DYNAMICS OF MACHINERY (ME3101PC)

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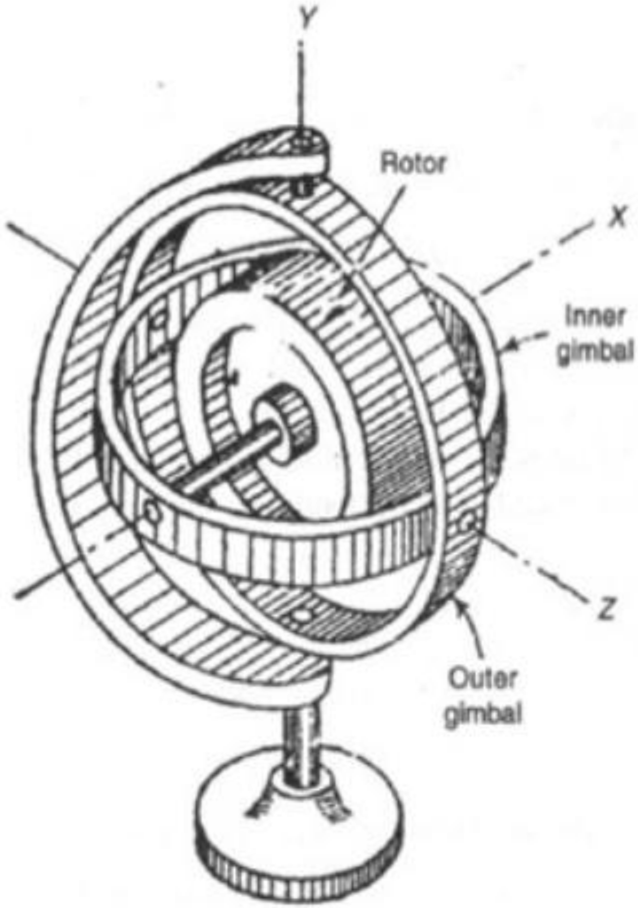
Gyroscope

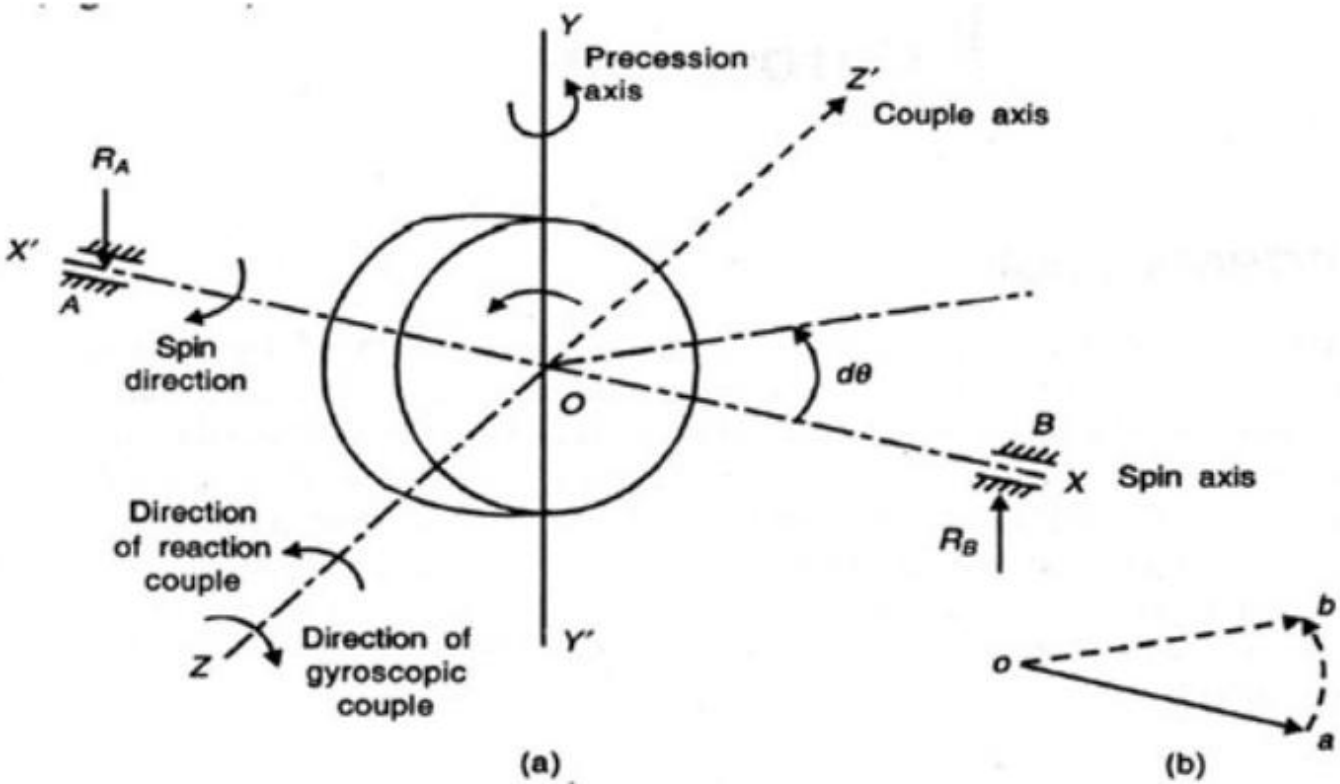
A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance to this motion is called gyroscopic effect.

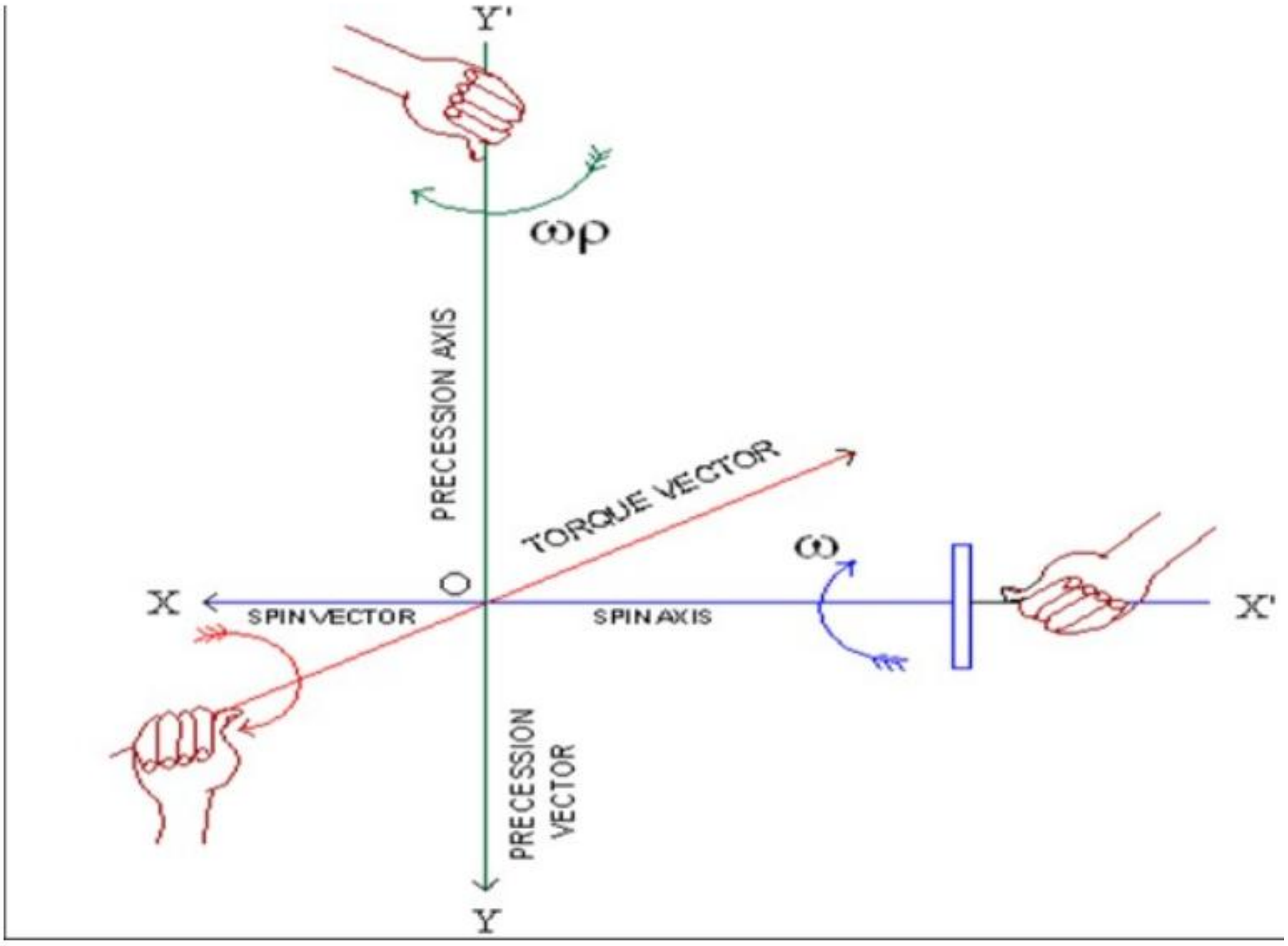


Gyroscope mechanism

GYROSCOPIC COUPLE

Consider a rotary body of mass *m having radius of gyration k mounted on the shaft* supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis .




Y'
ωρ
PRECESSION AXIS
TORQUE VECTOR
ω
O
X
SPIN VECTOR
SPIN AXIS
X'
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VECTOR
Y

