UGC AUTONOMOUS

I B.Tech I Sem **A.Y: 2022-23.**

Course Title: Linear Algebra and Calculus

Course Code: MA1101BS

Regulation: NR21

Course Objectives: To learn

• Types of matrices and their properties.Concept of a rank of the matrix and applying this concept to know the consistency and solving the system of linear equations.

• Concept of Eigen values and eigenvectors and to reduce the quadratic form to canonical form.

• Concept of Fourier series.

• Geometrical approach to the mean value theorems and their application to

the mathematical problems. Evaluation of improper integrals using Beta and Gamma functions.

• Partial differentiation, concept of total derivative .Finding maxima and minima of function of two and three variables.

Course Outcomes: After learning the contents of this paper the student must

be able to

• Write the matrix representation of a set of linear equations and to analyse the

solution of the system of equations

• Find the Eigen values and Eigen vectors. Reduce the quadratic form to canonical form using orthogonal transformations.

• Analyze the Fourier series.

• Solve the applications on the mean value theorems. Evaluate the improper integrals using Beta and Gamma functions

• Find the extreme values of functions of two variables with/ without constraints.

UNIT – I

MATRICES

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define rank of a matrix and give one example	L1	CO1	PO1
2	Define Hermitian and skew - Hermitian matrices.	L1	CO1	PO1
3	Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2 .	L2	CO1	PO2
4	State the different conditions in non - homogeneous system of equations .	L2	CO1	PO1
5	Find the rank of the matrix A=$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form.	L2	CO1	PO2
6	Define symmetric matrix and give a suitable example.	L1	CO1	PO1
7	Define an orthogonal matrix and give one example.	L1	CO1	PO1
8	Prove that $\frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$ is aunitary matrix.	L2	CO1	PO2
9	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$	L2	CO1	PO2
10	Show that the matrix $\frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix.	L2	CO1	PO1

S.NO	**Part –B (Long answer questions)**	BT	CO	PO
1(a)	Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 5 & 6 \end{bmatrix}$, by reducing it to the normal form.	L2	CO1	PO2
1(b)	Find the Inverse of a matrix $A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by using Gauss-Jordan method.	L3	CO1	PO2

2 (a)	Reduce the Matrix $A = \begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ into Echelon form. Hence find its Rank.	L2	CO1	PO2
2(b)	Examine for what values of p and q , so that the equations 2x+3y+5z = 9, 7x+3y+2z=8, 2x+3y+pz=q *have* (i) No solution (ii) Unique solution (iii) Infinitely many solutions.	L4	CO1	PO2
3(a)	Solve system of equations x+y+w= 0, y+z= 0 , x+y+z+w = 0 , x+y+2z= 0 .	L3	CO1	PO1
3(b)	Solve the equations 3x+y+2z=3, 2x-3y-z=-3, x+2y+z=4 using gauss elimination method.	L3	CO1	PO1
4	Solve the system of equations by gauss seidel method 20x+y-2z=17, 3x+20y-z=-18, 2x-3y+20z=25 .	L4	CO1	PO3
5(a)	Find the rank of the value of k , if the rank of the matrix A is 2 , where A= $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$	L2	CO1	PO1
5(b)	Show that the equations x+2y-z=3, 3x-y+2z =1 , 2x-2y+3z = 2, x-y+z = -1 are consistent and solve them.	L2	CO1	PO1
6	Solve $2x - 7y + 4z = 9,\ x + 9y - 6z = 1,$ $-3x + 8y + 5z = 6$ by LU-decomposition method.	L3	CO1	PO3

UNIT-II

Eigen values-Eigen vectors and Quadratic forms

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Define model and spectral matrices.	L1	CO2	PO1
2	Find the sum and product of the Eigen values of $A = \begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$	L2	CO2	PO1
3	Using Cayley Hamilton theorem find A^8, if A=$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.	L2	CO2	PO2
4	Find the Eigen values of A^{-1} where A= $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$	L2	CO2	PO1
5	Find the symmetric matrix corresponding to the quadratic form $x^2 + 6xy + 5y^2$.	L1	CO2	PO2
6	Find the characteristic roots of the matrix A=$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$	L2	CO2	PO1

7	Compute the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$	L2	CO2	PO1
8	Prove that zero is eigen value of a matrix iff it is singular.	L2	CO2	PO1
9	Find the Eigen values of $3A^3 + 5A^2 - 6A + 2I$ for the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$	L2	CO2	PO2
10	State Cayley – Hamilton theorem.	L1	CO2	PO1

