Q.P Code: 23MA101 Hall Ticket No. [ ]

NARSIMHAREDDY ENGINEERING COLLEGE

(UGC AUTONOMOUS)

I B.Tech I Semester (NR23) Regular Examination, January/February 2024

MATRICES AND CALCULUS

(Common to CE, EEE, ME, ECE, CSE, IT, CSE (CS), CSE (AI&ML))

Time : 3 hours **Maximum marks: 60**

Note:
- This question paper contains two parts, A and B
- Part A is compulsory which carries 10 marks (10 sub questions are two from each unit carry 1 Marks). Answer all questions in Part A
- Part B Consists of 5 Units. Answer one questiôn from each unit. Each question carries 10 Marks and may have a, b sub questions

Part-A **(10 Marks)**

Answer all questions

Q.No		Question	M	CO	BL
1)	a.	Define the Rank of the Matrix	1	CO1	L1
	b.	Explain Consistent and In-consistent	1	CO1	L2
	c.	State Cayley- Hamilton Theorem	1	CO2	L2
	d.	Define Index and Signature of the Quadratic form	1	CO2	L1
	e.	State Cauchy's Mean Value Theorem	1	CO3	L1
	f.	Obtain the Maclaurin's series expansion of the function $f(x)=e^x$	1	CO3	L3
	g.	If $u=\frac{x+y}{1-xy}$ and $\theta=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,\theta)}{\partial(x,y)}$	1	CO4	L2
	h.	Verify that $\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}$ when $u=\log(y\sin x+x\sin y)$	1	CO4	L3
	i.	Evaluate $\int_{y=0}^{[illegible]}\int_{x=0}^{[illegible]} xy\,dx\,dy$	1	CO5	L2
	j.	Evaluate $\int^{[illegible]}\int^{[illegible]} y\,dy\,dx$	1	CO5	L2

Part-B **(50 Marks)**

Answer all the Units

All Questions carry equal Marks

Q.No		Question	M	CO	BL
		UNIT-I			
2)	a.	Find the rank of the matrix $A=\begin{bmatrix}2&1&3&5\\4&2&1&3\\8&4&7&13\\8&4&-3&-1\end{bmatrix}$ by reducing it to Echelon form.	5	CO1	L3
	b.	Use the Gauss-Jordan method to compute the inverse of the matrix $A=\begin{bmatrix}1&0&4\\2&-2&1\\-1&1&-1\end{bmatrix}$	5	CO1	L3
		OR			
3)	a.	Find the rank of the matrix $A=\begin{bmatrix}2&1&3&5\\4&2&1&3\\8&4&7&13\\8&4&-3&-1\end{bmatrix}$ by reducing it to normal form.	5	CO1	L2
	b.	Solve the system of equations $10x+y+z=12$, $2x+10y+z=13$, $x+y+5z=7$ by using L-U decomposition method	5	CO1	L3

Q.No		Question	M	CO	BL
		UNIT-II			
4)	a.	Find the eigen values and corresponding eigen vectors of the matrix $A=\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$	5	CO2	L3
	b.	Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}8&-12&5\\15&-25&11\\24&-42&19\end{bmatrix}$ and hence find A^{-1} and $A^{[illegible]}$	5	CO2	L4
		OR			
5)	a.	Diagonalise the matrix and obtain the modal matrix for $A=\begin{bmatrix}-1&1&2\\0&-2&1\\0&0&-3\end{bmatrix}$ Hence find $A^{[illegible]}$	5	CO2	L3
	b.	Find the orthogonal transformation which transforms the quadratic form $3x^2-2y^2-z^2-4xy+12yz+8zx$ to canonical form and find the rank, index, signature and nature	5	CO2	L2
		UNIT-III			
6)	a.	Show that the Rolle's theorem is applicable for the function $f(x)=\log\left(\frac{x^2+ab}{x(a+b)}\right)$ in the interval $[a,b]$, $a>0, b>0$	5	CO3	L3
	b.	Expand the function $f(x,y)=e^x\log(1+y)$ in terms of x and y up to the terms of 3rd degree using Taylor's theorem	5	CO3	L4
		OR			
7)	a.	Show that the Lagrange's mean value theorem is applicable for the function $f(x)=x^3-x^2-5x+3$ in the interval $[0,4]$	5	CO3	L4
	b.	Show that $\int_0^1(\log 1/x)^{n-1}dx=\Gamma(n)$	5	CO3	L5
		UNIT-IV			
8)	a.	If $u=f(x^2+2yz, y^2+2zx)$ prove that $(y^2-zx)\frac{\partial u}{\partial x}+(x^2-yz)\frac{\partial u}{\partial y}+(z^2-xy)\frac{\partial u}{\partial z}=0$	5	CO4	L5
	b.	Verify if $u=2x-y+3z$, $v=2x-y-z$, $w=2x-y+z$ are functionally dependent and if so, find the relation between them	5	CO4	L6
		OR			
9)	a.	Examine the extrema of $f(x,y)=x^2+xy+y^2+\frac{1}{x}+\frac{1}{y}$	5	CO4	L4
	b.	Divide 24 into three points such that the continued product of the first, square of the second and cube of the third is maximum	5	CO4	L6
		UNIT-V			
10)	a.	Evaluate $\iint_R xy\,dx\,dy$ where R is the region bounded by the line $x+2y=2$, lying in the first quadrant	5	CO5	L4
	b.	Evaluate the following integral by transforming into polar coordinates $\int_0^a\int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2}\,dx\,dy$	5	CO5	L5
		OR			
11)	a.	By Changing the order of integration, evaluate $\int_0^1\int^{[illegible]}(x+y)\,dx\,dy$	5	CO5	L4
	b.	Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{1}{\sqrt{1-x^2-y^2-z^2}}\,dz\,dy\,dx$	5	CO5	L3

