KINEMATICS OF MACHINERY (ME2202PC)

2 ND YEAR B .TECH II - S E M

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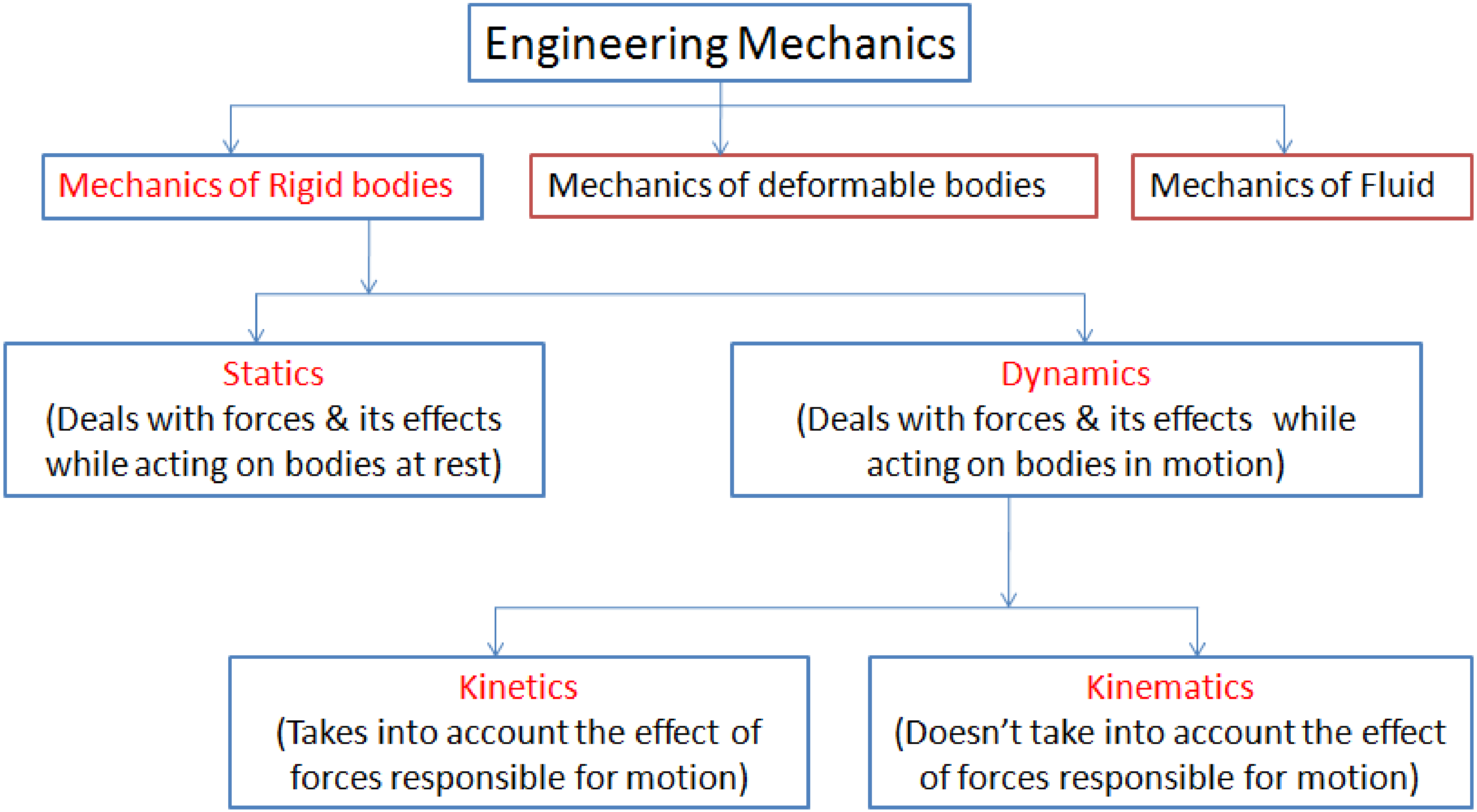
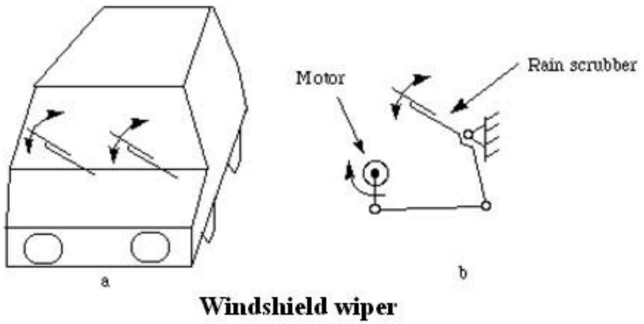
DEPARTMENT OF MECHANICAL ENGINEERING


Engineering Mechanics
Mechanics of Rigid bodies
Mechanics of deformable bodies
Mechanics of Fluid
Statics
(Deals with forces & its effects while acting on bodies at rest)
Dynamics
(Deals with forces & its effects while acting on bodies in motion)
Kinetics
(Takes into account the effect of forces responsible for motion)
Kinematics
(Doesn't take into account the effect of forces responsible for motion)

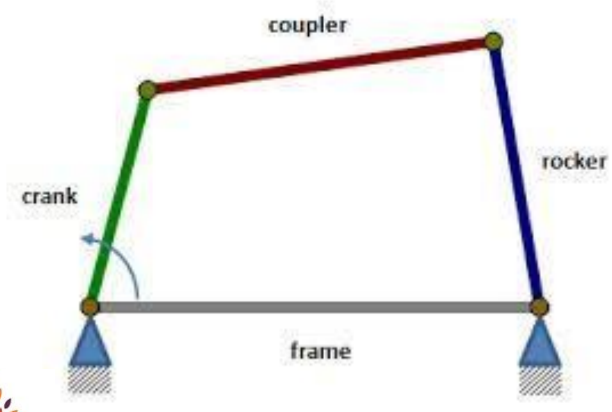
UNIT-I

BASICS



Mechanism:

- A number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others.
- A mechanism transmits and modifies a motion.
- Example: 4 bar mechanism, Slider crank mechanism



BASICS

Machine: (Combinations of Mechanisms)

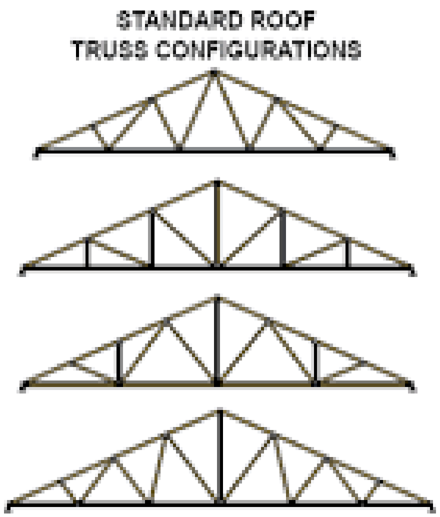
Transforms energy available in one form to another to do certain type of desired useful work.

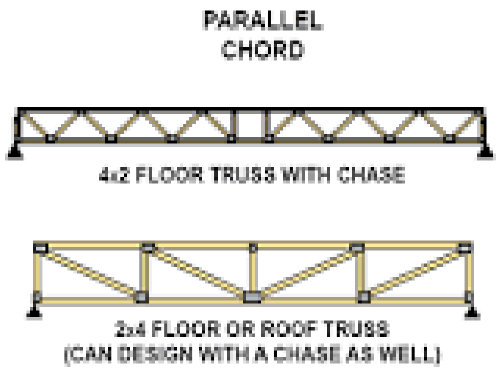
Lathe Machine

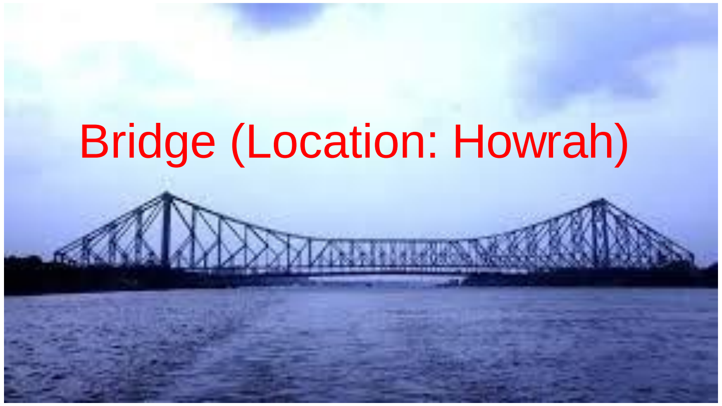
BASICS

Structure:

- ➢ Assembly of a number of resistant bodies meant to take up loads.
- ➢ No relative motion between the members







Truss

BASICS

Kinematic Link (element): It is a Resistant body i.e. transmitting the required forces with negligible deformation.

Types of Links

1. Rigid Link

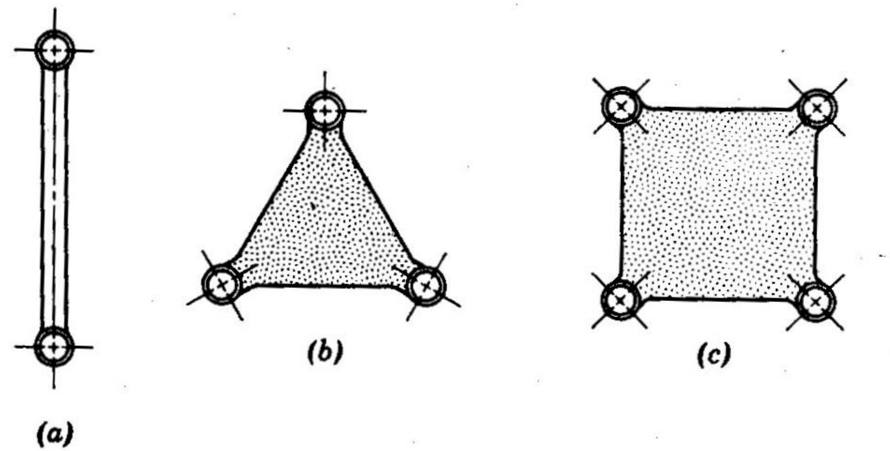
Doesn’t undergo deformation. Example: Connecting rod, crank

2. Flexible Link

Partially deformed link. Example: belts, Ropes, chains

3. Fluid Link

Formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only. Example: Jacks, Brakes



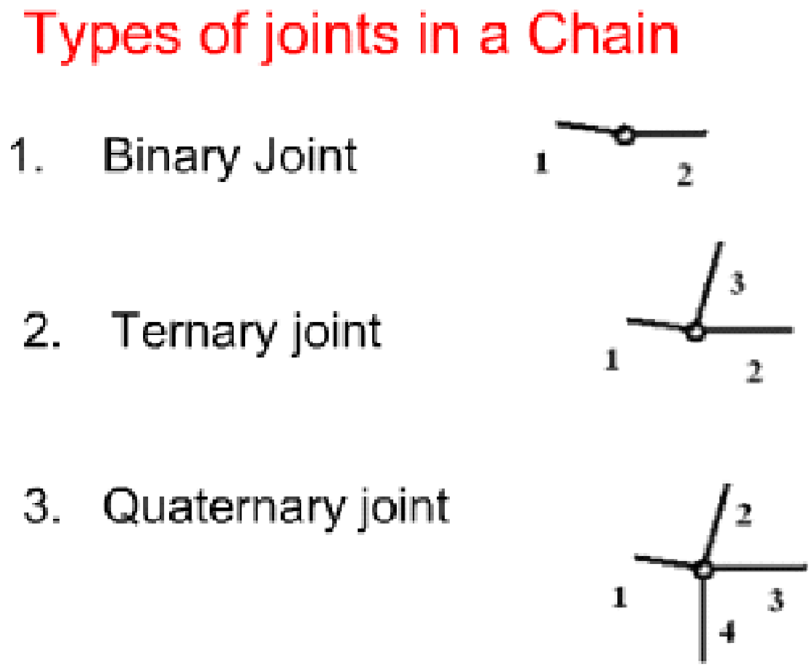
Binary link (2 vertices) **Ternary link (3 vertices)** **Quaternary link (4 vertices)**

BASICS

Kinematic Joint: Connection between two links by a pin

Types of Joints:

- Binary Joint (2 links are connected at the joint)
- Ternary Joint (3 links are connected)
- Quaternary Joint. (4 links are connected)

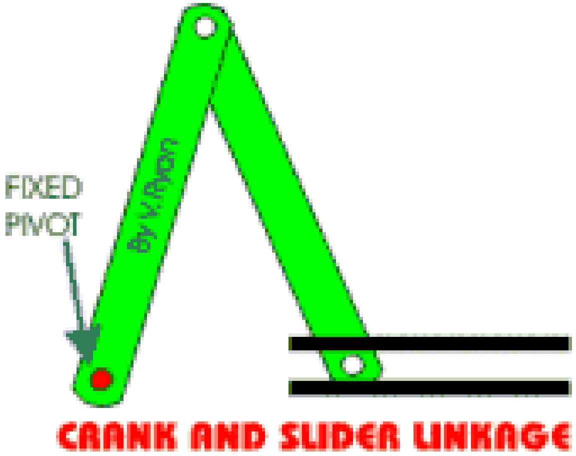


Note: if '**l**' number of links are connected at a joint, it is equivalent to (**l-1**) binary joints.

BASICS

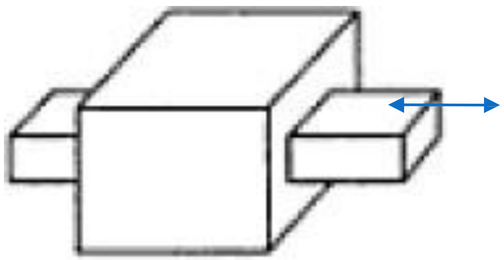
Kinematic Pair:

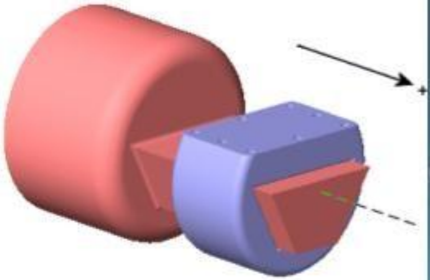
- The two links (or elements) of a machine, when in contact with each other, are said to form a pair.
- If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair



KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

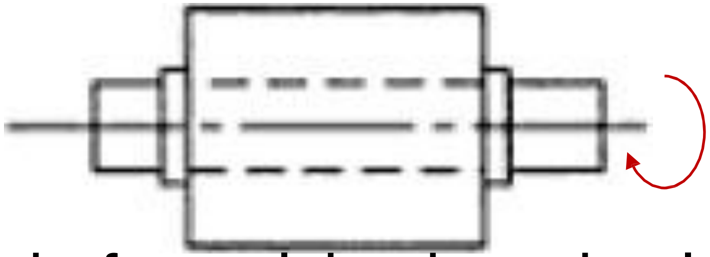
1. Sliding Pair





Rectangular bar in a rectangular hole

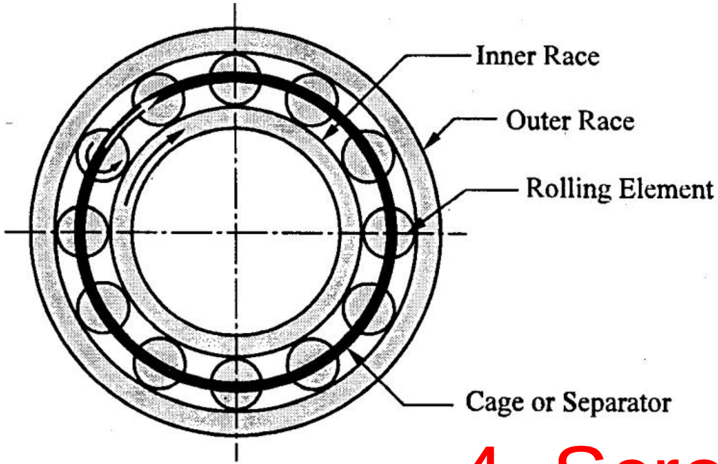
2. Turning or Revolving Pair



Collared shaft revolving in a circular hole

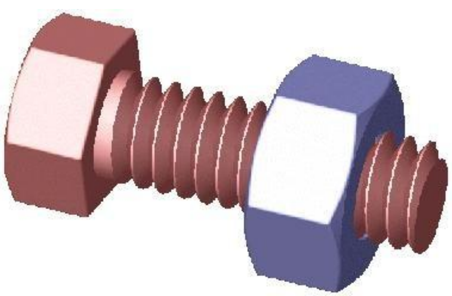
KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

3. Rolling Pair



Links of pairs have a rolling motion relative to each other.

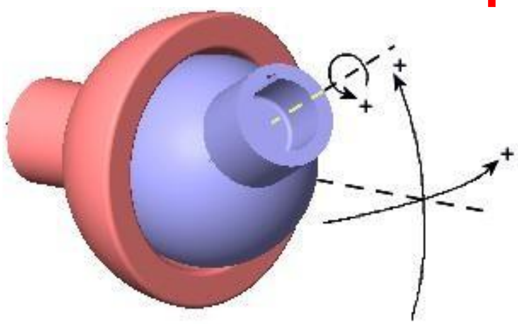
4. Screw or Helical Pair

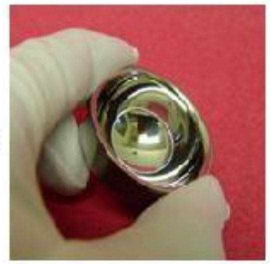


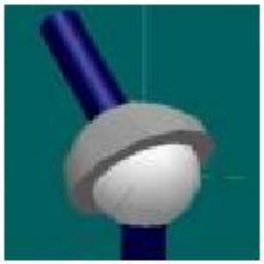
if two mating links have a turning as well as sliding motion between them.

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

5. Spherical Pair

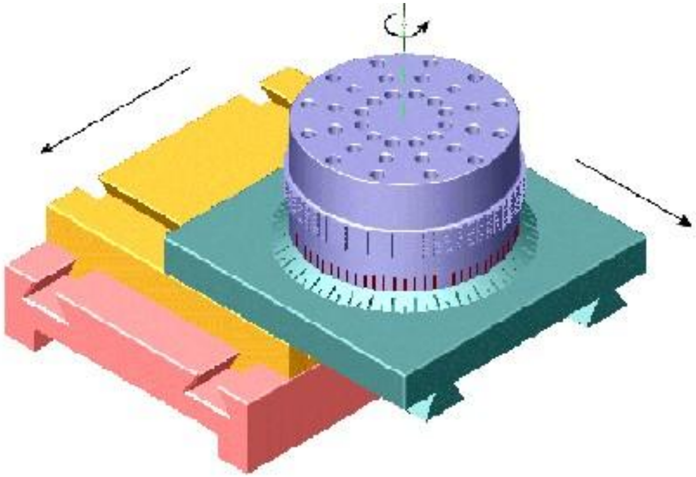






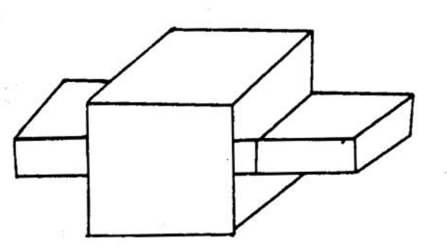
When one link in the form of a sphere turns inside a fixed link

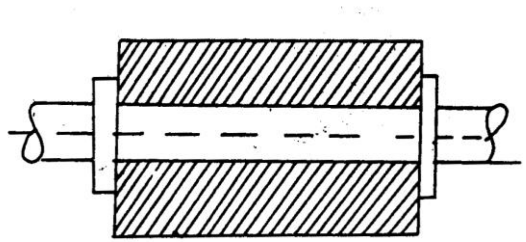
6. Planar Pair



KINEMATIC PAIRS ACCORDING TO TYPE OF CONTACT

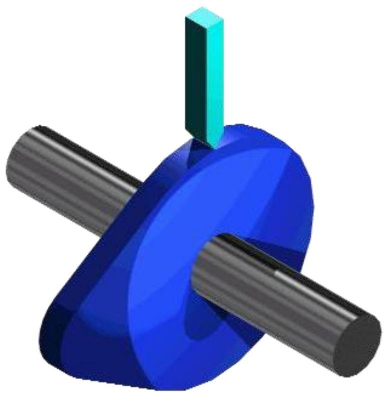
1. Lower Pair





The joint by which two members are connected has surface (Area) contact

2. Higher Pair

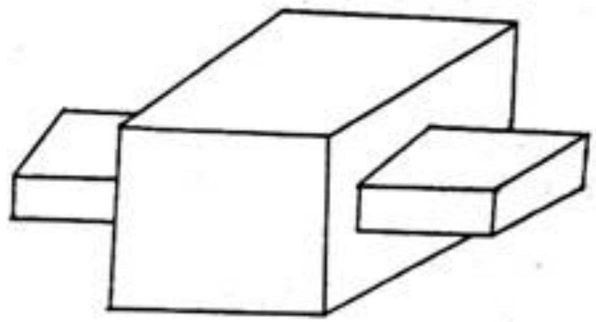


The contact between the pairing elements takes place at a point or along a line.

Toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs

KINEMATIC PAIRS ACCORDING TO TYPE OF CONSTRAINT

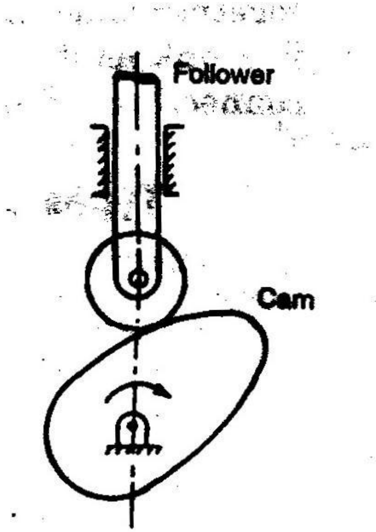
1. Closed Pair



Two elements of pair are held together mechanically to get required relative motion.

Eg. All lower pairs

2. Unclosed Pair

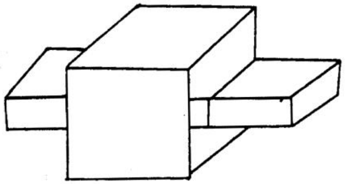


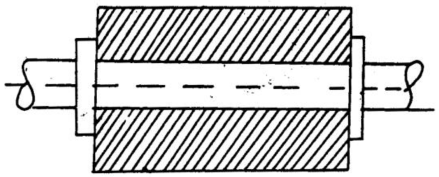
- Elements are not held mechanically.
- Held in contact by the action of external forces.

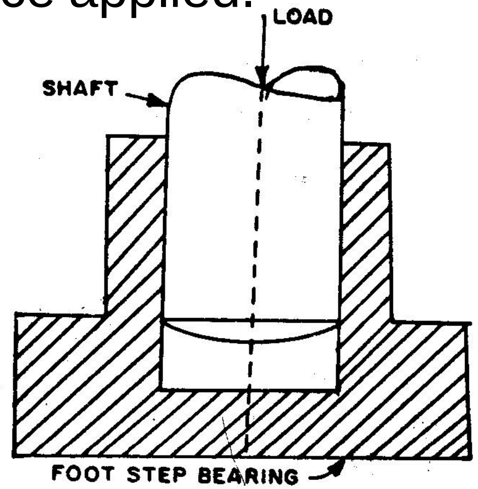
Eg. Cam and spring loaded follower pair

CONSTRAINED MOTION

1.Completely constrained Motion:
Motion in definite direction irrespective of the direction of the force applied.

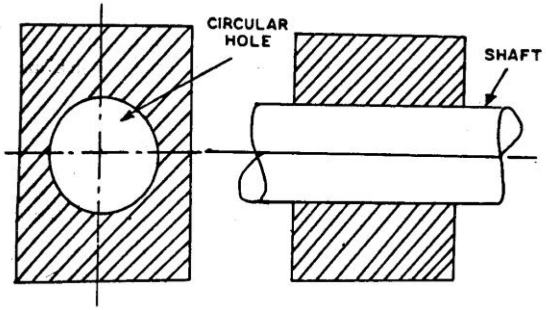






2. Successfully (partially) constrained Motion:

- Constrained motion is not completed by itself but by some other means.
- Constrained motion is successful when compressive load is applied on the shaft of the foot step bearing



3. Incompletely constrained motion:
Motion between a pair can take place in more than one direction.

Circular shaft in a circular hole may have rotary and reciprocating motion. Both are independent of each other.

KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative (i.e. completely or successfully constrained motion) to one another.

Example: 4 bar chain

The following relationship holds for kinematic chain

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

Where

p = number of lower pairs

l = number of links

j = Number of binary joints

KINEMATIC CHAIN

$$l = 2p - 4$$

$$j = \frac{3}{2} l - 2$$

If LHS > RHS, Locked chain or redundant chain; no relative motion possible.

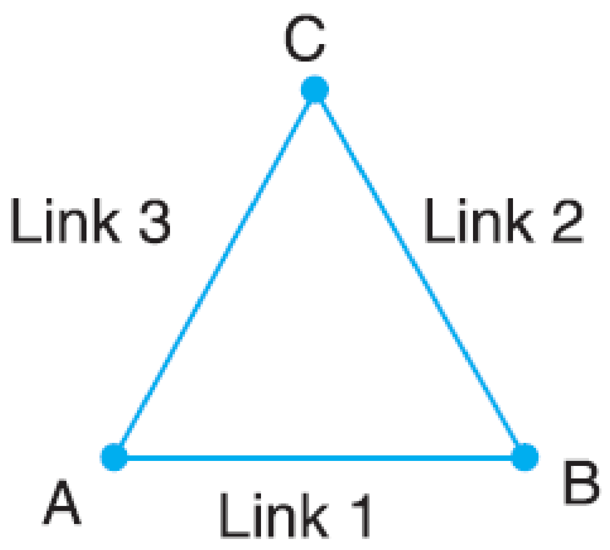
LHS = RHS, Constrained chain .i.e. motion is completely constrained

LHS < RHS, unconstrained chain. *i.e. the relative motion is not completely constrained.*

NUMERICAL EXAMPLE-1

Determine the nature of the chain

(K2:U)



$l = 3 \quad p = 3 \quad j = 3$

From equation

$$l = 2p - 4$$
$$= 2 \times 3 - 4 = 2$$

L.H.S. > R.H.S.

$$j = \frac{3}{2} l - 2$$
$$= \frac{3}{2} \times 3 - 2 = 2.5$$

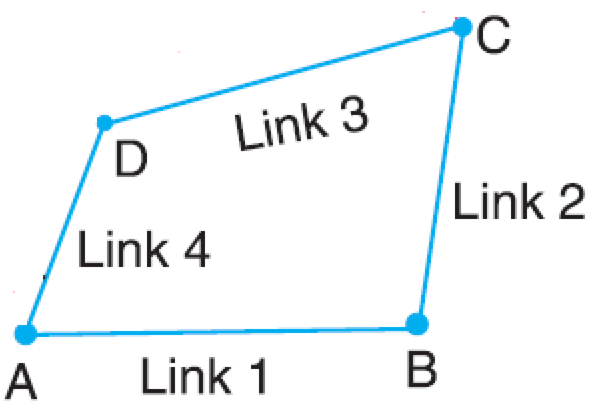
L.H.S. > R.H.S.

Therefore it is a locked Chain

EXERCISE

Determine the nature of the chains given below (K2:U)

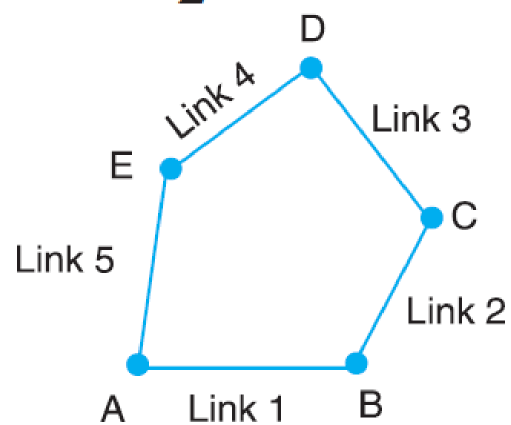
Hint: Check equations $l = 2p - 4$, $j = \frac{3}{2}l - 2$



$l = 4$, $p = 4$, and $j = 4$

L.H.S. = R.H.S.

constrained kinematic chain



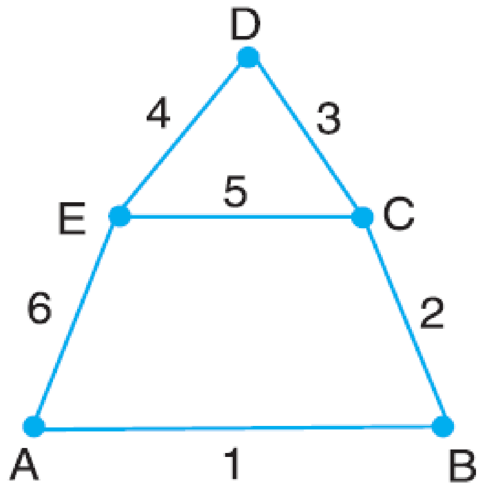
$l = 5$, $p = 5$, and $j = 5$

L.H.S. < R.H.S.

unconstrained chain

NUMERICAL EXAMPLE-2

Determine the nature of the chain (K2:U)



- **l** = 6
- j = 3 Binary joints (A, B & D) + 2 ternary joints (E & C)
- We know that, 1 ternary joint = (3-1) = 2 Binary Joints
- Therefore, j = 3 + (2*2) = 7.

$$j = \frac{3}{2} l - 2$$

$$= \frac{3}{2} \times 6 - 2 = 7$$

L.H.S. = R.H.S.

Therefore, the given chain is a kinematic chain or constrained chain.

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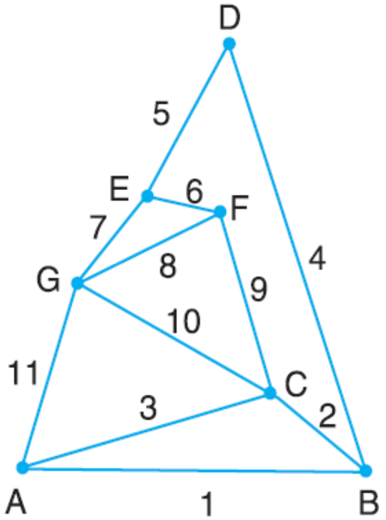
EXERCISE

Determine the number of joints (equivalent binary) in the given chains (K2:U)

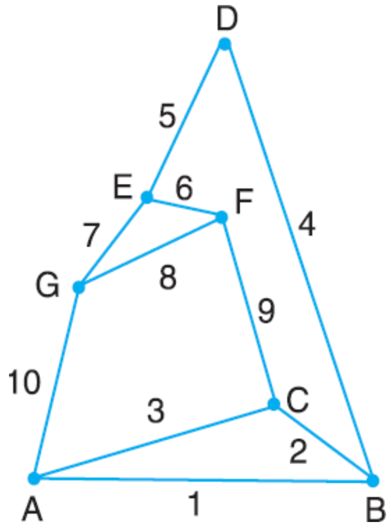


Number of Binary Joints = 1 (D)

No. of ternary joints = 4 (A, B, E, F)

No. of quaternary joints = 2 (C & G)

Therefore, j = 1 + 4 (2) + 2 (3) = 15



No. of Binary Joints = 1 (D)

No. of ternary joints = 6 (A, B, C, E,F, G)

j = 1 + 6 (2) = 13

KINEMATIC CHAIN

- For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.
- In this case to determine the nature of chain, the relation given by A.W. Klein, may be used

$$j + \frac{h}{2} = \frac{3}{2} l - 2$$

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

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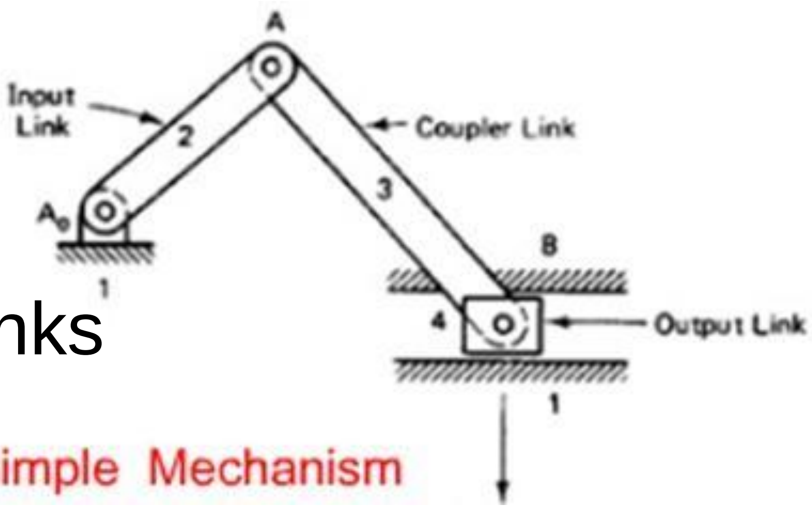
CLASSIFICATION OF MECHANISMS

Mechanism:

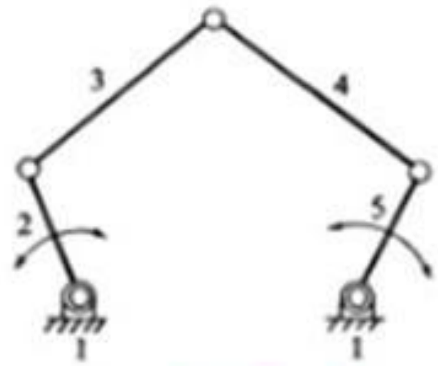
When one of the links of a kinematic chain is fixed, the chain is called Mechanism.

Types:

- Simple - 4 Links
- Compound - More than 4 links



Simple Mechanism

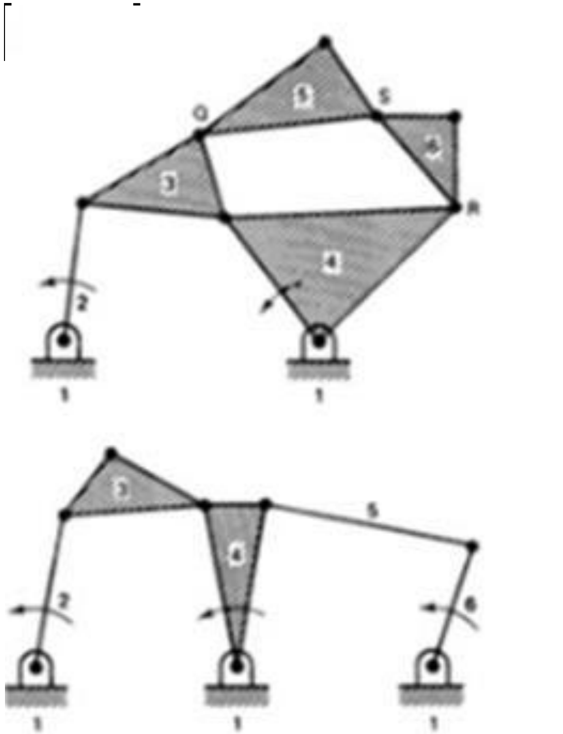


Compound Mechanism

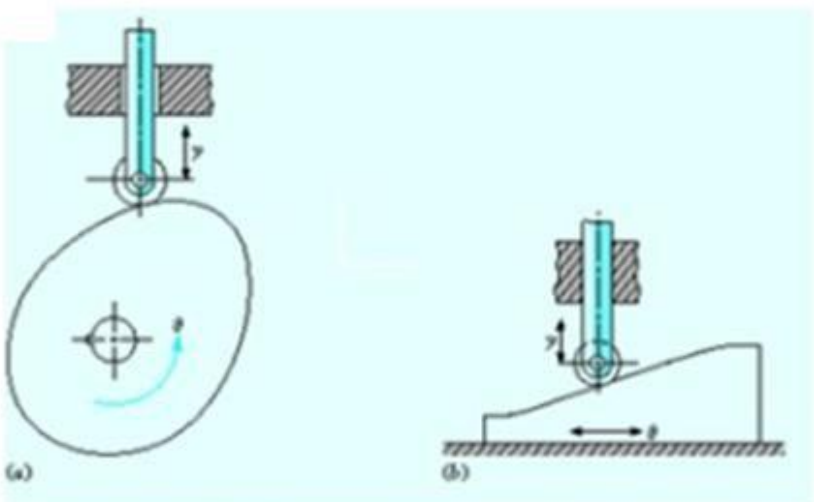
Classification of mechanisms

- Complex - Ternary or Higher order f
Links

- Planar - All links lie in the same plane



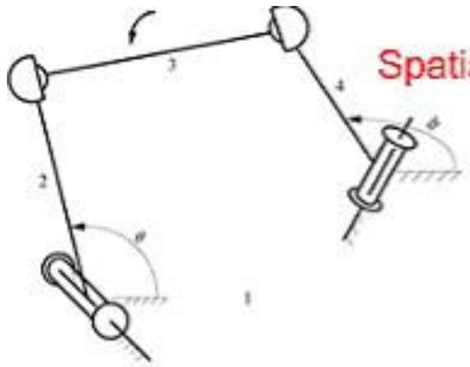
Complex Mechanism



Planar Mechanism

Classification of mechanisms

➢Spatial - Links of a mechanism lie in different planes



Spatial Mechanism

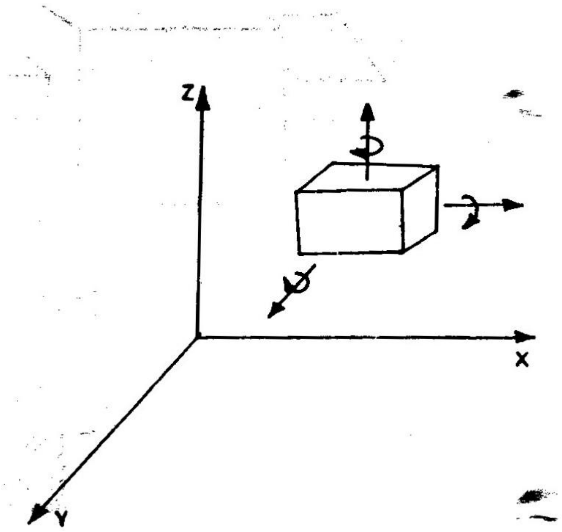
Parallel robot

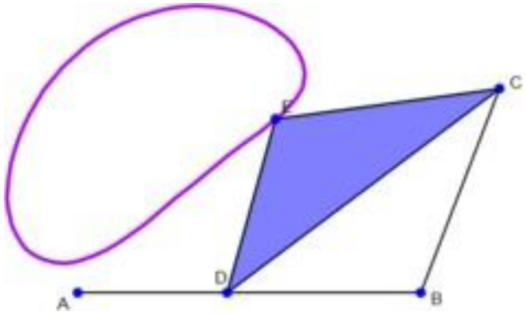
Machine

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a **machine**. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

DEGREES OF FREEDOM (DOF) / MOBILITY

It is the number of **independent coordinates** required to describe the **position of a body**.

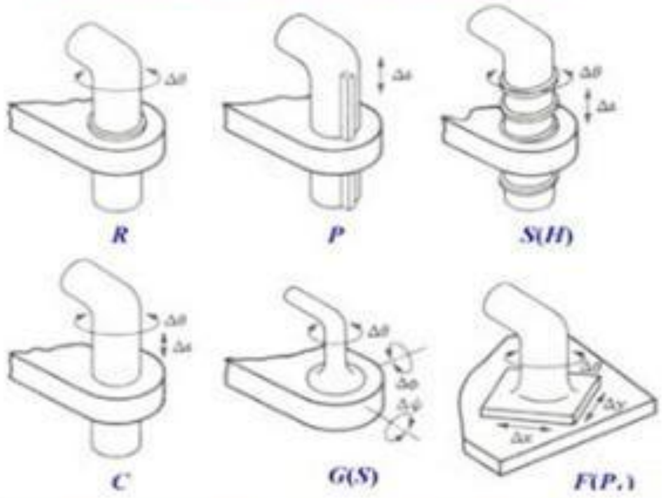




4 bar Mechanism has 1 DoF as the angle turned by the crank AD is fully describing the position of the every link of the mechanism

DOF

The Lower Pairs Joints



Pair	Symbol	Pair Variable	Degree of Freedom	Relative Motion
Revolute	R	$\Delta\theta$	1	Circular
Prism	P	Δs	1	Rectilinear
Screw	$S(H)$	$\Delta\theta$ or Δs ($\Delta s = h\Delta\theta$)	1	Helical
Cylinder	C	$\Delta\theta$ and Δs	2	Cylindric
Sphere	$G(S)$	$\Delta\theta, \Delta\phi, \Delta\psi$	3	Spheric
Flat	$F(P_L)$	$\Delta x, \Delta y, \Delta\theta$	3	Planar

DEGREES OF FREEDOM/MOBILITY OF A MECHANISM

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

KUTZBACH CRITERION

For mechanism having plane motion

$$\text{DoF} = n = 3(l - 1) - 2j - h$$

l = number of links

j = number of binary joints or lower pairs (1 DoF pairs)

h = number of higher pairs (i.e. *2 DoF pairs)*

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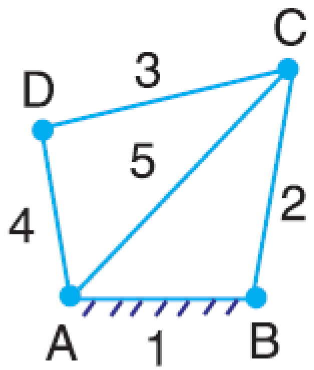
NUMERICAL EXAMPLE -1 &2

Determine the DoF of the mechanism shown below:

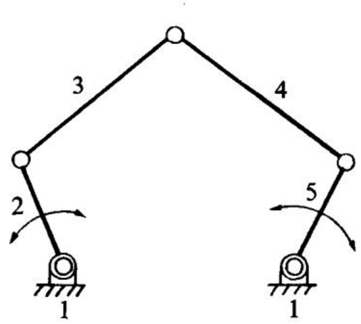


$$n = 3\,(l - 1) - 2\,j - h$$ **Kutzbach Criterion**

l = 5 ; j = 2+ 2* (3-1) = 6 ; h = 0

$$n = 3\,(5 - 1) - 2 \times 6 = 0$$

DoF = 0, means that the mechanism forms a structure



l = 5 ; j = 5; h = 0

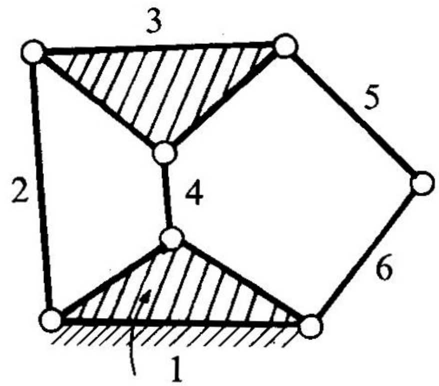
n = 3 (5-1) – 2*5 – 0 = 2

Two inputs to any two links are required to yield definite motions in all the links.

NUMERICAL EXAMPLE -3 &4

Determine the Dof for the links shown below:

$n = 3\,(l - 1) - 2\,j - h$ **Kutzbach Criterion**

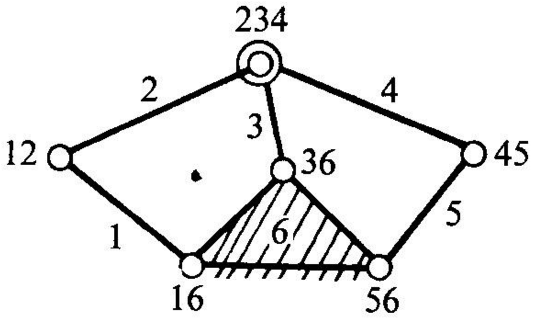


l = 6 ; j = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

i.e., one input to any one link will result in definite motion of all the links.



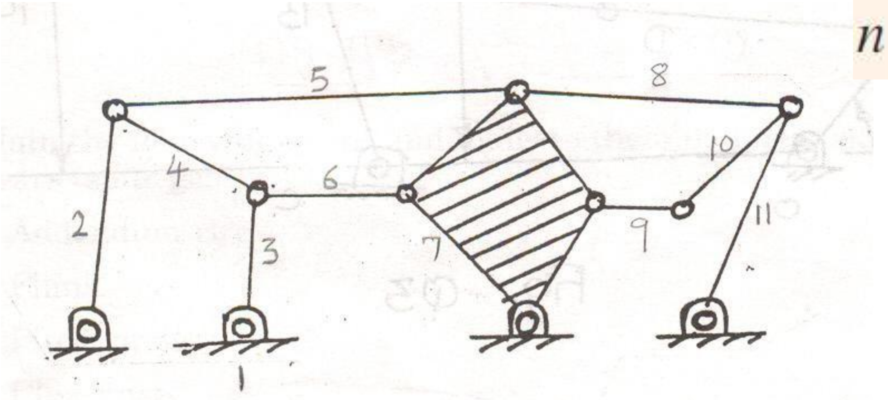
Note: at the intersection of 2, 3 and 4, two lower pairs are to be considered

l = 6 ; j = 5 + 1 (3-1) = 7 ; h = 0

n = 3 (6-1) – 2 (7) – 0 = 1

Dof = 1

NUMERICAL EXAMPLE - 5



$n = 3\,(l - 1) - 2\,j - h$

Kutzbach Criterion

l = 11 ; j = 7 + 4 (3-1) = 15 ; h = 0

n = 3 (11-1) – 2 (15) – 0 = 0

Dof = 0

Here, j= 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.

Summary

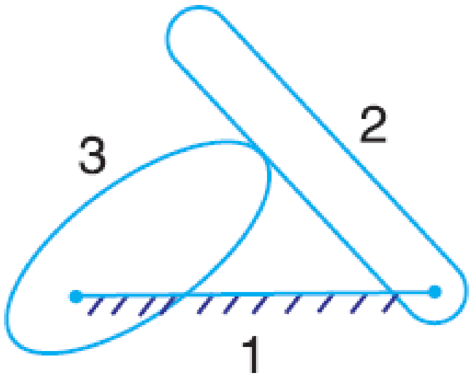
Dof = 0, Structure

Dof = 1, mechanism can be driven by a single input motion

Dof = 2, two separate input motions are necessary to produce constrained motion for the mechanism

Dof = -1 or less, redundant constraints in the chain and it forms a statically indeterminate structure

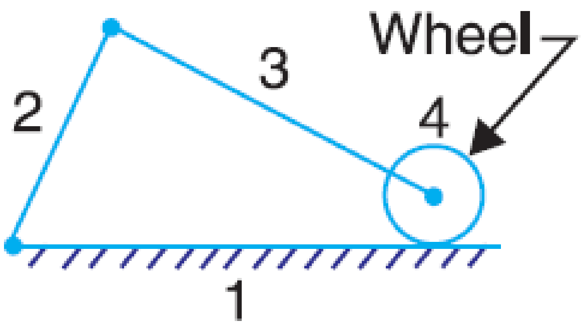
KUTZBACH CRITERION FOR HIGHER PAIRS



$l = 3, j = 2$ and $h = 1$

$n = 3\ (3 - 1) - 2 \times 2 - 1 = 1$

$n = 3\ (l - 1) - 2\ j - h$

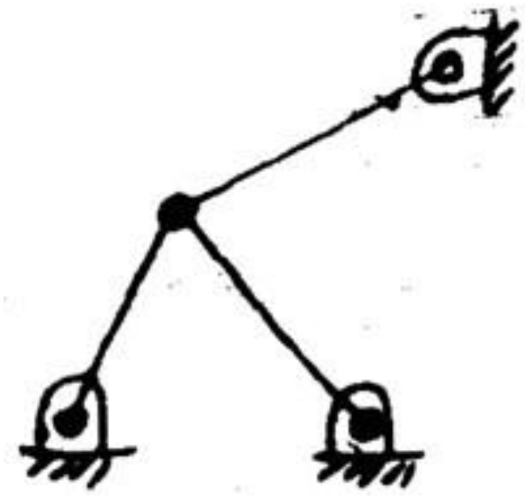


$l = 4,\ j = 3$ and $h = 1$

$n = 3\ (4 - 1) - 2 \times 3 - 1 = 2$

KUTZBACH CRITERION

$$n = 3\,(l - 1) - 2\,j - h$$



l = 4, j = 5, h=0

n = 3 (4-1) – 2 (5) – 0 = -1

Indeterminate structure

l = 3, j = 2, h=1

n = 3 (3-1) – 2 (2) – 1 = 1

GRUBLER'S CRITERION FOR PLANE MECHANISMS

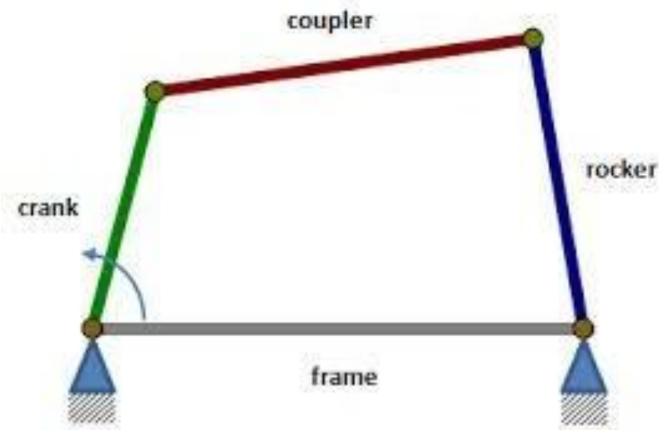
Kutzbach Criterion $n = 3\,(l - 1) - 2\,j - h$

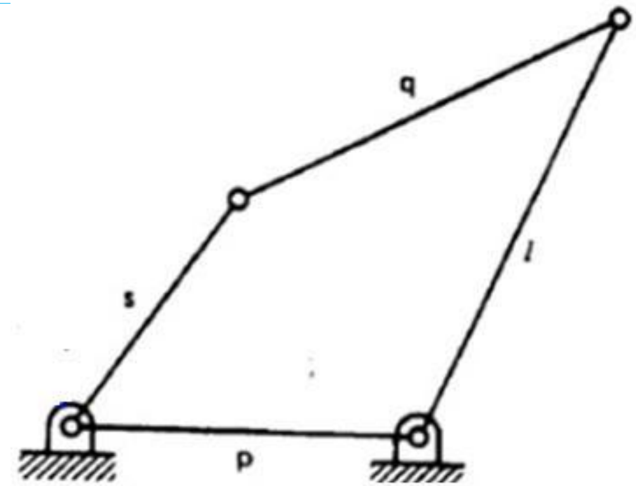
Grubler's criterion applies to mechanisms having 1 DoF.

Substituting n = 1 and h=0 in Kutzbatch equation, we can have Grubler's equation.

$$1 = 3\,(l - 1) - 2\,j \qquad \text{or} \qquad 3l - 2j - 4 = 0$$

GRASHOF’S LAW

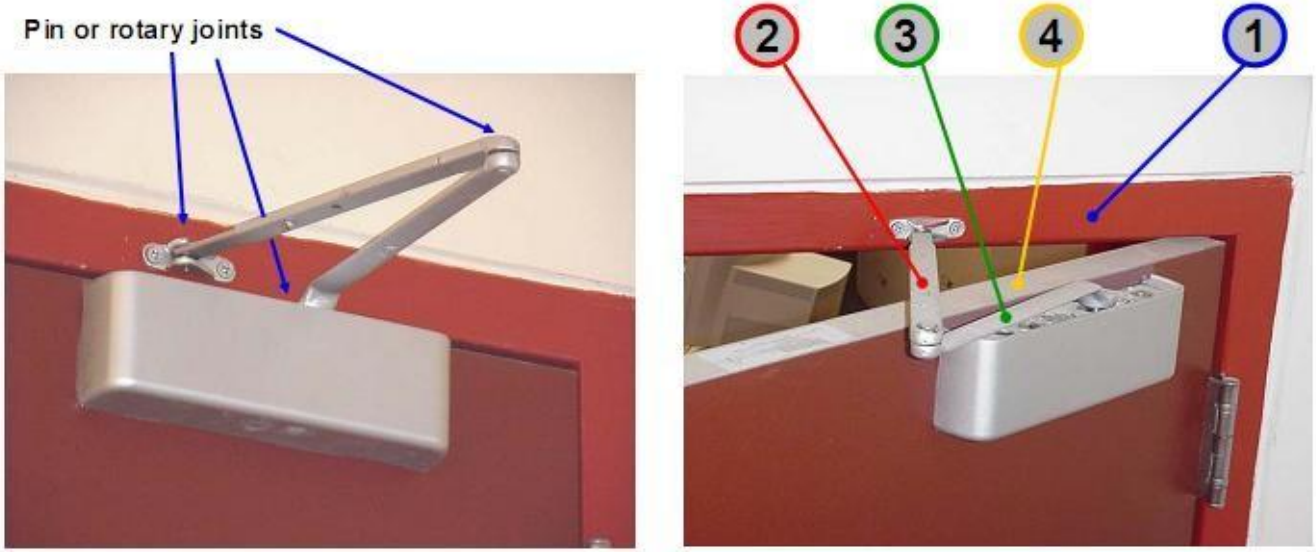




$$l + s \leq p + q$$

According to **Grashof ’s law for a four bar mechanism,** the sum of the shortest and longest link lengths <u>should not be greater than</u> the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

Example: 4 bar door damper linkage



1	= Wall	or	Link 1	This is the grounded (held still)
2	= Bar 2	or	Link 2	
3	= Bar 3	or	Link 3	
4	= Door	or	Link 4	

INVERSIONS OF MECHANISM

- A mechanism is one in which one of the links of a kinematic chain is fixed.

- Different mechanisms can be obtained by fixing different links of the same kinematic chain.

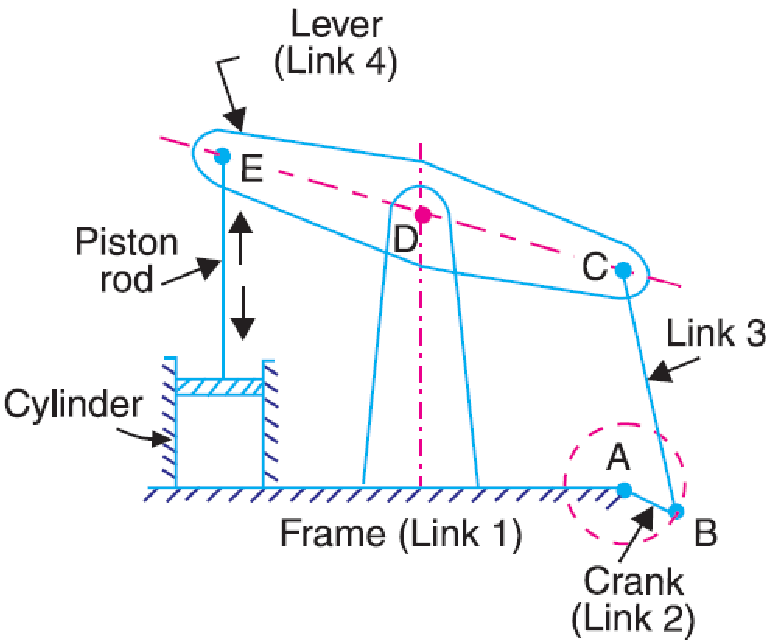
- It is known as inversions of the mechanism.

INVERSIONS OF FOUR BAR CHAIN

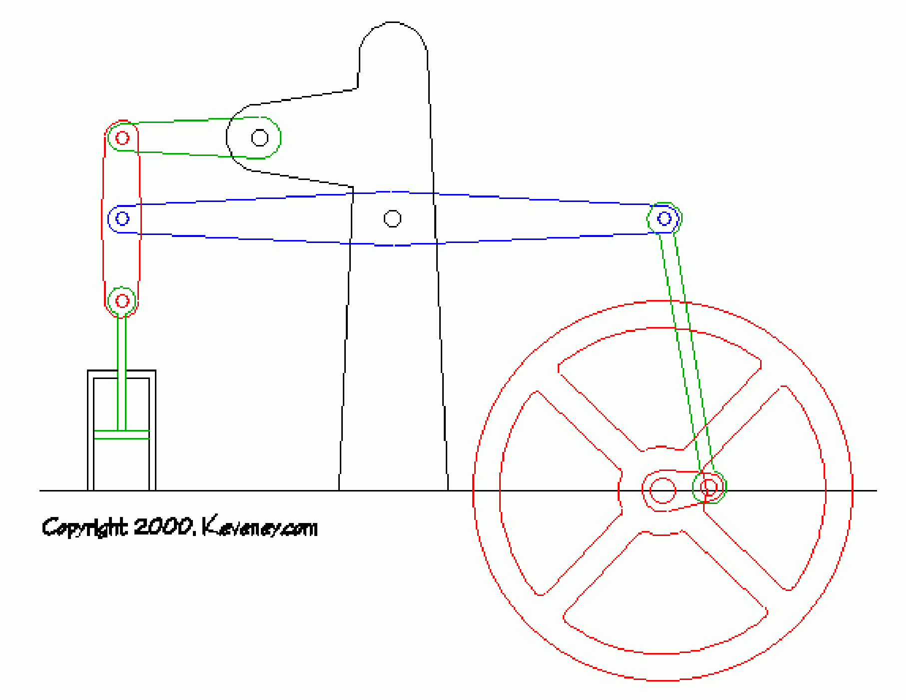
- Beam engine (crank and lever mechanism)
- Coupling rod of a locomotive (Double crank mechanism)
- Watt's indicator mechanism (Double lever mechanism)

INVERSIONS OF FOUR BAR CHAIN

1. Beam engine (crank and lever mechanism)



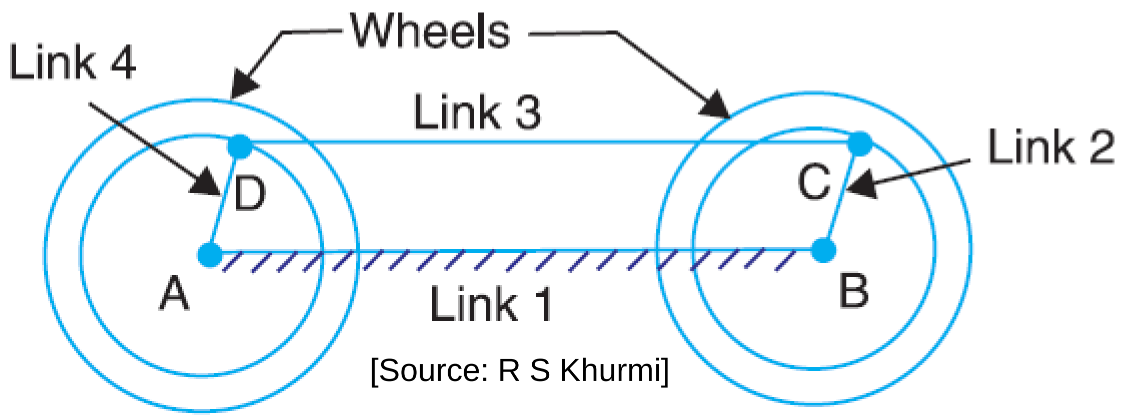
Beam engine.
[Source: R S Khurmi]



The purpose of this mechanism is to convert rotary motion into reciprocating motion.

INVERSIONS OF FOUR BAR CHAIN

2. *Coupling rod of a locomotive (Double crank mechanism).*

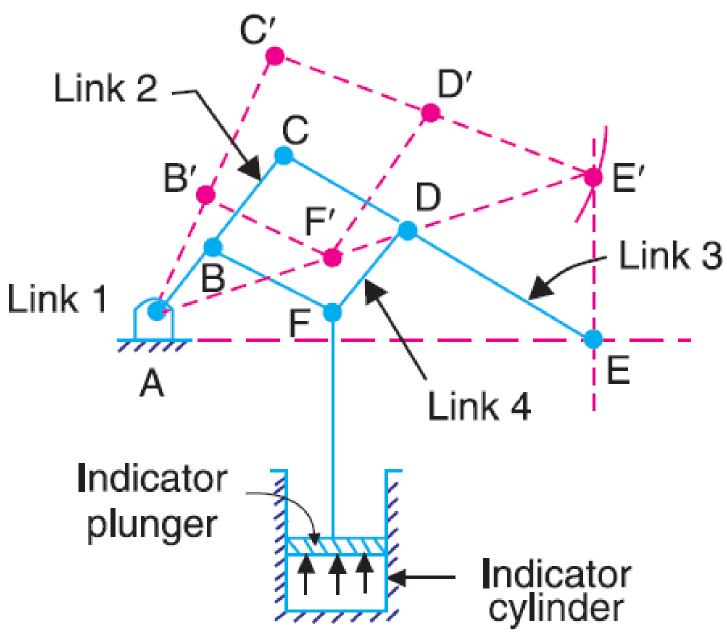


[Source: R S Khurmi]

- ➢links *AD and BC (having equal length) act as cranks and are connected* to the respective wheels.
- ➢The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them.
- ➢This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

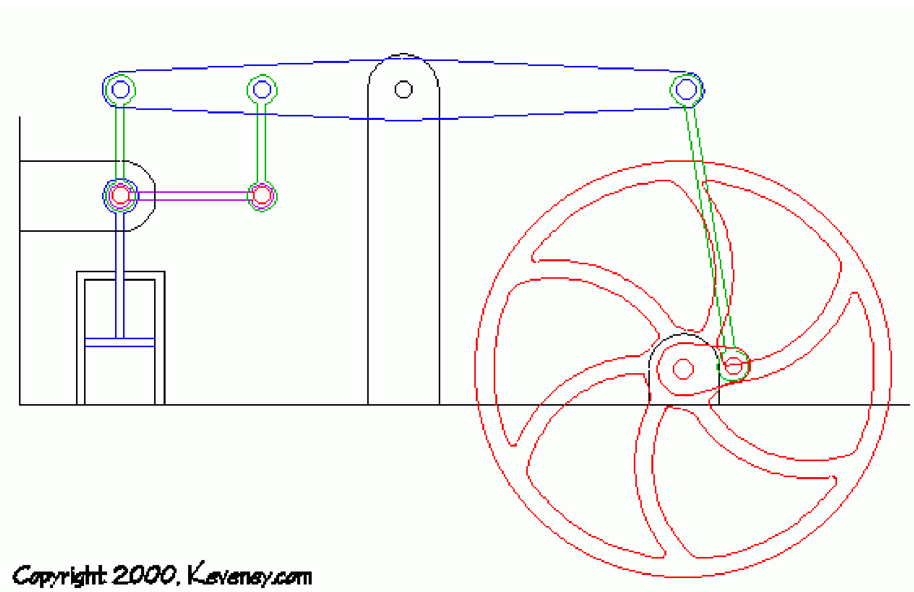
INVERSIONS OF FOUR BAR CHAIN

3.Watt's indicator mechanism (Double lever mechanism)



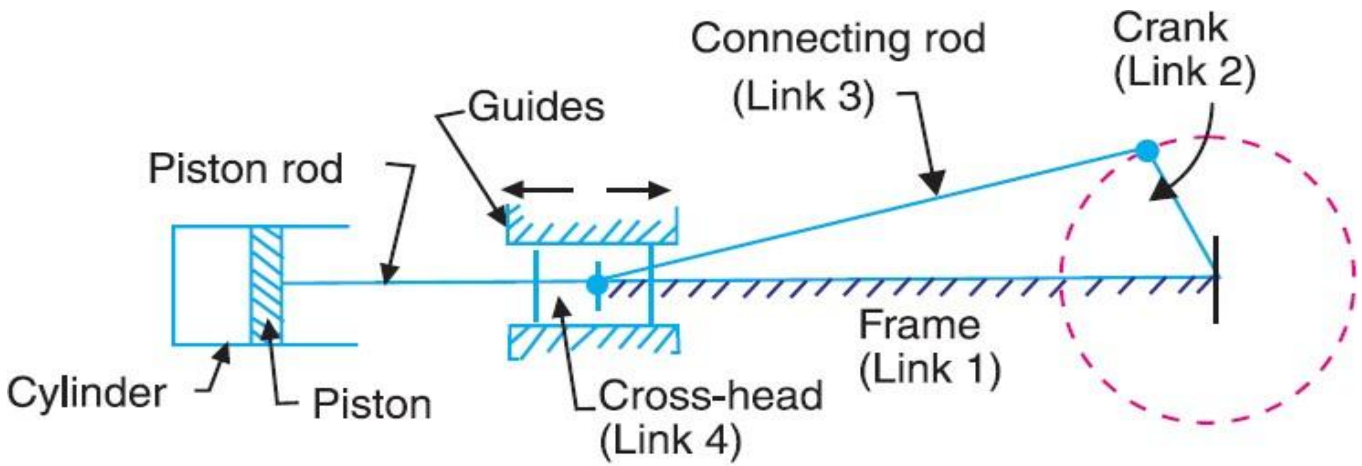
Watt's indicator mechanism.

