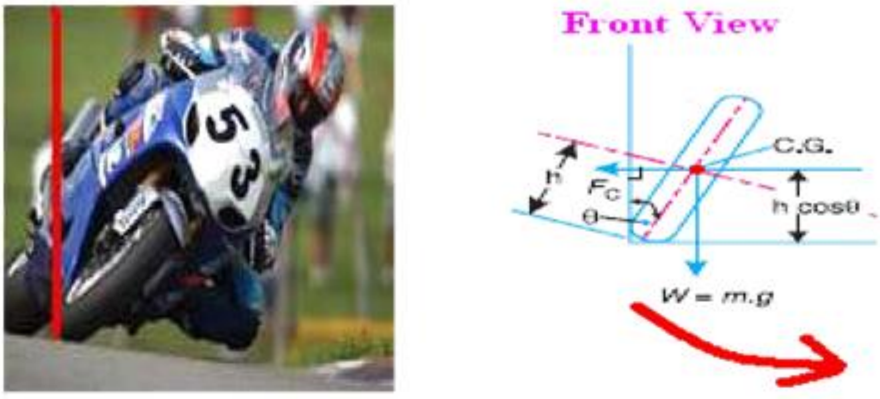
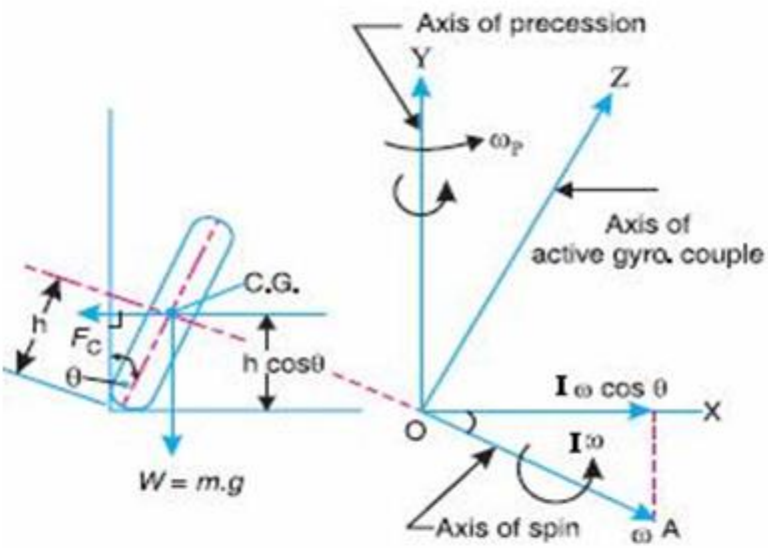


Fig.73



Let us consider the effect of the gyroscopic couple and centrifugal couple on the wheels.

1. Effect of Gyroscopic Couple

We know that, $V = {}_W \times \omega r_W$

$\omega_E = G . \omega_W$ or $\omega_E = G . v / r_W$

Angular momentum due to wheels = $2 \, I_W \, \omega_W$

Angular momentum due to engine and transmission = $I_E \, \omega_E$

Total angular momentum ($I \, x\omega$) = $2 \, I_W \, \omega_W \pm I_E \, \omega_E$

$$= \quad 2I_w \frac{v}{r_w} \pm I_E G \frac{v}{r_w}$$

$$= \frac{v}{r_w}(2I_w \pm GI_E)$$

Also, Velocity of precession = $\omega p = \frac{V}{R}$

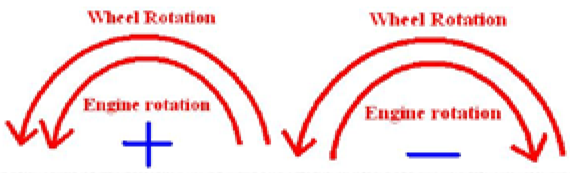
It is observed that, when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown of heel'. In other inclined words, to the horizontal axis of spin at a in Fig.73 Thus, the angular momentum vector I ω due to spin is represented by OA inclined to OX at an angle θ. But, the precession axis is in vertical. Therefore, the spin vector is resolved along OX.

Gyroscopic Couple,

$$C_g = (I\omega)\cos\theta \times \omega_p$$

$$C_g = \frac{v^2}{Rr_w}(2I_w \pm GI_E)\cos\theta$$

Note: When the engine is rotating in the same direction as that of wheels, then the positive sign is used in the above equation. However, if the engine rotates in opposite direction to wheels, then negative sign is used.



The gyroscopic couple will act over the vehicle outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown

Analysis:

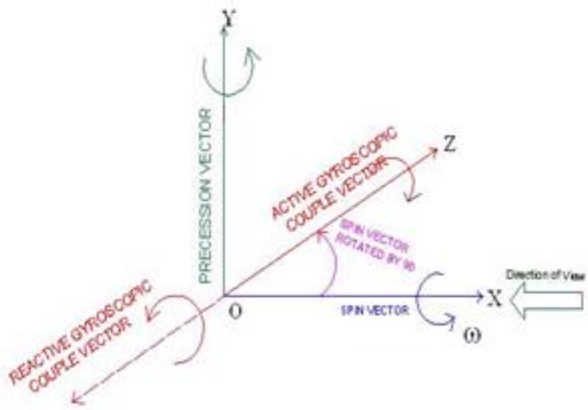


Fig.75

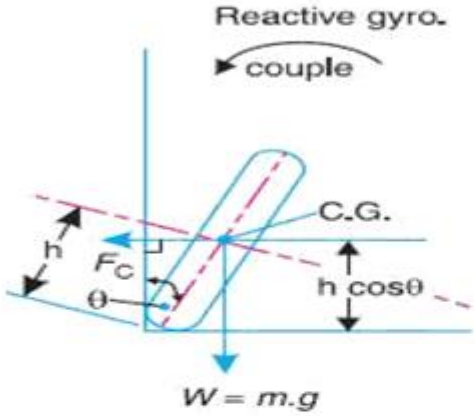


Fig.76

2. Effect of Centrifugal Couple

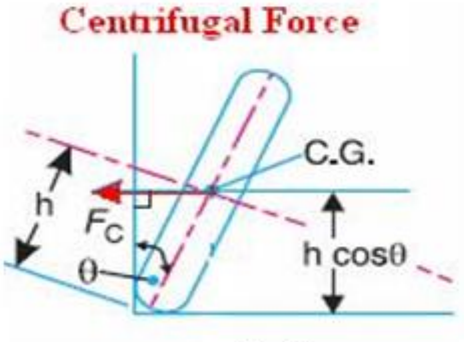


Fig. 77

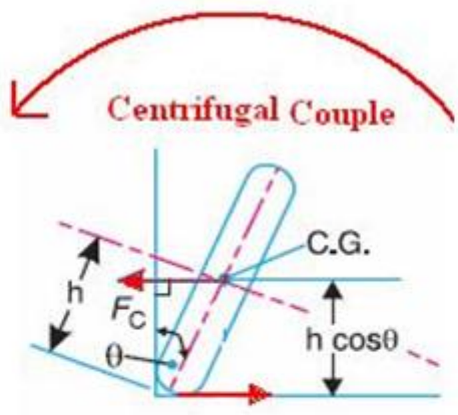
We have,

Centrifugal force, $F_c = \frac{mv^2}{R}$

Or

Centrifugal Couple, $C_c = F_c \times h\cos\theta$

$$= \frac{mv^2}{R} h\cos\theta$$



|

The Centrifugal couple will act over the two wheeler outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig.78

Therefore, the total Over turning couple: $C = C_g + C_c$

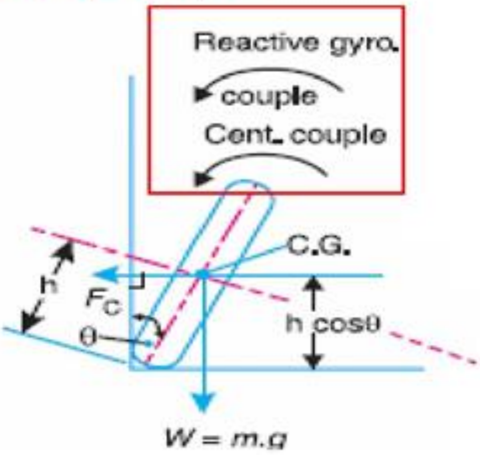


Fig.79

$$C = \frac{v^2}{Rr}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta$$

For the vehicle to be in equilibrium, overturning couple should be equal to balancing couple acting in clockwise direction due to the weight of the vehicle and rider.

$\therefore$ $C = mgh\sin\theta$

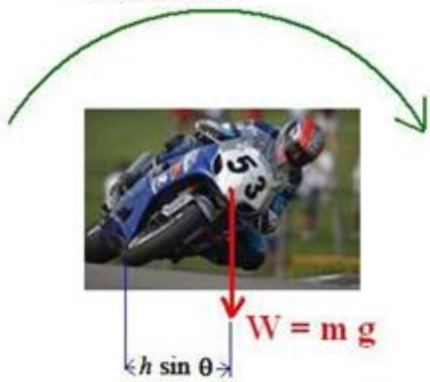


Fig.80

For the stability, overturning couple must be equal to balancing couple,

$$\boxed{\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R}h\cos\theta = mgh\sin\theta}$$

Therefore, from the above equation, the value of angle of heel (θ) may be determined, so that the vehicle does not skid. Also, for the given value of the maximum vehicle speed in the turn without skid may be determined.

Problem 5

A motorcycle and its rider together weighs 2000 N and their combined centre of gravity is 550 mm above the road when motorcycle is upright. Each wheel is of 580 mm diameter and has a moment of inertia of 1.0 kgm^2. The moment of inertia of rotating parts of engine is 0.15 kg m^2. The engine rotates at 5 times the speed of the vehicle and the same sense. Determine the angle of heel necessary when motorcycle is taking a turn over a track of 35 m radius at a speed of 60 kmph.

Solution:

Velocity of vehicle: $v = \frac{60 \times 1000}{3600} = 16.67$ m/s

Angular velocity of wheel: $\omega = \frac{2v}{d} = \frac{2 \times 16.67}{0.58} = 57.48$ rad/s

Angular velocity of precession: $\omega_p = \frac{v}{R} = \frac{16.67}{35} = 0.476$ rad/s

(i) Gyroscopic couple due to two wheels:

$C_W = 2I_W \omega \omega_p \cos\theta$
$= 2 \times 1.0 \times 57.48 \times 0.476 \times \cos\theta$
$= 54.72 \cos\theta$ Nm

(ii) Gyroscopic couple due to rotating parts of engine:

$C_E = I_E G \omega \omega_p \cos\theta$
$= 0.15 \times 5 \times 57.48 \times 0.476 \times \cos\theta$
$= 20.52 \cos\theta$ Nm

(iii) Centrifugal force due to angular velocity of die wheel:

$$F_c = \frac{mv^2}{R} = \frac{2000 \times 16.67^2}{9.81 \times 35} = 1618.7 \text{ N}$$

Centrifugal couple: $C_c = 1618.7 \times 0.55 \cos\theta$
$= 890.28 \cos\theta$ Nm

Total overturning couple: $C = C_W + C_e + C_c$
$= (54.72 + 20.52 + 890.28) \cos\theta$
$= 965.52 \cos\theta$ Nm

Balancing couple $= mgh \sin\theta$

$$= \frac{2000}{9.81} \times 9.81 \times 0.55 \sin\theta$$

$= 1100 \sin\theta$ Nm

For the stability of motorcycle, overturning couple should be equal to resisting couple.

∴ $1100 \sin\theta = 965.52 \cos\theta$

Or

$$\tan\theta = \frac{965.52}{1100} = 0.877$$

heel angle: $\theta = 41.27°$

Problem 6

A motor cycle with its rider has a mass of 300 kg. The centre of gravity of the machine and rider combined being 0.6 m above the ground with machine in vertical position. Moment of inertia of each wheel is 0.525 kg m^2 and the rolling diameter of 0.6 m. The engine rotates 6 times the speed of the road wheels and in the same sense. The engine rotating parts have a mass moment of inertia of 0.1686 kg m^2. Find (i) the angle of heel necessary if the vehicle is running at 60 km/hr round a curve of 30 m (ii) If the road and tyre friction allow for the angle of heel not to exceed 50^o, what is the maximum road velocity of the motor cycle.

Solution:

m = 300 kg, h = 0.6 m, I_W = 0.525 kg m^2, dw=0.6 m; r_W = 0.3 m, G = 6, I_E = 0.1686 m,
V= 60km/hr = 16.66 m/s, R = 30 m (i) θ=? (ii) θ= 50^o V=?

(i) Angle of heel,

We have,

$$\boxed{\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta = mgh\sin\theta}$$

$$\therefore \frac{16.66^2}{30}\left[\frac{2x0.525 + 6x0.1685}{0.3} + 300\,x\,0.6\right]\cos\theta = 300x9.81x0.6x\sin\theta$$

$$\theta = 45°$$

(ii) Given, $\theta = 50°$, V=?,

$$\boxed{\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta = mgh\sin\theta}$$

$$\therefore \frac{V^2}{30}\left[\frac{2\,x\,0.525 + 6\,x\,0.1685}{0.3} + 300\,x\,0.6\right]\cos 50 = 300\,x\,9.81\,x\,0.6\,x\,\sin 50$$

$$\therefore V = 66\ Kmph$$

1.6.2 Stability of Four Wheeled Vehicle negotiating a turn.

Stable condition

Unstable Condition

Fig.81

Consider a four wheels automotive vehicle as shown in Figure 82. The engine is mounted at the rear with its crank shaft parallel to the rear axle. The centre of gravity of the vehicle lies vertically above the ground where total weight of the vehicle is assumed to be acted upon.

Let

m = *Mass of the vehicle* (kg)

W = *Weight of the vehicle* (N) = $m.g$,

h = *Height of the centre of* gravity of the vehicle (m)

r_W = *Radius of the wheels (m)*

R = *Radius of track or* curvature (m)

I_W = *Mass moment of inertia of each wheel (kg-m²)*

I_E = *Mass moment of inertia of the rotating parts of the engine (kg-m²)*

ω_W = Angular velocity of the wheels (rad/s) ω_E = Angular velocity of the engine (rad/s)

G = *Gear ratio* = ω_E / ω_W,

v = *Linear velocity* $_W \times r\ _W$, *of the vehicle (m/s)* = ω x = Wheel track (m)

b = Wheel base (m)

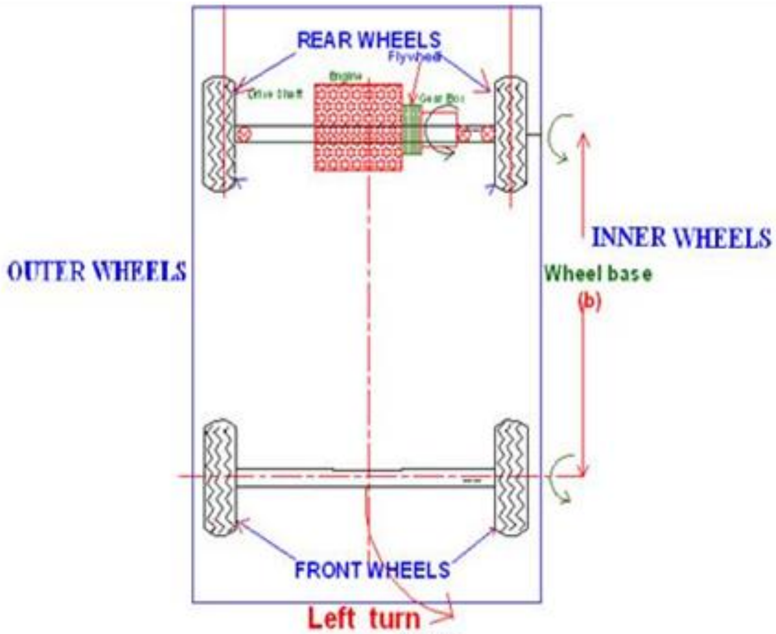


Fig.82

(i) **Reaction due to weight of Vehicle**

Weight of the vehicle. Assuming that weight of the vehicle *(mg)* is equally distributed over four wheels. Therefore, the force on each wheel acting downward is mg/4 and the reaction by the road surface on the wheel acts in upward direction.

$$R_w = \frac{mg}{4} \qquad 38$$

(ii) **Effect of Gyroscopic couple due to Wheel**

Gyroscopic couple due to four wheels is,

$$C_W = 4\, I_W \omega \omega_P$$

(iii) Effect of Gyroscopic Couple due to Engine

Gyroscopic couple due to rotating parts of the engine

$$C_E = I_E\, \omega \omega_P = I_E\, G\, \omega \omega_P$$

Therefore, total gyroscopic couple:

$$C_g = C_W + C_E = \omega \omega_P\, (4 I_W \pm I_E G)$$

When the wheels and rotating parts of the engine rotate in the same direction, then positive sign is used in the above equation. Otherwise negative sign should be considered.

Assuming that the vehicle takes a left turn, the reaction gyroscopic couple on the vehicle acts between outer and inner wheels.

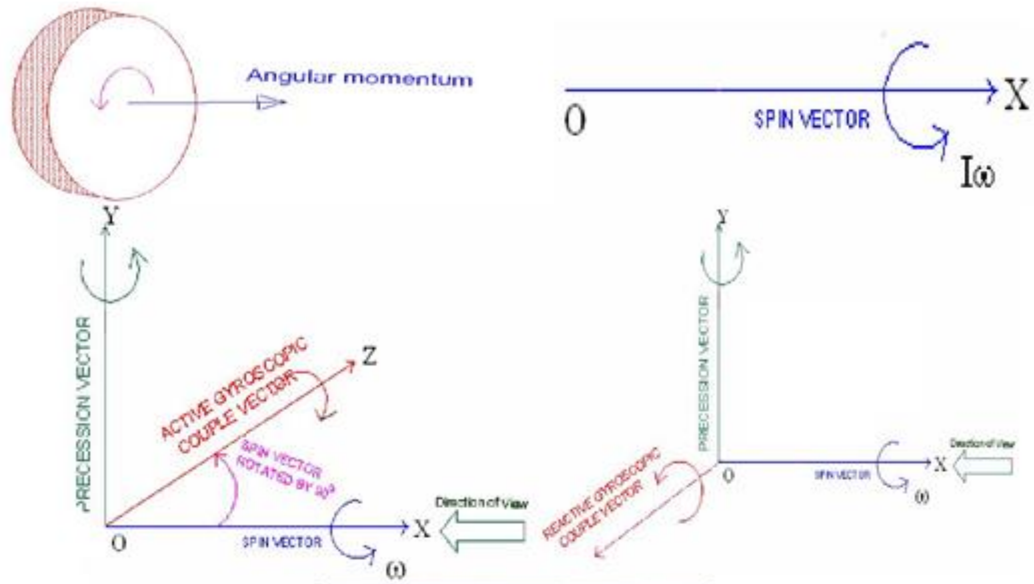


Fig.83

This gyroscopic couple tends to press the outer wheels and lift the inner wheels.

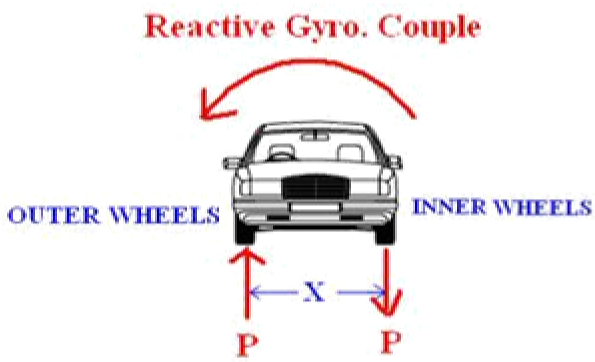


Fig.84

Due to the reactive gyroscopic couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be P Newtons, then,

$$\mathbf{P} \times X = C_g$$

$$\mathbf{P} = \frac{C_g}{x}$$

Road reaction on each outer/Inner wheel,

$$\frac{P}{2} = \frac{C_g}{2X}$$

(iii) **Effect of Centrifugal Couple**

When a vehicle moves on a curved path, a centrifugal force acts on the vehicle in outward direction through the centre of gravity of the vehicle (Fig)

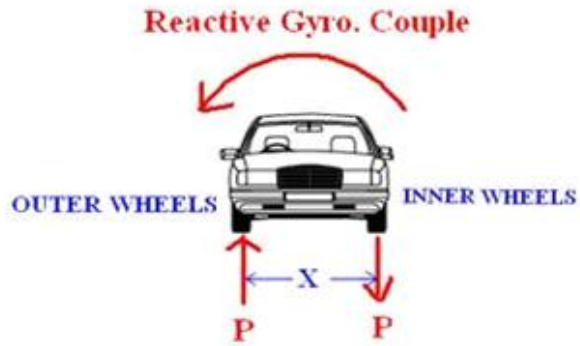


Fig.85

Centrifugal force,

$$F_c = m\omega_p^2 R = \frac{mv^2}{R}$$

This force forms a Centrifugal couple.

$$C_c = \frac{mv^2 h}{R}$$

This centrifugal couple tends to press the outer and lift the inner

Fig.86

Due to the centrifugal couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be F Newtons, then,

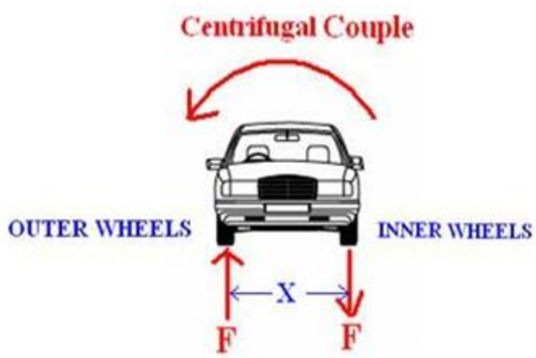


Fig.87

Road reaction on each outer/Inner wheel,

$$\frac{F}{2} = \frac{C_c}{2X}$$

The reactions on the outer/inner wheels are as follows,

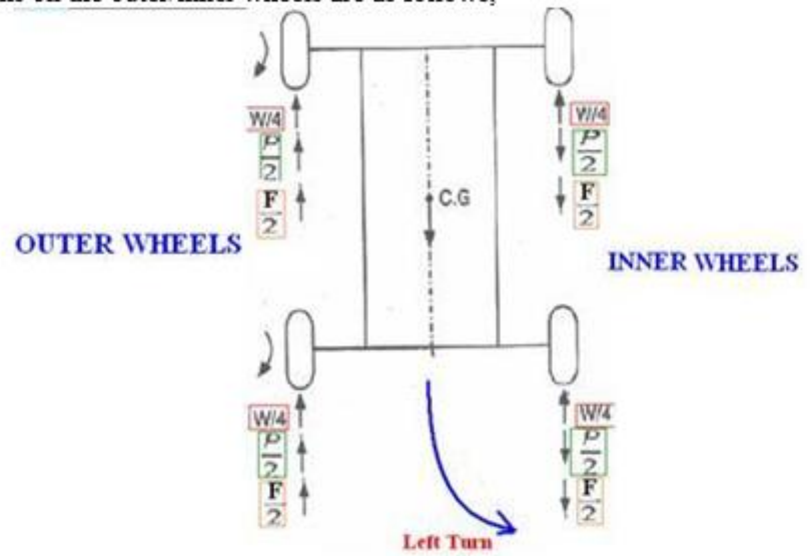


Fig.88

Total vertical reaction at each outer wheels

$$P_o = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

Total vertical reaction at each inner wheels

$$P_i = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

Problem 7

An automobile car is travelling along a track of 100 m mean radius. The moment of inertia of 500 mm diameter wheel is 1.8 kg m^2. The engine axis is parallel to the rear axle and crank shaft rotates in the same sense as the wheel. The moment of inertia of rotating parts of the engine is 1 kg m^2. The gear ratio is 4 and **the mass of** the vehicle is 1500 kg. If the centre of gravity of the vehicle is 450 mm above the road level and width of the track of the vehicle is 1.4 m, determine the limiting speed of the vehicle for condition that all four wheels maintain contact with the road surface.

Solution Let = limiting velocity of the vehicle.

Angular velocity: $\omega = \frac{v}{r} = \frac{v}{0.25}$ rad/s

Precession velocity: $\omega_p = \frac{v}{R} = \frac{v}{100}$ rad/s

(i) Reaction due to gyroscopic couple:

(a) Gyroscopic couple due to four wheels:

$$C_w = 4 I_w \omega \omega_p$$

$$= 4 \times 2 \times \frac{v}{0.25} \times \frac{v}{100} = 0.32\, v^2 \text{ Nm}$$

(b) Gyroscopic couple due to engine parts:

$$C_e = I_e G \omega \omega_p$$

$$= 1 \times 4 \times \frac{v}{0.25} \times \frac{v}{100} = 0.16\, v^2 \text{ Nm}$$

Reaction due to total gyroscopic couple on each outer wheel:

$$R_g = \frac{C_g}{2b} = \frac{0.48 v^2}{2 \times 1.5} = 0.16 v^2 \text{N} (\uparrow)$$

Reaction due to total gyroscopic couple on each inner wheel:

$$C_g = 0.16\ v^2 N\ (\downarrow)$$

(ii) Reaction due to centrifugal couple:

Centrifugal force: $$F_c = \frac{mv^2}{R} = \frac{1500 \times v^2}{100} = 15v^2\ \text{N}$$

Overturning couple due to centrifugal force:

$$C_c = F_c \times h$$

$$= 15\ v^2 \times 0.45 = 6.75\ v^2\ \text{Nm}$$

Vertical downward reaction on each inner wheel is:

$$R_c = \frac{C_c}{2b} = \frac{6.75v^2}{2 \times 1.5} = 2.25v^2\ \text{N}\ (\downarrow)$$

(iii) Reaction due to weight of the vehicle:

$$R_w = \frac{mg}{4} = \frac{1500 \times 9.81}{4} = 3678.75\ \text{N}\ (\uparrow)$$

The limiting condition to avoid lifting of inner wheels from the road surface is:

$$R_i = R_w - R_c - R_g > 0$$

Or

$$R_w > R_c + R_g$$

$$3678.75 \geq 2.25v^2 + 0.16\ v^2$$

Or

$$v = 39.07\ \text{m/s}\quad \text{or}\quad \mathbf{140.65\ kmph}$$

Force Analysis

Static Force Analysis

Introduction

A machine is a device that performs work and, as such, transmits energy by means mechanical force from a power source to a driven load. It is necessary in the design machine mechanisms to know the manner in which forces are transmitted from input to the output, so that the components of the machine can be properly size withstand the stresses that are developed. If the members are not designed to strong enough, then failure will occur during machine operation; if, on the other hand, the machine is over designed to have much more strength than required, then the machine may not be competitive with others in terms of cost, weight, size, power requirements, or other criteria. The bucket load and static weight loads may far exceed any dynamic loads due to accelerating masses, and a static-force analysis would be justified. An analysis that includes inertia effects is called a dynamic-force analysis and will be discussed in the next chapter. An example of an application where a dynamic-force analysis would be required is in the design of an automatic sewing machine, where, due to high operating speeds, the inertia forces may be greater than the external loads on the machine.

Another assumption deals with the rigidity of the machine components. No material is truly rigid, and all materials will experience significant deformation if the forces, either external or inertial in nature, are great enough. It will be assumed in this chapter and the next that deformations are so small as to be negligible and, therefore, the members will be treated as though they are rigid. The subject of mechanical vibrations, which is beyond the scope of this book, considers the flexibility of machine components and the resulting effects on machine behaviour. A third major assumption that is often made is that friction effects are negligible. Friction is inherent in all devices, and its degree is dependent upon many factors, including types of bearings, lubrication, loads, environmental conditions, and so on. Friction will be neglected in the first few sections of this chapter, with an introduction to the subject presented. In addition to assumptions of the types discussed above, other assumptions may be necessary, and some of these will be addressed at various points throughout the chapter.

The first part of this chapter is a review of general force analysis principles and will also establish some of the convention and terminology to be used in succeeding sections. The remainder of the chapter will then present both graphical and analytical methods for static-force analysis of machines.

Free-Body Diagrams:

Engineering experience has demonstrated the importance and usefulness of free-body diagrams in force analysis. A free-body diagram is a sketch or drawing of part or all of a system, isolated in order to determine the nature of forces acting on that body. Sometimes a free-body diagram may take the form of a mental picture; however, actual sketches are strongly recommended, especially for complex mechanical systems.

Generally, the first, and one of the most important, steps in a successful force analysis is the identification of the free bodies to be used. Figures 5.1B through 5.1E show examples of various free bodies that might be considered in the analysis of the four-bar linkage shown in Figure 5.1A. In Figure 5.1B, the free body consists of the three moving members isolated from the frame; here, the forces acting on the free body include a driving force or torque, external loads, and the forces transmitted:

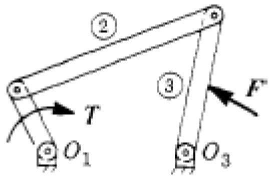


Figure 5.1(A) A four-bar linkage.

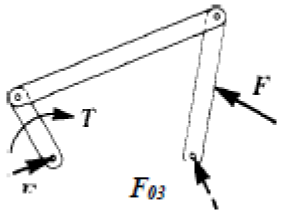


Figure 5.1(B) Free-body diagram of the three moving links

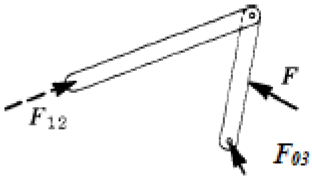


Figure 5.1(C) Free-body diagram of two connected links

Figure 5.1(D) Free-body diagram of a single link

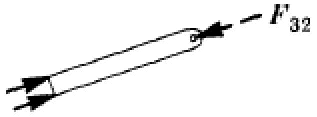


Figure 5.1(E) Free body diagram of part of a link.

► *5.1.2 Static Equilibrium:*

For a free body in static equilibrium, the vector sum of all forces acting on the body must be zero and the vector sum of all moments about any arbitrary point must also be zero. These conditions can be expressed mathematically as follows:

$$\sum F = 0 \qquad (5.1A)$$

$$\sum T = 0 \qquad (5.1B)$$

Since each of these vector equations represents three scalar equations, there are a total of six independent scalar conditions that must be satisfied for the general case of equilibrium under three-dimensional loading.

There are many situations where the loading is essentially planar; in which case, forces can be described by two-dimensional vectors. If the *xy* plane designates the plane of loading, then the applicable form of Eqs. 5.1A and 5.1B is:-

$$\sum F_x = 0 \qquad (5.2A)$$

$$\sum F_y = 0 \qquad (5.2B)$$

$$\sum T_z = 0 \qquad (5.2C)$$

Eqs. 5.2A to 5.2C are three scalar equations that state that, for the case of two-dimensional *xy* loading, the summations of forces in the *x* and *y* directions must individually equal zero and the summation of moments about any arbitrary point in the plane must also equal zero. The remainder of this chapter deals with two-dimensional force analysis. A common example of three-dimensional forces is gear forces.

5.1.3 Superposition:

The principle of superposition of forces is an extremely useful concept, particularly in graphical force analysis. Basically, the principle states that, for linear systems, the net effect of multiple loads on a system is equal to the superposition (i.e., vector summation) of the effects of the individual loads considered one at a time. Physically, linearity refers to a direct proportionality between input force and output force. Its mathematical characteristics will be discussed in the section on analytical force analysis. Generally, in the absence of Coulomb or dry friction, most mechanisms are linear for force analysis purposes, despite the fact that many of these mechanisms exhibit very nonlinear motions. Examples and further discussion in later sections will demonstrate the application of this principle.

5.1.4 Graphical Force Analysis:

Graphical force analysis employs scaled free-body diagrams and vector graphics in the determination of unknown machine forces. The graphical approach is best suited for planar force systems. Since forces are normally not constant during machine motion. Analyses may be required for a number of mechanism positions; however, in many cases, critical maximum-force positions can be identified and graphical analyses performed for these positions only. An important advantage of the graphical approach is that it provides useful insight as to the nature of the forces in the physical system.

This approach suffers from disadvantages related to accuracy and time. As is true of any graphical procedure, the results are susceptible to drawing and measurement errors. Further, a great amount of graphics time and effort can be expended in the iterative design of a machine mechanism for which fairly thorough knowledge of force-time relationships is required. In recent years, the physical insight of the graphics approach and the speed and accuracy inherent in the computer-based analytical approach have been brought together through computer graphics systems, which have proven to be very effective engineering design tools. There are a few special types of member loadings that are repeatedly encountered in the force analysis of mechanisms, These include a member subjected to two forces, a member subjected to three forces, and a member subjected to two forces and a couple. These special cases will be considered in the following paragraphs, before proceeding to the graphical analysis of complete mechanisms.

► *5.2.1 Analysis of a Two-Force Member:*

A member subjected to two forces is in equilibrium if and only if the two forces (1) have the same magnitude, (2) act along the same line, and (3) are opposite in sense. Figure 5.2A shows a free-body diagram of a member acted upon by forces F_1 and F_2 where the points of application of these forces are points A and B. For equilibrium the directions of F_1 and F_2 must be along line AB and F_1 must equal $-F_2$ graphical vector addition of forces F_1 and F_2 is shown in Figure 5.2B, and, obviously, the resultant net force on the member is zero when $F_1 = -F_2$. The resultant moment about any point will also be zero.

Thus, if the load application points for a two-force member are known, the line of action of the forces is defined, and it the magnitude and sense of one of the forces are known, then the other force can immediately be determined. Such a member will either be in tension or compression.

Figure 5.2(A) A two-force member. The resultant force and the resultant moment both equal Zero.

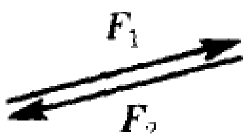


Figure 5.2(B) Force summation for a two-force member

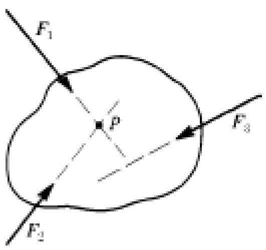
► *5.2.2 Analysis of a Three-Force Member:*

A member subjected to three forces is in equilibrium if and only if (1) the resultant of the three forces is zero, and (2) the lines of action of the forces all intersect at the same point. The first condition guarantees equilibrium of forces, while the second condition guarantees equilibrium of moments. The second condition can be under-stood by considering the case when it is not satisfied. See Figure 5.3A. If moments are summed about point *P*, the intersection of forces F_1 and F_2, then the moments of these forces will be zero, but F_3 will produce a nonzero moment, resulting in a nonzero net moment on the member. On the other hand, if the line of action of force F_3 also passes through point *P* (Figure 5.3B), the net moment will be zero. This common point of intersection of the three forces is called the point of concurrency.

A typical situation encountered is that when one of the forces, F_1, is known completely, magnitude and direction, a second force, F_2, has known direction but unknown magnitude, and force F_3 has unknown magnitude and direction. The graphical solution of this case is depicted in Figures 5.4A through 5.4C. First, the free-body diagram is drawn to a convenient scale and the points of application of the three forces are identified. These are points *A, B,* and *C*. Next, the known force F_1 is drawn on the diagram with the proper direction and a suitable magnitude scale. The direction of force F_2 is then drawn, and the intersection of this line with an extension of the line of action of force F_1 is the concurrency point *P*. For equilibrium, the line of action of force F_3 must pass through points *C* and *P* and is therefore as shown in Figure 5.4A.

The force equilibrium condition states that

$$F_1 + F_2 + F_3 = 0$$



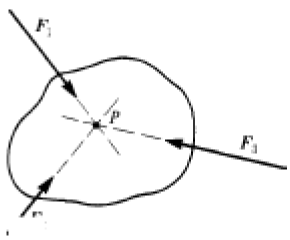


Figure 5.3(B) The three forces intersect at the same point *P*, called the *concurrency point,* and the net moment is zero.

Figure 5.3(A) The three forces on the member do not intersect at a common point and there is a nonzero resultant moment.

Since the directions of all three forces are now known and the magnitude of F_1 were given, this equation can be solved for the remaining two magnitudes. A graphical Solution follows from the fact that the three forces must form a closed vector loop, called a force polygon. The procedure is shown in Figure 5.4B. Vector ${}_1F$ is redrawn. From the head of this vector, a line is drawn in the direction of force F_2, and from the tail, a line is drawn parallel to F_3. The intersection of these lines closes the vector loop and determines the magnitudes of forces ${}_2F$ and F_3. Note that the same solution is obtained if, instead, a line parallel to ${}_3F$ is drawn from the head of F_1, and a line parallel to F_2 is drawn from the tail of F_1. See Figure 5.4C.

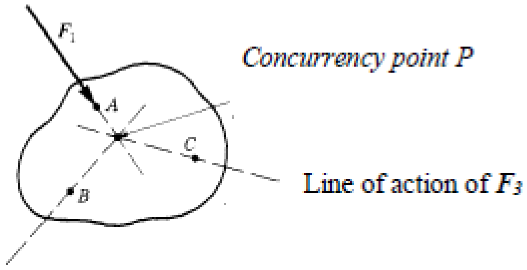


Figure 5.4(A) Graphical force analysis of a three- force member.

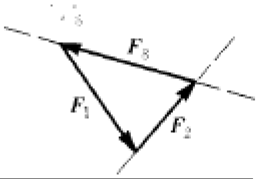


Figure 5.4(B) Force polygon for the three forces member.

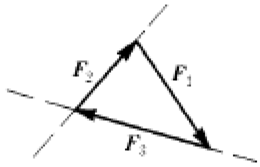


Figure 5.4(C) An equivalent force polygon for the three force member

This is so because vector addition is commutative, and, therefore, both force polygons are equivalent to the vector equation above. It is important to remember that, by the definition of vector addition, the force polygon corresponding to the general force equation

$$\sum F = 0$$

Will have adjacent vectors connected head to tail. This principle is used in identifying the sense of forces F_2 and F_3 in Figures 5.4B and 5.4C. Also, if the lines of action of F_1 and F_2 are parallel," then the point of concurrency is at infinity, and the third force F_3 must be parallel to the other two. In this case, the force polygon collapses to a straight line.

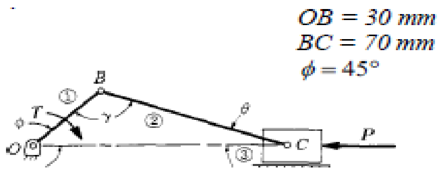
► 5.3.1 Graphical Force Analysis of the Slider Crank Mechanism:

The slider crank mechanism finds extensive application in reciprocating compressors, piston engines, presses, toggle devices, and other machines where force characteristics are important. The force analysis of this mechanism employs most of the principles described in previous sections, as demonstrated by the following example.

▼ EXAMPLE 5.1

Static-force analysis of a slider crank mechanism is discussed. Consider the slider crank linkage shown in Figure 5.5A, representing a compressor, which is operating at so low a speed that inertia effects are negligible. It is also assumed that gravity forces are small compared with other forces and that all forces lie in the same plane. The dimensions are OB = 30 mm and BC = 70 mm, we wish to find the required crankshaft torque T and the bearing forces for a total gas pressure force P = 40N at the instant when the crank angle $\phi = 45°$.

Figure 5.5(A) Graphical force analysis of a slider crank mechanism, which is acted on by piston force ***P*** and crank torque ***T***



SOLUTION

The graphical analysis is shown in Figure 5.5B. First, consider connecting rod 2. In the absence of gravity and inertia forces, this link is acted on by two forces only, at pins B and C. These pins are assumed to be frictionless and, therefore, transmit no torque. Thus, link 2 is a two-force member loaded at each end as shown. The forces F_{12} and F_{32} lie along the link, producing zero net moment, and must be equal and opposite for equilibrium of the link. At this point, the magnitude and sense of these forces are unknown.

Next, examine piston 3, which is a three-force member. The pressure force *P is* completely known and is assumed to act through the center of the piston (i.e., the pressure distribution on the piston face is assumed to be symmetric). From Newton's third law, which states that for every action there is an equal and opposite reaction, it follows that $F_{23} = -F_{32}$, and the direction of F_{23} is therefore known. In the absence of friction, the force of the cylinder on the piston, F_{03}, is perpendicular to the cylinder wall, and it also must pass through the concurrency point, which is the piston pin *C*. Now, knowing the force directions, we can construct the force polygon for member 3 (Figure 5.5B). Scaling from this diagram, the contact force between the cylinder and piston is $F_{03} = 12.70N$, acting upward, and the magnitude of the bearing force at *C* is $F_{23} = F_{32} = 42.0N$. This is also the bearing force at crankpin *B*, because $F_{12} = -F_{32}$. Further, the force directions for the connecting rod shown in the figure are correct, and the link is in compression.

Finally, crank 1 is subjected to two forces and a couple *T* (the shaft torque *T* is assumed to be a couple). The force at *B* is $F_{12} = -F_{21}$ and is now known. For force equilibrium, $F_{01} = -F_{21}$ as shown on the free-body diagram of link 1. However these forces are not collinear, and for equilibrium, the moment of this couple must be balanced by torque *T*. Thus, the required torque is clockwise and has magnitude

$$T = F_{21}h = (42.0N)(26.6mm) = 1120N.mm = 1.120N.m$$

It should be emphasized that this is the torque required for static equilibrium in the position shown in Figure 9.10A. If torque information is needed for a complete compression cycle, then the analysis must be repeated at other crank positions throughout the cycle. In general, the torque will vary with position.

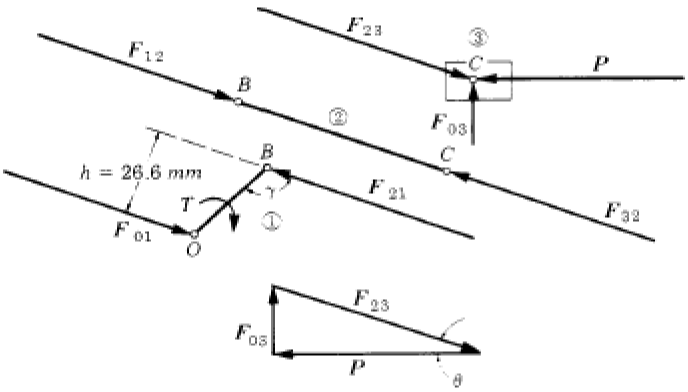


Figure 5.5(B) Static force balances for the three moving links, each considered as a free body

► *5.3.1 Graphical Force Analysis of the Four-Bar Linkage:*

The force analysis of the four-bar linkage proceeds in much the same manner as that of the slider crank mechanism. However, in the following example, we will consider the case of external forces on both the coupler and follower links and will utilize the principle of superposition.

▼ EXAMPLE 5.2

Static-force analysis of a four-bar linkage is considered. The link lengths for the four-bar linkage of Figure 5.6 A are given in the figure. In the position shown, coupler link 2 is subjected to force F_2 of magnitude *47 N*, and follower link 3 is subjected to force F_3, of magnitude *30 N*. Determine the shaft torque ***Ti*** on input link1 and the bearing loads for static equilibrium.

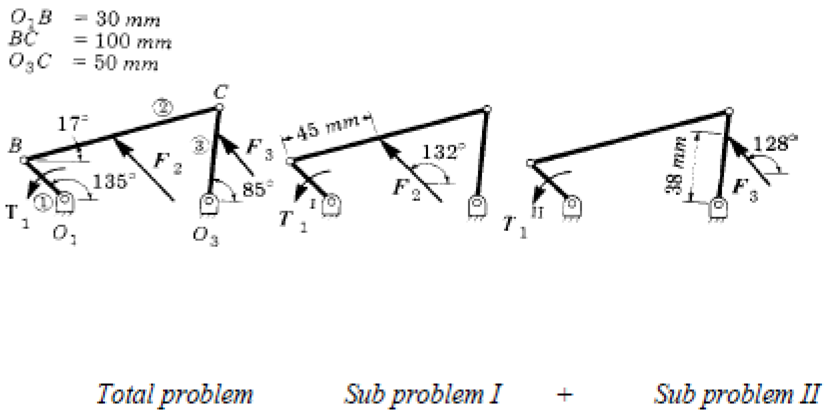


Figure 5.6(A) Graphical force analysis of a four-bar linkage, utilizing the principle of the superposition

SOLUTION

As shown in Figure 5.6A, the solution of the stated problem can be obtained by superposition of the solutions of sub problems *I* and *II*. In sub problem *I*, force F_3 is neglected, and in sub problem *II*, force F_2 is neglected. This process facilitates the solution by dividing a more difficult problem into two simpler ones.

The analysis of sub problem *I* is shown in Figure 5.6B, with quantities designated by superscript *I*. Here, member 3 is a two-force member because force F_3 is neglected. The direction of forces F_{23}^1 and F_{03}^1 are as shown, and the forces are equal and opposite (note that the magnitude and sense of these forces are as yet unknown), This information allows the analysis of member 2, which is a three-force member with completely known force F_2, known direction for F_{32}^1, and, using the concurrency point, known direction for F_{12}^1. Scaling from the force polygon, the following force magnitudes are determined (the force directions are shown in Figure (5.6B):

$$F_{32}^1 = F_{23}^1 = F_{03}^1 = 21.0N \qquad F_{12}^1 = F_{21}^1 = 36N$$

Link 1 is subjected to two forces and couple $T_1^{\ 1}$, and for equilibrium,

$$F_{03}^{11} = 29.0N \qquad F_{23}^{11} = F_{21}^{11} = F_{01}^{11}$$

And; $T_1^{\ 1} = F_{21}^1 h^1 = (36N)(11mm) = 396N.mm \quad CW$

The analysis of sub problem *II* is very similar and is shown in Figure 5.6C, where superscript II is used. In this case, link 2 is a two-force member and link 3 is a three-force member, and the following results are obtained:

$$F_{03}^{11} = 29N \qquad F_{23}^{11} = F_{21}^{11} = F_{01}^{11} = 17N$$

And; $T_1^{\ 11} = F_{21}^{11} h^{11} = (17N)(26mm) = 442N.mm \quad CW$

The superposition of the results of Figures 5.6B and 5.6C is shown in Figure 5.6D. The results must be added vectorially, as shown. By scaling from the free-body diagrams, the overall bearing force magnitudes are

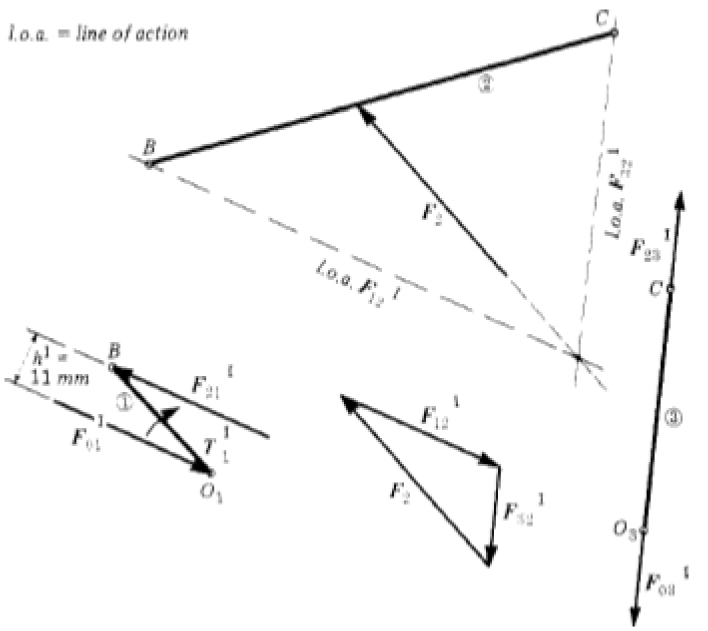


Figure 5.6B The solution of *sub problem I*

$F_{01} = 50N \qquad F_{23} = 31N$

$F_{12} = 50N \qquad F_{03} = 49N$

And the net crankshaft torque is

$$T_1 = T_1^{1} + T_1^{11} = 396N.mm + 442N.mm = 838N.mm \qquad CW$$

The directions of the bearing forces are as shown in the figure. These resultant quantities represent the actual forces experienced by the mechanism. It can be seen from the analysis that the effect of the superposition principle, in this example, was to create sub problems containing two-force members, from which the separate analyses could begin. In an attempt of a graphical analysis of the original problem without superposition, there is not enough intuitive force information to analyze three-force members 2 and 3, because none of the bearing force directions can be determined by inspection.

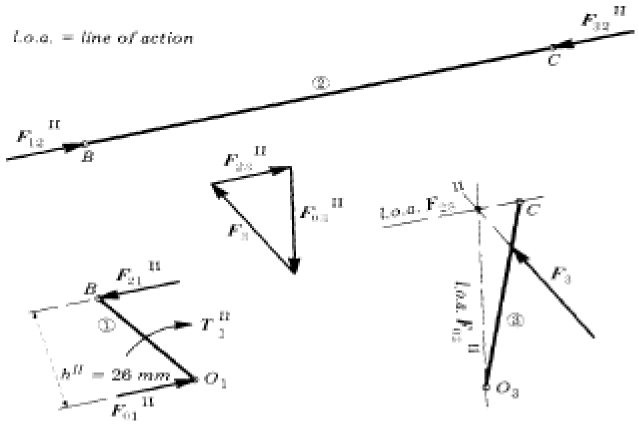


Figure 5.6C The solution of *sub problem II*

Dynamic Force Analysis

► *5.4.1 D'Alembert's Principle and Inertia Forces:*

An important principle, known as d'Alembert's principle, can be derived from Newton's second law. In words, d'Alembert's principle states that *the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium*.

$$F + (-ma_G) = 0 \qquad (5.3A)$$

$$T_{eG} + (-I_G \alpha) = 0 \qquad (5.3B)$$

The terms in parentheses in Eqs. 5.3A and 5.3B are called the reverse-effective force and the reverse-effective torque, respectively. These quantities are also referred to as inertia force and inertia torque. Thus, we define the inertia force F_i as

$$F_i = -ma_G \qquad (5.4A)$$

This reflects the fact that a body resists any change in its velocity by an inertia force proportional to the mass of the body and its acceleration. The inertia force acts through the center of mass G of the body. The inertia torque or inertia couple C_i is given by:

$$C_i = -I_G \alpha \qquad (5.4B)$$

As indicated, the inertia torque is a pure torque or couple. From Eqs. 5.4A and 5.4B, their directions are opposite to that of the accelerations. Substitution of Eqs. 5.4A and 5.4B into Eqs, 5.3A and 5.3B leads to equations that are similar to those used for static-force analysis:

$$\sum F = \sum F_e + F_i = 0 \qquad (5.5A)$$

$$\sum T_G = \sum T_{eG} + C_i = 0 \qquad (5.5B)$$

Where $\sum F$ refers here to the summation of external forces and, therefore, is the resultant external force, and $\sum T_{eG}$ is the summation of external moments, or resultant external moment, about the center of mass G. Thus, the dynamic analysis problem is reduced in form to a static force and moment balance where inertia effects are treated in the same manner as external forces and torques. In particular for the case of assumed mechanism motion, the inertia forces and couples can be determined completely and thereafter treated as known mechanism loads.

Furthermore, d'Alembert's principle facilitates moment summation about any arbitrary point P in the body, if we remember that the moment due to inertia force F_i must be included in the summation. Hence,

$$\sum T_P = \sum T_{eP} + C_i + R_{PG} \times F_i = 0 \qquad (5.5C)$$

Where; $\sum T_P$ is the summation of moments, including inertia moments, about point P. $\sum T_{eP}$ is the summation of external moments about P, C_i is the inertia couple defined by Eq. 5.4B, F_i is the inertia force defined by Eq. 5.4A, and R_{PG} is a vector from point P to point C. It is clear that Eq. 5.5B is the special case of Eq.5.5C, where point P is taken as the center of mass G (i.e., $R_{PG} = 0$).

For a body in plane motion in the xy plane with all external forces in that plane. Eqs. 5.5A and 5.5B become:

$$\sum F_x = \sum F_{ex} + F_{ix} = \sum F_{ex} + (-ma_{Gx}) = 0 \qquad (5.6A)$$

$$\sum F_y = \sum F_{ey} + F_{iy} = \sum F_{ey} + (-ma_{Gy}) = 0 \qquad (5.6B)$$

$$\sum T_G = \sum T_{eG} + C_i = \sum T_{eG} + (-I_G \alpha) = 0 \qquad (5.6C)$$

Where a_{Gx} and a_{Gy} are the x and y components of a_G. These are three scalar equations, where the sign convention for torques and angular accelerations is based on a right-hand xyz coordinate system; that is. Counterclockwise is positive and clockwise is negative. The general moment summation about arbitrary point P, Eq. 5.5C, becomes:

$$\begin{aligned} \sum T_P &= \sum T_{eP} + C_i + R_{PGx} F_{iy} - R_{PGy} F_{ix} \\ &= \sum T_{eP} + (-I_G \alpha) + R_{PGx}(-ma_{Gy}) - R_{PGy}(-ma_{Gx}) = 0 \end{aligned} \qquad (5.6D)$$

Where R_{PGx} and R_{PGy} are the x and y components of position vector R_{PG}. This expression for dynamic moment equilibrium will be useful in the analyses to be presented in the following sections of this chapter.

► *5.4.2 Equivalent Offset Inertia Force:*

For purposes of graphical plane force analysis, it is convenient to define what is known as the equivalent offset inertia force. This is a single force that accounts for both translational inertia and rotational inertia corresponding to the plane motion of a rigid body. Its derivation will follow, with reference to Figures 5.7A through 5.7D.

Figure 5.7A shows a rigid body with planar motion represented by center of mass acceleration a_C and angular acceleration α. The inertia force and inertia torque associated with this motion are also shown. The inertia torque $-I_G \alpha$ can be expressed as a couple consisting of forces Q and (- Q) separated by perpendicular

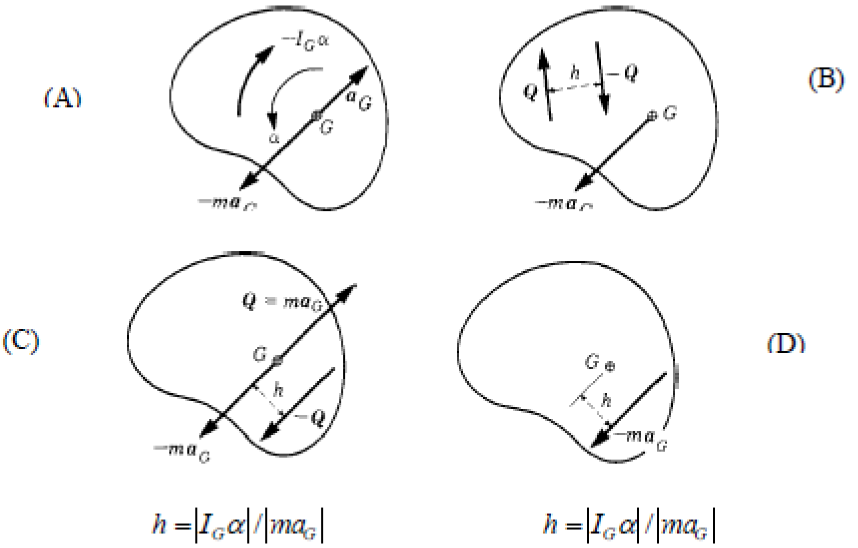


Figure 5.7 (A) Derivation of the equivalent offset inertia force associated with planer motion of a rigid body. (B) Replacement of the inertia torque by a couple. (C) The strategic choice of a couple. (D) The single force is equivalent to the combination of a force and a torque in figure 5.7(A)

Distance h, as shown in Figure 5.7B. The necessary conditions for the couple to be equivalent to the inertia torque are that the sense and magnitude be the same. Therefore, in this case, the sense of the couple must be clockwise and the magnitudes of Q and h must satisfy the relationship

$$|Q.h| = |I_G.\alpha|$$

Otherwise, the couple is arbitrary and there are an infinite number of possibilities that will work. Furthermore, the couple can be placed anywhere in the plane.

