SYSTEM OF FORCES

When two or more forces act on a body, they are called to for a system of forces.

Coplanar forces: The forces, whose lines of action lie on the same plane, are known as coplanar forces.

Collinear forces: The forces, whose lines of action lie on the same line, are known as collinear forces.

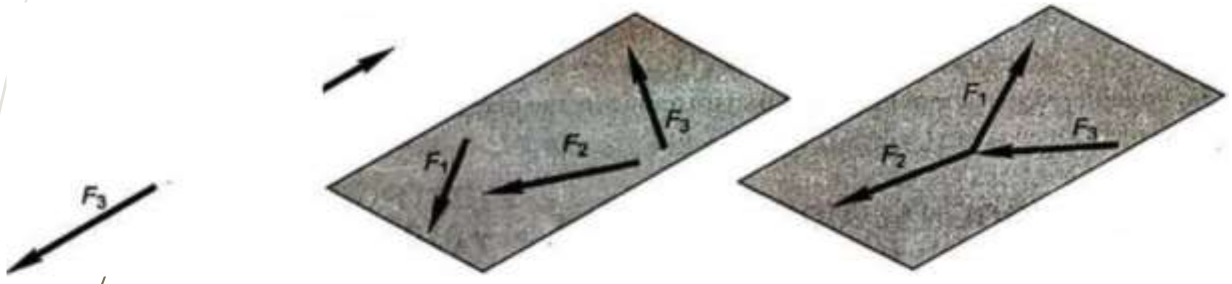
Concurrent forces: The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.

Coplanar concurrent forces: The forces, which meet at one point and their line of action also lay on the same plane, are known as coplanar concurrent forces.

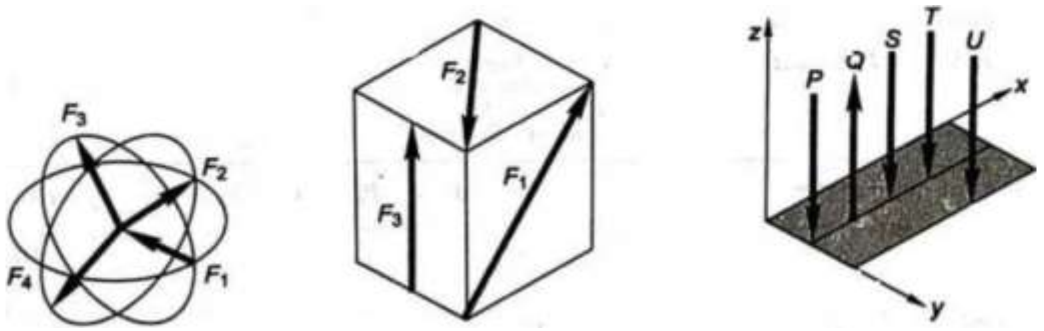
Coplanar non-concurrent forces: The forces, which do not meet at one point, but their lines of action lie on the same, are known as coplanar non-concurrent forces.

Non-Coplanar concurrent forces: The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

Non-Coplanar non-concurrent forces***:*** The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.



(a) Collinear **(i>) Coplaner Nonconcurrent** **(c) Coplaner Concurrent**

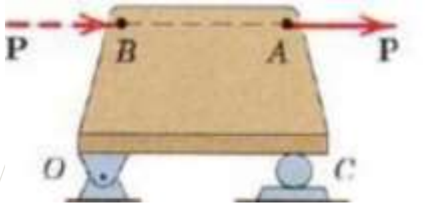


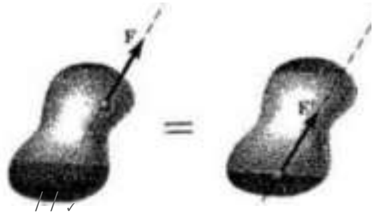
(d) **Non-coplaner concurrent (a) Non-coplaner Nonconcurrent (f) Parallel Forces**

1. **Like-[P-S-T-U]**
2. **Unlike [All]**
3. **Coplaner-[P-Q-S-7]**
4. **Non-Coplaner [All]**

Principle of transmissibility:

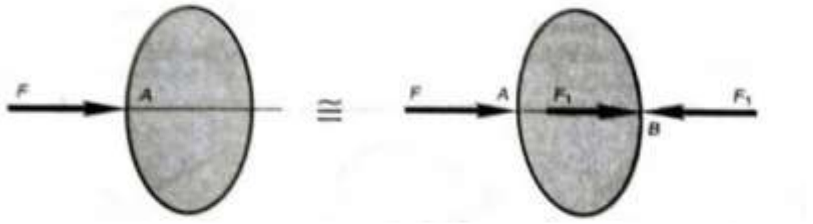
The principle of transmissibility states that a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a sliding vector, and we need specify only the magnitude, direction, and line of





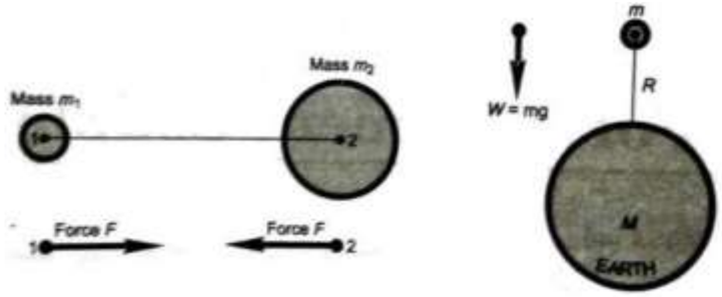
Principle of superposition:

The effect of a force on a body remains same or remains unaltered if a force system, which is in equilibrium, is added to or subtracted from it.



Law of Gravitation:

Magnitude of gravitational force of attraction between two particles is proportional to the product of their masses and inversely proportional to the square of the distance between their centers.



Law of parallelogram of force:

“Two force acting simultaneously on a body. If represented in magnitude and direction by the two adjacent side of a parallelogram then the diagonal of the parallelogram, from the point of intersection of above two forces, represents the resultant force in magnitude and direction”

As shown in fig P and Q are the forces acting on a body are taken as two adjacent sides of a parallelogram ABCD as shown in Fig. so diagonal AC gives the resultant ”R”.

The resultant can be determine by drawing the force with magnitude direction or mathematically is given as following:

$$R = VP^2 + Q^2 + 2PQcos0$$

$$Tana = \frac{Q\ SIN\ 0}{P+QCOS\ 0}$$

> Force:

"An agent which produces or tends to produce, destroys or tends to destroy motion of body is called force"

Unit: N, kN, Kg etc.

Quantity : Vector

Characteristics of Force:

1) Magnitude: Magnitude of force indicates the amount of force (expressed as N or kN) that will be exerted on another body
2) Direction: The direction in which it acts
3) Nature: The nature of force may be tensile or compressive
4) Point of Application: The point at which the force acts on the body is called point of application

Types of Forces:		System of Forces:	
	• Contact Force		• Coplanar Forces
	• Body force		• Concurrent forces
	• Point force and distributed force		• Collinear forces
	• External force and internal force		• Coplanar concurrent forces
	• Action and Reaction		• Coplanar non-concurrent forces
	• Friction force		• Non-coplanar concurrent forces
	• Wind force		• Non-coplanar non-concurrent forces
	• Hydrostatic force		• Like parallel forces
	• Cohesion and Adhesion		• Unlike parallel forces
	• Thermal force		• Spatial forces

> Principle of Individual Forces

1) Principle of transmissibility:

"If a force acts at a point on a rigid body, it may also be considered to act at any other point on its line of action, provided the point is rigidly connected with the body."

2) Principle of Superposition of forces:

"If two equal, opposite and collinear forces are added to or removed from the system of forces, there will be no change in the position of the body. This is known as principle of superposition of forces."

COPLANAR CONCURRENT FORCES

Resultant Force:

If number of Forces acting simultaneously on a particle, it is possible to find out a single force which could replace them or produce the same effect as of all the given forces is called resultant force.

Methods of Finding Resultant:-

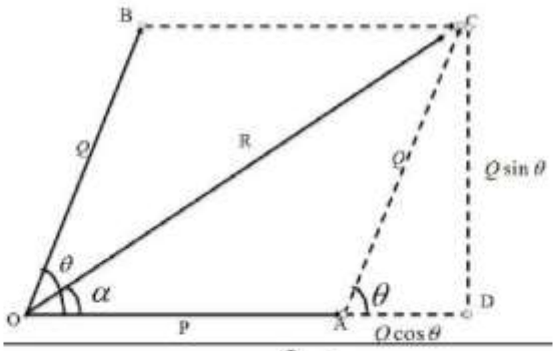
1) Parallelogram Law of Forces (For 2 Forces)
2) Triangle Law (For 2 Forces)
3) Lami"s theorem (For 3 forces)
4) Method of resolution (For more than 2 Forces)

[1] Parallelogram law of forces

$\boldsymbol{R} = \wedge \boldsymbol{P}^2 + Q^2 + \boldsymbol{2PQ} \cos 0$

$\tan a = \dfrac{\boldsymbol{Q} \sin 0}{\boldsymbol{P} + \boldsymbol{Q} \cos 0}$

Where, ***R*** = Resultant force 0 = angle between ***P*** and ***Q*** ***a*** = angle between ***P*** and ***R***

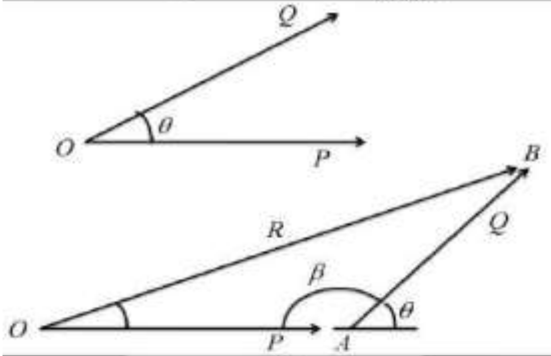


[2] Triangle law of forces

$\boldsymbol{R} = \wedge \boldsymbol{P}^2 + Q^2 - \boldsymbol{2PQ} \cos p$

Where, p = 180° -0 ***R*** =Resultant force 0 = angle between ***P*** and ***Q*** a = angle between ***P*** and ***R*** *a* = $\sin^{-1}$. ^{Q} sin p .

I*R* J

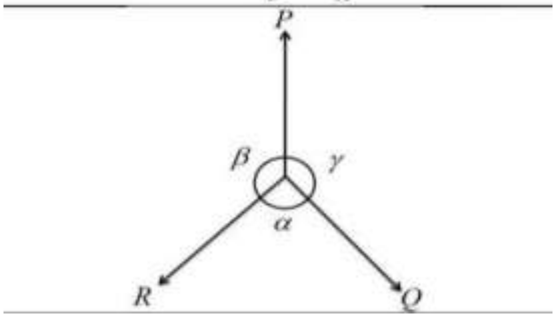


[3] Lami's theorem

P _ *Q* _ *R*

sina sin p sin y

Where, ***P,Q, R*** are given forces ***a*** = angle between Q and R P = angle between P and R y = angle between P and Q



[4] Resolution of concurrent forces

I***H*** = $\boldsymbol{P_1}$ cosQl + $\boldsymbol{P_2}$ cos02 + P_3 cos03 + $\boldsymbol{P_4}$ cos04

I***V*** = P sin 0i + P_2 sin 02 + P_3 sin03 + P_4 sin 04

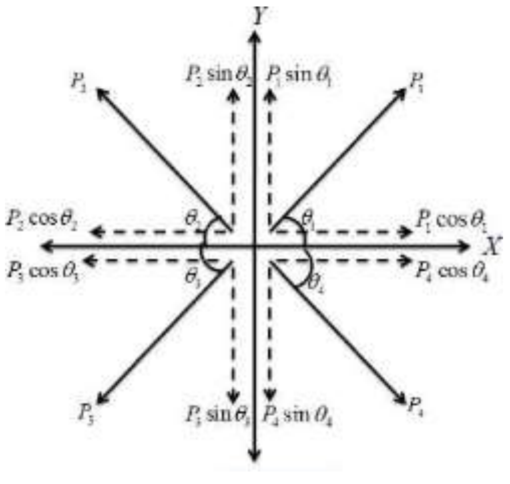
R = V(I H)$^2_+$ (IV)^2

tan0 = ^{IV}

I *H*

Where, $\boldsymbol{P_1}$, P_2 , $\boldsymbol{P_3}$, $\boldsymbol{P_4}$ are given forces o1,o2,o3,o4 are angle of accordingly P, P_2 , P_3 , $\boldsymbol{P_4}$ forces from X-axes ***R*** = Resultant of all forces

0 = angle of resultant with horizontal



> Equilibrium:

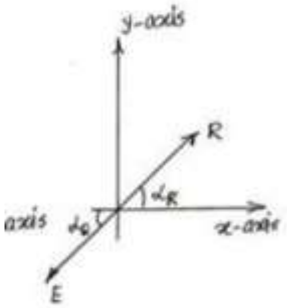
Equilibrium is the status of the body when it is subjected to a system of forces. We know that for a system of forces acting on a body the resultant can be determined. By Newton"s 2nd Law of Motion the body then should move in the direction of the resultant with some acceleration. If the resultant force is equal to zero it implies that the net effect of the system of forces is zero this represents the state of equilibrium. For a system of coplanar concurrent forces for the resultant to be zero hence

$$f = 0$$

$$1 \quad I\ {}^{f}y = {}^{0}$$

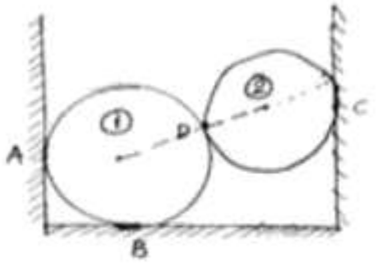
> Equilibrant:

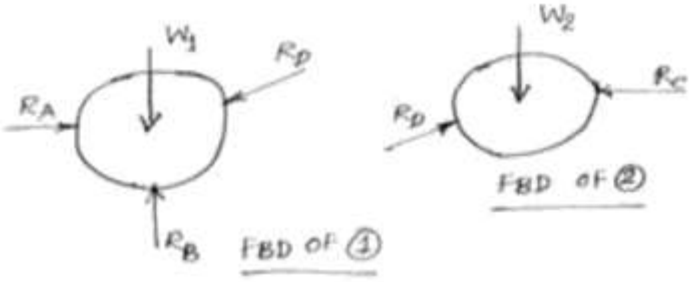
Equilibrant is a single force which when added to a system of forces brings the status of equilibrium. Hence this force is of the same magnitude as the resultant but opposite in sense. This is depicted in figure.



> Free Body Diagram:

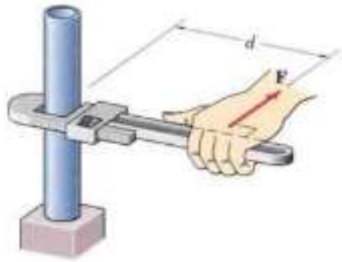
Free body diagram is nothing but a sketch which shows the various forces acting on the body. The forces acting on the body could be in form of weight, reactive forces contact forces etc. An example for Free Body Diagram is shown below.





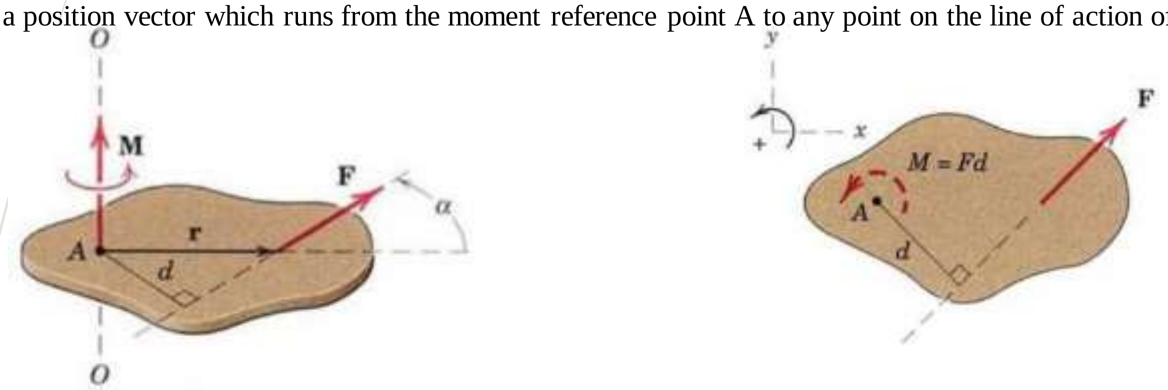
Moment

A force can tend to rotate a body about an axis which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the moment M of a force.



The moment M of a force Fa bout a point A is defined using cross product as $M_A = r \times F$

Where is a position vector which runs from the moment reference point A to any point on the line of action of F.



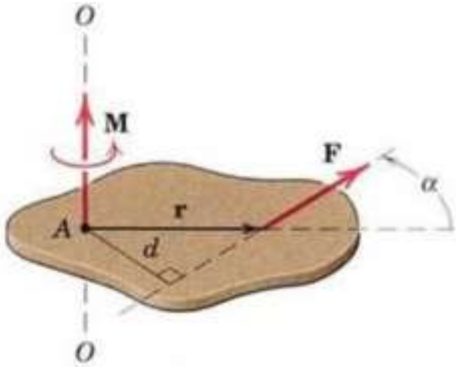
Note r x F = F x r.

Moment about a point A means here : Moment with respect to an axis normal to the plane and passing through the point A.

The magnitude M of the moment is defined as:

$$M (A) = F \times r \sin a = F \times d$$

Where disamoment arm and is defined as the perpendicular distance between the line of action of the force and the moment center.

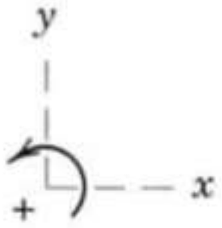


The moment M is a vector quantity. Its direction is perpendicular to the r-F-plane.

The sense of M depends on the direction in which F tends to rotate the body ^ right-hand rule

(+): counter clockwise rotation.

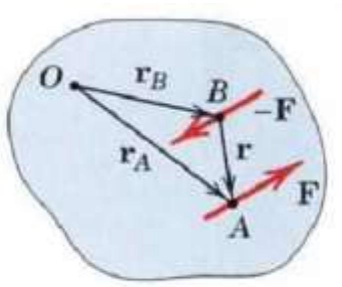
(-) :clock wise rotation.



Sign consistency with in a given problem is very important. The moment M may be considered sliding vector with a line of action coinciding with the moment axis.

Couple

The moment produced by two equal, opposite, parallel, and no collinear forces is called a couple. The force resultant of a couple is zero. Its only effect is to produce a tendency of rotation.



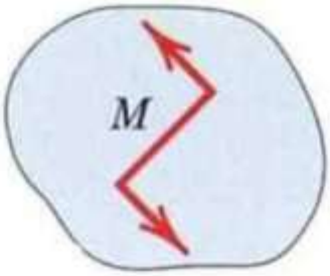
The moment M of a couple is defined as

$$\mathbf{M = id \ X \ F - r_a \ X \ F = (r_4 - r_{tf}) \ x \ F \quad M = r \ X \ F}$$

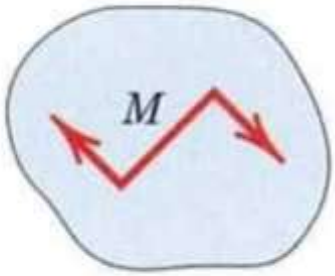
Where RA and RB are position vectors which run from point O to Arbitrary points A and B on the lines of action of F and -F.

The moment expression contains o reference to the moment center O and, therefore, is the same for all moment centers the moment of a couple is a free vector.

The sense of the moment M is established by the right-hand rule.

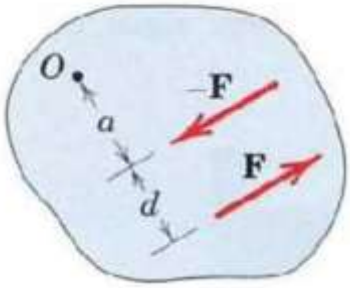


Counter clockwise couple (-)



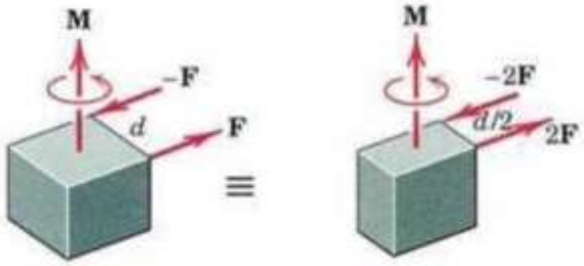
Clock wise couple(+)

The magnitude of the couple is independent of the distance.

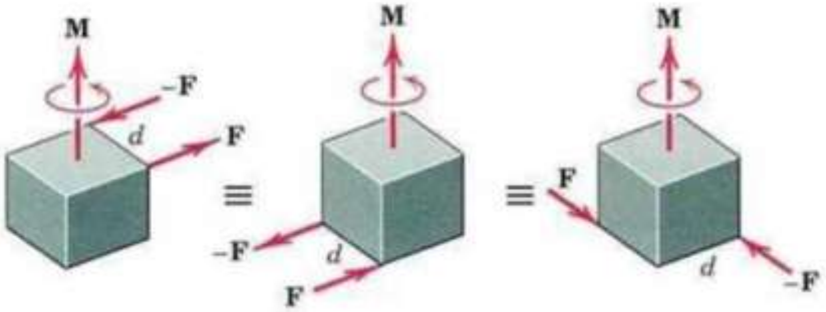


Equivalent Couples

Changing the values of F and does not change a given couple as long as the product Fd remains the same.



A couple is not affected if the forces act in a different but parallel plane.



F orce-CoupleSystems

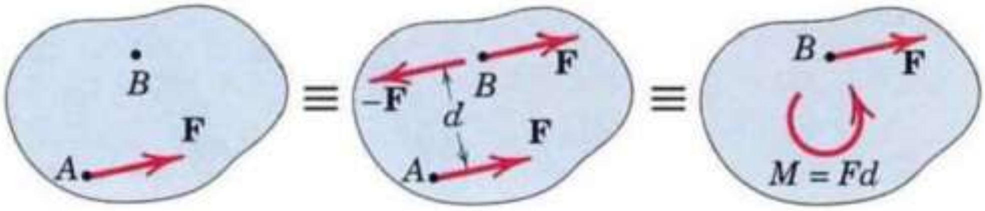
The effect of a force acting on a body is:

a) The tendency to push or pull the body in the direction of the force, and

b) To rotate the body about any fixed axis which does not intersect

The line of action of the force (force does not go through the mass center of the body).

We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.



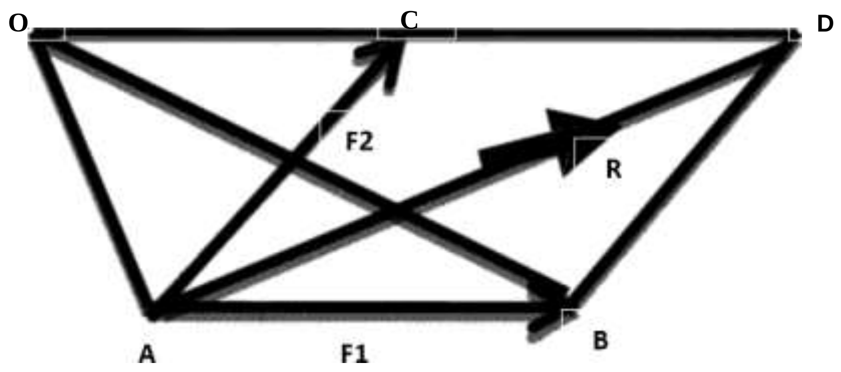
Also we can combine a given couple and a force which lies in the plane of the couple to produce a single, equivalent force.