[Source: R S Khurmi]



On any small displacement of the mechanism, the tracing point *E at the end of the link CE* traces out approximately a straight line

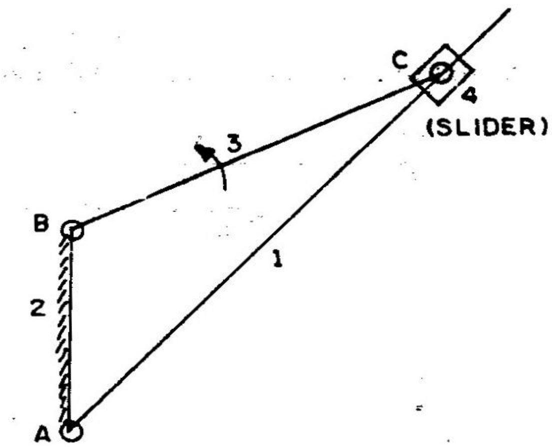
SINGLE SLIDER CRANK CHAIN



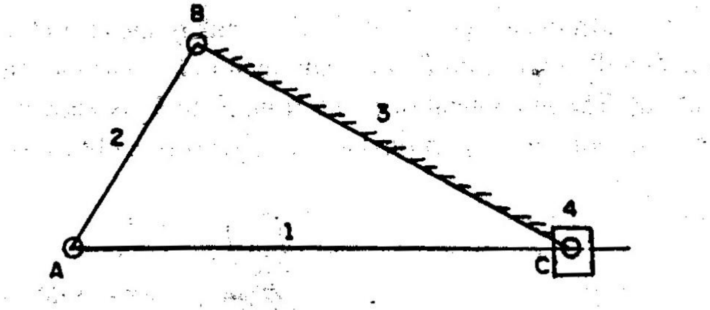
Single slider crank chain

Links 1-2, 2-3, 3-4 = Turning pairs; Link 4-1 = Sliding pair

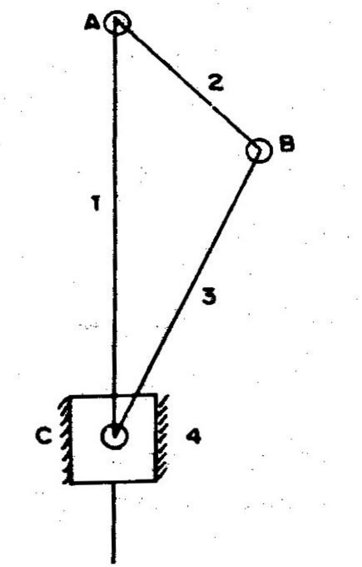
INVERSIONS OF SINGLE SLIDER CRANK CHAIN



crank fixed



connecting rod fixed

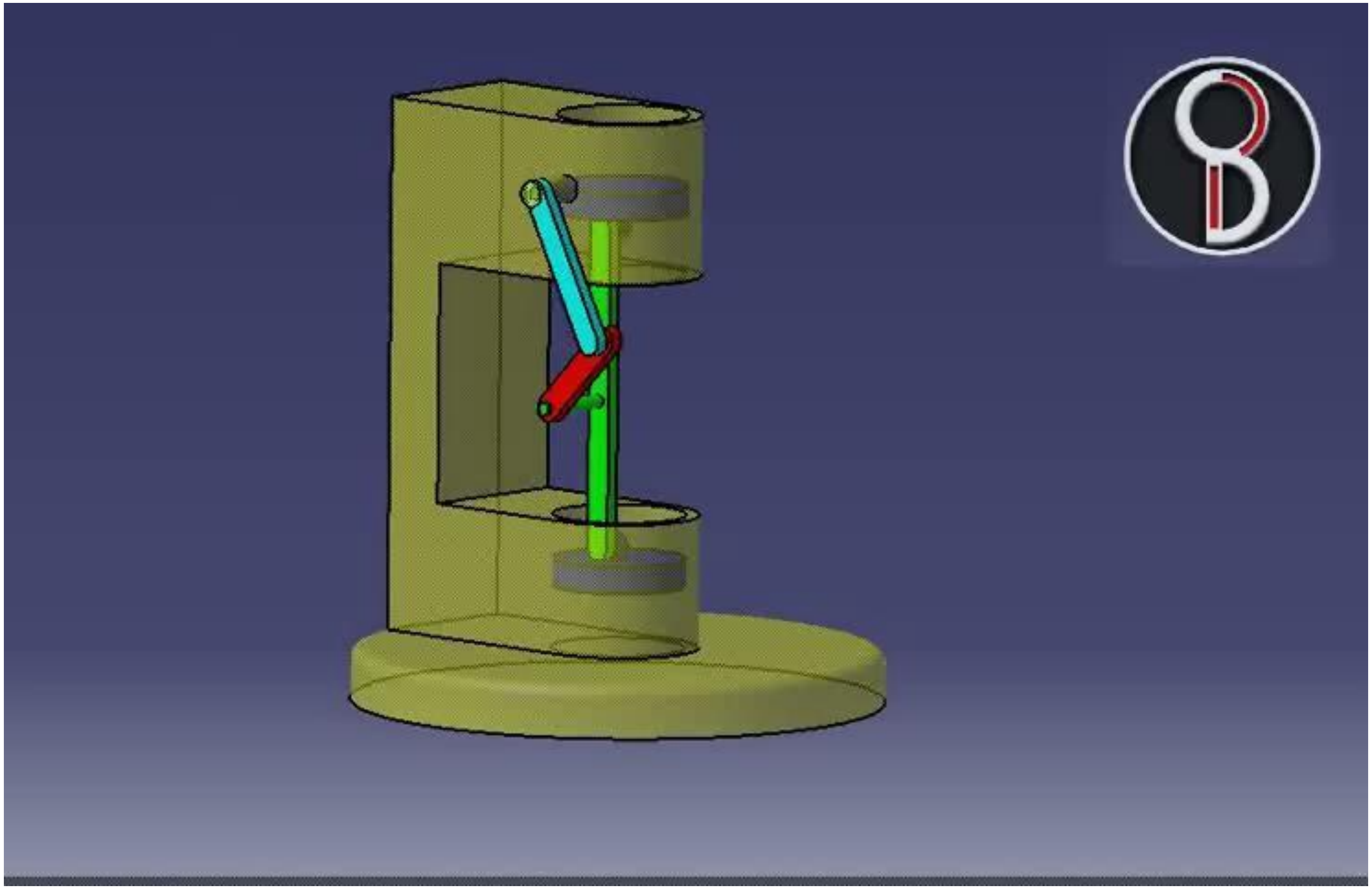


slider fixed

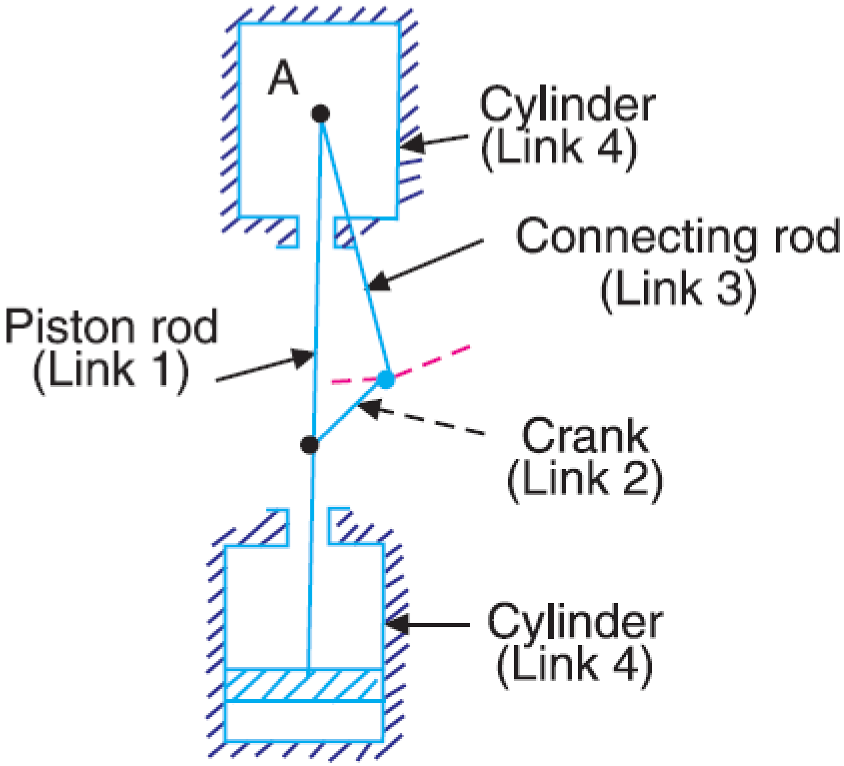
INVERSIONS OF SINGLE SLIDER CRANK CHAIN

- Pendulum pump or Bull engine
- Oscillating cylinder engine
- Rotary internal combustion engine (or) Gnome engine
- Crank and slotted lever quick return motion mechanism
- Whitworth quick return motion mechanism

PENDULUM PUMP OR BULL ENGINE



PENDULUM PUMP OR BULL ENGINE



[Source: R S Khurmi]

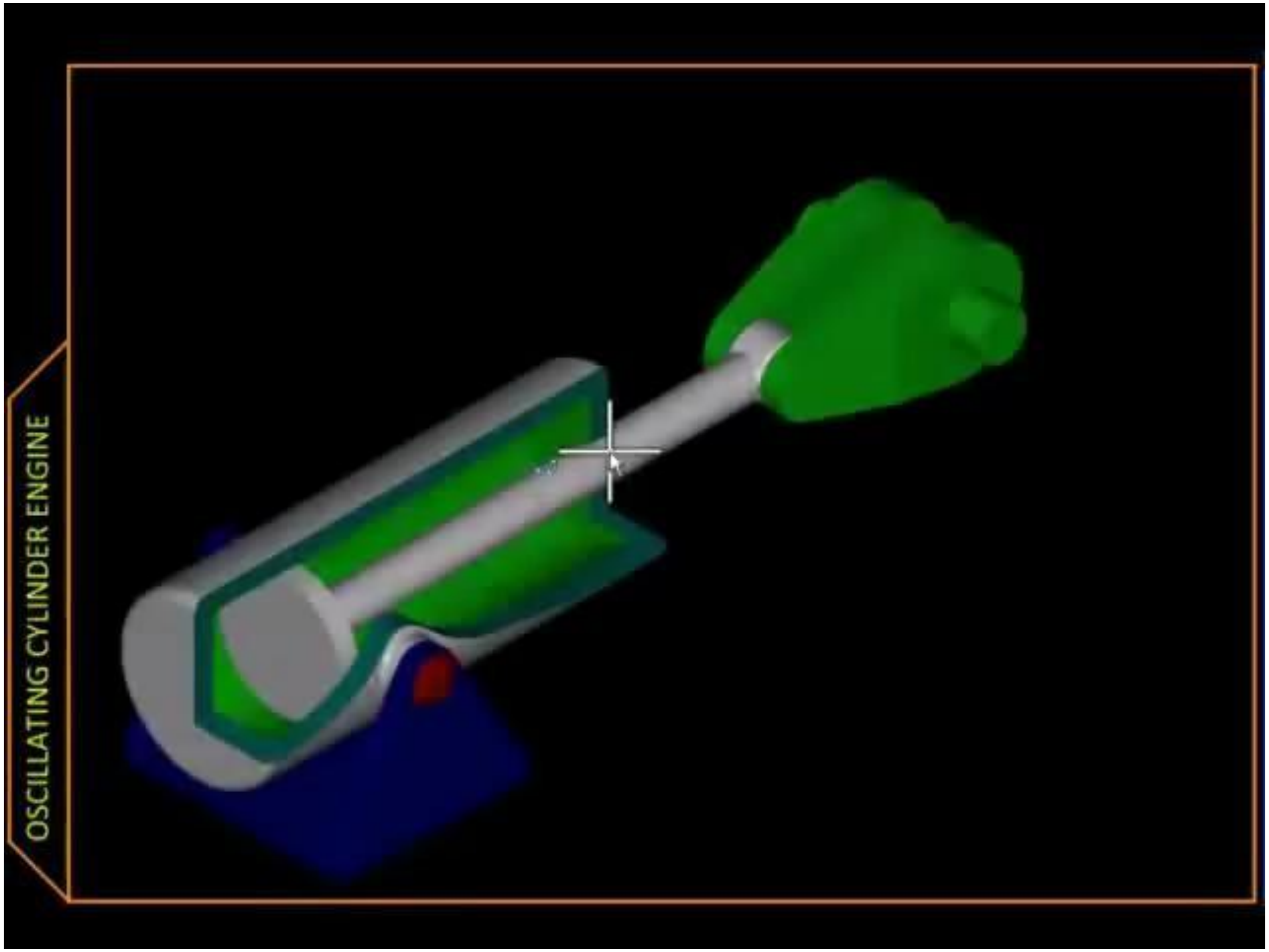
This inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair)

when the crank (link 2) rocks, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A.

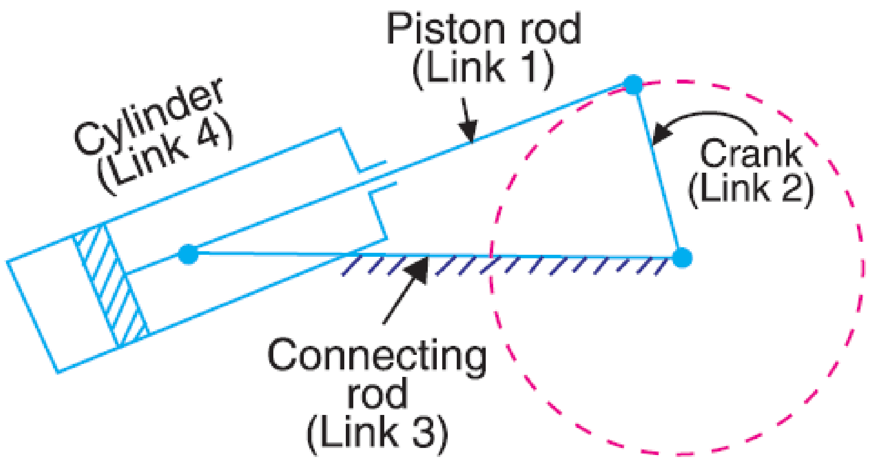
The piston attached to the piston rod (link 1) reciprocates.

It supplies water to a boiler.

OSCILLATING CYLINDER ENGINE



OSCILLATING CYLINDER ENGINE

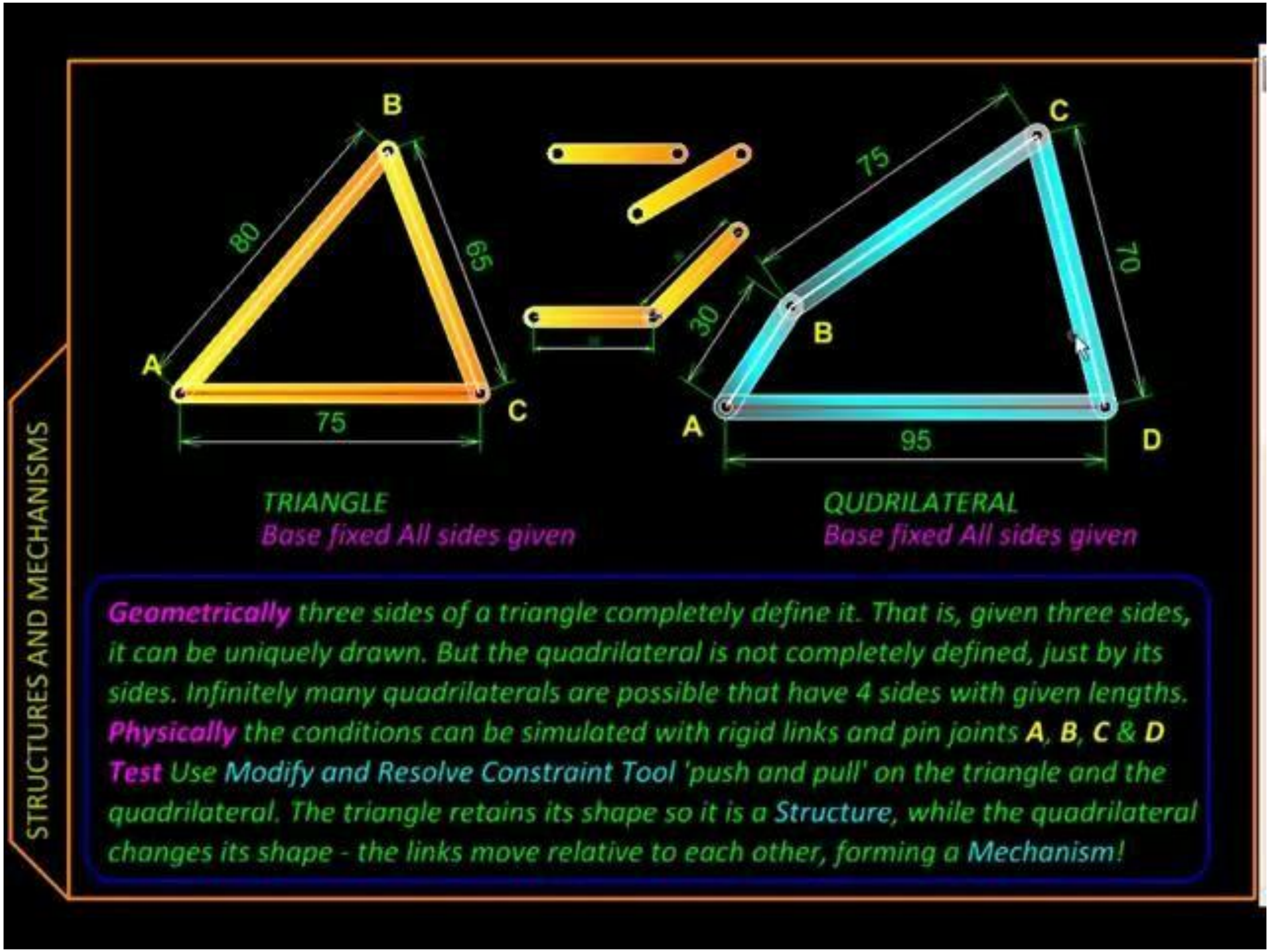


[Source: R S Khurmi]

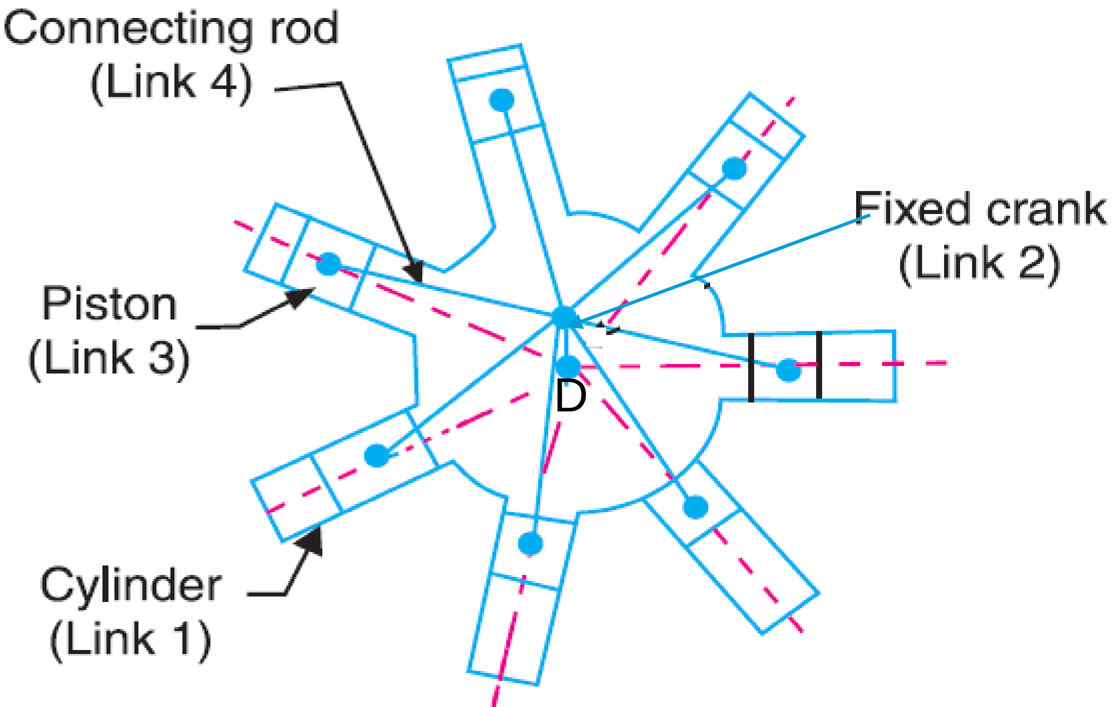
- used to convert reciprocating motion into rotary motion

- the link 3 (Connecting Rod) forming the turning pair is fixed.

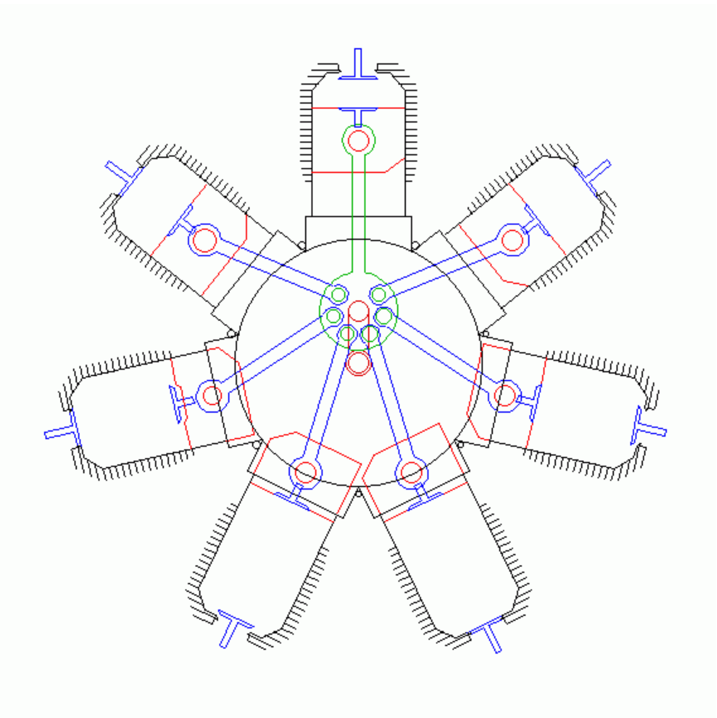
MULTI-CYLINDER RADIAL IC ENGINE



ROTARY INTERNAL COMBUSTION ENGINE (OR) GNOME ENGINE

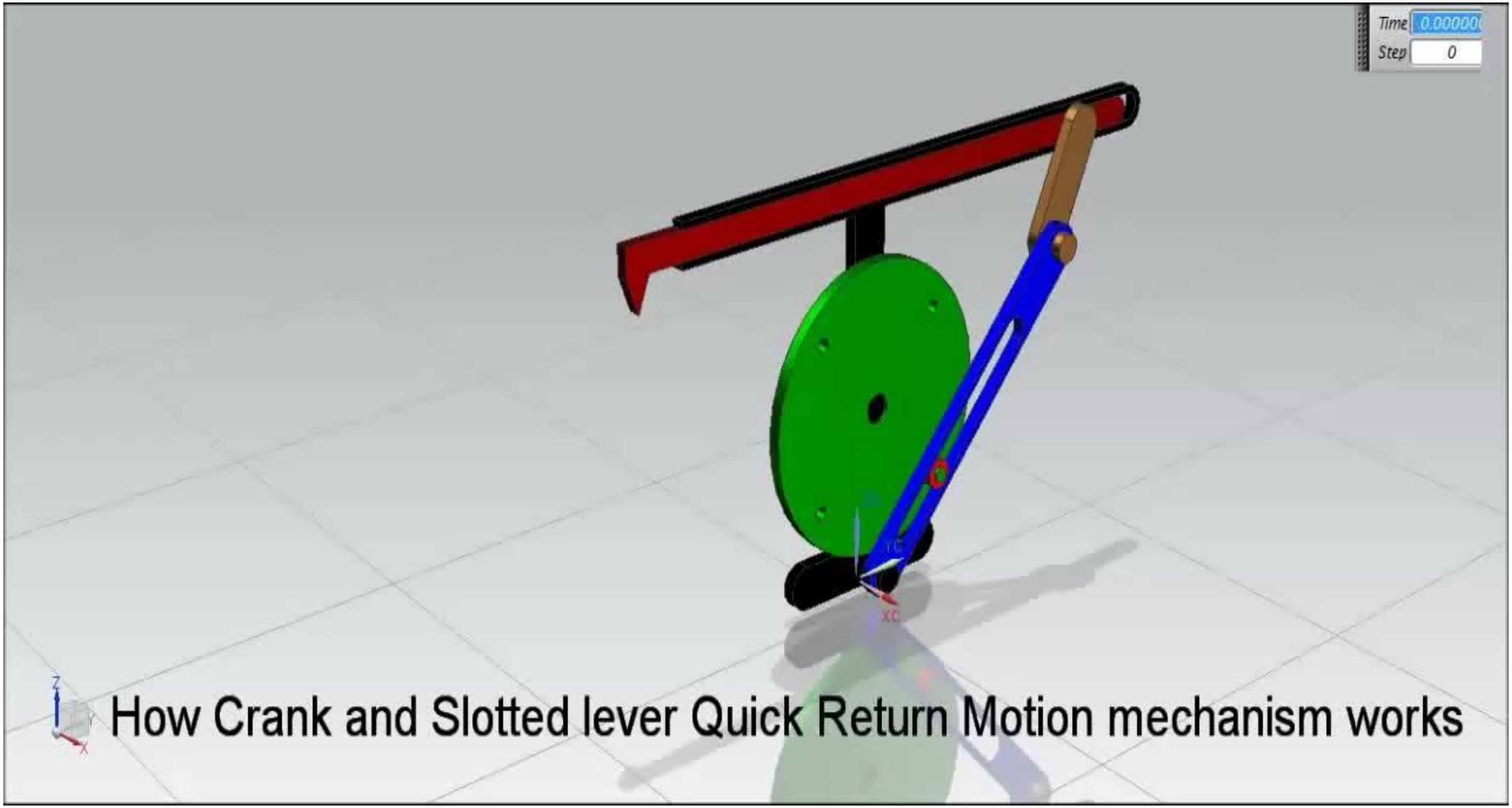


[Source: R S Khurmi]

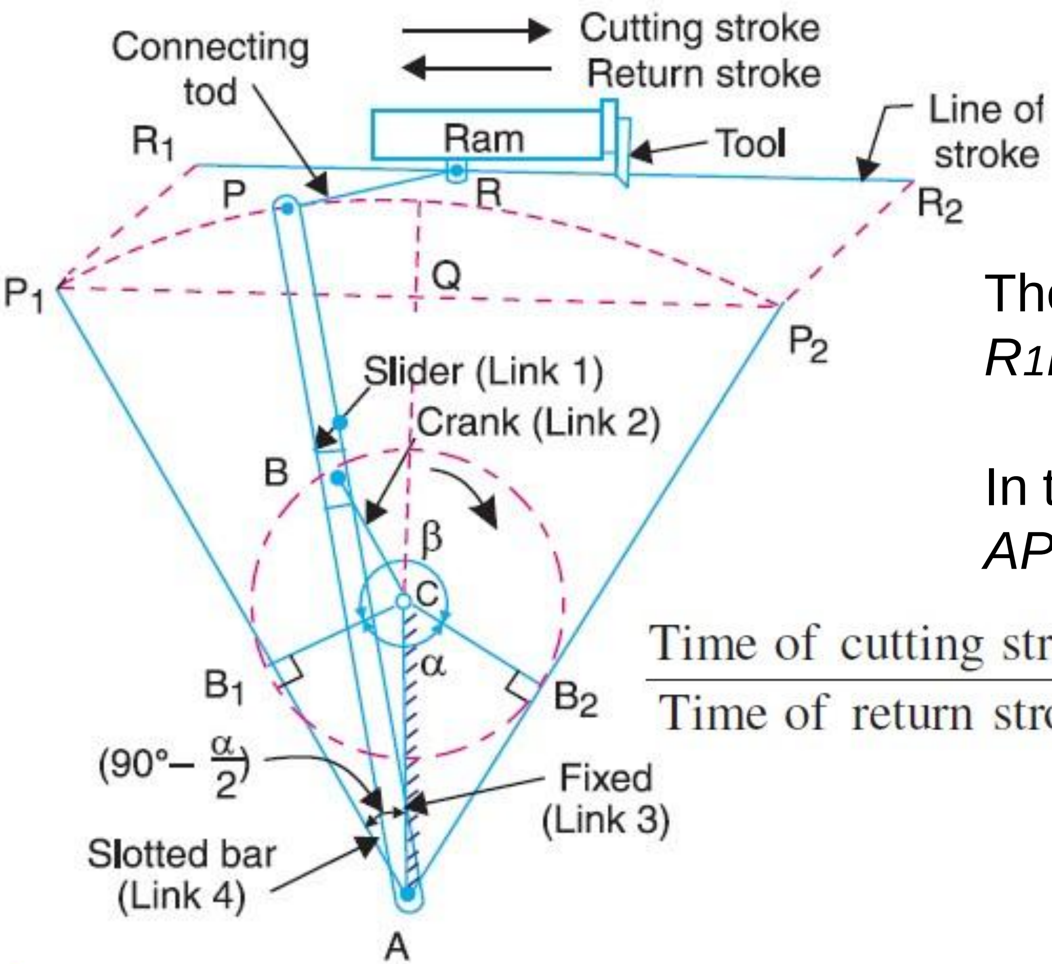


- Crank is fixed at center D
- Cylinder reciprocates
- Engine rotates in the same plane

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



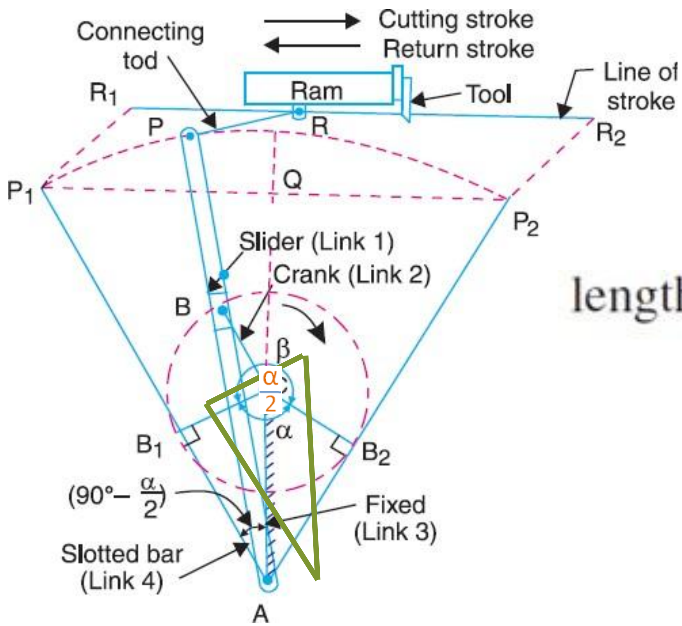
[Source: R S Khurmi]

The line of stroke of the ram (*i.e.* R_1R_2) is perpendicular to AC

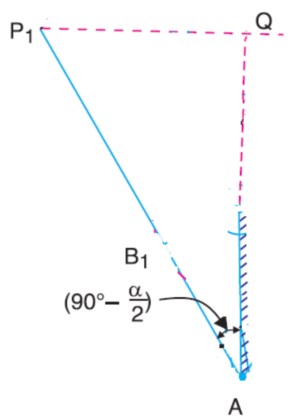
In the extreme positions, AP_1 and AP_2 are tangential to the circle

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

CRANK AND SLOTTED LEVER QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



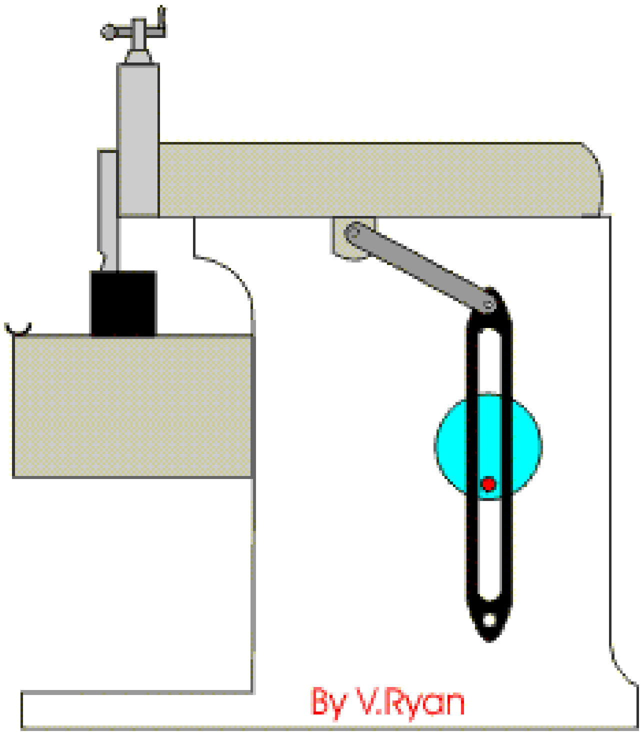
$$\text{length of stroke} = R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \quad \text{.....} (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \quad \text{...} \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

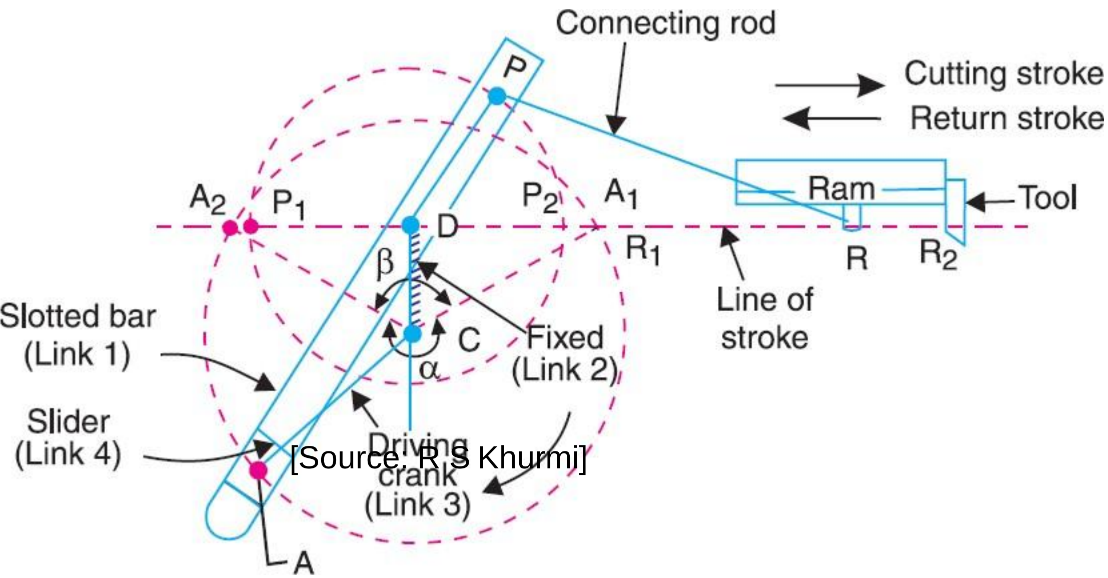
$$= 2AP \times \frac{CB}{AC} \quad \text{...} (\because CB_1 = CB)$$

Crank and slotted lever quick return mechanism is mostly used in shaping machines & slotting machines

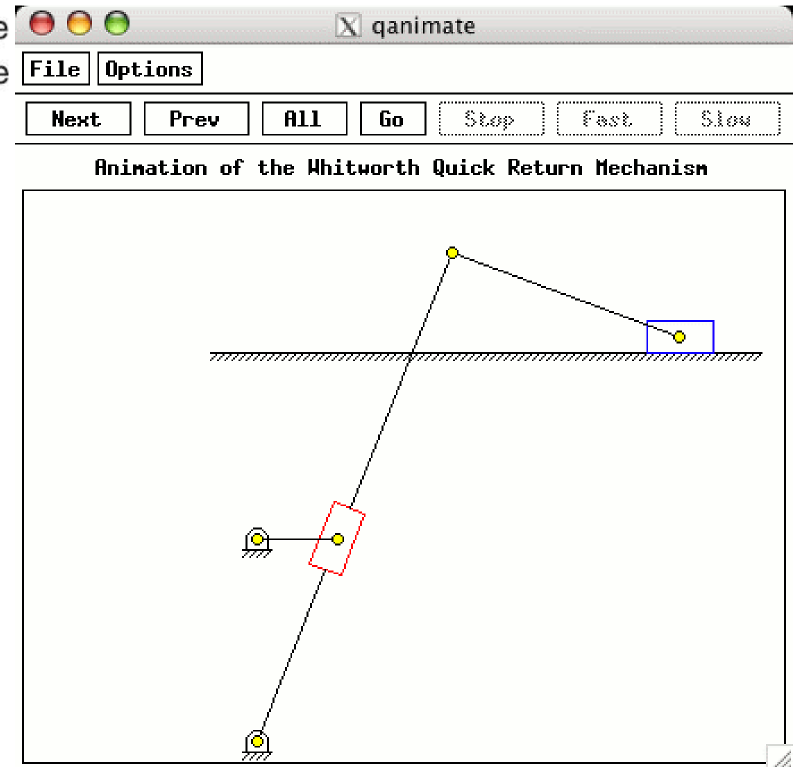


THE SHAPING MACHINE

WHITWORTH QUICK RETURN MOTION MECHANISM



[Source: R S Khurmi]



$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \quad \text{or} \quad \frac{360^\circ - \beta}{\beta}$$

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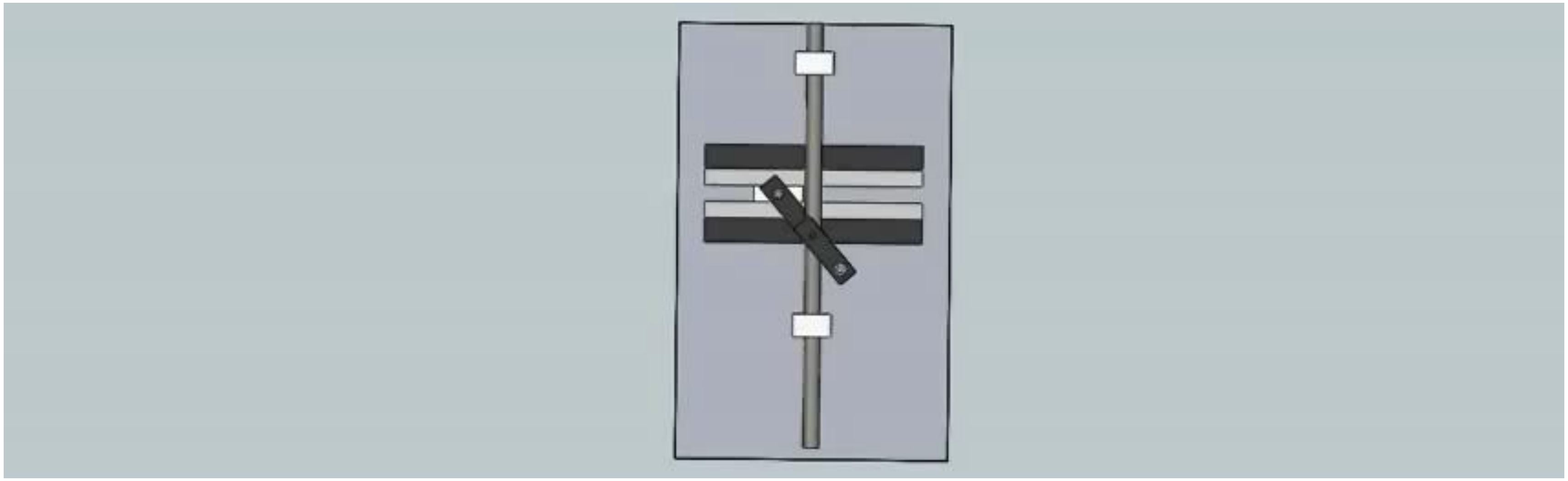
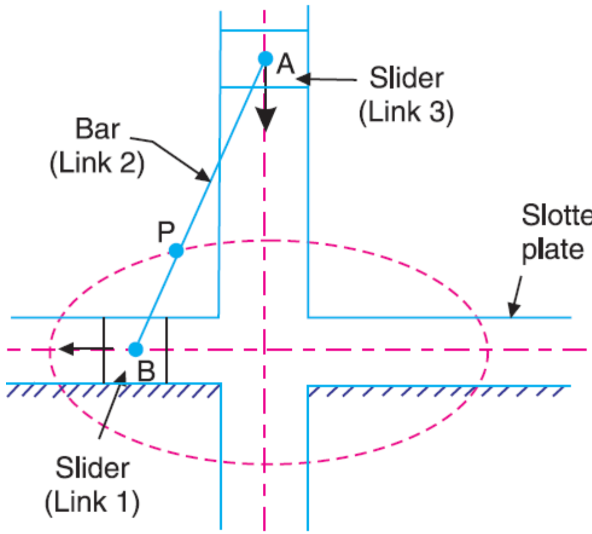
INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

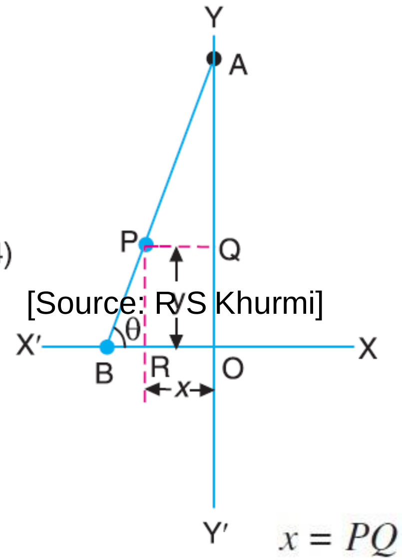
(1. ELLIPTICAL TRAMMELS)



INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(1. ELLIPTICAL TRAMMELS)





[Source: R/S Khurmi]

- used for drawing ellipses
- any point on the link 2 such as P traces out an ellipse on the surface of link 4
- AP - semi-major axis;
- BP - semi-minor axis

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

or

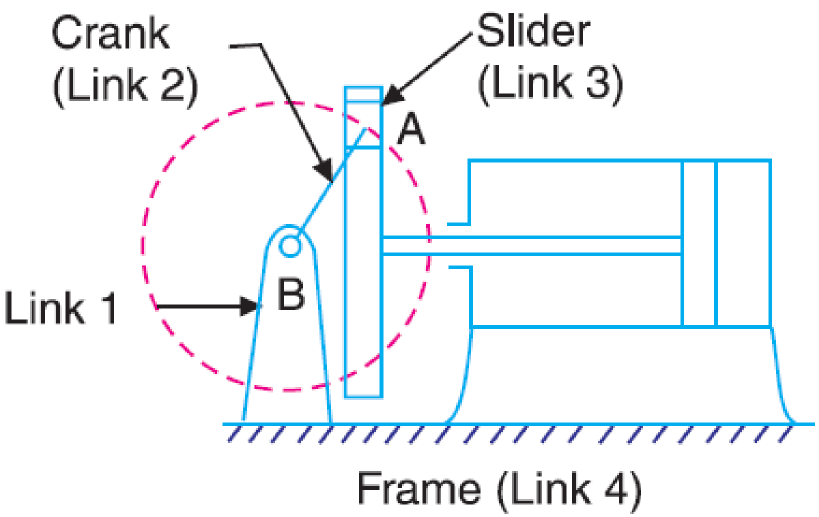
$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(2. SCOTCH YOKE MECHANISM)



Scotch yoke mechanism.

[Source: R S Khurmi]

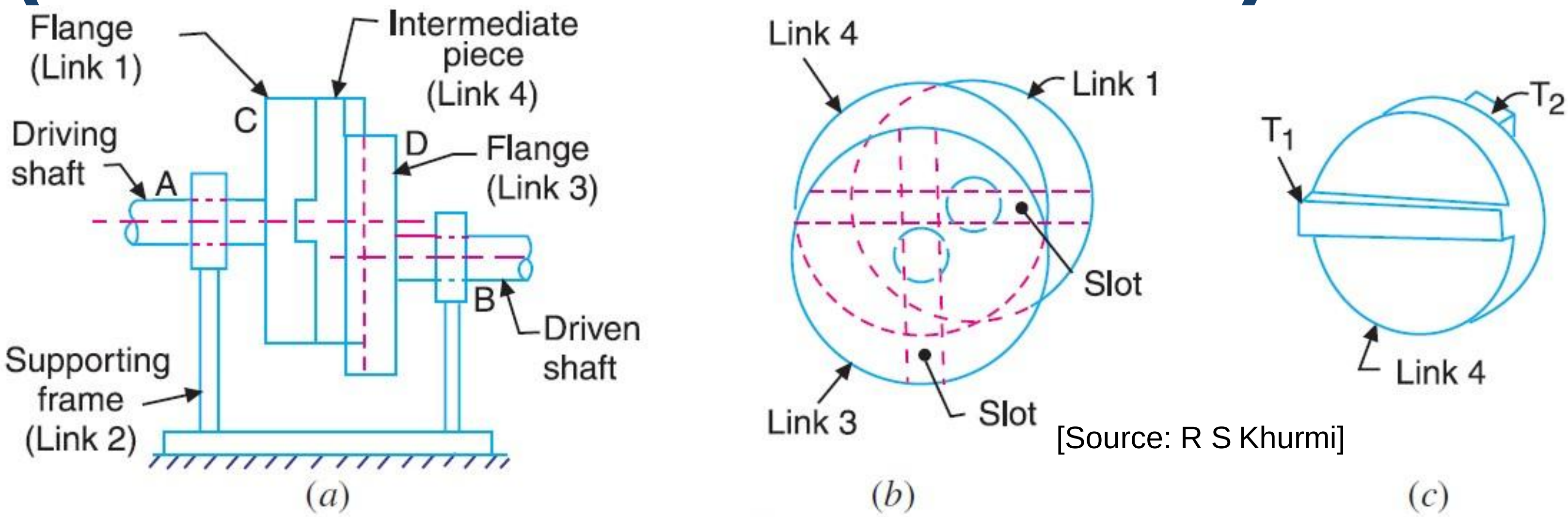
- This mechanism is used for converting rotary motion into a reciprocating motion.
- Link 1 is fixed.
- when the link 2 (crank) rotates about B as centre, reciprocation motion taking place.

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM'S COUPLING)

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

(3. OLDHAM'S COUPLING)



[Source: R S Khurmi]

Oldham's coupling.

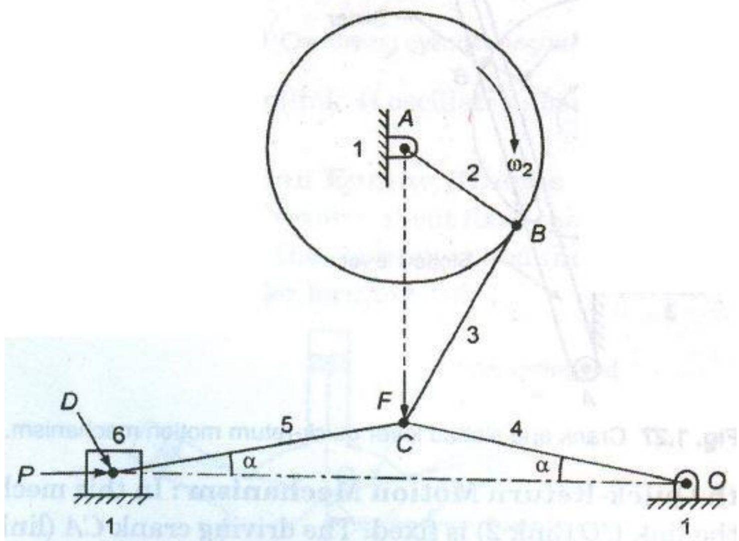
T_1 and T_2 two tongues (*i.e.* diametrical projections) on each face at right angles to each other

used for connecting two parallel shafts whose axes are at a small distance apart.

SOME COMMON MECHANISMS : TOGGLE MECHANISM

TOGGLE MECHANISM

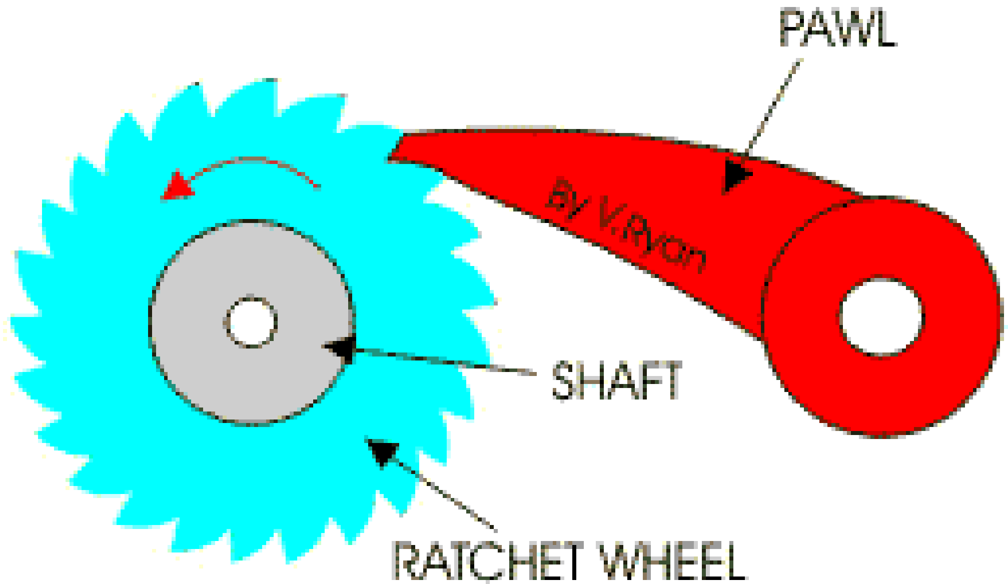
$$F = 2P \tan \alpha$$



- If α approaches to zero, for a given F, P approaches infinity.
- A <u>stone crusher</u> utilizes this mechanism to overcome a large resistance with a small force.
- It is used in numerous <u>toggle clamping devices</u> for holding work pieces.
- Other applications are: Clutches, Pneumatic riveters etc.,

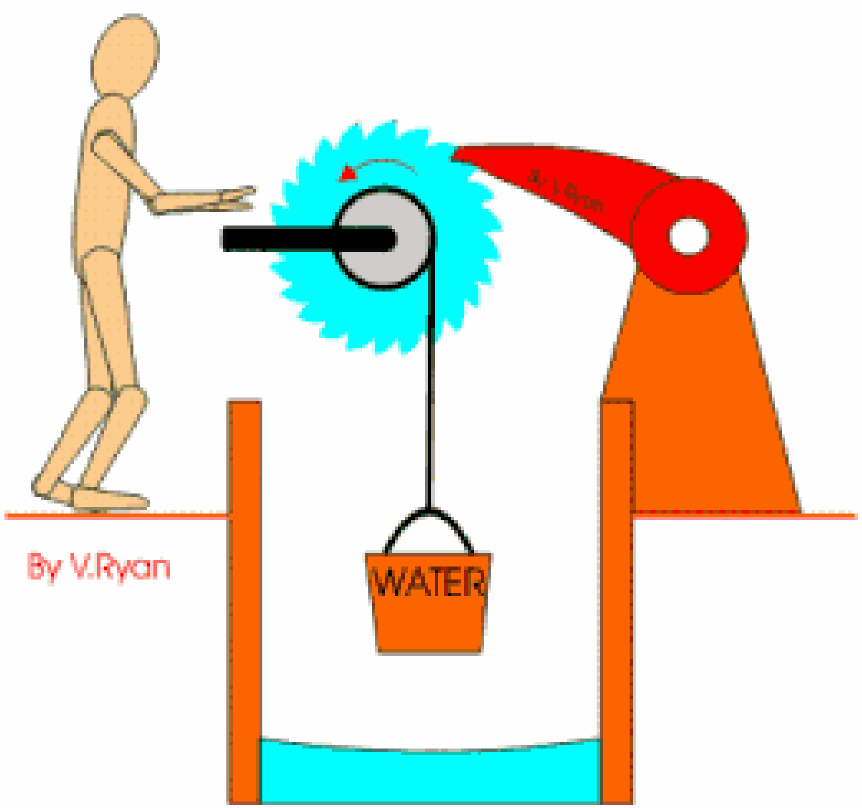
INTERMEDIATE MOTION MECHANISM

RATCHET AND PAWL MECH



- There are many different forms of ratchets and escapements which are used in:
- locks, jacks, clockwork, and other applications requiring some form of intermittent motion.

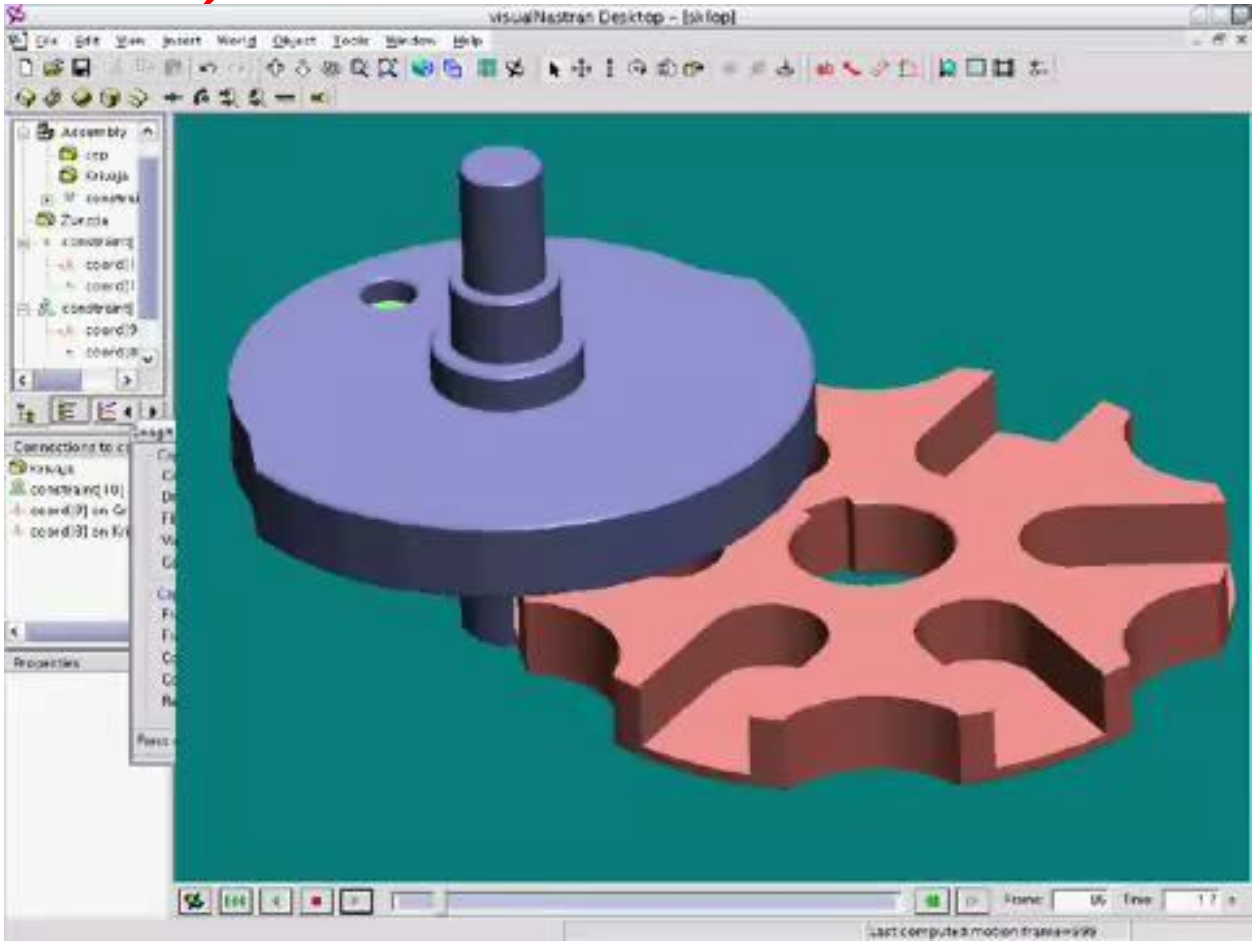
APPLICATION OF RATCHET PAWL MECHANISM



Used in Hoisting Machines as safety measure

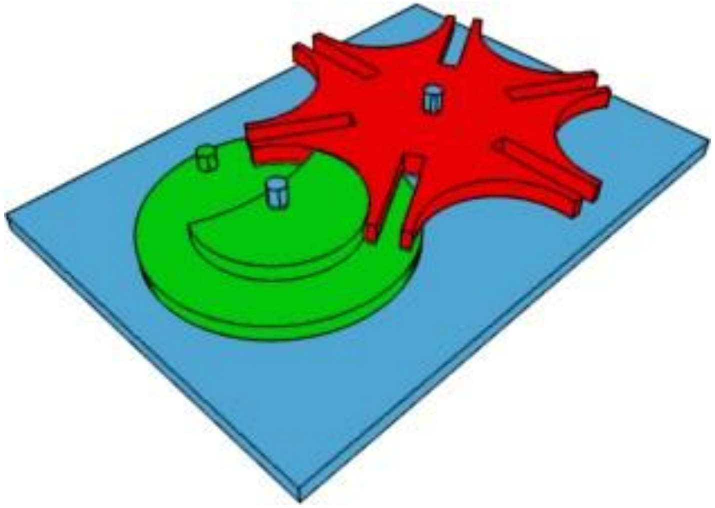
INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM (INDEXING MECHANISM)

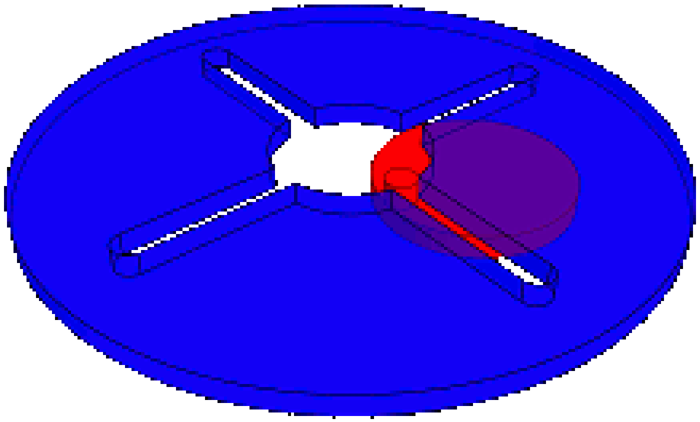


INTERMEDIATE MOTION MECHANISM

GENEVA MECHANISM

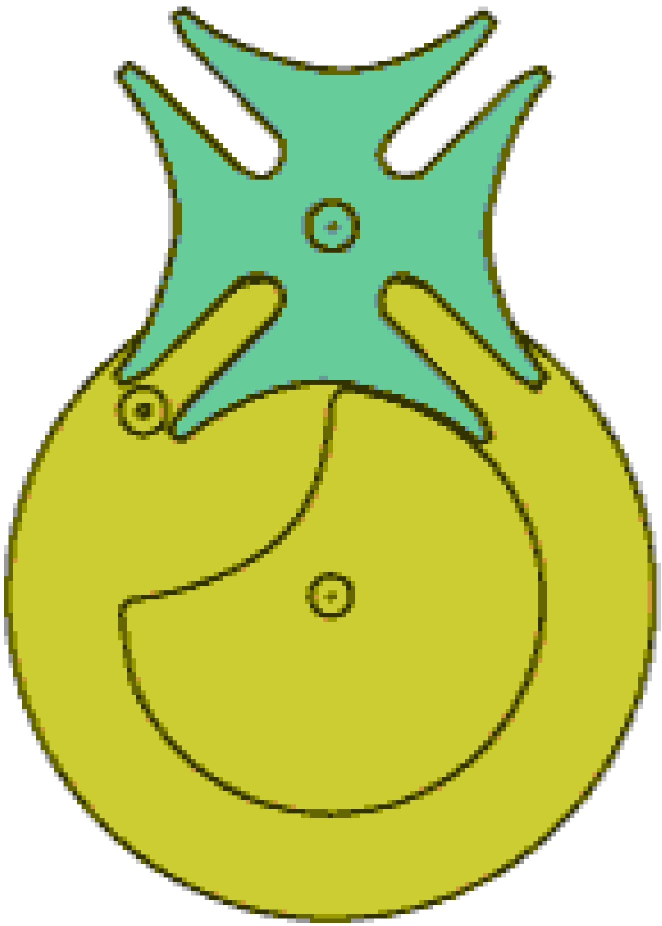


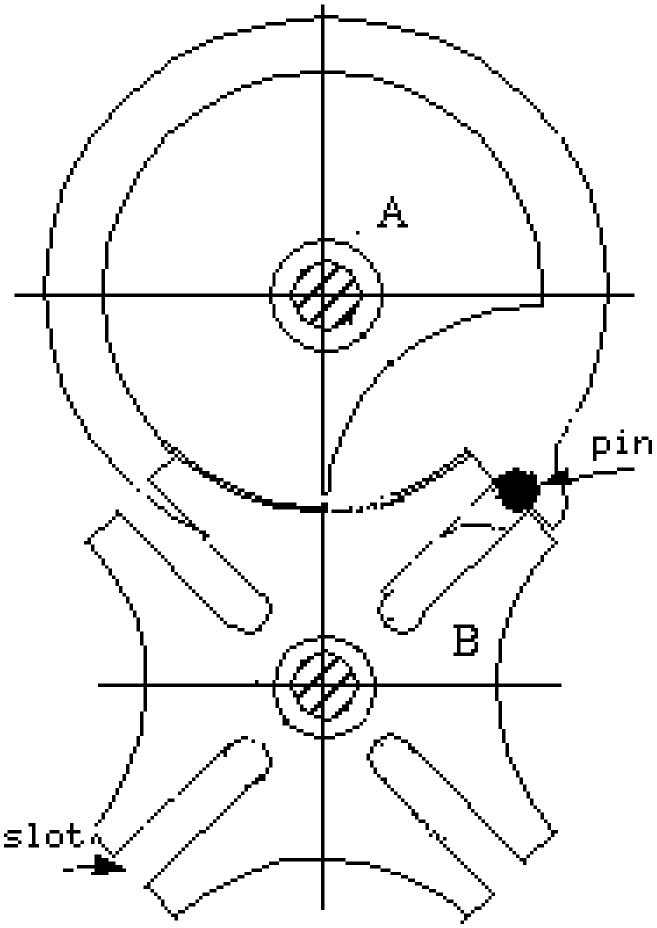
Animation showing a six-position external Geneva drive in operation



Animation showing an internal Geneva drive in operation.