Figure 5.7C shows a special case of the couple, where force vector Q is equal to ma_G and acts through the center of mass. Force $(-Q)$ must then be placed as shown to produce a clockwise sense and at a distance;

$$h = \frac{|I_G \alpha|}{|Q|} = \frac{|I_G \alpha|}{|ma_G|} \qquad (5.7)$$

Force Q will cancel with the inertia force $F_i = -ma_G$, leaving the single equivalent offset force shown in Figure 5.7D, which has the following characteristics:

1. The magnitude of the force is $|ma_G|$.
2. The direction of the force is opposite to that of acceleration α.
3. The perpendicular offset distance from the center of mass to the line of action of the force is given by Eq. 5.7.
4. The force is offset from the center of mass so as to produce a moment about the center of mass that is opposite in sense to acceleration a.

The usefulness of this approach for graphical force analysis will be demonstrated in the following section. It should be emphasized, however, that this approach is usually unnecessary in analytical solutions, where Eqs. 5.6A to 5.6D. Including the original inertia force and inertia torque, can be applied directly.

► *5.4.3 Dynamic Analysis of the Four-Bar Linkage:*

The analysis of a four-bar linkage will effectively illustrate most of the ideas that have been presented; furthermore, the extension to other mechanism types should become clear from the analysis of this mechanism.

▼ EXAMPLE 5.3

The four-bar linkage shown in Figure 5.8A has the dimensions shown in the figure where G refers to center of mass, and the mechanism has the following mass properties:

$$m_1 = 0.10kg \qquad I_{G1} = 20kg.mm^2$$
$$m_2 = 0.20kg \qquad I_{G2} = 400kg.mm^2$$
$$m_3 = 0.30kg \qquad I_{G3} = 20kg.mm^2$$

Determine the instantaneous value of drive torque T required to produce an assumed motion given by input angular velocity $\omega = 95 rad/s$ counterclockwise and input angular acceleration $a_1 = 0$ for the position shown in the figure. Neglect gravity and friction effects.

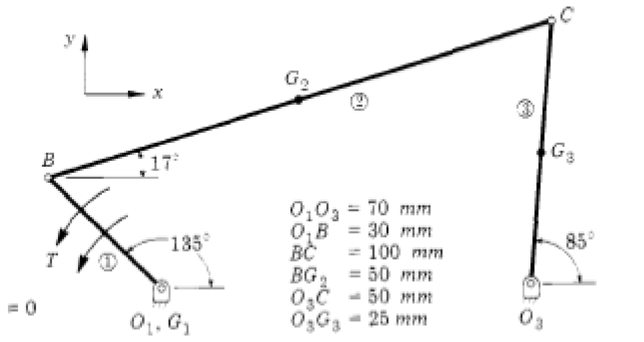


Figure 5.8(A) The four-bar linkage of Example 5.3

SOLUTION

This problem falls in the first analysis category that is given the mechanism motion, determine the resulting bearing forces and the necessary input torque. Therefore, the first step in the solution process is to determine the inertia forces and inertia torques. Thereafter, the problem can be treated as though it were a static-force analysis problem.

Kinematics analysis of the mechanism can be accomplished by using any of the methods presented in earlier chapters. Figure 5.8B shows a graphical analysis employing velocity and acceleration polygons. From the analysis, the following accelerations are determined:

$$a_{C1} = 0(Stationary\ Center\ of\ mass) \qquad \alpha_1 = 0(given)$$

$$a_{C2} = 235{,}000\angle 312°mm/Sec^2 \qquad \alpha_2 = 520 rad/s^2 \quad ccw$$

$$a_{C3} = 235{,}000\angle 308°mm/Sec^2 \qquad \alpha_3 = 2740 rad/s^2 \quad cw$$

Where the angles of the acceleration vectors are measured counterclockwise from the positive x direction shown in Figure 5.8A. From Eqs. 5.4A and 5.4B, the inertia forces and inertia torques are;

$$F_{i1} = 0$$

$$F_{i2} = -m_2 a_{G2} = 47{,}000\angle 132° kg.mm/s^2 = 47\angle 132° N$$

$$F_{i3} = -m_3 a_{G3} = 30{,}000\angle 128° kg.mm/s^2 = 30\angle 132° N$$

$$C_{i1} = 0$$

$$C_{i2} = -I_{G2}\alpha_2 = 208{,}000 kg.mm^2/s^2 cw = 208 N.mm \quad cw$$

$$C_{i3} = -I_{G3}\alpha_3 = 274{,}000 kg.mm^2/s^2 ccw = 274 N.mm \quad ccw$$

The inertia forces have lines of action through the respective centers of mass, and the inertia torqueses are pure couples.

The inertia forces have lines of action through the respective centres of mass, and the inertia torques are pure couples.

Velocity polygon

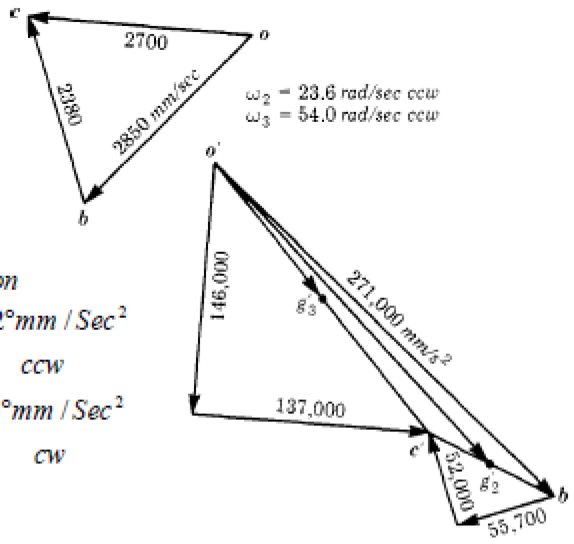


Figure 5.8(B) the velocity and acceleration analysis necessary for determination of inertia forces and inertia

Acceleration polygon

$a_{G2} = 235,000\angle 312° mm/Sec^2$

$\alpha_2 = 520 rad/Sec \quad ccw$

$a_{G3} = 100,000\angle 308° mm/Sec^2$

$\alpha = 2740 rad/Sec \quad cw$

GRAPHICAL SOLUTION

In order to simplify the graphical force analysis, we will account for the inertia torques by introducing equivalent offset inertia forces. These forces are shown in Figure 2.8C, and their placement is determined according to the previous section. For link 2, the offset force F_2 is equal and parallel to inertia force F_{12}. Therefore,

$$F_2 = 47\angle 132° N$$

It is offset from the center of mass G_2 by a perpendicular amount equal to

$$h_2 = \frac{|I_{G2}\alpha_2|}{|m_2 a_{G2}|} = \frac{208}{47} = 4.43 mm$$

And this offset is measured to the left as shown to produce the required clockwise direction for the inertia moment about point G_2. In a similar manner, the equivalent offset inertia force for link 3 is

$$F_3 = 30\angle 128° N \quad \text{at an offset distance} \quad h_3 = \frac{|I_{G3}\alpha_3|}{|m_3 a_{G3}|} = \frac{274}{30} = 9.13 mm$$

Where this offset is measured to the right from G_3 to produce the necessary counterclockwise inertia moment about G_3. From the values of h_2 and h_3 and the angular relationships, the force positions r_2 and r_3 in Figure 5.8C are computed to be

$$r_2 = BG_2 - \frac{h_2}{\cos(132° - 17° - 90°)} = 45.10 mm$$

$$r_3 = O_3G_3 + \frac{h_3}{\cos(90° + 85° - 128°)} = 38.40 mm$$

Now, we wish to perform a graphical force analysis for known forces F_2 and F_3. This has been done in Example Problem 9.2, and the reader is referred to that

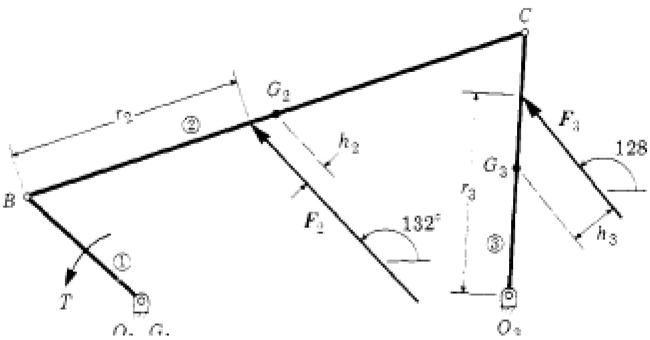


Figure 5.8(C) Equivalent offset inertia forces for members 2 and 3

Analysis. The required input torque was found to be $T = 383 N.mm$ *cw*

ANALYTICAL SOLUTION

Having determined the equivalent offset inertia forces F_2 and F_3 the analytical solution could proceed according to Example Problem 9, 6, which examined the same problem. However, it is not necessary to convert to the offset force, and here we will carry out the analytical solution in terms of the original inertia forces and inertia couples.

Figure 5.8D shows the linkage with the inertia torques and the inertia forces in *xy* coordinate form. Consistent with Figure 9.15A, we define the following quantities:

$$\ell_1 = 30mm \qquad \ell_2 = 100mm \qquad \ell_3 = 50mm$$

$$\phi_1 = 135° \qquad \phi_2 = 17° \qquad \phi_3 = 85°$$

$$r_1 = 0 \qquad r_2 = 50mm \qquad r_3 = 25mm$$

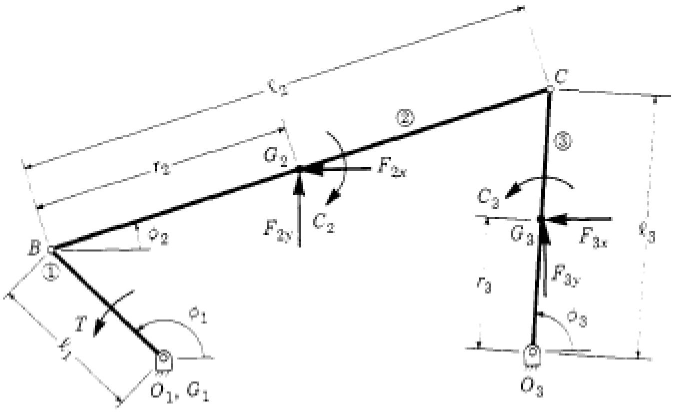
$$F_{2x} = 47\cos(132°) = -31.40N \qquad F_{2y} = 47\sin(132°) = 34.90N$$

$$F_{3x} = 30\cos(128°) = -18.50N \qquad F_{3y} = 30\sin(128°) = 23.60N$$

$$C_2 = -208N.mm \qquad C_3 = 274N.mm$$

$$F_{1x} = F_{1y} = C_1 = 0$$

Figure 5.8(D) Combinations of inertia forces and inertia torques for members 2 and 3



Where the differences are due to round off:

$$a_{11} = -49.8 \qquad a_{21} = 29.2 \qquad b_1 = -786$$

$$a_{12} = 4.36 \qquad a_{22} - 95.6 \qquad b_2 = -1920$$

Then,

$$F_{23} = 31.30N \qquad F_{12} = 50.30N$$

$$F_{03} = 49.20N \qquad F_{01} = 50.30N$$

And $\qquad T = -851N.mm$

Thus, it can be seen that the general analytical solution of the four-bar linkage presented in this Chapter for static-force analysis is equally well suited for dynamic-force analysis. Before leaving this example, a couple of general comments should be made.

First, the torque determined is the instantaneous value required for the prescribed motion, and the value will vary with position. Furthermore, for the position considered, the torque is opposite in direction to the angular velocity of the crank. This can be explained by the fact that the inertia of the mechanism in this position is tending to accelerate the crank in the counterclockwise direction, and, therefore, the required torque must be clockwise to maintain a constant angular speed. If a constant speed is to be maintained throughout the mechanism cycle, then there will be other positions of the mechanism for which the required torque will be counterclockwise. The second comment is that it may be impossible to find a mechanism actuator, such as an electric motor, that will supply the required torque versus position behavior. This problem can be alleviated, however, in the case of a "constant" rotational speed mechanism through the use of a device called a flywheel, which is mounted on the input shaft and produces a relatively large mass moment of inertia for crank 1. The flywheel can absorb mechanism torque and energy- variations with minima] speed fluctuation and. thus, maintains an essentially constant input speed. In such a case. The assumed-motion approach to dynamic-force analysis is appropriate.

► *5.4.3 Dynamic Analysis of the Slider-Crank Mechanism:*

Dynamic forces are a very important consideration in the design of slider crank mechanisms for use in machines such as internal combustion engines and reciprocating compressors. Dynamic-force analysis of this mechanism can be carried out in exactly the same manner as for the four-bar linkage in the previous section. Following such a process a kinematics analysis is first performed from which expressions are developed for the inertia force and inertia torque for each of the moving members, These quantities may then be converted to equivalent offset inertia forces for graphical analysis or they may be retained in the form of forces and torques for analytical solution, utilizing, in either case, the methods presented in this chapter. In fact, the analysis of the slider crank mechanism is somewhat easier than that of the four-bar linkage because there is no rotational motion and, in turn, no inertia torque for the piston or slider, which has translating motion only. The following paragraphs will describe an analytical approach in detail.

Figure 5.9A is a schematic diagram of a slider crank mechanism, showing the crank 1, the connecting rod 2, and the piston 3, all of which are assumed to be rigid. The center of mass locations are designated by letter **G**, and the members have masses m, and moments of inertia I_{Gi}, $i = 1, 2, 3$. The following analysis will consider the relationships of the inertia forces and torques to the bearing reactions and the drive torque on the crank, at an arbitrary mechanism position given by crank angle ϕ Friction will be neglected.

Figure 5.9B shows free-body diagrams of the three moving members of the linkage. Applying the dynamic equilibrium conditions. Eqs. 5.6A to 5.6D, to each member yields the following set of equations. For the piston (moment equation not included):

$$F_{23x} + (-m_3 a_{G3}) = 0 \qquad (5.8A)$$

$$F_{03y} + F_{23y} = 0 \qquad (5.8B)$$

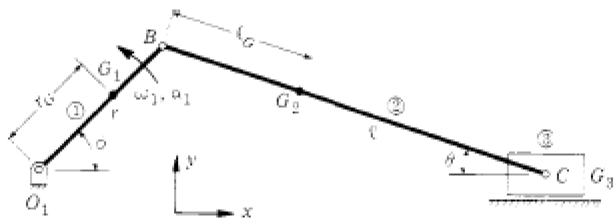


Figure 5.9(A) Dynamic-force analysis of a slider crank mechanism

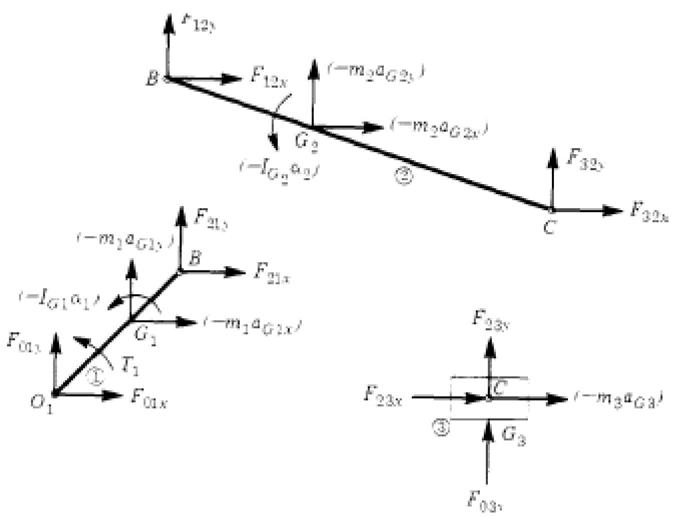


Figure 5.9(B) Free-body diagrams of the moving members

For the connecting rod (moments about point *B*):

$$F_{12x} + F_{32x} + (-m_2 a_{G2x}) = 0 \quad (5.8C)$$

$$F_{12y} + F_{32y} + (-m_2 a_{G2y}) = 0 \quad (5.8D)$$

$$F_{32x}\ell \sin\theta + F_{32y}\ell\cos\theta + (-m_2 a_{G2x})\ell_G \sin\theta + (-m_2 a_{G2y})\ell_G \cos\theta + (-I_{G2}\alpha_2) = 0 \quad (5.8E)$$

For the crank (moments about point O_1):

$$F_{01x} + F_{21x} + (-m_1 a_{G1x}) = 0 \quad (5.8F)$$

$$F_{01y} + F_{21y} + (-m_1 a_{G1y}) = 0 \quad (5.8G)$$

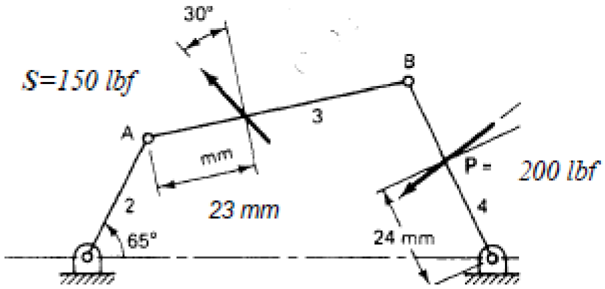
$$T_1 - F_{21x} r \sin\phi + F_{21y} r\cos\phi + (-m_1 a_{G1x}) r_G \sin\phi + (-m_1 a_{G1y}) r_G \cos\phi + (-I_{G1}\alpha_1) = 0 \quad (5.8H)$$

Where *T* is the input torque on the crank. This set of equations embodies both of the dynamic-force analysis approaches described in Newton's Laws. However, its form is best suited for the case of known mechanism motion, as illustrated by the following example.

Question 1:

The four-bar mechanism of Figure has one external force ***P = 200 Ibf*** and one inertia force ***S = 150 Ibf*** acting on it. The system is in dynamic equilibrium as a result of torque T_2 applied to link 2. *Find T_2* and *the pin forces*.

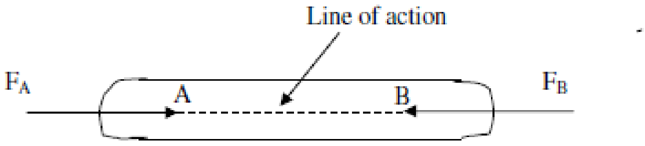
(a) Use the graphical method based on free-body diagrams.



O_2A = 30 mm
AB = 60 mm
O_4B = 45 mm
O_2O_4 = 90 mm

Very useful & important principles.

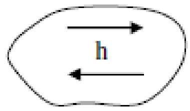
(i) Equilibrium of a body under the action of two forces only (no torque)



For body to the in Equilibrium under the action of 2 forces (only), the two forces must the equal opposite and collinear. The forces must be acting along the line joining A&B.

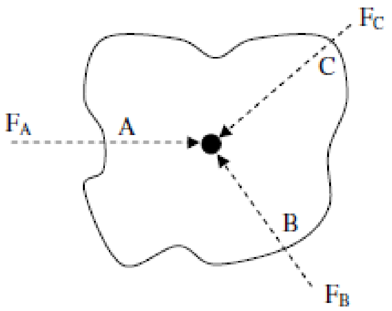
That is,

$F_A = -F_B$ (for equilibrium)



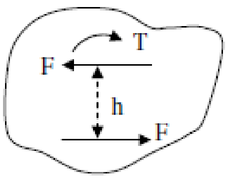
If this body is to be under equilibrium 'h' should tend to zero

(ii) Equilibrium of a body under the action of three forces only (no torque / couple)



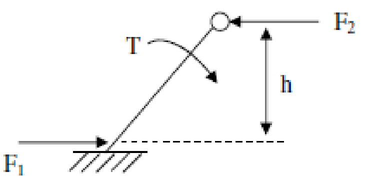
For equilibrium, the 3 forces must be concurrent and the force polygon will be a triangle.

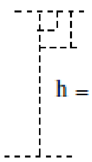
(iii) Equilibrium of a body acted upon by 2 forces and a torque.



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

Example:





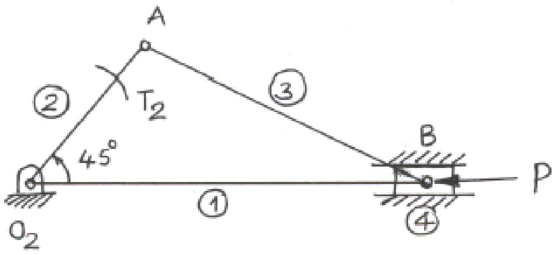
h = Perpendicular distance between F_1 & F_2

Free body diagram

The mass is separated from the system and all the forces acting on the mass are represented.

Problem No.1: Slider crank mechanism

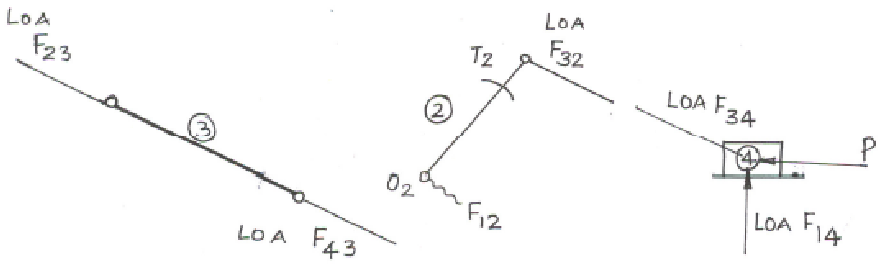
Figure shows a slider crank mechanism in which the resultant gas pressure 8 x 10^4 Nm^{-2} acts on the piston of cross sectional area 0.1 m^2. The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O_2. Determine forces acting on all the links (including the pins) and the couple on 2.



$$P = (8 \times 10^4) \times (0.1)$$

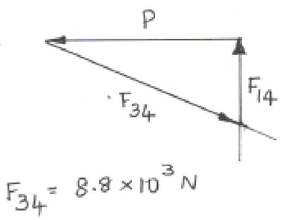
$$= 8 \times 10^3\ N$$

Free body diagram

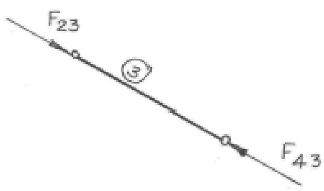


Force triangle for the forces acting on (4) is drawn to some suitable scale.

Magnitude and direction of P known and lines of action of F_{34} & F_{14} known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of F_{14} & F_{34}. Directions are also fixed.

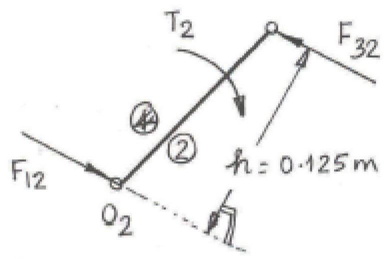


$$i.e,\ F_{23} = -F_{32}$$

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction

$$i.e.,\quad F_{43} = -F_{23} = 8.8 \times 10^3\ N$$

Also, F_{23} = - F_{32} (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque (stated in the problem), for equilibrium the two forces must be equal, parallel and opposite and their sense must oppose T_2.

There fore,

$$F_{32} = -F_{12} \; = 8.8 \times 10^3 \, N$$

F_{32} & F_{12} form a counter clock wise couple of magnitude,

$$(F_{23} \times h) = (F_{12} \times h) = (8.8 \times 10^3) \times 0.125 = 1100 Nm.$$

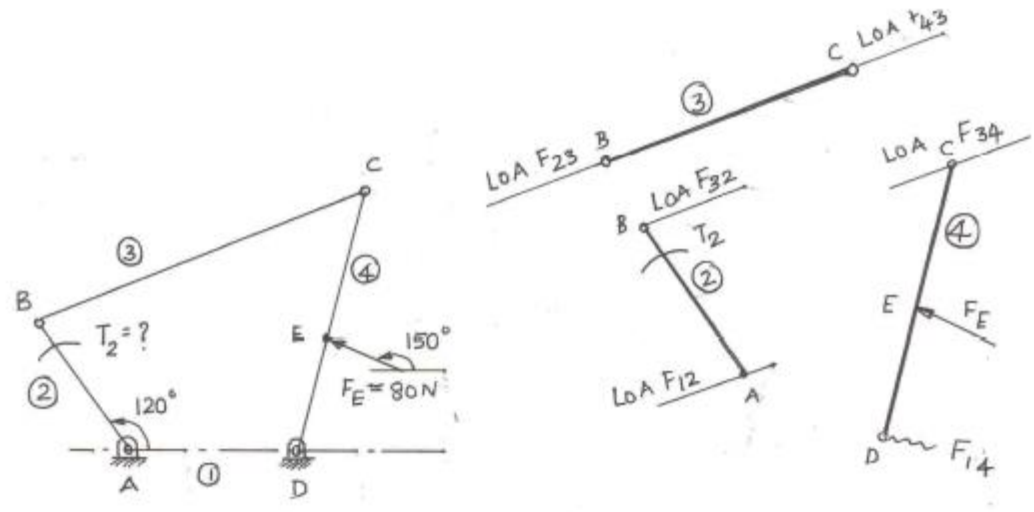
To keep 2 in equilibrium, T_2 should act clockwise and magnitude is 1100 Nm.
Important to note;

i) h is measured perpendicular to F_{32} & F_{12};
ii) always multiply back by scale factors.

Problem No 2. Four link mechanism.

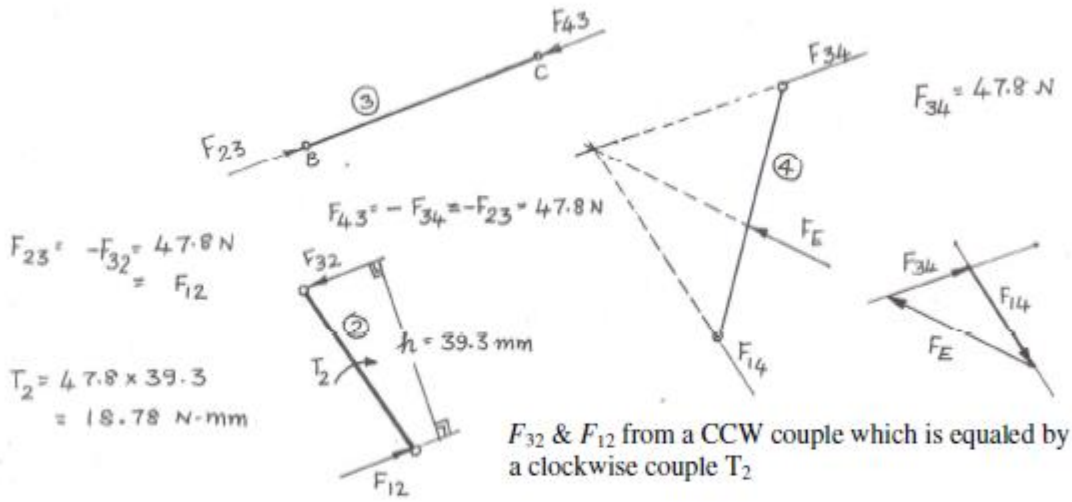
A four link mechanism is acted upon by forces as shown in the figure. Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium.

AD=50mm, AB=40mm, BC=100mm, Dc=75mm, DE= 35mm,



Link 3 is acted upon by only two forces F_{23} & F_{43} and they must be collinear & along BC.

Link 4 is acted upon by three forces F_{14}, F_{34} & F_4 and they must be concurrent. LOA F_{34} is known and F_E completely given.

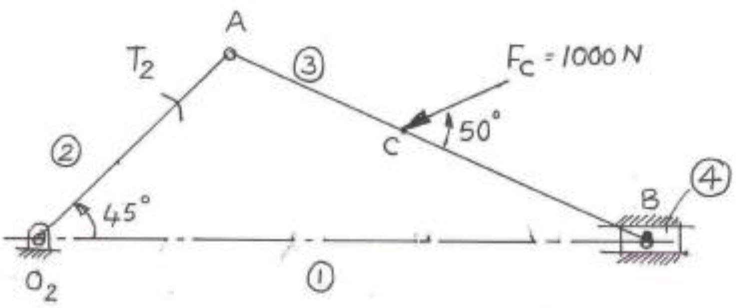


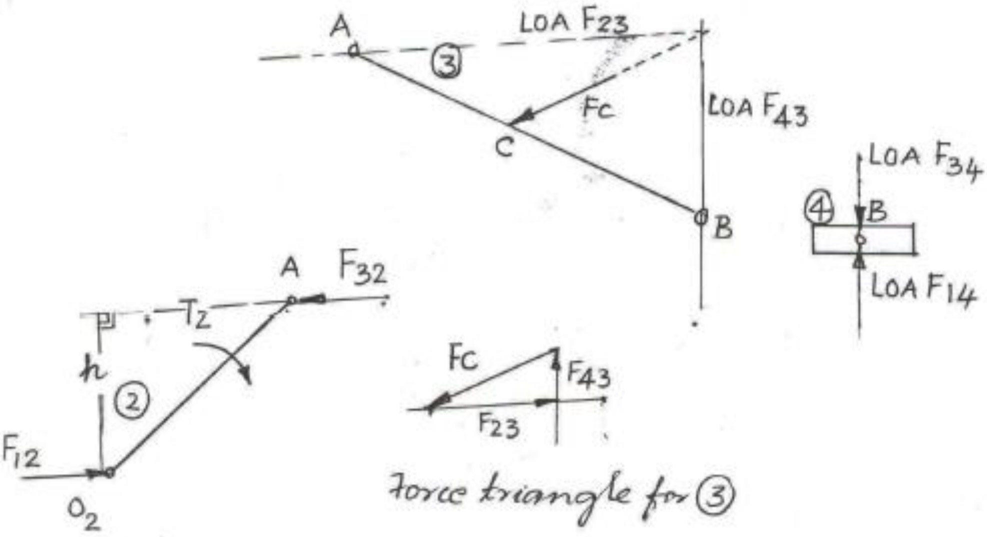
F_{32} & F_{12} from a CCW couple which is equaled by a clockwise couple T_2

Problem No 3.

Determine T_2 to keep the mechanism in equilibrium

AC=70mm,
AB=150mm,
O_2A= 40mm





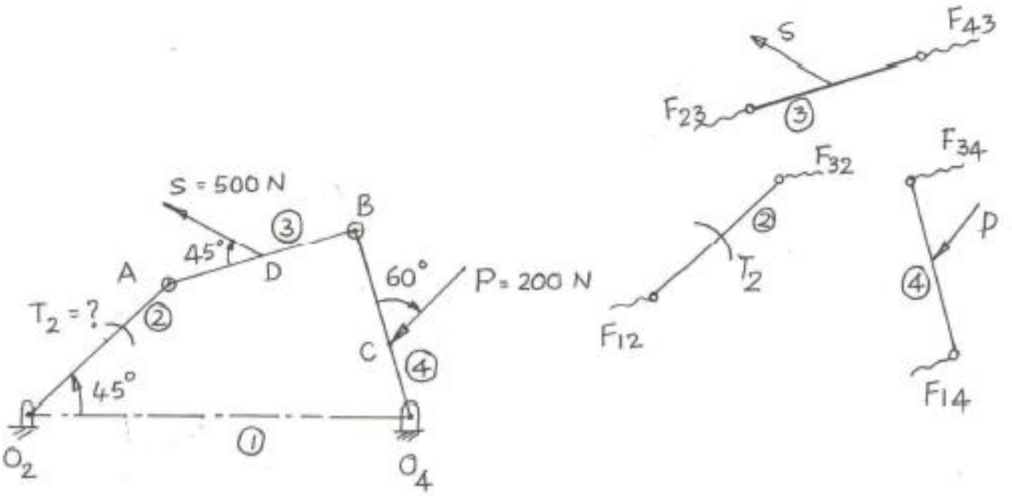
$$T_2 = F_{32} \times h = F_{12} \times h$$

F_{32} and F_{12} form a CCW couple and hence T_2 acts clock wise.

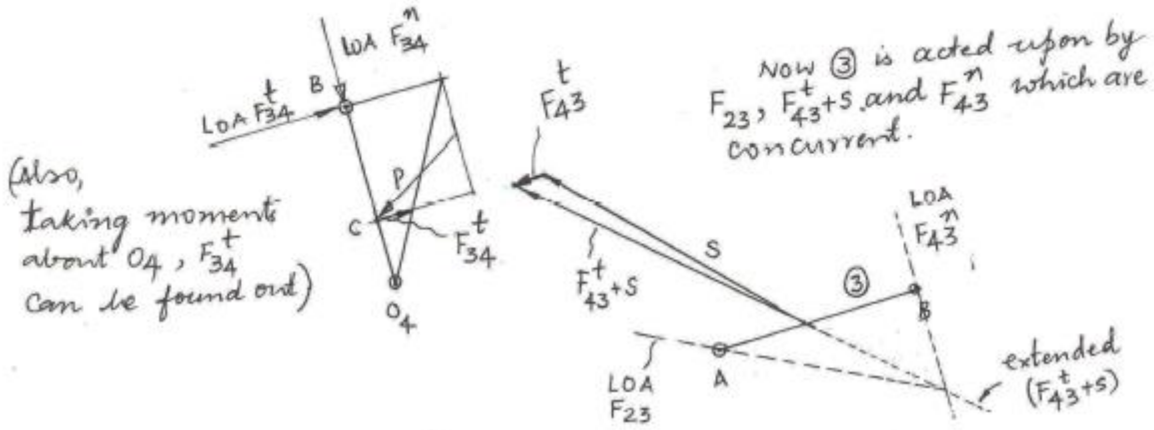
Problem No 4.

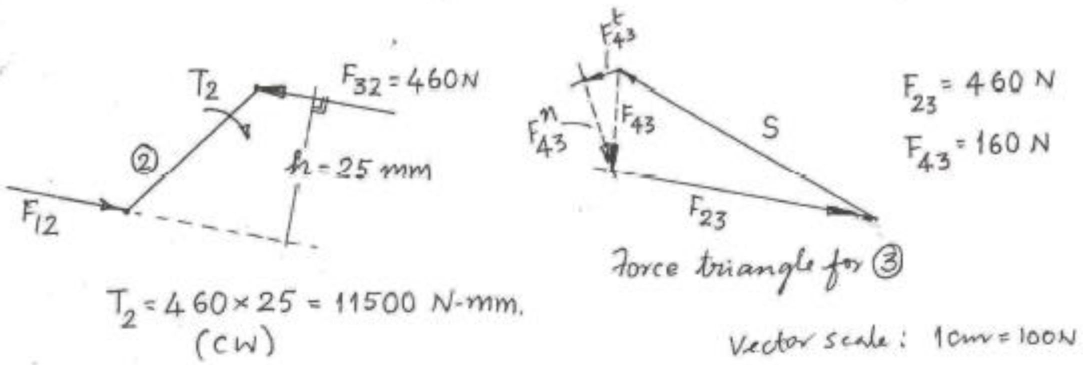
Determine the torque T_2 required to keep the given mechanism in equilibrium.

$O_2A = 30mm$, $= AB = O_4B$, $O_2O_4 = 60mm$, $A\hat{O_2}O_4 = 60^\circ$, $BC = 19mm$, $AD = 15mm$.



None of the links are acted upon by only 2 forces. Therefore links can't be analyzed individually.

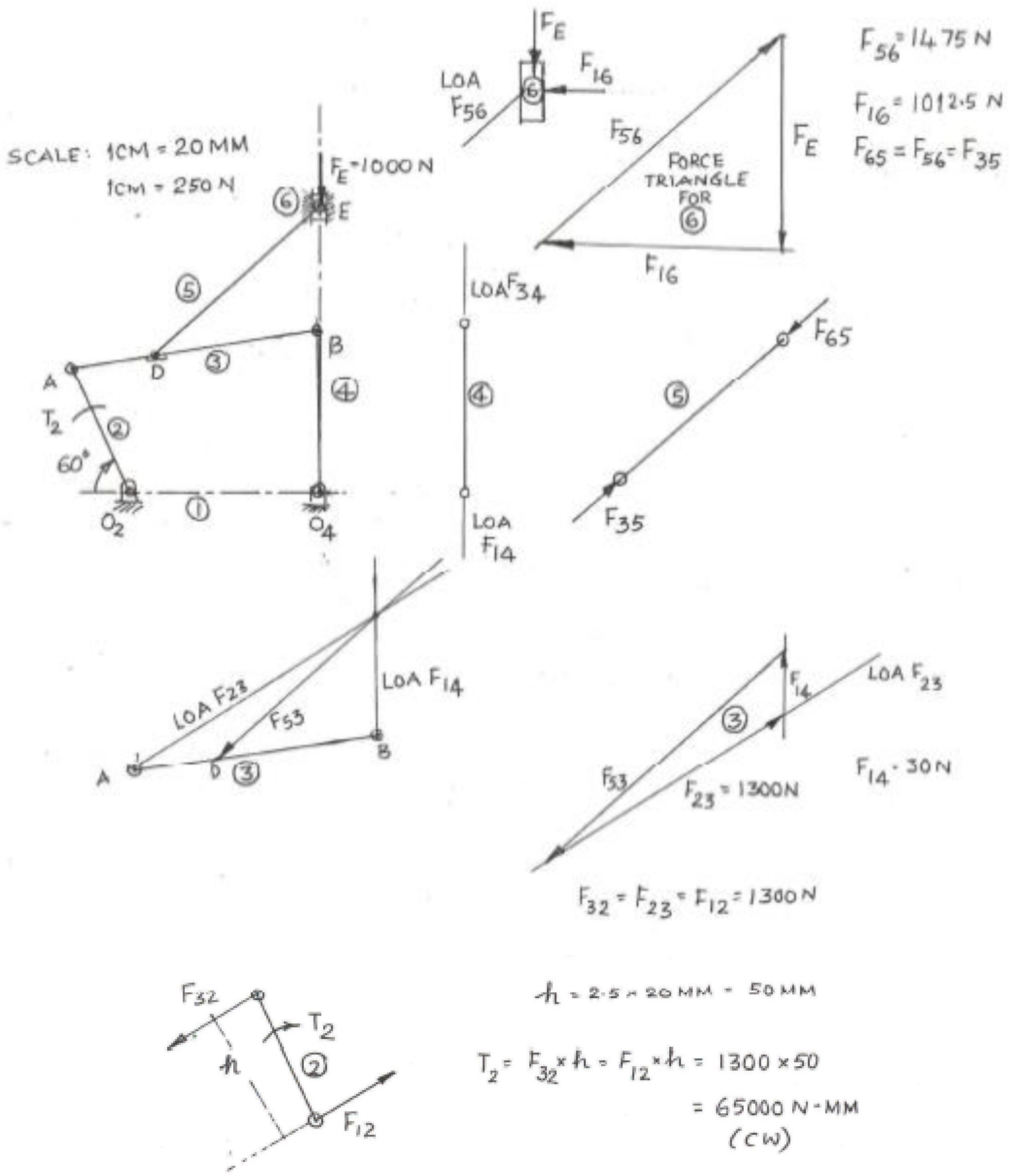




Problem No 5.

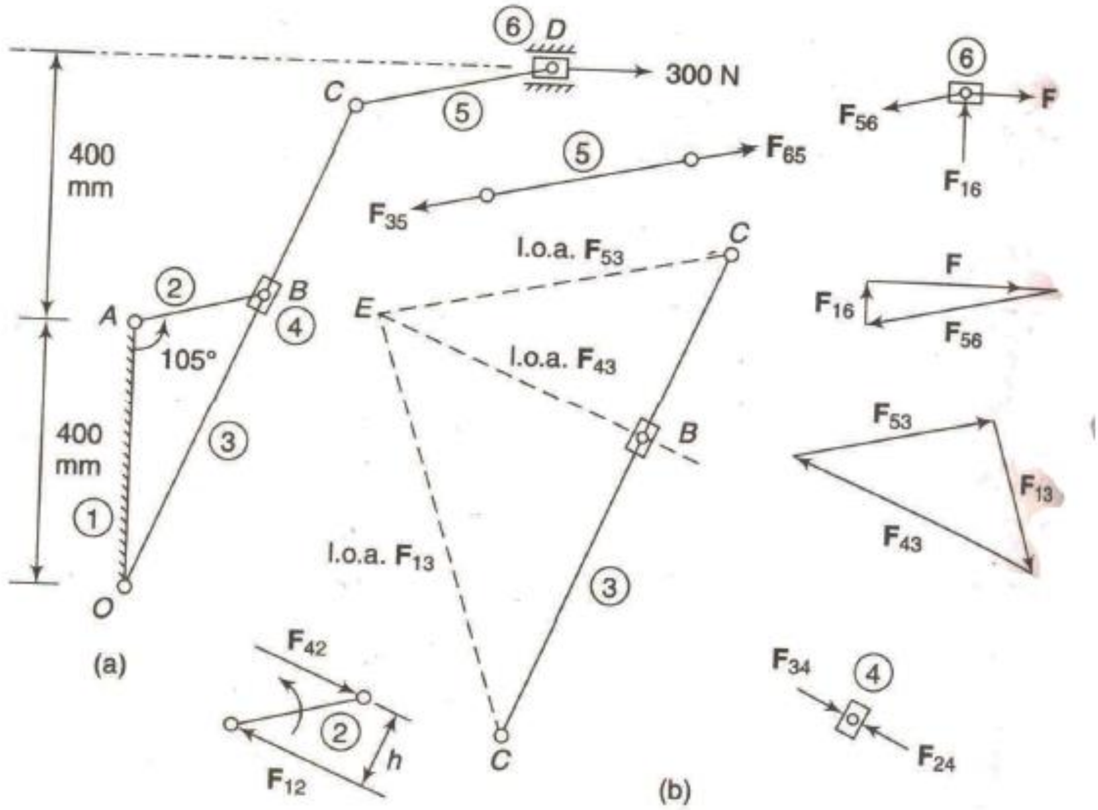
Determine the torque T_2 required to overcome the force F_E along the link 6.

AD=30mm, AB=90mm, O_4 B=60mm, DE=80mm, O_2 A=50mm, O_2 O_4 =70mm



Problem No 6

For the static equilibrium of the quick return mechanism shown in fig. 12.11 (a), determine the input torque T_2 to be applied on link AB for a force of 300N on the slider D. The dimensions of the various links are OA=400mm, AB=200mm, OC=800mm, CD=300mm



Than, torque on link 2,

$T_2 = F_{42} x\ h = 403x120 = 48\ 360$ N counter - clockwise

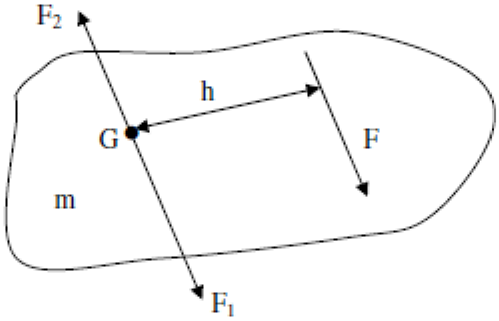
DYNAMIC FORCE ANALYSIS:

It is defined as the study of the force at the pin and guiding surfaces and the forces causing stresses in machine parts, such forces being the result of forces due to the motion of each part in the machine. The forces include both external and inertia forces. Inertia forces in high speed machines become very large and cannot be neglected, Ex: Inertia force of the piston of an automobile travelling at high speed might be thousand times the weight of the piston. The dynamic forces are associated with accelerating masses.

If each link, with its inertia force and force applied to the link can be considered to be in equilibrium, the entire system can also be considered to be in equilibrium.

Determination of force & couple of a link

(Resultant effect of a system of forces acting on a rigid body)



G = c .g point
F_1& F_2: equal and opposite forces acting through G (Parallel to F)

F: Resultant of all the forces acting on the rigid body.
h: perpendicular distance between F & G.
m = mass of the rigid body

Note: F_1=F_2 & opposite in direction; they can be cancelled with out affecting the equilibrium of the link. Thus, a single force 'F' whose line of action is not through G, is capable of producing both linear & angular acceleration of CG of link.

F and F_2 form a couple.

$T = F \times h = I\,\alpha = mk^2\,\alpha$ (Causes angular acceleration) . . . . . . (1)

Also, F_1 produces linear acceleration, f.

$F_1 = mf$

Using 1 & 2, the values of 'f' and 'α' can be found out if F_1, m, k & h are known.

D'Alembert's principle:

Final design takes into consideration the combined effect of both static and dynamic force systems. D'Alembert's principle provides a method of converting dynamics problem into a static problem.

Statement: The vector sum of all external forces and inertia forces acting upon a rigid body is zero. The vector sum of all external moments and the inertia torque, acting upon the rigid body is also separately zero. In short, sum of forces in any direction and sum of their moments about any point must be zero.

Inertia force and couple: Inertia: Tendency to resist change either from state of rest or of uniform motion Let 'R' be the resultant of all the external forces acting on the body, then this 'R' will be equal to the product of mass of the body and the linear acceleration of c.g of body. The force opposing this 'R' is the inertia force (equal in magnitude and opposite in direction).

(Inertia force is an Imaginary force equal and opposite force causing acceleration).

If the body opposes angular acceleration (α) in addition to inertia force R, at its cg, there exists an inertia couple Ig x α, Where Ig= M I about cg. The sense of this couple opposes α. i.e., inertia force and inertia couple are equal in magnitude to accelerating force and couple respectively but, they act in opposite direction.

Inertia force (F_i) = M x f,
(mass of the rigid body x linear acceleration of cg of body)

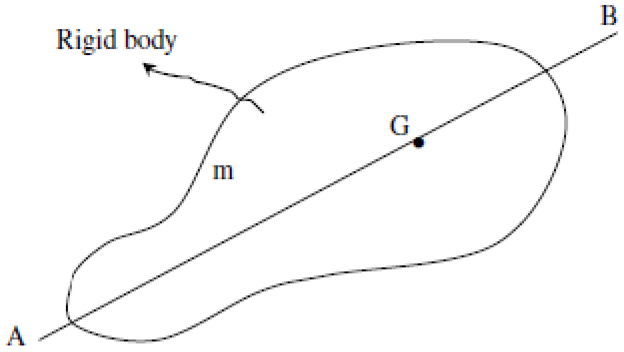
$$\text{Inertia couple } (C_i) = I \times \alpha, \left[\begin{array}{l}\text{MMI of the rigid body about an axis}\\ \text{perpendicular to the plane of motion}\end{array}\right] \left[\begin{array}{l}\text{Angular}\\ \text{acceleration}\end{array}\right]$$

Dynamic Equivalence:

The line of action of the accelerating force can also be determined by replacing the given link by a dynamically equivalent system. Two systems are said to be dynamically equivalent to one another, if by application of equal forces, equal linear and angular accelerations are produced in the two systems.

i.e., the following conditions must be satisfied;

i) The masses of the two systems must be same.
ii) The cg's of the two systems must coinside.
iii) The moments of inertia of the two systems about same point must be equal, Ex: about an axis through cg.

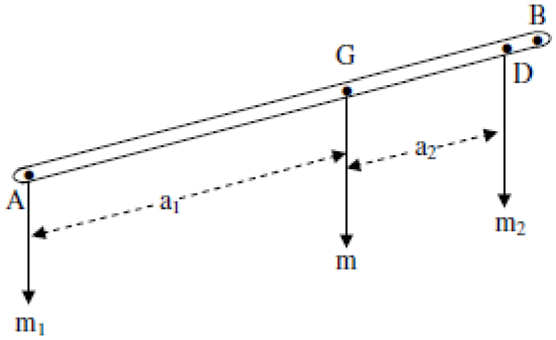


G = c.g.

m = mass of the rigid body

k_g = radius of gyration about an axis through G and perpendicular to the plane

Now, it is to be replaced by dynamically equivalent system.



m_1, m_2 – masses of dynamically equivalent system at a_1 & a_2 from G (respectively)

As per the conditions of dynamic equivalence,

$$m = m_1 + m_2 \quad \text{.. (a)}$$
$$m_1 a_1 = m_2 a_2 \quad \text{.. (b)}$$
$$mk_g^2 = m_1 a_1^2 + m_2 a_2^2 \quad \text{.. (c)}$$

Substituting (b) in (c),

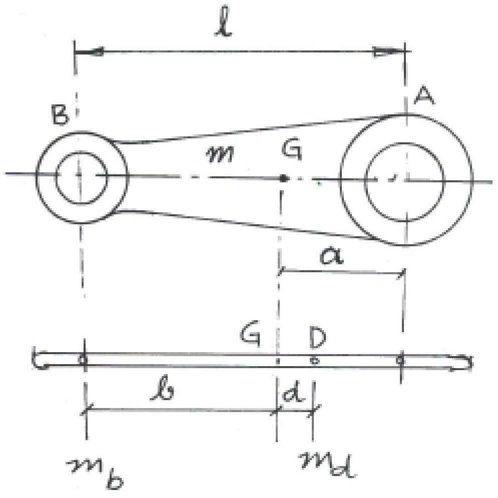
$$mk_g^2 = (m_2 a_2) a_1 + (m_1 a_1) a_2$$

$$= a_1 a_2 (m_2+m_1) = a_1 a_2 (m)$$

i.e., $k_g^2 = a_1 a_2$ $\qquad [I_g = mk_g^2 \; or \; k_g^2 = \frac{I_g}{m}]$

or $\frac{I_g}{m} = a_1 \, a_2$

Inertia of the connecting rod:



Connecting rod to be replaced by a massless link with two point masses m_b & m_d.

m = Total mass of the CR m_b& m_d point masses at B& D.

$$m_b + m_d = m \qquad -- \quad (i)$$

$$m_b \times b = m_d \times d \qquad -- \quad (ii)$$

Substituting (ii) in (i);

$$m_b + \left(m_b \times \frac{b}{d} \right) = m$$

$$m_b \left(1 + \frac{b}{d} \right) = m \qquad \text{or} \quad m_b \left(\frac{b+d}{d} \right) = m$$

$$\text{or } m_b = m \left(\frac{d}{b+d} \right) \qquad -- \quad (1)$$

Similarly; $$m_d = m \left(\frac{b}{b+d} \right) \qquad -- \quad (2)$$

Also; $I = m_b b^2 + m_d d^2$

$$= m \left(\frac{d}{b+d} \right) b^2 + m \left(\frac{b}{b+d} \right) d^2 \qquad [\mathit{from}\ (1)\ \&\ (2)]$$

$$I = mbd \left(\frac{b+d}{b+d} \right) = mbd$$

Then, $mk_g^2 = mbd$, (since $I = mk_g^2$)

$k_g^2 = bd$

The result will be more useful if the 2 masses are located at the centers of bearings A & B.

Let m_a = mass at A and dist. AG = a

Then,

$$m_a + m_b = m$$

$$m_a = m\left(\frac{b}{a+b}\right) = m\,\frac{b}{l}\ ; \quad Since\ (a+b=l)$$

Similarly, $$m_b = m\left(\frac{a}{a+b}\right) = m\,\frac{a}{l}\ ; \quad (Since,\ a+b=l)$$

$$I^1 = m_a{}^{a^2} + m_b{}^{b^2} = .\ .\ .\ . = mbd$$

(Proceeding on similar lines it can be proved)

Assuming; $a > d,\ I^1 > I$

i.e., by considering the 2 masses A & B instead of D and B, the inertia couple (torque) is increased from the actual value. i.e., there exists an error, which is corrected by applying a correction couple (opposite to the direction of applied inertia torque).

The correction couple,

$$\Delta T = \alpha_c (mab - mbd)$$

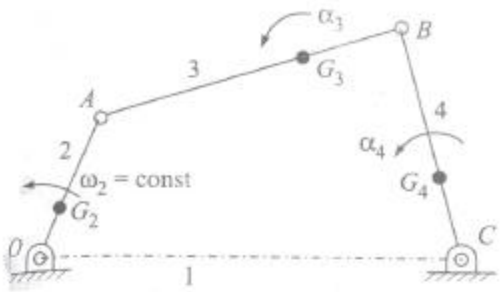
$$= mb\ \alpha_c\ (a-d)$$

$$= mb\ \alpha_c \left[(a+b) - (b+d)\right]$$

$$= mb\ \alpha_c\ (l-L) \qquad because\,(b+d=L)$$

As the direction of applied inertia torque is always opposite to the direction of angular acceleration, the direction of the correction couple will be same as that of angular acceleration i.e., in the direction of decreasing angle β.

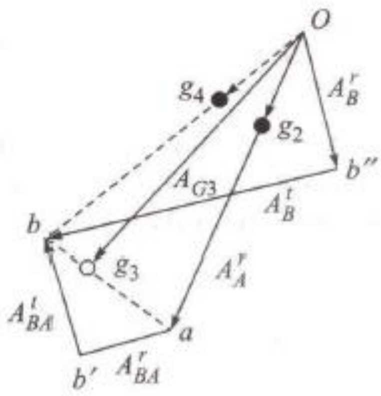
Dynamic force Analysis of a 4 – link mechanism.



OABC is a 4–bar mechanism. Link 2 rotates with constant ω_2. G_2, G_3 & G_4 are the cgs and M_1, M_2 & M_3 the masses of links 1, 2 & 3 respectively.

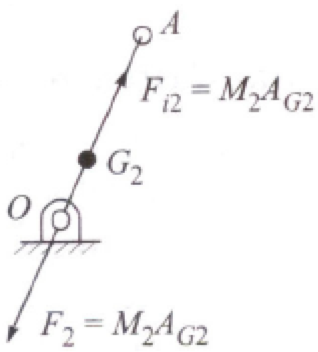
What is the torque required, which, the shaft at o must exert on link 2 to give the desired motion?

1. Draw the velocity & acceleration polygons for determing the linear acceleration of G_2, G_3 & G_4.
2. Magnitude and sense of α_3 & α_4 (angular acceleration) are determined using the results of step 1.



To determine inertia forces and couples

Link 2

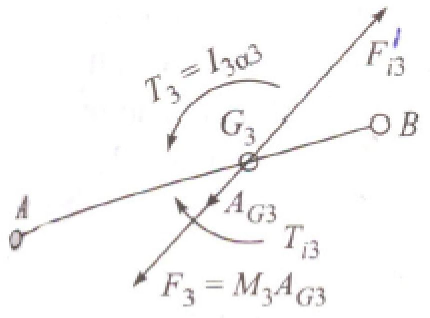


F_2 = accelerating force (towards O)

F_{i2} = inertia force (away from O)

Since ω_2 is constant, α_2 = 0 and no inertia torque involved.

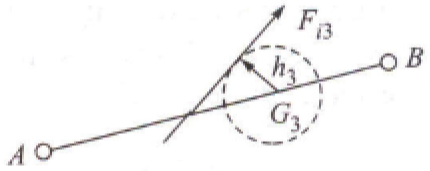
Link 3



Linear acceleration of G_3 (i.e., AG_3) is in the direction of Og_3 of acceleration polygon.

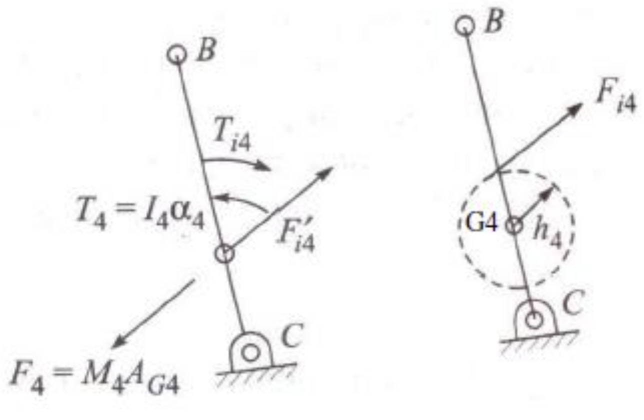
F_3 = accelerating force

Inertia force F'_{i3} acts in opposite direction. Due to α_3, there must be a resultant torque $T_3 = I_3\,\alpha_3$ acting in the sense of α_3 (I_3 is MMI of the link about an axis through G_3, perpendicular to the plane of paper). The inertia torque T_{i3} is equal and opposite to T_3.