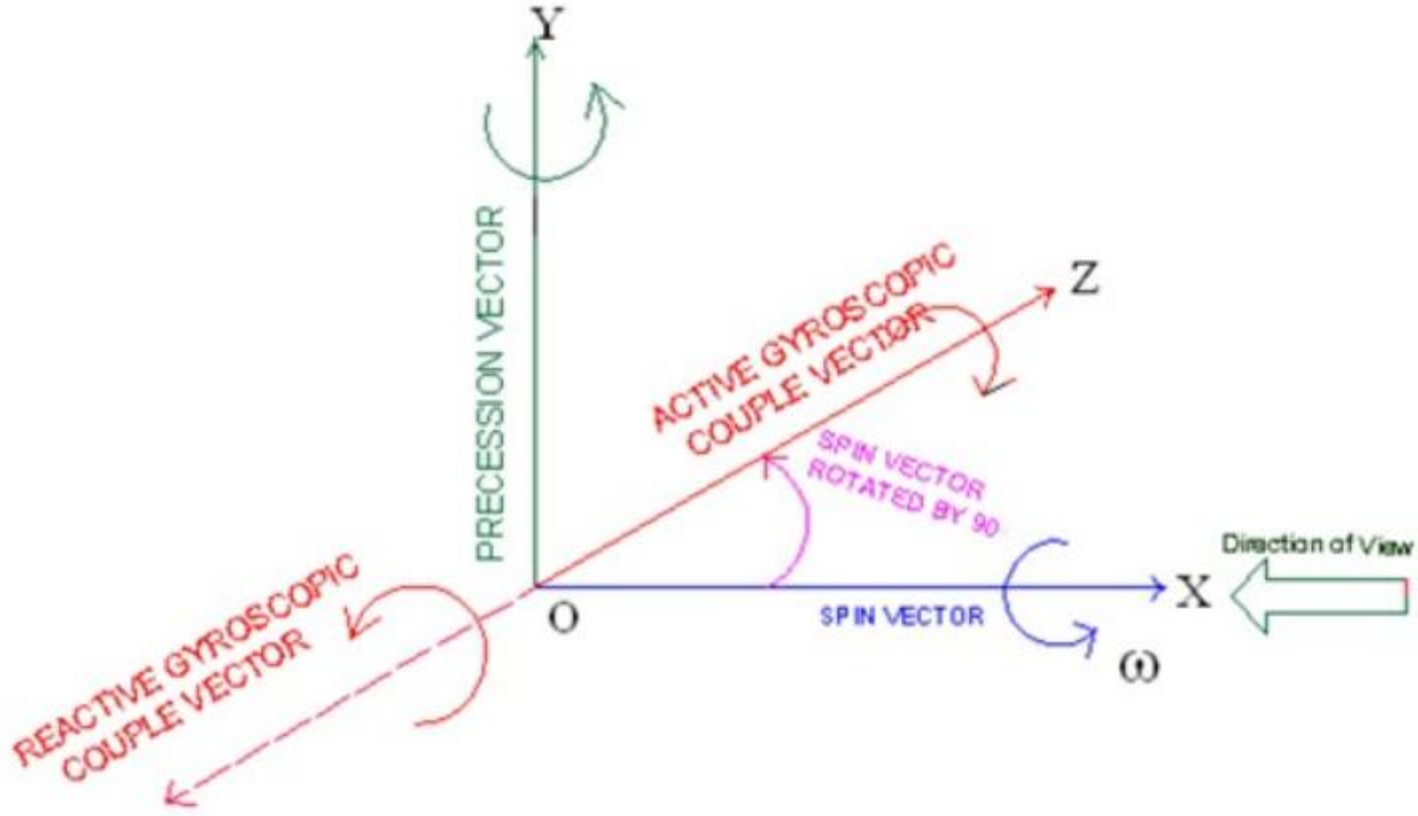


Fig. 5 Direction of active and reactive gyroscopic couple/torque vector

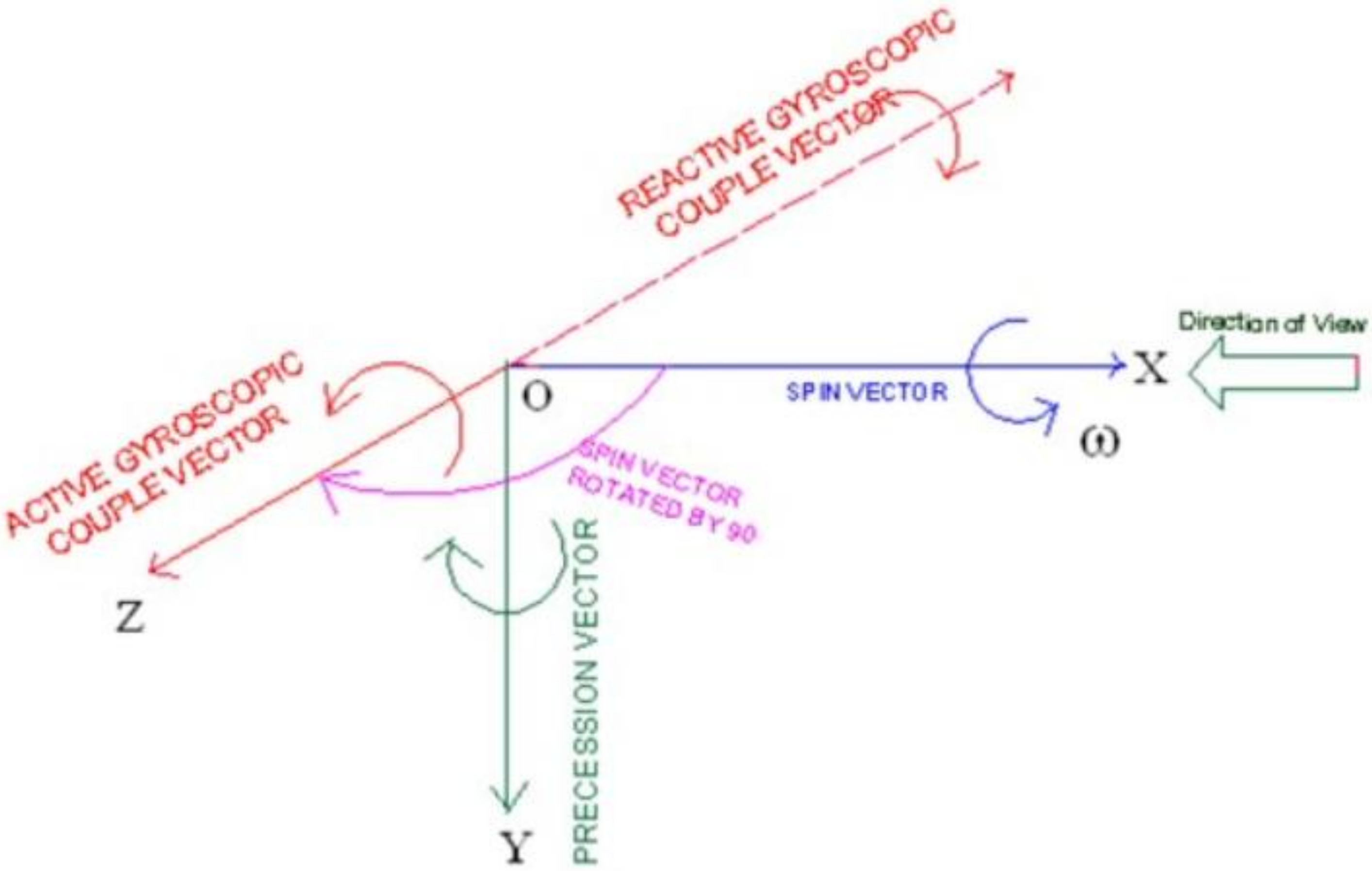


Fig. 6 Direction of active and reactive gyroscopic couple/torque vector

Problem 1

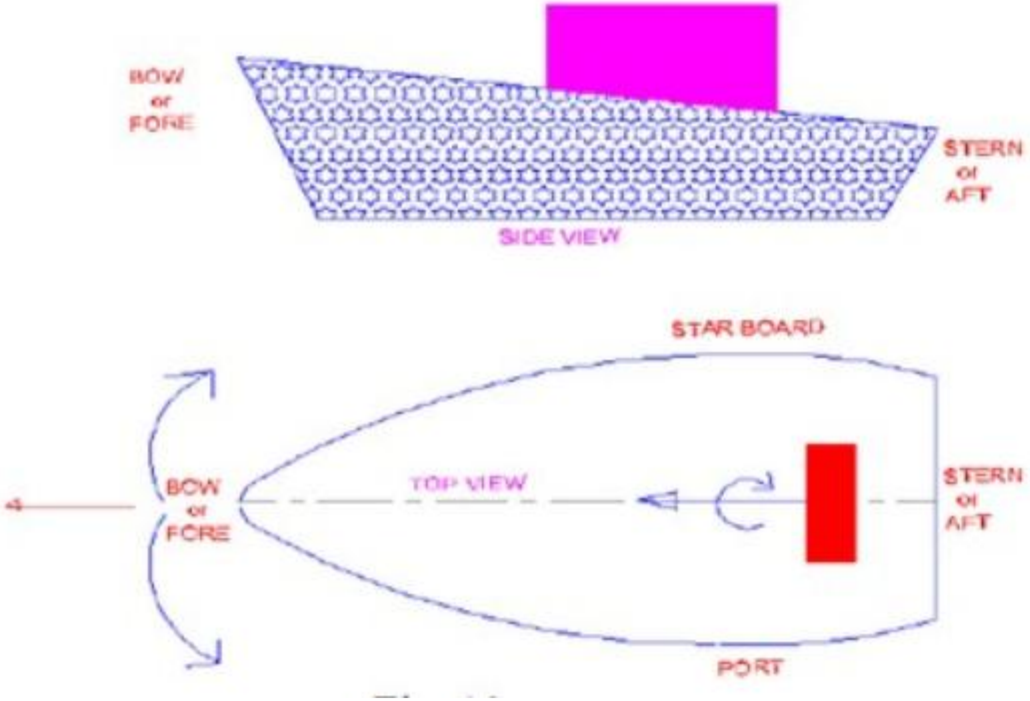
A disc of 5 kg mass with radius of gyration 70 mm is mounted at span on a horizontal shaft spins at 720 rpm in clockwise direction when viewed from the right hand bearing. If the shaft precesses about the vertical axis at 30 rpm in clockwise direction when viewed from the top, determine the reactions at each bearing due to mass of the disc and gyroscopic effect.

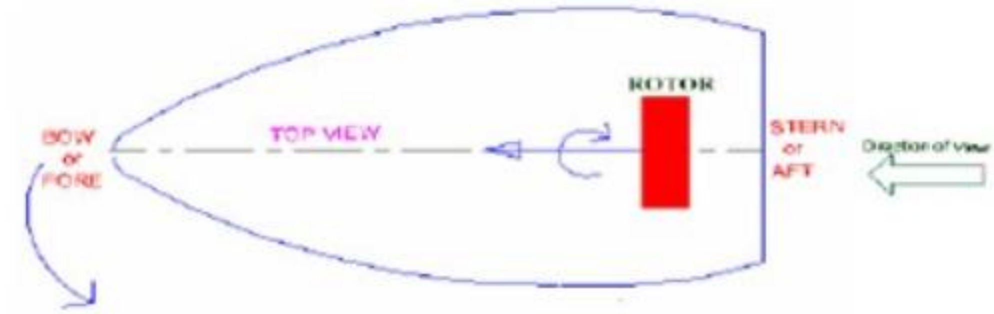
GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion:

(i) Steering—The turning of ship in a curve while moving forward
(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis
(iii) Rolling—Sideway motion of the ship about longitudinal axis.