INTERMITTENT MOTION MECHANISMS
GENEVA WHEEL MECHANISM





APPLICATIONS OF GENEVA MECHANISM

- Locating and locking mechanism
- Indexing system of a multi-spindle machine tool

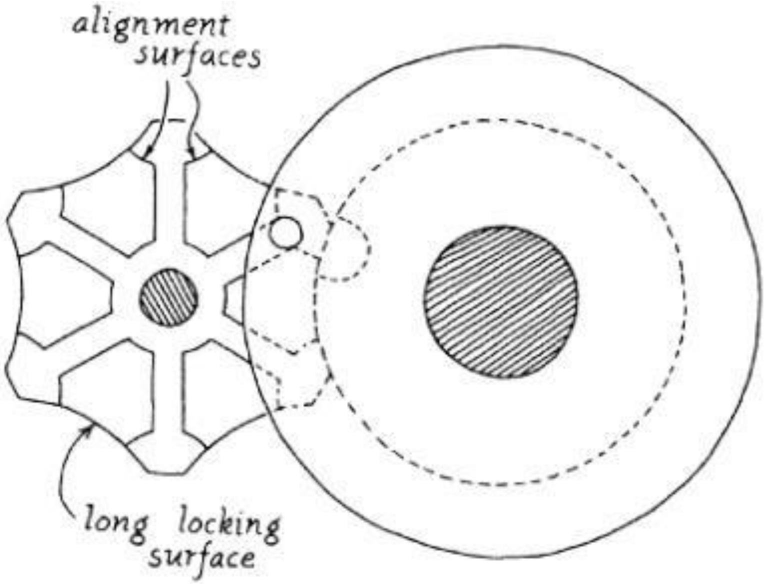
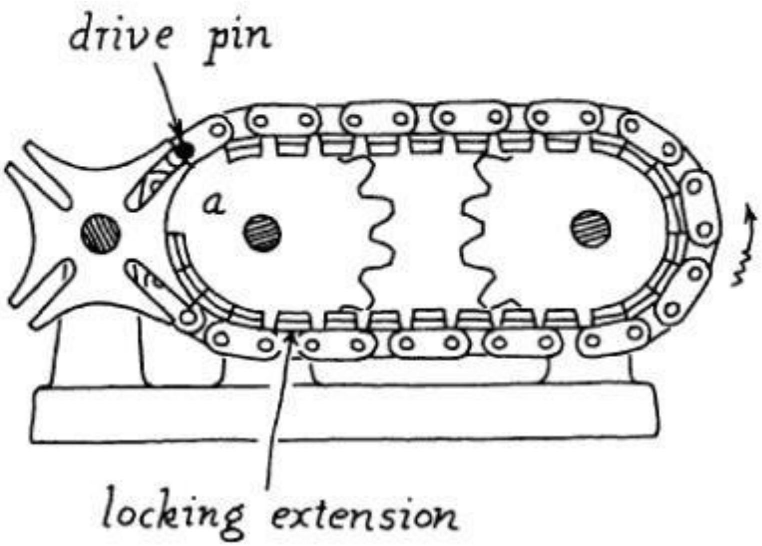


Fig. 9–15. Six-slot external Geneva used for light-duty instrument applications.

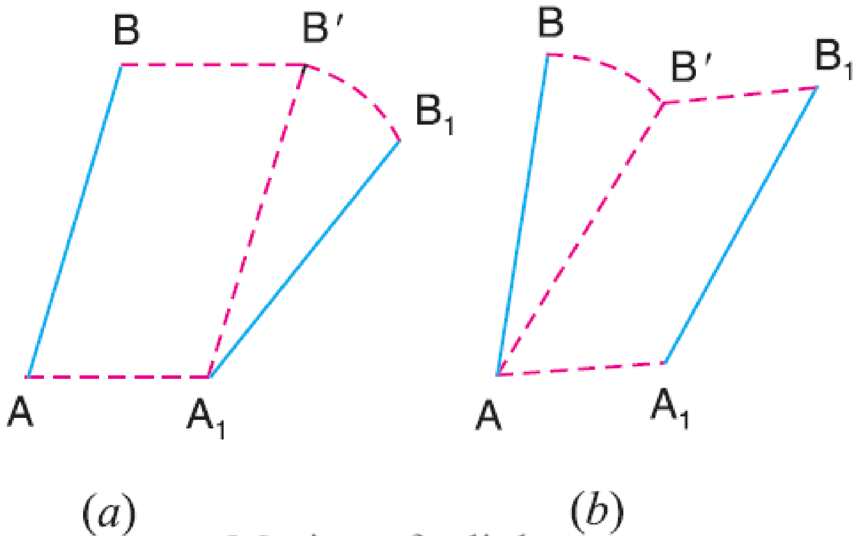


Drawing courtesy of PRODUCT ENGINEERING Magazine; June 8, 1964: pp. 67, 68

Fig. 9–18. Chain-mounted drive pins with blocks for locking during dwells.

UNIT-II

INTRODUCTION



Motion of a link.

Motion of link *AB to A*1*B*1 is an example of combined motion of rotation and translation, it being immaterial whether the motion of rotation takes first, or the motion of translation.

METHODS FOR DETERMINING THE VELOCITY OF A POINT ON A LINK

1. Relative velocity method

   Can be used in any configuration

2. Instantaneous centre method

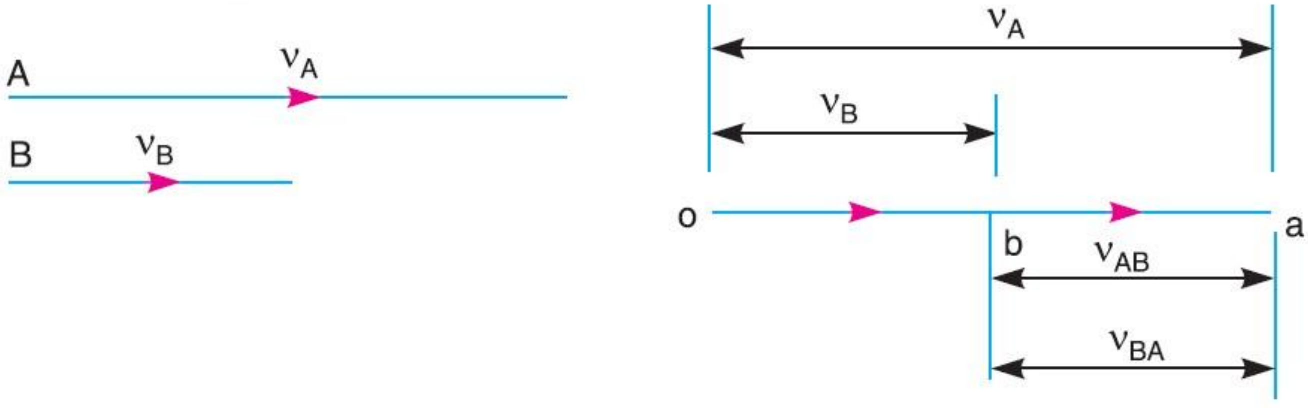
   convenient and easy to apply in simple mechanisms

RELATIVE VELOCITY METHOD

From Fig., the relative velocity of A with respect to B (*i.e.* v_{AB}) may be written in the vector form as follows :

$$\overline{ba} = \overline{oa} - \overline{ob}$$

Source : R. S. Khurmi



(*a*) (*b*)

Relative velocity of two bodies moving along parallel lines.

Similarly, the relative velocity of B with respect to A,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

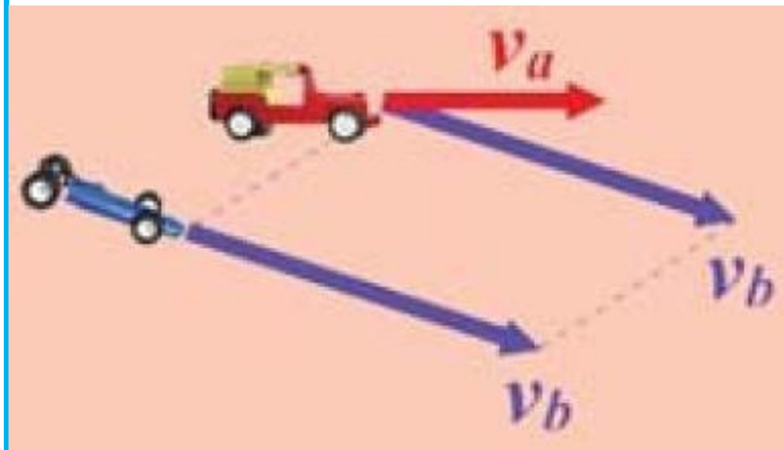
$$\overline{ab} = \overline{ob} - \overline{oa}$$

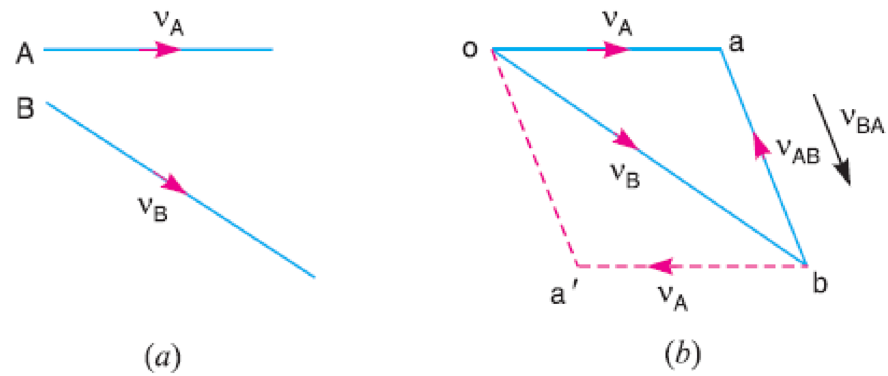
RELATIVE VELOCITY

v_{AB} = Vector difference of v_A and $v_B = \overline{v_A} - \overline{v_B}$

$\overline{ba} = \overline{oa} - \overline{ob}$

Source : R. S. Khurmi





Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of *B* with respect to *A* ,

v_{BA} = Vector difference of v_B and $v_A = \overline{v_B} - \overline{v_A}$

$\overline{ab} = \overline{ob} - \overline{oa}$

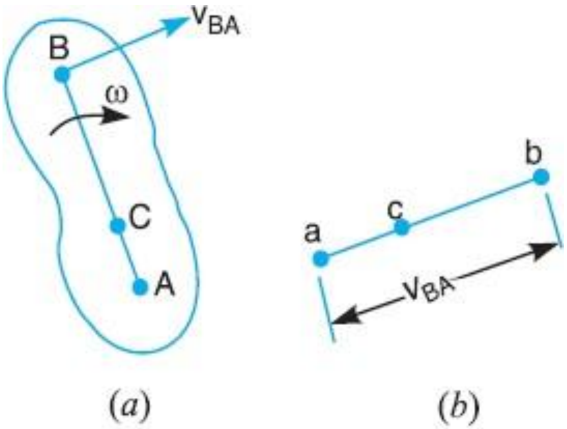
From above, we conclude that the relative velocity of point *A* with respect to *B* (v_{AB}) and the relative velocity of point *B* with respect *A* (v_{BA}) are equal in magnitude but opposite in direction, *i.e.*

$$v_{AB} = -v_{BA} \quad \text{or} \quad \overline{ba} = -\overline{ab}$$

Note: It may be noted that to find v_{AB}, start from point *b* towards *a* and for v_{BA}, start from point *a* towards *b*.

MOTION OF A LINK

Source : R. S. Khurmi



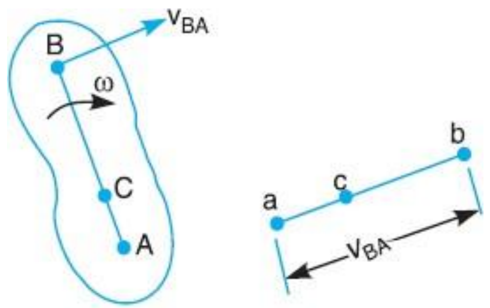
Motion of a Link.

- Let one of the extremities (B) of the link move relative to A, in a clockwise direction.
- No relative motion between A and B, along the line AB
- relative motion of B with respect to A must be perpendicular to AB.

Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.

MOTION OF A LINK

Source : R. S. Khurmi



(a) (b)

Motion of a Link.

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \quad \text{...(i)}$$

Similarly, the velocity of any point C on AB with respect to A,

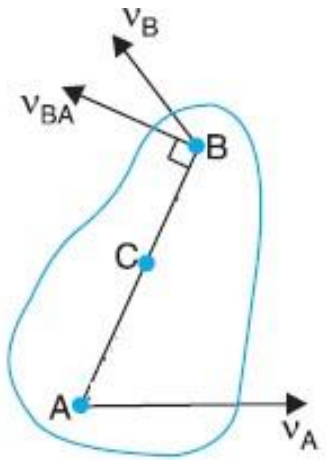
$$v_{CA} = \overline{ac} = \omega . AC \quad \text{...(ii)}$$

From equations (i) and (ii),

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \quad \text{...(iii)}$$

Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

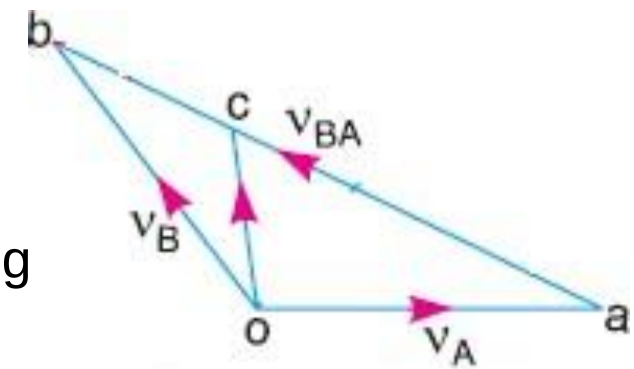
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



Motion of points on a link.

Source : R. S. Khurmi

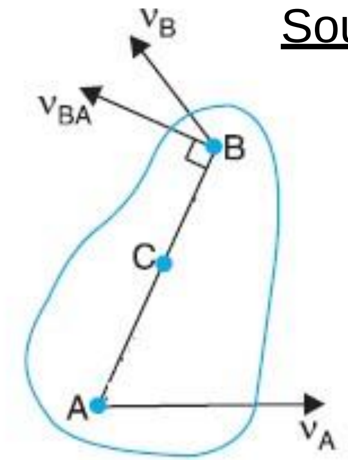
- V_A is known in magnitude and direction
- absolute velocity of the point *B i.e.* V_B is known in direction only
- V_B be determined by drawing the velocity diagram



Velocity diagram.

- With suitable scale, Draw oa = V_A
- Through a, draw a line perpendicular to AB
- Through o, draw a line parallel to V_B intersecting the line of V_{BA} at b
- Measure ob, which gives the required velocity of point B (V_B), to the scale
- ab = velocity of the link AB

VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD



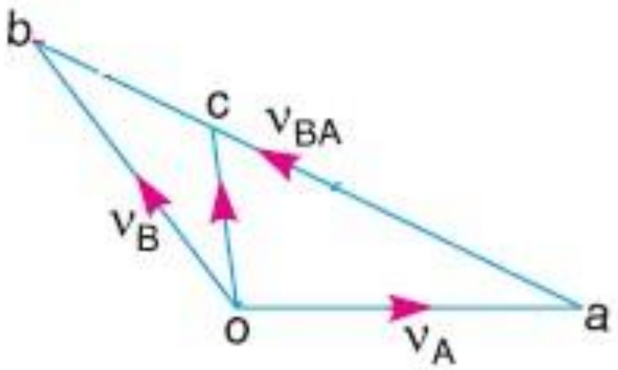
Motion of points on a link.

Source : R. S. Khurmi

- *How to find V_C ?*

Fix 'c' on the velocity diagram, using

$$\frac{ac}{ab} = \frac{AC}{AB}$$



Velocity diagram.

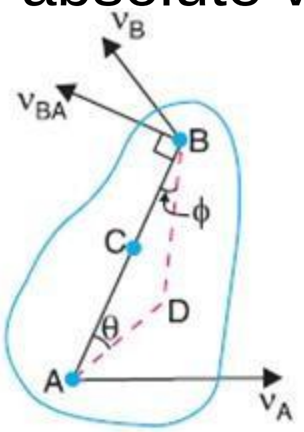
- oc = V_C = Absolute velocity of C

- the vector ac represents the velocity of C with respect to A i.e. V_{CA}.

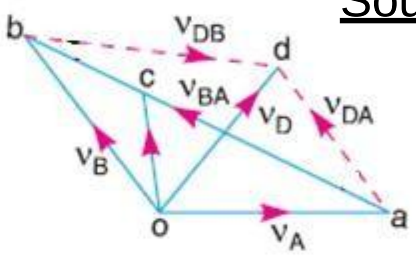
VELOCITY OF A POINT ON A LINK BY RELATIVE VELOCITY METHOD

How to find the absolute velocity of any other point D outside AB?

Source : R. S. Khurmi



(a) Motion of points on a link.



(b) Velocity diagram.

Construct triangle ABD in the space diagram

Completing the velocity triangle abd:

- Draw VDA perpendicular to AD;
- Draw VDB perpendicular to BD, intersection is ‘d’.
- od = absolute velocity of D.

The angular velocity of the link AB $= \omega_{AB} = \dfrac{v_{BA}}{AB} = \dfrac{ab}{AB}$

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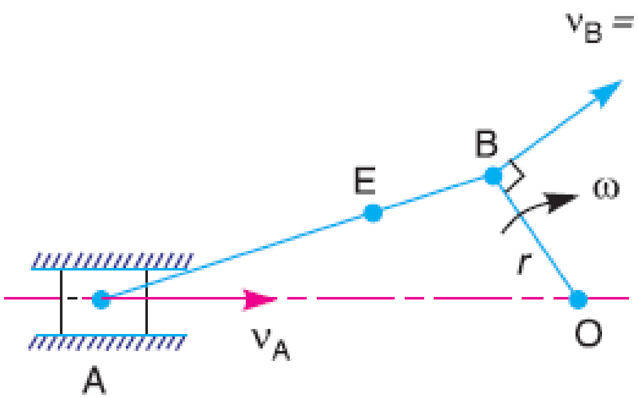
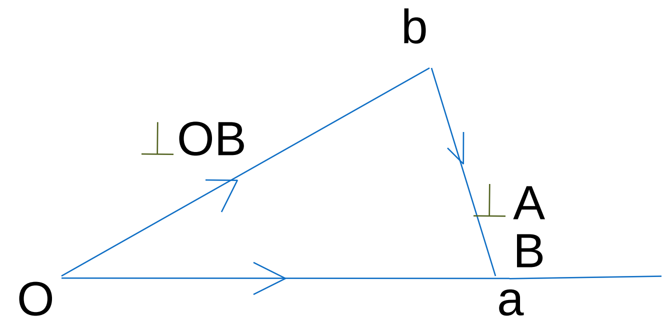
VELOCITIES IN SLIDER CRANK MECHANISM

Source : R. S. Khurmi

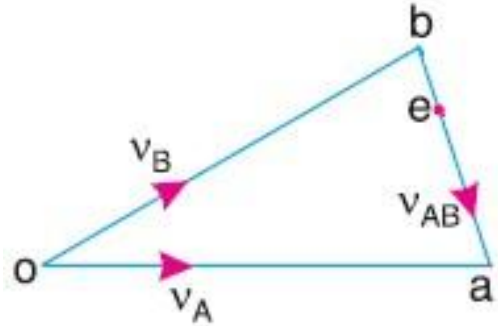


Slider crank mechanism.



Fix 'e', based on the ratio

$$be/ba = BE/BA$$



Velocity diagram.

V_E = length 'oe' = absolute vel. Of E

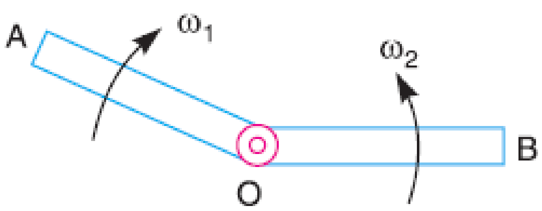
The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

RUBBING VELOCITY AT A PIN JOINT

The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.

Source : R. S. Khurmi



Links connected by pin joints.

Let ω_1 = Angular velocity of the link OA or the angular velocity of the point A with respect to O.

ω_2 = Angular velocity of the link OB or the angular velocity of the point B with respect to O, and

r = Radius of the pin.

According to the definition,

Rubbing velocity at the pin joint O

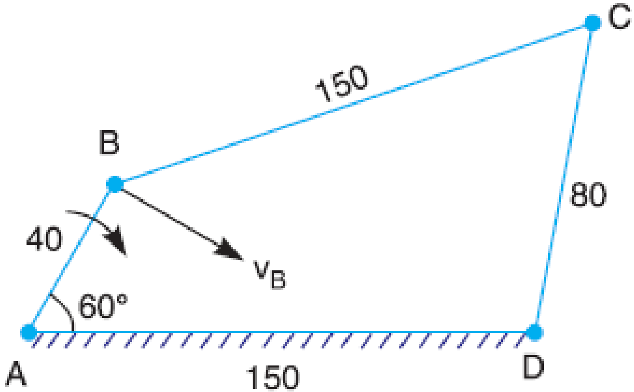
$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

NUMERICAL EXAMPLE-1

In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

Step-1 : Draw Space diagram with suitable scale



Space diagram (All dimensions in mm).

Step-2 : Identify Given data & convert it into SI units

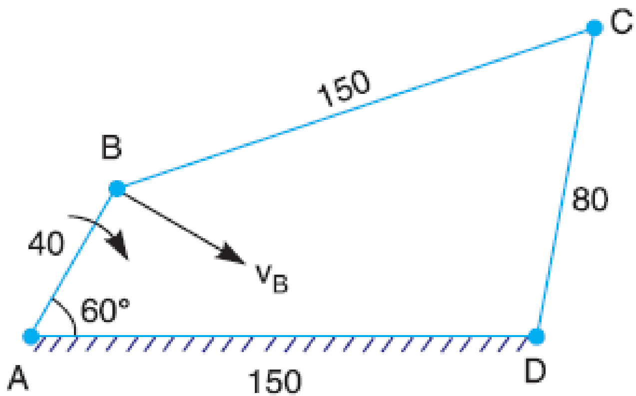
$N_{BA} = 120$ r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s

AB = 0.04 m ; BC = 0.15 m; CD = 0.08 m; AD = 0.15 m

Step-3 : Calculate V_B

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$

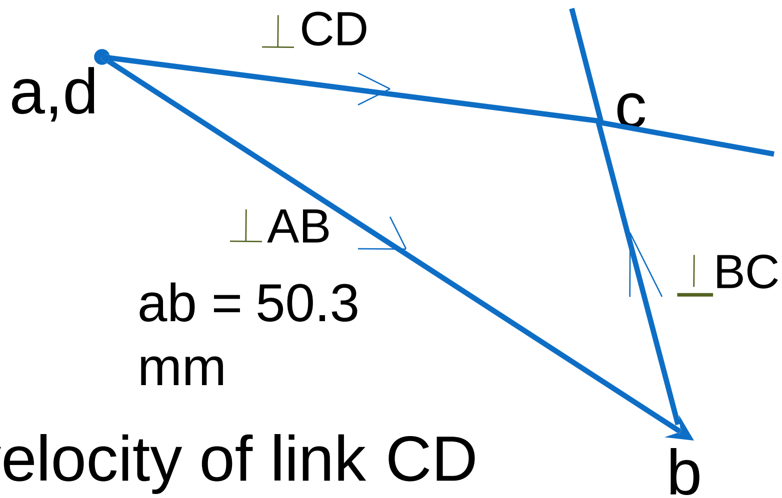
NUMERICAL EXAMPLE-1



Space diagram (All dimensions in mm).

Scale 1:100; i.e. V_B = 0.503 m/s = 50.3 mm

Source : R. S. Khurmi



Question: Find the angular velocity of link CD

V_{CD} = cd = 38.5 mm (by measurement) = 0.385 m/s , CD = 0.08 m

∴ Angular velocity of link *CD*,

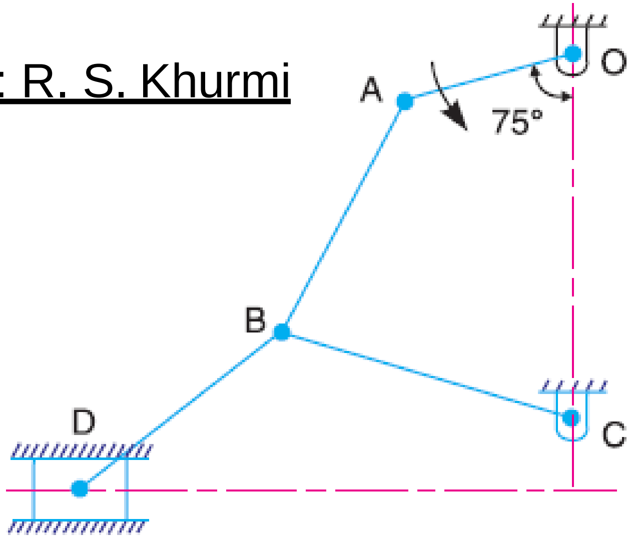
$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D) \textbf{ Ans.}$$

NUMERICAL EXAMPLE -2

In the given Fig., the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider m

path and OC is vertical.

Source : R. S. Khurmi



Find: V_D, ω_{BD}

Solution. Given: N_{AO} = 600 r.p.m. or

$\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s

Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

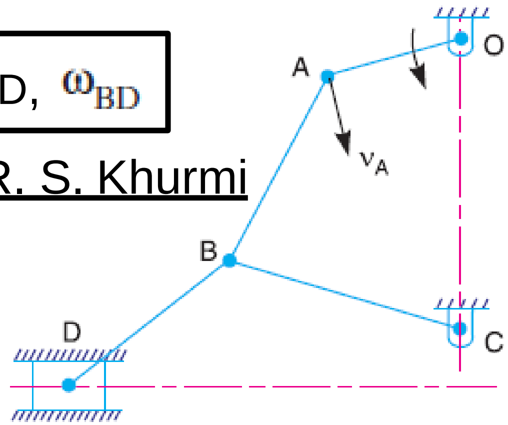
$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

. . . (Perpendicular to OA)

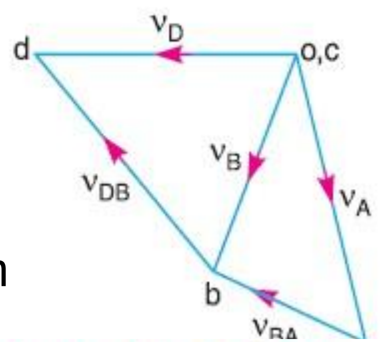
NUMERICAL EXAMPLE -2

Find: V_D, ω_{BD}

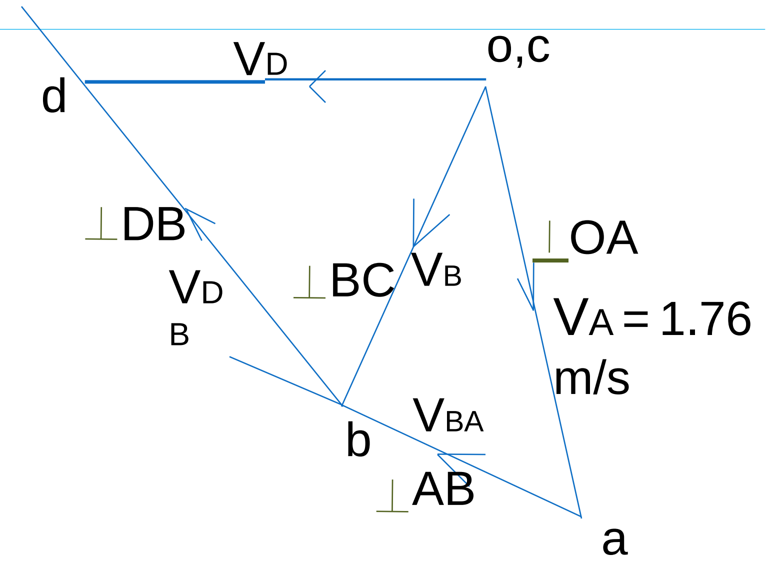
<u>Source : R. S. Khurmi</u>



(*a*) Space diagram.



Velocity diagram



Velocity diagram

By measurement, cd = od = $V_D = 1.6$ m/s

Angular velocity of the link BD

By measurement from velocity diagram, we find that velocity of *D* with respect to *B*,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link *BD* = 46 mm = 0.046 m, therefore angular velocity of the link *BD*,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B) \quad \textbf{Ans.}$$

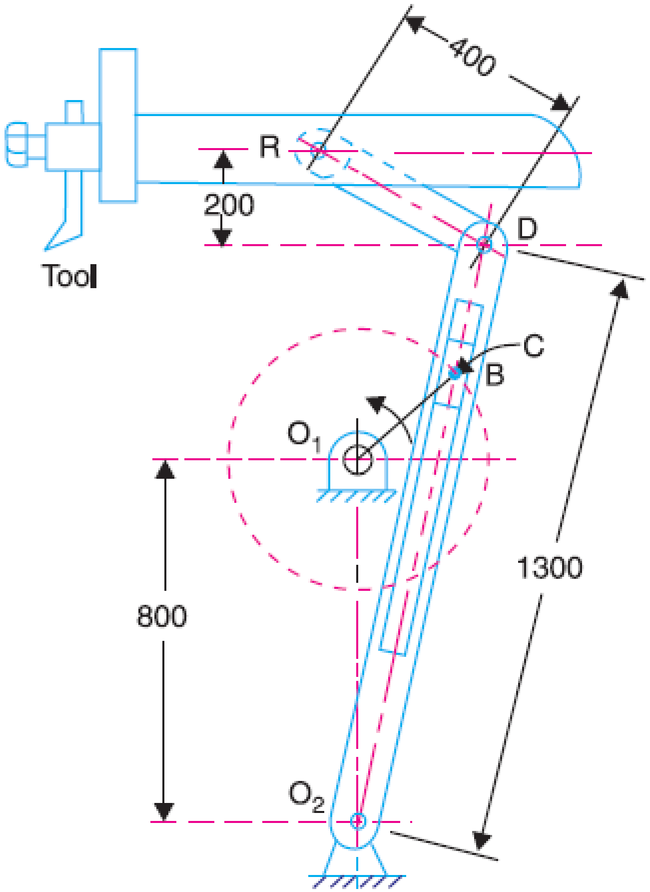
NUMERICAL EXAMPLE -3

A quick return mechanism of the crank and slotted lever type shaping machine is shown in the Fig. The dimensions of the various links are as follows :

O_1O_2 = 800 mm ; O_1B = 300 mm ; O_2D = 1300 mm ; DR = 400 mm.

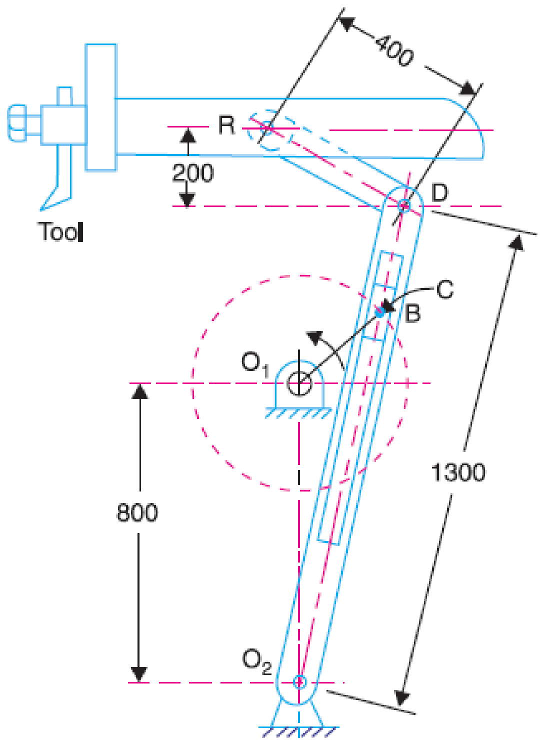
The crank O_1B makes an angle of 45° with the vertical and rotates at 40 r.p.m. in the counter clockwise direction.

Find : 1. velocity of the ram R, or the velocity of the cutting tool, and 2. angular velocity of link O_2D.

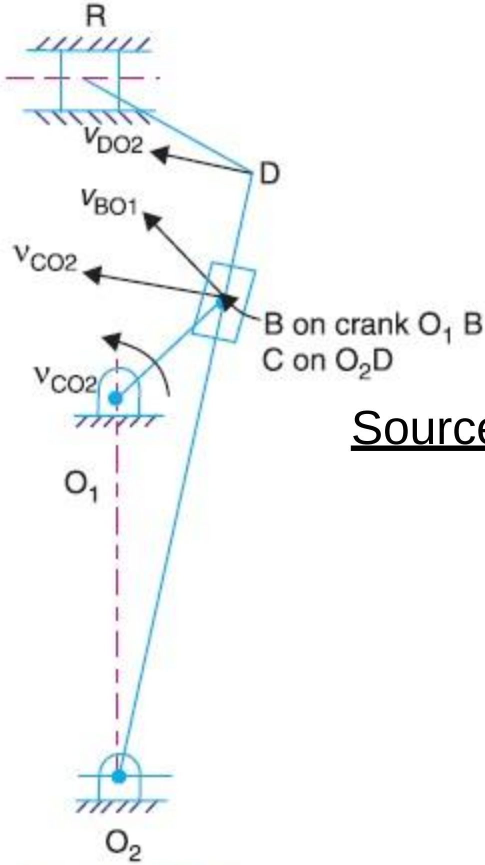


All dimensions in mm.

NUMERICAL EXAMPLE -3



All dimensions in mm.



Space diagram

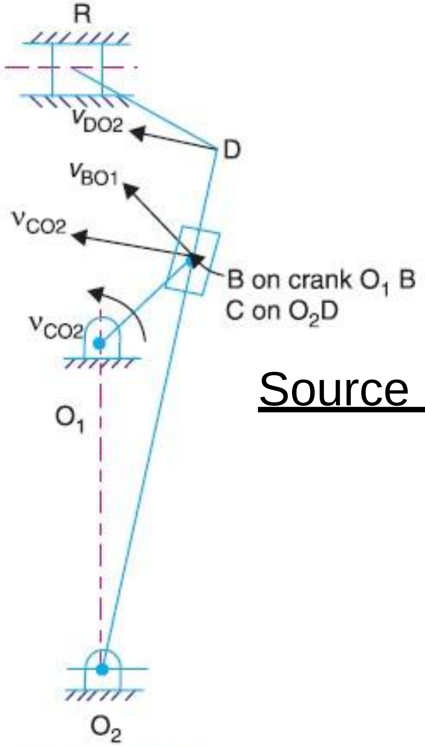
Source : R. S. Khurmi

Solution. Given: $N_{BO1} = 40$ r.p.m. or $\omega_{BO1} = 2\pi \times 40/60 = 4.2$ rad/s

$$v_{BO1} = v_B = \omega_{BO1} \times O_1B = 4.2 \times 0.3 = 1.26 \text{ m/s}$$

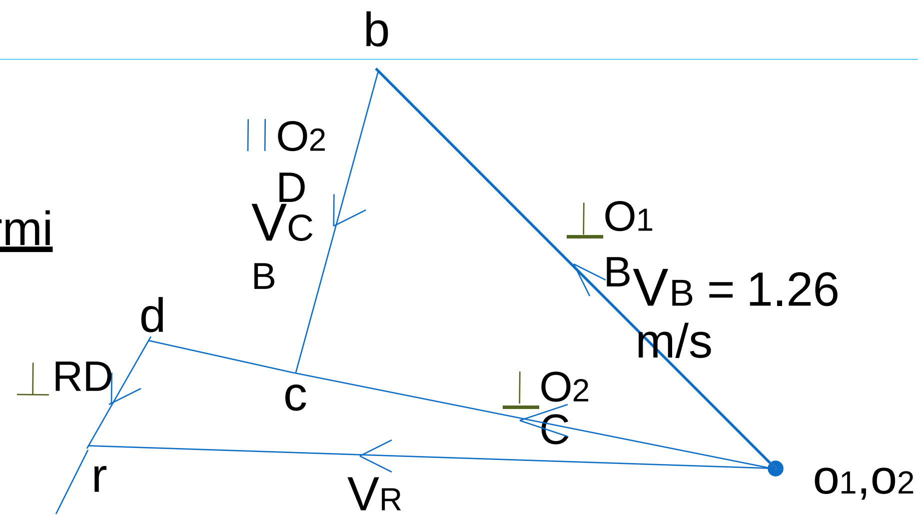
. . . (Perpendicular to O_1B)

AL EXAMPLE -3



Space diagram

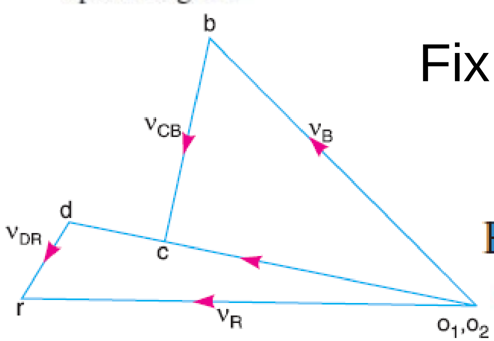
Source : R. S. Khurmi



Draw bc parallel to O_2D, to intersect at 'c'

Fix 'd' using the ratio $cd / o_2d = CD/O_2D$ ➡ $\frac{cd}{cd + co_2} = \frac{CD}{O_2D}$

Find V_R & ω_{DO_2}



Velocity diagram.

By measurement, velocity of the ram R, v_R = vector o_1r = 1.44 m/s **Ans.**

Angular velocity of link O_2D

By measurement, $v_{DO2} = v_D$ = vector o_2d = 1.32 m/s

$$\omega_{DO_2} = \frac{v_{DO2}}{O_2D} = \frac{1.32}{1.3} = 1.015 \text{ rad/s (Anticlockwise about } O_2) \quad \textbf{Ans.}$$

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TUTORIAL PROBLEM

Fig. 7.22 shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows :

OQ = 100 mm ; *OP* = 200 mm, *RQ* = 150 mm and *RS* = 500 mm.

The crank *OP* makes an angle of 60° with the vertical. Determine the velocity of the slider *S* (cutting tool) when the crank rotates at 120 r.p.m. clockwise.

Find also the angular velocity of the link *RS* and the velocity of the sliding block *T* on the slotted lever *QT*.

Source : R. S. Khurmi

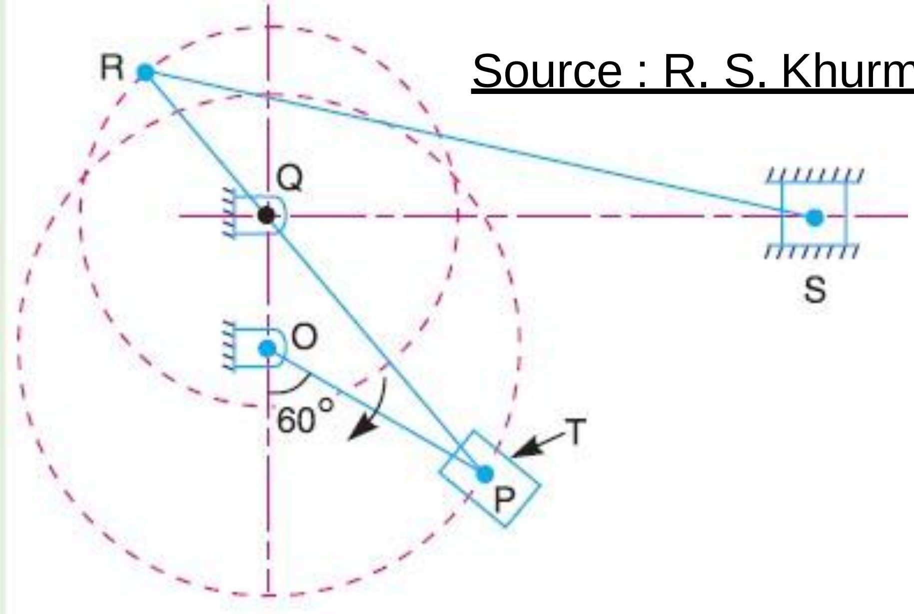
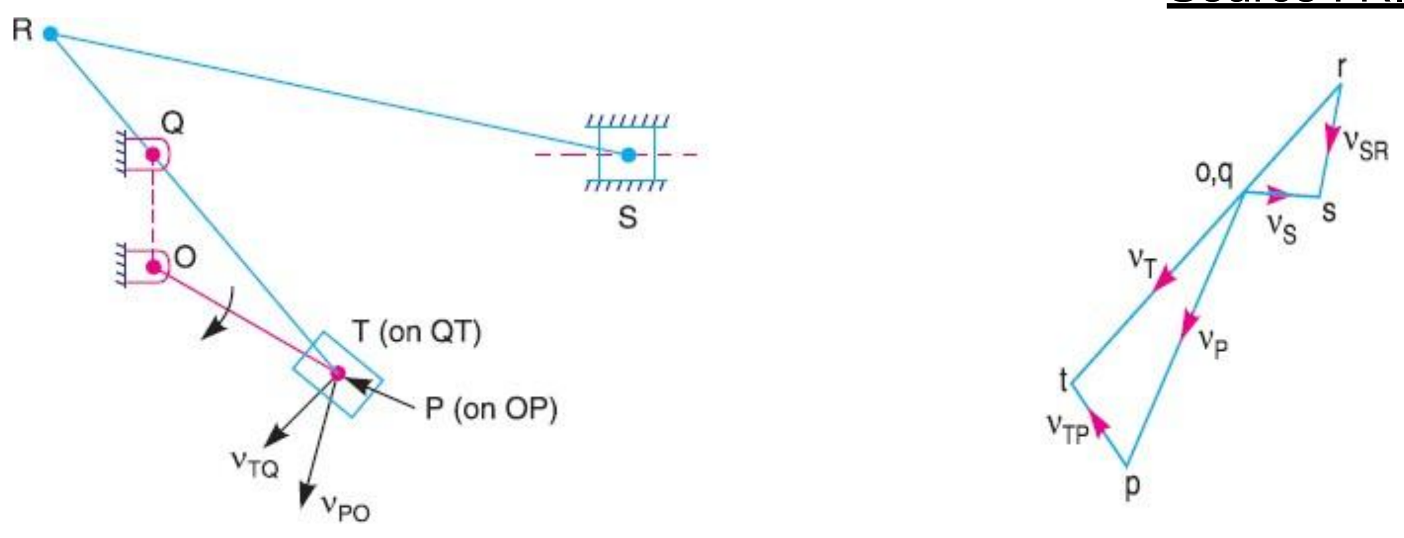


Fig. 7.22

TUTORIAL PROBLEM (SOLUTION)

Source : R. S. Khurmi



(*a*) Space diagram. (*b*) Velocity diagram.

v_S = vector os = 0.8 m/s **Ans.**

Angular velocity of link RS

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 1.92 \text{ rad/s} \quad \textbf{Ans.}$$

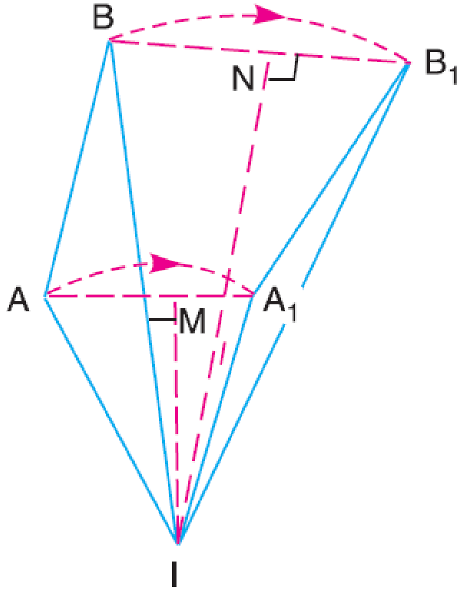
Velocity of the sliding block T on the slotted lever QT

v_{TP} = vector pt = 0.85 m/s **Ans.**

INSTANTANEOUS CENTRE METHOD

Translation of the link AB may be assumed to be a motion of pure rotation about some centre I, known as the instantaneous centre of rotation (also called centro or virtual centre).

The position of the centre of rotation must lie on the intersection of the right bisectors of chords *AA*1 and *BB*1. these bisectors intersect at *I as shown in Fig., which is the instantaneous centre of rotation or* virtual centre of the link *AB.*
(also called centro or virtual centre).



Instantaneous
centre of rotation.

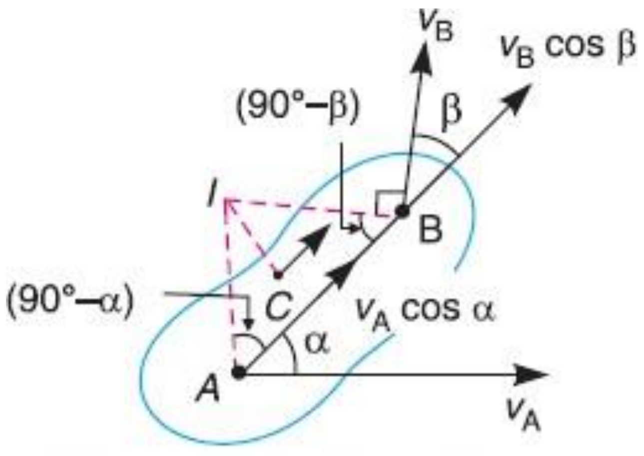
Source : R. S. Khurmi

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

V_A is known in Magnitude and direction V_B direction alone known

How to calculate Magnitude of V_B using instantaneous centre method ?

Source : R. S. Khurmi



Velocity of a point on a link.

Draw AI and BI perpendiculars to the directions V_A and V_B respectively to intersect at I, which is known as instantaneous centre of the link.

VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Since A and B are the points on a rigid link, there cannot be any relative motion between them along the line AB.

Now resolving the velocities along *AB*,

$$v_A \cos\alpha = v_B \cos\beta$$

or

$$\frac{v_A}{v_B} = \frac{\cos\beta}{\cos\alpha} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \ldots(i)$$

Applying Lami's theorem to triangle *ABI*,

$$\frac{AI}{\sin(90° - \beta)} = \frac{BI}{\sin(90° - \alpha)}$$

or

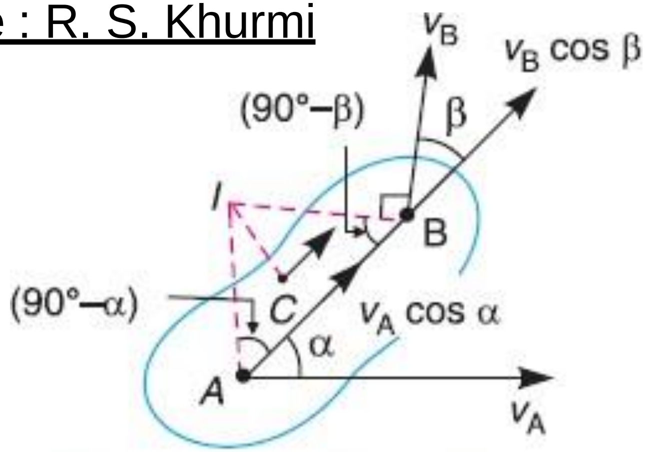
$$\frac{AI}{BI} = \frac{\sin(90° - \beta)}{\sin(90° - \alpha)} \quad \ldots(ii)$$

From equation (*i*) and (*ii*),

$$\frac{v_A}{v_B} = \frac{AI}{BI} \quad \text{or} \quad \frac{v_A}{AI} = \frac{v_B}{BI} = \omega \quad \ldots(iii)$$

where ω = Angular velocity of the rigid link.

Source : R. S. Khurmi



Velocity of a point on a link.

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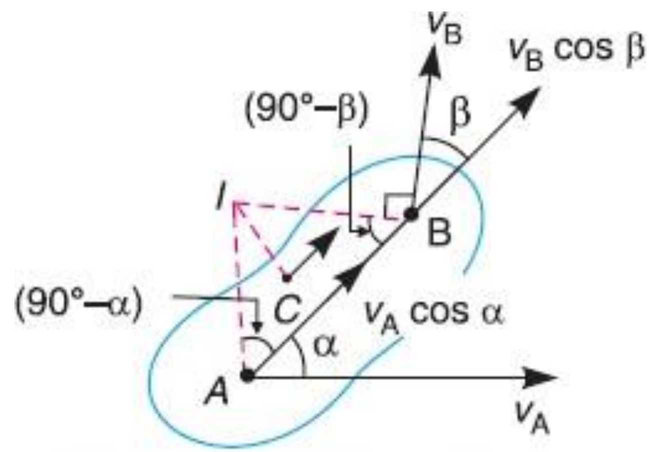
VELOCITY OF A POINT ON A LINK BY INSTANTANEOUS CENTRE METHOD

Source : R. S. Khurmi

If C is any other point on the link, then

$$\frac{v_A}{AI} = \frac{v_B}{BI} = \frac{v_C}{CI} \qquad \ldots(iv)$$

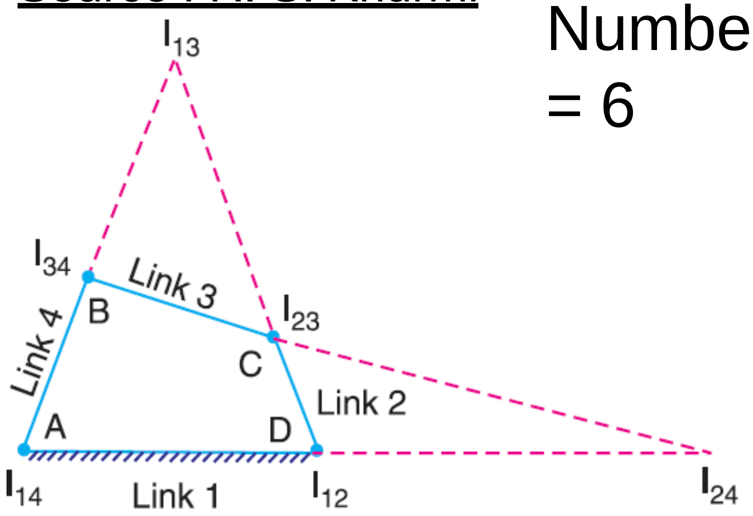


Velocity of a point on a link.

If V_A is known in magnitude and direction and V_B in direction only, then velocity of point B or any other point C lying on the same link may be determined (Using *iv*) in magnitude and direction.

TYPES OF INSTANTANEOUS CENTRES

Source : R. S. Khurmi



Types of instantaneous centres.

Number of Instantaneous Centres = N = 6

The instantaneous centres I_{12} and I_{14} ***fixed instantaneous centres***

The instantaneous centres I_{23} and I_{34} ***permanent instantaneous centres*** as they move when the mechanism moves, but the joints are of permanent nature.

I_{13} and I_{24} are ***neither fixed nor permanent instantaneous centres*** as they vary with the configuration of the mechanism.

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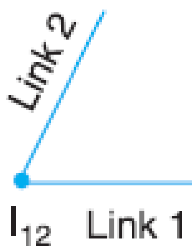
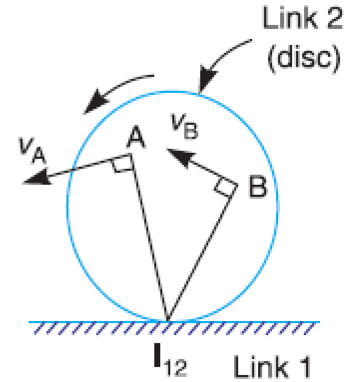
LOCATION OF INSTANTANEOUS CENTRES

Source : R. S. Khurmi

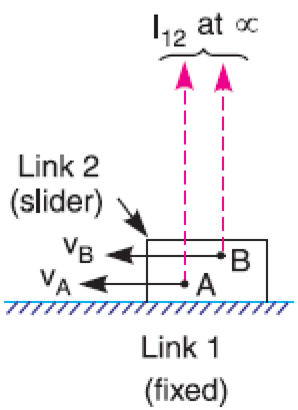


When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin



Pure rolling contact (i.e. link 2 rolls without slipping), the instantaneous centre lies on their point of contact.

$$\frac{v_A}{v_B} = \frac{I_{12}A}{I_{12}B}$$

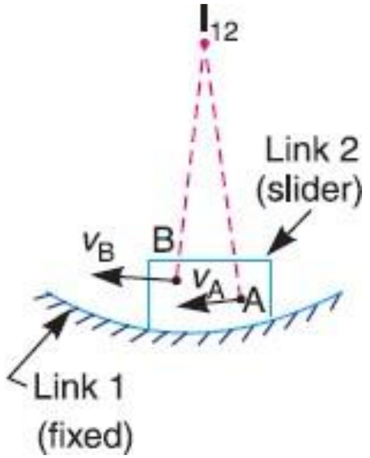


When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.

The instantaneous centre lies at infinity and each point on the slider have the same velocity.

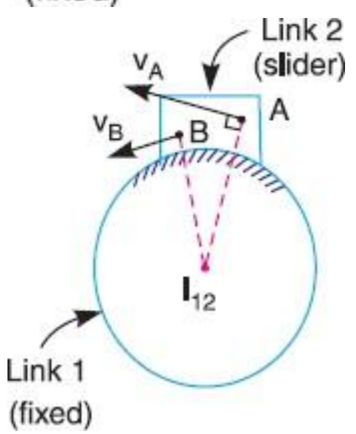
LOCATION OF INSTANTANEOUS CENTRES

When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact.



The instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

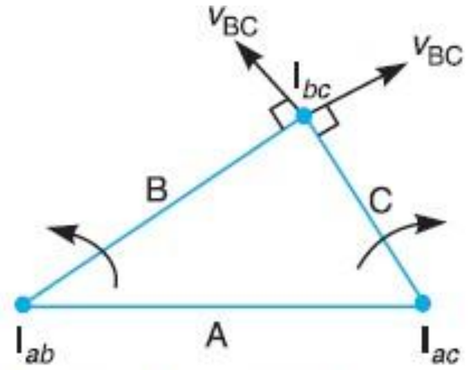
Source : R. S. Khurmi



When the link 2 (slider) moves on fixed link 1 having constant radius of curvature, the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.

ARONHOLD KENNEDY (OR THREE CENTRES IN LINE) THEOREM

It states that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line.



Aronhold Kennedy's theorem.

Source : R. S. Khurmi

the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab}\, I_{bc}$ and $I_{ac}\, I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}.

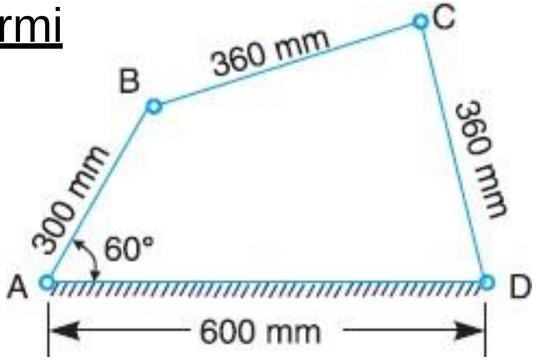
Thus the three instantaneous centres (I_{ab}, I_{ac} and I_{bc}) must lie on the same straight line.

The exact location of I_{bc} on line $I_{ab}\, I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

NUMERICAL EXAMPLE-1

In a pin jointed four bar mechanism, as shown in Fig. AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC

Source : R. S. Khurmi



Solution. Given : $N_{AB} = 100$ r.p.m or

$$\omega_{AB} = 2\,\pi \times 100/60 = 10.47 \text{ rad/s}$$

Since the length of crank $AB = 300$ mm $= 0.3$ m, therefore velocity of point B on link AB,

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

Location of instantaneous centres:

1. Find number of Instantaneous centres

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

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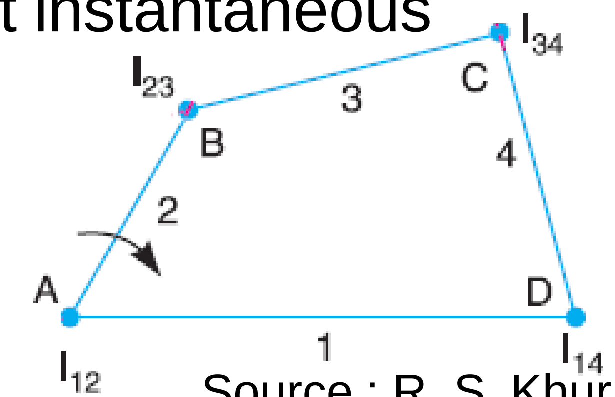
NUMERICAL EXAMPLE-1

2. List the Ins. centres

Links	1	2	3	4
Ins. Centres	12	13	14	
		23	24	
			34	

3. Draw configuration (space) diagram with suitable scale. And, Locate the fixed and permanent instantaneous centres by inspection

I_{12}, I_{14} – Fixed centres;
I_{23}, I_{34} – Permanent centres

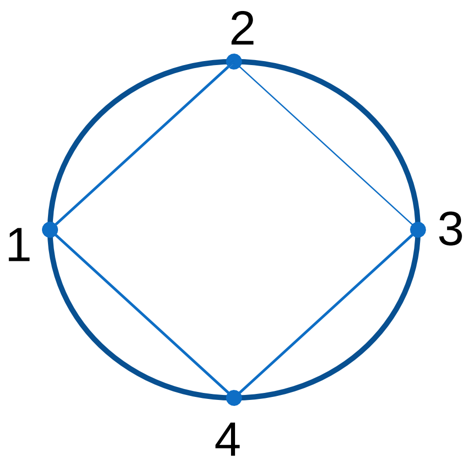


Source : R. S. Khurmi

How to locate I_{13}, I_{24} – Neither fixed nor Permanent centres

NUMERICAL EXAMPLE-1

4. Locate the neither fixed nor permanent instantaneous centres by Aronhold Kennedy’s theorem.



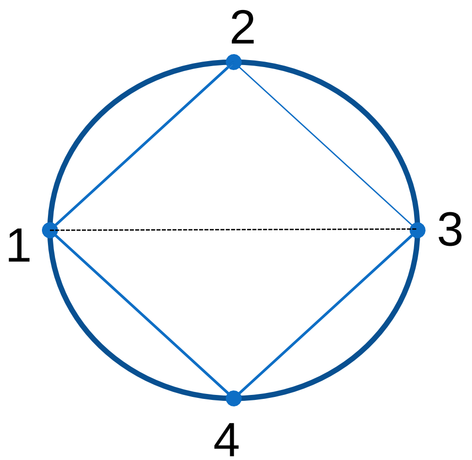
Source : R. S. Khurmi

Draw a circle with any arbitrary radius

At equal distance locate Links 1, 2, 3 & 4 as points on the circle.

NUMERICAL EXAMPLE-1

Locating I_{13}

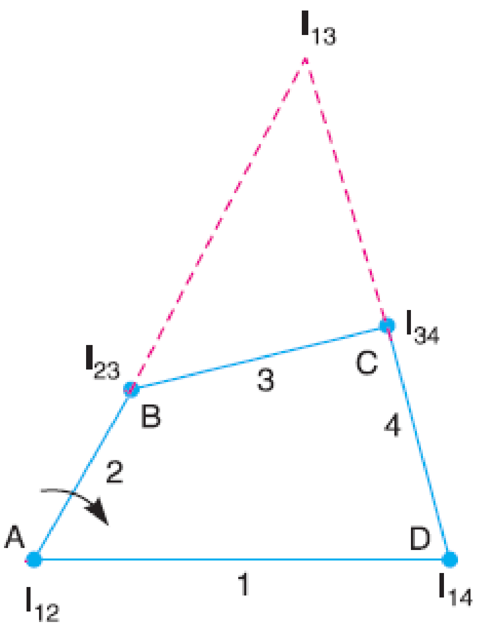


13 is common side to Triangle 134 & 123

I_{13} → I_{14} I_{34}

I_{13} → I_{12} I_{23}

Therefore, I_{13} lies on the intersection of the lines joining the points $I_{14}I_{34}$ & $I_{12}I_{23}$



Source : R. S. Khurmi

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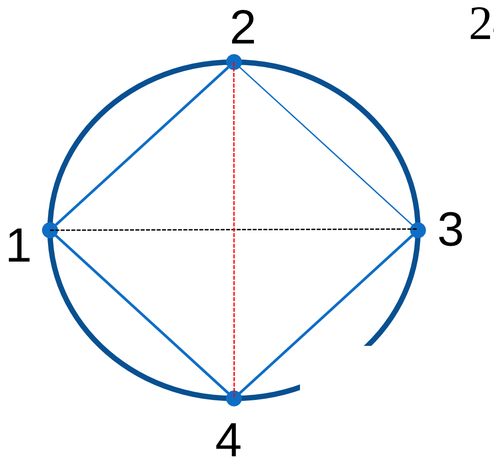
NUMERICAL EXAMPLE-1

Locating I_{24}

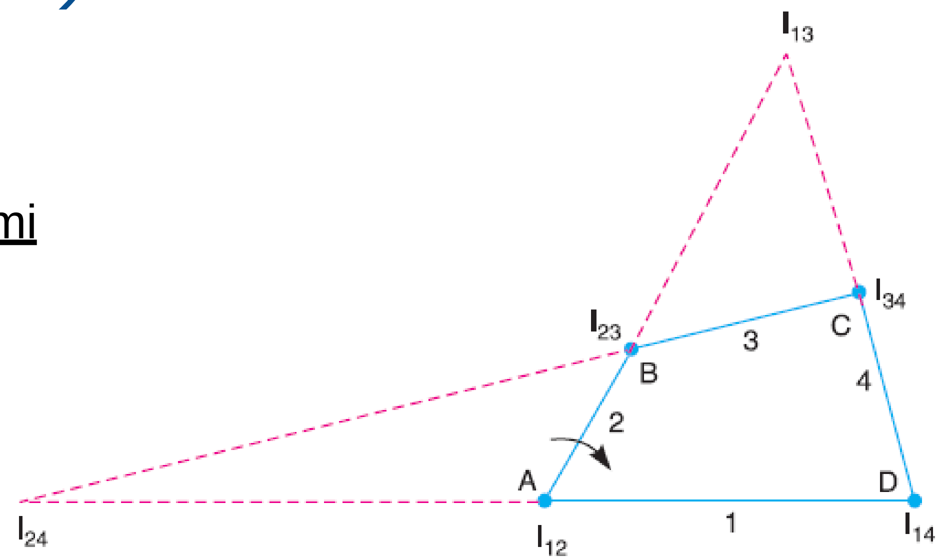


24 is common side to Triangle 124 & 234

I_{24} → I_{12} I_{14} ; I_{34} I_{23}

Therefore, I_{24} lies on the intersection of the lines joining the points $I_{12}I_{14}$ & $I_{34}I_{23}$

Source : R. S. Khurmi



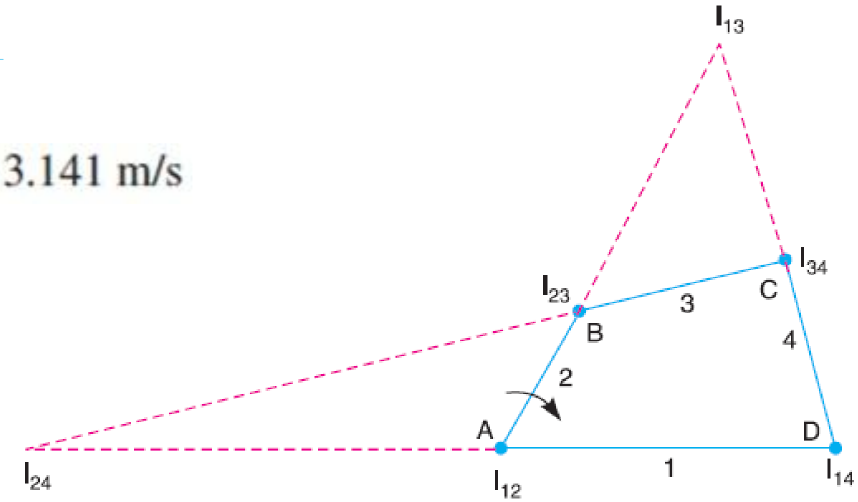
Thus all the six instantaneous centres are located.