Varignon's principle of moments:

If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

Proof:

For example, consider only two forces F1 and F2 represented in magnitude and direction by AB and AC as shown in figure below.



Let be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

By the parallelogram law of forces, the diagonal AD represents, in magnitude and direction, the resultant of two forces F1 and F2, let R be the resultant force.

By geometrical representation of moments

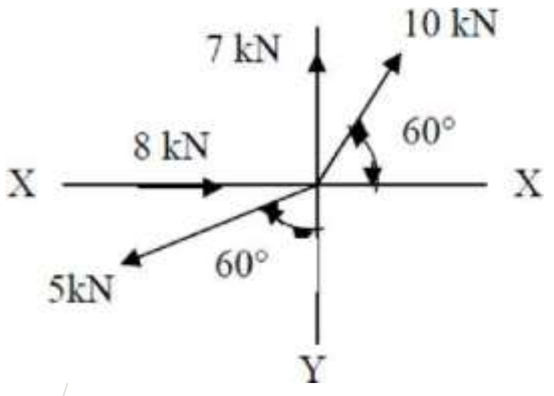
- The moment of force about O= 2 x Area of triangle AOB
- The moment of force about O= 2 x Area of triangle AOC
- The moment of force about O = 2 x Area of triangle AOD But,
- Area of triangle AOD = Area of triangle AOC + Area of triangle ACD
- Area of triangle ACD = Area of triangle ADB = Area of triangle AOB
- Area of triangle AOD = Area of triangle AOC + Area of triangle AOB

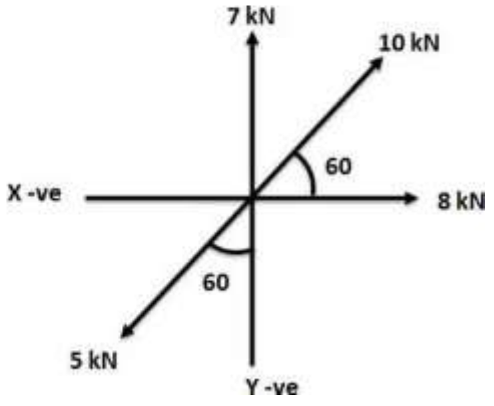
Multiplying throughout by 2, we obtain

2 x Area of triangle AOD= 2 x Area of triangle AOC + 2 x Area of triangle AOB

i.e. Moment of force R about O = Moment of force F1about O + Moment of force F2 about O

Example 1: - Find resultant of a force system shown in Figure





Answer:

1) Given Data

PI = 8 kN 01 = o

P2 = 10 kN 02 = 60

P3 = 7 kN 03 = 90

P4 = 5 kN

04 = 270 - 60 = 210

2) Summation of horizontal force

$ZH = P_1 \cos 0i + P_2 \cos 02 + P_3 \cos 03 + P_4 \cos 04 = 8.67kN(\wedge)$

3) Summation of vertical force

$ZV = P_1 \sin 0i + P_2 \sin 02 + P_3 \sin 03 + P_4 \sin 04 = 13.16kN\ (T)$

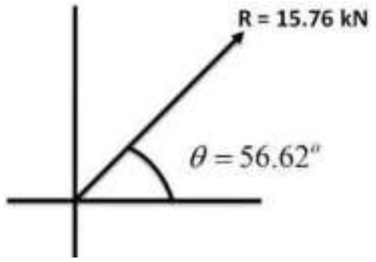
4) Resultant force

$R = yj(Z H)^2 + (ZV)^2 = 15.76kN$

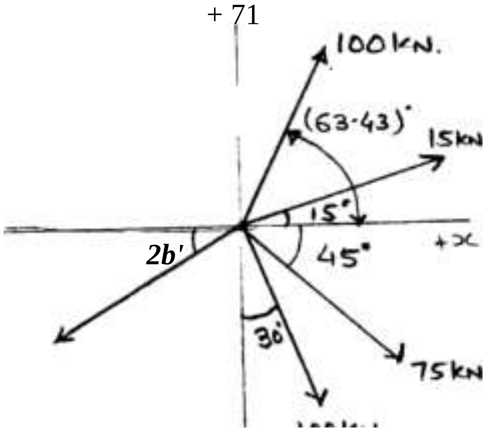
5) Angle of resultant

$$\tan 0 = \frac{ZV}{ZH} = 1.518$$

0 = 56.62



Example 2 Find magnitude and direction of resultant for a concurrent force system shown in Figure



Answer

1) Summation of horizontal force

^ (+Ve) ^ (-Ve)

2 H = +15 Cos 15° + 100Cos (63.43)° - 80Cos20° + 100Sin30° + 75Cos45° = +87.08 kN)

2) Summation of vertical force

t (+Ve) I (-Ve)

2 V = +15 Sin 15° + 100Sin (63.43)° - 80Sin20° + 100Cos30° + 75Sin45° = -73.68 kN (\ )

3) Resultant force

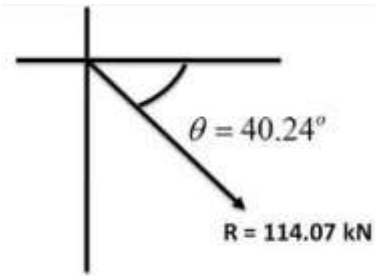
R = y/(X H)^2 + ***(Zvf =*** 114.07kN

4) Angle of resultant

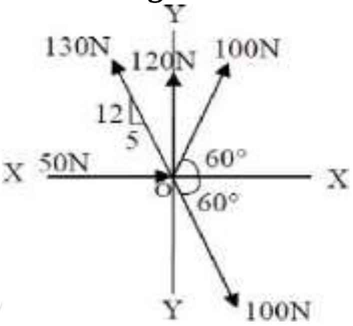
tan0 **IV** I 0.846
H

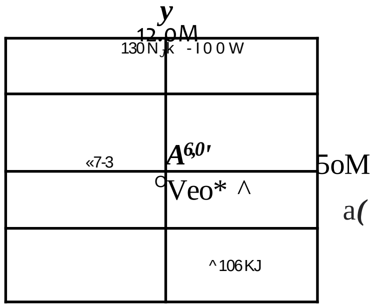
0 = 40.24

5) Angle of resultant with respect to positive x - axis



Example 3 Determine magnitude and direction of resultant force of the force system shown in fig.





Answer

tan p = . $\frac{12}{5}$ = 2.4 .R = 67.38⁰

1) Summation of horizontal force

^ (+Ve) ^ (-Ve)

Z H = +50 + 100 Cos 60° - 130Cos (67.38)° + 100Cos30° + 100Cos60° = +100 N)

2) Summation of vertical force

t (+Ve) I (-Ve)

Z V = +100Sin60° + 120 + 130Sin (67.38)° - 100Sin60° - 100Sin60° = +240 N (f *)*

3) Resultant force

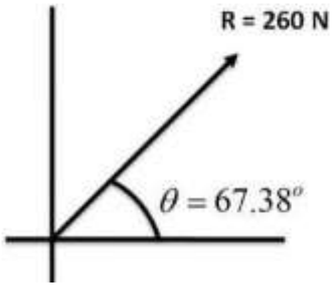
R* = *H*)² + *(ZV*)² = *260N

4) Angle of resultant

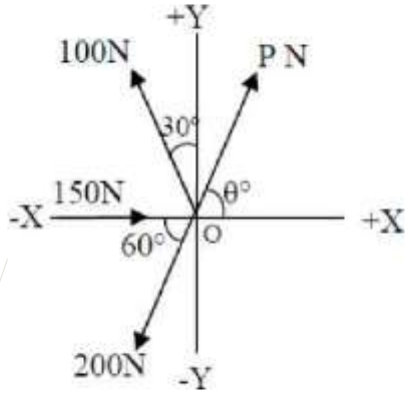
tan0 = $\frac{ZV}{Z\,H}$ = 2.4

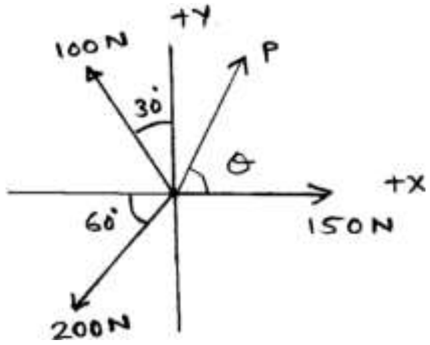
0 = 67.38°

5) Angle of resultant with respect to positive x - axis



Example: 4 A system of four forces shown in Fig. has resultant 50 kN along + X - axis. Determine magnitude and inclination of unknown force P.





Answer

As the R= 50N & directed along + X - axis.

X ***H* =** +50N and ***XV* = *0N***

Now, ***2 H*** = +150 + P Cos 9 - 100 Sin 30° - 200 Cos 60° = 50 N

P Cos9 = 50_(1)

Now, ***2*** V = +P Sin 9 - 100 Cos 30° - 200 Sin 60° = 0

• P Sin9 = 86.60_(2)

From Equation (1) & (2).

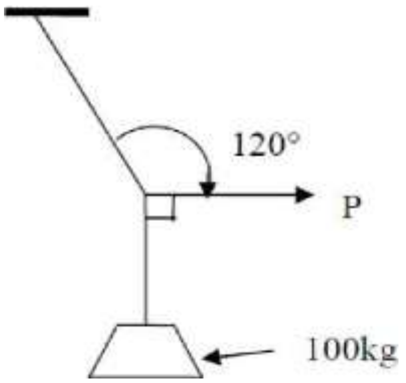
$$\tan 9 = \frac{86.60}{50}$$

tan 9 = 1.732

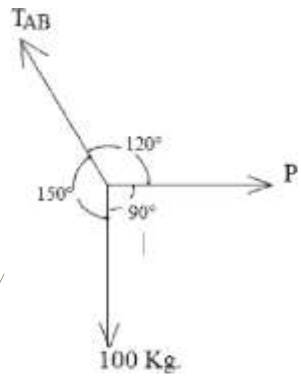
• 9 = 60°

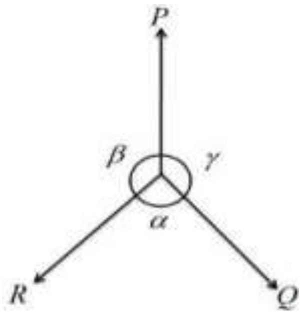
• P = 100 N

Example: 5 Find the magnitude of the force P, required to keep the 100 kg mass in the position by strings as shown in the Figure



Answer:





Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami"s theorem.

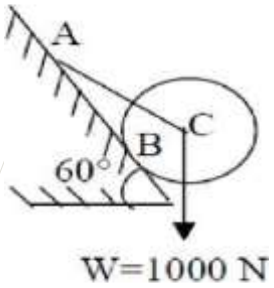
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\frac{P}{\sin 150} = \frac{T_{AB}}{\sin 90} = \frac{100}{\sin 120}$$

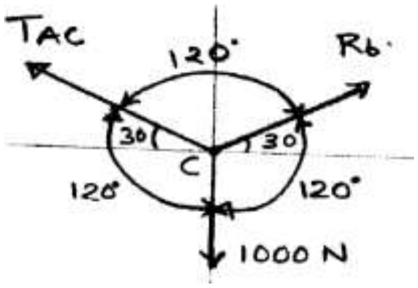
$P = 566.38$ N T_{AB}

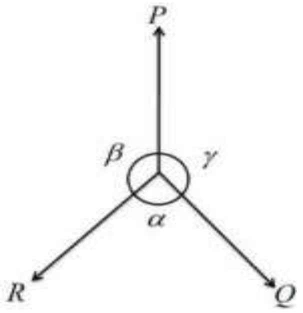
$= 1132.76$ N

Example: 6 A cylindrical roller 600mm diameter and weighing 1000 N is resting on a smooth inclined surface, tied firmly by a rope AC of length 600mm as shown in fig. Find tension in rope and reaction at B



Answer:





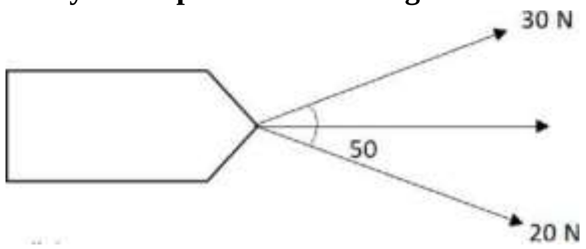
Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami"s theorem.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

sina sin P sin y , Tac Rb 1000

' ' Sin 120 Sin 120 Sin 120 Tac = 1000 N RB =1000N

Example: 7 A boat kept in position by two ropes as shown in figure. Find the drag force on the boat.



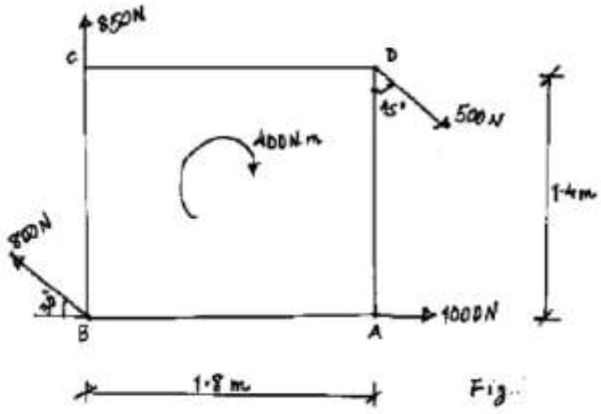
Answer:

According to law of parallelogram

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{20^2 + 30^2 + 2 \times 20 \times 30 \cos 50} = 45.5\ 1N$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{30 \sin 50}{20 + 30\cos 50} = 30.32^0$$

Example: 8 For a coplanar, non-concurrent force system shown in Fig. determine magnitude, direction and position with reference to point A of resultant force.



Answer

To find out magnitude & direction of R

Summation of horizontal force

EH = +500 Sin 45° - 800Cos 30° + 1000 = +660.73 N (^)

Summation of vertical force

2 V = -500 Cos45° + 850 + 800Sin30° = +896.45 N (|)

Resultant force

R* = *sj(E ***H***)^2 + ***(E***^{v})^2 = ***V*** (660.73)^2 + (896.45)^2 = 1113.64 ***N*** Angle of resultant

896.45

tan 0 = 660.73 0 = 53.61°

Here, we have to also locate the „R" @ pt. A Let the „R" is located at a dist^n x from A in the horizontal direction.

Now this dist^n „X" can be achived by using varignon"s principle.

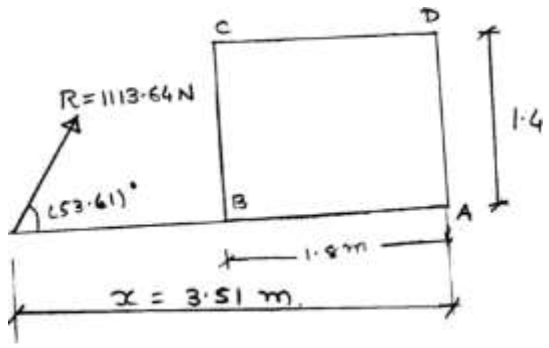
First, Take the moment @ A of all the forces.

MALL= + (500 Sin 45° X 1.4) + (850 X 1.8) + (800 Sin 30° X 1.8) + 400 = + 3144.97 N-m [1]_(1)

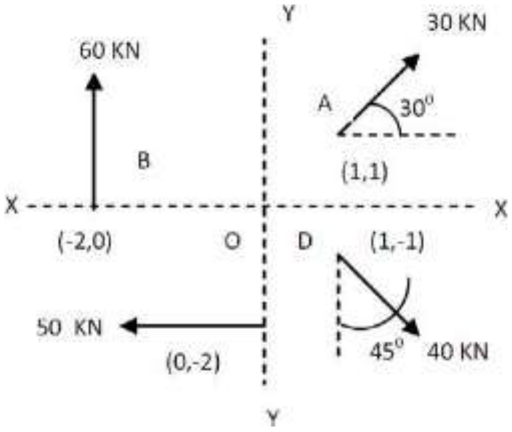
Now moment of „R" @ point „ A „

M$_R$ = + (R Sin ***Q. X)*** = + (£Fy. x) (1) 896.45. x (2)

= (2) 896.45 X = 3144.97 X = 3.51 m



Example: 9 Find magnitude, direction and location of resultant of force system with respect to point ‘O’ shown in fig.



Answer

Summation of horizontal forces

IH = +30 ***Cos*** 30°- 50 + 40Sm 45° = + 4.265 ***KN*** (^) Summation of vertical forces

IV = +30 ***Sin30° + 60 -*** 40 ***Cos*** 45° = +46.72 ***KN (*** |) Resultant force

R* = >*j(L H $)^2$ + $(Z^V)^2$ = V$(4.265)^2$ + $(46.72)^2$ = 46.91 Angle of ***KN***

resultant

tan 9 $\frac{46.72}{4.265}$

9 = 84.78

Now,as we requred to find out the position of „R" with respect to the point „O". Take the moment of all the forces @ point „O „ we have,

Mo= +(30 Cos30° X 1) - (30 Sin30° X 1) + (60 X 2)+ (50 X 2)- (40 Cos45° X 1) + (40 Sin45° X 1)

Mo= + 230.98 KN- unit (1)_(1)

Now, moment of „R" @ Pt. „O"

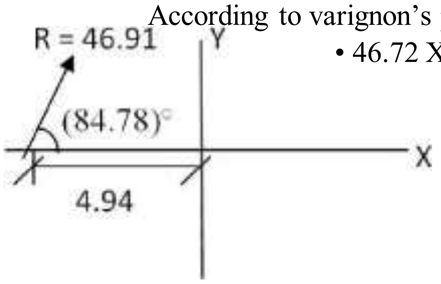
(considering „R" lies at a distance x from the point „O" in the horizontal direction)

MR = + (R Sin0 X) = (IFy.x)

M_R= +46.72.X _(2)

According to varignon’s principle

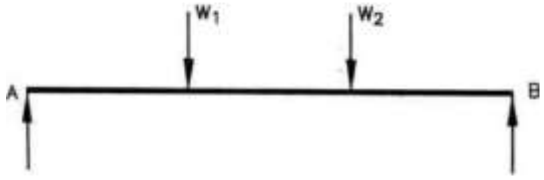
• 46.72 X = 230.98 • X = 4.94 unit



Types of load

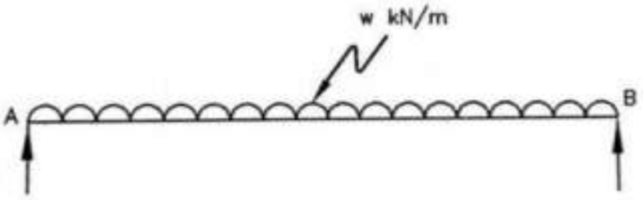
1) Point load
2) Uniformly distributed load
3) Uniformly varying load **Point load**

- Load concentrated on a very small length compare to the length of the beam, is known as point load or concentrated load. Point load may have any direction.
- For example truck transferring entire load of truck at 4 point of contact to the bridge is point load.



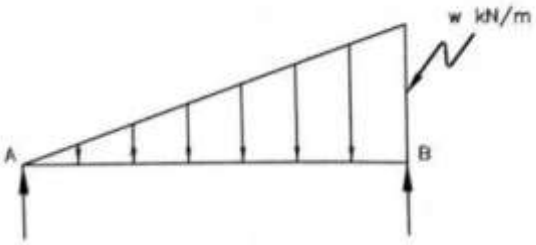
Uniformly distributed load

- Load spread uniformly over the length of the beam is known as uniformly distributed load.
- Water tank resting on the beam length
- Pipe full of water in which weight of the load per unit length is constant.



Uniformly varying load

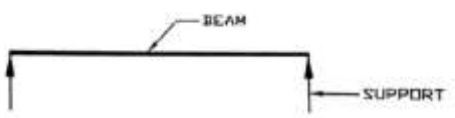
- Load in which value of the load spread over the length if uniformly increasing or decreasing from one end to the other is known as uniformly varying load. It is also called triangular load.



Type of beam

1) Simply supported beam
2) Cantilever beam
3) Fixed beam
4) Continuous beam
5) Propped cantilever beam **Simply supported beam**

• It is the beam which is rest on the support. Here no connection between beam and support.

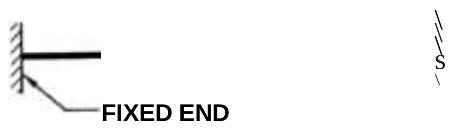


Cantilever beam

• If beam has one end fixed and other end free then it is known as cantilever beam

Fixed beam

• If both end of beam is fixed with support then it is called as fixed beam

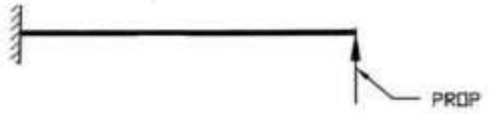


Continuous beam

• If beam has more than two span, it is called as continuous beam

Propped cantilever beam

• If one end of beam is fixed and other is supported with prop then it is known as propped cantilever beam.

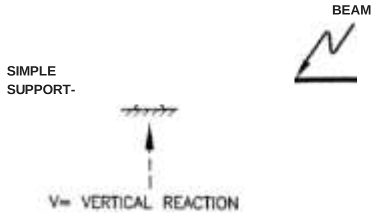


Type of support

1) Simple support
2) Roller support
3) Hinged support
4) Fixed support

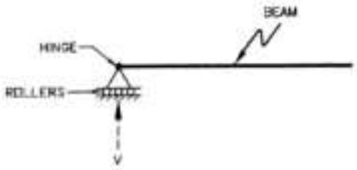
Simple support

• In this type of support beam is simply supported on the support. There is no connection between beam and support.Only vertical reaction will be produced.



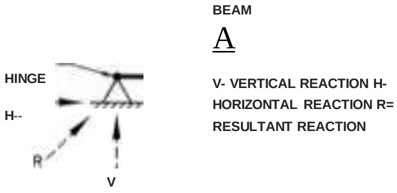
Roller support

- Here rollers are placed below beam and beam can slide over the rollers. Reaction will be perpendicular to the surface on which rollers are supported.
- This type of support is normally provided at the end of a bridge.



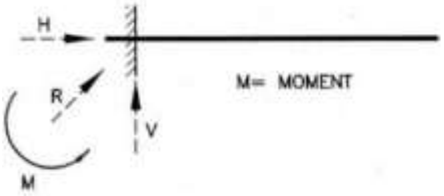
Hinged support

• Beam and support are connected by a hinge.Beam can rotate about the hingeReaction may be vertical, horizontal or inclined.

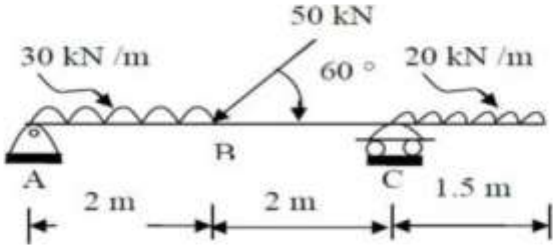


Fixed support

• Beam is completely fixed at end in the wall or support. Beam cannot rotate at end.Reactions may be vertical, horizontal, inclined and moment.



