NARSIMHA REDDY ENGINEERING COLLEGE
UGC AUTONOMOUS INSTITUTION
Maisammaguda (V), Kompally - 500100, Secunderabad, Telangana State, India
UGC - Autonomous Institute
Accredited by NBA & NAAC with 'A' Grade
Approved by AICTE
Permanently affiliated to JNTUH

COURSE TITLE: MATRICES AND CALCULUS

COURSE CODE: 23MA101

REGULATION: NR23

Course Objectives: To learn

1. Concept of a rank of the matrix and applying this concept to know the consistency and solving the system of linear equations.

2. Concept of Eigen values and Eigen vectors and to reduce the quadratic form to canonical form

3. Geometrical approach to the mean value theorems and their application to the mathematical problems and evaluation of improper integrals using Beta and Gamma functions.

4. Partial differentiation and finding maxima and minima of function of two or more variables.

5. Evaluation of multiple integrals and their applications

COURSE OBJECTIVES(CO'S):

C111.1	Solve the system of Linear Equations in various engineering problems.
C111.2	Find the Eigen values and Eigen vectors and reduce the quadratic form to canonical form using orthogonal transformations.
C111.3	Solve the applications on the mean value theorems and evaluate the improper integrals using Beta and Gamma functions.
C111.4	Find the extreme values of functions of two variables with/without constraints.
C111.5	Evaluate the multiple integrals and apply the concept to find Areas, Volumes.

UNIT – I MATRICES

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define rank of a matrix and give one example	L1	CO1	PO1
2	Define Echelon form.	L1	CO1	PO1

3	Find the value of k such that the rank of $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2.	L2	CO1	PO2
4	State the different conditions in non - homogeneous system of equations.	L2	CO1	PO1
5	Find the rank of the matrix A= $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form.	L2	CO1	PO2
6	Define symmetric matrix and give a suitable example.	L1	CO1	PO1
7	Define normal matrix.	L1	CO1	PO1
8	Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is aunitary matrix.	L2	CO1	PO2
9	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$	L2	CO1	PO2
10	Prove that the transpose of a unitary matrix is unitary.	L2	CO1	PO1

S.NO	Part –B (Long answer questions)	BT	CO	PO
1(a)	Reduce the Matrix $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ into Echelon form. Hence find its Rank.	L2	CO1	PO2
1(b)	Find the Inverse of a matrix $A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by using Gauss-Jordan method.	L3	CO1	PO2
2 (a)	Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$, by reducing it to the normal form.	L2	CO1	PO2
2(b)	Examine for what values of a and b ,so that the equations $x + y + z = 3, x + 2y + 2z = 6, x + 9y + az = b$ *have* (i) No solution (ii) Unique solution (iii) Infinitely many solutions.	L4	CO1	PO2

3(a)	Solve system of equations $4x+2y+z+3w=0, 6x+3y+4z+7w=0, 2x+y+w=0$	L3	CO1	PO1
3(b)	Solve the equations $x+y+z=6, 3x+3y+4z=20, 2x+y+3z=13$ using gauss elimination method.	L3	CO1	PO1
4	Solve the system of equations by gauss seidel method $x-y+2z=4, 3x+y+4z=6, x+y+z=1.$	L4	CO1	PO3
5	Show that the equations $3x+3y+2z=1, x+2y=4, 10y+3z=-2, 2x-3y-z=5$ are consistent and solve them.	L2	CO1	PO1
6	Solve $2x-7y+4z=9,\ x+9y-6z=1,$ $-3x+8y+5z=6$ by LU-decomposition method.	L3	CO1	PO3

UNIT-2: EIGEN VALUES-EIGEN VECTORS AND QUADRATIC FORMS

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define model and spectral matrices.	L1	CO2	PO1
2	Find the sum and product of the Eigen values of $A=\begin{bmatrix}2 & 3 & -2\\ -2 & 1 & 1\\ 1 & 0 & 2\end{bmatrix}$	L2	CO2	PO1
3	Define characteristic polynomial.	L2	CO2	PO2
4	Find the Eigen values of A^{-1} where $A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$	L2	CO2	PO1
5	Find the symmetric matrix corresponding to the quadratic form $3x^2+16xy-y^2$.	L1	CO2	PO2
6	Find the characteristic roots of the matrix $A=\begin{bmatrix}2 & 3 & 4\\ 0 & 4 & 2\\ 0 & 0 & 3\end{bmatrix}$	L2	CO2	PO1
7	Compute the Eigen values and Eigen vectors of $\begin{bmatrix}1 & 4\\ 3 & 2\end{bmatrix}$	L2	CO2	PO1
8	Prove that zero is eigen value of a matrix iff it is singular.	L2	CO2	PO1
9	If the eigen values of A are -1,1,3 then find the eigen values of A-3I and A^3	L2	CO2	PO2
10	State Cayley – Hamilton theorem.	L1	CO2	PO1