(i) Bow – It is the fore end of ship
(ii) Stern – It is the rear end of ship
(iii) Starboard – It is the right hand side of the ship looking in the direction of motion
(iv) Port – It is the left hand side of the ship looking in the direction of motion




ROTOR
BOW
or
FORE
TOP VIEW
STERN
or
AFT
Direction of View

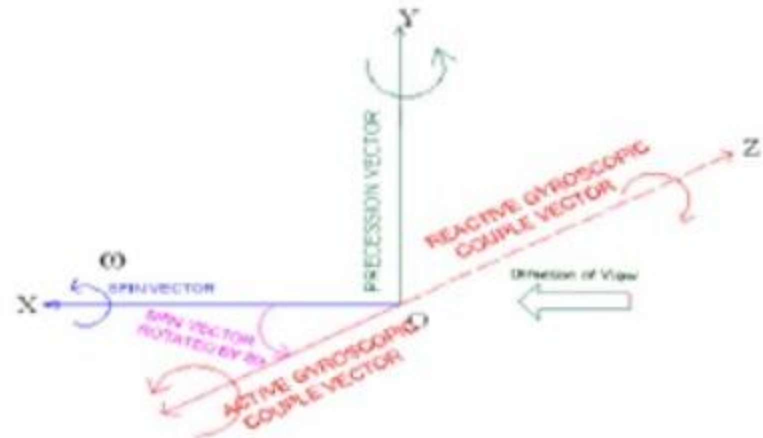
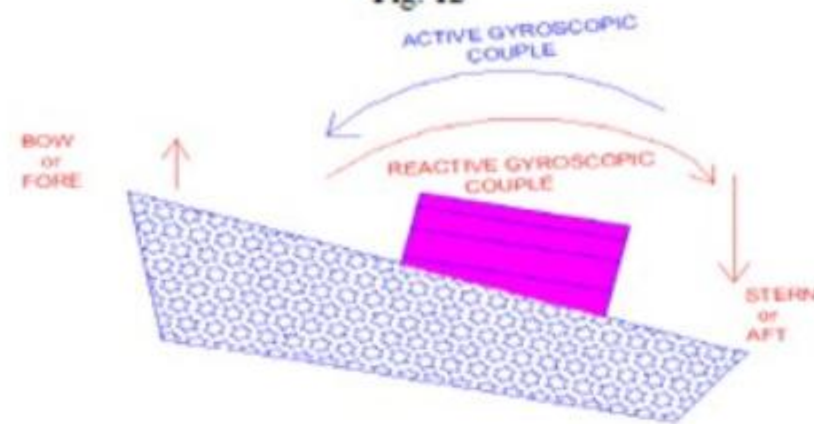

Y
Z
X
ω
O
PRECESSION VECTOR
REACTIVE GYROSCOPIC
COUPLE VECTOR
SPIN VECTOR
Direction of View
SPIN VECTOR
ROTATED BY 90
ACTIVE GYROSCOPIC
COUPLE VECTOR

Fig. 12


ACTIVE GYROSCOPIC
COUPLE
BOW
or
FORE
REACTIVE GYROSCOPIC
COUPLE
STERN
or
AFT

Problem 2

A turbine rotor of a ship has a mass of 3500 kg and rotates at a speed of 2000 rpm. The rotor has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from the stern (rear) end. Determine the magnitude of gyroscopic couple and its direction for the following conditions

(i) When the ship runs at a speed of 12 knots and steers to the left in a curve of 70 m radius
(ii) When the ship pitches 6° above and 6° below the horizontal position and the bow (Front) end is lowered. The pitching motion is simple harmonic with periodic time 30 sec.
(iii) When the ship rolls and at a certain instant, it has an angular velocity of 0.05 rad/s clockwise when viewed from the stern

Also find the maximum angular acceleration during pitching.

Problem 3

A ship is propelled by a rotor of mass of 2000 kg rotates at a speed of 2400 rpm. The radius of gyration of rotor is 0.4 m and spins clockwise direction when viewed from bow (front) end. Find the gyroscopic couple and its effect when:

(i) the ship takes left turn at a radius of 350 m with a speed of 35 kmph
(ii) the ship pitches with the bow rising at an angular velocity of 1 rad/s
(iii) the ship rolls at an angular velocity of 0.15 rad/s

Static force analysis.

If components of a machine accelerate, inertia is produced due to their masses. However, the magnitudes of these forces are small compares to the externally applied loads. Hence inertia effect due to masses are neglected. Such an analysis is known as static force analysis

- **What is inertia?**
- The property of matter offering resistance to any change of its state of rest or of uniform motion in a straight line is known as inertia.

conditions for a body to be in static and dynamic equilibrium?

- Necessary and sufficient conditions for static and dynamic equilibrium are
- Vector sum of all forces acting on a body is zero
- The vector sum of the moments of all forces acting about any arbitrary point or axis is zero.

Static force analysis and dynamic force analysis.

- If components of a machine accelerate, inertia forces are produced due to their masses. If the magnitude of these forces are small compared to the externally applied loads, they can be neglected while analysing the mechanism. Such an analysis is known as static force analysis.
- If the inertia effect due to the mass of the component is also considered, it is called dynamic force analysis.

D'Alembert's principle.

- D'Alembert's principle states that the inertia forces and torques, and the external forces and torques acting on a body together result in statical equilibrium.
- In other words, the vector sum of all external forces and inertia forces acting upon a system of rigid bodies is zero. The vector sum of all external moments and inertia torques acting upon a system of rigid bodies is also separately zero.

- **The principle of super position** states that for linear systems the individual responses to several disturbances or driving functions can be superposed on each other to obtain the total response of the system.
- The velocity and acceleration of various parts of reciprocating mechanism can be determined , both analytically and graphically.

Dynamic Analysis in Reciprocating Engines-Gas Forces

- **Piston efforts (F_p):** Net force applied on the piston , along the line of stroke In horizontal reciprocating engines.It is also known as effective driving force (or) net load on the gudgeon pin.

crank-pin effort.

- The component of F_Q perpendicular to the crank is known as crank-pin effort.

crank effort or turning movement on the crank shaft?

- It is the product of the crank-pin effort (F_T)and crank pin radius(r).

Forces acting on the connecting rod

- Inertia force of the reciprocating parts (F_1) acting along the line of stroke.
- The side thrust between the cross head and the guide bars acting at right angles to line of stroke.
- Weight of the connecting rod.
- Inertia force of the connecting rod (F_C)
- The radial force (F_R) parallel to crank and
- The tangential force (F_T) acting perpendicular to crank

15.12. Determination of Equivalent Dynamical System of Two Masses by Graphical Method

Consider a body of mass m, acting at G as shown in Fig. 15.15. This mass m, may be replaced by two masses m_1 and m_2 so that the system becomes dynamical equivalent. The position of mass m_1 may be fixed arbitrarily at A. Now draw perpendicular CG at G, equal in length of the radius of gyration of the body, k_G. Then join AC and draw CB perpendicular to AC intersecting AG produced in B. The point B now fixes the position of the second mass m_2.

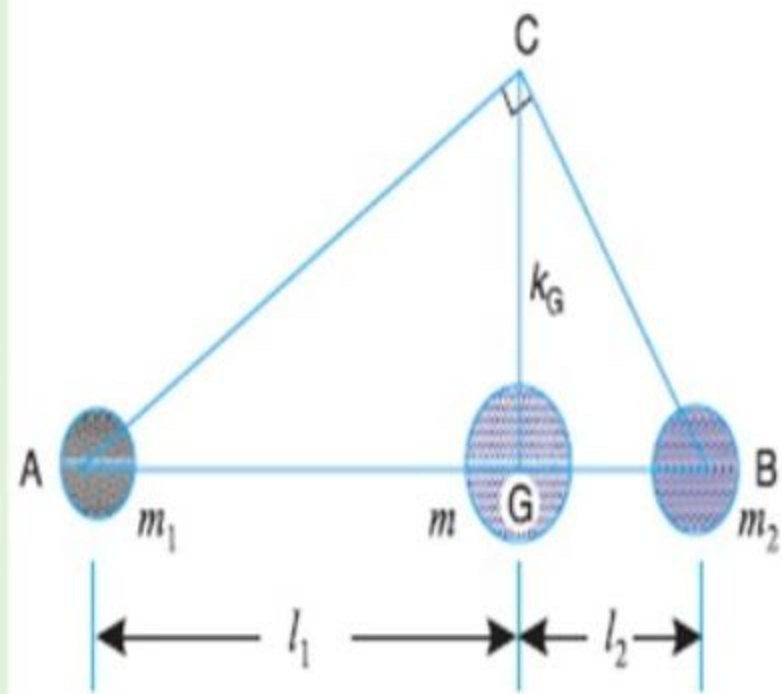


Fig. 15.15. Determination of equivalent dynamical system by graphical method.

A little consideration will show that the triangles ACG and BCG are similar. Therefore,

$$\frac{k_G}{l_1} = \frac{l_2}{k_G} \quad \text{or} \quad (k_G)^2 = l_1 . l_2$$

...(Same as before)

- **Determination of Equivalent Dynamical System of Two Masses by Graphical Method**
- Consider a body of mass *m, acting at G as*
- shown in fig 15.15. This mass *m, may be replaced*
- by two masses *m1 and m2 so that the system becomes* dynamical equivalent. The position of mass *m1 may* be fixed arbitrarily at *A. Now draw perpendicular CG at G, equal in length of the radius of gyration of* the body, *kG .Then join AC and draw CB* perpendicular to *AC intersecting AG produced in*
- *B. The point B now fixes the position of the second*
- mass *m2. The* triangles *ACG and BCG are similar. Therefore,*

Engine force Analysis:

Forces acting on the engine are weight of reciprocating masses & CR, gas forces, Friction & inertia forces (due to acceleration & retardation of engine elements)

i) Piston effort (effective driving force)

- Net or effective force applied on the piston.

In reciprocating engine:

The reciprocating parts (masses) accelerate during the first half of the stroke and the inertia forces tend to resist the same. Thus, the net force on the piston is reduced. During the later half of the stroke, the reciprocating masses decelerate and the inertia forces oppose this deceleration or acts in the direction of applied gas pressure and thus effective force on piston is increased.

In vertical engine, the weights of the reciprocating masses assist the piston during out stroke (down) this in creasing the piston effort by an amount equal to the weight of the piston. During the in stroke (up) piston effect is decreased by the same amount.

Force on the piston due to gas pressure; $F_P = P_1 A_1 - P_2$

P_1 = Pressure on the cover end, P_2 = Pressure on the rod
A_1 = area of cover end, A_2 = area of rod end, m = mass of the reciprocating parts.

Inertia force $(F_i) = m\ a$

$$= m.r\omega^2\left(Cos\theta + \frac{Cos2\theta}{n}\right)$$ (Opposite to acceleration of piston)

Force on the piston $F = F_P - F_i$

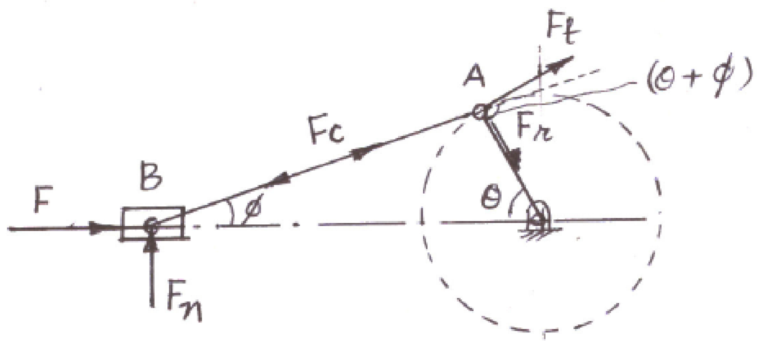
(if F_f frictional resistance is also considered)

$$F = F_P - F_i - F_i$$

In case of vertical engine, weight of the piston or reciprocating parts also acts as force.

$$\therefore\ F = F_P + mg - F_i - F_i$$

ii) Force (Thrust on the CR)



F_c = force on the CR

Equating the horizontal components;

$$F_c\, Cos\phi = F \;\; or \;\; F_c \; \frac{F}{Cos^2\phi}$$

iii) Thrust on the sides of the cylinder

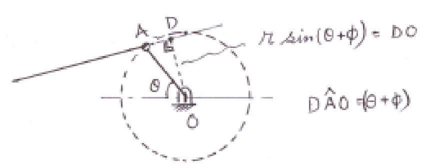
It is the normal reaction on the cylinder walls

$$F_n = F_c \sin\phi = F \tan\phi$$

iv) Crank effort (T)

It is the net force applied at the crank pin perpendicular to the crank which gives the required TM on the crank shaft.

$$F_t \times r = F_c\, r \sin(\theta + \phi)$$



$$F_t = F_c \sin(\theta + \phi)$$

$$= \frac{F}{\cos\phi} \sin(\theta + \phi)$$

v) Thrust on bearings (F_r)

The component of F_C along the crank (radial) produces thrust on bearings

$$F_r = F_c\, Cos(\theta+\phi) = \frac{F}{Cos\,\phi}\, Cos(\theta+\phi)$$

vi) Turning moment of Crank shaft

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)\times r \;=\; \frac{F_r}{\cos\phi}\,(\sin\theta + \cos\phi + \cos\theta\sin\phi)$$

$$= F\times r\left(\sin\theta + \cos\theta\,\frac{\sin\phi}{\cos\phi}\right)$$

Proved earlier

$$= F\times r\left(\sin\theta + \cos\theta\,\frac{\sin\theta}{n}\;\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$= F \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

Also,

$$r \sin(\theta + \phi) = OD \cos\phi$$

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi} . \, r \sin(\theta + \phi)$$

$$= \frac{F}{\cos\phi} . \, OD \cos\phi$$

$$T = F \times OD.$$

TURNING MOMENT DIAGRAMS AND FLYWHEEL

- A flywheel is nothing but a rotating mass which is used as an energy reservoir in a machine which absorbs the energy when the speed in more and releases the energy when the speed is less, thus maintaining the fluctuation of speed within prescribed limits.