NUMERICAL EXAMPLE-1

Find Angular velocity of the link BC

$$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141 \text{ m/s}$$

We know that:



Source : R. S. Khurmi

Let ω_{BC} = Angular velocity of the link BC.

Since B is also a point on link BC, therefore velocity of point B on link BC,

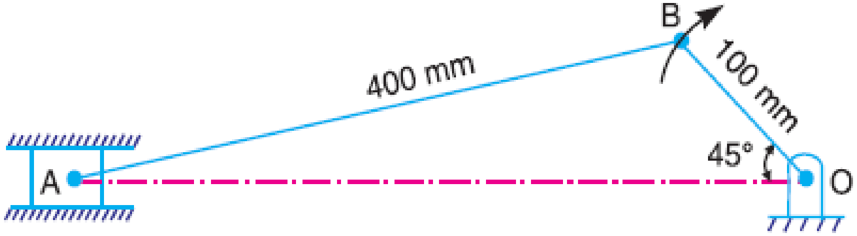
$$v_B = \omega_{BC} \times I_{13}B$$

By measurement, we find that $I_{13}B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13}B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

NUMERICAL EXAMPLE-2

Locate all the instantaneous centres of the slider crank mechanism as shown in the Fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: 1. Velocity of the slider A, and 2. Angular velocity



Source : R. S. Khurmi

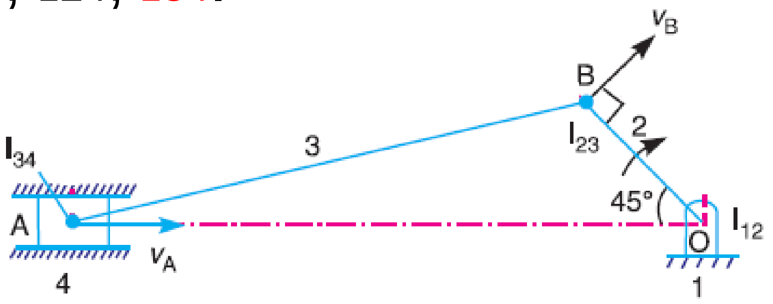
Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

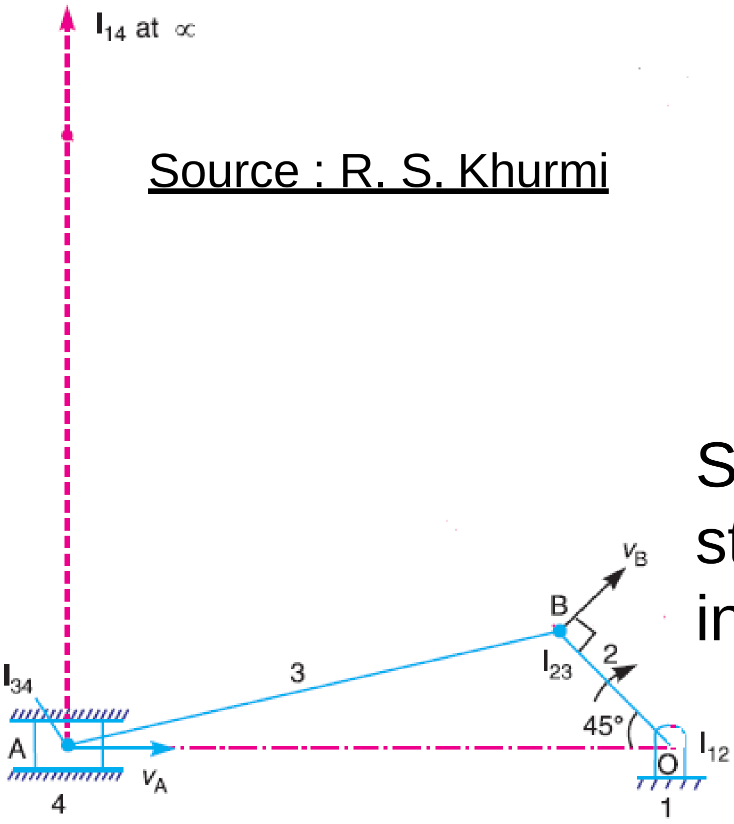
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

NUMERICAL EXAMPLE-2

- Draw configuration diagram with suitable scale.
- Locate Ins. Centres (Here, n = 4; No. of Ins. Centres N = 6)
- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.



By inspection Locate I_{12}, I_{23} & I_{34}.



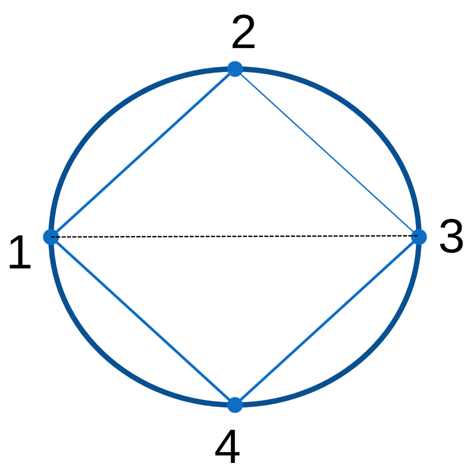
Source : R. S. Khurmi

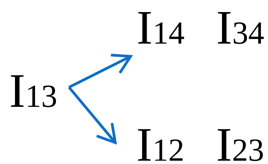
Since the slider (link 4) moves on a straight surface (link 1), I_{14}, will be at infinity.

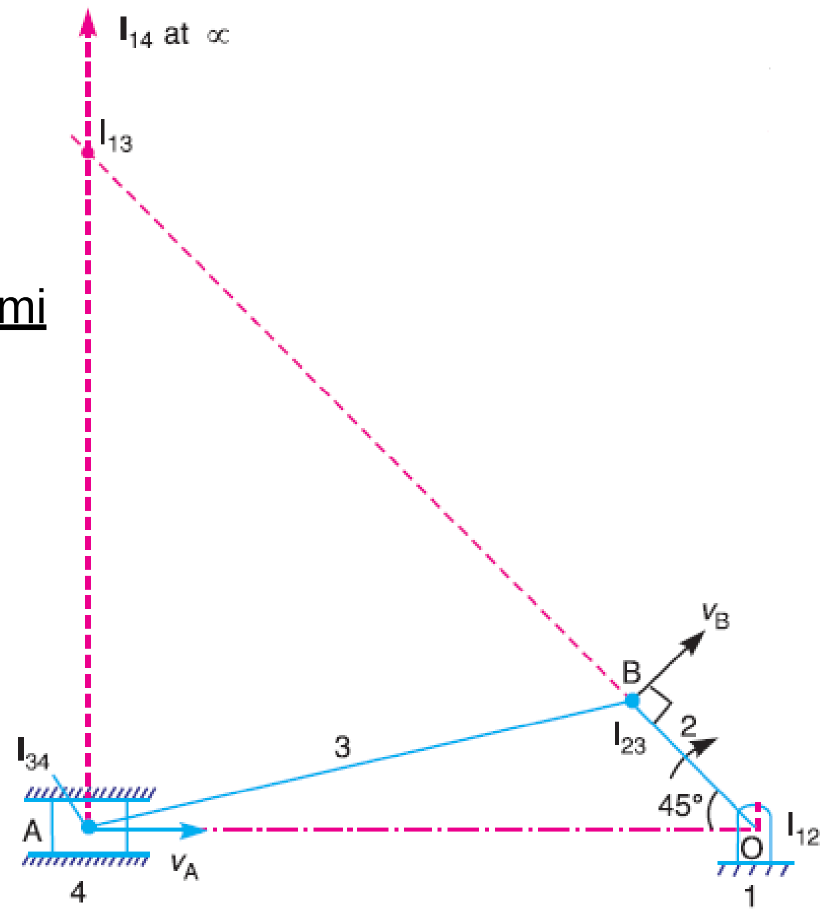
NUMERICAL EXAMPLE-2

- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{13}…… ?

Source : R. S. Khurmi

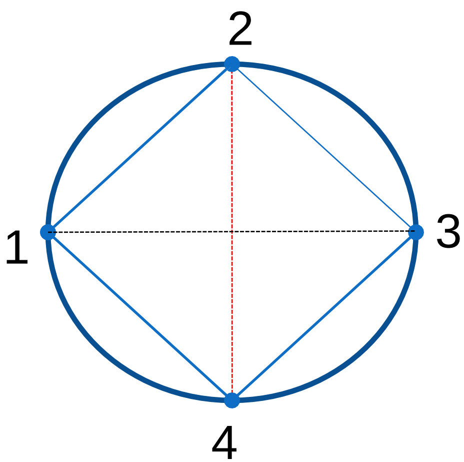






NUMERICAL EXAMPLE-2

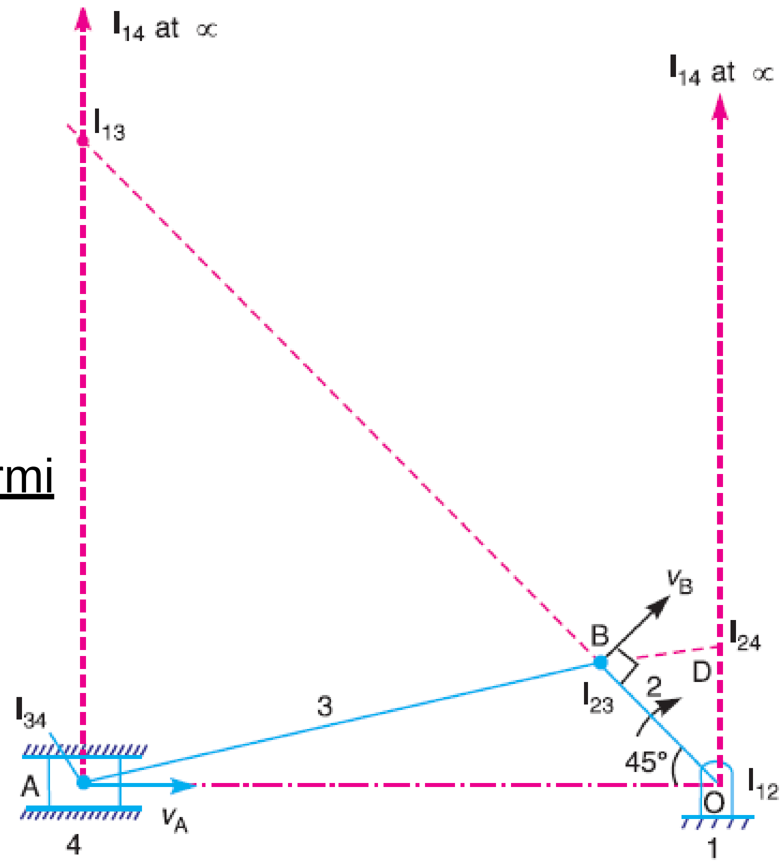
- Ins. Centers are I_{12}, I_{13}, I_{14}, I_{23}, I_{24}, I_{34}.
- Fixing I_{24}…… ?



I_{24} → I_{12} I_{14}

I_{24} → I_{34} I_{23}

Source : R. S. Khurmi

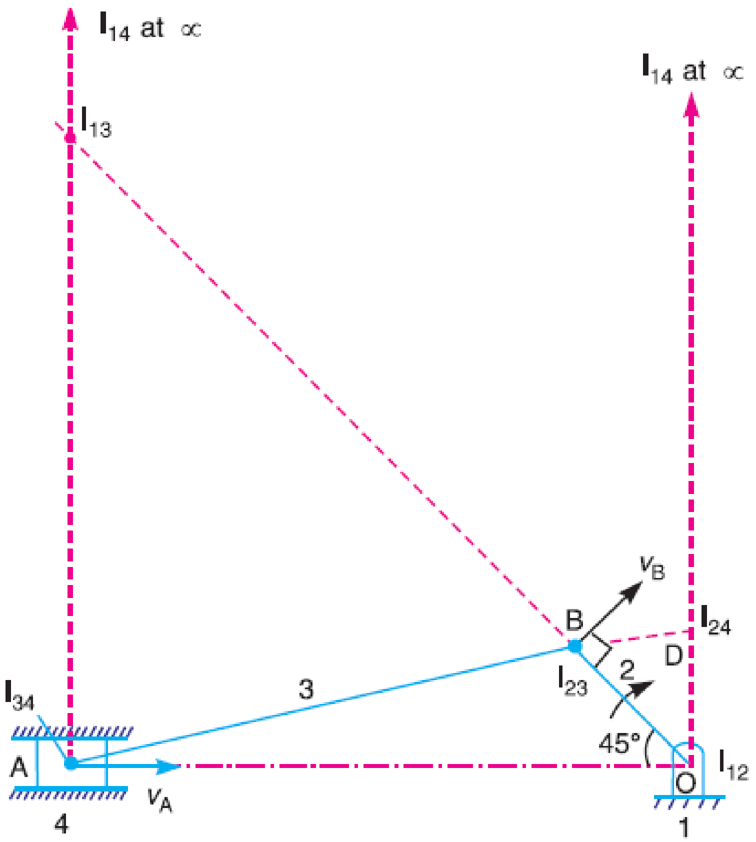


I_{14} can be moved to any convenient joint

NUMERICAL EXAMPLE-2

Solution:

Source : R. S. Khurmi



By measurement, we find that

$$I_{13}A = 460 \text{ mm} = 0.46 \text{ m} \text{ ; and } I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$$

1. *Velocity of the slider A*

Let v_A = Velocity of the slider A.

We know that $$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B}$$

or $$v_A = v_B \times \frac{I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocity of the connecting rod AB*

Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that $$\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B} = \omega_{AB}$$

$$\therefore \quad \omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s} \quad \textbf{Ans.}$$

VISIT THE FOLLOWING VIDEOS IN YOUTUBE

- https://www.youtube.com/watch?v=-tgruur8O0Q
- https://www.youtube.com/watch?v=WNh5Hp0lgms
- https://www.youtube.com/watch?v=ha2PzDt5SbE

EXERCISE-1

The mechanism of a wrapping machine, as shown in Fig. 6.18, has the following dimensions :

O_1A = 100 mm; AC = 700 mm; BC = 200 mm; O_3C = 200 mm; O_2E = 400 mm; O_2D = 200 mm and BD = 150 mm.

The crank O_1A rotates at a uniform speed of 100 rad/s. Find the velocity of the point E of the bell crank lever by instantaneous centre method.

Source : R. S. Khurmi

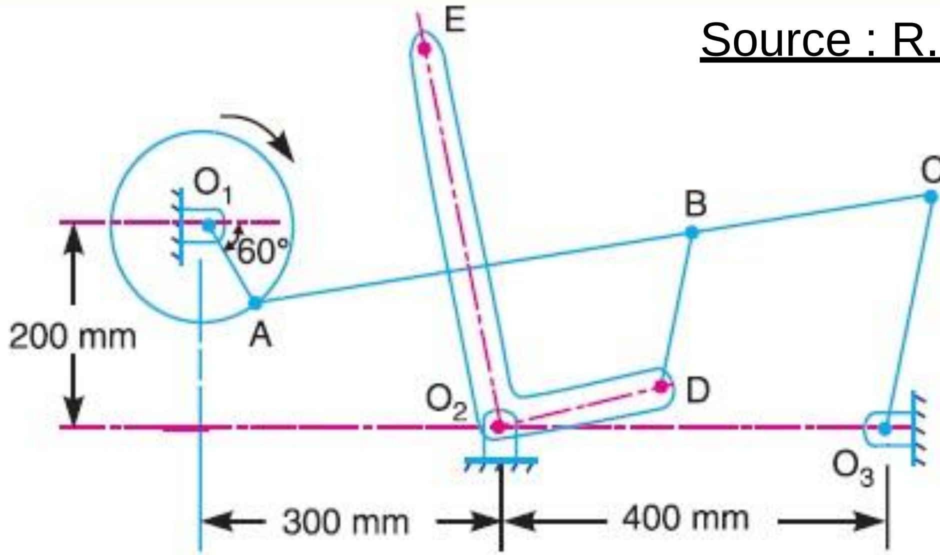
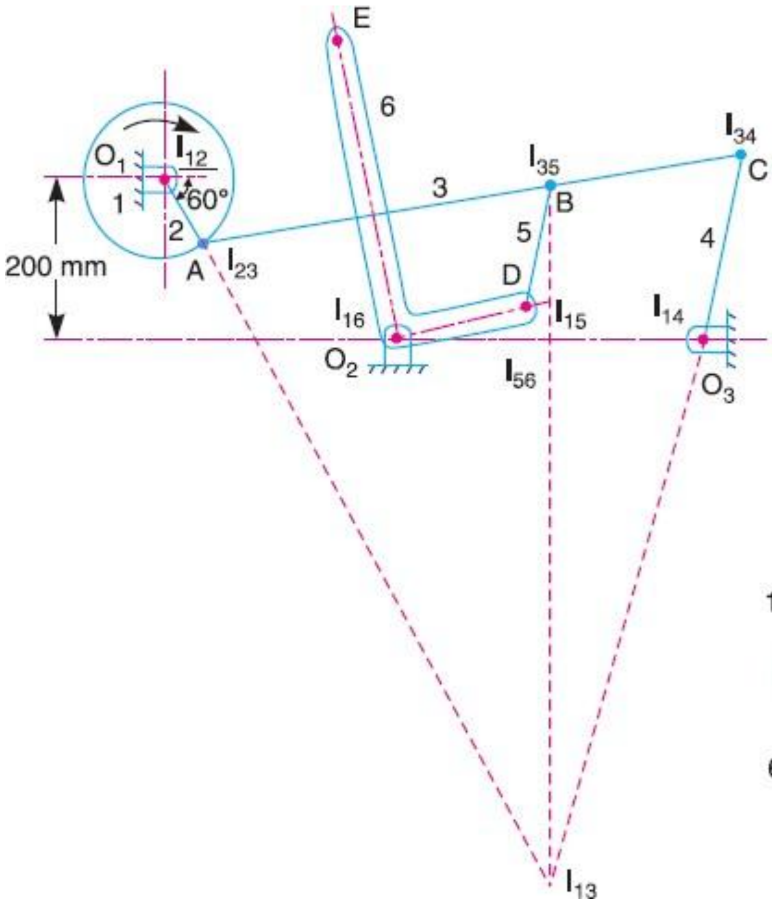


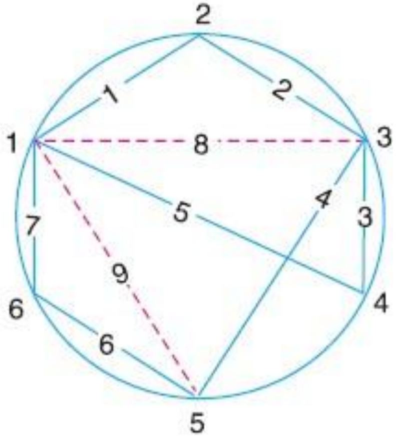
Fig. 6.18

EXERCISE-1: SOLUTION

$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$



Source : R. S. Khurmi



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EXERCISE-1: ANSWER

Velocity of point E on the bell crank lever

Let v_E = Velocity of point E on the bell crank lever,

v_B = Velocity of point B, and

v_D = Velocity of point D.

$$v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{10}{0.91} \times 0.82 = 9.01 \text{ m/s} \quad \textbf{Ans.}$$

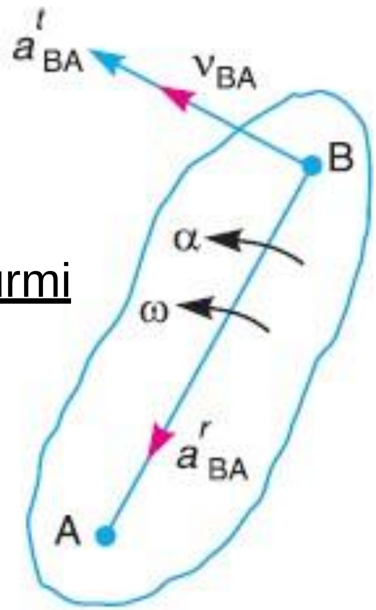
$$v_D = \frac{v_B}{I_{15}B} \times I_{15}D = \frac{9.01}{0.13} \times 0.05 = 3.46 \text{ m/s} \quad \textbf{Ans.}$$

$$v_E = \frac{v_D}{I_{16}D} \times I_{16}E = \frac{3.46}{0.2} \times 0.4 = 6.92 \text{ m/s} \quad \textbf{Ans.}$$

ACCELERATION IN MECHANISMS

Acceleration analysis plays a very important role in the development of machines and mechanisms

Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link AB.



Acceleration for a link.

Source : R. S. Khurmi

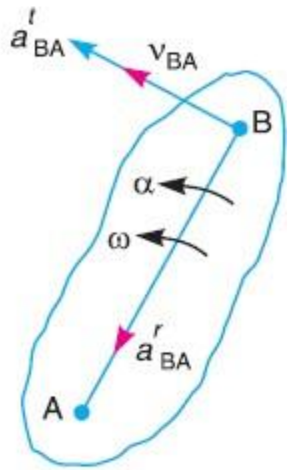
1. The centripetal or radial component of acceleration, which is perpendicular to the velocity (i.e. parallel to link AB) of the particle at the given instant.

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v^2_{BA} / AB \qquad \ldots \left(\because \omega = \frac{v_{BA}}{AB} \right)$$

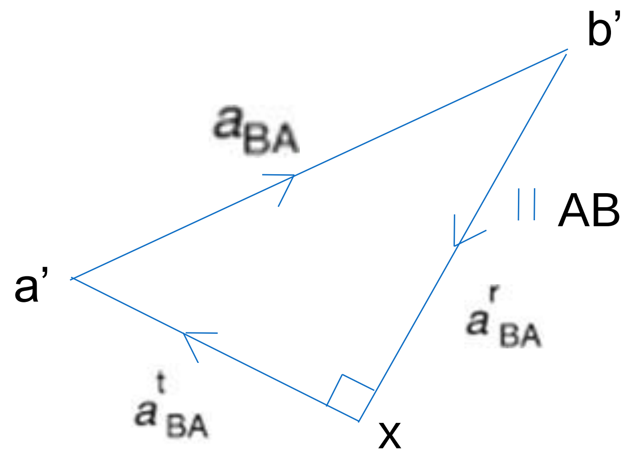
2. The tangential component, which is parallel to the velocity (i.e. Perpendicular to Link AB) of the particle at the given instant.

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

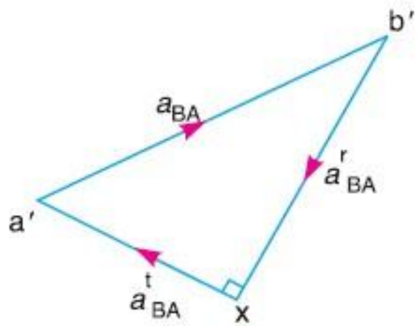
ACCELERATION DIAGRAM FOR A LINK



Acceleration for a link.



<u>Source : R. S. Khurmi</u>



Acceleration diagram.

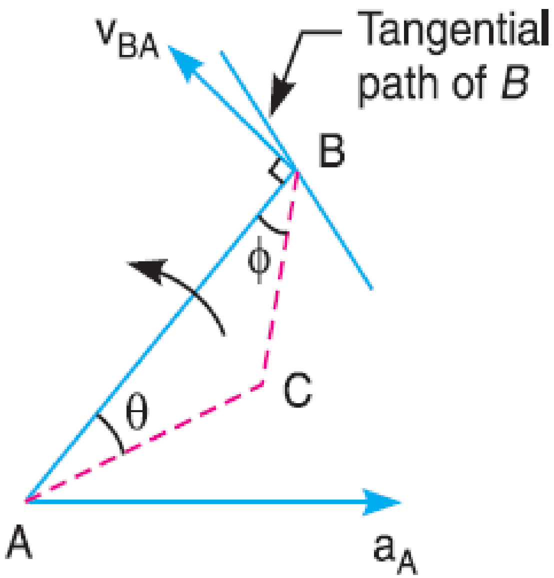
Total acceleration of B with respect to A is the vector sum of radial component and tangential component of acceleration

$$\vec{a}_{BA} = \vec{a}^{\,r}_{BA} + \vec{a}^{\,t}_{BA}$$

ACCELERATION OF A POINT ON A LINK

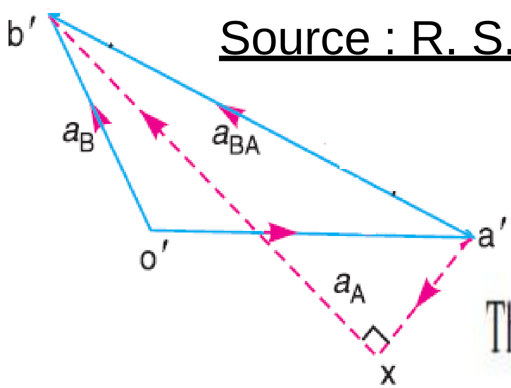
Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given.

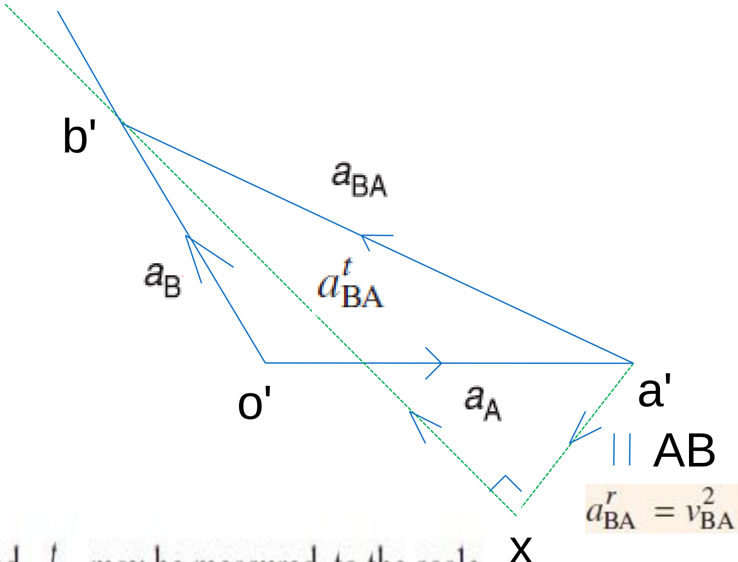
How to determine a_B ?
Draw acceleration diagram.



Points on a Link.

Source : R. S. Khurmi





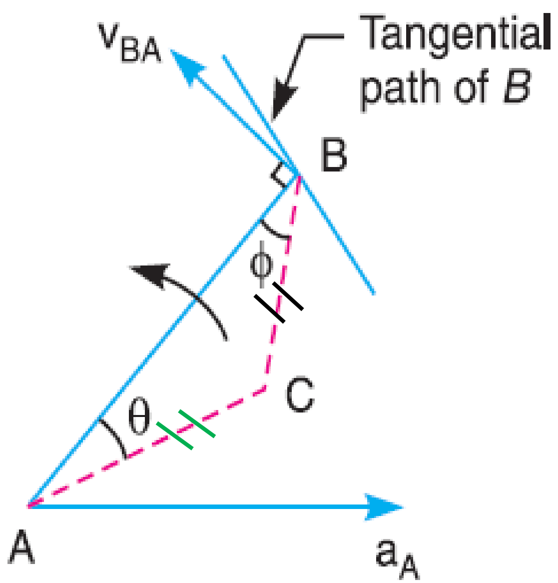
$a^r_{BA} = v^2_{BA} / AB$

The values of a_B, a_{BA} and a^t_{BA} may be measured, to the scale.

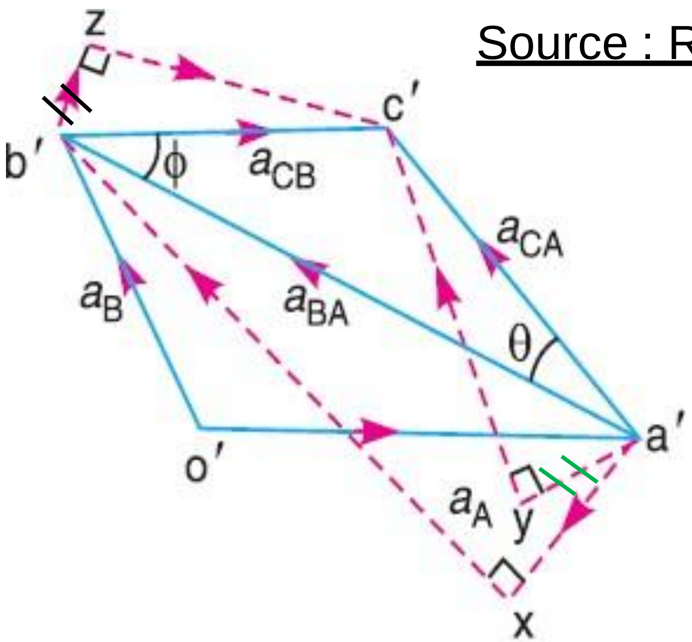
ACCELERATION OF A POINT ON A LINK

For any other point C on the link, draw triangle a'b'c' similar to triangle ABC.

Source : R. S. Khurmi



Points on a Link.

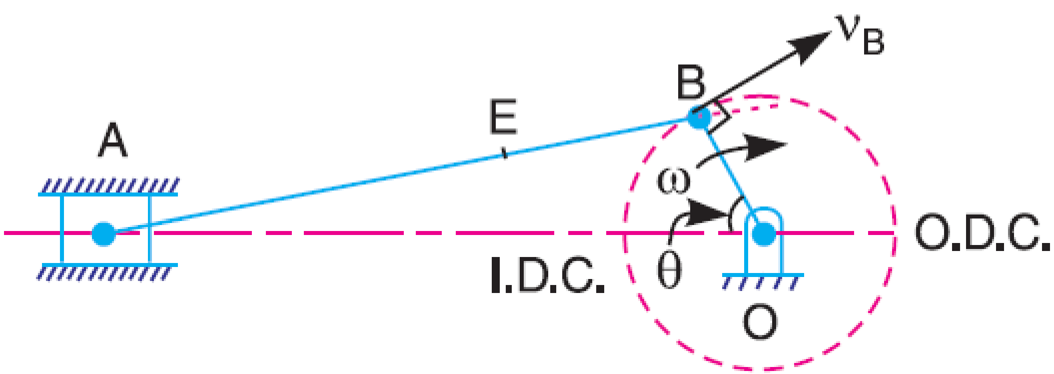


Acceleration diagram.

Mathematically, angular acceleration of the link *A B*,

$$\alpha_{AB} = a^t_{BA} \,/\, AB$$

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.

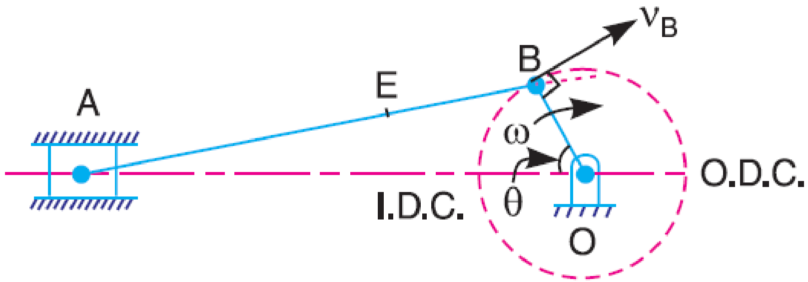
Source : R. S. Khurmi

$$v_{BO} = v_B = \omega_{BO} \times OB, \text{ acting tangentially at } B.$$

$$a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{OB}$$

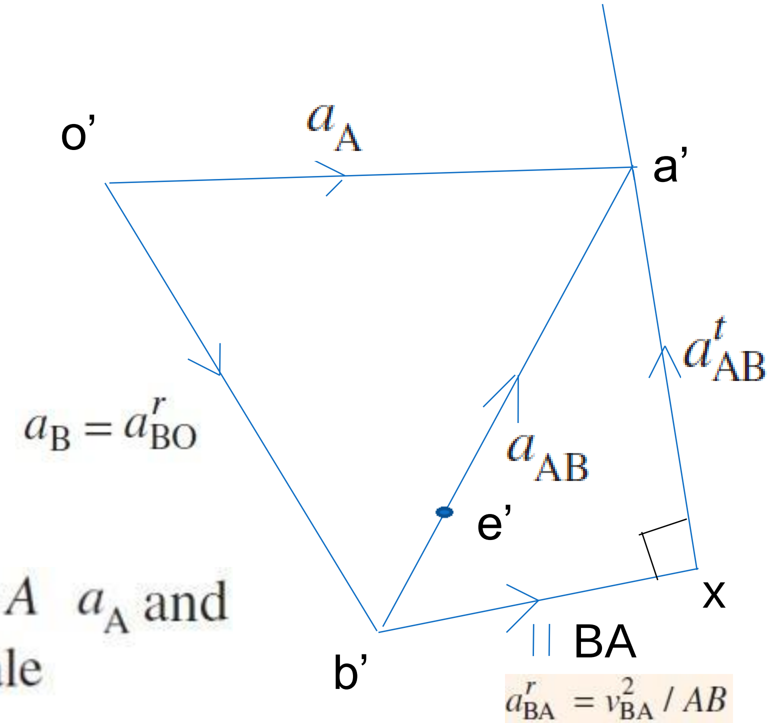
A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

ACCELERATION IN SLIDER CRANK MECHANISM



Slider crank mechanism.

Source : R. S. Khurmi



acceleration of the piston or the slider A a_A and a^t_{AB} may be measured to the scale

Point e' can be fixed using $a'e'/a'b' = AE/AB$

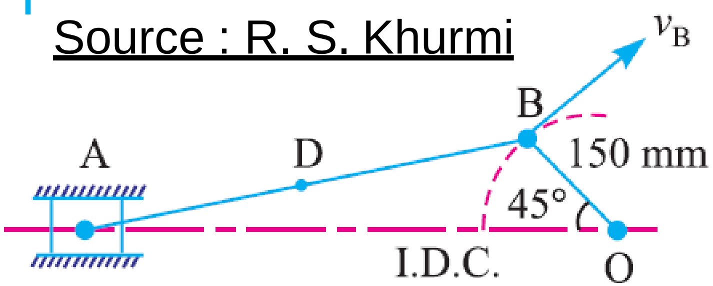
angular acceleration of AB, $\alpha_{AB} = a^t_{AB} / AB$

NUMERICAL EXAMPLE -1

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi



Space diagram.

Solution.

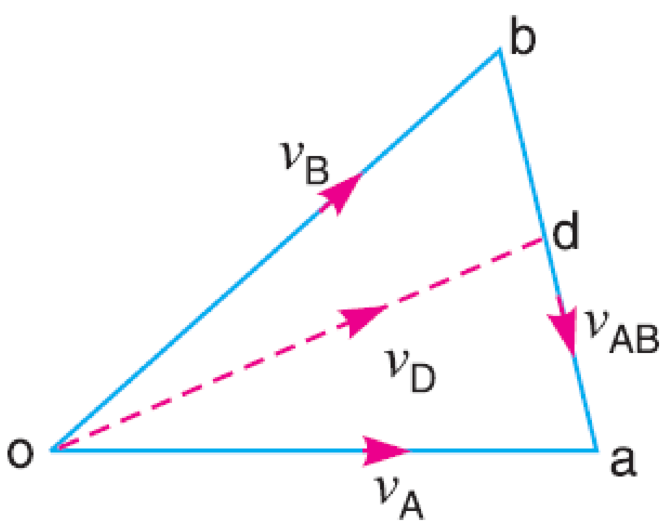
Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s;

OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

By measurement, v_{AB} = vector ba = 3.4 m/s

v_A = vector oa = 4 m/s



Velocity diagram.

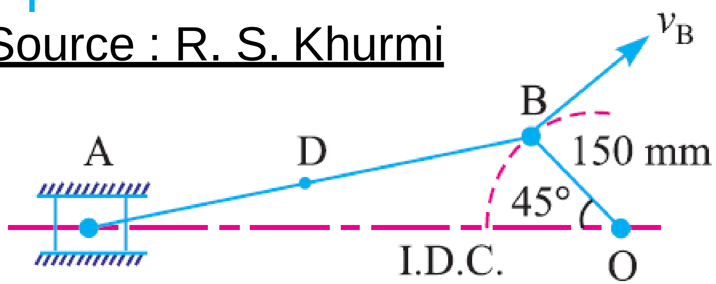
Since D is the midpoint of AB, d is also midpoint of vector ba.

velocity of the midpoint D

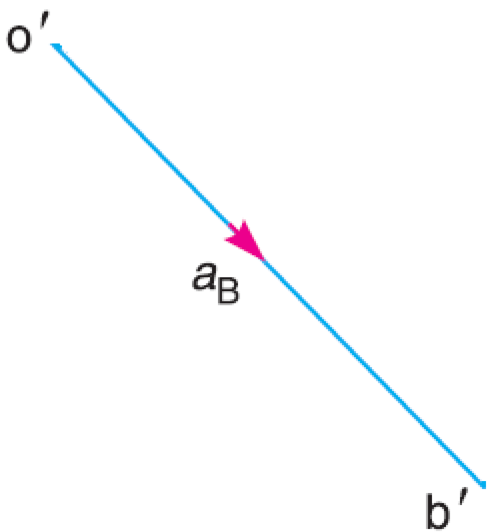
v_D = vector od = 4.1 m/s **Ans.**

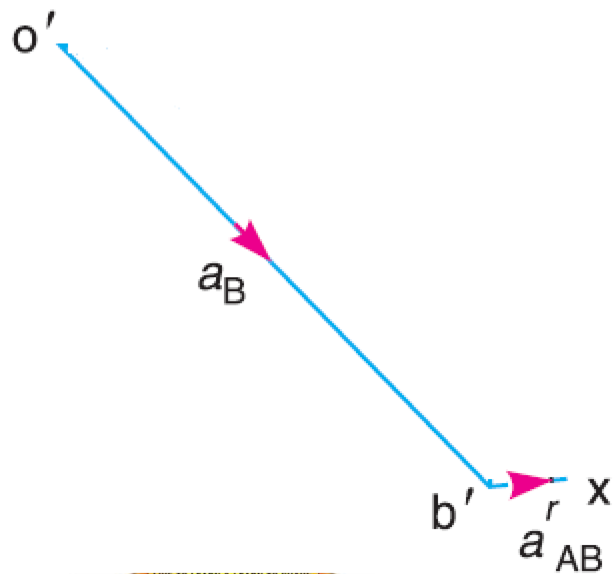
NUMERICAL EXAMPLE -1

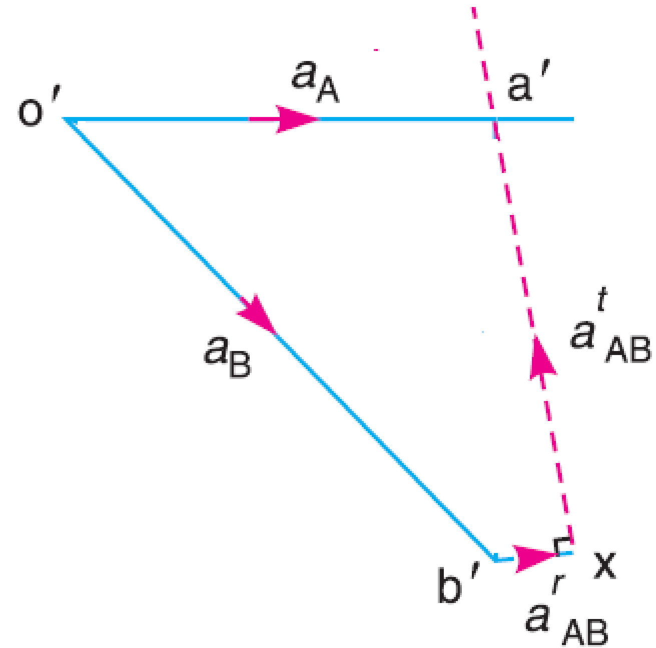
Source : R. S. Khurmi



Space diagram.



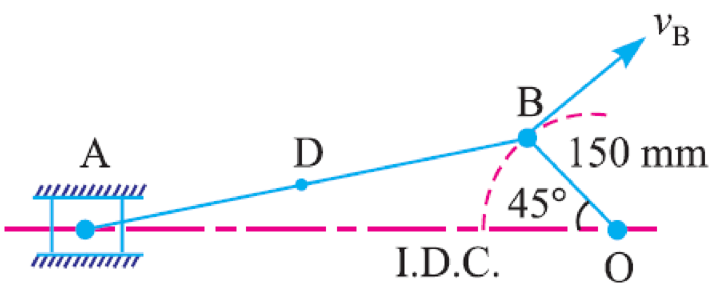




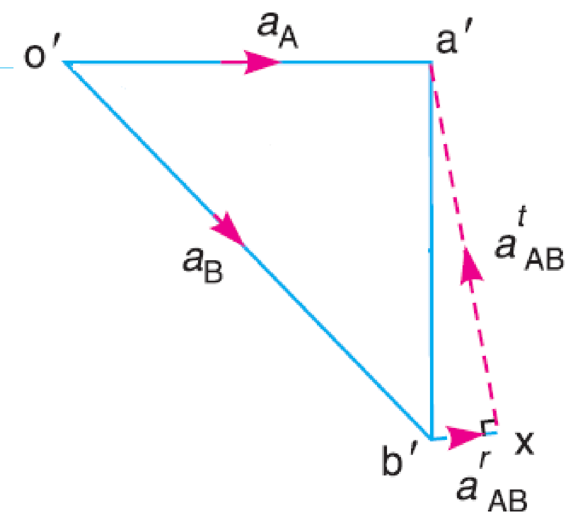
$$a^r_{BO} = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

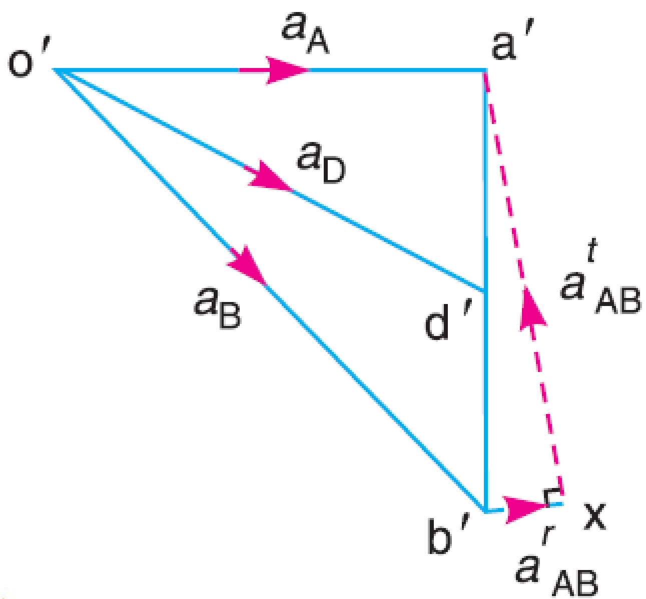
NUMERICAL EXAMPLE -1



Space diagram.



Source : R. S. Khurmi



By measurement, a_D = vector $o'\,d'$ = 117 m/s^2 **Ans.**

Angular velocity of the connecting rod

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, $a^t_{AB} = 103$ m/s^2

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \quad \textbf{Ans.}$$

TUTORIAL PROBLEM-1

The dimensions and configuration of the four bar mechanism, shown in Fig. 8.10, are as follows :

P_1A = 300 mm; P_2B = 360 mm; AB = 360 mm, and P_1P_2 = 600 mm.

The angle $AP_1P_2 = 60°$. The crank P_1A has an angular velocity of 10 rad/s and an angular acceleration of 30 rad/s², both clockwise. Determine the angular velocities and angular accelerations of P_2B, and AB and the velocity and acceleration of the joint B.

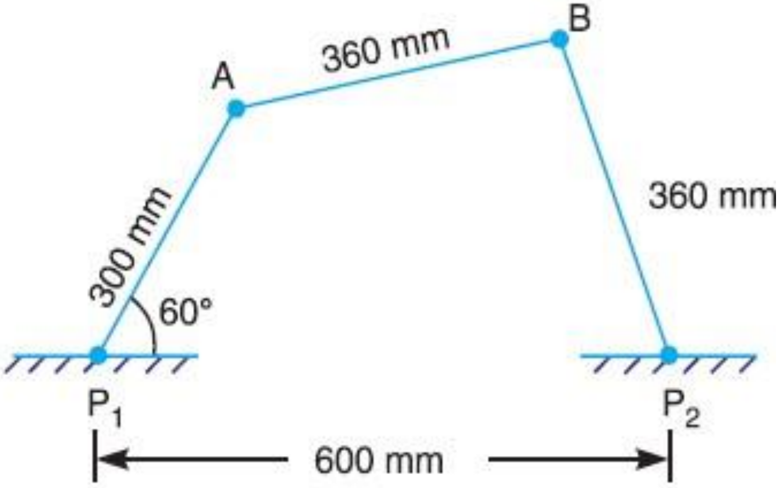


Fig. 8.10

Source : R. S. Khurmi

$v_{BP2} = v_B = 2.2$ m/s **Ans.**

$\omega_{P2B} = \frac{v_{BP_2}}{P_2B} = \frac{2.2}{0.36} = 6.1$ rad/s **Ans.**

$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{2.05}{0.36} = 5.7$ rad/s **Ans.**

$a_B = 29.6$ m/s² **Ans.**

$\alpha_{P2B} = \frac{a^t_{BP_2}}{P_2B} = \frac{26.6}{0.36} = 73.8$ rad/s² **Ans.**

$\alpha_{AB} = \frac{a^t_{BA}}{AB} = \frac{13.6}{0.36} = 37.8$ rad/s² **Ans.**

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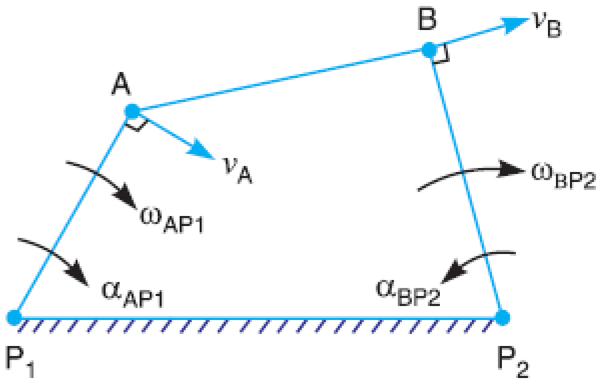
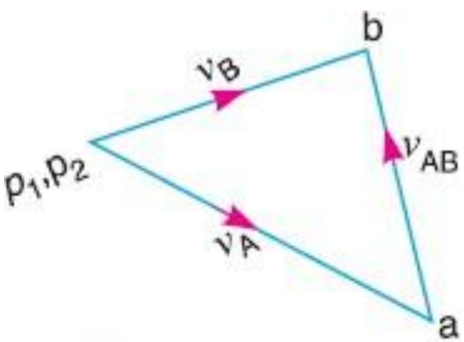
TUTORIAL PROBLEM-1

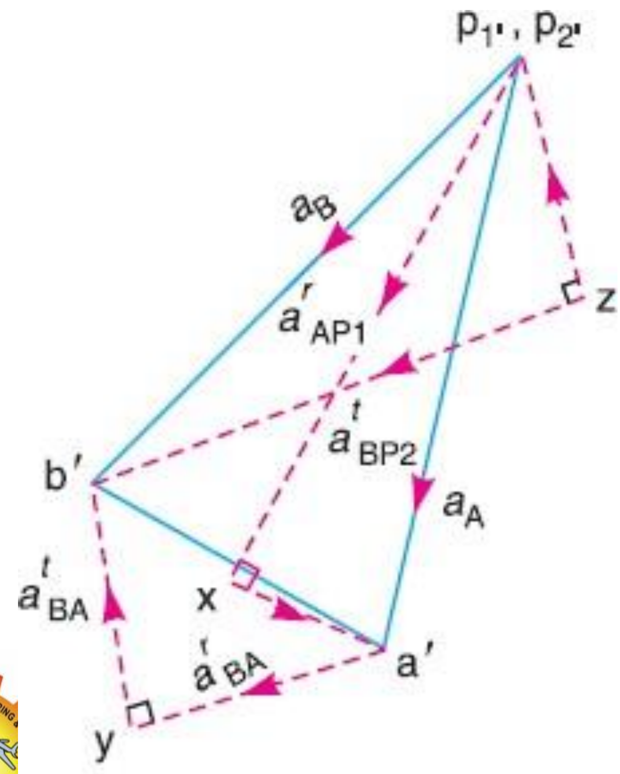
Source : R. S. Khurmi



Space diagram.

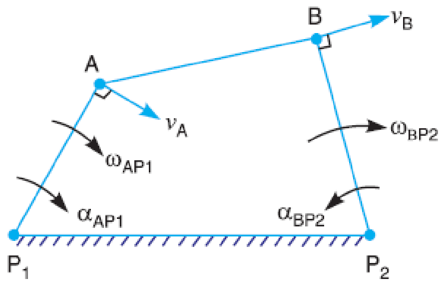


Velocity diagram.



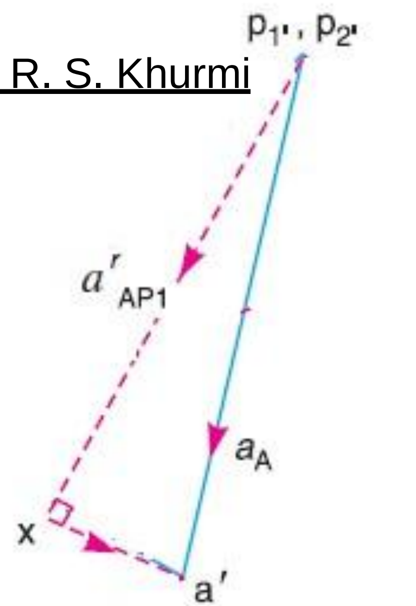
Acceleration diagram

RIAL PROBLEM-1



Space diagram.

Source : R. S. Khurmi

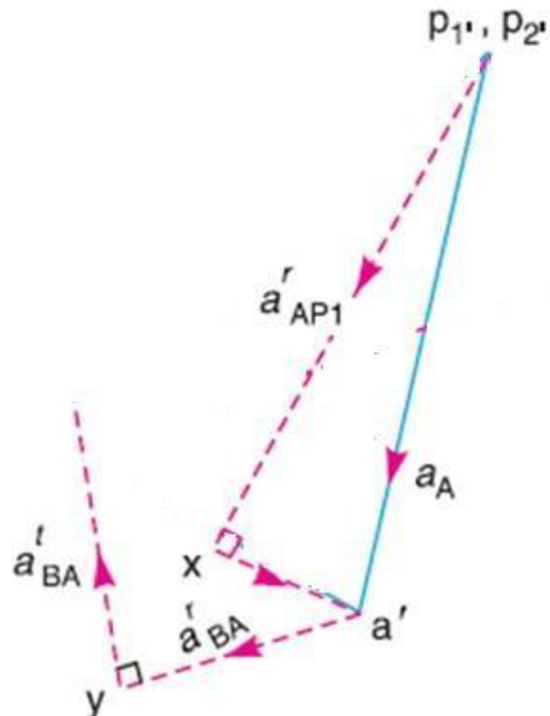


vector $p_1'x = a^r_{AP_1} = 30 \text{ m/s}^2$

vector $xa' = a^t_{AP_1} = 9 \text{ m/s}^2$

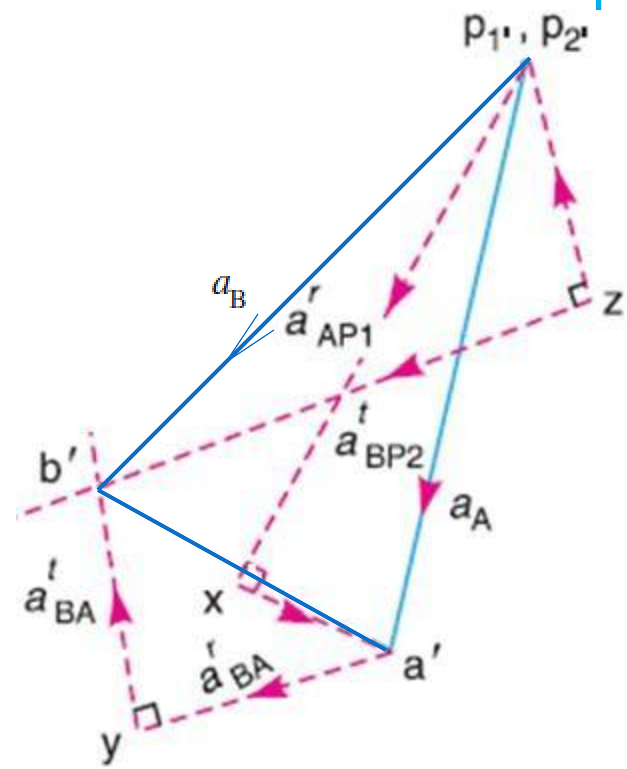
By measurement,

$a_A = a_{AP1} = 31.6 \text{ m/s}^2$



vector $a'y = a^r_{BA} = 11.67 \text{ m/s}^2$

a^t_{BA} magnitude is yet unknown



$p_2'z = a^r_{BP_2} = 13.44 \text{ m/s}^2$

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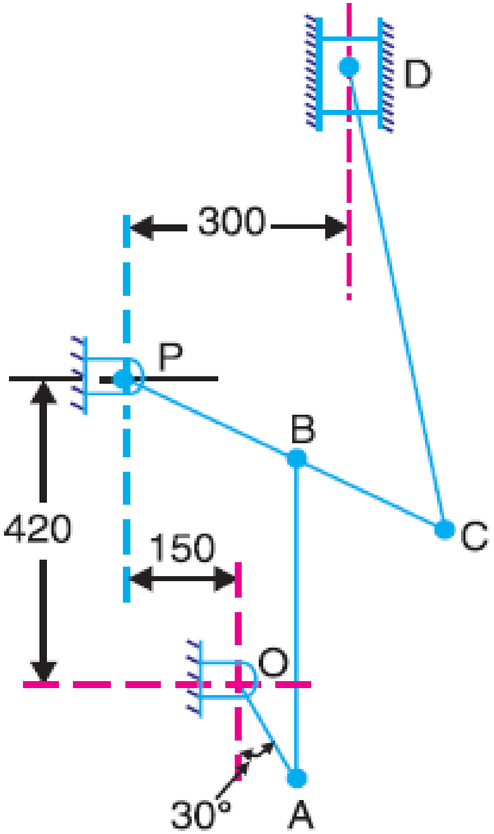
EXERCISE-1

Find out the acceleration of the slider D and the angular acceleration of link CD for the engine mechanism shown in Fig. 8.14.

The crank OA rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are: OA = 150 mm ; AB = 450 mm; PB = 240 mm ; BC = 210 mm ; CD = 660 mm.



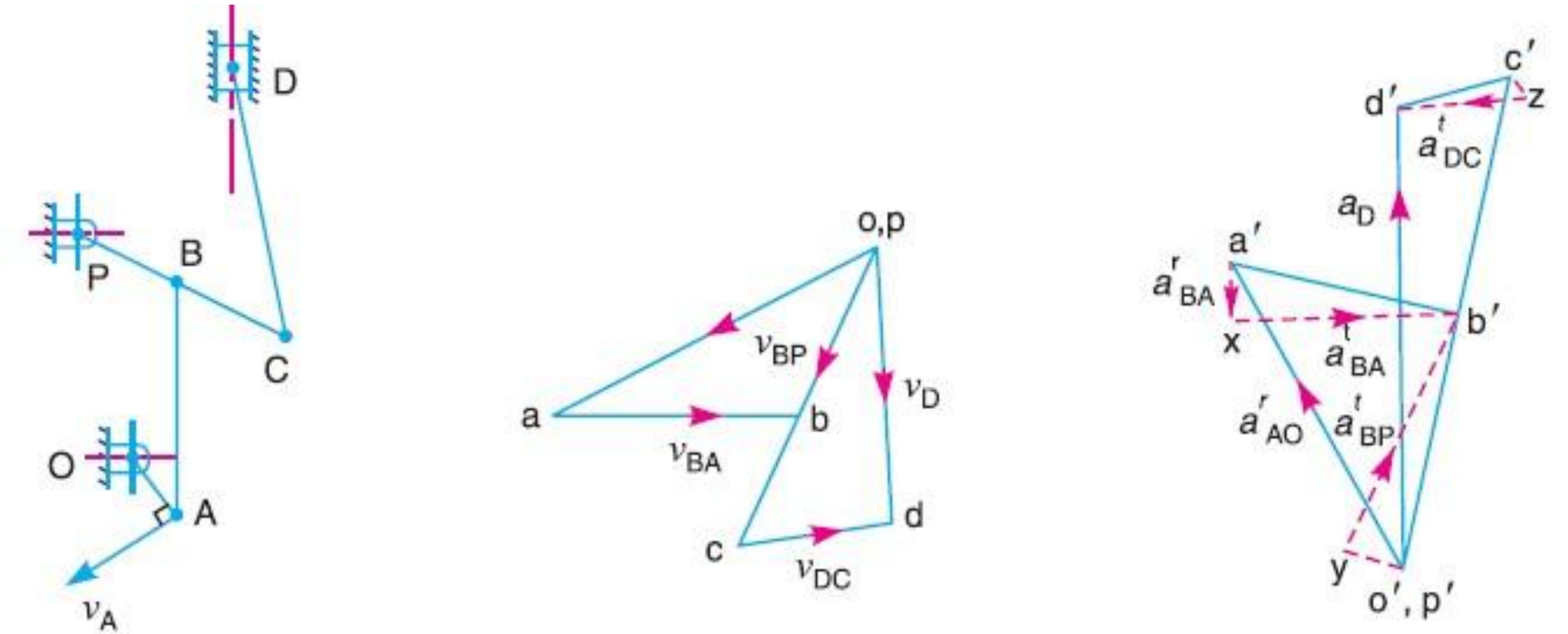
All dimensions in mm.

Fig. 8.14

Source : R. S. Khurmi

ANSWER

Source : R. S. Khurmi



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

a_D = vector $o'd'$ = 69.6 m/s² **Ans.**

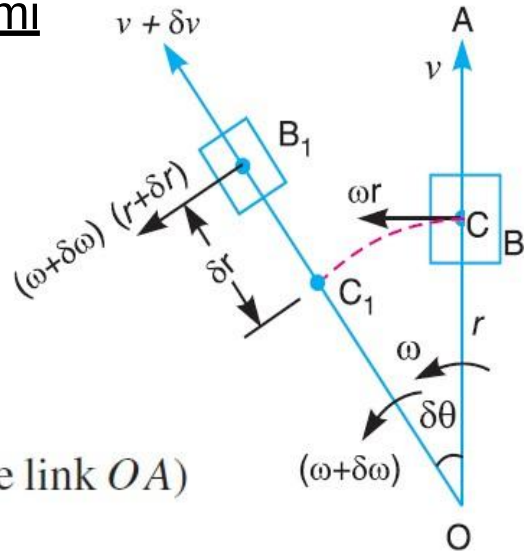
$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \quad \textbf{Ans.}$$

CORIOLIS COMPONENT OF ACCELERATION

Where?

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism

Source : R. S. Khurmi



Let ω = Angular velocity of the link OA at time t seconds.

v = Velocity of the slider B along the link OA at time t seconds.

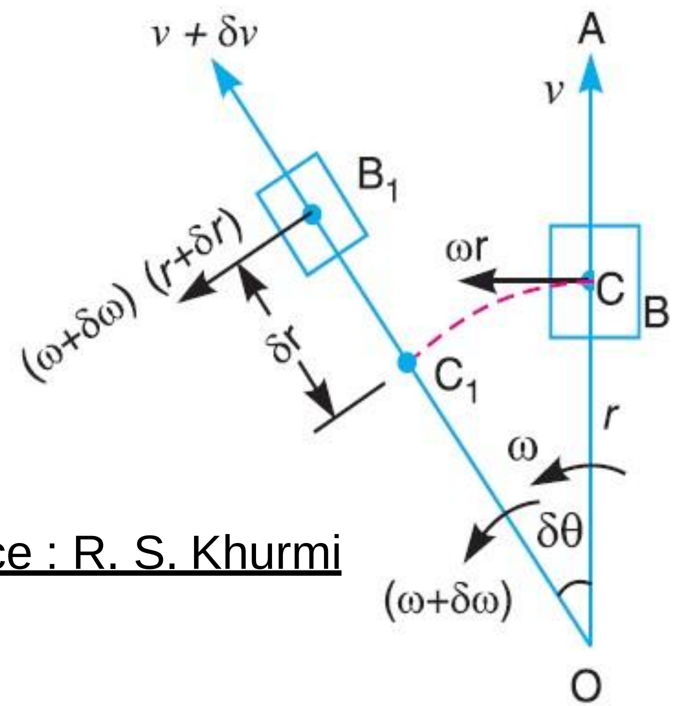
$\omega.r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\,(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.