S.NO	Part-B (Long answer questions)	BT	CO	PO
1	Find the Eigen values and Eigen vectors of a Matrix $$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$	L3	CO2	PO2
2(a)	Show that the matrix $A=\begin{bmatrix}3 & 2 & 2\\ 1 & 2 & 2\\ -1 & -1 & 0\end{bmatrix}$ satisfies its characteristic equation hence find A^2.	L2	CO2	PO2
2(b)	Verify Cayley Hamilton theorem for the matrix $$A=\begin{bmatrix}8 & -8 & 2\\ 4 & -3 & -2\\ 3 & -4 & 1\end{bmatrix}$$	L3	CO2	PO1
3(a)	Determine the eigen values and eigen vectors of $B=2A^2-\frac{1}{2}A+3I$ where $A=\begin{bmatrix}8 & -4\\ 2 & 2\end{bmatrix}$.	L2	CO2	PO1
3(b)	Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and hence find $B=A^5-4A^4-7A^3+11A^2-A-10I$.	L3	CO2	PO1
4	Diagonalize the matrix$A=\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$ and find A^8.	L3	CO2	PO3
5	Reduce the Quadratic form $10{x_1}^2+2{x_2}^2+5{x_3}^2-4x_1x_2-10x_1x_3+6x_2x_3$ into Canonical form and hence state nature, rank, index and signature of the Quadratic form.	L4	CO2	PO3
6	Diagonalize the matrix $A=\begin{bmatrix}7 & 4 & -4\\ 4 & -8 & -1\\ -4 & -1 & -8\end{bmatrix}$ by Orthogonal Reduction.	L4	CO2	PO3

UNIT-III: CALCULUS

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Verify Rolle's theorem for $f(x)=\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in[a,b], a>0,b>0.	L2	CO4	PO1
2	Verify Lagrange's mean value theorem for $f(x)=x^{\frac{1}{3}}\, in\, [-1,1]$	L2	CO4	PO2

3	Define beta and gamma functions.	L1	CO4	PO1
4	Find the value of $\Gamma(\frac{-5}{2})$	L2	CO4	PO1
5	Evaluate $\int_0^1 x^7 (1-x)^3\, dx$	L1	CO4	PO1
6	State Cauchy's mean value theorem.	L2	CO4	PO2
7	State the Taylor's theorem.	L1	CO4	PO2
8	Compute $B(\frac{9}{2},\frac{7}{2})$.	L1	CO4	PO2
9	Write the relation between Beta and Gamma function.	L1	CO4	PO2
10	Compute $\int_0^\infty e^{-2x}\, x^6 \quad dx$	L1	CO4	PO2
S.NO	**Part-B(Long answer questions)**	**BT**	**CO**	**PO**
1(a)	Verify Rolle's theorem for $f(x) = x^3 - 6x^2 + 11x - 6$ in[1,3].	L3	CO4	PO2
1(b)	If a<b, Prove that $\frac{b-a}{(1+b^2)} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{(1+a^2)}$ using Lagrange's mean value theorem.	L3	CO4	PO2
2(a)	Verify generalized mean value theorem for f(x) = e^x , g(x)= e^{-x} in [a,b] and find the value of c.	L3	CO4	PO2
2(b)	Prove that β (m,n) =$\beta(m+1,n) + \beta(m,n+1)$.	L3	CO4	PO3
3(a)	Evaluate $\int_0^\infty \frac{x}{1+x^6}\, dx$ in terms of Beta-Gamma function.	L3	CO4	PO2
3(b)	Expand the function $f(x,y) = e^x(\log(x+y))$ in terms of x and y up to the term of third degree using Taylor's theorem.	L2	CO4	PO2
4(a)	Prove that $\int_0^1 x^m log x^n dx = \frac{(-1)^n n!}{(m+1)^{(n+1)}}$ where n is a positive integer and m>-1.	L2	CO4	PO2
4(b)	Show that $\Gamma(n) = \int_0^1 (log\frac{1}{x})^{n-1}\ dx,\ n > 0$.	L2	CO4	PO2
5	Establish the relation between Beta and Gamma functions .	L3	CO4	PO2
6(a)	Show that $4\int_0^\infty \frac{x^2}{1+x^4}$ dx = $\sqrt{2}\,\pi$.	L4	CO4	PO2
6(b)	Evaluate $\int_0^\infty x^4 e^{=x^2} dx = \frac{3\sqrt{\pi}}{8}$ dx usingβ-Γ functions.	L4	CO4	PO2

UNIT-IV : MULTI VARIABLE CALCULUS (PARTIAL DIFFERENTIATION AND APPLICATIONS)

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	State Euler's theorem for homogeneous function in x and y.	L1	CO5	PO1
2	Define Jacobian for two variables.	L2	CO5	PO2
3	If x=u(1+v) , y=v(1+u) then prove that $\frac{\partial(x,y)}{\partial(u,v)}$ = 1+u+v .	L2	CO5	PO2
4	Write the working rule to find the maximum and minimum values of f(x,y) .	L2	CO5	PO1
5	Find $\frac{\partial^2 u}{\partial x \partial y}$ for the function u = $sin^{-1}\frac{x}{y}$.	L2	CO5	PO2
6	Find the first and second order partial derivatives of x^3+y^3-3axy and verify $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$	L2	CO5	PO2
7	Verify Euler's theorem for the function $ax^2 + 2hxy + by^2$.	L1	CO5	PO2
8	If $u=x^2-2y$, v= x+y+z , w=x-2y+3z find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	L1	CO5	PO2
9	Determine whether u and v are functionally dependent where u and v are defined by $u = sinx + cosy, v = cosx + siny$	L2	CO5	PO2
10	Find the stationary values of $f(x,y) = xy(a - x - y)$	L2	CO5	PO2

