

Power Electronics

UNIT- I

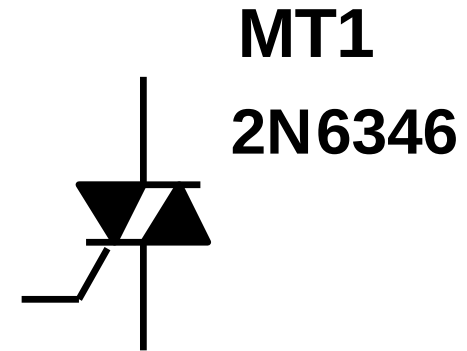
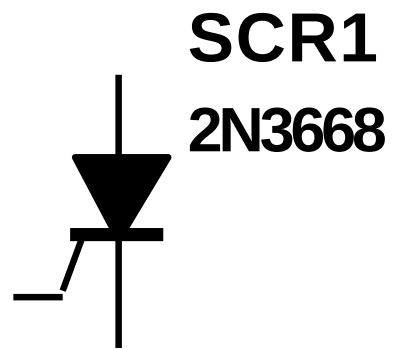
Power Switching Devices:

Concept of power electronics, scope and applications, types of power converters; Power semiconductor switches and their V-I characteristics - Power Diodes, Power BJT, SCR, Power MOSFET, Power IGBT; Thyristor ratings and protection, Series and parallel connections of SCRs, Two transistor analogy of SCR, methods of SCR commutation, UJT as a trigger source, gate drive circuits for BJT and MOSFETs.

Power Switching Devices

- Diodes
- Transistors
 - Power BJTs
 - Power MOSFETs
 - Insulated-Gate BJT
 - IGBT
 - Static Induction Transistors
 - SITs
- Thyristors
 - Force-Commutated
 - Line-Commutated
 - Gate Turn Off--GTO
 - RCT
 - Reverse-Conducting
 - GATI

Thyristor/Triac

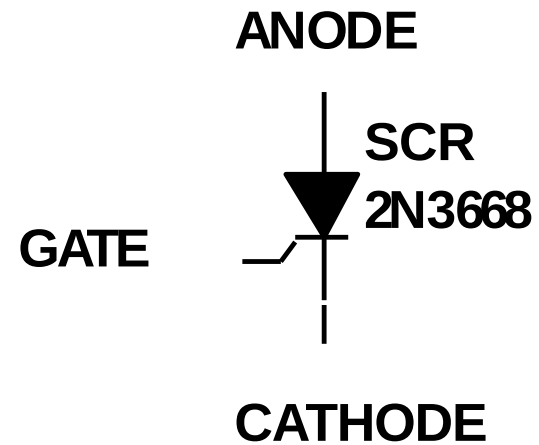


Power Electronic Circuits

- Diode Rectifiers (AC to Fixed DC)
- AC-DC Converters (Controlled Rectifiers)
- AC-AC Converters (AC Voltage Controllers)
- DC-DC Converters (DC Choppers)
- DC-AC Converters (Inverters)
- Static Switches

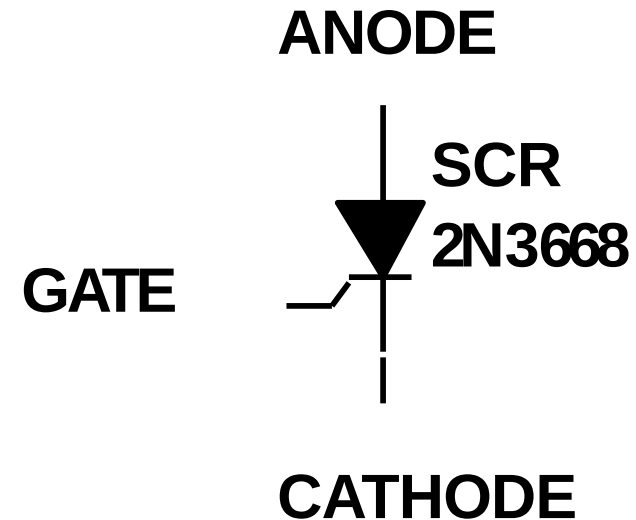
SCR / Thyristor

- Circuit Symbol and Terminal Identification



SCR / Thyristor

- Anode and Cathode terminals as conventional pn junction diode
- Gate terminal for a controlling input signal