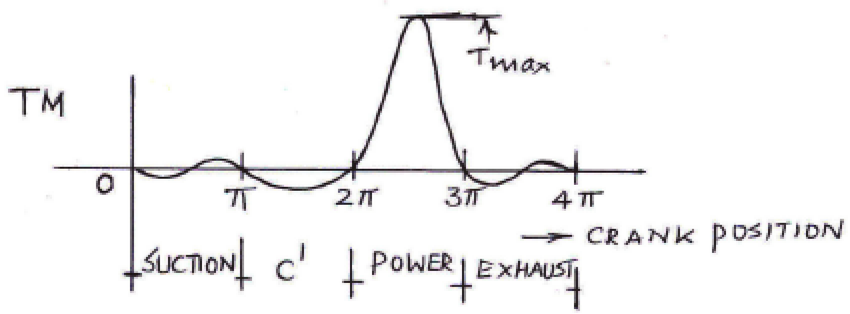
Difference between Governor and Flywheel:

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job. Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

Uses of turning moment Diagram

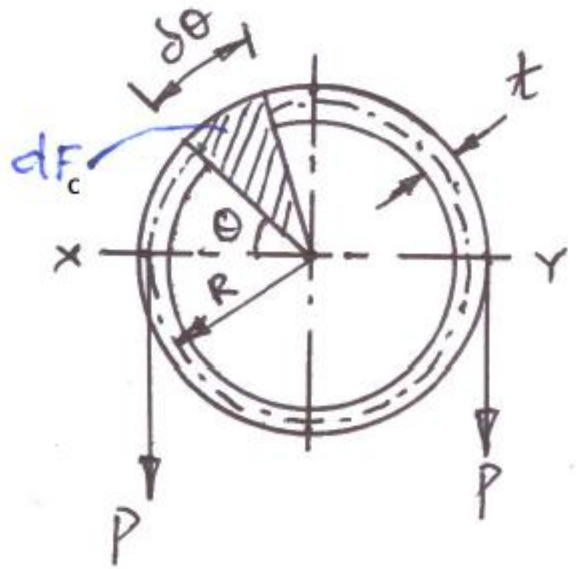
1) The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.
2) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us the find the fluctuation of energy.
3) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us the find diameter of the crank shaft.

TMD for a four stroke I.C. Engine



Size of fly wheel and hoop stress developed in a fly wheel.

Consider a rim of the fly wheel as shown in figure. Let D = mean diameter of rim, R = mean radius of rim, t = thickness of the fly wheel, A = cross sectional as area of rim in m^2 and ρ be the density of the rim material in Kg/m^3, N be the speed of the fly wheel in rpm, ω= angular velocity in rad/sec, V = linear velocity in m/σ, hoop stress in N/m^2 due to centrifugal force.



Consider small element of the rim. Let it subtend an angle $\delta\theta$ at the centre of flywheel.

Volume of the small element $= R\delta\theta.A$.

Mass of the small element $= dm = R\delta\theta.A\ \rho$

The centrifugal force on the small element

$$dF_C = dm\omega^2 R$$
$$= R\delta\theta.A\omega^2 R\,\rho$$
$$= R^2 A.\omega^2 \delta\theta\,\rho$$

Resolving the centrifugal force vertically

$$dF_C = dF_C Sin\theta$$

$$=\rho\ R^2 A\omega^2 Sin\theta.\ \delta\theta \quad ---(1)$$

Total Vertical upward force across diameter X & Y

$$= \int_0^{\pi} \rho R^2 A\omega^2 Sin\theta.\ \delta\theta$$
$$= \rho R^2 A\omega^2 \int_0^{\pi} Sin\theta.\ \delta\theta$$
$$2\rho\ = 2\rho A R^2\omega^2$$

This vertical upward force will produce tensile stress on loop stress developed & it is resisted by 2P.

We know that, $\sigma = P/A$

$$P = \sigma A$$

$$\therefore\ 2P = 2\sigma A$$

$$PAR^2\omega^2 = 2\sigma A$$

$\boxed{\sigma = \rho R^2\omega^2}$ % up to this deviation

Also,

Linear velocity V=Rx ω

$$\sigma = \delta V^2$$

$$\boxed{V = \sqrt{\sigma/\delta}}$$

Mass of the rim = volume x density

$$\boxed{m = \pi\, dA \times \rho}$$

Problem 1:

A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of the machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during ½ revolution and remains constant fore the following revolution. It then falls uniformly to 750 N-m during the next ½ revolution and remains constant for one revolution, the cycle being repeated thereafter.

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Solution.

Given: $N = 250\ r.p.m\ \ or\ \omega = 2\pi \times 250/60 = 26.2\, rad/s;\ m = 500 kg;\ k = 600\ \ m = 0.6$

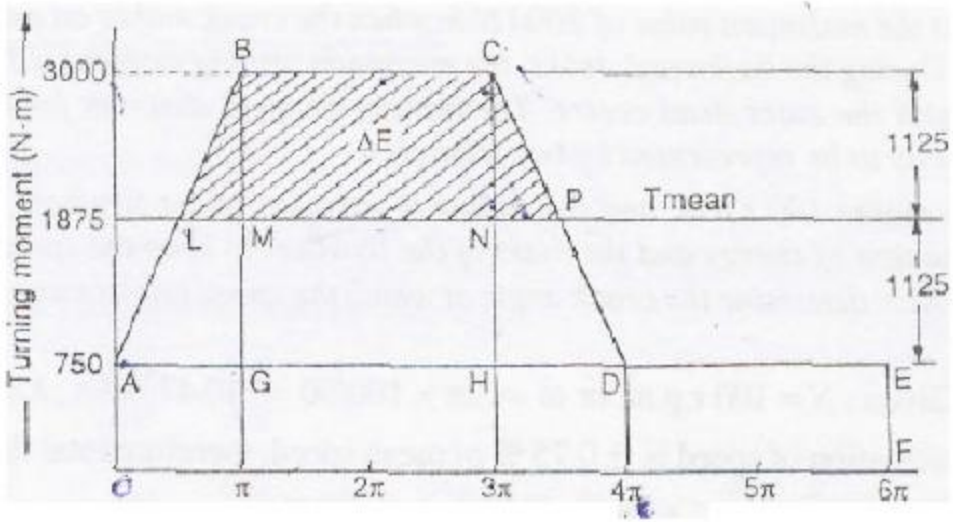
The turning moment diagram for the complete cycle is drawn.

The torque required for one complete cycle

$$= Area\ of\ \ figure\ OABCDEF$$
$$= Area\ \ OAEF + Area\ ABG + Area BCHG + Area\ CDH$$
$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$
$$= 6\pi \times 750 + \frac{1}{2} \times \pi (3000 - 750) + 2\pi (3000 - 750) + \frac{1}{2} \times \pi (3000 - 750)$$
$$= 11250\ \pi\ N - m$$

Torque required for one complete cycle $= T_{mean} \times \pi\ N - m$

$$\therefore T_{mean} = 11250\pi / 6\pi = 1875\ N - m$$



Power required to drive the machine, $P = T_{mean} \times \omega = 11875 \times 26.2 = 49125W = 49.125kW$.

To find Coefficient of fluctuation of speed, δ.

Find the values of *LM* and *NP*.

From similar triangles *ABG* and *BLM*,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad or \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad or \quad LM = 0.5\pi$$

From similar triangles *CHD* and *CNP*,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad or \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad or \quad NP = 0.5\pi$$

From the figure, we find that,

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

The area above the mean torque line represents the maximum fluctuation of energy. Therefore the maximum fluctuation of energy, ΔE

$$= \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\,\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\,\pi \times 1125 = 8837\ N - m$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.\delta = 500\,(0.6)^2\ (26.2)^2\ \delta = 123\ 559\ \delta$$

$$\boxed{\delta = 0.071}$$

Problem 2

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

Solution

θ, deg.	$T, N-m$
0	500
90	1500
180	500
270	1500
360	500

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T \, d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta) \, d\theta$$

$$= 2000\pi \ N - m$$

$$T_{mean} = \frac{W.D / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \ N - m$$

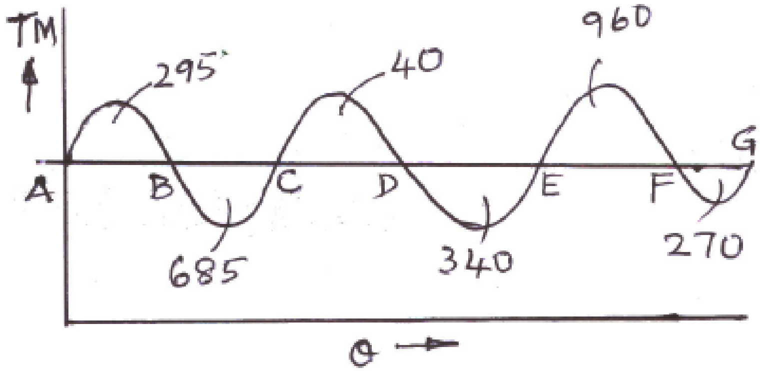
$$\text{Power developed } = T_{mean} \times \omega_{mean}$$

$$= 1000 \times \frac{2\pi \ N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

The TMD for a petrol engine is drawn to the following scale, turning moment, 1mm = 5Nm, crank 1mm = 1°. The TMD repeats itself at every half revolution of the engine & areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150mm. Calculate the maximum fluctuation of energy & co-efficient of fluctuation of speed when engine runs at 1800rpm



$$\text{Energy at } A = E$$
$$\text{Energy at } B = E + a_1 = E + 295$$
$$\text{Energy at } C = E + 295 - 685 = E - 390$$
$$\text{Energy at } D = E + 295 - 685 + 40 = E - 350$$
$$\text{Energy at } E = E - 350 - 340 = E - 690$$
$$\text{Energy at } F = E - 690 + 960 = E + 270$$
$$\text{Energy at } G = E + 270 - 270 = E$$
$$\therefore A = G$$

$$\text{Max Energy} = E + 295$$
$$\text{Min Energy} = E - 690$$

$$m = 36kg, k = 150mm, \quad N = 1800rpm$$

$$\text{Maximum Fluctuation of Energy } \Delta E = E + 295 - (E - 690) = 985mm^2$$

Scale: $1mm = 5Nm$ & $1mm = 1°$

$$Torque \times \theta = \frac{5}{180}\pi \times 1 = \frac{\pi}{36} Nm$$

$$\Delta E = 985 \times \frac{\pi}{36} = 85.95Nm$$

$$\Delta E = mk^2\omega^2\delta$$

$$86 = 36 \times 0.15^2 \times \left(\frac{2\Pi(1800)}{60}\right)^2 \delta$$

$$\delta = 0.003 \quad or \quad 0.3\%$$

The TMD for a multi cylinder engine has been drawn to a scale 1mm to 500Nm torque & 1mm to 6° of crank displacement. The intercepted area in order from one end is mm^2 are -30, 410, -280, 320, -330, 250, -360, +280, -260 mm^2 when engine is running at 800rpm. The engine has a stroke of 300mm & fluctuation of speed is not to exceed ±2% of the mean speed, determine

1. a suitable diameter & cross section of the fly wheel rim for a limiting value of the safe centrifugal stress of 7MPa. The material density may be assumed as 7200 kg/m^3. The width of the rim is to be 5times the thickness.