Example 1 Find out the support reactions for the beam.



Answer:

1)Now, Applying £ M = 0(1 +ve T -ve)
Now, Taking moment @ pt. A, we have,

+ (30 x 2 x 1) + (50 Sin 60 ° x 2) - (Rc x 4) = (20 x 1.5 x 4.75) = 0
Rc = 61.45 kN

2) Now £ Fy = 0

+ RAV - (30 X2) - (50 Sin60 °) +RC - (20 X 1.5) = 0 RAV
= 71.85 kN.

3) Now, £ Fx = 0

+ RAV - (50 Cos 60 °) = 0
RAV = 25.0 KN

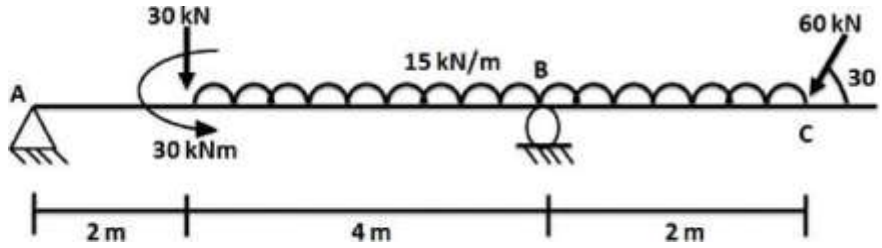
RA =V KV + RAH

R_A = 76.08 kN R
tan0 = .LAV

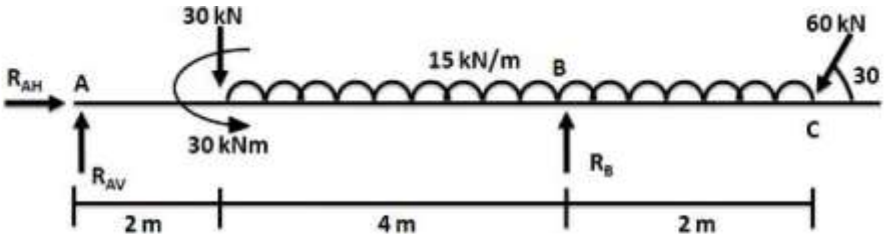
RA

6 = (70.81)

Example- 2 Determine the reactions at support A and B for the beam loaded as shown in figure



Answer:
The F.B.D. of the beam is shown below



1)Applying £ M = 0 ***1*** + ve ***T*** -ve
Take the moment @ pt. A, we have,

+ (30 X 2) - (30) - (R_B X 6) + (15 X 6 X 5) + (60 Sin 30 °) = 0 R_{AV} - 30 - (15 X 6) + R_B - (60 Sin30) = 0 R_B = 120 kN

2) I Fy = 0

R_{AV} = 30 kN

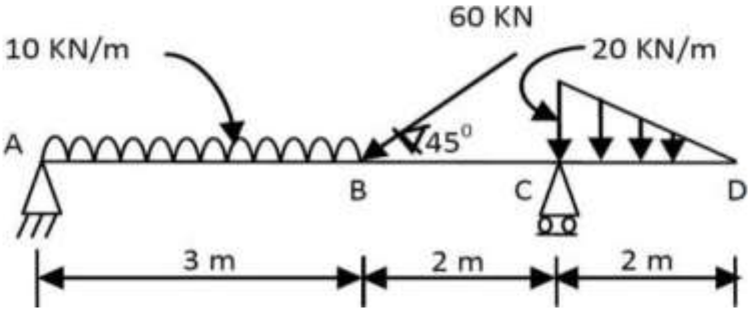
3) XFy = 0

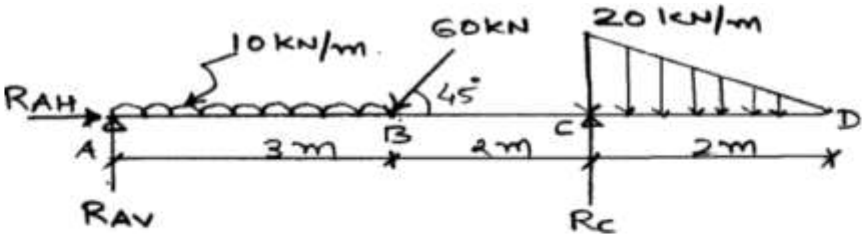
R_{AH} - 60 Cos 30 ° = 0

• R_{AH} = +51.96 kN Now, $\boldsymbol{R_A}$ = V^{R2}

y+ ^{R}***AH*** = 60 kN

Example: 3 Calculate reactions at support due to applied load on the beam as shown in Figure





Answer:

Showing the reactions at support.

1) Applying £ M = 0

Take the moment @ pt. A, we have,

+ (10 x 3 x 1.5) + (60 Sin 45 ° x 3) - (R_C x 5) + (1/2 x 20 x 2 x 5.66) = 0

R_C = 57.096 KN (|)

2) £ V = 0 t + Ve | - Ve

+ R_{AV} - (10 X 3) - (60 Sin 45 °) + R_C - (1/2 X 20 X 2) 0

Putting value of RC, we have.

R_{AV} = 35.33 KN

3) £ H = 0

R_{AH} - 60 Cos 45 ° = 0

RAH = 42.43 KN Now, RA

=VR$^{2'}$ + R^2

AH AV

= V(42 .43)^2 + (35.33)^2

=55.21 KN (^)

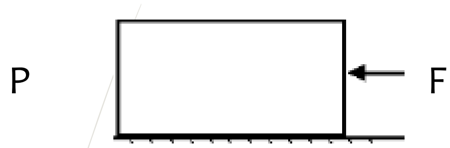
tan 0 = $\frac{R_{AV}}{R_{AH}}$ $\frac{\mathbf{35.33}}{\mathbf{42.43}}$

0 = (39.78)°

UNITII
FRICTION

> Friction or Friction Force: -

When a body slide or tens to slide on a surface on which it is resting, a resisting force opposing the motion is produced at the contact surface. This resisting force is called friction or friction force.



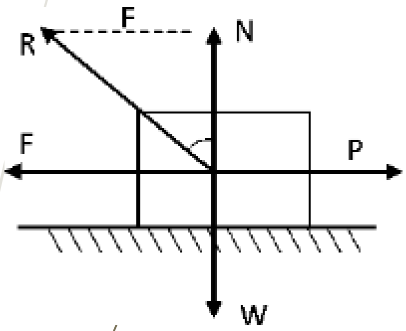
P = External force F = Friction force

Friction force (F) always act in the direction opposite to the movement of the body,

> Limiting Friction: -

When a body is at the verge of start of motion is called limiting friction or impending motion.

> Angle of Friction: -



The angle between normal reaction (N) and resultant force(R) is called angle of friction.

It is also called limiting angle of friction

The value of $ is more for rough surface as compared to smooth surface.

W = weight of block, F = Friction force

N= Normal reaction R = Resultant force

P= external force

> Coefficient of Friction (p): -

The ratio of limiting friction and Normal reaction is called coefficient of friction

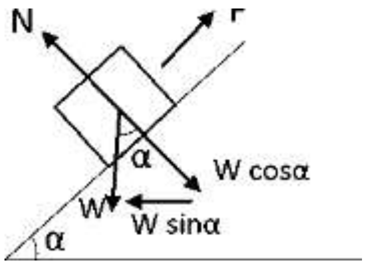
Fa N

F = pN

P = £ / ***N***

> Angle of Repose: -

With increase in angle of the inclined surface, the maximum angle at which, body starts sliding down the plane is called angle of response.



Consider a body, of weight W is resting on the plane inclined at angle (a) with horizontal.

Weight has two components

1. Parallel to the plane = w sina = F
2. Perpendicular to the plane = w cos a = N

p = **£** / ***N*** = w sin a / ***w*** cos a = tana - **1**

As we know that p = tan$ _

From equation 1 & 2

a = $

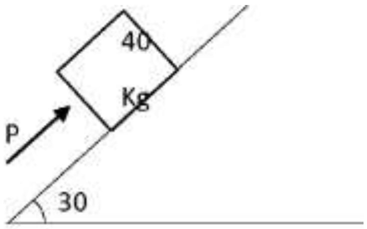
• Angle of friction = Angle of response = $.

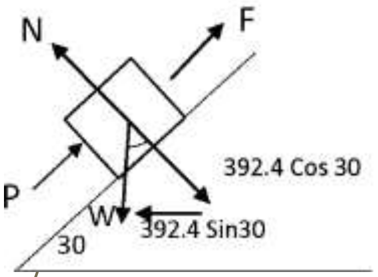
; UNIT II IFRICTIONi

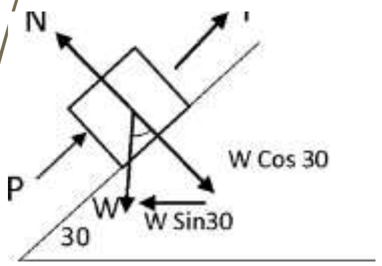
> Law of static friction: -

1. The friction force always acts in a direction, opposite to that in which the body tends to move.
2. The magnitude of friction force is equal to the external force.
3. The ratio of limiting friction (F) & normal reaction (N) is constant.
4. The friction force does not depend upon the area of contact between the two surfaces.
5. The friction force depends upon the roughness of the surfaces.

Example -1: A 40 Kg mass is placed on the inclined plane making angle of 30 with horizontal, as shown in figure. A push "P" is applied parallel to the plane. If coefficient of static friction between the plane & the mass is 0.25. Find the maximum & minimum value of P between which the mass will be in the equilibrium.







1. Weight of block

W = mg= 40* 9.81 = 392.4N

2. Minimum force (P) to maintain equilibrium.

- The force P is minimum, When the block is at point of sliding downwards.
- F will act upward along the plane

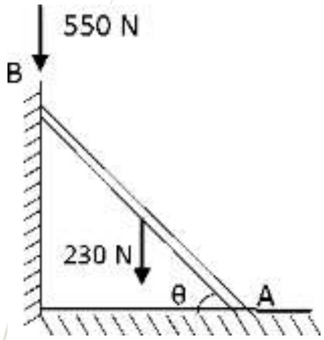
339.8

N

F = pN = 0.25*339.8 = 84.96N

Resolve force parallel to plane. P = F + 392.4 Sin30 = 281.16 N... Max of P

Example 2: A Uniform ladder AB weighting 230N & 4 m long is supported by vertical wall at top end B and by horizontal floor a t bottom end A as shown in figure. A man weighting 550N stood at the top of the ladder. Determine minimum angle of ladder AB with floors for the stability of ladders. Take coefficient of friction between ladder and wall as 1/3 & between ladder & Floor as 1/4.



pw **— 1/3, p**f **— 1/4.**

- Resolving force horizontally.

Rw **— Ff — p**f **Rf — 1/4 Rf.**

- Resolving Forces Vertically.

Rf **+ Rw — 550 + 230 R**f **+ p**w **R**w
— 780 Rf **— 1/3*1/4 R**f**—780 1.083**
Rf**—780**

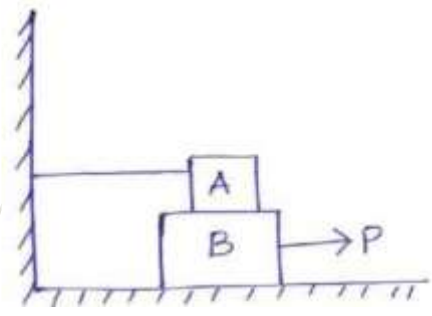
Rf **— 720.22N,**
Now,
Ff **— p**f **R**f **— 1/4 * 720.22 — 180.05 N R**w **— 1/4 R**f**— 1/4 * 720.22 — 180.05 N F**f **= P**w **R**w **F**f **— p**w **R**w **— 1/3 * 180.05 — 60 N Taking moment @ A.**
Rw *** (4 SinO) + F**w*** (4 Cos 9) — 550*4 Cos 9 +230 *2Cos 9**
• Dividing both side by Cos 0.
720.2 Tan 9 = 2420 9 —73.24°.

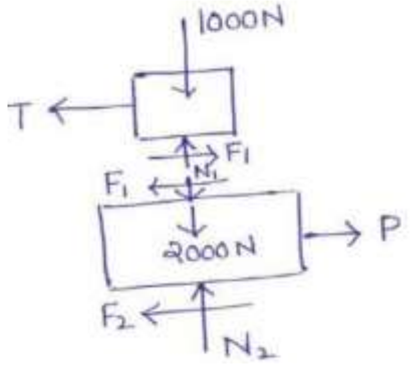
Problem 1: Block A weighing 1000N rests over block B which weighs 2000N as shown in figure. Block A is tied to wall with a horizontal string. If the coefficient of friction between blocks A and B is 0.25 and between B and floor is 1/3, what should be the value of P to move the block (B), if

(a) P is horizontal.

(b) P acts at 30° upwards to horizontal.

Solution: (a)





Considering block A,

ZV = 0

N1 =1000N

Since F1 is limiting friction,

_1= B = 0.25 N1

_ = 0.25N = 0.25x1000 = 250N

Z *H* = 0

_ - ***T*** = 0 ***T*** = _***1*** = 250N

Considering equilibrium of block B,

ZV = 0

N2 - 2000 - **N**1 = 0

N_2 = 2000 + Nj = 2000 +1000 = 3000N

_2 1

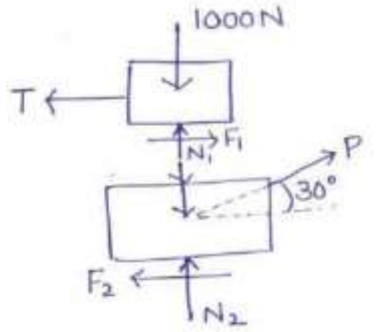
— = B = -***N***2 3

$_2$ = 0.3N_2 = 0.3x1000 = 1000N

Z H= **0**

P **=** ***F*** + F = 250 +1000 = 1250#

(b) When P is inclined:



Z^F = 0

N_2 - 2000 - F + P.sin30 = 0 ^

N2 + 0.5P = 2000 +1000 ^ N2

= 3000 - 0.5P

From law of friction,

F_2 $\frac{1}{3}$ N_2 $\frac{1}{3}$ (3000 - 0.5P) = 1000 - $\frac{05}{3}$ ***P***

Z H= 0

P cos 30 = F + F_2 , x

^ Pcos 30 = 250 + |1000 - 05 P|

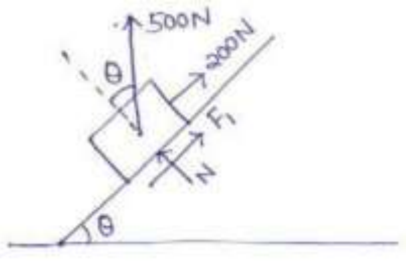
f 0 5 \ J

^ P| cos 30 + . P I = 1250

l J

^ P= 1210.43N

Problem 2: A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. The same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coefficient of friction between the inclined plane and the block.



$\sum V = 0$

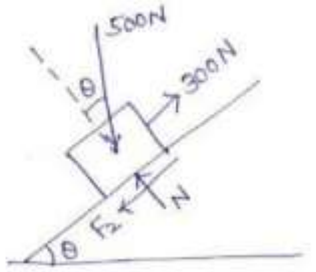
N = 500.cos0

F1 = p ***N*** = p.500 cos0

I *H*= **0**

200 + Fi = 500.sin0 ^ 200 + q.500 **(1)**

cos0 = 500.sin0



IV **=** 0

N **=** 500.cos**0**

F2 = **q**,N = **p**,.500.cos**0**

I *H*= **0**

500 sin0 + F2 = 300 ^ 500 sin0 + (2)

q.500 cos0 = 300 Adding Eqs. (1) and (2), we get

500 = 1000. sin9 sin 9 = 0.5 9 = 30°

Substituting the value of 9 in Eq. 2,

500 sin 30 + q.500 cos 30 = 300 50

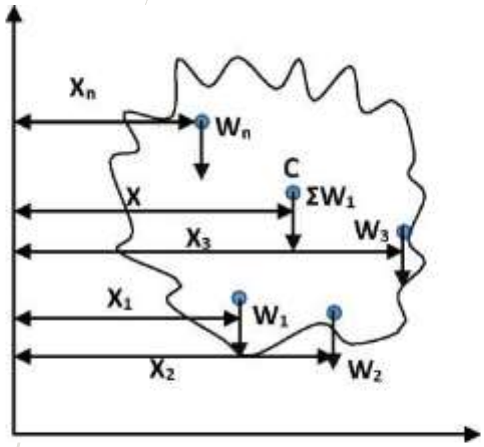
E = ■ — = 0.11547
500cos30

CENTROID AND CENTER OF GRAVITY UNIT III

Centre of Gravity

It is defined as an imaginary point on which entire, length, area or volume of body is assumed to be concentrated.

It is defined as a geometrical centre of object.



- The weight of various parts of body, which acts parallel to each other, can be replaced by an equivalent weight. This equivalent weight acts a point, known as centre of gravity of the body
- The resultant of the force system will algebraic sum of all parallel forces, there force

R = W1+W2+.........+Wn

- It is represented as weight of entire body.

W = R = I ", wi

- The location of resultant with reference to any axis (say y - y axis) can be determined by taking moment of all forces & by applying varignon"s theorem,
- Moment of resultant of force system about any axis = Moment of individual force about the same axis

R.^= W1 X1 +W2 X2 +.........+WnXm

we can write, Wlxl + W2x2 + ••• + Wnxm ***I wtXi***

* = ***n*** = I>

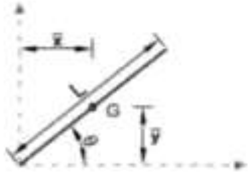
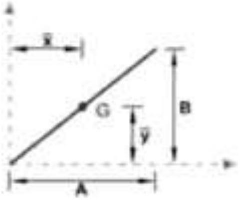
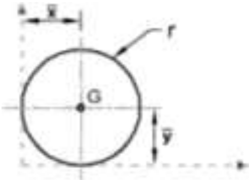
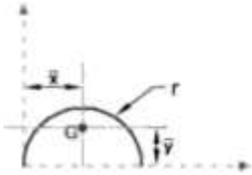
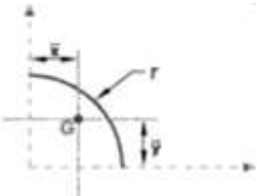
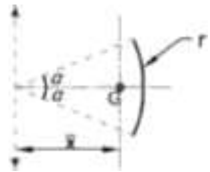
i

$$x = \frac{\int x\, dw}{\int dw}$$

Similarly,

$$y- \frac{1\ wiyi}{1\ Wi}$$

Line Element Centroid - Basic Shape

Element name	Geometrical Shape	Length	x	y
Straight line		L	*L* _cos *6 2*	*L* _sin *6 2*
Straight line		$^\wedge A^2 + B^2$	*A* 2	*B* 2
Circular wire		2 *nr*	r	r
Semi-circular		*nr*	r	*2 r* *n*
Quarter circular		*nr* *~2*	*2 r* *n*	*2 r* *n*
Circular arc		*2ra* (*a in radian*)	*r* sin *a* *a*	On Axis of Symmetry

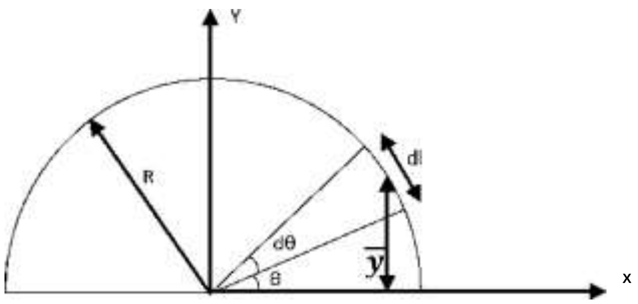
Here,

$$lix_{1+}l_2x_2 + \dots + l_n{}^x n\ ll + \quad 1\ liXi$$

$$k + it + \cdots In \quad 11$$

$$y = \frac{1\ im}{1l}$$

Centroid of semi - circular arc



> A semi-circular arc be uniform thin wire or a thin road, place it in such a way that y - axis is the axis of symmetry with this symmetry we havex=0.

Here
y
■ = ***sinO***
R

: Y = ***sindR***

dl
- = *ae*

dl = R. d0

Consider length of element is dl at an angle of 0 as shown in fig.

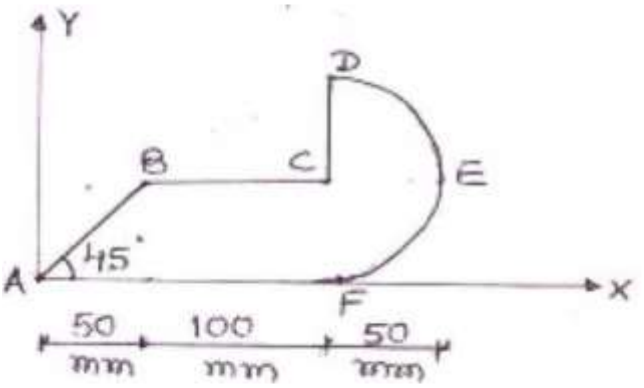
/ ***ydl*** <u>***f R*** **sin** ***6 R dP***</u> ***y f dl = f RdO***

R f **sin** ***G dG***
= ***fdd***

fQ sinG dG
fo de

2 R
y = ■
n

Example: 1. Determine the centroid of bar bent in to a shape as shown in figure.



Answer:

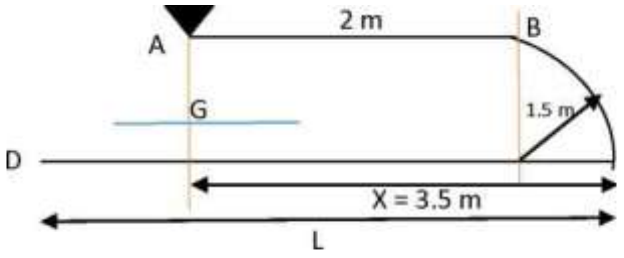
For finding out the centroid of given bar, let"s divide the bar in to 4 - element as AB, BC, CD, DEF

Member	Length	*x* mm	Y mm	Jx(mm²)	ly(mm²)
AB	***li*** = V50² + 50² = 70.71	x1 = (50/2) = 25	y1 = (50/2) = 25	***hxi*** = 1767.75	***hyi*** = 1767.75
BC	***I2*** = 100	x2 = (100/2) + 50 =100	KI ll o	***I2X2*** = 10000	o o o LD II
CD	***I3*** = 50	x3 = 50 +100 = 150	y3 = (50/2) + 50 = 75	***I3X3*** = 7500	***lsy3*** = 3750
DEF	***li*** = ***nr*** = 157.08	x4 = 50+100 + (2r/n) = 181.83	y4 = r = 50	***I1X4*** = 28561.85	***Uy4*** = 7853.95

X = ***llX±+l2X2+...+l^ Xn*** / ***I1+I2 + I3+—In*** **47829.6** / **377.79** 126.60 mm

y= ll yi + l2 y2+- +ln yn / ll +I2 +l3+ ln   18371.7 / 377.79 = 48.63 mm

Example-2. Calculate length of part DE such that it remains horizontal when ABCDE is hanged through as shown in figure.