CORIOLIS COMPONENT OF ACCELERATION

The tangential component of acceleration of the slider B with <u>respect to the coincident point C</u> on the link is known as coriolis component of acceleration and is always perpendicular to the link.



Source : R. S. Khurmi

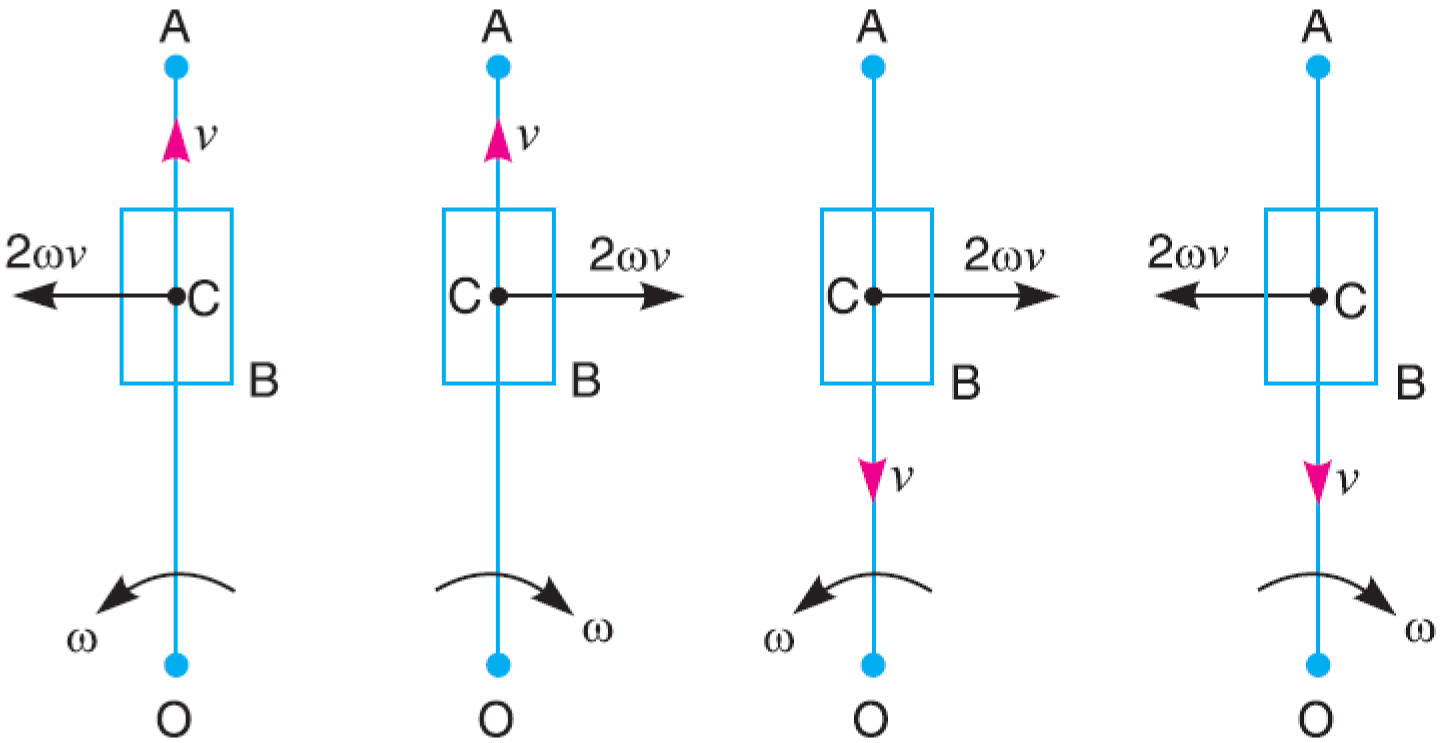
∴ Coriolis component of the acceleration of B with respect of C,

$$a^{c}_{BC} = a^{t}_{BC} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

CORIOLIS COMPONENT OF ACCELERATION



Direction of coriolis component of acceleration.

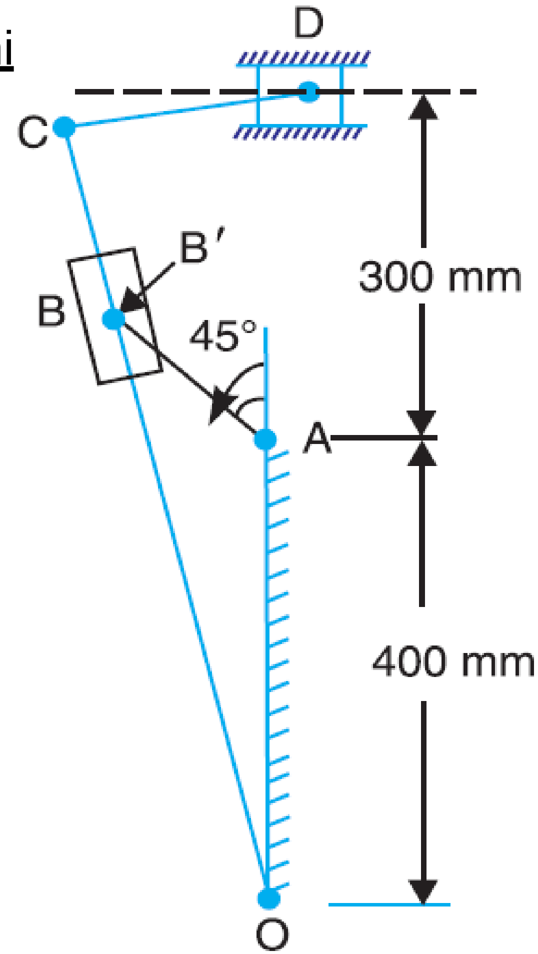
Source : R. S. Khurmi

NUMERICAL EXAMPLE -1

Source : R. S. Khurmi

A mechanism of a crank and slotted lever quick return motion is shown in the Fig. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever.

Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.



NUMERICAL EXAMPLE -1 (CONSTRUCTION OF VELOCITY DIAGRAM)

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\,\pi \times 120/60 = 12.57$ rad/s ; AB = 150 mm = 0.15 m; OC = 700 mm = 0.7 m; CD = 200 mm = 0.2 m

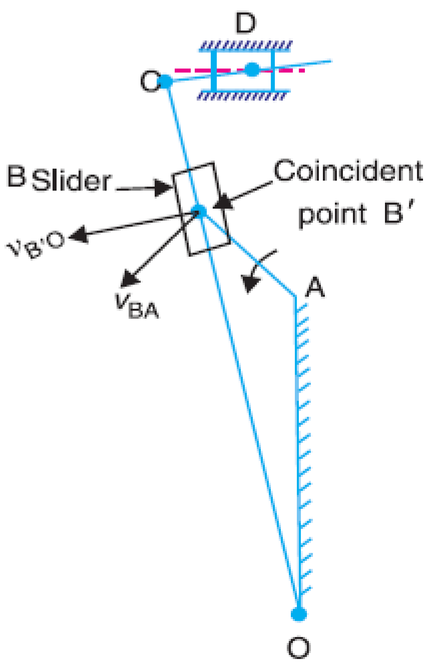
We know that velocity of B with respect to A,

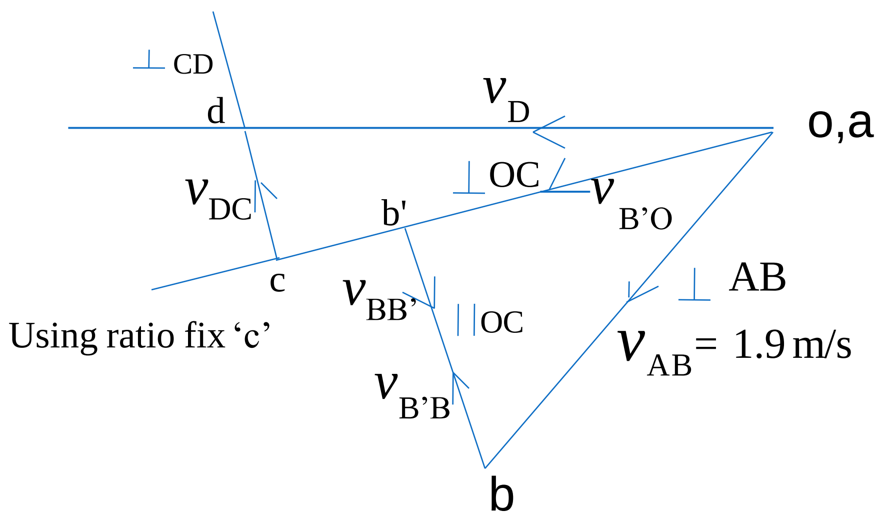
$$v_{BA} = \omega_{BA} \times AB$$

$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to AB)

<u>Source : R. S. Khurmi</u>





NUMERICAL EXAMPLE -1

v_D = vector od = 2.15 m/s **Ans.**

From velocity diagram by measurement :

From velocity diagram, we also find that

Velocity of B with respect to B',

$v_{BB'}$ = vector $b'b$ = 1.05 m/s

Velocity of D with respect to C,

v_{DC} = vector cd = 0.45 m/s

Velocity of B' with respect to O

$v_{B'O}$ = vector ob' = 1.55 m/s

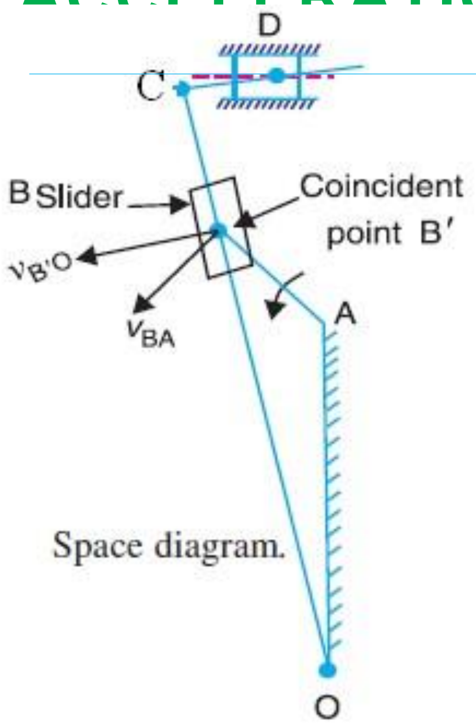
Velocity of C with respect to O,

v_{CO} = vector oc = 2.15 m/s

∴ Angular velocity of the link OC or OB',

$$\omega_{CO} = \omega_{B'O} = \frac{v_{CO}}{OC} = \frac{2.15}{0.7} = 3.07 \text{ rad/s}$$

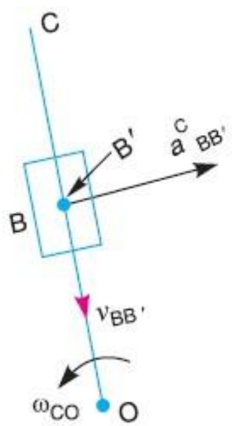
NUMERICAL EXAMPLE -1 (CONSTRUCTION OF ACCELERATION DIAGRAM)



Space diagram.

Link	Radial accel.	Tangen. accel.	Coriolis Accel.
AB	$a^r_{BA} = \omega^2_{BA} \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$	Zero	Nil
BB'	Direction $a^r_{BB'}$ wn.	-	$a^c_{BB'} = 2\omega.v = 2\omega_{CO}.v_{BB'} = 2 \times 3.07 \times 1.05 = 6.45$ m/
DC	$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$	Direction known a^t_{DC}	Nil
B'O	$a^r_{B'O} = \frac{v^2_{B'O}}{B'O} = \frac{(1.55)^2}{0.52} = 4.62 \text{ m/s}^2$	Direction known.	Nil

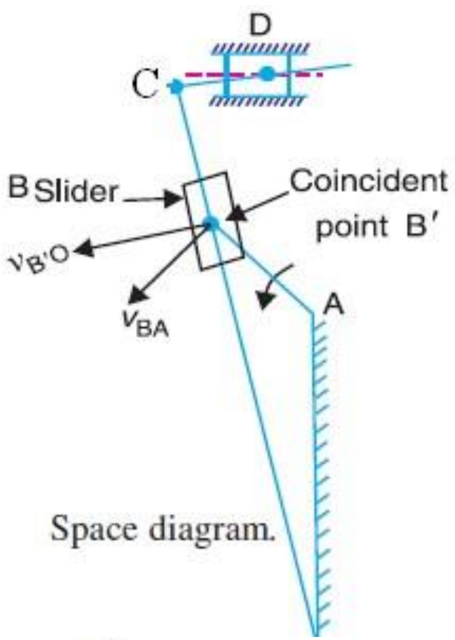
$a^t_{B'O}$



Direction of coriolis component.

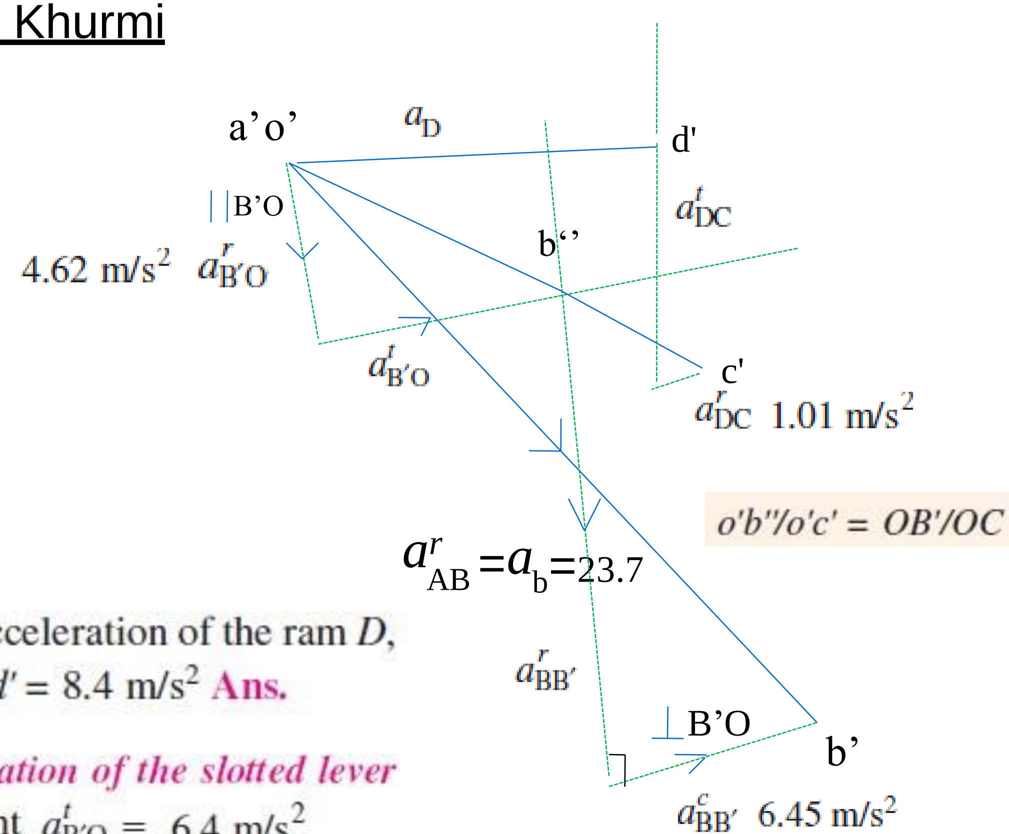
Source : R. S. Khurmi

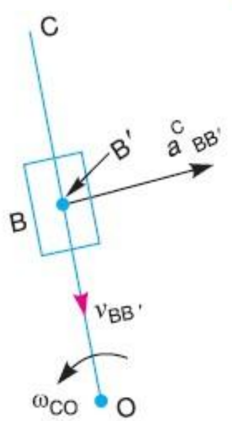
NUMERICAL EXAMPLE -1 (CONSTRUCTION OF ACCELERATION DIAGRAM)



Space diagram.

Source : R. S. Khurmi





Direction of coriolis component.

By measurement, acceleration of the ram D,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2 \textbf{ Ans.}$$

Angular acceleration of the slotted lever

By measurement $a^t_{B'O} = 6.4 \text{ m/s}^2$

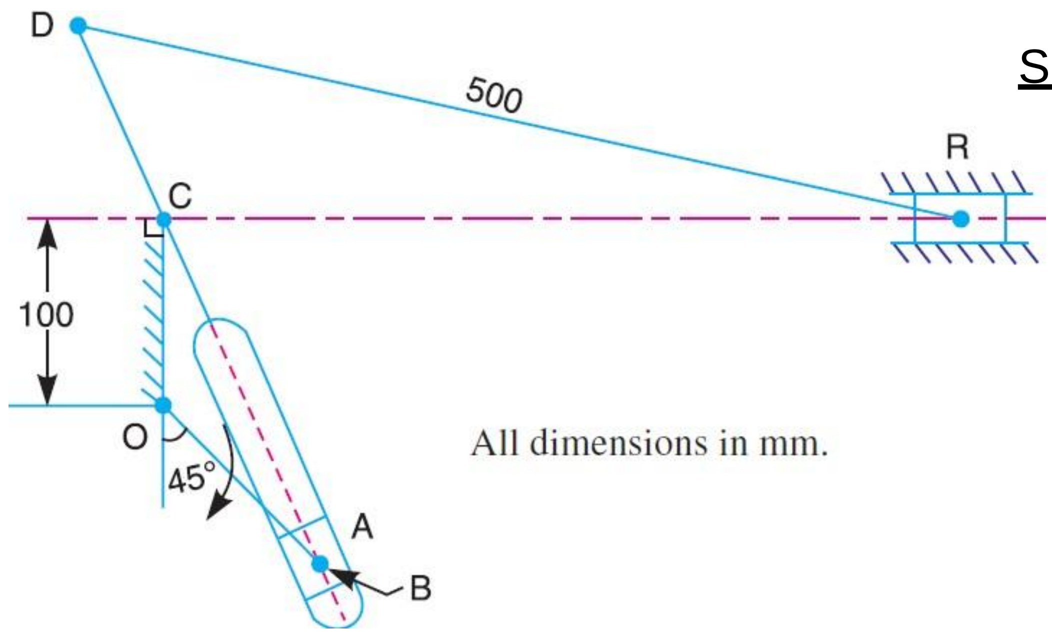
angular acceleration of the slotted lever,

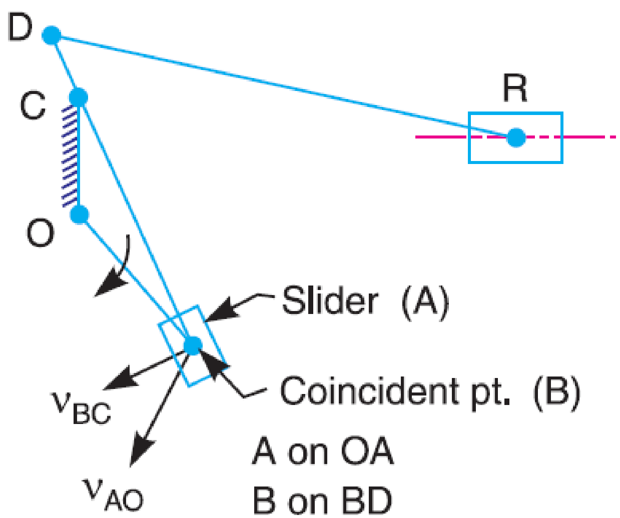
$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \textbf{ Ans.}$$

TUTORIAL PROBLEM

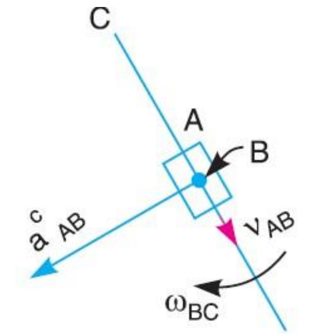
In a Whitworth quick return motion, as shown in the Fig., OA is a crank rotating at 30 r.p.m. in a clockwise direction. The dimensions of various links are : OA = 150 mm; OC = 100 mm; CD = 125 mm; and DR = 500 mm. Determine the acceleration of the sliding block R and the angular acceleration of the slotted lever CA.

Source : R. S. Khurmi



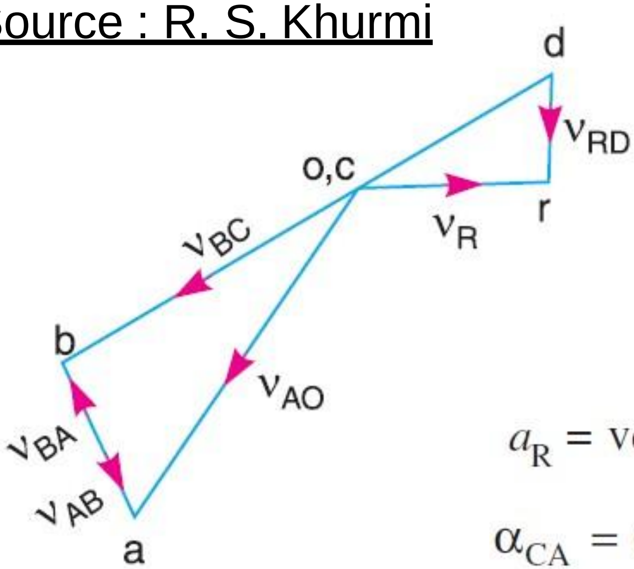


Space diagram.

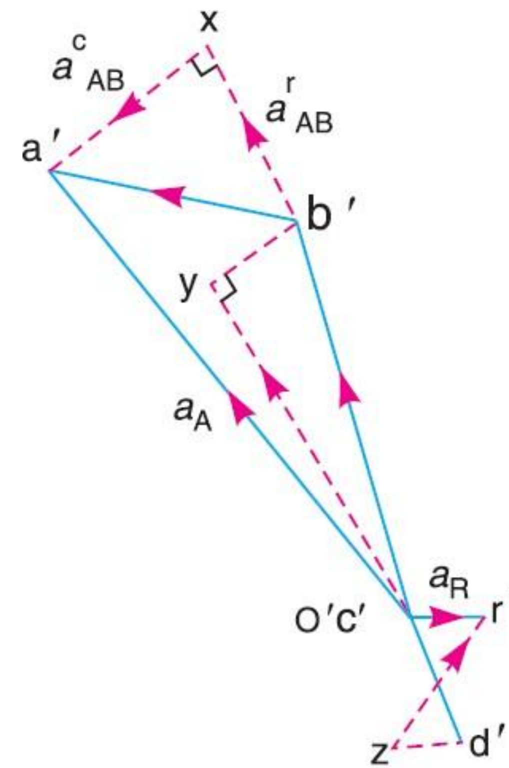


Direction of coriolis component.

<u>Source : R. S. Khurmi</u>



Velocity diagram.



Acceleration diagram.

a_R = vector $c'r'$ = 0.18 m/s^2 **Ans.**

$$\alpha_{CA} = \alpha_{BC} = \frac{a^t_{CB}}{BC} = \frac{0.14}{0.24} = 0.583 \text{ rad/s}^2$$ **Ans.**

UNIT-III

STRAIGHT LINE MOTION MECHANISMS

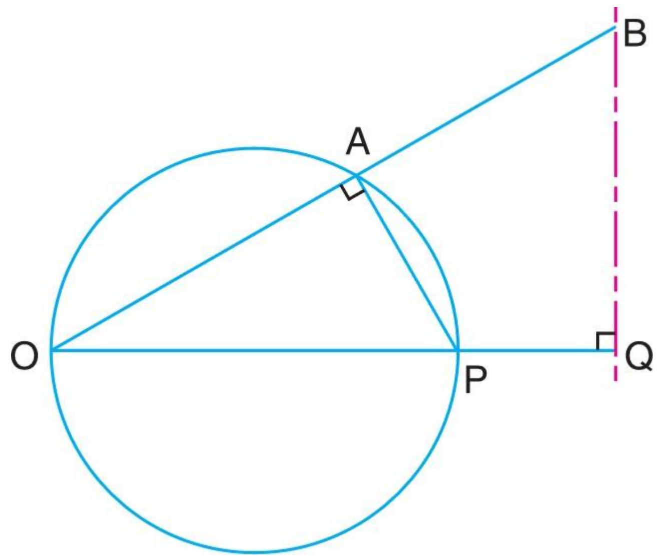
- One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line.
- The mechanisms used for this purpose are called straight line mechanisms. These mechanisms are of the following two types:
  - in which only turning pairs are used, an
  - in which one sliding pair is used.

These two types of mechanisms may produce **exact straight line motion or approximate straight line motion**, as discussed in the following articles.

EXACT STRAIGHT LINE MOTION MECHANISMS MADE UP OF TURNING PAIRS

- Let O be a point on the circumference of a circle of diameter OP.
- Let OA be any chord and B is a point on OA produced, such that,

$$OA \times OB = \text{constant}$$



Exact straight line motion mechanism

$$\frac{OA}{OP} = \frac{OQ}{OB}$$

$$OP \times OQ = OA \times OB$$

$$OQ = \frac{OA \times OB}{OP}$$

But OP is constant as it is the diameter of a circle, therefore, if OA×OB is constant, then OQ will be constant.
Hence the point B moves along the straight path BQ which is perpendicular to OP.

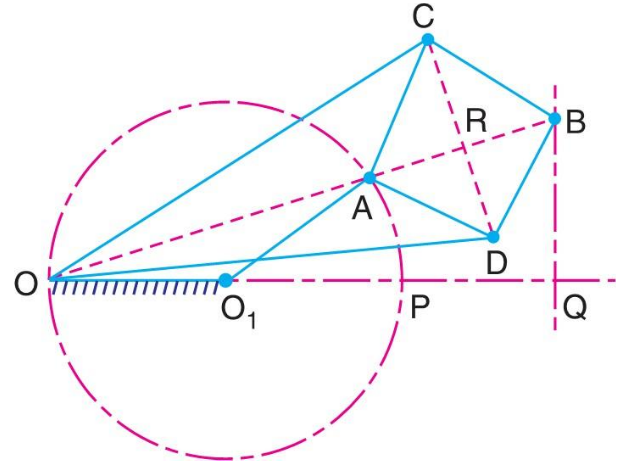
PEAUCELLIER MECHANISM

- It consists of a fixed link OO1 and the other straight links O1A , OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in Fig.
- The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O1A.

$$AC = CB = BD = DA \; ; \; OC = OD \; ; \text{ and } OO_1 = O_1A$$

$$OC^2 = OR^2 + RC^2 \qquad \ldots(i)$$

$$BC^2 = RB^2 + RC^2 \qquad \ldots(ii)$$



Subtracting equation (*ii*) from (*i*), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$

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HART'S MECHANISM

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.
- It consists of a fixed link OO1 and other straight links O1A , FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in Fig.
- The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio.
- A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to *FD and CE.
- Hence OAB is a straight line. It may be proved now that the product OA× OB is constant.

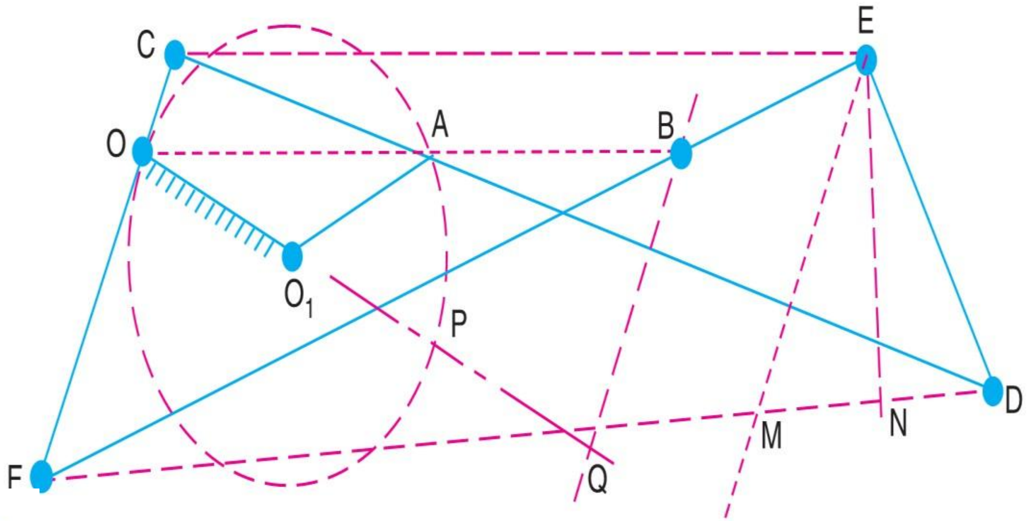
HART'S MECHANISM

From similar triangles *CFE* and *OFB*,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC}$$

and from similar triangles *FCD* and *OCA*

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC}$$



It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O1, then the point B will trace a straight line perpendicular to the diameter OP produced.

HART'S MECHANISM

Multiplying equations **(*i*)** and **(*ii*)**, we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of *OC*, *OF* and *FC* are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \qquad \ldots(iii)$$

$$\ldots\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

Now from point *E*, draw *EM* parallel to *CF* and *EN* perpendicular to *FD*. Therefore

$$FD \times CE = FD \times FM \qquad \ldots(\because CE = FM)$$

$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \qquad \ldots(\because MN = ND)$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles *FEN* and *EDN*)

$$= FE^2 - ED^2 = \text{constant} \qquad \ldots(iv)$$

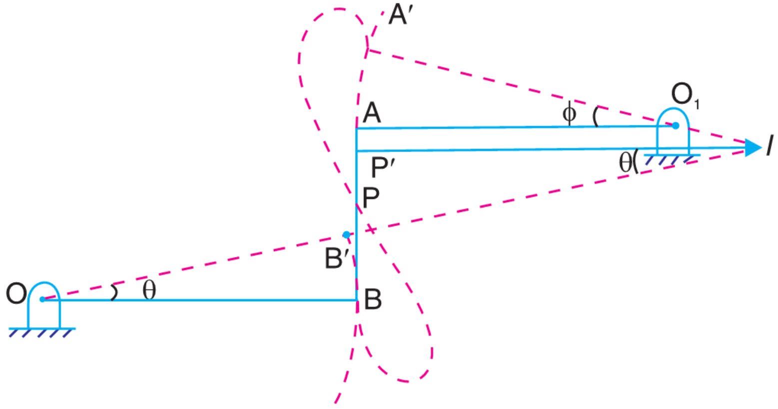
...($\because$ Length *FE* and *ED* are fixed)

From equations **(*iii*)** and **(*iv*)**,

$$OA \times OB = \text{constant}$$

APPROXIMATE STRAIGHT LINE MOTION MECHANISMS

- The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms. Following mechanisms to give approximate straight line motion, are important from the subject point of view:
- **Watt's mechanism:** It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.



$$\text{arc } BB' = \text{arc } AA' \quad \text{or} \quad OB \times \theta = O_1A \times \phi$$

$\therefore \quad OB / O_1A = \phi / \theta$

Also $\quad A'P' = IP' \times \phi$, and $B'P' = IP' \times \theta$

$\therefore \quad A'P' / B'P' = \phi / \theta$

From equations *(i)* and *(ii)*,

$$\frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP} \quad \text{or} \quad \frac{O_1A}{OB} = \frac{PB}{PA}$$

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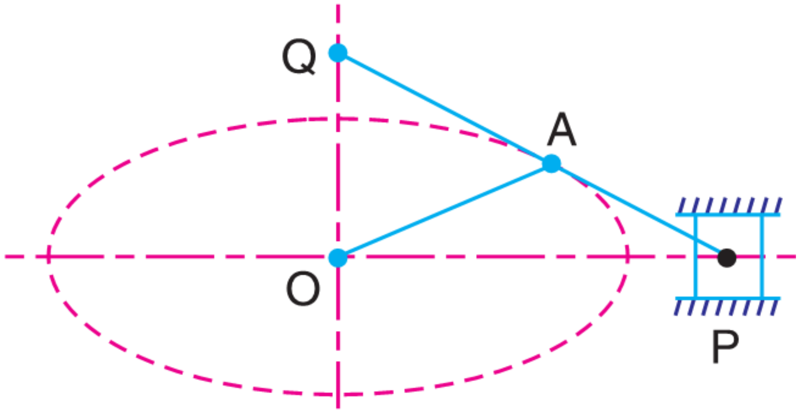
MODIFIED SCOTT-RUSSEL MECHANISM

- This mechanism is similar to Scott-Russel mechanism but in this case AP is not equal to AQ and the points P and Q are constrained to **move in the horizontal and vertical directions.**
- A little consideration will show that it forms an elliptical trammel, so that any point A on PQ traces an ellipse with semi-major axis AQ and semi-minor axis AP.

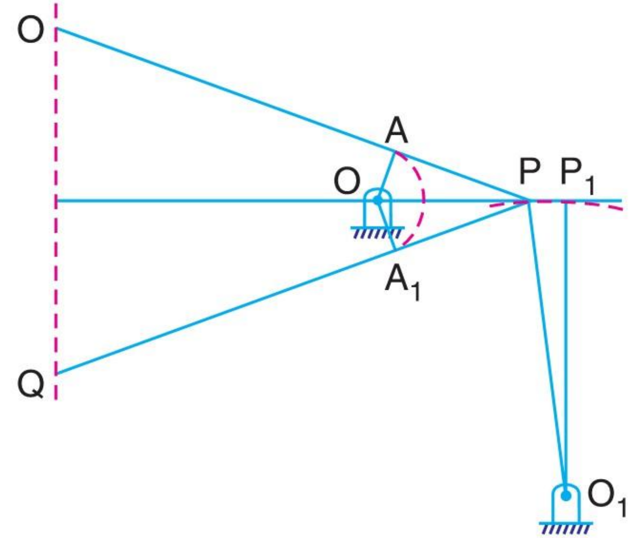
If the point A moves in a circle, then for point Q to move along an approximate straight line, the length OA must be equal (AP)2/AQ. This is limited to only small displacement of P.



Modified Scott-Russel mechanism

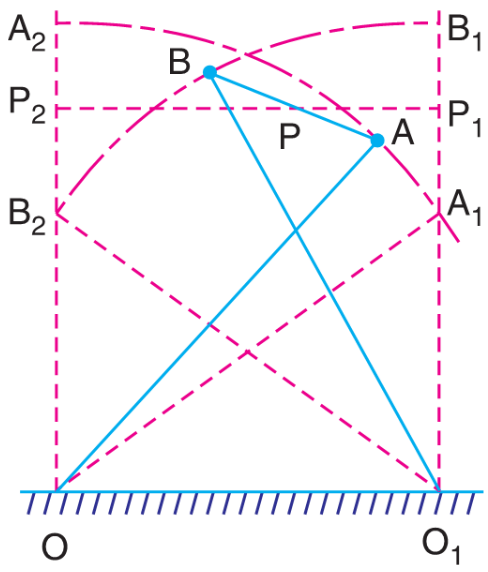
GRASSHOPPER MECHANISM

- This mechanism is a **modification of modified Scott-Russel's mechanism** with the difference that the point P does not slide along a straight line, but moves in a circular arc with centre O.
- It is a four bar mechanism and **all the pairs are turning pairs** as shown in Fig. In this mechanism, the centres O and O1 are fixed. The link OA oscillates about O through an angle AOA1 which causes the pin P to move along a circular arc with O1 as centre and O1P as radius. OA= (AP)2/AQ.



TCHEBICHEFF'S MECHANISM

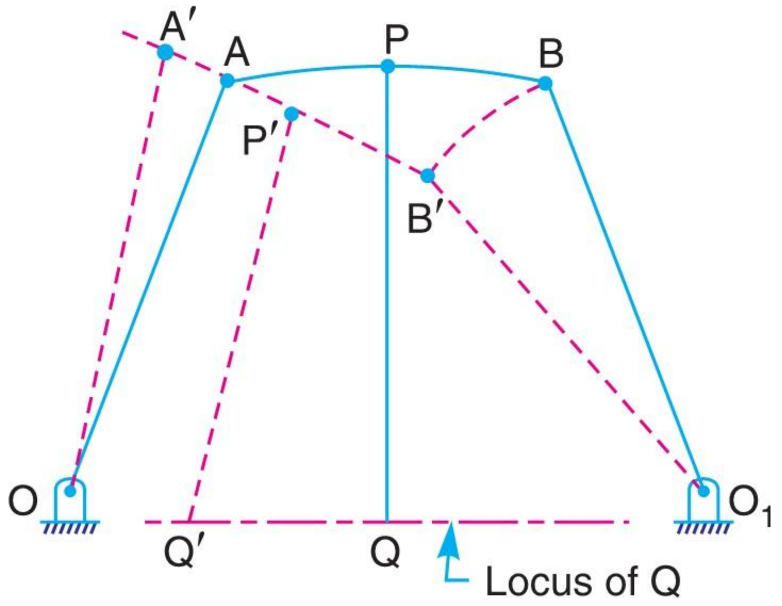
- It is a four bar mechanism in which the **crossed links OA and O1B are of equal length,** as shown in Fig. The point P, which is the mid-point of AB traces out an approximately straight line parallel to OO1.
- The proportions of the links are, usually, such that point P is exactly above O or O1 **in the extreme positions of the mechanism** i.e. when BA lies along OA or when BA lies along BO1.



It may be noted that the point P will lie on a straight line parallel to OO1, in the two extreme positions and in the mid position, if the **lengths of the links are in proportions AB: OO1: OA= 1 : 2 : 2.5.**

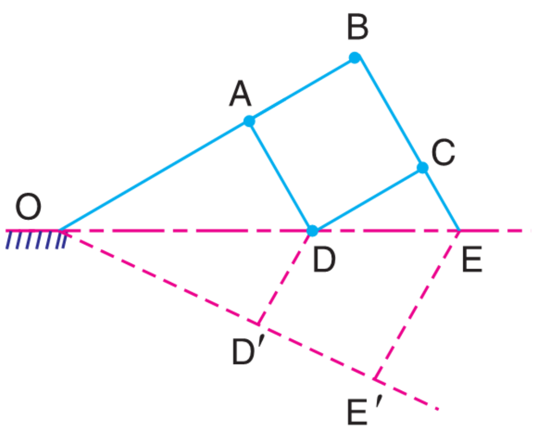
ROBERTS MECHANISM

- **It is also a four bar chain mechanism,** which, in its mean position, has the form of a trapezium.
- The links OA and O1 Bare of equal length and OO1 is fixed. A bar PQ is rigidly attached to the link AB at its middle point P.



PANTOGRAPH

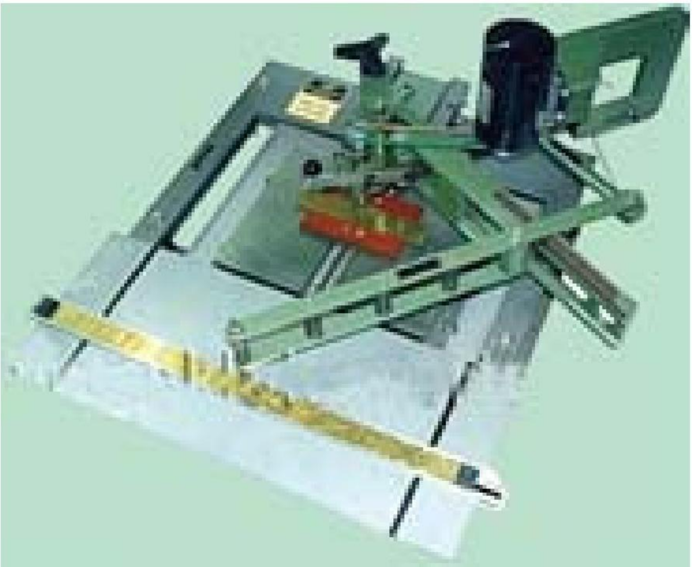
- A pantograph is an **instrument used to reproduce to an enlarged or a reduced scale** and as exactly as possible the path described by a given point.
- It consists of a jointed parallelogram ABCD as shown in Fig. It is made up of bars **connected by turning pairs.** The bars BA and BC are extended to O and E respectively, such that $OA/OB = AD/BE$
  - Thus, for all relative positions of the bars, the triangles OAD and OBE are similar and the points O, D and E are in one straight line.
  - It may be proved that point E traces out the same path as described by point D.



PANTOGRAPH

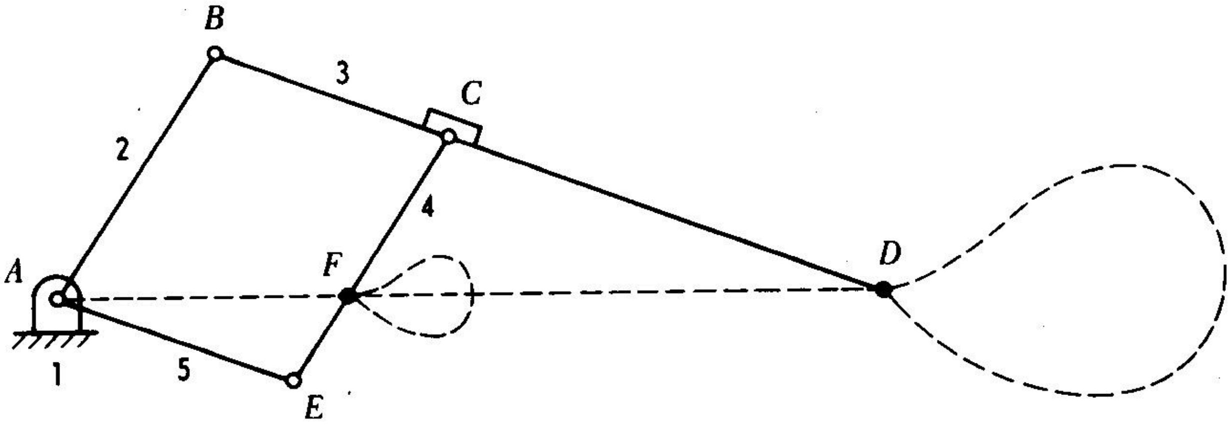
- From similar triangles OAD and OBE, we find that, $OD/OE = AD/BE$

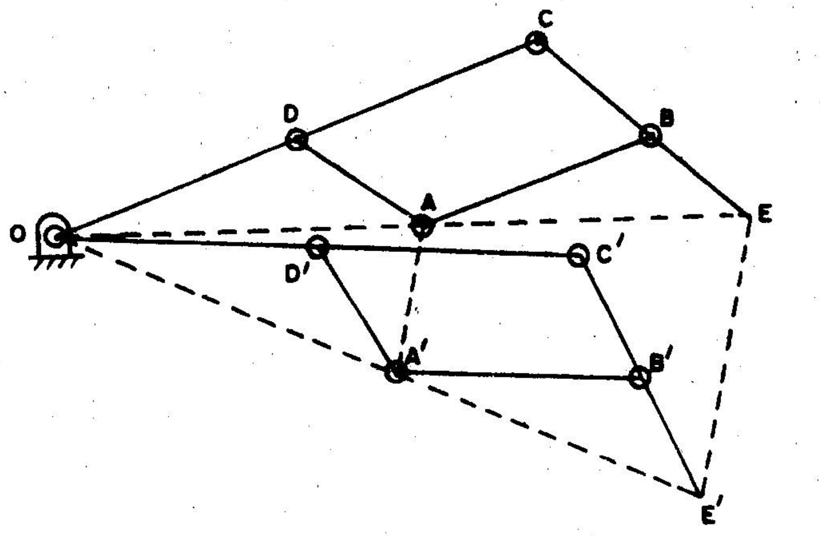
Let point O be fixed and the points D and E move to some new positions D′ and E′. Then $OD/OE = OD'/OE'$



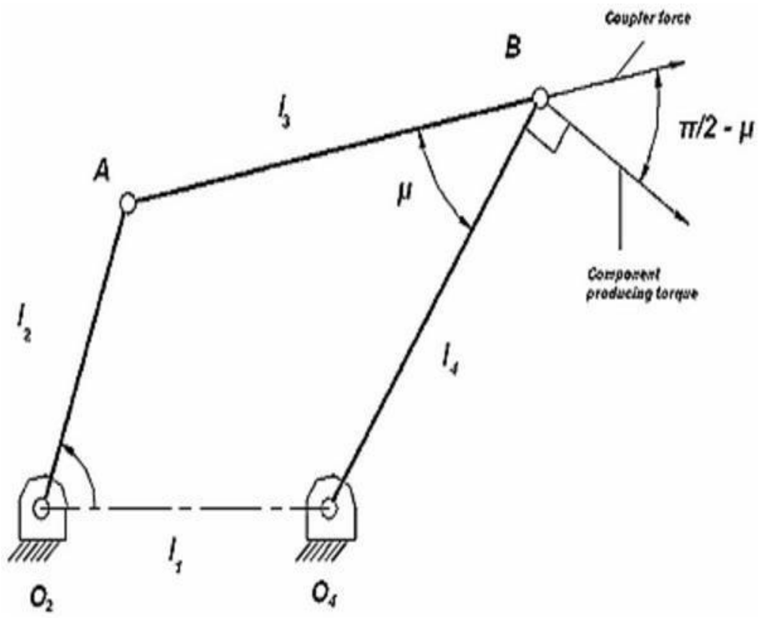
- A pantograph is mostly used for the reproduction of **plane areas and figures such as maps, plans etc., on enlarged or reduced scales.**
- It is, sometimes, used as an indicator rig in order to reproduce to a **small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine.** It is also used to guide cutting tools.
- A modified form of pantograph is used to collect power at the top of an electric locomotive.

PANTOGRAPH





TRANSMISSION ANGLE

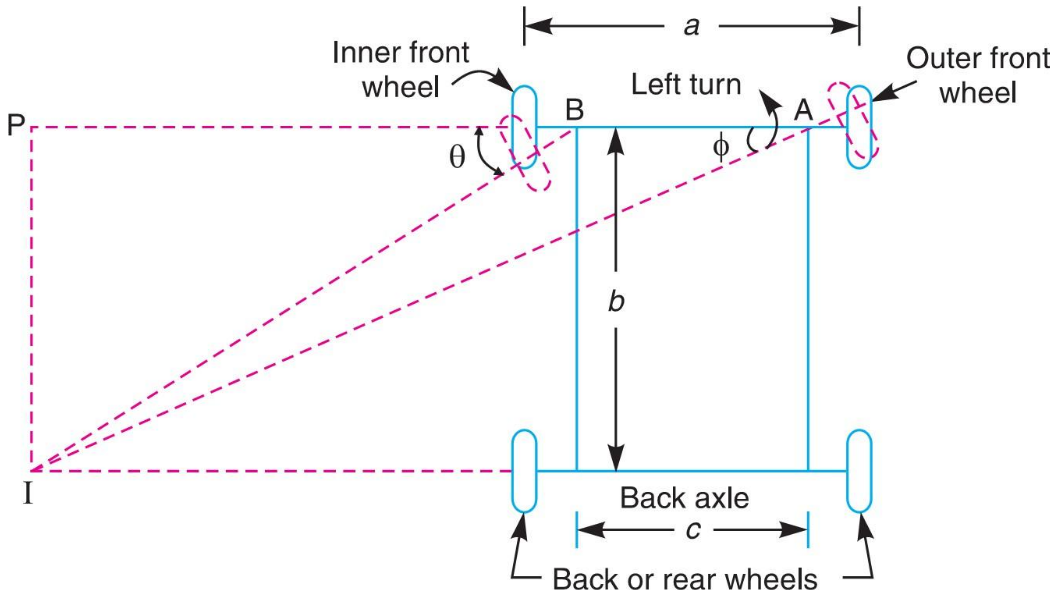


For a 4 R linkage, the transmission angle (μ) is defined as the acute angle between the coupler (AB) and the follower

For a given force in the coupler link, the torque transmitted to the output bar (about point O_4) is maximum when the angle μ between coupler bar AB and output bar BO_4 is π/2. Therefore, angle ABO_4 is called **transmission angle.**

STEERING GEAR MECHANISM

- The steering gear mechanism is used for changing the direction of two or more of the **wheel axles with reference to the chassis, so as to move the automobile in any desired path.**
- Usually the **two back wheels have a common axis, which is fixed in direction with reference to the chassis** and the steering is done by means of the front wheels.



STEERING GEAR MECHANISM

Thus, the condition for correct steering is that all the four wheels **must turn about the same instantaneous centre.** The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad \text{...}(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c/b$$

This is the **fundamental equation for correct steering.** If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

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DAVIS STEERING GEAR

- It is an **exact steering gear mechanism**. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These **constraints are connected to the slotted link** AM and BH by a sliding and a turning pair at each end.

a = Vertical distance between AB and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.

x = Distance moved by AC to $AC' = CC' = DD'$, and

α = Angle of inclination of the links AC and BD, to the vertical.

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DAVIS STEERING GEAR

From triangle $A\,A'\,C'$,

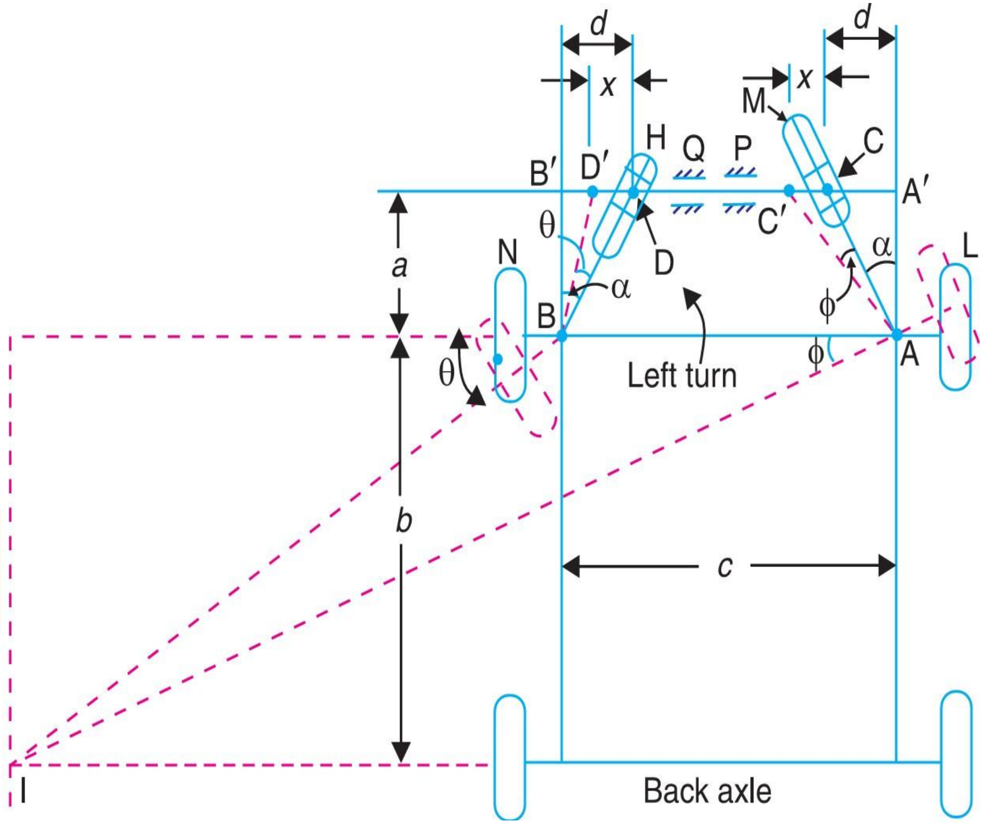
$$\tan(\alpha + \phi) = \frac{A'C'}{A\,A'} = \frac{d + x}{a}$$

From triangle $A\,A'C$,

$$\tan\alpha = \frac{A'C}{A\,A'} = \frac{d}{a}$$

From triangle $B\,B'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a}$$



We know that $\tan(\alpha + \phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}$

or

$$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

...[From equations (*i*) and (*ii*)]

DAVIS STEERING GEAR

$$(d + x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$a.\,d - d^2\tan\phi + a.\,x - d.x\tan\phi = a.d + a^2\tan\phi$$

$$\tan\phi\,(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x} \qquad \ldots(iv)$$

Similarly, from $\tan(\alpha - \theta) = \dfrac{d - x}{a}$, we get

$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x} \qquad \ldots(v)$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b} \qquad \ldots[\text{From equations } (iv) \text{ and } (v)]$$

$$\frac{2d.x}{a.x} = \frac{c}{b} \quad \text{or} \quad \frac{2d}{a} = \frac{c}{b}$$

$$2\tan\alpha = \frac{c}{b} \quad \text{or} \quad \tan\alpha = \frac{c}{2b} \qquad \ldots(\because d/a = \tan\alpha)$$

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ACKERMAN'S STEERING GEAR MECHANISM

- The Ackerman steering gear **mechanism is much simpler than Davis gear.** The difference between the Ackerman and Davis steering gears are:
- The whole mechanism of the **Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.**
- The Ackerman **steering gear consists of turning pairs,** whereas Davis steering gear consists of sliding members.
- The **shorter links BC and AD are of equal length** and are connected by hinge joints with front wheel axles.
- The **longer links AB and CD are of unequal length.**

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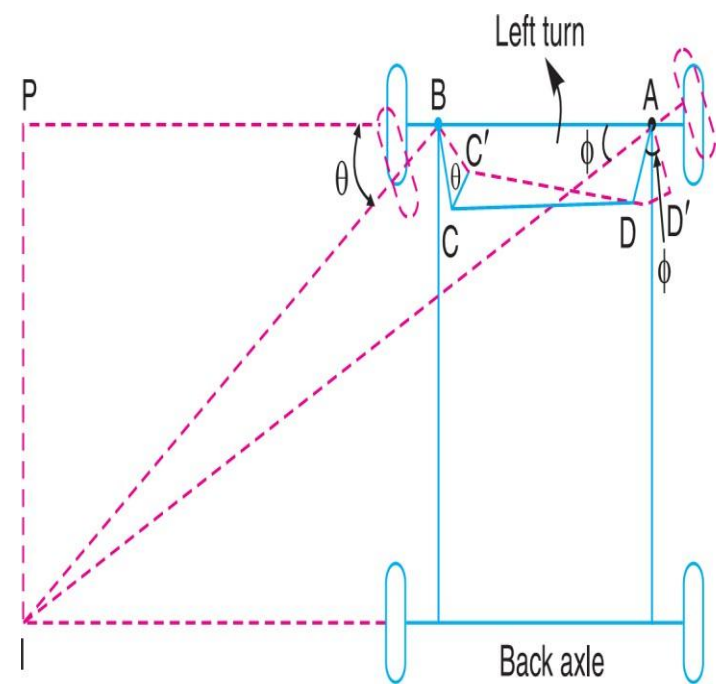
ACKERMAN'S STEERING GEAR MECHANISM

1. When the vehicle moves along a straight path, the **longer links AB and CD are parallel and the shorter links BC and AD are equally inclined** to the longitudinal axis of the vehicle, as shown by firm lines in Fig.

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the **front wheel axle intersect on the back wheel axle at I, for correct steering.**

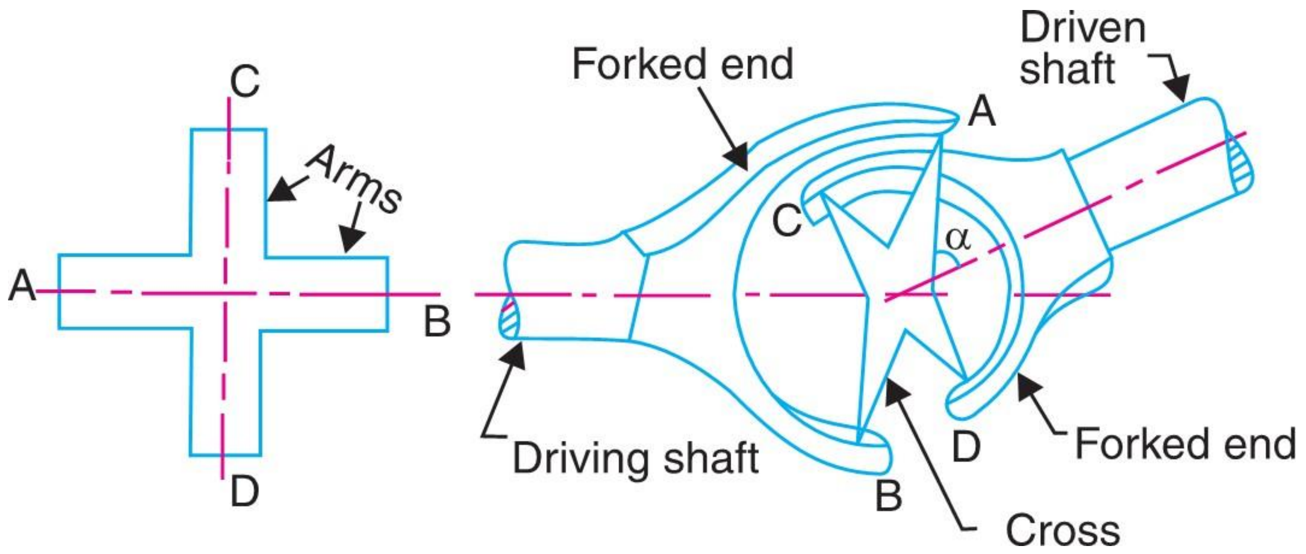
3. When the vehicle is **steering to the right, the similar position may be obtained.**

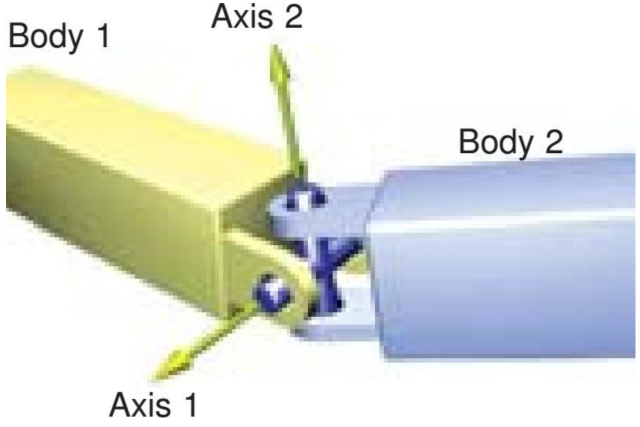


UNIVERSAL OR HOOKE’S JOINT

- A Hooke’s joint is **used to connect two shafts**, which are intersecting at a small angle, as shown in Fig.
- The end of each shaft is **forked to U-type and each fork provides two bearings for the arms of a cross**. The arms of the cross are perpendicular to each other.
- The motion is transmitted from the driving shaft to driven shaft through a cross. **The inclination of the two shafts may be constant**, but in actual practice it varies, when the motion is transmitted.
- The main application of the **Universal or Hooke’s joint is found in the transmission from the gear box to the differential or back axle** of the automobiles.
- It is also used for transmission of power to **different spindles of multiple drilling machine**. It is also used as a knee joint in milling machines.

UNIVERSAL OR HOOKE'S JOINT



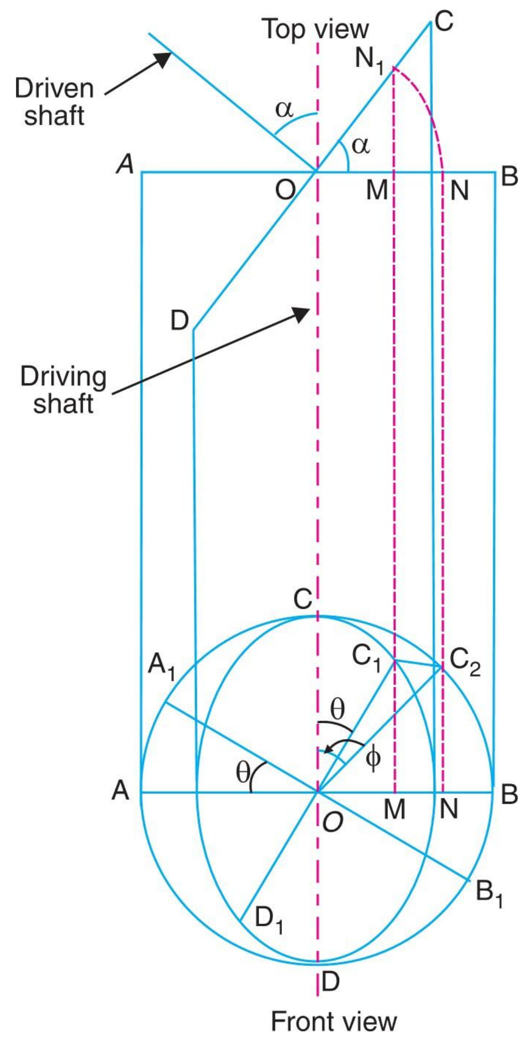


UNIVERSAL OR HOOKE'S JOINT

- The arms of the cross are perpendicular to each other. The motion is **transmitted from the driving shaft to driven shaft** through a cross. The **inclination of the two shafts may be constant,** but in actual practice it varies, when the motion is transmitted.
- The main application of the Universal or Hooke's joint is found in the **transmission from the gear box to the differential or back axle of the automobiles.**
- It is also used for **transmission of power to different spindles of multiple drilling machine.** It is also used as a knee joint in milling machines.

RATIO OF SHAFT VELOCITIES

- The top and front views connecting the two shafts by a universal joint are shown in Fig. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.
- Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A1B1 as shown in front view.
- A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse.



RATIO OF SHAFT VELOCITIES

- Therefore the arm CD takes new position C1D1 on the ellipse, at an angle θ. But the true angle must be on the circular path.
- To find the true angle, project the point C1 horizontally to intersect the circle at C2.
- Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ.

In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \quad \tan\theta = \frac{OM}{MC_1} \qquad \ldots(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \quad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1}$$

Dividing equation (*i*) by (*ii*),

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $\quad OM = ON_1 \cos\alpha = ON \cos\alpha$

RATIO OF SHAFT VELOCITIES

$$\therefore \quad \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

$$\tan\theta = \tan\phi \, . \cos\alpha$$

Let ω = Angular velocity of the driving shaft = $d\theta / dt$

ω_1 = Angular velocity of the driven shaft = $d\phi / dt$

Differentiating both sides of equation (*iii*),

$$\sec^2\theta \times d\theta / dt = \cos\alpha \, . \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \, . \sec^2\phi \times \omega_1$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha . \sec^2\phi} = \frac{1}{\cos^2\theta . \cos\alpha . \sec^2\phi}$$

RATIO OF SHAFT VELOCITIES

We know that $\sec^2\phi = 1 + \tan^2\phi = 1 + \dfrac{\tan^2\theta}{\cos^2\alpha}$ ...[From equation (*iii*)]

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta.\cos^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1 - \sin^2\alpha) + \sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta - \cos^2\theta.\sin^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{1 - \cos^2\theta.\sin^2\alpha}{\cos^2\theta.\cos^2\alpha} \quad ...(\because \cos^2\theta + \sin^2\theta = 1)$$

Substituting this value of $\sec^2\phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta.\cos\alpha} \times \frac{\cos^2\theta.\cos^2\alpha}{1 - \cos^2\theta.\sin^2\alpha} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha} \quad ...(v)$$

e: If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1 - \cos^2\theta.\sin^2\alpha}.$$

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UNIT-IV

CAMS

Cam - A mechanical device used to transmit motion to a follower by direct contact.

Cam – driver; Follower - driven

In a cam - follower pair, the cam normally rotates while the follower may translate or oscillate.