F_{i3} can replace the inertia force F'_{i3} and inertia torque T_{i3}. F_{i3} is tangent to circle of radius h_3 from G_3, on the top side of it so as to oppose the angular acceleration α_3. $h3 = \frac{I_3\alpha_3}{M_3AG_3}$

Link 4



$h4 = \frac{I_4\alpha_4}{M_4AG_4}$

Problem 1 :

It is required to carryout dynamic force analysis of the four bar mechanism shown in the figure.

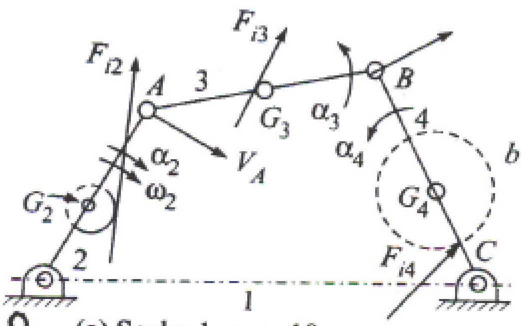
ω_2 = 20rad /s (cw), α_2 = 160 rad/s^2 (cw)

OA= 250mm, OG_2= 110mm, AB=300mm, AG_3=150mm, BC=300mm, CG_4=140mm, OC=550mm, $\angle AOC = 60°$

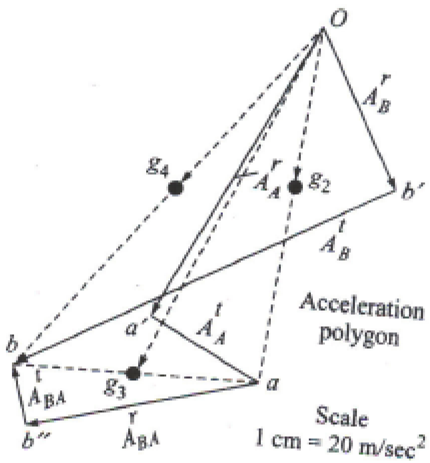
The masses & MMI of the various members are

Link	Mass, m	MMI (I_G, Kgm^2)
2	20.7kg	0.01872
3	9.66kg	0.01105
4	23.47kg	0.0277

Determine i) the inertia forces of the moving members
ii) Torque which must be applied to 2



(a) Scale: 1 cm = 10 cms



A) Inertia forces:

(i) (from velocity & acceleration analysis)

$V_A = 250\times 20;\ 5 m/s,\quad V_B = 4\ m/s,\ V_{BA} = 4.75\ m/s$

$a_A^r = 250\times 20^2;\ 100 m/s^2,\quad a_A^t = 250\times 160;\ 40 m/s^2$

Therefore;

$$A_B^r = \frac{V_B^2}{CB} = \frac{(4)^2}{0.3} = 53.33\ m/s^2$$

$$A_{BA}^r = \frac{V_{BA}^2}{B_A} = \frac{(4.75)^2}{0.3} = 75.21\ m/s^2$$

$$Og_2 = A_{G2} = 48\ m/s^2;\ Og_3 = AG_3 = 120 m/s^2$$

$$Og_4 = A_{G4} = 65.4\ m/s^2$$

$$\alpha_3 = \frac{A_{BA}^t}{AB} = \frac{19}{0.3} = 63.3\ rad/s^2$$

$$\alpha_4 = \frac{A_B^t}{CB} = \frac{129}{0.3} = 430\ rad/s^2$$

Inertia forces (accelerating forces)

$$F_{G2} = m_2\ A_{G2} = \frac{20.7}{9.81} \times 48 = 993.6\,N\ (in\ the\ direction\ of\ Og_2)$$

$$F_{G3} = m_3\ A_{G3} = 9.66 \times 120 = 1159.2\,N\ (in\ the\ direction\ of\ Og_3)$$

$$= F_{G4} = m_4\ A_{G4} = 23.47 \times 65.4 = 1534.94\,N\ (in\ the\ direction\ of\ Og_4)$$

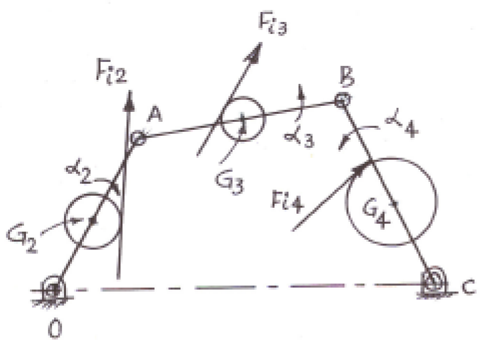
$$h_2 = \frac{I_{G2}(\alpha_2)}{F_2} = \frac{(0.01872 \times 160)}{993.6} = 3.01 \times 10^{-3}\,m$$

$$h_3 = \frac{I_{G3}(\alpha_3)}{F_3} = \frac{(0.01105 \times 63.3)}{1159.2} = 6.03 \times 10^{-4}\,m$$

$$h_4 = \frac{I_{G4}(\alpha_4)}{F_4} = \frac{(0.0277 \times 430)}{1534.94} = 7.76 \times 10^{-3}\,m$$

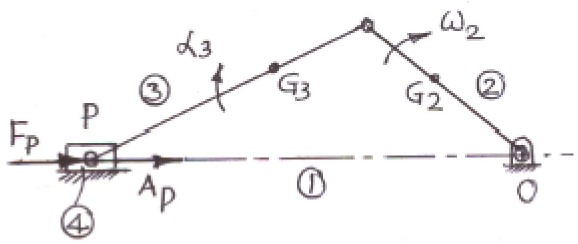
The inertia force F_{i2}, F_{i3} & F_{i4} have magnitudes equal and direction opposite to the respective accelerating forces and will be tangents to the circles of radius h_2, h_3 & h_4 from G_2, G_3 & G_4 so as to oppose α_2, α_3 & α_4.

$F_{i2} = 993.6\,N$, $F_{i3} = 1159.2\,N$ $F_{i4} = 1534.94\,N$



Further, each of the links is analysed for static equilibrium under the action of all external force on that link plus the inertia force.

Dynamic force analysis of a slider crank mechanism.



F_P = load on the piston

Link	mass	MMI
2	m_2	I_2
3	m_3	I_3
4	m_4	-

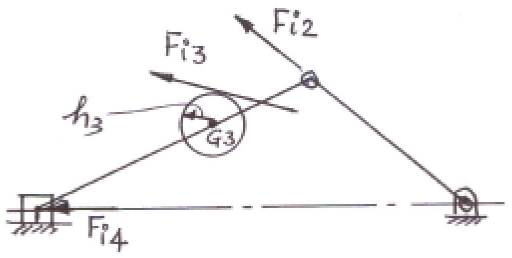
ω_2 assumed to be constant

Steps involved:

1. Draw velocity & acceleration diagrams
2. Consider links 3 & 4 together and single FBD written (elimination F_{34} & F_{43})
3. Since, weights of links are smaller compared to inertia forces, they are neglected unless specified.
4. Accelerating forces F_2 , F_3 & F_4 act in the directions of respective acceleration vectors Og_2, Og_3 & Og_p

Magnitudes: $F_2 = m_2 AG_2 \quad F_3 = m_3 AG_3 \quad F_4 = m_4 A_p$

$F_{i2} = F_2$, $F_{i3} = F_3$, $F_{i4} = F_4$ (Opposite in direction)



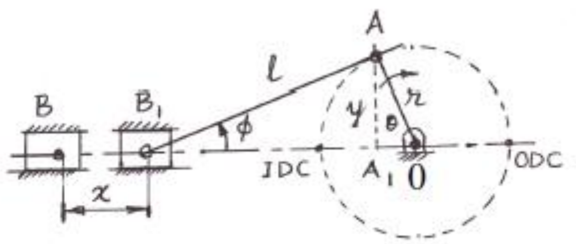
$$h_3 = \frac{I_3 \alpha_3}{M_3 \alpha_{g3}}$$

F_{i3} is tangent to the circle with h_3 radius on the RHS to oppose α_3

Solve for T_2 by solving the configuration for both static & inertia forces.

Dynamic Analysis of slider crank mechanism (Analytical approach)

Displacement of piston



x = displacement from IDC

$$x = BB_1 = BO - B_1O$$
$$= BO - (B_1A_1 + A_1O)$$
$$= (l + r) - (l\cos\phi + r\cos\theta) \qquad \left(\text{since}, \frac{l}{r} = n\right)$$
$$= (nr + r) - (rn\cos\phi + r\cos\theta)$$
$$= r[(n+1) - (n\cos\phi + \cos\theta)] \qquad \cos\phi = \sqrt{1 - \sin^2\phi}$$

$$= r\left|(n+1) - (\sqrt{n^2 - \sin^2\theta} + \cos\theta)\right|$$

$$= r\left|(1-\cos\theta) + (n - \sqrt{n^2 - \sin^2\theta})\right|$$

$$= \sqrt{1 - \frac{y^2}{l^2}}$$

$$= \sqrt{1 - \frac{(r\sin\theta)^2}{l^2}}$$

$(similary\ l >> r, \frac{l}{r} = n >> 1$ & max $value\, of \sin\theta = 1)$

$$= \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$\therefore \sqrt{n^2 - \sin^2\theta} \to \sqrt{n^2}\ or\ n),$

$$\boxed{x = r\ (1 - \cos\theta)}$$

$$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

This represents SHM and therefore Piston executes SHM.

Velocity of Piston:

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\ \frac{d\theta}{dt}$$

$$\frac{d}{d\theta}\left[r(1-\cos\theta) + n - (n^2 - \sin 2\theta)^{-\frac{1}{2}}\right]\frac{d\theta}{dt}$$

$$= r\left[0 + \sin\theta + 0 - \frac{1}{2}(n^2 - \sin 2\theta)^{-1/2}(-2\sin\theta\cos\theta)\right]\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

Since, $n^2 >> \sin^2\theta$,

$$\therefore\ v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

Since n is quite large, $\frac{\sin 2\theta}{2n}$ can be neglected.

$$\boxed{\therefore\ v = r\omega\sin\theta}$$

Acceleration of piston:

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\ \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\,(\sin\theta + \frac{\sin 2\theta}{2n})\right]\omega$$

$$= r\omega\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right]$$

$$= r\omega\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

If n is very large;

$$\boxed{a = r\omega^2\cos\theta}\ \textbf{(as in SHM)}$$

When θ = 0, at IDC,

$$a = r\omega^2\left(1 + \frac{1}{n}\right)$$

When θ = 180, at 0DC,

$$a = r\omega^2\left(-1 + \frac{1}{n}\right)$$

At θ = 180, when the direction is reversed,

$$a = r\omega^2\left(1 - \frac{1}{n}\right)$$

Angular velocity & angular acceleration of CR (α_c)

$$y = l\sin\phi = r\ \sin\theta$$

$$\sin\ \phi = \frac{\sin\ \theta}{n}$$

Differentiating w.r.t time,

$$\cos\ \phi\ \ \frac{d\phi}{dt} = \frac{1}{n}\cos\ \theta\ \ \frac{d\theta}{dt} \qquad \frac{d\phi}{dt} = \omega_c$$

$$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \qquad \frac{d\theta}{dt} = \omega$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$\omega_c = \omega \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}\left[\cos\theta\,(n^2 - \sin^2\theta)^{-\frac{1}{2}}\right]\omega$$

$$= \omega^2 \left[\cos\theta \frac{1}{2}(n^2 - \sin^2\theta)^{-\frac{3}{2}}(-2\sin\theta\,\cos\theta) + (n^2 - \sin^2\theta)^{-\frac{1}{2}}(-\sin\theta)\right]$$

$$= \omega^2 \sin^2\theta \left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

$$= -\,\omega^2 \sin\theta \left[\frac{(n^2 - 1)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

Negative sign indicates that, ϕ reduces (in the case, the angular acceleration of CR is CW)

UNIT-II

Engine Force Analysis and Turning Moment Diagram

Forces acting on the engine are weight of reciprocating masses & CR, gas forces, Friction & inertia forces (due to acceleration & retardation of engine elements).

i) Piston effort (effective driving force)

- Net or effective force applied on the piston.

In reciprocating engine:

The reciprocating parts (masses) accelerate during the first half of the stroke and the inertia forces tend to resist the same. Thus, the net force on the piston is reduced. During the later half of the stroke, the reciprocating masses decelerate and the inertia forces oppose this deceleration or acts in the direction of applied gas pressure and thus effective force on piston is increased.

In vertical engine, the weights of the reciprocating masses assist the piston during out stroke (down) this increasing the piston effort by an amount equal to the weight of the piston. During the in stroke (up) piston effect is decreased by the same amount.

Force on the piston due to gas pressure; $FP = P_1A_1 - P_2A_2$

P_1 = Pressure on the cover end,
P_2 = Pressure on the rod
A_1 = area of cover end,
A_2 = area of rod end,
m = mass of the reciprocating parts.

Inertia force (F_i) = m a

$$= m.r\omega^2\left(Cos\theta + \frac{Cos2\theta}{n}\right) \qquad \text{(Opposite to acceleration of piston)}$$

Force on the piston $F = F_P - F_i$

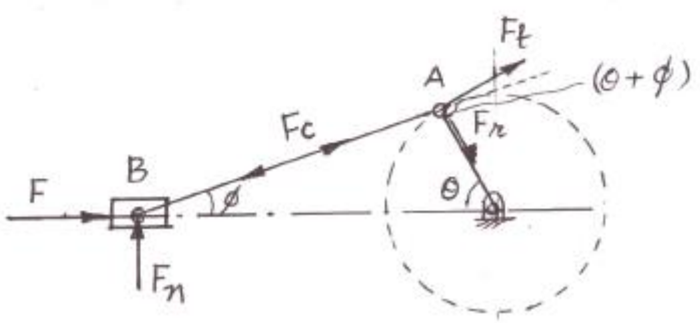
(if F_f frictional resistance is also considered)

$$F = F_P - F_i - F_i$$

In case of vertical engine, weight of the piston or reciprocating parts also acts as force.

$$\therefore F = F_P + mg - F_i - F_i$$

ii) Force (Thrust on the CR)



F_c = force on the CR

Equating the horizontal components;

$$F_c\, Cos\phi = F \ \ or \ \ F_c \frac{F}{Cos^2\phi}$$

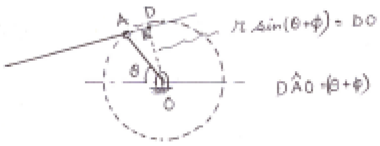
iii) Thrust on the sides of the cylinder

It is the normal reaction on the cylinder walls

$$F_s = F_c \sin\phi = F \tan\phi$$

iv) Crank effort (T)

It is the net force applied at the crank pin perpendicular to the crank which gives the required TM on the crank shaft.



$$F_t \times r = F_c\, r \sin(\theta+\phi)$$

$$F_t = F_c \sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)$$

v) Thrust on bearings (F_r)

The component of F_C along the crank (radial) produces thrust on bearings

$$F_r = F_c\, Cos(\theta+\phi) = \frac{F}{Cos\,\phi}\ Cos(\theta+\phi)$$

vi) Turning moment of Crank shaft

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)\times r \ = \frac{F_r}{\cos\phi}\ (\sin\theta + \cos\phi + \cos\theta\sin\phi)$$

$$= F\times r\left(\sin\theta + \cos\theta\frac{\sin\phi}{\cos\phi}\right)$$

Proved earlier

$$\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$= F\times r\left(\sin\theta + \cos\theta\,\frac{\sin\theta}{n}\ \ \frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$= F\times r\left(\sin\theta + \ \ \frac{\sin 2\,\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

Also,

$$r\sin(\theta+\phi) = OD\ \cos\phi$$

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}.\ r\ \sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}.\ OD\ \ \cos\phi$$

$$T = F \times OD.$$

Turning Moment Diagram of Flywheels

Introduction: The torque of an engine crank shaft varies considerably throughout the working cycle, due to variations in crank position, the P^r in the cylinder & inertia force on piston & connecting rod. If the

Turning Moment: Value of crank shaft torque, i.e., the turning moment T is plotted against crank angle θ, the diagram so obtained is turning moment dia.

Turning moment diagram is also known as crank-effort diagram, it is the graphical representation of the turning moment or crank-effort for various positions of the crank.

* **Turning moment Diagram for a single cylinder double Acting Steam Engine:**

A turning moment diagram for a single cylinder double acting steam engine is:

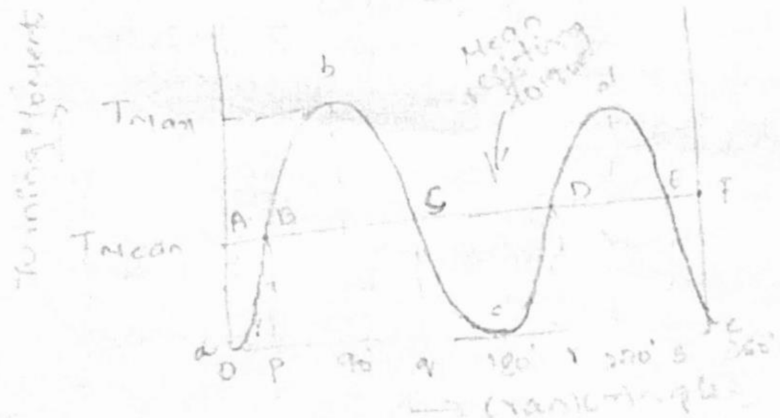


Fig ②: Turning moment diagram for a single cylinder double acting steam Engine.

The vertical ordinate represents Turning Moment & the horizontal ordinate represents crank angle.

Thus turning moment on crank shaft will be,

$$\boxed{T = F_p \times r \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right]}$$

moment is zero, .

where, $F_p \rightarrow$ Piston effort

$r \rightarrow$ radius of crank.

$n \rightarrow$ ratio of connecting rod length to radius of crank and

$\theta \rightarrow$ Angle turned by crank from inner dead centre.

From the fig@, we can say that 'T' → turning moment is zero, when the crank angle θ is zero. It is maximum when, the crank angle is $\left(\frac{180}{2}\right) = 90°$. Again it is zero when the crank angle is 180° and so on.

This is shown by curve 'abc' in fig., and it represents turning moment of out stroke. The curve 'cde' represents turning moment for in stroke.

NOTE: 1. when the turning moment is '+ve', i.e. when the engine torque is more than mean resisting torque, as shown between points B & C in fig@ the crank shaft accelerates and work is done by steam.

2. When the turning moment is '−ve', i.e. when the engine torque is less than the mean resisting torque, as shown between points C & D in fig@. the crank shaft retards and the work is done on the steam.

3. If $T \rightarrow$ Torque on crank shaft at any instant &

$T_{mean} \rightarrow$ Mean resisting Torque

Then accelerating torque on rotating parts of engine,

$= T - T_{mean}$.

4. If $(T - T_{mean})$ is '+ve', the flywheel accelerates

if $(T - T_{mean})$ is '−ve', then the flywheel retards.

(2)

ctuation of Energy:-

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in fig ⓐ. We see that the mean resisting torque line AF cuts the turning moment diagram at B, C, D & E. When the crank moves from a to p, the work done by the engine equal to the area aBP, whereas the energy required is represented by the a ABP. In the other words, the engine has done less work, ~~the remaining~~ than the requirement. This amount, i.e. required amount of energy is taken from the flywheel and hence the speed of flywheel decreases. Now the crank moves from 'p to q', the work done by the engine, is equal to the area PBbcq, where as the requirement of energy is represented by the area PBCQ. Therefore, the engine has done more work than the requirement. This excess energy stored in the flywheel, and the speed of flywheel increases while the crank moves from p to q.

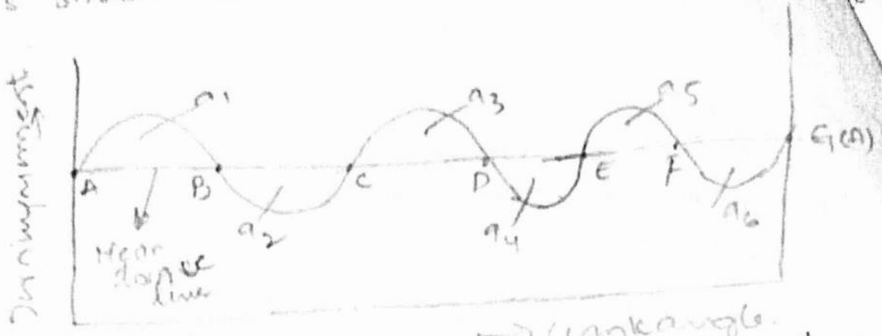
The variations of energy above and below the mean resisting torque line are called fluctuations of energy. The areas Bbc, CcD, DdE, etc. represent fluctuations of energy.

The difference between the ~~minimum~~ maximum and minimum energies is known as Maximum fluctuation of energy.

NOTE:- The area of the turning moment diagram is proportional to the work done per revolution as the work is the product of turning-moment & angle turned.

Determination of the fluctuation of energy:

A turning diagram for a multicylinder engine is shown as:



The horizontal line AG represents mean torque line

let, a_1, a_3, a_5 represents areas of above the mean torque line

a_2, a_4, a_6 " " below " " "

Let the energy in the flywheel at A = E.

we have, Energy at $B = E + a_1$

" " $C = E + a_1 - a_2$

" " $D = E + a_1 - a_2 + a_3$

" " $E = E + a_1 - a_2 + a_3 - a_4$

" " $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

" " $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (i.e. cycle repeat after G).

Suppose, the greatest energy is at B and least at E.

∴ Max. energy in flywheel, $= E + a_1$

min. " " " , $= E + a_1 - a_2 + a_3 - a_4$.

∴ Max. fluctuation of energy, ΔE,

ΔE = Max. Energy − Min. energy.

$= (E + a_1) - [E + a_1 - a_2 + a_3 - a_4]$

$= \cancel{E + a_1 - E}$

$$\boxed{\Delta E = a_2 - a_3 + a_4}$$

Co-effecient of Fluctuation of Energy [C_E]

- It may be defined as the, "ratio of maximum fluctuation of energy to the work done per cycle".

$$C_E = \frac{\text{max. fluctuatn of energy}}{\text{work done per cycle.}}$$

We may obtain work done per cycle in 2 methods:
work done per cycle, units → (N-m or J).

1. Work done per cycle = $T_{mean} \times \theta$.

where, T_{mean} → Mean torque &
θ → Angle turned (in radians), in one revolution.
= 2π, in case of steam engine & 2-stroke I.C. engines.
= 4π, in case of 4-stroke I.C. engines.

The mean torque can be obtained as,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where, P → Power Transmitted in watts,
N → Speed in rpm.
ω → Angular speed, $\frac{rad}{sec} = \frac{2\pi N}{60}$.

2. Work done per cycle,

$$= \frac{P \times 60}{n}.$$

where, n → no. of working strokes per minute,
$n = N$, in case of steam engines & 2 stroke I.C engines &
$n = \frac{N}{2}$, in case of 4-stroke I.C engines.

* FLY WHEEL:-

A flywheel is used in machines used efficient reservoirs, which stores the energy during the when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply.

In simple words, when the flywheel absorbs energy, its speed increases and when its releases the energy, its speed decreases.

We can say that, " A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation".

NOTE: The function of a governor, in an engine is entirely different from that of flywheel. The governor regulates the mean speed of an engine when there are variations in the load. Where as the flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by varying load.

Applications:-

Flywheels are provided in engines and fabricating machines such as presses, shearing machine rivetting machines, punching machines, steel rollers, crushers etc..

(A).

Co-effecient of Fluctuation of speed [C_s]:

The ratio of maximum fluctuation of speed to the mean speed is called co-effecient of fluctuation of speed.

"The difference b/w max. & min. speeds during a cycle is called max. fluctuation of speed."

Let, N_1 & N_2 are max. & min. speeds during cycle.

$N \rightarrow$ mean position

$$= \frac{N_1 + N_2}{2}.$$

$$\therefore \quad C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \ldots \text{(In terms of Angular speed)}$$

$$= \frac{V_1 - V_2}{V} = \frac{2(V_1 - V_2)}{V_1 + V_2} \quad \ldots \text{(In terms of linear speed)}.$$

NOTE: The reciprocal of co-effecient of friction of speed is known as "coefficient of steadiness" and is denoted by 'm'.

$$m = \frac{1}{C_s} = \frac{N}{(N_1 - N_2)}$$

* Energy Stored in a Flywheel:

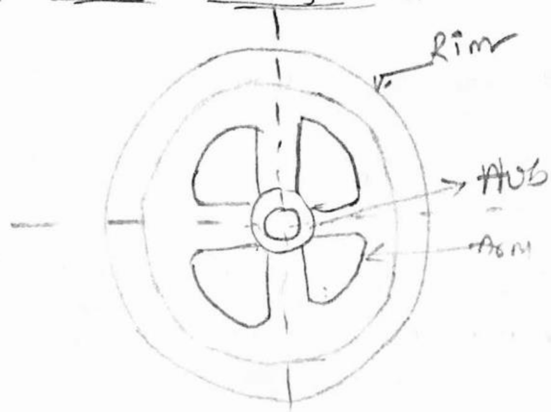


Fig: Flywheel.

We have already discussed that, when a flywheel absorbs energy, its speed increases and vice-versa.

Let, $m \rightarrow$ mass of flywheel.

$k \rightarrow$ radius of gyration.

$I \rightarrow$ mass moment of inertia, $= m \cdot k^2$.

N_1 & $N_2 \rightarrow$ Max. & Min. speeds during the cycle, in rpm.

ω_1 & $\omega_2 \rightarrow$ Max. & Min. angular speeds during the cycle, $\frac{rad}{sec}$.

$N_{mean} \Rightarrow$ Mean speed $= \frac{N_1+N_2}{2}$

$\omega \rightarrow$ Mean angular speed, $= \frac{\omega_1+\omega_2}{2}$.

$c_s \rightarrow$ co-efficient of fluctuation of speed $= \frac{N_1-N_2}{N}$ (or) $\frac{\omega_1-\omega_2}{\omega}$.

W.K.T, the mean kinetic energy of flywheel.

$$E = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} mk^2 \cdot \omega^2.$$

As the speed of flywheel changes from ω_1 to ω_2, max. fluc. A.e.

$$E_f = \text{Max. K.E} - \text{Min. K.E.}$$

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2 = \frac{1}{2} \times I \left[\omega_1^2 - \omega_2^2\right].$$

$$= \frac{1}{2} \times I \left[(\omega_1+\omega_2)(\omega_1-\omega_2)\right] = I\,\omega\,(\omega_1-\omega_2) \qquad \because \frac{\omega_1+\omega_2}{2} = \omega$$

multiply & divide by ω.

$$E_f = I\omega^2 \left[\frac{\omega_1-\omega_2}{\omega}\right]$$

$$E_f = I\omega^2 (c_s) = mk^2\omega^2 c_s.$$

$$= 2E c_s. \qquad \left[E = \frac{1}{2} \times I \cdot \omega^2\right].$$

'k' may be taken as 'r'. $k = R$

$$\boxed{\Delta E = m R^2 \omega^2 c_s}$$

$$= \boxed{mV^2 c_s = \Delta E} \qquad \therefore \boxed{V = \omega R} \text{ in m/s.}$$

$V \rightarrow$ linear velocity.

The mass of fly wheel of an engine is 6.5 tonnes and radius of gyration is 1.8m. It is found from the turning moment diagram that the fluctuation of energy $[E_f]$ is is 56 kN-m. If the mean speed of the engine is 120 rpm. find the max. & min. speeds.

Sol: Given Data:

$M = 6.5\,tonnes = 6500\,kgs.$

$K = 1.8\,m; \quad E_f = 56\,kN\text{-}m = 5600\,N\text{-}m. \; ; N = 120\,rpm.$

Let, N_1 = max speed ; N_2 = min. speed.

We K.T, $N = \frac{N_1 + N_2}{2}$

$$120 = \frac{N_1 + N_2}{2}$$

$$N_1 + N_2 = 240 \quad -(1).$$

$$E_f = \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2.$$

$$= \frac{1}{2} MK^2 [\omega_1^2 - \omega_2^2].$$

$$\frac{1}{2}(6500)(1.8)^2 \times \left[\left[\frac{2\pi N_1}{60}\right]^2 - \left[\frac{2\pi N_2}{60}\right]^2\right].$$

$$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right][N_1 - N_2][N_1 + N_2].$$

$$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right] \times 240 \times (N_1 - N_2)$$

$$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right]$$

$N_1 - N_2 = 0.2$

$N_1 + N_2 =$

$$N_1 + N_2 = 240$$
$$N_1 - N_2 = 0.2$$
$$2N_1 = 240.2$$
$$\boxed{N_1 = 120.1\,rpm}$$

$$N_1 + N_2 = 240$$
$$120.1 + N_2 = 240$$
$$N_2 = 240 - 120.1$$
$$\boxed{N_2 = 119.9\,rpm}$$

(2) The horizontal compound 2-stroke cylinder engine develops at 90 rpm. The co-effecient of fluctuation of energy, ... following from the turning moment diagram is to be 0.1 and ... crank angle diagram ... speed is to be at 0.5% of mean speed. The max ... of the flywheel required, if the radius of gyration is 2m.

Sol:- Given:

$P = 300\,KW$; $N = 90\,rpm$; $C_E = 0.1$; $C_s = \pm 0.5\%$ of N.

m → mass of flywheel

$$P = \frac{2\pi N T_{mean}}{60,000}$$

$$300 = \frac{2\pi \times 90 \times T_{mean}}{60,000}$$

$$\boxed{T_{mean} = 31830\ N\text{-}m}$$

$$C_E = \frac{E_f}{W.D/cycle}$$

$$0.1 = \frac{E_f}{T_{mean} \times 2\pi}$$

$$0.1 = \frac{E_f}{31830 \times 2\pi}$$

$$\boxed{E_f = 19999.9\,N\text{-}M}$$

$$C_s = \frac{N_1 - N_2}{N}$$

$N_1 = 90 - 0.5\%$ of $N \Rightarrow 90 - 0.5\% \times 90$

$\boxed{N_2 = 89.55}$ rpm.

$N_2 = 90 + 0.5\%$ of N.

$\boxed{N_1 = 90.45\ rpm}$

$$C_s = \frac{90.45 - 89.55}{90}$$

$\boxed{C_s = 0.01}$

$E_f = I\omega^2 C_s$.

$19999.9 = m k^2 \omega^2 C_s$.

$= m (2)^2 \left(\frac{2\pi \times 90}{60}\right)^2 \times 0.01$

$\boxed{m = 5628.9\,kg}$

T-θ-diagram Problem

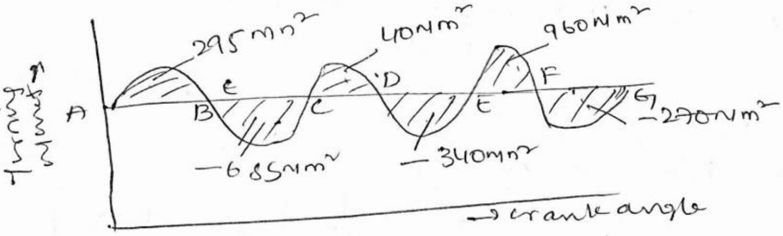
(6). Turning moment for a petrol engine is drawn through following scales: for torque 1 mm = 5 N-m, crank angle 1 mm = 1° = 1 × π/180. The turning moment diagram repeats itself at every revolution of engine and areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration 150 mm. Determine, coefficient of fluctuation of speed when the ranges at 1800 rpm.

Sol: Given Data:

for torque, 1 mm = 5 N-m

crank angle 1 mm = 1° = $\frac{1\times\pi}{180}$ = 0.0174 rad.

The areas are, +295, −685, +40, −340, +960, −270 Nm².



m = 36 kg.

K = 150 mm = 0.15 m; N = 1800 rpm. ; C_s = ?

Let energy at pt. A = E m

" " pt. B = E + 295 ⇒ Max. K.E

pt. C = E + 295 − 685 ⇒ E − 390.

pt. D = E + 295 − 685 + 40 ⇒ E − 350

pt. E = E + 295 − 685 + 40 − 340 ⇒ E − 690 ⇒ Min k.E

pt. F = E + 295 − 685 + 40 − 340 + 960 ⇒ E + 270

pt. G = E + 295 − 685 + 40 − 340 + 960 − 270 = E.

$\therefore E_f = \text{Max. K.E} - \text{Min. K.E.}$

$= (E+295) - (E-690)$

$= E + 295 - E + 690$

$E_f = 985 \text{ mm}^2$

$= 985 \times mm \times mm$

$= 985 \times 5 \times \frac{\pi}{180}$

$\boxed{E_f = 85.96 \text{ N-m}.}$

$E_f = I\omega^2 \times C_s.$

$85.96 = mk^2 \cdot \omega^2 \times C_s$

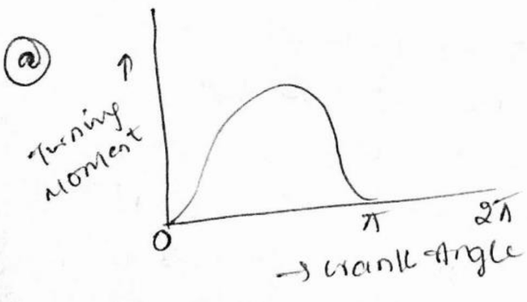
$85.96 = (36)(0.15)^2 \times \left[\frac{2\pi \times 1800}{60}\right]^2 \times C_s$

$C_s = 3 \times 10^{-3}$

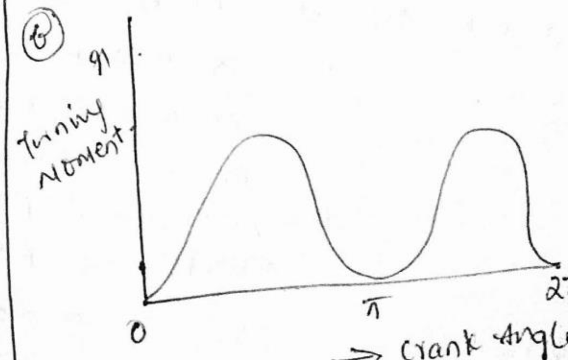
$\boxed{C_s = 0.003}$

$C_s = 0.3\%$ (or) $\boxed{C_s = \pm 0.15\%}$

* Turning moment diagrams of common engines:-



ⓐ Single acting steam engine



ⓑ Double acting steam engine.

diagram for 4-stroke IC Engine:

(7).

A Turning diagram for 4-stroke IC engine is shown in fig. w.k.t, in a four stroke IC engine, There is one stroke after the crank has turned through 2-revolutions. i.e: 720° (or) 4π.

Since, the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke.

∴ Negative loop is formed, as shown in fig,

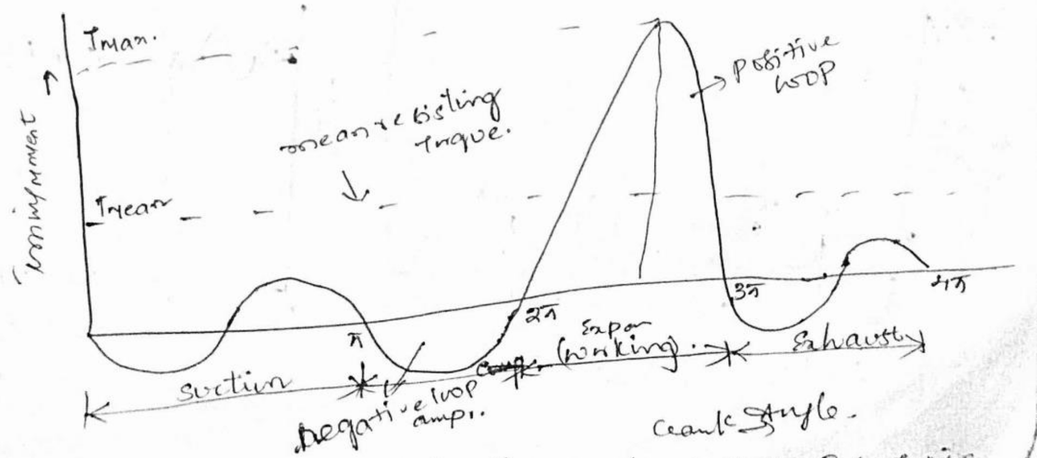
During the compression, the work is done ~~on~~ on the gases; therefore, higher negative loop is formed.

During the expansion stroke the fuel burns & the gases expands,

∴ a large +ve loop is obtained. In this stroke, the work is done by the gases.

During exhaust stroke, the work is done on the gases.

∴ There is a '-ve' loop during the exhaust stroke.



T-θ diagram for 4-stroke IC engine

Problem ⑧.

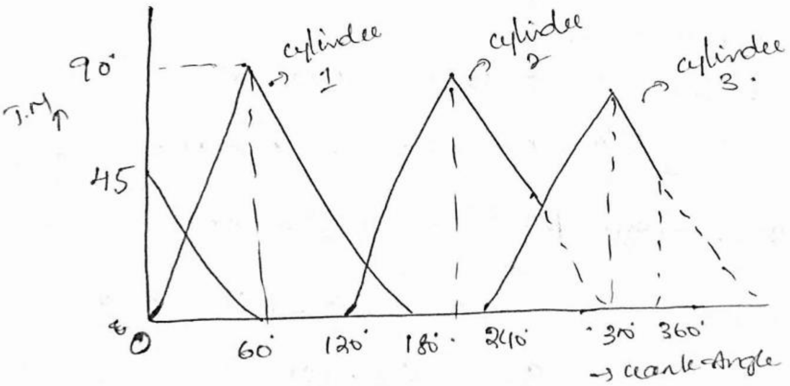
~~Single~~ 3 cylinder single acting engine has its crank set equally at 120° and its runs at 600 rpm. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque 90N-m at 60° from dead centre of corresponding crank. The torque on return stroke is sensibly zero. Determine:

1. Power developed; 2. coeffecient of fluctuation of speed, if the mass of flywheel is 12kg & has a radius of gyration of 80mm, 3. coeffecient of fluctuation of energy, &
4. Max. angular acceleration of flywheel.

Sol: Given $N = 600$ rpm ⓐ $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$

$\omega = 62.84$ rad/sec;

$T_{max} = 90$ N-m ; $m = 12$ kg ; $k = 80$ mm $= 0.08$ m.



ⓐ

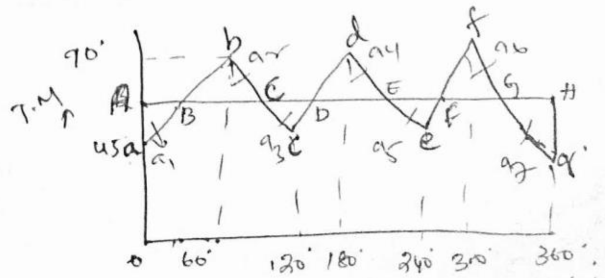


Fig ⓐ represents T-θ diagram for 3 cylinders in ... & Fig ⓑ represents resultant T-θ diagram for 3 cylinder

1. Power developed.

$IP = T_{mean} \times \omega$

WKT, $\frac{W.D}{cycle}$ = Area of 3 Δ les

$= 3 \times \frac{1}{2} \times \pi \times 90^{\circ}$

$= 424$ N-m

$T_{mean} = \frac{W.D/cycle}{crank Angle/cycle} = \frac{424}{2\pi} = ~~424~~ \; 67.5$ N-m

∴ $P = T_{mean} \times \omega = 67.5 \times 62.84 = 4240 W = 4.24 kW$.

2. coeff of fluctuation of speed:

Let, $C_s \rightarrow$ coeff. of fluctn of speed. $E_f = I\omega^2 C_s$.

So, that, ~~for~~ initially we have to find Max. fluctuation of energy.

from fig ⓑ we have to find,

$a_1 =$ Area of triangle $aAB = \frac{1}{2} AB \times Aa$. $\left(\because AB = 30° = \frac{\pi}{3} \right)$

$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45^{\circ}) = 5.89$ N-m $= a_7$.

$a_2 =$ Area of triangle $Bbc = \frac{1}{2} \times Bc \times bb'$. $\left(\because Bc = 60 = \pi/6 \right)$

$= \frac{1}{2} \times \frac{\pi}{3} \times (90 - 67.5)$

$a_2 = 11.78$ N-m.

$a_2 = a_3 = a_4 = a_5 = a_6$.

(9)

Let the total energy at $A = E$

then the energy at $B = E - 5.89$.

" " $C = E - 5.89 + 11.78 = E + 5.89$

" " $D = E + 5.89 - 11.78 = E - 5.89$

" " $E = E - 5.89 + 11.78 = E + 5.89$

" " $F = E + 5.89 - 11.78 = E - 5.89$

" " $G = E - 5.89 + 11.78 = E + 5.89$

" " $H = E + 5.89 - 5.89 = E$ = Energy at A.

$\therefore$ The max. Energy $= E + 5.89$

" Min. Energy $= E - 5.89$.

$\therefore$ $E_f = \Delta E$ = max. fluctuation of Energy

$= (E + 5.89) - [E - 5.89]$

$\Delta E = 11.78$ N-m.

W.K.T, max. fluctuation of energy

$E_f = \Delta E = I\omega^2 C_s$

$11.78 = mk^2 . \omega^2 C_s$

$11.78 = (12)(0.08)^2 \times (62.84)^2 C_s$

$\boxed{C_s = 0.04 \text{ (or) } 4\%}$

3. Coeff. of fluctuation of energy.

W.K.T, coeff. of fluctuation of energy,

$$C_E = \frac{\text{max. fluctuation of energy}}{\text{W.D/cycle}} = \frac{11.78}{424}$$

$= 0.0278$

$\boxed{C_E = 2.78\%}$

4. Max. angular acceleration of flywheel,

$\alpha \rightarrow$ angular acceleration.

$T_{max} - T_{mean} = I . \alpha . = mk^2 . \alpha$.

$90 - 67.5 = (12)(0.05)^2 . \alpha$

$\alpha = \frac{90 - 67.5}{0.077} \Rightarrow \boxed{\alpha = 292 \frac{rad}{sec^2}}$

② A single cylinder, single acting four stroke engine develops 20KW at 300 rpm. The work done by the gases during the expansion stroke is three times the work done the gases during compression stroke, the work done during suction stroke & exhaust stroke being negligible. If the total fluctuation of speed is not to ± 2% of mean speed & Turning moment diagram during compression & expansion is assumed to be Triangular in shape. find the moment of inertia of flywheel.

Sol: Given: $P = 20KW = 20\times10^3 W$; $N = 300$ rpm.

Total fluctuation of speed $(\omega_1-\omega_2)$ is not to exceed ± 2% of mean speed.

$\boxed{W_E = 3W_c}$

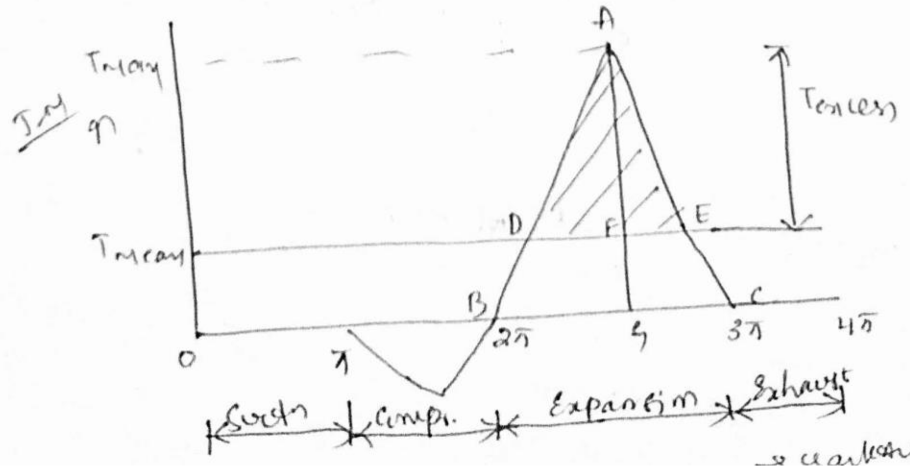
$\therefore \omega_1-\omega_2 = 4\omega$..

& co-effecient of fluctuation of speed,

$$C_S = \frac{\omega_1-\omega_2}{\omega} = 4\%$$

$$(i.e) \frac{\omega_1-\omega_2}{\omega} = 0.04$$

The following will be the $T-\theta$ diagram for the four stroke engine neglecting suction and exhaust strokes.



→ crank angle

(10).

W.K.T, for four stroke no. of working strokes / cycle.

$$n = \frac{N}{2} = \frac{300}{2} = 150.$$

$$\therefore \frac{W.D}{cycle} = P \times \frac{60}{n} = 20 \times 10^3 \times \frac{60}{150} \Rightarrow$$

$$\frac{W.D}{cycle} = 8000 \text{ N-m.} \quad (1).$$

Since workdone during suction & exhaust are negligible,

& net w.D/cycle ⇒

$$\Rightarrow W_E - W_c$$

$\because W_E = 3W_c.$

$$\boxed{W_c = \frac{W_E}{3}}$$

$$\Rightarrow \cancel{3W_c - W_c}$$

$$= W_E - \frac{W_E}{3}$$

$$w.D/cycle = \frac{2W_E}{3} \quad -(2).$$

Equating (1) & (2).

$$8000 = \frac{2}{3} W_E$$

$$W_E = 12000 \text{ N-m}$$

W.K.T, work done during expansion stroke $[W_E]$.

In order to get T_{max}.

12000 = Area of Δ^{le} ABC

$$= \frac{1}{2} BC \times AG$$

$$12000 = \pi \times AG.$$

$$\therefore T_{max} = AG = \frac{12000 \times 2}{\pi} = 7638 \text{ N-m.}$$

$$\text{Now } T_{mean} = FG = \frac{w.D/cycle}{\text{crank Angle/cycle}} = \frac{8000}{4\pi} = 637 \text{ N-m.}$$

$\therefore$ Tencess = $AF = AG - FG$

$= 7638 - 637$

Tencess = 7001 N-m.

Now, from similar $\triangle$les ADE & ABC.

$$\frac{DE}{BC} = \frac{AF}{FG}$$

(or) $DE = BC \times \frac{AF}{FG}$.

$= \pi \times \frac{7001}{637}$

$\therefore \boxed{DE = 2.88 \text{ rad.}}$

$\therefore$ The area above the mean represents max. fluctuation energy, $\therefore$ Max. fluctuation of Energy,

$\Delta E = E_f =$ Area of $\triangle^{le}$ ADE

$= \frac{1}{2} DE \times AF$

$= \frac{1}{2} \times 2.88 \times 7638$

$E_f = 10081$ N-m.

$\therefore$ Moment of inertia is to be calculated,

$E_f = I \omega^2 c_s$.

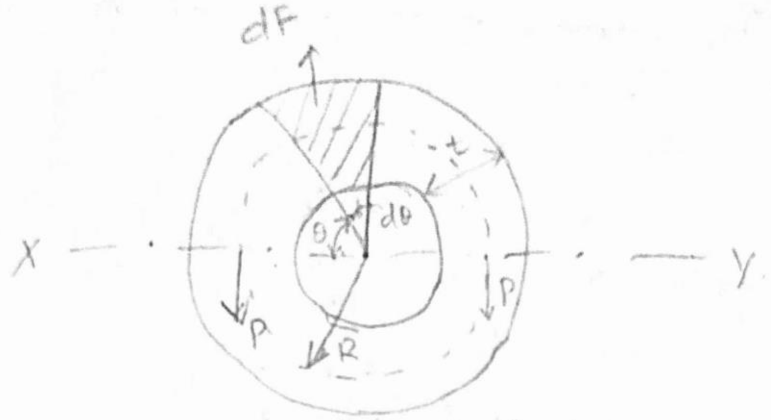
$10081 = I \cdot \left(\frac{2\pi 300}{60}\right)^2 \times 0.04$

$10081 = 39.5 I$

$\boxed{I = 255.2 \text{ kg-m}^2}$

(11).

Dimensions of the Flywheel Rim:-



Rim of a flywheel.

Consider a rim of the flywheel as shown,

Let, $D \rightarrow$ Mean Diameter of rim, mts.

$R \rightarrow$ Mean Radius of rim, mts.

$A \rightarrow$ cross-sectional area of rim, m^2.

$\rho \rightarrow$ Density of rim material, kg/m³.

$N \rightarrow$ Speed of the flywheel, rpm

$\omega \rightarrow$ Angular ~~speed~~ velocity of flywheel. rad/sec

$v \rightarrow$ linear velocity at mean radius in m/s

$= \omega \cdot R = \frac{\pi D N}{60}$, m/s.

$\sigma \rightarrow$ Tensile stress or hoop stress, N/m².
due to centrifugal force.

Consider a small element of the rim as shown shaded.

Let it subtends at an angle of $\delta\theta$ at the centre of flywheel.

Volume of small element.

$$dV = A \times R \cdot d\theta.$$

∴ mass of the small element,

$$dm = \text{Density} \times \text{Volume}$$

$$dm = \rho \times A \times R \cdot d\theta.$$

Centrifugal force on the element, acting radially outwards,

$$dF = dm \times R \cdot \omega^2$$

$$= \rho \cdot A \cdot R \times d\theta \cdot R \cdot \omega^2$$

$$dF = \rho \times A \cdot R^2 \omega^2 \cdot d\theta$$

Vertical component of force of the element of dF

$$= dF \cdot \sin\theta = \rho \cdot A \cdot R^2 \cdot \omega^2 \, d\theta \sin\theta$$

∴ Total vertical upward force tending to burst rim across the dia. x & y.

$$= \rho A \cdot R^2 \omega^2 \int_0^{\pi} \sin\theta \cdot d\theta. \qquad \cos\pi = -1 ; \cos 0 = 1$$

$$\left[\cos\pi - (-\cos 0)\right]$$

$$1 + 1 = 2$$

$$= \rho A R^2 \omega^2 \left[-\cos\theta\right]_0^{\pi} = 2\rho A R^2 \omega^2. \quad -(1)$$

The vertical upward force will produce hoop stress or circumferential force, & is resisted by 2P.

$$2P = 2 \cdot \sigma \cdot A. \quad -(2).$$

$$\boxed{A = b \times t}$$

Equating (1) & (2).

$$2\rho A R^2 \omega^2 = 2 \cdot \sigma \cdot A$$

$$\sigma = \rho R^2 \omega^2 = \rho v^2 \qquad \because v = R\omega ; \; \omega = \frac{v}{R}$$

$$\boxed{v = \sqrt{\frac{\sigma}{\rho}}}$$

We know mass of rim, $m = \text{Volume} \times \text{density} = \pi \cdot D A \cdot \rho$. ∴ $\boxed{A = \frac{m}{\pi D \rho}}$

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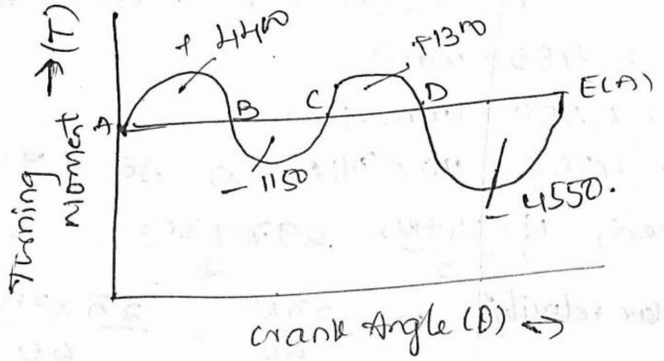
(A). In a turning moment dia, the areas above & below the mean torque line taken in order are 4400, 1150, 1300 & 4550 mm² resp. The scales of the turning moment diagram are;

Turning moment, 1mm = 100N-m: Crank Angle, 1mm = 1°.

find the mass of the fly wheel reqd. to keep the speed b/w 297 & 303 rpm. if the radius of gyration is 0.525 m.

Sol:



Sol: Given Data:

$N_1 = 297$ & $N_2 = 303$ rpm., $k = 0.525$m

Turning moment, 1mm = 100 N-m

Crank Angle, 1mm = 1° = $(1 \times \pi/180)$.

Let the total energy at, $A = E$,
The energies at diff. pt's.

at, $A = E$.

$B = E + 4410$.

$C = E + 4410 - 1150 = E + 3250$.

$D = E + 4410 - 1150 + 1310 = E + 4550$ (max.

$E = E + 4410 - 1150 + 1310 - 4550 = E$. (min eng)

W.K.T, max. fluctuatn of energy,

ΔE = max. energy − min. energy

$= E + 4550 - E = 4550\ mm^2$.

$\Delta E = 4550\ mm^2$

$= 4550 \times 1mm \times 1mm$

$= 4550 \times 110 \times \pi/180 \Rightarrow \Delta E = 7939.75\ N\text{-}m$.

mean speed, $N = \frac{N_1 + N_2}{2} = \frac{297 + 303}{2} = 300$ rpm.

mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} \Rightarrow \omega = 31.416$ rad/sec

coeff. of fluctuatn of speed, $C_s = \frac{N_1 - N_2}{N} = \frac{303 - 297}{300} = 0.02$.

W.K.T max. fluctuatn of energy,
$\Delta E = I\omega^2 C_s$.

$\Delta E = mk^2 \omega^2 C_s$.

$7939.75 = m(0.525)^2 \times (31.416)^2 \times (0.02)$

$\boxed{m = 1459.3\ Kg}$

Date..............................

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(3). In a machine, the intermittent operation demand the torque to be applied as follows:

- During the first half revolution, the torque increases from 1200 N-m to 3600 N-m.
- During the next one revolution, the torque remains constant.
- During the next one revolution, the torque decreases uniformly from 3600 N-m to 1200 N-m.
- During last 1½ revolution, the torque remains constant.

Thus a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 240 rpm. A flywheel of mass 2000 kg and radius of gyration 600 mm is fitted to shaft.

Determine: (i) The power of motor, f.

(ii) The total coefficient of fluctuation of speed of m/c shaft.

Sol: Given:

$N_{mean} = 200$ rpm; $m = 2000$ kg;

$K = 600\,mm = 0.6\,m$

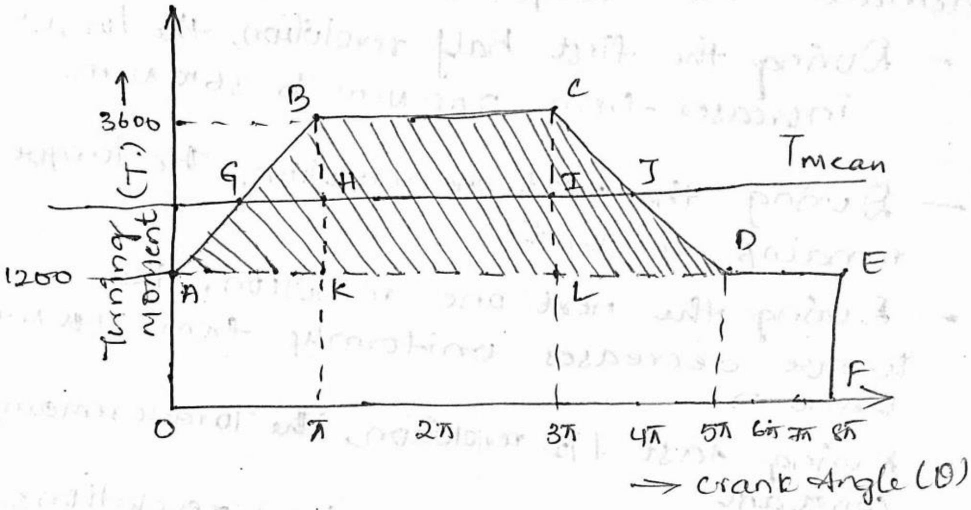
Mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60}$

$\omega = 20.94$ rad/sec.

Moment of inertia, $I = m.k^2$

$= (2000)(0.6)^2 = 720\,kg\text{-}m^2$.

The turning moment diagram for complete cycle is as below,



We know that the torque reqd. for one complete cycle

= Area of OABCDEFO.