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Q.P Code: MA1101BS Hall Ticket No: 22X01A0432

NARSIMHA REDDY ENGINEERING COLLEGE

(UGC AUTONOMOUS)

I B.Tech I Semester (NR21) Regular & Supplementary Examination, March 2023

LINEAR ALGEBRA & CALCULUS

(Common to CE, EEE, ME, ECE, CSE, CSE (CS), CSE (AI&ML), CSE (DS))

Time : 3 hours Maximum marks: 70

Note:
- This question paper contains two parts, A and B
- Part A is compulsory which carries 20 marks (10 sub questions are two from each unit carry 2 Marks). Answer all questions in Part A
- Part B Consists of 5 Units. Answer one question from each unit. Each question carries 10 Marks and may have a, b sub questions

Part-A **(20 Marks)**

Answer all questions

Q.No		Question	M	CO	BL
1)	a	Find the Rank of the Matrix $\begin{bmatrix}2&3&7\\3&-2&4\\1&-3&-1\end{bmatrix}$ to reduce Echelon Form	2	CO1	L1
	b	Prove that the matrix $A=\frac{1}{3}\begin{bmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{bmatrix}$ is orthogonal	2	CO1	L3
	c	Define Eigen Values and Eigen Vectors	2	CO2	L1
	d	Find the sum and product of the Eigen values of $\begin{bmatrix}2&3&-2\\-2&1&1\\1&0&2\end{bmatrix}$	2	CO2	L1
	e	Find the value of a_0, if $f(x)=x$ in $(0, 2\pi)$	2	CO3	L1
	f	Define Fourier series of a function f(x) in the interval $(c, c+2\pi)$.	2	CO3	L1
	g	State Rolle's Theorem	2	CO4	L1
	h	Define Gamma Function	2	CO4	L1
	i	Determine first and second order partial derivatives of $ax^2+2hxy+by^2$ and verify $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$	2	CO5	L3
	j	If $x=r\cos\theta, y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$	2	CO5	L1

Part-B **(50 Marks)**

Answer all the Units

All Questions carry equal Marks

Q.No		Question	M	CO	BL
		UNIT-I			
2)		Find the rank of the matrix $A=\begin{bmatrix}1&2&3&-1\\-2&-1&-3&-1\\1&0&1&1\\0&1&1&-1\end{bmatrix}$ by applying Normal form.	5	CO1	L3