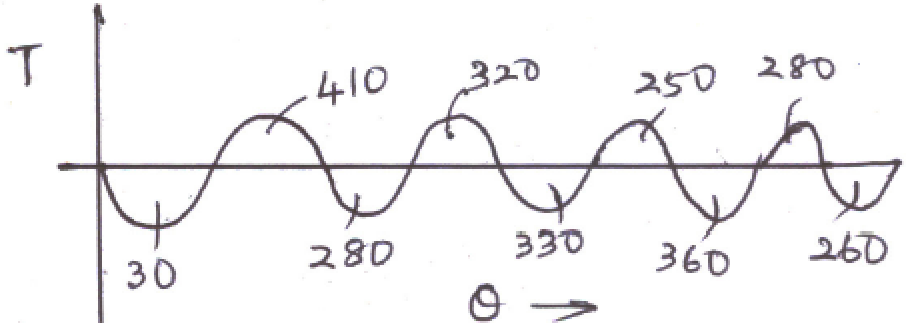
Solution:

$N = 800 \quad rpm$

$\pm\ 2\ \%means,\ \delta = 4\% = 0.04$

$\sigma = 7\,Mpa = 7\ N/m2$

$\rho = 7200\ kg/m^3$



Energy at $A = E$
Energy at $B = E - 30$
Energy at $C = E - 30 + 410 = E + 380$
Energy at $D = E + 380 - 280 = E + 100$
Energy at $E = E + 100 + 320 = E + 420$
Energy at $F = E + 420 - 330 = E + 90$
Energy at $G = E + 90 + 250 = E + 340$
Energy at $H = E + 340 - 360 = E - 20$
Energy at $I = E - 20 + 280 = E + 260$
Energy at $J = E + 260 - 260 = E$

$$\Delta E = E + 420 - (E - 30)$$
$$= 450 mm^2$$

$1mm = 500 Nm$, $1mm = 6°$ (0.1047 radians), $1mm^2 = 52.35 Nm$

$$\Delta E = 450 \times 52.35 = 23557.5 \text{ Nm}$$

$\sigma = \rho V^2$

$\Delta E = mr^2 \omega^2 \delta$

$7 \times 10^6 = 7200\, V^2$ $= mV^2 \delta$

$V = r\omega$

$V = 31.18\ m/s$

$$V = \frac{\pi DN}{60}, \quad D = 0.745 m$$

Cross sectional area $A = bt$

$A = (5t)t = 5t^2$

Fluctuation of energy $\Delta E = mV^2 \delta$

$$23.56 \times 10^3 = m(31.18)^2\ (0.04)$$

$$m = 605\ kg$$

$m = Volume \times Density$

$\pi DA \times \rho$

$605 = \pi(0.745)(5t^2)7200$

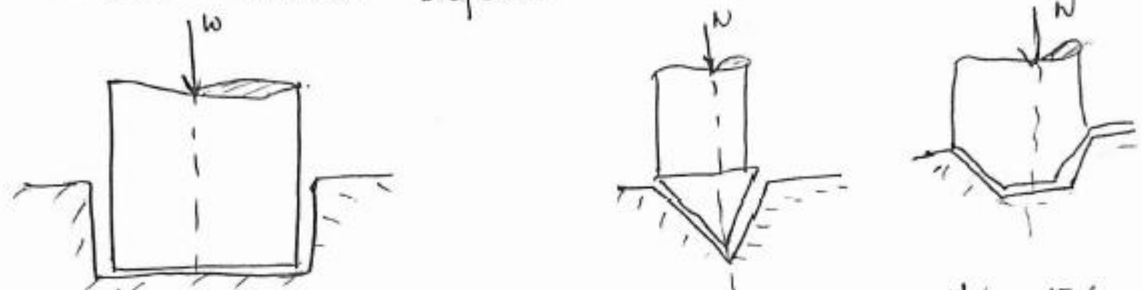
$t = 0.084 m$

Area $= 5t^2 = 0.035 m^2$

Pivot Bearing

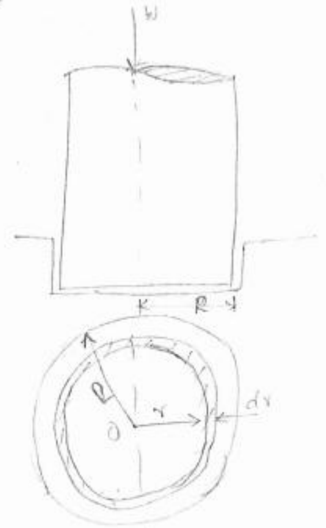
①

The rotating shafts are frequently subjected to axial thrust. These shafts can be kept in correct axial position if bearing surfaces are provided. The bearing surfaces which are flat (or) conical carry the axial thrust. The bearing surfaces placed at the end of a shaft are known as pivots. The pivot may flat, conical (or) truncated conical surfaces.



Flat (bearing) pivot   Conical pivot   Truncated pivot

* Flat Pivot :-



The bearing surface placed at the end of shaft is known as pivot. If the surface is flat as shown, then bearing surface is called flat-pivot (or) -foot-step. There will be friction along the surface of contact between shaft & bearing. The power lost can be obtained by calculating torque.

let, W → Axial load, (or) load transmitted to the bearing surface

R → Radius of pivot.

μ → co-effecient of friction.

p → Intensity of Pr, N/m².

T → Total frictional torque.

r → radius of ring

dr → thickness of ring.

Consider a circular ring of radius r & thickness dr as shown.

∴ Area of ring = $2\pi r \cdot dr$.

We will consider 2 cases; namely,

(i) Uniform Pressure over bearing surface &
(ii) Uniform wear over bearing surface

i) Case of Uniform Pr.:

When the Pr is assumed to be uniform over the bearing surface, then intensity of pressure is given by,

$$p = \frac{\text{Axial load}}{\text{Area of c/s}} = \frac{W}{\pi R^2} \quad -(1)$$

Now, the load transmitted to the ring & frictional torque on the ring,

Load transmitted to the ring,

$$dW = \text{Pr on ring} \times \text{Area of ring}$$
$$= p \times 2\pi r dr$$

frictional force on ring,

$$dF = \mu \times dW$$
$$= \mu \times \text{load on ring}$$
$$= \mu \times p \times 2\pi r \cdot dr$$

(Frictional torque on the ring) Moment of frictional force about the axis.

$$dT = \text{frictional force} \times \text{Radius of ring}$$
$$= dF \times r$$

$$\therefore dT = \mu \times p \times 2\pi \cdot r \cdot dr \cdot r$$
$$= \mu \cdot p \times 2\pi r^2 \cdot dr \quad -(a)$$

Now, the total frictional torque will be obtained by integrating above eq. (a).

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi \mu p r^2 \cdot dr$

$= 2\pi \mu p \int_0^R r^2 \cdot dr$

$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_0^R$

$= \frac{2}{3} \cdot \mu \pi p R^3$

$= \frac{2}{3} \pi \times \mu \times R^3 \times \frac{W}{\pi R^2}$ $\quad \left[\because p = \frac{W}{\pi R^2}\right]$

$$\boxed{T = \frac{2}{3} \mu W R}$$

$\therefore$ Power lost in friction $= T \times \omega$

$= \frac{T \times 2\pi N}{60}$

$= \frac{2\pi N T}{60}$

(ii) In case of Uniform Wear :- For uniform wear of bearing surface, the load transmitted to the various circular rings should be same.

But load transmitted to any circular ring is equal to the product of pressure & area of ring. Area of ring is directly proportional to the radius of ring. Hence for uniform wear, the product of Pr & radius should be constant. i.e. $P \times r =$ constant.

For Uniform wear, $p \times r =$ constant

i.e. $p \times r = C$.

$\therefore$ $p = \frac{C}{r}$ — (a).

load transmitted to the ring,

$= p_r \times \text{Area of ring}$

$= p \times 2\pi r \cdot dr$

$= \frac{C}{r} \times 2\pi \cdot r \cdot dr$

$dW = 2\pi C \cdot dr \quad - (6).$

Total load transmitted to the bearing, is obtained by integrating from '0 to R'

∴ Total load transmitting to the bearing,

$W = \int_0^R dW$

$= \int_0^R 2\pi C\, dr = 2\pi C \int_0^R dr = 2\pi C [r]_0^R$

$W = 2\pi CR$

$C = \frac{W}{2\pi R}$

Now frictional ~~torque~~ force on the ring,

$dF = \mu \times \text{load on ring} = \mu \times dW$

$= \mu \times 2\pi C dr$

Hence frictional torque on the ring,

$dT = \text{Frictional force on ring} \times \text{radius}$

$= dF \times r$

$= \mu \times 2\pi C \cdot dr \cdot r$

$= \mu \cdot 2\pi C \cdot r \cdot dr$

∴ Total frictional torque, $T = \int_0^R dT$

$= \int_0^R \mu 2\pi C \cdot r \cdot dr$

$= 2\pi \cdot C \cdot \mu \cdot \int_0^R r\, dr$

$= 2\pi C \cdot \mu \left[\frac{r^2}{2}\right]_0^R = 2\pi C \cdot \mu \left[\frac{R^2}{2}\right]$

$= 2\pi \times \frac{W}{2\pi R} \times \mu \left[\frac{R^2}{2}\right]$

$T = \frac{1}{2} \mu W R$

∴ Power lost in friction, $P = \frac{2\pi N T}{60}$

Problem: Find the power lost in friction assuming (i) uniform Pr. & (ii) Uniform wear. When a vertical shaft of 100mm dia. rotating at 150rpm rests on a flat end-foot step bearing. The coefficient of friction is equal to 0.05 & shaft carries a vertical load of 15 kN.

Sol:

Given:

Dia, $D = 100mm \Rightarrow 0.1m$ $\therefore R = \frac{0.1}{2} = 0.05m$

$N = 150$ rpm ; co-efficient of friction, $\mu = 0.05$

load, $W = 15 KN = 15\times10^3 N$.

i) Power lost in friction assuming uniform pressure.