ANSWER :

- here, we want to determine length of DC = l such that DC remains horizontal, for that centroidal axisis passes through "A".
- Reference axis is passing through c as shown in figure.

Part	Shape	Length	*x* mm	Zx(m^2)
AB	Straight line	***h* — 2**	***Xl*** — 1.5- -2 **2**	* II Ln
BC	Semi-circular arc	***2 nr*** ***li*** — " 4	***X2*** ***2 r*** — ***1.5 -*** ***n***	***I2X2*** — 1.284
CD	Straight line	***Is* — *l***	1 *3 — 2	***l²*** ***I3X3*** — —

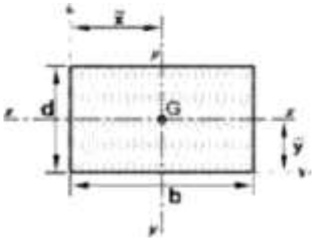
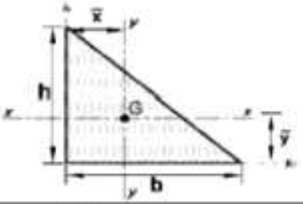
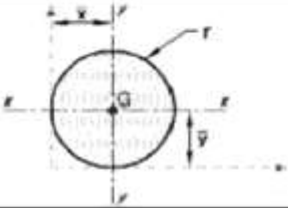
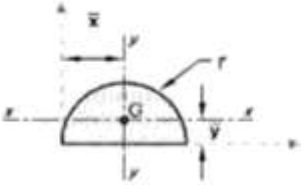
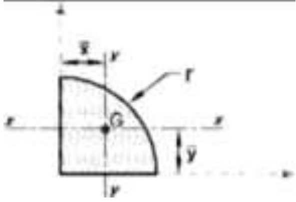
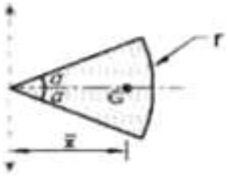
X *lx* 0.5*1*² +6.284 £ / 1 4.356+1 3.5

... 15.246+3.5 1= 0.51²+6.284

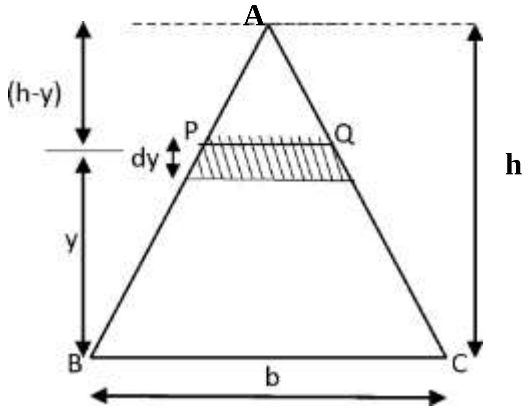
... 0.5Z²—3.5 1-8.962=0 ... 1

=8.993m

Area(Lamina) Element Centroid- Basic Shape

Element name	Geometrical Shape	Area	x	y
Rectangle		**bd**	$\frac{b}{2}$	$\frac{d}{2}$
Triangle		$\frac{1}{2}bh$	$\frac{b}{3}$	$\frac{h}{3}$
Circle		nr^2	**r**	**r**
Semicircle		$\frac{nr^2}{2}$	**r**	$\frac{4r}{3n}$
Quarter circle		$\frac{nr^2}{4}$	$\frac{4r}{3n}$	$\frac{4r}{3n}$
Circular segment		ar^2 (*a in radian*)	$\frac{2r\sin a}{3a}$	**On Axis of Symmetry**

Centroid of a triangle area



- Place one side of the triangle on any axis, say $x - x$ axis as shown in fig.
- Consider a differential strip of width „dy" at height y, by similar triangles AABC & ACDB

DE - h~Y AB h

. . . DE = (1-£)b
h

= (bfb)
h

- Now, area of strip,

dA = (b-^b) dy
h

- Now, we have

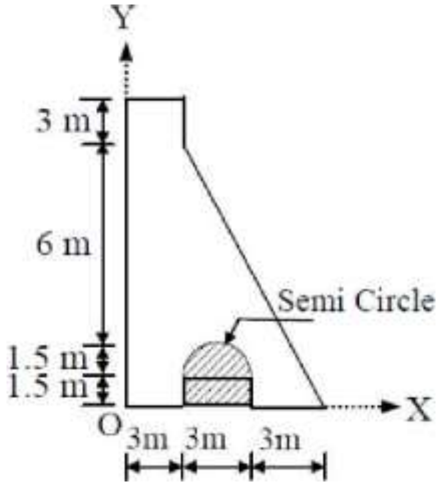
fy^dA fydA y
f dA = A

... ^Ar=f_0^h y dA
=f^h y(by — b_ y2)dy
0 h

1 X b X h X y = bh2_ —
2 2 3

y=^h
3

Example-3. Determine co-ordinates of centroid with respect to ‘o’ of the section as shown in figure.



Answer:

Let divide the given section in to 4 (four) pare

(1) : Rectangular (3 X 12)

(2) : Triangle (6 x 9)

(3) : Rectangular (3 x 1.5)

(4) : Semi - circular (r = 1.5 m)

Sr. no	Shape	Area (m^2)	*x* (m)	Y(m)	***Ax*** (m^3)	***Ay*** (m^3)
1	Rectangle	***Ai _ 12X3*** =36	3 * _ 2 _ 1.5	12 *y1* _ ***T*** _ 6	***A1X1 _ 54***	***A!y1 _ 216***
2	Triangle	1 ***Ai _ - X6X9*** =27	*2 _ 3 + . 6 3 _ 5	***9*** *y2* _ 3 _ 3	***A2X2 _*** 135	***Aryi _*** 81
3	Rectangle	***As _*** -3X1.5 _ ***-4.5***	*3 _ 3 + 1.5 _ 4.5	***1.5*** ***ys _*** - _ ***o.*** 75	A3^3 _ ***-20.25***	A3J3 _ ***-3.375***
4	Semi-circle	***WF2*** A' _ - 2 =- 3.53	***X4*** ***_ 3 + 1.5 _ 4.5***	4r y4 = L5+ _ 2.134	***A4X4*** _ -15. 88	A4J4 i _ ***-7.*** 53

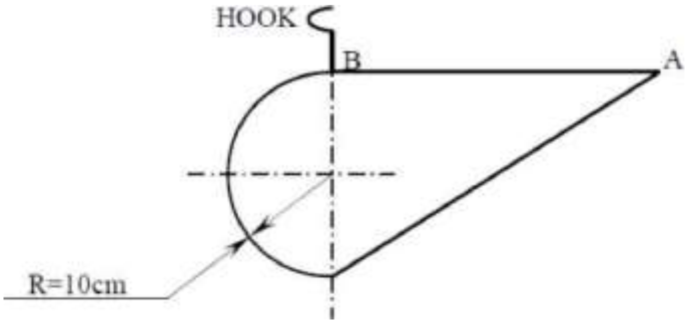
x = = 2.78 mm

^{E} ***A*** ***A!+A2+A3+-+***A_n

~Y= = ^{A}***iyi+***^{A}***2yz+-+***^{A}***nyn*** _ 5.20 mm

E A ***Ai+***A_2***+***A_3 + -+A_n

Example 4 A lamina of uniform thickness is hung through a weight less hook at point B such that side AB remains horizontal as shown in fig. determine the length AB of the lamina.



Answer:

Let, length AB=L, for remains horizontal of given lamina moment of areas of lamina on either side of the hook must be equal.

.■ . $A_1X_1 = A_2X_2$

2

■ ■ ■ (1 x L x 20) (- X L) = (12. x n) (i££=i°2)

2 3 2 3n J

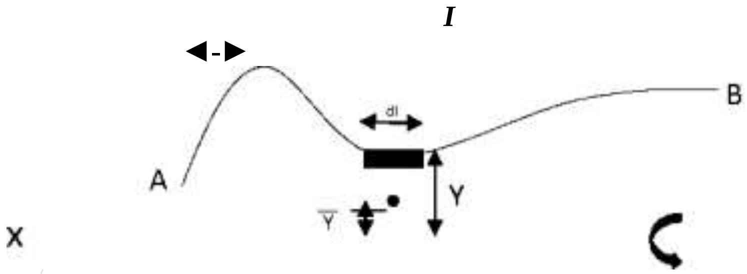
$20l^2$

. ■ . . = 157.08 x 4.244

6

. . . L =14.14 cm

<u>**Pappus Guldinus first theorem**</u>



- This theorem states that, "the area of surface of revolution is equal to the product of length of generating curves & the distance travelled by the centroid of the generating curve while the surface is being generated".
- As shown in fig. consider small element having length dl & at 'y' distance from $x - x$ axis.
- Surface area dA by revolving this element dA= 2ny.dl (complete revolution)

^ Now, total area,

. . A = / *dA* =/ 2 *nydl= 2nJ ydl*

. . A = *2ny l*

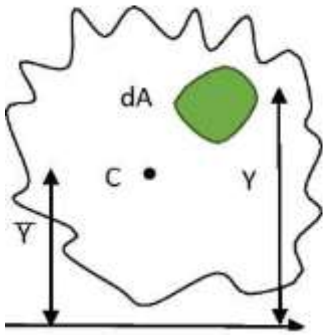
> When the curve rotate by an angle '0'

. . A = 2ny *12-* = ^{0} *yl*

<u>**Pappus guldinus second theorem**</u>

- This the rem states that, "the volume of a body of revolution is equal to the product of the generating area & distance travelled by the centroid of revolving area while rotating around its axis of rotation.
- Consider area 'dA' as shown in fig. the volume generated by revolution will be

dv= Q n Y .dA



- Now, the total volume generated by lamina,

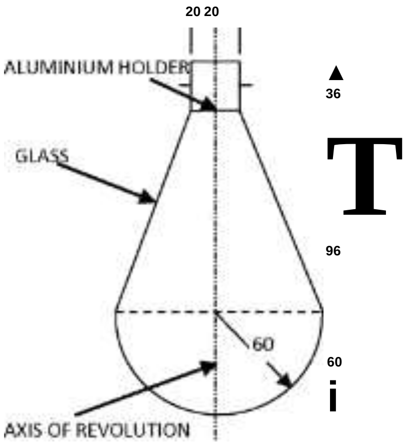
V=J *dv* =/ *2nydA*

= *2nyA* (completed revolution)

- When the area revolves about '0' angle volume will be

$V=2nyA \frac{0}{} = 0\, yA$

Example-5. Find surface area of the glass to manufacture an electric bulb shown in fig using first theorem of Pappu's Guldinus.



Line	length	x mm	!^(mm²)
AB	L1=20	20 = T= i0	200
BC	L2=36	xi = 20	720
CD	L3=V40² + 96² =104	40 x — 20 + — 40 3 2	4160
DE	L4=s£ 2 =94.25	2r x — = 38.20 4 n	36000

X = $\frac{2\,Lx}{L}$ = 34.14mm

Surface area = L9 ***x* =** 254.25 x 2***n*** x34.14 =

54510.99mm²

MOMENT OF INERTI'A
■ UNIT IV !

Introduction

- The moment of force about any point is defined as product of force and perpendicular distance between direction of force and point under consideration. It is also called as first moment of force.
- In fact, moment does not necessary involve force term, a moment of any other physical term can also be determined simply by multiplying magnitude of physical quantity and perpendicular distance. Moment of areas about reference axis has been taken to determine the location of centroid. Mathematically it was defined as,

Moment = area x perpendicular distance.

$$M = (A \times y)$$

- If the moment of moment is taken about same reference axis, it is known as moment of inertia in terms of area, which is defined as,

Moment of inertia = moment x perpendicular distance.

$$I_A = (M \times y) = A.y \times y = A\,y^2$$

- Where I_A is area moment of inertia, A is area and 'y' is the distance been centroid of area and reference axis. On similar notes, moment of inertia is also determined in terms of mass, which is defined as,

$$I_m = mr^2$$

- Where 'm' is mass of body, 'r' is distance between center of mass of body and reference axis and I_m is mass of moment of inertia about reference axis. It must be noted here that for same area or mass moment of inertia will be change with change in location of reference axis.

> Theorem of na.ra.llel Axis: -

- It states, "If the moment of inertia of a plane area about an axis through its center of gravity is denoted by **I**$_G$, then moment of inertia of the area about any other axis AB parallel to the first and at a distance 'h' from the center of gravity is given by,

$$I_{AB} = I_G + ah^2$$

- Where I_{AB} = moment of inertia of the area about AB axis I_G = Moment of inertia of the area about

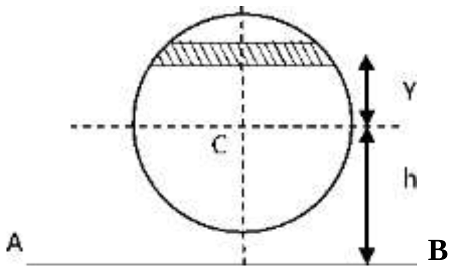
centroid
a = Area of section

h = Distance between center of gravity (centroid) of the section and axis AB.

Proof: -

- Consider a strip of a circle, whose moment of inertia is required to be found out a line 'AB' as shown in figure.

Let d**a** = Area of the strip. y = Distance of the strip from the C.G. of the section
h = Distance between center of gravity of the section and the 'AB 'axis.



- We know that moment of inertia of the whole section about an axis passing through the center of gravity of the section.

$$= day^2$$

- And M.I of the whole section about an axis passing through centroid.

$$I_G = E da\ y^2$$

- Moment of inertia of the section about the AB axis

$$I_{AB} = Eda(h+y)^2$$

$$= Eda\ (h^2 + 2hy + y^2)$$

$$= ah^2 + I_G$$

- It may be noted that $Edah^2 = ah$ and $Ey^2da = I_G$ and $Eday$ is the algebraic sum of moments of all the areas, about an axis through center of gravity of the section and is equal ay, where y is the distance between the section and the axis passing through the center of gravity which obviously is zero.

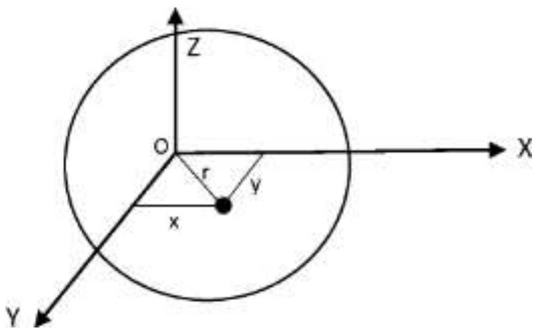
> Theorem of Perpendicular Axis: -

- It states, If I_{XX} and I_{YY} be the moment of inertia of a plane section about two perpendicular axis meeting at 'o' the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by,

$$I_{ZZ} = I_{ZZ} + I_{YY}$$

Proof: -

- consider a small lamina (P) of area 'da' having co-ordinates as ox and oy two mutually perpendicular axes on a plane section as shown in figure.
- Now, consider a plane OZ perpendicular ox and oy. Let (r) bethe distance of the lamina (p) from z-z axis such that op = r.



- From the geometry of the figure,we find that,

$$r^2 = x^2 + y^2$$

- We know that the moment of inertia of the lamina 'p' about x-x axis,

$$I_{xx} = da \cdot y^2$$

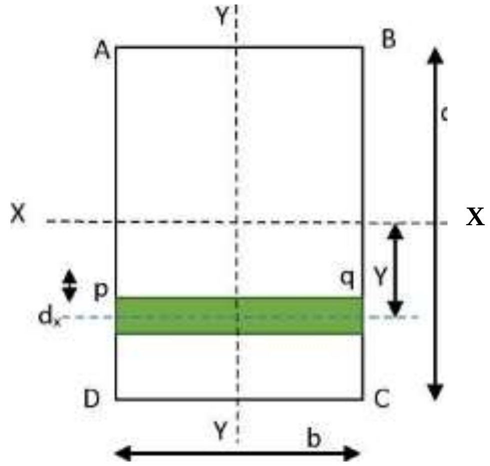
Similarly, $I_{yy} = da\, x^2$ and $I_{zz} = da\, r^2$

$$= da\,(x^2 + y^2)$$

$$= da\, x^2 + da\, y^2$$

$$I_{zz} = I_{zz} + I_{YY}$$

> **Moment of Inertia of a Rectangular Section: -**



Y

- Consider a rectangular section ABCD as shown in fig. who se moment of inertia is required to be found out.
- Let, b = width of the section d = Depth of the section
- Now, consider a strip PQ of thickness dy parallel to x-x axis and at a distance y-from it as shown in fig.

Area of strip = b.dy

- We know that moment of inertia of the strip about x-x axis,

= Area x y^2

= (b.dy) y^2

- Now, moment of inertia of the whole section may be found out by integrating the about equation for the whole length of the lamina i.e. from -d/2 to +d/2

IXX = (+d/2 ***b. y²d*** v

'-d/2

+d/2

IXX ~6J-d/2 • ***y²dy***

y* 3 *+d/2

[3] ***'-d/2***

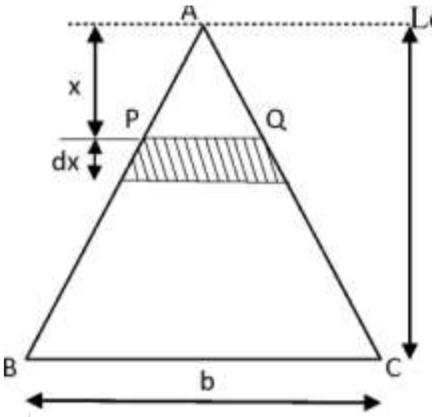
***bd* ³ 12**

Similarly, IYY ***db*** ³ **12**

If it is square section,

$I_{XX} = I_{YY}$ ***b⁴*** ***4*** _ ***d***

12 12



Let, b = Base of the triangular section.

h = height of the triangular section.

Now, consider a small strip PQ of thickness 'dx' at a distance from the vertex A as shown in figure, we find that the two triangle APQ and ABC are similar.

$$\frac{PQ}{BC} = \frac{x}{h} \qquad \text{or } PQ = \frac{BC \cdot x}{h} = \frac{b \cdot x}{h}$$

We know that area of the strip PQ = $\frac{b \cdot x}{h}$ dx

And moment of inertia of the strip about the base BC

= Area x (Distance)^2

= $\frac{bx}{h}$ dx (h-x)^2

- **Now, moment of inertia of the whole triangular section may be found out by integrating the above equation for the above equation for the whole height of the triangle i.e. from 0 to h.**

IBC = J$_0^h$ V(h - x)^2 dx

= ~ f^h (h^2 + x^2 + 2hx) xdx

h 0

_b x^{2}y ^{2}, x^4, 2hx^3 ■ h

= h$^{l'}$ r$^+$ T$^+$ 3"Jo

bh ^{3}

BC 12

- **We know that the distance between center of gravity of the triangular section and Base BC,**

$$d = \frac{h}{3}$$

- **so, Moment of the inertia of the triangular section about an axis through its center through its center of gravity parallel to x-x axis,**

IG = IBC - ad^2

$$\frac{bh^3}{12} \quad \left(\frac{bh}{3} \wedge \frac{}{3}\right)^2$$

$$I_G = \frac{bh^3}{36}$$

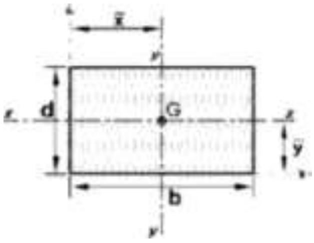
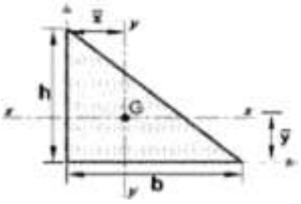
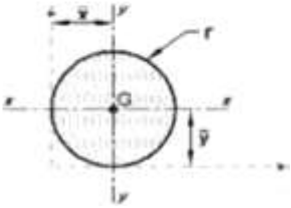
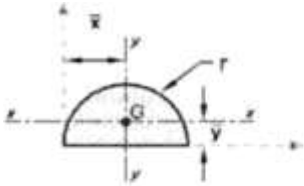
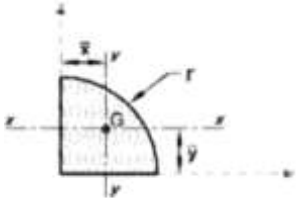
Note: - The moment of inertia of section about an axis through its vertex and parallel to the base.

Itop = IG + ad^2 ***bh*** ^{3} ***bh 2 h***

=16 + (T)(T)^2

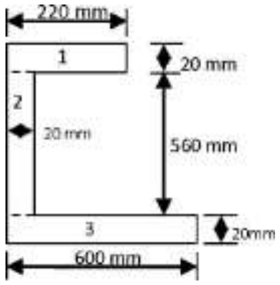
9bh ^{3} 36 ***bh*** ^{3}

Area (Lamina) Element - Moment of Inertia (Basic Shape)

Element name	Geometrical Shape	Area		*yy*
Rectangle		bd	***bd³*** ***~Y2***	***db***³ ~12
Triangle		$\frac{1}{2}$ ***bh***	***bh³*** ***~36***	***hb³*** ***~36***
Circle		***nr***²	***nd⁴*** ***~64***	***nd⁴*** ***~64***
Semicircle		***nr***² ~	0.11 ***r⁴***	***nd⁴*** 128
Quarter circle		***nr²*** -	0.055 ***r⁴***	0.055 ***r⁴***

d= diameter

Example - 1: Find out moment of inertia at horizontal and vertical centroid axes, top and bottom edge of the given lamina.