= Area OAEF + Area ABK + Area BCLK + Area CDL

$$= (OF \times OA) + \left[\tfrac{1}{2} \times AK \times BK\right] + \left[KL \times CL\right] + \left[\tfrac{1}{2} \times DL \times CL\right]$$

$$= \left[8\pi \times 1200\right] + \left[\tfrac{1}{2} \times \pi \times (3600-1200)\right] + \left[2\pi \times (3600-1200)\right] + \left[\tfrac{1}{2} \times 2\pi \times (3600-1200)\right].$$

$$= 30159.28 + 3769.91 + 15079.6 + 7539.8$$

$$T = 56548.61 \text{ N-m.} \quad \text{—(a)}$$

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If T_{mean} is the mean torque, then torque required for 1 complete cycle

$$\Rightarrow \quad = T_{mean} \times 8\pi. \quad -(b).$$

Equating eq^n (a) & (b),

$$56548.61 = T_{mean} \times 8\pi$$

$$T_{mean} = 2250 \text{ N-m}.$$

(i) Power of the motor :

W.K.T, Power, $P = \frac{2\pi N T}{60}$

$$P = \frac{2\pi \times 200 \times 2250}{60}$$

$$P = 47123.8 \text{ W}$$

$$= 47.124 \text{ kW}.$$

(ii) Total coeffecient of fluctuation of Speed [C_S]:-

$$\Delta E = I\omega^2 C_S.$$

we need to find out the fluctuation of energy ΔE.

for that first need to find out the values of GH & IJ.

from similar triangles ABK & GBH, we get,

$$\frac{GH}{AK} = \frac{BH}{BK} \quad \text{or} \quad \frac{GH}{\pi} = \frac{(3600-2250)}{(3600-1200)}$$

$$\text{or} \quad GH = 1.767 \text{ rad.}$$

from similar triangles CIJ and CLD.

$$\frac{IJ}{LD} = \frac{CI}{CL}$$

$$\frac{IJ}{2\pi} = \frac{(3600-2250)}{(3600-1200)}$$

$$IJ = 3.534 \text{ rad.}$$

W.K.T the area ~~above~~ above mean torque line represents max. fluctuatn of energy [ΔE]

$\therefore \quad \Delta E$ = Area of GBCJ = Area GBH + Area BCIH + Area CJI.

$$= \left[\tfrac{1}{2} \times GH \times BH\right] + \left[HI \times CI\right] + \left[\tfrac{1}{2} \times IJ \times CI\right].$$

$$= \left[\tfrac{1}{2} \times 1.767 \times (3600-2250)\right] + \left[2\pi \times (3600-2250)\right] + \left[\tfrac{1}{2} \times 3.534 \times (3600-2250)\right]$$

$$\Delta E = 12060.475 \text{ N-m.}$$

& also $\Delta E = I\omega^2 C_s$.

$$12060.475 = 720(20.94)^2 \times C_s.$$

$\therefore$ coefficient of fluctuatn of speed,

$$C_s = 0.0382 \quad \text{or} \quad 3.8\%$$

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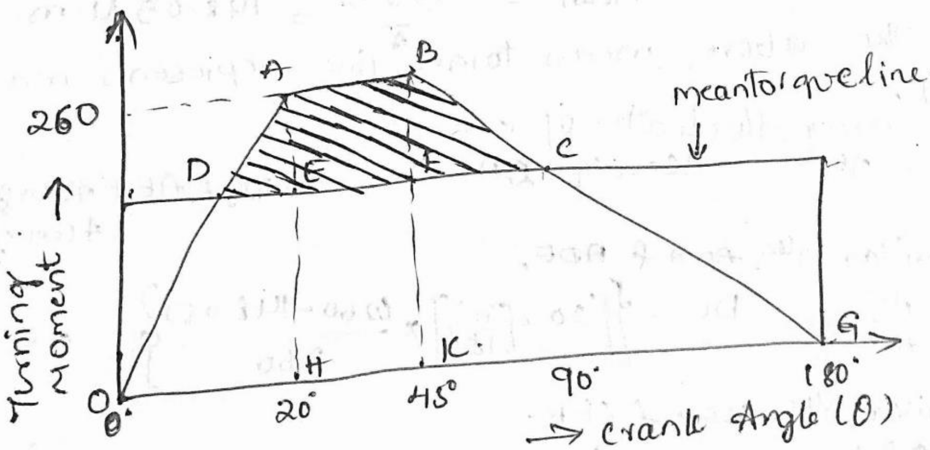
(4) The variation of crank shaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° & 180° and as 260 N-m for crank angles 20° & 45°, the intermediate portions of torque graph being straight lines. The cycle is being repeated in every half revolution. The average speed is 600 rpm. Supposing that engine drives a machine at constant torque, determine the mass of flywheel of radius of gyration 250 mm, which must be provided so that total variation of speed shall be one percent.

Sol: Given Data:-

$N = 600$ rpm; $K = 250$ mm $= 0.25$ m

$C_s = 1\% = 0.01$

The turning moment diagram,



Work done for half revolution

= Area of turning moment diagram

= Area of OABG.

= [Area of OAH] + [Area of HABK] + [Area of KBG]

$$= \left[\frac{1}{2}\left(20^\circ \times \frac{\pi}{180}\right) \times 260\right] + \left[(45^\circ - 20^\circ)\left[\frac{\pi}{180}\right) \times 260\right] + \left[\frac{1}{2} \times (180 - 45^\circ)\left[\frac{\pi}{180}\right] \times 260\right].$$

= 465.13 N-m.

If T_{mean} is the mean torque, then work done corresponding of mean torque for half revolution, is given by,

$$T_{mean} \times \pi = \text{work done} = 465.13.$$

$$T_{mean} = \frac{465.13}{\pi} = 148.05 \text{ N-m.}$$

Since the above mean torque line represents max. fluctuation of energy,

∴ max. fluctuation of energy,

ΔE = Area of ΔABC = Area of ΔAE + Area of EABF + Area of CFB.

From similar △les AOH & ADE,

$$\frac{DE}{OH} = \frac{AE}{AH} \Rightarrow DE = \left\{\left[20^\circ \times \left[\frac{\pi}{180}\right]\right] \times \frac{(260 - 148.05)}{260}\right\}. = 0.15 \text{ rad.}$$

From similar △les, BGK & CFB.

$$\frac{FC}{KG} = \frac{BFB}{BK} \Rightarrow FC = \left\{\left[(180 - 45^\circ)\left[\pi/180\right]\right] \times \frac{(260 - 148.05)}{260}\right\} = 1.01 \text{ rad.}$$

$$\therefore \Delta E = \left[\frac{1}{2} \times 0.15 \times 119.95\right] + [0.4363 \times 111.95] \times \left[\frac{1}{2} \times 111.95 \times 1.014\right]$$

ΔE = 114 N-m.

W.K.T, $\Delta E = m \cdot k^2 \omega^2 C_s$.

$114 = m \cdot (0.25)^2 \cdot (62.83)^2 \cdot (0.01)$;

$$\underline{m = 46.2 \text{ kg}}$$

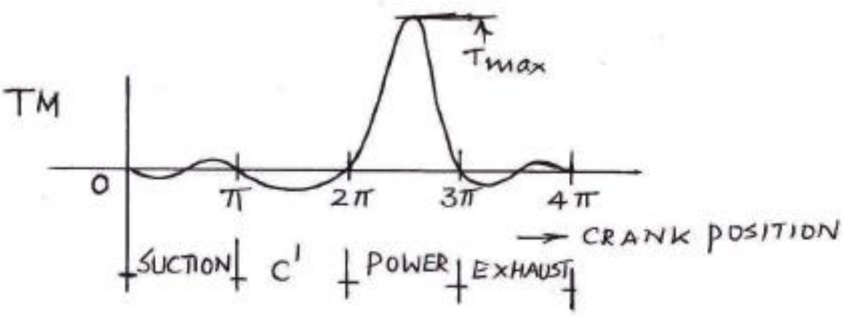
Difference between Governor and Flywheel:

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job.
Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

Crank effort diagrams or Turing Moment diagrams:
It is the graphical representation of turning moment or crank effort for the various positions of the crank. The TM is plotted on ‘y’ axis and crank angle on ‘x’ axis.

Uses of turning moment Diagram:
1) The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.
2) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us the find the fluctuation of energy.
3) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us the find diameter of the crank shaft.

TMD for a four stroke I.C. Engine



We know that four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions (4p or 720°). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke therefore a negative loop is formed as shown in figure. During compression stroke the work done on engine the gases therefore a higher negative loop is obtained. During expansion or working stroke the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases; therefore a negative loop is firmed.

Problem 2

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

Solution

θ, deg.	$T, N-m$
0	500
90	1500
180	500
270	1500
360	500

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T \, d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta)\, d\theta$$

$$= 2000\pi \ N-m$$

$$T_{mean} = \frac{W.D / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \ N-m$$

$$\text{Power developed } = T_{mean} \times \omega_{mean}$$

$$= 1000 \times \frac{2\pi \ N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

Problem: 3

The turning moment curve for an engine is represented by the equation,

$T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:

1. Power developed by the engine;
2. Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not the exceed 1% of mean speed which is 180 r.p.m. and
3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.

Solution:

Given, $T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m ;

N = 180 r.p.m. or $\omega = 2\pi \times 180/60 = 18.85$ rad/s

Since the total fluctuation of speed $(\omega_1 - \omega_2)$ is 1% of mean speed (ω), coefficient of fluctuation of speed,

$$\delta = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. Power developed by the engine.

Work done per revolution

$$= \int_0^{2\pi} T d\theta = \int_0^{2\pi} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta) d\theta$$

$$= \left[20000\ \theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20000 \times 2\pi = 40\ 000\ \pi\ N-m$$

Mean resisting torque of the engine,

$$T_{mean} = \frac{Work\ done\ per\ revolution}{2\pi} = \frac{40\ 000\ \pi}{2\pi} = 20000\ N-m$$

Power developed by the engine

$$= T_{mean}.\omega = 20\ 000 \times 18.85 = 377\ 000 W = 377 kW.$$

2. Moment of inertia of the flywheel

The turning moment diagram for one stroke (i. e. half revolution of the crankshaft) is shown in the Fig. Since at points *B* and *D*, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

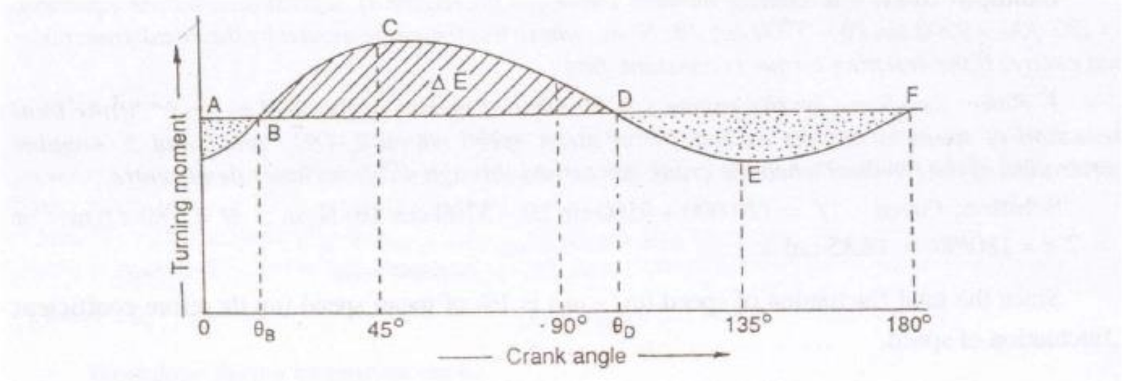
$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000$$

or $$9500 \sin 2\theta = 5700 \cos 2\theta$$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$\therefore$ $$2\theta = 31^\circ \; or \theta = 15.5^\circ$$

$\therefore$ $$\text{i.e.,} \theta_B = 15.5^\circ \; and \; \theta_D = 90^\circ + 15.5^\circ = 105.5^\circ$$



Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta$$

$$= \int_{15.5^\circ}^{105.5^\circ} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\;000)\, d\theta$$

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta \;\; = \left[-\frac{9500 \sin 2\theta}{2} - \frac{5700 \cos 2\theta}{2} \right]_{15.5^\circ}^{105.5^\circ} = 11078 \; N - m$$

Maximum fluctuation of energy (ΔE),

$$11\,078 = I.\omega\;\; \delta \;\; = I(18.85)^2 \, 0.01 = 3.55\, I$$

$$I = 11078/3.55 = 3121 \text{ kg-m}^2.$$

3. Angular acceleration of the flywheel

Let α = Angular acceleration of the flywheel, and

θ = Angle turned by the crank from inner dead centre = 45°... (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. Excess torque at any instant,

$$T_{excess} = T - T_{mean}$$

$$20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\ 000$$

$$9500 \sin 2\theta - 5700 \cos 2\theta$$

$\therefore$ Excess torque at 45°= 9500 sin 90° - 5700 cos 90° =9500Nm

We also know that excess torque= $I.\alpha$ = 3121 xα

From equations (i) and (ii),

α = 9500 / 3121 = 3.044 rad/s^2.

Problem 5: The equation of the turning moment diagram of a three crank engine is 21000+7000 sin3θ Nm. Where θ in radians is the crank angle. The moment of inertia of the flywheel is $4.5 \times 10^3\ Nm^2$ and the mean engine speed is 300 rpm. Calculate the power of the engine and the total percentage fluctuation of speed of the flywheel (i) if the resisting torque is constant (ii) if the resisting torque is 21000 + 3000 sinθ Nm.

a) $T_m = 21000\,Nm.$

$$\text{Power} = \frac{2\pi \times 21000 \times 300}{60} = 660\,kW.$$

b) (i) $$\Delta E = \int_0^{\frac{\pi}{3}} 7000 \sin 3\theta d\theta = 4666.7\ Nm.$$

$$\therefore \text{Total percent fluctuation of speed} = \frac{100\ \Delta E}{I\omega^2_{mean}}$$

$$= \frac{100 \times 4666.7 \times 9.8}{45 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.04\%$$

(ii) Engine torque = load torque, at crack angles given by

7000 sin3θ = 3000 sinθ

i.e., 2.33 (3sinθ - 4 sin$^3\theta$) = sinθ

One solution is sinθ =0, i.e., θ = 0 and 180º, and the other is sinθ = ±0.803, i.e., θ = 53°24' or 126°36' between 0° and 180º. The intersections are shown in figure and the areas between the curves represent increase or decrease of total energy. The numerically longest is between θ = 53° 24' and 126º36'.

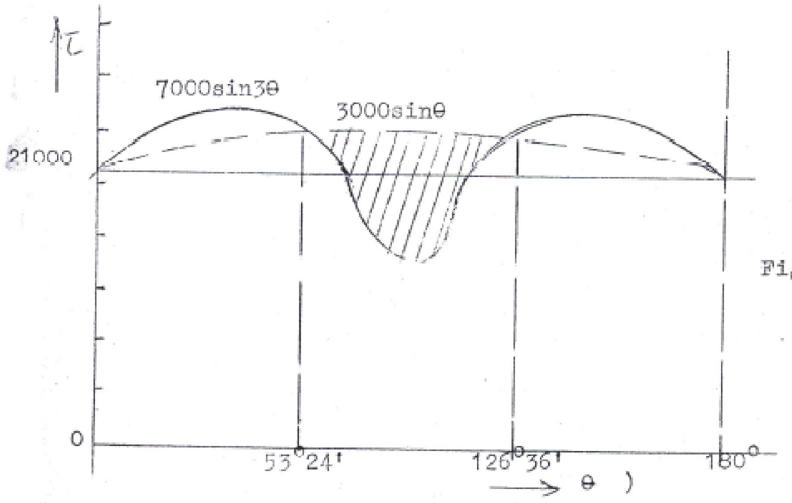


Fig.66.

$$i.e.,\ \Delta E = \int_{53°24'}^{126°36'} (7000\ \sin\ 3\theta - 3000\ \sin\ \theta)\ d\theta$$

=7960 Nm.

Therefore, the total (percentage) fluctuation of speed $\frac{100\ \Delta E}{I\omega^2_{mean}}$

$$= \frac{100 \times 7960 \times 9.8}{4.5 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.65\%$$

A machine punching 3.8 cm dia hole in a 3.2 cm thick plate does 600 J of work / sq. cm of sheared area. The punch has a stroke of 10.2 cm and punches 6 holed / min. The maximum speed of the flywheel at its radius of gyration is 27.5 m/s. Find the mass of the flywheel so that its speed at the same radius does not fall below 24.5 m/s. Also determine the power of the motor, driving this machine.

d = 3.8cm, t = 3.2 cm, A= 38.2 cm^2

Energy required / punch = 600 x 38.2 = 22.920 J

$$\text{Assuming, } \frac{(\theta_2 - \theta_1)}{(2\pi)} = \frac{t}{2S} = \frac{3.2}{20.4}$$

$$\therefore (\Delta K_E)_{\max} = E\left[1 - \frac{t}{2S}\right] = \frac{1}{2} I\left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$= 22.920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} mk^2\left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$V_{\max} = k\ \omega_{\max} = 27.5 m/s$$

$$V_{\min} = k\ \omega_{\min} = 24.5 m/s$$

We get,

$$22920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} m\left(27.5^2 - 24.5^2\right) = \frac{1}{2} m\ 158$$

$$\therefore\ m = 244 kg.$$

The energy required / minute is $6 \times 22920 J$

$$\therefore\ Motor\ \ power\ = \frac{6 \times 22920}{1000 \times 60} k\omega = 2.292 kW$$

A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute.

Solution.

Given: $P = 3$ *kW; m*= 150 *kg; k* = 0.6m; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$rad/s

Speed of the flywheel immediately after riveting

Let ω_2 = Angular speed of the flywheel immediately after riveting.

We know that, energy supplied by the motor,

$$E_2 = 3 \ kW = 3000 \ W = 3000 \ N - m/s \qquad (\because 1W = 1 \ N - m/s)$$

But, energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\ 000 \ N - m$$

$\therefore$ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\ 000 - 3000 = 7000 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150\ (0.6)^2 \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

$$\therefore \qquad (\omega_2)^2 = 987.2 - 7000/27 = 728 \ or \ \omega_2 = 26.98 \ rad/s$$

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60 / 2\pi = 257.6 \ r.p.m.$$

Number of rivets that can be closed per minute.

Since, the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\ 000} \times 60 = 18 \ rivets$$

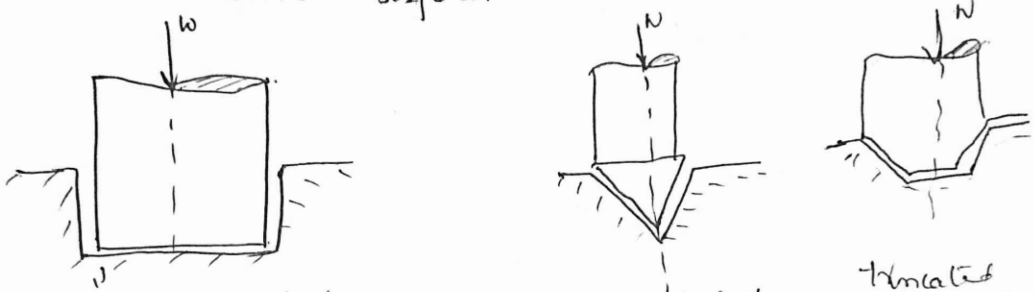
UNIT-III

Friction, Brakes & Dynamometers

PIVOT BEARING

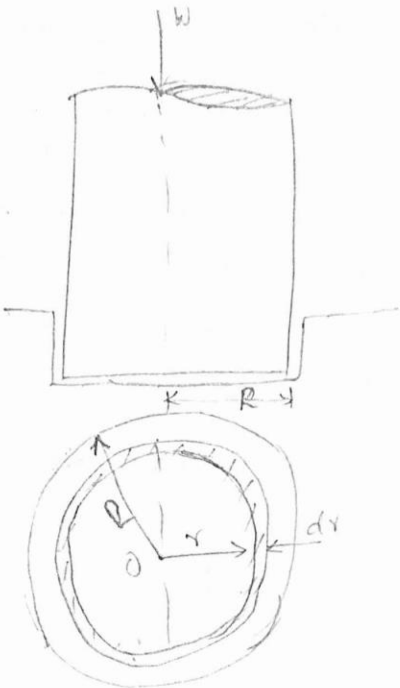
(1)

The rotating shafts are frequently subjected to axial thrust. These shafts can be kept in correct axial position if bearing surfaces are provided. The bearing surfaces which are flat (or) conical carry the axial thrust. The bearing surfaces placed at the end of a shaft are known as pivots. The pivot may flat, conical (or) truncated conical surfaces.



Flat (bearing) pivot   Conical pivot   Truncated pivot

* **Flat Pivot :-**



The bearing surface placed at the end of shaft is known as pivot. If the surface is flat as shown, then bearing surface is called flat-pivot (or)-foot-step. There will be friction along the surface of contact between shaft & bearing. The power lost can be obtained by calculating torque.

let, W → Axial load, (or) load transmitted to the bearing surface

R → Radius of pivot.

μ → co-effecient of friction.

p → Intensity of Pr, N/mm².

T → Total frictional torque.

r → radius of ring

dr → thickness of ring.

③

Consider a circular ring of ~~thickness~~ Radius 'r' & thickness dr as shown.

$\therefore$ Area of ring $= 2\pi r \cdot dr$.

We will consider 2 cases; namely:

(i) Uniform Pressure over bearing surface &

(ii) Uniform wear over bearing surface

(i) Case of Uniform Pr.:

When the Pr is assumed to be uniform over the bearing surface, then intensity of pressure is given by,

$$p = \frac{\text{Axial load}}{\text{Area of c/s}} = \frac{W}{\pi R^2} \quad -(1)$$

Now, the load transmitted to the ring & frictional torque on the ring,

Load transmitted to the ring,

$$dW = \text{Pr on ring} \times \text{Area of ring}$$
$$= p \times 2\pi r\, dr$$

frictional force on ring,

$$dF = \mu \times dW$$
$$= \mu \times \text{load on ring}$$
$$= \mu \times p \times 2\pi r\, dr$$

But,

~~&~~ Frictional torque on the ring) Moment of frictional force about that axis,

$$dT = \text{Frictional force} \times \text{Radius of ring}$$
$$= dF \times r.$$

$$\therefore \quad dT = \mu \times p \times 2\pi \cdot r \cdot dr \cdot r$$
$$= \mu \cdot p \times 2\pi r^2 \cdot dr \quad -(a)$$

Now, the total frictional torque will be obtained by integrating above eq. ⓐ.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi \mu p r^2 \cdot dr$

$= 2\pi \mu p \int_0^R r^2 \cdot dr$

$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_0^R$

$= \frac{2}{3} \mu \pi p R^3$

$= \frac{2}{3} \pi \times \mu \times R^{\cancel{3}} \times \frac{W}{\pi \cancel{R^2}}$ $\qquad \left[\because p = \frac{W}{\pi R^2}\right]$

$$\boxed{T = \frac{2}{3} \mu W R}$$

$\therefore$ Power lost in frictn $= T \times \omega$

$= \frac{T \times 2\pi N}{60}$

$= \frac{2\pi N T}{60}$

(ii) In case of Uniform Wear :- For uniform wear of bearing surface, the load transmitted to the various circular rings should be same.

But load transmitted to any circular ring is equal to the product of pressure & area of ring. Area of ring is directly proportional to the radius of ring. Hence for uniform wear, the product of pr radius should be constant. i.e. $p \times r =$ constant.

For uniform wear, $p \times r =$ constant

i.e. $p \times r = C$.

$\therefore p = \frac{C}{r}$ — ⓐ.

(5)

Load transmitted to the ring,

$= p_r \times$ Area of ring

$= p \times 2\pi r \cdot dr$

$= \frac{C}{r} \times 2\pi \cdot r \cdot dr$

$dW = 2\pi C \cdot dr \quad -(6).$

Total load transmitted to the bearing, is obtained by integrating from '0 to R'

∴ Total load transmitting to the bearing,

$W = \int_0^R dW$

$= \int_0^R 2\pi C\, dr = 2\pi C \int_0^R dr = 2\pi C [r]_0^R$

$W = 2\pi C R$

$\boxed{C = \frac{W}{2\pi R}}$

Now frictional ~~torque~~ force on the ring,

$dF = \mu \times$ load on ring $= \mu \times dW$

$= \mu \times 2\pi C dr$

Hence frictional torque on the ring,

$dT =$ Frictional force on ring × radius

$= dF \times r$

$= \mu \times 2\pi C \cdot dr \cdot r$

$= \mu \cdot 2\pi C \cdot r \cdot dr$

∴ Total frictional torque, $T = \int_0^R dT$

$= \int_0^R \mu 2\pi C \cdot r \cdot dr$

$= 2\pi \cdot C \cdot \mu \cdot \int_0^R r\, dr$

$= 2\pi C \cdot \mu \left[\frac{r^2}{2}\right]_0^R = 2\pi C \cdot \mu \left[\frac{R^2}{2}\right]$

$= 2\pi \times \frac{W}{2\pi R} \times \mu \left[\frac{R^2}{2}\right]$

$\boxed{T = \frac{1}{2} \mu W R}$

∴ Power lost in friction, $P = \frac{2\pi N T}{60}$

Problem ①:- Find the power lost in friction assuming (i) uniform Pr. & (ii) Uniform wear. When a vertical shaft of 100mm dia. rotating at 150rpm rests on a flat end-foot step bearing. The coeffecient of friction is equal to 0.05 & shaft carries a vertical load of 15 kN.

Sol:-

Given:-

Dia, $D = 100mm \Rightarrow 0.1m$ $\therefore R = \frac{0.1}{2} = 0.05m$

$N = 150$ rpm ; co-effecient of friction, $\mu = 0.05$

load, $W = 15 kN = 15 \times 10^3 N$.

(i) Power lost in friction assuming uniform pressure,

For uniform Pr.,

$$T = \frac{2}{3} \mu W R$$

$$T = \frac{2}{3}(0.05)(15 \times 10^3)(0.05)$$

$$T = 25 \text{ N-m}$$

Power lost, $P = \frac{2\pi N T}{60} \Rightarrow \frac{2\pi 150 \times 25}{60}$

$$\boxed{P = 392.7 W}$$

(ii) For uniform wear,

$$T = \frac{1}{2} \mu W R.$$

$$= \frac{1}{2} \times 0.05 \times 15 \times 10^3 \times 0.05$$

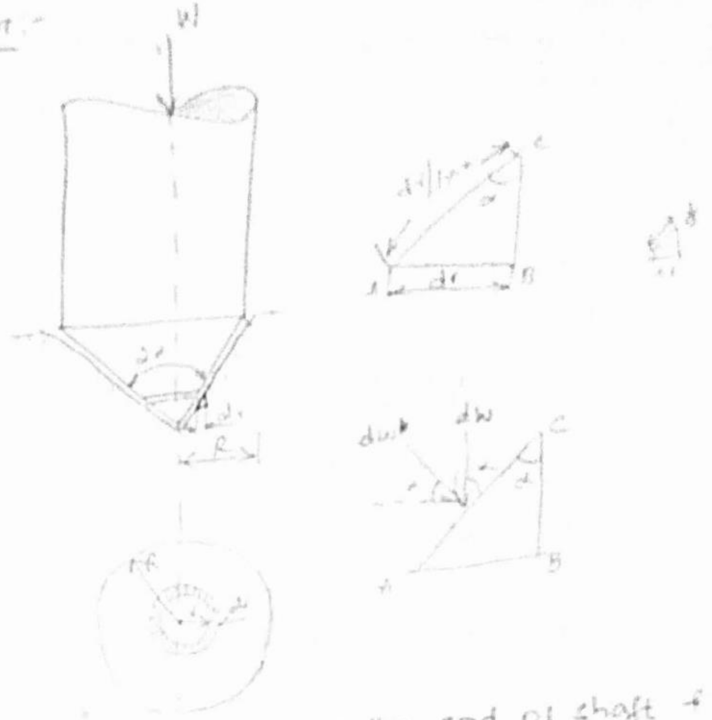
$$T = 18.75 Nm$$

$$P = \frac{2\pi N T}{60}$$

$$P = \frac{2\pi \times 150 \times 18.75}{60}$$

$$\boxed{P = 294.5 W}$$

CONICAL PIVOT :-



The bearing surface placed at the end of shaft & having a conical surface, is known as conical pivot bearing as shown.

let, $W \rightarrow$ Axial load
$\mu \rightarrow$ co-efficient of friction
$\alpha \rightarrow$ semi-angle of cone
$p \rightarrow$ P_r intensity to the cone surface

consider a circular ring of radius 'r' & thickness dr. The actual thickness of the sloping ring will be $dr/\sin\alpha$ as shown in which $AB = dr$. (on enlarged scale)

angle $ACB = \alpha$ and sloping length of ring,

$$\sin\alpha = \frac{AB}{AC} \Rightarrow AC = \frac{AB}{\sin\alpha}$$

$$AC = dr/\sin\alpha$$

$\therefore$ Area of ring along conical surface

$= 2\pi r \times$ actual thickness of sloping ring.

$= 2\pi r \times dr/\sin\alpha$

Now assuming 2 cases
(i) Uniform Pressure
(ii) Uniform Wear

(i) for Uniform Pressure :-

Load acting on the circular ring, normal to the conical surface.

$\therefore$ load on the ring normal to conical surface,

$$dW = P_r \times \text{Area of ring along conical surface}$$

$$dW^* = p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

Vertical component of above load,

$$dW = \left[p \times 2\pi r \times \frac{dr}{\sin\alpha}\right] \cdot \sin\alpha$$

$$\therefore \quad dW = p \times 2\pi r \cdot dr.$$

$\therefore$ Total vertical load transmitted to bearing

$$W = \int_0^R p \times 2\pi r \, dr$$

$$= p \times 2\pi \int_0^R r \, dr$$

$$= p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \left[\frac{R^2}{2}\right] \;:) \quad p\pi R^2 = W \quad (a)$$

i.e., $W = p\pi R^2$

$$p = \frac{W}{\pi R^2} \quad (b)$$ [This eq (b) shows that P_r intensity is independent on angle of conical surface].

Now the frictional force on the conical ring,

$dF = \mu \times$ load on ring normal to conical surface

$$= \mu \times dW^*$$

$$dF = \mu \times p \times 2\pi r \times dr / \sin\alpha]$$

(7)

Moment of this frictional force about the shaft [dT]

= Frictional torque on ring

= Frictional force × Radius => $dF \times r$

$$= \mu \times \left[p \times 2\pi r \cdot \frac{dr}{\sin\alpha} \right] \cdot r$$

$$= \mu p \cdot 2\pi \cdot r^2 \frac{dr}{\sin\alpha} \quad \text{--- (c)}$$

Total moment of the frictional force about shaft axis or total frictional torque on conical surface is obtained by integrating.

∴ Total frictional torque,

$$T = \int_0^R dT$$

$$= \int_0^R \mu p \cdot 2\pi r^2 \cdot dr / \sin\alpha$$

$$= \frac{\mu p \cdot 2\pi}{\sin\alpha} \int_0^R r^2 \cdot dr = \frac{2\pi \mu \cdot p}{\sin\alpha} \left[\frac{r^3}{3} \right]_0^R$$

$$T = \frac{2\pi \mu p}{\sin\alpha} \left(\frac{R^3}{3} \right)$$

$$= \frac{2\cancel{\pi} \mu}{\sin\alpha} \times \frac{W}{\cancel{\pi} R^{\cancel{2}}} \left[\frac{R^3}{3} \right]$$

$$\boxed{T = \frac{2}{3} \frac{\mu W R}{\sin\alpha}}$$

∴ Power lost in friction $= \frac{2\pi N T}{60}$

$$\boxed{P = \frac{2\pi N T}{60}}$$

(ii) Case of Uniform Wear: — (10)

For the uniform wear, the load transmitted to various circular rings to be constant:

$$p \times r = C.$$

$$p = \frac{C}{r}$$

The total vertical load transmitted to the bearing.

$$= \int_0^R p \times 2\pi r \times dr.$$

$$= \int_0^R \frac{C}{r} \times 2\pi r \times dr$$

$$= \int_0^R C\, 2\pi\, dr = 2\pi C \int_0^R dr = 2\pi C\, [r]_0^R$$

$$W = 2\pi C\, [R]$$

But ~~total~~ vertical load transmitted to bearing is also equal to W

$$\therefore W = 2\pi C R$$

$$C = W/2\pi R.$$

Now the frictional torque on ring is given by

$$dT = \text{Frictional force} \times \text{Radius}$$

$$= dF \times r.$$

$$= \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \times r$$

$$= \mu\, p \times 2\pi r^2 \times dr/\sin\alpha$$

$$= \mu \cdot \frac{C}{r} \times 2\pi r^2 \times dr/\sin\alpha$$

$$= \mu C\, 2\pi r\, dr/\sin\alpha \Rightarrow \mu \times \frac{W}{\cancel{2\pi} R} \times \cancel{2\pi} r \cdot \frac{dr}{\sin\alpha}$$

$$dT = \frac{\mu W\, r\, dr/\sin\alpha}{R}$$

Total frictional torque,

$$T = \int_0^R dT$$

$$= \int_0^R \frac{\mu W}{R} \cdot r\, dr/\sin\alpha$$

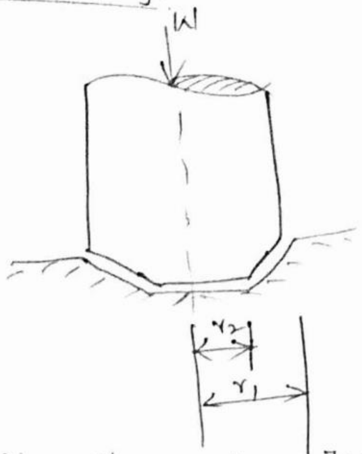
$$= \frac{\mu W}{R \sin\alpha} \int_0^R r \cdot dr \Rightarrow \frac{\mu W}{R \sin\alpha} \left[\frac{r^2}{2}\right]_0^R = \frac{\mu W}{R \sin\alpha} \left[\frac{R^2}{2}\right]$$

$$\boxed{T = \frac{1}{2} \frac{\mu W R}{\sin\alpha}}$$

Power lost in friction, $P = 2\pi N T/60$

(17).

* Truncated Pivot Bearing:-



The above fig. shows truncated conical pivot of external & internal radii r_1 & r_2 respectively.

(i) Case of Uniform Pressure:-

vertical load transmitted to the bearing

$$dW = p \times 2\pi r \times dr \quad -(a)$$

For total vertical load, integrating with limits r_2 to r_1.

$$W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= p\, 2\pi \int_{r_2}^{r_1} r\,dr = p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$W = p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$p = \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \quad -(b).$$

frictional force, $\mu \times dW$ normal to the surface $= \mu \times p \times 2\pi r\, dr / \sin\alpha$

Frictional torque on the rings.

$$dT = \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \times r.$$

Total frictional torque, $T = \int_{r_2}^{r_1} dT$

$$T = \int_{r_2}^{r_1} \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \cdot r$$

$$= \frac{2\pi \times \mu \cdot p}{\sin\alpha} \int_{r_2}^{r_1} r^2 dr$$

$$= \frac{2\pi \mu \cdot p}{\sin\alpha} \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = \frac{2\pi \times \mu \cdot p}{\sin\alpha} \left[\frac{(r_1^3 - r_2^3)}{3}\right]$$

$$= \frac{2\cancel{\pi} \times \mu}{3\sin\alpha} \cdot \frac{W}{\cancel{\pi}(r_1^2 - r_2^2)} \cdot (r_1^3 - r_2^3)$$

$$\therefore \boxed{T = \frac{2}{3} \frac{\mu W}{\sin\alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

Power lost in friction, $P = \frac{2\pi N T}{60}$.

(ii) Uniform Wear:

$$p \times r = C.$$

$$p = C/r.$$

Vertical load transmitted, $dW = p \times 2\pi r\, dr$

Total vertical load, $W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$

$$= \int_{r_2}^{r_1} \frac{C}{\cancel{r}} \cdot 2\pi \cancel{r}\, dr \Rightarrow 2\pi C \int_{r_2}^{r_1} dr = 2\pi C \left[r\right]_{r_2}^{r_1}$$

$$W = 2\pi C [r_1 - r_2]$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

total frictional torque, $T = \int_{r_2}^{r_1} 2\pi \mu \times C \times r \times dr / \sin\alpha$

$$T = \frac{2\pi \mu \cdot C}{\sin\alpha} \int_{r_2}^{r_1} r \cdot dr \Rightarrow T = \frac{1}{\sin\alpha} \cdot 2\pi \cdot \mu \cdot C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

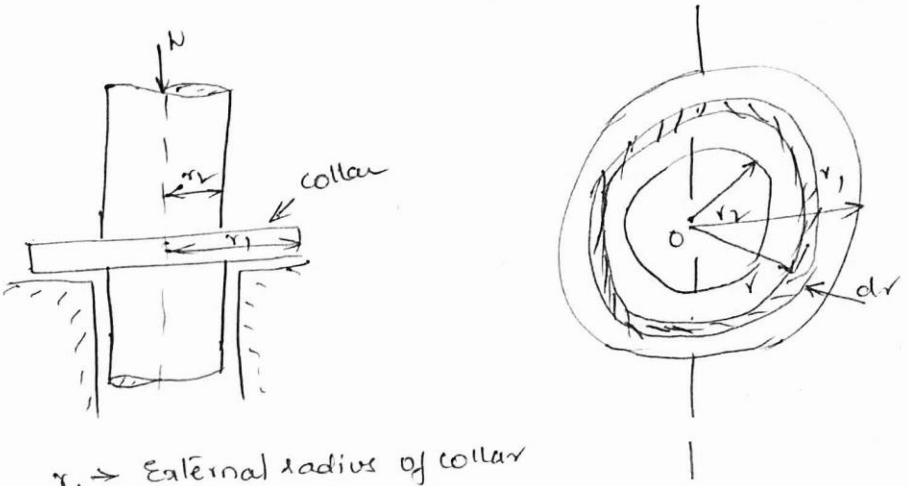
$$\Rightarrow T = \frac{1}{\sin\alpha} \cancel{2\pi} \cdot \mu \cdot \frac{W}{\cancel{2\pi}(r_1 - r_2)} \left[\frac{(r_1^2 - r_2^2)}{\cancel{2}}\right]$$

$$\boxed{T = \frac{1}{2} \frac{\mu W (r_1 + r_2)}{\sin\alpha}}$$

Power lost in friction, $\boxed{P = \frac{2\pi N T}{60}}$

(13).

Flat collar:- The bearing surface provided at any position on the shaft (but not at the end) to carry axial thrust is known as collar. Collar bearings are also known as thrust bearings



let, $r_1 \rightarrow$ External radius of collar

$r_2 \rightarrow$ Internal radius of collar

$p \rightarrow$ intensity of Pr.

$W \rightarrow$ Axial load or total load transmitted to bearing surface

$\mu \rightarrow$ co-effecient of friction

$T \rightarrow$ Total frictional torque.

consider a circular ring of radius 'r' & thickness dr

$\therefore$ Area of ring, $= 2\pi r \cdot dr$.

load on ring, $=$ Pr $\times$ Area of ring

$= p \times 2\pi \cdot r \cdot dr$.

Friction Force on ring, $= \mu \times$ load on ring

$= \mu \times p\, 2\pi r\, dr$.

Friction torque $=$ frictn. force $\times$ Radius

$= p \cdot \mu \times 2\pi r\, dr \times r$

$= 2\pi \mu p\, r^2 \cdot dr$.

$\therefore$ total frictional torque, $T = \int_{r_2}^{r_1} dT$

$$T = \int^{r_1} 2\pi \mu \cdot p r^2 \cdot dr.$$

(i) Uniform Pressure:-

$$p = \text{constant}.$$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} \text{load on ring } (dW)$$

$$= \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= p \times 2\pi \int_{r_2}^{r} r\, dr. \Rightarrow p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$\Rightarrow p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right] \Rightarrow p \times \pi \left[r_1^2 - r_2^2\right] = W$$

$$p = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}.$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu \cdot p r^2\, dr$$

$$= 2\pi \mu \cdot p \int_{r_2}^{r_1} r^2 \cdot dr$$

$$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi \mu p \cdot \left(\frac{r_1^3 - r_2^3}{3}\right)$$

$$= \frac{2}{3} \pi \mu \cdot \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \cdot \left[r_1^3 - r_2^3\right] \Rightarrow \boxed{T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

power lost in friction, $P = \frac{2\pi N T}{60}$

15

ii) for Uniform Wear:

$$p \times r = \text{constant}$$

$$p \times r = C$$

$$p = C/r$$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} \text{load in ring} = \int_{r_2}^{r_1} dW$$

$$W = \int_{r_2}^{r_1} p \times 2\pi r \, dr$$

$$W = \int_{r_2}^{r_1} \frac{C}{r} \cdot \times 2\pi \, dr \cdot r$$

$$= 2\pi \cdot C \int_{r_2}^{r_1} dr$$

$$= 2\pi C \, [r]_{r_2}^{r_1} \Rightarrow 2\pi C [r_1 - r_2] = W$$

$$\Rightarrow C = \frac{W}{2\pi [r_1 - r_2]}$$

Total frictional torque

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} dF \times r$$

$$= \int_{r_2}^{r_1} 2\pi \mu p \cdot r^2 \cdot dr$$

$$= 2\pi \mu \cdot \int_{r_2}^{r_1} \frac{C}{r} \cdot r^2 \cdot dr$$

$$= 2\pi \mu C \int_{r_2}^{r_1} r \, dr = \pi \mu C [r_1^2 - r_2^2] = T$$

$$= \pi \mu \cdot \frac{W}{2\pi (r_1 - r_2)} [r_1^2 - r_2^2]$$

$$\boxed{T = \frac{1}{2} \mu W (r_1 + r_2)}$$

Power lost in frictn, $P = \frac{2\pi N T}{60}$

Problems

1. A conical pivot with an angle of cone is 120°, supports a vertical shaft of dia 300 mm. It is subjected to load of 20 kN. The coeff. of friction is 0.05 & the speed of shaft is 210 rpm. Calculate the power lost in friction assuming (i) Uniform Pr., (ii) Uniform wear.

A load angle 350 N

Sol: **Given**

$2\alpha = 120°;\quad \alpha = 60°.$

$D = 300mm;\ R = 150mm;\ R = 0.15m;$

$W = 20\ kN = 20\times10^3 N;\quad \mu = 0.05$

$N = 210\ rpm$

(i) Considering Uniform Pr.

$$T = \frac{2}{3}\frac{\mu W R}{\sin\alpha}$$

$$= \frac{2}{3}\times\frac{(0.05)(20\times10^3)\times(0.15)}{\sin(60°)}$$

$$\Rightarrow T = 115.53\ N\text{-}m;$$

Power lost in friction, $P = \frac{2\pi N T}{60} \Rightarrow \frac{2\pi\times210\times115.53}{60}$

$$\boxed{P = 2540.6\ W}\ \text{Ans}$$

(ii) Considering Uniform Wear:

$$T = \frac{1}{2}\frac{\mu W R}{\sin\alpha}$$

$$= \frac{1}{2}\times\frac{(0.05)(20\times10^3)(0.15)}{\sin(60°)}$$

$$T = 86.60\ N\text{-}m$$

Power lost in friction,

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi\times210\times86.60}{60}$$

$$\boxed{P = 1904\ W}$$

(17)

A load of 25 kN is supported by a conical pivot with angle of cone as 120°. The intensity of Pr is not to exceed 350 kN/m². The external radius is 2 times the internal radius. The shaft is rotating at 180 rpm & $\mu = 0.05$. Find the power absorbed in friction assuming uniform pr.

Sol: Given:

$W = 25\,kN = 25\times10^3\,N$;

$2\alpha = 120°$; $\alpha = 60°$; $p = 350\,kN/m^2 = 350\times10^3\,N/m^2$

$r_1 = 2r_2$; $N = 180\,rpm$; $\mu = 0.05$.

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

$$350\times10^3 = \frac{25\times10^3}{\pi\left[(2r_2)^2 - r_2^2\right]} \Rightarrow \quad 350\times\pi r_2^2 = 250$$

$r_2 = 0.087\,m$; $r_1 = 2r_2$

$r_1 = 2(0.087) \Rightarrow r_1 = 0.174\,m$.

For Uniform pr.:

$$T = \frac{2}{3}\frac{\mu W}{\sin\alpha}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3}\times\frac{(0.05)(25\times10^3)}{\sin 60°}\left[\frac{(0.174)^3 - (0.087)^3}{(0.174)^2 - (0.087)^2}\right]$$

$T = 195.37\,N\text{-}m$

Power absorbed, $P = \dfrac{2\pi NT}{60} \Rightarrow$

$$= \frac{2\pi\times180\times195.37}{60}$$

$P = 3682.6\,W \Rightarrow \boxed{P = 3.68\,kW}$

* NOTE:-

(1) If the axial load on the bearing is too great, then the bearing pr. on the collar will become more than limiting bearing pr. which is appr. equal to $400 kN/m^2$. Hence, to reduce the intensity of pr. on collar, two (or) more collars are used.

If $n \rightarrow$ no. of collars in multi-collar bearing, then

(i) $n = \dfrac{\text{Total load}}{\text{load permissible on one collar}}$

(ii) p = Intensity of uniform pr.

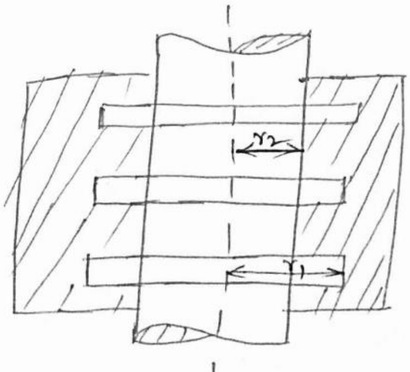
$= \dfrac{\text{Load}}{\text{No. of collars} \times \text{Area of one-collar}}$

$= \dfrac{W}{n \times \pi (r_1^2 - r_2^2)}$

(iii) Total torque transmitted remains constant i.e;

$$T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right].$$

(2) The frictional torque for uniform pr. is greater than that of uniform wear. Hence for safe design of bearing surfaces when power lost in frictn is to be determined & no assumption is mentioned, then assume uniform pr. But when power transmitted is to be determined & no assumptn is given, then assume uniform wear.



multi-collar Bearing

Q. (20)

In a thrust bearing, the external & internal radii of contact surfaces are 210mm & 160mm respectively. The total axial load is 60KN & coefficient of friction = 0.05. The shaft is rotating 380rpm intensity of pressure is not to exceed 350KN/m². calculate:

(i) power lost in overcoming the friction &

(ii) no. of collars reqd. for thrust bearing

Sol:- **Given:-**

External radius, $r_1 = 210mm = 0.21m$

Internal radius, $r_2 = 160mm = 0.16m$

$W = 60KN = 60\times10^3 N$; $\mu = 0.05$

$N = 380rpm$; $p = 350KN/m^2$

$= 350\times10^3 N/m^2$.

Here, the power lost in overcoming the friction is to be determined. Also no assumptn is given, hence it is safe to assume uniform pr.

(i) Powerlost in overcoming frictn.

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$T = 558.378 N\text{-}m$

$$\therefore P = \frac{2\pi NT}{60} = \frac{2\pi\times380\times558.3}{60} \Rightarrow \boxed{P = 22219.8W}$$

(ii) No. of collars reqd:

$$\text{No. of collars, } n = \frac{\text{Total load}}{\text{load per collar}}$$

load per collar, we have, $p = \dfrac{W^*}{\pi(r_1^2 - r_2^2)}$

W^* is load per collar, $W^* = p\times[\pi(r_1^2 - r_2^2)]$

$W^* = 20341.8N$

$$\therefore \text{No. of collars } n = \frac{60\times10^3}{20341.8} = 2.95$$

$\simeq$ 3 collars

Problems

1. In a collar thrust bearing the external & internal radii are 250mm & 150mm respectively. The total axial load is 50kN & Shaft is rotating at 150 rpm. The co-effecient of friction is equal to 0.05. Find the power lost in friction assuming uniform pressure.

single Pla... ke...

Sol:- Given:-

External radius = 250mm = 0.25 m = r_1

Internal radius = 150mm = 0.15 m = r_2

$W = 50KN = 50\times10^3$ N ; N = 150 rpm

$\mu = 0.05$;

For uniform pr. total frictional torque,

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3}\, 0.05\times 50\times10^3\left[\frac{(0.25)^3 - (0.15)^3}{(0.25)^2 - (0.15)^2}\right]$$

$T = 510.42$ N-m

∴ power lost in friction, $P = \frac{2\pi NT}{60}$

$$= \frac{2\pi(150)(510.42)}{60}$$

$P = 8017.6$ W

$P = 8.01$ kW

(21)

Single Plate clutch:

let, $r_1 \rightarrow$ External radius of friction lining on clutch plate

$r_2 \rightarrow$ Internal radius of friction lining

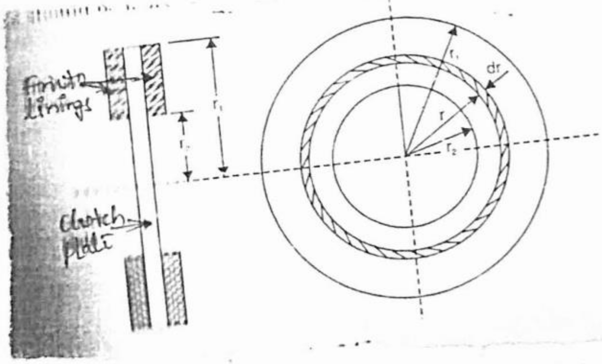
$p \rightarrow$ Intensity of Pr.

$W \rightarrow$ Total Axial load

$\mu \rightarrow$ Coeffecient of friction.

$T \rightarrow$ Torque transmitted.

The theory of single plate clutch is also based on same principle as that of collar bearing. In case of bearing, the power lost due to friction should be reduced & hence the value of coeffecient of friction should decrease. But in case of clutch the power transmitted by friction linings should be more & hence co-effecient of friction should be increased.



Also in case of a new clutch, the intensity of Pr is approximately uniform over the entire surface where as in an old clutch uniform wear theory is more approximate.

Consider a circular ring of radius 'r' & thickness dr as shown.

Area of ring, $dA = 2\pi r\,dr$

Axial load on ring, $dW = p \times$ Area of ring

$= p \times 2\pi r\,dr$

Frictional force on ring,

$dF = \mu \times$ load on ring

$= \mu \times p \times 2\pi r\,dr$

Frictional torque on ring, $dT = dF \times r$

$= \mu \times p \times 2\pi r\,dr \times r$

$= \mu \times p \times 2\pi r^2\,dr$

(i) For Uniform Pressure:

$p =$ constant

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}.$$

∴ Total frictional torque.

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \times p \times 2\pi r^2 . dr$$

$$= 2\pi \mu . p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi \mu . p \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$= \frac{2}{3} \mu . \pi . \frac{W}{\pi(r_1^2 - r_2^2)} . (r_1^3 - r_2^3)$$

$$= \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]. \quad -(a)$$

(23)

Total frictional torque acting on friction surface can also be expressed in terms of mean radius (R_M) of friction surface as,

$$T = \mu \cdot W \times R_M \qquad -(b).$$

comparing eq's (a) & (b),

$$R_M = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

In a single clutch plate, there are 2 friction surfaces, one on each side of the frictional plate, hence, total torque on the clutch plate is given by,

$$T^* = 2T.$$

$$= \cancel{2\times\left[\frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\right]} \quad 2\times\left[\frac{2}{3}\cdot\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\right]$$

Where, $T^* \Rightarrow$ Total frictional torque on clutch plate.

(ii) Uniform Wear:-

$$p \times r = \text{constant}$$

$$p = c/r.$$

W.K.T, axial load on ring

$$dW = p \times 2\pi r\, dr$$

$\therefore$ Total axial load is given by integrating above eq.

$$W = \int_{r_2}^{r_1} p \times 2\pi r \cdot dr$$

$$= \int_{r_2}^{r_1} c \cdot 2\pi\, dr$$

$$= 2\pi c\,[r]_{r_2}^{r_1} \Rightarrow 2\pi c\,[r_1 - r_2] \Rightarrow C = \frac{W}{2\pi(r_1 - r_2)}$$

The frictional torque on friction surface.

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \cdot p \times 2\pi r^2 \cdot dr$$

$$= \int_{r_2}^{r_1} \mu \cdot \frac{C}{r} \times 2\pi r^2 \, dr = \mu \cdot C \cdot 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$T = \frac{1}{2} \mu W (r_1 + r_2)$$

$$T = \mu W R_m$$

$R_m = \frac{1}{2}(r_1 + r_2)$: mean Radius.

∴ Total torque on a single clutch plate, is given by

$$T^* = 2T$$

$$= 2 \times \left[\frac{\mu W}{2} (r_1 + r_2)\right].$$

NOTE:- (i) For power transmission by friction through a clutch, uniform wear theory gives safer result. Hence, uniform wear should be assumed in case of friction clutch, unless it is specified otherwise.

Calculate the power transmitted by a single plate clutch at a speed of 2000rpm., if the outer & inner radii of frictn surfaces are 150mm & 100mm respectively. The max. intensity of pr. at any pt. of contact surface not to exceed 0.8×10^5 N/m², Take both sides of plate as effective & coeffecient of frictn = 0.3. Assume uniform wear.

Sol:- Given:-

Speed, $N = 2000$ rpm

$r_1 = 150mm = 0.15m$ | $r_2 = 100mm = 0.1m$ | $p_{max} = 0.8\times10^5$ N/m²; $\mu = 0.3$; No. of effective sides = 2

For Uniform Wear, we have,

$$p\times r = \text{constant}$$

$$(\text{or}) \quad p_1\times r_1 = p_2\times r_2 = C.$$

As for Uniform wear, the product of pressure & radius is constant, hence pressure will be more where radius is less. Therefore, at inner radius, the pr. will be more.

$$\therefore \quad p_{max}\times r_2 = C. \quad (\because r_2 \text{ inner radius})$$

$$0.8\times10^5\times0.1 = C \Rightarrow C = 0.8\times10^4$$

$$W = 2\pi C(r_1 - r_2)$$

$$W = 2\pi(0.8\times10^4)(0.15-0.1) = 2513.27\,N.$$

The torque due to both active surfaces

$$T^* = 2\left[\frac{\mu W}{2}(r_1+r_2)\right]$$

$$= 2\left[\frac{(0.3)(2513.27)}{2}(0.15+0.1)\right]$$

$$T^* = 188.49\,N\text{-}m$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi\times2000\times(188.49)}{60} \Rightarrow P = 39477.25\,W$$

$$\boxed{P = 39.477\,kW}\ \text{Ans}$$

(2) The external radius of a friction plate of single clutch having both sides as effective, is 150mm. The power transmitted is 20KW at a speed of 1000rpm. The maximum intensity of pr at any pt. of contact surface is 0.8×10^5 N/m². If the coefficient of frictn is 0.30 then find: (i) The internal radius of frictn plate; (ii) Axial thrust with which the frictn surfaces are held together.

Sol:- Given:-

External radius, $r_1 = 150mm = 0.15m$

Power transmitted, $P = 20KW = 20 \times 10^3$ W

$N = 1000$ rpm

Max. Pr. $p_{max} = 0.8 \times 10^5$ N/m². $\mu = 0.3$

~~Find~~ (i) Internal radius,

(ii) Axial Thrust, W

Since, nothing is mentioned to what to assume, f or power transmission in the problem it is clear that it is safer to assume uniform through clutch and hence it is wear.

for uniform wear, $p \times r = c$.

hence, pr. will be max. where radius is minimum.

$$p_{max} \times r_2 = c.$$

$$(0.8 \times 10^5) \times r_2 = c.$$

$$P = \frac{2\pi N T}{60}$$

$$20 \times 10^3 = \frac{2\pi \times 1000 \times T}{60} \Rightarrow T = 190.986 \text{ N-m} \quad \text{--- (i)}$$

Now using eq. of uniform wear.