Q.No		Question	M	CO	BL
		Discuss for what the values of λ and μ the equations $x+2y+z=8, 2x+2y+2z=13, 3x+4y+\lambda z=\mu$ have (i) no solution (ii) unique solution (iii) infinitely many solutions.	5	CO1	L6
		OR			
3)		Solve the system of equations using Gauss Jordan method $x-y-z=0, x-3y+2z=0, 2x-y+z=0$	5	CO1	L6
		Solve the system of equations by Gauss Seidel method $20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25$.	5	CO1	L6
4)	a	Find the Eigen values and corresponding Eigen vectors of the matrix $A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$	5	CO2	L4
	b	Reduce the matrix $A=\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix}$ to the diagonal form and hence Determine A^8	5	CO2	L5
		OR			
5)	a	Verify Cayley – Hamilton theorem for the Matrix $A=\begin{bmatrix}1&4\\2&3\end{bmatrix}$ and Determine its Inverse. Also express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A.	5	CO2	L5
	b	Find the orthogonal transformation which transforms the quadratic form $x^2+3y^2+3z^2-2yz$ to canonical form and Determine the rank, index, signature and nature.	5	CO2	L5
6)	a	Find the Fourier series representation of the function $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in $(0, 2\pi)$ Hence Prove that $\frac{1}{1^2}+\frac{1}{2^2}+\cdots+=\frac{\pi^2}{12}$	5	CO3	L5
	b	Find the Fourier series representation of the function $f(x)=\begin{cases}-\pi & for\ -\pi<x<0\\ x & for\ 0<x<\pi\end{cases}$ Also Prove that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$	5	CO3	L5
		OR			
7)	a	Express the given function as a Fourier series expansion $f(x)=\|x\|$ in $(-\pi, \pi)$	5	CO3	L5
	b	Determine the half range sine series for the functions $f(x)=e^{ax}$ in $0<x<\pi$	5	CO3	L5

		UNIT-IV			
8)	a.	Find 'c' of Cauchy's mean value theorem for $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ in [a, b] for $0<a<b$.	5	CO4	L4
	b.	Prove that $1-\frac{a}{b}<\log\frac{b}{a}<\frac{b}{a}-1, 0<a<b$ by using Lagrange's mean value theorem.	5	CO4	L5
		OR			
9)	a.	Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m,n>0.	5	CO4	L5
	b.	Prove that $\beta(m,n)=\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}dy$	5	CO4	L5
		UNIT-V			
10)	a.	If $U=\frac{1}{\sqrt{x^2+y^2+z^2}}, x^2+y^2+z^2\neq 0$ then prove that $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=0$	5	CO5	L5
	b.	Prove that the functions $u=x+y+z$, $v=x^2+y^2+z^2-2xy-2yz-2zx$ and $w=x^3+y^3+z^3-3xyz$ are functionally related and find the relation between them.	5	CO5	L5
		OR			
11)	a.	Prove that the function $f(x, y)=x^3+y^3-63(x+y)+12xy$ is maximum at (-7, -7) and minimum at (3, 3).	5	CO5	L5
	b.	Divide 24 into three points such that the continued product of the first, square of the second and cube of the third is maximum.	5	CO5	L6

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Hall Ticket No.: 2 [redacted] X d 1 A 0 5 7 7

Question Paper Code:MA1101BS
Set 2

NARSIMHA REDDY ENGINEERING COLLEGE

(UGC-AUTONOMOUS)

B.TECH I YEAR I SEMESTER SUPPLEMENTARY EXAMINATIONS, SEPT-2022

(Regulation: NR21)

LINEAR ALGEBRA & CALCULUS

(Common to CIVIL, EEE, ME, ECE, CSE, CSE(CS), CSE(AI&ML), CSE(DS))

Time: 3 hours Max. Marks: 70

Answer any Five Questions

All Questions carry Equal Marks

		Marks	Bloom's Level
1.	a. Reduce the Matrix $A=\begin{bmatrix}2&1&3&5\\4&2&1&3\\8&4&7&13\\8&4&-3&-1\end{bmatrix}$ into Echelon form. Hence find its Rank	7	L4
	b. Solve the equations $x+3y-2z=0, 2x-y+4z=0, x-11y+14z=0$.	7	L3
2.	State Cayley-Hamilton theorem and Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$ and find A^{-1} and A^{2}.	14	L1
3.	Expand $f(x)=\left(\frac{\pi-x}{2}\right)^2, 0<x<2\pi$ in a Fourier series. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=\frac{\pi^2}{6}$	14	L4
4.	a. Verify Rolle's theorem for $f(x)=\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in the interval [a,b].	7	L4
	b. If $f(x)=\log x$ and $g(x)=x^2$ in $[a,b]$, then prove that $\frac{\log b-\log a}{b-a}=\frac{a+b}{2c^2}$ using Cauchy's mean value theorem	7	L5
5.	a. Find the maximum and minimum values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$.	7	L1
	b. If $U=\log(x^3+y^3+z^3-3xyz)$, then Prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 U=\frac{-9}{(x+y+z)^2}$	7	L4
6.	a. Investigate for what values of a,b the equations $x+2y+3z=4, x+3y+4z=5, x+3y+az=b$ have (i) No solution (ii) Unique solution (iii) infinitely many solutions.	7	L1
	b. Solve $2x-7y+4z=9, x+9y-6z=1, -3x+8y+5z=6$ by LU-decomposition method	7	L3
7.	a. Evaluate (i) $\int_0^{\frac{\pi}{2}} Sin^5\theta Cos^7\theta d\theta$ using β-Γ functions	7	L5
	b. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$	7	L4
8.	Use Gauss-Seidel Iteration method to solve the system $10x+y+z=12, 2x+10y+z=13, 2x+2y+10z=14$	14	L3

