23ME403: Fluid Mechanics and Hydraulic Machines

Topic: Fluid statics

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UNIT 1

FLUID STATICS

UNIT – I

Fluid Statics:

- Dimensions and Units.
- Physical Properties of fluids- Specific gravity, viscosity, surface tension.
- Vapour pressure and their influence on fluid motion.
- Atmospheric, gauge and vaccum pressure.
- Measurement of pressure- Piezometer, U-tube and Differential manometers.

INTRODUCTION

- Fluid Mechanics is basically a study of:
  - Physical behavior of fluids and fluid systems and laws governing their behavior.
  - Action of forces on fluids and the resulting flow pattern.
- Fluid is further sub-divided in to liquid and gas.
- The liquids and gases exhibit different characteristics on account of their different molecular structure.

FLUID MECHANICS COVER MANY AREAS LIKE:

- Design of wide range of hydraulic structures (dams, canals, weirs etc) and machinery (Pumps, Turbines etc).
- Design of complex network of pumping and pipe lines for transporting liquids. Flow of water through pipes and its distribution to service lines.
- Fluid control devices both pneumatic and hydraulic.
- Design and analysis of gas turbines and rocket engines and air- craft.
- Power generation from hydraulic, stream and Gas turbines.
- Methods and devices for measurement of pressure and velocity of a fluid in motion.

UNITS AND DIMENSIONS:

- A dimension is a name which describes the measurable characteristics of an object such as mass, length and temperature etc. a unit is accepted standard for measuring the dimension. The dimensions used are expressed in four fundamental dimensions namely Mass, Length, Time and Temperature.
- Mass (M) – Kg
- Length (L) – m
- Time (T) – S
- Temperature (t) – ^{0}C or K (Kelvin)

UNITS AND DIMENSIONS:

- **Density:** Mass per unit volume= kg/m^3
- **Newton:** Unit of force expressed in terms of mass and acceleration, according to Newton's 2nd law motion. Newton is that force which when applied to a mass of 1 kg gives an acceleration $1 m/Sec^2$. F=Mass x Acceleration = kg - m/sec^2 = N.
- **Pascal:** A Pascal is the pressure produced by a force of Newton uniformly applied over an area of 1 m^2. Pressure = Force per unit area = N/ m^2 = Pascal or P_a.
- **Joule:** A joule is the work done when the point of application of force of 1 Newton is displaced Work = Force per unit = N/m = J or Joule.
- **Watt:** A Watt represents a work equivalent of a Joule done per second.
- Power = Work done per unit time = J/ Sec = W or Watt.

DEFINITIONS

Density or Mass Density:

- The density or mass density of a fluid is defined as the ratio of the mass of the fluid to its volume. Thus the mass per unit volume of the fluid is called density.
- It is denoted by ρ .
- The unit of mass density is Kg/m^3

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

- The value of density of water is $1000 Kg/m^3$.

DEFINITIONS

- **Specific weight or Specific density:** It is the ratio between the weights of the fluid to its volume. The weight per unit volume of the fluid is called weight density and it is denoted by **w**.

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{\text{Mass of fluid} \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}} = \rho \times g$$

- **Specific volume:** It is defined as the volume of the fluid occupied by a unit mass or volume per unit mass of fluid is called Specific volume.

$$\text{Specific volume} = \frac{\text{Volume of the fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume of the fluid}}} = \frac{1}{\rho}$$

- Thus the Specific volume is the reciprocal of Mass density. It is expressed as m^3/kg and is commonly applied to gases.

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DEFINITIONS

- **Specific Gravity:** It is defined as the ratio of the Weight density (or density) of a fluid to the Weight density (or density) of a standard fluid. For liquids the standard fluid taken is water and for gases the standard liquid taken is air. The Specific gravity is also called relative density. It is a dimension less quantity and it is denoted by **s**.

- $$S \text{ (for liquids)} = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

- $$S \text{ (for gases)} = \frac{\text{weight density of gas}}{\text{weight density of air}}$$

PROBLEMS

1. Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7

Sol: i) Density of a liquid = S

$$\rho = 0.7 \times 1000 = 700 \text{ kg/m}^3$$

ii) Specific weight $w = \rho \times g = 700 \times 9.81 = 6867 \text{ N/m}^3$

iii) Weight (w) Volume = 1 liter = 0.001 m^3

We know that, specific weight $w = \dfrac{\text{weight of fluid}}{\text{volume of the fluid}}$

Weight of petrol = w× volume of petrol

= 6867 × 0.001

= 6.867 N

TOPICS TO BE COVERED

- Properties of fluids
- Viscosity
- Kinematic Viscosity
- Newton’s Law of viscosity
- Types of Fluids
- Surface tension

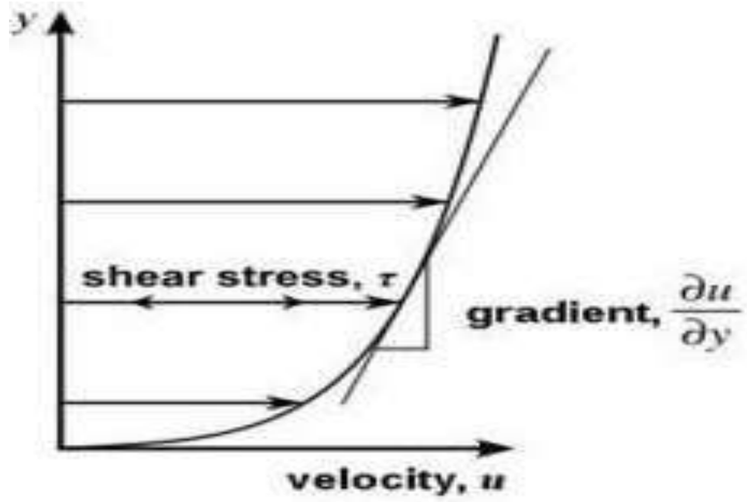
LECTURE 2

Properties of Fluids

VISCOSITY

DEFINITION

- It is defined as the property of a fluid which offers resistance to the movement of one layer of the fluid over another adjacent layer of the fluid.
- When the two layers of a fluid, at a distance ‘dy’ apart, move one over the other at different velocities, say u and u+du.
- The viscosity together with relative velocities causes a shear stress acting between the fluid layers.



VISCOSITY

- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
- This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by symbol τ (tau).

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

- Where μ is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ represents the rate of shear strain or rate of shear deformation or velocity gradient.
- From the above equation, we have $\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$
- Thus, viscosity is also defined as the shear stress required producing unit rate of shear strain.

KINEMATIC VISCOSITY

- It is defined as the ratio between dynamic viscosity and density of fluid.
- It is denoted by symbol Ɒ (nu)

$$Ɒ = \frac{\text{viscosity}}{\text{density}} = \frac{\mu}{\rho}$$

- The unit of viscosity in CGS is called poise and is equal to dyne- see/ cm^2
- The unit of kinematic viscosity is m^2 /sec

- Thus one stoke = cm^2/sec = $\left(\frac{1}{100}\right)^2$ m^2/sec= 10^{-4} m^2/sec

NEWTON'S LAW OF VISCOSITY

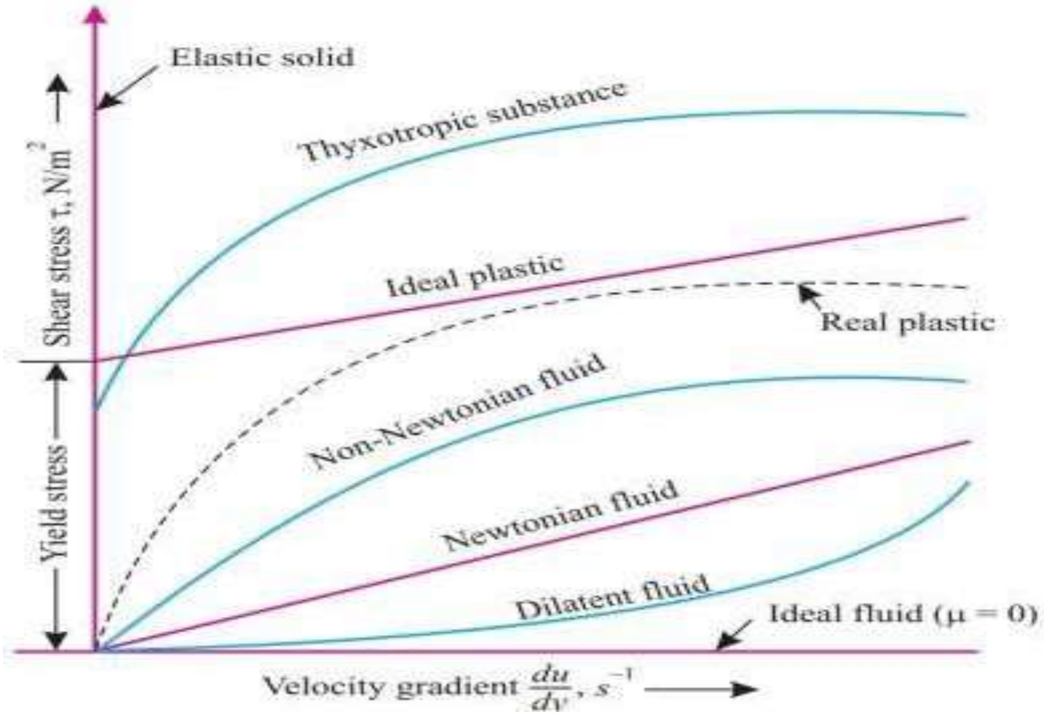
- It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain.
- The constant of proportionality is called the co- efficient of viscosity.
- It is expressed as:

$$\tau = \mu \frac{du}{dy}$$

- Fluids which obey above relation are known as NEWTONIAN fluids and fluids which do not obey the above relation are called NON- NEWTONIAN fluids.

TYPES OF FLUIDS

- The fluids may be classified in to the following five types.
- ➢ Ideal fluid
- ➢ Real fluid
- ➢ Newtonion fluid
- ➢ Non-Newtonion fluid
- ➢ Ideal plastic fluid



TYPES OF FLUIDS

- **Ideal fluid**: A fluid which is compressible and is having no viscosity is known as ideal fluid. It is only an imaginary fluid as all fluids have some viscosity.
- **Real fluid**: A fluid possessing a viscosity is known as real fluid. All fluids in actual practice are real fluids.
- **Newtonian fluid**: A real fluid, in which the stress is directly proportional to the rate of shear strain, is known as Newtonian fluid.
- **Non-Newtonian fluid**: A real fluid in which shear stress is not proportional to the rate of shear strain is known as Non-Newtonian fluid.
- **Ideal plastic fluid:** A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain is known as ideal plastic fluid.

SURFACE TENSION

- Surface tension is defined as the tensile force acting on the surface of a liquid is contact with a gas or on the surface behaves like a membrane under tension.
- The magnitude of this force per unit length of free surface will have the same value as the surface energy per unit area.
- It is denoted by σ (sigma).
- In MKS units it is expressed as Kg f/m while in SI units as N/m

SURFACE TENSION ON LIQUID DROPLET

- Consider a small spherical droplet of a liquid of radius ‘r’ on the entire surface of the droplet, the tensile force due to surface tension will be acting

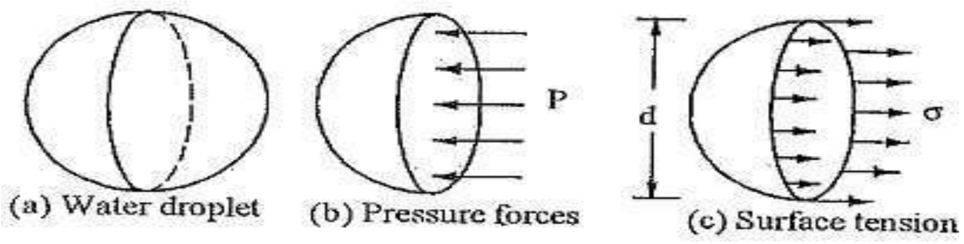
- Let σ = surface tension of the liquid

  p = pressure intensity inside the droplet (In excess of outside pressure intensity)

  d = Diameter of droplet

- Let, the droplet is cut in to two halves. The forces acting on one half (say left half) will be

FORCES ON DROPLET



SURFACE TENSION ON LIQUID DROPLET

- Tensile force due to surface tension acting around the circumference of the cut portion = σ ×circumference = σ × πd
- Pressure force on the area $\frac{\pi}{4}d^2 = p \times \frac{\pi}{4}d^2$
- These two forces will be equal to and opposite under equilibrium conditions i.e.

$$p \times \frac{\pi}{4}d^2 = \sigma\,\pi d$$

$$p = \frac{\sigma\,\pi d}{\frac{\pi}{4}d^2}$$

$$p = \frac{4\sigma}{d}$$

SURFACE TENSION ON HOLLOW BUBBLE

- A hallows bubble like soap in air has two surfaces in contact with air, one inside and other outside.
- Thus, two surfaces are subjected to surface tension.

$$p \times \frac{\pi}{4} d^2 = 2(\sigma \pi d)$$

$$p = \frac{8\sigma}{d}$$

SURFACE TENSION ON LIQUID JET

- Consider a liquid jet of diameter ‘d’ length ‘L’
- Let, p = pressure intensity inside the liquid jet above the outside pressure

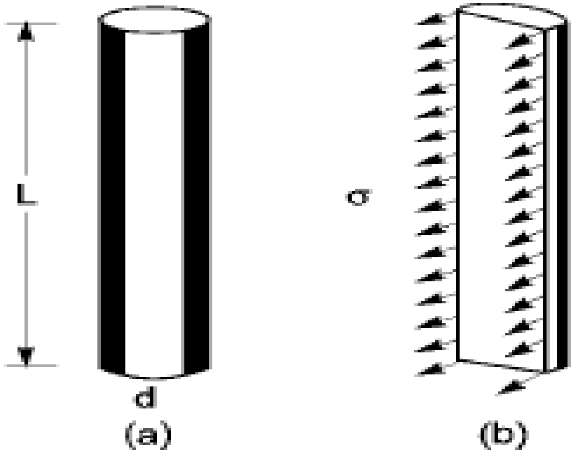
  σ = surface tension of the liquid
- Consider the equilibrium of the semi- jet .
- Force due to pressure = p × area of the jet = p × L × d semi-

•Force due to surface tension = σ × 2L Equating

above forces we get,

$$p \times L \times d = \sigma \times 2L$$

$$p = \frac{2\sigma}{d}$$



TOPICS TO BE COVERED

- Capillarity
- Expression for Capillary rise
- Expression for Capillary fall
- Vapor pressure
- Variation of vapor pressure with temperature

LECTURE 3

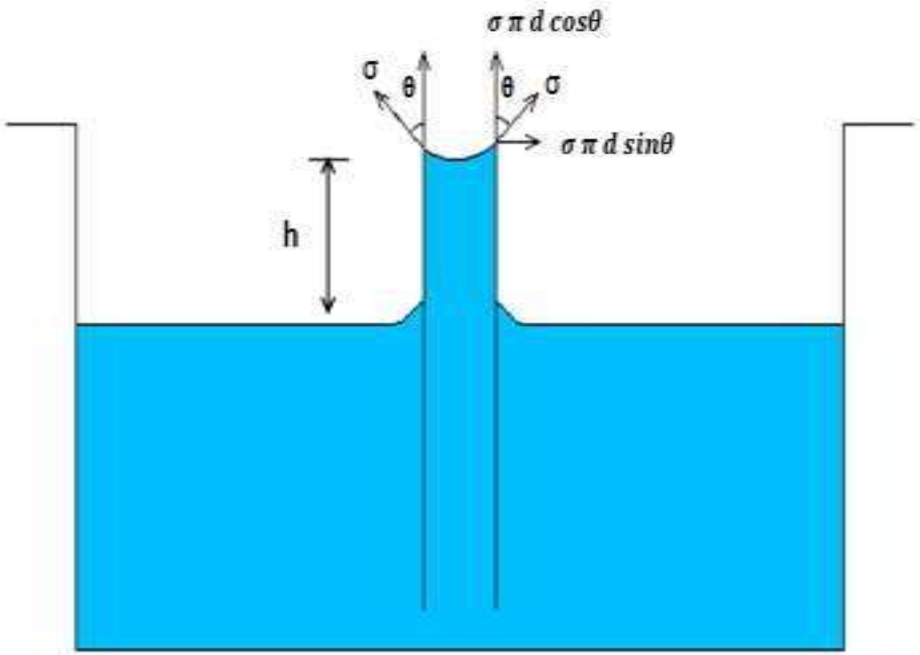
Concept of Vapor pressure & its variation

CAPILLARITY

- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.
- The rise of liquid surface is known as capillary rise, while the fall of the liquid surface is known as capillary depression.
- It is expressed in terms of ‘cm’ or ‘mm’ of liquid.
- Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

EXPRESSION FOR CAPILLARY RISE

- Consider a glass tube of small diameter ‘d’ opened at both ends and is inserted in a liquid.
- The liquid will rise in the tube above the level of the liquid outside the tube.
- Let ‘h’ be the height of the liquid in the tube.
- Under a state of equilibrium, the weight of the liquid of height ‘h’ is balanced by the force at the surface of the liquid in the tube.



EXPRESSION FOR CAPILLARY RISE

- But, the force at the surface of the liquid in the tube is due to surface tension.
- Let σ = surface tension of liquid

  θ = Angle of contact between the liquid and glass tube
- The weight of the liquid of height 'h' in the tube

  $= (\text{area of the tube} \times h) \times \rho \times g = \frac{\pi}{4} d^2 \times h \times \rho \times g$

  Where 'ρ' is the density of the liquid.
- The vertical component of the surface tensile force = $(\sigma \times \text{circumference}) \times \cos\theta = \sigma \times \pi d \times \cos\theta$

EXPRESSION FOR CAPILLARY RISE

For equilibrium, $\frac{\pi}{d} d^2 \times h \times \rho \times g = \sigma \pi d \cos\theta$,

$$h = \frac{\sigma \pi d \cos\theta}{\frac{\pi}{d} d^2 \times \rho \times g}$$

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

The value of θ is equal to ‘0’ between water and clean glass tube, then cos θ = 1, then

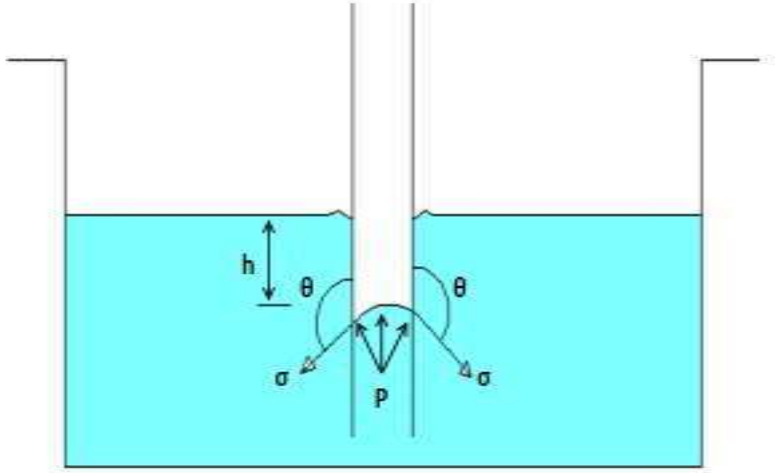
$$h = \frac{4\sigma}{\rho g d}$$

EXPRESSION FOR CAPILLARY FALL

- If the glass tube is dipped in mercury, the Level of mercury in the tube will be lower than the general level of the outside liquid.

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

- The value θ of for glass & mercury 128^0



VAPOUR PRESSURE

- A change from the liquid state to the gaseous state is known as Vaporizations.
- The vaporization (which depends upon the prevailing pressure and temperature condition) occurs because of continuous escaping of the molecules through the free liquid surface.
- Consider a liquid at a temp. of 20°C and pressure is atmospheric is confined in a closed vessel.
- This liquid will vaporize at 100°C, the molecules escape from the free surface of the liquid and get accumulated in the space between the free liquid surface and top of the vessel.
- These accumulated vapours exert a pressure on the liquid surface. This pressure is known as vapour pressure of the liquid or pressure at which the liquid is converted in to vapours.

VAPOUR PRESSURE

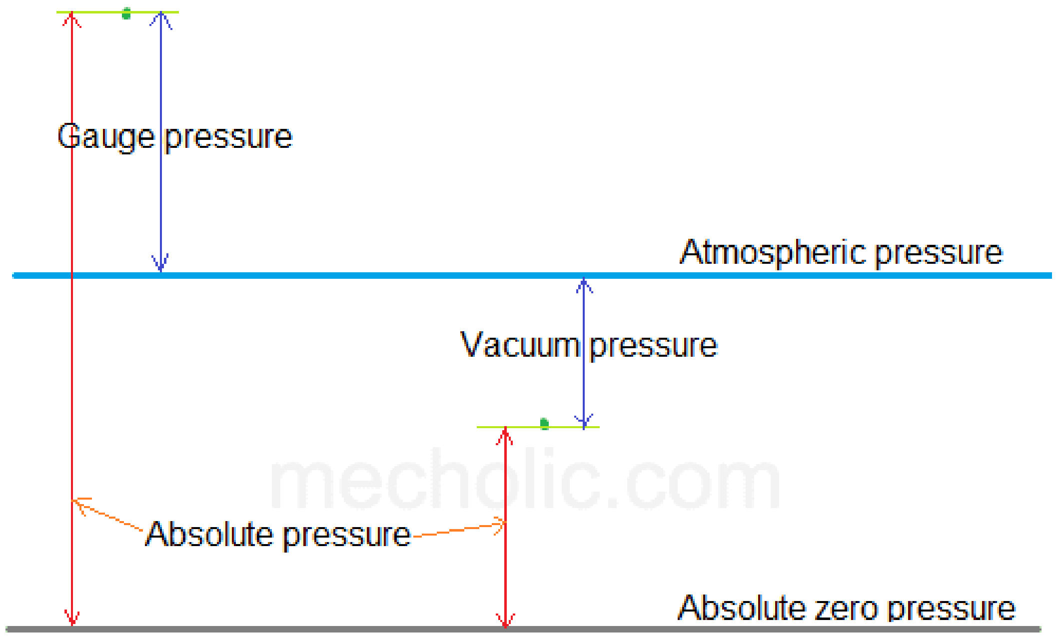
- Consider the same liquid at 20° C at atmospheric pressure in the closed vessel and the pressure above the liquid surface is reduced by some means; the boiling temperature will also reduce.
- If the pressure is reduced to such an extent that it becomes equal to or less than the vapour pressure, the boiling of the liquid will start, though the temperature of the liquid is 20°C.
- Thus, the liquid may boil at the ordinary temperature, if the pressure above the liquid surface is reduced so as to be equal or less than the vapour pressure of the liquid at that temperature.

VARIATION OF VAPOUR PRESSURE WITH TEMPERATURE

- The vapour pressure of a liquid varies with its temperature.
- As the temperature of a liquid or solid increases its vapour pressure also increases.
- Conversely, vapour pressure decreases as the temperature decreases.

PRESSURE MEASURING SYSTEM

- The pressure on a fluid is two measure in different systems.
- In one system, it is measured above the absolute zero or complete vacuum and it is called the Absolute pressure.
- In other system, pressure is measured above the atmospheric pressure and is called Gauge pressure.



DEFINITIONS

- **ABSOLUTE PRESSURE:** It is defined as the pressure which is measured with reference to absolute vacuum pressure.
- **GAUGE PRESSURE:** It is defined as the pressure, which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric on the scale is marked as zero.
- **VACUUM PRESSURE:** It is defined as the pressure below the atmospheric pressure

  i) Absolute pressure = Atmospheric pressure+ gauge pressure

  $p_{ab} = p_{atm} + p_{guage}$

  Vaccum pressure= atmospheric pressure - Absolute pressure

- The atmospheric pressure at sea level at 15^0C is 10.13N/cm^2 or 101.3KN/m^2 in S I Units and 1.033 Kg f/cm^2 in M K S System.
- The atmospheric pressure head is 760mm of mercury or 10.33m of water.

MEASUREMENT OF PRESSURE

- The pressure of a fluid is measured by the fallowing devices.
  - Manometers
  - Mechanical gauges.

- **Manometers:** Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of fluid. They are classified as:
  - Simple Manometers
  - Differential Manometers

MEASUREMENT OF PRESSURE

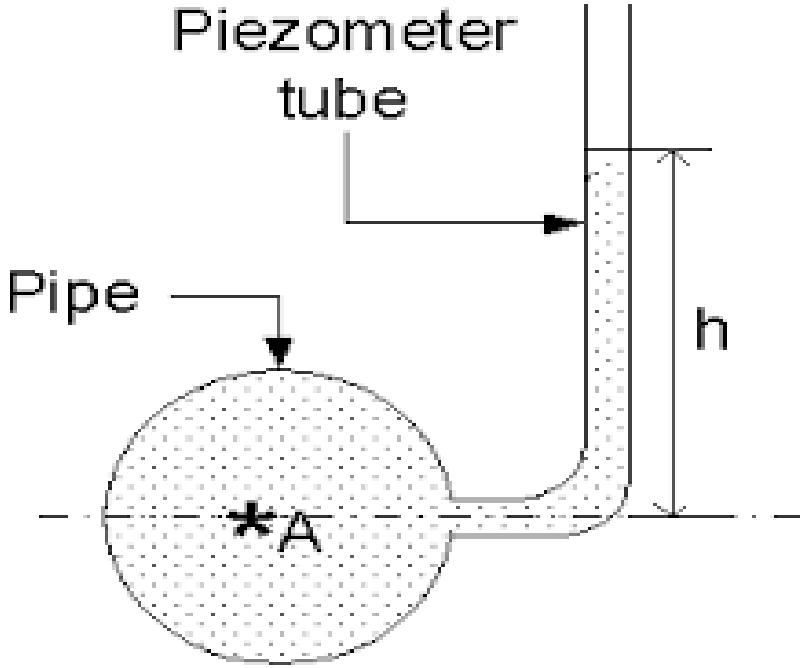
- **Mechanical Gauges:** These are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight.
- The commonly used Mechanical pressure gauges are:
    - Diaphragm pressure gauge
    - Bourdon tube pressure gauge
    - Dead - Weight pressure gauge
    - Bellows pressure gauge.

SIMPLE MANOMETER

- A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and the other end remains open to the atmosphere.
- The common types of simple manometers are:
  - Piezometer.
  - U-tube manometer (for gauge & vaccum pressure)
  - Single column manometer
    - Vertical Single column manometer
    - Inclined Single column manometer

PIEZOMETER

- It is a simplest form of manometer used for measuring gauge pressure.
- One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere.
- The rise of liquid in the Piezometer gives pressure head at that point A.
- The height of liquid say water is ‘h’ in piezometer tube, then



Pressure at A = ρ g h $\frac{N}{m^2}$

U- TUBE MANOMETER

- It consists of a glass tube bent in u-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.
- The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.
- **For Gauge Pressure:** Let B is the point at which pressure is to be measured, whose value is p. The datum line A – A

U- TUBE MANOMETER

- Let

h_1 = height of light liquid above datum line

h_2 =height of heavy liquid above datum line

S_1 = sp. gravity of light liquid

ρ_1= density of light liquid

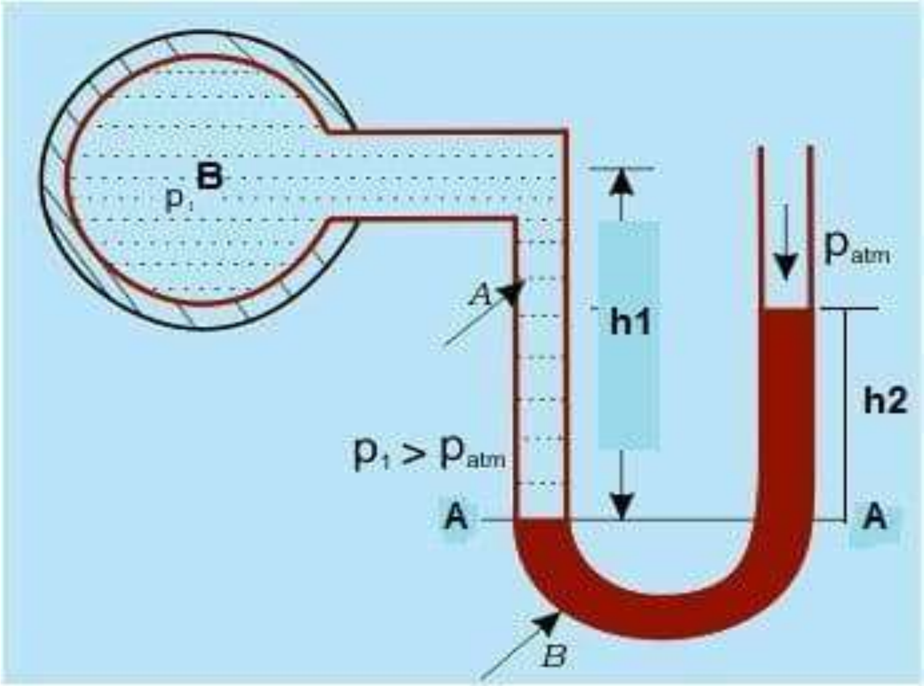
= 1000 S_1

S_2 = sp. gravity of heavy liquid

ρ_2= density of heavy liquid

= 1000 S_2

GAUGE PRESSURE



U- TUBE MANOMETER

- As the pressure is the same for the horizontal surface. Hence the pressure above the horizontal datum line A - A in the left column and the right column of U - tube manometer should be same.
- Pressure above A—A ion the left column = p + $\rho_1 gh_1$
- Pressure above A - A in the left column = gh_2

ρ_2

- Hence equating the two pressures p + $\rho_1 gh_1$ = ρ_2 gh_2

p =$\rho_2 gh_2$ - $\rho_1 gh_1$

U- TUBE MANOMETER

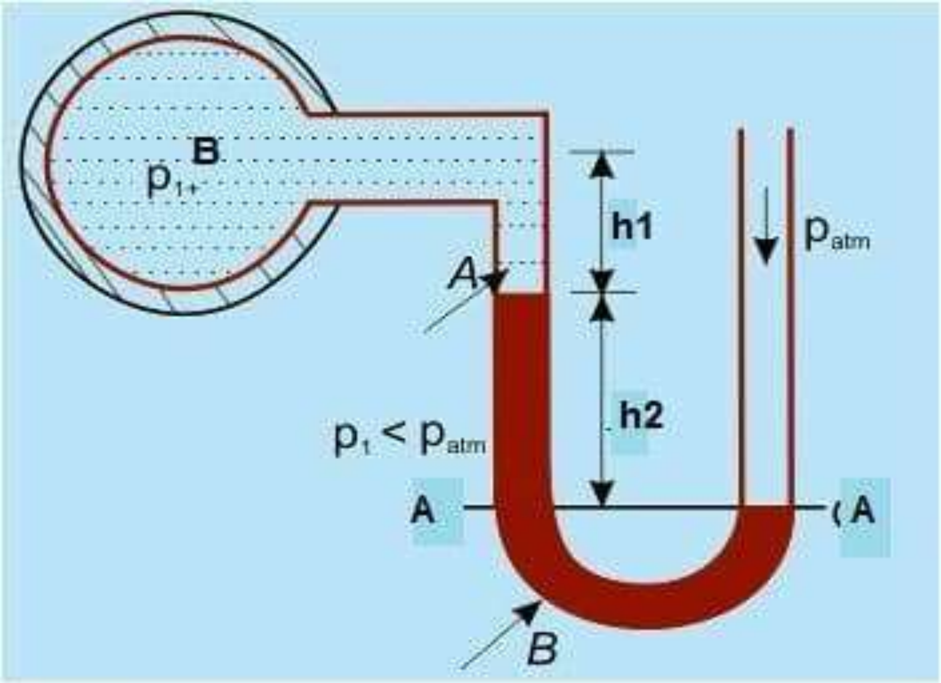
- For measuring vacuum pressure, the level of heavy fluid in the manometer will be as shown in fig.
- Pressure above AA in the left column = ρ_2 g h_2 + ρ_1g h_1+ P
- Pressure head in the right

  column above AA = 0
- Therefore, equating two pressures we get,

  ρ_1 g h_2 + ρ_1g h_1 + P = 0

p =$-(\rho_2 gh_2 + \rho_1 gh_1)$

VACCUM PRESSURE



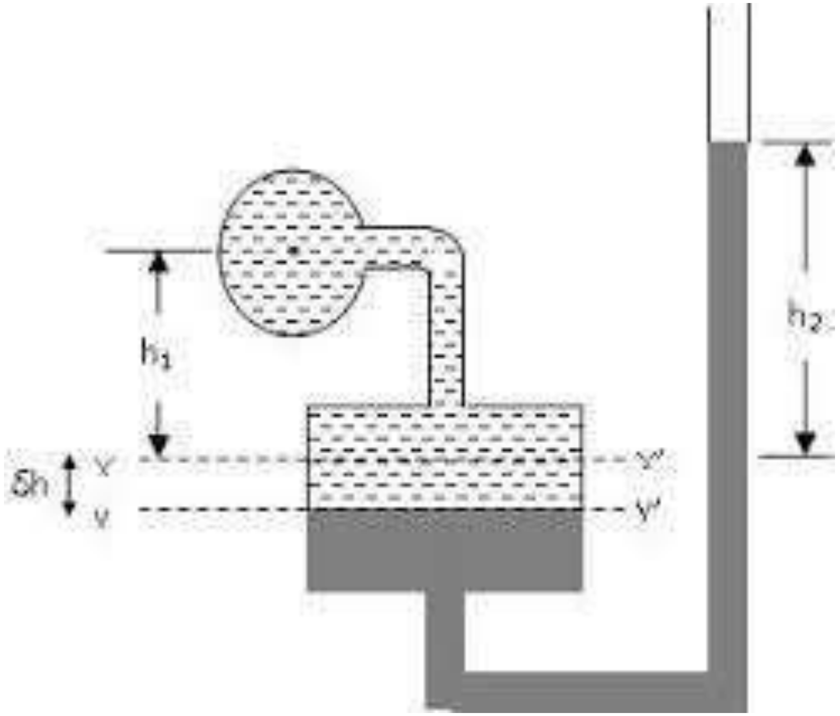
SINGLE COLUMN MANOMETER

- Single column manometer is a modified form of a U- tube manometer in which a reservoir, having a large cross sectional area (about. 100 times) as compared to the area of tube is connected to one of the limbs (say left limb) of the manometer.
- Due to large cross sectional area of the reservoir for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of the liquid in the other limb.
- The other limb may be vertical or inclined.
- Thus, there are two types of single column manometer
    - ➢ Vertical Single Column Manometer
    - ➢ Inclined Single Column Manometer

VERTICAL SINGLE COLUMN MANOMETER

- Let X – X be the datum line in the reservoir and in the right limb of the manometer, when it is connected to the

  pipe, when the Manometer is connected to the pipe, due to high pressure at A.
- The heavy pressure in the reservoir will be pushed downwards and will rise in the right limb.



DERIVATION

- Let, Δ h= fall of heavy liquid in the reservoir

  h_2= rise of heavy liquid in the right limb

  h_1= height of the centre of the pipe above X - X p_A= Pressure at A, which is to be measured.

  A = Cross- sectional area of the reservoir

  a = cross sectional area of the right limb S_1= Specific. Gravity of liquid in pipe

  S_2= sp. Gravity of heavy liquid in the reservoir and right limb

  ρ_1 = density of liquid in pipe

  ρ_2 = density of liquid in reservoir

- Fall of heavy liquid reservoir will cause a rise of heavy liquid level in the right limb

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a \times h_2}{A} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots .(1)$$

- Now consider the datum line Y - Y
- Then the pressure in the right limb above Y - Y = $\rho_2 \times g \times (\Delta h + h_2)$
- Pressure in the left limb above Y – Y = $\rho_1 \times g \times (\Delta h + h_1) + P_A$
- Equating the pressures, we have

$$\rho_2 g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A \quad p_A = \rho_2 \times g \times (\Delta h + h_2) - \rho_1 \times g \times (\Delta h + h_1)$$

$$= \Delta h\,(\rho_2 g \ \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g$$

- But, from eq. (1) $\Delta h = \frac{a \times h_2}{A}$
- $P_A = \frac{a \times h_2}{A} (\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g$
- As the area A is very large as compared to a, hence the ratio $\frac{a}{A}$ becomes very small and can be neglected Then,

$$p_A = h_2 \rho_2 g - h_1 \rho_1 g \quad \text{… … … … … … (2)}$$

INCLINED SINGLE COLUMN MANOMETER

- The manometer is more sensitive.
- Due to inclination the distance moved by heavy liquid in the right limb will be more.
- Let L= length of heavy liquid moved in the rite limb

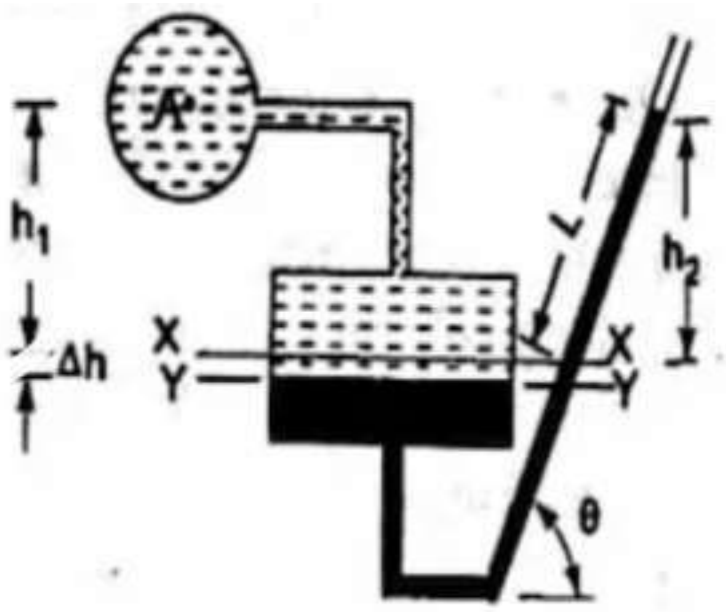
  θ = inclination of right Limb with horizontal.