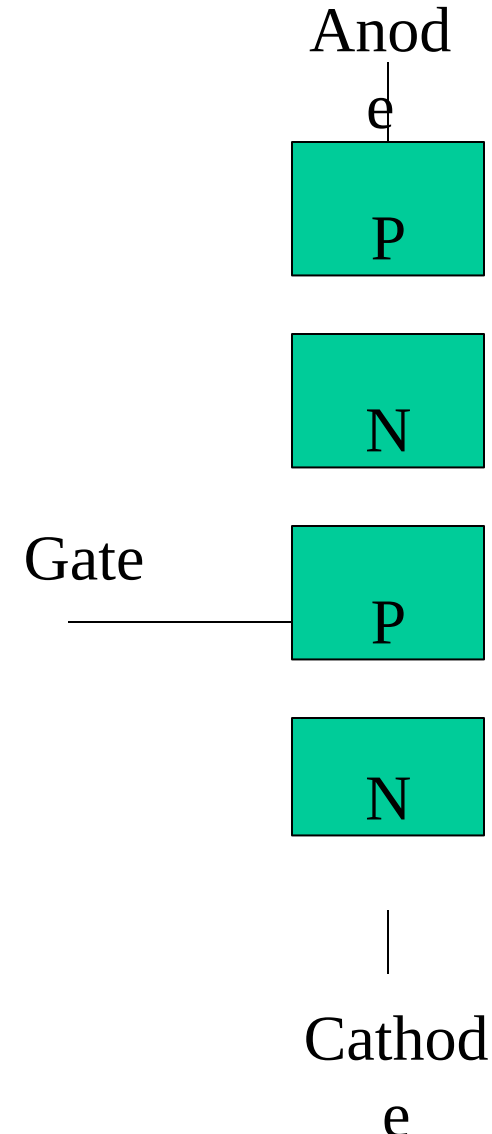


SCR/ Thyristor

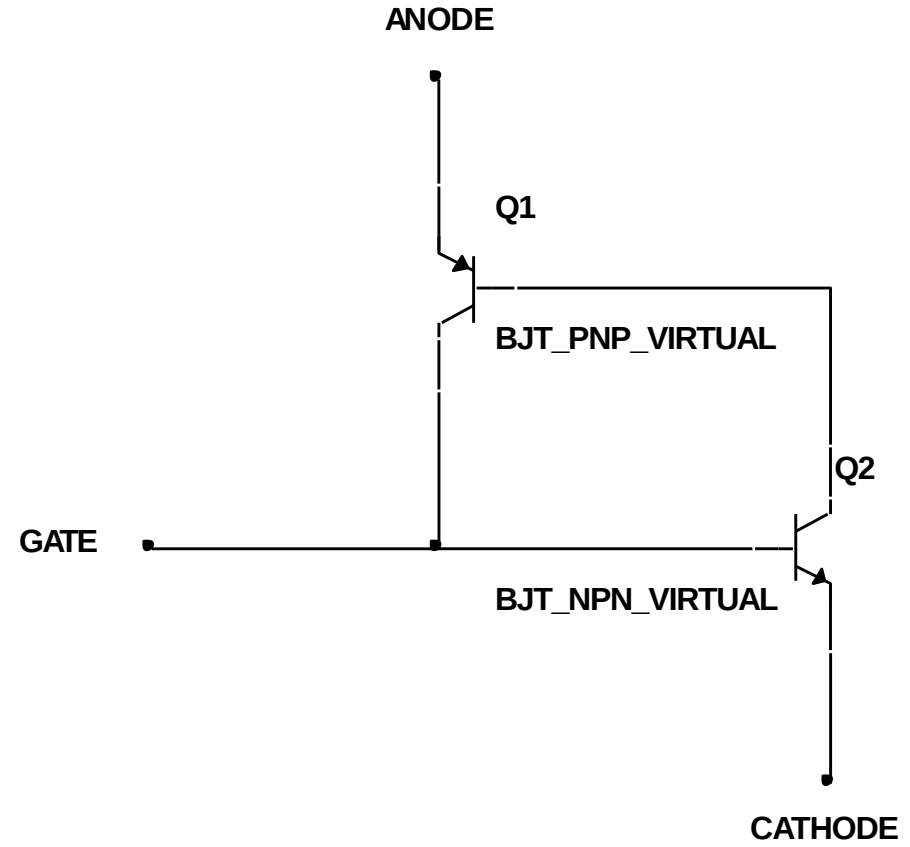
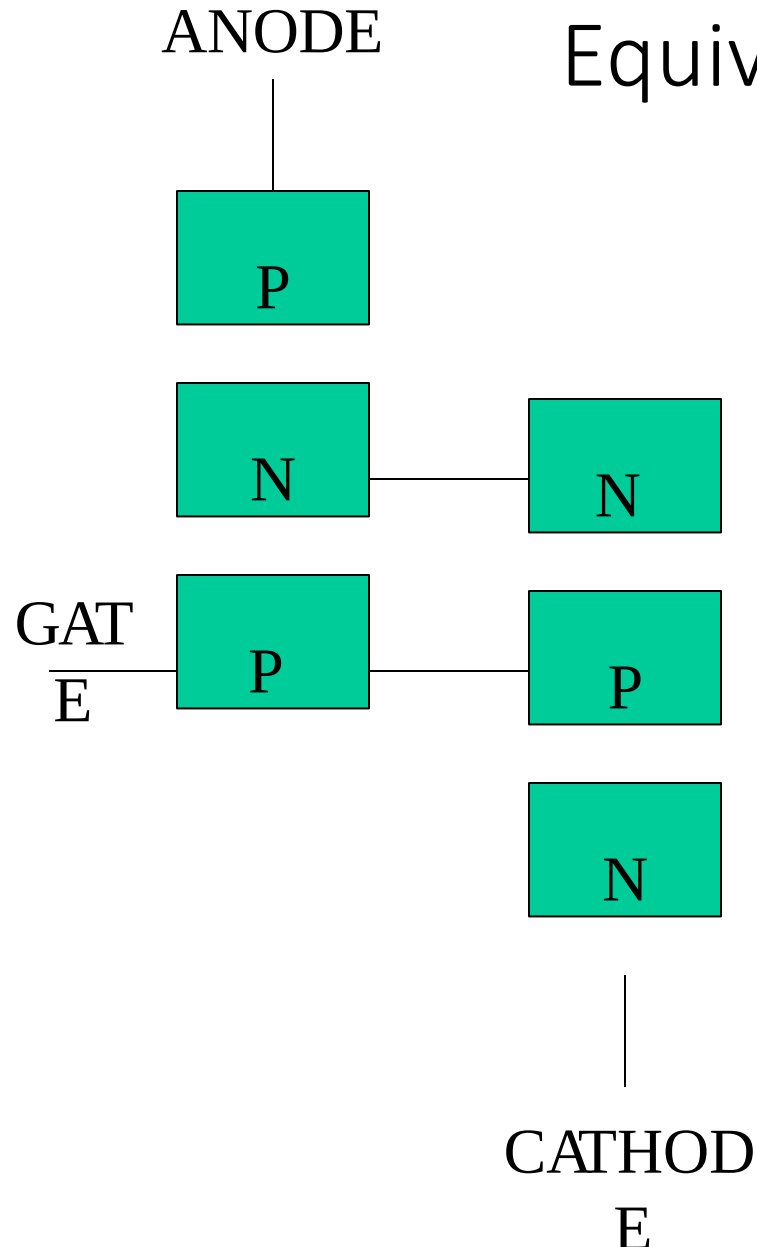
- An SCR (Thyristor) is a “controlled” rectifier (diode)
- Control the conduction under forward bias by applying a current into the Gate terminal
- Under reverse bias, looks like conventional pn junction diode

SCR / Thyristor

- 4-layer (pnpn) device
- Anode, Cathode as for a conventional pn junction diode
- Cathode Gate brought out for controlling input