Answer: -

1) <u>centroid of given lamina</u>

Let's divide the given lamina in to three Rectangle

(1) Top rectangle 200 x 20 mm^2

(2) Middle rectangle 20 x 600 mm^2

(3) Bottom rectangle 580 x 20 mm^2

Sr no	Shape	Area (mm^2)	X (mm)	Y (mm)	AX (mm^2)	AY (mm^2)
1	**1**	**A_1 = 200 x20 = 4000**	**X_1 = 20+2^{00}. = 120 2**	**Y_1 = 20+560+20 2 = 590**	**A1X1= 480,000**	**A_1Y_1 = 2,36,0000**
2	**2**	**A_2 = 600 x 20 =12000**	**X_2 = 20 = 10** 2	**Y_2 = ^{600} = 300** 2	**A_2X_2 = 1,20,000**	**A_2Y_2 = 3,60,0000**
3	**3**	**A_3 = 580 x 20 =11600**	**X_3 = 580 + 20 = 3 10** 2	**Y_3 =** 20 = 10 2	**A_3X_3 = 35,96,000**	**A_3Y_3 = 116000**
		EA =27600			**XAX = 4196000**	**XAY = 6076000**

y = M. = $\frac{6076000}{}$ = 220.15 mm

XA 27600

X = = 4196000. = 152.03 mm

XA 27600

(2) Moment of inertia about centroid horizontal axis: -

Sr No	Area (mm^2)	h (mm)	Ah^2 (mm^4)	I_G (mm^4)	$I_{xx} = I_G + Ah^2$
1	**A_1 = 4000**	**hj = yt - = 2 369.85**	2 **A1h1 = 5.4716 x 10^8**	**$I_{G1} = b_1h_1^3/12$ = 1.33334x10^5**	**I_1 = 5.4729 x 10^8**
2	**A_2 = 12000**	**h_2 = yt - — = 79.85** 2	2 **A2h2 = 7.6512 x 10^7**	**$I_{G2} = b_2h_2^3/12$ = 3.6 x 10^8**	**I_2 = 4.3651 x 10^8**
3	**A_3 = 11600**	**h3= yb - =** 2 **210.15**	2 **A3h3 = 5.1229 x 10^8**	**$I_{G3} = b_3h_3^3/12$ = 3.8667 x 10^5**	**I_3 = 5.1268 x 10^8**

Now, Moment of inertia at centroid horizontal axis $I_{XX} = I_1 + I_2 + I_3$

= 1.4965 x 10^9 mm^4

(3) Moment of inertia about centroid verticalaxis: -

Shape No	Area (mm^2)	h (mm)	Ah^2 (mm^4)	I_G (mm^4)	$I_{yy} = I_G + Ah^2$
1	A_1 = 4000	$h_1 = X_i - X_1$ = 32.03	$A_1h_1^2 = 4.1036 \times 10^6$	$I_{G1} = d_1b_1^3/12 = 1.33334 \times 10^7$	$I_1 = 1.7437 \times 10^7$
2	A_2 = 12000	$h_1 = X_i - X_2$ = 142.03	$A_2h_2^2 = 2.4207 \times 10^8$	$I_{G2} = d_2b_2^3/12 = 4 \times 10^5$	$I_2 = 2.4247 \times 10^8$
3	A_3 = 11600	$h_1 = X_3 - X_i$ = 310	$A_3h_3^2 = 1.1148 \times 10^9$	$I_{G3} = d_3b_3^3/12 = 3.2519 \times 10^8$	$I_3 = 1.4399 \times 10^9$

Now, Moment of inertia at centroidal axis

$I_{yy} = I_I + I_2 + I_3$

$= 1.6998 \times 10^9$ mm^4

(4) Moment of inertia about top edge of horizontal axis: -

Shape no	Area (mm^2)	h (mm)	Ah^2 (mm^4)	I_G (mm^4)	$I_{tt} = I_G + Ah^2$
1	A_1 = 4000	$h_1 = d_1/2 = 10$	$A_1h_1^2 = 4 \times 10^5$	$I_{G1} = b_1d_1^3/12 = 1.33334 \times 10^5$	$I_1 = 5.3334 \times 10^5$
2	A_2 = 12000	$h_2 = d_2/2 = 300$	$A_2h_2^2 = 1.08 \times 10^9$	$I_{G2} = b_2d_2^3/12 = 3.6 \times 10^9$	$I_2 = 1.44 \times 10^9$
3	A_3 = 11600	$h_3 = d_3/2 = 590$	$A_3h_3^2 = 4.038 \times 10^9$	$I_{G3} = b_3d_3^3/12 = 3.8667 \times 10^5$	$I_3 = 4.0384 \times 10^9$

Now, Moment of inertia at top edge of horizontal axis

$I_{tt} = I_i + I_2 + I_3$

$= 5.4789 \times 10^9$ mm^4

(5) Moment of inertia about bottom edge of horizontal axis: -

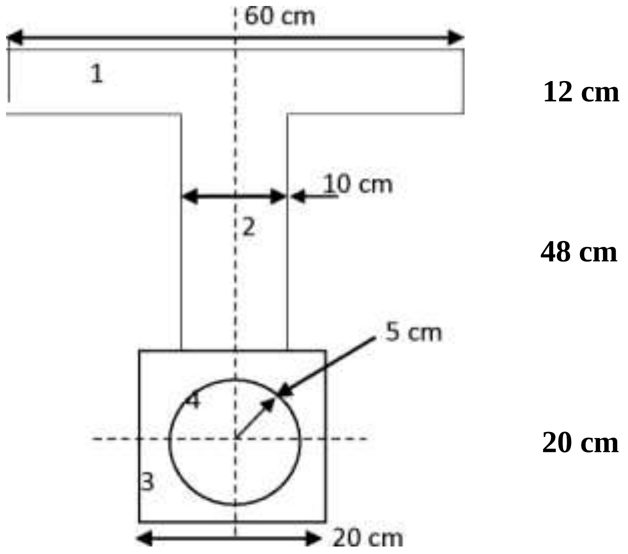
Shape no	Area (mm^2)	h (mm)	Ah^2 (mm^4)	I_G (mm^4)	$I_{bb} = I_G + Ah^2$
1	A_1 = 4000	$h_1 = d - ^2 2$ = 590	$A_1h_1^2 = 1.3924 \times 10^9$	$I_{G1} = b_1d_1^3/12 = 1.33334 \times 10^5$	$I_1 = 1.3925 \times 10^9$
2	A_2 =12000	h_2 = — 2 = 300	$A_2h_2^2 = 1.08 \times 10^9$	$I_{G2} = b_2d_2^3/12 = 3.6 \times 10^5$	$I_2 = 1.44 \times 10^9$
3	A_3 = 11600	h = 43 2 = 10	$A_3h_3^2 = 1.16 \times 10^6$	$I_{G3} = b_3d_3^3/12 = 3.8667 \times 10^5$	$I_3 = 1.5467 \times 10^6$

Now, Moment of inertia at bottom edge of horizontal axis

$I_{tt} = I_i + I_2 + I_3$

$= 2.834 \times 10^9$ mm^4

Example-2: Determine moment of inertia of a section shown in figure about horizontal centroid axis.



Answer: -

(1) Centroid of given lamina

Let's divide the given lamina in to four part

- (i) Top rectangular 60 x 12 cm^2
- (ii) Middle rectangular 10 x 48 cm^2
- (iii) Bottom square 20 x 20 cm^2
- (iv) Deduct circle of radius 5 cm from bottom square

SR NO.	Shape	Area (cm^2)	Y (cm)	AY (cm^3)
1	1	A_1 = 60 x 12 = 720	Y_1 = 20+48+12 = 2 74	A_1Y_1 = 34560
2	2	A_2 = 10 x 48 = 480	Y_2 = 20 + 48-= 300 2	A_2Y_2 = 21120
3	3	A_3 = 20 x 20 = 400	Y_3 = 20 = 10 2	A_3Y_3 = 4000
4	4	$A_4 = -nr^2 = -78.54$	Y_4 = 20 = 10 2	A_4Y_4 = -785.4
		LA = 1521.46		LAY = 58894.6

$$y = \frac{LAY}{LA} = \frac{58894.6}{1521.46} = 38.70 \text{ cm}$$

(2) Moment of inertia about centroid horizontal axis: -

Shape no	Area (cm^2)	h (cm)	Ah^2 (cm^4)	I$_G$ (cm^4)	I$_{XX}$ = I$_G$ + Ah^2
1	A_1 = 720	h1 = yt - 41 = 35.3 2	A_1h^2 = 897.1 x 10³	I_{G1} = b1h1³/ 12 = 8640	I_1 = 905824.8
2	A_2 = 480	h2 = yt - 41 = 17.3 2	A_2h^2 = 143.65 x 10³ 2 2	I_{G2} = b2h2³/ 12 = 92160	I_2 = 235819.2
3	A_3 = 400	h3= yb - 42 = 28.7 2	$A_3h_3^2$ = 329.4 x 10³	I_{G3} = b3h3³/ 12 = 13333.34	I_3 = 342809.34
4	A_4 = 78.54	H4= 28.7	$A_4h_4^2$ = -64.6 x 10³	I_{G3} = Md⁴/ 64 = -490.8	I_3 = -65183.48

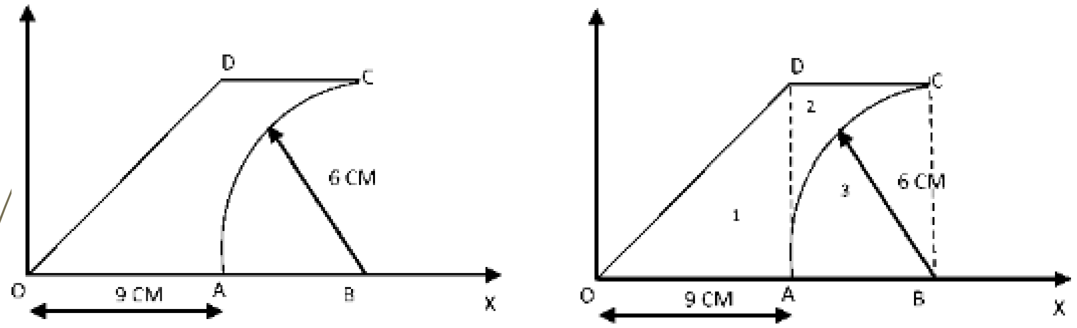
Now, Moment of inertia at centroid horizontal axis

$I_{XX} = I_i + I_2 + I_3$

$= 1.419 \times 10^6$ cm⁴

Example-3: - Find the moment of inertia about Y-axis and X-axis for the area shown in fig-

Y Y



(1) Moment of inertia about x- axis (o-x line)

Sr No	Area (cm^2)	h (cm)	Ah^2 (cm^4)	I$_G$ (cm^4)	I$_{OX}$ = I$_G$ + Ah^2
1	A_1 = '/$_2$ bh = 4000	h1 = i = 2 3	A1h1²= 108	I_{G1} = bh³/ 36 = 54	I_1 = 162
2	A_2 = d x d = 12000	h2 = d = 3 2	A2h2²= 324	I_{G2} = d⁴/ 12 = 108	I_2 = 432
3	A_3 = «r²=11600 4	h3= il = 2.55 3 M	A3h3 = 183.35	I_{G3} = 0.055r⁴ = 71.28	I_3 = 254.62

Now, Moment of inertia at centroid horizontal axis

$I_{XX} = I_i + I_2 + I_3$

$= 339.37$ cm^4

(2) Moment of inertia about y- axis (OY - line)

Shape no	Area (cm^2)	h (cm)	Ah^2 (cm^4)	I$_G$ (cm^4)	I$_{OY}$ = I$_G$ + Ah^2
1	t-' (N II <	h1 = 6	A1h 3^2 = 972	I_{G1} = b^{3}h/ 36 = 121.5	I_1 = 1093.5
2	A2 = 12	h2 = 12	A2h$_2$^{2} = 5184	I_{G2} = d^4/ 12 = 108	I_2 = 5292
3	A3 = 12.45	h3= 12.45	A3h 3“ = 4381.9	I_{G3} = 0.055r^4 = 71.28	I_3 = 4456.35

Now, Moment of inertia at centroid horizontal axis

$I_{XX} = I_i + I_2 - I_3 = 1929.15$ cm^4

UNIT V

KMEMATICS AND *123

CONCEPT OF MOTION

***A body is said to be in motion if it changes its position with respect to its surroundings.* The nature of path of displacement of various particles of a body determines the type of motion. The motion may be of the following types :**

1. **Rectilinear translation**
2. **Curvilinear translation**
3. **Rotary or circular motion.**

Rectilinear translation is also known as *straight line motion.* Here particles of a body move in straight parallel paths. *Rectilinear means forming straight lines and translation means behaviour.* Rectilinear translation will mean behaviour by which straight lines are formed. Thus, when a body moves such that its particles form parallel straight paths the body is said to have rectilinear translation.

In a *curvilinear translation the particles of* a body move *along circular arcs or curved paths.*

***Rotary* or *circular motion* is a special case of curvilinear motion where particles of a body move along *concentric circles* and the *displacement* is *measured in terms of angle in radians or revolutions*.**

DEFINITIONS

1. Displacement. If a particle has rectilinear motion with respect to some point which is assumed to be fixed, its displacement is its total change of position during any interval of time. The point of reference usually assumed is one which is at rest with respect to the surfaces of the earth.

The unit of displacement is same as that of distance or length. In M.K.S. or S.I. system it is *one metre.*

2. Rest and motion. A body is said to be at *rest* at an instant (means a small interval of time) if its position with respect to the surrounding objects remains unchanged during that instant.

A body is said to be in *motion* at an instant if it changes its position with respect to its surrounding objects during that instant.

Actually, nothing is absolutely at rest or absolutely in motion : *all rest or all motion is relative*

***3. only.* Speed. The speed of body is defined as *its rate of change of its position with respect to its surroundings irrespective of direction.* It is a *scalar quantity.* It is measured by distance covered per unit time.**

Mathematically, speed

Distance covered _ *S* Time taken *t* Its units are m/sec or km/ hour.

4. Velocity. The velocity of a body is *its rate of change of its position with respect to its surroundings in a particular direction.* It is a *vector quantity.* It is measured by the distance covered in a *particular direction* per unit time.

i.e., **Velocity** $= \frac{\text{Distance covered (in a particular direction)}}{\text{Time taken}}$

$$v = \frac{S}{t}$$

Its units are same as that of speed *i.e.,* m/sec or km/hour.

5. Uniform velocity. *If a body travels equal distances in equal intervals of time in the same direction it is said to be moving with a uniform or constant velocity.* If a car moves 50 metres with a constant velocity in 5 seconds, its velocity will be equal to,

$\frac{50}{5} = 10$ **m/s.**

6. Variable velocity. *If a body travels unequal distances in equal intervals of time, in the same direction, then it is said to be moving with a variable velocity* or if it is changes either its speed or its direction or both shall again be said to be moving with a variable velocity.

7. Average velocity. *The average or mean velocity of a body is the velocity with which the distance travelled by the body in the same interval of time, is the same as that with the variable velocity*.

If *u* = initial velocity of the body

***v* = final velocity of the body *t* = time taken**

***S* = distance covered by the body**

Then average velocity $\frac{u + v}{}$

and $S = $ F $\frac{u + v}{}$ $\times t$

H 2 K

8. Acceleration. The *rate of change of velocity of a body is called its acceleration.* When the velocity is increasing the acceleration is reckoned as *positive,* when decreasing as *negative.* It is represented by *a* or *f.*

If *u* = initial velocity of a body in m/sec

***v* = final velocity of the body in m/sec**

***t* = time interval in seconds, during which the change has occurred,**

Then acceleration, $a = \sim$ u m/sec

***t* sec**

or $\frac{v \sim u}{}$ 2

$a =$ m/sec^2 t

From above, it is obvious that if *velocity of the body remains constant, its acceleration will be zero.*

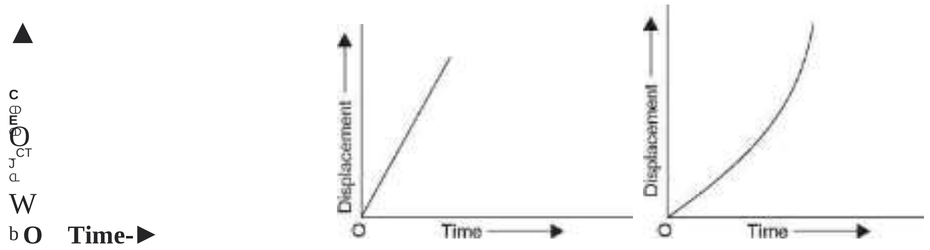
9. Uniform acceleration. ***If the velocity of a body changes by equal amounts in equal intervals of time, the body is said to move with uniform acceleration.***

10. **Variable acceleration.** ***If the velocity of a body changes by unequal amount in equal intervals of time, the body is said to move with variable acceleration.***

DISPLACEMENT-TIME GRAPHS

Refer to Fig (a). The graph is parallel to the time-axis indicating that the *displacement is not changing with time*. The *slope of the graph is zero*. The body has no velocity and is at rest.

Refer to Fig. (b). The displacement increases linearly with time. The displacement increases by equal amounts in equal intervals of time. *The slope of the graph is constant.* In other words, the body is moving with a *uniform velocity*.



(a) (b) (c)

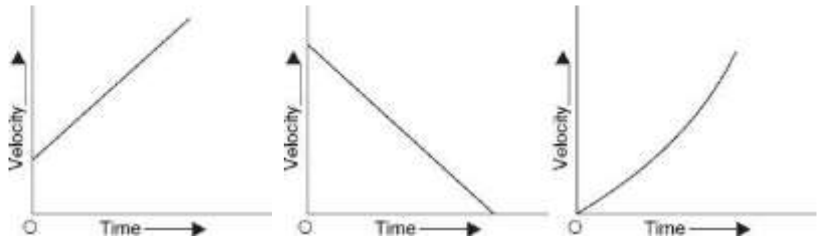
Fig. Displacement-time graphs

Refer to Fig. (*c*). The displacement time graph is a *curve*. This means that the displacement is not changing by equal amounts in equal intervals of time. The slope of the graph is different at different times. In other words, the velocity of the body is changing with time. The motion of the body is accelerated.

7.4. VELOCITY-TIME GRAPHS

Refer to Fig. (*a*). The velocity of the body increases linearly with time. The slope of the graph is constant, i.e., velocity changes by equal amounts in equal intervals of time. In other words, the *acceleration of the body is constant.* Also, at time *t* = 0, the velocity is finite. Thus, the body, *moving with a finite initial velocity, is having a constant acceleration*.

Refer to Fig. (b). The body has a finite initial velocity. As the time passes, the velocity decreases linearly with time until its final velocity becomes zero, *i.e.* it comes to rest. Thus, the body has a *constant deceleration* (or retardation) since the *slope of the graph is negative.*



(a) (b) (c)

Fig. Velocity-time graphs

Refer to Fig. (c). The velocity-time graph is a *curve.* The slope is therefore, different at different times. In other words, the *velocity is not changing at a constant rate. The body does not have a uniform acceleration since the acceleration is changing with time.*

EQUATIONS OF MOTION UNDER UNIFORM ACCELERATION First Equation of Motion. *Relation between u, v, a and t.*

Let us assume that a body starts with an initial velocity *u* and acceleration *a.* After time t, it attains a velocity v. Therefore, the change in velocity in *t* seconds = *v - u.* Hence, the change in velocity in one second $\frac{v - u}{t}$. By definition, this is equal to the acceleration *a.*

Thus, $$a = \frac{v - u}{t}$$

or $at = v - u$

or $v = u + at$

Second Equation of Motion. ***Relation between S, u, a and t.***

Let a body moving with an initial uniform velocity *u* is accelerated with a uniform acceleration *a* for time t. After time *t* its final velocity is v. The distance *S* which the body travels in time *t* is determined as follows :

Now, since the acceleration is uniform, *i.e.,* the velocity changes by an equal amount in equal intervals of time, it is obvious that the average velocity is just the average of initial and final velocities.

Average velocity =

H 2 K

∴ Distance travelled = average velocity x time

F ***u* + v** I

S* = x *t

F 2 K ***+ at*** I **x *t***

or ***S* =** F ***u + u*** K **(V *v = u + at)***

H 2 **atl x *t***

H 2 K

i

or ***S = ut + at*²** 2

Third Equation of Motion. ***Relation u, v, a and S.*** **We know, that**

verage	velocity x time	
F *u* + v I	x *t*	
.H 2 K		
F *u* + v I	F v — *u I*	F.
2 2 v² - u H 2 K	H *a* K	H

*u*I

t *v*

K

a

2a

v² - u² = 2aS

DISTANCE COVERED IN nth SECOND BY A BODY MOVING WITH UNIFORM ACCELERATION

Let u = initial velocity of the body a = acceleration

then

S_nth = d	istance covered in nth second	
S =	'distance covered I J	1 distance covered in (n - 1)
nth	H in n second, s_n K $S = un + \frac{1}{2}an^2$ H	second, s_{n-1}

Using the relation,

$S_n = un + \frac{1}{2}an^2$ ('■ t = n)

and

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$= u(n-1) + \frac{1}{2}a(n^2 - 2n + 1)$$

$$S_{nth} = S_n - S_{n-1} = un + \frac{1}{2}an^2 - u(n-1) + \frac{1}{2}a(n^2 - 2n + 1)$$

H K N 2 Q

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - a/2$$

$$= u + an - a/2$$

$$S_{nth} = u + a/2(2n - 1)$$

1. *A car accelerates from a velocity of 36 km/hour to a velocity of 108 km/hour in a distance of 240 m. Calculate the average acceleration and time required.*

Sol. Initial velocity,

$$u = 36 \text{ km/hour}$$

$$= \frac{36 \times 1000}{60 \times 60} = 10 \text{ m/sec}$$

Final velocity,

$$v = 108 \text{ km/hour} = \frac{108 \times 1000}{60 \times 60} = 30 \text{ m/sec}$$

Distance, S = 240 m.

Average acceleration, a = ?

Using the relation,

$$v^2 - u^2 = 2aS$$

$(30)^2 - (10)^2 = 2 \times a \times 240$ or $900 - 100 = 480\,a$

or $a = \frac{800}{480} = 1.67$ m/sec^2. (Ans.)

Time required, t = ?

$$v = u + at$$

$$30 = 10 + 1.67 \times t$$

$$t = \frac{(30 - 10)}{1.67} = 11.97 \text{ sec. (Ans.)}$$

2. A body has an initial velocity of 16 m/sec and an acceleration of 6 m/sec^2. Determine its speed after it has moved 120 metres distance. Also calculate the distance the body moves during 10th second.

Sol. Initial velocity, u = 16 m/sec
Acceleration, a = 6 m/sec^2
Distance, S = 120 metres
Speed, v = ?

Using the relation,

$$v^2 - u^2 = 2aS$$

$$v^2 - (16)^2 = 2 \times 6 \times 120$$

or

$$v^2 = (16)^2 + 2 \times 6 \times 120 = 256 + 1440 = 1696$$

v = 41.18 m/sec. (Ans.)

Distance travelled in 10th sec ; S_{10th} = ?

Using the relation,

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

$$S_{10th} = 16 + \frac{6}{2}(2 \times 10 - 1) = 16 + 3(20 - 1)$$

= 73 m. (Ans.)

3. *On turning a corner, a motorist rushing at 15 m/sec, finds a child on the road 40 m ahead. He instantly stops the engine and applies brakes, so as to stop the car within 5 m of the child, calculate : (i) retardation, and (ii) time required to stop the car.*

Sol. Initial velocity, u = 15 m/sec
Final velocity, v = 0
Distance, S = 40 - 5 = 35 m.
(i) Retardation, a = ?

Using the relation,

$$v^2 - u^2 = 2aS$$

$$0^2 - 15^2 = 2 \times a \times 35$$

a = - 3.21 m/sec^2. (Ans.)

[- ve sign indicates that the acceleration is negative, *i.e.*, retardation]

(ii) Time required to stop the car, t = ?

Using the relation,

$$v = u + at$$

$$0 = 15 - 3.21 \times t \quad (\because a = -3.21 \text{ m/sec}^2)$$

$$t = \frac{15}{3.21} = 4.67 \text{ s. (Ans.)}$$

4. A burglar's car had a start with an acceleration 2 m/sec^2. A police vigilant party came after 5 seconds and continued to chase the burglar's car with a uniform velocity of 20 m/sec. Find the time taken, in which the police will overtake the car.

Sol. Let the police party overtake the burglar's car in t seconds, after the instant of reaching the spot.

Distance travelled by the burglar's car in t seconds, Si :

Initial velocity, *u* = 0

Acceleration, *a* = 2 m/sec²

Time, *t* = (5 + t) sec.