  H_2 = vertical rise of heavy liquid in
the right limb above X - X = L sinθ
- From above eq.(2), the pressure at A is

$$p_A = h_2 \rho_2 g - h_1 \rho_1 g$$

- Substituting the value of h_2

$$p_A = L\sin\theta\, \rho_2 g - h_1 \rho_1 g$$



DIFFERENTIAL MANOMETERS

- Differential manometers are the devices used for measuring the difference of pressure between two points in a pipe or in two different pipes.
- A differential manometer consists of a U-tube, containing heavy

  liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.
- The common types of U- tube differential manometers are:
  - U- Tube differential manometer
  - Inverted U- tube differential manometer

U-TUBE DIFFERENTIAL MANOMETER

TWO POINTS A AND B ARE AT DIFFERENT LEVELS AND ALSO CONTAINS LIQUIDS OF DIFFERENT SP.GR.

- These points are connected to the U - Tube differential manometer.
- Let the pressure at A and B are p_A and p_B
- Let h = Difference of mercury levels in the u - tube

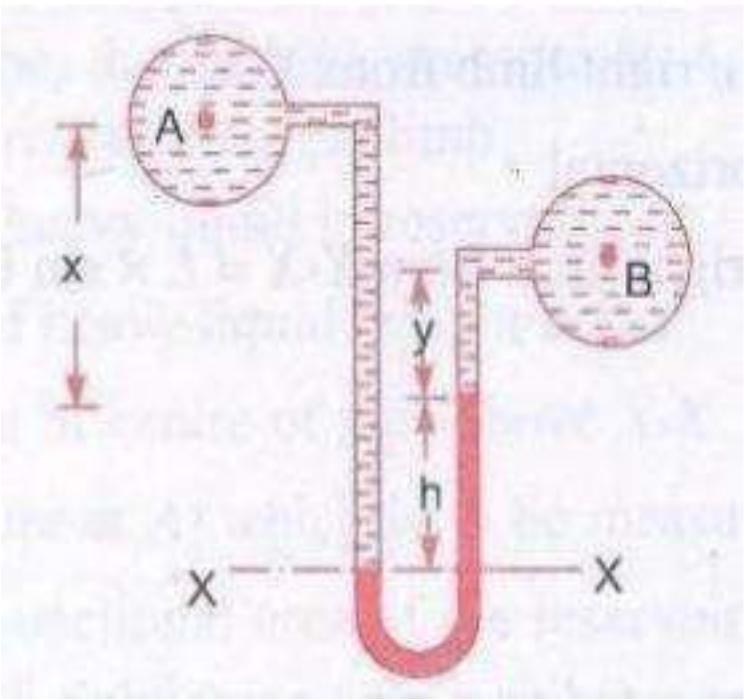
y = Distance of centre of B from the mercury level in the right limb

x = Distance of centre of A from the mercury level in the left limb

ρ_1 = Density of liquid A

ρ_2 = Density of liquid B

ρ_g= Density of heavy liquid or mercury



- Taking datum line at X - X
- Pressure above X - X in the left limb $=\rho_1 g(h + x) + p_A$ (where p_A = Pressure at A)
- Pressure above X - X in the right limb $= \rho_g\, g\, h + \rho_2 g\, y + P_B$

  (where p_B = Pressure at B)
- Equating the above two pressures, we have

$$\rho_1 g(h + x) + p_A = \rho_g\, g\, h + \rho_2\, g\, y + p_B$$

$$p_A - p_B = \rho_g\, g\, h + \rho_2 g\, y - \rho_1 g\,(h + x)$$

$$= h\, g\,(\rho_g - \rho_1) + \rho_2 g\, y - \rho_1 g\, x$$

- Therefore, Difference of Pressures at A and B is

$$h\, g\,(\rho_g - \ell_1) + \rho_2\, g\, y - \rho_1 g\, x$$

U-TUBE DIFFERENTIAL MANOMETER

TWO POINTS A AND B ARE AT SAME LEVELS AND ALSO CONTAINS SAME LIQUIDS OF SP.GR.

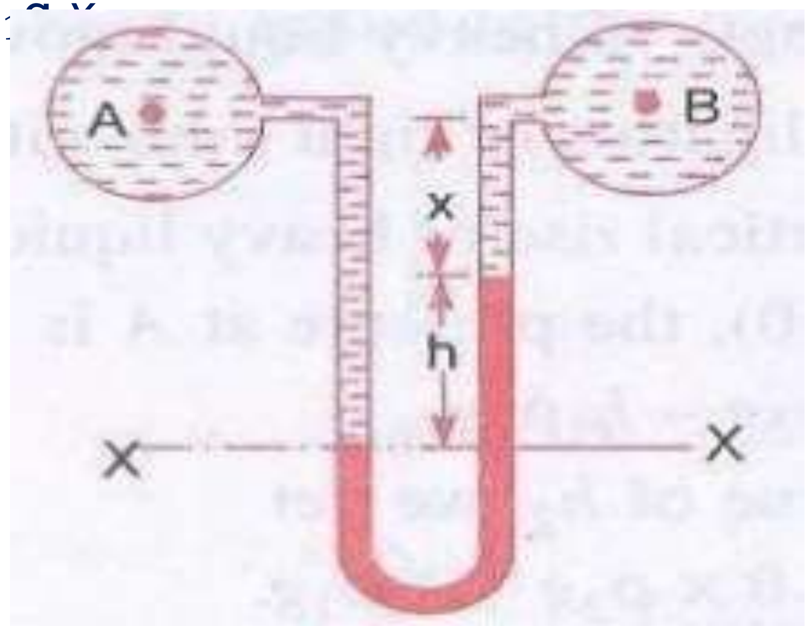
- Then pressure above X - X in the right limb = $\rho_g g h + \rho_1 g x + P_B$
- Pressure above X–X in the left limb = $\rho_1 g (h + x) + P_A$
- Equating the two pressures

$$\rho_g g h + \rho_1 g x + p_B = \rho_1 g (h + x) + p_A$$

$$P_A - P_B = \rho_g g h + \rho_1 g x - \rho_1 g (h + x)$$

$$= g h (\rho_g - \rho_1)$$

- Therefore, Difference of pressure at A and B = $g h (\rho_g - \rho_1)$



APPLICATIONS

- Specific pressure monitoring applications
- Visual monitoring of air and gas pressure for compressors.
- Vacuum equipment and specialty tank applications such as medical gas cylinders, fire extinguishers.
- In power plants, mercury absolute manometer have been used to check condenser efficiency by monitoring vacuum at several points of the condenser
- Used for the research of atmosphere of other planets.
- And many more applications such as in whether studies, research labs, gas analysis and in medical equipment's.
- Never operate damage equipment.
- Meter and its tubing should be free from any breaking and blockage.

- In addition, since the tubes in many manometers are made of glass and can be easily broken, it is important to use care in handling these manometers.
- Electronic manometers do not measure water pressures; under these conditions they will fail. Do not exceed 10 PSI input pressure.
- Some types of liquids used in manometers are toxic and can be damaging to the environment. Therefore, when using manometers to measure or indicate pressure, do not connect any manometer to a pressure that has the potential to exceed the range of the manometer. This could cause the liquid to be forced out of the tube

PROBLEM-1

- Two horizontal plates are placed 1.25 cm apart, the space between them being filled with oil of viscosity 14 poises. Calculate shear stress in oil, if the upper plate is moved velocity of 2.5 m/sec.

Sol: Given Data

Distance between the plates, dy= 1.25 cm = 0.0125m

Viscosity, μ = 14 poise = $\frac{14}{10}$ N s/m²

Velocity of upper plate, u = 2.5 m/sec

Shear stress, $\tau = \mu \frac{du}{dy}$

Where du = change of velocity between plates = u - 0

= u = 2.5 m/sec

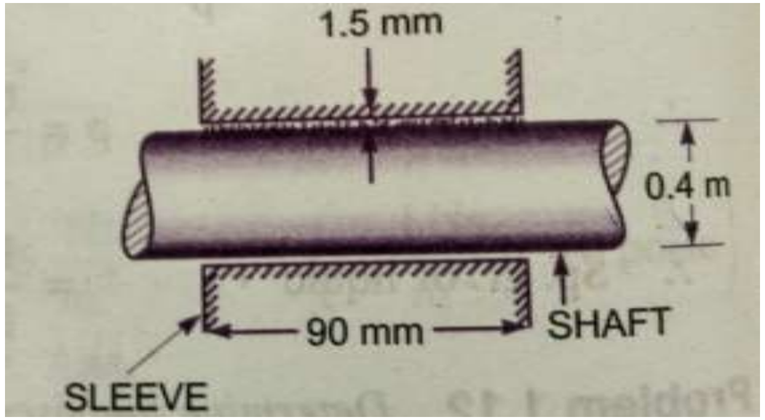
$\tau = \frac{14}{10} \times \frac{2.5}{0.0125} =$ **280 N/m²**

PROBLEM-2

- The dynamic viscosity of oil used for lubrication between a shaft and sleeve is 6 poise. The shaft dia. is 0.4m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90mm. The thickness foil film is 1.5mm

Sol: Given Data,

Viscosity, μ = 6 poise = $\frac{6}{10}\frac{N\,s}{m^2} = 0.6\frac{N\,s}{m^2}$



Dia. of shaft, D = 0.4M

Speed of shaft, N = 190 rpm Sleeve

length, L = 90mm = 90 Thickness of a film,

t = 1.5mm

$= 1.5\times 10^{-3}$ m

Tangential velocity of shaft = u = $\frac{\pi DN}{60} = \frac{\pi \times 0.4 \times 190}{60}$ = 3.98 m/sec

Using the relation $\tau = \mu \frac{du}{dy}$

Where du = change of velocity = u - 0 = u = 3.98 m/sec dy = change of distance = t = 1.5

Then, Shear stress on the shaft

$$\tau = 0.6 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

We know that, Shear force on the shaft, F = shear stress × Area

=1592 × πDL= 180.05 N

Torque on the shaft, T = Force × $\frac{D}{2}$ = 36.01 Nm

Therefore, Power lost, P = $\frac{2\pi NT}{60}$ =

716.48 W

PROBLEM-8

- The right limb of a simple U – tube manometer containing mercury is open to the atmosphere, while the left limb is connected to a pipe in which a fluid of sp.gr.0.9 is flowing. The centre of pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe, if the difference of mercury level in the two limbs is 20 cm.

Sol: Given data,

Sp.gr. of liquid, S_1= 0.9

Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/ m^3 Sp.gr. of mercury, S_2 = 13.6

Density of mercury, $\rho_2 = 13.6 \times 1000 = 13600$ kg/m^3 Difference of mercury level, h_2 = 20cm = 0.2m

Height of the fluid from A – A, h_1 = 20 – 12 = 8cm = 0.08 m

Let 'P' be the pressure of fluid in pipe

Equating pressure at A – A, we get

$p + \rho_1 g h_1 = \rho_2 g h_2$

$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times 0.2$

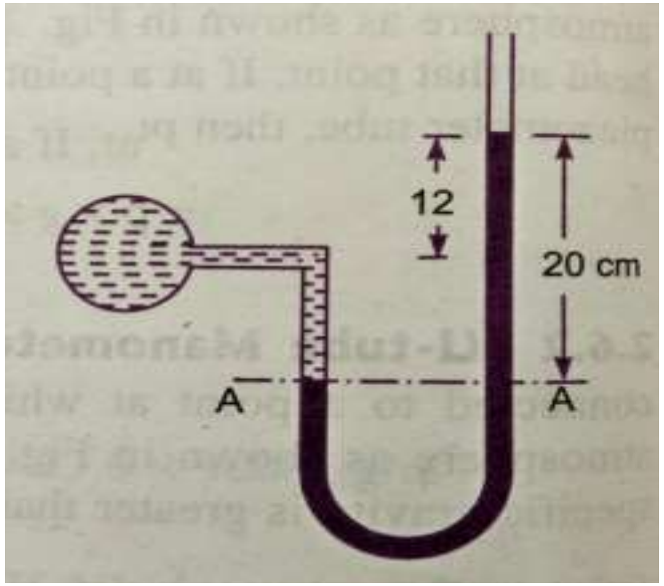
$p = 13.6 \times 1000 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$

$p = 26683 - 706$

$p = 25977\ N/m^2$

$p = 2.597\ N/cm^2$

Therefore, Pressure of fluid P= 2.597 N/ cm^2



PROBLEM-9

- A simple U – tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. 0.8 And having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm. and the height of the fluid in the left tube from the centre of pipe is 15cm below.

Sol: Given data,

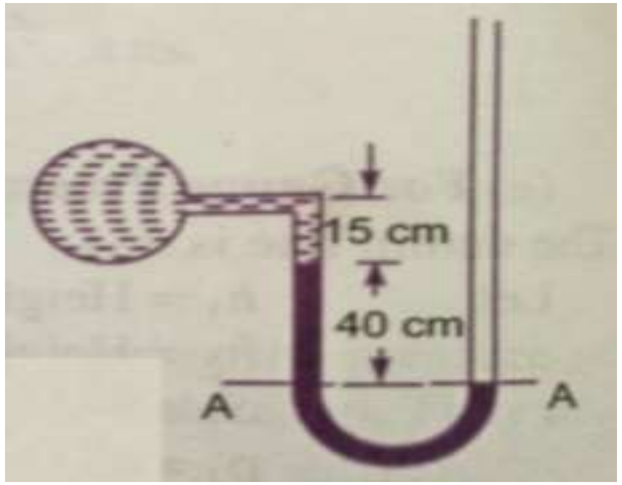
Sp.gr of fluid, $S_1 = 0.8$ Sp.gr. of mercury, $S_2 = 13.6$

Density of the fluid $\rho_1 = S_1 \times 1000 = 0.8 \times 1000$

$= 800\ kg/m^3$

Density of mercury $\rho_2 = 13.6 \times 1000$ Difference of mercury level h_2 = 40cm = 0.4m

Height of the liquid in the left limb = 15cm =0.15m



Let the pressure in the pipe = p

Equating pressures above datum line A- A

$\rho_2 g h_2 + \rho_1 g h_1 + P = 0$ $P = -[\rho_2 g h_2 + \rho_1 g h_1]$

$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$

= 53366.4 + 1177.2

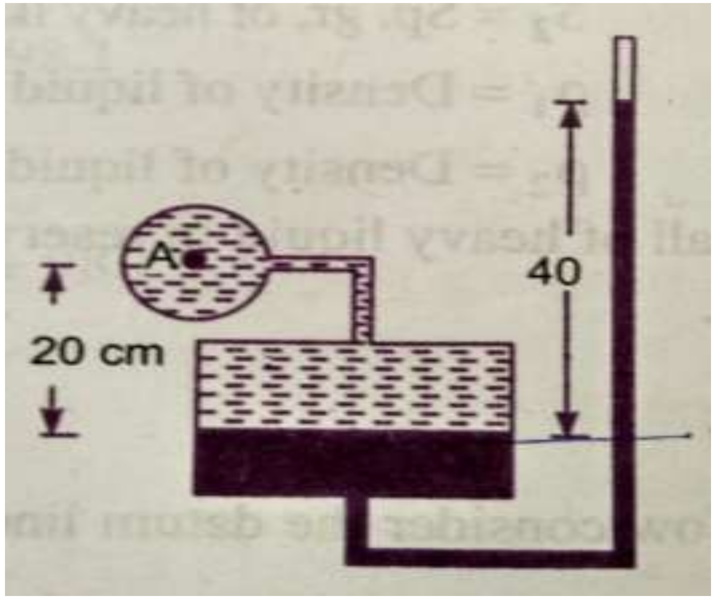
= -54543.6 N/m^2

Therefore, the vacuum pressure in pipe,

P = - 5.454 N/cm^2

PROBLEM-10

- A single column manometer is connected to the pipe containing liquid of sp.gr.0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube of manometer. sp.gr. of mercury is 13.6. Height of the liquid from the centre of pipe is 20cm and difference in level of mercury is 40cm.



Sol: Given data,

Sp.gr. of liquid in pipe, S_1 = 0.9 Density, ρ_1 = 900 kg/ m^3

Sp.gr. of heavy liquid, S_2 = 13.6 Density, ρ_2 = 13600

$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a}$ — = 100

Height of the liquid, h_1 = 20cm = 0.2m

Rise of mercury in the right limb, h_2 = 40cm = 0.4m

Pressure in pipe A

$$p_A = \frac{A}{a} \times h_2[\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

$$= \frac{1}{100} \times 0.4\,[13600 \times 9.81 - 900 \times 9.81] + 0.4 \times 13600 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}[133416 - 8829] + 53366.4 - 1765.8$$

$$= 533.664 + 53366.4 - 1765.8$$

$$= 52134 \text{ N/m}$$

Therefore, Pressure in pipe A= **5.21 N/ cm²**

PROBLEM-11

- A pipe contains an oil of sp.gr.0.9. A differential manometer is connected at the two points A and B shows a difference in mercury level at 15cm. find the difference of pressure at the two points.

Sol: Given data,

Sp.gr. of oil S_1 = 0.9: density ρ_1 = 0.9 x 1000 = 900 kg/m^3

Difference of level in the mercury h = 15cm = 0.15 m

Sp.gr. of mercury = 13.6, Density = 13.6 × 1000 = 13600 kg/m^3 The difference of

pressure $p_A - p_B = g \times h \times (\rho_g - \rho_1)$

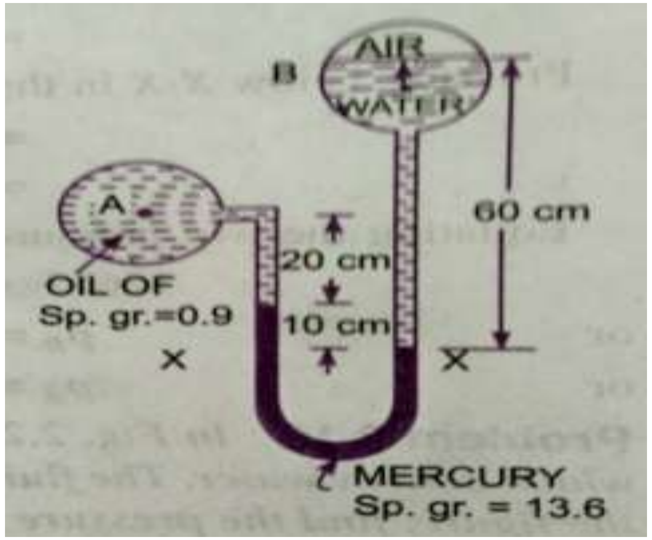
= 9.81 x 0.15 (13600 – 900)

= 18688 N/m^2

Therefore, **$p_A - p_B$ = 18688 N/ m^2**

PROBLEM-12

- A differential manometer is connected at two points A and B .At B air pressure is 9.81 N/cm^2. Find absolute pressure at A.



Sol: Given data,

Density of air = 0.9 × 1000 = 900 kg/m^3

Density of mercury = 13.6 × 10^3 kg/ m^3 Pressure at B= 9.81 N/cm^2

= 98100 N/m^2 Let pressure at A is p_A

Taking datum as X – X

Pressure above X – X in the right limb

= 1000 × 9.81 × 0.6 + p_B = 5886 + 98100 = 103986

Pressure above X – X in the left limb

= 13.6 × 10^3 × 9.81 × 0.1 +0900 × 9.81 × 0.2 + p_A

= 13341.6 +1765.8 +p_A

Equating the two pressures heads, we get 103986 =

$13341.6 + 1765.8 + p_A$

$103986 = 15107.4 + p_A$

$p_A = 103986 - 15107.4$

$= 88878.6\ N/m^2$

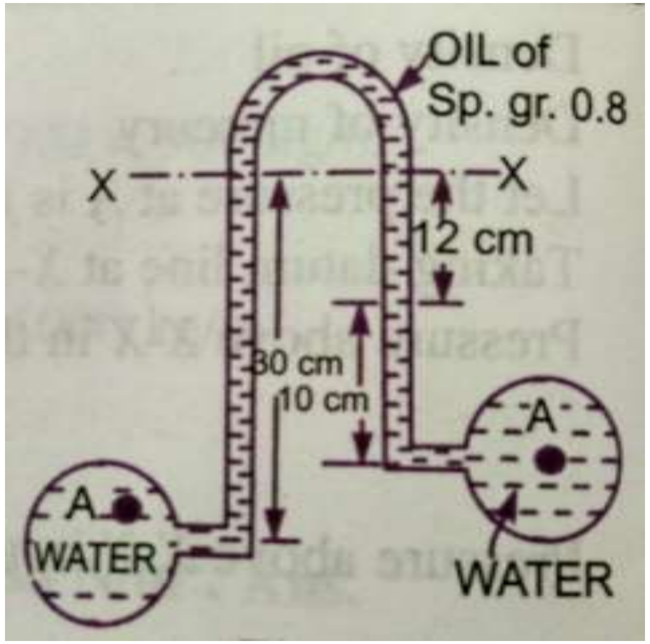
Therefore, Pressure at A,

$\mathbf{p_A = 8.887\ N/cm^2}$

PROBLEM-8

- Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp.gr. 0.8 is connected. The pressure head in the pipe A is 2m of water. Find the pressure in the pipe B for the manometer readings shown in fig.

Sol: Given data,



Pressure head at A $= \frac{p_A}{\rho g}$ = 2m ~~of~~ water

$p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620$ N/m^2

Pressure below X – X in the left limb

$= p_A - \rho_1 g h_1$

$= 19620 - 1000 \times 9.81 \times 0.3 = 16677$ N/m^2

Pressure below X – X in the right limb

$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= p_B - 981 - 941.76 = p_B - 1922.76$

Equating the two pressures, we get, $16677 = p_B - 1922.76$

$p_B = 16677 + 1922.76$ $p_B = 18599.76$ N/m^2

-

Therefore, Pressure at B, $\mathbf{p_B = 1.859}$ **N/**$\mathbf{cm^2}$

ASSIGNMENT QUESTIONS

- Two large planes are parallel to each other and are inclined at 30° to the horizontal with the space between them filled with a fluid of viscosity 20 cp. A small thin plate of 0.125 m square slides parallel and midway between the planes and reaches a constant velocity of 2 m/s. The weight of the plate is 1 N. Determine the distance between the plates.
- A U- tube mercury manometer is used to measure the pressure of oil flowing through a pipe whose specific gravity is 0.85. The center of the pipe is 15 cm below the level of mercury. The mercury level difference in the manometer is 25 cm, determine the absolute pressure of the oil flowing through the pipe. Atmospheric pressure is 750 mm of Hg.
- A single column vertical manometer is connected to a pipe containing oil of specific gravity 0.9. The area of the reservoir is 80 times the area of the manometer tube. The reservoir contains mercury of sp. gr. 13.6. The level of mercury in the reservoir is at a height of 30 cm below the center of the pipe and difference of mercury levels in the reservoir in the right limb is 50 cm. find the pressure in the pipe.

- Find the height through which water rises by capillary action in a glass tube of 2mm bore if the surface tension ate the prevailing temperature is
  0.075 N/m.
- The space between two parallel square plates each of side 0.8m is filled with an oil of specific gravity 0.8. If the space between the plates is 12.5mm and the upper plate which moves with velocity of 1.25m/s requires a force of 51.2 N. Determine (i) Dynamic viscosity of oil in poise
  (ii) Kinematic viscosity in stokes.
- List all the properties of fluid and derive Newton’s law of viscosity.
- Explain atmospheric, gauge and vaccum pressure with the help of a neat
  sketch.

UNIT – II

Fluid Kinematics:

- Stream Line, Path line, streak lines and stream tubes.
- Classification of flows- steady & unsteady, uniform & non-uniform, laminar & turbulent, rotational & irrotational flows.
- Equation of continuity- 1D flow.

Fluid Dynamics:

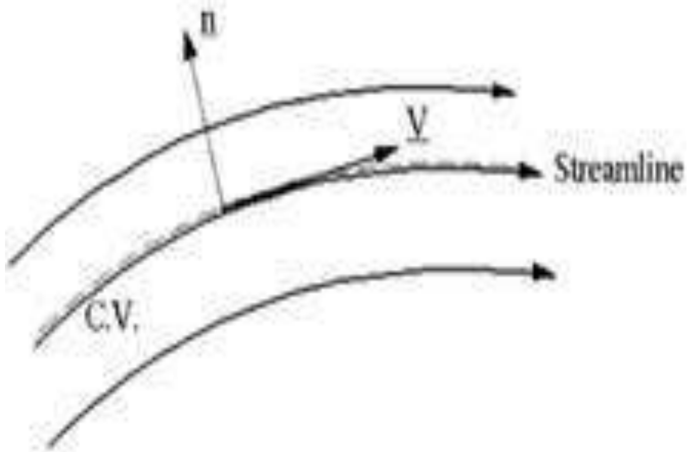
- Surface & body forces- Euler's and Bernouli's equation for flow along a stream line.
- Momentum equation & its application on force on pipe bend.

FLUID KINEMATICS

- Kinematics is defined as a branch of science which deals with motion of particles without considering the forces causing the motion.
- The velocity at any point in a flow field at any time is studied in this.
- Once the velocity is known, then the pressure distribution and hence the forces acting on the fluid can be determined.

STREAM LINE

- A stream line is an imaginary line drawn in a flow field such that the tangent drawn at any point on this line represents the direction of velocity vector.
- From the definition it is clear that there can be no flow across stream line.
- Considering a particle moving along a stream line for a very short distance 'ds' having its components dx , dy and dz, along three mutually perpendicular co-ordinate axes.
- Let the components of velocity vector Vs along x, y and z directions be u, v and w respectively.



- The time taken by the fluid particle to move a distance ‘ds’ along the stream line with a velocity Vs is:

$$t = \frac{ds}{Vs}$$

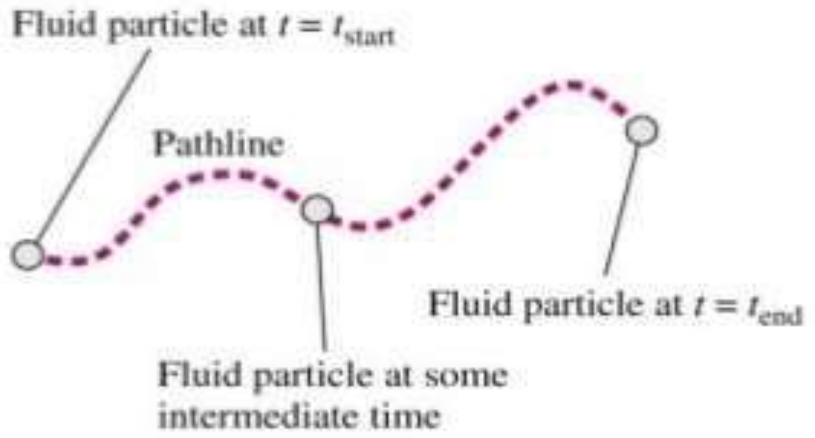
which is same as $t = \frac{dX}{u} = \frac{dy}{v} = \frac{ds}{w}$

- Hence the differential equation of the steam line may be written as:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{ds}{w}$$

PATH LINE

- A path line is locus of a fluid particle as it moves along.
- In other words a path line is a curve traced by a single fluid particle during its motion.
- A stream line at time t_1 indicating the velocity vectors for particles A and B.
- At times t_2 and t_3 the particle A occupies the successive positions.
- The line containing these various positions of A represents its **Path line**

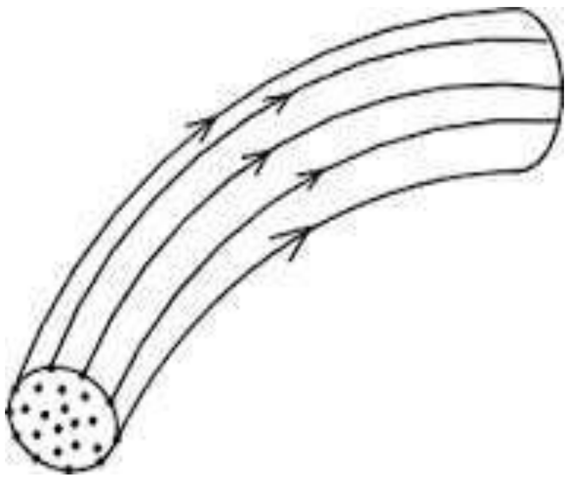


STREAK LINE

- When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path fallowed by dye or smoke is called the **streak line**.
- Thus the streak line connects all particles passing through a given point.
- In steady flow, the stream line remains fixed with respect to co-ordinate axes.
- Stream lines in steady flow also represent the path lines and streak lines.
- In unsteady flow, a fluid particle will not, in general, remain on the same stream line (except for unsteady uniform flow).
- Hence the stream lines and path lines do not coincide in unsteady non-uniform flow.

STREAM TUBE

- If stream lines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate.
- Such a surface bounded by stream lines is known as **Stream tube**.
- From the definition of stream tube, it is evident that no fluid can cross the bounding surface of the stream tube.
- This implies that the quantity of fluid entering the stream tube at one end must be the same as the quantity leaving at the other end.
- The Stream tube is assumed to be a small cross-sectional area, so that the velocity over it could be considered uniform.



CLASSIFICATION OF FLOWS

- The fluid flow is classified as:
  - Steady and unsteady flows.
  - Uniform and Non-uniform flows.
  - Laminar and Turbulent flows.
  - Compressible and incompressible flows.
  - Rotational and Irrotational flows.
  - One, two and three dimensional flows.

STEADY & UNSTEADY FLOW

- Steady flow is defined as the flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.
- Thus for a steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} = 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} = 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} = 0$$

- Un-Steady flow is the flow in which the velocity, pressure, density at a point changes with respect to time.
- Thus for un-steady flow, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x,y,z} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x,y,z} \neq 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x,y,z} \neq 0$$

UNIFORM & NON-UNIFORM FLOW

- Uniform flow is defined as the flow in which the velocity at any given time does not change with respect to space. (i.e. the length of direction of flow)

- For uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} = 0$

  Where, ∂V = Change of velocity

  ∂s = Length of flow in the direction of - S

- Non-uniform is the flow in which the velocity at any given time changes with respect to space.

- For Non-uniform flow $\left(\frac{\partial V}{\partial s}\right)_{t=const} \neq 0$

LAMINAR & TURBULENT FLOW

- Laminar flow is defined as the flow in which the fluid particles move along well-defined paths or stream line and all the stream lines are straight and parallel.
- Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called streamline flow or viscous flow.
- Turbulent flow is the flow in which the fluid particles move in a zigzag way.
- Due to the movement of fluid particles in a zigzag way, the eddies formation takes place, which are responsible for high energy loss.

LAMINAR & TURBULENT FLOW

- For a pipe flow, the type of flow is determined by a non-dimensional number $\left(\frac{VD}{v}\right)$ called the Reynolds number.

  Where D = Diameter of pipe.

  V = Mean velocity of flow in pipe.

  v = Kinematic viscosity of fluid.

- If the Reynolds number is lessthan2000, the flow is called Laminar flow.
- If the Reynolds number is more than 4000, it is called Turbulent flow.
- If the Reynolds number is between 2000 and 4000 the flow may be Laminar or Turbulent flow.

COMPRESSIBLE & INCOMPRESSIBLE FLOW

- Compressible flow is the flow in which the density of fluid changes from point to point or in other words the density is not constant for the fluid.
- For compressible flow, $\rho \neq$ Constant.
- In compressible flow is the flow in which the density is constant for the fluid flow.

- Liquids are generally incompressible, while the gases are compressible.
- For incompressible flow, $\rho =$ Constant

ROTATIONAL & IRROTATIONAL FLOW

- Rotational flow is a type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

- And if the fluid particles, while flowing along stream lines, do not rotate about their own axis, the flow is called Irrotational flow.

ONE DIMENSION FLOW

- **1D flow** is a type of flow in which flow parameter such as velocity is a function of time and one space co-ordinate only, say '*x*'.
- For a steady one- dimensional flow, the velocity is a function of one space co-ordinate only.
- The variation of velocities in other two mutually perpendicular directions is assumed negligible.
- Hence for one dimensional flow **u = f(x), v = 0 and w = 0**

Where u, v and w are velocity components in x, y and z directions respectively.

TWO DIMENSION FLOW

- **2D flow** is the type of flow in which the velocity is a function of time and two space co-ordinates, say x and y.
- For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.
- The variation of velocity in the third direction is negligible.
- Thus for two dimensional flow **$u = f_1 (x, y)$, $v = f_2 (x, y)$ and $w = 0$.**
- **3D flow** is the type of flow in which the velocity is a function of time and three mutually perpendicular directions.
- But for a steady three-dimensional flow, the fluid parameters are functions of three space co-ordinates (x, y, and z) only.
- Thus for **three- dimensional flow $u = f_1 (x, y, z)$, $v = f_2 (x, y, z)$, $z = f_3 (x, y, z)$.**

RATE OF FLOW OR DISCHARGE (Q)

- It is defined as the quantity of a fluid flowing per second through a section of pipe or channel.
- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of the liquid flowing cross the section per second. or compressible fluids.
- The rate of flow is usually expressed as the weight of fluid flowing across the section.
- Thus (i) For liquids the unit of Q is m^3/sec or Litres/sec.

  (ii) For gases the unit of Q is Kgf/sec or Newton/sec.
- **The discharge Q = A×V**

  Where, A = Area of cross-section of pipe.

  V= Average velocity of fluid across the section.

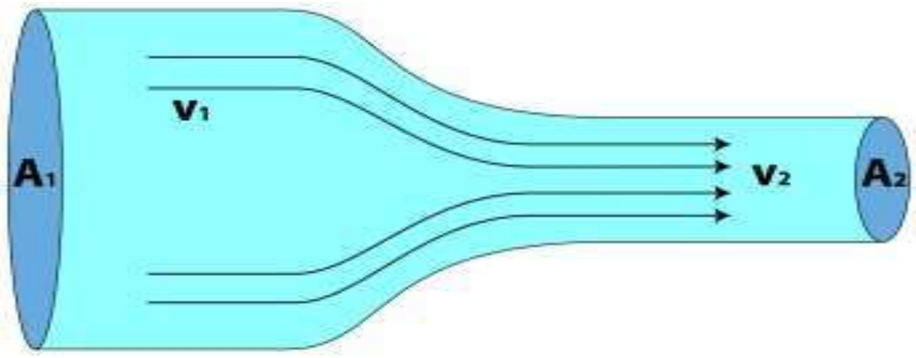
EQUATION OF CONTINUITY

- The equation based on the principle of conservation of mass is called Continuity equation.
- Thus for a fluid flowing through the pipe at all cross- sections, the quantity of fluid per second is constant.
- Consider two cross- sections of a pipe.
- Let V_1 = Average velocity at cross- section 1-1

  ρ_1 = Density of fluid at section 1-1

  A_1 = Area of pipe at section 1-1

  And V_2, ρ_2, A_2 are the corresponding values at section 2-2



- Then the rate flow at section 1-1= $\rho_1 A_1 V_1$
- Rate of flow at section 2-2 = $\rho_2 A_2 V_2$
- According to law of conservation of mass, Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\boldsymbol{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

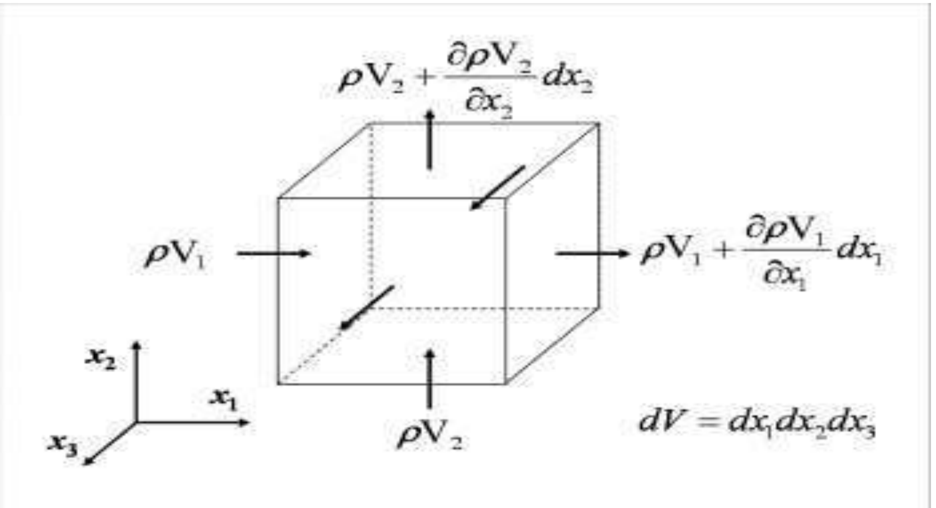
- This equation is applicable to the compressible as well as incompressible fluids and is called **“Continuity equation”**.
- If the fluid is incompressible, then $\rho_1=\rho_2$ and the continuity equation reduces to

$$\mathbf{A_1 V_1 = A_2 V_2}$$

EQUATION OF CONTINUITY FOR 3D FLOW

- Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z.
- Let u, v and w are the inlet velocity components in x, y and z directions respectively.
- Mass of fluid entering the face ABCD per second = ρ × velocity in x - direction × Area of ABCD

$$= \rho \times u \times (dy \times dz)$$



- Then the mass of fluid leaving the face EFGH per second

$$= \rho \times u \times (dy \times dz) + \frac{\partial}{\partial x}(\rho\, u\, dy\, dz\, dx)$$

- Gain of mass in x- direction = Mass through ABCD - Mass through EFGH per second = $\rho\, u\, dy\, dz - \rho\, u\, dy\, dz - \frac{\partial}{\partial x}(\rho\, u\, dy\, dz\, dx)$

$$= -\frac{\partial}{\partial x}(\rho\, u\, dy\, dz\, dx)$$

$$= -\frac{\partial}{\partial x}(\rho\, u)\, dx dy dz \qquad \_\_\_\_\_\_\_\_\_\_ (1)$$

- Similarly gain of mass in y- direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz \qquad \_\_\_\_\_\_\_\_\_\_ (2)$$

- Similarly gain of mass in z- direction

$$= -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz \qquad \_\_\_\_\_\_\_\_\_\_ (3)$$

- Net gain of mass = $-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\,dy\,dz$ (4)
- Since mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element.
- But the mass of fluid in the element is ρ dxdydz and its rate of increase with time is $\frac{\partial}{\partial t}(\rho.\ dx.\ dy.\ dz.)$ or $\frac{\partial \rho}{\partial t}.\ dx.\ dy.\ dz.$ (5)
- Equating the two expressions (4) & (5)

$$-\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right) dx\,dy\,dz = \frac{\partial \rho}{\partial t}.\ dx.\ dy.\ dz.$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (6)$$

- This equation is applicable to
  - Steady and unsteady flow
  - Uniform and non- uniform flow , and
  - Compressible and incompressible flow.
- For steady flow $\frac{\partial \rho}{\partial t} = 0$ and hence equation (6) becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad \text{———(7)}$$

- If the fluid is incompressible, then ρ is constant and the above equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{———(8)}$$

- This is the continuity equation in three - dimensional flow.