For uniform Pr.,

$T = \frac{2}{3}\mu W R$

$T = \frac{2}{3}(0.05)(15\times10^3)(0.05)$

$T = 25$ N-m

Power lost, $P = \frac{2\pi NT}{60} \Rightarrow \frac{2\pi 150\times25}{60}$

$P = 392.7 W$

ii) For uniform wear,

$T = \frac{1}{2}\mu W R.$

$= \frac{1}{2}\times 0.05\times15\times10^3\times0.05$

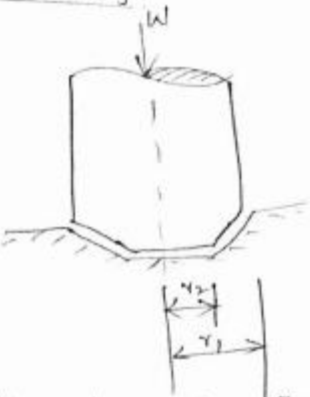
$T = 18.75 Nm$

$P = \frac{2\pi NT}{60}$

$P = \frac{2\pi\times150\times18.75}{60}$

$P = 294.5 W$

* Truncated Pivot Bearing:-



The above fig. shows truncated conical pivot of external & internal radii r_1 & r_2 respectively.

(i) Case of Uniform Pressure :-

vertical load transmitted to the bearing

$$dW = p \times 2\pi r \times dr \quad - (a)$$

For total vertical load, integrating with limits r_2 to r_1.

$$W = \int_{r_2}^{r_1} p \times 2\pi r \, dr$$

$$= p\, 2\pi \int_{r_2}^{r_1} r\,dr = p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$W = p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$p = \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \quad - (b).$$

(frictional force, $\mu \times dW$ normal to the surface $= \mu \times p \times 2\pi r\,dr/\sin\alpha$)

frictional torque on the rings.

$$dT = \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \times r.$$

Total frictional torque, $T = \int_{r_2}^{r_1} dT$

$$T = \int_{r_2}^{r_1} \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \cdot r$$

$$= \frac{2\pi \times \mu \cdot p}{\sin\alpha} \int_{r_2}^{r_1} r^2\, dr$$

$$= \frac{2\pi\, \mu \cdot p}{\sin\alpha} \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = \frac{2\pi \times \mu \cdot p}{\sin\alpha} \left[\frac{(r_1^3 - r_2^3)}{3}\right]$$

$$= \frac{2\pi \times \mu}{3 \sin\alpha} \cdot \frac{W}{\pi (r_1^2 - r_2^2)} \cdot (r_1^3 - r_2^3)$$

$$\therefore \boxed{T = \frac{2}{3} \frac{\mu W}{\sin\alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

Power lost in friction, $P = \frac{2\pi N T}{60}$.

(ii) Uniform Wear:

$$p \times r = C$$

$$p = C/r$$

Vertical load transmitted, $dW = p \times 2\pi r\, dr$

Total vertical load, $W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$

$$= \int_{r_2}^{r_1} \frac{C}{r} \cdot 2\pi r\, dr \Rightarrow 2\pi C \int_{r_2}^{r_1} dr = 2\pi C \left[r\right]_{r_2}^{r_1}$$

$$W = 2\pi C [r_1 - r_2]$$

$$C = \frac{W}{2\pi (r_1 - r_2)}$$

total frictional torque, $T = \int_{r_2}^{r_1} 2\pi \mu \times C \times r \times dr / \sin\alpha$

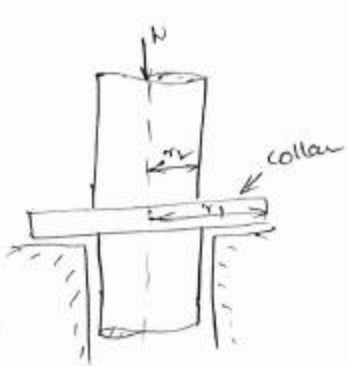
$$T = \frac{2\pi\, \mu \cdot C}{\sin\alpha} \int_{r_2}^{r_1} r \cdot dr \Rightarrow T = \frac{1}{\sin\alpha} \cdot 2\pi \cdot \mu \cdot C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

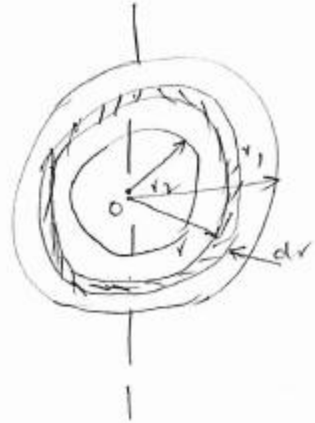
$$\Rightarrow T = \frac{1}{\sin\alpha} \cdot 2\pi \cdot \mu \cdot \frac{W}{2\pi (r_1 - r_2)} \left[\frac{(r_1^2 - r_2^2)}{2}\right]$$

$$\boxed{T = \frac{1}{2} \frac{\mu W (r_1 + r_2)}{\sin\alpha}}$$

Power lost in friction, $\boxed{P = \frac{2\pi N T}{60}}$

Flat collar:- The bearing surface provided at any position on the shaft (but not at the end) to carry axial thrust is known as collar. Collar bearings are also known as thrust bearings.





let,

- $r_1 \rightarrow$ External radius of collar
- $r_2 \rightarrow$ Internal radius of collar
- $p \rightarrow$ intensity of Pr.
- $W \rightarrow$ Axial load or total load transmitted to bearing surface
- $\mu \rightarrow$ co-efficient of friction
- $T \rightarrow$ Total frictional torque.

Consider a circular ring of radius 'r' & thickness dr

$\therefore$ Area of ring, $= 2\pi r \cdot dr$.

load on ring, $= Pr \times$ Area of ring

$= p \times 2\pi \cdot r \cdot dr$.

Friction torque $=$ friction force $\times$ Radius

$= p \cdot \mu \times 2\pi r\, dr \times r$

$= 2\pi \mu p\, r^2 \cdot dr$.

$\therefore$ total frictional torque,

$$T = \int_{r_2}^{r_1} dT$$

$$T = \int^{r_1} 2\pi\mu \cdot p r^2 \cdot dr.$$

(i) Uniform Pressure:-

$p =$ constant.

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} \text{load on ring } (dW)$$

$$= \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= p \times 2\pi \int_{r_2}^{r} r\, dr \Rightarrow p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$\Rightarrow p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right] \Rightarrow p \times \pi \left[r_1^2 - r_2^2\right] = W$$

$$p = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}.$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi\mu \cdot p r^2\, dr$$

$$= 2\pi\mu \cdot p \int^{r_1} r^2 \cdot dr$$

$$= 2\pi\mu \cdot p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p \cdot \left(\frac{r_1^3 - r_2^3}{3}\right)$$

$$= \frac{2}{3} \pi \mu \cdot \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \cdot \left[r_1^3 - r_2^3\right] \Rightarrow \boxed{T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

Power lost in friction, $P = \frac{2\pi N T}{60}$

ii). For Uniform Wear:

$p \times r =$ constant

$p \times r = C$

$p = C/r$

Total load transmitted to the bearing,

$W = \int_{r_2}^{r_1}$ ~~load on ring~~ $= \int_{r_2}^{r_1} dW$

$W = \int_{r_2}^{r_1} p \times 2\pi r \, dr$

$W = \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi \, dr \cdot r$

$= 2\pi \cdot C \int_{r_2}^{r_1} dr$

$= 2\pi C \left[r \right]_{r_2}^{r_1} \Rightarrow 2\pi C [r_1 - r_2] = W$

$C = \frac{W}{2\pi [r_1 - r_2]}$

Total frictional torque

$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} dF \times r$

$= \int_{r_2}^{r_1} 2\pi \mu p \cdot r^2 \cdot dr$

$= 2\pi \mu \int_{r_2}^{r_1} \frac{C}{r} \cdot r^2 \cdot dr$

$= 2\pi \mu C \int_{r_2}^{r_1} r \, dr = \pi \mu C [r_1^2 - r_2^2] = T$

$= \pi \mu \cdot \frac{W}{2\pi (r_1 - r_2)} [r_1^2 - r_2^2]$

$$T = \frac{1}{2} \mu W (r_1 + r_2)$$

Power lost in friction, $P = \frac{2\pi N T}{60}$

STATIC AND DYNAMIC BALANCING

When man invented the wheel, he very quickly learnt that if it wasn't completely round and if it didn't rotate evenly about it's central axis, then he had a problem!
What the problem he had?
The wheel would vibrate causing damage to itself and it's support mechanism and in severe cases, is unusable.
A method had to be found to minimize the problem. The mass had to be evenly distributed about the rotating centerline so that the resultant vibration was at a minimum.

UNBALANCE:

The condition which exists in a rotor when vibratory force or motion is imparted to its bearings as a result of centrifugal forces is called unbalance or the uneven distribution of mass about a rotor's rotating centreline.

BALANCING:

Balancing is the technique of correcting or eliminating unwanted inertia forces or moments in rotating or reciprocating masses and is achieved by changing the location of the mass centres.

The objectives of balancing an engine are to ensure:

1. That the centre of gravity of the system remains stationery during a complete revolution of the crank shaft and
2. That the couples involved in acceleration of the different moving parts balance each other.

Types of balancing:

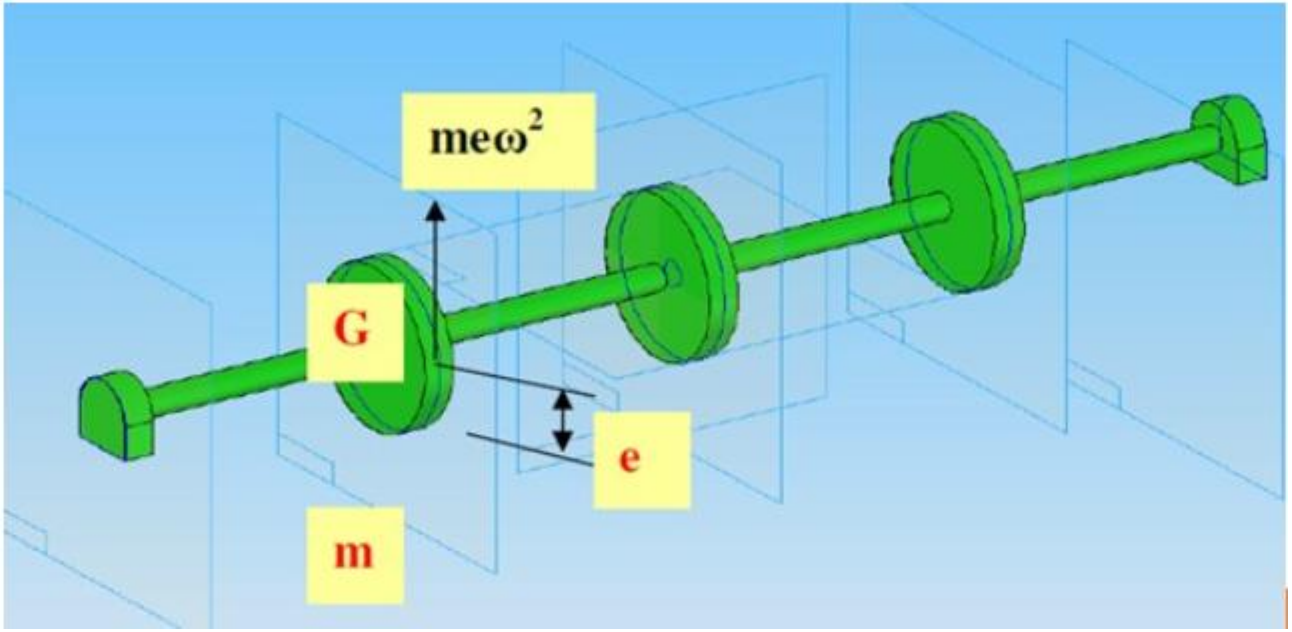
a) **Static Balancing:**
i) Static balancing is a balance of forces due to action of gravity.
ii) A body is said to be in static balance when its centre of gravity is in the axis of rotation.

b) **Dynamic balancing:**
i) Dynamic balance is a balance due to the action of inertia forces.
ii) A body is said to be in dynamic balance when the resultant moments or couples, which involved in the acceleration of different moving parts is equal to zero.
iii) The conditions of dynamic balance are met, the conditions of static balance are also met.