CLASSIFICATION OF CAMS (BASED ON SHAPE)

- **Disk or plate cams**
- Cylindrical Cam
- Translating cam

CLASSIFICATION OF CAMS (BASED ON SURFACE IN CONTACT)

- Knife edge follower
- Roller follower
- Flat faced follower
- Spherical follower

CAM NOMENCLATURE

Base circle : smallest circle of the cam profile.

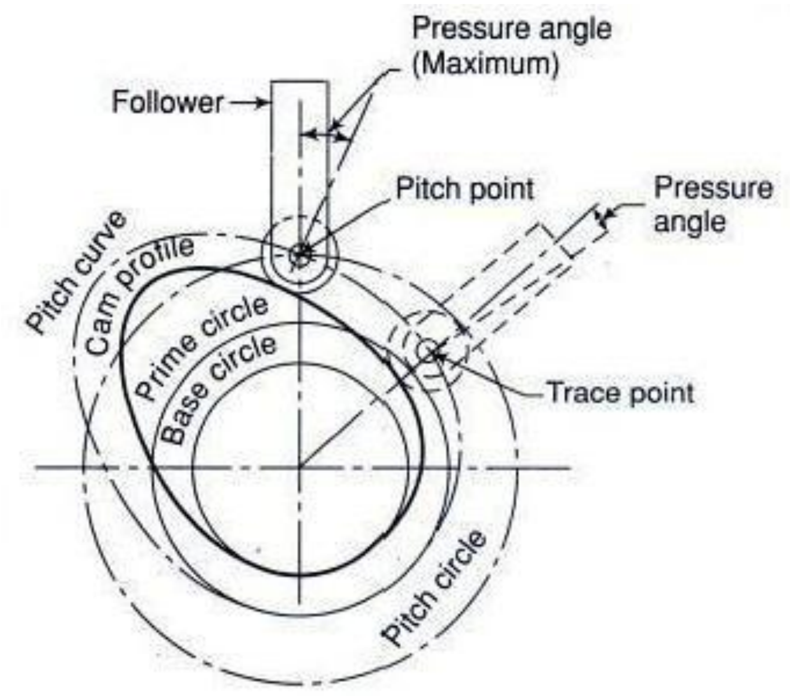
Trace point :

Reference point on the follower

Which generates the pitch curve.

Pressure angle:

Angle between the direction of the follower motion and a normal to the pitch curve



CAM NOMENCLATURE

Pitch point: Point on the pitch curve having the maximum pressure angle.

Pitch circle: circle drawn through the pitch points.

Pitch curve: curve generated by the trace point

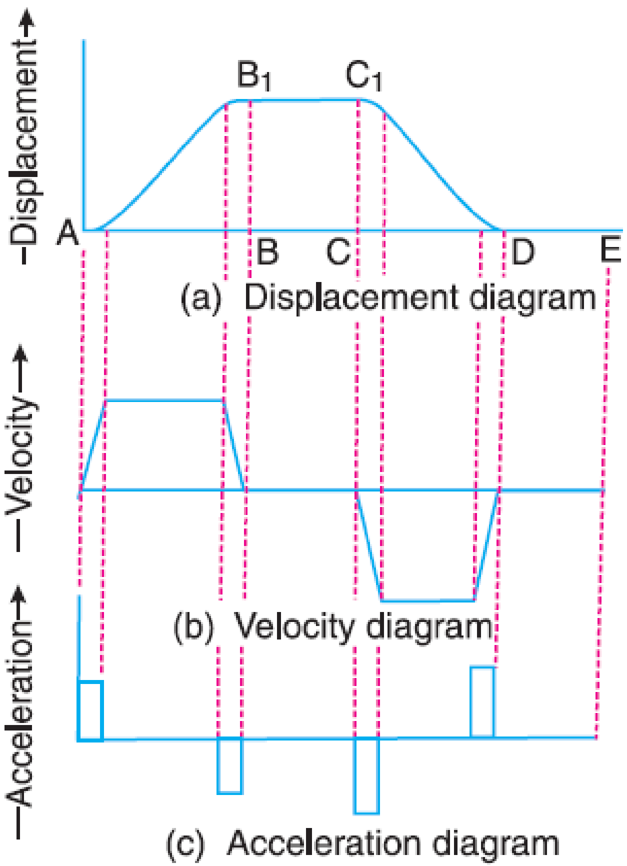
Prime circle: It is tangent to the pitch curve.

Lift or stroke: maximum travel of the follower from its lowest position to the Top most position.

MOTION OF THE FOLLOWER

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation,
4. Cycloidal motion

UNIFORM VELOCITY



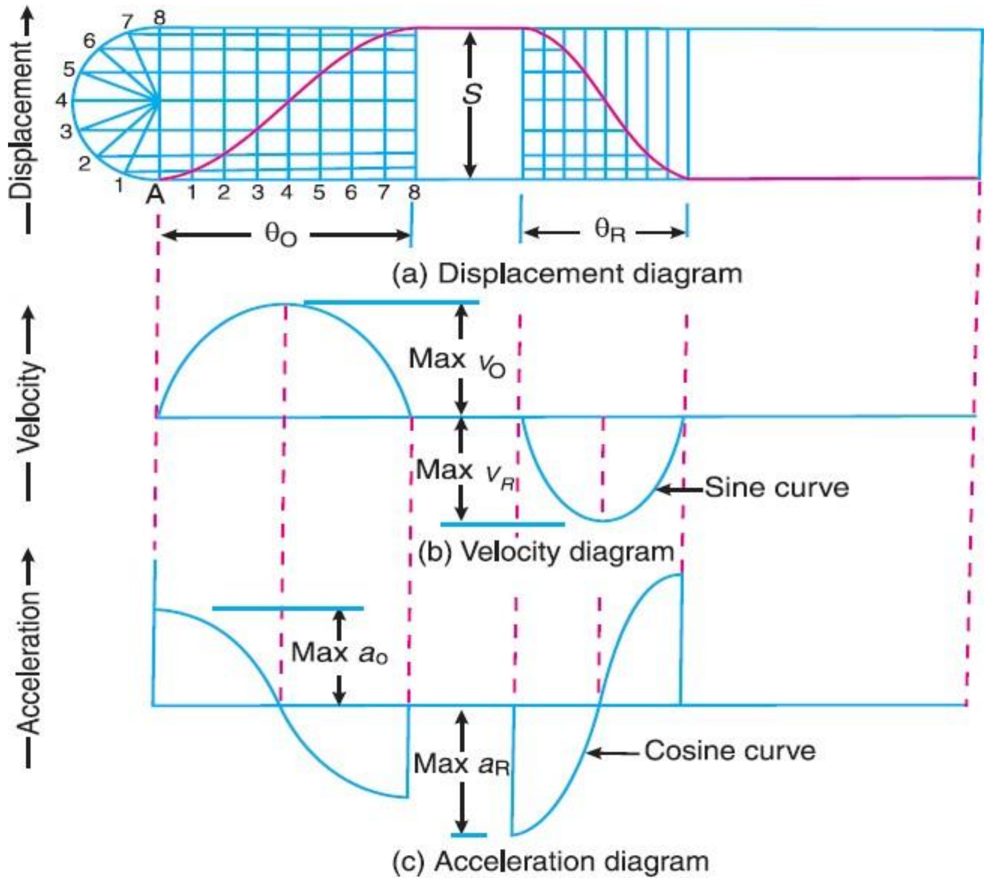
Modified displacement, velocity and acceleration diagrams when the follower moves with uniform velocity.

[This Figure is taken from Book authored by R S Khurmi]

➢The sharp corners at the beginning and at the end of each stroke are rounded off by the parabolic curves in the displacement diagram.

➢The parabolic motion results in a very low acceleration of the follower for a given stroke and cam speed.

FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)



[This Figure is taken from Book authored by R S Khurmi]

- Draw a semi-circle on the follower stroke as diameter.
- Divide the semi-circle into any number of even equal parts (say eight).

FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)

Outward stroke in SHM is equivalent to π ;
Meanwhile CAM is making θ
At any instant of time 't', angular disp. = θ = ωt

$$\text{SHM, } y = \frac{S}{2}\left(1 - \cos\frac{\pi\theta}{\theta_o}\right)$$

$$V = \frac{dy}{dt} = \frac{dy}{d\theta}\frac{d\theta}{dt} = \frac{dy}{d\theta}\,\omega = \frac{\pi\omega S}{2\theta_o}\,\mathrm{Sin}\,\frac{\pi\theta}{\theta_o}$$

For Max. outward velocity $V_o = \dfrac{\pi\omega S}{2\theta_o}$

FOLLOWER MOVES WITH SIMPLE HARMONIC MOTION (SHM)

Similar manner, acceleration can be found by taking time derivative of velocity.

(OR)

$$a_O = a = \frac{(v\ )^2}{OP} = \left(\frac{\pi\omega.S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2\omega^2.S}{2(\theta_O)^2}$$

Similarly, maximum velocity of the follower on the return stroke,

$$v_R = \frac{\pi\omega.S}{2\theta_R}$$

maximum acceleration of the follower on the return stroke,

$$a_R = \frac{\pi^2\omega^2.S}{2(\theta_R)^2}$$

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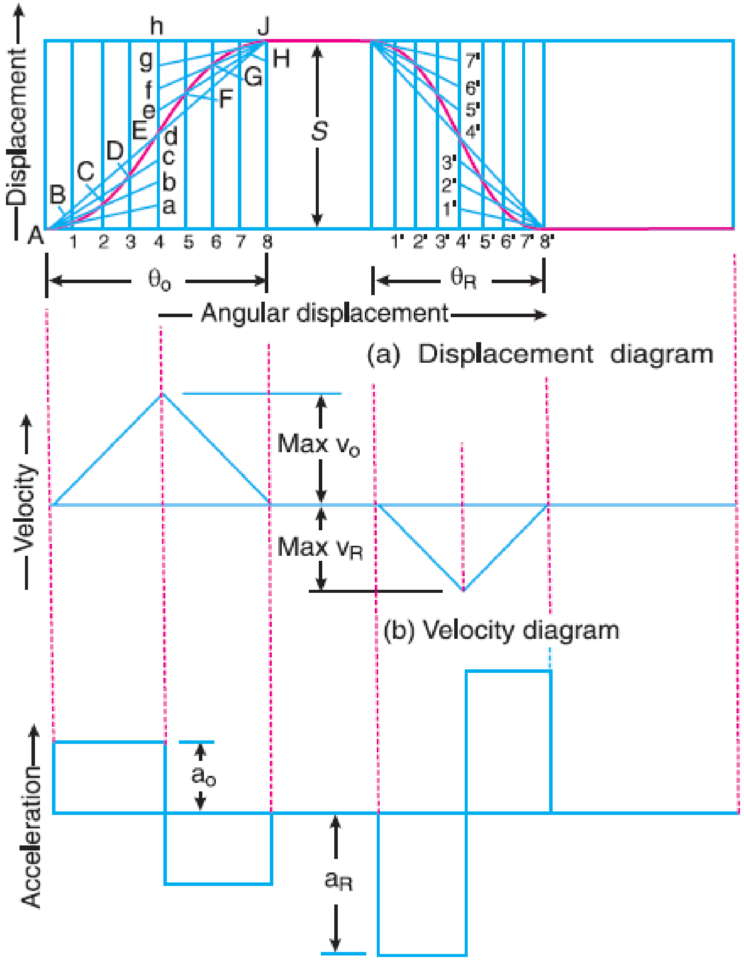
FOLLOWER MOVES WITH UNIFORM ACCELERATION AND RETARDATION



(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

[This Figure is taken from Book authored by R S Khurmi]

maximum velocity of the follower during outstroke,

$$v_O = \frac{S}{t_O / 2} = \frac{2\omega.S}{\theta_O}$$

maximum velocity of the follower during return stroke,

$$v_R = \frac{2\omega.S}{\theta_R}$$

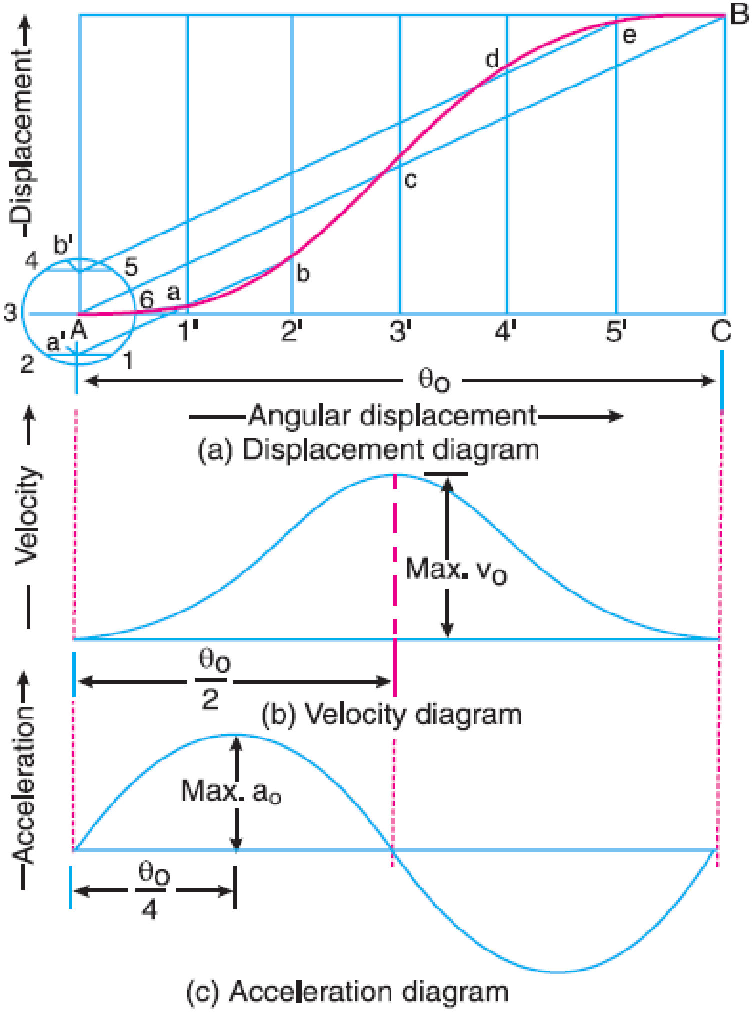
Maximum acceleration of the follower during outstroke,

$$a_O = \frac{v_O}{t_O / 2} = \frac{2 \times 2\,\omega.S}{t_O.\theta_O} = \frac{4\,\omega^2.S}{(\theta_O)^2}$$

maximum acceleration of the follower during return stroke,

$$a_R = \frac{4\,\omega^2.S}{(\theta_R)^2}$$

FOLLOWER MOVES WITH CYCLOIDAL MOTION



[This Figure is taken from Book authored by R S Khurmi]

cycloid is a curve traced by a point on a circle when the circle rolls without slipping on a straight line

Radius of the circle r = S / 2π

Where S = stroke

Max. Velocity of the follower during outward stroke $= v_O = \frac{2\omega S}{\theta_O}$

Max. Velocity of the follower during return stroke $= v_R = \frac{2\omega S}{\theta_R}$

FOLLOWER MOVES WITH CYCLOIDAL MOTION

maximum acceleration of the follower during outstroke,

$$a_O = \frac{2\pi\omega^2.S}{(\theta_O)^2}$$

maximum acceleration of the follower during return stroke,

$$a_R = \frac{2\pi\omega^2.S}{(\theta_R)^2}$$

SUMMARY

Type	Max Outstroke Velocity	Max return stroke Velocity	Max Outstroke acceleration	Max return stroke acceleration
SHM	$\frac{\pi\omega S}{2\theta_O}$	$\frac{\pi\omega S}{2\theta_R}$	$\frac{\pi^2\omega^2 .S}{2(\theta_O)^2}$	$\frac{\pi^2\omega^2 .S}{2(\theta_R)^2}$
Uniform Acceleration and Retardation	$\frac{2\omega S}{\theta_O}$	$\frac{2\omega S}{\theta_R}$	$\frac{4\omega^2 .S}{(\theta_O)^2}$	$\frac{4\omega^2 .S}{(\theta_R)^2}$
Cycloidal Motion	$\frac{2\omega S}{\theta_O}$	$\frac{2\omega .S}{\theta_R}$	$\frac{2\pi\omega^2 .S}{(\theta_O)^2}$	$\frac{2\pi\omega^2 .S}{(\theta_R)^2}$

NUMERICAL EXAMPLE -1

A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.
2. Dwell for the next 30°.
3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.
4. Dwell during the remaining 180°.

Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

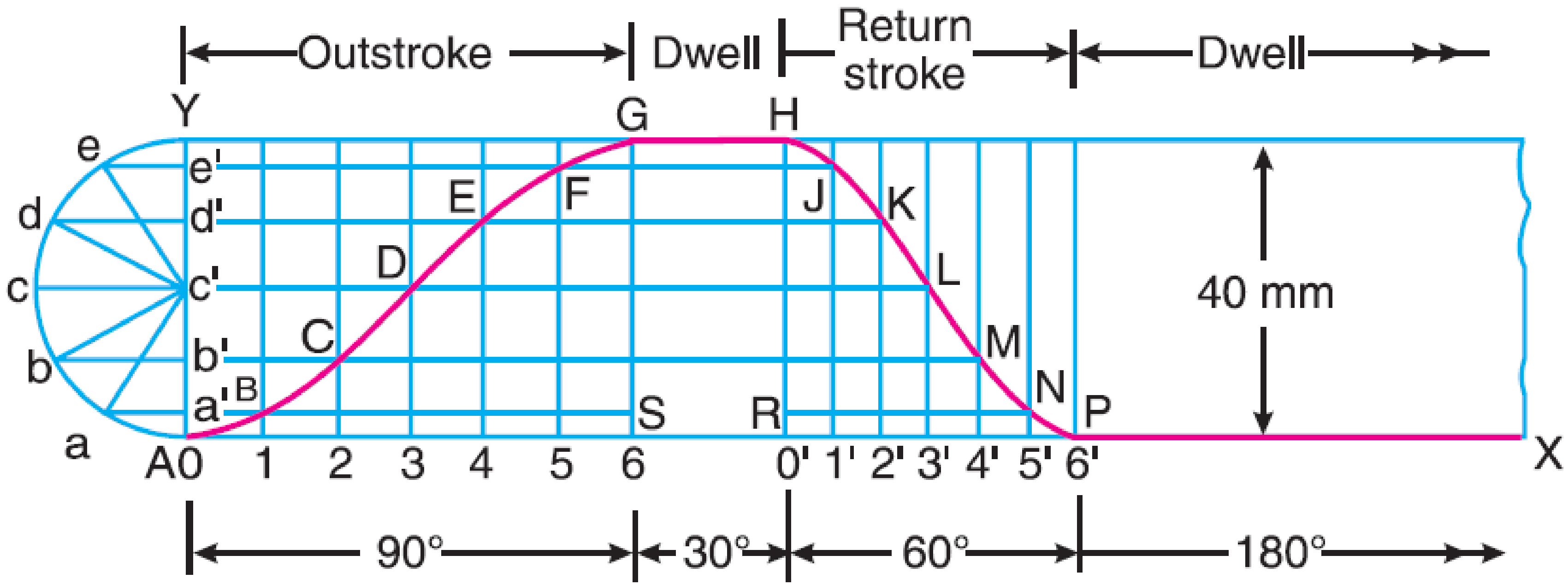
(b) the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

NUMERICAL EXAMPLE -1

Given : $S = 40$ mm $= 0.04$ m; $\theta_O = 90° = \pi/2$ rad $= 1.571$ rad

$\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ; $N = 240$ r.p.m.

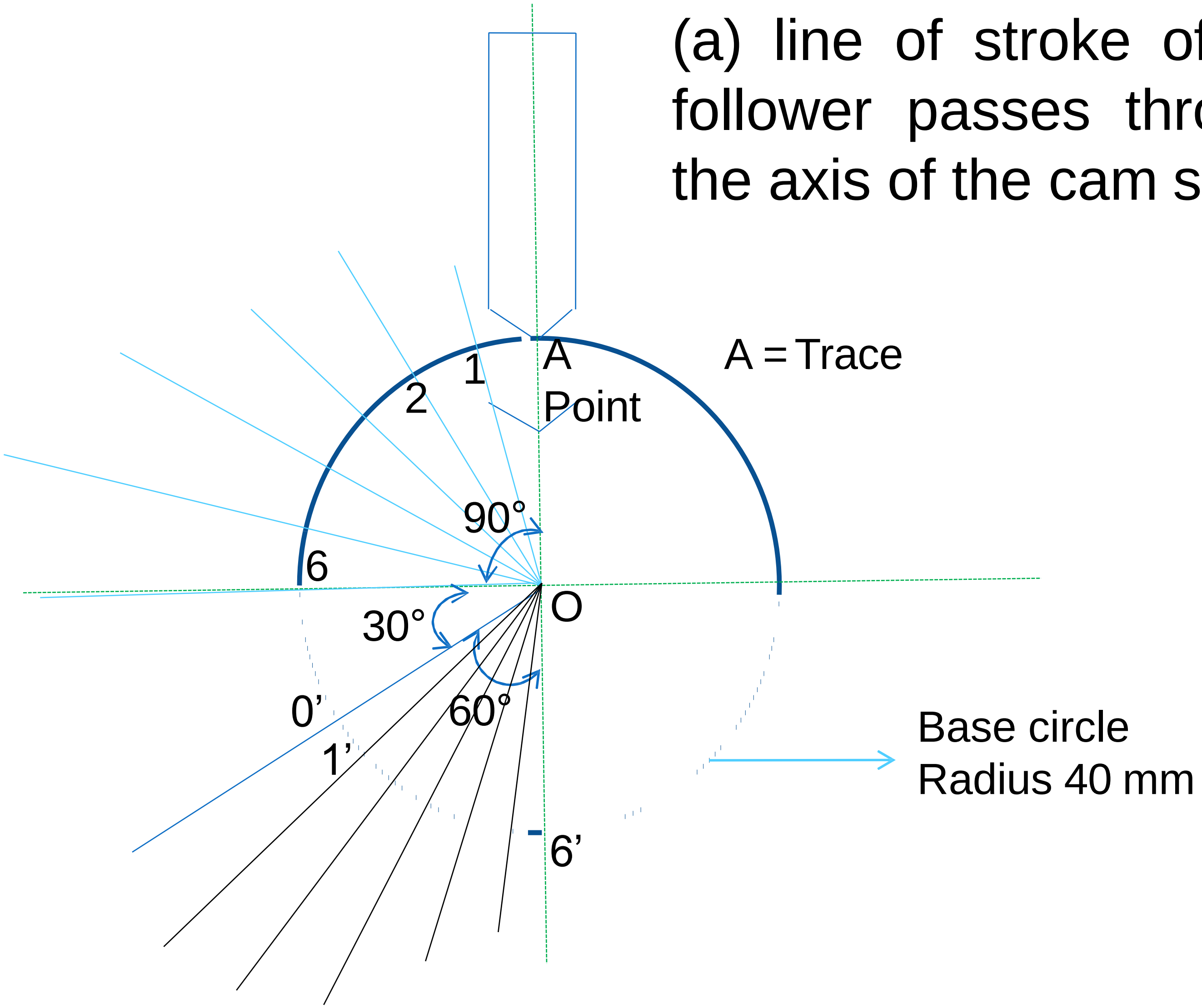


[This Figure is taken from Book authored by R S Khurmi]

Draw horizontal line AX = 360° to any convenient scale

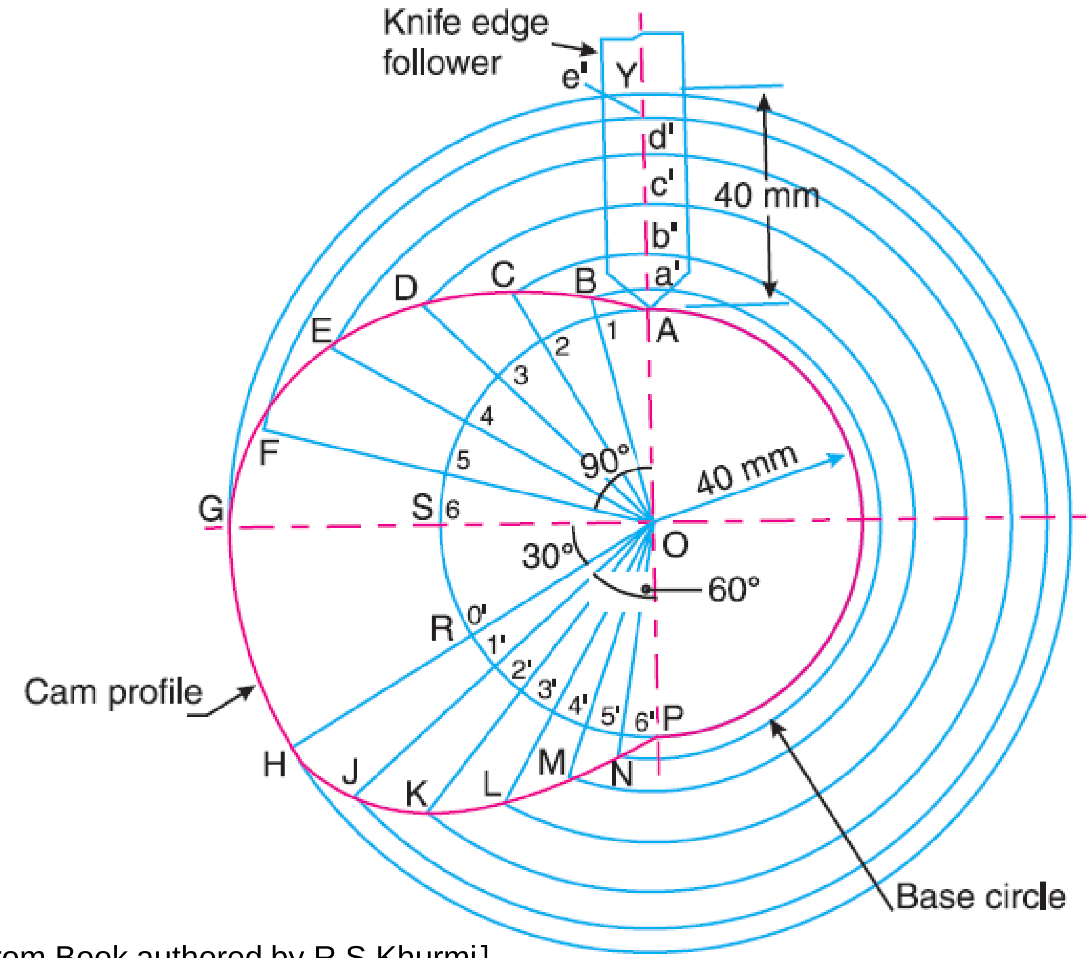
NUMERICAL EXAMPLE -1

(a) line of stroke of the follower passes through the axis of the cam shaft



NUMERICAL EXAMPLE -1

Line of stroke of the follower passes through the axis of the cam shaft



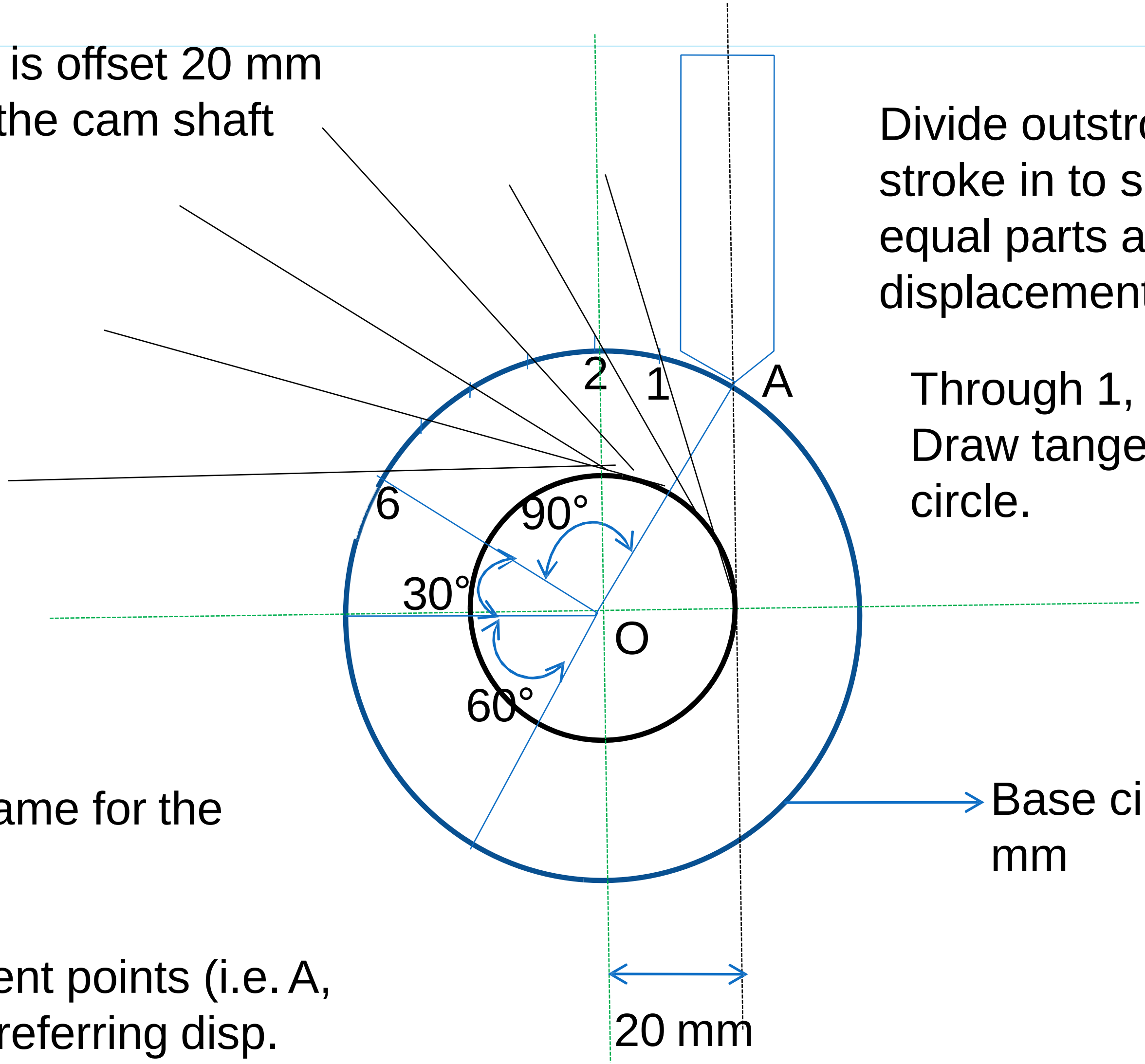
[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE -1

(b) line of stroke is offset 20 mm from the axis of the cam shaft

Divide outstroke and return stroke in to same number of equal parts as in displacement diagram

Through 1, 2, .. & 6, Draw tangent to offset circle.



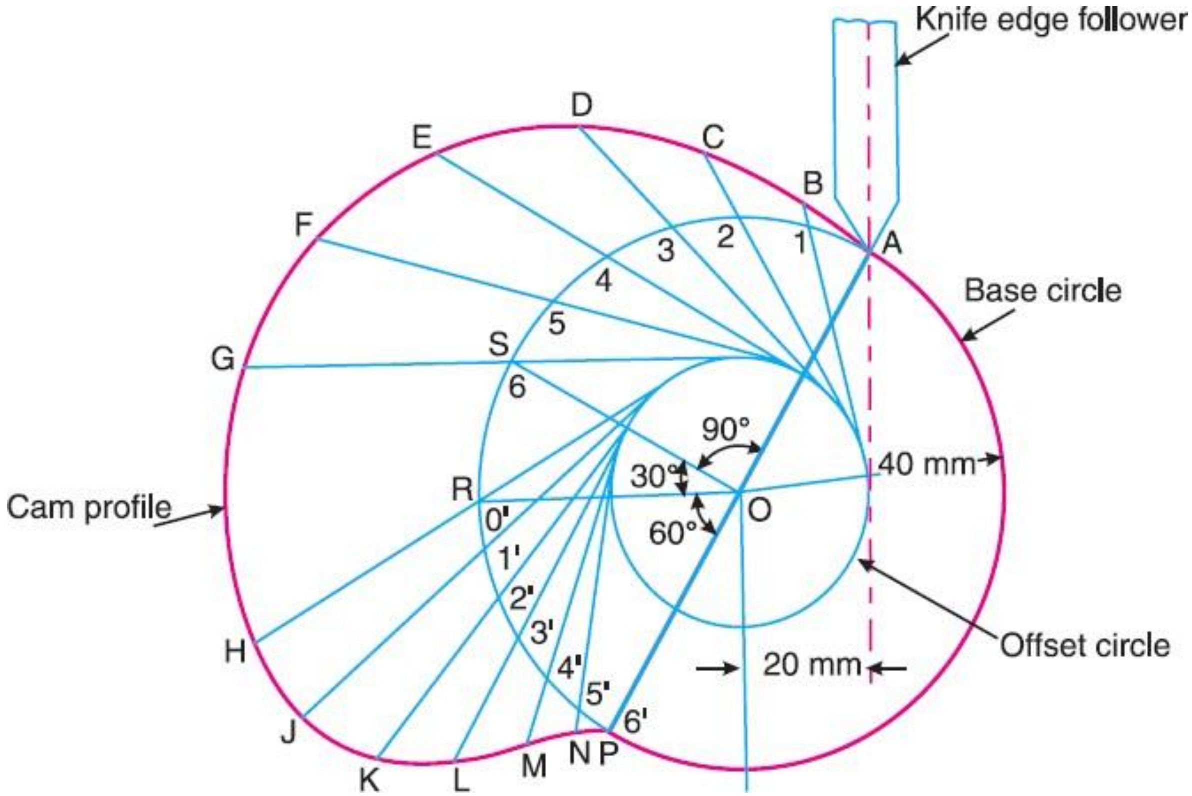
Base circle Radius 40 mm

Repeat the same for the return stroke.

Mark the salient points (i.e. A, B, C .. G) by referring disp. diagram

NUMERICAL EXAMPLE -1

line of stroke is offset 20 mm from the axis of the cam shaft



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NUMERICAL EXAMPLE -1

Maximum velocity of the follower during its ascent and descent

We know that $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14$ rad/s

$$v_O = \frac{\pi\omega.S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s}$$ **Ans.**

$$v_R = \frac{\pi\omega.S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s}$$ **Ans.**

Maximum acceleration of the follower during its ascent and descent

$$a_O = \frac{\pi^2\omega^2.S}{2(\theta_O)^2} = \frac{\pi^2(25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2$$ **Ans.**

$$a_R = \frac{\pi^2\omega^2.S}{2(\theta_R)^2} = \frac{\pi^2(25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2$$ **Ans.**

NUMERICAL EXAMPLE-2

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

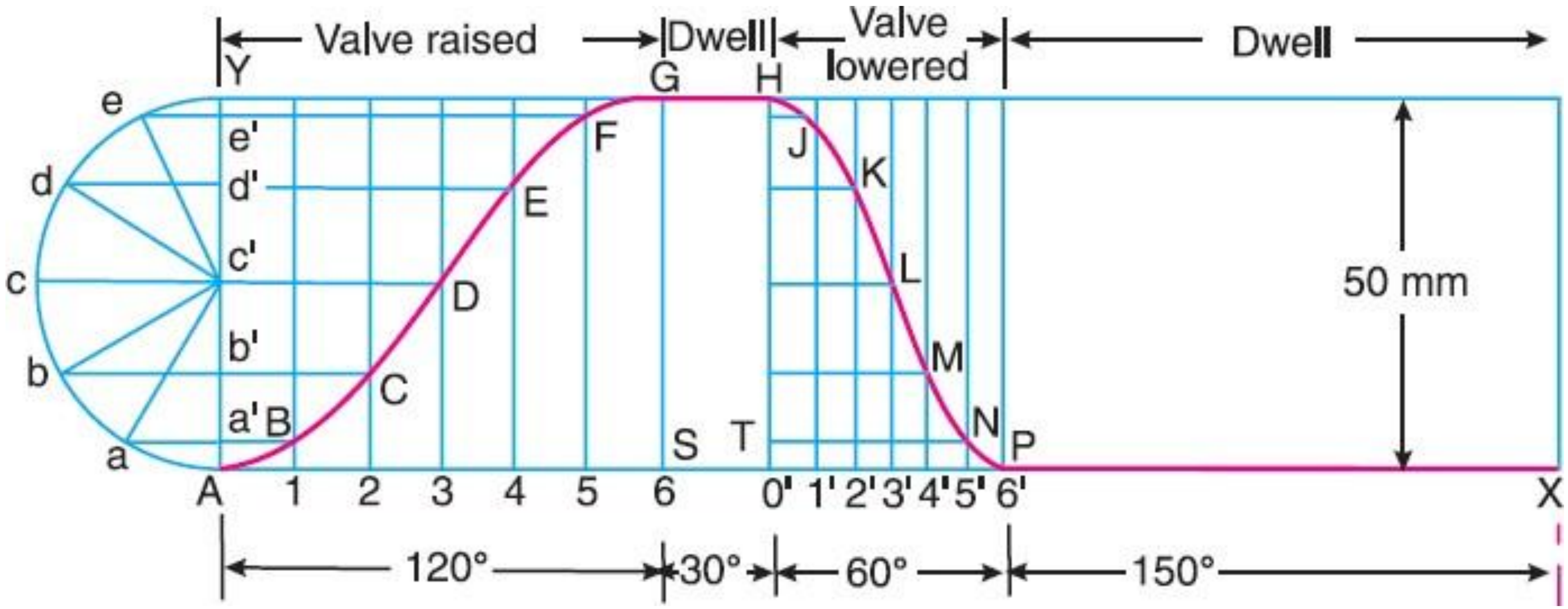
1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30°;
3. To lower the valve during next 60°; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm. Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

NUMERICAL EXAMPLE-2

Given : $S = 50$ mm $= 0.05$ m ; $\theta_O = 120° = 2\pi/3$ rad $= 2.1$ rad ; $\theta_R = 60° = \pi/3$ rad $= 1.047$ rad ; $N = 100$ r.p.m.

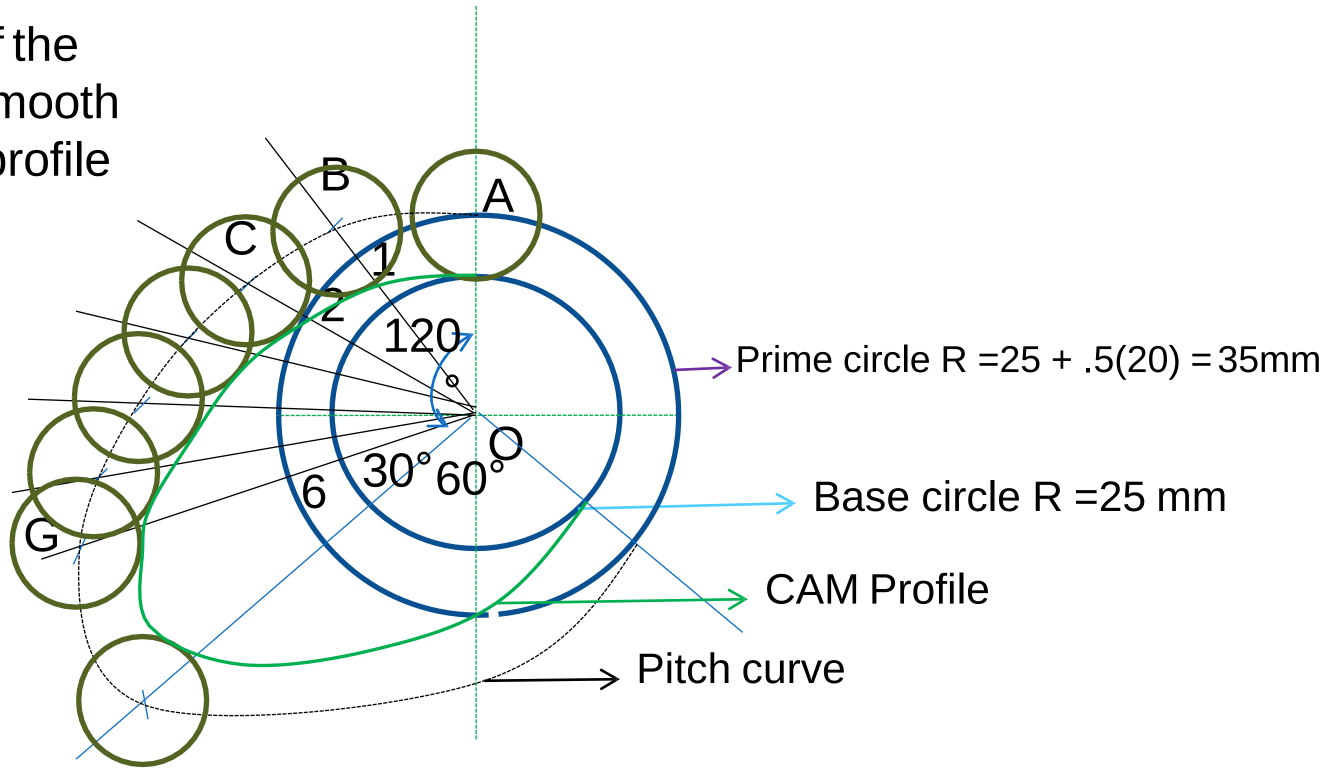


[This Figure is taken from Book authored by R S Khurmi]

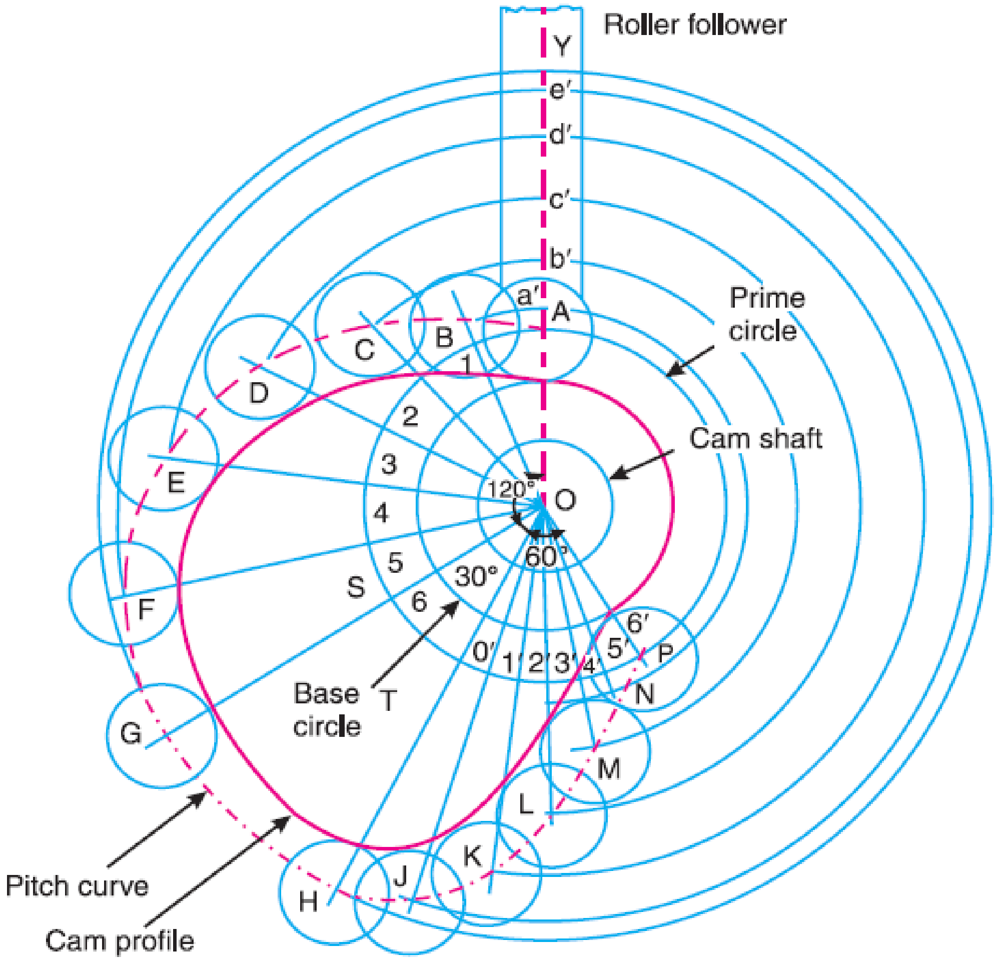
NUMERICAL EXAMPLE-2

Draw circle by keeping B as center &
r = roller radius
Similarly from C, D, .. G.

Join the bottoms of the circles with a smooth curve to get CAM profile



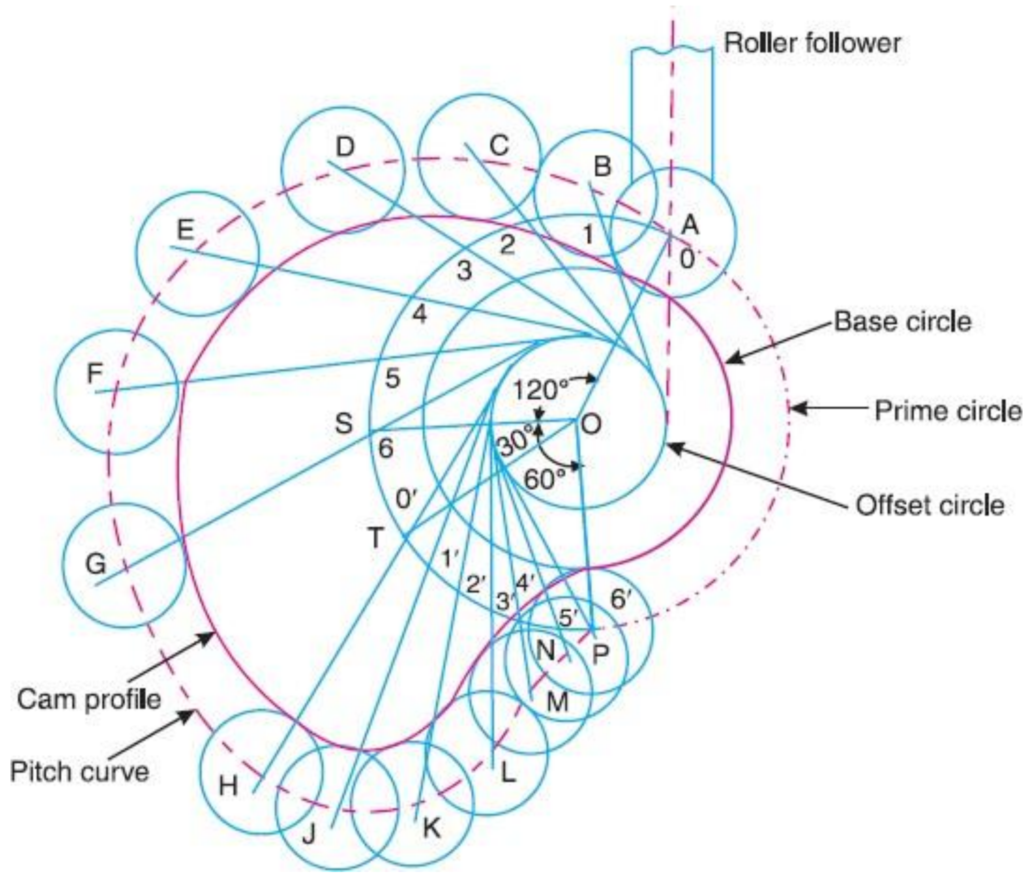
NUMERICAL EXAMPLE-2



[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE-2

When the line of the stroke is offset 15 mm from the axis of the cam shaft



[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE-2

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

maximum velocity of the valve rod to raise valve,

$$v_O = \frac{\pi \omega S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

maximum velocity of the valve rod to lower the valve,

$$v_R = \frac{\pi \omega S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

maximum acceleration of the valve rod to raise the valve,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_0)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(2.1)^2} = 6.13 \text{ m/s}^2 \text{ Ans.}$$

maximum acceleration of the valve rod to lower the valve,

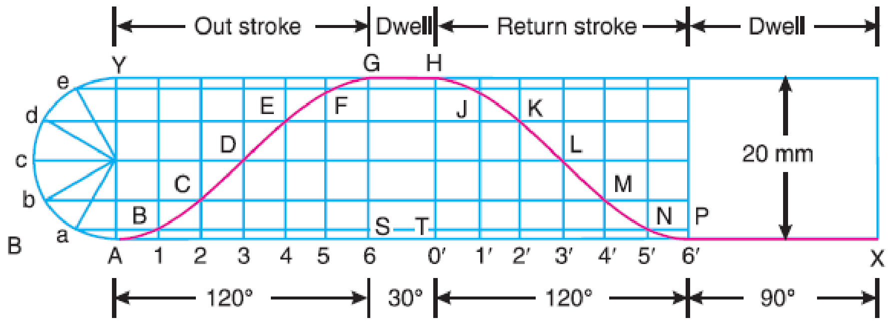
$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2 \text{ Ans.}$$

NUMERICAL EXAMPLE -3

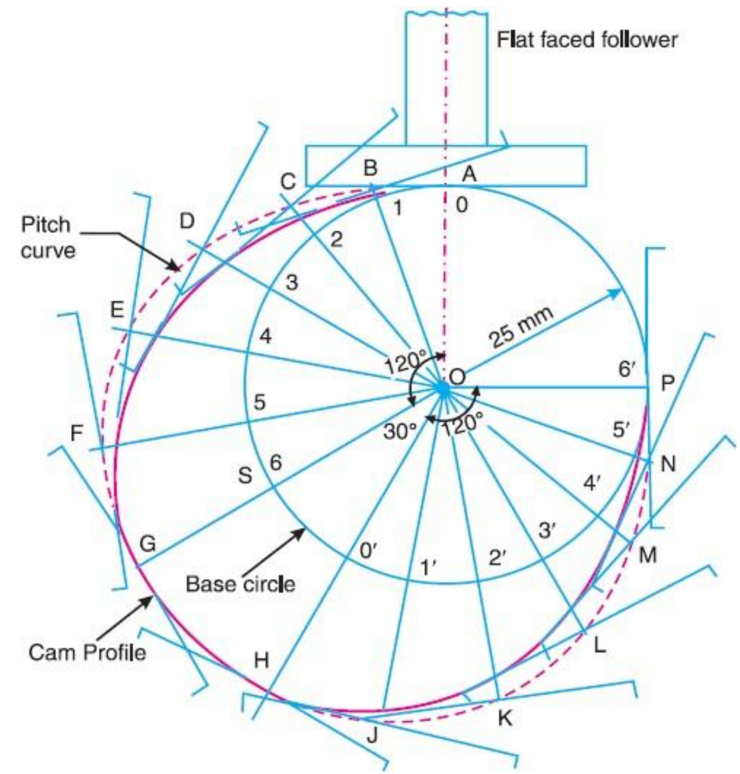
A cam drives a flat reciprocating follower in the following manner :

During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.

NUMERICAL EXAMPLE -3



[This Figure is taken from Book authored by R S Khurmi]



[This Figure is taken from Book authored by R S Khurmi]

- Construction procedure is Similar to knife edge / roller follower.
- Pitch circle is drawn by transferring distances 1B, 2C, 3D etc., from displacement diagram to the profile construction.
- Now at points B, C, D . . . M, N, P, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.
- CAM profile is the curve drawn tangentially to the flat side of the follower .

NUMERICAL EXAMPLE -4

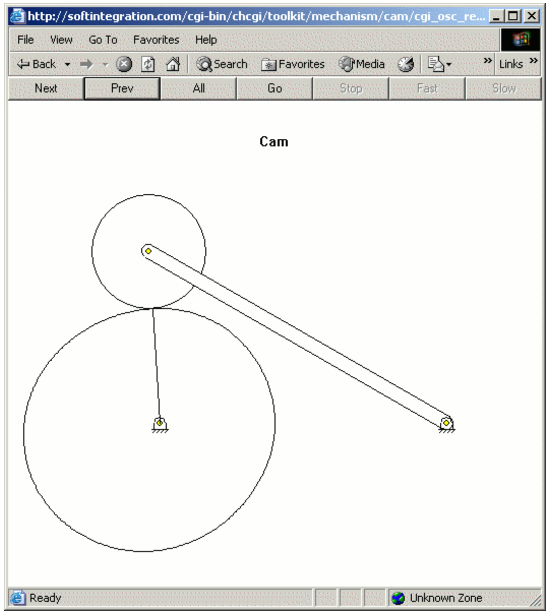
Draw a cam profile to drive an oscillating roller follower to the specifications given below :

(a).Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;

(b).Follower to return to its initial position during next 120° rotation of the cam ;

(c) Follower to dwell during the next 120° of cam rotation.

The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.



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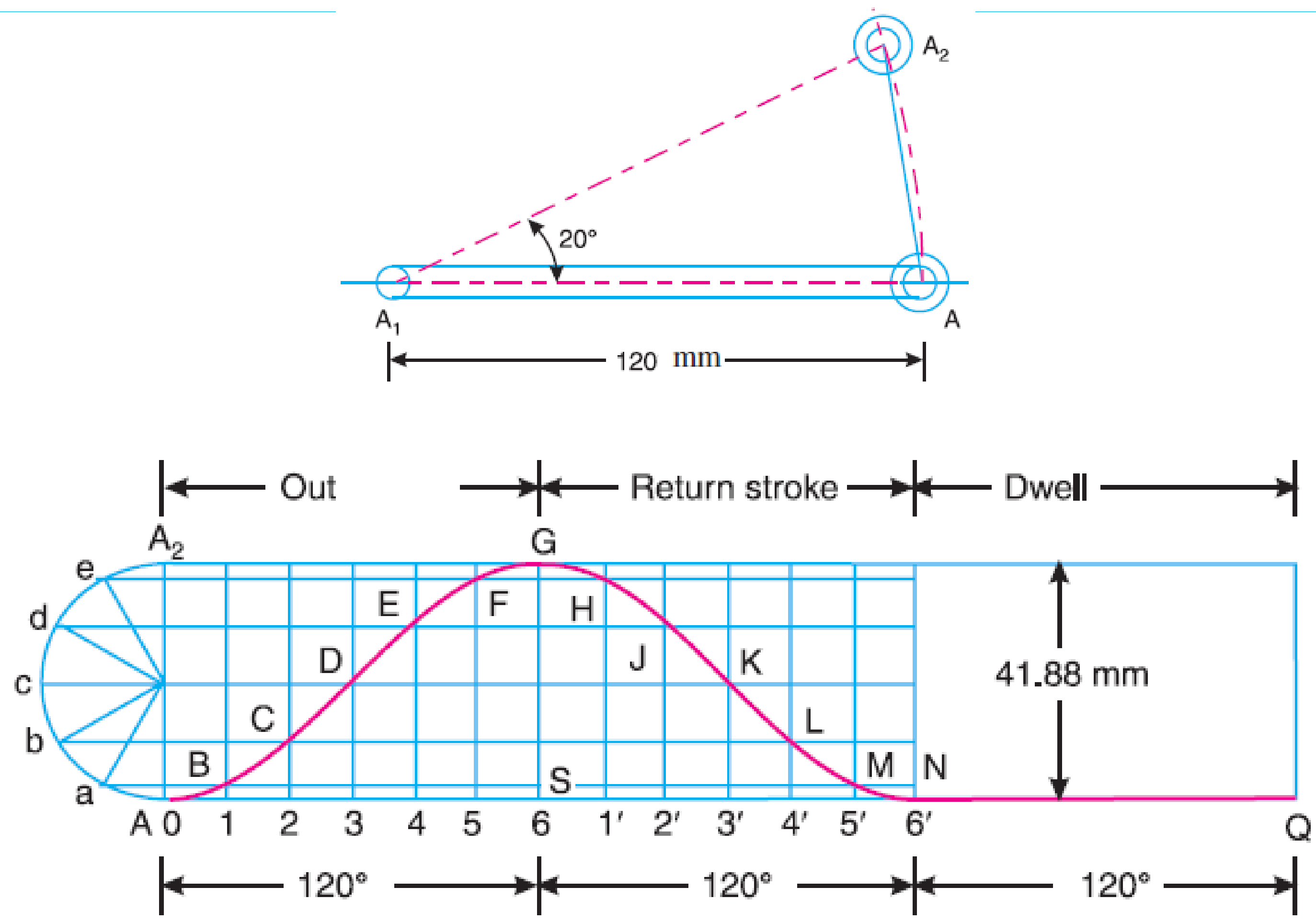
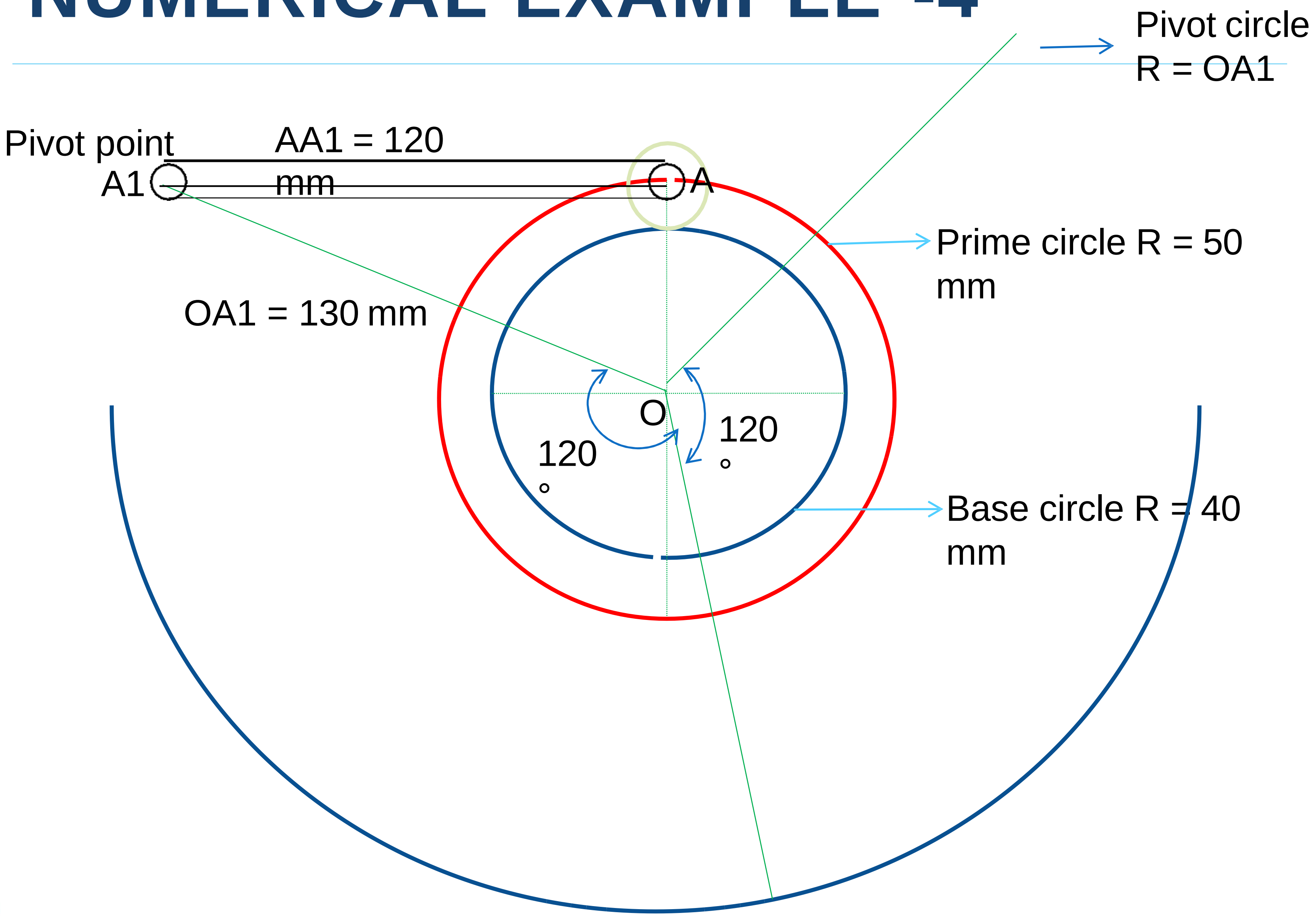
NUMERICAL EXAMPLE -4



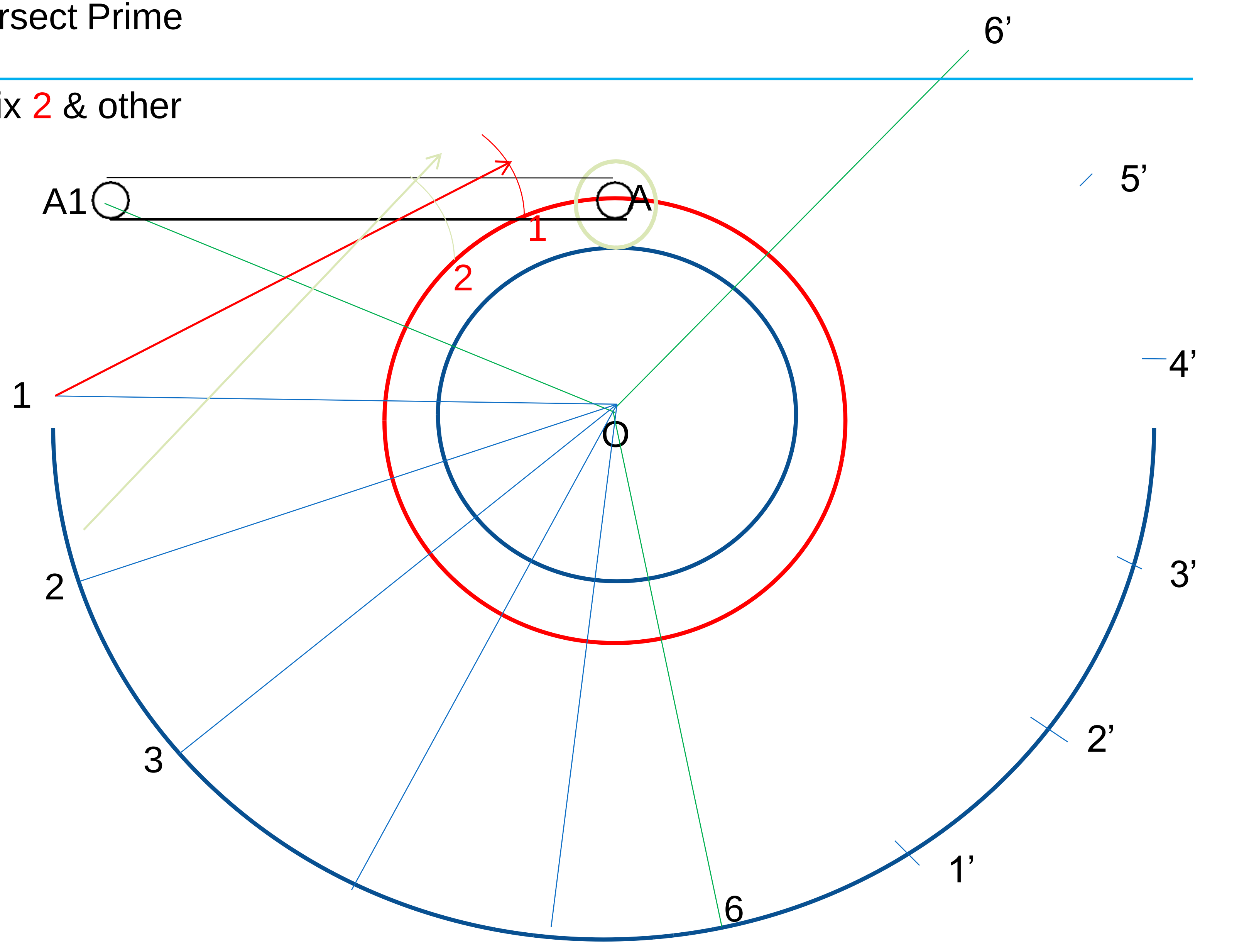
[This Figure is taken from Book authored by R S Khurmi]

NUMERICAL EXAMPLE -4



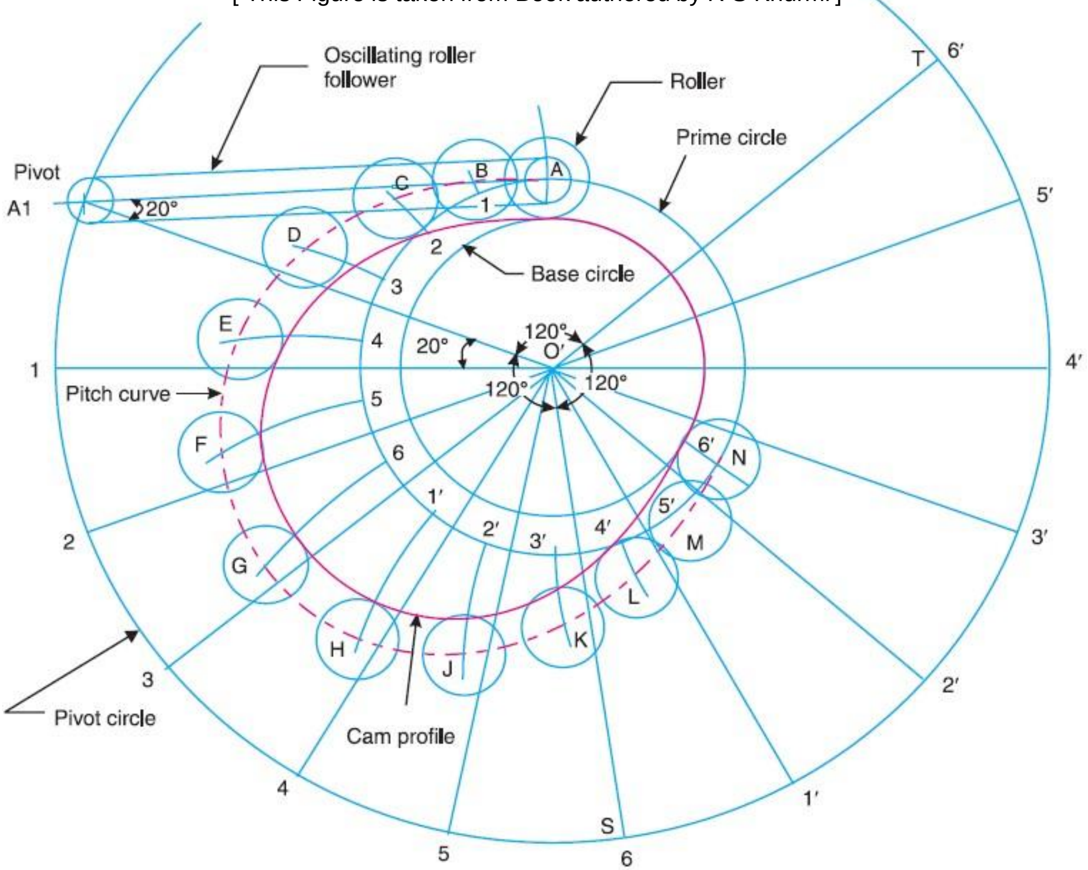
1 as center, AA1 = 120 mm as radius, make an arc to intersect Prime circle at 1

Similarly fix 2 & other points



NUMERICAL EXAMPLE -4

[This Figure is taken from Book authored by R S Khurmi]



Set off the distances 1B, 2C, 3D... 4L, 5M along the arcs drawn equal to the distances as measured from the displacement diagram

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NUMERICAL EXAMPLE -4

- The curve passing through the points A, B, C....L, M, N is known as pitch curve.
- Now draw circles with A, B, C, D....L, M, N as centre and radius equal to the radius of roller.
- Join the bottoms of the circles with a smooth curve as shown in Fig.
- This is the required CAM profile.