FLUID DYNAMICS

- A fluid in motion is subjected to several forces, which results in the variation of the acceleration and the energies involved in the flow of the fluid.
- The study of the forces and energies that are involved in the fluid flow is known as Dynamics of fluid flow.
- The various forces acting on a fluid mass may be classified as:
  - Body or volume forces
  - Surface forces
  - Line forces.

FORCES

- **Body forces:** The body forces are the forces which are proportional to the volume of the body.

  Examples: Weight, Centrifugal force, magnetic force, Electromotive force etc.

- **Surface forces:** The surface forces are the forces which are proportional to the surface area which may include pressure force, shear or tangential force, force of compressibility and force due to turbulence etc.

- **Line forces:** The line forces are the forces which are proportional to the length.

  Example: surface tension.

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Example: surface tension.

- The dynamics of fluid flow is governed by Newton's second law of motion which states that the resultant force on any fluid element must be equal to the product of the mass and acceleration of the element and the acceleration vector has the direction of the resultant vector.
- The fluid is assumed to be in compressible and non- viscous.

$$\sum F_x = M.\ a$$

Where $\sum F$ represents the resultant external force acting on the fluid element of mass **M** and **a** is total acceleration.

- Both the acceleration and the resultant external force must be along same line of action.
- The force and acceleration vectors can be resolved along the three reference directions x, y and z and the corresponding equations may be expressed as ;

$$\sum F_x = M.\ a_x$$

$$\sum F_y = M.\ a_y$$

$$\sum F_z = M.\ a_z$$

Where $\sum F_x$, $\sum F_y$ and $\sum F_z$ are the components of the resultant force in the x, y and z directions respectively, and a_x , a_y and a_z are the components of the total acceleration in x, y and z directions respectively.

FORCES ACTING ON FLUID IN MOTION

- The various forces that influence the motion of fluid are due to gravity, pressure, viscosity, turbulence and compressibility.
- The gravity force ‘Fg’ is due to the weight of the fluid and is equal to Mg . The gravity force per unit volume is equal to “ρ g”.
- The pressure force ‘Fp’ is exerted on the fluid mass, if there exists a pressure gradient between the two points in the direction of the flow.
- The viscous force ‘Fv’ is due to the viscosity of the flowing fluid and thus exists in case of all real fluids.
- The turbulent flow ‘Ft’ is due to the turbulence of the fluid flow.
- The compressibility force ‘Fc’ is due to the elastic property of the fluid and it is important only for compressible fluids.

FORCES ACTING ON FLUID IN MOTION

- If a certain mass of fluid in motion is influenced by all the above forces, then according to Newton's second law of motion
- The net force $F_x = M. a_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$
- If the net force due to compressibility(F_c) is negligible, the resulting net force

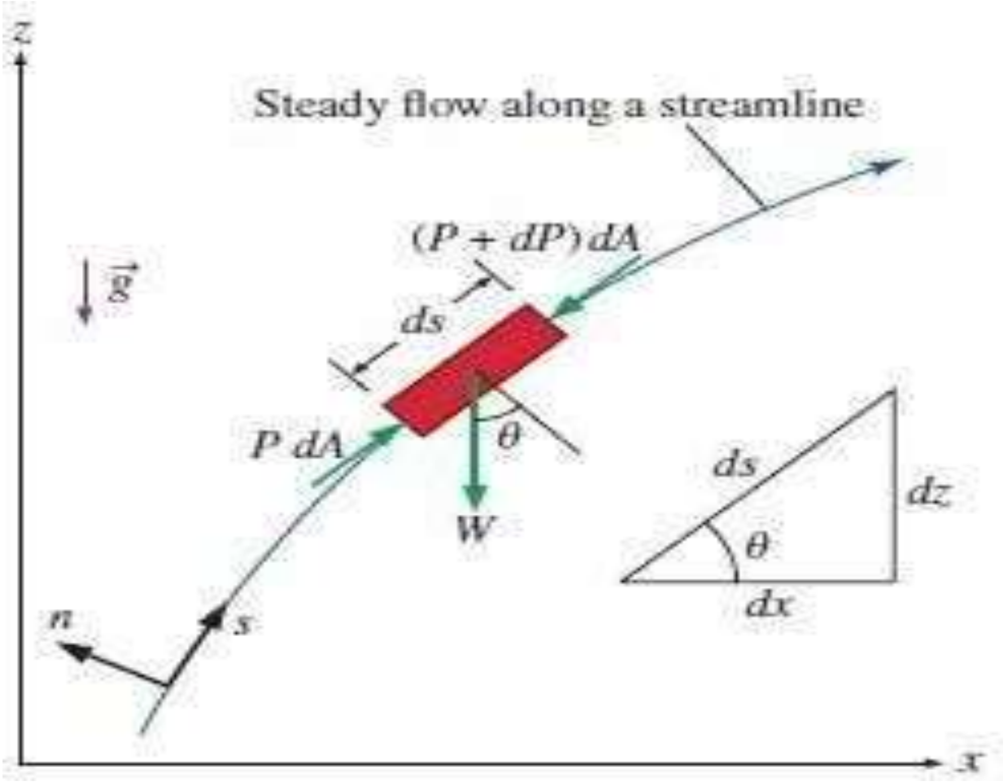
$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and the equation of motions are called **Reynolds's equations of motion.**

- For flow where (Ft) is negligible, the resulting equations of motion are known as **Navier – Stokes equation.**
- If the flow is assumed to be ideal, viscous force (F_v) is zero and the equations of motion are known as **Euler's equation of motion.**

EULER'S EQUATION OF MOTION

- In this equation of motion the forces due to gravity and pressure are taken in to consideration.
- This is derived by considering the motion of the fluid element along a stream-line as:
- Consider a stream-line in which flow is taking place in s- direction.
- Consider a cylindrical element of cross-section dA and length



ds.

- The forces acting on the cylindrical element are:
  - Pressure force pdA in the direction of flow.
  - Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$
  - Weight of element ρ g dA.ds
- Let θ is the angle between the direction of flow and the line of action of the weight of the element.
- The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element× acceleration in the direction of s.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dAds \cos\theta = \rho dAds \times a_s \quad \text{———} \quad (1)$$

Whereas is the acceleration in the direction of s.

- Now, $a_s = \frac{dv}{dt}$ where 'v' is a function of s and t.

$$a_s = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

- If the flow is steady, then $\frac{\partial v}{\partial t} = 0$. So, $a_s = \frac{v\partial v}{\partial s}$
- Substituting the value of a_s in equation (1) and simplifying, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho g dAds\cos\theta = \rho dAds \times \frac{\partial v}{\partial s}$$

- Dividing by ρ dA.ds, $-\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) - g\cos\theta = \frac{v\partial v}{\partial s}$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\cos\theta + \frac{v\partial v}{\partial s} = 0$$

But we have $\cos\theta = \frac{dz}{ds}$

$$\left(\frac{1}{\rho}\right) \times \left(\frac{\partial p}{\partial s}\right) + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0$$

$$\mathbf{\frac{\partial p}{\rho} + gdz + vdv = 0}$$

∴ This equation is known as Euler's equation of motion.

BERNOULLI'S EQUATION

- Bernoulli's equation is obtained by integrating the Euler's equation of motion as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = Constant$$

- If the flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

- The above equation is Bernoulli's equation in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid or pressure head.

$\frac{v^2}{2g}$ = Kinetic energy per unit weight of fluid or Kinetic head.

z = Potential energy per unit weight of fluid or Potential head.

ASSUMPTIONS OF BERNOULLI’S EQUATION

- The following are the assumptions made in the derivation of Bernoulli’s equation.

➢ The fluid is ideal i.e. Viscosity is zero.

➢ The flow is steady.

➢ The flow is incompressible.

➢ The flow is Irrotational.

MOMENTUM EQUATION

- It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass equal to the change in the momentum of the flow per unit time in that direction.
- The force acting on a fluid mass 'm' is given by Newton's second law of motion.

$$F = m \times a$$

- Where a is the acceleration acting in the same direction as force F.

But $a = \frac{dv}{dt}$

$F = m\frac{dv}{dt} = \frac{d(mv)}{dt}$ (Since m is a constant and can be taken inside differential)

$$F = \frac{d(mv)}{dt}$$

The above equation is known as the momentum principle.

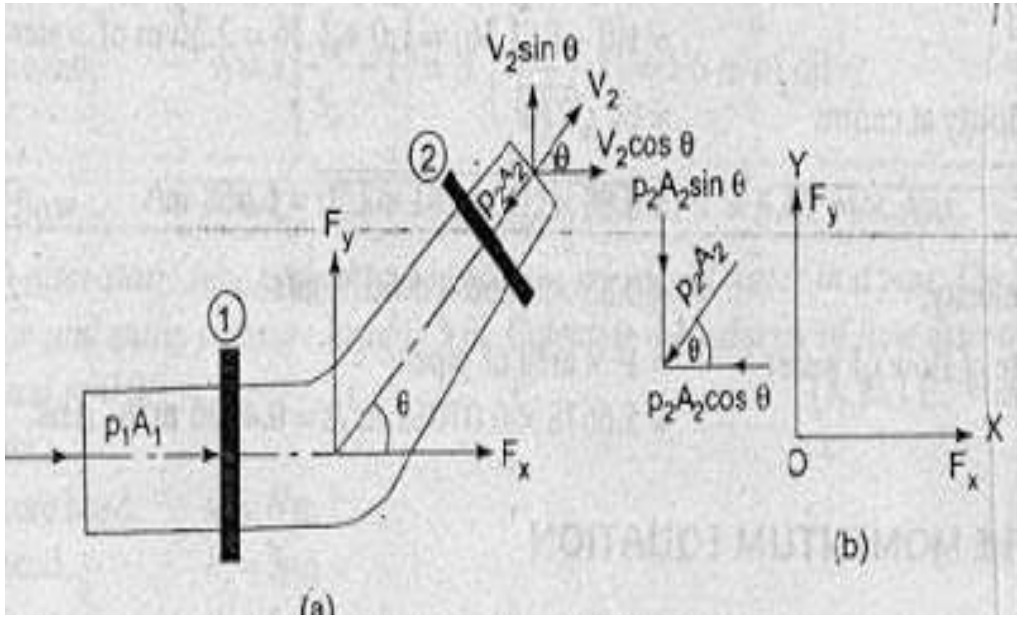
F. dt = d(mv)

The above equation is known as the impulse momentum equation.

- It states that the impulse of a force 'F' acting on a fluid mass m in a short interval of time 'dt' is equal to the change of momentum 'd(mv)' in the direction of force.

FORCE EXERTED BY A FLOWING FLUID ON A PIPE-BEND:

- The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe bend.
- Consider two sections (1) and (2) as above
- Let v_1= Velocity of flow at section (1)
- P_1= Pressure intensity at section (1)
- A_1 = Area of cross-section of pipe at section (1)
- And V_2, P_2, A_2 are corresponding values of Velocity, Pressure, Area at section (2)



- Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in *x* and *y* directions respectively.
- Then the force exerted by the bend on the fluid in the directions of *x* and *y* will be equal to F_X and F_Y but in the opposite directions.
- Hence the component of the force exerted by the bend on the fluid in the *x* – direction = $-F_x$ and in the direction of *y* = $-F_y$.
- The other external forces acting on the fluid are $p_1 A_1$ and $p_2 A_2$ on the sections (1) and (2) respectively. Then the momentum equation in x-direction is given by
- Net force acting on the fluid in the direction of x = Rate of change of momentum in x -direction

$$= p_1 A_1 - p_2 A_2 \cos\theta - F_x = \text{(Mass per second) (Change of velocity)}$$

$= \rho Q$ (Final velocity in x-direction – Initial velocity in x-direction)

$= \rho Q (V_2 \cos\theta - V_1)$

$F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1 A_1 - p_2 A_2 \cos\theta$ ________(1)

- Similarly the momentum equation in y-direction gives

$0 - p_2 A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0)$

$F_y = \rho Q (-V_2 \sin\theta) - p_2 A_2 \sin\theta$ ________________(2)

- Now the resultant force (F_R) acting on the bend

$$F_R = \sqrt{F_x^2 + F_y^2}$$

- And the angle made by the resultant force with the horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x}$$

PROBLEM 1

- A pipe through which water is flowing is having diameters 20cms and 10cms at cross-sections 1 and 2 respectively. The velocity of water at section 1 is 4 m/sec. Find the velocity head at section 1 and 2 and also rate of discharge?

<u>Sol:</u> Given data

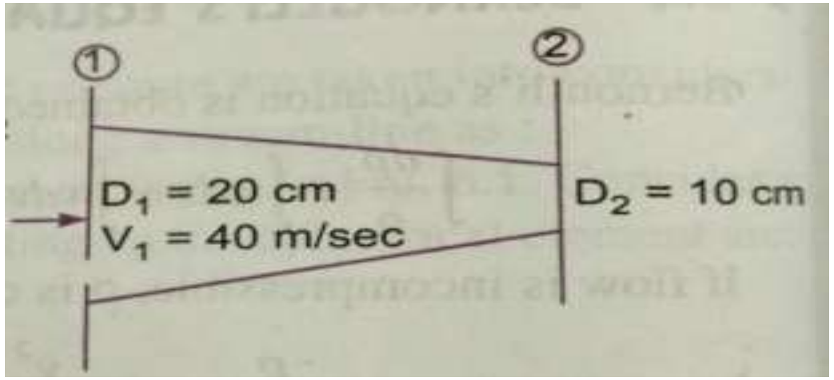
D_1 = 20cms = 0.2m

$A_1 = \frac{\pi}{4} \times (0.2)^2 = 0.0314 m^2$

V_1 = 4 m/sec

D_2 = 10 cm = 0.1 m

$A_2 = \frac{\pi}{4} \times (0.1)^2 = 0.007854 m^2$



i) Velocity head at section 1

$$\frac{V_1^2}{2g} = \frac{4\times4}{2\times9.81} = 0.815m$$

ii) Velocity head at section 2 $\frac{V_2^2}{2g}$

To find V_2, apply continuity equation, $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314\times4}{0.00785} = 16m/sec$$

- Velocity head at section 2
- $$\frac{V_2^2}{2g} = \frac{16\times16}{2\times9.81} = 13.047m$$
- iii) Rate of discharge

$Q = A_1 V_1 = A_2 V_2$

$= 0.0314 \times 4 = 0.1256\ m^3/sec$

Q = 125.6 Liters/sec

PROBLEM 2

- Water is flowing through a pipe of 5cm dia. Under a pressure of 29.43N/cm^2 and with mean velocity of 2 m/sec. find the total head or total energy per unit weight of water at a cross-section, which is 5m above datum line.

Sol: Given data

Dia. of pipe, d = 5cm = 0.05m

Pressure, P = 29.43N/cm^2 = 29.43 x 10^4N/m^2

Velocity, V = 2 m/sec

Datum head, Z = 5m

Total head = Pressure head + Kinetic head + Datum head

$$\text{Pressure head} = \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30m$$

Kinetic head $= \dfrac{V^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204m$

Datum head = Z = 5m

$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = 30 + 0.204 + 5 = 35.204m$$

Total head = 35.204m

PROBLEM 3

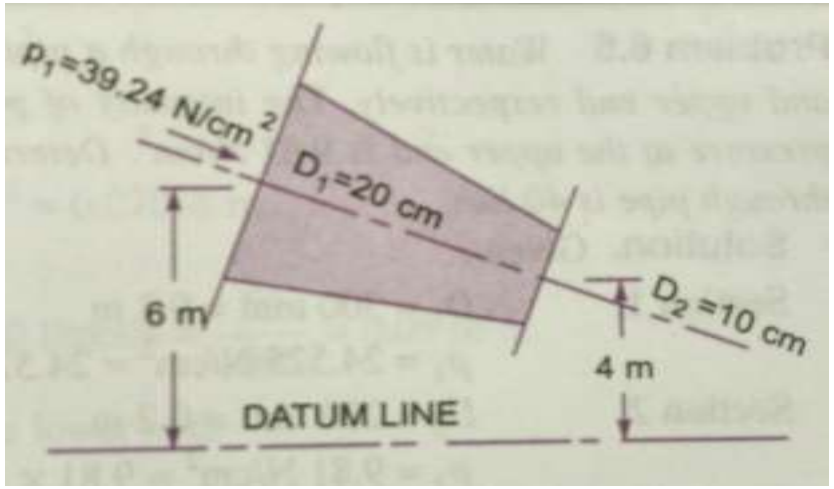
- Water is flowing through a pipe having diameters 20cms and 10cms at sections 1 and 2 respectively. The rate of flow through pipe is 35 liters/sec. The section 1 is 6m above the datum and section 2 is 4m above the datum. If the pressure at section 1 is 39.24n/cm^2. Find the intensity of pressure at section 2?

<u>Sol:</u> Given data

At section 1, D_1 = 20cm =0.2m

$$A_1 = \left(\frac{\pi}{4}\right) \times (0.2)^2 = 0.0314 m^2$$

P_1 = 39.24N/cm^2 =39.24 x 10^{4}N/m^2 Z_1 = 6m



At section 2, D_2 =10cm = 0.1m

$$A_2 = \left(\frac{\pi}{4}\right) \times (0.1)^2 = 0.0007854 m^2$$

Z_2 = 4m

Rate of flow Q = 35 lt/sec = (35/1000) m^3/sec = 0.035 m^3/sec

$$Q = A_1V_1 = A_2V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 m/sec, \quad V_2 = \frac{Q}{A_2} = \frac{0.035}{0.007854} = 4.456 m/sec$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$= \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114^2)}{2\times9.81} + 6 = \frac{P_2}{(1000\times9.81)} + \frac{(4.456^2)}{2\times9.81} + 4$$

$$= 40 + 0.063 + 6 = \frac{P_2}{9810} + 1.102 + 4$$

$$= 46.063 = \frac{P_2}{9810} + 5.102$$

$$= \frac{P_2}{9810} = 46.063 - 5.102 = 41.051$$

Therefore P_2 = 41.051 ×9810 =402710 N/m^2

P_2 = 40.271N/cm^2

PROBLEM 4

- Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525N/cm^2 and the pressure at the upper end is 9.81N/cm^2. Determine the difference in datum head if the rate of flow through is 40lit/sec?

<u>Sol:</u> Given data

Section 1, D_1= 300mm =0.3m

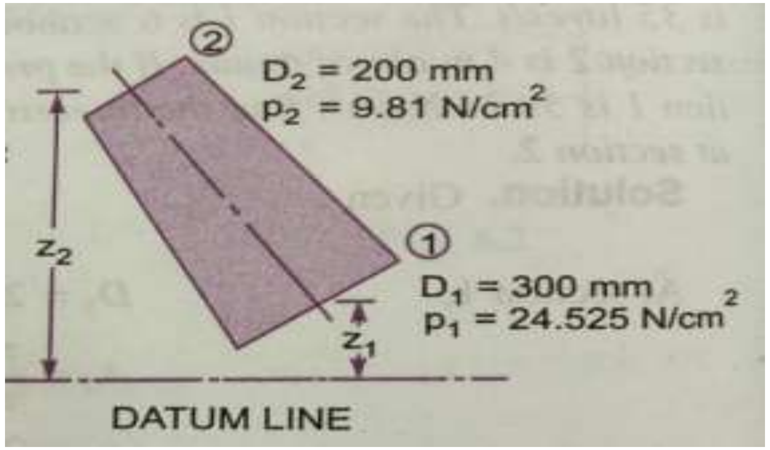
$A_1 = \left(\frac{\pi}{4}\right) \times (0.3)^2$ A_1 = 0.07065 m^2

P_1= 24.525N/cm^2 = 24.525 × 10^{4}N/m^2

Section 2, D_2 = 200mm 0.2m

$A_2 = \left(\frac{\pi}{4}\right) \times (0.2)^2 = 0.0314\ m^2$

P_2 = 9.81N/cm^2 = 9.81 × 10^4 N/m^2



Rate of flow , $Q = 40$ lit/Sec $= 40/1000 = 0.04\ m^3/sec$

$Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.04}{0.07065} = 0.566 \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.04}{00.0314} = 1.274 \text{m/sec}$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$= \frac{(24.525\times10^4)}{(1000\times9.81)} + \frac{(0.566)^2}{2\times9.81} + Z_1 = \frac{(9.81\times10^4)}{1000\times9.81} + \frac{(1.274^2)}{(2\times9.81)} + Z_2$$

$= 25 + 0.32 + Z_1 = 10 + 1.623 + Z_2$

$= Z_2 - Z_1 = 25.32 - 11.623 = 13.697$ or say 13.70m

The difference in datum head = **$Z_2 - Z_1 = 13.70m$**

PROBLEM 1

- The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50lts/sec. the pipe has a slope of 1 in 30. Find the pressure at the lower end, if the pressure at the higher level is 19.62N/cm^2?

<u>Sol:</u> Given data

Length of pipe L = 100m

Dia. At the upper end D_1 = 600mm = 0.6

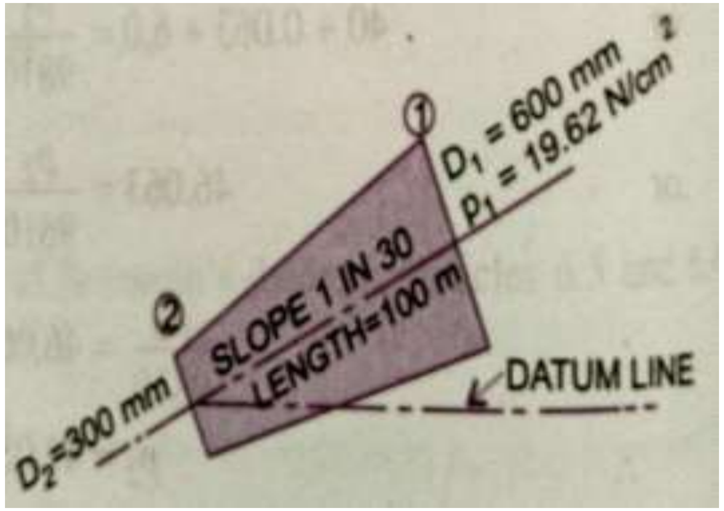
$$A_1 = \left(\frac{\pi}{4}\right) \times (0.6)^2 = 0.2827 m^2$$

P_1 = 19.62N/cm^2 = 19.62 × 10^4 N/m^2

Dia. at the lower end D_2 = 300mm = 0.3m

$$A_2 = \left(\frac{\pi}{4}\right) \times (0.3)^2 = 0.07065 m^2$$

Rate of flow Q = 50 Lts/sec = $\frac{50}{1000}$ = 0.05 m^3/sec



Let the datum line is passing through the centre of the lower end.

Then $Z_2 = o$

As slope is 1 in 30 means $Z_1 = \left(\frac{1}{30}\right) \times 100 = \left(\frac{10}{3}\right)$ m

We also know that, $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.05}{0.2827} = 0.177 \text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.05}{0.07065} = 0.707 \text{m/sec}$$

Applying Bernoulli's equation at sections 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$= \frac{19.62\times10^4}{1000\times9.81} + \frac{(0.177)^2}{2\times9.81} + \frac{10}{3} = \frac{P_2}{1000\times9.81} + \frac{(0.707)^2}{2\times9.81} + 0$$

$$= 20 + 0.001596 + 3.334 = \frac{P_2}{9810} + 0.0254$$

$$= 23.335 = \frac{P_2}{9810} + 0.0254$$

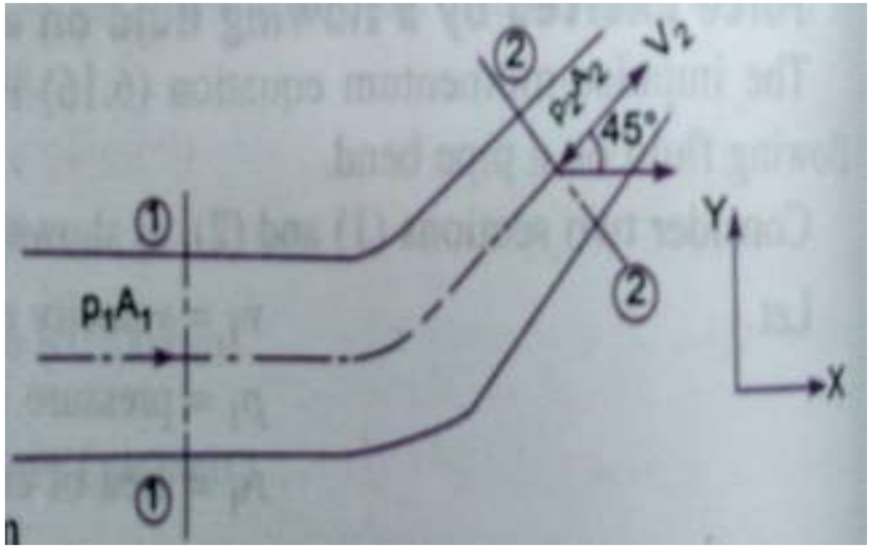
$$= \frac{P_2}{9810} = 23.335 - 0.0254 = 23.31$$

$$= P_2 = 23.31 \times 9810 = 228573 N/m^2$$

P_2 = 22.857N/cm^2

PROBLEM 2

- A 45° reducing bend is connected to a pipe line, the diameters at inlet and out let of the bend being 600mm and 300mm respectively. Find the force exerted by the water on the bend, if the intensity of pressure at the inlet to the bend is 8.829N/cm^2 and rate of flow of water is 600 Lts/sec.



<u>Sol: </u>Given data

Angle of bend θ = 45°

Dia. at inlet D_1 = 600mm = 0.6m

$$A_1 = \left(\frac{\pi}{4}\right) \times (0.6)^2 = 0.2827\ m^2$$

Dia. at outlet D_2 = 300mm = 0.3m

$$A_2 = \left(\frac{\pi}{4}\right) \times (0.3)^2 = 0.07065\ m^2$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$, then $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$

$$= \frac{8.829\times10^4}{1000\times9.81} + \frac{(2.122^2)}{2\times9.81} = \frac{P_2}{1000\times9.81} = \frac{(8.488^2)}{2\times9.81}$$

$$= 9 + 0.2295 = \frac{P_2}{9810} + 3.672$$

$$= \frac{P_2}{9810} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$P_2 = 5.5575 \times 9810 = 5.45 \times 10^4$ N/m^2

Force exerted on the bend in X and Y – directions

$F_x = \rho\, Q\, (V_1 - V_2 \text{ Cos } \theta) + P_1 A_1 - P_2 A_2 \text{ Cos } \theta$

$= 1000 \times 0.6\ (2.122 - 8.488 \text{ Cos}45°) + 8.829 \times 10^4 \times 0.2827 - 5.45 \times 10^4 \times 0.07065 \times \text{Cos}45°$

$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2 = 19911.4\text{N}$

$F_x = 19911.4$ N

$F_y = \rho\, Q\, (-V_2 \text{ Sin } \theta) - P_2 A_2 \text{ Sin } \theta$

$= 1000 \times 0.6\ (-8.488 \text{ Sin}45°) - 5.45 \times 10^4 \times 0.07068 \text{ Sin}45°$

$= -3601.1 - 2721.1 = -6322.2\text{N}$

(- ve sign means Fy is acting in the down ward direction)

$F_y = -6322.2$N

Therefore the Resultant Force F_R = $F_x{}^2 + F_y{}^2$ = $(19911.4)^2$+ $(-6322.2)^2$ = 20890.9N

F_R = 20890.9 N

The angle made by resultant force with X - axis is Tan θ = $\frac{F_x}{F_y}$

= (6322.2/19911.4) = 0.3175

θ= $\tan^{-1}$0.3175= $17^0 36'$

APPLICATIONS

- For sizing of pumps: Volute in the casing of centrifugal pumps converts velocity of fluid into pressure energy by increasing area of flow. The conversion of kinetic energy into pressure is according to Bernoulli's equation.
- Carburetor works on principle of Bernoulli's principle: the faster the air moves, the lower its static pressure and higher its dynamic pressure.
- Application of the Momentum Equation

➢ Force due to the flow of fluid round a pipe bend.

➢ Force on a nozzle at the outlet of a pipe.

➢ Impact of a jet on a plane surface.

ASSIGNMENT QUESTIONS

- A pipe line 300m long has a slope of 1 in 100 and tapers from 1.2m diameter at the high end to 0.6m at the low end. The discharge through the pipe is 5.4m^3/min. If the pressure at the high end is 70kPa, find the pressure at the lower end. Neglect losses.
- A pipe 1 of 450mm in diameter branches into two pipes (2&3) of diameter 300mm and 200mm. If average velocity in 450mm diameter pipe is 3/s. Find (i) Discharge through 450mm diameter pipe. (ii) Velocity in 200mm diameter pipe if the velocity in 300mm pipe is 2.5m/s.
- A pipe line ABC 200m long is laid on an upward slope 1 in 40. The length of the portion AB is 100m and its diameter is 100mm. At B the pipe section suddenly enlarges to 200mm diameter and remains so for the remainder of its length BC, 100m. A flow of 0.0m^3/s is pumped into the pipe at its lower end A and is discharged at the upper end C into closed tank. The pressure at the supply end is 200kN/m^2. What is the pressure at C?

- A 45^0 reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 40cm and 20cm. Find the force exerted by water on the bend if the intensity of pressure at inlet of bend is 21.58N/cm^2. The rate of flow of water is 500litres/s.
- Derive Bernoulli's equation from Euler's equation of motion. What are the assumptions made in deriving Bernoulli's theorem?

UNIT – III

Boundary Layer Concept:

- Definition- thickness.
- Characterization along thin plate.
- Laminar and turbulent boundary layers (No Derivation).

Closed Conduit Flow:

- Reynold’s experiment, Darcy Weisbach equation.
- Major & minor losses, pipes in series & parallel.
- Total energy line and hydraulic gradient line.
- Measurement of flow- Pitot tube, Venturimeter & Orificemeter.

COURSE OUTLINE

UNIT -3

LECTURE	LECTURE TOPIC	KEY ELEMENTS	Learning objectives
1	Introduction to boundary layer concept	Characterization. Laminar & Turbulent	Understanding concepts (B2)
2	Closed conduit flow	Reynolds experiment	Understanding type of flow (B2)
3	Darcy Weisbach equation	Major losses Friction factor	Apply law of conservation of mass (B3)
4	Minor losses	Pipes in series & parallel	Analyze different types of losses in pipe flow (B4)
5	Problems on major & minor losses		
6	Total Energy Line and Hydraulic Gradient Line	HGL & TEL	Understanding concepts (B2)
7	Measurement of flow	Pitot tube Venturimeter	Evaluate flow through venturimeter (B5)
8	Measurement of flow	Orificemeter	Evaluate flow through orificemeter (B5)
9	Problems on Venturimeter & orificemeter		

- The qualitative picture of the boundary-layer growth over a flat plate is shown in Fig.
- A laminar boundary layer is initiated at the leading edge of the plate for a short distance and extends to downstream.
- The transition occurs over a region, after certain length in the downstream followed by fully turbulent boundary layers.
- For common calculation purposes, the transition is usually considered to occur at a distance where the Reynolds number is about 500,000. With air at standard conditions, moving at a velocity of 30m/s, the transition is expected to occur at a distance of about 250mm.

- A typical boundary layer flow is characterized by certain parameters as given below
  - Boundary thickness
  - Free stream flow (no viscosity)
  - Concepts of displacement thickness

BOUNDARY LAYER THICKNESS

- It is known that no-slip conditions have to be satisfied at the solid surface: the fluid must attain the zero velocity at the wall.
- Subsequently, above the wall, the effect of viscosity tends to reduce and the fluid within this layer will try to approach the free stream velocity.
- Thus, there is a velocity gradient that develops within the fluid layers inside the small regions near to solid surface.

- The *boundary layer thickness* is defined as the distance from the surface to a point where the velocity is reaches 99% of the free stream velocity.

- Thus, the velocity profile merges smoothly and asymptotically into the free stream as shown in Fig. 2.

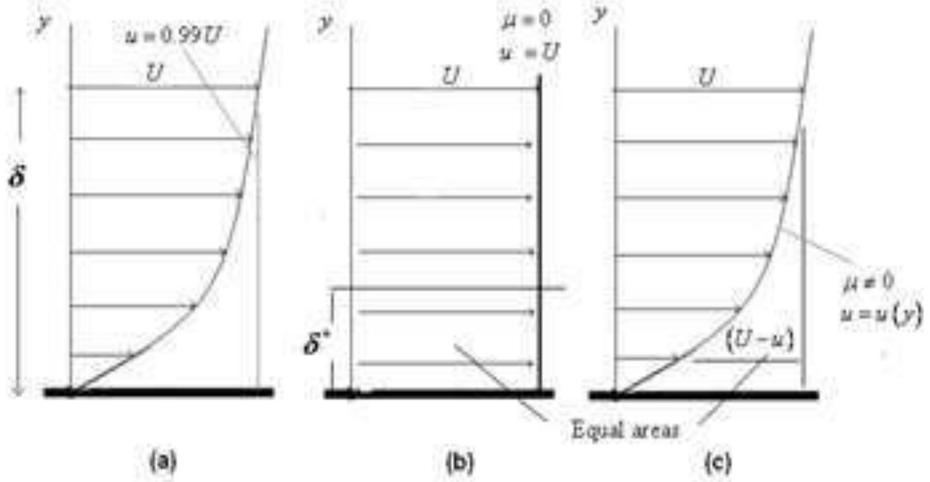


Fig. 5.7.3: (a) Boundary layer thickness; (b) Free stream flow (no viscosity); (c) Concepts of displacement thickness.

- It consists of a constant head tank filled with water, a small tank containing dye, a horizontal glass tube provided with a bell- mouthed entrance a and regulating valve.
- The water was made to flow from the tank through the glass tube in to the atmosphere and the velocity if flow was varied by adjusting the regulating valve.
- The liquid dye having the same specific weight as that of water was introduced in to the flow at the bell – mouth through a small tube.
- From the experiments it was disclosed that when the velocity of flow was low, the dye remained in the form of a straight line and stable filament passing through the glass tube so steady that it scarcely seemed to be in motion with increase in the velocity of flow a critical state was reached at which the filament of dye showed irregularities and began of waver.

- Further increase in the velocity of flow the fluctuations in the filament of dye became more intense and ultimately the dye diffused over the entire cross-section of the tube, due to intermingling of the particles of the flowing fluid
- Reynolds's deduced from his experiments that at low velocities the intermingling of the fluid particles was absent and the fluid particles moved in parallel layers or lamina, sliding past the adjacent lamina but not mixing with them, which is the laminar flow.
- At higher velocities the dye filament diffused through the tube it was apparent that the intermingling of fluid particles was occurring in other words the flow was turbulent.

- The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as Critical Velocity.
- The state of flow in between these types of flow is known as transitional state or flow in transition.
- Reynolds discovered that the occurrence of laminar and turbulent flow was governed by the relative magnitudes of the inertia and the viscous forces.
- At low velocities the viscous forces become predominant and flow is viscous.
- At higher velocities of flow the inertial forces predominance over viscous forces.

REYNOLD'S NUMBER

- Reynolds related the inertia to viscous forces and arrived at a dimension less parameter.

$$R_e \; or \; N_e = \frac{\text{inertia force}}{\text{viscous force}} = \frac{F_i}{F_v}$$

- According to Newton's 2nd law of motion, the inertia force F_i is given by

F_i = mass × acceleration

= ρ × volume × acceleration (ρ = mass density)

= ρ × L^3 × $\frac{L}{T^2}$ = ρ L^2 V^2 ---- (1) (L = Linear dimension)

- Similarly viscous force F_V is given by Newton's 2nd law of velocity as

$F_V = \tau \times$ area　　　　τ = shear stress

$$= \mu \frac{dv}{dy} \times L^2 = \mu VL \quad \text{----------------} \quad (2)$$

V = Average Velocity of flow

μ = Viscosity of fluid

$$R_e \text{ or } N_R = \frac{\rho L^2 V^2}{\mu VL} = \frac{\rho VL}{\mu}$$

- In case of pipes L = D

- In case of flow through pipes

$$R_e = \frac{\rho D V}{\mu} \text{ or } \frac{VD}{\nu}$$

Where μ/ρ = kinematic viscosity of the flowing liquid ν

- The Reynolds number is a very useful parameter in predicting whether the flow is laminar or turbulent.

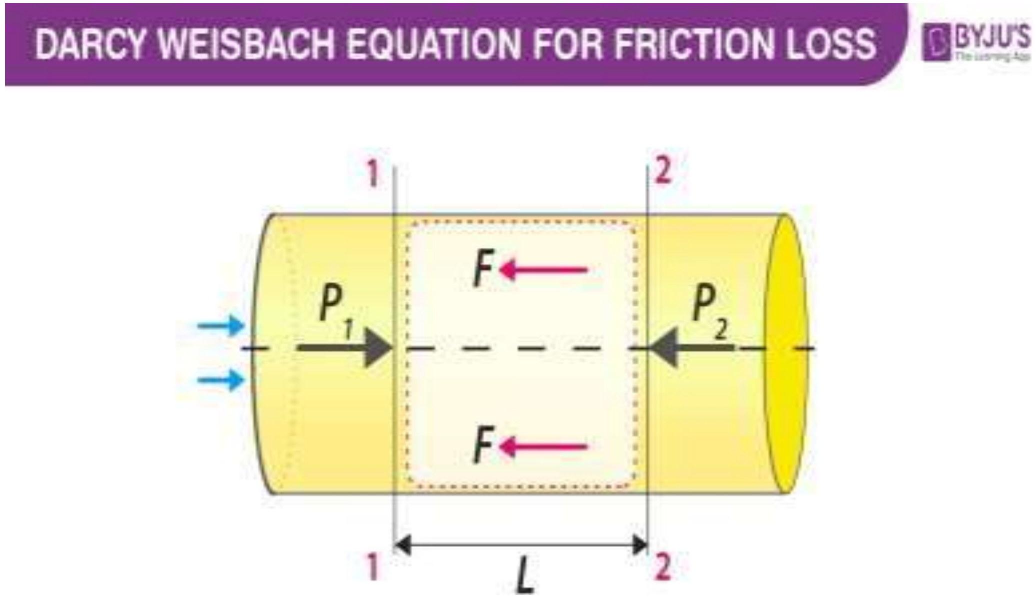
➢ R_e < 2000 viscous / laminar flow

➢ R_e → 2000 to 4000 transient flow

➢ R_e > 4000 Turbulent flow

FRICTIONAL LOSS IN PIPE FLOW – DARCY WEISBACK EQUATION

- When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero.
- The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity.
- This viscous action causes loss of energy, which is known as frictional loss.