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi (0.8 \times 10^5) \times r_2 \times (0.15 - r_2)$$

$$= 502654.8 r_2 (0.15 - r_2)$$

The frictional torque due to both sides active surfaces

$$T = 2 \times \left[\frac{\mu W}{2} (r_1 + r_2) \right]$$

$$= 2 \times \left[\frac{0.3 \times 502654.8 r_2 (0.15 - r_2)}{2} (0.15 + r_2) \right]$$

$$T = 150796.44 r_2 (0.15^2 - r_2^2) \quad \text{— (ii)}$$

Equating eq. (i) & (ii)

$$190.986 = 150796.44 r_2 (0.15^2 - r_2^2)$$

$$r_2^3 - 0.0225 r_2 + 0.0012665 = 0$$

The above is the cubic eq. can be solved by trial & error method. i.e. LHS should be zero.

let, $r_2 = 0.095$ m, then LHS = -0.0000137

$r_2 = 0.1$ m, then LHS = +ve.

$0.1 > r_2 > 0.095$

i.e. ~~0.1 > r_2 > 0.095~~

let us consider $r_2 = 0.097$

$r_2 = -0.0000137$

i.e. r_2 is slightly more than 0.097

let, $r_2 = 0.0974$ m = 97.4 mm

(ii) Axial thrust (W)

$$W = 502654.8 r_2 (0.15 - r_2)$$

$$= 502654.8 (0.0974)(0.15 - 0.0974)$$

$$W = 2575.22 \text{ N}$$

(3) The external & internal radii of a friction clutch of disc type are 90mm & 50mm respectively. Both sides of friction clutch are effective & coeffecient of friction is equal to 0.25. The friction clutch is used to rotate a machine from a shaft which is rotating at a constant speed of 240 rpm. The moment of inertia of rotating parts of the machine is 5.5 kg-m². The intensity of pr is not to exceed 0.8×10^5 N/m². Assuming uniform wear, determine the time reqd. for the machine to attain the full speed when the clutch is suddenly applied. Also determine the energy lost in slipping of clutch.

Sol:- **Given:-**

External radius, $r_1 = 90mm = 0.09m$

Internal radius, $r_2 = 50mm = 0.05m$

No. of effective sides = 2

Coefficient of frictn, $\mu = 0.25$;

Constant speed of driving shaft, $N = 240rpm$

M.O.I of M/c parts = $5.5\ kg\text{-}m^2$.

Max. Pr, $p = 0.8\times10^5\ N/m^2$

Theory assumed = uniform wear.

(i) Time Required for the machine to attain full speed of 240rpm:-

The driving shaft is rotating at a constant speed, Whereas the machine is at rest. But when the clutch is engaged, the machine will attain its full speed not immediately but after some time. Let this time 't' sec

Initially let us find Axial load & frictional torque for uniform wear.

$$p\times r = C.$$

$$\therefore\ p_{max}\times r_2 = C.$$

$$0.8\times10^5\times0.05 = C$$

$$\Rightarrow C = 4000$$

$$W = 2\pi C(r_1 - r_2)$$

$$W = 2\pi\, 4000(0.09 - 0.05)$$

$$W = 1005.31\,N$$

The frictional torque due to both active surface.

$$T^* = 2\left[\frac{\mu W}{2} \times (r_1 + r_2)\right]$$

$$= 2\left[\frac{(0.25)(1005.31)}{2} \times (0.09 + 0.05)\right]$$

$$T^* = 35.186\ N\text{-}m$$

Now, angular acceleration, when total torque is 35.186 N-m

$$\text{Torque} = M.O.I \times \text{Angular acceleration}$$

$$= I \times \alpha$$

$$35.186 = 5.5 \times \alpha$$

$$\alpha = 6.397\ rad/s^2$$

The m/c starts from rest. After some time the final angular speed of m/c will be corresponding to speed of shaft

$$\therefore \text{final angular speed},\ \omega = \frac{2\pi N}{60} = \frac{2\pi 240}{60} = 8\pi\ rad/s.$$

Let $t \rightarrow$ time reqd. $\boxed{v = u + at}$

using, $\omega = \omega_0 + \alpha t$

Initial angular speed. $\omega_0 = 0$.

$$8\pi = 0 + 6.397\, t$$

$$t = 3.928\ sec\ \text{Ans}$$

(ii) Energy lost in slipping of clutch:-

The driving shaft is rotating at a uniform speed of 240rpm i.e: uniform angular velocity 8πrad/s. Let us find the angles turned by driving shaft & the driven shaft.

Angle turned by driving shaft,

$$\theta_1 = \omega \times t$$

$$\theta_1 = 8\pi \times 3.928 = 98.72 \text{ rad}$$

The angle turned by driven shaft (n) m/c.

$$\theta_2 = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \left(\because s = ut + \tfrac{1}{2}at^2\right)$$

$$= 0 \times 3.928 + \frac{1}{2}(6.397)(3.928)^2$$

$$\theta_2 = 49.35 \text{ rad}$$

Energy lost in friction due to clutch slip

= Frictional torque × Angle of slip

$$= T \times (\theta_1 - \theta_2)$$

$$= 35.186 \times (98.72 - 49.35)$$

$$= 1737.13 \text{ N-m}$$

(31)

Multi-Plate Clutch:-

Let, $r_1 \rightarrow$ External radius of friction lining in friction plate

$r_2 \rightarrow$ Internal radius " " " " "

$W \rightarrow$ Axial load

$p \rightarrow$ Intensity of pr.

$n_1 \rightarrow$ no. of friction plates in driving shaft

$n_2 \rightarrow$ " " discs in driven shaft

Then, no. of active surfaces (or) friction surfaces will be given as,

$$\boxed{n = n_1 + n_2 - 1}$$

Total torque transmitted is given by

$$T = n \times \mu \cdot W \times R_m$$

1) A multi-clutch has six plates (friction rings) on the driving shaft & six plates on driven shaft. The external radius of the friction surface is 115mm whereas the internal radius is 80mm. Assuming uniform wear & $\mu = 0.1$, find the power transmitted at 2000rpm. Axial intensity of pr is not to exceed $0.16 N/mm^2$.

Sol:- Given:- no. of frictn plates, $n_1 = 6$

no. of discs, $n_2 = 6$.

$\therefore$ no. of active surfaces, $n = n_1 + n_2 - 1$

$= 6 + 6 - 1 \Rightarrow 11$

External radius of frictn surface, $r_1 = 115mm = 0.115m$

internal " " " , $r_2 = 80mm = 0.08m$

$\mu = 0.1$; $N = 2000rpm$

Max. intensity of pr, $p_{max} = 0.16 N/mm^2$

$= 0.16 \times 10^6 N/m^2$

Total torque transmitted

$T = n \times \mu \times W \times R_m$

Where, $R_m \to$ mean Radius.

$R_m = \frac{r_1 + r_2}{2} = \frac{0.115 + 0.08}{2}$

$R_M = 0.0975m$

For Uniform Wear,

$p \times r = C$

$p_{max} \times r_2 = C$

$0.16 \times 10^6 \times 0.08 = C$

$\Rightarrow C = 128 \times 10^2$

W.K.T, $W = 2\pi C (r_1 - r_2)$

$= 2\pi \cdot 128 \times 10^2 (0.115 - 0.08)$

$W = 2814.867 N$

$\therefore T = \mu \cdot W \cdot R_M \times n$

$= 0.1 \times 2814.867 \times 0.0975 \times 11$

$T = 301.89 N\text{-}m$

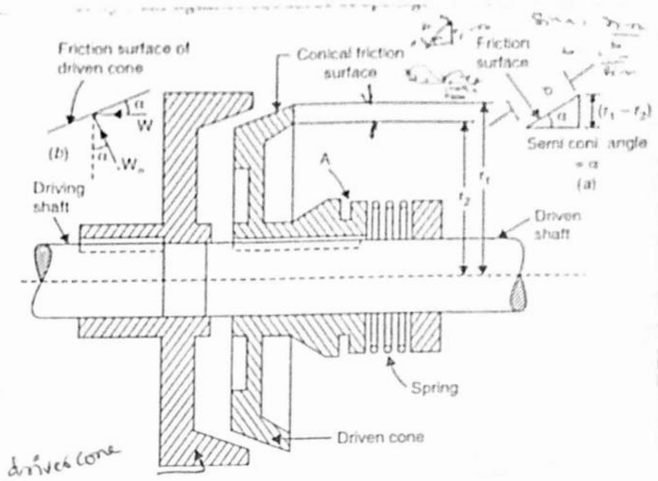
$\therefore$ Power transmitted, $P = \frac{2\pi N T}{60}$

$= \frac{2\pi \times 2000 \times 301.89}{60}$

$P = 63.22 kW$

(b) 33

Cone Clutch:-



Let $r_1 \rightarrow$ External radius of friction surface.
$r_2 \rightarrow$ Internal " " " " .
$\alpha \rightarrow$ Semi cone angle
$W \rightarrow$ Total axial load.
$R_m \rightarrow$ Mean Radius
$\mu \rightarrow$ coefficient of friction.
$b \rightarrow$ width of contact surface (or) width of cone face.

$$= \frac{(r_1 - r_2)}{\sin\alpha}$$

similar to that of conical surface

(i) In case of Uniform Pressure:

$$T = \frac{2}{3} \cdot \frac{\mu W}{\sin\alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right].$$

(ii) uniform Wear:

$$T = \frac{1}{2} \frac{\mu W}{\sin\alpha} (r_1 + r_2)$$

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(iii) Driving torque based on Mean Radius:-

let, $P_n \rightarrow$ intensity of pr at mean radius normal to frictn surface

$W_n \rightarrow$ Total load normal to frictn surface

$= p_n \times (2\pi R_m \times b)$

W = component of W_n in axial dirn.

$= W_n \times \sin\alpha$

$$T = \frac{1}{2} \cdot \frac{\mu W}{\sin\alpha} (r_1 + r_2)$$

$$= \mu \times \frac{W}{\sin\alpha} \left[\frac{r_1 + r_2}{2}\right]$$

$$\boxed{T = \mu \times W_n \times R_m}$$

$\because W_n = \frac{W}{\sin\alpha}$; $R_m = \frac{r_1 + r_2}{2}$.

The above eq. gives the torque interms of W_n & R_m

Problem 35

A cone clutch of cone angle 30° is used to transmit a power of 10 kW at 800 rpm. The intensity of Pr b/w the contact surfaces is not to exceed 85 kN/m². The width of conical friction surface is half of mean radius. If co-efficient of friction = 0.15, then find the dimensions of contact surfaces. Assume uniform wear. Also find the axial load or force reqd. to hold the clutch while transmitting the power. What is the width of friction surface?

Sol:- Given:-

Cone angle, $2\alpha = 30°$; $\alpha = 15°$

Power $P = 10\,kW = 10\times10^3\,W$; $N = 800\,rpm$.

Max. Pr ; $P_{max} = 85\,kN/m^2 = 85\times10^3\,N/m^2$

Width, $b = \frac{1}{2}\times$ mean Radius $= \frac{1}{2}\times R_m$

$= \frac{1}{2}\frac{(r_1+r_2)}{2}$; $\mu = 0.15$

(i) Dimensions of contact surfaces i.e: r_1 & r_2:

W.K.T, $P = \frac{2\pi NT}{60}$ — (i)

$T = 119.366\,N\text{-}m$

width 'b' given as $= \frac{1}{2} R_m = \frac{1}{2}\frac{(r_1+r_2)}{2} = b$ — (a)

But $b = \frac{r_1-r_2}{\sin\alpha} = \frac{r_1-r_2}{\sin 15°} = \frac{r_1-r_2}{0.2588}$ — (b)

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Equating ⓐ & ⓑ

$$\frac{r_1+r_2}{4} = \frac{r_1-r_2}{0.2588}$$

$$r_1 = 1.138 r_2$$

For uniform wear, $p \times r = c$

$$p_{max} * r_2 = C.$$

$$85 \times 10^3 \times r_2 = C.$$

The value of 'W' uniform wear is given by.

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi \times 85 \times 10^3 \times r_2 (r_1 - r_2)$$

$$= 534070 r_2 (r_1 - r_2)$$

The frictional torque for uniform wear

$$T = \frac{1}{2} \cdot \frac{\mu W}{\sin\alpha} (r_1 + r_2)$$

$$= \frac{\frac{1}{2} \times (0.15)(534070 r_2 (r_1 - r_2))}{\sin\alpha} (r_1 + r_2)$$

$$= 154762 r_2 (r_1 + r_2)(r_1 - r_2).$$

Sub. T from eq (i) in above eq.

$$119.366 = 154762 r_2 (r_1^2 - r_2^2)$$

$$\because r_1 = 1.138 r_2$$

$$119.366 = 154762 r_2 \left[(1.138 r_2)^2 - r_2^2\right]$$

$$r_2 = 0.138m = 138mm$$

$$r_1 = 0.157m = 157mm$$

Sub. values of r_1 & r_2 $\quad W = 534070 r_2 (r_1 - r_2)$

$$W = 1400.3N$$

width of friction surface, $b = \frac{r_1 - r_2}{\sin\alpha} = 73.4mm.$

(37)

(2). A cone clutch of semi-cone angle 15° is used to transmit a power of 30 kW at 800 rpm. The mean frictional surface radius is 150 mm. The normal intensity of pressure at the mean radius is not to exceed 0.15 N/mm². The coefficient of friction is 0.2. Assuming uniform wear. Determine: (i) Width of contact surface &

(ii) Axial load needed to engage the clutch.

Sol:- Given:-

$\alpha = 15°$; $P = 30\,kW = 30\times10^3\,W$;

$N = 800\,rpm$; $R_m = 150\,mm = 0.15\,m$; $p_n = 0.15\,N/mm^2 = 0.15\times10^6\,N/m^2$

$\mu = 0.2$;

$$P = \frac{2\pi N T}{60}$$

$$30\times10^3 = \frac{2\pi\times800\times T}{60}$$

$$T = 358.1\,N\text{-}m$$

W.K.T,

$$T = \mu \cdot W_n \times R_m$$

$$358.1 = 0.2\times W_n\times0.15$$

$$W_n = 11936.67\,N.$$

But, $W_n \Rightarrow$ Total load normal to friction surface of cone.

$$W_n = p_n\times(2\pi R_m\times b)$$

Sub,... W_n to get 'b'

$$11936.67 = 0.15\times10^6(2\pi(0.2)\times b)$$

$$b = 0.084\,m \text{ (or) } 84\,mm$$

To get Axial load

$$W = W_n\times\sin\alpha$$

$$= 11936.67\times\sin15°$$

$$W = 3089.4\,N$$

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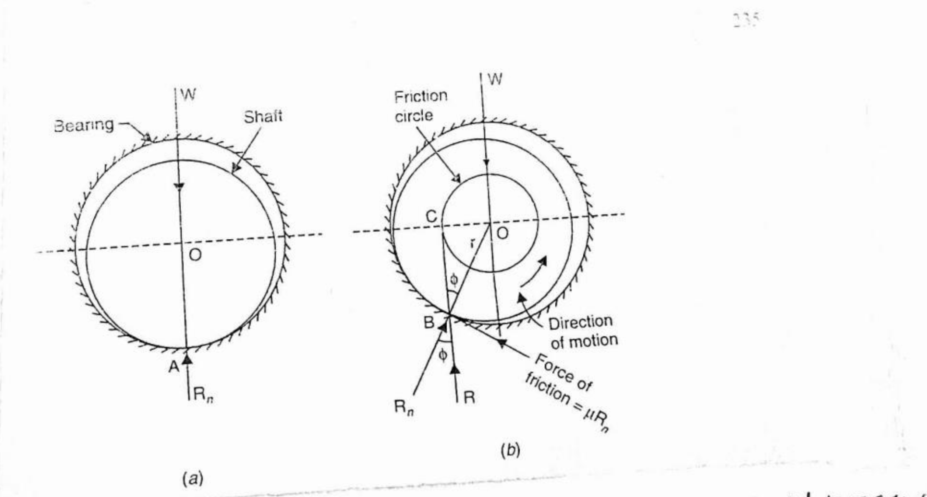
* Greasy friction of a Journal:

The following diagram shows a shaft inside a bearing. When the shaft is at rest in the bearing, the weight of shaft, W passes through centre of gravity at 'O'. A contact of shaft & bearing is maintained at pt. 'A' as shown in fig. (a).



The contact point 'A' is known as seat of pressure for bearing. The reaction of bearings acts at 'A' & is in line with W in the vertically upward dirn.

When the shaft is rotating because of clearance seat of pressure will roll or climb up the bearing in opposite dirn to that of rotation at pt. 'B' as shown. Metal to metal contact exists at pt. 'B' & greasy friction condition is applicable. as oil film is having very thin layer of lubricant.

(18)

The climbing or rolling up will stop when following three forces are in equilibrium:

(i) Wt. of shaft W, acting vertically downwards.

(ii) Normal Reactn R_N at B, which is radial & passes through the pt. 'O' as shown.

(iii) Frictional force, tangential to shaft at B & acting in opp. dirn of motn of shaft.

$$F = \mu \cdot R_N$$

The frictional force & normal reactn can be combined in to single resultant force R which is inclined at ϕ. Hence the shaft is in equilibrium now under following forces:

1. Weight of shaft W, acting vertically downwards.
2. Single resultant reactn R.

For equilibrium, R must be equal to W, & must act vertically ↑. R & W are equal & parallel. & they form a couple. This couple is called as frictn couple.

Moment of friction couple

$$= W \times \perp \text{ distance b/w } R \& W$$

$$= W \times OC.$$

$$= W \times r \sin\phi.$$

The angle ϕ is very small, $\sin\phi = \tan\phi$.

$$\therefore W \times r \tan\phi$$

$$= W \times r \times \mu. \qquad \because \tan\phi = \mu$$

This frictn couple acts in a dirn opposite to dirn of rotatn as is clear. This frictn couple opposes the driving ~~shaft~~ torque on shaft. And it will be equal to driving torque for equilibrium.

(41)

* Friction Circle :-

The circle of radius equal to $OC = r \tan\phi$
$= r \times \mu$

is known as "friction circle". This radius of friction circle, which is equal to $r \times \mu$, will be constant as the values of 'r' & μ are constant. Hence the radius of friction circle is independent of load or weight of shaft.

Power loss in Friction:

frictional torque, T = Moment of friction couple

$= W \times OC$

$= W \times r \times \mu$

Power lost in friction,

$= T \times \omega$

$= (W \times r \times \mu) \times \omega$

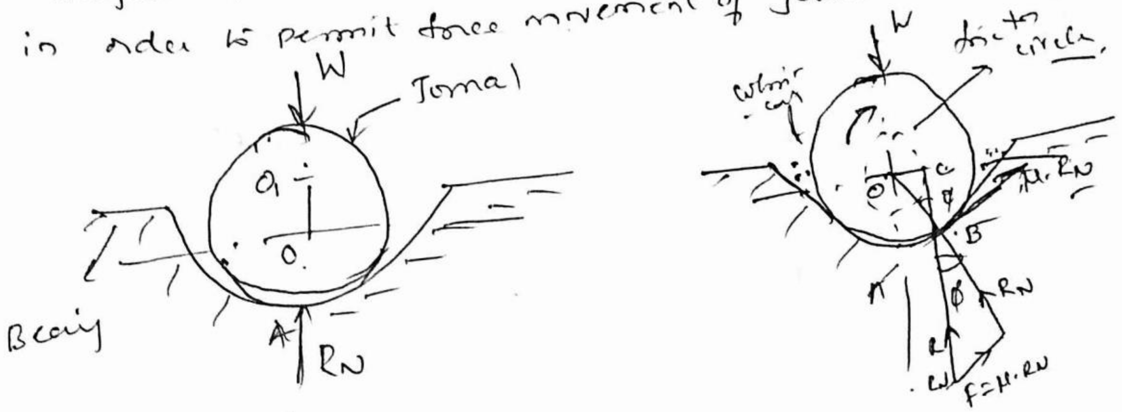
$= (W \times r \times \mu) \cdot \left(\frac{V}{r}\right)$

$= W \times \mu \cdot V$ Watts

$= \frac{\mu W V}{1000}$ kW.

* friction Circle:-

A journal bearing forms a turning pair. The fixed outer element of turning pair is called a bearing. The inner element which fits the bearing is called journal. The journal is slightly less in dia than that of bearing, in order to permit free movement of journal in bearing.



When bearing is not lubricated then there is line contact b/w 2 elements. the load W & normal Reac line RN of bearing acts through centre. Reactn RN acts vertically upwards at pt. A. This pt. called pt of contact of shaft.

Now consider a shaft rotating inside a bearing in c.w dirn. Increase frictn. ∴ the reactn, R doesnot act vertically upwards, but acts at another pt. of contact 'B'. This is due to fact that the shaft rotates. F = μ.RN acts at circumference of shaft which has a tendency to rotate the shaft in opp. dirn of motion & this shifts pt. A to pt B.

(Resultant of F & RN = R)

φ → Angle b/w R & RN

μ → .

T → friction torque

r → rad. of shaft

for uniform motn, resultant force acting on shaft must be zero. & resultant T.m should be zero

$R = W$ & $T = W \times OC = W \times OB \sin\phi$

$= W r \sin\phi$.

∴ since ϕ is very small tob.. $\sin\phi = \tan\phi$.

$T = W \cdot r \tan\phi = \mu \cdot W r$.

If shaft rotatin with angle velocity ω .. then power lost

$P = T \cdot \omega$

NOTE:- ① . if a circle is drawn with centre O &. radius as OC = then the circle is called a friction circle.

② force Reaction R by mechanism of a tangent to friction circle
turning pair acts along

Brakes & Dynamometers.

Brake:

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of machine.

The capacity of a brake depends upon the following factors:

1. The unit pressure between the braking surfaces.
2. The co-effecient of friction between braking surfaces.
3. The peripheral velocity of brake drum
4. The projected area of friction surfaces, &
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The materials used for the brake lining should have the following characteristics:

1. The co-effecient of frictn should remain constant, with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have adequate mechanical strength.
6. It should not affected by moisture (or) oil.

(2)

Types of Brakes:

The brakes, according to the means used for transforming the energy by braking elements are classified as:

1. Hydraulic Brakes e.g., pumps (or) hydrodynamic brake.
2. Electric Brake. e.g., Generators.
3. Mechanical Brake.

Hydraulic & electric brakes cannot bring the member to rest and are largely used where large amount of energy is to be transformed.

These brakes are also used for retarding (or) controlling the speed of vehicle for down-hill travel.

Mechanical brakes according to the direction of acting force, may be divided in following 2 groups:

a) Radial brakes &
b) Axial brakes.

a) Radial Brakes:- In these brakes, the force acting on the brake drum is in radial direction. These may subdivided in to external brakes & internal brakes. According to the shape of the friction elements, these brakes may block (or) shoe brakes & band brakes.

In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes & cone brakes.

* Single Block (or) Shoe Brake:-

It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made up of a softer material than the rim of wheel. This type of brake is commonly used in trains, and tram cars.

The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retards the rotation of wheel. The block is pressed against wheel by a force applied to one end of lever is pivoted on a fixed fulcrum O.

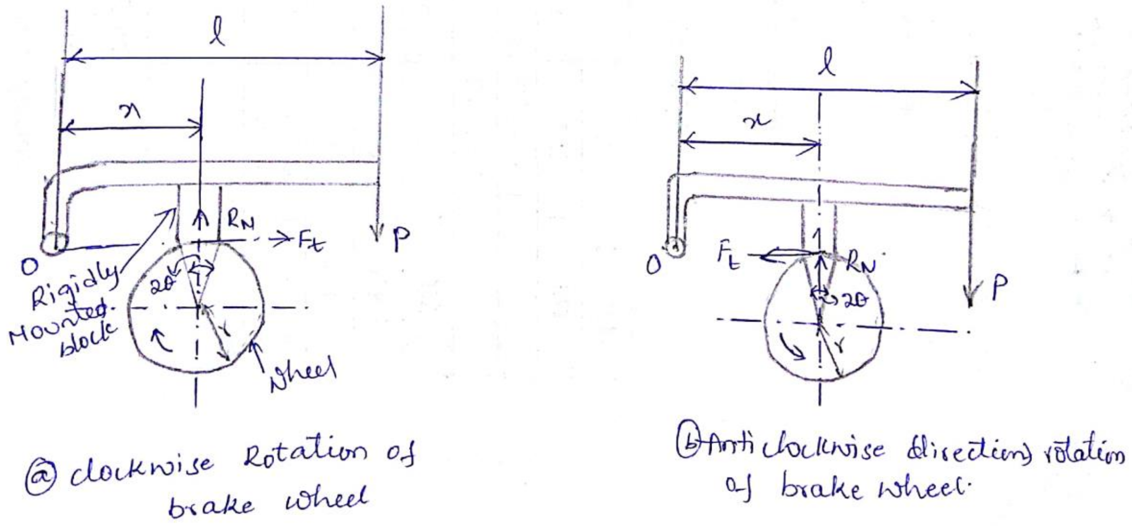


Fig:) Line of action of F_t passes through the fulcrum

4

Let, $P \rightarrow$ force applied at the end of lever,

$R_N \rightarrow$ Normal force pressing the brake block on wheel.

$r \rightarrow$ radius of wheel.

$2\theta \rightarrow$ Angle of contact surface of block.

$\mu \rightarrow$ co-effecient of friction.

$F_t \rightarrow$ Tangential braking force or frictional force acting at the surface of the block & wheel.

If the angle of contact is less than 60°, then it may be assumed that normal pressure between the block & wheel is uniform. In such cases, tangential breaking force on wheel, $F_t = \mu \cdot R_N$ — ①

& Braking Torque, $T_B = F_t \cdot r = \mu \cdot R_N \cdot r$ — ②.

Let us consider the following cases:

Case 1: When the line of action of tangential braking force passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in fig 1(a), then for equilibrium, taking moments about fulcrum (O),

then,

$$R_N \times x = P \times l.$$

$$R_N = \frac{P \times l}{x}.$$

Then, $T_B = \mu \cdot R_N \cdot r$.

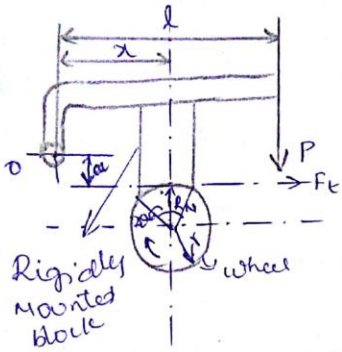
$$= \mu \cdot \frac{P \times l}{x} \cdot r$$

$$\boxed{T_B = \frac{\mu \times P \times l \times r}{x}}$$

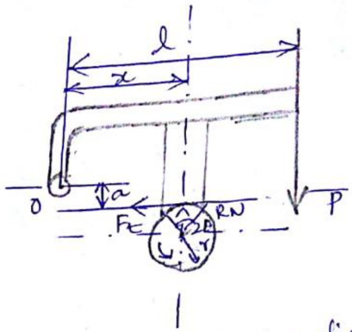
The braking torque in this case will also be same for anticlockwise direction.

(5).

Case 2: When the line of action of tangential braking force passes through a distance 'a' below the fulcrum 'O', and the brake wheel rotates clockwise as shown in fig 2(a).



@ clockwise rotation of Brake Wheel.



(b) Anti-clockwise direction of Brake wheel.

Now, when the brake wheel rotates in clockwise direction, then for equilibrium, taking moments about the fulcrum O,

$$R_N \times x + F_t \times a = P \times l$$

$$R_N \times x + \mu \cdot R_N \times a = P \cdot l.$$

$$R_N [x + \mu \cdot a] = P \cdot l$$

$$\boxed{R_N = \frac{P \cdot l}{x + \mu \cdot a}}$$

& Braking torque, $T_B = \mu \cdot R_N \cdot r$.

$$= \mu \cdot \frac{P \cdot l}{(x + \mu \cdot a)} \cdot r.$$

$$\boxed{T_B = \frac{\mu \cdot P l \cdot r}{(x + \mu a)}}$$

When the brake wheel rotating in A.C.W then for equilibrium, taking moments about fulcrum then,

$$R_N \times x = F_t \times a + P \times l.$$

$$R_N \times x = \mu R_N \times a + P \times l.$$

$$R_N \times x - \mu R_N \times a = P \times l.$$

$$R_N [x - \mu a] = P \times l$$

$$R_N = \frac{P \times l}{x - \mu a}.$$

then Braking torque,

$$T_B = \mu \cdot R_N \cdot r.$$

$$= \mu \left[\frac{P \times l}{x - \mu a}\right] \cdot r$$

$$\boxed{T_B = \frac{\mu \times P \times l \cdot r}{(x - \mu a)}}$$

Case 3: When the line of action of the tangential braking force (F_t) passes through a distance 'a' above the fulcrum O,

Now, the brake wheels rotates in the clockwise direction then for equilibrium moments about fulcrum O,

$$R_N \cdot x = P \cdot l + F_t \cdot a$$

$$R_N \cdot x = P \cdot l + \mu \cdot R_N \cdot a$$

$$R_N \cdot x - \mu \cdot R_N \cdot a = P \cdot l$$

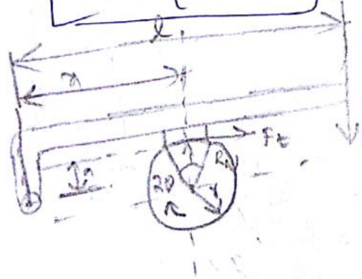
$$R_N [x - \mu \cdot a] = P \cdot l$$

$$R_N = \frac{P \cdot l}{[x - \mu a]}$$

Then $T_B = \mu \cdot R_N \cdot r$

$$= \mu \frac{P \cdot l}{x - \mu a} \cdot r$$

$$\boxed{T_B = \left(\frac{\mu \cdot P \cdot l \cdot r}{x - \mu a}\right)}$$



a) clockwise rotation of a wheel

When the brake wheels rotates in counter clockwise direction, then for equilibrium taking moments along fulcrum O,

$$R_N \cdot x + F_t \cdot a = P \times l$$

$$R_N \cdot x + \mu \cdot R_N \times a = P \times l$$

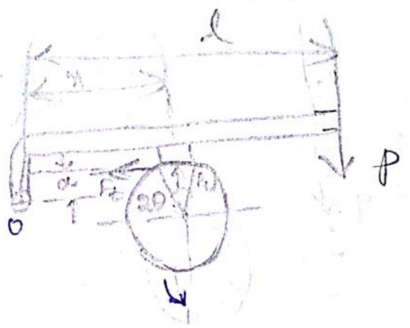
$$R_N \{x + \mu \cdot a\} = P \times l$$

$$R_N = \frac{P \times l}{x + \mu a}$$

Then, $T_B = F_t \cdot r = \mu \cdot R_N \cdot r$

$$= \mu \cdot \frac{P \times l}{x + \mu a} \cdot r$$

$$\boxed{T_B = \left(\frac{\mu \cdot P \times l \cdot r}{x + \mu a}\right)}$$



(b) Anticlockwise rotation of a brake wheel.

Fig: Line of Action of F_t passes below fulcrum.

NOTE :

1. From the above we see that when the brake wheel is rotates anticlockwise in case 2 and when it rotates in clockwise in case 3, the equations are same.

(a) Here, the frictional force helps to apply the ~~brake~~ brake. Such type of brakes are said to self-energising brake.

(b) When the frictional force is ~~not enough~~ i.e., greater enough to apply the brake with no external force, then the brake is said to be self-locking brake.

No external force is needed to apply the ~~brake~~ brake & hence the brake is self locking.

∴ the condition will be,

$$\boxed{x \leq \mu.a.}$$

2. The brake should be self energising & not self locking.

3. In order to avoid self locking & to prevent the brake from grabbing, x is kept greater than $(\mu.a)$.

4. If A_b is the projected bearing area of shoe brake, then the bearing pressure on the shoe,

$$P_b = \frac{R_N}{A_b}$$

where, $P_b \rightarrow$ bearing pressure.

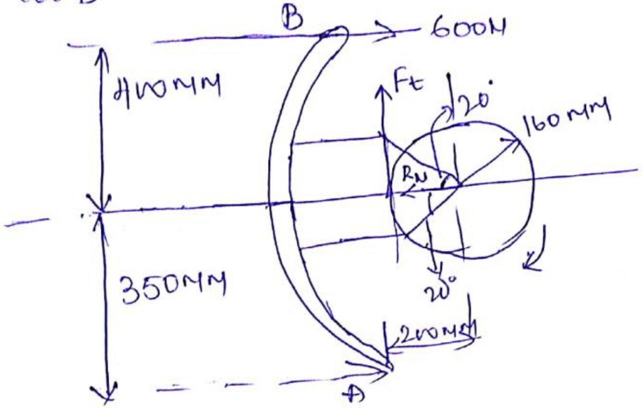
$A_b \rightarrow$ width of shoe x projected length of shoe

$= w(2r\sin\theta)$

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) & produces bending of shaft. To overcome this drawback, double shoe brake is used.

Problem (8).

(1) Following fig shows a brake applied to a drum by a lever AB which is pivoted at a fixed point A & rigidly fixed to the shoe. The radius of drum is 160MM. The coeff. of friction at brake lining is 0.3. If the drum rotates in C.W, find the braking torque due to horizontal force of 600N at B



Sol: Given, $r = 160MM = 0.16M$; $\mu = 0.3$; $P = 600N$.

Since the angle subtended by the shoe at the drum is $40°$.

Let, $R_N \rightarrow$ Normal force pressing the block of brake drum

$F_t \rightarrow$ Tangential force $= \mu \cdot R_N \Rightarrow F_t = \mu \cdot R_N$

$R_N = \frac{F_t}{\mu}$.

Taking moments about point A,

$$R_N \times 350 + F_t(200-160) = 600(410+350)$$

$$\frac{F_t}{\mu} \times 350 + F_t(40) = 600(750)$$

$$\frac{F_t}{0.3} \times 350 + 40F_t = 450 \times 10^3.$$

$$F_t\left[\frac{350}{0.3} + 40\right] = 450 \times 10^3.$$

$$\therefore \boxed{F_t = 372.8N}$$

We know that,

$$T_B = F_t \cdot r$$

$$= 372.8 \times 0.16$$

$$\boxed{T_B = 59.6 N\text{-}M}$$

(9).

(2). A bicycle and rider of mass 100kg are travelling at the rate of 16km/hr on a level road. A brake is applied to the rear wheel which is 0.9m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100N & μ=0.05

Sol: Given Data:

m=100kg; v= 16km/hr

v= 4.4m/sec

D= 0.9m; R_N= 100N; μ=0.05.

(i) Distance travelled by a bicycle before it comes to rest

Let, x → distance travelled by the bicycle before it comes to rest.

W.K.T, Tangential breaking force acting at the point of contact of brake & wheel.

$F_t = \mu \cdot R_N$

$= 0.05 \times 100 = 5N$.

and Work done, ⇒ $F_t \times x = 5 \times x$ N-m.

In order to bring the bicycle to rest, Workdone against friction must be equal to be Kinetic energy.

Kinetic Energy, $K.E = \frac{mv^2}{2} = \frac{(100)(4.4)^2}{2}$ $KJ = N \cdot \frac{m}{sec}$ N-m

$= 986$ N-m

∴ Work done = K E

$5 \times x = 986$

$x = \frac{986}{5}$

$\boxed{x = 197.2m}$

(ii) no. of revolutions made by bicycle

distance travelled = πDN

x = πDN

197.2 = π(0.9)N

$\boxed{N = 70}$ revolutions

∴ 70 revolutions made by bicycle before it comes to rest.

(10)

* Pivoted Block (or) Shoe Brake:-

In the last case we have discussed that, if the angle of contact is less than 60°, then the normal pressure between block & wheel is uniform.

If the angle of contact is greater than 60° between block & wheel, then the unit normal pressure to the surface of contact is less at the ends than at centre. In such cases, block is pivoted to lever. as shown, instead being rigidly attached to lever. This gives uniform wear of a brake lining in the direction of applied force.

The braking torque for a pivoted block when $2\theta > 60°$ will be,

$$T_B = F_t \times r$$
$$= \mu' R_N \cdot r$$

Where, $\mu' \rightarrow$ Equivalent frictional force, $= \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

$\mu \rightarrow$ actual friction

These brakes have more life time & may provide a higher braking torque.

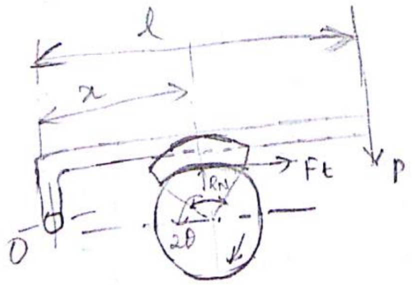
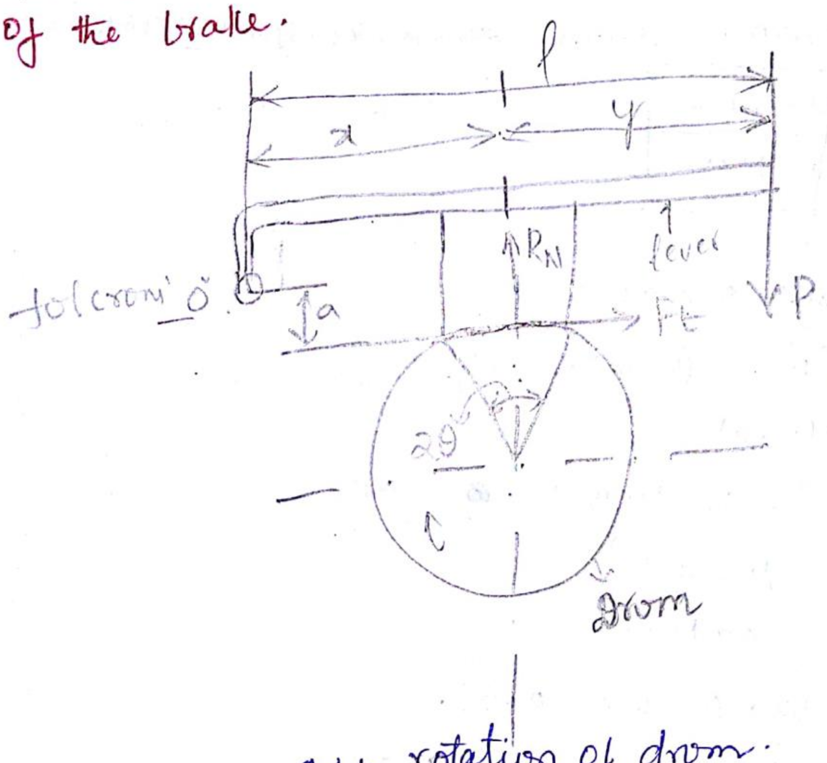


fig: pivoted block

The diameter of the brake drum of a single block shown in fig. is 1m. It sustains 240N-m of torque at 400rpm. The coeffecient of friction is 0.32. Determine the required force to be applied when the rotation of drum is (a) clockwise; (b) counter-cw. and the angle of contact is (i) 35° ; & (ii) 100°;
Given that, $l = 800mm$; $x = 150$ mm; $a = 25mm$.
Also find the new value of 'a' for self-locking of the brake.



C.W. rotation of drum.

Sol:- Given Data:-

$d = 1m$; $T_B = 240 N\text{-}m$. $l = 800mm = 0.8m$.

$r = 0.5m$; $\mu = 0.32$; $x = 150mm = 0.15m$;

$a = 25mm = 0.025m$.

W.K.T: Braking Torque, $T_B = \mu \cdot R_N \cdot r$

$240 = 0.32 \times R_N \cdot 0.5$

$\therefore R_N = 1500N$

(i) when angle of contact, $2\theta = 35°$.

(a) when the rotatn of drum is ~~clockwise~~ Anti clockwise.

$$\boxed{T_B = \frac{\mu \cdot P \cdot l \cdot r}{x - \mu a}}$$

$P = ?$

$$240 = \frac{0.32 \times P \times 0.8 \times 0.5}{[0.15 - (0.32 \times 0.025)]}$$

$P = 266.25 N$. Ans

(b) When the rotatn of drum is C.W:

$$T_B = \frac{\mu \cdot P \cdot l \cdot r}{x + \mu a}$$

$$240 = \frac{(0.32) P (0.8)(0.5)}{[0.15 + (0.32 \times 0.025)]}$$

$P = 296.25N$ Ans

(c) New value of 'a' for self-locking brake:-

For self-locking, the external force must be zero: ie: 'p' must be zero and thus the condition is,

$$x \leq \mu . a$$

or

$$x = \mu . a$$

$$\therefore a = \frac{x}{\mu}$$

$$a = \frac{0.15}{0.32} = 0.468m \Rightarrow \boxed{a = 468mm} \text{ Ans}$$

(ii) When $2\theta = 100°$:

Since, the angle of contact is more than 60°: then the coeffecient of friction [μ] is replaced by, μ'.

$$\therefore \mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.32 \times \sin 50}{100 \times \frac{\pi}{180} + \sin 100}$$

$$\therefore \mu' = 0.359.$$

(a) When the rotation of drum is clockwise :-

$$\tau_B = \frac{\mu' . P . l . r}{x + \mu' a}$$

$$240 = \frac{0.359 \times P . (0.8)(0.5)}{[0.15 + (0.359)(0.025)]}$$

$\therefore \quad P = 265.7 N.$

(b) When the rotation of drum is A.C.W.

$$\tau_B = \frac{\mu' . p . l . r}{x - \mu' a.}$$

$$240 = \frac{(0.359) . P . (0.8)(0.5)}{[0.15 - (0.359)(0.025)]}$$

$P = 235.7 N.$

(c) New value of 'a' for self locking of brake:-

Again the condition is,

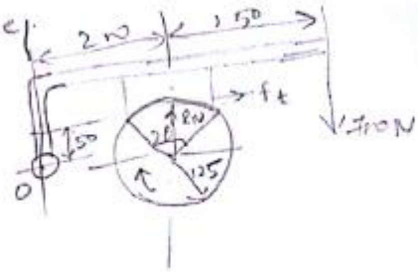
$$x = \mu' . a.$$

$$a = \frac{x}{\mu'} = \frac{0.15}{0.359} = 0.417 m$$

$$\therefore \boxed{a = 417 mm}$$

(1) A Single block brake as shown in fig. (11)

The diameter of the drum is 250mm and the angle of contact is 90°. If the operating force of 700N is applied at the end of lever & coefficient of friction between drum & lining is 0.35. Determine the torque that may be transmitted by the block brake.



Sol: Given Data:

$d = 250\,mm$; $r = 125\,mm$

$2\theta = 90°$; $P = 700\,N$; $\mu = 0.35$.

$T_B = F_t \times r$

$\therefore F_t = \mu' \cdot R_N$.

$\mu' \to$ Equivalent frictional force, $\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

$$\mu' = \frac{4 \times 0.35 (\sin 45°)}{90 + \sin 90} = 0.385.$$

Taking moments about fulcrum O, we have,

$R_N \times 200 = 700(200+250) + F_t \times 50$

$\dfrac{F_t}{\mu'} \times 200 = 315000 + F_t \times 50.$

$\dfrac{F_t}{0.385} \times 200 = 315000 + F_t \times 50$

$520 F_t = 315000 + F_t \times 50$

$520 F_t - 50 F_t = 315000$

$\boxed{F_t = 670\,N}$

Now Torque transmitted by a brake may be,

$T_B = F_t \times r$

$= 670 \times 125$

$= 83750\ N\text{-}mm$

$\boxed{T_B = 83.75\ N\text{-}m}$

*Simple Band Brake:

(5)

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of circumference of drum. A simple band brake in which one end of the band is fixed to fixed point (or) pin (or) fulcrum of lever and the other end is attached to lever at a distance b from fulcrum.

When a force P is applied to the lever at C, the lever turns about fulcrum pin 'O' & tightens the band on the drum and hence the brakes are applied. The friction between the band the drum provides brake force. The force 'P' on the lever at C may be determined as: &

$T_1 \rightarrow$ Tension in the tight side of band,

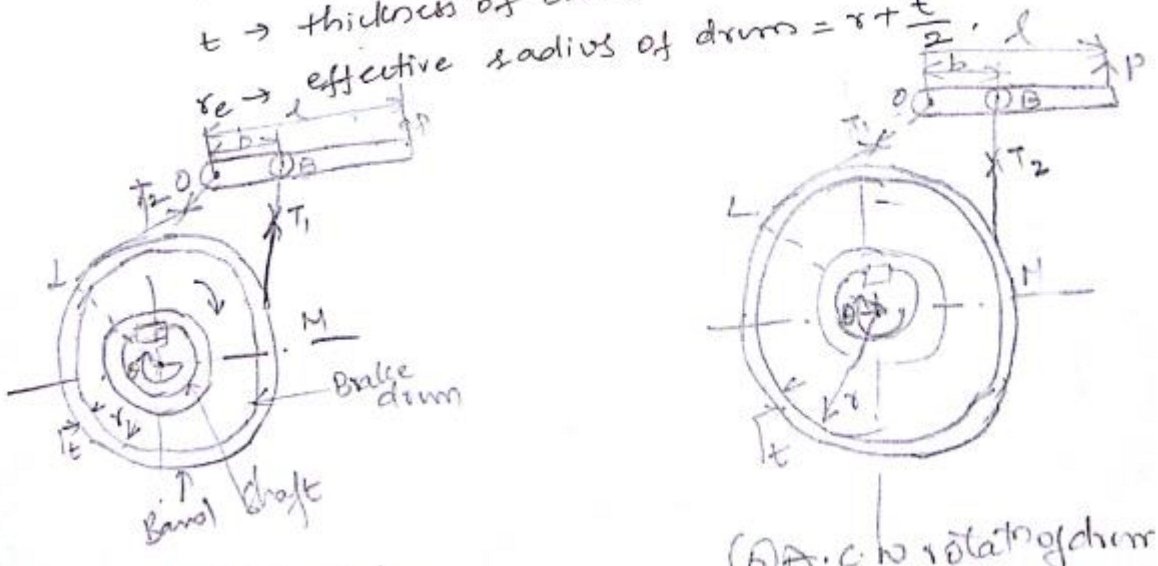
$T_2 \rightarrow$ Tension in the slack side of band.

$\theta \rightarrow$ Angle of lap of band on drum.

$r \rightarrow$ radius of drum,

$t \rightarrow$ thickness of band &

$r_e \rightarrow$ effective radius of drum $= r + \frac{t}{2}$.



(a) C.W rotation of drum

(b) A.C.W rotation of drum

(14)

W.K.T the ~~relation~~ of limiting ratio of tensions,

$$\frac{T_1}{T_2} = e^{\mu\theta}. \quad (or) \quad 2.3 \log\left[\frac{T_1}{T_2}\right] = \mu.\theta.$$

& braking force on drum $= T_1 - T_2$.

$\therefore$ Braking Torque, $T_B = (T_1 - T_2)\, r$ . . . (neglecting thickness of band)

$= (T_1 - T_2)\, r_e$ . . (considering thickness of band)

Now considering equilibrium of lever OBC. It may be noted that when the drum rotates in c.w dirn,

→ the end of the band attached to the fulcrum will be the slack side with tension T_2 and the band attached to the lever will be tight side with tension T_1

→ On the other hand, when the drum rotates in A.C.W, dirn, the end of the band attached to the fulcrum will be the tight side and the other end attached to the lever will be the slack side.

Now taking moments about O, we have,

$P.l = T_1 B.$ . . for c.w

$P.l = T_2 \cdot B$ . . . for A.C.W.

NOTE: 1. When the brake band is attached to lever, then the force (P) must act in upward direction in order to tighten the band.

2. If the permissible tensile stress (σ) for the material of band is known, then the max. tension in the band is given by,

$$T_1 = \sigma . w . t$$

$\sigma \to$ permissible tensile stress

$w \to$ width of band &

$t \to$ thickness " .

Problems

1. A band brake acts on the $\frac{3}{4}$th of circumference of a drum of 450mm dia which is keyed to the shaft. The band brakes provides a braking torque of 225N-m. One end of the band is attached to a fulcrum pin of lever and the other end to a pin 100mm from the fulcrum. If the operating force is applied at 500mm from the fulcrum and the coeffecient of friction is 0.25. Find the operating force when the drum rotates in (a) Anti clockwise dir^n and (b) clockwise dir^n.

Sol: Given Data:

$d = 450mm$

$r = 225mm = 0.225m$;

$\tau_B = 225 N\text{-}m$

$b = OB = 100mm$; $= 0.1m$;

$l = 500mm$

$l = 0.5m$.

$\mu = 0.25$.

Let, $P \Rightarrow$ Operating force.

~~Operating~~

We know that,

Angle of wrap, $\theta = \frac{3}{4}$th of circumference

$= \frac{3}{4} \times 360$.

$= 270^\circ \times \pi / 180^\circ$

$\theta = 4.713$ rad.

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ii. Operating force when drum rotates in Anticlockwise:-

Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates in Anticlockwise, then, the end of the band attached to O will be tight side with tension T_1 and the end of the band attached to other end B, will be slack side with tension T_2.

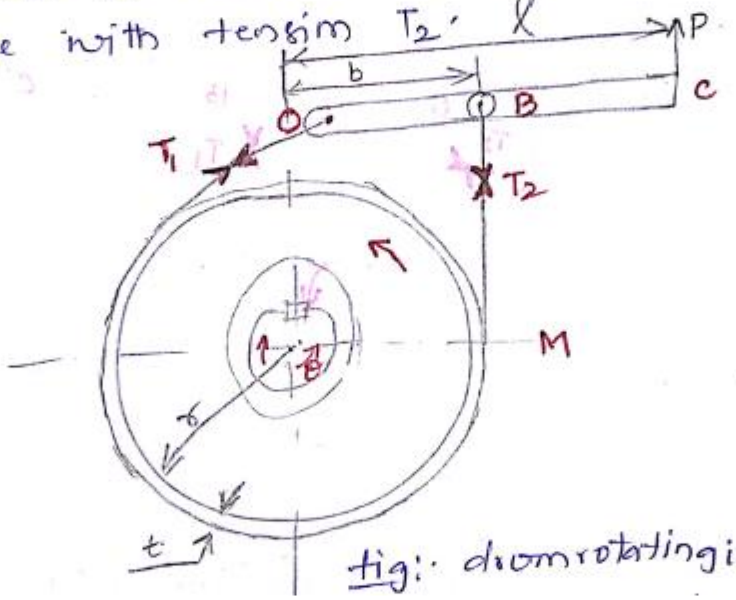


fig:- drum rotating in A.C.W

Now taking moments about fulcrum O, we have.

$$P \times l = T_2 \cdot b.$$

$$P \cdot 0.5 = 444 \times 0.1$$

$$\therefore \boxed{P = 88.8 N}$$ Ans

② Operating force when the drum rotates in C.W:-

As we know that, the operating force acts in upward dirⁿ. and the drum is rotating in clockwise dirⁿ, then the end of the band attached to the fulcrum 'O' will be slack side, with tension T_2 and the other end of the band attached to 'B' will be tight side with tight side T_1.

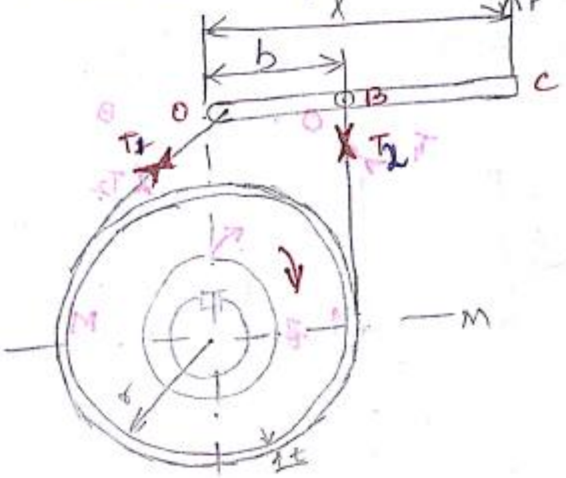


fig: Drum rotating in C.W

Now taking moments about 'O',

$$P.l = T_1.b.$$

$$P.0.5 = 1443.8 \times 0.1$$

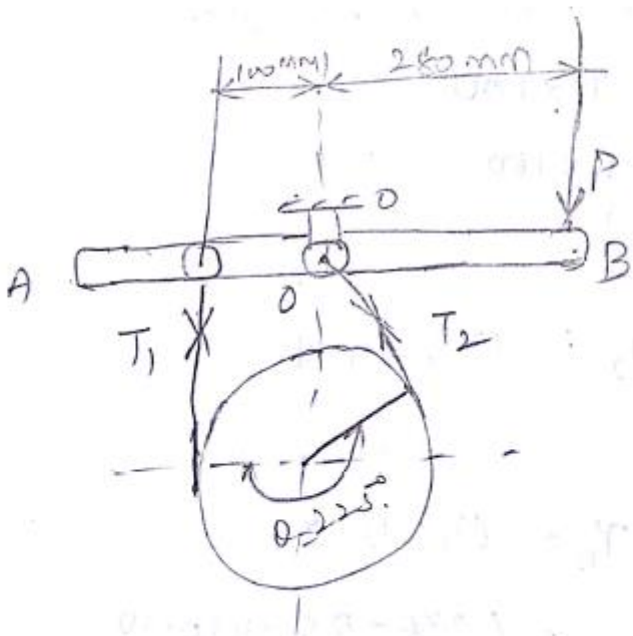
$$\boxed{P = 288.76N} \text{ Ans}$$

1) A simple band brake shown in fig. applied to a shaft carrying a flywheel of mass of 250kg and of radius of gyration 300mm. The shaft speed is 200 rpm. The drum diameter is 200 mm & coefficient of friction is 0.25. The angle of lap of band on drum is 225°.

: Determine: (i) The brake torque when a force of 120N is applied at lever end.

(ii) The no. of turns of flywheel before it comes to rest, and.

(iii) The time taken by the flywheel ~~before~~ To come to rest.



Sol: Given Data:-

$D = 200\,mm$
$R = 100\,mm$
$r = 0.10\,m$

$m = 250\,kg$; $k = 300\,mm = 0.3\,m$; $N = 200\,rpm$;
$\mu = 0.25$; $\theta = 225° = 225° \times \pi/180 = 3.92\,rad$; $P = 120\,N$.

(i) Brake Torque applied at lever end;

Tension ratio, $2.3 \log\left[\frac{T_1}{T_2}\right] = \mu \cdot \theta$.

$$2.3 \log\left[\frac{T_1}{T_2}\right] = (0.25)(3.92)$$

$$\log\left[\frac{T_1}{T_2}\right] = 0.426$$

$$\frac{T_1}{T_2} = 2.67 \Rightarrow T_1 = 2.67\,T_2 \quad \text{— (a)}$$

Taking moments about fulcrum O, we get,

$$P \times 280 = T_1 \times 100$$

$$120 \times 280 = T_1 \times 100$$

$$\boxed{T_1 = 336\,N}$$

$T_1 = 2.67\,T_2$
$336 = 2.67\,T_2$

$T_2 = 125.84\,N$.

$\therefore$ Braking torque, $\tau_B = (T_1 - T_2)\,r$.

$$= (336 - 125.84)\,0.10$$

$$\boxed{\tau_B = 21.01\,N\text{-}m}$$

(ii) No. of turns of flywheel before it comes to rest,

$K.E = \cancel{\ldots} \cdot \frac{1}{2} I \omega^2. \qquad \because \omega = \frac{2\pi N}{60}$

$\omega = \frac{2\pi \times 210}{60}$

$= \frac{1}{2} m k^2 \cdot \omega^2$

$= \frac{1}{2} \times 250 \times (0.33)^2 \times \left[\frac{2\pi \times 200}{60}\right]^2.$

$K.E = 4934.80 \text{ N-m}.$

This kinetic energy is used to overcome the work done due to braking torque.