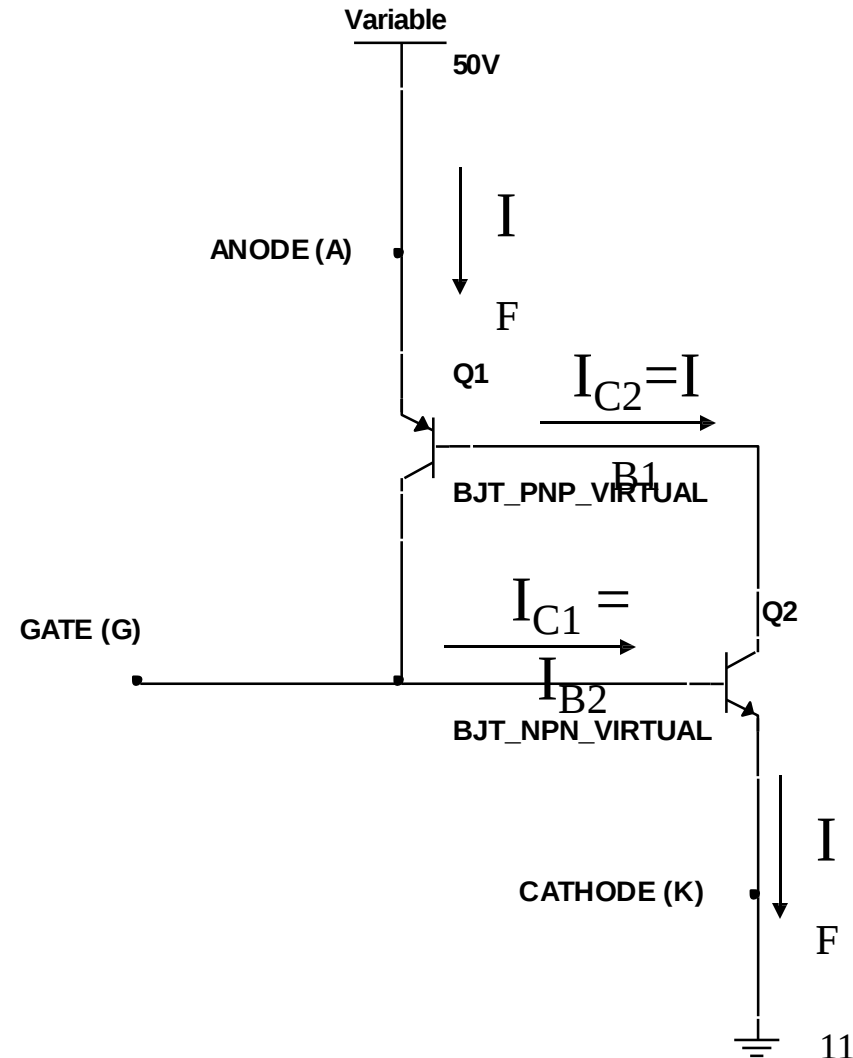
Equivalent Circuit



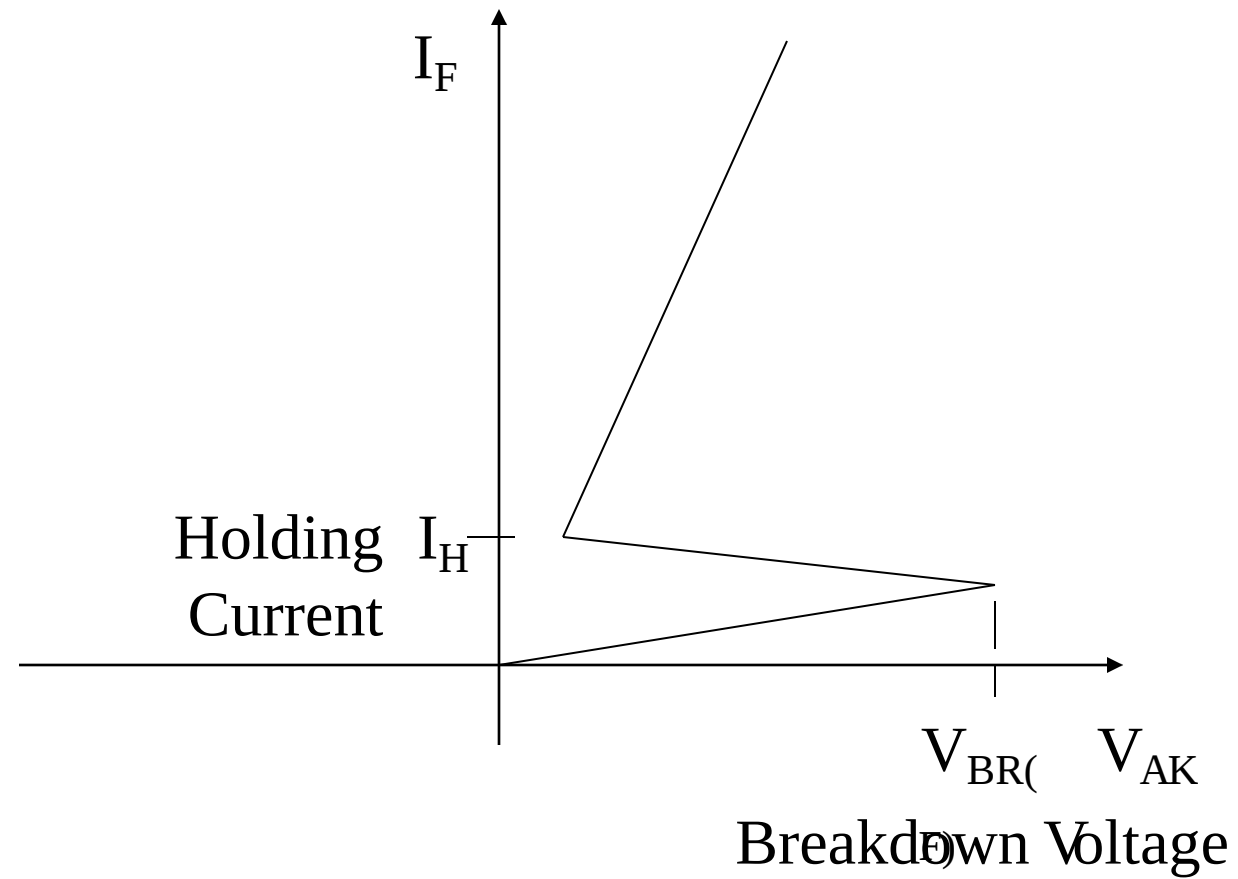
Apply Biasing

With the Gate terminal OPEN, both transistors are OFF. As the applied voltage increases, there will be a “breakdown” that causes both transistors to conduct (saturate) making $I_F > 0$ and $V_{AK} = 0$.