DERIVATION

- Consider a uniform horizontal pipe having steady flow. Let 1-1, 2-2 are two sections of pipe.

  Let P_1 = Pressure intensity at section 1-1

  V_1 = Velocity of flow at section 1-1

  L = Length of pipe between section 1-1 and 2-2 d = Diameter of pipe

  f' = Fractional resistance for unit wetted area per a unit velocity

  h_f = Loss of head due to friction

- And P_2,V_2 = are values of pressure intensity and velocity at section 2-2

- Applying Bernoulli's equation between sections 1-1 and 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = Z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$ Or

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \qquad \text{——— (1)}$$

- But h_f is head is lost due to friction and hence the intensity of pressure will be reduced in the direction flow by frictional resistance.

- Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity × Wetted Area × (velocity)^2

  $F_1 = f' \times \pi d L \times V^2$ [∵Wetted area $= \pi d \times L$, Velocity = $V = V_1 = V_2$]

  $F_1 = f' \times p L V^2$ ——————(2) [∵ πd = perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are

Pressure force at section 1-1 = $P_1 \times A$ Pressure (where A = area of pipe)

force at section 2-2 = $P_2 \times A$ Frictional force = F_1

- Resolving all forces in the horizontal direction, we have

  $P_1 A - P_2 A - F_1 = 0$

  $(P_1 - P_2)A = F_1 = f' \times p \times L \times V^2$ from equation – (2)

  $P_1 - P_2 = \dfrac{f^F \times p \times L \times V^2}{A}$ But from equation (1) $P_1 - P_2 = \rho g h_f$

- Equating the value of $P_1 - P_2$, we get

$$\rho g h_f = \frac{f' \times p \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{------(3)}$$

- In the equation (3) $\frac{P}{A} = \frac{\text{Wetted Permiter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d}$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$ Where f is known as co-efficient of friction.

- Equation (4) becomes as $h_f = \frac{4f}{d} \times \frac{LV^2}{2g}$

$$h_f = \frac{4fLV^2}{2gd}$$

- This Equation is known as Darcy - Weisbach equation, commonly used for finding loss of head due to friction in pipes
- Then f is known as a friction factor or co-efficient of friction which is a dimensionless quantity. f is not a constant but, its value depends upon the roughness condition of pipe surface and the Reynolds number of the flow.

MINOR LOSSES IN PIPES

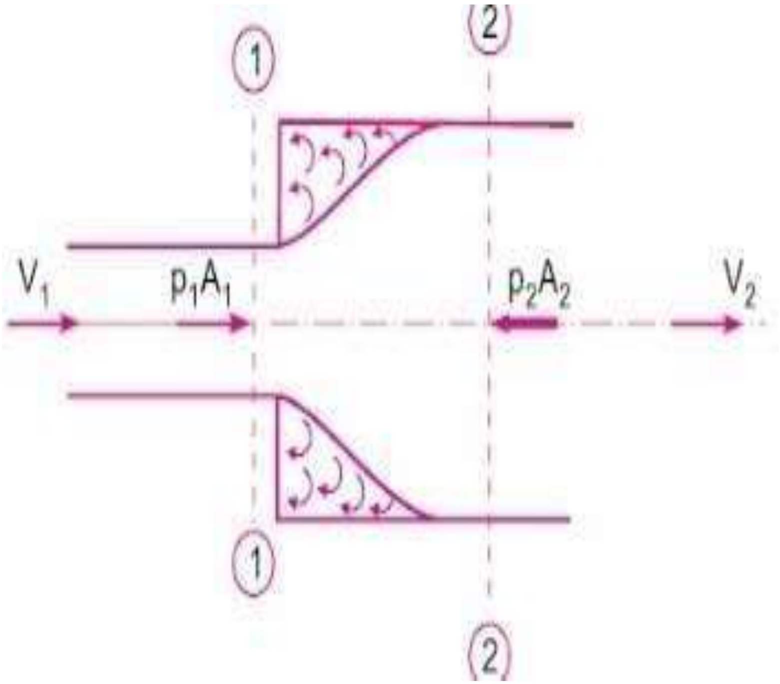
- The loss of energy due to friction is classified as a major loss, because in case of long pipe lines it is much more than the loss of energy incurred by other causes.
- The minor losses of energy are caused on account of the change in the velocity of flowing fluids (either in magnitude or direction).
- In case of long pipes these loses are quite small as compared with the loss of energy due to friction and hence these are termed as ‘’minor losses ‘’
- Which may even be neglected without serious error, however in short pipes these losses may sometimes outweigh the friction loss.
- Some of the losses of energy which may be caused due to the change of velocity are:

MINOR LOSSES IN PIPES

- Loss of energy due to sudden enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$
- Loss of energy due to sudden contraction, $h_c = 0.5 \frac{V_2^2}{2g}$
- Loss of energy at the entrance to a pipe, $h_i = 0.5 \frac{V^2}{2g}$
- Loss of energy at the exit from a pipe, $h_o = \frac{V^2}{2g}$
- Loss of energy due to gradual contraction or enlargement, $h_l = \frac{k(V_1 - V_2)^2}{2g}$
- Loss of energy in the bends, $h_b = \frac{kV^2}{2g}$
- Loss of energy in various pipe fittings, $h_l = \frac{kV^2}{2g}$

LOSS OF HEAD DUE TO SUDDEN ENLARGEMENT

- Consider a liquid flowing through a pipe which has sudden enlargement.
- Consider two sections 1-1 and 2-2 before and after enlargement.
- Due to sudden change of diameter of the pipe from D_1 to D_2.
- The liquid flowing from the smaller pipe is not able to fallow the abrupt change of the boundary.
- Thus the flow separates from the boundary and turbulent eddies are formed.
- The loss of head takes place due to the formation of these eddies



- Let $p' =$ Pressure intensity of the liquid eddies on the area ($A_2 - A_1$)

  h_e = loss of head due to the sudden enlargement.

- Applying Bernoulli's equation at section 1-1 and 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Loss of head due to sudden enlargement}$$

- But, $z_1 = z_2$ as pipe is horizontal

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\text{Or } h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad (1)$$

- The force acting on the liquid in the control volume in the direction of flow

$$F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2$$

- But experimentally it is found that $p' = p_1$

$$F_x = p_1A_1 + p_1(A_2 - A_1) - p_2A_2$$

$$= p_1A_2 - p_2A_2$$

$$= (p_1 - p_2)A_2 \quad \text{———————— (2)}$$

- Momentum of liquid/ second at section 1-1 = mass × velocity

$$= \rho A_1V_1 \times V_1$$

$$= \rho A_1{V_1}^2$$

- Momentum of liquid/ second at section 2-2 $\rho A_2V_2 \times V_2 = \rho A_2{V_2}^2$
- Change of momentum/second $= \rho A_2{V_2}^2 - \rho A_1{V_1}^2$ ——— (3)

- But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

Or $$A_1 = \frac{A_2V_2}{V_1}$$

∴ Change of momentum/sec $= \rho A_2 V_2^2 - \rho \times \frac{A_2V_2}{V_1} \times V_1^2$

$$= \rho A_2V_2^2 - \rho A_1V_1V_2$$

$$= \rho A_2 \left(V_2^2 - V_1V_2\right) \qquad (4)$$

- Now the net force acting on the control volume in the direction of flow must be equal to rate of change of momentum per second. Hence equating equation (2) and equation (4)

$$\left(p_1 - p_2\right) A_2 = \rho A_2 \left(V_2^2 - V_1V_2\right)$$

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1V_2$$

- Dividing both sides by 'g' we have

$$\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

Or $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$

- Substituting in equation (1)

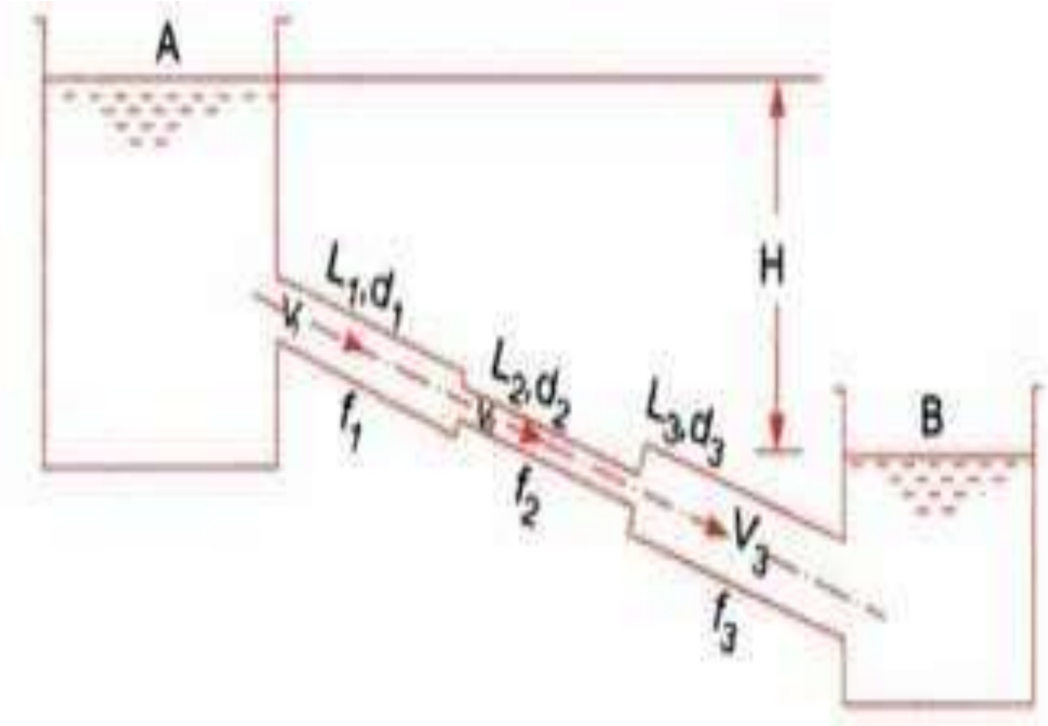
$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = \frac{2V_2^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_1^2 + V_2^2 - 2V_1 V_2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

PIPES IN SERIES

- If a pipe line connecting two reservoirs is made up of several pipes of different diameters d_1, d_2, d_3, etc. and lengths L_1, L_2, L_3 etc. all connected in series (i.e. end to end), then the difference in the liquid surface levels is equal to the sum of the head losses in all the sections.
- Further the discharge through each pipe will be same.



$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{0.5V_3^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3}$$

- Also, $Q = \left(\frac{\pi \times d_1^2}{4}\right) \times V_1 = \left(\frac{\pi \times d_2^2}{4}\right) \times V_2 = \left(\frac{\pi \times d_3^2}{4}\right) \times V_3$
- However if the minor losses are neglected as compared with the loss of head due to friction in each pipe, then

$$H = \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

- The above equation may be used to solve the problems of pipe lines in series.

- There are two types of problems which may arise for the pipe lines in series. Viz.

➢ Given a discharge Q to determine the head H and

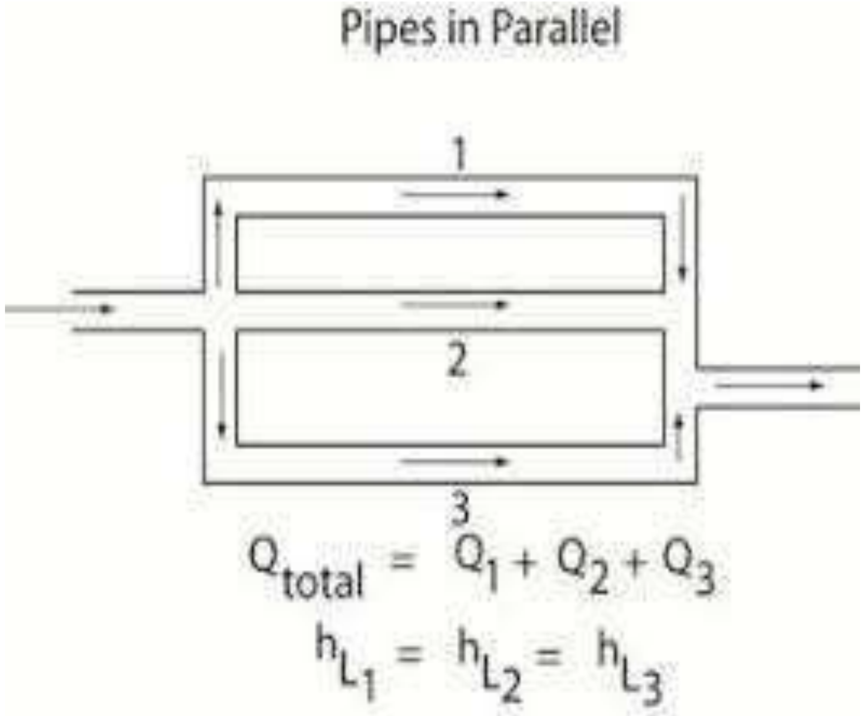
➢ Given H to determine discharge Q.

- If the co-efficient of friction is same for all the pipes i.e. $f_1 = f_2 = f_3$, then

$$H = \frac{4f_1}{2g}\left[\frac{L_1 V_1^2}{d_1} + \frac{L_2 V_2^2}{d_2} + \frac{L_3 V_3^2}{d_3}\right]$$

PIPES IN PARALLEL

- When a main pipeline divides in to two or more parallel pipes, which may again join together downstream and continue as main line, the pipes are said to be in parallel.
- The pipes are connected in parallel in order to increase the discharge passing through the main.
- It is analogous to parallel electric current in which the drop in potential and flow of electric current can be compared to head loss and rate of discharge in a fluid flow respectively.



- The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes.
- Thus, $Q = Q_1 + Q_2$
- The flow of liquid in pipes (1) and (2) takes place under the difference of head between the sections A and B and hence the loss of head between the sections A and B will be the same whether the liquid flows through pipe (1) or pipe (2).
- Thus if D_1, D_2 and L_1, L_2 are the diameters and lengths of the pipes (1) and (2) respectively, then the velocities of flow V_1 and V_2 in the two pipes must be such as to give

$$h_f = \frac{f L_1 V_1^2}{2 g d_1} = \frac{f L_2 V_2^2}{2 g d_2}$$

- Assuming same value of f for each parallel pipe

$$\frac{L_1 V_1^2}{2 g d_1} = \frac{L_2 V_2^2}{2 g d_2}$$

PROBLEM 1

- In a pipe of diameter 350 mm and length 75m water is flowing at a velocity of 2.8 m/s. Find the head lost due to friction by using Darcy Weisbach equation?

Solution. Diameter of the pipe, $D = 350 \text{ mm} = 0{\cdot}35 \text{ m}$

Length of the pipe, $L = 75 \text{ m}$

Velocity of flow, $V = 2{\cdot}8 \text{ m/s}$

Chezy's constant, $C = 55$

Kinematic viscocity of water, $\nu = 0{\cdot}012 \text{ stoke} = 0{\cdot}012 \times 10^{-4} \text{ m}^2/\text{s}$.

Head lost due to friction, h_f :

(*i*) Darcy-Weisbach fomula :

Darcy-Weisbach formula is given by:

$$h_f = \frac{4 fLV^2}{D \times 2g}$$

where, f = coefficient of friction (a function of Reynolds number, Re)

$$Re = \frac{V \times D}{\nu} = \frac{2{\cdot}8 \times 0{\cdot}35}{0{\cdot}012 \times 10^{-4}} = 8{\cdot}167 \times 10^5$$

$$\therefore \quad f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(8.167 \times 10^5)^{1/4}} = 0.00263$$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4 \times 0.00263 \times 75 \times (2.8)^2}{0.35 \times 2 \times 9.81} = \mathbf{0.9 \text{ m}} \quad \textbf{(Ans.)}$$

$$\frac{p_1}{w} + \frac{0.566^2}{2 \times 9.81} = \frac{p_2}{w} + \frac{2.26^2}{2 \times 9.81} + 0.0978$$

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} = \frac{2.26^2}{2 \times 9.81} + 0.0978 - \frac{0.566^2}{2 \times 9.81}$$

$$= 0{\cdot}26 + 0{\cdot}0978 - 0{\cdot}016 = 0{\cdot}3418$$

Hence, $p_1 - p_2 = w \times 0{\cdot}3418 = 9{\cdot}81 \times 0{\cdot}3418$

$= \mathbf{3{\cdot}35\ kN/m^2}$ **(Ans.)**

(*ii*) **Neglecting minor losses :**

We know that, $$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g}$$ ...[Eqn. (12·10)]

$$18 = \frac{V_1^2}{2g}\left(\frac{4 \times 0.0075 \times 450}{0·3} + \frac{4 \times 0.0078 \times 255 \times 2.25^2}{0·2} + \frac{4 \times 0.0072 \times 315 \times (0.5625)^2}{0·4}\right)$$

$$= \frac{V_1^2}{2g}(45 + 201.4 + 7.176) = 253.57 \times \frac{V_1^2}{2g}$$

or, $$V_1 = \sqrt{\frac{18 \times 2 \times 9.81}{253.57}} = 1.18 \text{ m}$$

∴ Discharge, $Q = A_1V_1 = (\pi/4) \times 0·3^2 \times 1·18 = \mathbf{0·0834\ m^3/s}$ **(Ans.)**

$$= \frac{1200}{(0.75)^5} + \frac{750}{(0.6)^5} + \frac{600}{(0.45)^5}, \text{ we have:}$$

$$\frac{L}{(0.45)^5} = 5056.8 + 9645 + 32515.4 = 47217.2$$

or, $$L = 47217{\cdot}2 \times (0{\cdot}45)^5 = \mathbf{871{\cdot}3\ m} \quad \textbf{(Ans.)}$$

(*ii*) **Equivalent diameter, *D* :**

Length of the equivalent pipe, $L = 2550$ m (*Given*)

Now, $$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

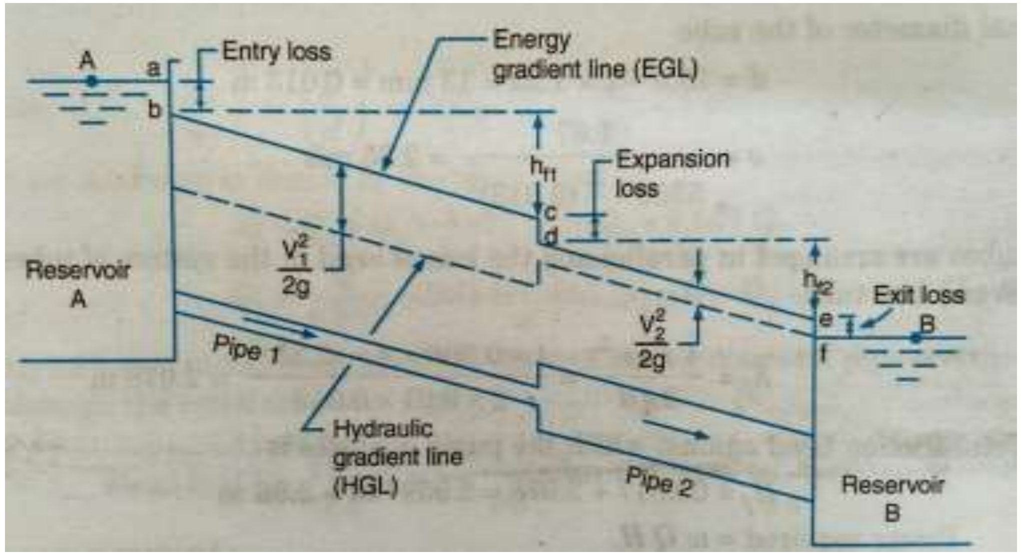
or, $$\frac{2550}{D^5} = \frac{1200}{(0.75)^5} + \frac{750}{(0.6)^5} + \frac{600}{(0.45)^5}$$

$$= 5056.8 + 9645 + 32515.4 = 47217.2$$

or, $$D = \left(\frac{2550}{47217.2}\right)^{1/5} = 0.5578 \text{ m or } \mathbf{557.8\ mm} \textbf{ (Ans.)}$$

HYDRAULIC GRADIENT LINE AND TOTAL ENERGY LINE

- Consider a long pipe line carrying liquid from a reservoir A to reservoir B.
- At several points along the pipeline let piezometers be installed.



- The liquid will rise in the to piezometers certain heights corresponding to the pressure intensity at each section.
- The height of the liquid surface above the axis of the pipe in the piezometer at any section will be equal to the pressure head (p/w) at that section.

- On account of loss of energy due to friction, the pressure head will decrease gradually from section to section of pipe in the direction of flow.
- If the pressure heads at the different sections of the pipe are plotted to scale as vertical ordinates above the axis of the pipe and all these points are joined by a straight line, a sloping line is obtained, which is known as Hydraulic Gradient Line (H.G.L).
- Since at any section of pipe the vertical distance between the pipe axis and Hydraulic gradient line is equal to the pressure head at that section, it is also known as pressure line.
- Moreover if Z is the height of the pipe axis at any section above an arbitrary datum, then the vertical height of the Hydraulic gradient line above the datum at that section of pipe represents the piezometric head equal to (p/w + z)..

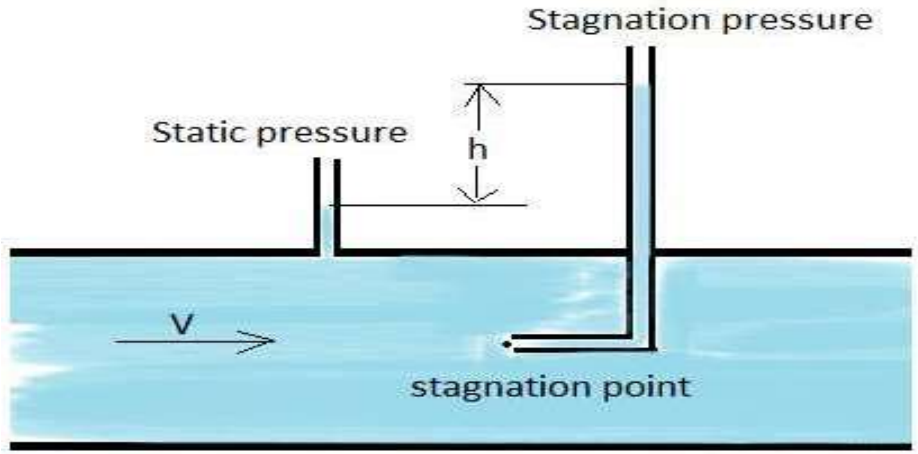
- Sometimes the Hydraulic gradient line is also known as piezometric head line.
- At the entrance section of the pipe for some distance the Hydraulic gradient line is not very well defined.
- This is because as liquid from the reservoir enters the pipe, a sudden drop in pressure head takes place in this portion of pipe.
- Further the exit section of pipe being submerged, the pressure head at this section is equal to the height of the liquid surface in the reservoir B and hence the hydraulic gradient line at the exit section of pipe will meet the liquid surface in the reservoir B.

- At the entrance section of the pipe there occurs some loss of energy called "Entrance loss" equal to $h_L = 0.5\left(\frac{V^2}{2g}\right)$ and hence the energy grade line at this section will lie at a vertical depth equal to $0.5\left(\frac{V^2}{2g}\right)$ below the liquid surface in the reservoir A.

PITOT TUBE

- A Pitot tube is a simple device used for measuring the velocity of flow.
- The basic principle used in this is that if the velocity of flow at a

particular point is zero, reduced to which is known as stagnation point, the pressure there is increased due to conversion of the kinetic energy in to pressure energy and by measuring the increase in pressure energy at this point,the velocity of flow may be determined.



- Simplest form of a pitot tube consists of a glass tube, large enough for capillary effects to be negligible and bent at right angles.
- A single tube of this type is used for measuring the velocity of flow in an open channel.
- The tube is dipped vertically in the flowing stream of fluid with its open end A directed to face the flow and other open end projecting above the fluid surface in the stream.
- The fluid enters the tube and the level of the fluid in the tube exceeds that of the fluid surface in the surrounding stream. This is so because the end A of the tube is a stagnation point, where the fluid is at rest, and the fluid approaching end A divides at this point and passes around tube.

- Since at stagnation point the kinetic energy is converted in to pressure energy, the fluid in the tube rises above the surrounding fluid surface by a height, which corresponds to the velocity of flow of fluid approaching end A of the tube.
- The pressure at the stagnation point is known as stagnation pressure.
- Consider a point 1 slightly upstream of end A and lying along the same horizontal plane in the flowing stream of velocity V.
- Now if the point 1 and A are at a vertical depth of h_O from the free surface of fluid and h is the height of the fluid raised in the pitot tube above the free surface of the liquid.

- Then by applying Bernoulli's equation between the point 1 and A, neglecting loss of energy,

we get $$h_o = \frac{V^2}{2g} = h_o + h$$

- (h_o + h) is the stagnation pressure head at a point A, which consists of static pressure head h_O and dynamic pressure head h.
- Simplifying the expression,

$$\frac{V^2}{2g} = h \quad \text{Or} \quad v = \sqrt{2gh} \qquad \text{\_\_\_\_\_\_(1)\_\_\_\_\_\_}$$

- This equation indicates that the dynamic pressure head h is proportional to the square of the velocity of flow close to end A.
- Thus the velocity of flow at any point in the flowing stream may be determined by dipping the Pitot tube to the required point and measuring the height 'h' of the fluid raised in the tube above the free surface.

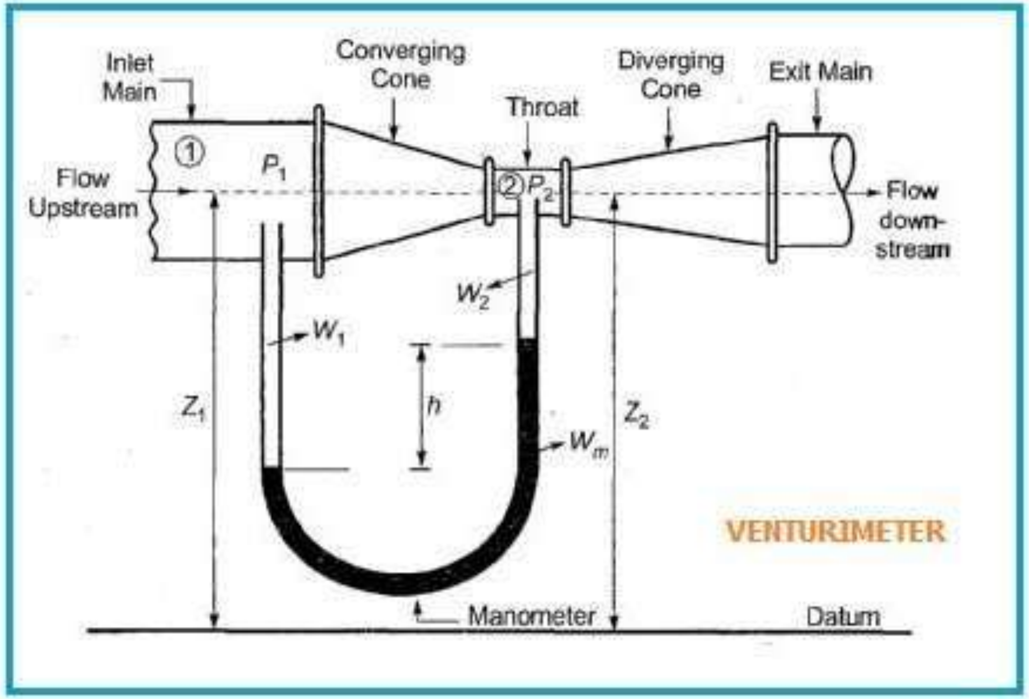
- The velocity of flow given by the above equation (1) is more than actual velocity of flow as no loss of energy is considered in deriving the above equation.
- When the flow is highly turbulent the Pitot tube records a higher value of h, which is higher than the mean velocity of flow.
- In order to take in to account the errors due to the above factors, the actual velocity of flow may be obtained by introducing a co- efficient C or C_V called Pitot tube co-efficient.
- So the actual velocity is given by

$$v = C\sqrt{2gh}$$

(Probable value of C is 0.98)

VENTURIMETER

- A venturimeter is a device used for measuring the rate of flow of fluid through a pipe.
- The basic principle on which venturimeter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of the pressure difference enables the determination of the discharge through the pipe.



- A venture meter consists of (1) an inlet section, followed by a converging cone (2) a cylindrical throat and (3) a gradually divergent cone.
- The inlet section of venture meter is the same diameter as that of the pipe which is followed by a convergent cone.
- The convergent cone is a short pipe, which tapers from the original size of the pipe to that of the throat of the venture meter.
- The throat of the venture meter is a short parallel – sided tube having its cross-sectional area smaller than that of the pipe.
- The divergent cone of the venture meter is a gradually diverging pipe with its cross-sectional area increasing from that of the throat to the original size of the pipe.
- At the inlet section and the throat i.e sections 1 and 2 of the venture meter pressure gauges are provided.

DERIVATION

- Let a_1 and a_2 be the cross-section areas at inlet and throat sections, at which P_1 and P_2 the pressures and velocities V_1 and V_2 respectively.
- Assuming the flowing fluid is incompressible and there is no loss of energy between section 1 and 2 and applying Bernoulli's equation between sections 1 and 2, we get,

$$\frac{P_1}{\omega} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\omega} + \frac{v_2^2}{2g} + z_2$$

Where ω is the specific weight of flowing fluid.

- If the venturi meter is connected in a horizontal pipe, then $Z_1 = Z_2$,

then

$$\frac{P_1}{\omega} + \frac{v_1^2}{2g} = \frac{P_2}{\omega} + \frac{v_2^2}{2g}$$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

- In the above expression $\left(\frac{P_1}{\omega} - \frac{P_2}{\omega}\right)$ *is the pressure difference* between the pressure heads at section 1 and 2 , is known as venture head and is denoted by h

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$Q_{th} = a_1 v_1 = a_2 v_2 \quad ,$$

$$v_1 = \frac{Q_{th}}{a_1} \quad , \quad v_2 = \frac{Q_{th}}{a_2}$$

$$h = \frac{Q_{th}^2}{2g}\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right)$$

$$Q_{th} = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$Q = C_d Q_{th} = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

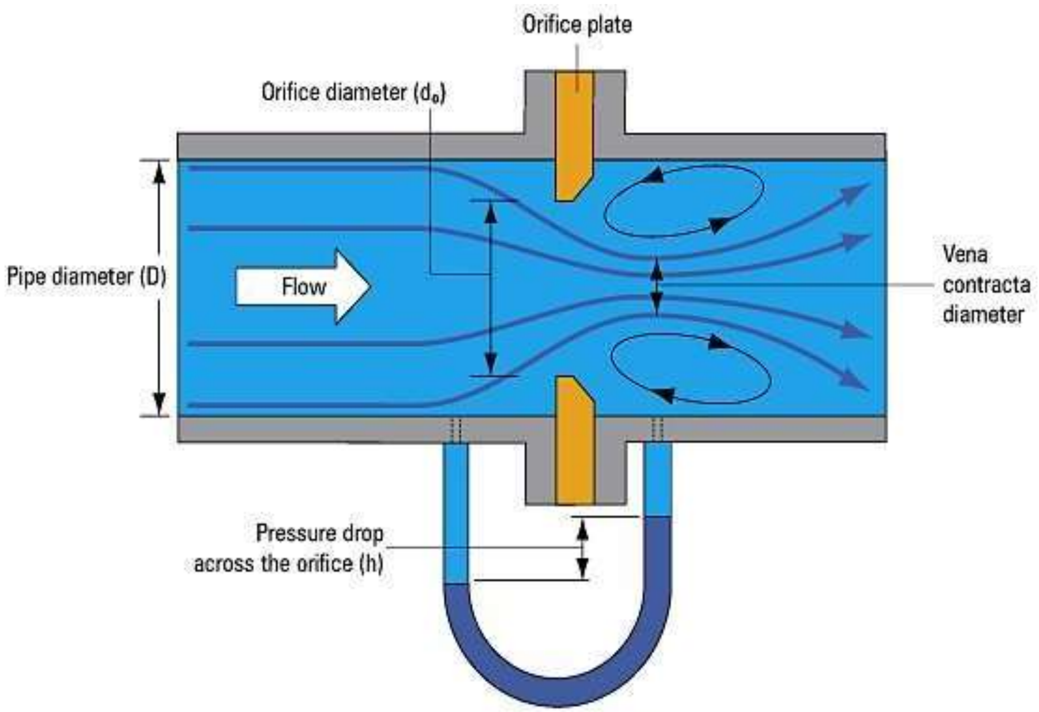
$$= C_d C \sqrt{h} \quad \left(\because C = \frac{a_1 a_2 \sqrt{2g}}{\sqrt{a_1^2 - a_2^2}}\right)$$

$$\boldsymbol{Q_{actual} = C_d C \sqrt{h}}$$

C_d = Co-efficient of discharge < 1

ORIFICEMETER

- An orifice meter is a simple device for measuring the discharge through pipes.
- Orifice meter also works on the same principle as that of venture meter i.e by reducing cross-sectional area of the flow passage, a pressure difference between the two sections is developed and the measurement of the pressure difference enables the determination of the discharge through the pipe.



- Orifice meter is a cheaper arrangement and requires smaller length and can be used where space is limited.

$$Q = Ca_0a_1 \frac{\sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

- This gives the discharge through an orifice meter and is similar to the discharge through venture meter.
- The co-efficient C may be considered as the co-efficient of discharge of an orifice meter.
- The co-efficient of discharge for an orifice meter is smaller than that for a venture meter.
- This is because there are no gradual converging and diverging flow passages as in the case of venture meter, which results in a greater loss of energy and consequent reduction of the co-efficient of discharge for an orifice meter.

PROBLEM 1

- At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

<u>Sol:</u> Given Data

Dia. of smaller pipe D_1= 240mm =0.24m

Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.24)^2$

Dia. of larger pipe D_2 = 480mm = 0.48m

Area $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.48)^2$

Rise of hydraulic gradient i.e. $\left(Z_2 + \frac{P_2}{\rho g}\right) - \left(Z_1 + \frac{P_1}{\rho g}\right)$ = 10mm = $\frac{10}{1000} m = \frac{1}{100} m$

Let the rate of flow = Q

Applying Bernoulli's equation to both sections i.e smaller and larger sections

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$ + Head loss due to enlargement________(1)

But head loss due to enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$ ——— (2)

From continuity equation, we have $A_1 V_1 = A_2 V_2$ $\quad V_1 = \frac{A_2 V_2}{V_1}$

$$V_1 = \frac{\frac{\pi}{4} D_2^2 \; V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.48}{0.24}\right)^2 V_2 = 2^2 \; V_2 = 4V_2$$

Substituting this value in equation (2), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_e and V_1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)$$

But Hydraulic gradient rise = $\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right) = \left(\frac{1}{100}\,m\right)$

$$\frac{6V_2^2}{2g} = \left(\frac{1}{100}\,m\right) \qquad V_2 = \sqrt{\frac{2\times 9.81}{6\times 100}} = 0.1808 = 0.181\,m/sec$$

Discharge, **Q** = $A_2 V_2 = \frac{\pi}{4} D_2^2 V_2$

$$= \frac{\pi}{4}(0.48)^2 \times 0.181 = 0.03275\,m^3/sec$$

Q = 32.75Lts/sec

PROBLEM 2

- A 150mm dia. pipe reduces in dia. abruptly to 100mm dia. If the pipe carries water at 30lts/sec, calculate the pressure loss across the contraction. Take co-efficient of contraction as 0.6

<u>Sol:</u> Given Data

Dia. of larger pipe, D_1 = 150mm = 0.15m

Area of larger pipe, $A_1 = \frac{\pi}{4}(0.15)^2 = 0.01767m^2$

Dia. of smaller pipe, D_2 = 100mm = 0.10m

Area of smaller pipe, $A_2 = \frac{\pi}{4}(0.10)^2 = 0.007854m^2$

Discharge, Q = 30 lts/sec = 0.03m³/sec Co-efficient

of contraction, C_C = 0.6

From continuity equation, we have $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\ m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\ m/sec$$

Applying Bernoulli's equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \quad \_\_\_\_\_\_ \quad (1)$$

But $Z_1 = Z_2$ and h_c the head loss due to contraction is given by the equation

$$h_c = \left(\frac{V_2^2}{2g}\right)\left[\left(\frac{1}{C_c} - 1\right)\right]^2 = \frac{(3.82^2)}{2\times 9.81}\left[\left(\frac{1}{0.6} - 1\right)\right]^2 = 0.33$$

Substituting these values in equation (1), we get

$$\frac{P_1}{\rho g} + \frac{(1.697)^2}{2\times 9.81} = \frac{P_2}{\rho g} + \frac{(3.82)^2}{2\times 9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271\ m\ of\ Water$$

$$P_1 - P_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 = 0.909 \times 10^4\ N/m^2$$

= 0.909 N/cm²

Pressure loss across contraction = **$P_1 - P_2 = 0.909 N/cm^2$**

PROBLEM 3

- A pitot tube is placed in the centre of a 300mm pipe line has one end pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe, if the pressure difference between the two orifices is 60mm of water. Co-efficient of Pitot tube C_V =0.98

Sol: Given Data

Diameter of pipe = 300mm =0.3m

Difference of pressure head h = 60mm of water = 0.06m of water

Mean velocity $\bar{V}$ = 0.80 × central velocity

Central velocity = $C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times 0.06} = 1.063 \frac{m}{sec}$

Mean velocity =0.8 × 1.063 = 0.8504m/sec

Discharge, Q = Area of pipe × Mean velocity = A × $\bar{V}$

= $\frac{\pi}{4}(0.3)^2 \times 0.8504$

Q = 0.06m³/sec

PROBLEM 4

- Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100mm. Co- efficient of pitot tube C = 0.98 and sp.gr.of oil =0.8.