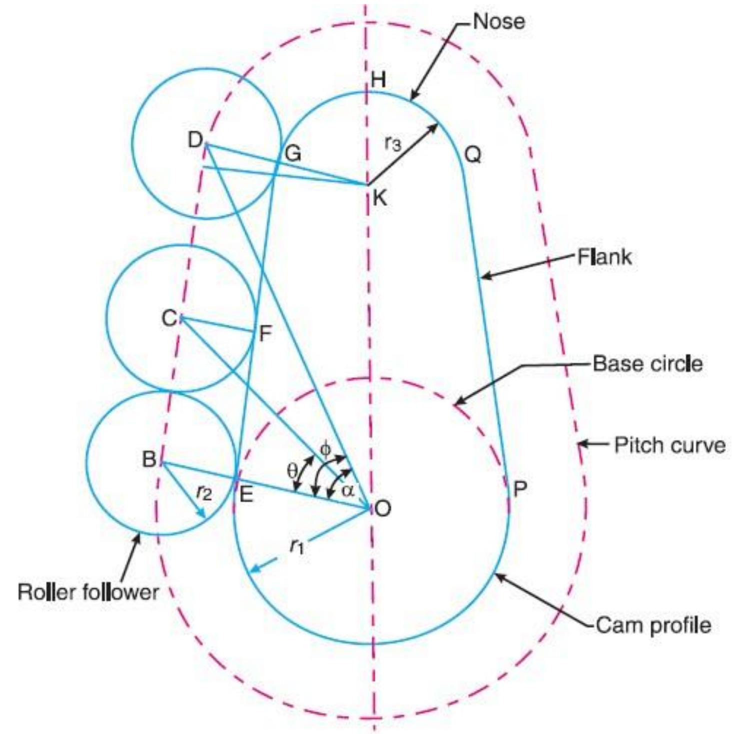
CAMS WITH SPECIFIED CONTOURS

In the previous sessions, we have discussed the design of the profile of a cam when the follower moves with the specified motion - the shape of the cam profile obtained may be difficult and costly to manufacture.

In actual practice, the cams with specified contours (cam profiles consisting of circular arcs and straight lines are preferred) are assumed and then motion of the follower is determined.

CAMS WITH SPECIFIED CONTOURS

- When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as a tangent cam.
- Used for operating the inlet and exhaust valves of IC engines



Tangent cam with reciprocating roller follower having contact with straight flanks.

[This Figure is taken from Book authored by R S Khurmi]

RADIUS OF CURVATURE

- It is a mathematical property of a function. No matter how complicated the a curve's shape may be, nor how high the degree of the function, it will have always an instantaneous radius of curvature at every point of the curve.

- When they are wrapped around their prime or base circle, they may be concave, convex or flat portions.

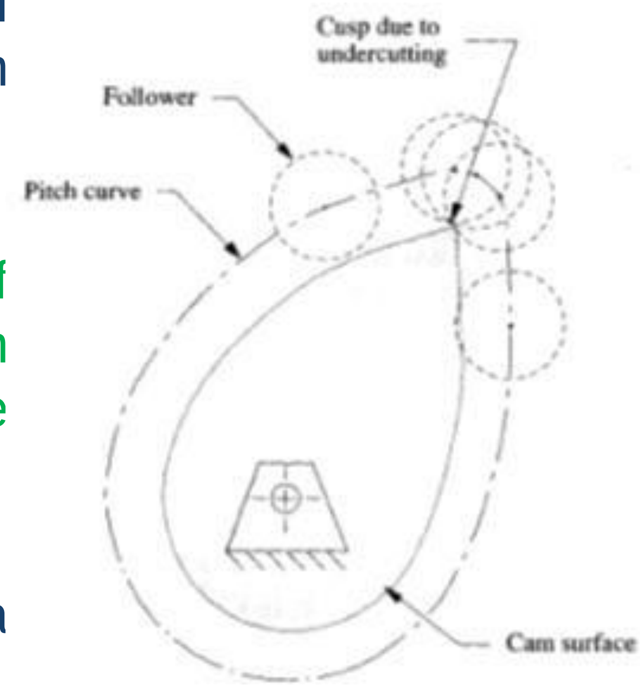
- Both, the pressure angle and the radius of curvature will dictate the minimum size of the cam and they must be checked.

RADIUS OF CURVATURE

- Undercutting: The roller follower radius R_f is larger than the smallest positive (convex) local radius. No sharp corners for an acceptable cam design.

- The golden rule is to keep the absolute value of the minimum radius of curvature of the cam pitch curve at least 2 or 3 times large as the radius of the follower.

- Radius of curvature can not be negative for a flat-faced follower.



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MANUFACTURING CONSIDERATIONS

Materials: Hard materials as high carbon steels, cast iron. Sometimes made of brass, bronze and plastic cams (low load and low speed applications).

Production process: rotating cutters. Numerical control machinery. For better finishing, the cam can be ground after milling away most of the unneeded material. Heat treatments are usually required to get sufficient hardness to prevent rapid wear.

Geometric generation: actual geometries are far from been perfect. Cycloidal function can be generated. Very few other curves can.

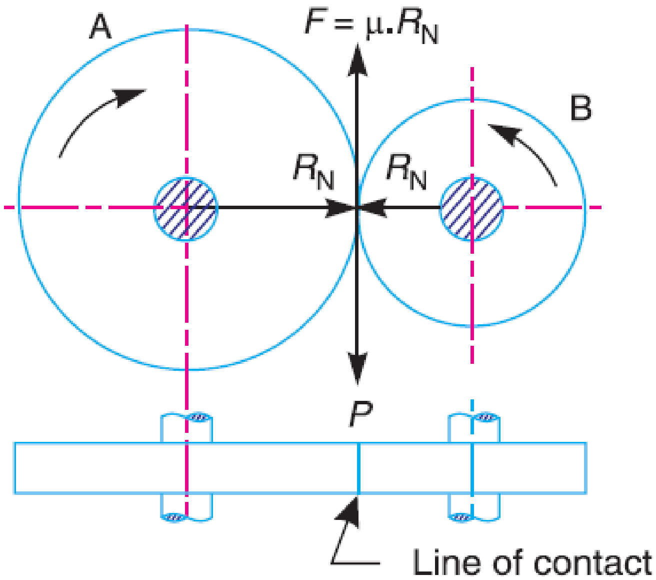
UNIT-V

POWER TRANSMISSION SYSTEMS

Belt/Rope Drives - Large center distance of the shafts

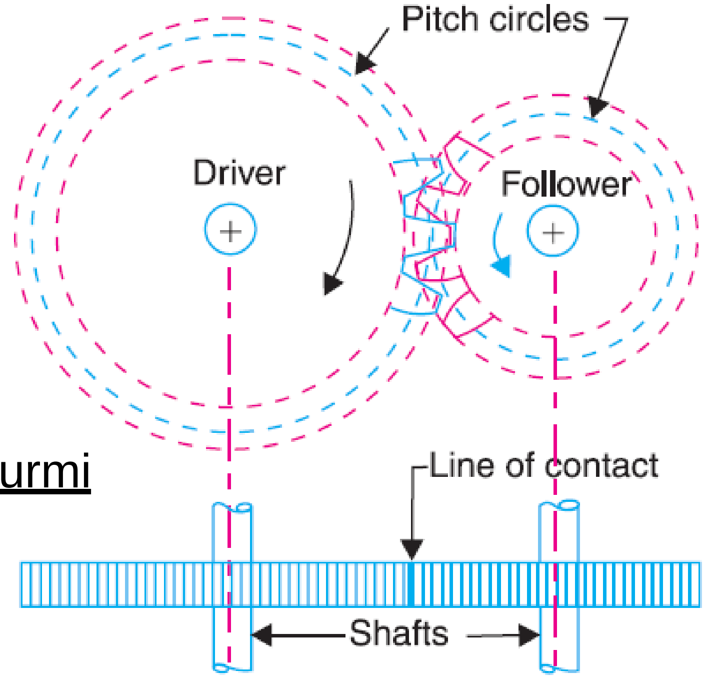
Chain Drives - Medium center distance of the shafts

Gear Drives - Small center distance of the shafts

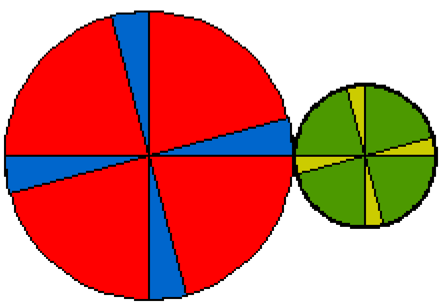


Friction wheels

Source: R. S. Khurmi



Toothed wheels.



when the tangential force (*P*) *exceeds the frictional resistance* (*F*), *slipping will take place between the two* wheels. Thus the friction drive is <u>not a positive drive.</u>

ADVANTAGES AND DISADVANTAGES OF GEAR DRIVE

The following are the advantages and disadvantages of the gear drive as compared to belt, rope and chain drives :

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

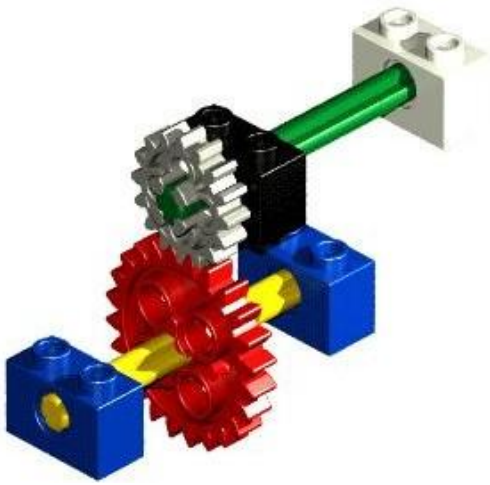
Disadvantages

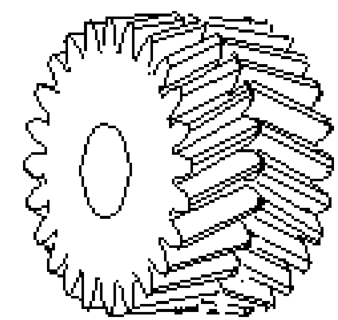
1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

CLASSIFICATION OF TOOTHED WHEELS

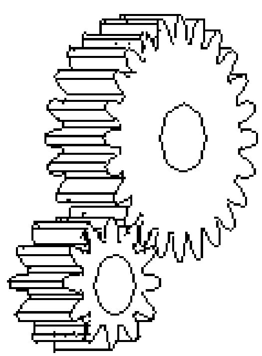
1. *According to the position of axes of the shafts*

(a) Parallel

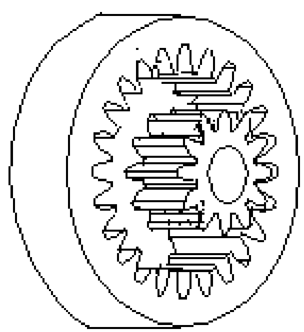




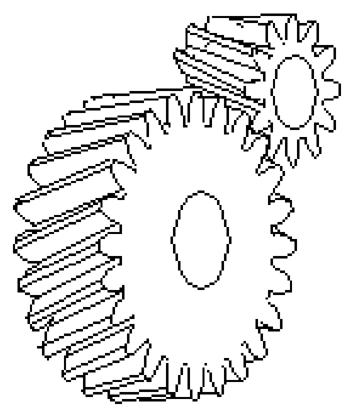
Heringbone gears (Double Helical gears)



External contact



Internal contact

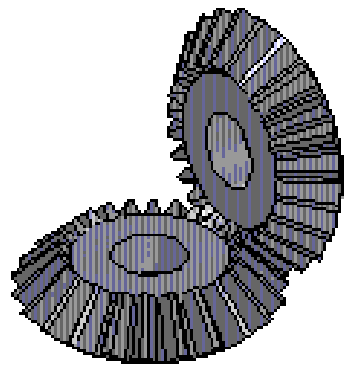


Parallel Helical gears

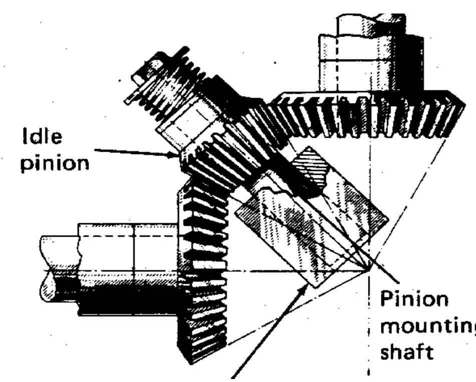
CLASSIFICATION OF TOOTHED WHEELS

1. *According to the position of axes of the shafts* **(b) Intersecting** *(Bevel Gears)*

Spiral bevel gears

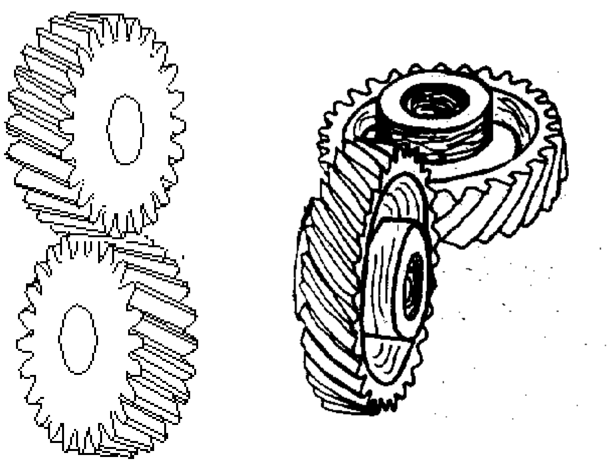


Straight bevel gears

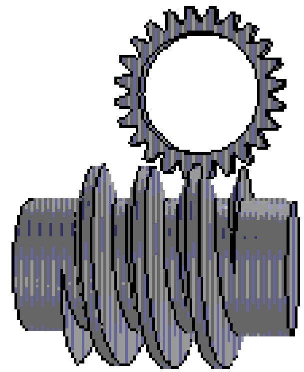


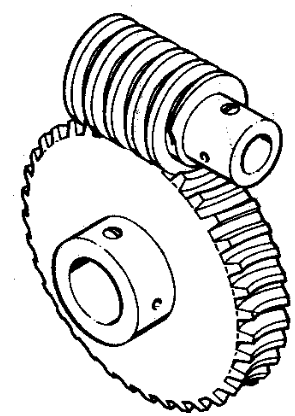
CLASSIFICATION OF TOOTHED WHEELS

1. *According to the position of axes of the shafts* **(c) Non-intersecting and non-parallel**



Crossed-helical gears





Worm & Worm Wheel

CLASSIFICATION OF TOOTHED WHEELS

2. According to the peripheral velocity of the gears

(a) Low velocity (velocity less than 3 m/s)

(b) Medium velocity (between 3 to 15 m/s)

(c) High velocity (More than 15 m/s)

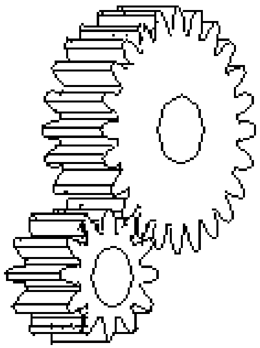
CLASSIFICATION OF TOOTHED WHEELS

3. According to the type of gearing

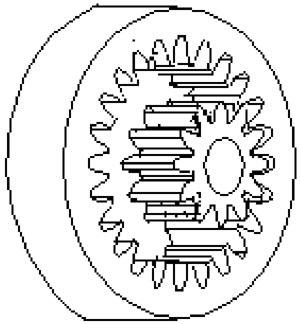
(a) External gearing

(b) Internal gearing

(c) Rack and pinion



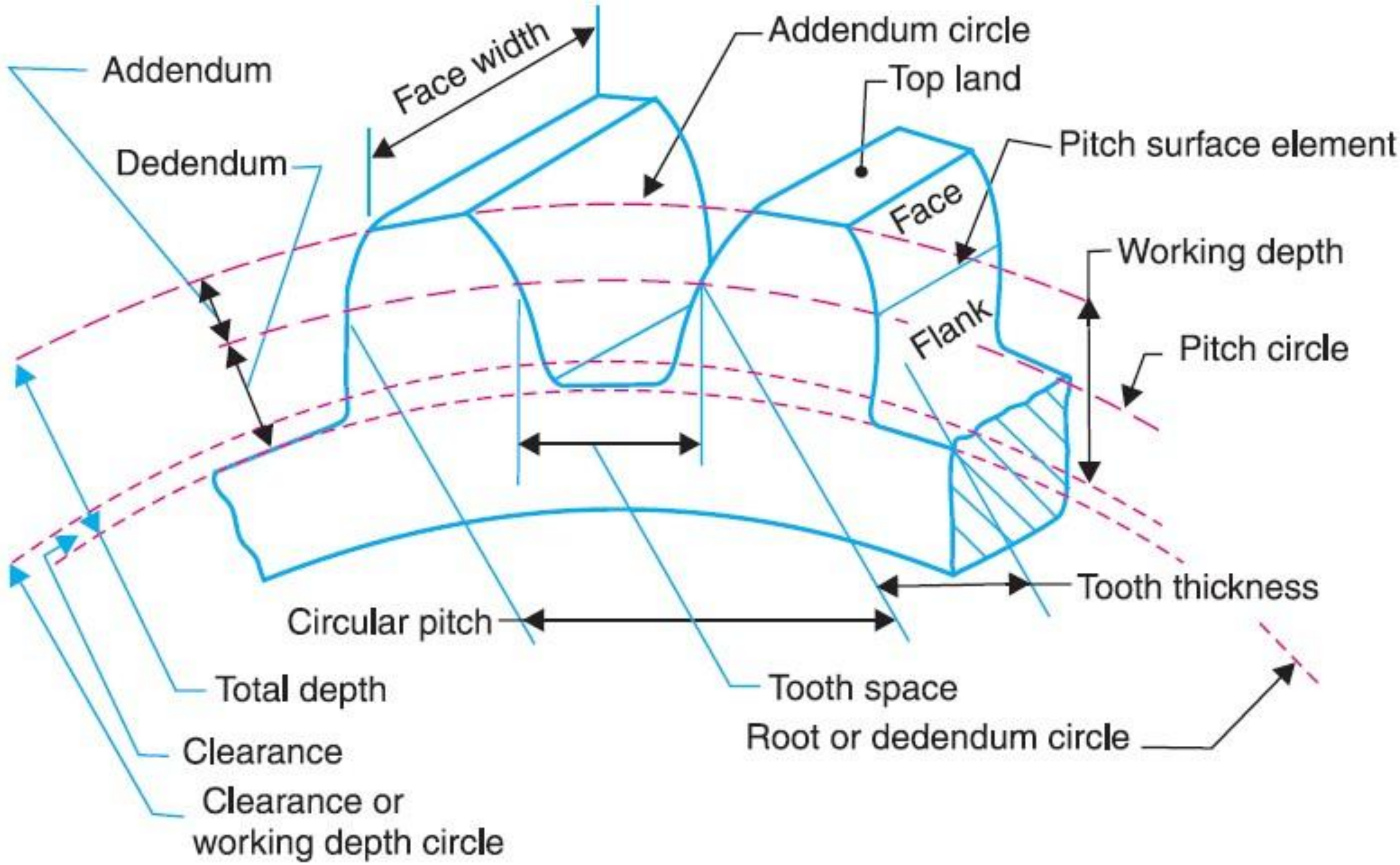
External gearing



Internal gearing

Rack and pinion

TERMINOLOGY : SPUR GEAR



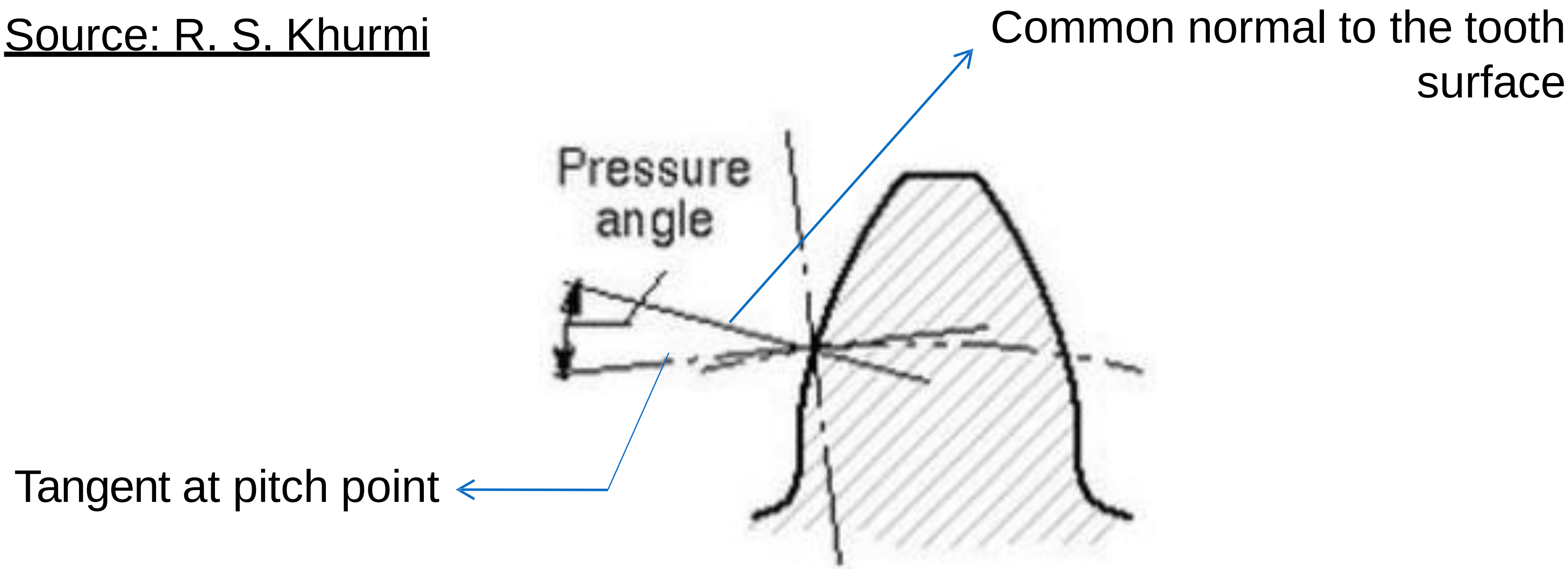
Source: R. S. Khurmi

TERMINOLOGY : SPUR GEAR

Pressure angle or angle of obliquity:

It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ø. The standard pressure angles are 14.5° and 20° .

Source: R. S. Khurmi



TERMINOLOGY : SPUR GEAR

Circular pitch: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c .

$$p_c = \pi D/T$$

D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

Note: Two gears will mesh together correctly, if the two wheels have the same circular pitch.

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

TERMINOLOGY : SPUR GEAR

Diametral pitch.

It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d. Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because \; p_c = \frac{\pi D}{T} \right)$$

T = Number of teeth, and

D = Pitch circle diameter.

Module.

It is the ratio of the pitch circle diameter in millimeters to the number of teeth.

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20.

TERMINOLOGY : SPUR GEAR

Backlash: It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle.

Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion

FORMULAE

$$Center\ distance = \begin{pmatrix} Teeth\ on\ pinion \\ + \\ Teeth\ on\ Gear \end{pmatrix} \frac{Circular\ pitch}{2 \times \pi}$$

$$= \frac{(Teeth\ on\ pinion + Teeth\ on\ Gear)}{2 \times Diametral\ pitch}$$

$$\text{Base Circle Diameter} = \text{Pitch Diameter} \times \text{Cos}\ \phi$$

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FORMULAE SPECIFIC TO GEARS WITH STANDARD TEETH

Addendum = 1 ÷ Diametral Pitch
= 0.3183 × Circular Pitch

Dedendum = 1.157 ÷ Diametral Pitch
= 0.3683 × Circular Pitch

Working Depth = 2 ÷ Diametral Pitch
= 0.6366 × Circular Pitch

Whole Depth = 2.157 ÷ Diametral Pitch
= 0.6866 × Circular Pitch

FORMULAE SPECIFIC TO GEARS WITH STANDARD TEETH

- **Clearance** = 0.157 ÷ Diametral Pitch
  = 0.05 × Circular Pitch

- **Outside Diameter** = (Teeth + 2) ÷ Diametral Pitch
  = (Teeth + 2) × Circular Pitch ÷ π

- **Diametral Pitch** = (Teeth + 2) ÷ Outside Diameter

GEAR MATERIALS

Selection of materials depends upon strength and service conditions like wear, noise etc.,

- Metallic materials (cast iron, steel (plain carbon steel or alloy steel) and bronze)

- Non- Metallic materials – reduces noise (wood, compressed paper and synthetic resins like nylon)

Note: phosphor bronze is widely used for worm gears in order to reduce wear of the worms

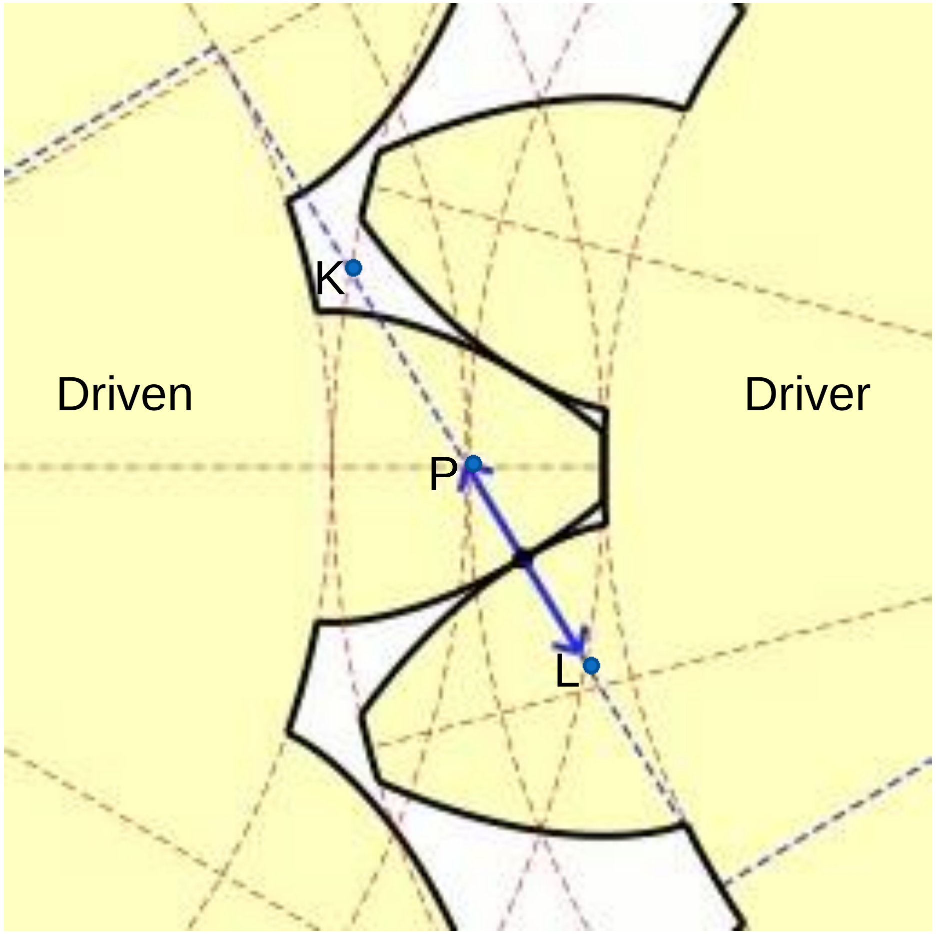
LAW OF GEARING

Involute Gear

The moving point 'P' is Pitch point.

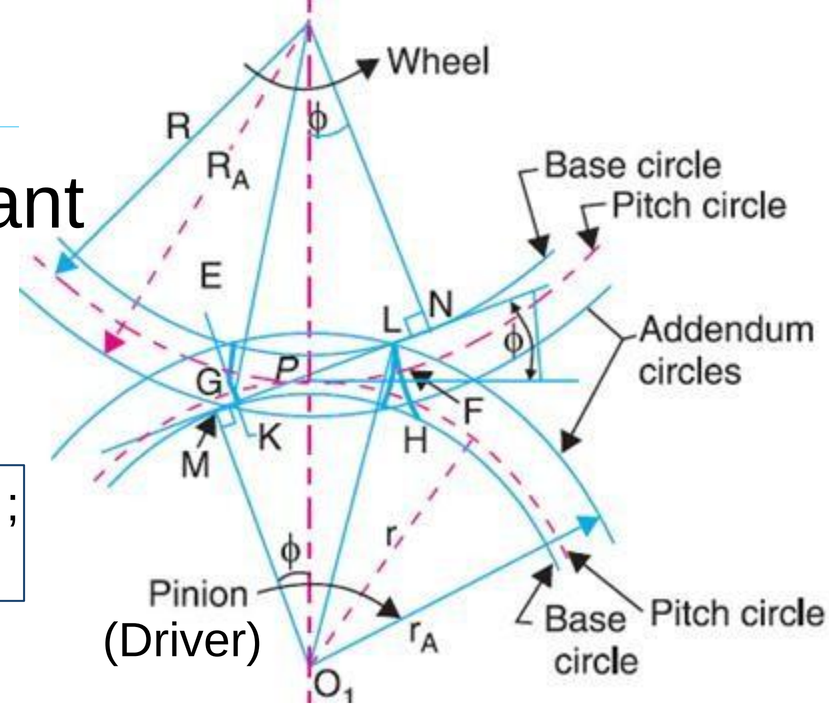
The profiles which give constant Velocity ratio & Positive drive is known as Conjugate profiles

KL – Length of path of contact
KP – Path of approach
PL – Path of recess



LAW OF GEARING

Source: R. S. Khurmi



Angular velocity Ratio is constant

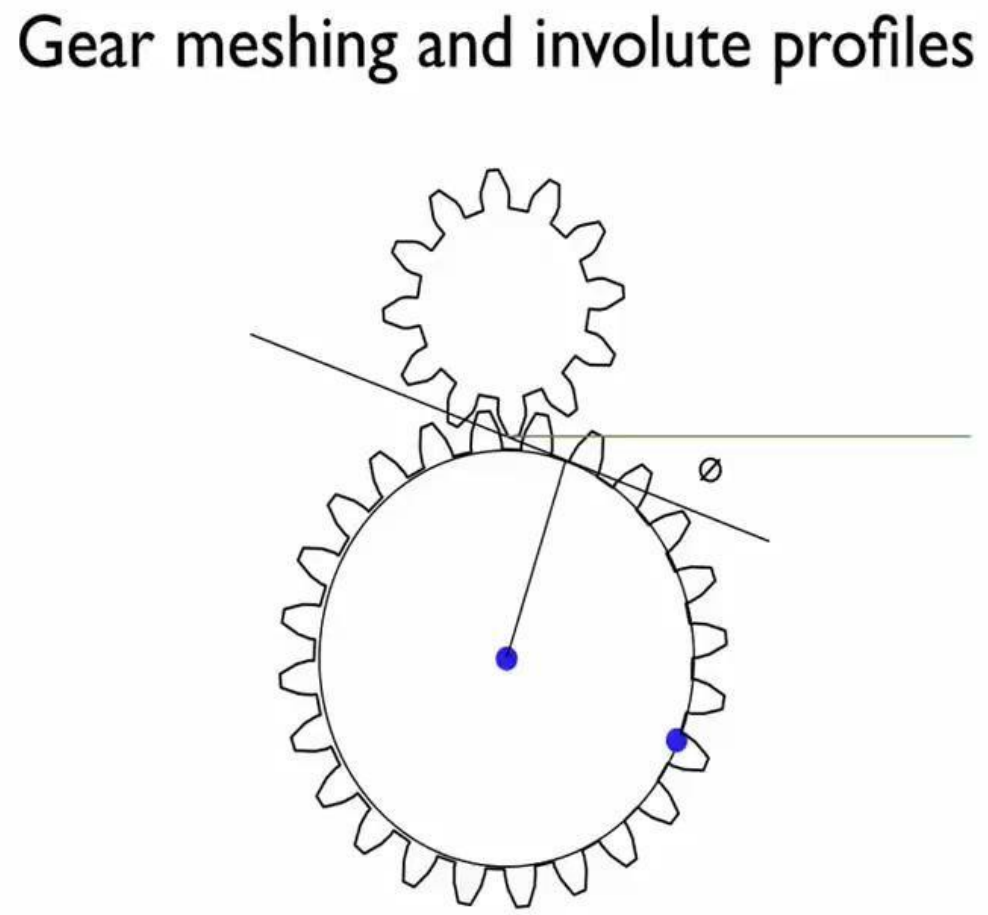
$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

D – pitch circle diameter ;
T – Number of Teeth

- In order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. i.e. the common normal at the point of contact between a pair of teeth must always pass through the pitch point.
- This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing

INVOLUTE TOOTH PROFILE



[https://www.youtube.com/watch?v=4QM0juVXW54]

COMPARISON BETWEEN INVOLUTE AND CYCLOIDAL GEARS

S. No	Involute Gears	Cycloidal Gears
1.	Advantage of the involute gears is that the centre distance for a pair of involute gears can be varied within limits without affecting velocity ratio	Not true
2.	Pressure angle, from the start of the engagement of teeth to the end of the engagement, remains constant (smooth running and less wear of gears)	pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts decreasing and again becomes maximum at the end of engagement (less smooth running of gears)
3.	The face and flank of Involute teeth are generated by a single curve. Hence, easy to manufacture.	double curves (i.e. epi-cycloid and hypo-cycloid) . Hence, difficult to manufacture.

COMPARISON BETWEEN INVOLUTE AND CYCLOIDAL GEARS

S.No	Involute Gears	Cycloidal Gears
4.	Less strong	Cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch
5.	Occurs	Interference does not occur
6.	Less weighted	outweighed

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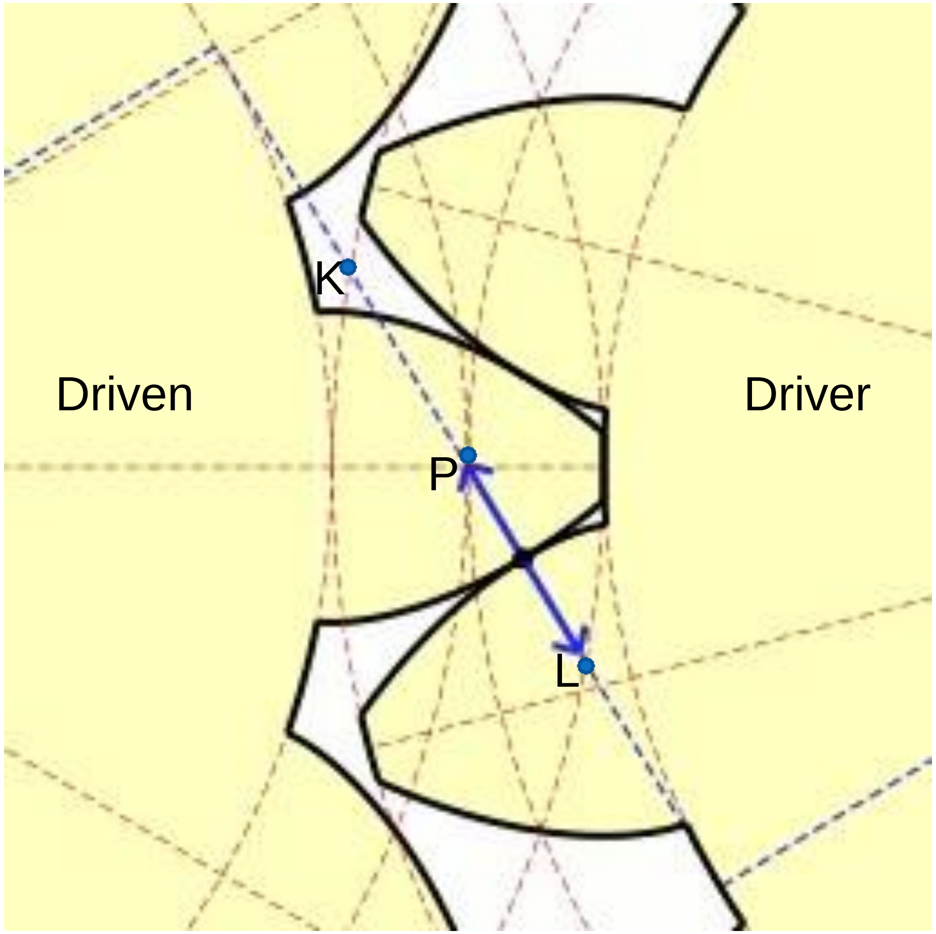
STANDARD PROPORTIONS OF GEAR SYSTEMS

S. No.	*Particulars*	$14\frac{1}{2}°$ *composite or full depth involute system*	*20° full depth involute system*	*20° stub involute system*
1.	Addenddm	1 *m*	1 *m*	0.8 *m*
2.	Dedendum	1.25 *m*	1.25 *m*	1 *m*
3.	Working depth	2 *m*	2 *m*	1.60 *m*
4.	Minimum total depth	2.25 *m*	2.25 *m*	1.80 *m*
5.	Tooth thickness	1.5708 *m*	1.5708 *m*	1.5708 *m*
6.	Minimum clearance	0.25 *m*	0.25 *m*	0.2 *m*
7.	Fillet radius at root	0.4 *m*	0.4 *m*	0.4 *m*

The increase of the pressure angle from $14\frac{1}{2}°$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base.

LENGTH OF PATH OF CONTACT

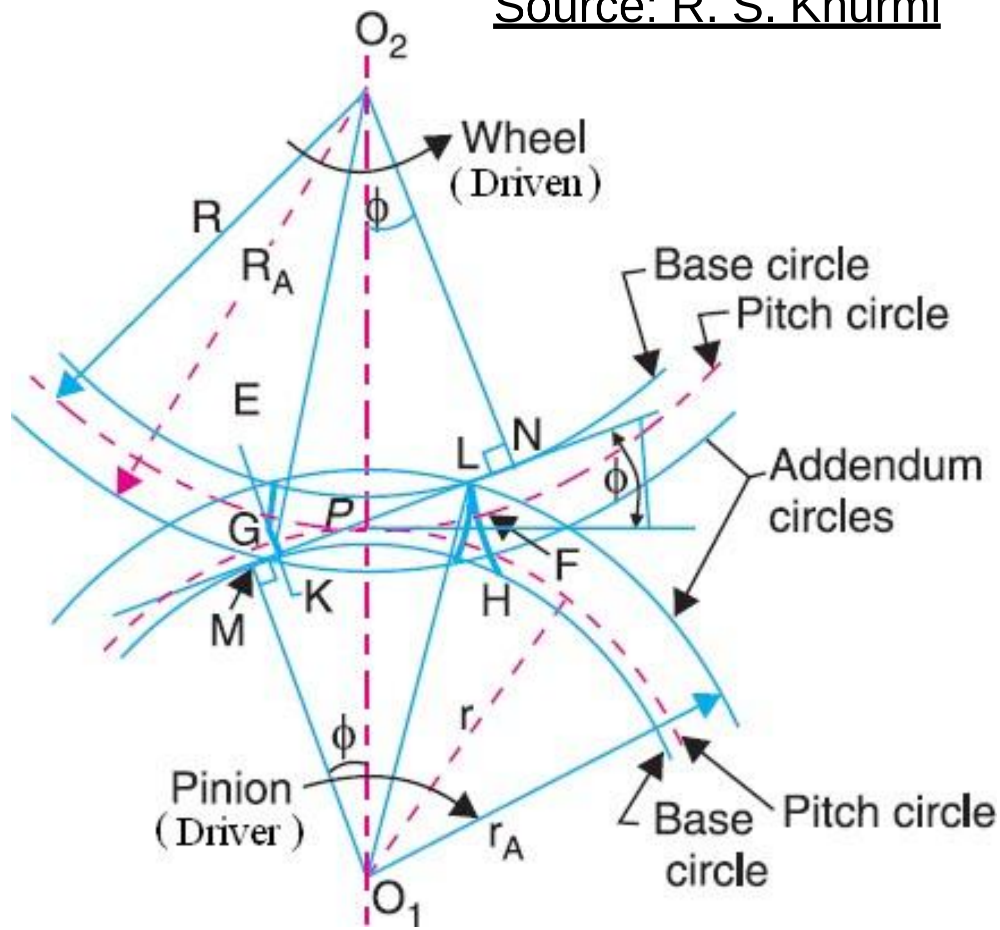
KL – Length of path of contact
KP – Path of approach
PL – Path of recess



LENGTH OF PATH OF CONTACT

Source: R. S. Khurmi

- contact between a pair of involute teeth begins at K ends at L
- MN is the common normal at the point of contact
- MN is also the common tangent to the base circles



KP – Path of approach
PL – Path of recess
KL – Length of path of contact

LENGTH OF PATH OF CONTACT

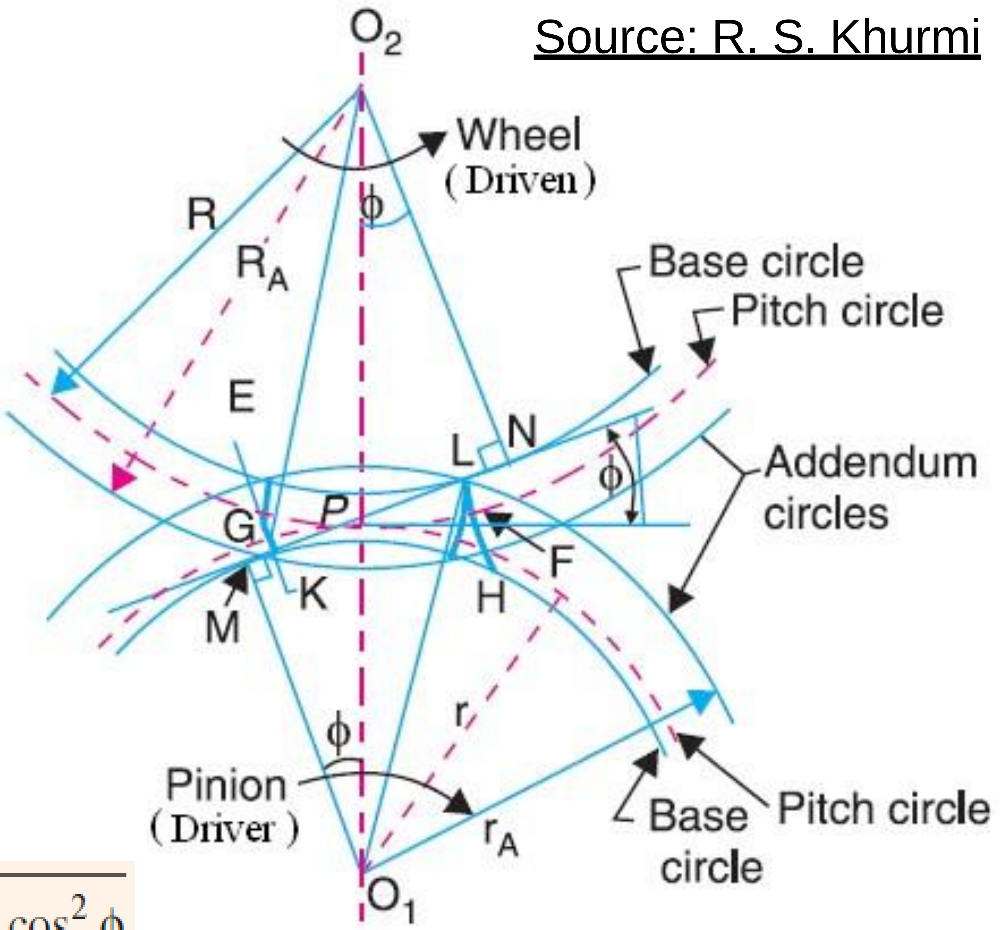
Source: R. S. Khurmi

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.



From Triangle O_1MP, $O_1M = r\cos\phi$

Triangle O_2NP, $O_2N = R\cos\phi$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

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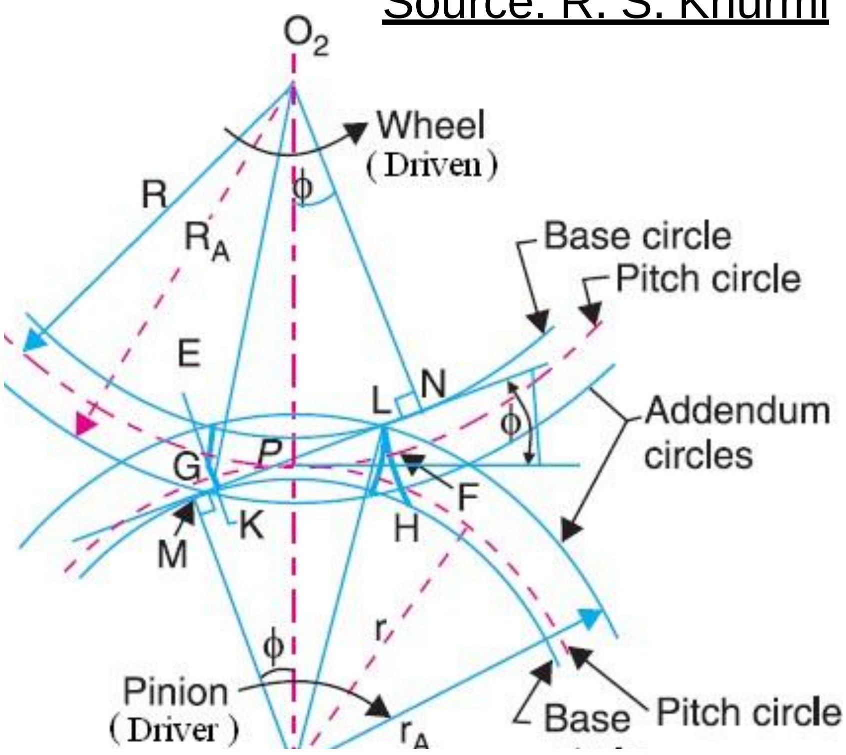
LENGTH OF PATH OF CONTACT

Source: R. S. Khurmi

∴ Length of the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi$$



Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$ and

$$MP = O_1P\sin\phi = r\sin\phi$$

∴ Length of path of recess, PL

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

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LENGTH OF ARC OF CONTACT

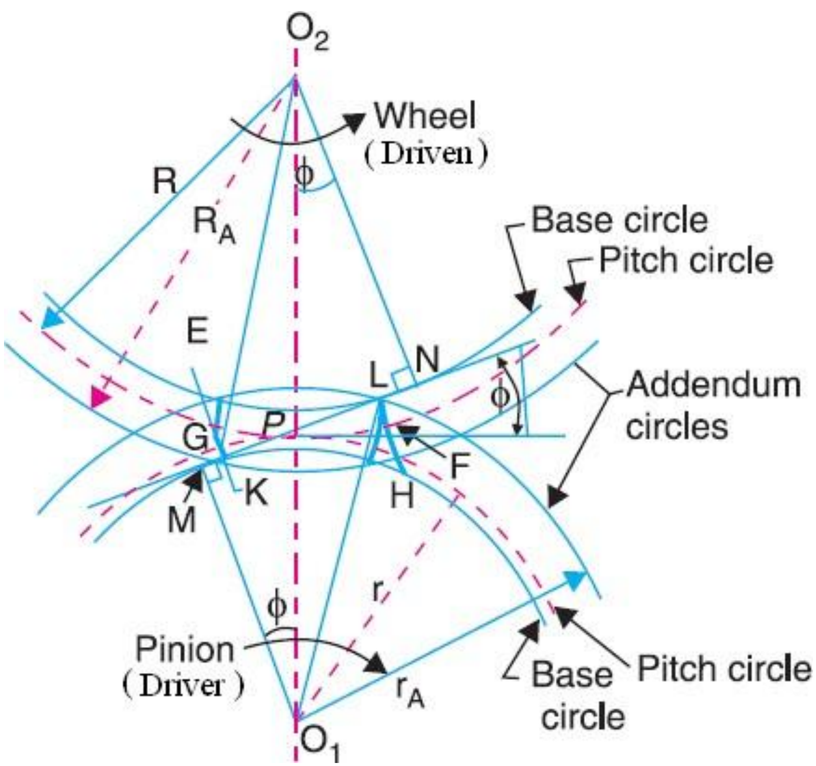
Source: R. S. Khurmi

- arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth

- Arc of contact is *EPF or GPH.*

- The arc *GP is known as arc of approach*

- The arc *PH is called arc of recess*



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LENGTH OF ARC OF CONTACT

We know that the length of the arc of approach (arc GP)

$$= \frac{\text{Length of path of approach}}{\cos\phi} = \frac{KP}{\cos\phi}$$

the length of the arc of recess (arc PH)

$$= \frac{\text{Length of path of recess}}{\cos\phi} = \frac{PL}{\cos\phi}$$

$$\text{Length of the arc of contact} = \text{arc } GP + \text{arc } PH = \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

CONTACT RATIO
(NUMBER OF PAIRS OF TEETH IN CONTACT)

- It is defined as the ratio of the length of the arc of contact to the circular pitch.

$$\text{Contact ratio} = \frac{\text{Length of the arc of contact}}{p_c}$$

$$p_c = \text{Circular pitch} = \pi m, \text{ and}$$

$$m = \text{Module.}$$

The contact ratio, usually, is not a whole number. For example, if the contact ratio is 1.6, it does not mean that there are 1.6 pairs of teeth in contact. It means that there are alternately one pair and two pairs of teeth in contact and on a time basis the average is 1.6

Larger the contact ratio, more quietly the gears will operate

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NUMERICAL EXAMPLE -1

The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.

Given : $T = t = 40$; $\phi = 20°$; $m = 6$ mm

Length of arc of contact $= 1.75\ p_c$

We know that the circular pitch,

$$p_c = \pi\, m = \pi \times 6 = 18.85 \text{ mm}$$

Length of arc of contact $= 1.75\ p_c = 1.75 \times 18.85 = 33$ mm

Length of path of contact = Length of arc of contact × cos ϕ = 33 cos 20° = 31 mm

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NUMERICAL EXAMPLE -1

We know that pitch circle radii of each wheel,

$$R = r = m.T / 2 = 6 \times 40/2 = 120 \text{ mm}$$

length of path of contact $=31 = \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi$

$$= 2\left[\sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi\right] \quad ...(\because R = r, \text{ and } R_A = r_A)$$

$$R_A = 126.12 \text{ mm}$$

addendum of the wheel,

$$= R_A - R = 126.12 - 120 = 6.12 \text{ mm}$$ **Ans.**

NUMERICAL EXAMPLE -2

A pair of gears, having 40 and 20 teeth respectively, are rotating in mesh, the speed of the smaller being 2000 r.p.m.

Determine the velocity of sliding between the gear teeth faces at the point of engagement, at the pitch point, and at the point of disengagement if the smaller gear is the driver. Assume that the gear teeth are 20° involute form, addendum length is 5 mm and the module is 5 mm.

Also find the angle through which the pinion turns while any pairs of teeth are in contact.

NUMERICAL EXAMPLE -2

Solution. Given : $T = 40$; $t = 20$; $N_1 = 2000$ r.p.m. ; $\phi = 20°$; addendum = 5 mm ; $m = 5$ mm

We know that angular velocity of the smaller gear,

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 2000}{60} = 209.5 \text{ rad/s}$$

angular velocity of the larger gear, $\omega_2 = 104.75$ rad/s $\quad ...\left(\because \frac{\omega_2}{\omega_1} = \frac{t}{T}\right)$

Pitch circle radius of the smaller gear, $r = m.t / 2 = 5 \times 20/2 = 50$ mm

$R = m.T / 2 = 5 \times 40/2 = 100$ mm

Radius of addendum circle of smaller gear, $r_A = r + \text{Addendum} = 50 + 5 = 55$ mm

larger gear, $R_A = R + \text{Addendum} = 100 + 5 = 105$ mm

length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(105)^2 - (100)^2\cos^2 20°} - 100\sin 20°$$

$$= 12.65 \text{ mm}$$

length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20°} - 50\sin 20°$$

$$= 11.5 \text{ mm}$$

NUMERICAL EXAMPLE -2

Velocity of sliding at the point of engagement

We know that velocity of sliding at the point of engagement *K*,

$$v_{SK} = (\omega_1 + \omega_2)\ KP = (209.5 + 104.75)\ 12.65 = 3975 \text{ mm/s} \quad \textbf{Ans.}$$

Velocity of sliding at the pitch point

Since the velocity of sliding is proportional to the distance of the contact point from the pitch point, therefore the velocity of sliding at the pitch point is zero. **Ans.**

Velocity of sliding at the point of disengagement

We know that velocity of sliding at the point of disengagement *L*,

$$v_{SL} = (\omega_1 + \omega_2)\ PL = (209.5 + 104.75)\ 11.5 = 3614 \text{ mm/s} \quad \textbf{Ans.}$$

Angle through which the pinion turns

$$= \text{Length of arc of contact} \times \frac{360°}{\text{Circumference of pinion}}$$

$$= 25.7 \times \frac{360°}{314.2} = 29.45° \quad \textbf{Ans.}$$

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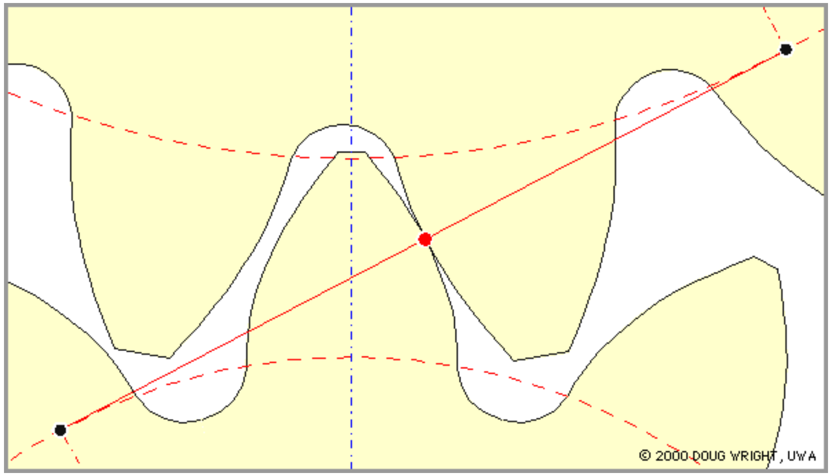
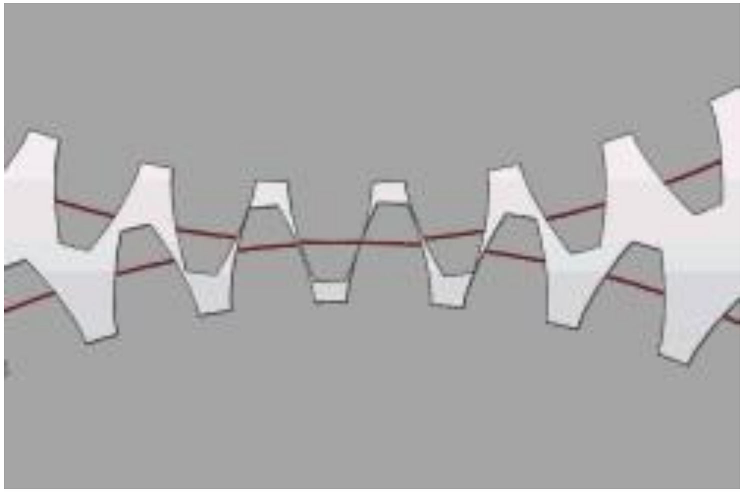
INTERFERENCE AND UNDERCUTTING

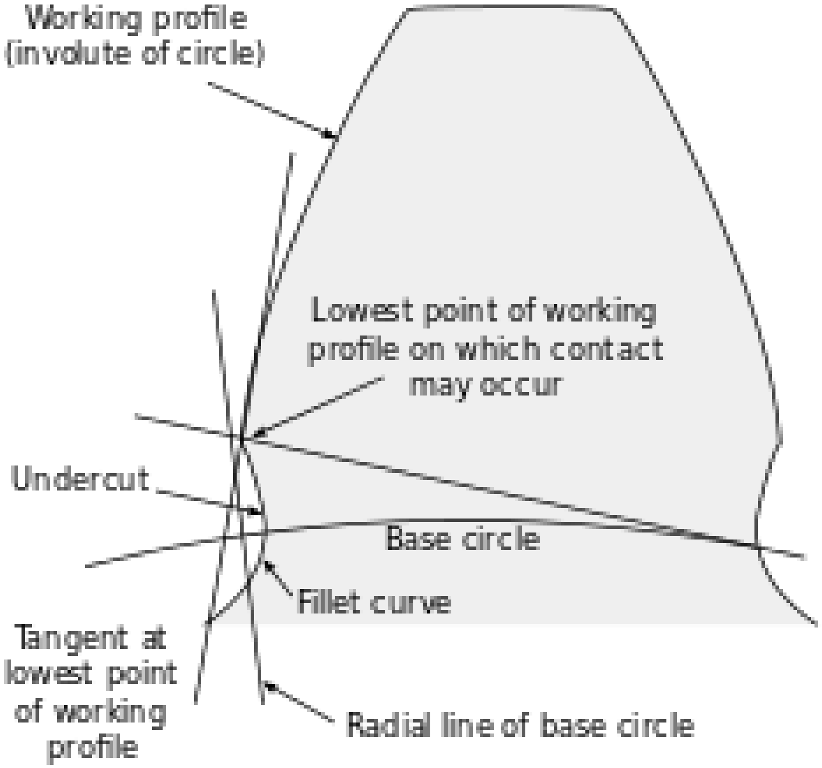


Full fit involute (Conjugate Profile)



Full fit involute (Conjugate Profile)

Source: R. S. Khurmi



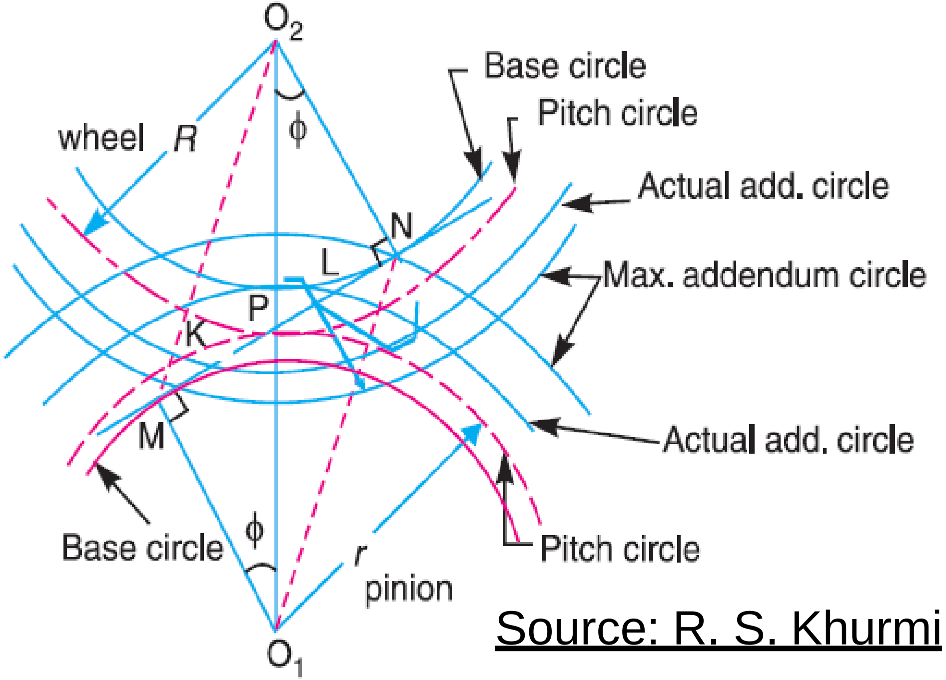
INTERFERENCE AND UNDERCUTTING

- if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N.