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Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, March/April - 2023

MATRICES AND CALCULUS

(Common to CE, ME, ECE, EIE, AE, MIE, CSE(AI&ML), CSE(IOT), AI&DS, AI&ML)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) What is the value of 'k' if the rank of the matrix $\begin{bmatrix}1 & 2 & 3\\ 2 & k & 7\\ 3 & 6 & 10\end{bmatrix}$ is 2? [1]

b) Find the value of 'a' for which the equations have infinite number of solutions: [1]
x+y+z=1; ax-ay+3z = 5; 5x-3y +az =6.

c) If the eigenvalues of A= $\begin{bmatrix}5 & -2 & 0\\ -2 & 6 & 2\\ 0 & 2 & 7\end{bmatrix}$ are 3, 6 and 9, then what are the eigenvalues of adj A? [1]

d) What is the nature of the quadratic form $2xy + 6xz - 4yz$? [1]

e) Find the value of the constant in Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ defined on [a, b], $0 < a < b$. [1]

f) Find the Taylor's expansion of $f(x) = e^x$ around x=1. [1]

g) If $u = \frac{y}{x}, v = xy$, then find $J\left(\frac{u,v}{x,y}\right)$. [1]

h) Find the stationary point's of the function $x^2+2xy+2y^2+2x+2y$. [1]

i) Evaluate $\int_0^1 \int_2^4 xy dx dy$. [1]

j) In the integral $\int_0^4 \int_x^4 f(x,y) dx dy$, write the limits after changing the order of integration. [1]

PART-B

(50 Marks)

2.a) Find rank of matrix $A = \begin{bmatrix}1 & 0 & 2 & 1\\ 0 & 1 & -2 & 1\\ 1 & -1 & 4 & 0\\ 2 & 2 & 8 & 0\end{bmatrix}$ by reducing it into normal form.

b) Solve the system of equations:
$x+y+54z=110;\ 27x+6y-z=85;\ 6x+15y+2z=72$ using Gauss-Seidel method. [5+5]

OR

3.a) Solve the system of equations by Gauss -elimination method
$5x+y+z+w=4$, $x+7y+z+w=12$, $x+y+6z+w=-5$,
$x+y+z+4w=-6$.

b) Find the inverse of $A=\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$ using Gauss-Jordan method. [5+5]

4.a) Find the eigenvalues and the corresponding eigen vectors of $\begin{bmatrix}6&-2&2\\-2&3&1\\2&-1&3\end{bmatrix}$.

b) Using Cayley Hamilton theorem for the matrix A= $\begin{bmatrix}1&-3&1\\6&3&1\\1&3&1\end{bmatrix}$, find A^4. [5+5]

OR

5. Reduce the quadratic form $3x^2+2y^2+3z^2-2xy-2yz$ to the canonical form by orthogonal transformations and find rank, index, signature, nature of the quadratic form. [10]

6.a) Find the region in which f(x) = $1-4x-x^2$ is increasing and the region in which it is decreasing using Mean Value Theorem.

b) Find the volume of the solid generated by the revolution of the area bounded by y = x^2 and y = x about y – axis. [5+5]

OR

7.a) Expand $\tan^{-1} x$ in powers of (x – 1) up to the term containing fourth degree.

b) Evaluate $\int_0^1 x^4\left(\log\frac{1}{x}\right)^3 dx$. [5+5]

8.a) If u = $\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

b) If x + y + z = u, y + z = uv, z = uvw, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. [5+5]

OR

9.a) If x = $e^r\sec\theta$, y = $e^r\tan\theta$, prove that $\frac{\partial(x,y)}{\partial(r,\theta)}\cdot\frac{\partial(r,\theta)}{\partial(x,y)}=1$.