<u>Sol:</u> Given Data

Difference of mercury level x = 100mm = 0.1m

Sp.gr. of oil = 0.8, C_V = 0.98

Difference of pressure head $h = x\left[\left(\frac{S_m}{S}\right) - 1\right] = 0.1\left[\left(\frac{13.6}{0.8} - 1\right)\right]$

= 1.6 m of oil

Velocity of flow = $C_V\sqrt{2 \times g \times h} = 0.98\sqrt{2 \times 9.81 \times 1.6}$

V = 5.49m/sec

PROBLEM 5

- A pitot tube is used to measure the velocity of water in a pipe. The stag nature pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow. Co-efficient of pitot tube is 0.98.

Sol: Given Data

Stag nature pressure head, h_S =6m Static pressure head,

h_t =5m

$h = h_S - h_t = 6 - 5 = 1m$

Velocity of flow, $V = C_V \sqrt{2 \times g \times h}$

$= 0.98\sqrt{2 \times 9.81 \times 1}$

V = 4.34 m/sec

PROBLEM 6

- A pitot tube is inserted in a pipe of 300mm diameter. The static pressure in the pipe is 100mm of mercury (Vacuum). The stagnation pressure at the centre of the pipe is recorded by Pitot tube is 0.981N/cm^2. Calculate the rate of flow of water through the pipe. The mean velocity of flow is 0.85 times the central velocity C_V = 0.98
- Sol: Given Data

Diameter of pipe d = 0.3m

Area of pipe a = $\frac{\pi}{4}(0.3)^2$ =0.07068m^2

Static pressure head = 100mm of mercury = $\frac{100}{1000} \times 13.6$ =1.36m of water

Stagnation pressure head = $\frac{0.981 \times 10^4}{1000 \times 9.81} = 1\ m$

h= Stagnation pressure head - static pressure head = 1 - (- 1.36) = 2.36m

Velocity at centre = $C_V \sqrt{2gh} = 0.98 \sqrt{2 \times 9.81 \times 2.36}$ = 6.668m/sec

Mean velocity $\bar{V}$ = 0.85× 6.668 = 5.6678m/sec

Rate of flow of water = $\bar{V}$ × Area of pipe

= 5.6678 × 0.07068

Q = 0.4006m³/sec

APPLICATIONS

- The Venturimeter which has long been used in hydraulics is here applied to the measurement of volume flow of blood through vessels.
- In fluid flow, **friction loss** (or skin friction) is the loss of pressure or "head" that occurs in pipe or duct flow due to the effect of the fluid's viscosity near the surface of the pipe or duct.
- In mechanical systems such as internal combustion engines, the term refers to the power lost in overcoming the friction between two moving surfaces, a different phenomenon.
- Rigorous calculation of the pressure loss for flowing gases, based on as properties, flow, and piping configuration (pipe length, fittings, and valves). Results can be printed out or "cut and paste" into other applications.

ASSIGNMENT QUESTIONS

- Two reservoirs are connected by a pipe line consisting of two pipes, one of 15cm diameter and length 6m and the other diameter 22.5cm and 16m length. If the difference of water levels in the two reservoirs is 6m. Calculate the discharge and draw the energy gradient line. Take f=0.004
- A venturimeter has an area ratio of 9 to 1, the larger diameter being 300 mm. During the flow, the recorded pressure head in the larger section is 6.5m and that at the throat 4.25m. Take C d = 0.99, compute the discharge through the meter.
- A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300litres/sec. Find the head lost due to friction for a length of 50m of the pipe.
- Derive Darcy Weisbach equation.

- The rate of flow of water through a horizontal pipe is $0.254 m^3/s$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm. The pressure intensity in the smaller pipe is $11.772 N/cm^2$. Determine i) Loss of head due to sudden contraction ii) pressure intensity in the larger pipe iii) power lost due to friction
- Explain boundary layer concept in detail.

UNIT – IV (SYLLABUS)

Basics of Turbo Machinery:

- Hydrodynamic force of jets on stationary
- Jet on moving flat,
- jet on inclined, and curved vanes.

Hydraulic Turbines:

- Classification of turbines
- impulse and reaction turbines,
- Pelton wheel turbine,
- Francis turbine and Kaplan turbine
- draft tube.
- Performance of hydraulic turbines: characteristic curves, cavitation, surge tank, water hammer

COURSE OUTLINE

UNIT -4

LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	Introduction to hydrodynamic force on jets	Derivation of force on Stationary- Flat, inclined & curved plate	Evaluate force exerted (B5)
2	Hydrodynamic force on jets	Derivation of force on Moving- Flat, inclined & curved plates	Evaluate force exerted (B5)
3	Example Problems on force on jets for stationary & moving plates		
4	Classification of turbines	Impulse & Reaction turbines	Understanding the types of turbines (B2)
5	Pelton wheel Turbine	Working principle, derivation of work done & η	Understand working principle (B2) Evaluate the efficiency (B5)
6	Francis & Kaplan turbine	Working principle, derivation of work done & η	Understand working principle (B2) Evaluate the efficiency (B5)
7	Hydraulic Design- Draft Tube theory	Functions & η	Evaluate the efficiency (B5)
8	Example Problems on turbines		
9	Geometric similarity	Derivation of Unit & specific quantities	Evaluate the unit quantities (B5)

HYDRO-DYNAMIC FORCE OF JETS:

- The liquid comes out in the form of a jet from the outlet of a nozzle, the liquid is flowing under pressure.
- If some plate, which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate.
- This force is obtained by Newton’s second law of motion or from Impulse – Momentum equation.

$$F = ma$$

The following cases of impact of jet i.e. the force exerted by the jet on a plate will be considered.

1) Force exerted by the jet on a stationary plate, when

   a) Plate is vertical to the jet.
   b) Plate is inclined to the jet and
   c) Plate is curved

2) Force exerted by the jet on a moving plate, when

   a) Plate is vertical to the jet.
   b) Plate is inclined to the jet.
   c) Plate is curved.

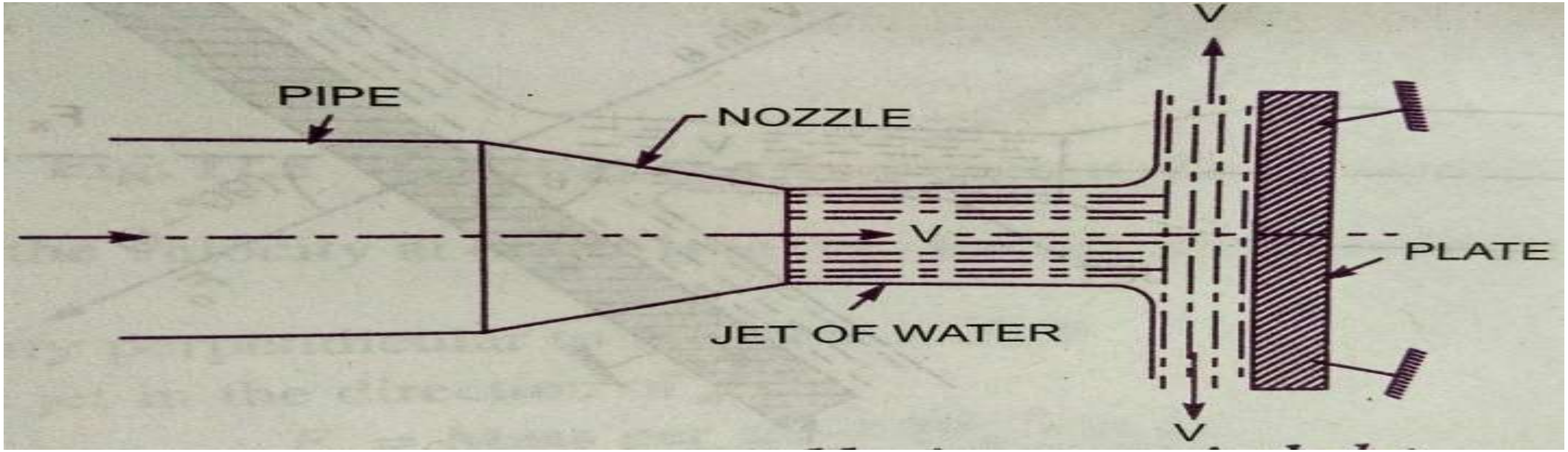
JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate. Let V= Velocity of jet.

d = Diameter of jet.

a = Area of cross-section of jet. = d^2

The jet of water after striking the plate will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will be deflected through 90.



- The force exerted by the jet on the plate in the direction of jet,

F_x = Rate of change of momentum in the direction of force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$$

$$= \frac{Mass}{Time}(\text{Initial velocity} - \text{Final velocity})$$

$$= (Mass/sec) \times (\text{Velocity of jet before striking} - \text{Final velocity of jet after striking})$$

$$= \rho \, a \, V \, (V - 0)$$

$$F_x = \rho a V^2$$

- For deriving the above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated, then final velocity minus initial velocity is to be taken

JET ON A STATIONARY INCLINED FLAT PLATE

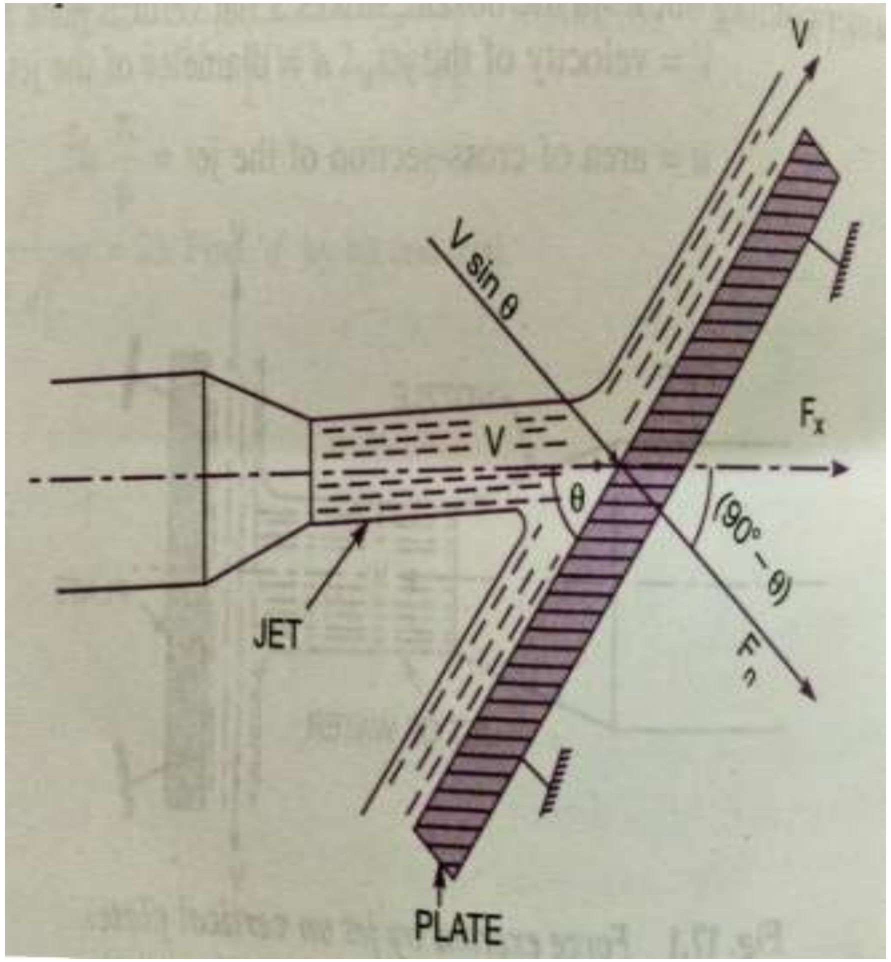
Let a jet of water coming out from the nozzle, strikes an inclined flat plate.

V = Velocity of jet in the direction of X

= Angle between the jet and plate.

a = Area of cross-section of jet.

Mass of water per second striking the plate = p a v



Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n.

Then

F_n = Mass of jet striking per second*(Initial velocity of jet before striking in the direction of n – Final velocity of jet after striking in the direction of n)

$= \rho a V(V\sin\theta - 0) = \rho a V^2 \sin\theta$ ———— (1)

This force can be resolved in two components, one in the direction of the jet and the other perpendicular to the direction of flow.

Then we have F_x = Component of F_n in the direction of flow.

$$F_x = F_n \cos(90 - \theta) = F_n \sin\theta - \rho a V^2 \sin\theta \times \sin\theta$$

$$F_x = \rho a V^2 sin^2\theta \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

And F_y = Component of F_n in the direction perpendicular to the flow.

$$F_y = F_n \sin(90 - \theta) = F_n \cos\theta = \rho a v^2 \sin\theta \times \cos\theta$$

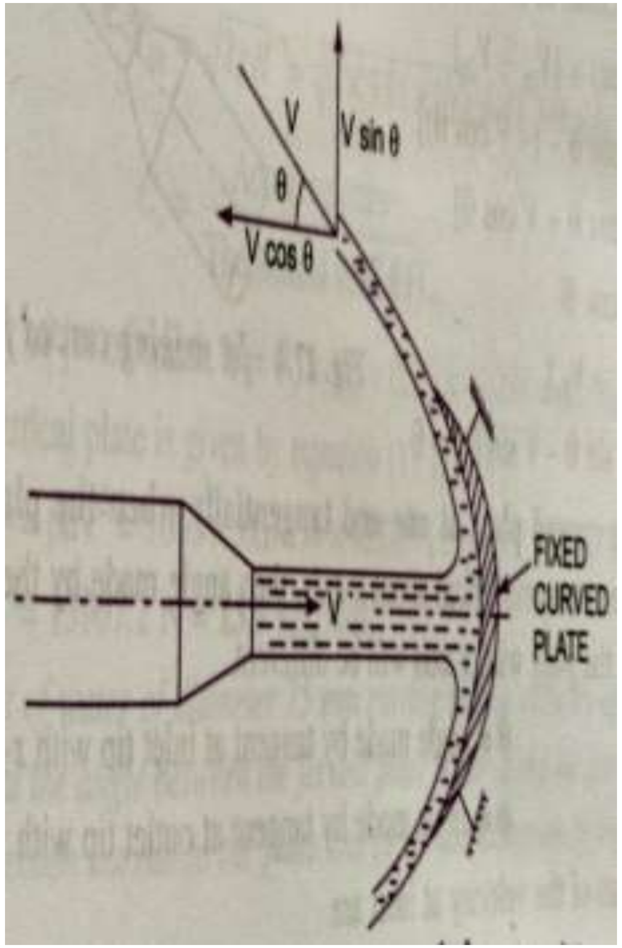
$$F_y = \rho a v^2 sin\theta cos\theta \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

JET STRIKES THE CURVED PLATE AT THE CENTRE

Component of velocity in the direction of jet $= -V\cos\theta$

(-ve sign is taken as the velocity at out let is in the opposite direction of the jet of water coming out from nozzle.)

Component of velocity perpendicular to the jet $= V\sin\theta$



Force exerted by the jet in the direction of the jet

$$F_x = \text{Mass per sec}\,(V_{1x} - V_{2x})$$

Where V_{1x} = Initial velocity in the direction of jet $= V$

V_{2x} = Final velocity in the direction of jet $= -V\cos\theta$

$$F_x = \rho aV[V - (-V\cos\theta)] = \rho aV[V + V\cos\theta] = \rho aV^2(1+\cos\theta) \text{ ——— } (1)$$

Similarly $F_y = \text{Mass per second}\,(V_{1y} - V_{2y})$

Where V_{1y} = Initial velocity in the direction of y $= 0$

V_{2y} = Final velocity in the direction of y $= V\sin\theta$

$$F_y = \rho aV[0 - V\sin\theta] = -\rho aV^2\sin\theta \text{ ——— } (2)$$

JET STRIKES THE CURVED PLATE AT ONE END TANGENTIALLY WHEN THE PLATE IS SYMMETRICAL

Let V = Velocity of jet of water.

θ = Angle made by the jet with x-axis at the inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the out let tip of the curved plate will be equal to V. The force exerted by the jet of water in the direction of x

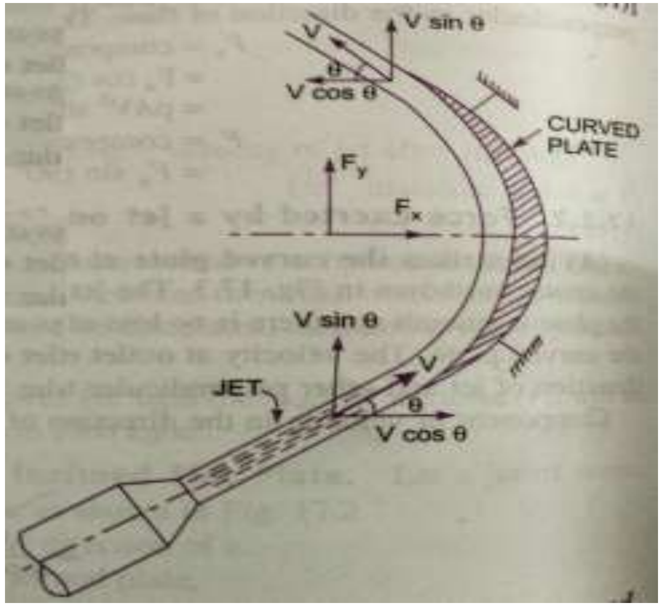
and y are

$$F_x = (\text{mass/sec}) \times (V_{1x} - V_{2x})$$

$$= \rho a V[V \cos\theta - (-V\cos\theta)]$$

$$= 2\rho a V^2 \cos\theta$$

$$F_y = \rho a V(V_{1y} - V_{2y}) = \rho a V[V\sin\theta - V\sin\theta] = 0$$



JET STRIKES THE CURVED PLATE AT ONE END TANGENTIALLY WHEN THE PLATE IS UN-SYMMETRICAL

When the curved plate is unsymmetrical about *x*- axis, then the angles made by tangents drawn at inlet and outlet tips of the plate with *x*- axis will be different.

Let θ = Angle made by tangent at the inlet tip with *x*-axis.

$\emptyset$ = Angle made by tangent at the outlet tip with *x*-axis The two components of velocity

at inlet are

$V_{1x} = V\cos\theta$ and $V_{1y} = V\sin\theta$ The two components of velocity

at outlet are

$V_{2x} = -V\cos\emptyset$ and $V_{2y} = V\sin\emptyset$

The forces exerted by the jet of water in the directions of *x* and *y* are:

$$F_x = \rho aV(V_{1x} - V_{2x}) = \rho aV(V\cos\theta + V\cos\emptyset)$$

$$= \rho a V^2(\cos\theta + \cos\emptyset)$$

$$F_y = \rho aV(V_{1y} - V_{2y}) = \rho aV[V\sin\theta - V\sin\emptyset] = \boldsymbol{\rho}\, a\, V^2(\sin\boldsymbol{\theta} - \sin\emptyset)$$

FORCE ON FLAT VERTICAL PLATE MOVING IN THE DIRECTION OF JET

Let V = Velocity of jet.

a = Area of cross-section of jet.

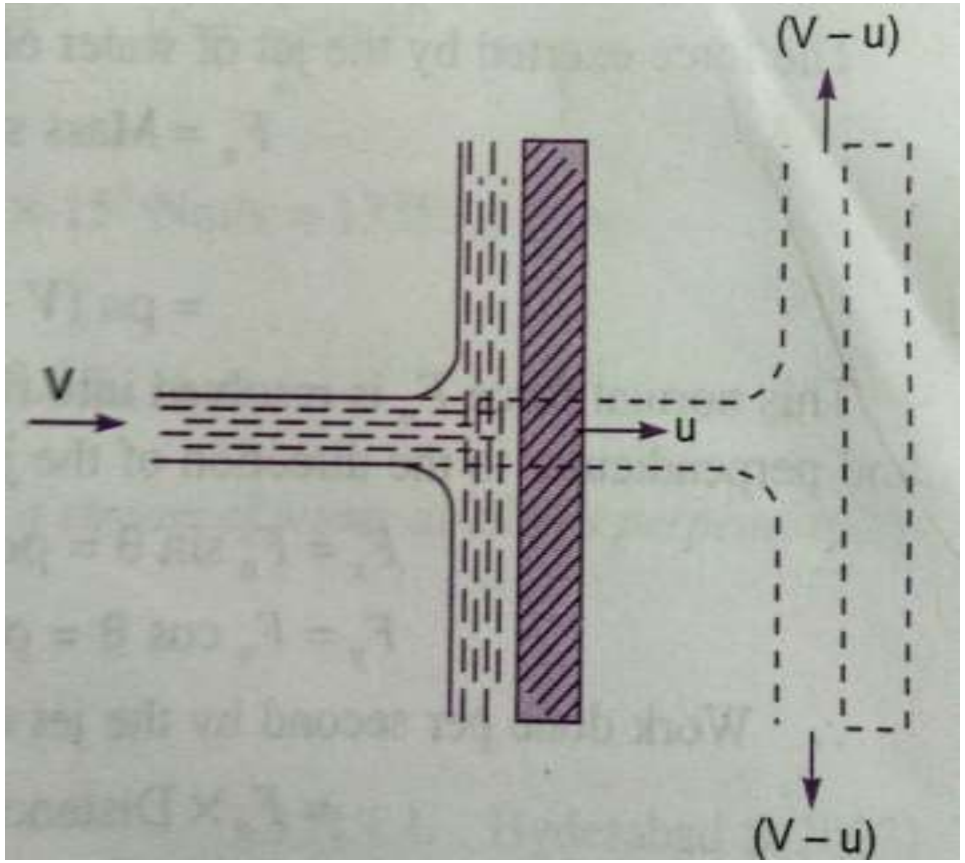
u = Velocity of flat plate.

- In this case, the jet does not strike the plate with a velocity v, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus velocity of the plate.

Hence relative velocity of the jet with respect to plate = $V - u$

Mass of water striking the plate per second= $= a(V - u)$

Force exerted by the jet on the moving in the direction of the plate



F_x = mass of water striking per second velocity with which water strikes - velocity) (Initial Final

$= a(V - u)[(V - u) - 0] = a(V - u)^2$

Since final velocity in the direction of jet is zero.

In this, case the work will be done by the jet on the plate, as the plate is moving. For stationary plates, the work done is zero.

The work done per second by the jet on the plate =

$$= F_x\, u \quad = a(v - u)^2\, u \text{ ------------ } (2)$$

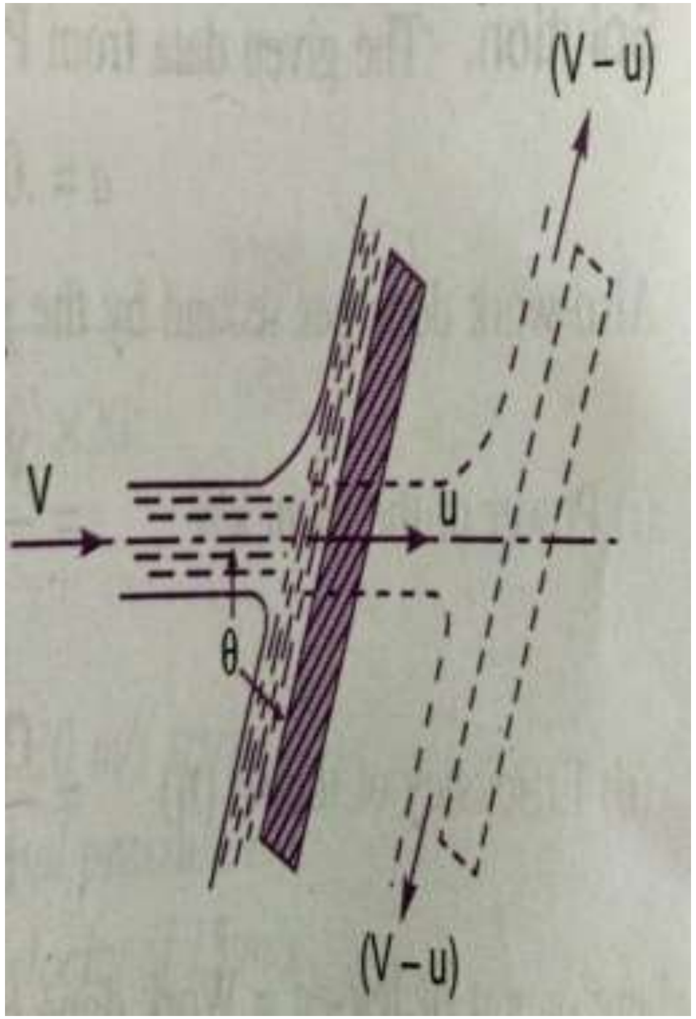
In the above equation (2), if the value of for water is taken in S.I units (i.e.1000kg/m^3) the work done will be in N m/s. The term is equal to Watt (W).

FORCE ON INCLINED PLATE MOVING IN THE DIRECTION OF JET

Let V = Absolute velocity of water.

u = Velocity of plate in the direction of jet.

a = Cross-sectional area of jet



θ = Angle between jet and plate.

Relative velocity of jet of water = $V - u$

The velocity with which jet strikes = $V - u$

Mass of water striking per second = $\rho a (V - u)$

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V - u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

F_n = Mass striking per sec x (Initial velocity in the normal direction with which jet strikes – final velocity)

$= \rho a (V - u) [(V - u) \sin\theta - 0]$

$= \boldsymbol{\rho} a (V - u)^2 \sin\boldsymbol{\theta}$

This normal force F_n is resolved in to two components, namely F_x and F_y in the direction of jet and perpendicular to the direction of jet respectively.

$$F_x = F_n \sin\theta = \rho a (V - u)^2 \sin^2\theta$$

$$F_y = F_n \cos\theta = \rho a (V - u)^2 \sin\theta \cos\theta$$

Work done per second by the jet on the plate

$= F_x \times$ Distance per second in the direction of x

$= F_x \times u = \rho a (V - u)^2 \sin^2\theta \times u$

$= \boldsymbol{\rho} a (V - u)^2 u \sin^2\boldsymbol{\theta}$ N m/sec

FORCE ON THE CURVED PLATE WHEN THE PLATE IS MOVING IN THE DIRECTION OF JET

Let V=absolute velocity of jet.

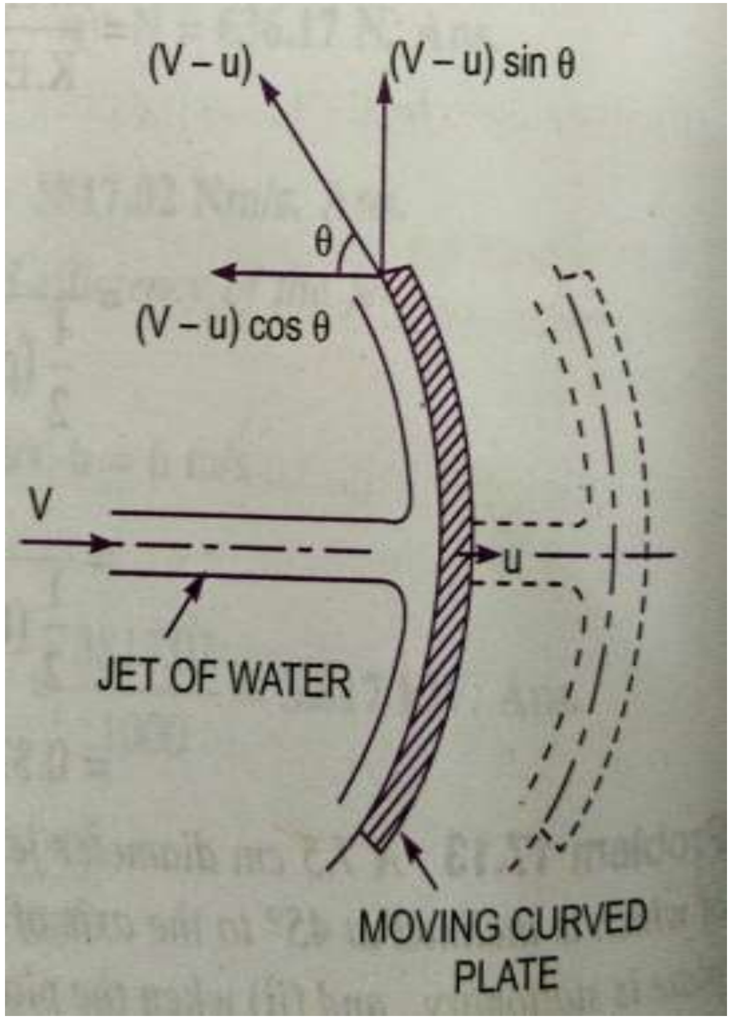
a=area of jet.

u=Velocity of plate in the direction of jet.

Relative velocity of jet of water or the velocity with which jet strikes the curved plate=V−u

If the plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the

jet will be leaving the curved vane=(V−u)



This velocity can be resolved in to two components, one in the direction of jet and the other perpendicular to the direction of jet.

Component of the velocity in the direction of jet = - (V – u) Cosθ

(-ve sign is taken as at the out let, the component is in the opposite direction of the jet). Component of velocity in the

direction perpendicular to the direction of jet = (V – u) sinθ

Mass of water striking the plate = ρa × velocity with which jet strikes the plate.

$$= \rho a \ (V - u)$$

∴Force exerted by the jet of water on the curved plate in the direction of jet F_x

F_x = Mass striking per sec [Initial velocity with which jet strikes the plate in the direction of jet −Final velocity]

$$= \rho a(V - u)[(V - u) - (-(V - u)\cos\theta)]$$

$$= \rho a(V - u)^2[1 + \cos\theta] \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Work done by the jet on the plate per second

= F_x ×Distance travelled per second in the direction of x

$$= F_x \times u = \rho a(V - u)^2 [1 + \cos\theta] \times u$$

$$= \rho a(V - u)^2 \times u[1 + \cos\theta]$$

1. Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of the water at the centre of the nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.

ANS:GIVEN :Diameter of nozzle d = 100mm =0.1m

Area of nozzle = $\frac{\pi}{4} \times (0.1)^2 = 0.00785 m^2$

Head of water H = 100m

Co-efficient of velocity= $C_v = 0.95$

Theoretical velocity of jet of water $V_{th} = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 100} = 44.294$ m/sec

$$\text{But} \quad C_v = \frac{\text{Actual velocity}}{\text{Theoritical velocity}}$$

$\therefore$ Actual velocity of jet of water $= C_v \times V_{th} = 0.95 \times 44.294 = \mathbf{42.08\ m/sec}$

Force exerted on a fixed vertical plate

$$F = \rho a V^2 = 1000 \times 0.007854 \times (42.08)^2$$

F = 13907.2 N = 13.9 kN

2. A jet of water of diameter 75mm moving with a velocity of 25m/sec strikes a fixed plate in such a way that the angle between the jet and plate is 60. Find the force exerted by the jet on the plate

I. In the direction normal to the plate and

II. In the direction of the jet. ANS:Given:

$$\text{Diameter of the jet } d = 75mm = 0.075m$$

$$\text{Area of the jet} = \frac{\pi}{4} \times (0.075)^2 = 0.004417 m^2$$

$$\text{Velocity of jet } V = 25 m/sec$$

$$\text{Angle between jet and plate } \theta = 60^\circ$$

i) The force exerted by the jet of water in the direction normal to the plate

$$F_n = \rho a V^2 \sin\theta = 1000 \times 0.004417 \times (25)^2 \sin 60^0$$

$$F_n = 2390.7 N$$

i) The force in the direction of jet

$$F_x = \rho a V^2 \sin^2\theta = 1000 \times 0.004417 \times (25)^2 \times \sin^2 60^0$$

$$F_x = 2070.4 N$$

3. A jet of water of dia.50mm moving with a velocity of 40m/sec strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of jet, if the direction of jet is deflected through an angle of 120 at the out let of curved plate.

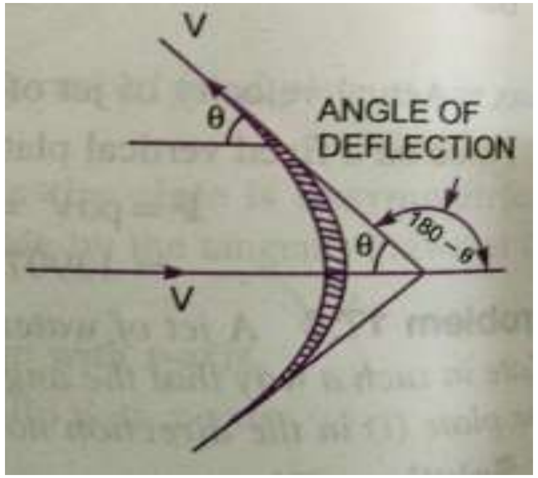
Ans) **Given:**

Dia. of jet d = 0.05m

Area of jet $a = \frac{\pi}{4} \times (0.05)^2 = 0.001963 m^2$

Velocity of jet V = 40m/sec

Angle of deflection $= 180 - \theta = 180 - 120 = 60°$



Force exerted by the jet on the curved plate in the direction of jet

$F_x = \rho a V^2 [1 + cos\theta]$

$F_x = 1000 \times 0.001963 \times (40)^2 \times [1 + \cos 60^0]$

$\boldsymbol{F_x = 4711.15\, N}$

4. A jet of water of dia. 75mm moving with a velocity of 30m/sec strikes a curved fixed plate tangentially at one end at an angle of 30 to the horizontal. The jet leaves the plate at an angle of 20^0 to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.

Given: Dia. of jet d = 75mm = 0.075m,

Area of jet $a = \frac{\pi}{4} \times (0.075)^2 = 0.004417\ m^2$

Velocity of jet V = 30m/sec

Angle made by the jet at inlet tip with the horizontal $\theta = 30°$

Angle made by the jet at out let tip with the horizontal $\emptyset = 20°$ The force exerted by the jet of water on the plate in horizontal direction F_x

$$F_x = \rho a V^2[\cos\theta + \cos\emptyset]$$

$$= 1000 \times 0.004417[\cos 30^0 + \cos 20^0] \times (30)^2$$

$$\boldsymbol{F_x = 7178.2\ N}$$

The force exerted by the jet of water on the plate in vertical direction F_y

$$F_y = \rho a V^2[sin\ \theta - sin\ \emptyset]$$

$$= 1000 \times 0.004417[\sin 30^0 - \sin 20^0] \times (30)^2$$

$$\boldsymbol{F_y = 628.13\ N}$$

5. A nozzle of 50mm dia. delivers a stream of water at 20m/sec perpendicular to the plate that moves away from the plate at 5m/sec. Find:

i) The force on the plate.

ii) The work done and

iii) The efficiency of the jet.

Ans)

Given: Dia. of jet $d = 50mm = 0.05m$,

Area of jet $a = \frac{\pi}{4}(0.05)^2 = 0.0019635\, m^2$

Velocity of jet $V = 20m/sec$,

Velocity of plate $u = 5m/sec$

i) The force on the plate

$$F_x = \rho a(V-u)^2$$

$$F_x = 1000 \times 0.0019635 \times (20-5)^2$$

$$\boldsymbol{F_x = 441.78N}$$

ii) The work done by the jet $= F_x \times u$

$$= 441.78 \times 5$$

$$= \mathbf{2208.9\ Nm/s}$$

iii) The efficiency of the jet $\eta = \dfrac{Output\ of\ jet}{Input\ of\ jet}$

$$= \frac{Work\ done/sec}{K.E\ of\ jet/sec} = \frac{F_x \times u}{\frac{1}{2} m v^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho a V) V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$$= \mathbf{0.3377} = \mathbf{33.77\%}$$

6. A 7.5cm dia. jet having a velocity of 30m/sec strikes a flat plate, the normal of which is inclined at 45 to the axis of the jet. Find the normal pressure on the plate: (i) When the plate is stationary and (ii) When the plate is moving with a velocity of 15m/sec away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Given: Dia. of the jet d = 7.5cm = 0.075m

Area of jet a = $\frac{\pi}{4}$ $(0.075)^2$ = 0.004417m^2

Angle between jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet V = 30m/sec

i) When the plate is stationary, the normal force F_n on the plate is

$$F_n = \rho a V^2 \sin\theta = 1000 \times 0.004417 \times (30)^2 \times \sin 45^0$$

$$= \mathbf{2810.96\ N}$$

ii)When the plate is moving with a velocity of 15m/sec away from the jet, the normal force on the plate F_n

$$F_n = \rho a (V-u)^2 \sin\theta = 1000 \times 0.004417 \times (30-15)^2 \times \sin 45^0$$

= **702.74 N**

iii) The power and efficiency of the jet, when the plate is moving is obtained as

Work done /sec by the jet

= Force in the direction of jet x Distance moved by plate in the direction of jet/sec

$= F_x \times u$ Where $F_x = F_n \sin\theta = 702.74 \times \sin 45^0 = 496.9\ N$

Work done/ sec = 496.9 × 15 = 7453.5Nm/s

∴ Power in kW = $\frac{Work\ done\ /sec}{1000} = \frac{7453.5}{1000}$ = **7.453 kW**

Efficiency of jet $= \frac{Out\ put}{Input} = \frac{Work\ done\ per\ sec}{K.E\ of\ jet\ per\ sec}$

$= \frac{7453.5}{\frac{1}{2}(\rho a V) \times V} = \frac{7453.5}{\frac{1}{2}\rho a V^3}$

$= \frac{7453.5}{\frac{1}{2} \times 1000 \times 0.004417 \times 30^3}$

= 0.1249 ≃ 0.125 =12.5%

7. A jet of water of dia. 7.5cm strikes a curved plate at its centre with a velocity of 20m/sec. the curved plate is moving with a velocity of 8m/sec in the direction of the jet. The jet is deflected through an angle of 165. Assuming plate is smooth, find

1. Force exerted on the plate in the direction of jet.
2. Power of jet.
3. Efficiency of jet.

Given: Dia. of jet d = 7.5cm = 0.075m

Area of jet $a = \frac{\pi}{4}(0.075)^2 = 0.004417 m^2$

Velocity of jet V = 20m/sec

Velocity of plate u = 8m/sec

Angle made by the relative velocity at the out let of the plate $\theta = 180° - 165° = 15°$

i) Force exerted by the jet on the plate in the direction of jet

$$F_x = \rho a (V - u)^2 (1 + \cos\theta)$$

$$F_x = 1000 \times 0.004417 \times (20 - 8)^2 [1 + \cos 15^0]$$

$$= \mathbf{1250.38 N}$$

ii) Work done by the jet on the plate per second

$$= F_x \times u$$

$$= 1250.38 \times 8$$

$$= \mathbf{10003.04\ Nm/s}$$

$\therefore$ Power of jet = $\frac{10003.04}{1000}$ = 10kW

iii)Efficiency of the jet = $\frac{Out\ put}{Input} = \frac{Work\ done\ per\ sec}{K.E\ of\ jet\ per\ sec}$

$$= \frac{1250.38 \times 8}{\frac{1}{2}\rho a V . V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times 0.004417 \times (20)20^3}$$

= 0.564 = 56.4%

8. A jet of water having a velocity of 40m/sec strikes a curved vane, which is moving with a velocity of 20m/sec. The jet makes an angle of 30 with the direction of motion of vane at inlet and leaves at an angle of 90 to the direction of motion of vane at out let. Draw velocity triangles at inlet and outlet and determine vane angles at inlet and outlet, so that the water enters and leaves the vanes without shock.