BALANCING OF ROTATING MASSES

When a mass moves along a circular path, it experiences a centripetal acceleration and a force is required to produce it. An equal and opposite force called centrifugal force acts radially outwards and is a disturbing force on the axis of rotation. The magnitude of this remains constant but the direction changes with the rotation of the mass.

In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of rotor lies on the axis of rotation of the shaft. When this does not happen, there is an eccentricity and an unbalance force is produced. This type of unbalance is common in steam turbine rotors, engine crankshafts, rotors of compressors, centrifugal pumps etc.

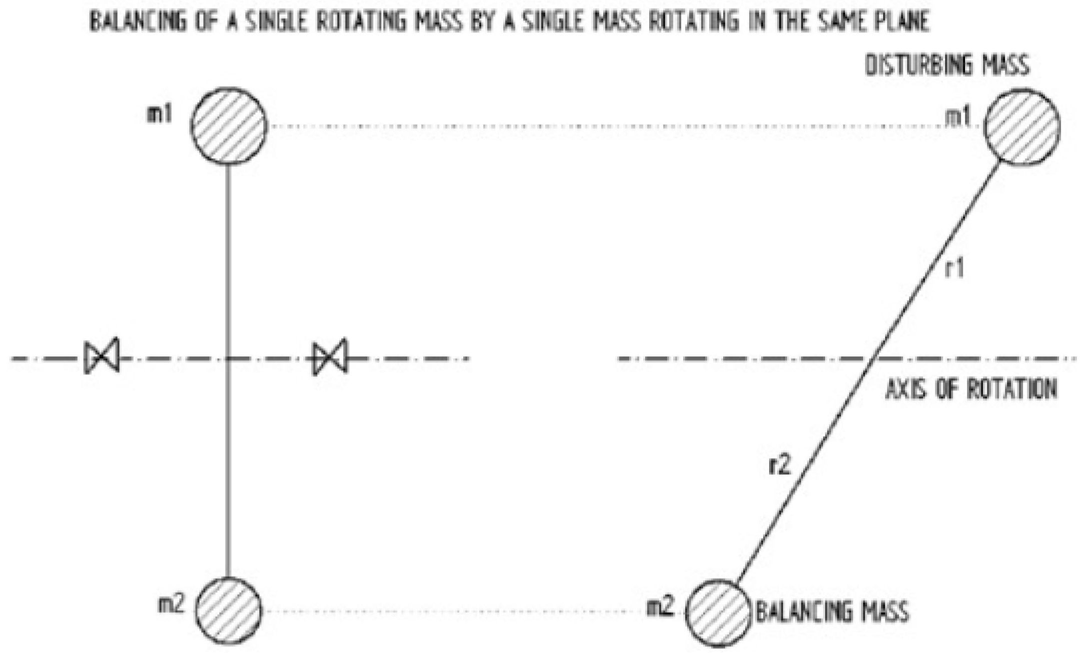


The unbalance forces exerted on machine members are time varying, impart vibratory motion and noise, there are human discomfort, performance of the machine deteriorate and detrimental effect on the structural integrity of the machine foundation.

Balancing involves redistributing the mass which may be carried out by addition or removal of mass from various machine members. Balancing of rotating masses can be of

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of several masses rotating in the same plane
4. Balancing of several masses rotating in different planes

BALANCING OF A SINGLE ROTATING MASS BY A SINGLE MASS ROTATING IN THE SAME PLANE



Consider a disturbing mass m1 which is attached to a shaft rotating at rad/s.

r = radius of rotation of the mass m

The centrifugal force exerted by mass m1 on the shaft is given by,

F = m r c 1 1

This force acts radially outwards and produces bending moment on the shaft. In order to counteract the effect of this force Fc1 , a balancing mass m2 may be attached in the same plane of rotation of the disturbing mass m1 such that the centrifugal forces due to the two masses are equal and opposite.

BALANCING OF A SINGLE ROTATING MASS BY TWO MASSES ROTATING

There are two possibilities while attaching two balancing masses:

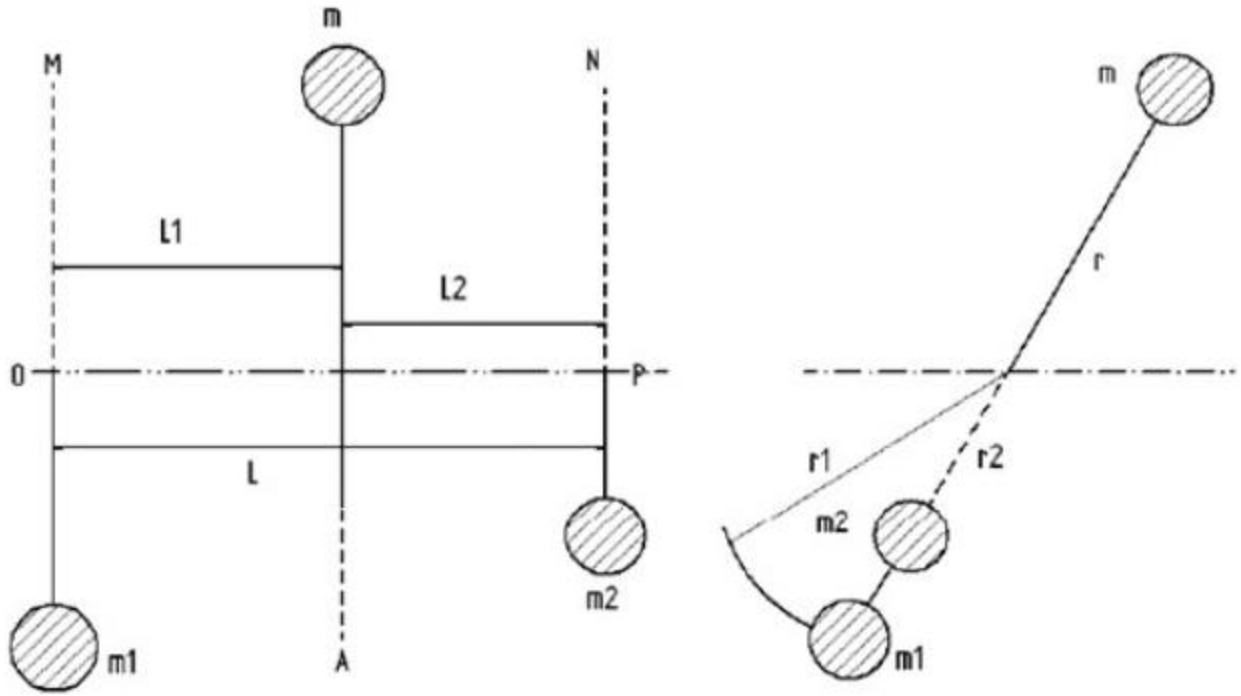
1. The plane of the disturbing mass may be in between the planes of the two balancing masses.

2. The plane of the disturbing mass may be on the left or right side of two planes containing the balancing masses.

In order to balance a single rotating mass by two masses rotating in different planes which are parallel to the plane of rotation of the disturbing mass i) the net dynamic force acting on the shaft must be equal to zero, i.e. the centre of the masses of the system must lie on the axis of rotation and this is the condition for static balancing ii) the net couple due to the dynamic forces acting on the shaft must be equal to zero, i.e. the algebraic sum of the moments about any point in the plane must be zero. The conditions i) and ii) together give dynamic balancing.

THE PLANE OF THE DISTURBING MASS LIES IN BETWEEN THE PLANES OF THE TWO BALANCING MASSES.

The plane of the disturbing mass lies inbetween the planes of the two balancing masses



Consider the disturbing mass m lying in a plane A which is to be balanced by t rotating masses m_1 and m_2 lying in two different planes M and N which are parallel the plane A as shown.

Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, M and N respectively. Let L_1, L_2 and L be the distance between A and M, A and N, and M and N respectively Now,

The centrifugal force exerted by the mass m in plane A will be,

$$F_c = m\ \omega^2 r \text{ ------------------(1)}$$

Similarly,

The centrifugal force exerted by the mass m_1 in plane M will be,

$$F_{c1} = m_1 \omega^2 r_1 \text{------------------(2)}$$

And the centrifugal force exerted by the mass m_2 in plane N will be,

$$F_{c2} = m_2 \omega^2 r_2 \text{ -------------------(3)}$$

For the condition of static balancing,

$$F_c = F_{c1} + F_{c2}$$

$$\text{or } m\omega^2 r = m_1 \omega^2 r_1 + m_2 \omega^2 r_2$$

$$\text{i.e. } m r = m_1 r_1 + m_2 r_2 \text{ -----------------(4)}$$

Now, to determine the magnitude of balancing force in the plane 'M' or the dynamic force at the bearing 'O' of a shaft, take moments about ' P ' which is the point of intersection of the plane N and the axis of rotation.

Therefore,

$$F_{c1} x L = F_c x L_2$$

$$\text{or } m_1 \omega^2 r_1 \, x \, L = m\omega^2 r \, x L_2$$

Similarly, in order to find the balancing force in plane 'N' or the dynamic force at the bearing 'P' of a shaft, take moments about ' O ' which is the point of intersection of the plane M and the axis of rotation.