Using the relation,

$$S = ut + \frac{1}{2}at^2$$

S = 0 + ¹ x 2 x (5 + t)² ¹ 2

= (5 + t)² ...(/)

Distance travelled by the police party, S_2 :

Uniform velocity, *v* = 20 m/sec.

Let *t* = time taken to overtake the burglar's car

" Distance travelled by the party,

S2 = *v* x *t* = 20t ...(n)

For the police party to overtake the burglar's car, the two distances Si and S2 should be equal.

***i.e.*, Si = S2**

(5 + t)² = 20t 25 + t² + 10t = 20t t² - 10t + 25 = 0

+ 10 + ^00 - 100

" =2 or *t* = 5 sec. (Ans.)

5. A car starts from rest and accelerates uniformly to a speed of 80 km/hour over a distance of 500 metres. Calculate the acceleration and time taken.

If a further acceleration raises the speed to 96 km/hour in 10 seconds, find the acceleration and further distance moved.

The brakes are now applied and the car comes to rest under uniform retardation in 5 seconds. Find the distance travelled during braking.

Sol. Considering the first period of motion :

Initial velocity, *u* = 0

$$\text{Velocity attained, } v = \frac{80 \times 1000}{60 \times 60} = 22.22 \text{ m/sec.}$$

Distance covered, *S* = 500 m

If *a* is the acceleration and *t* is the time taken,

Using the relation :

$$v^2 - u^2 = 2aS$$

$$(22.22)^2 - 0^2 = 2 \times a \times 500$$

$a = \frac{(22,22)}{2 \times 500} = 0.494$ m/sec². (Ans.)

Also, $v = u + at$

$22.22 = 0 + 0.494 \times t$ 22 22

• $t = \frac{2222}{0,494} = 45$ sec. (Ans.)

Now considering the second period of motion,

Using the relation,

$v = u + at$

where $v = 96$ km/hour $= \frac{96 \times 1000}{60 \times 60} = 26.66$ m/sec

$u = 80$ km/hour $= 22.22$ m/sec t

$= 10$ sec

• $26.66 = 22.22 + a \times 10$

• $a = \frac{26.66 - 22.22}{10} = 0.444$ m/sec². (Ans.)

To calculate distance covered, using the relation

$S = ut + \frac{1}{2} at^2$

$= 22.22 \times 10 + \frac{1}{2} \times 0.444 \times 10^2$

$= 222.2 + 22.2 = 244.4$

• $S = 244.4$ m. (Ans.)

During the period when brakes are applied :

Initial velocity, $u = 96$ km/hour $= 26.66$ m/sec

Final velocity, $v = 0$

Time taken, $t = 5$ sec.

Using the relation,

$v = u + at$ $0 =$

$26.66 + a \times 5$

• $a = \frac{-26.66}{5} = -5.33$ m/sec².

(-ve sign indicates that acceleration is negative *i.e.*, retardation) Now using the relation,

$v^2 - u^2 = 2aS$

$0^2 - (26.66)^2 = 2 \times -5.33 \times S$

$\frac{26.66^2}{2 \times 5.33}$ 66.67 m.

• **Distance travelled during braking = 66.67 m. (Ans.)**

6. Two trains A and B moving in opposite directions pass one another. Their lengths are 100 m and 75 m respectively. At the instant when they begin to pass, A is moving at8.5 m/sec with a constant acceleration of 0.1 m/sec ^{2} and B has a uniform speed of 6.5 m/sec. Find the time the trains take to pass.

Sol. Length of train *A* = 100 m Length of train *B* = 75 m

Total distance to be covered

= 100 + 75 = 175 m

Imposing on the two trains *A* and B, a velocity equal and opposite to that of B.

Velocity of train *A* = (8.5 + 6.5) = 15.0 m/sec

and velocity of train *B* = 6.5 - 6.5 = 0.

Hence the train *A* has to cover the distance of 175 m with an acceleration of 0.1 m/sec^2 and an initial velocity of 15.0 m/sec.

Using the relation,

$$S = ut + \frac{1}{2} at^2$$

$175 = 15t + \frac{1}{2} \times 0.1 \times t^2$

$3500 = 300t + t^2$

or $t^2 + 300t - 3500 = 0$

$\frac{-300 \pm 90000 + 14000}{} = \frac{-300 \pm 322.49}{} = 2 = 2 = 11.24$ sec.

Hence the trains take 11.24 seconds to pass one another. (Ans.)

7. The distance between two stations is 2.6 km. A locomotive starting from one station, gives the train an acceleration (reaching a speed of 40 km/h in 0.5 minutes) until the speed reaches 48 km/hour. This speed is maintained until brakes are appliedapdibjaffleisibmPugh£toBᵉpeafcrfffi thi$_s$ sgffindystation under a negative acceleration of 0.9 m/sec

Sol. Considering the motion of the locomotive *starting from the first station.*

Initial velocity *u* = 0

Final velocity *v* = 40 km/hour

$$\frac{40 \times 1000}{60 \times 60} \quad 11.11 \text{ m/sec.}$$

Time taken, *t* = 0.5 min or 30 sec.

Let '*a*' be the resulting acceleration.

Using the relation,

$$v = u + at$$

$$11.11 = 0 + 30a$$

$$a = \frac{11.11}{30} = 0.37 \text{ m/sec}^2.$$

Let $t1$ = time taken to attain the speed of 48 km/hour

$$\frac{48 \times 1000}{60 \times 60} = 13.33 \text{ m/sec.}$$

Again, using the relation,

$v = u + at$

13.33 = 0 + 0.37ti

$t_1 = \frac{13.33}{0.37}$ = 36 sec. , CO

and the distance covered in this interval is given by the relation,

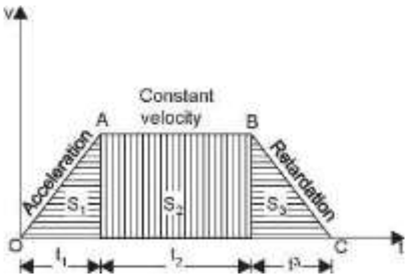
$S = ut + ^1 at^2$ 1 1 2

1

= 0 +— x 0.37 x 36^2 = 240 m.

2

Now, considering the motion of the *retarding period* before the locomotive comes to rest at the second station (i.e., portion *BC* in Fig. 7.3).



Now, u = 13.33 m/sec v = 0

a = - 0.9 m/sec^2 t = t3 be

Let the time taken

Using the relation, $v = u + at$ 0 = 13.33 - 0.9 ta

$t_3 = \frac{13.33}{0.9}$ = 14.81 sec ...(ii)

and distance covered,

*S*3 = average velocity x time

F 13.33 + 0 l X 14.81 = 98.7 m

H 2 K

Distance covered with constant velocity of 13.33 m/sec,

S_2 = total distance between two stations - (S_x + S_2) = (2.6 x 1000) - (240 + 98.7) = 2261.3 m.

.•. Time taken to cover this distance,

t_2 = $\frac{226L3}{13.33}$ = 169.6 sec ...(iii)

Adding (Z), (ii) and *(iii)*

Total time taken,

$T = t_1 + t_2 + t_3$

= 36 + 169.6 + 14.81 = 220.41 sec. (Ans.)

8. Two trains A and B leave the same station on parallel lines. A starts with a uniform acceleration of 0.15 m/sec^2 and attains a speed of 24 km/hour when the steam is required to keep speed constant. B leaves 40 seconds after with uniform acceleration of 0.30 m/sec^2 to attain a maximum speed of 48 km/hour. When will B overtake A ?

Sol. Motion of train *A* :

Uniform acceleration, a_1 = 0.15 m/sec^2

Initial velocity, U_1 = 0

Final velocity, V1 = 24 km/hour

$$= \frac{24 \times 1000}{60 \times 60} = \frac{20}{3} \text{ m/sec.}$$

Let t1 be the time taken to attain this velocity (in seconds).

Using the relation,

$$v = u + at$$

$$\frac{20}{3} = 0 + 0.15t_1$$

$$t_1 = \frac{20}{3 \times 0.15} = 44.4 \text{ sec.}$$

Also, distance travelled during this interval,

$$S_1 = ut_1 + \frac{1}{2} at_1^2$$

$$= 0 + \frac{1}{2} \times 0.15 \times 44.4^2$$

$$= 148 \text{ m.}$$

Motion of train B :

Initial velocity, U2 = 0

Acceleration, a_2 = 0.3 m/sec^2

Final velocity, V2 = 48 km/hr

$$= \frac{48 \times 1000}{60 \times 60} = \frac{40}{3} \text{ m/sec.}$$

Let t2 be the time taken to travel this distance, say S2.

Using the relation,

$$v = u + at$$

$$\frac{40}{3} = 0 + 0.3t_2$$

$$t_2 = \frac{40}{3 \times 0.3} = 44.4 \text{ sec}$$

and $$S_2 = u_2 t_2 + \frac{1}{2} a_2 t_2^2$$

$$= 0 + \frac{1}{2} \times 0.3 \times (44.4)^2$$

$$= 296 \text{ m.}$$

Let the train *B* overtake the train *A* when they have covered a distance *S* from the start. And let the train *B* take *t* seconds to cover the distance.

Thus, time taken by the train *A* = (t + 40) sec.

Total distance moved by train *A*,

S = 148 + distance covered with constant speed 20

S = 148 + [(t + 40) - *t*]

1 3

= 148 + [t + 40 - 44.4] 20/3

= 148 + (t - 4.4) x 20 -(f)

[{(t + 40) - t1} is the time during which train *A* moves with constant speed]

Similarly, total distance travelled by the train B,

S = 296 + distance covered with constant speed 40

= 296 + (t - 44.4) x 40/3 ...(ii)

Equating (*/*) and *(ii)*,

148 + (t - 4.4) 20/3 = 296 + (t - 44.4) x 40/3

148 + 20/3 t - 88/3 = 296 + 40/3 t - 1776/3

F 40/3 - 20/3 t = 148 - 296 + 1776/3 - 88/3 2 K

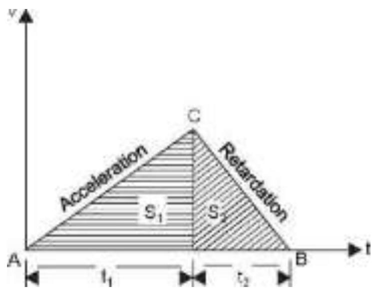
H

or *t* = 62.26 sec.

Hence, the train *B* overtakes the train *A* after 62.26 sec. of its start. (Ans.)

9. Two stations A and B are 10 km apart in a straight track, and a train starts from A and comes to rest at B. For three quarters of the distance, the train is uniformly accelerated and for the remainder uniformly retarded. If it takes 15 minutes over the whole journey, find its acceleration, its retardation and the maximum speed it attains.

Sol. Refer to Fig. 7.4.



Distance between *A* and B,

S = 10 km = 10,000 m

Considering the motion in the first part :

Let u1 = initial velocity = 0 a_1 = acceleration t_1 = time taken S_x = distance travelled.

Using the relation,

$$S = ut + \frac{1}{2}at^2$$

...(*i*)

$$S_1 = 0 + \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_1 t_1^2$$

$$7500 = \frac{1}{2}a_1 t_1^2$$ ...(ii)

[∵ S1 = 3/4 x 10,000 = 7500 m]

Also, for the second retarding part

U_2 — initial velocity

= final velocity at the end of first interval —

$0 + a_1 t_1 — a_1 t_1$

Hence V2 — final velocity at the end of second part

$= U_2 - a_2 t_2$

$= a_1 t_1 - a_2 t_2$

— 0, because the train comes to rest

$a_1 t_1 = a_2 t_2$

or $\frac{a_1}{a_2} = \frac{t_2}{t_1}$ —(iif)

Also, S_2 — distance travelled in the second part = average velocity x time

F aiti + 0 I

— G x t

rrl 2

. t ...(iv)

2 2

Adding (i) and (iv),

$S_1 + S_2 = \frac{a_1 t_1^2}{2} + \frac{a_1 t_1}{2} \cdot t_2$

$= \frac{a_1 t_1}{2}(t_1 + t_2)$

or $S_1 + S_2 = \frac{a_1 t_1}{2} \times 900$ (∵ $t_1 + t_2$ — 15 min. = 900 sec)

$10{,}000 = \frac{a_1 t_1}{2} \times 900$ (∵ $S_1 + S_2$ = 10 km = 10,000 m)

or $a_1 t_1 = \frac{20{,}000}{900} = \frac{200}{9}$

But $a_1 t_1$ = maximum velocity

Hence max. velocity $= \frac{200}{9}$ = 22.22 m/sec (Ans.)

Also, from eqn. (ii)

$7500 = \frac{1}{2} \times 22.22 \times t_1$

or $t_1 = \frac{7500}{11.11} = 675$ sec

$t_2 = 900 - 675 = 225$ sec

Now from eqn. (iii),

$\frac{a_1}{a_2} = \frac{t_2}{t_1} = \frac{225}{675} = \frac{1}{3}$

$3a_1 — a_2$.

AISO, v_{max} = 22'22 = a1t1

9999

a = —!— = 0.0329 m/sec^2. (Ans.)

1 675

and $a_2 = 3a_x$

= 3 x 0.0329

= 0.0987 m/sec^2. (Ans.)

MOTION UNDER GRAVITY

It has been seen that bodies falling to earth (through distances which are small as compared to the radius of the earth) and entirely unrestricted, increase in their velocity by about 9.81 m/sec for every second during their fall. This acceleration is called the acceleration due to gravity and is conventionally denoted by *'g'*. Though the value of this acceleration varies a little at different parts of the earth's surface but the generally adopted value is 9.81 m/sec^2.

For downward motion For upward motion

$a = +g$ | $v = u + gt$

I $h = ut + \frac{1}{2} gt^2$

$v^2 - u^2 = 2gh$

A $a = -g$ $v = u - gt$ 1

$h = ut - \frac{1}{2} gt^2$

$v^2 - u^2 = -2gh$.

SOME HINTS ON THE USE OF EQUATIONS OF MOTION

(i) If a body starts from rest, its initial velocity, $u = 0$ (ii) If a body comes to rest ; its final velocity, $v = 0$

(iii) When a body is thrown upwards with a velocity u, time taken to reach the maximum height = – and velocity on reaching the maximum height is zero *(i.e.,* $v = 0$) . This value of t is g obtained by equating $v = u - gt$ equal to zero.

(iv) Greatest height attained by a body projected upwards with a velocity $u = \frac{u^2}{2g}$ ' obtained by substituting $v = 0$ in the equation $v^2 - u^2 = -2gh$.

(v) Total time taken to reach the ground = ^ , the velocity on reaching the ground being $12gh$.

g v

(V $v^2 - u^2 = 2gh$ or $v^2 - 0^2 = 2gh$ or $v = 2gh$)

(vi) The velocity with which a body reaches the ground is same with which it is thrown upwards.

10. A stone is dropped from the top of tower 100 m high. Another stone is projected upward at the same time from the foot of the tower, and meets the first stone at a height of 40 m. Find the velocity, with which the second stone is projected upwards.

Sol. Motion of the first particle :

Height of tower = 100 m

Initial velocity, $u = 0$

Height, $h = 100 - 40 = 60$ m.

Let t be the time (in seconds) when the two particles meet after the first stone is dropped from the top of the tower.

Refer to Fig. 7.5.

Top of tower $u = 0$ ▲ • First particle

Using the relation,

$$h = ut + \frac{1}{2} gt^2$$

$$60 = 0 + \frac{1}{2} \times 9.81\, t^2$$

| u:

$$t = \sqrt{\frac{120}{9.81}} = 3.5 \text{ sec.}$$

M 1

Second particle

Motion of the second particle :

Height, $h = 40$ m

Time, $t = 3.5$ sec.

Let u be the initial velocity with which the second particle has been projected upwards. Using the relation,

$$h = ut - \frac{1}{2} gt^2$$ (V Particle is projected upwards)

$$40 = u \times 3.5 - \frac{1}{2} \times 9.81 \times 3.5^2$$

$$3.5u = 40 + \frac{1}{2} \times 9.81 \times 3.5^2$$

$$u = 28.6 \text{ m/sec. (Ans.)}$$

11. A body projected vertically upwards attains a maximum height of 450 m. Calculate the velocity of projection and compute the time of flight in air. At what altitude will this body meet a second body projected 5 seconds later with a speed of 140 m/sec ?

Sol. Maximum height attained by the body

= 450 m

Let u = initial velocity of the body

v = final velocity of the body = 0

Using the relation,

$v^2 - u^2 = -2gh$ (V body is thrown upwards)

$0^2 - u^2 = -2 \times 9.81 \times 450$ u = 94 m/sec. (Ans.)

Let 't be the time taken by the body in reaching the highest point from the point of projection. Then, using the relation,

$v = u - gt$ $0 = 94 - 9.81t$

$$t = \frac{94}{9.81} = 9.6 \text{ sec.}$$

.•. Total time of flight in air

= 2 x 9.6 = 19.2 sec. (Ans.)

(V The body will take the same time in returning also)

Let the second body meet the first body at a height '*h*' from the ground. Let '*t* be the time taken by the first body.

Then, time taken by the second body

= (t - 4) sec.

Considering the motion of first body

$h = ut - \frac{1}{2} gt^2$

$= 94t - \frac{1}{2} \times 9.81t^2$ ...(i)

Considering the motion of the second body

$h = 140 (t - 5) - \frac{1}{2} \times 9.81 (t - 5)^2$ ...(ii)

Equating (i) and (ii), we get

$94t - \frac{1}{2} \times 9.81t^2 = 140 (t - 5) - \frac{1}{2} \times 9.81 (t - 5)^2$

$188t - 9.81t^2 = 280 (t - 5) - 9.81 (t - 5)^2$

$188t - 9.81t^2 = 280t - 1400 - 9.81 (t - 5)^2$

$188t - 9.81t^2 = 280t - 1400 - 9.81t^2 + 98.1t - 245.25$

From which $t = 8.65$ sec.

Putting this in eqn. (i), we get

$h = 94 \times 8.65 - \frac{1}{2} \times 9.81 \times 8.65^2$

= 813.3 - 367 = 446.3 m.

Hence, the second body will meet the first one at a height of 446.3 m from the ground. (Ans.)

12. Two stones are thrown vertically upwards one from the ground with a velocity of 30 m/sec and another from a point 40 metres above with a velocity of 10 m/sec. When and where will they meet

Sol. Refer to Fig.

Let the two stones meet after '*t* seconds from their start at a height of 5 metres from the ground.

Motion of first stone :

u = initial velocity = 30 m/sec *h* = vertical distance travelled *t* = time taken

First stone Second stone

h

10 m/sec

30 m/sec

T"

Second stone

First stone

Using the relation, $h = ut - \frac{1}{2} gt^2$

(∨ stone is thrown upwards) x

$h = 30t - \frac{1}{2} 9.81t^2$ -(0

Motion of second stone :

Vertical distance travelled

$h' = h - 40$ $u = 10$ m/sec.

Again using the relation,

$h = ut +\sim gt^2$ 2

1

(h - 40) = 10t - -x 9.8t^2 ...(ii)

2

Subtracting *(ii)* from (/),

40 = 20t

$t = 2$ sec. (Ans.)

Substituting this value in eqn. (i), we get

1

$h = 30 \times 2 - \cdot \times 9.81 \times 2^2 = 40.38$ m. (Ans.)

2

Hence, the two stones meet after 2 seconds at 40.38 m from the ground.

13. A stone is thrown from the ground vertically upwards, with a velocity of 40 m/sec. After 3 seconds another stone is thrown in the same direction and from the same place. If both of the stones strike the ground at the same time, compute the velocity with which the second stone was thrown.

Sol. Motion of first stone :

u = velocity of projection = 40 m/sec v = velocity at the maximum height = 0

t = time taken to reach the maximum height = ?

Using the relation,

$v = u - gt$ (V stone is moving upward)

0 = 40 - 9.81t

or $t = \frac{40}{9.81} = 4$ sec.

Therefore, total time taken by the first stone to return to the earth = 4 + 4 = 8 sec (because the time taken to reach the maximum height is same as that to come down to earth).

Therefore, the time taken by the second stone to return to the earth = 8 - 3 = 5 sec. or time taken to reach the maximum height = 5/2 = 2.5 sec.

Motion of second stone :

u = velocity of projection = ? v = final velocity at max. height = 0 t = time taken to reach the max. height

Using the relation,

$v = u - gt$

$0 = u - 9.81 \times 2.5$

$u = 9.81 \times 2.5 = 24.5$ m/sec.

Hence, the velocity of projection of second stone

= 24.5 m/sec. (Ans.)

14. A body, falling freely under the action of gravity passes two points 15 metres apart vertically in 0.3 seconds. From what height, above the higher point, did it start to fall.

Sol. Refer to Fig. 7.7.

Let the body start from *O* and pass two points *A* and *B*, 15 metres apart in 0.3 second after traversing the distance OA.

Let $OA = h$

Considering the motion from O to A,

Initial velocity, $u = 0$

Using the relation,

15 m

1 B

$$h = ut + \frac{1}{2}gt^2 \text{ (V the body is falling downward)}$$

$$h \quad 0 + \sim\frac{1}{2}gt^2 \qquad \text{(i)}$$

Considering the motion from O to B.

Initial velocity, $u = 0$

Time taken, $t = (t + 0.3)$ sec.

Again, using the relation, $$h + 15 = 0 + \frac{1}{2}g\,(t + 0.3)^2 \qquad ...(ii)$$

Subtracting, (*i*) from (*ii*), $$15 = \frac{1}{2}g(t + 0.3)^2 - \frac{1}{2}gt^2$$

$$30 = g(t^2 + 0.6t + 0.09) - gt^2$$
$$30 = gt^2 + 0.6\,gt + 0.09\,g - gt^2$$
$$0.6\,gt = 30 - 0.09\,g$$

$$t = \frac{30}{0.6g} - \frac{0.09g}{0.6g} = 5.1 - 0.15 = 4.95 \text{ sec.} \qquad \text{(iii)}$$

Substituting the value of *t* in eqn. (i), we get

$$h = \frac{1}{2} \times 9.81 \times (4.95)^2 = 120.2 \text{ m. (Ans.)}$$

15. A stone dropped into a well is heard to strike the water after 4 seconds. Find the depth of the well, if the velocity of sound is 350 m/sec.

Sol. Initial velocity of stone, $u = 0$

Let t = time taken by stone to reach the bottom of the well, and h = depth of the well Using the relation,

$$h = ut + \frac{1}{2}gt^2$$

$$h = 0 + \frac{1}{2} \times 9.8t^2 = 4.91^2 \qquad ...(i)$$

Also, the time taken by the sound to reach the top

<u>Depth of the well</u> Velocity of sound

$$h\ 4.91^2 \quad \ldots(h)$$
$$350 \quad 350$$

Total time taken = time taken by the stone to reach the bottom of the well

+ time taken by sound to reach the ground = 4 seconds (given)

$$t + \frac{4.9t'^2}{350} = 4$$

or $4.91^2 + 350t - 1400 = 0$

or $t \quad \dfrac{-350 + \sqrt{(350)^2 + 4 \times 4.9 \times 1400}}{2 \times 4.9}$

$$\frac{-350 + 387.2}{9.8} = 3.8 \text{ sec}$$

/. $t = 3.8$ sec.

Substituting the value in eqn. (/), we get

$$h = 4.9 \times (3.8)^2 = 70.8 \text{ m}$$

Hence, the depth of well = 70.8 m. (Ans.)