Ans)

Given: Velocity of jet $V_1 = 40 m/sec$

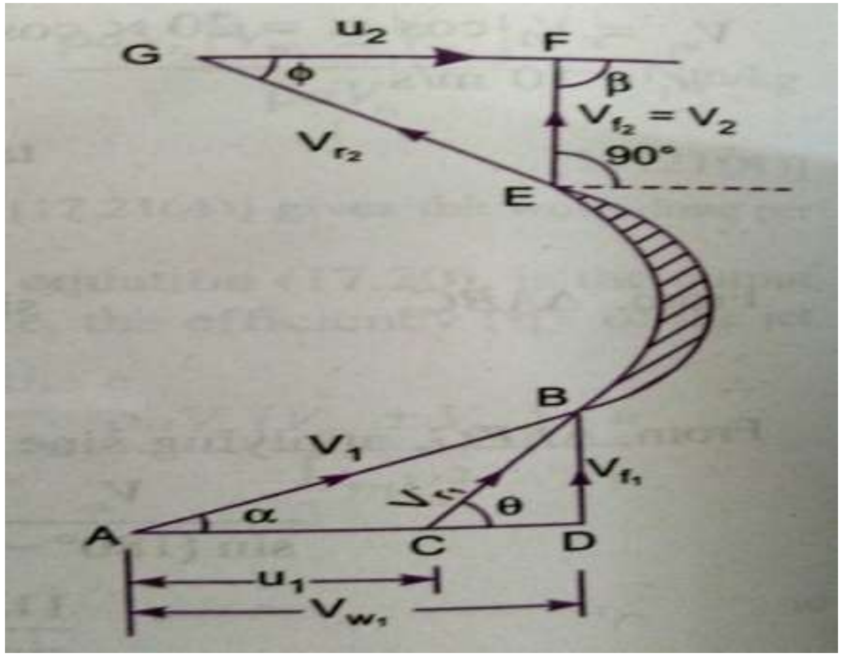
Velocity of vane $u_1 = 20 m/sec$

Angle made by jet at inlet $\alpha = 30°$

Angle made by leaving jet $= 90°$

$\therefore \beta = 180° - 90° = 90°$

$u_1 = u_2 = u = 20 m/sec$



Vane angles at inlet and outlet are θ and $\emptyset$

From Δ BCD we have $\tan\theta = \frac{BD}{CD} = \frac{BD}{AD-AC} = \frac{V_{f1}}{V_{w1}-u_1}$

Where $V_f = V_1 \sin\alpha = 40 \times \sin 30^0 = 20 m/s$

$V_{w_1} = V_1 \cos\alpha = 40 \times \cos 30^0 = 34.64\ m/s$

$u_1 = 20 m/s$

$\therefore \quad \tan\theta = \frac{20}{34.64-20} = \frac{20}{14.64} = 1.366 = \tan 53.79^0$

$\therefore \quad \boldsymbol{\theta = 53.79^0\ or\ 53^0 47.4'}$

Also from Δ BCD we have $\sin\theta = \frac{V_{f1}}{V_{r1}}$ or $V_{r1} = \frac{V_{f1}}{\sin\theta} = \frac{20}{\sin 53.79^0} = 24.78\ m/s$

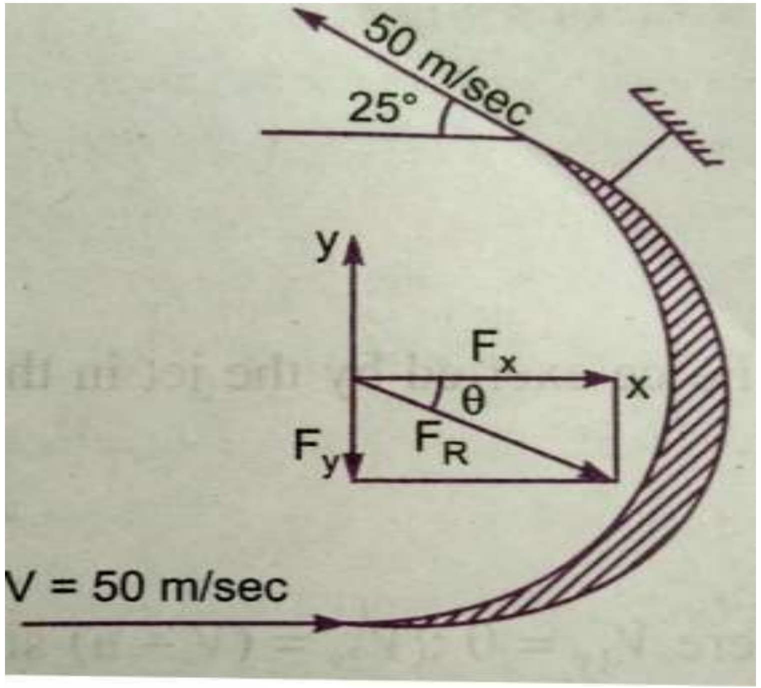
$\therefore \quad V_{r_1} = 24.78 m/s$

But $V_{r_2} = V_{r_1} = 24.78$

Hence, From Δ EFG, $\cos\emptyset = \frac{u_2}{V_{r2}} = \frac{20}{24.78} = 0.8071 = \cos 36.18^0$

$\emptyset = \boldsymbol{36.18^0\ or\ 36^0\ \ 10.8'}$

9. A stationary vane having an inlet angle of zero degree and an outlet angle of 25, receives water at a velocity of 50m/sec. Determine the components of force acting on it in the direction of jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of the flow



Given: Velocity of jet V = 50 m/sec

Angle at outlet = 25°

For the stationary vane, the force in the direction of jet.

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

Where $V_{1x} = 50$ m/sec, $V_{2x} = -50 \cos 25^0 = -45315$

∴ Force in the direction of jet per unit weight of water F_x

$$F_x = \frac{Mass/\sec[50-(-45.315)]}{Weight\ of\ water\ /sec} = \frac{Mass/\sec[50+45.315]}{(mass/sec)\times g}$$

$$= \frac{95.315}{9.81} = 9.716\ N/N$$

Force exerted by the jet in perpendicular direction to the jet per unit weight of flow

$V_{1y} = 0 \qquad V_{2y} = 50 \sin 25^0$

$$F_y = \frac{Mass/\sec(V_{1y} - V_{2y})}{g \times\ mass\ per\ sec}$$

$$= \frac{(0-50\ Sin\ 25°)}{g} = \frac{-50\ Sin\ 25°}{9.81}$$

= - 2.154 N

- ve sign means the force F_y is acting in the downward direction.

∴ Resultant Force per unit weight of water $F_R = \sqrt{F_x^{\,2} + F_y^{\,2}}$

$$F_R = \sqrt{(9.716)^2 + (2.154)^2} = \mathbf{9.952N}$$

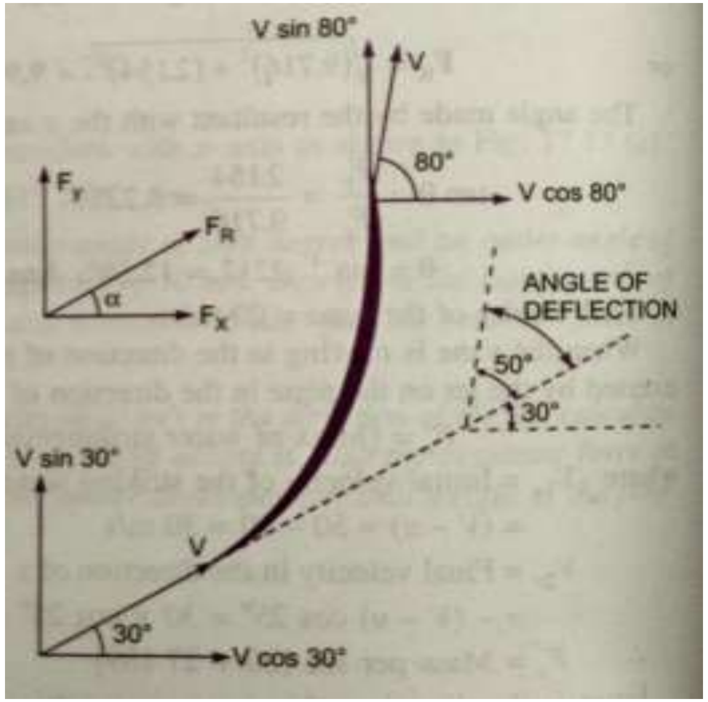
The angle made by the Resultant Force with x- axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{2.154}{9.716} = 0.2217$$

$$\boldsymbol{\theta = tan^{-1}0.2217 = 12.50°}$$

10. A jet of water diameter 50mm moving with a velocity of 25m/sec impinges on a fixed curved plate tangentially at one end at an angle of 30 to the horizontal. Calculate the resultant force of the jet on the plate, if the jet is deflected through an angle of 50. Take g=10m/sec^2.

Given:



Dia. of jet d = 50mm = 0.05m,

Area of jet $a = \frac{\pi}{4}(0.05)^2 = 0.0019635\, m^2$

Velocity of jet V = 25m/sec,

Angle made by the jet at inlet with horizontal $\theta = 30°$ Angle of deflection = 50°

Angle made by the jet at the outlet with horizontal $\emptyset$

$$\emptyset = \theta + angle\ of\ deflection = 30° + 50° = 80°$$

The Force exerted by the jet of water in the direction of x

$$F_x = \rho a V(V_{1x} - V_{2x})$$

Where $\rho = 1000$

$a = \frac{\pi}{4}(0.05)^2$ V = 25 m/s

$V_{1x} = V\cos 30^0 = 25\cos 30^0$

$V_{2x} = V\cos 80^0 = 25\cos 80^0$

$$F_x = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\cos 30^0 - 25\cos 80^0] = \mathbf{849.7\ N}$$

The Force exerted by the jet of water in the direction of y

$$F_y = \rho a V(V_{1y} - V_{2y})$$

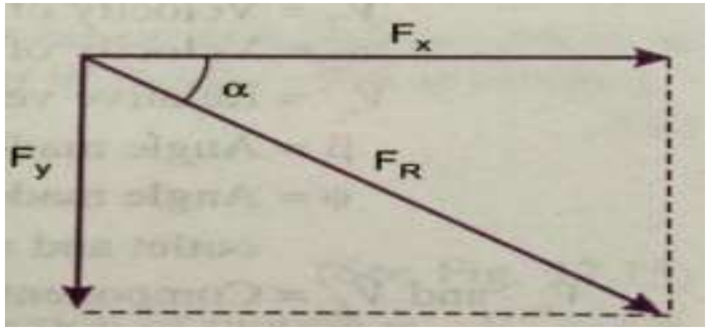
$$F_y = 1000 \times \frac{\pi}{4}(0.05)^2 \times 25[25\sin 30^0 - 25\sin 80^0] = \mathbf{-594.9\ N}$$

- ve sign shows that Force F_y is acting in the downward direction.

The Resultant force $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(849.7)^2 + (594.9)^2}$

= **1037N**



Angle made by the Resultant Force with the Horizontal

$$\tan\alpha = \frac{F_y}{F_x} = \frac{594.9}{849.7} = 0.7$$

$$\alpha = \boldsymbol{tan^{-1}0.7 = 35°}$$

CLASSIFICATION OF HYDRAULIC TURBINES:

The Hydraulic turbines are classified according to the type of energy available at the inlet of the turbine, direction of flow through the vanes, head at the inlet of the turbine and specific speed of the turbine. The following are the important classification of the turbines.

1. According to the type of energy at inlet:

   (a) Impulse turbine and

   (b) Reaction turbine

2. According to the direction of flow through the runner:

   (a) Tangential flow turbine

   (b) Radial flow turbine.

   (c) Axial flow turbine

   (d) Mixed flow turbine.

3. According to the head at inlet of the turbine:

(a) High head turbine

(b) Medium head turbine and

(c) Low head turbines.

4. According to the specific speed of the turbine:

(a) Low specific speed turbine

(b) Medium specific speed turbine

(c) High specific speed turbine.

- If at the inlet of turbine, the energy available is only kinetic energy, the turbine is known as **Impulse turbine**.
- As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **Reaction turbine**.
- As the water flows through runner, the water is under pressure and the pressure energy goes on changing in to kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water.

- If the water flows along the tangent of runner, the turbine is known as **Tangential flow turbine**.

- If the water flows in the radial direction through the runner, the turbine is called **Radial flow turbine.**

- If the water flows from outward to inwards radially, the turbine is known as **Inward** radial flow turbine, on the other hand

- if the water flows radially from inward to outwards, the turbine is known as **outward** radial flow turbine.
- If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called **axial flow** turbine
- If the water flows through the runner in the radial direction but leaves in the direction parallel to the axis of rotation of the runner, the turbine is called **mixed flow** turbine.

PELTON WHEEL (TURBINE)

- It is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and out let of turbine is atmospheric. This turbine is used for high heads and is named after L.A.Pelton an American engineer.
- The water from the reservoir flows through the penstocks at the out let of which a nozzle is fitted. The nozzle increases the kinetic energy of the water flowing through the penstock. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner.

MAIN PARTS OF PELTON WHEEL TURBINE

- Nozzle and flow regulating arrangement (spear)
- Runner and Buckets.
- Casing and
- Breaking jet

1. **Nozzle and flow regulating arrangement:** The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle. The spear is a conical needle which is operated either by hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward in to the nozzle, the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.
2. **Runner with buckets:** It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided in to two symmetrical parts by a dividing wall, which is known as splitter.

3. **Casing:** The function of casing is to prevent the splashing of the water and to discharge the water to tailrace. It also acts as safeguard against accidents. It is made of Cast Iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. **Breaking jet:** When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero. But the runner due to inertia goes on revolving for a long time. To stop the runner in a short time, a small nozzle is provided, which directs the jet of water on the back of the vanes. This jet of water is called Breaking jet.

VELOCITY TRIANGLES AND WORK DONE FOR PELTON WHEEL

- The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts.
- These parts of the jet, glides over the inner surfaces and comes out at the outer edge.
- The splitter is the in let tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangle is drawn at the splitter and outer velocity triangle is drawn at the outer edge of the bucket.

Let

H = Net head acting on the Pelton Wheel

$$= H_g - h_f$$

Where H_g = Gross Head

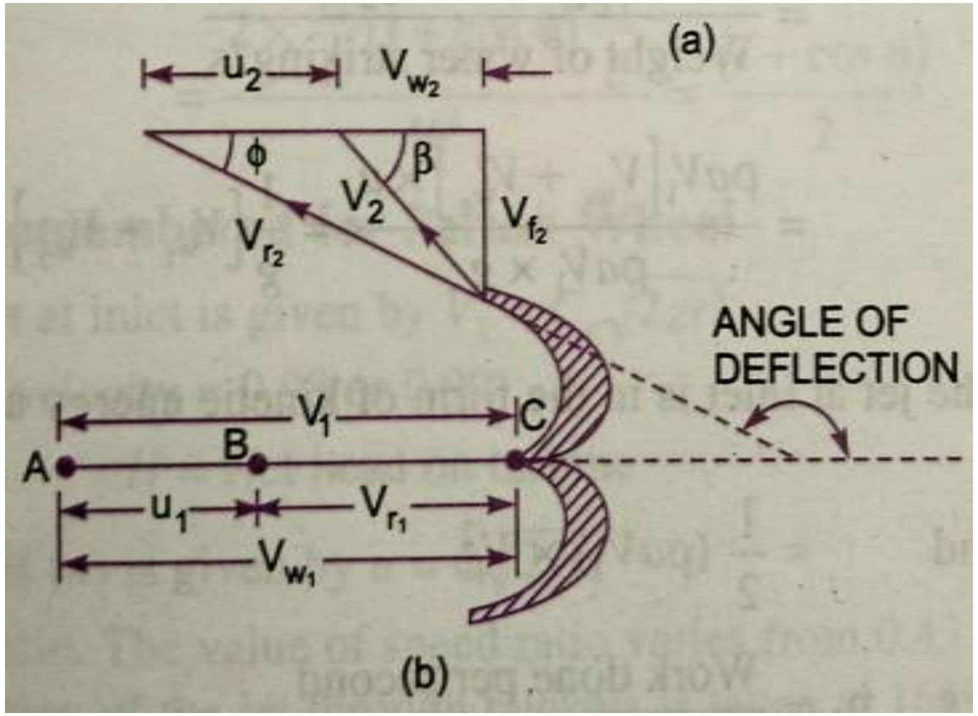
$$h_f = \frac{4fLV^2}{D^* \times 2g}$$

Where D^* =diameter of penstock, D = Diameter of wheel, d = Diameter of Jet,

N = Speed of the wheel in r.p.m

Then V_1 = Velocity of jet at inlet $= \sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}$$



The Velocity Triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1 \qquad \alpha = 0^0 \qquad and \ \theta = 0^0$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \ and \ V_{w_2} = V_{r_2} \cos \emptyset - u_2$$

The force exerted by the Jet of water in the direction of motion is

$$F_x = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \qquad (1)$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation (1) 'a' is the area of the jet = $\frac{\pi}{4} d^2$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u \qquad Nm/s$$

Power given to the runner by the jet $= \dfrac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \ kW$

Work done/s per unit weight of water striking/s $= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{Weight\ of\ water \frac{striking}{s}}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\rho a V_1 \times g} = \frac{1}{g}[V_{w_1} + V_{w_2}] \times u \qquad \text{\_\_\_\_\_\_\_\_(3)}$$

The energy supplied to the jet at inlet is in the form of kinetic energy

$\therefore$ K.E. of jet per second $= \frac{1}{2} mV^2 = \frac{1}{2}(\rho a V_1) \times V_1{}^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \frac{Work\ done\ per\ second}{K.E. of\ jet\ per\ second}$

$$= \frac{\rho a V_1 [V_{w_1} + V_{w_2}] \times u}{\frac{1}{2}(\rho a V_1) \times V_1{}^2}$$

$$= \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1{}^2} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Now $V_{w_1} = V_1$ and $V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

And $V_{w_2} = V_{r_2} \cos \emptyset - u_2$

$= V_{r_2} \cos \emptyset - u$

$= (V_1 - u) \cos \emptyset - u$

Substituting the values of V_{w_1} and V_{w_2} in equation (4)

$$\eta_h = \frac{2[V_1+(V_1-u)\cos\varnothing-u]\times u}{V_1^{\,2}} = \frac{2[V_1-u+(V_1-u)\cos\varnothing]\times u}{V_1^{\,2}}$$

$$= \frac{2(V_1-u)[1+\cos\varnothing]u}{V_1^{\,2}} \quad \text{\_\_\_\_\_\_\_\_\_\_ (5)}$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad or \quad \frac{d}{du}\left[\frac{2u(V_1-u)[1+\cos\varnothing]}{V_1^{\,2}}\right] = 0$$

Or $\frac{(1+\cos\varnothing)}{V_1^{\,2}} \frac{d}{du}(2uV_1 - 2u^2) = 0$

Or $\frac{d}{du}[2uV_1 - 2u^2] = 0 \qquad \left(\because \frac{1+\cos\phi}{V_1^{\,2}} \neq 0\right)$

Or $2V_1 - 4u = 0$ Or $u = \frac{V_1}{2}$ ______ (6) ______

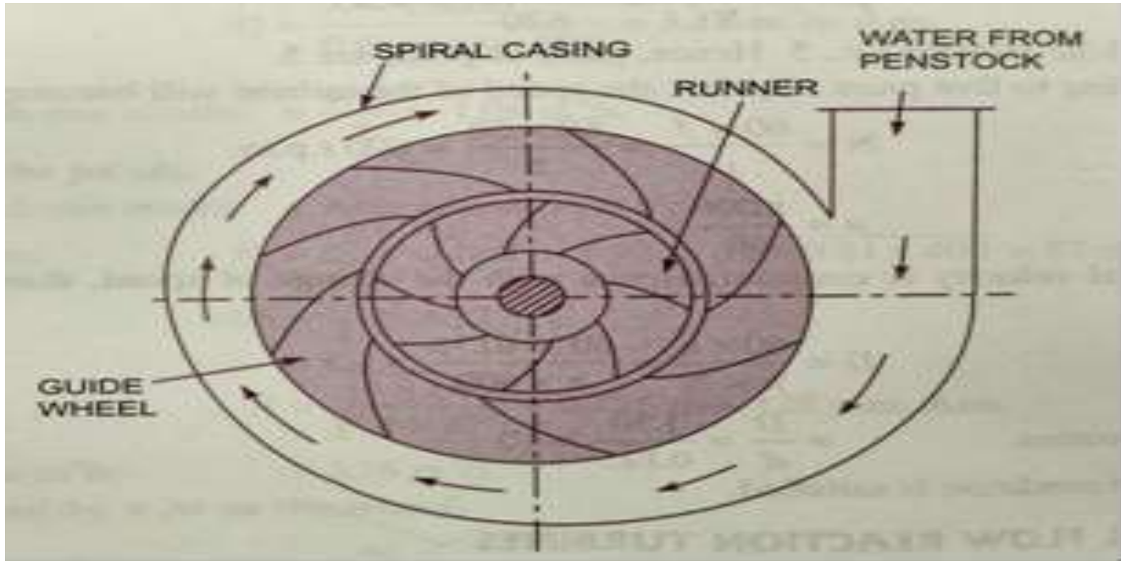
Equation (6) states that hydraulic efficiency of a Pelton wheel will be maximum when the velocity of the wheel is half the velocity of the jet water at inlet. The expression for maximum efficiency will be obtained by substituting the value of $u = \frac{V_1}{2}$ in equation (5)

$$\text{Max. } \eta_h = \frac{2\left(V_1-\frac{V_1}{2}\right)(1+\cos\varnothing)\times\frac{V_1}{2}}{V_1^{\,2}} = \frac{2\times\frac{V_1}{2}(1+\cos\varnothing)\frac{V_1}{2}}{V_1^{\,2}} = \frac{(1+\cos\varnothing)}{2}$$

RADIAL FLOW REACTION TURBINE

- In the Radial flow turbines water flows in the radial direction.
- The water may flow radially from outwards to inwards (i.e. towards the axis of rotation) or from inwards to outwards.
- If the water flows from outwards to inwards through the runner, the turbine is known as inwards radial flow turbine. And if the water flows from inwards to outwards, the turbine is known as outward radial flow turbine.

Main parts of a Radial flow
Reaction turbine:
Casing
Guide mechanism
Runner and
Draft tube



1. **Casing:** in case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross-section one of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The water enters the runner at constant velocity throughout the circumference of the runner.

2. **Guide Mechanism:** It consists of a stationary circular wheel all around the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

3. **Runner:** It is a circular wheel on which a series of radial curved vanes are fixed. The surfaces of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runners are made of cast steel, cast iron or stain less steel. They are keyed to the shaft.

4. **Draft - Tube:** The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit can't be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging the water from the exit of the turbine to the tail race. This tube of increasing area is called draft-tube.

5. **Inward Radial Flow Turbine:** In the inward radial flow turbine, in which case the water from the casing enters the stationary guiding wheel. The guiding wheel consists of guide vanes which direct the water to enter the runner which consists of moving vanes. The water flows over the moving vanes in the inward radial direction and is discharged at the inner diameter of the runner. The outer diameter of the runner is the inlet and the inner diameter is the outlet.

VELOCITY TRIANGLES AND WORK DONE BY WATER ON RUNNER:

Work done per second on the runner by water

$$= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} u_1 \pm V_{w_2} u_2] \text{\_\_\_\_\_\_\_\_}(1) \qquad (\because aV_1 = Q)$$

The equation represents the energy transfer per second to the runner.

Where V_{w_1} = Velocity of whirl at inlet Velocity of

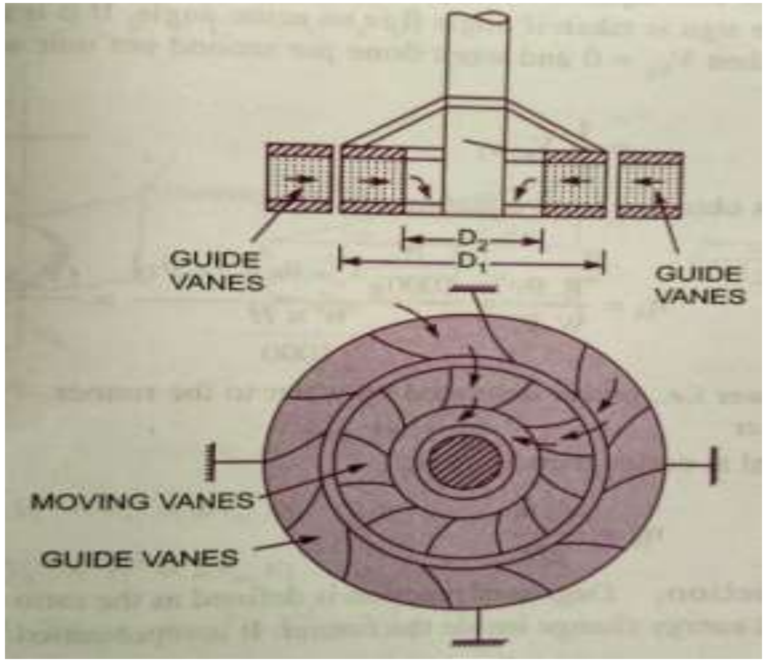
V_{w_2} = whirl at outlet Tangential velocity at inlet

u_1 = , Where Outer dia. Of runner, Tangential

$= \frac{\pi D_1 \times N}{60}$ velocity at outlet D_1 =

u_2 =

$= \frac{\pi D_2 \times N}{60}$



Where D_1 = Inner dia. Of runner,

N = Speed of the turbine in r.p.m.

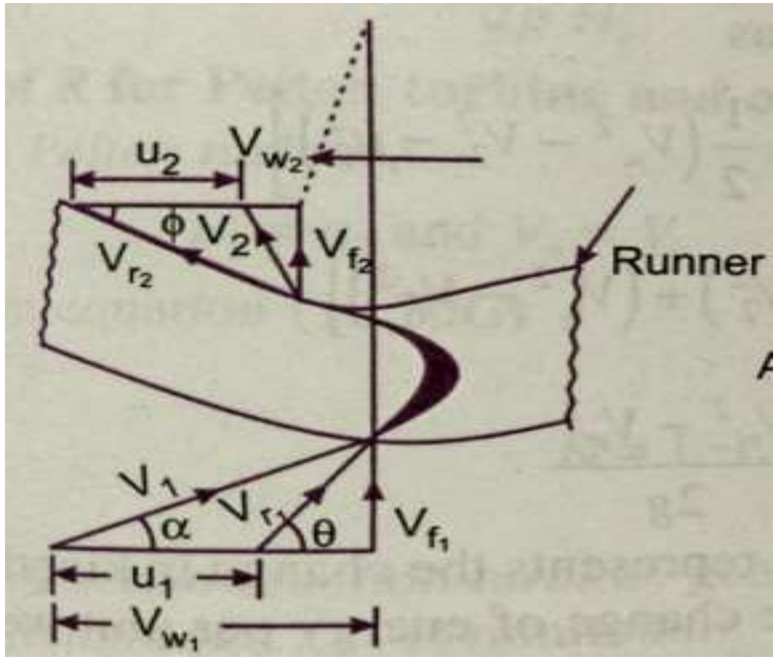
The work done per second per unit weight of water per second

$$= \frac{\text{work done per second}}{\text{weight of water striking per second.}}$$

$$= \frac{\rho Q [V_{w_1} u_1 \pm V_{w_2} u_2]}{\rho Q \times g}$$

$$= \frac{1}{g} [V_{w_1} u_1 \pm V_{w_2} u_2] \quad \text{———(2)}$$

Equation (2) represents the energy transfer per unit weight/s to the runner. This equation is known by **Euler's equation.**



In equation +ve sign is taken if β is an acute angle,

-ve sign is taken if β is an obtuse angle.

If $\beta = 90^0$ then $V_{w_2} = 0$ and work done per second per unit weight of water striking/s

Work done $= \frac{1}{g} V_{w_1} u_1$

Hydraulic efficiency $\eta_h = \frac{R.P.}{W.P.} = \frac{Power\ delivered\ to\ runner}{Power\ supplied\ at\ inlet}$

$$= \frac{\frac{W}{1000 \times g}\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{\frac{W \times H}{1000}} = \frac{\left(V_{w_1} u_1 \pm V_{w_2} u_2\right)}{gH} \qquad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

Where R.P. = Runner Power i.e. power delivered by water to the runner

W.P. = Water Power

If the discharge is radial at outlet, then $V_{w_2} = 0$

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

1. A pelton wheel has a mean bucket speed of 10m/s with a jet of water flowing at the rate of 700lts/sec under a head of 30 m. the buckets deflect the jet through an angle of 160^0 calculate the power given by the water to the runner and hydraulic efficiency of the turbine? Assume co-efficient of velocity=0.98

ANS)

Given:

Speed of bucket $u = u_1 = u_2 = 10m/s$

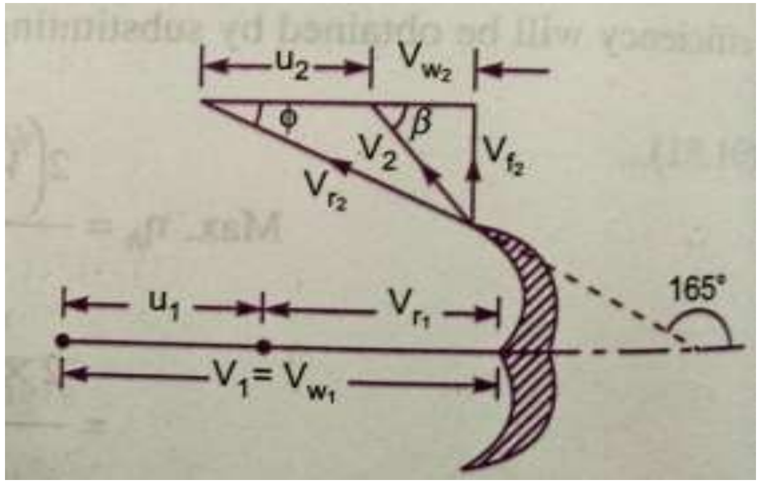
Discharge Head of water Q = 700lt/sec=0.7m^3 /s H = 30m

Angle deflection = 160^0

$\therefore Angle \quad \emptyset = 180 - 160 = 20^0$

Co-efficient of velocity The $C_v = 0.98$

velocity of jet $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77m/s$



$$V_{r_1} = V_1 - u_1 = 23.77 - 10 = 13.77 m/s$$

$$V_{w_1} = V_1 = \mathbf{23.77 m/s}$$

From the outlet velocity triangle

$$V_{r_2} = V_{r_1} = \frac{13.77m}{s}$$

$$V_{w_2} = V_{r_1} \cos \varnothing - u_2$$

$$= 13.77 \cos 20^0 - 10 = \mathbf{2.94 m/s}$$

Work done by the jet/sec on the runner is given by equation

$$= \rho a V_1 [V_{w_1} + V_{w_2}] \times u$$

$$= 1000 \times 0.7[23.77 + 2.94] \times 10$$

$$= \mathbf{186970 Nm/s}$$

Power given to the turbine $= \frac{186970}{1000} = \mathbf{186.97 kW}$

The hydraulic efficiency of the turbine is given by equation

$$\eta_h = \frac{2[V_{w_1} + V_{w_2}] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77} = \mathbf{0.9454}$$

Or = **94.54%**

2. A reaction turbine works at 450rpm under a head of 120m. its diameter at inlet is 120cm and flow area is 0.4m^2. The angles made by absolute and relative velocities at inlet are 20^0 and 60^0 respectively, with the tangential velocity. Determine

i) Volume flow rate **ii)** the power developed

iii) The hydraulic efficiency. Assume whirl at outlet is zero.

Ans **Given:** Speed of turbine N=450rpm H=

Head 120m

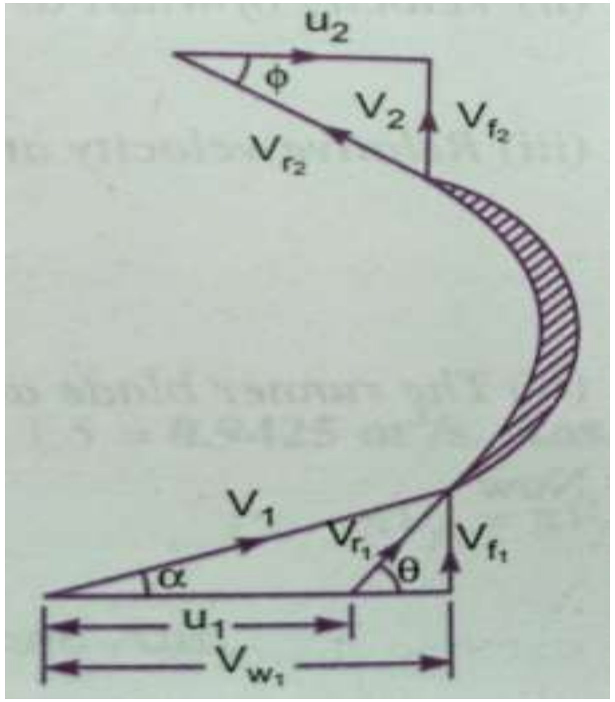
Diameter of inlet Flow D$_1$ = 120cm=1.2m

area $\pi D_1 \times B_1 = 0.4m^2$

Angle made by absolute velocity Angle made $\alpha = 20^0$

by relative velocity Whirl at outlet $\theta = 60^0$

Tangential velocity of the turbine at inlet $V_{w_2} = 0$



From inlet triangle $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 m/s$

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$\text{Tan } 20^\circ = \frac{V_{f_1}}{V_{w_1}} = 0.364,$$

$$V_{f_1} = 0.364 V_{w_1} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

Also $$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27} \qquad (\because \tan\theta = tan60^\circ = 1.732)$$

$$1.732 = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27}$$

$$0.364 V_{w_1} = 1.732(V_{w_1} - 28.27)$$

$$0.364 V_{w_1} = 1.732 V_{w_1} - 28.27 \times 1.732$$

$$V_{w_1}(1.732 - 0.364) = 48.96$$

$$V_{w_1} = \frac{48.96}{1.732 - 0.364} = \mathbf{35.79 m/s}$$

From equation (1) $$V_{f_1} = 0.364 V_{w_1} = 0.364 \times 35.79 = \mathbf{13.027 m/s}$$

i) Volume flow rate is given by equation as $Q = \pi D_1 B_1 V_{f_1}$

$$Q = 0.4 \times 13.027 = \mathbf{5.211 m^3/sec} \qquad (\because \pi D_1 B_1 = 0.4 m^2)$$

ii)Work done per second on the turbine is given by equation

$$= \rho Q [V_{w_1} \times u_1]$$

$$= 1000 \times 5.211[35.79 \times 28.27] = 5272.402 Nm/s$$

Power developed in $$kW = \frac{work\ done\ per\ sec}{1000} = \frac{5272402}{1000} = \mathbf{5272.402 kW}$$

iii) The hydraulic efficiency is given by equation

$$\eta_h = \frac{V_{w_1} \times u_1}{g \times H} = \frac{35.79 \times 28.27}{9.81 \times 120} = 0.8595$$

= ***85.95%***

FRANCIS TURBINE

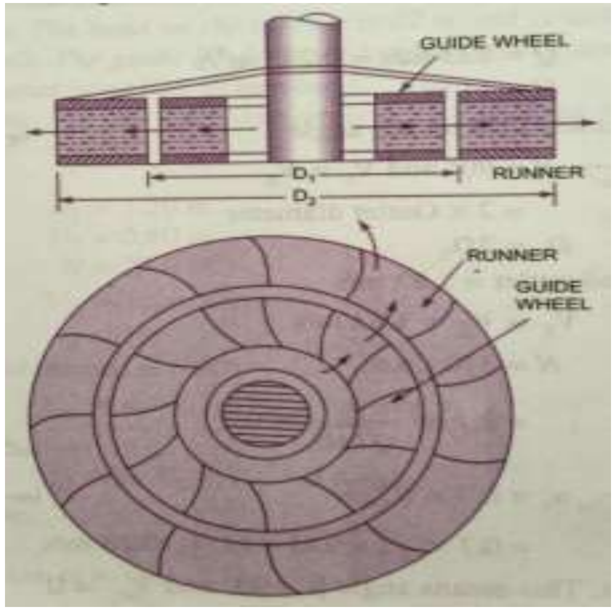
- The inward flow reaction turbine having radial discharge at outlet is known as Francis Turbine. The water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus the Francis turbine is a mixed flow type turbine.
- The work done by water on the runner per second will be

$$= \rho Q[V_{w_1} u_1]$$

Theworkdonepersecondperunitweightofwaterstriking/sec= $\frac{1}{g}[V_{w_1} u_1]$

Hydraulicefficiency

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$



Important relations for Francis turbines:

1. The ratio of width of the wheel to its diameter is given as $n = \frac{B_1}{D_1}$. The value of n varies from 0.10 to 0.40
2. The flow ratio is given as

   Flow ratio $= \frac{V_{f_1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30

3. The speed ratio $= \frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9

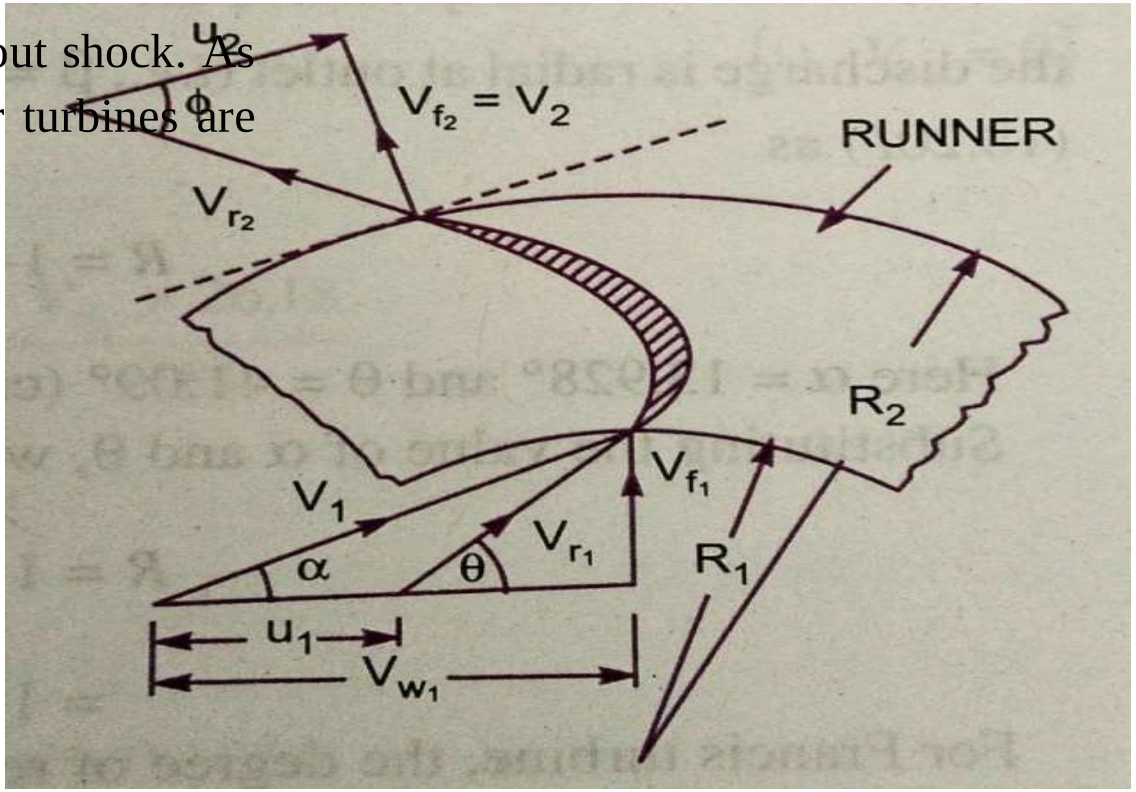
OUTWARD RADIAL FLOW REACTION TURBINE:

In this case as the inlet of the runner is at the inner diameter of the runner, the tangential velocity at inlet will be less than that of an outlet. i.e.