S.NO	**Part-B (Long answer questions)**	BT	CO	PO
1	Find the Eigen values and Eigen vectors of a Matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$	L3	CO2	PO2
2(a)	Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation hence find A^{-1}.	L2	CO2	PO2
2(b)	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$	L3	CO2	PO1
3(a)	Find the Eigen values and eigen vector of the hermitian matrix $\begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$	L2	CO2	PO1
3(b)	Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence find A^4 .	L3	CO2	PO1
4	Diagonalize the matrix$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$	L3	CO2	PO3
5	Reduce the Quadratic form $3{x_1}^2 + 3{x_2}^2 + 3{x_3}^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into Canonical form and hence state nature, rank, index and signature of the Quadratic form.	L4	CO2	PO3
6	Diagonalize the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by Orthogonal Reduction.	L4	CO2	PO3

UNIT-III
Fourier series

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	If f(x) = x , $-\pi<x<\pi$, find b_n .	L2	CO3	PO1
2	Define Fourier series of a function f(x) in the interval $(c,c+2\pi)$.	L1	CO3	PO1
3	Define Fourier series for even and odd functions.	L1	CO3	PO1
4	If f(x)= $\left(\frac{\pi-x}{2}\right)^2$, $0<x<2\pi$ find a_0 value.	L2	CO3	PO2
5	If f(x)=x^3 , $0<x<\pi$, find a_0 value .	L2	CO3	PO1
6	Find a_1 , if f(x)= x $\sin x$ where $0<x<2\pi$	L2	CO3	PO1
7	If f(x)=$\begin{cases} 1, 0<x<\pi \\ 0, \pi<x<2\pi \end{cases}$, find a_0 value .	L2	CO3	PO1
8	Define half rage Fourier series.	L1	CO3	PO1
9	If f(x)= $\pi - x$ in $[0,\pi]$, find a_0 value .	L2	CO3	PO1
10	Define periodic function and give suitable examples.	L2	CO3	PO1

S.NO	**Part-B(Long answer questions)**	BT	CO	PO
1	Find the Fourier Series of the period 2π for the function f(x)= $x^2 - x$ in $(-\pi , \pi)$ and Hence deduce the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-$ ……….= $\frac{\pi^2}{12}$.	L4	CO3	PO2
2	Find the Fourier expansion of f(x) = x $\cos x$, $0<x<2\pi$	L3	CO3	PO2
3	Find the Fourier series to represent the function is given by $f(x) = \begin{cases} 0, \ for \ -\pi<x<0 \\ x^2, \ for \ 0<x<\pi \end{cases}$	L3	CO3	PO2
4	Find the Fourier series to represent the function f(x) = $\lvert sinx \rvert$, $-\pi<x<\pi$.	L3	CO3	PO2
5	Find the half range cosine series for the function f(x)= x in the range , $0<x<\pi$ and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..=\frac{\pi^2}{8}$.	L4	CO3	PO2
6	Find the half range sine series for the function f(x)= $x(\pi - x)$ in $0<x<\pi$. Hence deduce that $\frac{1}{1^2}-\frac{1}{3^2}+\frac{1}{5^2}-$ ……….= $\frac{\pi^2}{32}$.	L4	CO3	PO2

UNIT-IV

Calculus (Mean value theorems and Beta & Gamma functions)

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	Verify Rolle's theorem for f(x)= $2x^3+x^2-4x-2$ in $[-\sqrt{3},\sqrt{3}]$.	L2	CO4	PO1
2	Verify Lagrange's mean value theorem for f(x) =$\log_e x$ in [1,e] .	L2	CO4	PO2
3	Define beta and gamma functions.	L1	CO4	PO1
4	Find the value of $\Gamma(\frac{1}{2})$	L2	CO4	PO1
5	Evaluate $\int_0^1 x^5 (1-x)^3 \, dx$	L1	CO4	PO1
6	Find c of Cauchys mean value theorem for f(x)=$\sqrt{x}$ and g(x)=$\frac{1}{\sqrt{x}}$ in [a,b] where 0<a<b.	L2	CO4	PO2
7	Using Rolles theorem show that g(x)= $8x^3-6x^2-2x+1$ has a zero between 0 and 1 .	L1	CO4	PO2
8	Prove that $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx = \frac{1}{5}B(\frac{2}{5},\frac{1}{2})$	L1	CO4	PO2
9	Find the value of $\Gamma(\frac{5}{2})$	L1	CO4	PO2
10	Compute $\int_0^\infty e^{-x} x^3 \, dx$	L1	CO4	PO2