- When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel

The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as interference

The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.



Source: R. S. Khurmi

Interference in involute gears.

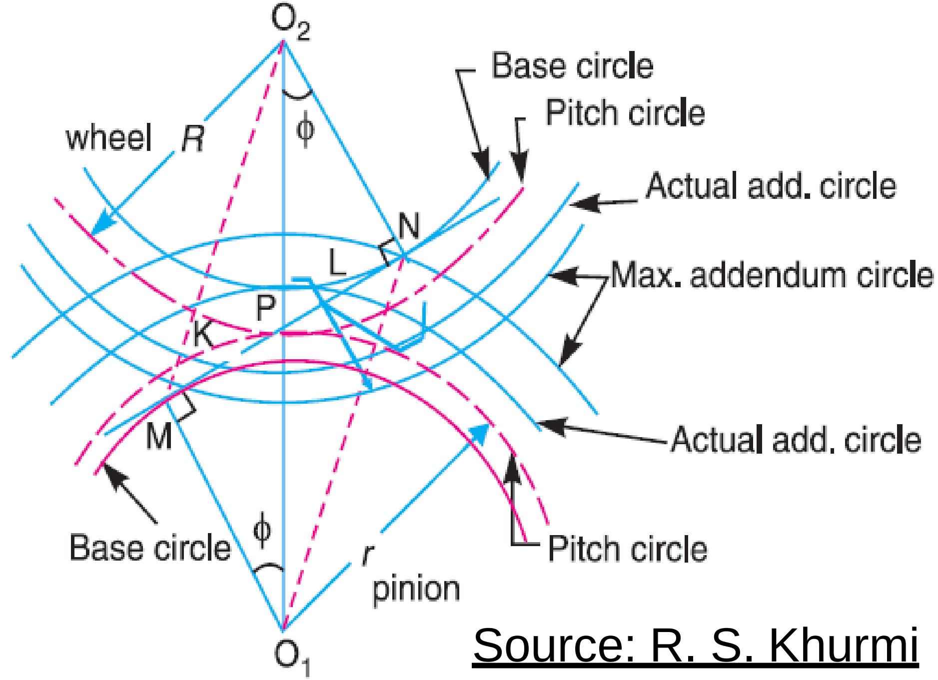
INTERFERENCE AND UNDERCUTTING

Similarly, if the radius of the addendum circle of the wheel increases beyond O2M, the tip of tooth on wheel will cause interference with the tooth on pinion.

The points M and N are called interference points.

Obviously, interference may be avoided if the path of contact does not extend beyond interference points.

The limiting value of the radius of the addendum circle of the pinion is O1N and of the wheel is O2M.



Source: R. S. Khurmi

Interference in involute gears.

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INTERFERENCE AND UNDERCUTTING

~~To avoid interference:~~ Maximum length of path of approach, $MP = r \sin \phi$

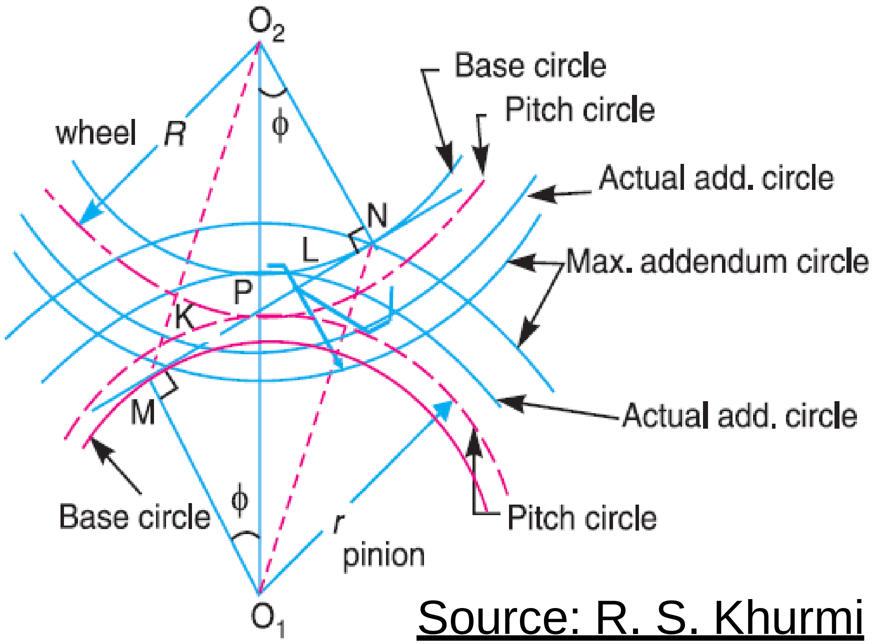
maximum length of path of recess, $PN = R \sin \phi$

$\therefore$ Maximum length of path of contact,

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

maximum length of arc of contact =

$$\frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$



Source: R. S. Khurmi

Interference in involute gears.

NUMERICAL EXAMPLE -3

Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.

Solution. Given : $t = 20$; $T = 40$; $m = 10$ mm ; $\phi = 20°$

$r = 100$ mm

$R = 200$ mm

Find pitch circle radius using $r = m.t/2$

the line of contact on each side of the pitch point (*i.e.* the path of approach and the path of recess) has half the maximum possible length, therefore

Path of approach, $KP = \frac{1}{2} MP$

$$\sqrt{(R_A)^2 - R^2 \cos^2\phi} - R\sin\phi = \frac{r.\sin\phi}{2} \Rightarrow R_A = 206.5 \text{ mm}$$

NUMERICAL EXAMPLE -3

∴ Addendum height for larger gear wheel

$$= R_A - R = 206.5 - 200 = 6.5 \text{ mm } \textbf{Ans.}$$

Now path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi = \frac{R.\sin \phi}{2} \Longrightarrow r_A = 116.2 \text{ mm}$$

Addendum height for smaller gear wheel $= r_A - r = 6.2$ mm **Ans.**

Length of the path of contact $= KP + PL = \frac{1}{2} MP + \frac{1}{2} PN = \frac{(r + R)\sin \phi}{2} = 51.3$ mm **Ans.**

Length of the arc of contact $= \dfrac{\text{Length of the path of contact}}{\cos \phi} = \dfrac{51.3}{\cos 20°} = 54.6$ mm **Ans.**

Contact ratio

circular pitch, $P_c = \pi m = \pi \times 10 = 31.42$ mm

$$\text{Contact ratio} = \frac{\text{Length of the path of contact}}{p_c} = 1.74 \textbf{ Ans.}$$

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

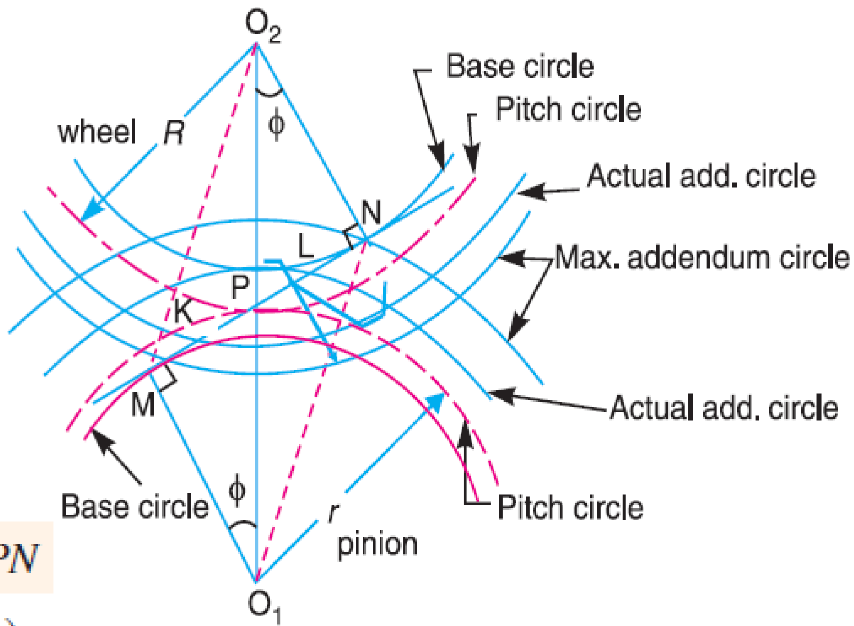
$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2 \left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R \sin^2\phi}{r}\right]$$

$$= r^2 \left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$



Interference in involute gears.

Source: R. S. Khurmi

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

$$(O_1N)^2 = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

∴ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_P.m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

$$\text{addendum of the pinion} = O_1N - O_1P$$

$$A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2}$$

$$\ldots(\because O_1P = r = m.t/2)$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G + 2)\sin^2\phi} - 1}$$

MINIMUM NUMBER OF TEETH ON THE PINION IN ORDER TO AVOID INTERFERENCE

S. No.	*System of gear teeth*	*Minimum number of teeth on the pinion*
1.	$14\frac{1}{2}°$ Composite	12
2.	$14\frac{1}{2}°$ Full depth involute	32
3.	20° Full depth involute	18
4.	20° Stub involute	14

NUMERICAL EXAMPLE -4

A pair of spur gears with involute teeth is to give a gear ratio of 4 : 1. The arc of approach is not to be less than the circular pitch and smaller wheel is the driver. The angle of pressure is 14.5°. Find : 1. the least number of teeth that can be used on each wheel, and 2. the addendum of the wheel in terms of the circular pitch ?

Solution. *Given* : $G = T/t = R/r = 4$; $\phi = 14.5°$

1. ***Least number of teeth on each wheel***

Let t = Least number of teeth on the smaller wheel *i.e.* pinion,

T = Least number of teeth on the larger wheel *i.e.* gear, and

r = Pitch circle radius of the smaller wheel *i.e.* pinion.

the maximum length of the arc of approach

$$= \frac{\text{Maximum length of the path of approach}}{\cos\phi} = \frac{r\sin\phi}{\cos\phi} = r\tan\phi$$

$$\text{circular pitch, } p_c = \pi m = \frac{2\pi r}{t} \qquad \ldots\left(\because m = \frac{2r}{t}\right)$$

NUMERICAL EXAMPLE -4

Since the arc of approach is not to be less than the circular pitch, therefore

$$r\ \tan\phi = \frac{2\pi r}{t} \quad \text{or} \quad t = \frac{2\pi}{\tan\phi} = \frac{2\pi}{\tan 14.5°} = 24.3 \text{ say } 25 \textbf{ Ans.}$$

$$T = G.t = 4 \times 25 = 100 \textbf{ Ans.} \qquad ...(\because G = T/t)$$

2. Addendum of the wheel

addendum of the wheel

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{m \times 100}{2}\left[\sqrt{1 + \frac{25}{100}\left(\frac{25}{100} + 2\right)\sin^2 14.5°} - 1\right]$$

$$= 0.85\ m = 0.85 \times p_c/\pi = 0.27\ p_c \textbf{ Ans.}$$

$$...(\because m = p_c/\pi)$$

GEAR TRAINS

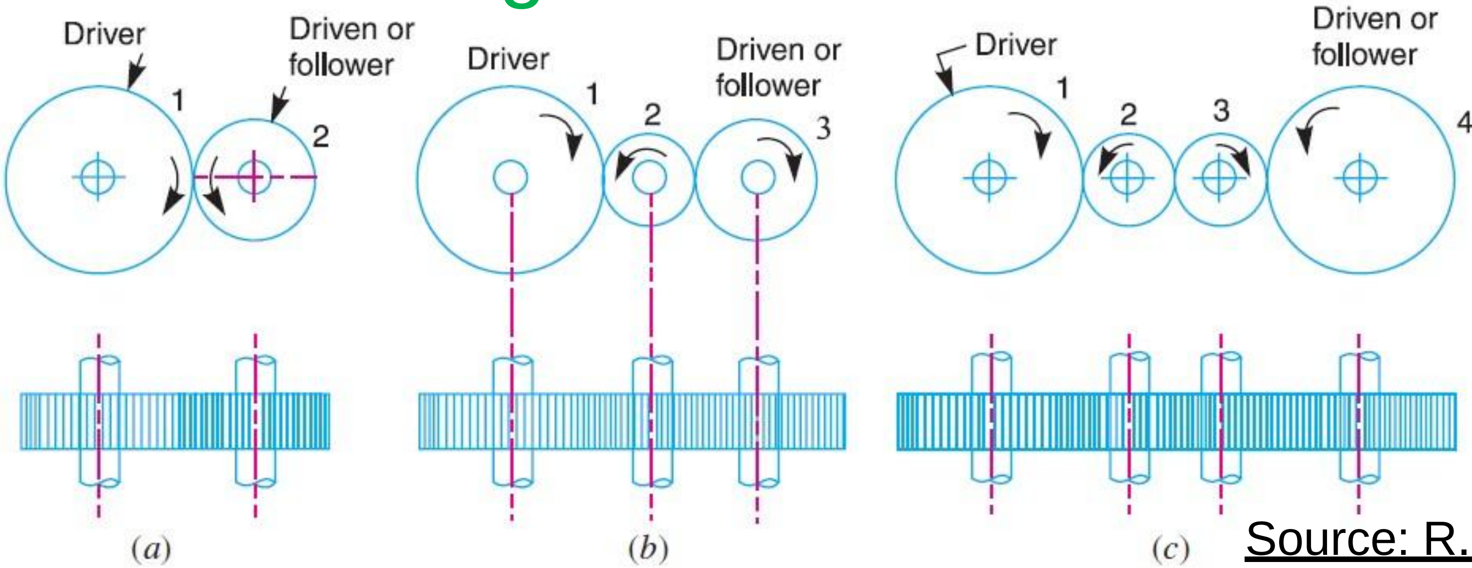
Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

SIMPLE GEAR TRAIN

One gear on each shaft



Source: R. S. Khurmi

If the distance between the two gears is large, intermediate gears employed. If the number of intermediate gears are odd, the motion of both the Gears is like. If Even - unlike direction

N_1 = Speed of gear 1(or driver) in r.p.m., N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and T_2 = Number of teeth on gear 2.

The speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

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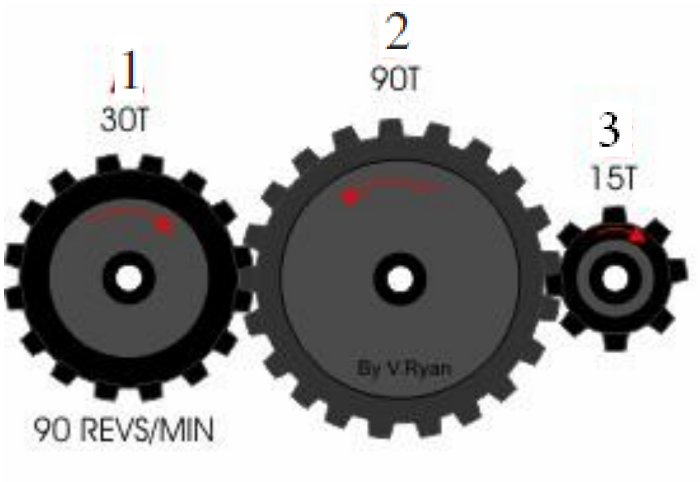
SIMPLE GEAR TRAIN

The ratio of the speed of the driven to the speed of the driver is known as train value of the gear train



$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

speed ratio for gear 1 & 2 $\quad \frac{N_1}{N_2} = \frac{T_2}{T_1}$

speed ratio for gear 2 & 3 $\quad \frac{N_2}{N_3} = \frac{T_3}{T_2}$

The speed ratio of the gear train is obtained by multiplying the above two equations

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \quad \text{or} \quad \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

SIMPLE GEAR TRAIN

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

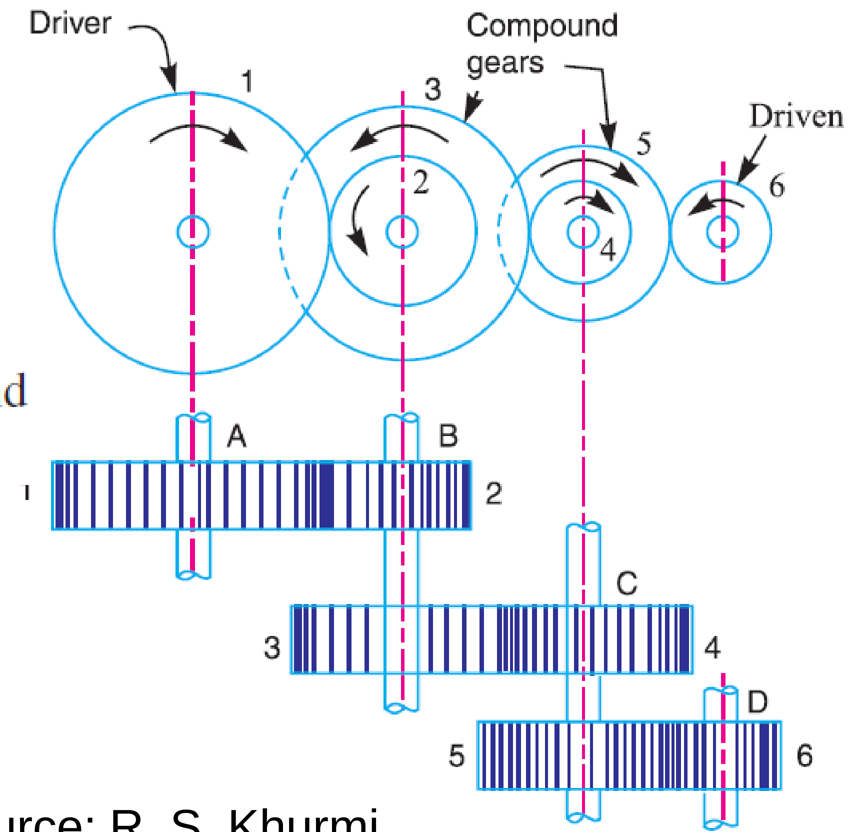
The intermediate gears are called idle gears, as they do not effect the speed ratio or train value of the system.

The idle gears are used

- To connect gears where a large centre distance is required, and
- To obtain the desired direction of motion of the driven gear (i.e. clockwise or anticlockwise).

COMPOUND GEAR TRAIN

More than one gear on a shaft



$N_2, N_3 \ldots, N_6$ = Speed of respective gears in r.p.m., and
$T_2, T_3 \ldots, T_6$ = Number of teeth on respective gears.

speed ratio 1 and 2 $\quad \dfrac{N_1}{N_2} = \dfrac{T_2}{T_1}$

speed ratio 3 and 4, $\quad \dfrac{N_3}{N_4} = \dfrac{T_4}{T_3}$

gears 5 and 6, speed ratio $\quad \dfrac{N_5}{N_6} = \dfrac{T_6}{T_5}$

Source: R. S. Khurmi

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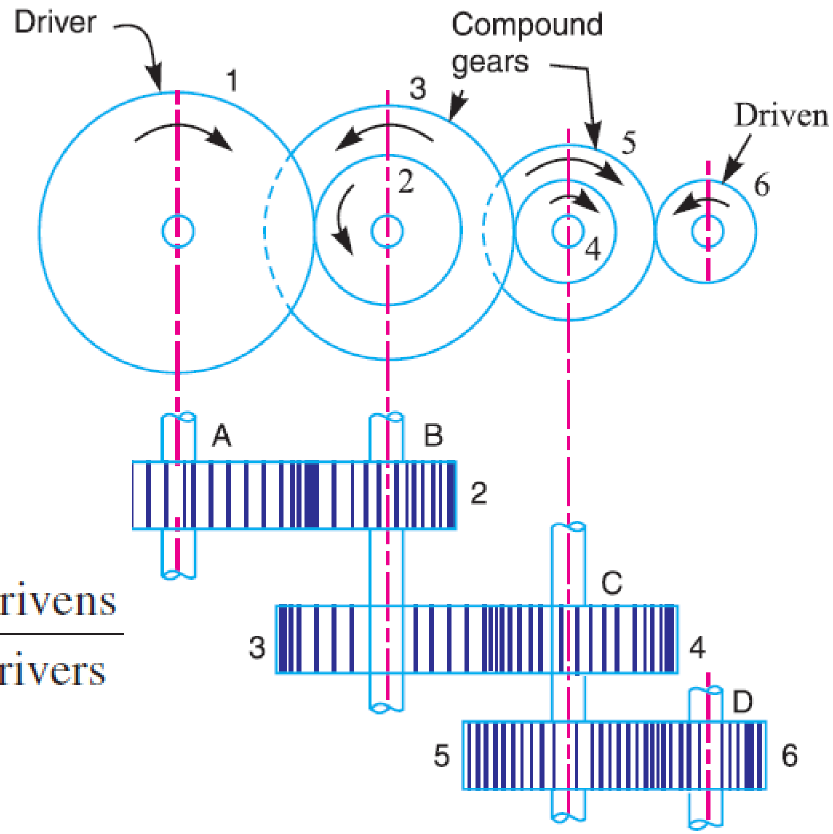
COMPOUND GEAR TRAIN

The speed ratio of compound gear train is obtained by

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

or $$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$



$$\text{Speed ratio} = \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

$$\text{Train value} = \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}}$$

$$= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the drivens}}$$

Source: R. S. Khurmi

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COMPOUND GEAR TRAIN

Advantage of Compound Gear Train over simple gear train:

- a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.
- If a simple gear train is used to give a large speed reduction, the last gear has to be very large.

Design of Spur Gears

x = Distance between the centres of two shafts,

N_1 = Speed of the driver,

T_1 = Number of teeth on the driver,

d_1 = Pitch circle diameter of the driver,

N_2, T_2 and d_2 = Corresponding values for the driven

p_c = Circular pitch.

$$x = \frac{d_1 + d_2}{2}$$

speed ratio

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

NUMERICAL EXAMPLE -1

Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 r.p.m. and the other at 120 r.p.m. Design the gears, if the circular pitch is to be 25 mm.

Given : $x = 600$ mm ; $N_1 = 360$ r.p.m. ; $N_2 = 120$ r.p.m. ; $p_c = 25$ mm

d_1 = Pitch circle diameter of the first gear, and

d_2 = Pitch circle diameter of the second gear. $T_2 = 3T_1 = 114$ ($\because$ Speed ratio =3)

speed ratio, $\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{360}{120} = 3$ or $d_2 = 3d_1$ ...(*i*)

$$x = 600 = \frac{1}{2}(d_1 + d_2) \quad \text{...(ii)}$$

From (*i*) and (*ii*), $d_1 = 300$ mm, and $d_2 = 900$ mm

Number of teeth on the first gear,

$$T_1 = \frac{\pi d_2}{p_c} = \frac{\pi \times 300}{25} = 37.7 \quad = 38$$

Now the exact pitch circle diameter of the first gear,

$$d_1' = \frac{T_1 \times p_c}{\pi} = \frac{38 \times 25}{\pi} = 302.36 \text{ mm}$$

the exact pitch circle diameter of the second gear,

$$d_2' = \frac{T_2 \times p_c}{\pi} = \frac{114 \times 25}{\pi} = 907.1 \text{ mm}$$

Exact distance between the two shafts,

$$x' = \frac{d_1' + d_2'}{2} = 604.73 \text{ mm}$$

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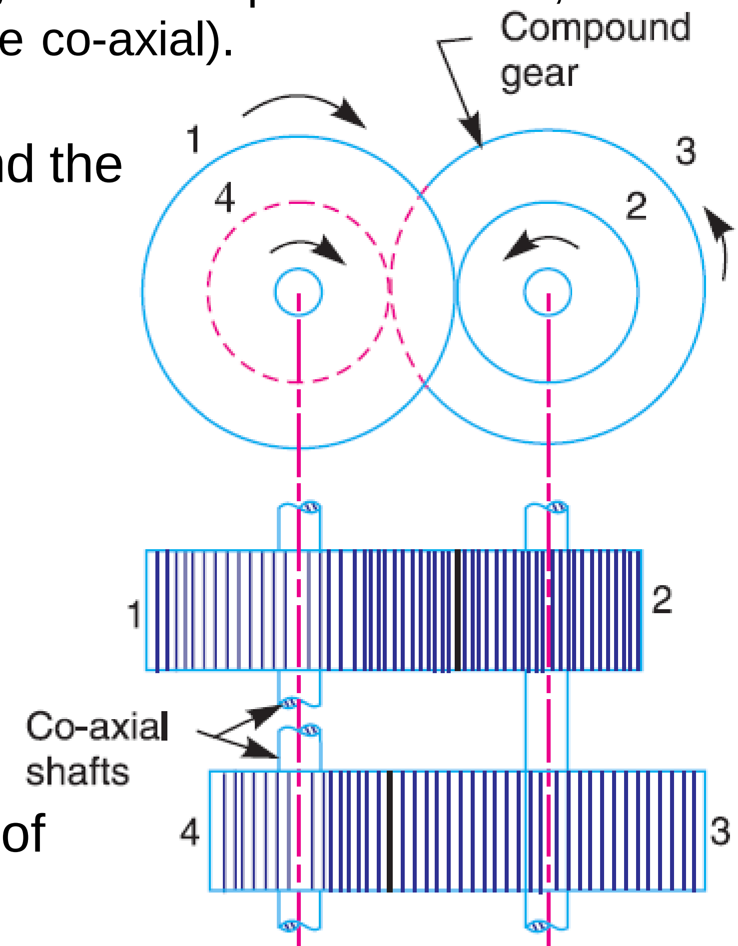
REVERTED GEAR TRAIN

Used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).



Source: R. S. Khurmi

The axes of the first gear (i.e. first driver) and the last gear (i.e.Last driven are co-axial)

Let T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1, and

N_1 = Speed of gear 1 in r.p.m.

Similarly,

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears, and

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

The distance between the centres of the shafts of gears 1 and 2 and the gears 3 and 4 are same

$$r_1 + r_2 = r_3 + r_4 \quad \ldots(i)$$

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REVERTED GEAR TRAIN

$$T_1 + T_2 = T_3 + T_4 \qquad \ldots(ii)$$

We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \quad \text{or} \quad r = \frac{m.T}{2},$$

$$r_1 = \frac{m.T_1}{2}\ ;\ r_2 = \frac{m.T_2}{2}\ ;\ r_3 = \frac{m.T_3}{2}\ ;\ r_4 = \frac{m.T_4}{2}$$

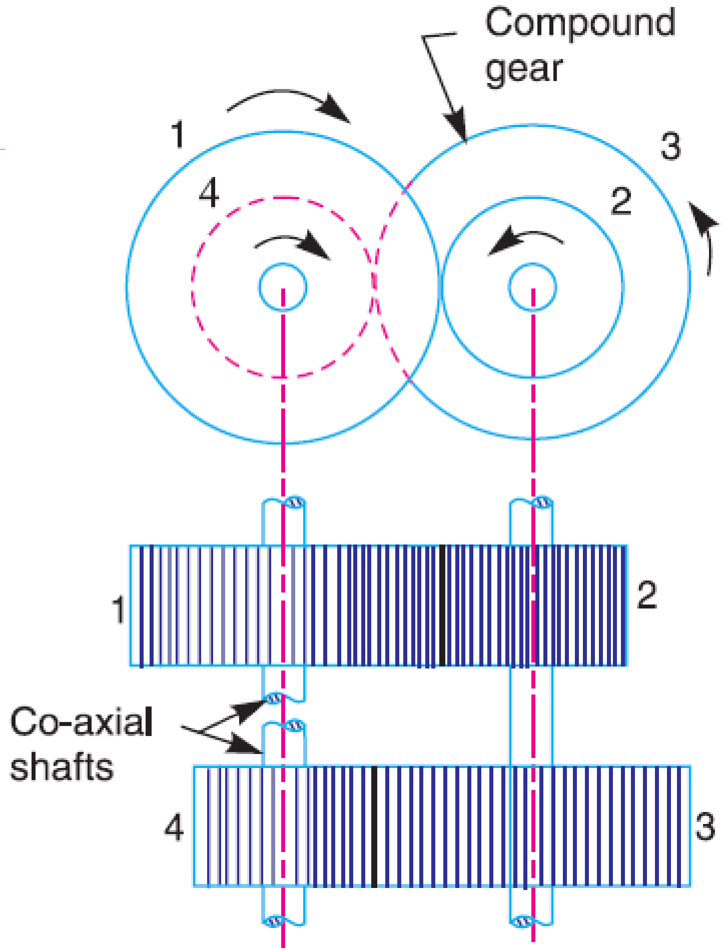
from equation $r_1 + r_2 = r_3 + r_4$

$$\frac{m.T_1}{2} + \frac{m.T_2}{2} = \frac{m.T_3}{2} + \frac{m.T_4}{2}$$

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \qquad \ldots(iii)$$

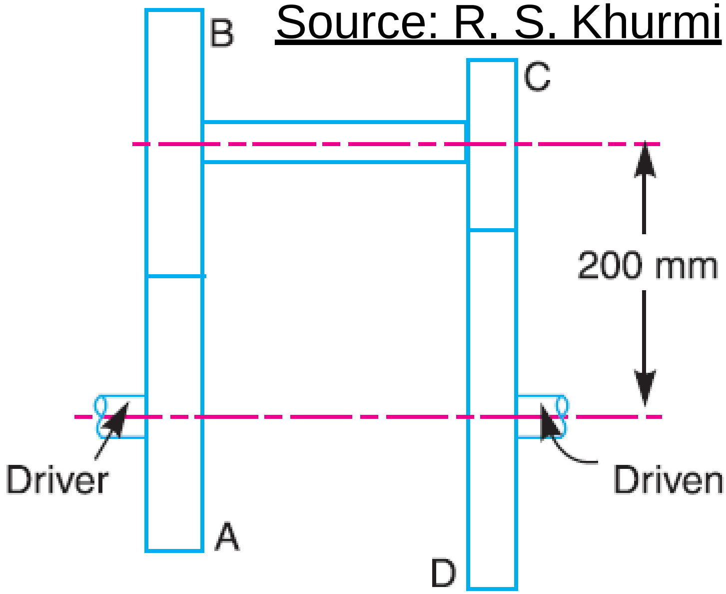


Source: R. S. Khurmi

From equations (i), (ii) and (iii), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily

NUMERICAL EXAMPLE-2

The speed ratio of the reverted gear train, as shown in the figure is to be 12. The module of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.



Source: R. S. Khurmi

Solution. Given : Speed ratio, $N_A/N_D = 12$; $m_A = m_B = 3.125$ mm ; $m_C = m_D = 2.5$ mm

Let N_A = Speed of gear A,

T_A = Number of teeth on gear A,

r_A = Pitch circle radius of gear A,

N_B, N_C, N_D = Speed of respective gears,

T_B, T_C, T_D = Number of teeth on respective gears, and

r_B, r_C, r_D = Pitch circle radii of respective gears.

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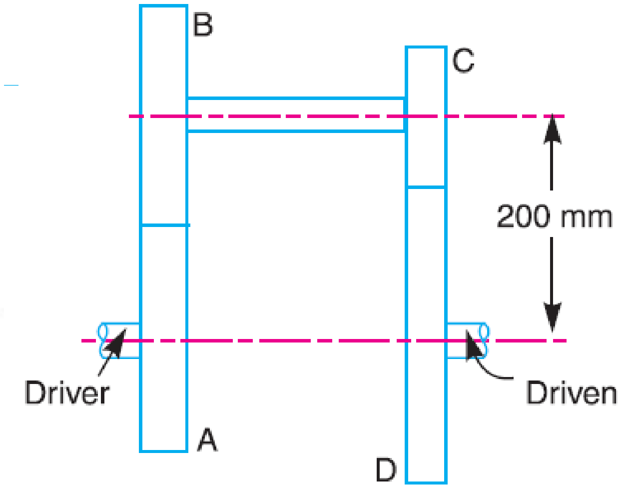
NUMERICAL EXAMPLE-2

We know that speed ratio $= \dfrac{\text{Speed of first driver}}{\text{Speed of last driven}} = \dfrac{N_A}{N_D} = 12$



Also $\quad \dfrac{N_A}{N_D} = \dfrac{N_A}{N_B} \times \dfrac{N_C}{N_D} \quad$ ...($N_B = N_C$, being on the same shaft)

For $\dfrac{N_A}{N_B}$ and $\dfrac{N_C}{N_D}$ to be same, each speed ratio should be $\sqrt{12}$ so that

$$\frac{N_A}{N_D} = \frac{N_A}{N_B} \times \frac{N_C}{N_D} = \sqrt{12} \times \sqrt{12} = 12$$

therefore $\dfrac{N_A}{N_B} = \dfrac{N_C}{N_D} = \sqrt{12} = 3.464 \Rightarrow \dfrac{T_B}{T_A} = \dfrac{T_D}{T_C} = 3.464 \quad$ ...(*i*)

Source: R. S. Khurmi

We know that the distance between the shafts

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

$$\frac{m_A . T_A}{2} + \frac{m_B . T_B}{2} = \frac{m_C . T_C}{2} + \frac{m_D . T_D}{2} = 200 \quad \ldots \left(\because r = \frac{m.T}{2}\right)$$

$$3.125\,(T_A + T_B) = 2.5\,(T_C + T_D) = 400 \quad \ldots(\because m_A = m_B, \text{ and } m_C = m_D)$$

$\therefore \quad T_A + T_B = 400 / 3.125 = 128 \quad$ ...(*ii*)

$T_C + T_D = 400 / 2.5 = 160 \quad$ ...(*iii*)

REVERTED GEAR TRAIN

From equation (*i*), $T_B = 3.464\ T_A$. Substituting this value of T_B in equation (*ii*),

$$T_A + 3.464\ T_A = 128 \quad \text{or} \quad T_A = 128 / 4.464 = 28.67 \text{ say } 28 \quad \textbf{Ans.}$$

and

$$T_B = 128 - 28 = 100 \quad \textbf{Ans.}$$

Again from equation (*i*), $T_D = 3.464\ T_C$. Substituting this value of T_D in equation (*iii*),

$$T_C + 3.464\ T_C = 160 \quad \text{or} \quad T_C = 160 / 4.464 = 35.84 \text{ say } 36 \quad \textbf{Ans.}$$

and

$$T_D = 160 - 36 = 124 \quad \textbf{Ans.}$$

Note : The speed ratio of the reverted gear train with the calculated values of number of teeth on each gear is

$$\frac{N_A}{N_D} = \frac{T_B \times T_D}{T_A \times T_C} = \frac{100 \times 124}{28 \times 36} = 12.3$$

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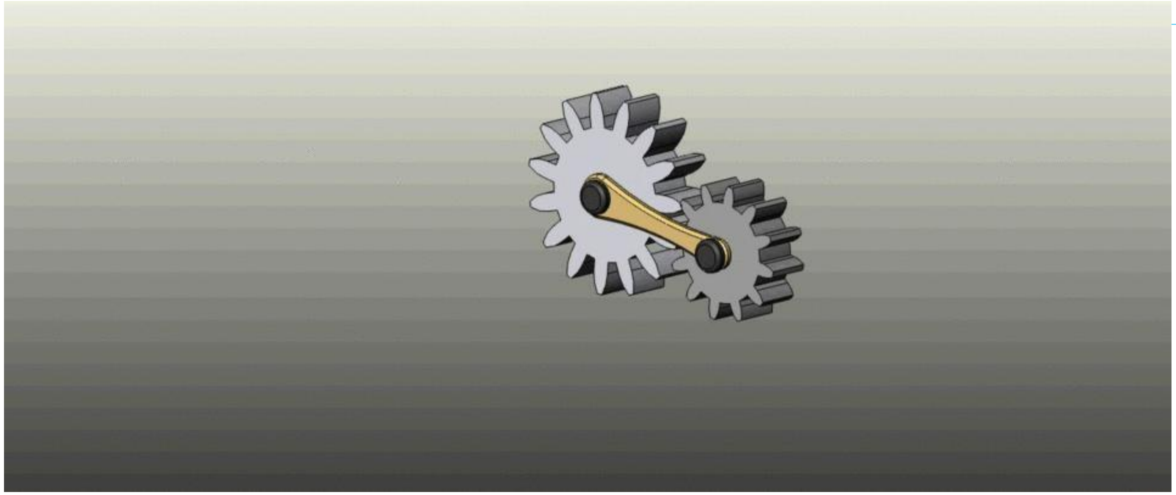
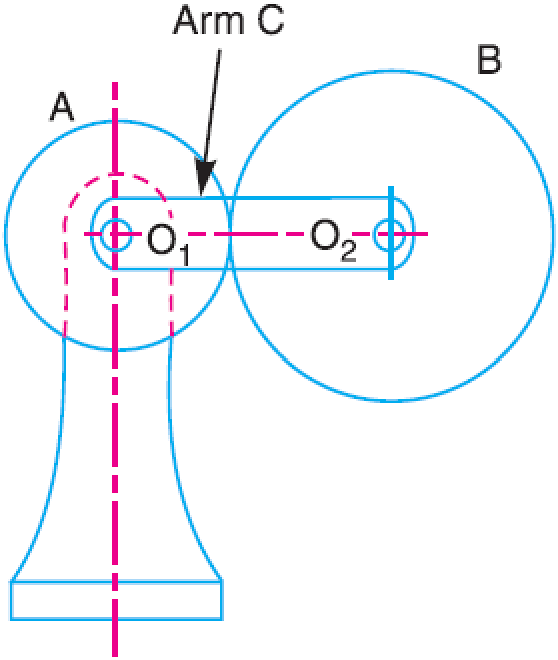
EPICYCLIC GEAR TRAIN



Source: R. S. Khurmi



Epicyclic gear train.

In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.

Gear A and the arm C have a common axis at O_1 about which they can rotate

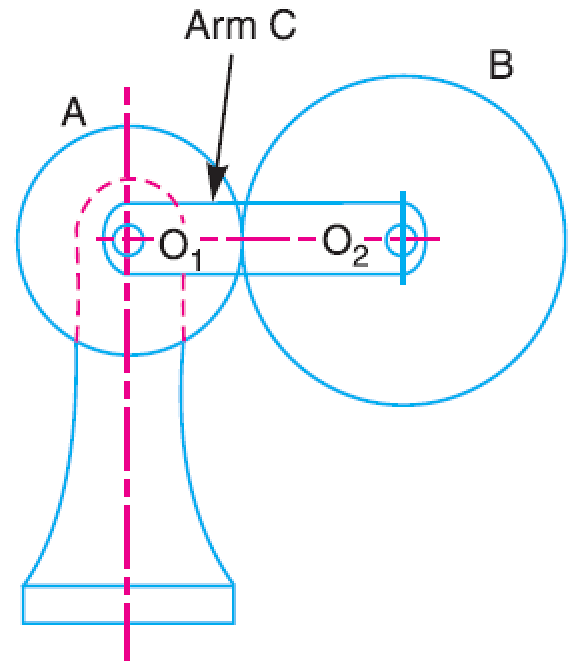
The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate.

Source: R. S. Khurmi

EPICYCLIC GEAR TRAIN

If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa,.

If gear A is fixed and the arm is rotated about the axis of gear A (i.e. O_1), the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic.



Epicyclic gear train.

- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in comparatively lesser space.

- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles,
- hoists, pulley blocks, wrist watches etc.,

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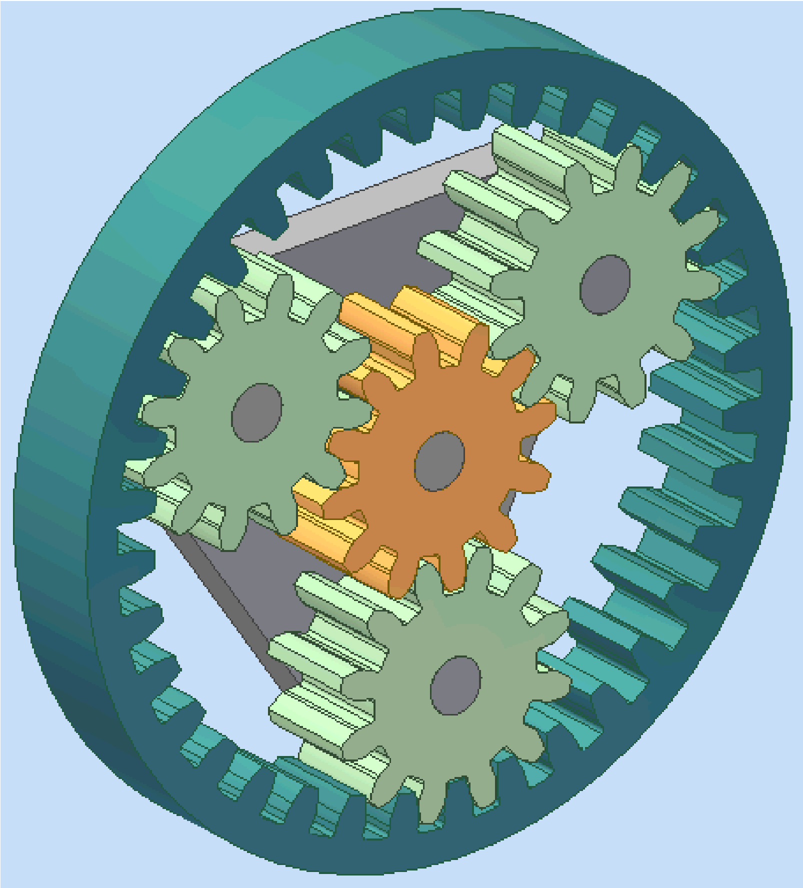
VELOCITY RATIOS IN EPICYCLIC GEAR TRAIN

The following two methods used for finding out the velocity ratio of an epicyclic gear train.



1. Tabular method
2. Algebraic method.

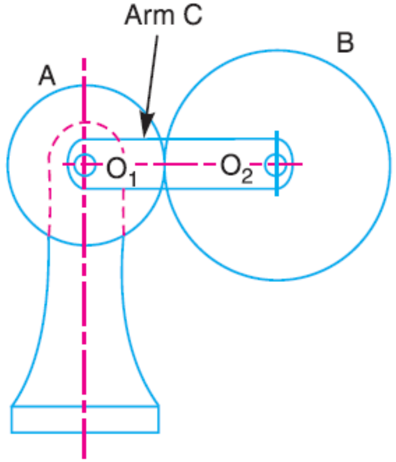
VELOCITY RATIOS IN EPICYCLIC GEAR

We know that $N_B / N_A = T_A / T_B$. Since $N_A = 1$ revolution, therefore $N_B = T_A / T_B$.

When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B revolutions, clockwise.

Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make $(-T_A / T_B)$ revolutions.

Source: R. S. Khurmi



Epicyclic gear train.

Tabular method

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through $+ x$ revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+ y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \frac{T_A}{T_B}$

Velocity Ratios in Epicyclic Gear Train

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_A}{T_B}$

when two conditions about the <u>motion of rotation of any two elements are known</u>, then the <u>unknown speed of the third element may be obtained</u> by substituting the given data in the third column of the fourth row.

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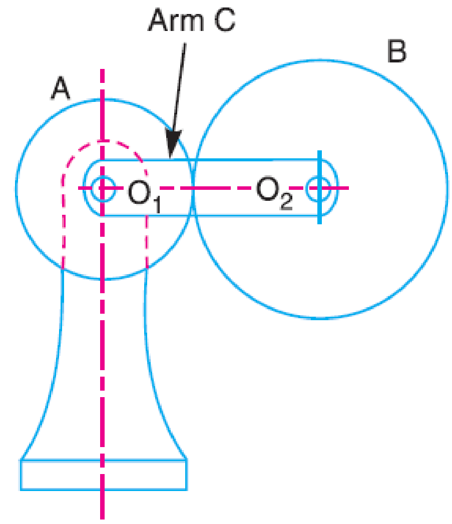
Velocity Ratios in Epicyclic Gear Train (Algebraic method)

- The motion of each element of the epicyclic train relative to the arm is set down in the form of equations

Source: R. S. Khurmi



Epicyclic gear train.

- The number of equations depends upon the number of elements in the gear train

- But the two conditions are, usually, supplied in any epicyclic train *viz. some element is fixed* and the other has specified motion

- These two conditions are sufficient to solve all the equations

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Velocity Ratios in Epicyclic Gear Train
(Algebraic method)

Let the arm C be fixed in an epicyclic gear train as shown in the figure

The speed of the gear A relative to the arm C $= N_A - N_C$

speed of the gear *B* relative to the arm *C* $= N_B - N_C$

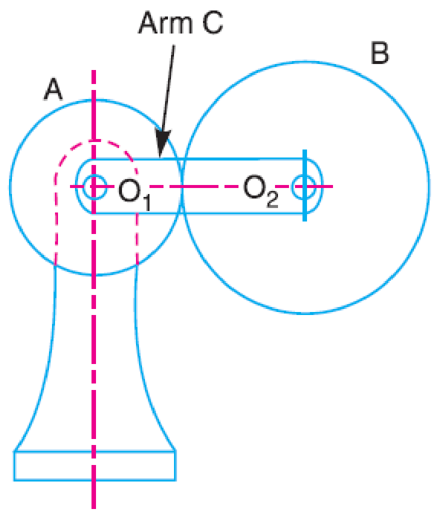
Since the gears *A* and *B* are meshing directly, they will revolve in ***opposite*** directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since the arm *C* is fixed, $N_C = 0$. $\longrightarrow$ $\dfrac{N_B}{N_A} = -\dfrac{T_A}{T_B}$

If the gear *A* is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \Rightarrow \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$



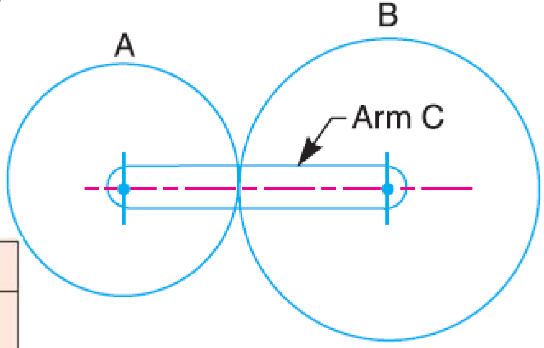
Epicyclic gear train.

Source: R. S. Khurmi

Note : The tabular method is easier and hence mostly used in solving problems on epicyclic gear train.

NUMERICAL EXAMPLE-3

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?



Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Source: R. S. Khurmi

NUMERICAL EXAMPLE-3

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table, $y = +150$ r.p.m.

Also the gear A is fixed, therefore $x + y = 0$

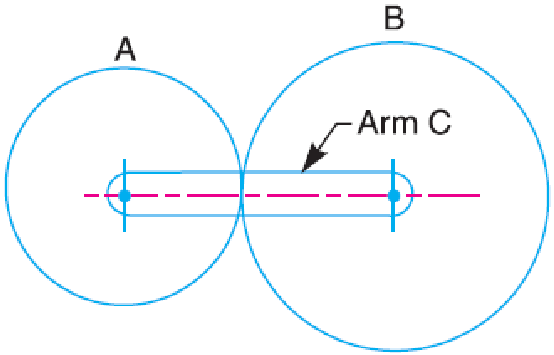
or $x = -y = -150$ r.p.m.

$\therefore$ Speed of gear B, $N_B = y - x \times \frac{T_A}{T_B}$

$$= 150 + 150 \times \frac{36}{45} = +270 \text{ r.p.m.}$$

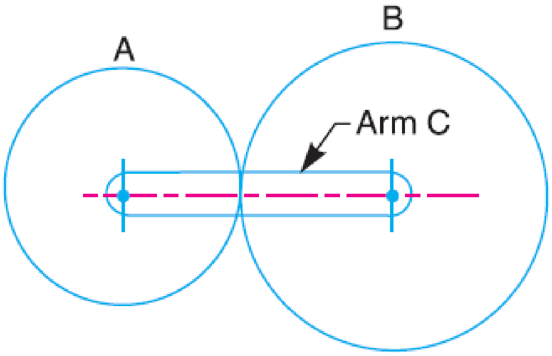
$= 270$ r.p.m. (anticlockwise) **Ans.**

Source: R. S. Khurmi



NUMERICAL EXAMPLE-3

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$



Source: R. S. Khurmi

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

∴ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

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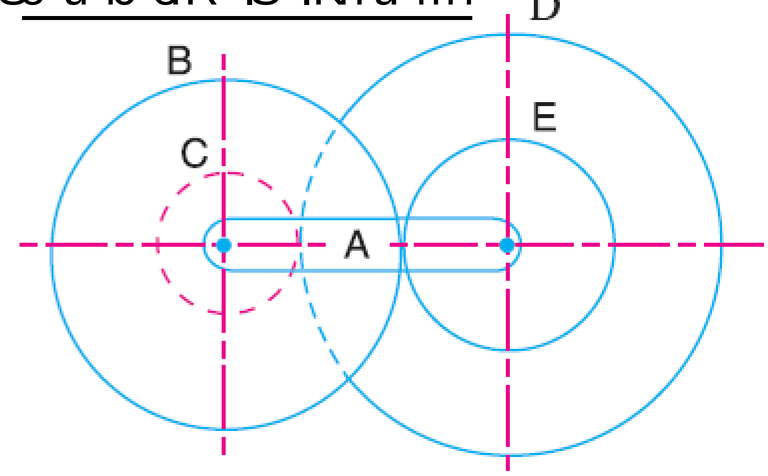
NUMERICAL EXAMPLE-4

In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.

Source R. S. Khurmi



Given : $T_B = 75$; $T_C = 30$; $T_D = 90$;

$N_A = 100$ r.p.m. (clockwise)

Find the number of teeth on gear (T_E)

$$T_B + T_E = T_C + T_D$$

$$\therefore \quad T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$$

NUMERICAL EXAMPLE-4

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D*-*E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D*-*E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear *B* is fixed, $y - x \times \frac{T_E}{T_B} = 0$

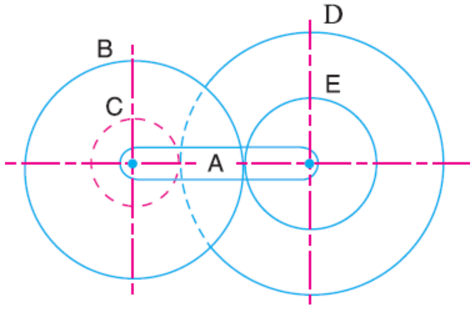
$\therefore\ y - x \times \frac{45}{75} = 0 \longrightarrow y - 0.6x = 0$ ...(*i*)

Also the arm *A* makes 100 r.p.m. clockwise, therefore

$y = -100$ ...(*ii*)

Substituting (*ii*) in equation (*i*), we get

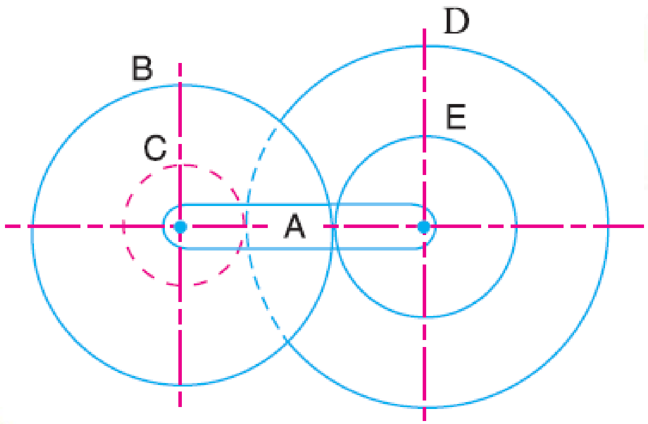
$x = -100 / 0.6 = -166.67$



Source: R. S. Khurmi

NUMERICAL EXAMPLE-4

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x \times \frac{T_E}{T_B}$	$y-x \times \frac{T_D}{T_C}$



From the fourth row of the table, speed of gear *C*,

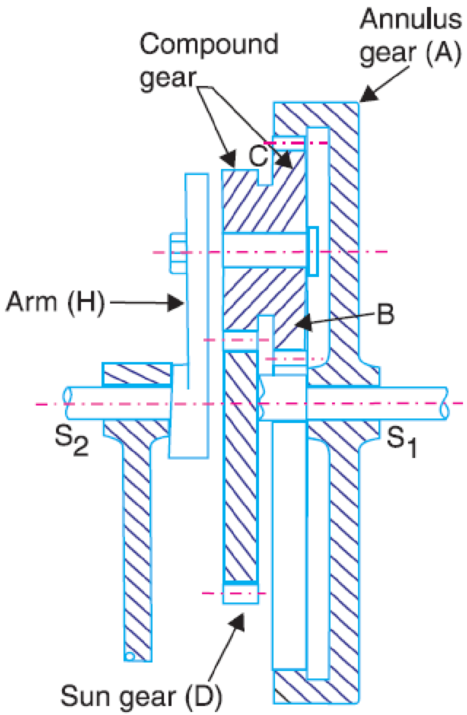
$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

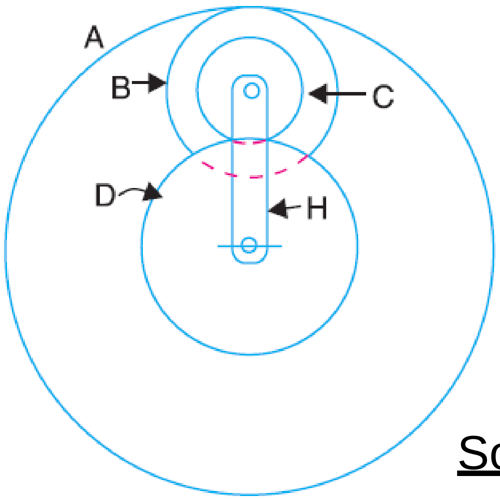
$$= 400 \text{ r.p.m. (anticlockwise)} \textbf{ Ans.}$$

Source: R. S. Khurmi

COMPOUND EPICYCLIC GEAR TRAIN: SUN AND PLANET GEAR

COMPOUND EPICYCLIC GEAR TRAIN: SUN
T GEAR

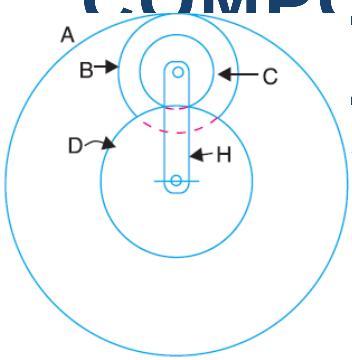




- ➢The annulus gear A meshes with the gear B
- ➢the sun gear D meshes with the gear C.
- ➢when the annulus gear is fixed, the sun gear provides the drive
- ➢when the sun gear is fixed, the annulus gear provides the drive.
- ➢In both cases, the arm acts as a follower.

Source: R. S. Khurmi

COMPOUND EPICYCLIC GEAR TRAIN: SUN
LANET GEAR



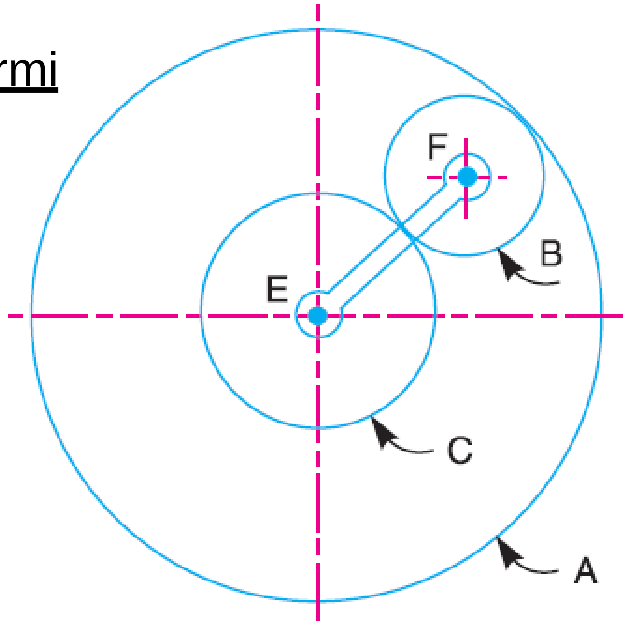
Source: R. S. Khurmi

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear *D* rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear *D* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

NUMERICAL EXAMPLE-5

An epicyclic gear consists of three gears A, B and C as shown in the Figure. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.

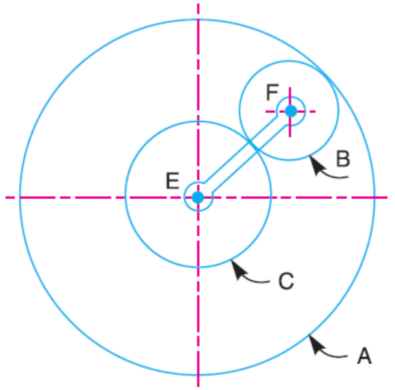
Source: R. S. Khurmi



Given : $T_A = 72$; $T_C = 32$; Speed of arm $EF = 18$ r.p.m.

NUMERICAL EXAMPLE-5

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear *C* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear *C* rotates through + *x* revolutions	0	+ *x*	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add + *y* revolutions to all elements	+ *y*	+ *y*	+ *y*	+ *y*
4.	Total motion	+ *y*	*x* + *y*	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$



Source: R. S. Khurmi

Speed of gear C

the speed of the arm is 18 r.p.m. therefore, $y = 18$ r.p.m.

and the gear *A* is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \longrightarrow 18 - x \times \frac{32}{72} = 0 \longrightarrow x = 40.5$$

$\therefore$ Speed of gear *C* $= x + y = 40.5 + 18$

$= +\,58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

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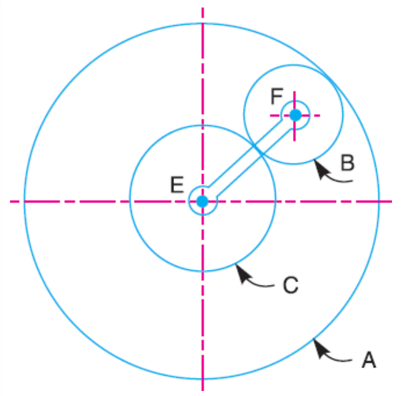
NUMERICAL EXAMPLE-5

Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$



Source: R. S. Khurmi

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears

from the geometry of Fig. $d_B + \frac{d_C}{2} = \frac{d_A}{2}$ or $2\,d_B + d_C = d_A$

Since the number of teeth are proportional to their pitch circle diameters,

$2\,T_B + T_C = T_A$ or $2\,T_B + 32 = 72$ or $T_B = 20$

$\therefore$ Speed of gear B $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8$ r.p.m.

= 46.8 r.p.m. in the opposite direction of arm. **Ans.**

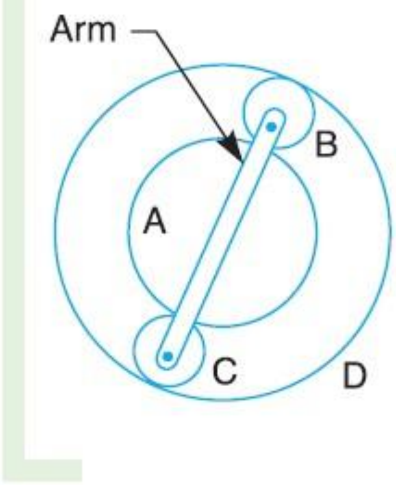
NUMERICAL EXAMPLE-6

An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make :

1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and

2. when A makes one revolution clockwise and D is stationary ?

The number of teeth on the gears A and D are 40 and 90 respectively.



Source: R. S. Khurmi

Given : $T_A = 40$; $T_D = 90$

find the number of teeth on gears B and C (*i.e.* T_B and T_C).

from the geometry of the figure, $d_A + d_B + d_C = d_D$ or $d_A + 2\,d_B = d_D$ ...($\because$ $d_B = d_C$)

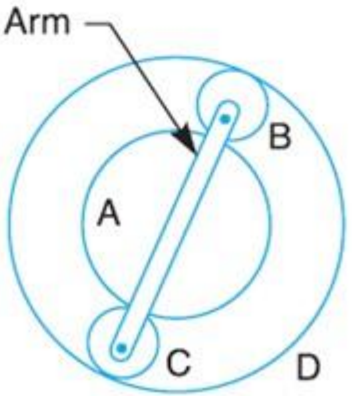
Since the number of teeth are proportional to their pitch circle diameters,

$T_A + 2\,T_B = T_D$ or $40 + 2\,T_B = 90$

$T_B = 25$, and $T_C = 25$ ...($\because$ $T_B = T_C$)

NUMERICAL EXAMPLE-6

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through – x revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add – y revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$



Source: R. S. Khurmi

1. *Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise*

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad ...(ii)$$

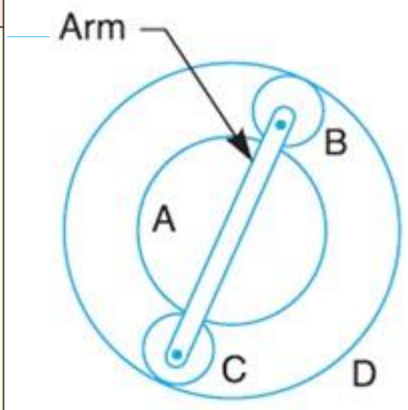
From equations *(i)* and *(ii)*, $x=1.04$ and $y=-0.04$

$$\therefore \quad \text{Speed of arm}=-y=-(-0.04)=+0.04$$

$$=0.04 \text{ revolution anticlockwise} \quad \textbf{Ans.}$$

NUMERICAL EXAMPLE-6

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear *A* rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B} \times \frac{T_B}{T_D} = +\frac{T_A}{T_D}$
2.	Arm fixed, gear *A* rotates through – *x* revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$	$+x \times \frac{T_A}{T_D}$
3.	Add – *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x \times \frac{T_A}{T_B} - y$	$x \times \frac{T_A}{T_D} - y$



2. Speed of arm when A makes 1 revolution clockwise and D is stationary

Source: R. S. Khurmi

Since the gear *A* makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad \text{...(iii)}$$

Also the gear *D* is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0 \quad \text{or} \quad x \times \frac{40}{90} - y = 0$$

$$\therefore \quad 40x - 90y = 0 \quad \text{or} \quad x - 2.25y = 0 \qquad \text{...(iv)}$$

From equations (*iii*) and (*iv*),

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308 = 0.308$ revolution clockwise **Ans.**

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THANK YOU