$u_1 < u_2$ As $D_1 < D_2$

All the working conditions flow through the runner blades without shock. As such eddy losses which are inevitable in Francis and propeller turbines are almost completely eliminated in a Kaplan turbine.

The discharge through the runner is obtained as



$$Q = \frac{\pi}{4}\left(D_0^{\,2} - D_b^{\,2}\right) \times V_{f_1}$$

Where D_0 = outer diameter of the runner

D_b = Diameter of the hub Velocity of

V_{f_1} = flow at inlet

IMPORTANT POINTS FOR KAPLAN TURBINE:

The peripheral velocity at inlet and outlet are equal.

$$u_1 = u_2 = \frac{\pi D_0 N}{60}$$

Where D_0 = Outer diameter of runner.

2. Velocity of flow at inlet and outlet are equal.

$$V_{f_1} = V_{f_2}$$

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}\left(D_0^{\,2} - D_b^{\,2}\right)$$

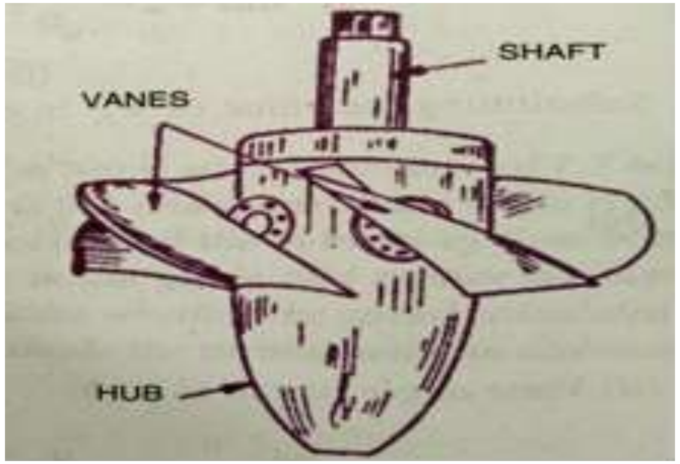
AXIAL FLOW REACTION TURBINE

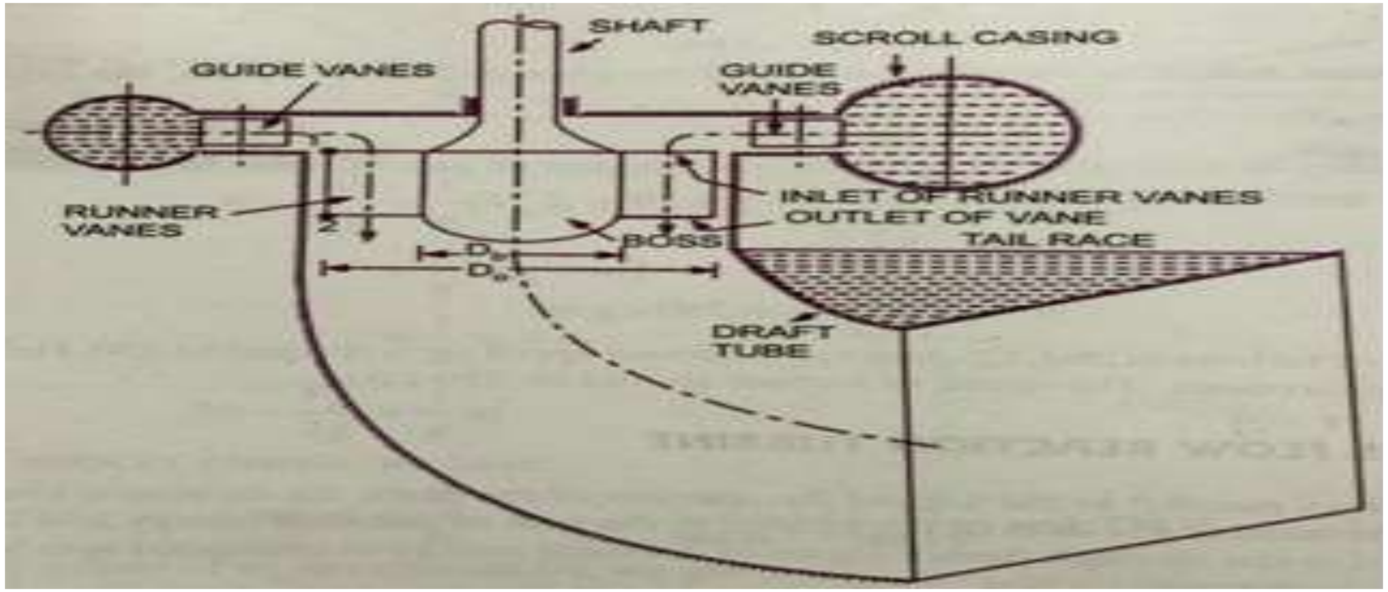
1. Propeller Turbine
2. Kaplan Turbine

KAPLAN TURBINE

- The main parts of the Kaplan turbine are:

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes or runner of the turbine
4. Draft tube





WORKING PROPORTIONS OF KAPLAN TURBINE

The main dimensions of Kaplan Turbine runners are similar to Francis turbine runner. However the following are main deviations,

i. Choose an appropriate value of the ratio $n = \frac{d}{D}$, where d in hub or boss diameter and D is runner outside diameter.

   The value of n varies from 0.35 to 0.6

ii. The discharge Q flowing through the runner is given by

$$Q = \frac{\pi}{4}(D^2 - d^2)V_f = \frac{\pi}{4}(D^2 - d^2)\psi\sqrt{2gH}$$

   The value of flow ratio ψ for a Kaplan turbine is 0.7

iii. The runner blades of the Kaplan turbine are twisted, the blade angle being greater at the outer tip than at the hub. This is because the peripheral velocity of the blades being directly proportional to radius. It will valy from section to section along the blade, and hence in order to have shock free entry and exit of water over the blades with angles varying from section to section will have to be designed.

A Francis turbine with an overall efficiency of 75% is required to produce 148.25kW power. It is working under a head of 7.62m. The peripheral velocity=0.26 and the radial velocity of flow at inlet is 0.96 . The wheel runs at 150rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge determine

i) The guide blade angle **ii)** The wheel vane angle at inlet

iii) The diameter of the wheel at inlet, and **iv)** Width of the wheel at inlet

Ans

Given: Overall efficiency Head $\eta_0 = 75\% = 0.75$

H=7.62rpm

Power Produced S.P. = 148.25kW Speed N= 150rpm

Hydraulic loses =22% of energy

Peripheral velocity $u_1 = 0.26\sqrt{2gh} = 0.26\sqrt{2 \times 9.81 \times 7.62} = 3.179 m/s$

Discharge at outlet = Radial

$V_{w_2} = 0 \qquad V_{f_2} = V_2$

The hydraulic efficiency $\eta_h = \frac{Total\ head\ at\ inlet - Hydraulic\ losses}{Heat\ at\ inlet}$

$$= \frac{H - 0.22H}{H} = 0.78$$

But $\eta_h = \frac{V_{w_1} u_1}{gH}$, $\frac{V_{w_1} u_1}{gH} = 0.78$

$$V_{w_1} = \frac{0.78 \times g \times H}{u_1} = \frac{0.78 \times 9.81 \times 7.62}{3.179} = \mathbf{18.34 m/s.}$$

i) The guide blade angle i.e. α From inlet velocity triangle

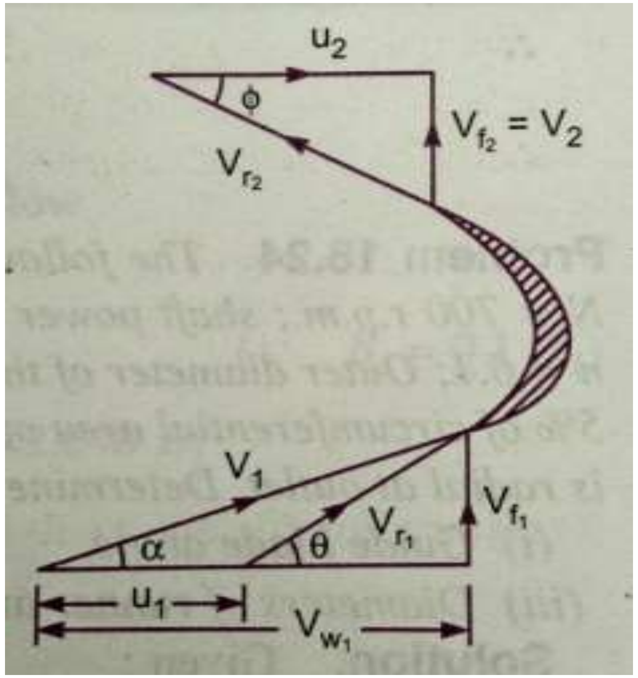
$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$$\alpha = \tan^{-1}(0.64) = \mathbf{32.619^0} \qquad or\ \mathbf{32^0 37'}$$

ii) The wheel angle at inlet (θ)

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} 0.774$$

$$\theta = \tan^{-1}(0.774) = \mathbf{37.74^0} \qquad or\ \mathbf{37^0 44.4'}$$



iii) The diameter of wheel at inlet (D_1)

Using relation

$$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 150} = 0.4047m$$

iv)Width of the wheel at inlet But (B_1)

$$\eta_0 = \frac{SP}{WP} = \frac{148.25}{WP}$$

$$WP = \frac{W \times H}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$$\eta_0 = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_0} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 m^3/s \quad (\because \eta_0 = 75\%)$$

Using equation

$$Q = \pi D_1 B_1 \times V_{f_1}$$

$$2.644 = \pi \times 0.4047 \times B_1 \times 11.738$$

$$B_1 = \frac{2.644}{\pi \times 0.4047 \times 11.738} = \mathbf{0.177m}$$

- A Kaplan turbine runner is to be designed to develop 7357.5kW shaft power. The net available head is 5.50m. Assume that the speed ratio is 2.09 and flow ratio is 0.68 and the overall efficiency is 60%. The diameter of boss is of the diameter of runner. Find the diameter of the runner, its speed and specific speed

Given: Shaft power P = 7357.5kW

Head H = 5.5m Speed ratio =

$$\frac{u_1}{\sqrt{2gH}} = 2.09$$

Flow ratio

$$\therefore \quad u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.5} = 21.71 m/s$$

$$= \frac{V_{f_1}}{\sqrt{2gH}} = 0.68$$

Overall Efficiency Diameter of boss

Using the relation

$$\therefore V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.5} = 7.064 m/s$$

$$\eta_0 = 60\% = 0.60$$

$$D_b = \frac{1}{3} \times D_0$$

$$\eta_0 = \frac{Shaft\ power}{water\ power} = \frac{7357.5}{\frac{\rho g Q H}{1000}}$$

$$0.60 = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

Discharge $$Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27\ m^3/s$$

Using equation for discharge

$$Q = \frac{\pi}{4}\left[{D_0}^2 - {D_b}^2\right] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4}\left[{D_0}^2 - \left(\frac{D_0}{3}\right)^2\right] \times V_{f_1}$$

$$227.27 = \frac{\pi}{4} \times \frac{8}{9} {D_0}^2 \times 7.064$$

$${D_0}^2 = 227.27 \times \frac{4}{\pi} \times \frac{9}{8} \times \frac{1}{7.064}$$

$$D_0 = \mathbf{6.788m}$$

$$D_b = \frac{1}{3} D_0 = \frac{6.788}{3} = \mathbf{2.262m}$$

Using the relation $$u_2 = \frac{\pi D_0 N}{60} \qquad (\because\ u_1 = u_2)$$

$$N = \frac{60 \times u_1}{\pi D_0} = \frac{60 \times 21.71}{\pi \times 6.788} = \mathbf{61.08\ rpm}$$

The specific speed $$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{61.08 \times \sqrt{7357.5}}{(5.5)^{5/4}} = \mathbf{622\ rpm}$$

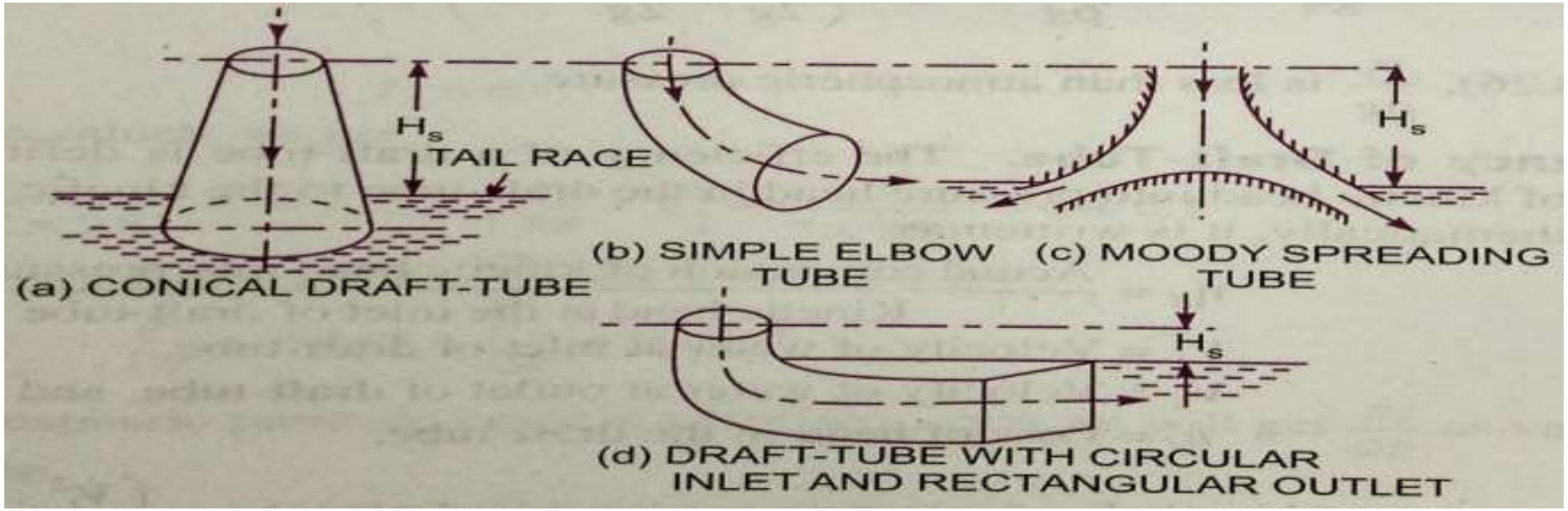
DRAFT TUBE:

- The draft tube is a pipe of gradually increasing area, which connects the outlet of the runner to the tail race.
- It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft tube.
- One end of the draft tube is connected to the outlet of the runner and the other end is submerged below the level of water in the tail race.

TYPES OF DRAFT TUBE

- **Types of Draft Tube:**

1. Conical Draft Tube
2. Simple Elbow Tubes
3. Moody Spreading tubes
4. Elbow Draft Tubes with Circular inlet and rectangular outlet



DRAFT TUBE THEORY

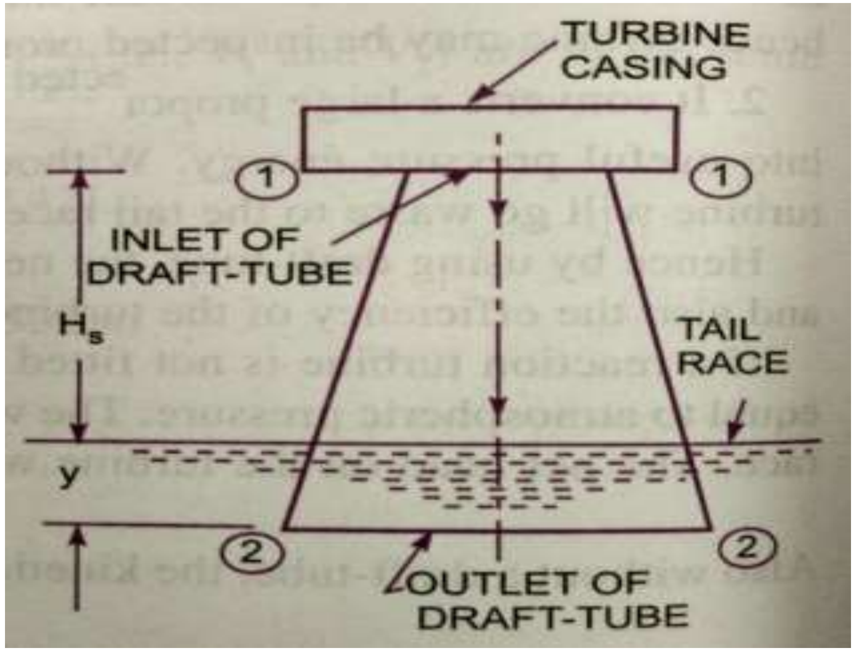
Consider a conical draft tube

H_s =Vertical height of draft tube above tail race

Y= Distance of bottom of draft tube from tail race.

Applying Bernoulli's equation to inlet section 1-1 and outlet section 2-2of the draft tube and taking section 2-2 a datum, we get

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + (H_s + y) = \frac{p_2}{\rho g} + \frac{V_2{}^2}{2g} + 0 + h_f$$



Where h_f = loss of energy between section 1-1 and 2-2.

But $\frac{p_2}{\rho g}$ = Atmospheric Pressure + y = $\frac{p_a}{\rho g} + y$

Substituting this value of $\frac{p_2}{\rho g}$ in equation (1) we get

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + (H_s + y) = \frac{p_a}{\rho g} + y + \frac{V_2{}^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} + \frac{V_1{}^2}{2g} + H_s = \frac{p_a}{\rho g} + \frac{V_2{}^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} + \frac{V_2{}^2}{2g} + h_f - \frac{V_1{}^2}{2g} - H_s$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left[\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g} - h_f\right] \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Efficiency of Draft Tube: the efficiency of a draft tube is defined as the ratio of actual conversion of kinetic head in to pressure in the draft tube to the kinetic head at the inlet of the draft tube.

$$\eta_d = \frac{Actual\ conversion\ of\ Kinetic\ head\ in\ to\ Pressure\ head}{Kintic\ head\ at\ the\ inlet\ of\ draft\ tube}$$

Let V_1 = Velocity of water at inlet of draft tube Velocity of

V_2 =water at outlet of draft tube Loss of head in the draft

h_f = tube

Theoretical conversion of Kinetic head into Pressure head in

Draft tube $= \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right]$

Actual conversion of Kinetic head into pressure head $= \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f$

Now Efficiency of draft tube $\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{\frac{V_1^2}{2g}}$

GEOMETRIC SIMILARITY

- The geometric similarity must exist between the model and its proto type. the ratio of all corresponding linear dimensions in the model and its proto type are equal.

Let length of model

$L_m =$

Breadth of model Diameter of model

$b_m =$

Area of model Volume of model

$D_m =$

$A_m =$

$\forall_m =$

And $L_p, b_p, D_p, A_p, \forall_p$ Corresponding values of the proto type.

For geometrical similarity between model and prototype, we must have the relation,

$$\frac{L_P}{L_m} = \frac{b_P}{b_m} = \frac{D_P}{D_m} = L_r$$

Where L_r is called scale ratio.

For area's ratio and volume's ratio the relation should be,

$$\frac{A_P}{A_m} = \frac{L_P \times b_P}{L_m \times b_m} = L_r \times L_r = L_r^{\,2}$$

$$\frac{\forall_P}{\forall_m} = \left(\frac{L_P}{L_m}\right)^3 = \left(\frac{b_P}{b_m}\right)^3 = \left(\frac{D_P}{D_m}\right)^3 = L_r^{\,3}$$

PERFORMANCE OF HYDRAULIC TURBINES

- In order to predict the behavior of a turbine working under varying conditions of head, speed, output and gate opening , the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected.

The three important unit quantities are:

1. Unit speed,
2. Unit discharge, and
3. Unit power

1. Unit Speed: it is defined as the speed of a turbine working under a unit head. It is denoted by (N_u). The expression of unit speed (N_u) is obtained as:

Let N = Speed of the turbine under a head H

H = Head under which a turbine is working

u = Tangential velocity.

The tangential velocity, absolute velocity of water and head on turbine are related as:

$u \propto V$ Where $V \propto \sqrt{H}$

$\propto \sqrt{H}$ ___________(1)

Also tangential velocity (*u*) is given by

$u = \frac{\pi D N}{60}$ Where D = Diameter of turbine.

For a given turbine, the diameter (D) is constant

$u \propto N$ Or $N \propto u$ $\quad N \propto \sqrt{H} \quad (\because From\ (1),\ u \propto \sqrt{H})$

$\therefore \quad N = K_1 \sqrt{H}$ ——— (2) Where K_1 is constant of proportionality.

If head on the turbine becomes unity, the speed becomes unit speed or

When $H = 1, \ N = N_u$

Substituting these values in equation (2), we get

$$N_u = K_1 \sqrt{1.0} = K_1$$

Substituting the value of K_1 in equation (2)

$$N = N_u \sqrt{H} \quad or \quad N_u = \frac{N}{\sqrt{H}} \quad \text{(I)}$$

2. Unit Discharge: It is defined as the discharge passing through a turbine, which is working under a unit head (i.e. 1 m). It is denoted by 'Q_u' the expression for unit discharge is given as:

Let H = head of water on the turbine

Q = Discharge passing through turbine when head is H on the turbine.

a = Area of flow of water

The discharge passing through a given turbine under a head *'H'* is given by, Q=Area of flow

Velocity $\times$

But for a turbine, area of flow is constant and velocity is proportional to $\sqrt{H}$

$$Q \propto velocity \propto \sqrt{H}$$

Or $Q = K_2\sqrt{H}$ ________(3)

Where K_2 is constant of proportionality

If H = 1, $Q = Q_u$ (By definition)

Substituting these values in equation (3) we get

$$Q_u = K_2\sqrt{1.0} = K_2$$

Substituting the value of K_2 in equation (3) we get

$$Q = Q_u\sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$ ______________**(II)**

3. Unit Power: It is defined as the power developed by a turbine working under a unit head (i.e. under a head of 1m). It is denoted by $'P_u'$. The expression for unit power is obtained as:

Let H= Head of water on the turbine
P= Power developed by the turbine under a head of H Q= Discharge through turbine under a head H

The overall efficiency (η_0) is given as

$$\eta_0 = \frac{Power\ developed}{Water\ power} = \frac{P}{\frac{pgQH}{1000}}$$

$$P = \eta_0 \times \frac{pgQh}{1000}$$

$$\propto\ Q \times H \quad \propto\ \sqrt{H} \times H \qquad (\because\ Q \propto \sqrt{H})$$

$$\propto\ H^{\frac{3}{2}}$$

$P = K_3 H^{3/2}$ __________ (4) Where K_3 is a constant of proportionality

When H=1 m, $P = P_u$

$$\therefore \quad P_u = K_3 (1)^{3/2} \quad = K_3$$

Substituting the value of K_3 in equation (4) we get

$$P = P_u H^{\frac{3}{2}}$$

$P_u = \frac{P}{H^{3/2}}$ ________________ **(III)**

UNIT QUANTITIES

Use of Unit Quantities $(N_u , : Q_u , P_u)$

If a turbine is working under different heads, the behaviour of the turbine can be easily known from the values of the unit quantities i.e. from the value of unit speed, unit discharge and unit power.

Let H_1, H_2, H_3, -------- are the heads under which a turbine works, N_1, N_2, N_3, --------- are the corresponding speeds,

Q_1, Q_2, Q_3, -------- are the discharge and

P_1, P_2, P_3, --------- are the power developed by the turbine.

Using equation I, II, III respectively,

$$\left.\begin{aligned} N_u &= \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}} = \frac{N_3}{\sqrt{H_3}} \\ Q_u &= \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}} = \frac{Q_3}{\sqrt{H_3}} \\ P_u &= \frac{P_1}{H_1{}^{3/2}} = \frac{P_2}{H_2{}^{3/2}} = \frac{P_3}{H_3{}^{3/2}} \end{aligned}\right\} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_(IV)}$$

Hence, if the speed, discharge and power developed by a turbine under a head are known, then by using equation (IV) the speed, discharge, power developed by the same turbine a different head can be obtained easily.

CHARACTERISTIC CURVES OF HYDRAULIC TURBINES:

- Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions can be known. These curves are plotted from the results of the tests performed on the turbine under different working conditions.

1) Speed (N)
2) Head (H)
3) Discharge (Q)
4) Power (P)
5) Overall Efficiency ( ) and
6) Gate opening.

CAVITATION :

Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid in a region, where the pressure of the liquid falls below its vapour pressure and the sudden collapsing of these vapour bubbles in a region of higher pressure.

Precaution against Cavitation:

•The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5m of water.

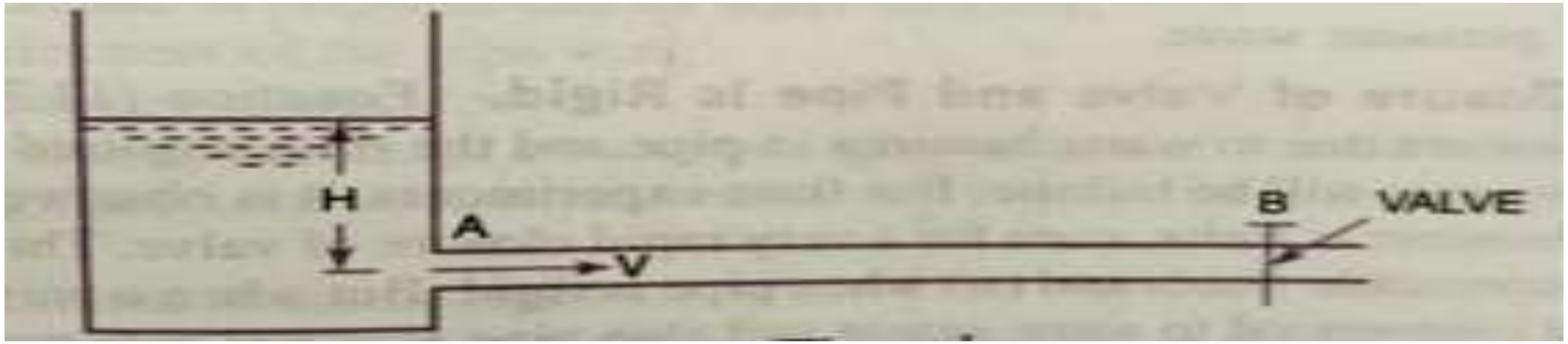
•The special materials or coatings such as Aluminum-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation

•The metallic surfaces are damaged and cavities are formed on the surfaces.
Due to sudden collapse of vapour bubbles, considerable noise and vibrations are produced.

•The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by the water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and efficiency decreases.

WATER HAMMER



The pressure rise due to water hammer depends up on:

1. Velocity of flow of water in pipe.
2. The length of pipe.
3. Time taken to close the valve.
4. Elastic properties of the material of the pipe.

The following cases of water hammer in pipes will be considered.

1. Gradual closure of valve
2. Sudden closure of valve considering pipe in rigid
3. Sudden closer of valve considering pipe elastic.

APPLICATIONS

- Pelton wheels are preferred turbine for hydro power, when the available water source has relatively high hydraulic head at low flow rates.
- Pelton wheels are made in all sizes. For maximum power and efficiency, the wheel and turbine system is designed such way that the water jet velocity is twice the velocity of the rotating buckets.
- There exist in multi ton Pelton wheel mounted on vertical oil pad bearing in hydroelectric power.
- Kaplan turbines are widely used throughout the world for electrical power production.
- Inexpensive micro turbines on Kaplan turbine model are manufactured for individual power production with as little as two feet of head.
- Large Kaplan turbines are individually designed for each site to operate at the highest possible efficiency., typically over 90%. They are very expensive to design, manufacture and install, but operate for decades.
- Francis turbines are used for pumped storage, where a reservoir is filled by the turbine (acting as a pump) driven by the generator acting as a large electrical motor during periods of low power demand.

ASSIGNMENT QUESTIONS

- A Pelton wheel is to be designed for the following specifications. Power= 735.75 kW S.P head=200m, Speed=800rpm, overall efficiency=0.86 and jet diameter is not to exceed one-tenth the wheel diameter. Determine: (i) Wheel diameter, (ii) the no of jets required and (iii) diameter of the jet. Take Cv=0.98 and speed ratio=0.45.
- A francis turbine with an overall efficiency of 70% is required to produce 147.15 kW. It is working under a head of 8m. The peripheral velocity=0.30 and the radial velocity of flow at inlet is 0.96. The wheel runs at 200 rpm and the hydraulic losses in the turbine are 20% of the available energy. Assume radial discharge, determine: (i) the guide blade angle, (ii) the wheel vane angle at inlet (iii) the diameter of wheel at inlet and (iv) width of wheel at inlet.
- One of the Kaplan turbine, installed at Ganguwal power house is rated at 25000 kW when working under 30 m of head at 180 rpm. Find the diameter of the runner, if the overall efficiency of the turbine is 0.91. Assume flow ratio of 0.65 and diameter of runner hub equal to 0.3 times the external diameter of runner. Also find specific speed of the turbine.

- A turbine develops 1000 kW under a head of 16 m at 200 rpm, while discharging 9 cubic metres of water/sec. Find the unit power and unit discharge of the wheel.
- A reaction turbine works at 500rpm under a head of 100m. The diameter of turbine at inlet is 100cm and flow area is 0.365m 2 . The angles made by absolute and relative velocities at inlet are 15
  0 and 60degrees respectively with the tangential velocity. Determine: (i) The volume flow rate (ii) the power developed (iii) efficiency. Assume the whirl velocity as zero and unit discharge of the wheel.

UNIT – V

Centrifugal Pumps:

- Classification, working- work done.
- Manometric head and efficiencies.
- Specific speed.
- Performance characteristic curves, NPSH.

Reciprocating Pumps:

- Working, discharge.
- Slip and Indicator diagram.

COURSE OUTLINE

LECTURE	LECTURE TOPIC	KEY ELEMENTS	Learning objectives
1	Introduction to Pumps	Classification of centrifugal pumps	Understand types of pumps (B2)
2	Centrifugal Pump- work done	Manometric Head & η	Evaluate η of pump (B5)
3	Specific Speed of Centrifugal Pump	Performance	Understand the performance of pump (B2)
4	Characteristic curves of pump NPSH	Performance	Understand the performance of pump (B2) Evaluate NPSH (B5)
5	Problems on Centrifugal pumps		
6	Reciprocating pumps	Working, Q & η	Evaluate η of pump (B5)
7	Slip & Indicator diagram	Positive displacement	Evaluate slip of pump (B5) Understand Indicator diagram (B2)
8	Problems on Reciprocating pumps		

INTRODUCTION TO CENTRIFUGAL PUMPS

- The hydraulic machines which convert the mechanical energy in to hydraulic energy are called pumps.
- The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted in to pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.
- The centrifugal pump acts as a reversed of an inward radial flow reaction turbine. This means that the flow in centrifugal pumps is in the radial outward directions.
- The centrifugal pump works on the principle of forced vertex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place.

MAIN PARTS OF A CENTRIFUGAL PUMP

- **Impeller:** The rotating part of a centrifugal pump is called impeller. It consists of a series of backward curved vanes.

➢ The impeller is mounted on a shaft which is connected to the shaft of an electric motor

- **Casing:** It is similar to the casing of a reaction turbine. It is an air tight passage surrounding the impeller and is designed in such a way that the kinetic energy of the water discharged at the outlet of the impeller is converted in to pressure energy before the water leaves the casing and enters the delivery pipe.
- The following three types of the casing are commonly adopted.

➢ Volute

➢ Vortex

➢ Casing with guide blades

- **Suction pipe with foot valve and a strainer**: A pipe whose one end is connected to the inlet of the pump and other end dips in to water in a sump is known as suction pipe.
- A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe.
- The foot valve opens only in the upward direction.
- A strainer is also fitted at the lower end of the suction pipe.
- **Delivery pipe:** A pipe whose one end is connected to the outlet of the pump and the other end delivers the water at the required height is known as delivery pipe.

HEADS OF A CENTRIFUGAL PUMPS

- **Suction Head $\boldsymbol{(h_s)}$** : It is the vertical height of the centre line of centrifugal pump, above the water surface in the tank or sump from which water is to be lifted. This height is also called suction lift $'h_s'$.
- **Delivery Head $\boldsymbol{(h_d)}$** : The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by $'h_d'$.
- **Static Head $\boldsymbol{(H_s)}$** : The sum of suction head and delivery head is known as statics head $'H_s'$.
- $$H_s = h_s + h_d$$
- **Manometric Head $\boldsymbol{(H_m)}$** : Manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by H_m.

a) H_m = Head imparted by the impeller to the water - Loss of head in the pump = $\frac{V_{w_2} u_2}{g}$ − Loss of head in impeller and casing

$= \frac{V_{w_2} u_2}{g}$ ... ... If loss of head in pump is zero.

b) H_m = Total head at outlet of pump - Total head at the inlet of the pump = $\left(\frac{P_0}{\rho g} + \frac{V_0^2}{2g} + Z_0\right) - \left(\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$

Where $\frac{P_0}{\rho g}$ = Pressure head at outlet of the pump = h_d

$\frac{V_0^2}{2g}$ = Velocity head at outlet of the pump

= Velocity head in delivery pipe $= \frac{V_d^2}{2g}$

Z_0 = Vertical height of the outlet of the pump from datum line, and

$\frac{P_i}{\rho g}, \frac{V_i^2}{2g}, Z_i$ = Corresponding values of pressure head, velocity head and datum head at the Inlet of the pump, i.e. h_s, $\frac{V_s^2}{2g}$ and Z_s respectively.

c) $H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$

Where h_s = Suction head,

h_d = Delivery head,

h_{f_s} = Frictional head loss in suction pipe,

h_{f_d} = Frictional head loss in delivery pipe

V_d = Velocity of water in delivery pipe.

EFFICIENCIES OF CENTRIFUGAL PUMP

- **Manometric Efficiency η_{man}** ( ): The ratio of the Manometric head to the head imparted by the impeller to the water is known as

$$Manometric\ Efficiency\ \eta\ (_{man}) = \frac{Manometric\ head}{Head\ imparted\ by\ impeller\ to\ water}$$

$$= \frac{H_m}{\left(\frac{V_{w2}u_2}{g}\right)} = \frac{gH_m}{V_{w2}u_2}$$

- **Mechanical Efficiency η_m** : The power at the shaft of the centrifugal pump is more the power available at the impeller of the pump. The ratio of the power available at the impeller to the power at the shaft of the centrifugal pump is known as mechanical efficiency.

$$\eta_m = \frac{Power\ at\ the\ impeller}{Power\ at\ the\ shaft}$$

- The power at the impeller in kW $= \frac{Work\ done\ by\ impeller\ per\ second}{1000}$

$$= \frac{W}{g} \times \frac{V_{w2} \times u_2}{1000}$$

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} \times u_2}{1000}\right)}{S.P}$$

Where S.P. = Shaft power.

- **Overall Efficiency** (η_0) **:** It is defined as the ratio of power output of the pump to the power input to the pump.