$\therefore$ Kinetic energy of flywheel $= T_B \times 2\pi n.$

$4934.80 = 21.01 \times 2\pi (n)$

$\therefore n = 37.38$ revolutions.

$\boxed{n \simeq 37}.$

(iii) Time taken by the flywheel to come to rest:-

Time taken $= \frac{n}{N} = \frac{37}{200} = 0.1868 \text{ min}$

$= 11.208$ sec's

* Internal Expanding Brake:-

This type of brake is provided internally on the brake drum. In olden days, band brakes were used in automobiles, which were exposed to dirt and water. Their heat dissipatn capacity is also poor. In these days, band brakes were replaced by internal expanding brakes, which have atleast one self-energising shoe per wheel. This results in increased friction, giving great breaking power.

Working Principle:-

This consists of two shoes S_1 and S_2: the outer surfaces of which are lined with some friction materials. The shoes S_1 and S_2 are pivoted about the fulcrums O_1 and O_2 respectively. The other ends of the shoes are in contact with cam.

When the cam rotates, the shoes push outwards against the rim of drum. The friction between brake linings and drum, produces braking torque, reducing the speed of drum. The shoes are normally held in off-position by a tension spring. The drum encloses the entire brake mechanism, protecting the brake lining from dust & moisture.

For anticlockwise rotation of drum, the left hand shoe is known as primary (or) leading shoe, while the right hand side shoe is known as secondary (or) trailing shoe.

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Let,

$\alpha \Rightarrow$ Angle of Inclination of plane to horizontal;

$m \rightarrow$ mass of vehicle;

$W \rightarrow$ Weight of vehicle; $W = m \cdot g$,

$h \Rightarrow$ height of C·G of vehicle above road surface.

$x \rightarrow$ Perpendicular distance of C·G from rear axle.

$L \Rightarrow$ Wheel base of vehicle.

$R_A \Rightarrow$ total normal reaction between the ground and front wheels.

$R_B \Rightarrow$ total normal reaction between the ground and rear wheels.

$\mu \Rightarrow$ coeffecient of friction b/w the types of road surfaces.

$a \rightarrow$ Retardation of vehicle.

$F_A \rightarrow \mu \cdot R_A \Rightarrow$ Total braking force acting at the front wheels due to application of brakes and.

$F_B \Rightarrow \mu \cdot R_B \rightarrow$ Total braking force acting at rear wheels due to application of brakes.

Substituting F_A and R_A in (iii)

$$\mu \cdot R_A \cdot h + (mg\cos\alpha - R_A)x = R_A(L - x)$$

$$\mu \cdot R_A \cdot h + mg\cos\alpha \cdot x = R_A \cdot L$$

$$\therefore R_A = \frac{mg\cos\alpha \cdot x}{(L - \mu \cdot h)} \quad \text{—(A)}$$

and $R_B = mg\cos\alpha - R_A$

$$= mg\cos\alpha - \frac{mg\cos\alpha \cdot x}{L - \mu \cdot h}$$

$$= mg\cos\alpha\left[1 - \frac{x}{L - \mu \cdot h}\right]$$

$$\therefore R_B = mg\cos\alpha\left[\frac{L - \mu h - x}{L - \mu \cdot h}\right] \rightarrow \text{(B)}$$

From equation (i) the retardation of vehicle is given by,

$$a = \frac{F_A + mg \cdot \sin\alpha}{m} = \frac{\mu \cdot R_A + mg\sin\alpha}{m}$$

Substituting the value of R_A,

$$\cancel{a = \frac{\mu \cdot mg\cos\alpha \cdot x + mg\sin\alpha}{(L - \mu \cdot h)\, m}}$$

$$\Rightarrow a = \frac{\mu \cdot R_A}{m} + \frac{mg\sin\alpha}{m}$$

$$\Rightarrow a = \frac{\mu \cdot mg\cos\alpha \cdot x}{(L-\mu h)\cdot m} + \frac{mg\sin\alpha}{m}$$

$$\boxed{a = \frac{\mu g\cos\alpha \cdot x}{L-\mu \cdot h} + g\sin\alpha.} \quad -(C).$$

NOTE:- 1. When the vehicles moves on a level track, then, $\alpha = 0$.

$$\therefore \boxed{a = \frac{\mu g x}{L-\mu \cdot h}}$$

2. When the vehicle moves down the plane, then equation (i) becomes.

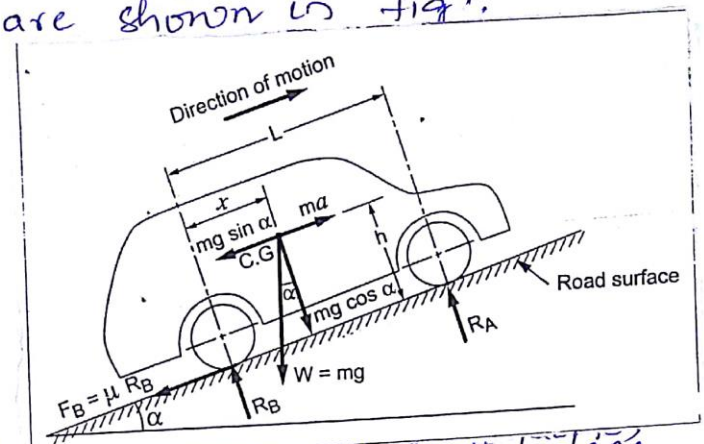
$$F_A - mg\sin\alpha = m \cdot a$$

$$\therefore a = \frac{F_a}{m} - g\sin\alpha = \frac{\mu \cdot R_A}{m} - g\sin\alpha$$

$$\therefore \boxed{a = \frac{\mu g\cos\alpha \cdot x}{L-\mu \cdot h} - g\sin\alpha.}$$

Case(ii):- When the brakes applied to the rear wheels only:-

Consider a car moving up an inclined plane. For equilibrium of vehicle, the various forces acting on vehicle are shown in fig:.



Resolving the forces parallel to the plane,

$$F_B + mg \cdot \sin\alpha = m \cdot a \rightarrow (i)$$

Resolving the forces $\perp$ to the plane,

$$R_A + R_B = mg \cos\alpha \rightarrow (ii)$$

Now, taking moments about C.G,

$$F_B \cdot h + R_B \cdot x = R_A [L - x] \rightarrow (iii)$$

W.k.t,

$F_B = \mu \cdot R_B$ and.

$R_A = mg \cos\alpha - R_B$ &

$R_B = mg \cos\alpha - R_A$.

(23)

Substituting the values of R_A & R_B in (iii).

$$\mu \cdot R_B \cdot h + R_B \cdot x = (mg\cos\alpha - R_B)(L - x)$$

$$R_B \cdot \mu \cdot h + R_B \cdot x = mg\cos\alpha [L - x] - R_B [L - x]$$

$$R_B \cdot \mu \cdot h + \cancel{R_B \cdot x} + R_B \cdot L - \cancel{R_B \cdot x} = mg\cos\alpha [L - x]$$

$$R_B [\mu \cdot h + L] = mg\cos\alpha [L - x]$$

$$\therefore R_B = \frac{mg\cos\alpha [L - x]}{[L + \mu \cdot h]}.$$

and, $R_A = mg\cos\alpha - R_B$

$$= mg\cos\alpha - \frac{mg\cos\alpha [L - x]}{L + \mu \cdot h}$$

$$= mg\cos\alpha \left[1 - \frac{(L - x)}{L + \mu \cdot h} \right]$$

$$= mg\cos\alpha \left[\frac{\cancel{L} + \mu \cdot h - \cancel{L} + x}{L + \mu \cdot h} \right]$$

$$R_A = \frac{mg\cos\alpha (x + \mu \cdot h)}{L + \mu \cdot h}.$$

From eq.(i) retardation of vehicle is,

$$a = \frac{F_B + m \cdot g \sin\alpha}{m}$$

$$= \frac{F_B}{m} + \frac{mg\sin\alpha}{m}$$

$$= \frac{\mu \cdot R_B}{m} + g\sin\alpha$$

$$= \frac{\mu \cdot mg\cos\alpha(L-x)}{(L+\mu h) \cdot m} + g\sin\alpha$$

$$\boxed{a = \frac{\mu g\cos\alpha[L-x]}{(L+\mu \cdot h)} + g\sin\alpha.}$$

NOTE:- 1. When the vehicle moves on a level track, then $\alpha = 0$.

$$R_B = \frac{m \cdot g[L-x]}{L+\mu \cdot h}; \quad R_A = \frac{mg(x+\mu h)}{L+\mu \cdot h}.$$

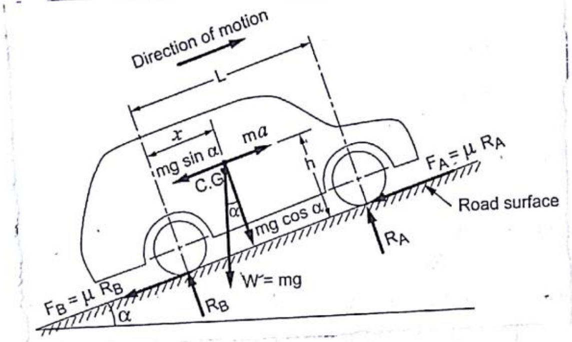
$$\therefore \quad a = \frac{\mu g[L-x]}{L+\mu \cdot h}$$

2. When vehicle moves in a down ward to the plane, then.

$$a = \frac{\mu \cdot g\cos\alpha[L-x]}{L+\mu \cdot h} - g\sin\alpha$$

Case iii :- When the brakes are applied to all the four wheels:-

consider a car moving up an inclined plane. For the equilibrium of the vehicle, the various forces acting upon the vehicle as shown.



Resolving the forces parallel to the plane, we get,

$$F_A + F_B + mg\sin\alpha = m \cdot a \quad - (i).$$

Resolving the forces ⊥ar to the plane,

$$R_A + R_B = mg\cos\alpha \quad - (ii)$$

Taking moments about C.G, we have,

$$(F_A + F_B) \cdot h + R_B \cdot x = R_A [L - x]. \quad -(iii).$$

W.K.T;

$$F_A = \mu \cdot R_A.$$

$$F_B = \mu \cdot R_B$$

$$R_B = mg\cos\alpha - R_A$$

$$R_A = mg\cos\alpha - R_B$$

Substituting the values in eq (iii) we get,

$$\mu (R_A+R_B).h + (mg\cos\alpha - R_A)x = R_A[L-x].$$

$$\Rightarrow \mu[R_A + mg\cos\alpha - \cancel{R_A}].h + [mg\cos\alpha - R_A].x = R_A[L-x].$$

$$\Rightarrow h\cancel{\mu}.R_A + \mu mg\cos\alpha h - \cancel{R_A.h\mu} + mg\cos\alpha x - \cancel{R_A x} = R_A L - \cancel{R_A x}$$

$$\Rightarrow \mu mg\cos\alpha h + mg\cos\alpha . x = R_A . L .$$

$$\therefore R_A = \frac{m.g\cos\alpha(\mu.h+x)}{L}.$$

$$\therefore R_B = mg\cos\alpha - R_A$$

$$= mg\cos\alpha - \left[\frac{mg\cos\alpha[\mu.h+x]}{L}\right]$$

$$= mg\cos\alpha\left[1 - \frac{(\mu.h+x)}{L}\right]$$

$$R_B = m.g\cos\alpha\left[\frac{L-\{\mu h+x\}}{L}\right].$$

From eq (i) Retardation 'a' of the vehicle will be,

$$F_A + F_B + m \cdot g \sin\alpha = m \cdot a.$$

$$\mu \cdot R_A + \mu \cdot R_B + mg \sin\alpha = m \cdot a$$

$$\mu (R_A + R_B) + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu \left[\frac{mg \cdot \cos\alpha (\mu \cdot h + x)}{L} + \frac{mg \cdot \cos\alpha (L - \mu \cdot h - x)}{L} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu\, mg \cos\alpha \left[\frac{\mu \cdot h + x}{L} + \frac{L - \mu \cdot h - x}{L} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu \cdot m \cdot g \cos\alpha \left[\frac{\cancel{\mu h} + \cancel{x} + \cancel{L} - \cancel{\mu h} - \cancel{x}}{\cancel{L}} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \quad \mu \cdot mg \cos\alpha + mg \sin\alpha = m \cdot a$$

$$\Rightarrow \quad \therefore a = \mu g \cos\alpha + g \sin\alpha$$

$$\boxed{a = g [\mu \cos\alpha + \sin\alpha]}$$

NOTE:- 1. When the vehicle moves on a level track, $\alpha = 0$.

$$\therefore \boxed{a = \mu \cdot g}$$

2. When the vehicle moves ~~to the~~ downward to the plane, $\boxed{a = g [\mu \cos\alpha - \sin\alpha]}$

Problems

1. A truck has 3.15m wheel base and the centre of gravity is 1.28m in the front of the rear axle and 0.9m above the ground level. The coeffecient of adhesion between types & roads is 0.6 and the brakes are applied to rear wheels only. What is the minimum distance in which the truck can be stopped on a level road when travelling at 48km/hr? If the weight of truck is 8tons., find the Rn on each wheel during braking.

∴ Given; $L = 3.15m$; $x = 1.28m$; $h = 0.9m$; $\mu = 0.6$;
$u = 48 km/hr \Rightarrow u = 13.3 m/s$; $m = 8 tons = 8000kg$.

⇒ minimum distance travelled by truck before it comes to rest.

Let, $S \Rightarrow$ Distance travelled by truck.

$v^2 - u^2 = 2as$

$s = \frac{u^2}{2a}$

W.K.T, $\alpha = 0$ [∵ at level road] and brakes applied to rear wheels only.

$$\therefore \quad a = \frac{\mu \cdot g [L - x]}{L + \mu \cdot h}$$

$$a = \frac{0.6 \times 9.81 [3.15 - 1.28]}{3.15 + (0.6 \times 0.9)}$$

$$a = 2.98 m/s^2$$

∴ Distance travelled. $S = \frac{u^2}{2a}$

$$S = \frac{(13.3)^2}{2 \times 2.98} = 29.65 \ m$$

Pressure on each wheel during braking:-

Let, $R_A \rightarrow$ Normal reaction between ground & front wheels.

$R_B \rightarrow$ Normal reaction between ground & rear wheels.

W.K.T when brakes are applied to rear wheels only,

then, $R_A = \dfrac{m \cdot g\,(x + \mu \cdot h)}{L + \mu \cdot h}$

$= \dfrac{(8000)(9.81)(1.28 + (0.6)(0.9))}{3.15 + (0.6)(0.9)}$

$R_A = 38708.3\ N$

Pressure on each front wheel, $\dfrac{R_A}{2} = 19354.14\ N$. Ans

For Rear wheels,

$R_B = \dfrac{m \cdot g\,(L - x)}{L + \mu \cdot h} = \dfrac{8000\,(9.81)\,[3.15 - 1.28]}{3.15 + (0.6)(0.9)}$

$\therefore R_B = 39771.70\ N$

Pressure on each Rear wheel, $\dfrac{R_B}{2} = 19885.86\ N$ Ans

Summary

Table 5.1 summarises the expressions used for determining the retardation of the vehicle for different cases.

Table 5.1.

Sl. No.	Case	Vehicle moves up an inclined plane	Vehicle moves on a level track	Vehicle moves down the inclined plane
1.	Brakes are applied to front wheels only	$a = \frac{\mu g \cos\alpha \times x}{(L - \mu h)} + g \sin\alpha$	$a = \frac{\mu g x}{L - \mu h}$	$a = \frac{\mu g \cos\alpha \times x}{L - \mu h} - g \sin\alpha$
2.	Brakes are applied to rear wheels only	$a = \frac{\mu g \cos\alpha (L - x)}{L + \mu h} + g \sin\alpha$	$a = \frac{\mu g (L - x)}{L + \mu h}$	$a = \frac{\mu g \cos\alpha (L - x)}{L + \mu h} - g \sin\alpha$
3.	Brakes are applied to all four wheels	$a = g(\mu \cos\alpha + \sin\alpha)$	$a = g \cdot \mu$	$a = g(\mu \cos\alpha - \sin\alpha)$

* DYNAMOMETERS:

A dynamometer is a brake incorporating a device to measure the frictional resistance applied. This is used to for measuring the driving force or torque transmitted and also the power developed by machine.

Types of Dynamometer:

Dynamometer

- I. Absorbtion dynamometer
  - 1. Prony brake Dynamometer
  - 2. Rope brake dynamometer
- II. Transmission Dynamometer
  - 1. Belt Transmission dynamometer
  - 2. Epicyclic dynamometer
  - 3. Torsion Dynamometer.

* Absorption Dynamometer:

→ In absorption type dynamometer, the entire power developed by the prime mover is absorbed by frictional resistance of the brake and is transformed to heat during the process of measurement.

→ These dynamometers are suitable for measuring output of machines of moderate powers.

→ Some examples: (a) Prony brake dynamometer &
(b) Rope brake dynamometer.

5.8.1. Prony Brake Dynamometer

The prony brake dynamometer is the simplest form of absorption dynamometer. A typical form of prony brake dynamometer is shown in Fig.5.23. It is suitable for engine tests in laboratory.

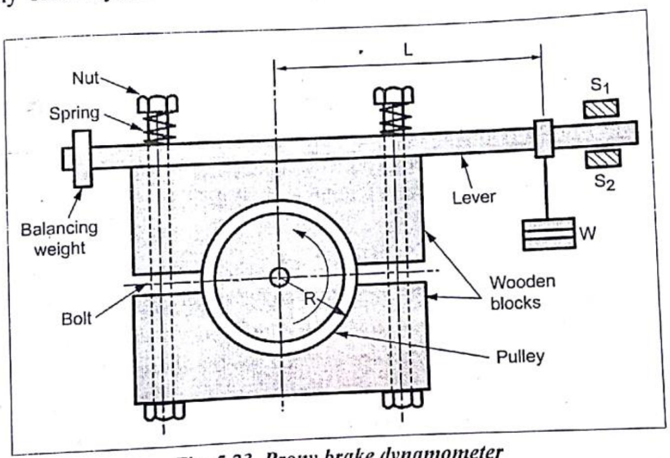


Fig. 5.23. Prony brake dynamometer

Arrangement

It consists of two wooden blocks placed around a pulley fixed to the shaft of the prime mover, whose power is to be measured. The blocks are clamped by means of bolts and nuts. The pressure of the blocks over the pulley is adjusted with the help of nut-helical spring-bolt arrangement. The upper block is attached with a long lever which carries a weight W at its one end. A counter/balancing weight is placed at the other end of the lever to balance the brake when unloaded. Two stoppers S_1 and S_2 are provided to limit the motion of lever.

Working

The friction between the blocks and the pulley tends to rotate the blocks in the direction of shaft rotation. Power is absorbed due to friction. However, the motion is prevented by the suspended weight W provided at the end of lever. The lever remains in horizontal position for the required speed of the engine.

Therefore for measuring the power of the engine, (*i*) attach a known weight W at the end of the lever, and (*ii*) tighten the nuts until the shaft runs at a constant speed and the lever is in horizontal position. At this instant, the moment due to weight W will balance the moment of the frictional resistance between the blocks and the pulley.

Power of the Prime mover:

Let, $W \rightarrow$ weight at the end of lever.

$R \rightarrow$ Radius of pulley.

$L \rightarrow$ Horizontal distance of weight from centre of pulley.

$N \rightarrow$ Speed of shaft, rpm

$F \rightarrow$ frictional resistance b/w blocks & pulley.

W.K.T, braking torque on shaft i.e., the moment of frictional resistance,

$$T = W \cdot L = F \cdot R.$$

$\therefore$ Brake power of engine, = Braking Torque × Angular Speed

$$= T \cdot \omega.$$

$$= \frac{T \times 2\pi N}{60}$$

$$\boxed{P = \frac{W \cdot L \times 2\pi N}{60} \text{ Watt}}$$

Braking power of prime mover is independent on (i) radius of pulley.

(ii) coeffecient of fricⁿ.

(iii) Pr exerted by tightening of the nuts.

(18)

Prob:

① In a prony brake dynamometer, The spring balance reading is 200N, Radius of brake drum is 300mm & distance between The drum axis & hinge of the blocks is 600mm. Determine the pr exerted on drum by tightening the screw, tangential force acting on brake drum & the o/p power of prime mover if recd speed is 300 rpm. Take $\mu = 0.25$.

Sol:

$W = 200N$; $R = 300mm = 0.3m$; $L = 600mm = 0.6m$

$N = 300 rpm$; $\mu = 0.25$.

Tangential force,

$$F = \mu \cdot R_N = \mu \cdot W = 0.25 \times (200) = 50N$$

Power of prime mover,

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times N \times W \cdot L}{60}$$

$$= \frac{2\pi \times 300 \times 200 \times 0.6}{60}$$

$$P = \underline{3.77 kW}$$

Unit-IV

Governors & Balancing of Masses

UNIT - VI ①

GOVERNORS

The function of the governor is to regulate the mean speed of an engine, when there are variations in the load.
i.e: ~~when there are variations~~
when the load increases, its speed decreases. then, the supply of working fluid is increased.
on the other hand, when the load decreases, its speed increases, ~~therof~~ thers less working fluid is required to supply.

Governor automatically controls the supply of working fluid to the engine with varying load conditions & keeps the mean speed within certain limits.

The governors may be classified into:
(1) centrifugal governor:
(2) Inertia governor.

① Centrifugal governor:-
These governors further may be classified as

- Centrifugal Governors
  - Pendulum type
    - Watt Governor
  - Loaded Type
    - Dead weight Governors
      - Porter Governor
      - Proell Governor
    - Spring controlled gove
      - Hartnell governor
      - Hartung governor
      - Wilson-Hartnell governor
      - Pickering governor

(2)

are worked on the balancing of the . gal force on rotating balls by an equal and site radial force, known as "controlling force".
consists of 2 balls which are known as governor balls.

→ These balls revolve with in the spindle. The upper ends of the arms are pivoted to the spindle. So that the balls may rise up or fall down as they revolve about vertical axis.

→ The arms are connected by the links to the sleeve which is keyed to the spindle.

→ This sleeve revolves with the spindle, but can slide up and down. The balls rises up when the spindle speed increases and falls when the spindle speed decreases.

→ In order to limit the travel of sleeve in upward & downward direction, two stops are provided on the spindle.

→ The supply of working fluid increases, when the sleeve falls and it decreases when the sleeve raises.

→ If the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls moves inwards and sleeves comes downwards.

→ If the load on the engine decreases, the engine and the governor speed increases. This results in the increase in centrifugal force of balls. Thus the balls moves upwards and sleeve raises upwards.

*NOTE: The controlling force is provided by the action of gravity as in Watt Governor "OR" by a spring in case of "Hartnell Governor"

(3)

Used in Governors:-

...ght of the Governor: It is the vertical (or) radial distance from the centre of the ball to the (height) point where the axes of arms intersect on the spindle axes. It is denoted by 'h'.

Equilibrium Speed:- It is the speed at which the governor balls, arms etc.. are in complete equilibrium of the sleeve doesnot move downwards (or) upwards.

3. **Sleeve Lift:** It is the vertical distance which the sleeve travels due to change in equilibrium speed.

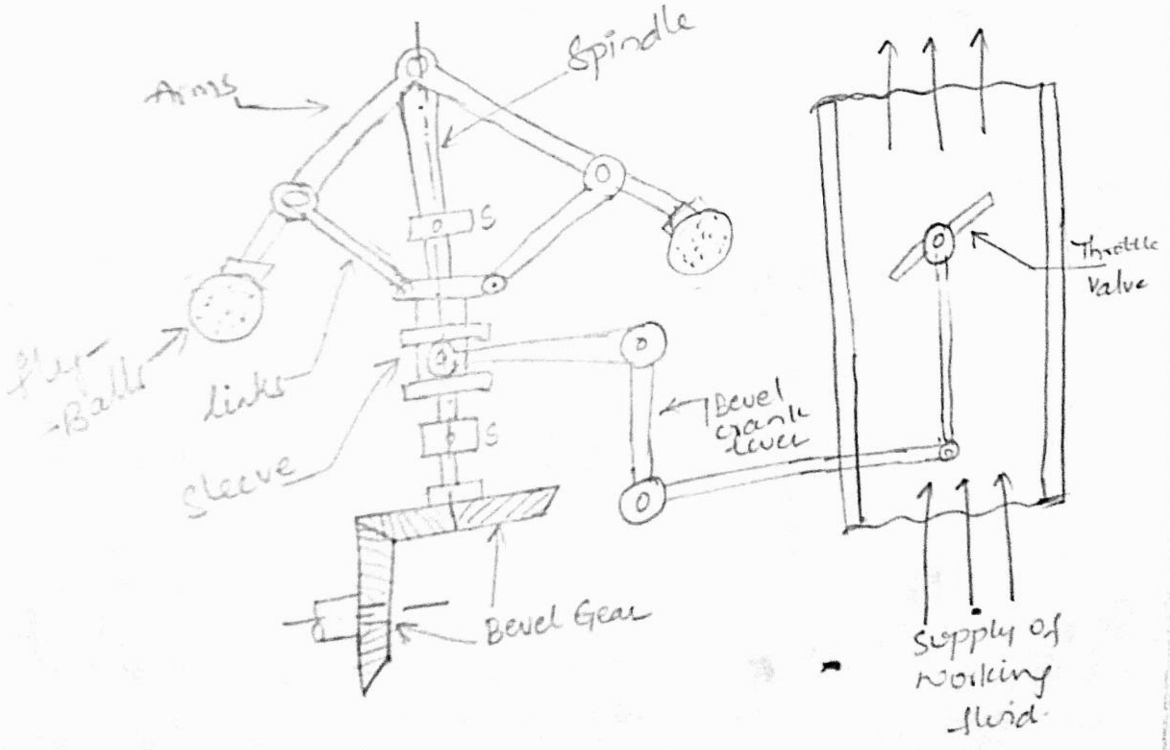


fig: Centrifugal Governor.

...overnor:

(1)

- The simplest form of a centrifugal governor is a ... governor.
- It is basically a conical pendulum with links attached to sleeve of negligible mass. The arms of governor may be connected to the spindle in 3 ways;
  - (i) The pivot P, may be on the spindle axis as shown in fig ⓐ.
  - (ii) The pivot P, may be offset from the axis and the arms intersects at O, when they are produced as shown in fig ⓑ.
  - (iii) The pivot P, may be offset, but the arms cross the axis at O, as shown in fig ⓒ.

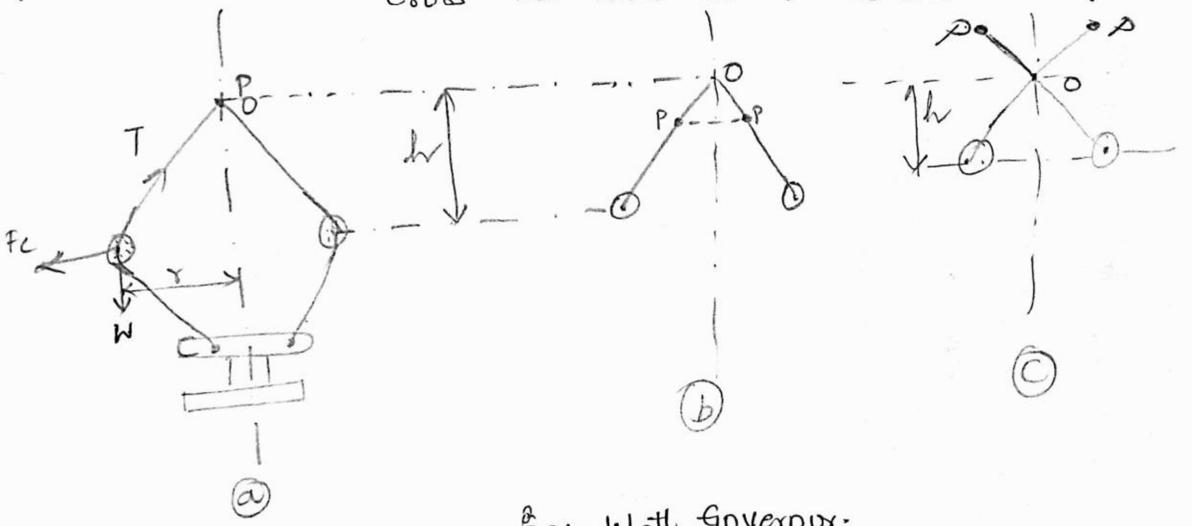


fig: Watt Governor.

Here, m → mass of the ball in kg.
W → weight of the ball in newtons = m.g.
T → Tension in arm in newtons.
ω → Angular velocity of the arm and the ball about spindle axis, in rad/sec.
r → radius of the path of rotation of the ball i.e., horizontal distance from the centre of ball to the spindle axis, in m.
Fc → centrifugal force acting on ball in N = $m\omega^2 r$.
h → height of the governors in m.

...c is assumed that the weight of arms, links (5).
the sleeve are negligible when compared to weight
the balls. Now, the ball is in equilibrium
condition under the action of

(i) The centrifugal force (F_c) acting on ball.
(ii) The tension (T) in the arm. and.
(iii) The weight of the ball.

Taking the moments about point O, we have,

$$F_c \times h = W \times r$$

$$\because F_c = m r w^2 \text{ \& } W = mg.$$

$$\cancel{m}\cancel{r} w^2 \times h = \cancel{m} \cdot g \cdot \cancel{r}$$

$$h = \frac{g}{w^2} \quad -(1).$$

W.K.T, $\omega = \dfrac{2\pi N}{60}$.

$$h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2} \Rightarrow h = \frac{9.81}{\left(\frac{2\pi N}{60}\right)^2}$$

$$\therefore \boxed{h = \frac{895}{N^2} \text{ mts}} \quad -(2).$$

NOTE:- from the above expression we can notice that, h is inversely proportional to the N^2.
i.e; the height of governor decreases at (larger) high speeds. The governor may only work relatively at low speeds i.e; from 60 to 80 rpm.

Problem (6)

Calculate the vertical height of a watt governor when it rotates at 60 rpm. Also find the change in vertical height when its speed increases to 61 rpm.

Sol: Given data:

$N_1 = 60$ rpm & $N_2 = 61$ rpm.

Initial height:

W.K.T, $h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2}$

$= 0.248$ m.

final height, $h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2}$

$= 0.24$ m.

∴ change in vertical height,

$= h_1 - h_2$

$= 0.248 - 0.24$

$= 0.008$ m

$= 8$ mm.

Problems

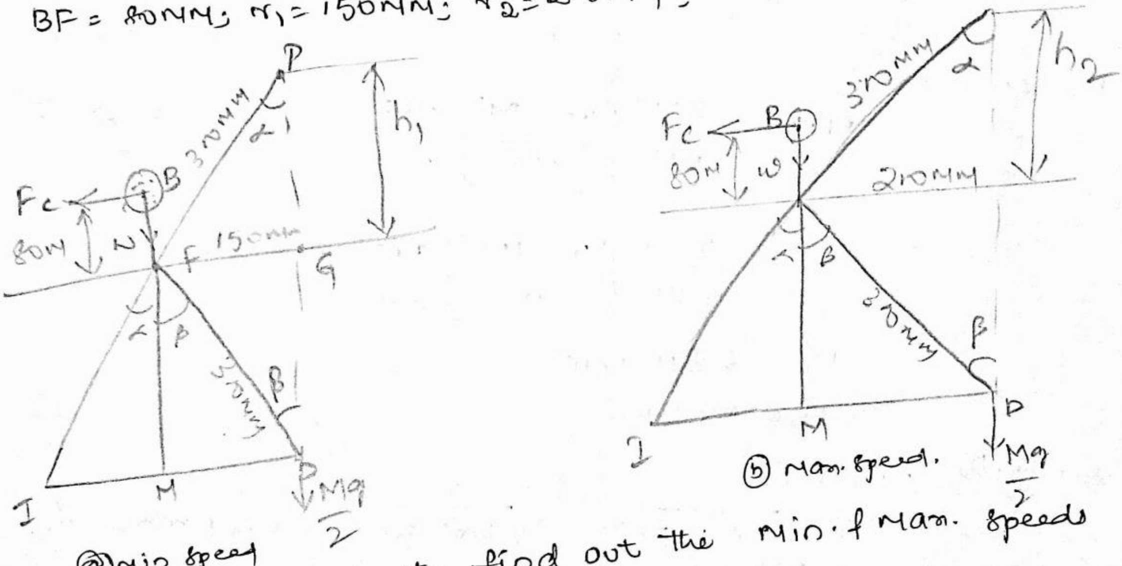
(16).

A Porter governor has equal arms of length 300 mm. The upper & lower ends of arms are pivoted on the axis of governor. The extension arms of the lower links are each 80 mm long & parallel to the axis when the radii of rotation of the balls are 150 mm & 200 mm. The mass of each ball is 10 kg. and the mass of central load is 100 kg. Determine the range of speed of governor.

Sol: Given Data:

PF = DF = 300 mm;

BF = 80 mm; $r_1 = 150$ mm; $r_2 = 200$ mm; m = 10 kg; M = 100 kg.



Here, we need to find out the Min. & Max. speeds of the governor

i.e.; $N_1 \rightarrow$ Min. speed of governor, when $r_1 = 150$ mm

$N_2 \rightarrow$ Max. speed of governor, when $r_2 = 200$ mm

Before that we have to find out the height's of the governor.

$h_1 = PG.$

By applying pythogorus theorem.

$PG^2 = PF^2 - FG^2$

$= (300)^2 - (150)^2$

$PG = 259.8 \simeq 260\,mm$

$\therefore \boxed{PG = 0.26m = h_1}$

& $FM = PG = h_1$

$= 0.26m.$

$\therefore BM = BF + FM \Rightarrow 80 + 260$

$= 340\,mm \Rightarrow \boxed{BM = 0.34m}$

We know that,

$$N_1^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\frac{895}{h_1} \qquad [\because \alpha = \beta \text{ or } r = 1].$$

$$= \frac{0.26}{0.34}\left[\frac{10+100}{10}\right]\frac{895}{0.26}$$

$N_1^2 = 28929.15$

$\therefore N_1 = 170\,rpm.$

Similarly, $h_2 = PG$

$h_2 \Rightarrow PG = \sqrt{PF^2 - FG^2} = \sqrt{300^2 - 200^2} = 224\,mm = h_2.$

$FM = h_2 = 0.224m = 224\,mm.$

$BM = BF + FM = 80 + 224 = 304\,mm = 0.304m.$

Now, $N_2^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\frac{895}{h_2}$

$= \frac{0.224}{0.304}\left[\frac{100+10}{10}\right]\frac{895}{0.224}$

$N_2 = 180\,rpm.$

Range of Speed $\Rightarrow N_2 - N_1 = 180 - 170$

$= 10\,rpm.$

(11).

Following particulars refer to a proell governor with open arms:

→ Length of all arms = 200 mm;

→ Distance of pivot of arms from the axis of rotation = 40 mm

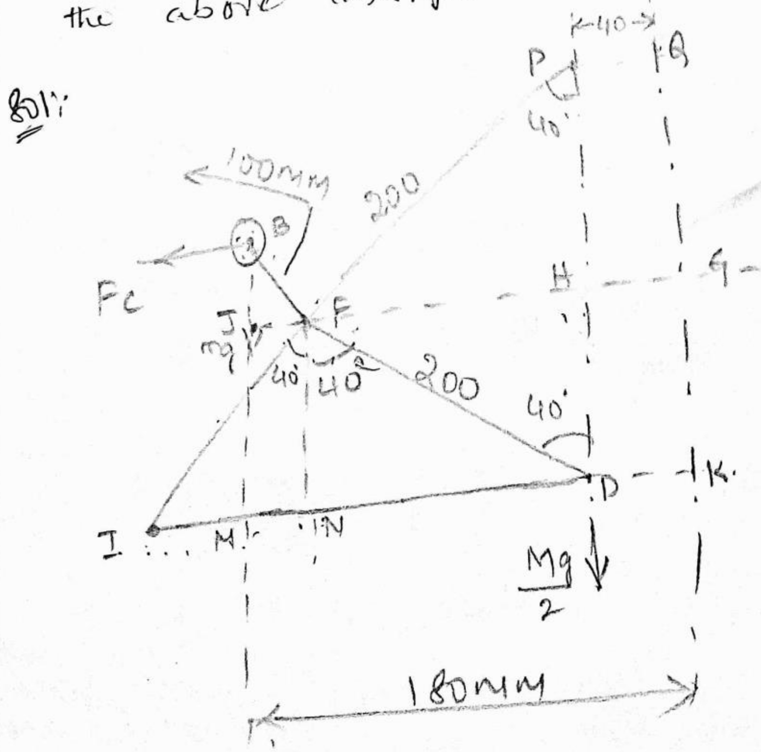
→ length of extension of lower arms to which each ball is attached = 100 mm;

→ mass of each ball = 6 kg.

→ mass of central load = 150 kg.

If the radius of rotation of balls is 180 mm when the arms are inclined at an angle of 40° to the axis of rotation, find the equilibrium speed for the above configuration.

Sol:



(12).

Given Data:

$DF = 200mm$;

$\phi = DK = 40\ mm = HG$.

$BF = 100mm$;

$m = 6kg$; $M = 150kg$;

$r = JG = 180mm$; $\alpha = \beta = 40°$.

Let, N → Equilibrium speed.

Taking the moments about I,

$$F_c \times BM = m \cdot g * IM + \frac{M \cdot g}{2} \cdot ID. \quad -(1).$$

calculating BM, IM & ID values.

from the equilibrium position of the governor,

consider Δ^{le} PFH,

$$\cos\alpha = \frac{PH}{PF} \Rightarrow PH = PF \times \cos 40°.$$

$$= 200 \times \cos 40°.$$

$$\boxed{PH = 153.2mm}$$

Similarly, $\sin\alpha = \frac{FH}{PF} \Rightarrow FH = PF \times \sin 40°$

$$= 200 \times \sin 40°$$

$$\boxed{FH = 128.5\ mm}$$

Now, $JF = JG - HG - FH$

$$= 180 - 40 - 128.5$$

$$JF = 11.5\ mm$$

Now from Δ^{le} BJF

applying pythogorus theorem,

$$BJ^2 = BF^2 - JF^2$$

$$= 100^2 - 11.5^2$$

$$BJ = 99.3\ mm.$$

(13).

$BM = BJ + JM \qquad [JM = HD = PH]$

$= BJ + PH$

$BM = 99.3 + 153.2$

$\boxed{BM = 252.5\ mm}$

Now, $IM = IN - MN$

$= FH - JF$

$IM = 128.6 - 11.5$

$\boxed{IM = 117.1\ mm}$

$ID = IN + ND$

$= 2 \times IN \qquad [\because IN = FH]$

$= 2 \times FH$

$ID = 2 \times 128.6 \Rightarrow 257.2\ mm$

Sub.. BM, ID, IM in eq. (1).

$F_c \times BM = m \cdot g \times IM + \frac{M \cdot g}{2} \times I \cdot D$

$m r \omega^2 \times 252.5 = (6)(9.81)(117.1) + \frac{150 \times 9.81}{2} \times (257.2)$.

$(6)(180)\left[\frac{2\pi N}{60}\right]^2 \times (252.5) = 196127.4$

$(6 \times 0.18)\left[\frac{2\pi N}{60}\right]^2 = 776.74$

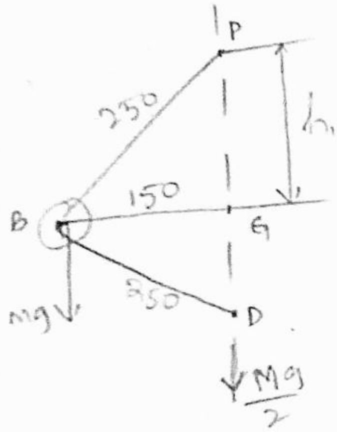
$N = 256\ rpm$

Problems

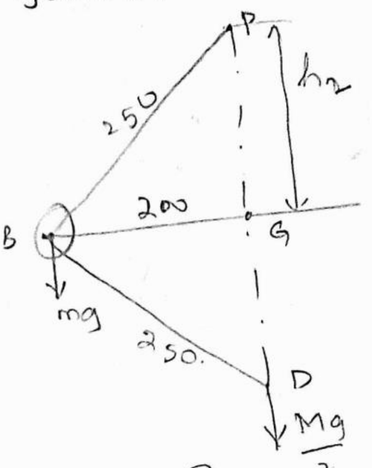
(16)

1. A porter governor has equal arms each 250 mm long & pivoted on axis of rotation. Each ball has a mass of 5 kg and the mass of central load on sleeve is 15 kg. The radius of rotation of ball is 150 mm when the governor begins to lift & 200 mm when the governor is at maximum speed. Find the min. & max. speeds and range of speed of governor.

Sol:



(a) min. pos.



(b) Max. pos

Given Data:

$BP = BD = 250\,mm$; $m = 5\,kg$; $M = 15\,kg$;

$r_1 = 150\,mm = 0.15\,m$; $r_2 = 200\,mm = 0.2\,m$

case (i) Minimum speed, when, $r_1 = BG = 0.15\,m$

Let, N_1 = min. speed.

First we have to ht. of governor (h_1).

$h_1 = PG^2 = PB^2 - BG^2 \Rightarrow \sqrt{(250)^2 - (150)^2}$

$\Rightarrow h_1 = 0.2\,m$.

W.K.T, $N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1}$

$= \frac{5+15}{5} \times \frac{895}{0.2}$

$N_1 = 133.8$ rpm.

case (ii) Max. speed, $BG = 200\,m$

$r_2 = 0.2\,m$.

$h_2 = PG^2 = \sqrt{(250)^2 - (200)^2}$

$h_2 = 0.15\,m$.

$N_2^2 = \frac{m+M}{m} \times 895/h_2$

$= \frac{5+15}{5} \times \frac{895}{0.15}$

$N_2 = 154.5$ rpm.

$\therefore$ Range of speed,

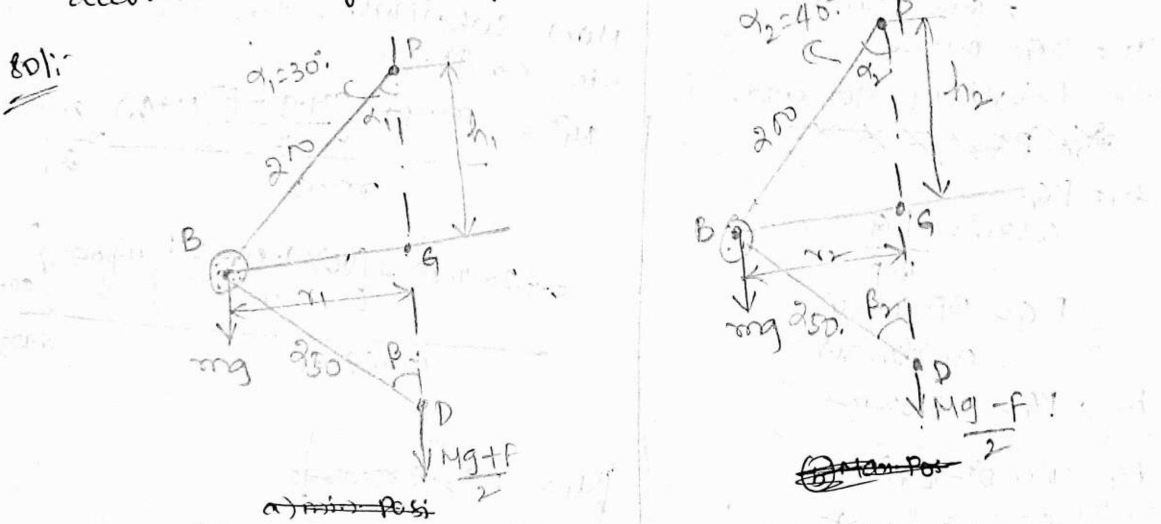
$N_2 - N_1 = 154.5 - 133.8$

$= 20.7$ rpm.

(17)

...ngine governor of the Porter type, the upper & lower arms ...00mm ~~long~~ & 250 mm respectively & pivoted on the axis of ...ation. The mass of central load is 15 kg and the mass of each ball is 2 kg & friction of sleeve together with resistance of operating gear is equal to a load of 24N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° & 40°. find, taking friction into account, range of speed of governor.

Sol:-



Given Data:

$BP = 200mm$; $BD = 250mm$;

$BP = 0.2m$; $BD = 0.25m$;

$M = 15kg$; $m = 2kg$; $F = 24N$; $\alpha_1 = 30°$; $\alpha_2 = 40°$.

Let, $N_1 \rightarrow$ min. speed and

$N_2 \rightarrow$ max. speed.

case(i):-

We know that, when the sleeve ~~moves downwards~~ move downwards, then the min. speed of the frictional force acts upwards, then the min. speed of governor may be,

$$\boxed{N_1^2 = \frac{m.g + \left[\frac{M.g - F}{2}\right](1+q_1)}{m.g} \times \frac{895}{h_1}} \quad -(1).$$

Here, height, F, q are to be calculated and also the value of β.

→ Minimum radius of rotation,

$$r_1 = BG$$

$$\sin\alpha_1 = \frac{BG}{BP}$$

$$BG = BP \cdot \sin\alpha_1$$

$$= (200)(\sin 30)$$

$$r_1 = BG = 0.1\,m$$

→ Now height of governor,

$$h_1 = PG$$

$$\cos 30^\circ = \frac{PG}{BP}$$

$$PG = BP \cdot \cos 30$$

$$= (200)(\cos 30)$$

$$h_1 = PG = 0.1732\,m$$

$$DG = \sqrt{(BD)^2 - (BG)^2}$$

$$= \sqrt{(0.25)^2 - (0.1)^2}$$

$$DG = 0.23\,m$$

$$\tan\beta_1 = \frac{BG}{PG} = \frac{0.1}{0.23}$$

$$\tan\beta_1 = 0.4348$$

$$\tan\alpha_1 = \tan 30 = 0.5774$$

$$\therefore q = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4348}{0.5774}$$

$$q_1 = 0.753$$

Now substitute h_1, q_1, F, ... in eq. (1).

$$N_1^2 = \frac{m \cdot g + \left[\frac{M \cdot g - F}{2}\right](1+q_1)}{m \cdot g} \times \frac{895}{h_1}$$

$$= \frac{(2 \times 9.81) + \left[\frac{(15)(9.81) - 24}{2}\right](1+0.753)}{(2)(9.81)} \times \frac{895}{0.1732}$$

$$\boxed{N_1 = 183.3\,rpm}$$

(18)

When the sleeve moves upwards, the frictional force acts downwards, then the max speed of governor will be,

$$\boxed{N_2^2 = \frac{m.g + \left[\frac{M.g + F}{2}\right](1+q_2)}{m.g} \times \frac{895}{h_2}} \quad (2).$$

Now calculating the values of h_2, β_2 & q_2,

Now, $\alpha_2 = 40°$.

Similarly,

→ radius of rotation,

$r_2 = BG$

$\sin\alpha_2 = \frac{BG}{BP}$

$BG = BP \sin\alpha_2$

$= (200)(\sin 40°)$

$r_2 = BG = 0.1268m$

→ Height of governor,

$h_2 = PG$

$\cos\alpha_2 = \frac{PG}{BP}$

$PG = BP \cos\alpha_2$

$= (200) \cos 40°$

$PG = 0.1532m$

Now, $DG = \sqrt{(BD)^2 - (BG)^2}$

$= \sqrt{(0.25)^2 - (0.1268)^2}$

$\boxed{DG = 0.2154m}$

$\tan\beta_2 = \frac{BG}{DG} = \frac{0.1268}{0.2154}$

$\tan\beta_2 = 0.59.$

$\tan\alpha_2 = \tan 40° = 0.839$

$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.59}{0.839}$

$q_2 = 0.703.$

Now sub.. q_2, h_2.. in (2).

$$N_2^2 = \frac{(2\times9.81) + \left[\frac{(15\times9.81)+24}{2}\right](1+0.703)}{2\times9.81} \times \frac{895}{0.1532}$$

$\boxed{N_2 = 222 \text{ rpm}}$

∴ Range of speed,

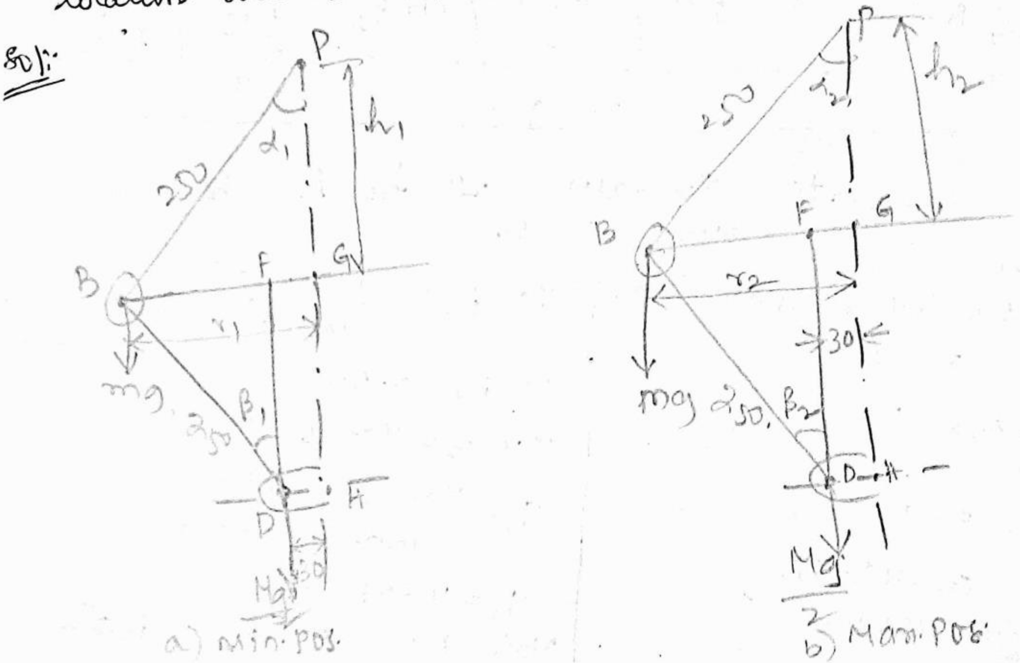
$N_2 - N_1$

$= 222 - 183.3$

$= 38.7$ rpm

(3) A porter governor has all four arms 250 mm long. The upper arms are attached to axis of rotation & the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg & the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm & 200 mm. Determine the range of speed.

Sol:



Given Data:

BD = BP = 250 mm = 0.25 m;

DH = 30 mm; m = 5 kg;

M = 50 kg; r_1 = 150 mm & r_2 = 200 mm

Let, $N_1 \rightarrow$ Min. speed.

$N_2 \rightarrow$ max. speed.

W.K.T,

$$\boxed{N_1^2 = \frac{m + \frac{M}{2}(1+q_1)}{m} \times \frac{895}{h_1}} \quad (1)$$

here, we have to calculate h_1 & q_1.

(19).

of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$$

$$= \sqrt{(250)^2 - (150)^2}$$

$$h_1 = PG = 200\,mm$$

$$h_1 = 0.2\,m$$

$$BF = BG - \cancel{FG}$$

$$= 150 - 30$$

$$\boxed{BF = 120\,mm}$$

$$\tan\alpha_1 = \frac{BG}{PG} = \frac{150}{200} = 0.75.$$

$$DF = \sqrt{(250)^2 - (120)^2}$$

$$DF = 219\,mm.$$

$$\tan\beta_1 = \frac{BF}{DF}$$

$$= \frac{120}{219}$$

$$\tan\beta_1 = 0.548$$

$$q_1 = \frac{\tan\beta_1}{\tan\alpha_1}$$

$$q_1 = \frac{0.548}{0.75}$$

$$q_1 = 0.731$$

$$N_1^2 = \frac{m + \frac{M}{2}\cdot g(1+q_1)}{m} \times \frac{895}{h_1}$$

$$= \frac{5 + \frac{50}{2}(1+0.731)}{5} \times \frac{895}{0.2}$$

$$N_1 = 208\,rpm.$$

Case(ii):-

$$\boxed{N_2^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2}}$$

→ Height of the governor,

$h_2 = PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(250)^2 - (200)^2}$

$h_2 = 150\text{ mm} = 0.15\text{ m}$

$BF = BG - FG$

$= 200 - 30 = 170\text{ mm} = 0.17\text{ m}.$

$DF = \sqrt{(DB)^2 - (BF)^2}$

$= \sqrt{(250)^2 - (170)^2}$

$DF = 183\text{ mm}$

$\tan\beta_2 = \frac{BF}{DF} = \frac{170}{183} \Rightarrow \tan\beta_2 = 0.93$

$\tan\alpha_2 = \frac{BG}{PG} = \frac{200}{150} \Rightarrow \tan\alpha_2 = 1.33$

$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.93}{1.33} \Rightarrow q_2 = 0.7.$

Now,

$$N_2^2 = \frac{5 + \frac{50}{2}(1+0.7)}{5} \times \frac{895}{0.15}$$

$N_2 = 238\text{ rpm}.$

$\therefore$ The range of speed, $= N_2 - N_1$

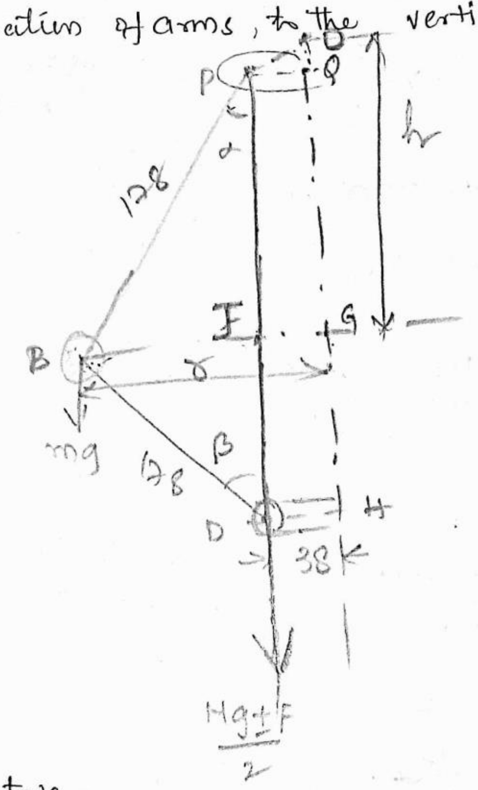
$= 238 - 208$

$= 30\text{ rpm}.$

(3) (20).

Arms of a porter governor are 178mm long and are pivoted at a distance of 38mm from the axis of rotation. The mass of each ball is 1.15kg and mass of sleeve is 20kg. The governor sleeve begins to rise at 280rpm. When the links are at an angle of 30° to the vertical. Assuming the frictional force to be constant, determine the minimum & maximum speed of rotation when the inclination of arms to the vertical is 45°.

Sol:-



Given Data:-

$BP = BD = 178mm$; $DH = JG = 38mm$; $N = 280rpm$;

$m = 1.15kg$; $M = 20kg$; $\alpha = \beta = 30°$.

case(i):-

First we find the friction when inclination will be 30°.