Therefore,

$$F_{c2} \times L = F_c \times L_1$$

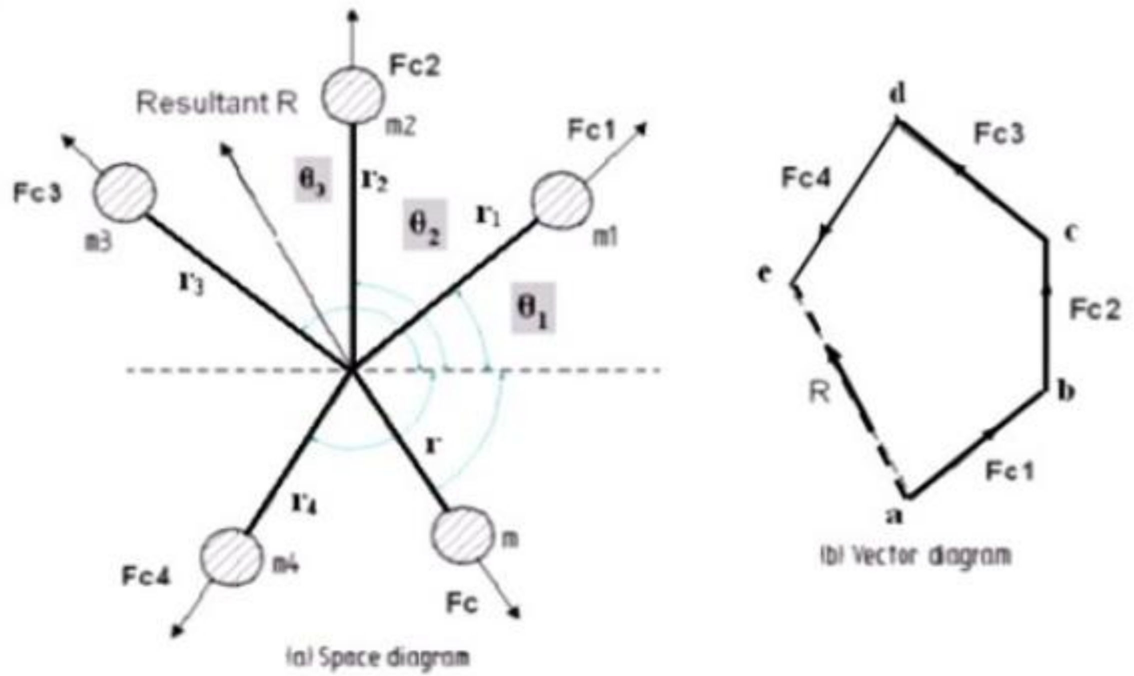
$$\text{or} \quad m_2 \omega^2 r_2 \times L = m \omega^2 r \times L_1$$

Therefore,

$$m_2 r_2 L = m r L_1 \quad \text{or} \quad m_2 r_2 = m r \frac{L_1}{L} \text{---------}(6)$$

For dynamic balancing equations (5) or (6) must be satisfied along with equation (4).

Balancing Multi-cylinder Engines, Balancing V-engines



BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE

Consider a rigid rotor revolving with a constant angular velocity ω rad/s. A number of masses say, four are depicted by point masses at different radii in the same transverse plane.

If m_1, m_2, m_3 and m_4 are the masses revolving at radii r_1, r_2, r_3 and r_4 respectively in the same plane.
The centrifugal forces exerted by each of the masses are F_{c1}, F_{c2}, F_{c3} and F_{c4} respectively.
Let F be the vector sum of these forces. i.e.

$$F = F_{c1} + F_{c2} + F_{c3} + F_{c4}$$
$$= m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 \text{----------} (1)$$

The rotor is said to be statically balanced if the vector sum F is zero. If the vector sum F is not zero, i.e. the rotor is unbalanced, then introduce a counterweight (balance weight) of mass 'm' at radius 'r' to balance the rotor so that,

$$m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 + m \omega^2 r = 0 \text{----------} (2)$$

or

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 + m r = 0 \text{-----------------} (3)$$

The magnitude of either 'm' or 'r' may be selected and the other can be calculated.
In general, if $\sum \mathbf{m}_i \mathbf{r}_i$ is the vector sum of $\mathbf{m}_1 \mathbf{r}_1$, $\mathbf{m}_2 \mathbf{r}_2$, $\mathbf{m}_3 \mathbf{r}_3$, $\mathbf{m}_4 \mathbf{r}_4$ etc, then,

$$\sum m_i r_i + mr = 0 \text{---------} (4)$$

1. Analytical Method:

Procedure:

Step 1: Find out the centrifugal force or the product of mass and its radius of rotation exerted by each of masses on the rotating shaft, since ω^2 is same for each mass, therefore the magnitude of the centrifugal force for each mass is proportional to the product of the respective mass and its radius of rotation.

Step 2: Resolve these forces into their horizontal and vertical components and find their sums. i.e.,

Sum of the horizontal components

$$= \sum_{i=1}^{n} m_i r_i \cos\theta_i = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + \text{---------}$$

Sumof the vertical components

$$= \sum_{i=1}^{n} m_i r_i \sin\theta_i = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + \text{---------}$$

Step 3: Determine the magnitude of the resultant centrifugal force

$$R=\sqrt{\left(\sum_{i=1}^{n} m_i r_i \cos\theta_i\right)^2+\left(\sum_{i=1}^{n} m_i r_i \sin\theta_i\right)^2}$$

Step 4: If θ is the angle, which resultant force makes with the horizontal, then

$$\tan\theta=\frac{\sum_{i=1}^{n} m_i r_i \sin\theta_i}{\sum_{i=1}^{n} m_i r_i \cos\theta_i}$$

Step 5: The balancing force is then equal to the resultant force, but in opposite direction.
Step 6: Now find out the magnitude of the balancing mass, such that

$$R=mr$$

Where, m = balancing mass and r = its radius of rotation

2. Graphical Method:

Step 1:
Draw the space diagram with the positions of the several masses, as shown.

Step 2:
Find out the centrifugal forces or product of the mass and radius of rotation exerted by each mass.

Step 3:
Now draw the vector diagram with the obtained centrifugal forces or product of the masses and radii of rotation. To draw vector diagram take a suitable scale.
Let ab, bc, cd, de represents the forces F_{c1}, F_{c2}, F_{c3} and F_{c4} on the vector diagram.
Draw 'ab' parallel to force F_{c1} of the space diagram, at 'b' draw a line parallel to force F_{c2}. Similarly draw lines cd, de parallel to F_{c3} and F_{c4} respectively.

Step 4:
As per polygon law of forces, the closing side 'ae' represents the resultant force in magnitude and direction as shown in vector diagram.

Step 5:
The balancing force is then , equal and opposite to the resultant force.

Step 6:

Determine the magnitude of the balancing mass (m) at a given radius of rotation (r), such that,

$$F_c = m\omega^2 r$$

or

$$mr = \text{resultant of } m_1 r_1, m_2 r_2, m_3 r_3 \text{ and } m_4 r_4$$

GOVERNORS

Governors

- Engine Speed control

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Governors

- Governors serve three basic purposes:
- Maintain a speed selected by the operator which is within the range of the governor.
- Prevent over-speed which may cause engine damage.
- Limit both high and low speeds.

Governors

- Generally governors are used to maintain a fixed speed not readily adjustable by the operator or to maintain a speed selected by means of a throttle control lever.

- In either case, the governor protects against overspeeding.

How does it work?

- If the load is removed on an operating engine, the governor immediately closes the throttle.
- If the engine load is increased, the throttle will be opened to prevent engine speed form being reduced.

Example

- The governor on your lawnmower maintains the selected engine speed even when you mow through a clump of high grass or when you mow over no grass at all.

Hunting

- Hunting is a condition whereby the engine speed fluctuate or is erratic usually when first started.
- The engine speeds up and slows down over and over as the governor tries to regulate the engine speed.
- This is usually caused by an improperly adjusted carburetor.

Stability

- Stability is the ability to maintain a desired engine speed without fluctuating.
- Instability results in hunting or oscillating due to over correction.
- Excessive stability results in a dead-beat governor or one that does not correct sufficiently for load changes.

Sensitivity

- Sensitivity is the percent of speed change required to produce a corrective movement of the fuel control mechanism.

- High governor sensitivity will help keep the engine operating at a constant speed.

Summary

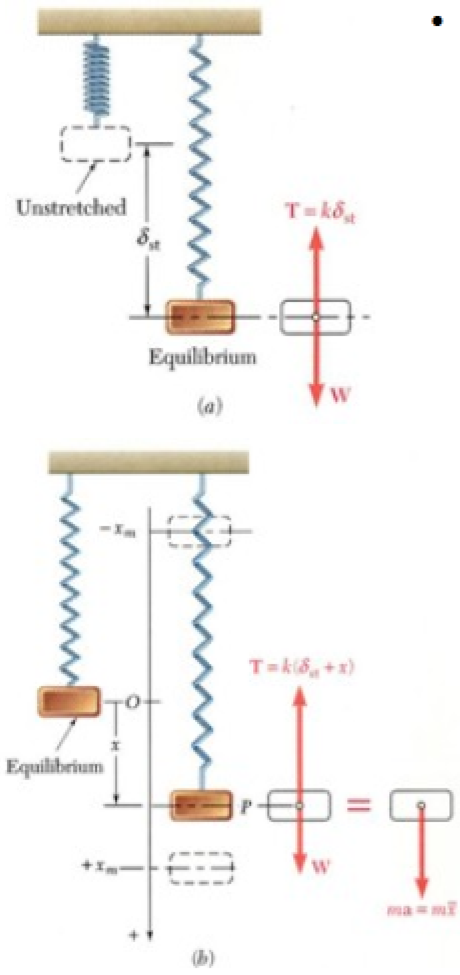
- Small engine governors are used to:
  - Maintain selected engine speed.
  - Prevent over-speeding.
  - Limit high and low speeds.

SUMMARY

- The governor must have stability and sensitivity in order to regulate speeds properly. This will prevent hunting or erratic engine speed changes depending upon load changes.

- *Mechanical vibration* is the motion of a particle or body which oscillates about a position of equilibrium. Most vibrations in machines and structures are undesirable due to increased stresses and energy losses.

Free Vibrations of Particles. Simple Harmonic Motion

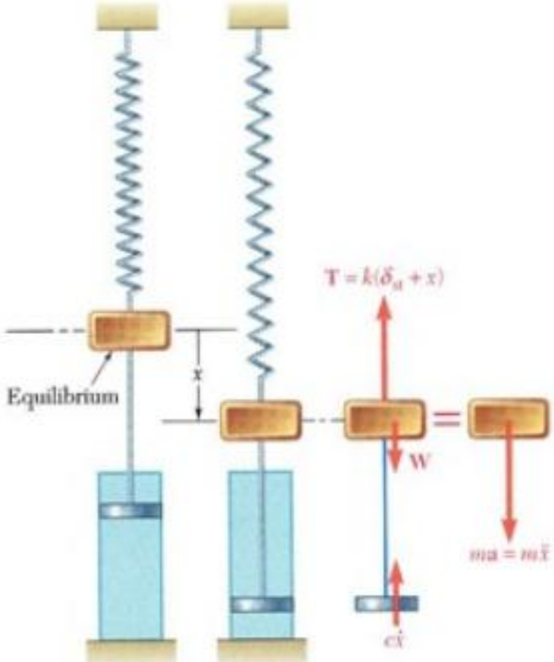


- If a particle is displaced through a distance x_m from its equilibrium position and released with no velocity, the particle will undergo *simple harmonic motion*,

$$ma = F = W - k(\delta_{st} + x) = -kx$$

$$m\ddot{x} + kx = 0$$

Damped Free Vibrations



- All vibrations are damped to some degree by forces due to *dry friction*, *fluid friction*, or *internal friction*.

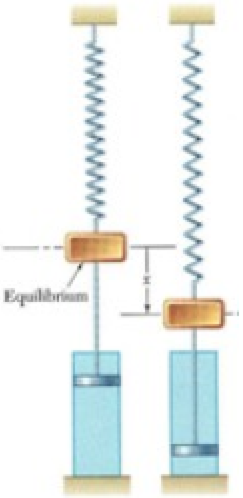
Damped Free Vibrations

- Characteristic equation,

$$m\lambda^2 + c\lambda + k = 0 \qquad \lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$c_c = 2m\omega_n =$ critical damping coefficient

Damped Forced Vibrations



$$m\ddot{x} + c\dot{x} + kx = P_m \sin \omega_f t \qquad x = x_{complementary} + x_{particular}$$