The power output of the pump in kW $= \frac{Weight\ of\ water\ lifted \times H_m}{1000} = \frac{WH_m}{1000}$

The power input to the pump = Power supplied by the electric motor

= S.P. Of the pump

$$\therefore \quad \eta_0 = \frac{\left(\frac{WH_m}{1000}\right)}{S.P.}$$

$$\eta_0 = \eta_{man} \times \eta_m$$

- The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90^0 with the direction of motion of the impeller at inlet.

- Hence angle $\alpha = 90^0$ and $V_{w_1} = 0$ for drawing the velocity triangles the same notations are used as that for turbines.
    - Let N = Speed of the impeller in r.p.m.

$D_1 =$ Diameter of impeller at inlet

$u_1 =$ Tangential velocity of impeller at inlet $= \frac{\pi D_1 N}{60}$

$D_2 =$ Diameter of impeller at outlet

$u_2 =$ Tangential velocity of impeller at outlet $= \frac{\pi D_2 N}{60}$

V_1 = Absolute velocity of water at inlet.

V_{r_1} = Relative velocity of water at inlet

α = Angle made by absolute velocity V_1 at inlet with the direction of motion of vane

θ = Angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane

And V_2, V_{r_2}, β and $\emptyset$ are the corresponding valves at outlet.

- As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha =$

90^0 and $V_{w_1} = 0$.

- A centrifugal pump is the reverse of a radially inward flow reaction turbine. But in case of a radially inward flow reaction turbine, the work done by the water on the runner per second per unit weight of the water striking per second is given by equation.

$$= \frac{1}{g}\left[V_{w_1}u_1 - V_{w_2}u_2\right]$$

∴Work done by the impeller on the water per second per unit weight of water striking/second

$$= -[\text{workdone in case of a turbine}]$$

$$= -\left[\frac{1}{g}\left(V_{w_1}u_1 - V_{w_2}u_2\right)\right]$$

$$= \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$= \frac{1}{g}V_{w_2}u_2 \qquad \text{———— (1)} \qquad (\because V_{w_1} = 0)$$

- Work done by the impeller on water per second= $\frac{W}{g} \times V_{w_2}u_2$

  Where W = Weight of water $= \rho \times g \times Q$, Q = Volume of water

Q = Area × Velocity of flow

$$= \pi D_1 B_1 \times V_{f_1}$$

$$= \pi D_2 B_2 \times V_{f_2}$$

- Where B_1 and B_2 are width of impeller at inlet and outlet and V_{f_1} And V_{f_2} are velocities of flow at inlet and outlet
- **Head imparted to the water by the impeller or energy given by impeller to water per unit weight per second**

$$\mathbf{H} = \frac{1}{g} V_{w_2} u_2$$

SPECIFIC SPEED OF CENTRIFUGAL PUMP

- The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump, which would deliver one cubic meter of liquid per second against a head of one meter.
- It is denoted by $'N_s'$.
- The discharge Q for a centrifugal pump is given by the relation

$$Q = Area \times Velocity\ of\ flow$$

$= \pi D \times B \times V_f$ Or Q a $D \times B \times V_f$ ______(1) Where D = Diameter of the impeller of the pump and

B = Width of the impeller

- We know that B a D
- From equation (1) we have Q a $D^2 \times V_f$ __________(2)

- We also know that the tangential velocity is given by

$$u = \frac{\pi D N}{60} \text{ a } DN \qquad \text{(3)}$$

- Now the tangential velocity (u) and velocity of flow V_f are related to Manometric head H_m as u a $(V_f$ a$)$ H_m $\sqrt{\quad}$ (4)

- Substituting the value of (u) in equation (3), we get

$$\sqrt{H_m} \text{ a } DN \quad \text{Or} \quad D \text{ a } \frac{\sqrt{H_m}}{N}$$

- Substituting the values of D in equation (2)

$$Q \text{ a } \frac{H_m}{N^2} \times V_f$$

$$\text{a } \frac{H_m}{N^2} \times \sqrt{H_m} \qquad [\because From\ eq\ (4)\ V_f \text{ a } \sqrt{H_m}]$$

$$\text{a } \frac{H_m^{3/2}}{N^2}$$

$$Q = K \frac{H_m^{3/2}}{N^2} \qquad \text{(5)}$$

Where K is a constant of proportionality

- If $H_m = 1m$ and $Q = 1m^3/sec$ N becomes N_s
- Substituting these values in equation (5), we get

$$1 = K \frac{1^{3/2}}{H_m} = \frac{K}{N_s^{\ 2}}$$

$$\therefore \qquad K = N_s^{\ 2}$$

- Substituting the value of K in equation (5), we get

$$Q = N_s^{\ 2} \frac{H_m^{\ 3/2}}{N^2} \ or \qquad N_s^{\ 2} = \frac{N^2 Q}{H_m^{\ 3/2}}$$

$$\boldsymbol{N_s = \frac{N\sqrt{Q}}{H_m^{\ 3/4}}}$$

MULTISTAGE CENTRIFUGAL PUMPS

1. TO PRODUCE HIGH HEADS

- If a high head is to be developed, the impellers are connected in series (or on the
  same shaft) while for discharging large quantity of liquid, the impellers (or pumps) are connected in parallel.

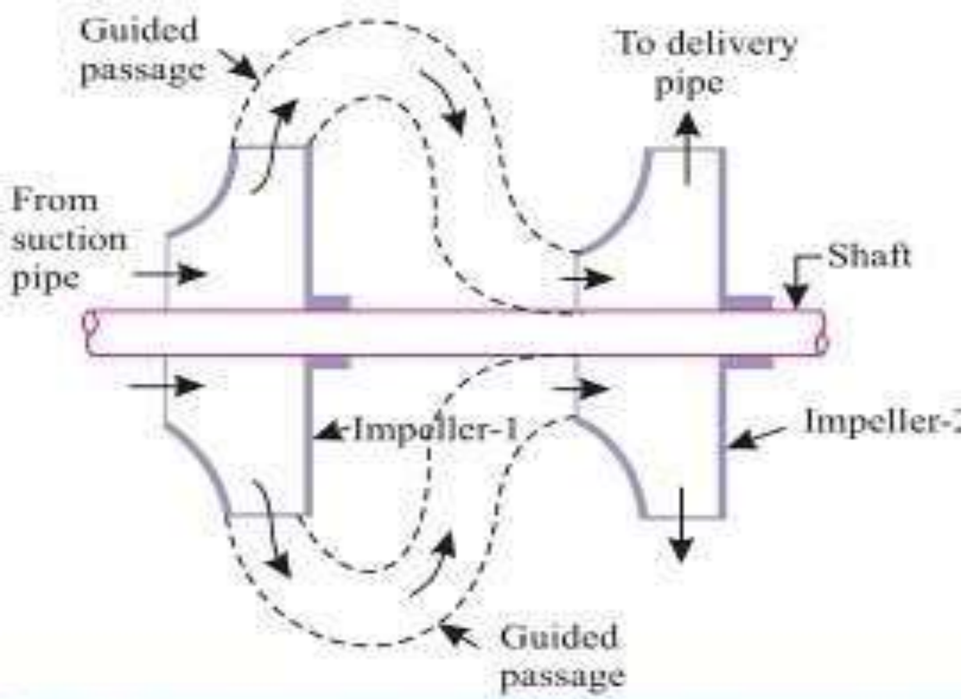


Fig. 3.23. Two-stage pump-impellers in series.

- For developing a high head, a number of impellers are mounted in series on the same shaft.
- The water from suction pipe enters the 1st impeller at inlet and is discharged at outlet with increased pressure.
- The water with increased pressure from the outlet of the 1st impeller is taken to the inlet of the 2nd impeller with the help of a connecting pipe.
- At the outlet of the 2nd impeller the pressure of the water will be more than the water at the outlet of the 1st impeller.
- Thus if more impellers are mounted on the same shaft, the pressure at the outlet will be increased further.
- Let n= Number of identical impellers mounted on the same shaft,

  $H_m =$ Head developed by each impeller.
- Then total Head developed $= n \times H_m$
- The discharge passing through each impeller is same.

MULTISTAGE CENTRIFUGAL PUMPS

2. TO PRODUCE HIGH DISCHARGE

- For obtaining high discharge, the pumps should be connected in parallel.
- Each of the pumps lifts the water from a common sump and discharges water to a common pipe to which the delivery pipes of each pump is connected.
- Each of the pumps is working against the same head.

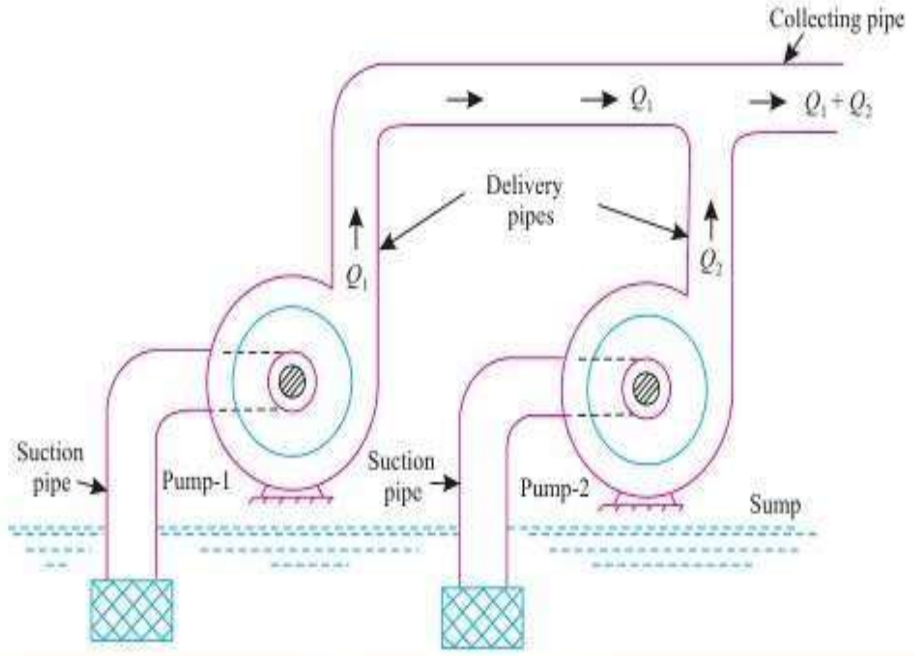


Fig. 3.24. Pumps in parallel.

- Let n = Number of identical pumps arranged in parallel.

  Q = Discharge from one pump.

  $\therefore$ Total Discharge $= \boldsymbol{n} \times \boldsymbol{Q}$

PERFORMANCE CHARACTERISTIC CURVES

- The characteristic curves of a centrifugal pump are defined as those curves which are plotted from the results of a number of tests on the centrifugal pump.

- These curves are necessary to predict performance of the behavior and the pump, when the pump is working under different flow rate, head and speed.
- The following are the important characteristic curves for the pumps:

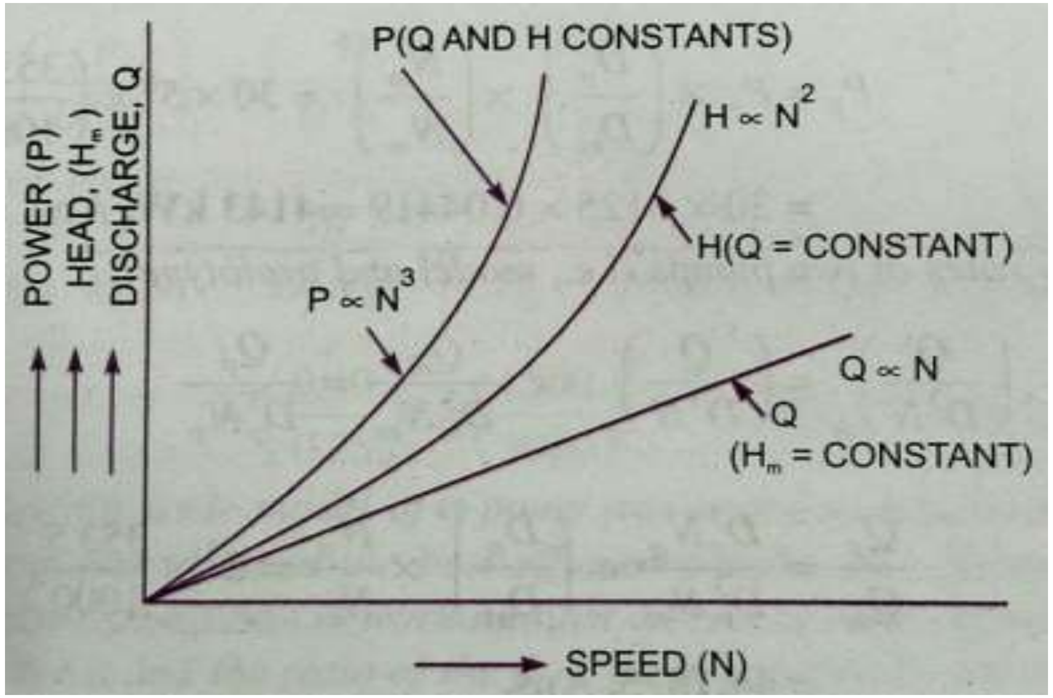
➢ Main characteristic curves.

➢ Operating characteristic curves and

➢ Constant efficiency or Muschel curves at different flow rate, head and speed.

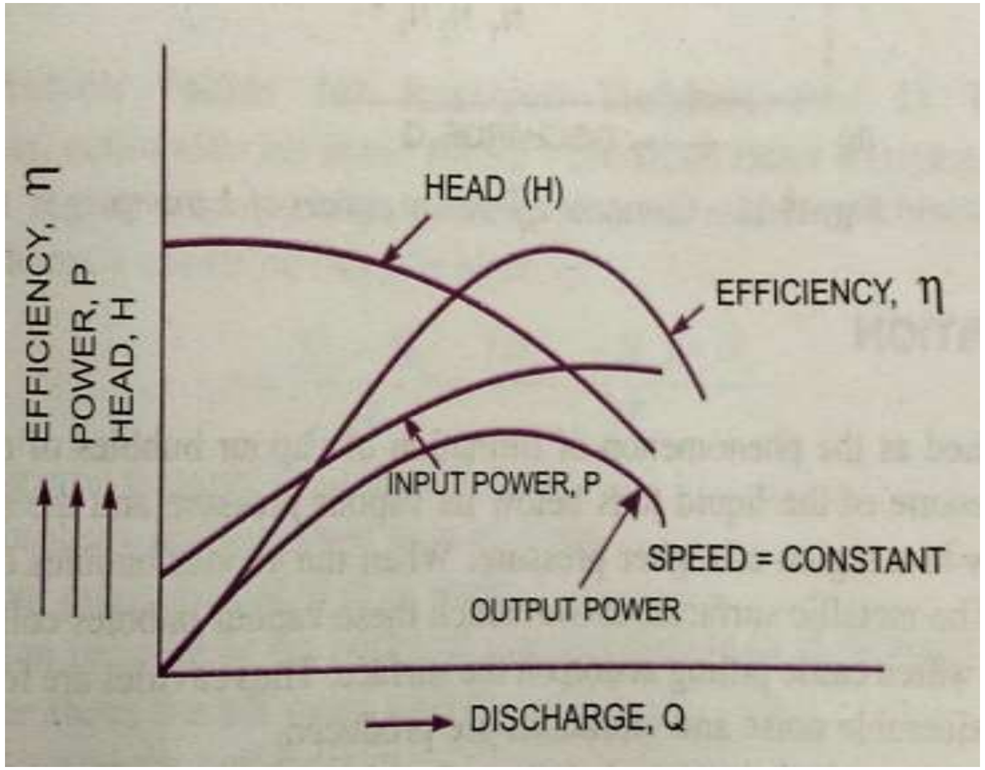
MAIN CHARACTERISTIC CURVES

- The main characteristic curves of a centrifugal pump consists of a head (Manometric head H_m) power and discharge with respect to speed.
- For plotting curves of Manometric head versus speed, discharge is kept constant. For plotting curves of discharge versus speed, Manometric head (H_m) is kept constant.
- For plotting curves power versus speed, Manometric head and discharge are kept constant.



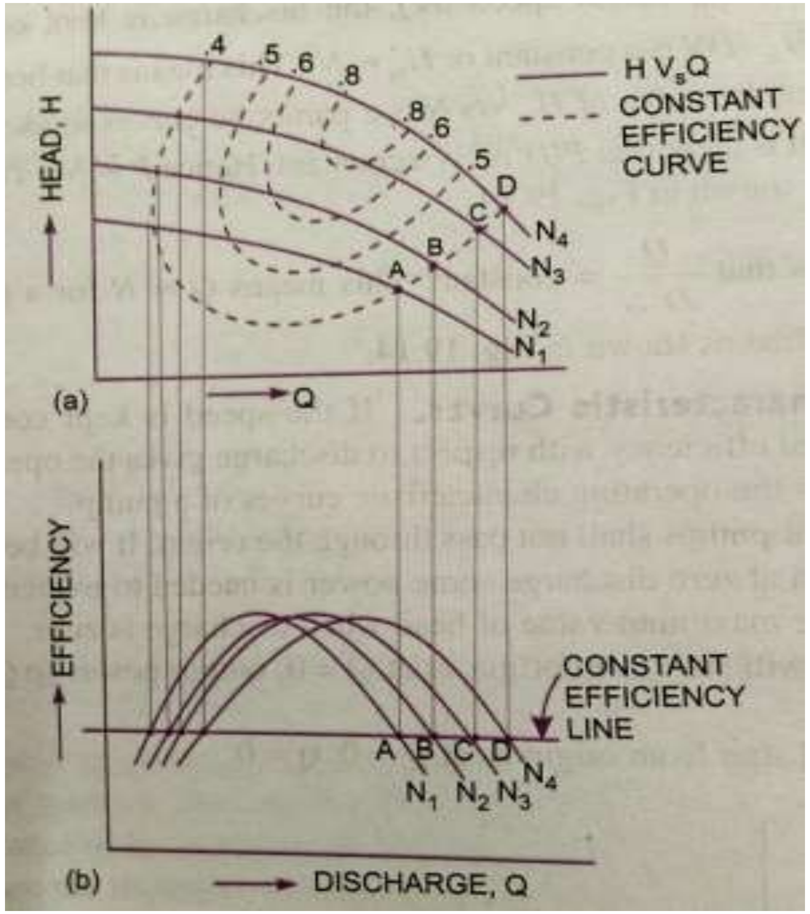
OPERATING CHARACTERISTIC CURVES

- If the speed is kept constant, the variation of Manometric head, power and efficiency with respect to discharge gives the operating characteristics of the pump.
- The input power curve for pumps shall not pass through the origin. It will be slightly away from the origin on the y-axis, as even at zero discharge some power is needed to overcome mechanical losses.
- The head curve will have maximum value of head when the discharge is zero.



CONSTANT EFFICIENCY OR MUSCHEL CURVES

- For obtaining constant efficiency curves for a pump, the head versus discharge curves and efficiency v/s discharge curves for different speeds are used.
- By combining these curves $(H \sim Q\ curves\ \eta \sim$



NET POSITIVE SUCTION HEAD (NPSH)

- The term NPSH is very commonly used selection of a pump. The minimum suction conditions are specified in terms NPSH.
- It is defined as the absolute pressure head at the inlet to the pump minus the vapour pressure head plus velocity head.

∴ NPSH = Absolute pressure head at inlet of pump - vapour pressure head (absolute units) + Velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \quad \text{—— (1)}$$

(∵

- $NPSH = \frac{p_a}{\rho g} - \frac{v_s^2}{2g} - h_s - h_{f_s} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

$$\left(\because \frac{p_a}{\rho g} = H_a = Atmospheric\ pressure\ head, \right)$$

$$= \left[\left(H_a - h_s - h_{f_s} - H_v \right) \right] \quad (2)$$

$$\because \frac{p_v}{\rho g} = H_v =$$

External diameter of the impeller, $D_2 = 450 \text{ mm} = 0.45 \text{ m}$

Speed of impeller, $N = 1440$ r.p.m.

Velocity of flow, $V_{f1} = V_{f2} = 2.5$ m/s

Vane angle at outlet, $\phi = 25°$

(*i*) Inlet vane angle, θ:

Tangential velocity of impeller at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1440}{60} = 15.08 \text{ m/s}$$

From velocity triangle at *inlet,* we have:

$$\tan\theta = \frac{V_{f1}}{u_1}, \text{ or, } \tan\theta = \frac{2.5}{15.08} = 0.1658$$

$$\therefore \quad \theta = \tan^{-1} 0.1658 = \mathbf{9.4° \ (Ans.)}$$

(*ii*) The angle, absolute velocity of water at exit makes with the tangent, β:

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0{\cdot}45 \times 1440}{60} = 33.93 \text{ m/s}$$

From velocity triangle at *outlet,* we have:

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\phi}, \text{ or, } V_{w2} = 33.93 - \frac{2.5}{\tan 25°} = 28.57 \text{ m/s}$$

Now,
$$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{2.5}{28.57} = 0.0875$$

$$\therefore \quad \beta = \tan^{-1} 0.0875 = \mathbf{5° \ (Ans.)}$$

(*iii*) Work done per N of water:

$$\textit{Work done per N of water} = \frac{V_{w2} u_2}{g} = \frac{28.57 \times 33.93}{9.81} = \mathbf{98.81 \text{ Nm} \ (Ans.)}$$

$$\therefore \quad V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times 0.05} = 3.0 \text{ m/s}$$

Manometric efficiency, $\eta_{mano} = \dfrac{gH_{mano}}{V_{w2} u_2}$

$$\text{or,} \quad 0.75 = \frac{9.81 \times 25}{V_{w2} \times 18.98}, \quad \text{or,} \quad V_{w2} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23 \text{ m/s}$$

From velocity triangle at *outlet*, we have:

$$\tan \phi = \frac{V_{f2}}{u_2 - V_{w2}} = \frac{3.0}{18.98 - 17.23} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = \mathbf{59.74^\circ} \ \textbf{(Ans.)}$$

or, $31.41 - V_{w2} = \frac{3.0}{\tan 40°}$, or, $V_{w_2} = 31.41 - \frac{3.0}{\tan 40°} = 27.83$ m/s

Substituting the values in eqn (*i*), we get the work done by the impeller

$$= \frac{9.81 \times 0.2356}{9.81} \times 27.83 \times 31.41 = \mathbf{205.95\ kNm}\ \textbf{(Ans.)}$$

$$(\because\ w = 9.81\ \text{kN/m}^3)$$

(*iii*) Manometric efficiency, (η_{mano}):

Manometric efficiency is given by eqn. (3.9) as:

$$\eta_{mano} = \frac{gH_{mano}}{V_{w2}u_2} = \frac{9.81 \times 48}{27.83 \times 31.41} = 0.5386 \text{ or } \mathbf{53.86\%}\ \textbf{(Ans.)}$$

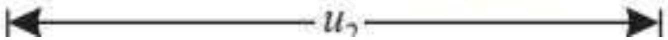
(*iii*) Angle made by the absolute velocity at outlet with the direction of motion, β:

From velocity triangle at *outlet*, we have:

$$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{5.05}{14.08} = 0.3586 \quad \therefore\ \beta = \tan^{-1}(0.3586) = \mathbf{19.7°}\ \textbf{(Ans.)}$$

(*iv*) Rate of flow through the pump, Q:

$$Q = \pi D_2 B_2 \times V_{f2} = \pi \times 0.48 \times 0.06 \times 5.05 = \mathbf{0.457\ m^3/s}\ \textbf{(Ans.)}$$



From velocity triangle at *inlet*, we have:

$$\tan\theta = \frac{V_{f1}}{u_1} = \frac{2.5}{8.455} = 0.2957$$

$\therefore$ $$\theta = \tan^{-1}(0.2957) = \mathbf{16.47^\circ}\ \textbf{(Ans.)}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.48 \times 1000}{60} = 25.13 \text{ m/s}$$

From velocity triangle at outlet (Fig. 3.11), we have:

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}, \text{ or, } \tan 45° = \frac{2.4}{25.13 - V_{w2}}, \text{ or, } 25.13 - V_{w2} = \frac{2.4}{\tan 45°} = 2.4$$

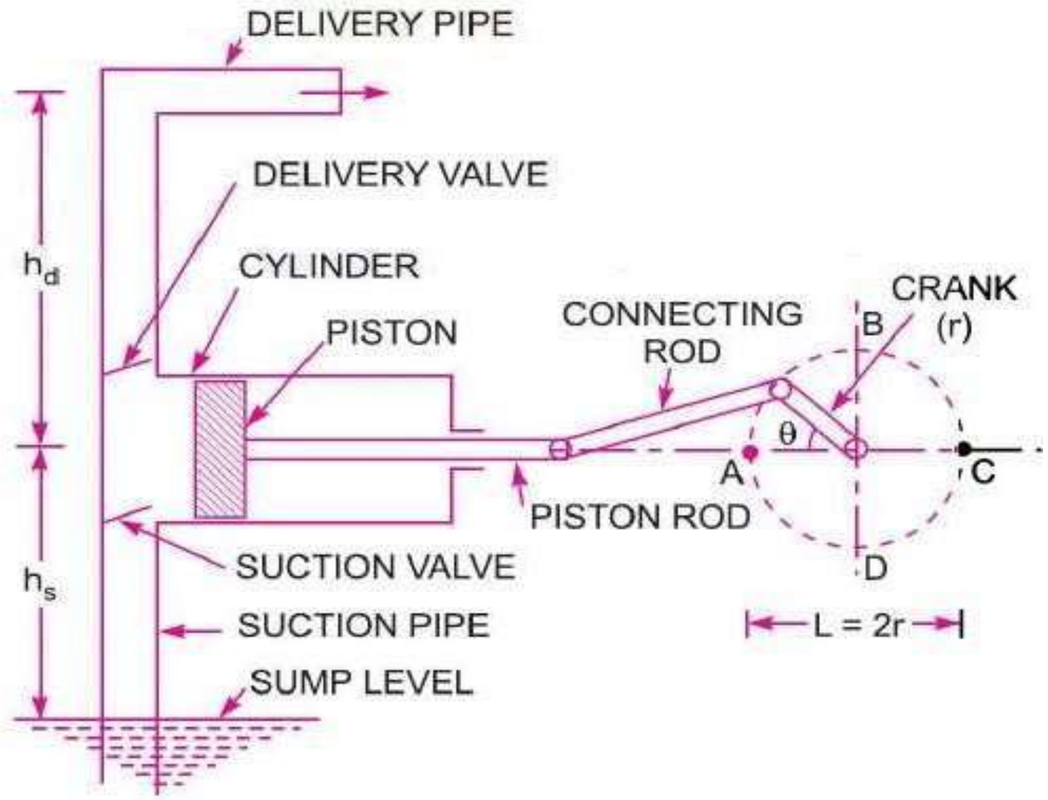
$$\therefore \quad V_{w2} = 25.13 - 2.4 = 22.73 \text{ m/s}$$

Substituting the values in the above equation, we get:

$$\eta_{mano} = \frac{9.81 \times 25.03}{22.73 \times 25.13} = 0.43 \text{ or } \mathbf{43\%} \ \textbf{(Ans.)}$$

RECIPROCATING PUMPS

- The mechanical energy is converted in to hydraulic energy (pressure energy) by sucking the liquid in to a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy) the pump is known as reciprocating pump.



- A single acting reciprocating pump consists of a piston, which moves forwards and backwards in a close fitting cylinder.
- The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod.
- The crank is rotated by means of an electric motor.
- Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder.
- The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only.
- Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe only.

- When the crank starts rotating, the piston moves to and fro in the cylinder. When the crank is at A the piston is at the extreme left position in the cylinder.
- As the crank is rotating from A to C (i.e. from $\theta = 0 \ to \ 180^0$) the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder.
- But on the surface of the liquid in the sump atmospheric pressure in acting, which is more than the pressure inside the cylinder.
- Thus the liquid is forced in the suction pipe from the sump.
- This liquid opens the suction valve and enters the cylinder.

- When crank is rotating from C to A (i.e. from $\theta = 180^0 \, to \, 360^0$), the piston from its extreme right position starts moving towards left in the cylinder.
- The movement of the piston towards the left increases the pressure on the liquid inside the cylinder more than atmospheric pressure.
- Hence the suction valve closes and delivery valve opens.
- The liquid is forced in to the delivery pipe and is raised to the required height.

DISCHARGE THROUGH A RECIPROCATING PUMP

- Consider a single acting reciprocating pump.

Let D = Diameter of cylinder

A = Cross-sectional area of piston or cylinder $= \frac{\pi}{4} D^2$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke $= 2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

- Volume of water delivered in one revolution or Discharge of water in one revolution = Area × Length of stroke $= A \times L$

- Number of revolutions per second = $\frac{N}{60}$
- Discharge of pump per second Q = Discharge in one revolution ×

  No. of revolutions per sec $= A \times L \times \frac{N}{60}$

  $$= \frac{ALN}{60}$$

- Weight of water delivered per second $W = \rho \times g \times Q$

  $$= \frac{\rho g A L N}{60} \qquad \text{———— (1)}$$

WORK DONE BY RECIPROCATING PUMP

- Work done per second = Weight of water lifted per second ×Total height through which water is lifted = $W \times (h_s + h_d)$ [(2)]

  Where $(h_s + h_d)$ = Total height through which water is lifted

- From equation (1) weight of water is given by

$$W = \frac{\rho g A L N}{60}$$

- Substituting the value of W in equation (2), we get
- Work done per second $= \frac{\rho g A L N}{60} \times (h_s + h_d)$
- Power required to drive the pump in kW

$$P = \frac{Work\ done\ per\ second}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$P = \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000}\ kW$$

SLIP OF RECIPROCATING PUMP

- Slip of a pump is defined as the difference between the theoretical discharge and actual discharge of a pump.
- The actual discharge of pump is less than the theoretical discharge due to leakage.
- The difference of the theoretical discharge and actual discharge is known as slip of the pump.
- Hence $slip = Q_{th} - Q_{act}$
- But slip is mostly expressed as percentage slip

$$Percentage\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \quad \because \left(\frac{Q_{act}}{Q_{th}} = C_d\right)$$

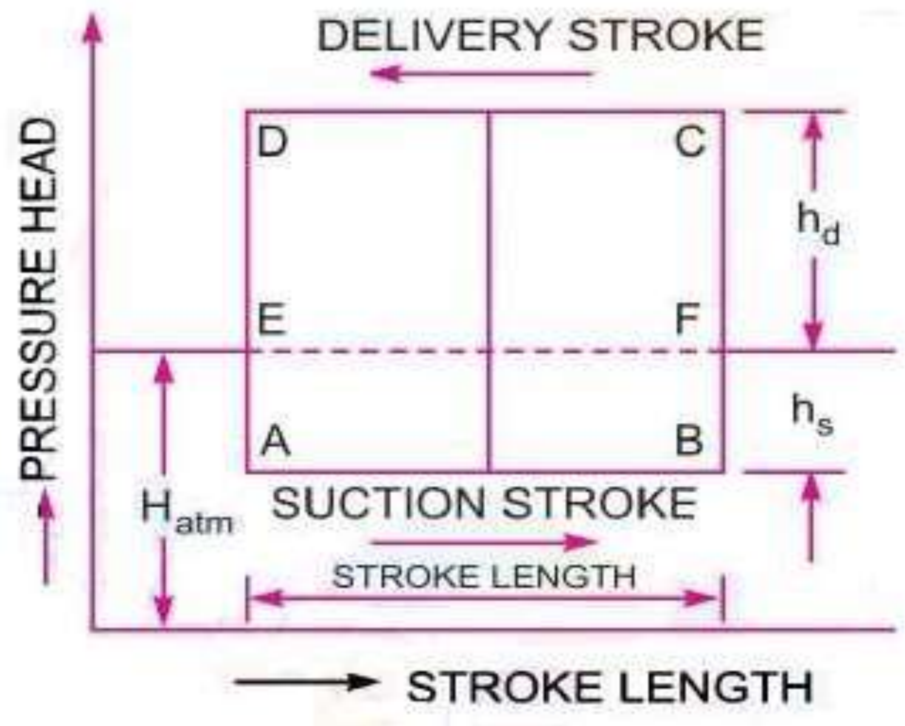
Where C_d = Co-efficient of discharge.

NEGATIVE SLIP OF THE RECIPROCATING PUMP

- Slip is equal to the difference of theoretical discharge and actual discharge.
- If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve.
- In that case the slip of the pump is known as negative slip.
- Negative slip occurs when the delivery pipe is short, suction pipe is long and pump is running at high speed.

INDICATOR DIAGRAM

- The indicator diagram for a reciprocating pump is defined the graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for one complete revolution of the crank.
- As the maximum distance travelled by the piston is equal to the stroke length and hence the indicator diagram is a graph between pressure head and stroke length of the piston for one complete revolution.
- The pressure head is taken as ordinate and stroke length as abscissa.



IDEAL INDICATOR DIAGRAM

- The graph between pressure head in the cylinder and the stroke length of piston for one complete revolution of the crank under ideal conditions is known as ideal indicator diagram.
- Line EF represents the atmospheric pressure head equal to 10.3 meters of water.
- Let $H_m =$ Atmospheric pressure head = 10.3 m of water L = Length of the stroke

  $h_s =$ Suction head and

  $h_d =$ Delivery head
- During suction stroke, the pressure head in the cylinder is constant and equal to suction head h_s , which is below the atmospheric pressure head H_{atm} by a height of h_s.

( )

( )

- The pressure head during suction stroke is represented by a horizontal line AB which is below the line EF by a height of $'h_s'$
- During delivery stroke, the pressure head in the cylinder is constant and equal to delivery head (h_d), which is above the atmospheric head by a height of $'h_d'$.
- Thus the pressure head during the delivery stroke is represented by a horizontal line CD, which is above the line EF by a height of h_d.
- Thus for one complete revolution of crank, the pressure head in the cylinder is represented by the diagram ABCD.
- This diagram is known as ideal indicator diagram.

- The work done by the pump per second $= \frac{\rho g A L N}{60} \times (h_s + h_d)$

$$= K \times L (h_s + h_d)$$

$$\propto L \times (h_s + h_d) \quad \text{\_\_\_\_(1)}$$

Where $K = \frac{\rho g A N}{60} = constant$

- Area of Indicator diagram $= AB \times BC = AB \times (BF + FC) = L \times (h_s + h_d)$
- Substituting this value in equation (1), we get

Work done by pump $\propto$ **Area of Indicator diagram**

PROBLEM 1

- A single acting reciprocating pump running at 50 r.p.m. delivers $0.01\ m^3/s$ of water. The diameter of piston is 200mm and stroke length in 400mm. Determine **i)** The theoretical discharge of pump **ii)** Co-efficient of discharge **iii)** Slip and the percentage slip of pump.

Sol: Given data

The speed of the pump, N = 50 rpm Actual discharge,

$Q_{act} = 0.01\ m^3/s$ Dia. Of piston, D = 200mm = 0.2m

Area, $A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.2)^2 = 0.031416\ m^2$

i) The theoretical discharge, $Q_{th} = \frac{ALN}{60} = \frac{0.031416 \times 0.4 \times 50}{60}$

$= \mathbf{0.01047 m^3/s}$

ii) The Co-efficient of discharge, $C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047}$

$= \mathbf{0.955}$

iii) Slip, $Q_{th} - Q_{act} = 0.01047 - 0.01$

$= \mathbf{0.00047 m^3/s}$

Percentage Slip = $\left(\frac{Q_{th}-Q_{act}}{Q_{th}}\right) \times 100 = \frac{0.01047-0.01}{0.01047} \times 100$

$= \mathbf{4.489\%}$

PROBLEM 2

- A double acting reciprocating pump, running at 40 r.p.m. is discharging 1.0m3 of water per minute. The pump has a stroke of 400 mm. the diameter of piston is 200 mm. the delivery and suction head are 20m and 5m respectively. Find the slip of the pump and power required to drive the pump.

Sol: Given data,

Speed of the pump, N = 40 r.p.m.

Actual discharge, $Q_{act} = 1\,m^3/s = \frac{1}{60} = 0.01666 m^3/s$

Stroke, L = 400mm = 0.4m

Diameter of piston, D = 200 mm = 0.2m

∴ $Area, A = \frac{\pi}{4} D^2 = \frac{\pi}{4} - (0.2^2) = 0.031416 m^2$

Suction Head, $h_s = 5m$

Delivery head, $h_d = 20\ m$

- Theoretical discharge for double acting pump

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times 0.31416 \times 0.4 \times 40}{60}$$

$$= \mathbf{0.01675 m^3/s}$$

- Slip= $Q_{th} - Q_{act} = 0.01675 - 0.1666$

$$= \mathbf{0.00009 m^3/s}$$

- Power required to drive the double acting pump

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

$$= \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (5+20)}{60{,}000}$$

$$= \mathbf{4.109 kW}$$

APPLICATIONS

- Centrifugal pumps are used in variety of applications. Almost 70- 80% centrifugal pumps are used in industry or for domestic purpose.
- Water supply and irrigation
- Chemical, food, Petrochemical industries
- Mining, domestic appliances
- Reciprocating pumps are mainly used in oil and gas industry.
- They can also be used sugar industries, soap and detergent industries, water treatment plants and Food & beverages.
- Have huge demand in cryogenic applications.

ASSIGNMENT QUESTIONS

- A centrifugal pump delivers water to a height of 22 m at a speed of 800 rpm. The velocity of flow is constant at a speed of 2m/s and the outlet vane angle is 450. If the pump discharges 225 litres of water / second, find the diameter of the impeller and width of the impeller.
- A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200 rpm works against a total head of 75m. The velocity of flow through the impeller is constant and is equal to 3m/s. The vanes are set back at an angle of 300 at outlet. If the outer diameter of the impeller is 600 mm and width at outlet is 50 mm, determine (i) vane angle at inlet (ii) work done per second by impeller (iii) manometric efficiency.
- What is the working principle of a reciprocating pump? Explain its working with the help of an indicator diagram.

- A single acting reciprocating pump having cylinder diameter of 150 mm and stroke 300 mm is used to raise water through a total height of 30m. Find the power required to drive the pump, if the crank rotates at 60 rpm.
- A fluid is to be lifted against a head of 120m. The pumps that run at a speed of 1200 rpm with rated capacity of 300 litres/sec are available. How many pumps are required to pump the water if specific speed is 700.