→ Radius of rotation will be,

$r = BG = BJ + JG$

$BJ = BP \sin\alpha$

$BJ = 178 \times \sin 30°$

$BG = 178 \times \sin 30 + 38$

$\therefore$ ~~BJ = 2~~

$r = BG = 127mm$

$r = BG$

$= BJ + JG$

$= BP \sin\alpha + JG$

$= 178 \sin 45^\circ + 38$

$\boxed{r = 164\,mm}$

and height of the governor,

$$h = \frac{BG}{\tan\alpha}$$

$$= \frac{164}{\tan 45^\circ}$$

$h = 164\,mm = 0.164\,m.$

Let, $N_1 \rightarrow$ min. speed of rotatn &

$N_2 \rightarrow$ max. speed of rotatn.

w.k.t,

$$N_1^2 = \frac{m.g + [M.g - F]}{m.g} \times \frac{895}{h}$$

$$= \frac{(1.15)(9.81) + \{(20 \times 9.81) - 10\}}{(1.15)(9.81)} \times \frac{895}{0.164}$$

$N_1 = 309$ rpm.

and height of Governor,

$$h = \frac{BG}{\tan\alpha}$$

$$h = 127/\tan 30^\circ.$$

$$h = 220\,mm = 0.22\,m.$$

W.K.T,

$$N^2 = \frac{m.g + [M.g \pm F]}{m.g} \times \frac{895}{h} \qquad [\because \alpha = \beta = 30^\circ;\ q = 1].$$

$$(280)^2 = \frac{(1.15 \times 9.81) + [(20)(9.81) \pm F]}{(1.15 \times 9.81)} \times \frac{895}{0.22}$$

$$\pm F = \frac{(280)^2 \times (1.15) \times (9.81) \times 0.22}{895} - (1.15 \times 9.81) - (20 \times 9.81)$$

$$\boxed{F = 10\,N}$$

When the inclination of the arms to the vertical is 45°. i.e; $\alpha = \beta = 45^\circ$. with considering frictional force, $F = 10N$.

$$N_1^2 = \frac{m.g + [M.g \pm F]}{m.g} \times \frac{895}{h} \quad \because \left[\begin{matrix}\alpha = \beta = 45^\circ. \\ q = 1\end{matrix}\right].$$

$$N_2^2 = \frac{m.g + [M.g + F]}{m.g} \times \frac{895}{h}$$

and $$N_2^2 = \frac{m \cdot g + [M \cdot g + F]}{m \cdot g} \times \frac{895}{h}$$

$$= \frac{(1.15)(9.81) + [(20)(9.81) + 10]}{(1.15)(9.81)} \times \frac{895}{0.164}$$

$N_2 = 324$ rpm

and range of speed, $= N_2 - N_1$

$= 324 - 309$

$= 15$ rpm.

(22)

Governor:-

A Hartnell Governor is a spring loaded governor. It consists of two bell crank levers pivoted at points O, O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB & a roller at the end of horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on sleeve.

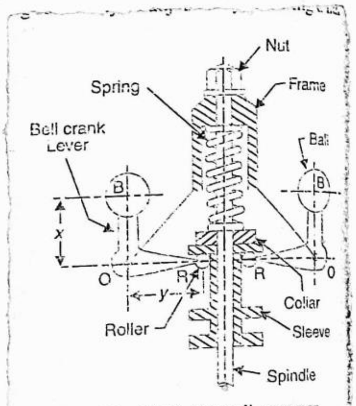


Fig. 18.18. Hartnell governor.

Let, $m \rightarrow$ mass of each ball in kg,

$M \rightarrow$ mass of sleeve, in kg

$r_1 \rightarrow$ minimum radius of rotation, mts.

$r_2 \rightarrow$ maximum radius of rotation, mts.

$\omega_1 \rightarrow$ Angular speed of governor at min. radius, rad/s.

$\omega_2 \rightarrow$ Angular speed of governor at max. radius, rad/s.

$S_1 \rightarrow$ Spring force exerted on the sleeve at ω_1, in N.

$S_2 \rightarrow$ Spring force exerted on the sleeve at ω_2, in N

$F_{c1} \rightarrow$ centrifugal force at ω_1, N $F_{c1} = m r \omega_1^2$

$F_{c2} \rightarrow$ centrifugal force at ω_2, N, $F_{c2} = m r \omega_2^2$.

$S \rightarrow$ Stiffness of the spring

$x \rightarrow$ length of the vertical or ball arm of lever, mts.

$y \rightarrow$ length of horizontal or sleeve arm of lever, mts.

$r \rightarrow$ Distance of fulcrum 'O' from the governor axis (or) radius of rotation when the governor is in mid position.

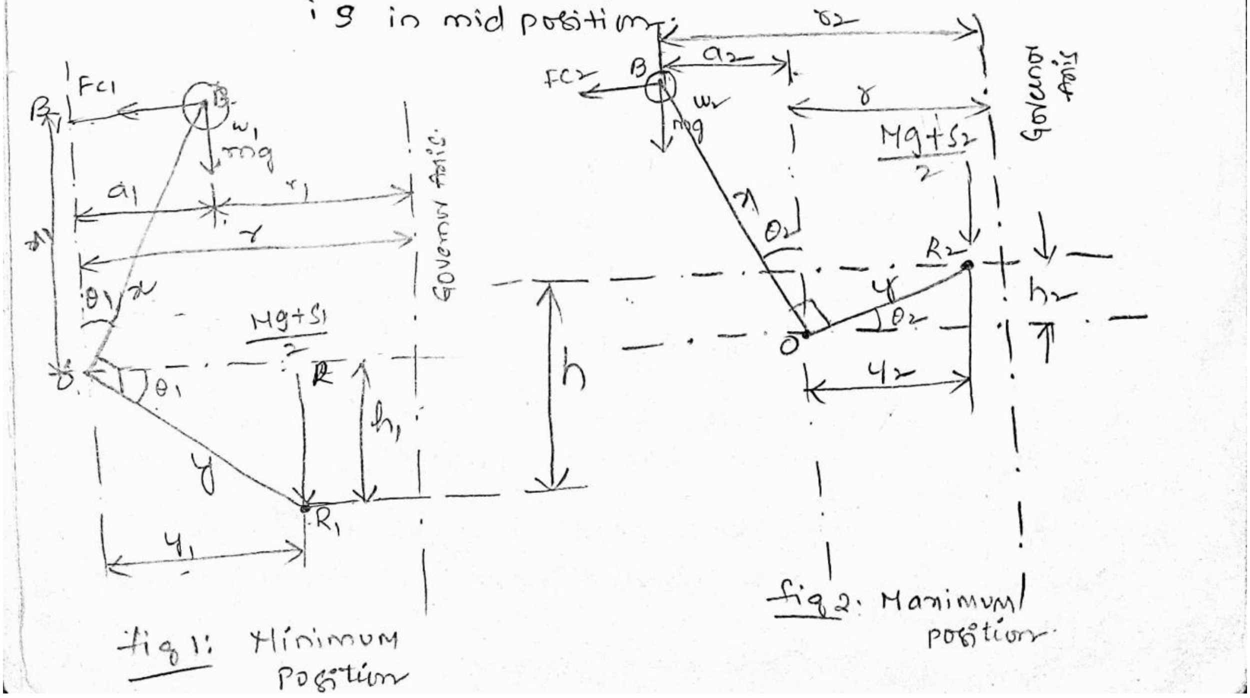


fig 1: Minimum Position

fig 2: Maximum position

(53).

...nsider the forces acting at one ball crank lever. The minimum & maximum position are shown in fig ① & fig ②.

Let, $h \rightarrow$ the compression of the spring when radius of rotation changes from r_1 to r_2.

* For minimum position : i.e; when the radius of rotation changes from ~~...~~ r to r_1, as shown in fig ①, the compression of the spring or lift of sleeve h_1 is given by,

$\theta = \frac{Arc}{Radius} = \frac{RR_1}{OR_1} = \frac{BB_1}{OB}$

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \text{— ①.}$$

For maximum position, i.e; when the radius of rotation changes from r to r_2, as shown in fig ②, the compression of spring, or lift of sleeve h_2,

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \text{— ②}$$

Adding Eq. ① + ②.

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x}$$

Sleeve lift, h (or) $\boxed{h = (r_2 - r_1) \times \frac{y}{x}}$ — ③.

Now for minimum position, taking moments about 'O',

$$\frac{M \cdot g + S_1}{2} \times y_1 = F_{C1} \times r_1 - m \cdot g \times a_1 \quad -(4)$$

$$\left(M \cdot g + S_1 = \frac{2}{y_1}\left[F_{C1} \times r_1 - m \cdot g \times a_1\right] \quad -(4) \right)$$

Now taking moments again 'O' for maximum position,

$$\frac{M \cdot g + S_2}{2} \times y_2 = F_{C2} \times r_2 + m \cdot g \times a_2 \quad -(5)$$

$$\left(Mg + S_2 = \frac{2}{y_2}\left[F_{C2} \times r_2 + m \cdot g \times a_2\right] \quad -(5) \right)$$

Substracting eq. (4) from eq. (5).

$$S_2 - S_1 = \frac{2}{y_2}\left[F_{C2} \times r_2 + m \cdot g \times a_2\right] - \frac{2}{y_1}\left[F_{C1} \times r_1 - m \cdot g \times a_1\right]$$

Let, $S_2 - S_1 = h \cdot S$ & w.k.t,

$$h = (r_2 - r_1) \cdot \frac{y}{r}$$

$$\boxed{S = \frac{S_2 - S_1}{h} = \frac{S_2 - S_1}{(r_2 - r_1)} \times \frac{r}{y}}$$

Neglecting obliquity effect of the arms (i.e; $r_1 = r_2 = r$ & $y_1 = y_2 = y$) and the moment due to weight of ball, (m.g.),

(5) (24).

minimum position,

$$\frac{M \cdot g + s_1}{2} \times y = F_{C_1} \times r \quad \text{or}$$

$$M \cdot g + s_1 = \frac{2}{y}[F_{C_1} \times r]. \quad -(6)$$

for maximum position,

$$\frac{Mg + S_2}{2} \times y = F_{C_2} \times r \quad \text{or.}$$

$$Mg + S_2 = \frac{2}{y}[F_{C_2} \times r]. \quad -(7).$$

$Mg + S_2 - Mg + S_1$

substituting (6) ~~(5)~~ from (7),

$$S_2 - S_1 = \frac{2}{y}[F_{C_2} - F_{C_1}] \times r$$

$$\boxed{S_2 - S_1 = 2[F_{C_2} - F_{C_1}] \times \frac{r}{y}} \quad \rightarrow (8)$$

~~let~~ let; $S_2 - S_1 = h \cdot s$; $h = (r_2 - r_1)\left(\frac{y}{r}\right)$.

~~h.s~~ , sub. .. in eq (8).

$$h \cdot s = 2[F_{C_2} - F_{C_1}] \times \frac{r}{y}$$

$$(r_2 - r_1) \cdot \frac{y}{r} \cdot s = 2[F_{C_2} - F_{C_1}] \times \frac{r}{y}$$

$$\boxed{s = 2\left[\frac{F_{C_2} - F_{C_1}}{r_2 - r_1}\right] \times \left[\frac{r}{y}\right]^2} \quad -(9).$$

$s \rightarrow$ stiff

NOTE:-

1. When the friction is taken into account, the weight of the sleeve $M.g$ may be replaced by $M.g \pm F$.

2. The centrifugal force for any intermediate position i.e; between minimum & Maximum position, at a radius of rotation 'r' may be obtained as;

Since, the stiffness for a given spring is constant for all positions, therefore for minimum & intermediate position,

$$S = 2\left[\frac{F_C - F_{C_1}}{r - r_1}\right]\left[\frac{x}{y}\right]^2 \quad - \text{(10)}$$

& for intermediate & Maximum position,

$$S = 2\left[\frac{F_{C_2} - F_C}{r_2 - r}\right]\left[\frac{x}{y}\right]^2.$$

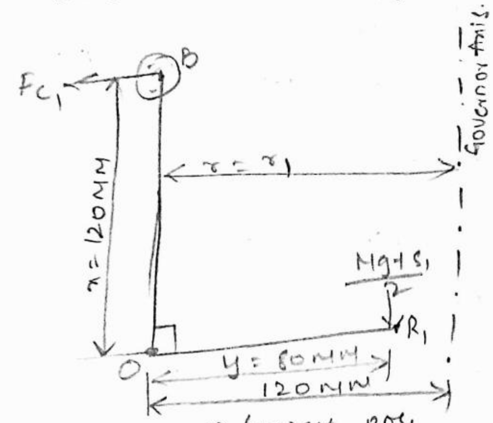
$\therefore$ from eq's (8), (9) & (10).

$$\frac{F_{C_2} - F_{C_1}}{r_2 - r_1} = \frac{F_C - F_{C_1}}{r - r_1} = \frac{F_{C_2} - F_C}{r_2 - r}$$

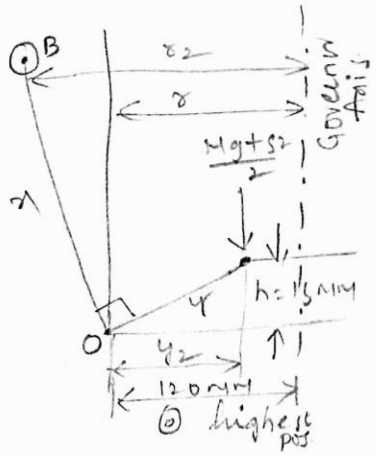
Problem (25)

A Hartnell governor having a central sleeve spring & 2 right angled bell crank levers moves between 290 r.p.m & 310 rpm for a sleeve lift of 15 mm. The sleeve arms and ball arms are 80 mm & 120 mm resp. The levers are pivoted at 120 mm from governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine: 1. loads on the spring at the lowest and highest equilibrium speeds & 2. stiffness of spring.

Sol:



a) lowest pos.



(b) highest pos

Given Data:

$N_1 = 290$ rpm

$\omega_1 = \frac{2\pi \times 290}{60}$

$\omega_1 = 30.4 \frac{rad}{sec}$

$N_2 = 310$ rpm

$\omega_2 = \frac{2\pi \times 310}{60}$

$\omega_2 = 32.5 \frac{rad}{sec}$

$h = 15$ mm

$h = 0.015$ m ; $y = 80$ mm

$y = 0.08$ m

$x = 120$ mm $= 0.12$ m ; $r = 120$ mm

$r = 0.12$ m ;

$m = 2.5$ kg.

1. Loads on the spring at lowest & highest equilibrium speeds.

Let, S_1 = Spring load at lowest equilibrium speed.

S_2 " " " highest " " .

Since, the ball arms are parallel to governor axis at lowest equilibrium speed (i.e; $N_1 = 290$ rpm)

$\therefore r = r_1 = 120$ mm $= 0.12$ m.

W.K.T, F_c at min. speed,

$F_{c_1} = m r_1 \omega_1^2 = (2.5) \times (0.12)(30.4)^2 = 277\,N$

Before finding F_c for max. speed, i.e; $F_{c_2} = m r_2 \omega_2^2$.
we have to know r_2.

Firstly let us calculate 'r_2' i.e. radius of rotation at highest speed.

W.K.T, $h = (r_2 - r_1) \cdot \frac{y}{x}$

$r_2 = (h + r_1) \times \frac{x}{y}$

$= (0.015 + 0.12) \times \frac{0.12}{0.08}$

$r_2 = 0.1425\,m.$

Now, $F_{c_2} = m r_2 \omega_2^2 = (2.5) \times (0.1425)(32.5)^2$

$F_{c_2} = 376\,N.$

Neglecting obliquity effect ~~of arms~~ moment due to weights,

for min. posi. $(M.g) + S_1 = 2 F_{c_1} \times \frac{x}{y}$

$S_1 = 2 \times 277 \times \frac{0.12}{0.08} = 831\,N.$

for max. pos. $M.g + S_2 = 2 F_{c_2} \times \frac{x}{y}$

$= 2 \times 376 \times \frac{0.12}{0.08}$

$S_2 = 1128\,N$

2. stiffness of spring:

$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8\ N/mm$

Head of the Department

Principal

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Governor:

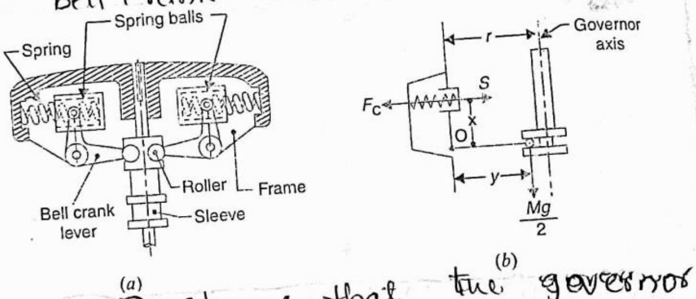
A spring controlled governor of the Hartung type is shown in fig. In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of governor when the rollers at the horizontal arm press against the sleeve.

Let, $S \rightarrow$ spring force.

$F_c \rightarrow$ centrifugal force.

$M \rightarrow$ mass on the sleeve &

x & $y \rightarrow$ lengths of vertical & horizontal arm of bell crank lever respectively.



The fig. (a) & (b) shows that the governor in mid-position. Neglecting the effect of obliquity of arms, taking moments about fulcrum 'O',

$$\boxed{F_c \times x = S \times x + \frac{Mg}{2} \times y.}$$

Problem

1. In a spring-controlled governor of the Hartung type, the length of the ball and sleeve arms are 80mm and 120mm respectively. The total travel of the sleeve is 25mm. In the mid position, each spring is compressed by 50mm and the radius of rotation of mass centres is 140mm. Each ball has a mass of 4kg and the spring has a stiffness of 10kN/m of compression. The equivalent mass of the governor gear at the sleeve is 16kg. Neglecting the moment due to revolving masses when the arms are inclined, determine the ratio of the range of speed to the mean speed of the governor, find, also, the speed in the mid-position.

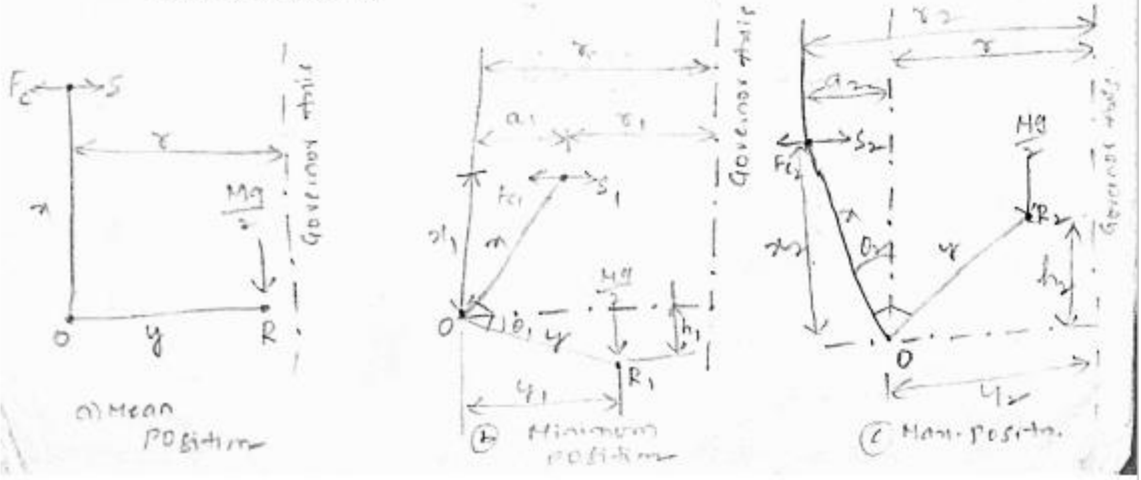
Sol: Given, $x = 80mm = 0.08m$; $y = 120mm = 0.12m$; $h = 25mm$, $h = 0.025m$

$r = 140mm = 0.14m$; $m = 4kg$; $M = 16kg$;

$S = 10\ kN/m = 10\times10^3\ N/m$; initial compression $= 50mm = 0.05m$.

* Mean Speed of the Governor:-

Let us find out the mean speed of the governor, i.e., the speed when the governor is in mid-position.



$\omega \rightarrow$ Mean Angular speed in rad/s (27).

$N \rightarrow$ Mean Speed, rpm.

centrifugal force acting on ball spring,

$$F_c = mr\omega^2 = 4 \times \omega^2 \times 0.14$$

$$F_c = 0.56\omega^2 \text{ N.}$$

Spring Force, S = Stiffness X Initial compression.

$$S = 10 \times 10^3 \times 0.05$$

$$S = 500 \text{ N.}$$

Now taking moments about pt. O, neglecting ~~amount~~ moment due to revolving masses, we have,

$$F_c \times x = S \times x + \frac{Mg}{2} \times y.$$

$$0.56\omega^2 \times 0.08 = 500 \times 0.08 + \frac{16 \times 9.81}{2} \times 0.12$$

$$0.56\omega^2 = \frac{49.42}{0.08}$$

$$0.56\omega^2 = 617.7$$

$$\omega^2 = 1103.00$$

$$\omega = 33.2 \text{ rad/s.}$$

$$\frac{2\pi N}{60} = 33.2$$

$$N = 317 \text{ rpm.}$$

* Ratio of Range of Speed to Mean Speed.

Let, $\omega_1 \rightarrow$ min. angular speed $\frac{rad}{s}$, at min. radius of rotation r_1.

$\omega_2 \rightarrow$ Max. angular speed, at max. radius of rotation. r_2.

N_1 & $N_2 \rightarrow$ corresponding min & Max. speeds, r.p.m.

first let us find the minimum speed, N_1.

from fig (b).

$$\frac{r-r_1}{h_1} = \frac{x}{y} \qquad \therefore h_1 = h/2.$$

$$\frac{0.14-r_1}{\frac{0.025}{2}} = \frac{0.08}{0.12}$$

$$\boxed{r_1 = 0.132\,m}.$$

w.k.t, centrifugal force at minimum position.

$$F_C = m r_1 \omega_1^2$$

$$= (4)(0.132)(\omega_1^2)$$

$$F_C = 0.528\,\omega_1^2 \ N$$

Spring force at the min. pos.

$$S_1 = [\text{initial compr.} - (r-r_1)] \times \text{stiffness}$$

$$= [0.05 - (0.14 - 0.132)] \times 10 \times 10^3.$$

$$S_1 = 420N.$$

min. radius

(28).

Taking moments about 'O', neglecting obliquity of arms.

$$F_{c_1} \times x = S_1 \times x + \frac{M \cdot g}{2} \cdot y_1$$

$$0.528\omega_1^2 \times 0.08 = (420) \times (0.08) + \frac{16 \times 9.81}{2} \times 0.12$$

$$\omega_1^2 = 1019$$

$$\omega_1 = 31.92.$$

$$\frac{2\pi N_1}{60} = 31.92$$

$$N_1 = 304.83 \text{ rpm}.$$

Similarly for max. position,

max. speed N_2, $\frac{r_2 - r}{h_2} = \frac{x}{y}$ | $h_2 = h/2$

$r_2 = 0.148\text{m}$

$F_{c_2} = m r_2 \omega_2^2$; $F_c = 4 \times (0.148)\,\omega_2^2 \Rightarrow F_c = 0.592\,\omega_2^2$.

+ Spring force, $S_2 = [\text{Initial Compr.} + (r_2 - r)] \times \text{Sti}$

$$= [0.05 + (0.148 - 0.14)] \times 10 \times 10^3$$

$$S_2 = 580\text{N}.$$

Taking moments about 'O',

$$F_c \times x = S_2 \times x + \frac{Mg}{2} \times y.$$

$$0.592\omega_2^2 \times 0.08 = 580 \times 0.08 + \frac{16 \times 9.81}{2} \times 0.12$$

$$\omega_2 = 34.32$$

$$N_2 = 327.7 \text{ rpm}$$

Range of speed, $N_2 - N_1 = 327.7 - 304.83$

$= 22.8$ rpm.

∴ Ratio Range of speed to mean speed,

$$\frac{22.8}{317} = 0.07 = 7\%$$

* Sensitiveness of Governor:-

Consider 2 governors A & B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that is sufficient to change the energy supplied to the engine by required amount. For this reason, the sensitiveness is defined, as the "ratio of difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed."

Let, $N_1 \rightarrow$ minimum equilibrium speed.

$N_2 \rightarrow$ maximum equilibrium speed.

$N \rightarrow$ Mean equilibrium speed $= \frac{N_1+N_2}{2}$.

Sensitiveness of governor, $= \frac{N_2-N_1}{N} = \frac{2(N_2-N_1)}{N_1+N_2}$.

$$= \frac{2(\omega_2-\omega_1)}{(\omega_1+\omega_2)} \rightarrow \text{(In terms of angular speed).}$$

* ~~Power~~ Effort of Governor:

The effort of a governor is the mean force exerted at sleeve for given % change of speed. It may be noted that when G is running steadily, ~~there~~ there is no force at sleeve. But when the speed changes there is a resistance at the sleeve which opposes the motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a max. value to zero while governor moves into its new positn of equilibrium

~~P = Effort × height~~

Power of the G

(29)

* Stability of Governors:

A governor is said to be stable, when for every speed within the working range there is only radius of rotatn of governor balls at which the governor is in equilibrium. for a stable governor if the ~~governor~~ equilibrium speed ↑, the radius of rotatn of ↑.

NOTE: for an unstable governor, radius of rotatn decreases for ↑ in speed.

* Isochronous

Governor is said to be isochronous when equilibrium speed is constant (range of speed '0') for all radii of rotatn of balls within working range, neglecting fricn. The isochronism is the stage of infinite sensitivity.

$N_2 - N_1 = 0$. Hartnell G: $\frac{Mg+S_1}{Mg+S_2} = \frac{r_1}{r_2}$

Porter governor (vertical isochronous) $h_1 = h_2$ & $(N_1 = N_2)$

no practical use for isochronism. because the sleeve moves to its extreme positn immediately the speed deviates from its isochronous speed.

* **Hunting:**

A governor is said to be hunt, if the speed of engine fluctuates continously above & below the mean speed. This is caused by too sensitive Governor which changes fuel supply by a large amount when a small change in speed of rotation takes place.

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* **Power of a Governor:-**

Power of the governor is the work done at the sleeve for a given % change of speed. It is the product of the mean value of the effort & the distance through which the sleeve moves.

Mathematically, Power = Mean Effort × lift of sleeve.

* **Effort & Power of a Porter Governor**

The effort & power of a porter governor may be discussed below:

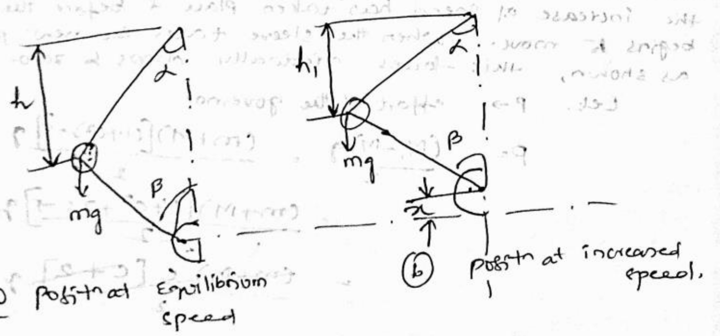
Let, $N \rightarrow$ Equilibrium speed corresponding to the configuration as shown

$c \Rightarrow$ % increase in speed.

$\therefore$ increase in speed = $c \cdot N$

$\therefore$ increased speed = $N + c \cdot N = N(1+c)$

The equilibrium position of the governor at the increased speed



(a) Position at Equilibrium speed

(b) Position at increased speed.

Assuming $\alpha = \beta$ $\therefore$ $q = 1$

then $$h = \frac{m+M}{m} \times \frac{895}{N^2} \quad -(1)$$

When the increase of speed takes place, a downward force P will have to exert on sleeve in order to prevent the sleeve from rising. If the speed increases to $N(1+c)$ & ht. of the governor remains same, the load on the sleeve increases to $M_1 g$

$$\therefore \quad h = \frac{m+M_1}{m} \times \frac{895}{(1+c)^2 N^2} \quad -(2)$$

Equating eq's (1) & (2)

$$m+M = \frac{m+M_1}{1+c^2}$$

$$M_1 = (m+M)(1+c^2) - m$$

substituting M on b.s.

$$M_1 - M = (m+M)(1+c^2) - m - M$$

$$M_1 - M = (m+M)\left[(1+c^2)-1\right] \quad -(3)$$

A little consideration will show that, $(M_1-M)g$ is the downward force which must be applied in order to prevent the sleeve from rising as speed increases. It is the same force which acts on the governor sleeve immediately after the increase of speed has taken place & before the sleeve begins to move. When the sleeve takes the new position, this forces gradually drops to zero.
as shown,

Let, $P \rightarrow$ effort of the governor

$$P = \frac{(M_1-M)g}{2} = \frac{(m+M)\left[(1+c^2)-1\right]g}{2}$$

$$= \frac{(m+M)\left[\cancel{1}+c^2+2c-\cancel{1}\right]g}{2}$$

$$= \cancel{\frac{(m+M)\,c\,[c+2]\,g}{2}}$$

$$\cancel{P = c\,(m+M)\,2g.}$$

neglecting c^2 being very small

$$= \frac{(m+M)(2c)\,g}{2}$$

$$\boxed{P = c\,(m+M)\,g} \quad -(4)$$

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If the frictional force F acts at the sleeve, then

$$P = c\,[m\cdot g + Mg \pm F]$$

W.K.T, Power is the product of governor effort & sleeve lift

Let $x \rightarrow$ sleeve lift.

$\therefore$ Governor power $= P \times x$ — (5)

If the height of the governor at speed N is h & at an increased speed (1+c)N is h_1 then,

$$x = 2(h - h_1) \quad \text{— (6)}$$

$\therefore \quad h = \frac{m+M}{m} \times \frac{895}{N^2}$ & $h_1 = \frac{m+M}{m} \cdot \frac{895}{(1+c)^2 N^2}$

$$\frac{h_1}{h} = \frac{\cancel{\left(\frac{m+M}{m}\right)} \frac{895}{\cancel{N^2}(1+c)^2}}{\cancel{\left(\frac{m+M}{m}\right)} \frac{895}{\cancel{N^2}}} = \frac{1}{(1+c)^2}$$

$$\frac{h_1}{h} = \frac{1}{(1+c)^2} \Rightarrow h_1 = \frac{h}{(1+c)^2}$$

$\therefore$ sub. h_1 in (6) eq.

$$x = 2\left[h - \frac{h}{(1+c)^2}\right] = 2h\left[1 - \frac{1}{(1+c)^2}\right]$$

$$= 2h\left[\frac{\cancel{1} + c^2 + 2c - \cancel{1}}{1 + c^2 + 2c}\right]$$

neglecting c^2

$$x = 2h\left[\frac{2c}{1+2c}\right] \quad \text{— (7)}$$

sub. x in eq (5).

$\therefore$ Governor Power $= c(m+M)\cdot g \times 2h\left[\frac{2c}{1+2c}\right]$

$$= \left(\frac{4c^2}{1+2c}\right)(m+M)\cdot g h$$

Prob (1) A porter governor has equal arms each 250mm long & pivoted on the axis of rotatn. Each ball has a mass of 5kg. & central mass 25kg The radius of rotatn of the ball is 150mm when the governor begins to lift & 200mm at max. speed. find the range of speed, sleeve lift, governor effort & power of governor when:

1. friction is considered as 10N

(i) when frictn neglected, 2. " is neglected.

Sol: $N_2 - N_1 = 25$ rpm

sleeve lift,

$$x = 2(h_1 - h_2) = 2(200 - 150) = 0.1\,m$$

~~Power~~ effort of governor,

$$P = C(m+M)g.$$

$$\boxed{C\,N_1 = N_2 - N_1} \Rightarrow CN_1 = 25$$

$$C = \frac{25}{164} = 0.152.$$

$$\therefore P = 0.152(5+25)9.81 = 44.7\,N$$

Power of Governor,

$$= P \cdot x = 44.7 \times 0.1 = 4.47\,N\text{-}m.$$

* when frictn is considered

$$N_2 - N_1 = 31.4\,rpm$$

$$C \cdot N_1 = N_2 - N_1$$

$$C = \frac{31.4}{161} \Rightarrow C = 0.195$$

Governor effort, $P = C\left[mg + Mg + F\right]$

$$= 0.195\left[(5 \times 9.81) + (25 \times 9.81 + 10)\right]$$

$$= 57.4\,N$$

Power $= P \times x$

$= 57.4 \times 0.1$

Power 5.74 N-m

Balancing of Masses

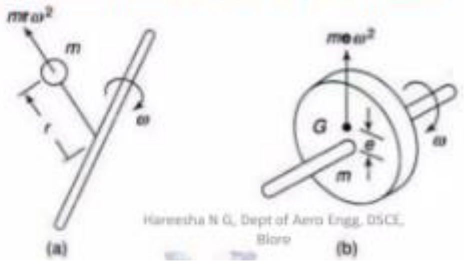
Unit 4: Balancing of Rotating Masses

- **Static and dynamic balancing**
- **Balancing of** single rotating mass by balancing masses in same plane and in different planes.
- Balancing of several rotating masses by balancing masses in same plane and in different planes.

What is Balancing ?

- Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses.
- Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

- A particle or mass moving in a circular path experiences a centripetal acceleration and a force is required to produce it.
- An equal and opposite force acting radially outwards acts on the axis of rotation and is known as centrifugal force .
- This is a disturbing force on the axis of rotation, the magnitude of which is constant but the direction changes with the rotation of the mass.
- In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of the rotor lies on the axis of the shaft.
- When the centre of mass does not lie on the axis or there is an eccentricity, an unbalanced force is produced



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Why Balancing is necessary?

- The high speed of engines and other machines is a common phenomenon now-a-days.
- It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible.
- If these parts are not properly balanced, the dynamic forces are set up.
- These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Balancing of Rotating Masses

- Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.
- In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass.
- This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite.
- The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

Balancing of Rotating Masses

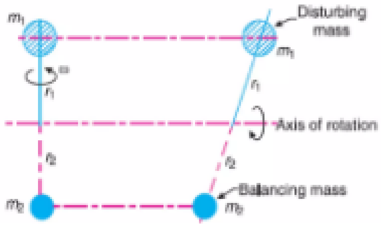
- The following cases are important from the subject point of view:
  1. Balancing of a single rotating mass by a single mass rotating in the same plane.
  2. Balancing of a single rotating mass by two masses rotating in different planes.
  3. Balancing of different masses rotating in the same plane.
  4. Balancing of different masses rotating in different planes.

Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

- Consider a disturbing mass *m*1 attached to a shaft rotating at ω rad/s as shown in Fig.
- Let *r*1 be the radius of rotation of the mass *m*1 (*i.e. distance between the axis of rotation of the shaft* and the centre of gravity of the mass *m*1).
- We know that the centrifugal force exerted by the mass *m*1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

- This centrifugal force acts radially outwards and thus produces bending moment on the shaft.
- In order to counteract the effect of this force, a balancing mass (*m*2) may be attached in the same plane of rotation as that of disturbing mass (*m*1) such that the centrifugal forces due to the two masses are equal and opposite.



Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Notes : **1.** The product $m_2 . r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.

2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

- In the previous arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings.
- Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for *static balancing*.
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1) and (2) together give *dynamic balancing.***

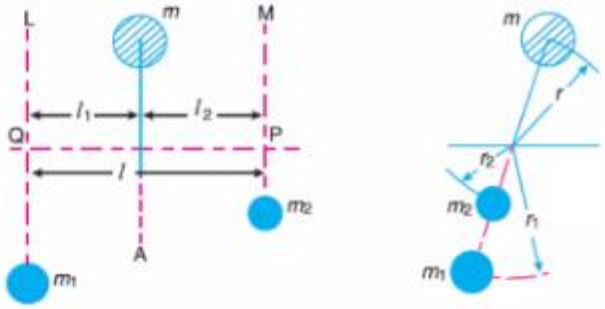
Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

- The following two possibilities may arise while attaching the two balancing masses :
  1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
  2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

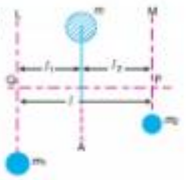
1. *When the plane of the disturbing mass lies in between the planes of the two balancing masses*

- Consider a disturbing mass *m lying in a plane A to be balanced by two rotating masses m*1 and *m*2 lying in two different planes *L and M as shown in Fig.*
- *Let r, r*1 and *r*2 be the radii of rotation of the masses in planes *A, L and M respectively.*

Let l_1 = Distance between the planes A and L,
l_2 = Distance between the planes A and M, and
l = Distance between the planes L and M.



We know that the centrifugal force exerted by the mass m in the plane A,



$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

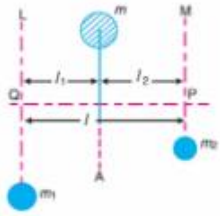
$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (ii)$$

Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

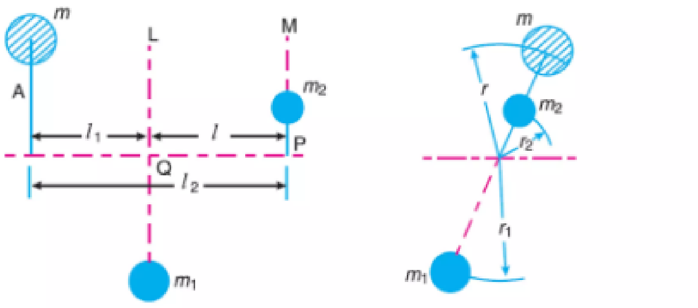
$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (iii)$$

- It may be noted that equation (*i) represents the condition for static balance, but in order to* achieve dynamic balance, equations (*ii) or (iii) must also be satisfied.*

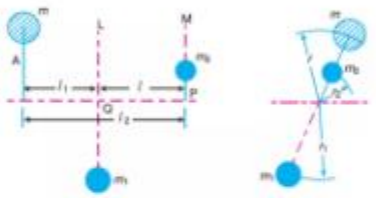


When the plane of the disturbing mass lies on one end of the planes of the balancing masses

- In this case, the mass *m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig.*



As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*



$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \qquad \ldots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (v)$$

... [Same as equation (*ii*)]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (vi)$$

... [Same as equation (*iii*)]

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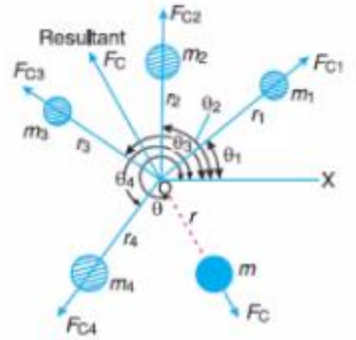
Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

- The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram.

1. *Analytical method*

- The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :
  1. First of all, find out the centrifugal force exerted by each mass on the rotating shaft.
  2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV. We know that

     Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

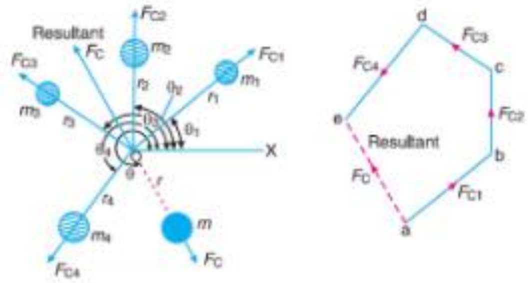
where m = Balancing mass, and

r = Its radius of rotation.

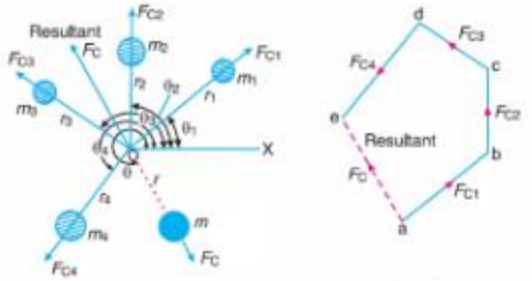
2. Graphical method

- The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (*a*).
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).



2. Graphical method



(*a*) Space diagram. (*b*) Vector diagram.

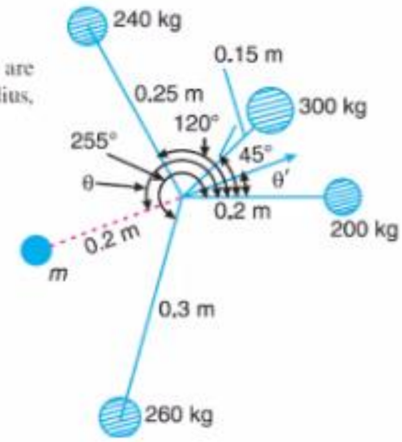
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 21.4 (*b*).
5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

Example 21.1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.*

Solution. Given : m_1 = 200 kg ; m_2 = 300 kg ; m_3 = 240 kg ; m_4 = 260 kg ; r_1 = 0.2 m : r_2 = 0.15 m ; r_3 = 0.25 m ; r_4 = 0.3 m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m



Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

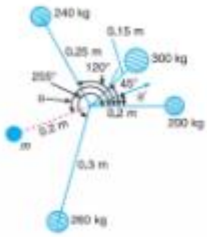
$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

1. *Analytical method*

The space diagram is shown in Fig.



Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$
$$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$$
$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$
$$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$$
$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2 / 0.2 = 116 \text{ kg} \quad \textbf{Ans.}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48° \quad \textbf{Ans.}$$

2. *Graphical method*

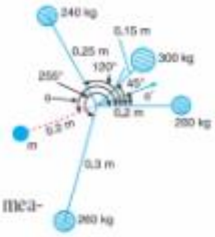
The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses

2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

$$m_1.r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$
$$m_2.r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$
$$m_3.r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$
$$m_4.r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

Take: 10kg-m=1cm



3. Now draw the vector diagram with the above values, to some suitable scale.

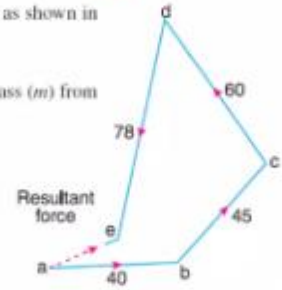
The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.

4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. Since the balancing force is proportional to *m.r*, therefore

$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m} \quad \text{or} \quad m = 23/0.2 = \mathbf{115 \text{ kg}} \textbf{ Ans.}$$

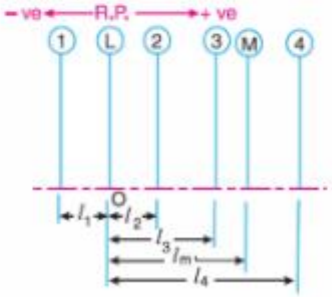
By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal mass of 200 kg,

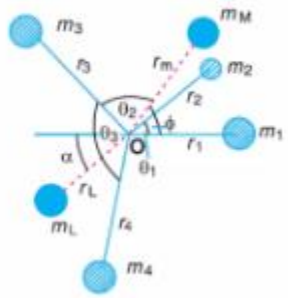
$$\theta = 201° \textbf{ Ans.}$$



Balancing of Several Masses Rotating in Different Planes

- When several masses revolve in different planes, they may be transferred to a *reference plane (briefly written as R.P.), which may be defined as the plane passing through a* point on the axis of rotation and perpendicular to it.
- The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane.

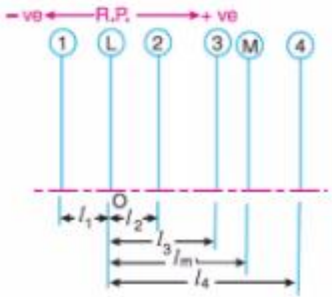


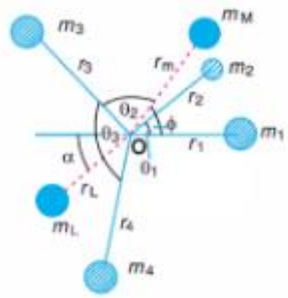


(b) Angular position of the masses.

Balancing of Several Masses Rotating in Different Planes

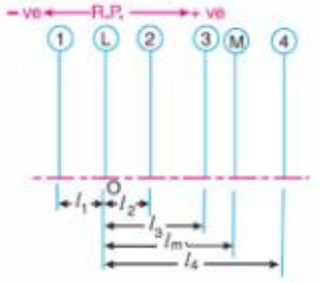
- In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :
  1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
  2. The couples about the reference plane must balance, *i.e. the resultant couple must be zero.*



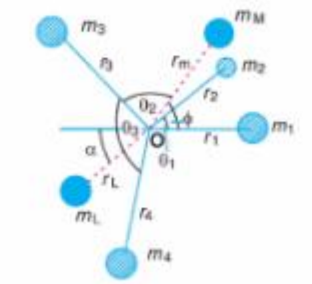


Balancing of Several Masses Rotating in Different Planes

- Let us now consider four masses $m1$, $m2$, $m3$ and $m4$ revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. (*a*).
- *The relative angular positions of these masses are shown in the end view [Fig. (b)].*



(*a*) Position of planes of the masses.

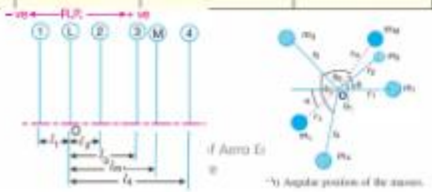


(*b*) Angular position of the masses.

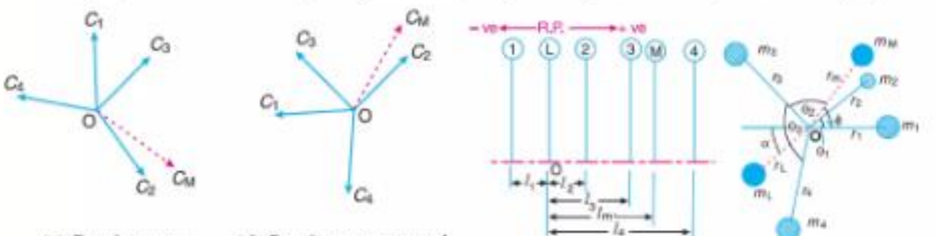
The magnitude of the balancing masses m_L and m_M in planes *L and M may be obtained as* discussed below :

1. Take one of the planes, say L as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive***.
2. Tabulate the data as shown in Table The planes are tabulated in the same order in which they occur, reading from left to right.

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is proportional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. 21.7 (c). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.



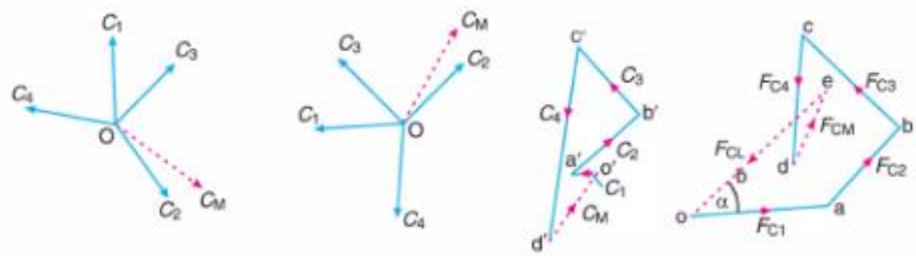
(c) Couple vector. (d) Couple vectors turned counter clockwise through a right angle. (b) Angular position of the masses.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (d). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. 21.7 (e). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane M may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (b).

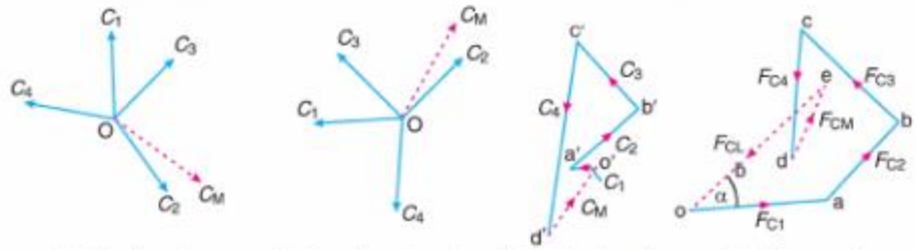


(c) Couple vector. (d) Couple vectors turned counter clockwise through a right angle. (e) Couple polygon. (f) Force polygon.

6. Now draw the force polygon as shown in Fig. 21.7 (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L . r_L$, therefore,

$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

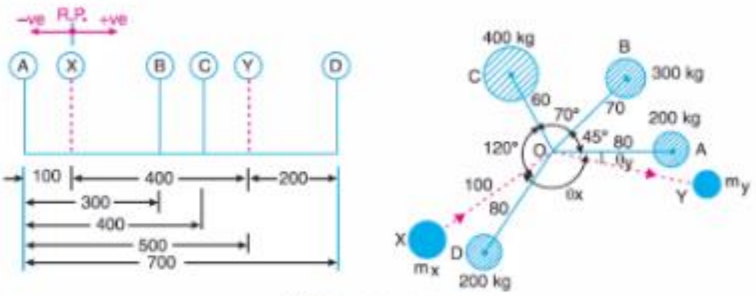
From this expression, the value of the balancing mass m_L in the plane *L* may be obtained



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

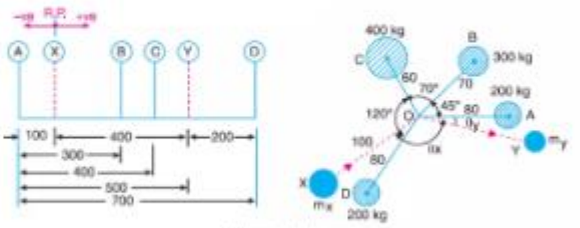
Example 21.2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; $r_X = r_Y$ = 100 mm = 0.1 m



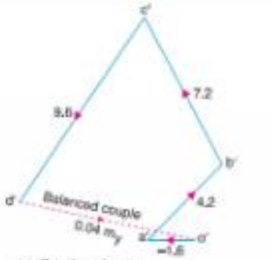
All dimensions in mm.

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	200	0.08	16	− 0.1	− 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

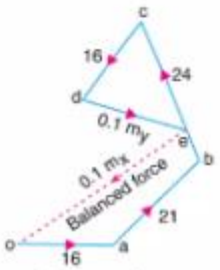


All dimensions in mm.

(a) Position of planes. (b) Angular position of masses.



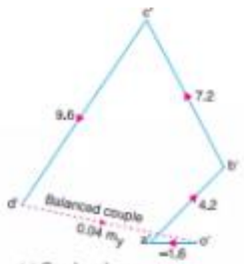
(c) Couple polygon.



(d) Force polygon.

$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$ or $m_Y = 182.5$ kg **Ans.**

$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m}$ or $m_X = 355$ **kg Ans.**



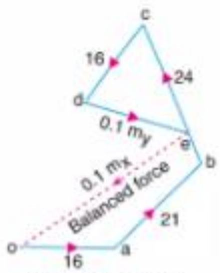
(c) Couple polygon.

$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$ or $m_Y = 182.5$ kg **Ans.**

$0.1\, m_X = \text{vector } eo = 35.5 \text{ kg-m}$ or $m_X = 355$ **kg Ans.**

$\theta_Y = 12°$ in the clockwise direction from mass m_A

$\theta_X = 145°$ in the clockwise direction from mass m_A



(d) Force polygon.

Example 21.3. *Four masses A, B, C and D as shown below are to be completely balanced.*

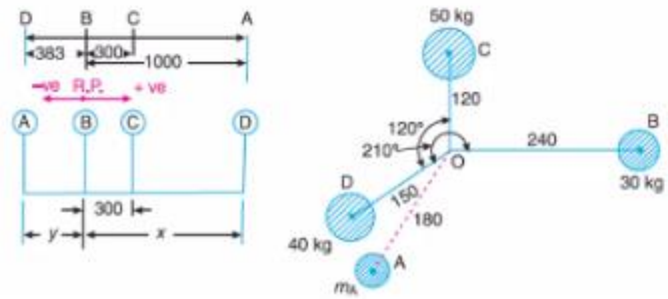
	A	B	C	D
Mass (kg)	—	30	50	40
Radius (mm)	180	240	120	150

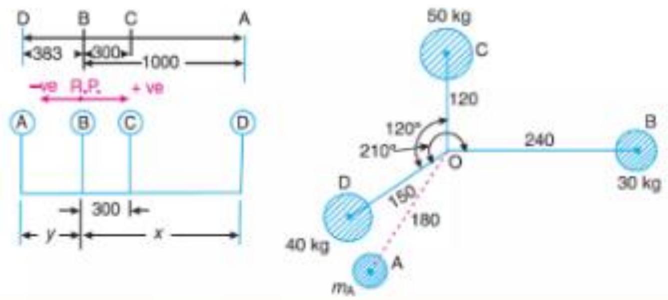
The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

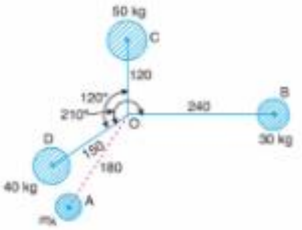
Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$





Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 m_A y
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

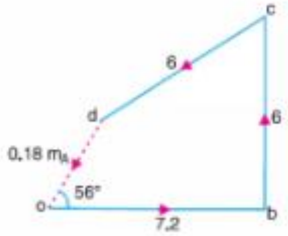
Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent.force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from plane B* (l) m (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 m_A.y
B (*R.P*)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

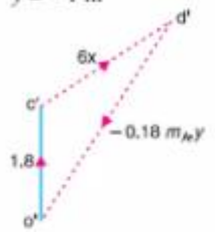


$6\,x$ = vector $c'\,d'$ = 2.3 kg-m^2 or x = 0.383 m

– 0.18 $m_A.y$ = vector $o'\,d'$ = 3.6 kg-m^2

– 0.18 × 20 y = 3.6 or y = – 1 m



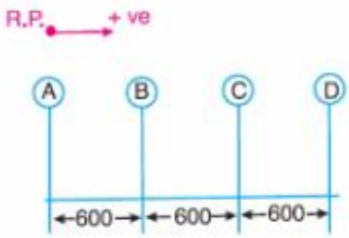


(*d*) Couple polygon.

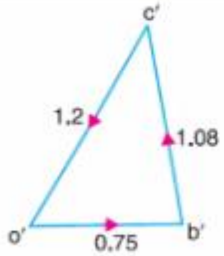
Example 21.4. *A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.*

Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

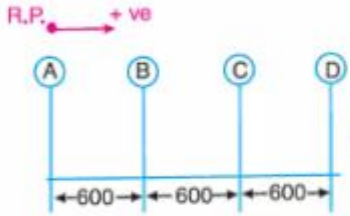
Solution. Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ; r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg

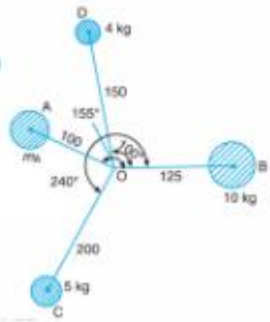


Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08



(c) Couple polygon.



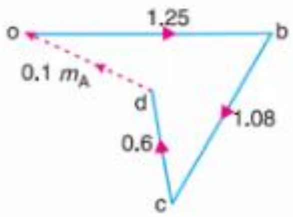


$\angle BOA = 155°$ **Ans.**

$\angle BOC = 240°$ **Ans.**

$\angle BOD = 100°$ **Ans.**

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08



(d) Force polygon.

$0.1\ m_A = 0.7$ kg-m^2 or $m_A = 7$ kg **Ans.**

Unit-V

VIBRATIONS

1

When elastic bodies such as spring, a beam, & a shaft are displaced from equilibrium position by the applicatn of external forces & released, they execute a vibratory motion.

Terms:

1. Period of Vibratn (or) time period: It is the time interval after which the motion is repeated itself. in 's'.

2. cycle: It is the motn completed after during one time period.

3. frequency: It is no. of cycles described in one second. Hz.

* Types of Vibratory Motions:

1. Free (or) Natural Vibratns: When no external force acts on body, after giving it an initial displacement, then body is said to be under natural (or) force vibratn.

'Natural (or) free frequency'

2. Forced Vibratn: When the body vibrates under the influence of external force, then that vibratn is said to be forced vibratn.

natural Freq = Forced frequency then resonance takes place.

3. Damped Vibratn: When there is a reductn in amplitude over every cycle of vibratn, that vibratn is said to be Damped vibratn.