b) The Temperature T at any point (x, y, z) in space is $T=400xyz^2$. Find the highest temperature at the surface of the unit sphere $x^2+y^2+z^2=1$. [5+5]

10.a) By changing the order of integration, evaluate $\int_0^a\int_0^{\sqrt{a^2-x^2}}\sqrt{a^2-x^2-y^2}\,dydx$.

b) Find the volume bounded by the cylinder $x^2+y^2=4$ and the plane $z=0, y+z=4$. [5+5]

OR

11.a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dydx$ by changing into polar coordinates.

b) Using spherical polar coordinates, evaluate $\iiint \frac{xyz\; dxdydz}{\sqrt{x^2+y^2+z^2}}$ taken over the volume bounded by the sphere $x^2+y^2+z^2=a^2$ in the first octant. [5+5]

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Code No: 181AN **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year I Semester Examinations, March/April - 2023
MATRICES AND CALCULUS
(Common to EEE, CSE, IT, CSIT, CE(SE), CSE(CS), CSE(DS), CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Are the system of equations $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30$ Consistent? [1]

b) Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. [1]

c) If λ be a eigen value of a matrix A (non-zero matrix). Is λ^{-1} is an eigen value of A^{-1}? [1]

d) The eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigenvalues of S^2? [1]

e) Does Lagrange mean value theorem for the function $f(x) = \log_e x$ in the interval [1, e] apply? [1]

f) Write the Beta function β (m, n) in terms of Sine and Cosine. [1]

g) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ where $u = e^x \sin y \ \ and \ \ v = x + \log \sin y$. [1]

h) If $u = x^y$, then find $\frac{\partial u}{\partial y}$. [1]

i) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. [1]

j) Change the order of integration for $\int_0^a \int_0^y f(x,y)dx\,dy$. [1]

PART - B

(50 Marks)

2.a) Use either the Gaussian Elimination or the Gauss Jordan method to solve

$$x + 2y - 3z = 9$$
$$2x - y + z = 0$$
$$4x - y + z = 4$$

b) For what values of n will the equations:
$x = y + z = 1;\ \ x + 2y + 4z = n;\ \ x + 4y + 10z = n^2$ be consistent. Solve them completely in each case. [5+5]

OR

3. Using Gauss-seidel method solve the following system of equations
$5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20.$ [10]

4.a) For matrix $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in normal form and hence find rank of matrix A.

b) Diagonalise the Hermitian matrix $A = \begin{bmatrix} 2 & 1-2i \\ 1+2i & -2 \end{bmatrix}$ to unitarily similar diagonal matrix. [5+5]

OR

5.a) Find a matrix P that transforms $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form and hence find A^4.

b) Using Cayley Hamilton Theorem, find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Also find A^{-1}. [5+5]

6.a) State Rolle's Theorem. And Test the validity of the theorem for the functions in the interval mentioned against them $(x-2)^2$ in [-1,5].

b) Show that $\Gamma(n) = \int_0^1 \left(log\frac{1}{x}\right)^{n-1} n > 0.$ [5+5]

OR

7.a) Prove that $\beta(n,n) = \dfrac{\sqrt{\pi}.\,\Gamma(n)}{2^{2n-1}.\Gamma\left(n+\frac{1}{2}\right)}$.

b) The curve $y^2(a+x) = x^2(3a-x)$ revolves about the axis by x. Find the volume generated by the loop. [5+5]

8.a) Determine if the following function is functionally dependent. If they are functionally dependent, then find a functional relation between them.
$u = x\sqrt{1-y^2} + y\sqrt{1-x^2},\ \ v = \sin^{-1}x + \sin^{-1}y$

b) Calculate $\frac{\partial(u,v)}{\partial(r,\theta)}$ if $u = 2axy,\ v = a(x^2 - y^2)$ where $x = r\cos\theta,\ y = r\sin\theta.$ [5+5]

OR

9.a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1.$ [5+5]

10.a) Evaluate $\iint \frac{dxdy}{x^4 + y^4}$, over the region bounded by $y \geq x^2$, $x \geq 1$.

b) If a denotes the radius of the base, h the altitude of a right circular cone, express the volume as a triple integral and evaluate it. [5+5]

OR

11.a) Find the volume of the solid bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 2y$.

b) A triangular prism is formed by planes whose equations are $ay = bx, y = 0$ and $x = a$. Find the volume of the prism between the planes $z = 0$ and the surface $z = c + xy$. [5+5]

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