S.NO	**Part-B(Long answer questions)**	BT	CO	PO
1(a)	If $U = \log(x^3 + y^3 + z^3 - 3xyz)$ prove that $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 U = \frac{-9}{(x+y+z)^2}$.	L3	CO5	PO2
1(b)	If $x = rcos\theta, y = rsin\theta$ the prove that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$ $and \frac{1}{r}\frac{\partial x}{\partial \theta} = r\frac{\partial \theta}{\partial x}$.	L3	CO5	PO2
2(a)	If $u = 2xy, v = x^2 - y^2, x = rcos\theta, y = rsin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$	L3	CO5	PO2
2(b)	If $u = x + y + z, v = x + y$ and $z = z, find$ $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	L2	CO5	PO2
3(a)	If x=uv , y = $\frac{u}{v}$ find $\frac{\partial(x,y)}{\partial(u,v)}$.	L2	CO5	PO2
3(b)	Determine whether the function $u = \frac{x+y}{x-y}, v = \frac{xy}{(x-y)^2}$ are dependent. If so, find the relation between them.	L3	CO5	PO3
4	Find the maximum and minimum values of the function$f(x,y) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.	L4	CO5	PO1
5(a)	Find the maximum and minimum values of the function	L4	CO5	PO2

	$f(x,y) = 2(x^2 - y^2) - x^4 + y^4$			
5(b)	A rectangular open at the top is to have volume of 32cu.feet. find the dimensions of the box requiring least material for its construction.	L3	CO5	PO3
6	Divide 24 into three points such that the continued product of the first, square of the second and cube of the third is maximum.	L4	CO5	PO3

UNIT-V: MULTIPLE INTEGRALS

S.No		Questions	BT	CO	PO
Part – A (Short Answer Questions)					
1		Evaluate $\int_0^2 \int_0^3 xy\,dx\,dy$	L1	CO3	PO1
2		Evaluate $\int_0^2 \int_0^x ydx\,dy$	L2	CO3	PO2
3		Evaluate $\int_0^1 \int_0^2 y^2dy\,dx$	L1	CO3	PO1
4		Evaluate $\int_0^1 \int_0^{\frac{\pi}{2}} rsin\theta drd\theta$	L1	CO3	PO1
5		Write spherical polar coordinates.	L3	CO3	PO2
6		Find $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dxdydz$	L3	CO3	PO2
7		Find $\int_0^1 \int_1^2 \int_2^3 xyzdxdydz$	L1	CO3	PO1
8		Write the limits after change of order of integration to the integral $\int_0^\infty \int_0^x xe^{\frac{-x^2}{y}}\,dy\,dx.$	L3	CO3	PO2
9		Shade the region bounded by the $y = x^2\ and\ x = y^2$.	L3	CO3	PO2
10		Convert the following integral to polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)dy\,dx.$	L3	CO3	PO2
Part – B (Long Answer Questions)					
11	a)	Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$	L4,L5	CO3	PO3
	b)	Evaluate $\oiint ydxdy$ where R is the region bounded by the parabolas $y^2 = 4x\ and\ x^2 = 4y$.	L3	CO3	PO2

12	a)	Evaluate $\iint (x^2+y^2)$dxdy in the positive quadrant for which x+y ≤1.	L3	CO3	PO2
	b)	Evaluate $\int_0^{\frac{\pi}{4}} \int_0^{a\sin\theta} \frac{r}{\sqrt{a^2-r^2}} dr d\theta$.	L3	CO3	PO2
13	a)	Evaluate $\iint r^3 dr d\theta$ over the area included between the circles r=2sinθ and r=4sinθ.	L4,L5	CO3	PO3
	b)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx$ by change of order of integration.	L3	CO3	PO2
14	a)	Evaluate$\iiint (xy+yz+zx) dx\, dy\, dz$ where V is the region of the space bounded by x=0,x=1,y=0,y=2,z=0,z=3.	L3	CO3	PO2
15		Find the volume of the tetrahedron bounded by the planes x=0,y=0,z=0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$	L2,L3	CO3	PO2
16		Using cylindrical coordinates, find the volume of the sphere $x^2+y^2+z^2=a^2$.	L2,L3	CO3	PO2

*Blooms Taxonomy Level (BT) (L1-Remembering; L2- Understanding, L3-Applying;L4-Analyzing; L5-Evaluating; L6-Creating)

Course Out comes (CO)

Program Out comes (PO).

Prepared by : G SANDHYA RANI

ASSISTANT PROFESSOR

FME DEPARTMENT