VARIABLE ACCELERATION

16. The equation of motion of a particle is $S = -6 - 5t^2 + t^3$

where S is in metres and t in seconds.

Calculate : (i) *The displacement and the acceleration when the velocity is zero.*

(ii) *The displacement and the velocity when the acceleration is zero.*

Sol. The equation of motion is

$$S = -6 - 5t^2 + t^3 \quad \ldots\text{(given)} \ldots\text{(i)}$$

Differentiating both sides,

ds or $v = -10t + 3t^2$ *dt*

.2 $v = -10t + 3t^2$.(*ii*)

Again, differentiating both sides, *dv*

or $a = -10 + 6t$ *dt*

2 $a = -10 + 6t$

Now, *(i)* When the velocity is zero, ...(iii)

$v = -10t + 3t^2 = 0$ 2 $t(3t - 10) = 0$

$t = {}^{10} = 3.33$ sec. 3

(ignoring $t = 0$ which means start)

Substituting this value in eqns. (i) and (iii),

S = displacement

$= -6 - 5 \times 3.33^2 + 3.333 = -6 - 55.44 + 36.92 = -24.52$ m. (Ans.)

The negative sign indicates that distance is travelled in the other direction.

Also, a = acceleration

1 n

$= -10 + 6 \times — = 10$ m/sec^2. (Ans.)

3

(ii) When the acceleration is zero

$a = -10 + 6t = 0$

$6t = 10$

or $t = {}^{10} = {}^{5} = 1.67$ sec.

6 3

Substituting this value in eqns. (i) and (ii), we get S = displacement

$= -6 - 5t^2 + t^3 = -6 - 5 \times (1.67)^2 + (1.67)^3 = -6 - 13.94 + 4.66 = -15.28$ m. (Ans.)

The -ve sign again means that the distance is travelled in the other direction.

Also, $v = -10t + 3t^2$

$= -10 \times 1.67 + 3 \times (1.67)^2 = -16.7 + 8.36 = -8.34$ m/sec. (Ans.)

17. If a body be moving in a straight line and its distance S in metres from a given point in the line after t seconds is given by the equation

$S = 20t + 3t^2 - 2t^3$.

Calculate : (a) *The velocity and acceleration at the start.*

(b) The time when the particle reaches its maximum velocity.

(c) The maximum velocity of the body.

Sol. The equation of motion is

$S = 20t + 3t^2 - 2t^3$	...(i)
Differentiating both sides	
$— = v = 20 + 6t - 6t^2$	...(ii)
dt	
Again, differentiating	
d²S dv $= = a = 6 - 12t$	...(*iii*)
dt² dt	
(a) At start, $t = 0$ Hence from eqns. *(ii)* and (iii),	
$v = 20 + 0 - 0 = 20$ m/sec. (Ans.) $a = 6 - 12 \times 0 = 6$ m/sec. (Ans.)	

(b) When the particle reaches its maximum velocity

$a = 0\ 6 - 12t = 0$

i.e., $t = 0.5$ sec. (Ans.)

(b) The maximum velocity of the body

When $t = 0.5$ sec.

$$v_{max} = 20 + 6t - t^2 = 20 + 6 \times 0.5 - 6 \times 0.5^2$$

$$= 20 + 3 - 1.5$$

$$= 21.5 \text{ m/sec. (Ans.)}$$

SELECTED QUESTIONS EXAMINATION PAPERS

18. Two trains A and B leave the same station on parallel lines. A starts with uniform acceleration of 0.15 m/s² *and attains a speed of 24 km/hour when the steam is reduced to keep the speed constant. B leaves 40 seconds after with a uniform acceleration of 0.30 m/s² to attain a maximum speed of 48 km/hour. When will B overtake A ?*

Sol. Motion of train A:

Uniform acceleration, $a_x = 0.15$ m/s² Initial velocity, u1 = 0

Final velocity, v1 = 24 km/h

$= \frac{24 \times 1000}{60 \times 60} = \frac{20}{3}$ m/sec

Let t1 be the time taken to attain this velocity (in seconds)

Using the relation:

$$v = u + at$$

$$\frac{20}{3} = 0 + 0.15 \times t_1$$

$$t_1 = \frac{20}{3 \times 0.15} = 44.4 \text{ sec}$$

Also, distance travelled during this interval,

$$s_1 = u_1 t_1 + \frac{1}{2} a t_1^2$$

$$= 0 + \frac{1}{2} \times 0.15 \times 44.4^2 = 148$$

Motion of train B:

Initial velocity, $u_2 = 0$

Acceleration, $a = 0.3$ m/sec²

Final velocity, $v_2 = 48$ km/h $= \frac{48 \times 1000}{60 \times 60} = \frac{40}{3}$ m/sec

Let t_2 be taken to travel this distance, say s_2 Using the relation:

$$v = u + at$$

$\frac{40}{2} = 0 + 0.3 \times t$ 3

$t = \frac{40}{2 \quad 3 \times 0.3} = 44.4$ s

and $S2 = U2t2 + \frac{1}{2} a2t^2$ 2

$= 0 + \frac{1}{2} \times 0.3 \times (44.4)^2 = 296$ m

Let the train *B* overtake the train *A* when they have covered a distance *s* from the start. And let the train *B* take *t* seconds to cover the distance.

Thus, time taken by the train *A* = (t + 40) sec.

Total distance moved by train A.

s = 148 + distance covered with constant speed = 148 + [(t + 40) - ti] x 20/3 = 148 + [t + 40 - 44.4] x 20/3 = 148 + (t - 4.4) x 20/3 (i)

[{(t + 40) - t2} is the time during which train *A* moves with constant speed].

Similarly, total distance travelled by the train B,

s = 296 + distance covered with constant speed = 296 + (t - 44.4) x 40/3 ...(ii)

Equating (/) and *(ii)*

148 + (t - 4.4) x 20/3 = 296 + (t - 44.4) x 40/3 148

$+ \frac{20}{3} t - \frac{88}{3} = 296 + \frac{40}{3} t - \frac{1776}{3}$

F 40 - 201 t = 148 - 296 + ^ - 88

H 3 3 K 3 3

t = 62.26 s

Hence, *train B, overtakes train A after 62.26 s of its start.* (Ans.)

19. A cage descends a mine shaft with an acceleration of 1 m/s ^{2}. After the cage has travelled 30 m, stone is dropped from the top of the shaft. Determine: (i) the time taken by the stone to hit the cage, and (ii) distance travelled by the cage before impact.

Sol. Acceleration of cage,

$a = 1$ m/s^2

Distance travelled by the shaft before dropping of the stone = 30 m (i)

Time taken by the stone to hit the cage = ?

Considering motion of the stone.

Initial velocity, $U = 0$

Let *t* = time taken by the stone to hit the cage, and h1 = vertical distance travelled by the stone before the impact.

Using the relation,

$h = ut + \frac{1}{2} gt^2$

$h = 0 + \frac{1}{2} \times 9.8\, t^2 = 4.9\, t^2$.(*i*)

Now let us consider motion of the cage for 30 m Initial velocity, $u = 0$

Acceleration, $a = 1.0$ m/s².

Let t = time taken by the shaft to travel 30 m

Using the relation,

$s = ut +\sim at^2$ 2

$$30 = 0 + ^1 \times 1 \times (t')^2 2$$

$t = 7.75$ s.

It means that cage has travelled for 7.75 s before the stone was dropped. Therefore total time taken by the cage before impact = (7.75 + t).

Again using the relation:

1

$s = ut + \quad at^2$

2 i

$s_i = 0 + ^1 X 1 \times (7.75 + t)^2$ ...(ii)

In order that stone may hit the cage the two distances must be equal *i.e.*, equating (i) and (//).

$4.9 t^2 = ^1 \times (7.75 + t)^2 2$

$4.9 = 0.5 (60 + t^2 + 15.5 t)$ or $9.8 = t^2 + 15.5 t + 60$

or $t^2 + 15.5 t - 50.2 = 0$

$$t = \frac{- 15.5 \pm \text{\^{}}(15.5)2 + 4 \times 50.2}{2} \qquad \frac{— 15.5 \pm V441.05}{2}$$

$$\frac{— 15.5 \pm 21.0}{2} = 2.75 \text{ s}$$ (neglecting -ve sign)

$t = 2.75$ s. (Ans.)

(ii) Distance travelled by the cage before impact = ?

Let S2 = distance travelled by the cage before impact.

We know total time taken by the cage before impact.

= 7.75 + 2.75 = 10.5 s.

Now using the relation,

$$s = ut + \frac{1}{2} at^2$$

$$= 0 + — \times 1 \times (10.5)^2 = 55.12 \text{ m } 2$$

Hence distance travelled by the cage before impact = 55.12 m. (Ans.)

8.9. D' ALEMBERT'S PRINCIPLE

D' Alembert, a French mathematician, was the first to point out that on the lines of *equation of static equilibrium, equation of dynamic equilibrium can also be established by introducing inertia force in the direction opposite the acceleration in addition to the real forces on the plane.*

Static equilibrium equations are :

$$ZH \text{ (or } P_x) = 0,\; ZV \text{ (or } ZP_y) = 0,\; ZM = 0$$

Similarly when different external forces act on a system in motion, the algebraic sum of all the forces (including the *inertia force)* is zero. This is explained as under :

We know that, $P = ma$ **(Newton's second law of motion)**

or $P - ma = 0$ **or** $P + (- ma) = 0$

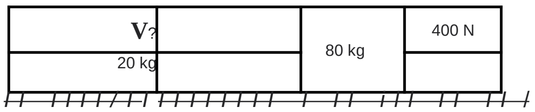
The expression in the block (- ma**) is the *inertia force* and negative sign signifies that it acts in a direction opposite to that of acceleration/retardation** a**.**

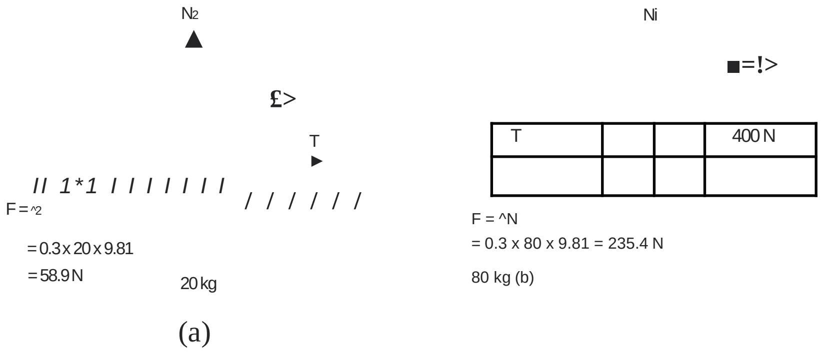
It is also known as the *"principle of kinostatics"*.

Example 8.15. ***Two bodies of masses 80 kg and 20 kg are connected by a thread and move along a rough horizontal surface under the action of a force 400 N applied to the first body of mass 80 kg as shown in Fig. 8.6. The co-efficient of friction between the sliding surfaces of the bodies and the plane is 0.3.***

Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.

Sol. Refer to Figs. 8.5 and 8.6





Acceleration of the bodies, a :

As per *D' Alembert's principle for dynamic equilibrium condition the algebraic sum of all the active forces acting on a system should be zero.*

The various forces acting on the bodies are :

(i) Force applied = 400 N

(ii) Inertia force = (80 + 20) *a*

(iii) Frictional force = 0.3 x 80 x 9.81 + 0.3 x 20 x 9.81

= 235.4 + 58.9 = 294.3 N 400 - (80 - 20) *a* = 294.3 = 0

or $\frac{400 - 294.3}{(80 + 20)}$ 1.057 m/s^2. (Ans.)

Tension in the thread between the two masses, *T :*

Considering free body diagrams of the masses 80 kg and 20 kg separately as shown in Fig. (a) and (b).

Applying D' Alembert's principle for Fig. 8.6 (a), we get 400 - *T* - 80 x 1.057 - 0.3 x 80 x 9.81 = 0 .-. T = 80 N. (Ans.)

Now, applying D' Alembert's principle for Fig. 8.6 (b), we get *T* - 0.3 x 20 x 9.81 - 20 x 1.057 = 0 .-. T = 80 N. (Ans.)

It may be noted that the same answer is obtained by considering the two masses separately.

MOTION OF A LIFT

Consider a lift (elevator or cage etc.) carrying some mass and moving with a uniform acceleration.

Let *m* = mass carried by the lift in kg,

W (= *m.g)* = weight carried by the lift in newtons, *a* = uniform acceleration of the lift, and *T* = tension in the cable supporting the lift.

There could be the following *two* cases :

(i) When the lift is moving *upwards,* and (ii) When the lift is moving *downwards.*

1. Lift moving upwards :

Refer to Fig. 8.7.

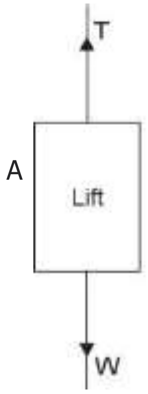


Fig. 8.7. Lift moving upwards.

Fig. 8.8. Lift moving downwards.

The net upward force, which is responsible for the motion of the lift

= *T - W = T - m.g* ...(*i*)

Also, this force = mass x acceleration

= *m.a* —(ii)

Equating (i) and (ii), we get

T - m.g ■ *m.a*

T = *m.a + m.g = m(a + g)* ...(8.4)

2. Lift moving downwards :

Refer to Fig. 8.8.

Net downward force responsible for the motion of the lift

= *W - T = m.g - T* ...(i)

Also, this force = mass x acceleration = *m.a* -(if)

Equating (i) and (ii), we get

$$m.g - T = m.a$$

$$T = m.g - m.a = m(g - a)$$

16. ***An elevator cage of mass 900 kg when empty is lifted or lowered vertically by means of a wire rope. A man of mass 72.5 kg is standing in it. Find :***

(a) ***The tension in the rope,***

(b) The reaction of the cage on the man, and

(c) The force exerted by the man on the cage, for the following two conditions :

(i) when the cage is moving up with an acceleration of 3 m/s² and

(ii) when the cage is moving down with a uniform velocity of 3 m/s.

Sol. Mass of the cage, M = 900 kg Mass of the man, m = 72.5 kg.

(i) Upward acceleration, a = 3 m/s^2

(a) Let T be the tension in the rope in newtons The various forces acting on the cage are :

1. Tension, T of the rope acting vertically upwards.

2. Total mass = $M + m$, of the cage and the man acting vertically downwards.

As the cage moves upwards, $T > (M + m)g$

Net accelerating force = $T - (M + m)g = (m + m)a$

.'. $T - (M + m)g = (M + m)a$ ...(i)

Substituting the given values, we get

T - (900 + 72.5) 9.81 = (900 + 72.5) x 3 T = 12458 N. (Ans.)

(b) Let 'R be the reaction of the cage on the man in newtons.

Considering the various forces, the equation of motion is

$R - mg = m.a$ —(ii]

or $R = mg + ma = m(g + a)$

= 72.5 (9.81 + 3) = 928.7 N. (Ans.)

(c) The force exerted by the man on the cage must be equal to the force exerted by the cage on the man (Newton's third law of motion).

Force exerted by the man on the cage = 928.7 N. (Ans.)

(ii) When the cage moves with a uniform velocity 3 m/s :

When the cages moves with a uniform velocity, acceleration is equal to zero.

(a) Tension in the rope, T :

Putting a = 0 in eqn. (i), we get

$T - (M + m)g = (M + m) \times 0 = 0$ $T = (M + m)g$

= (900 + 72.5) x 9.81 = 9540 N. (Ans.)

(b) Also from equation (ii)

When $a = 0$,

$R = mg + m \times 0 = mg$

= 72.5 x 9.81 = 711.2 N. (Ans.)

(c) Force exerted by the man on the cage

= force exerted by the cage on the man = 711.2 N. (Ans.)

17. *An elevator of mass 500 kg is ascending with an acceleration of 3 m/s²*.

During this ascent its operator whose mass is 70 kg is standing on the scales placed on the floor. What is the scale reading ? What will be total tension in the cables of the elevator during his motion ? Sol. Mass of the elevator, $M = 500$ kg Acceleration, $a = 3\ m/s^2$

Mass of the operator, $m = 70$ kg

Pressure (R) exerted by the man, when the lift moves upward with an acceleration of $3\ m/s^2$,

$R = mg + ma = m (g + a)$

= 70 (9.81 + 3) = 896.7 N. (Ans.)

Now, tension in the cable of elevator

$T = M (g + a) + m (g + a)$

$= (M + m) (g + a)$

= (500 + 70) (9.81 + 3) = 7301.7 N. (Ans.)

MOTION OF TWO BODIES CONNECTED BY A STRING PASSING OVER A SMOOTH PULLEY

Fig. 8.9 shows two bodies of weights Wi and W2 respectively hanging vertically from a *weightless* and *inextensible* string, passing over a *smooth pulley*. Let T be the common tension in the string. If the *pulley were not smooth, the tension would have been different in the two sides of the string.*

Let W_1 be greater than W_2 and a be the acceleration of the bodies and their motion as shown.

Consider the motion of body 1:

Forces acting on it are : Wi (downwards) and T (upwards).

.'. Resulting force = Wi - T (downwards) ...(i)

Since this weight is moving downward, therefore, force acting on this weight

$$= \frac{W_1}{g} a \quad \text{...(ii)}$$

Equating (i) and (ii)

$$W_1 - T = \frac{W_1}{g} a \quad \text{.(1)}$$

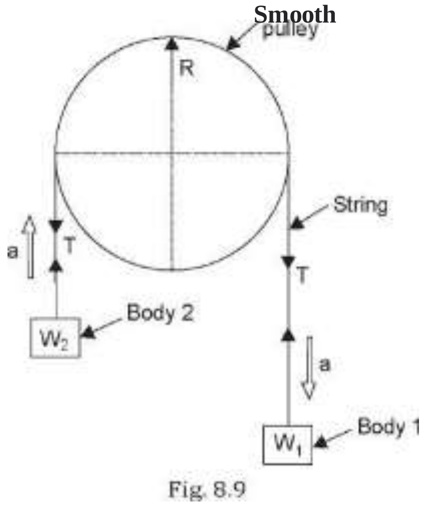


Fig. 8.9

Now consider the motion of body 2:

Forces acting on it are : T (upwards) W_2 (downwards) .

Resultant force = $T - W_2$ ...(*iii*)

Since the body is moving upwards therefore force acting on the body

$$\frac{W_2}{g} \cdot a. \qquad (iv)$$

Equating *(iii)* and (iv)

$$T - W_2 = \frac{W_2}{g} a \qquad ...(2)$$

Now adding eqns. (1) and (2), we get

F W1+Z21

$$W_x - W2 = \quad J \quad a$$

a =F|Z^|j

from which,

g

From equation (2),

$$T - W_2 = \frac{W_9}{g} a$$

$$T = W + Z\ a = W_2\ G1 + a$$

H gK

2 g g

Substituting the value of *'a'* from equation (8.6), we get

$$T = W\ |_1 + j|\ W1 \sim W2 \ .\ g\ |P$$

H Wi+W2K g PQ

from which, $T = \frac{2\ W1\ Wz}{W_1 + W_2}$

Reaction of the pulley,

$$R = T + T = 2T$$

$$\frac{4W^\wedge W_2}{W_1 + W_2}$$

Example 8.18. ***Two bodies weighing 45 N and 60 N are hung to the ends of a rope, passing over a frictionless pulley. With what acceleration the heavier weight comes down ? What is the tension in the string ?***

Sol. Weight of heavier body, $W_1 = 60$ **N**

Weight of lighter body, $W_2 = 45$ **N**

Acceleration of the system, a **= ?**

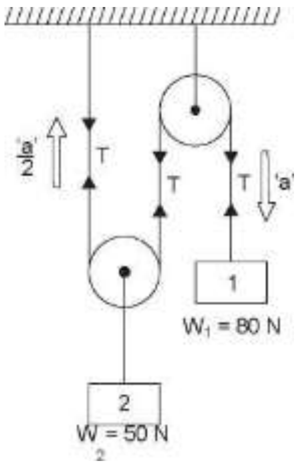
Using the relation,

$$a = \frac{g^Z l\_^Z 2\}}{(W_1 + W_2)} = \frac{981(60 \quad 45)}{(60 + 45)} = 1.4 \text{ m/s}^2. \text{ (Ans.)}$$

Tension in the string, T = ?

Using the relation,

$$T = \frac{2\ W1\ W2}{W1 + W2} = \frac{2 \times 60 \times 45}{(60 + 45)} = 51.42 \text{ N. (Ans.)}$$



Example 8.19. *A system of frictionless pulleys carries two weights hung by inextensible cords as shown in Fig. . Find :*

(i) The acceleration of the weights and tension in the cords.

(ii) The velocity and displacement of weight '1' after 5 seconds from start if the system is released from rest.

Sol. **Weight, *Wi* = 80 N**

Weight, W2 = 50 N

Let *T* = tension (constant throughout the cord, because pulleys are *frictionless*, and *cord is continuous).*

When weight W1 travels unit distance then weight W2 travels half the distance. Acceleration is proportional to the distance.

If *a* = acceleration of weight W1 then, a/2 = acceleration of weight W2.

It is clear from the figure that weight W1 moves downward and weight *W*2 moves upward.

(i) Acceleration of weights, T = ?

Consider the motion of weight W_1**:**

$$W_1 - T = \frac{W_1}{g} a$$

$$80 - T = \frac{80}{g} \times a \qquad \text{...(i)}$$

Consider the motion of weight W_2 **:**

$$2T - W = \frac{Wn}{g}$$

$$2T - 50 = \frac{50}{g} \times \frac{a}{2} \qquad \text{...(ii)}$$

Multiplying eqn. (i) by 2 and adding eqns. (i) and (ii), we get

$$110 = \frac{185}{g} a$$

$$a = \frac{110 \times 9.81}{185} = 5.8 \text{ m/s}^2$$

Hence acceleration of W_1 **= 5.8 m/s². (Ans.) and acceleration of W2 = 5.8/2 = 2.9 m/s². (Ans.)**

Substituting the value of *'a'* in eqn. (i), we get

$$80 - T = \frac{80}{9.81} \times 5.8$$

.-. T **= 32.7 N. (Ans.)**

(*ii*) Velocity and displacement of weight *W*1 after 5 sec. = ?

$u = 0$, $a = 5.8$ m/s², $t = 5$ s

.•. $v = u + at = 0 + 5.8 \times 5 = 29$ **m/s. (Ans.)**

and $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5.8 \times 5^2 = 72.5$ **m. (Ans.)**

MOTION OF TWO BODIES CONNECTED AT THE EDGE OF A HORIZONTAL SURFACE

Fig. 8.11 shows two bodies of weights *Wi* and W2 respectively connected by a light inextensible string. Let the body 1 hang free and body 2 be placed on a rough horizontal surface. Let the body 1 move downwards and the body 2 move along the surface of the plane. We know that the velocity and acceleration of the body will be the same as that of the body 2, therefore tension will be same throughout the string. Let p be the co-efficient of friction between body 2 and the horizontal surface.