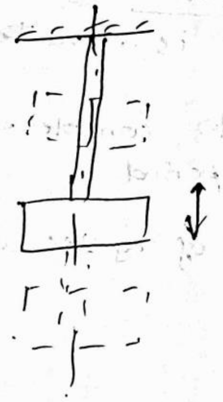
(2)

Types of ~~Free~~ Vibration:

1. Longitudinal Vibration
2. Transverse "
3. Torsional "

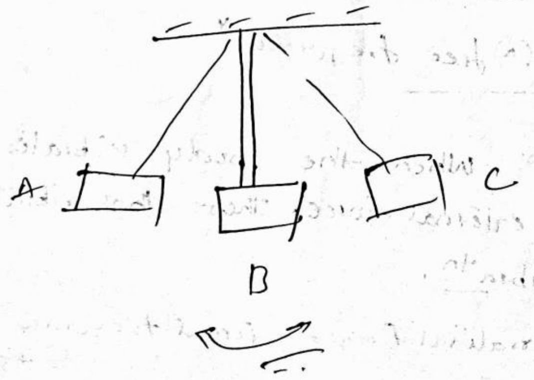
1. Longitudinal Vibration:

When the particles of shaft (or) disc moves parallel to axis of shaft, that vibration are known as Longitudinal.



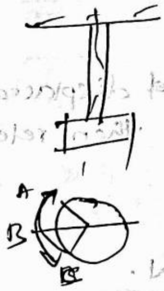
2. Transverse Vibration:

When the particles of shaft (or) disc move approximately ⊥ to axis of shaft



3. Torsional:

when the particles of a shaft moves in a circle about the axis of shaft.

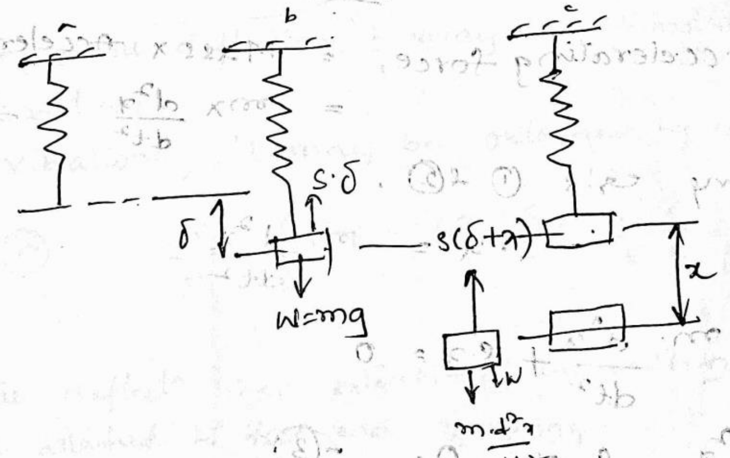


Natural frequency of Free Longitudinal Vibrations:

1. ~~Energy method~~ Equilibrium Method:

→ Accelerating force ↓

→ stiffness ↑



$mg = S \cdot \delta$

let,

S → stiffness of spring.

m → mass of body suspended from spring

W → wt. of body in N

δ → static deflection of spring, m due to weight.

x → displacement given to body by external force.

$$W = mg = s \cdot \delta \quad \text{—(a)}$$

In the equilibrium positn, $W = mg$;

$$\delta = \frac{mg}{s}$$

$$\frac{\delta}{g} = \frac{m}{s}$$

$$\frac{s}{m} = g/\delta$$

∵ the mass is now ~~replaced~~ displaced from its equilibrium positn by a distance of 'x' & then released after time 't'.

then Restoring force,

$$s(\delta + x) = W.$$

$$\cancel{s(\delta+x) - W};\ W - s(\delta + x)$$

$$\Rightarrow W - s\delta - s \cdot x$$

Restoring force.

$$\cancel{s(\delta+x) - W = 0}$$

$$\cancel{s\delta + s \cdot x - s\delta}$$

$$s \cdot \delta - s \cdot \delta - s \cdot x = s \cdot x$$

$$W - s(\delta + x)$$

$$- s \cdot x \quad \text{—(1)}$$

Upward force consider "−ve"

& Accelerating force, $= \text{Mass} \times \text{Acceleration}.$

$$= m \times \frac{d^2x}{dt^2} \quad \text{—(2)}$$

Equating eq's (1) & (2).

$$-s \cdot x = m \cdot \frac{d^2x}{dt^2}$$

$$m \cdot \frac{d^2x}{dt^2} + s \cdot x = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} \cdot x = 0. \quad \text{—(3)}$$

W.K.T fundamental eq. of SHM is.

$$\frac{d^2x}{dt^2} + \omega^2 \times x = 0 \quad \text{—(4)}$$

(5)

Comparing eq (3) & (4)

$$\omega^2 = \frac{s}{m}$$

$$\omega = \sqrt{\frac{s}{m}}$$

∴ time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$

Natural frequency,

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

NOTE: The value of static deflection 'δ' may be found out from given conditions.

For longitudinal vibrations, it may be obtained by relatn:

$$\frac{\text{stress}}{\text{strain}} = E \quad (\text{or}) \quad \frac{W}{A} \times \frac{l}{\delta} = E \quad (\text{or}) \quad \delta = \frac{W \cdot l}{E \cdot A}$$

$\delta \rightarrow$ static deflectn i.e. extension (or) compression.

$W \rightarrow$ load attached to free end of spring

$l \rightarrow$ length of spring

$E \rightarrow$ Young's modulus

$A \rightarrow$ c/s nal area of spring

⑥

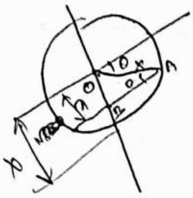
Rayleigh's Method:

In this method, the max. K.E at mean position (P.E = 0) is equal to the maximum P.E at extreme position (K.E = 0)

Assuming the motion executed by the vibration to SHM,

then, $OB = OA \sin\theta$, $= X \sin\omega t$

$x = X \sin \omega t$ — ⓐ



$x \rightarrow$ displacement of body from mean position after time 't' secs.

$X \rightarrow$ max. displacement from mean position to extreme position.

Now, diff. eq ⓐ

$$\frac{dx}{dt} = \omega \times X \cos \omega t$$

at mean position, $t = 0$ ∴ Max. velocity at mean position,

$$V = \frac{dx}{dt} = \omega \cdot X.$$

∴ Max. K.E at mean position,

$$= \frac{1}{2} m V^2 = \frac{1}{2} \cdot m\, \omega^2 X^2 \quad — ①$$

& Max. P.E at extreme position,

$$\left[\frac{0 + s \cdot X}{2}\right] \cdot X = \frac{1}{2} s X^2 \quad — ②$$

P.E = mean force × displacement

Equating eq's ① & ②

$$\cancel{\tfrac{1}{2}}\, m \omega^2 \cancel{X^2} = \cancel{\tfrac{1}{2}}\, s \cancel{X^2}$$

$$m\omega^2 = s$$

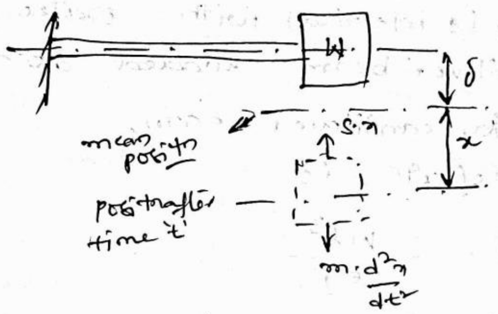
$$\omega = \sqrt{\frac{s}{m}}$$

∴ time period, $t_p = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{s}}$

$$f_n = \frac{1}{t_p} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

(7)

* Natural Frequency of Transverse Vibrations:



Consider a shaft of negligible mass, whose end is fixed & other end carries a body of weight W.

Let, $S \rightarrow$ stiffness of shaft.

$\delta \rightarrow$ static deflection due to weight of body

$x \rightarrow$ Displacement of body from mean position after time 't'.

$m \Rightarrow$ Mass of body $= W/g$.

W.k.t, restoring force, $= -Sx$. —(a).

& Accelerating force, $= m \times \frac{d^2x}{dt^2}$ —(b)

Equating (1) & (2) eq's.

$$\frac{d^2x}{dt^2} + \frac{S}{m} \cdot x = 0.$$

$$\therefore \boxed{t_p = 2\pi\sqrt{\frac{m}{S}}}$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{S}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}}$$

(8)

NOTE: The shape of the curve, in which the vibrating shaft deflects, is identical with static deflection curve of a cantilever beam loaded at end.

~~It has been~~ for cantilever beam, static deflection is,

$$\delta = \frac{Wl^3}{3EI}$$

$W \rightarrow$ load at free end

$l \rightarrow$ length of shaft

$E \rightarrow$ Young's modulus ~~form~~ of shaft

$I \rightarrow$ Moment of inertia of shaft.

(9)

Problem

1. A cantilever ~~beam~~ shaft of 50mm dia. & 300mm long has a disc of mass 100kg at its free end. The young's modulus for the shaft mtrl is 200GN/m². Determine the frequency of longitudinal & transverse vibrations.

Sol: Given:

$d = 50mm = 0.05\ m$; $l = 300mm = 0.3m$

$m = 100kg$; $E = 200\ GN/m^2 = 200\times10^9\ N/m^2$.

W.K.T c/s nal area of shaft.

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(0.05)^2 = 1.96\times10^{-3}\ m^2.$$

& Moment of inertia,

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}(0.05)^4 = 0.3\times10^{-6}\ m^4.$$

Frequency of longitudinal vibratn.

→ $\delta = \frac{W.l}{A.E}$; $\delta = 0.751\times10^{-6}\ m$.

∴ frequency of longitudinal vibratn, $f_n = \frac{0.4985}{\sqrt{\delta}}$

$f_n = 575\ Hz$

→ Frequency of transverse vibratn:

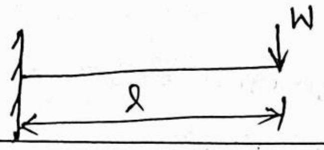
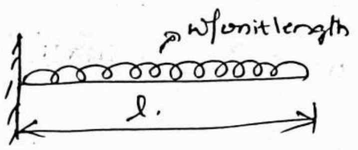
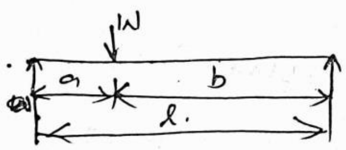
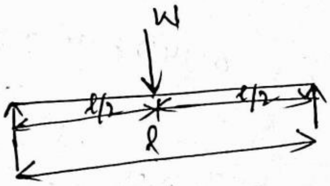
$\delta = \frac{W.l^3}{3.EI}$: $\delta = 0.147\times10^{-3}\ m$

$$f_n = \frac{0.4985}{\sqrt{\delta}} = 41\ Hz$$

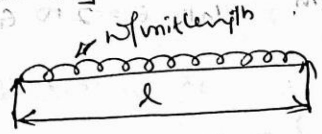
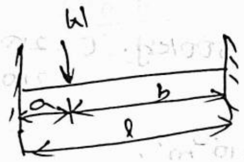
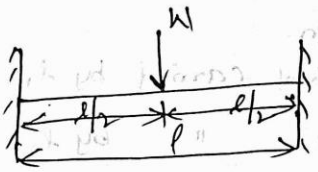
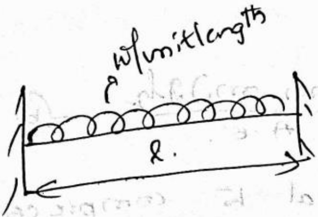
Pl. allow for xerox

10

* Values of static deflection for various types of beams & under various load conditions:

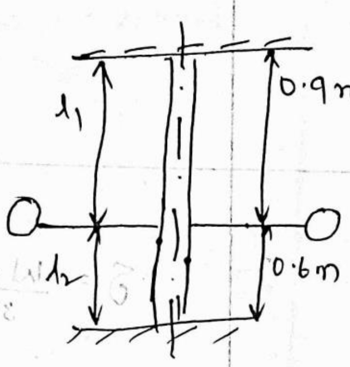
S.No.	Type of Beam	Deflection [δ]
1.	Cantilever Beam with pt. load W at free end. 	$\delta = \dfrac{Wl^3}{3EI}$ (at free end)
2.	cantilever Beam with UDL w/unit length 	$\delta = \dfrac{wl^4}{8EI}$ (at free end)
3.	Simply supported Beam with an eccentric pt. load W. 	$\delta = \dfrac{Wa^2b^2}{3EIl}$ (at pt. load)
4.	Simply supported beam with central point load. 	$\delta = \dfrac{Wl^3}{48EI}$ (at centre

11

S.No	Type of Beam	Deflection [δ]
5.	Simply supported beam with uniformly distributed load 	$\delta = \frac{5}{384} \times \frac{Wl^4}{EI}$ (at centre)
6.	Fixed Beam with an eccentric pt. load W 	$\delta = \frac{W a^3 b^3}{3EI\, l^3}$ (at pt. load)
7.	Fixed Beam with central load W 	$\delta = \frac{Wl^3}{192EI}$ (at centre)
8.	Fixed Beam with UDL 	$\delta = \frac{Wl^4}{384EI}$ (at centre)

Problems (12)

1. A flywheel is mounted on a vertical shaft as shown. The both ends of shaft is fixed & its dia is 50mm. The flywheel has a mass of 500 kg. Find the natural frequencies of longitudinal & transverse vibratn. Take $E = 210\ GN/m^2$.



Sol:- Given, $d = 50mm = 0.05m$; $m = 500kg$; $E = 210 \times 10^9 N/m^2 = 210 \times 10^9 N/m^2$.

W.K.T, c/snal area of shaft,

$$A = \pi/4\, d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} m^2.$$

& M.I of shaft, $I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6} m^4$.

* Natural frequency of longitudinal vibratn.

let, $m_1 \rightarrow$ mass of the flywheel carried by l_1

$m - m_1 \rightarrow$ " " " " by l_2.

Extension of length l_1

$$= \frac{W_1 l_1}{A \cdot E} = \frac{m_1 \cdot g \cdot l_1}{A \cdot E} \quad \text{— ⓐ}$$

Compression of length l_2

$$= \frac{(W - W_1) \cdot l_2}{A \cdot E} = \frac{(m - m_1) g\, l_2}{A \cdot E} \quad \text{— ⓑ}.$$

$\because$ Extension of length l_1 is equal to compression of length l_2

$\therefore$ Equating eq's ⓐ & ⓑ

(13)

$m_1 l_1 = (m - m_1) \cdot l_2$

$m_1 \times 0.9 = (500 - m_1) \cdot 0.6$

$m_1 = 200\,kg.$

$f_n = \dfrac{0.4965}{\sqrt{\delta}}$

$f_n = 235\,Hz$

∴ Extension of l_1

$$\delta = \frac{m_1 \cdot g \cdot l_1}{A \cdot E} = \frac{200 \times 9.81 \times 0.9}{1.96 \times 10^{-3} \times 210 \times 10^{9}}$$

$\delta = 4.29 \times 10^{-6}\,m$

* Natural frequency of Transverse vibration.

$$\delta = \frac{W a^3 b^3}{3 E I l^3} = \frac{500 \times 9.81\,(0.9)^3\,(0.6)^3}{3 \cdot 210 \times 10^{9} \times 0.307 \times 10^{-6} \times (1.5)^3}$$

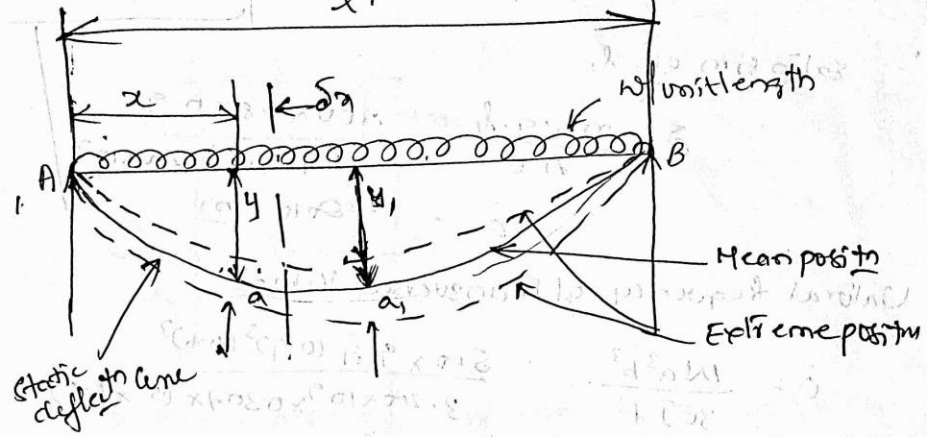
$= 1.24 \times 10^{-3}\,m$

$$f_n = \frac{0.4965}{\sqrt{\delta}} = 14.24\,Hz$$

(19)

* Natural frequency of Free Transverse Vibrations due to UDL acting over a simply supported shaft.

Consider a shaft AB carrying UDL of w/unit length l.



Let, $y_1 \rightarrow$ static deflection at middle of shaft

$a_1 \rightarrow$ Amplitude of vibration at " " "

$w_1 \rightarrow$ UDL per unit static deflection at middle of shaft $= w/y_1$

Let us consider an elementary section of shaft at a distance x from 'A' & length δx.

Let, $y \rightarrow$ static deflection at distance x from 'A'

$a \rightarrow$ Amplitude of its vibration.

$\therefore$ Work done on this small section.

$$= \frac{1}{2} \times w_1 \times a_1 \cdot \delta x \cdot a = \frac{1}{2} \cdot \frac{w}{y_1} \times a_1 \cdot \delta x \cdot a$$

$$= \frac{1}{2} \cdot w \times \frac{a_1}{y_1} \cdot a \cdot \delta x$$

$\because$ the max. P.E at extreme position is equal to amount of work done to move the shaft from mean position to one of its extreme position.

Max. P.E at extreme position

$$\therefore \int_0^l \frac{1}{2} \times w_1 \times \frac{a_1}{y_1} \times a \cdot dx. \quad \text{--- (1)}$$

Assuming that the shape of curve of a vibrating shaft is similar to static deflection curve of a beam

$$\therefore \frac{x_1}{y_1} = \frac{x}{y} = \text{constant},\ c \quad (\text{or}) \quad \frac{x_1}{y_1} = c$$

$$\& \ x_1 = y \cdot c$$

sub. x in eq(1).

$$\int_0^l \frac{1}{2} \cdot w \times c \cdot y \cdot c \cdot dx = \frac{1}{2} \cdot w \times c^2 \int_0^l y \cdot dx. \quad \text{—(2)}$$

$\because$ the max. velocity at mean position, $\omega \cdot x_1$, where ω is circular frequency of vibration, Therefore,

$$\text{Max. KE} = \int_0^l \frac{1}{2} \frac{w \cdot dx}{g} (\omega \cdot x)^2 = \frac{w}{2g} \times \omega^2 \cdot c^2 \int_0^l y^2 \cdot dx \quad \text{—(3)}$$

Equating eq's (2) & (3)

$$\frac{1}{2} \cdot w \times c^2 \int_0^l y \, dx = \frac{w}{2g} \times \omega^2 \cdot c^2 \int_0^l y^2 \cdot dx$$

$$\therefore \omega^2 = \frac{g \int_0^l y \, dx}{\int_0^l y^2 \cdot dx} \quad (\text{or}) \quad \omega = \sqrt{\frac{g \int_0^l y \, dx}{\int_0^l y^2 \cdot dx}}$$

When shaft is simply supported, then static deflection from A to x is

$$\boxed{y = \frac{w}{24EI} \left(x^4 - 2lx^3 + l^3 x\right)}$$

Where $w \Rightarrow$ ~~or~~ UDL,

$E \Rightarrow$ Young's modulus

$I \Rightarrow$ M.I. of shaft

$\cancel{\delta_s = \frac{5 w l^4}{384 E I}}$

Natural frequency due to UDL,

$$f_n = \frac{\omega}{2\pi} = \frac{\pi^2}{2\pi}\sqrt{\frac{EIg}{wl^4}} = \frac{\pi}{2}\sqrt{\frac{EIg}{wl^4}} \quad -(1)$$

W.K.T

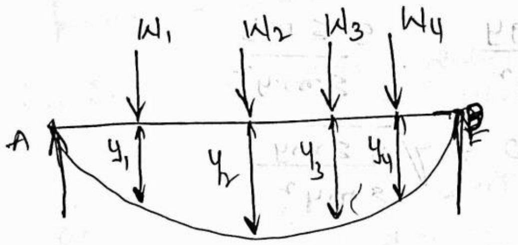
$$\delta_s = \frac{5 w l^4}{384 EI} \quad (\approx) \quad \frac{EI}{wl^4} = \frac{5}{384\delta_s}$$

$\therefore$ Eq (1) can be written as,

$$f_n = \frac{\pi}{2}\sqrt{\frac{5g}{384\delta_s}} = \frac{0.5615}{\sqrt{\delta_s}} \text{ Hz.}$$

(17)

* Natural frequency of free vibrations for a shaft subjected to no. of pt. loads:



consider a shaft AB of negligible mass loaded with point loads, W_1, W_2, W_3, W_4 in N. let m_1, m_2, m_3 & m_4 be the corresponding masses.

I Rayleigh's Method:

let y_1, y_2, y_3 & y_4 ... be total deflection under loads $W_1, W_2, W_3, W_4, \ldots$

W.K.T Max. P.E

$$= \frac{1}{2} m_1 \cdot g \cdot y_1 + \frac{1}{2} m_2 g y_2 + \frac{1}{2} m_3 g y_3 + \frac{1}{2} m_4 g y_4 + \ldots$$

$$= \frac{1}{2} \Sigma m g y$$

& Min. K.E.

$$= \frac{1}{2} m_1 (\omega \cdot y_1)^2 + \frac{1}{2} m_2 (\omega \cdot y_2)^2 + \frac{1}{2} m_3 (\omega \cdot y_3)^2 + \frac{1}{2} m_4 (\omega \cdot y_4)^2 + \ldots$$

$$= \frac{1}{2} \times \omega^2 \cdot \left[m_1 \cdot y_1^2 + m_2 y_2^2 + m_3 y_3^2 + m_4 y_4^2 + \ldots \right]$$

$$= \frac{1}{2} \cdot \omega^2 \cdot \Sigma m y^2.$$

Where, $\omega \Rightarrow$ circular frequency of vibration

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Equating Max. P.E & min. K.E

$$\frac{1}{2}\omega^2 \cdot \Sigma m \cdot y^2 = \frac{1}{2}\Sigma m \cdot g \cdot y$$

$$\therefore \quad \omega^2 = \frac{\Sigma mgy}{\Sigma my^2} = \frac{g\Sigma my}{\Sigma m \cdot y^2}$$

$$\omega = \sqrt{\frac{g\Sigma my}{\Sigma my^2}}$$

$$\therefore \quad \cancel{f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\Sigma my^2}{g\Sigma my}}}$$

$$\therefore \quad \boxed{f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g\Sigma my}{\Sigma my^2}}}$$

2. <u>Dunkerly's Method:</u>

The natural frequency of transverse vibration for a shaft carrying no. of pt. loads & UDL is obtained by Dunkerly's empirical formula.

According to this,

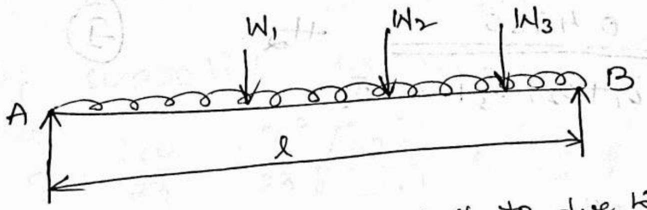
$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} + \cdots + \frac{1}{(f_{ns})^2}$$

$f_n \rightarrow$ Natural frequency of transverse vibration of shaft carrying pt. load & UDL

$f_{n1}, f_{n2}, f_{n3} \ldots \rightarrow$ Natural frequency of T.V. of each pt. load.

$f_{ns} \rightarrow$ Natural frequency of T.V. of UDL or due to mass of shaft

(19)



Let, $\delta_1, \delta_2, \delta_3 \ldots \Rightarrow$ static deflectn due to load $W_1, W_2, W_3, \ldots$ when considered separately

$\delta_s \rightarrow$ static deflectn due to UDL (or) due to mass of shaft.

W.K.T the natural frequency of transverse vibratn, due to load W_1;

$$\Rightarrow f_{n1} = \frac{0.4985}{\sqrt{\delta_1}} \text{ Hz}$$

||ly,

$$f_{n2} = \frac{0.4985}{\sqrt{\delta_2}} \text{ Hz}$$

$$f_{n3} = \frac{0.4985}{\sqrt{\delta_3}} \text{ Hz}$$

For sns. beam carry UDL

$\cancel{\delta_s = \frac{5Wl^4}{384EI}}$

$\cancel{\frac{EI}{Wl^4} = \frac{5}{384\delta_s}}$

$\cancel{f_n = \frac{\pi}{2}}$

Also Natural frequency of T.V due to UDL,

$$f_{ns} = \frac{0.5615}{\sqrt{\delta_s}} \text{ Hz}$$

Therefore, according to Dunkerly's method

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_n)^3} + \ldots + \frac{1}{(f_{ns})^2}$$

$$= \frac{\delta_1}{(0.4985)^2} + \frac{\delta_2}{(0.4985)^2} + \frac{\delta_3}{(0.4985)^2} + \ldots + \frac{\delta_s}{(0.5615)^2}$$

$$= \frac{1}{(0.4985)^2}\left[\delta_1 + \delta_2 + \delta_3 + \ldots + \frac{\delta_s}{1.27}\right]$$

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$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3+\cdots+\delta_s}} \text{ Hz} \quad \text{(I)}$$

NOTE:

1. When there is no UDL (or) mass of shaft Then

$$\delta_s = 0$$

∴ eq (I) Becomes,

$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3+\cdots}} \text{ Hz.}$$

2. The value of δ_1, δ_2, δ_3 etc. for a simply supported shaft may be obtained as

$$\delta = \frac{W a^2 b^2}{3EIl}$$

$\delta \rightarrow$ static deflectn due to load W,

a and b $\rightarrow$ Distances of loads from ends

$E \rightarrow$ Young's modulus for mtl of shaft

$I \rightarrow$ moment of inertia of shaft.

$l \rightarrow$ length of shaft

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for Simply supported beam carrying UDL,

$$f_n = \frac{\omega}{2\pi} = \frac{\pi^2}{2\pi}\sqrt{\frac{EIg}{wl^4}} = \frac{\pi}{2}\sqrt{\frac{EIg}{wl^4}}$$

Static deflectn, $\delta_s = \frac{5wl^4}{384EI}$

$$\frac{EI}{wl^4} = \frac{5}{384\delta_s}$$

$$f_n = \frac{\pi}{2}\sqrt{\frac{5g}{384\delta_s}} \Rightarrow \boxed{f_n = \frac{0.5615}{\sqrt{\delta_s}}} \text{ Hz}$$

Problem

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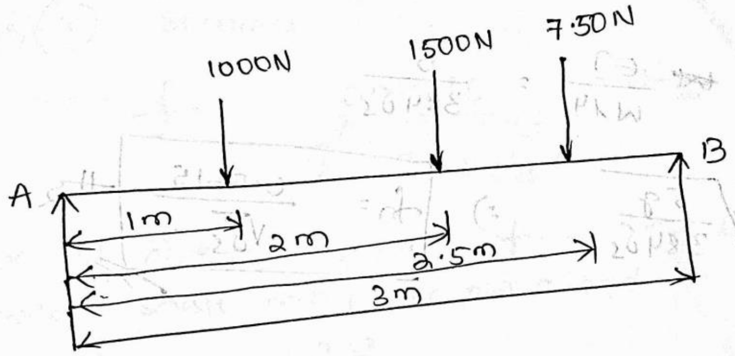
1. A shaft 50mm diameter & 3 metres long is simply supported at ends & carries three loads of 1000N, 1500N, 750N at 1m, 2m & 2.5m from left support. The Young's modulus of shaft mtl is 200 GN/m². Find the frequency of T.V.

Sol:



Given:

$d = 50mm = 0.05m$; $l = 3m$; $W_1 = 1000N$; $W_2 = 1500N$;

$W_3 = 750N$; $E = 200\ GN/m^2$

$= 200 \times 10^9\ N/m^2$;

W.K.T, M.I of shaft,

$$I = \pi/64 \times d^4 = \frac{\pi}{64} \times (0.05)^4$$

$$I = 0.307 \times 10^{-6}\ m^4.$$

& static deflection to pt. load W,

$$\delta = \frac{W a^2 b^2}{3EIl}$$

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∴ Static deflection due to 1000 N,

$$\delta_1 = \frac{1000 \times (1)^2 (2)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3}$$

$$\delta_1 = 7.24 \times 10^{-3}\ m$$

" ",

$$\delta_2 = \frac{1000 \times (2)^2 (1)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3}$$

$$= \cancel{2.42 \times 10^{-3}\ m}\ 10.86 \times 10^{-3}\ m$$

&

$$\delta_3 = \frac{1000 \times (2.5)^2 (0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3}$$

$$= \cancel{0.4}\ 2.12 \times 10^{-3}\ m$$

W.K.T. frequency of T.V

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$f_n = 3.5\ Hz$$

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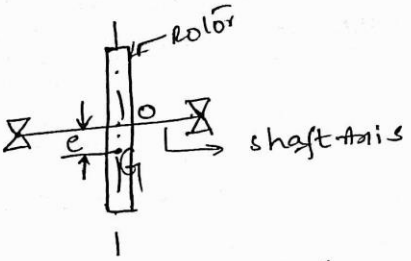
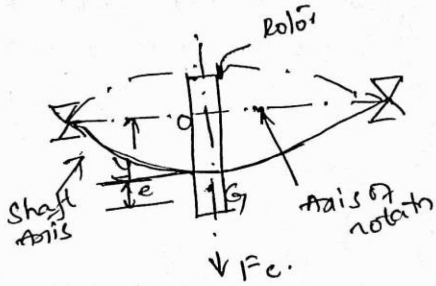
* Critical (or) Whirling Speed of a Shaft:

The speed at which the shaft runs so that the additional deflection of shaft from the axis of rotation becomes infinite, is known as critical (or) whirling speed.



When shaft is stationary

(a)



When Shaft is rotating

(b).

Consider a shaft of negligible mass carrying rotor. The point 'O' is on the shaft axis & G is the C.G of rotor. When the shaft is stationary, the centre line of bearing & axis of shaft coincides. Fig (b) shows the shaft when rotating about axis of rotatn at a uniform speed ω rad/sec.

Let,

$m \rightarrow$ Mass of rotor.

$e \rightarrow$ initial distance of C.G of rotor from centre line of bearing (or) shaft axis, when shaft is stationary

$y \rightarrow$ Additnal deflectn of centre of gravity of rotor when the shaft starts rotating at ω rad/sec

$s \rightarrow$ Stiffness of the shaft i.e; the load reqd. per unit deflection of shaft.

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Since the shaft is rotating at ω; ∴
centrifugal force acting radially outwards through G causing the shaft to deflectn

$$F_c = m\cdot\omega^2(y+e)$$

The shaft behaves like a spring,
∴ The force resisting the deflection y,

$$= s\cdot y$$

for equilibrium,

$$m\cdot\omega^2(y+e) = s\cdot y$$

$$m\cdot\omega^2 y + m\omega^2 e = s\cdot y$$

$$y(s - m\omega^2) = m\omega^2 e$$

$$y = \frac{m\cdot\omega^2 e}{(s-m\omega^2)} = \frac{\omega^2 e}{\left[\frac{s}{m}-\omega^2\right]}$$

W.K.T, circular frequency

$$\omega_n = \sqrt{\frac{s}{m}}$$

$$\therefore y = \frac{\omega^2 e}{(\omega_n^2-\omega^2)}$$

A little consideratn show that when $\omega > \omega_n$, the value y will be negative, then shaft rotates in opposite dirn.

~~In order to have the value of y always +ve.~~

$$y = \pm\frac{\omega^2 e}{(\omega_n)^2-(\omega^2)} = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2-1} = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2-1}$$

$$\omega_n = \omega_c$$

$\omega_c \rightarrow$ critical (or) whirling speed.

$$\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}} \text{ Hz}$$

~~If Nc is~~

(27)

If N_c is the critical (or) whirling speed,

$$2\pi N_c = \sqrt{\frac{g}{\delta}} \quad (or) \quad N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$N_c = \frac{0.4985}{\sqrt{\delta}} \text{ rps.}$$

Natural frequency,

$$f_n = \frac{1}{t_p} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$= \frac{0.4985}{\sqrt{\delta}}.$$

Hence, the critical (or) whirling speed is ~~the~~ same as natural frequency of transverse vibration. unit: "rps"

NOTE:

1. When the c.g of the rotor lies b/w centre line of shaft & centre line of bearing, 'e' is taken -ve. On the other hand, if the c.g of rotor does not lie b/w the centre line of shaft & centre line of bearing. the value 'e' is +ve can be taken.

2. To determine the critical speed of a shaft which may be subjected to pt. loads, UDL (or) combination of both, find natural frequency of T.V which is equal to critical speed of a shaft in r.p.s. The Dunkerley's Method D.M may be used for calculating frequency.

3. A shaft supported in short bearings (or) ball bearings is assumed to simply supported shaft while shaft supported in long bearings is assumed to have both ends fixed.

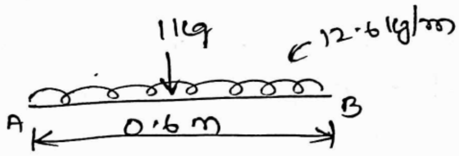
Problem (58)

1. calculate the whirling speed of a shaft 20mm dia. and 0.6m long carrying a mass of 1kg at its midpt. The density of shaft mtl is 40 Mg/m^3, Young's modulus 200 GN/m^2. Assume shaft to be freely supported.

sol: Given:

$d = 20mm = 0.02m$; $l = 0.6m$; $m_1 = 1kg$; $\rho = 40\ Mg/m^3$
$= 40 \times 10^6\ g/m^3$.
$= 40 \times 10^3\ kg/m^3$.

$E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$.



W.K.T,

M.I, $I = \frac{\pi}{64} \times d^4 = \pi/64\,(0.02)^4$
$= 7.855 \times 10^{-9}\ m^4$

since the density of shaft mtl is $40 \times 10^3\ kg/m^3$.

$\therefore$ Mass of shaft / mtr length

m_s = Area × length × density
$= \frac{\pi}{4}(0.02)^2 \times (\cancel{0.6}) \times 40 \times 10^3 = 12.6\ kg/m$

W.K.T ; static deflectn due to 1kg of mass at centre,

$$\delta = \frac{Wl^3}{48EI} = \frac{1 \times 9.81 \times (0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 28 \times 10^{-6}\ m$$

& static deflectn due to mass of shaft,

$$\delta_s = \frac{5Wl^4}{384EI} = \frac{5 \times 12.6 \times 9.81\ (0.6)^4}{384 \times (200 \times 10^9) \times 7.855 \times 10^{-9}}$$

$$= 0.133 \times 10^{-3}\ m$$

(29)

∴ frequency of Transverse Vibratn.

$$f_n = \frac{0.4985}{\sqrt{\delta + \frac{\delta_s}{1.27}}}$$

$$= \frac{0.4985}{\sqrt{28\times10^{-6} + \frac{0.133\times10^{-3}}{1.27}}}$$

$f_n = 43.3\,Hz$

Let, $N_c \rightarrow$ whirling speed.

$\therefore\ N_c = 43.3$ r.p.s

$= 43.3 \times 60 = 2598$ rpm.

---

(2) A shaft 1.5m long, supported in flexible bearings at the ends carries two wheels each of 50kg mass. One wheel is situated at the centre of the shaft and other at a distance of 375mm from centre towards left. The shaft is hollow of external dia. 75mm & internal diameter 40mm. The density of shaft mtl is 7700 kg/m³ & its modulus of elasticity is 210 GN/m². Find the lowest whirling speed of shaft, taking in to account the mass of shaft.

Sol:- $l = 1.5m$; $m_1 = m_2 = 50kg$; $d_1 = 75mm = 0.075m$; $d_2 = 40mm = 0.04m$;

$\rho = 7700\,kg/m^3$; $E = 210\times10^9\,N/m^2$

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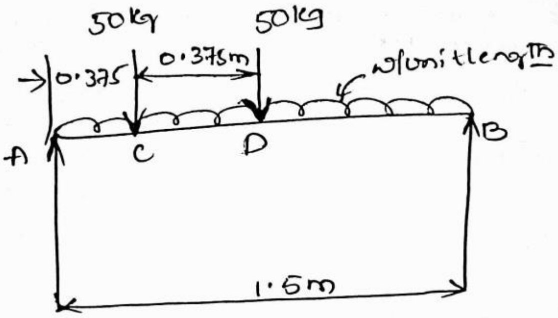

W.K.T. M.I of shaft, $I = \frac{\pi}{64}\left[d_1^4 - d_2^4\right]$

$$= \frac{\pi}{64}\left[(0.075)^4 - (0.04)^4\right] = 1.4\times10^{-6}\ m^4.$$

since the density of shaft 7700 kg/m³

∴ Mass per unit ~~metre~~ length:

$m_s =$ Area × length × density

$$= \frac{\pi}{4}\left[(0.075)^2 - (0.04)^2\right]\times 1\times 7700$$

$m_s = 24.34\ kg/m$

W.K.T, static deflectn due to load W,

$$= \frac{Wa^2b^2}{3EIl} = \frac{m\cdot g\cdot a^2b^2}{3EIl}$$

δ_1 at 'C' $= \frac{m_1\, g\cdot a^2 b^2}{3EIl} = \frac{50\times9.81\times(0.375)^2\times(1.125)^2}{3\times210\times10^{9}\times1.4\times10^{-6}\times1.5}$

$\delta_1 = 70\times10^{-6}\ m$

δ_2 at D : $\frac{m_2\, g\, a^2b^2}{3EIl} = \frac{50\times9.81\times(0.75)^2(0.75)^2}{3\times200\times10^{9}\times1.4\times10^{-6}\times1.5}$

$\delta_2 = 123\times10^{-6}\ m$

static deflectn due to UDL,

$\delta_s = \frac{5}{384}\times\frac{wl^4}{EI} = \frac{5}{384}\times\frac{24.34\times9.81\times(1.5)^4}{210\times10^{9}\times1.4\times10^{-6}} = 56\times10^{-6}\ m$

W.K.T for T.V $f = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\frac{\delta_s}{1.27}}} = \frac{0.4985}{\sqrt{70\times10^{-6}+123\times10^{-6}+\frac{56\times10^{-6}}{1.27}}}$

$= 32.4\ Hz$

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9. A vertical shaft of 5mm dia is 200mm long & it is supported in long bearings at its ends. A disc of mass 50kg is attached to centre of shaft. Neglecting any increase in stiffness due to attachment of disc to shaft, find the critical speed of rotatn & max. bending stress when shaft is rotating at 75% of critical speed. The centre of disc is 0.25 mm from geometric axis of shaft. $E = 200\ GN/m^2$.

Sol: Given:

$d = 5mm = 0.005\ m$; $l = 200mm = 0.2m$; $m = 50kg$;

$e = 0.25mm = 0.25 \times 10^{-3}\ m$ | $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$.

Critical speed of Rotation:

W.K.T, M.I of shaft,

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.005)^4 = 30.7 \times 10^{-12}\ m^4$$

Since the shaft is supported in long bearings, it is assumed to be fixed at both ends.

W.K.T, static deflection at centre of shaft ~~which~~ is fixed at both ends,

$$\delta = \frac{Wl^3}{192EI} = \frac{(9.81)(50)(0.2)^3}{192 \times 200 \times 10^9 \times 30.7 \times 10^{-12}}$$

$$\delta = 3.33 \times 10^{-3}\ m$$

$$N_c = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{3.33 \times 10^{-3}}} \Rightarrow N_c = 8.64\ rps \Rightarrow 518.4\ rpm$$

(57)

* **Maximum Bending stress:**

Let, $\sigma \rightarrow$ Max. Bending stress

$N \rightarrow$ speed of shaft

$= 75\%$ of N_c

$= 0.75 N_c$

When the shaft starts rotating, the additional dynamic load (W_1) to which the shaft is subjected may be obtained by,

$$\frac{M}{I} = \frac{\sigma}{y_1} \quad \text{(or)} \quad M = \frac{\sigma \cdot I}{y_1} \quad \text{— (a)}.$$

W.K.T, a shaft fixed at both ends & carrying pt. load (W_1) at centre the max. bending moment,

$$M = \frac{W_1 \cdot l}{8} \quad \text{— (b)}$$

Equating eq's (a) & (b)

$$\frac{\sigma \cdot I}{y_1} = \frac{W_1 \cdot l}{8} \qquad \because (y_1 = d/2)$$

$$W_1 = \frac{\sigma \times 30.7 \times 10^{-12} \times 8}{\left(\frac{0.005}{2}\right) \times 0.2} \Rightarrow W_1 = 0.49 \times 10^{-6} \sigma . N$$

$\therefore$ Additional deflectn due to load W_1,

$$y = \frac{W_1}{W} \times \delta = \frac{0.49 \times 10^{-6} \cdot \sigma}{50 \times 9.81} \times 3.33 \times 10^{-3}$$

$$= 3.327 \times 10^{-12} \sigma .$$

W.K.T,

$$y = \frac{\pm e}{\left[\frac{\omega_c}{\omega}\right]^2 - 1} = \frac{\pm e}{\left[\frac{N_c}{N}\right]^2 - 1}$$

$$3.327 \times 10^{-12} \sigma = \frac{\pm 0.25 \times 10^{-3}}{\left[\frac{N_c}{0.75 N_c}\right]^2 - 1} = \pm 0.32 \times 10^{-3}$$

$$\sigma = 0.0962 \times 10^{9} \text{ N/m}^2$$

$$= 96.2 \times 10^{6} \text{ N/m}^2$$

$$\boxed{\sigma = 96.2 \times M \text{ N/m}^2}$$

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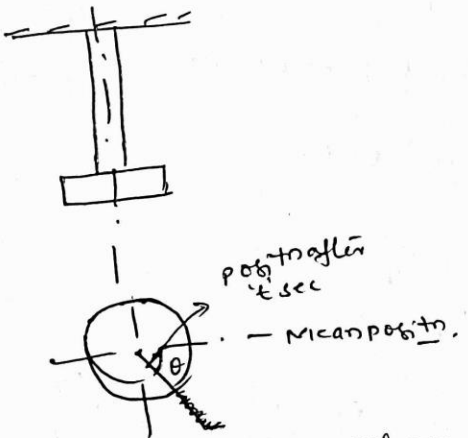
* Torsional Vibrations:

When the particles of a shaft or disc move in circle about axis of shaft, then the vibrations are known as "Torsional Vibrations"

* Natural Frequency of Free Torsional Vibrations:



Consider a shaft of negligible mass whose one end is fixed & other end carrying disc.

let, $\theta \rightarrow$ Angular displacement of shaft from mean posi'tn after 't' sec's

$m \rightarrow$ mass of disc, kg

$I \rightarrow$ Mass M.I of disc, kg-m²

$k \rightarrow$ radius of Gyratn, m

$q \rightarrow$ Torsional stiffness of shaft in N-m

∴ Restoring force, $= q \cdot \theta$ —(i)

& accelerating force, $= I \times \frac{d^2\theta}{dt^2}$ —(ii)

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Equating Eq's (i) & (ii)

$$I \cdot \frac{d^2\theta}{dt^2} = -q \cdot \theta$$

$$I \cdot \frac{d^2\theta}{dt^2} + q \cdot \theta = 0.$$

$$\frac{d^2\theta}{dt^2} + \frac{q}{I} \cdot \theta = 0.$$

From fundamental eq of SHM

$$\frac{d^2\theta}{dt^2} + \omega^2 \cdot x = 0.$$

$$\omega^2 = \frac{q}{I}$$

$$\omega = \sqrt{q/I}$$

$\therefore$ time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

Natural frequency, $\boxed{f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}}$

NOTE: The value of torsional stiffness 'q' may be obtained from torsion equatn.

$$\frac{T}{J} = \frac{C \cdot \theta}{l} \quad (\text{or}) \quad \frac{T}{\theta} = \frac{C \cdot J}{l}.$$

$$\boxed{q = \frac{C \cdot J}{l}} \qquad \therefore \boxed{\frac{T}{\theta} = q}$$

where, $C \to$ Modulus of rigidity for shaft mtl.

$J \to$ Polar m.I of shaft c/s.

$= \frac{\pi}{32} d^4$; $d \to$ dia. of shaft.

$l \to$ length of shaft.

Problem

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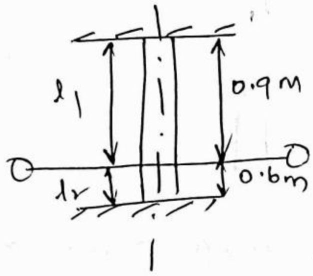
① A flywheel is mounted on a vertical shaft as shown. The both ends of a shaft are fixed & its diameter is 50mm. The flywheel has a mass of 500kg & its radius of gyration is 0.5m. find the natural frequency of torsional vibrations, if modulus of rigidity is 80 GN/m².



Sol: Given:

$d = 50mm = 0.05m$ | $m = 500kg$ | $k = 0.5m$
$G = 80\,GN/m^2 = 80 \times 10^9\,N/m^2$.

W.K.T, Polar moment of inertia,

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{32}(0.05)^4 = 0.6 \times 10^{-6}\,m^4$$

∴ Torsional stiffness, for length l_1

$$q_1 = \frac{C \cdot J}{l_1} = \frac{80 \times 10^9 \times 0.6 \times 10^{-6}}{0.9}$$

$$q_1 = 56 \times 10^3\,N\text{-}m$$

$$q_2 = \frac{C \cdot J}{l_2} = \frac{80 \times 10^9 \times 0.6 \times 10^{-6}}{0.6} = 84 \times 10^3\,N\,m$$

$$q = q_1 + q_2$$

Mass m.I, $I = m \cdot k^2 = 500 \times (0.5)^2 = 125\,kg\text{-}m^2$

Natural freq $f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}} = \frac{1}{2\pi}\sqrt{\frac{140 \times 10^3}{125}}$

$$f_n = 5.32\,Hz$$

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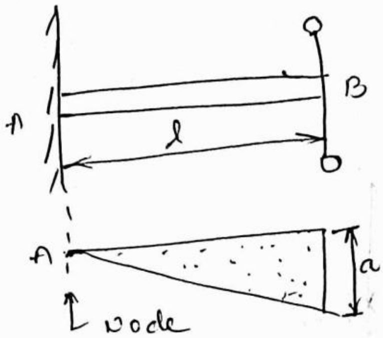
Free Torsional Vibrations of Single Rotor System:



Natural frequency for a shaft fixed at one end at other end carrying a single rotor,

is. $f_n = \frac{1}{2\pi}\sqrt{\frac{q}{I}} = \frac{1}{2\pi}\sqrt{\frac{C \cdot J}{I \cdot l}}$.

$\because \left(q = \frac{C \cdot J}{l} \right)$.

Where, $C \rightarrow$ modulus of rigidity for shaft mtl.

$J \rightarrow$ Polar moment of inertia of a shaft

$= \pi/32 \; d^4$

$d \rightarrow$ diameter of shaft

$l \rightarrow$ length of shaft.

$m \rightarrow$ mass of rotor.

$k \rightarrow$ radius of gyration.

$I \rightarrow$ Mass M.I of rotor $= m \cdot k^2$.

A little consideration will show that the amplitude of vibration is zero at 'A'. at max. 'B'. It may be noted that the point (or) section of shaft whose amplitude of torsional vibration is zero, is known as "node". In other words, at the node the shaft is unaffected due to vibration.

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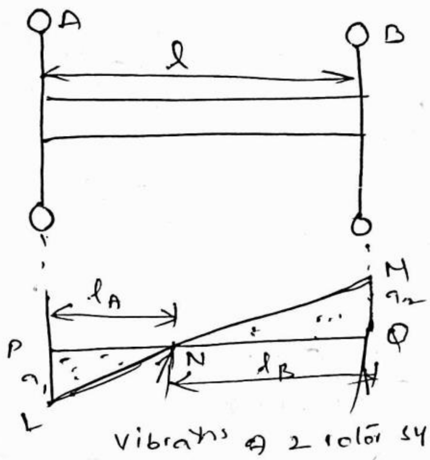
* Free Torsional Vibration of Two Rotor System:-



Vibrations of 2 rotor system.

Consider a 2 rotor system. It consists of a shaft with 2 rotors at its ends. In this system the torsional vibrations occurs only when the 2 rotors A & B move in opposite direction. If A moves ACW then B should CW & vice versa but with same frequency.

From the above fig. Node lies at pt 'N', & that pt. may be considered as a fixed end & shaft may be considered as 2 shafts NP & NQ each fixed to one of its end & carrying rotors at free ends.

Let,
- $l \rightarrow$ length of shaft
- $l_A \rightarrow$ length of part NP.
- $l_B \rightarrow$ length of part NQ.
- $I_A \rightarrow$ Mass MI of rotor A
- $I_B \rightarrow$ " " " B
- $d \rightarrow$ diameter of shaft
- $J \rightarrow$ Polar MI of shaft &
- $C \rightarrow$ modulus of rigidity.

∴

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∴ Natural frequency of Torsional vibratn for rotor A,

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{C \cdot J}{l_A \cdot I_A}} \quad \text{— (i)}$$

||ly,

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{C \cdot J}{l_B \cdot I_B}} \quad \text{— (ii)}$$

$\because$ $f_{nA} = f_{nB}$

$$\therefore \frac{1}{2\pi}\sqrt{\frac{C \cdot J}{l_A \cdot I_A}} = \frac{1}{2\pi}\sqrt{\frac{C \cdot J}{l_B \cdot I_B}}$$

(or) $l_A \cdot I_A = l_B \cdot I_B$ — (iii)

$$\therefore l_A = \frac{l_B \cdot I_B}{I_A}$$

We also know that, $l = l_A + l_B$ — (iv)

from eq's (iii) & (iv) we can find l_A & l_B & substituting l_A & l_B in eq's (i) & (ii) to find the corresponding f_n's

The line LNM is known as elastic line

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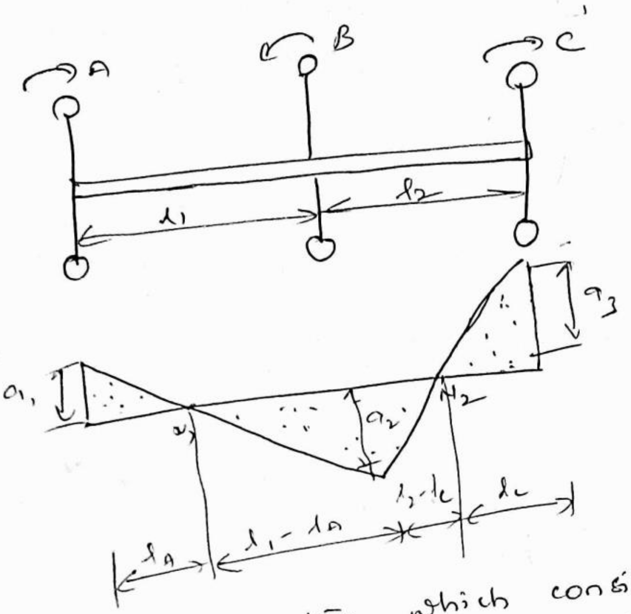
* Free Torsional vibration of 3 rotor system:



Consider 3 rotor system which consist of a shaft & 3 rotors A, B & C. where 2 rotors attached to the end of shaft and other is attached in b/w the shaft. Torsional vibration may occur in 2 ways either 1 node (or) 2 node. In each case the 2 rotors rotate in one dirn & other rotate in opp. dirn. with same frequency.

$l_1 \rightarrow$ distance of

.

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$J \rightarrow$ polar M.I

$C \rightarrow$ Modulus of rigidity for a shaft mtl.

W.K.T

$$f_{nA} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_A \cdot I_A}}$$

$$f_{nB} = \frac{1}{2\pi}\sqrt{\frac{CJ}{I_B}\left[\frac{1}{(l_1 - l_A)} + \frac{1}{(l_2 - l_C)}\right]}$$

$$f_{nC} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_C \cdot I_C}}$$

$\therefore f_{nA} = f_{nB} = f_{nC}$

$$\frac{1}{2\pi}\sqrt{\frac{CJ}{l_A \cdot I_A}} = \frac{1}{2\pi}\sqrt{\frac{CJ}{l_C \cdot I_C}}$$

$$l_A \cdot I_A = l_C \cdot I_C$$

(40)

When amplitude of vibration of rotor A (a_1) is known then amplitude of a_2 (θ) of rotor B

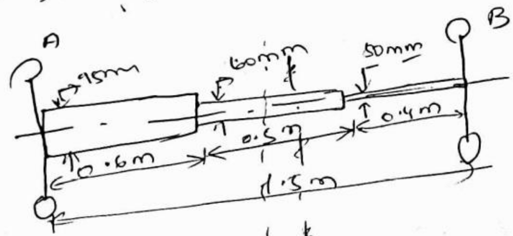
$$\frac{a_2}{l_A - l_1} = \frac{a_1}{l_A} \quad \therefore a_2 = \frac{l_A - l_1}{l_A} \cdot a_1$$

& amplitudes of rotor 'c'

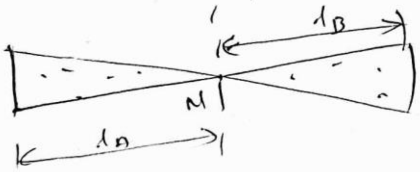
$$a_3 = \frac{l_C}{l_C - l_2} \times a_2$$

* A steel shaft 1.5m long is 95mm in dia for first 0.6m of its length & 60mm in dia 0.5m of length & 50mm of dia for remaining 0.4m of its length. The shaft carries 2 flywheels at 2 ends the first having a mass of 900kg & 0.85m 'K' located at 95mm dia end & second having a mass of 700kg & 0.55m 'K' located at other end. Determine the location of node & f_n of free torsional vibration of system. $C = 80\,GN/m^2$

Sol:



Given:- $L = 1.5m$
$l_1 = 0.6m$ $d_1 = 95mm$
$l_2 = 0.5m$ $d_2 = 60mm$
$l_3 = 0.4m$ $d_3 = 50mm$
$m_A = 900kg$; $m_B = 700kg$
$C = 80 \times 10^9\,N/m^2$



W.K.T, length of equivalent shaft $l = l_1 + l_2\left[\frac{d_1}{d_2}\right]^4 + l_3\left[\frac{d_1}{d_3}\right]^4$ $l = 8.95m$

(41)

Location of node:

$l_A \rightarrow$ Distance of node from flywheel A

$l_B \rightarrow$ " " " " B.

$I_A = m_A k^2 = 650\ kg\text{-}m^2$

$I_B = m_B k^2 = 212\ kg\text{-}m^2$

W.k.t, $l_A \cdot I_A = l_B \cdot I_B$ ⇒ $l_A = \frac{l_B \cdot I_B}{I_A}$

$l_A = 0.326\, l_B$

$l_A + l_B = l.$

$0.326\, l_B + l_B = 8.95\,m$

$l_B = 6.75\,m$. | $l_A = 2.2\,m$.

Hence node lies at a distance of 2.2 m from flywheel A & 6.75 m from flywheel B.

∴ Original position of shaft from flywheel A:

$l_1 + (l_A - l_1)\left[\frac{d_1}{d_2}\right]^4 = 0.655\,m.$

Natural frequency:

$J = \pi/32\,(d_1)^4 = 8\times10^{-6}\,m^4$

$f_n = f_{nA} = f_{nB}$

$= \frac{1}{2\pi}\sqrt{\frac{CJ}{I_A \cdot l_A}} = \frac{1}{2\pi}\sqrt{\frac{80\times10^{9}\times8\times10^{-6}}{2.2\times650}} \Rightarrow f_n = 3.37\,Hz.$