S.NO	**Part-B(Long answer questions)**	BT	CO	PO
1(a)	Verify Rolle's theorem for $f(x) = (x-a)^m(x-b)^n$ where m, n are positive integers in [a , b].	L3	CO4	PO2
1(b)	Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}(\frac{3}{5}) > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.	L3	CO4	PO2
2(a)	Verify generalized mean value theorem for f(x) = e^x , g(x)= e^{-x} in [3,7] and find the value of c.	L3	CO4	PO2
2(b)	Prove that β (m,n) =$\beta(m+1,n) + \beta(m,n+1)$.	L3	CO4	PO3
3(a)	Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$ in terms of Beta function.	L3	CO4	PO2
3(b)	Evaluate $\int_0^1 x^7 (1-x)^5 \, dx$ by using β-Γ functions.	L2	CO4	PO2
4(a)	Evaluate $\int_0^{\frac{\pi}{2}} Sin^6\theta \; Cos^7\theta \, d\theta$ using β-Γ functions .	L2	CO4	PO2
4(b)	Show that $\Gamma(n) = \int_0^1 (log\frac{1}{x})^{n-1} \, dx, \; n > 0$.	L2	CO4	PO2
5	Establish the relation between Beta and Gamma functions .	L3	CO4	PO2

6(a)	Show that $4\int_0^{\infty} \frac{x^2}{1+x^4}\, dx = \sqrt{2}\,\pi$.	L4	CO4	PO2
6(b)	Evaluate $\int_0^1 x^3\sqrt{(1-x)}\, dx$ using β-Γ functions .	L4	CO4	PO2

UNIT-V

Multi variable Calculus (Partial Differentiation and Applications)

S.NO	Questions	BT	CO	PO
	Part – A(Short answer questions)			
1	State Euler's theorem for homogeneous function in x and y.	L1	CO5	PO1
2	Determine whether the functions u = e^{x} siny , v = e^{x} cosy are functionally dependent or not.	L2	CO5	PO2
3	If x=u(1+v) , y=v(1+u) then prove that $\frac{\partial(x,y)}{\partial(u,v)}$ = 1+u+v .	L2	CO5	PO2
4	Write the working rule to find the maximum and minimum values of f(x,y) .	L2	CO5	PO1
5	Verify $\frac{\partial^2 u}{\partial x\partial y} = \frac{\partial^2 u}{\partial y\partial x}$ for the function u = $\tan^{-1}\frac{x}{y}$.	L2	CO5	PO2
6	Find the first and second order partial derivatives of x^3+y^3-3axy and verify $\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$	L2	CO5	PO2
7	Verify Euler's theorem for the function xy+yz+zx .	L1	CO5	PO2
8	If u=x^2-2y , v= x+y+z , w=x-2y+3z find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	L1	CO5	PO2
9	Verify if u= 2x-y+3z , v= 2x-y-z ,w=2x-y+z are functionally dependent and if ,so find the relation between them.	L2	CO5	PO2
10	Find the maximum an minimum values of f(x,y)=$x^3+3xy^2-3x^2-3y^2+4$	L2	CO5	PO2

S.NO	**Part-B(Long answer questions)**	BT	CO	PO
1(a)	If z= $\log(e^x + e^y)$ $show\ that\ rt - s^2$=0 , where r=$\frac{\partial^2 z}{\partial x^2}$, t=$\frac{\partial^2 z}{\partial y^2}$,s=$\frac{\partial^2 z}{\partial x\, \partial y}$	L3	CO5	PO2
1(b)	If $\sin u = \frac{x^2y^2}{x^2+y^2}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = Sin2u.$	L3	CO5	PO2

2(a)	If u=x+y+z , y+z= uv , z=uvw show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ = u^2 v.	L3	CO5	PO2
2(b)	If u = $x^2 - y^2$, v= 2xy where x=r cos θ ,y = r sinθ show that $\frac{\partial(u,v)}{\partial(r,\theta)}$ = $4r^3$.	L2	CO5	PO2
3(a)	If x= $\frac{u^2}{v}$, y = $\frac{v^2}{u}$ find $\frac{\partial(u,v)}{\partial(x,y)}$.	L2	CO5	PO2
3(b)	If x= u$\sqrt{(1-v^2)}$+v$\sqrt{(1-u^2)}$ and y= $\sin^{-1} u + \sin^{-1} v$ then show that x and y are functionally related , also find the relationship.	L3	CO5	PO3
4	Find the maximum and minimum values of the function $f(x,y) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$.	L4	CO5	PO1
5(a)	Find the maximum and minimum values of the function f(x, y)= $x^3y^2(1-x-y)$.	L4	CO5	PO2
5(b)	Find the maximum and minimum values of x+y+z subject to $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$ by Lagrange's method of undetermined multipliers.	L3	CO5	PO3
6	Find the dimensions of the rectangular parallelepiped box open at top of max capacity whose surface area is 256 sq. inches.	L4	CO5	PO3

*Blooms Taxonomy Level(BT) (L1-Remembering; L2- Understanding:L3-Applying;L4-Analyzing;L5-Evaluating;L6-Creating)

Course Outcomes(CO)

Program Outcomes(PO)

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