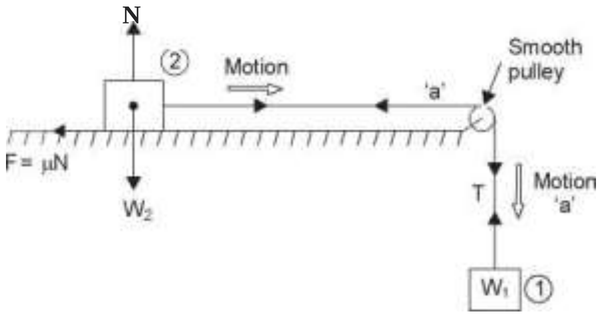


Fig. 8.11

Normal reaction at the surface, *N* = W2 force of friction, *F* = pN = pW_2

and Let *a* = acceleration of the system

***T* = tension in the string.**

Consider the motion of body 1 :

Forces acting on it are : W1 (downwards) and *T* (upwards)

Resultant force = W1 - *T* **■■■(f)**

Since the body is moving downwards, therefore force acting on this body

$$= \frac{W_1}{g} \cdot a \quad \text{-(if)}$$

Equating (f) and *(if)*,

$$W1 - T = \frac{W_1}{g} a \quad ...(1)$$

Now consider the motion of body 2 :

Forces acting on it are : *T* (towards right), Force of friction *F* (towards left).

.'. Resultant force = *T* - *F* = *T* - p W2 *...(iii)*

Since, the body is moving horizontally with acceleration, therefore force acting on this body

$$= \frac{W_2}{g} \cdot a \quad \text{-Civ)}$$

Equating *(iif)* and (iv), we get

$$T - pW = \frac{W_2}{g} a \quad ...(2)$$

Adding equations (1) and (2), we get

$W_1 - qW_2 = \frac{W_1}{g} a + \frac{W_2}{g} a$

or $W_1 - qW_2 = \frac{a}{g}(W_1 + W_2)$

or $a = \left(\frac{W_1 - qW_2}{W_1 + W_2}\right) g$

Substituting this value of *'a'* in equation (1), we get

$W_1 - T = \frac{W_1}{g} \times \left(\frac{W_1 - qW_2}{W_1 + W_2}\right) g$

$T = W_1 \left[1 - \frac{W_1 - qW_2}{W_1 + W_2}\right]$

$= W_1 \left[\frac{W_1 + W_2 - W_1 + qW_2}{W_1 + W_2}\right]$

i.e., $T = \frac{W_1 W_2 (1 + q)}{W_1 + W_2}$

For *smooth horizontal surface* ; putting q = 0 in equations (8.9) and (8.10), we get

$a = \frac{W_1 \cdot g}{W_1 + W_2}$

and $T = \frac{W_1 W_2}{W_1 + W_2}$

20. *Find the acceleration of a solid body A of weight 8 N, when it is being pulled by another body of weight 6 N along a smooth horizontal plane as shown in Fig. 8.12.*

Sol. Refer to Fig.

Weight of body B, W1 = 6 N

Weight of body A, W2 = 8 N

Acceleration of body, *a* = ?

Tension in the string, *T* = ?

A 8 N

6 N B

Equation of motion for body *B*

$6 - T = \frac{6}{g} a$ ...(i)

Equation of motion for body *A*

$T = \frac{8}{g} a$ ...(ii)

235 Adding (i) and (ii), we get

$6 = \frac{14}{g} . a$

$a = \frac{6 \times 9.81}{14} = 4.2 \text{ m/s}^2$. (Ans.)

Substituting this value of *a* in (i), we get

$6 - T = \frac{6}{9.81} \times 4.2$

$T = 3.43$ N. (Ans.)

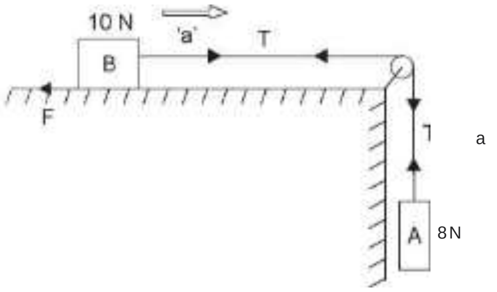
21. *Two blocks shown in Fig. have weights A = 8 N and B = 10 N and co-efficient of friction between the block A and horizontal plane,* p *= 0.2.*

If the system is released, from rest and the block A falls through a vertical distance of 1.5 m, what is the velocity acquired by it ? Neglect the friction in the pulley and extension of the string.

Sol. Refer to Fig. 8.13.

Considering vertical string portion:

$8 - T = \frac{8}{g} . a$ ...(*i*)



Considering horizontal string portion :

$T - F = \frac{10}{g} . a$

or $T - pN = \frac{10}{g} . a$

or $T - 0.2 \times 10 = \frac{10}{g} a$ (V $N_B = W_B = 10$ newtons)

or $T - 2 = \frac{10}{g} a$ ...(ii)

Adding (*i*) and (*ii*)

$6 = \frac{18 a}{g}$

$a = \frac{6 \times 9.81}{18} = 3.27 \text{ m/s}^2$

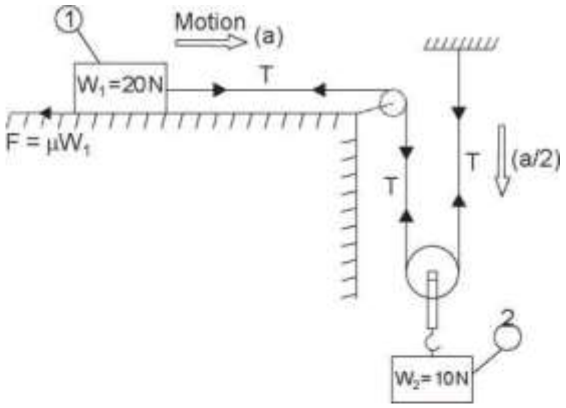
Now using the relation :

$v^2 - u^2 = 2as$ or $v^2 - u^2 = 2 \times 3.27 \times 1.5$ $v = 3.13$ m/s

Hence the velocity acquired by weight *A* = 3.13 m/s. (Ans.)

22. *A body '1' of weight 20 N is held on a rough horizontal table. An elastic string connected to the body '1' passes over a smooth pulley at the end of the table and then under a second smooth pulley carrying a body '2' of weight 10 N as shown in Fig. 8.14. The other end of the string isfixed to a point above the second pulley. When the 20 N body is released, it moves with an accelera- tion of g/5. Determine the value of co-efficient of friction between the block and the table.*

Sol. Weight of body '1', W_1 = 20 N Weight of body '2', W2 = 10 N Acceleration of body '1' $a = g/5$ Let T = tension in string in newtons, and U = co-efficient of friction between block and the table.



Considering the motion of body '1' :

$$T - uW_1 = \frac{W_1}{g} a$$

or $\quad T$- u x 20 = $\frac{20}{g} \times \frac{g}{5}$ = 4 ...(i)

Considering the motion of body '2' :

A little consideration will show that the acceleration of the body '2' will be half of that of the body '1' *i.e.*, g/10.

Now,

$$W_2 - 2T = \frac{W_2}{g} \times \frac{a}{2}$$

or

$$10 - 2T = \frac{10}{g} \times \frac{g}{10} = 1 \qquad \text{...(ii)}$$

Now multiplying eqn. (i) by 2 and adding eqns. (i) and (ii), we get

$$10 - 40u = 9$$

$$40u = 1 \text{ or } u = 0.025. \text{ (Ans.)}$$

Example 8.23. *A string passing across a smooth table at right angle to two opposite edges has two masses M_1 and M_2 ($M_1 > M_2$) attached to its ends hanging vertically as shown in Fig. 8.15. If a mass M be attached to the portion of the string which is on the table, find the acceleration of the system when left to itself.*

Sol. Refer to Fig. 8.15.

T2 ◀ M ▶ Ti

(3 /////// / / /////// String

T₂ Ti

M₂ M₁

Let T_1 and T_2 be the tensions in the two portions of the strings.

Acceleration of the system, a = ?

We know that

$$W_1 = M_1 g, \; W_2 = M_2 g$$

Equations of motion are :

$$M_1 g - T_1 = M_1 a \qquad \text{...(i)}$$

$$T_1 - T_2 = M \cdot a \qquad \text{...(ii)}$$

$$T_2 - M_2 g = M_2 \cdot a \qquad \text{...(iii)}$$

Adding (*i*), (*ii*) and (*iii*), we get

$$M_1 g - M_2 g = a (M_1 + M + M_2)$$

$$a = \left(\frac{M_1 - M_2}{M_1 + M + M_2} \right) \times g. \qquad \text{(Ans.)}$$

MOTION OF TWO BODIES CONNECTED BY A STRING ONE END OF WHICH ISHANGING FREE AND THE OTHER LYING ON A ROUGH INCLINED PLANE

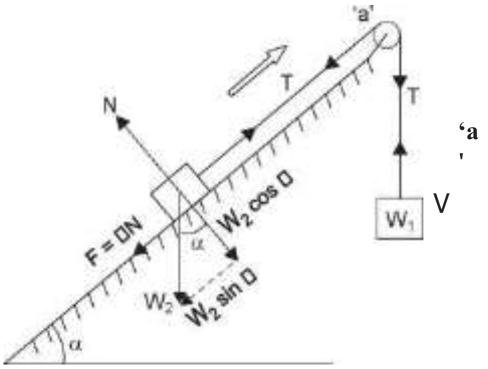


Fig. 8.16 shows two bodies of weight W_1 and W_2 respectively connected by a light inextensible string. Let the body 1 of weight W1 hang free and body 2 of weight W2 be placed on an inclined rough surface. The velocity and acceleration of the body 1 will be the same as that of body 2. Since the string is inextensible, therefore, tension will be same throughout.

Let *a* = acceleration of the system a = inclination of the plane p = co-efficient of friction between body and the inclined surface *T* = tension in the string.

Consider the motion of body 1 :

Forces acting on it are : *W*1 (downwards), *T* (upwards)

Resultant force = W1 - *T* ■■■(0

Since the body is moving downwards, therefore force acting on the body

$$\frac{W\text{-i}}{g} a \quad \text{...(ii)}$$

Equating (/) and *(if)* $W_1 - T = \frac{W,}{g} . a$ **...(1)**

Now consider the motion of body 2 :

Normal reaction at the surface,

N = W2 cos a

.'. Force of friction, *F* = pN = pW2 cos a The forces acting on the body 2 as shown are :

***T* (upwards), *W* sin a (downwards) and *F* = pW2 cos *a* (downwards)**

.•. Resultant force = *T* - W_2 sin *a* - pW2 cos *a* ■■■(iii)

Since, this body is moving along the inclined surface with acceleration therefore force acting on this body

$$\frac{Wy}{g} a \quad \text{...(iv)}$$

Equating (iii) and (iv), we get

$$T - W \sin a - pW2 \cos a = \frac{W}{g} a \quad \text{...(2)}$$

Adding equations (1) and (2), we get

$$W_1 - W_2 \sin a - pW_2 \cos a = \frac{a}{g}(W_1 + W_2)$$

$$a = \frac{g\,(W_1 - W_2 \sin\alpha - \mu W_2 \cos\alpha)}{W_1 + W_2}$$

Substituting this value of '*a*' in equation (1), we get

W - T ■ 1 W_1 / g a

T = W - Wia = W Gi - a|

■ wiWN+W2 sin a - uW2 cos a O / W1 + W2

| Wi + W- ~ Wi + W; sin a + uWL cos a |

1N I Wi + W2 Q

L1 + sin a + u cos a U

■ W, 1W- M - p / N ^+W2 Q

i.e.,

$$T = \frac{W_1 W_2 (1 + \sin\alpha + \mu\cos\alpha)}{W_1 + W_2}$$

For *smooth inclined surface* ; putting u = 0 in equations (8.13) and (8.14).

$$a = \frac{g\,(W_1 - W_2 \sin\alpha)}{W_1 + W_2}$$

and

$$T = \frac{W_1 W_2 (1 + \sin\alpha)}{W_1 + W_2}$$

Example 8.24. *A body weighing 8 N rests on a rough plane inclined at 15° to the horizontal. It is pulled up the plane, from rest, by means of a light flexible rope running parallel to the plane. The portion of the rope, beyond the pulley hangs vertically down and carries a weight of 60 N at the end. If the co-efficient of friction for the plane and the body is 0.22, find:*

(i) *The tension in the rope,*

(ii) *The acceleration in m/s², with which the body moves up the plane, and*

(iii) The distance in metres moved by the body in 2 seconds, starting from rest.

Sol. Refer to Fig.

Let *T* newton be the tension in the string and *a* m/s² the acceleration of the system.

Considering motion of 60 N weight (W_1):

$$60 - T = \frac{60}{g} \cdot a \qquad ...(i)$$

Considering motion of 8 N weight (W_2):

$$T - W_2 \sin\alpha - F = \frac{W_2}{g} \cdot a$$

$$T - 8\sin\alpha - \mu N = \frac{8}{g} \cdot a$$

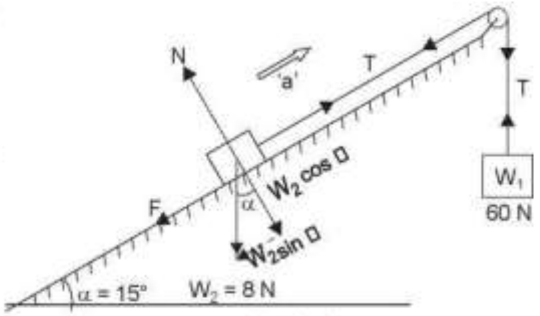


Fig. 8.17

$$T - 8 \sin a - 0.22 \times 8 \cos a = \frac{8}{g} . a \quad (\because N = W_2 \cos a = 8 \cos a) \quad ...(ii)$$

Adding *(i)* and *(ii)*

$$60 - 8 \sin a - 0.22 \times 8 \cos a = \frac{68}{g} . a$$

$$60 - 8 \sin 15° - 1.76 \cos 15° = \frac{68}{9.81} \times a$$

$$60 - 2.07 - 1.7 = \frac{68}{9.81} \times a$$

.'. $a = 8.11 \ m/s^2$. (Ans.)

Substituting this value of *'a'* in equation (i), we get

$$T = 60 - \frac{60}{9.81} \times 8.11 = 10.39 \text{ N. (Ans.)}$$

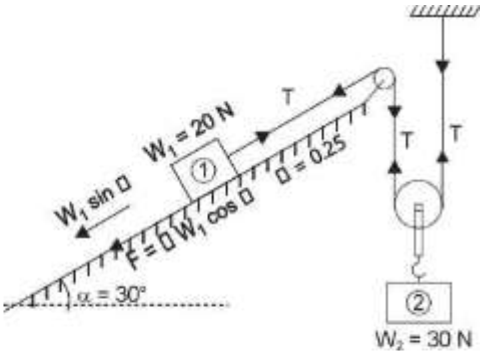
Distance moved in 5 seconds, s = ?

Initial velocity, $u = 0$ Time, $t = 2$ s.

Using the relation : $s = ut + \frac{1}{2} at^2$

$$s = 0 + \frac{1}{2} \times 8.11 \times 2^2 = 16.22 \text{ m. (Ans.)}$$

Example 8.25. ***Determine the resulting motion of the body '1' assuming the pulleys to be smooth and weightless as shown in Fig. . If the system starts from rest, determine the velocity of the body '1' after 5 seconds.***



Sol. Weight of body '1', W1 = 20 N Weight of body '2', W2 = 30 N

Let T = tension in the string, and a = acceleration of the body '1'. Considering the motion of body '1' :

$$T - W \sin a - \mu W_1 \cos a = \frac{W_1}{g} a$$

or $$T - 20 \sin 30° - 0.25 \times 20 \cos 30° = \frac{20}{g} \times a$$

or $$T - 10 - 4.33 = \frac{20}{g} a$$

or $$T - 14.33 = \frac{20}{g} a \quad ...(i)$$

Considering the motion of body '2' :

A little consideration will show that the acceleration of body '2' will be half the acceleration of body '1' (*i.e* a/2).

$$30 - 2T = \frac{30}{g} \times \frac{a}{2} = \frac{15}{g} a \quad \text{...(ii)}$$

Multiplying eqn. (i) by 2 and adding eqns. (i) and (ii), we get

$$1.34 = \frac{55}{g} a$$

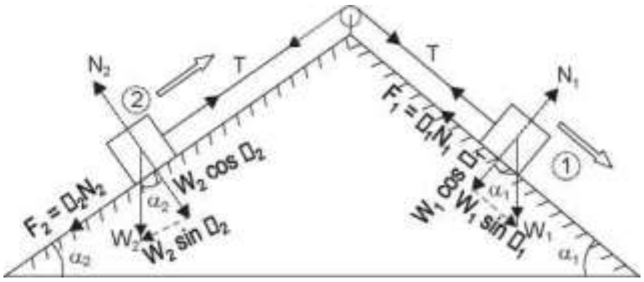
$$\frac{1.34 \times g}{55} = \frac{1.34 \times 9.81}{55} = 0.239 \text{ m/s}^2$$

Velocity of body 1' after 5 sec., if the system starts from rest,

$$v = u + at = 0 + 0.239 \times 5 = 1.195 \text{ m/s. (Ans.)}$$

8.14. MOTION OF TWO BODIES CONNECTED OVER ROUGH INCLINED PLANES

Fig. shows two bodies of weight W_1 and W2 respectively resting on the two inclined planes with inclinations 0,1 and 02 respectively.



Let *a* = acceleration of the system

P1 = co-efficient of friction between body 1 and the inclined plane 1 and P2 = co-efficient of friction between body 2 and the inclined plane 2. Consider the motion of body 1 :

Normal reaction at the surface,

$$= W \cos 0$$

Force of friction, $F_1 = p_1 N_1 = p_1 W_1 \cos a_1$

The forces acting on body 1 are :

***T* (upwards), force of friction F1 (upwards) and W1 sin 01 (downwards) as shown in Fig. 8.19. .'.**

Resultant force = $W_1 \sin a_x - T - p_x W_x \cos a_x$ **...(i)**

Since this body is moving downwards, the force acting on this body

$$\frac{W1}{g} \quad \text{...(ii)}$$

Equating (z) and *(ii)*

$$W_1 \sin a_1 - T - p_1 W_1 \cos a_1 = \frac{W_1}{g} . a \quad \text{...(1)}$$

Now consider motion of body 2 :

Normal reaction at the surface,

$$N_2 = W_2 \cos a_2$$

Force of friction, $F_2 = p_2 N_2 = \text{^}2\ W \cos a$

The ***forces acting on body 2 are :***

T **(upwards), force of friction of F2 (downwards) and** ***W*****2 sin a2 (downwards) as shown in Fig.**

Resultant force = $T - W_2 \sin a_2 - p_2 W \cos a_2$ **...(iii)**

Since the body is moving upwards, the force acting on the body

$$\frac{W_7}{g} a$$ **...(iv)**

Equating ***(iii)*** **and (iV)** **...(2)**

$$T - W \sin a - p\ W \cos a = \frac{W_2}{g} a$$

2 2 2 2 2

Adding eqns. (1) and (2), we get $W a_x - p_2 W \cos a_2$ $\frac{a}{g}(W + W)$

sin a - $W_2 \sin a_2 - p_x W_x \cos$ **a (W^sin ai - W-^sin a2 - m W^cos ai - U2 W^cos a;) j-g**

$$W_1 + W_2$$

Substituting this value of ***'a'*** **in equation (1), we get** ***W*** **sin a -** ***T*** **- m** ***W*** **cos a**

Wi x ***g*** **(W^sin ai - W^sin a2 - m W_^cos ai - m2 W^cos a2)**

$$g \quad W_1 + W_2$$

$$T = (W_1 \sin a_1 - p_i W_1 \cos a_1)$$

Wi(Wisin ai - W^sin a2 - m W^cos ai - m2 W^cos a2)

$$W_1 + W_2 i$$

or $T =$ **[(W + W) (W sin a - pW cos a) -** ***W*** **(W sin a**

(Wi + W2) i 2 i i i i i i i

- $W_2 \sin a_2 - p_i W_i \cos a_i - p_2 W_2 \cos a_2$**)] i**

(Wi + W2) **x [W sin a i - p*W2* cos** $a_i + W_i W_2 \sin a_i$

- p ***W W*** **cos a -** W^2 **sin a +** ***W W*** **sin a**

i i 2 i i i i 2 2

+ p W^2 **cos a + p** ***W W*** **cos a]**

i i i i 2 i 2 2

$W_i + W_2$ *(W W* **sin a +** ***W W*** **sin a - p** ***W W*** **cos a + p** ***W W*** **cos a)**

i 2 i i 2 2 i i 2 i 2 i 2 2

L $W_1 W_2$ (sin ai + sin a?) - W_t W_2 (u1 cos ai - u? cos a?) O

=MN

$W_1 + W_2$ Q

1

$W_1 + W_2$ [WJW (sin a + sin a_2) - WW fu cos a - U cos a)]

i.e., T = $\frac{W_1 W?}{W_1 + W?}$ fsin a 1 + sin a_2 - u_1 cos a_1 + p? cos a?) .(8.18)

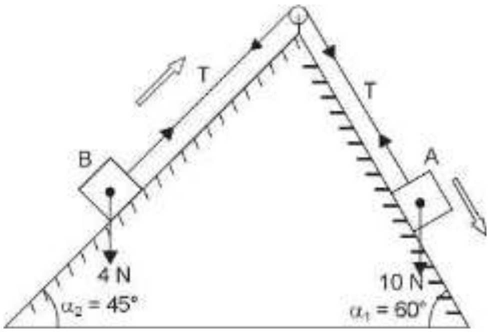
For *smooth inclined plane* : putting pi = 0 and p = 0 in equations (8.17) and (8.18), we get

a = $\frac{g\ (W\text{^}\sin a1 - W?\sin a?)}{W_1 + W?}$ ...(8.19)

and $\frac{W_1\ W_z}{W + W_2}$ (sin a1 + sin $a_?$) ...(8.?0)

26. Blocks A and B weighing 10 Nand 4 N respectively are connected by a weightless rope passing over a frictionless pulley and are placed on smooth inclined planes making 60° and 45° with the horizontal as shown in Fig. . Determine :

(i) The tension in the string and (ii) Velocity of the system 3 seconds after starting from rest.



Sol. Refer to Fig.

Let *'T '* be the tension in the rope and *'a'* the acceleration of the system.

(i) Tension, T = ?

For block A :

Resolving forces *parallel* to the plane :

$$10 \sin 60° - T = \frac{10}{g} . a \quad ...(i)$$

For block B:

Resolving forces parallel to the plane,

$$T - 4 \sin 45° = \frac{4}{g} . a \quad ---(ii)$$

Adding (i) and (ii), we get

$$10 \sin 60° - 4 \sin 45° = \frac{14}{g} . a$$

$$8.66 - ?.83 = \frac{14}{9.81} \times a$$

$$a = 4.08 \text{ m/s?}$$

Substituting this value of equations *'a'* in (i), we get

$$10 \sin 60° - T = \frac{10}{9.81} \times 4.08$$

$$T = 10 \sin 60^\circ - \frac{10}{9.81} \times 4.08$$

$= 8.66 - 4.16 = 4.5$ N. (Ans.)

(ii) Velocity after 3 seconds, v = ?

Using the relation : $v = u + at$

$= 0 + 4.08 \times 3$ (V $u = 0$)

$= 12.24$ m/s. (Ans.)