$$V_{\text{Breakdown}} = V_{\text{BR(F)}}$$



Volt-Ampere Characteristic



Apply a Gate Current

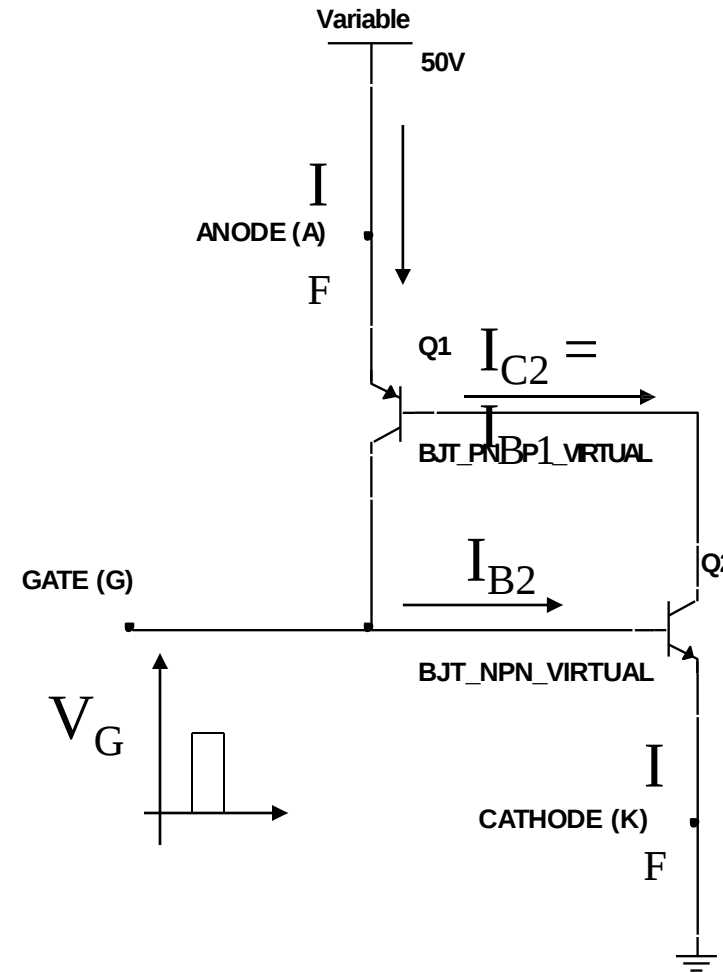
For $0 < V_{AK} < V_{BR(F)}$,

Turn Q_2 ON by applying a current into the Gate

This causes Q_1 to turn ON, and eventually both transistors SATURATE

$$V_{AK} = V_{CEsat} + V_{BEsat}$$

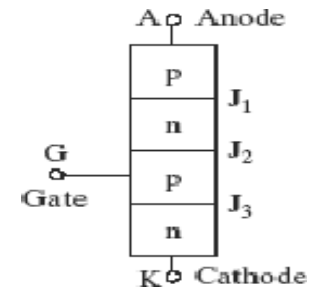
If the Gate pulse is removed, Q_1 and Q_2 still stay ON!



How do you turn it OFF?

- Cause the forward current to fall below the value of the “holding” current, I_H
- Reverse bias the device

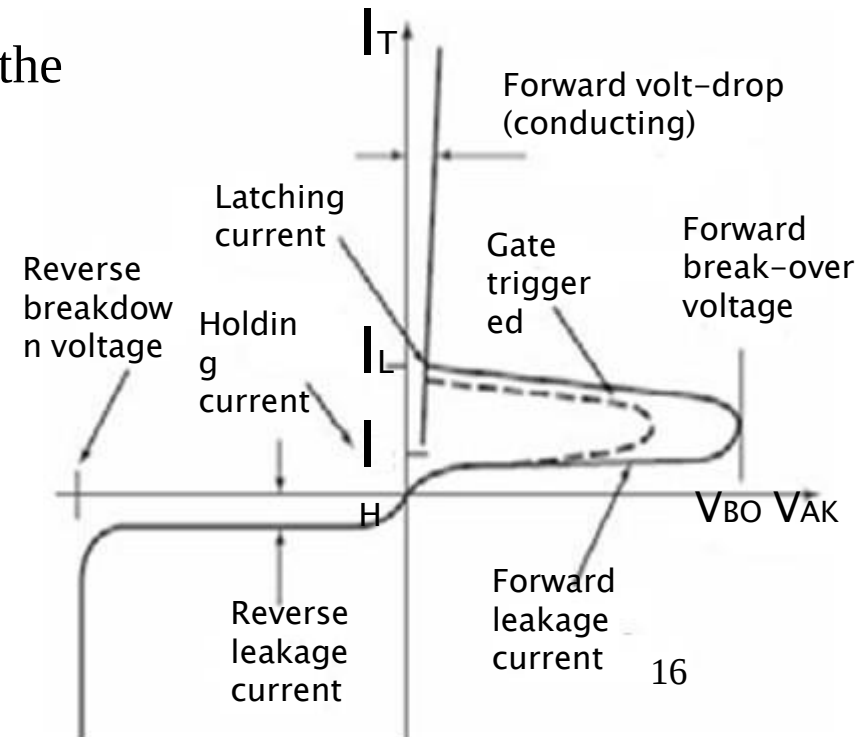
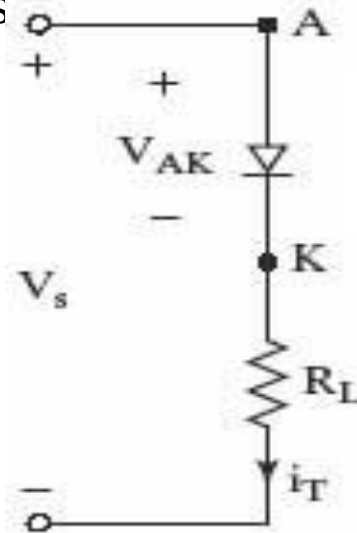
Characteristics of thyristors



- When the anode is at a positive potential V_{AK} with respect to the cathode with no voltage applied at the gate, junctions J_1 and J_3 are forward biased, while junction J_2 is reverse biased. As J_2 is reverse biased, no conduction takes place.
- Now if V_{AK} is increased beyond the breakdown voltage V_{BO} of the thyristor, avalanche breakdown of J_2 takes place and the thyristor starts conducting.
- If a positive potential V_G is applied at the gate terminal with respect to the cathode, the breakdown of the junction J_2 occurs at a lower value of V_{AK} . By selecting an appropriate value of V_G , the thyristor can be switched into the on state suddenly.

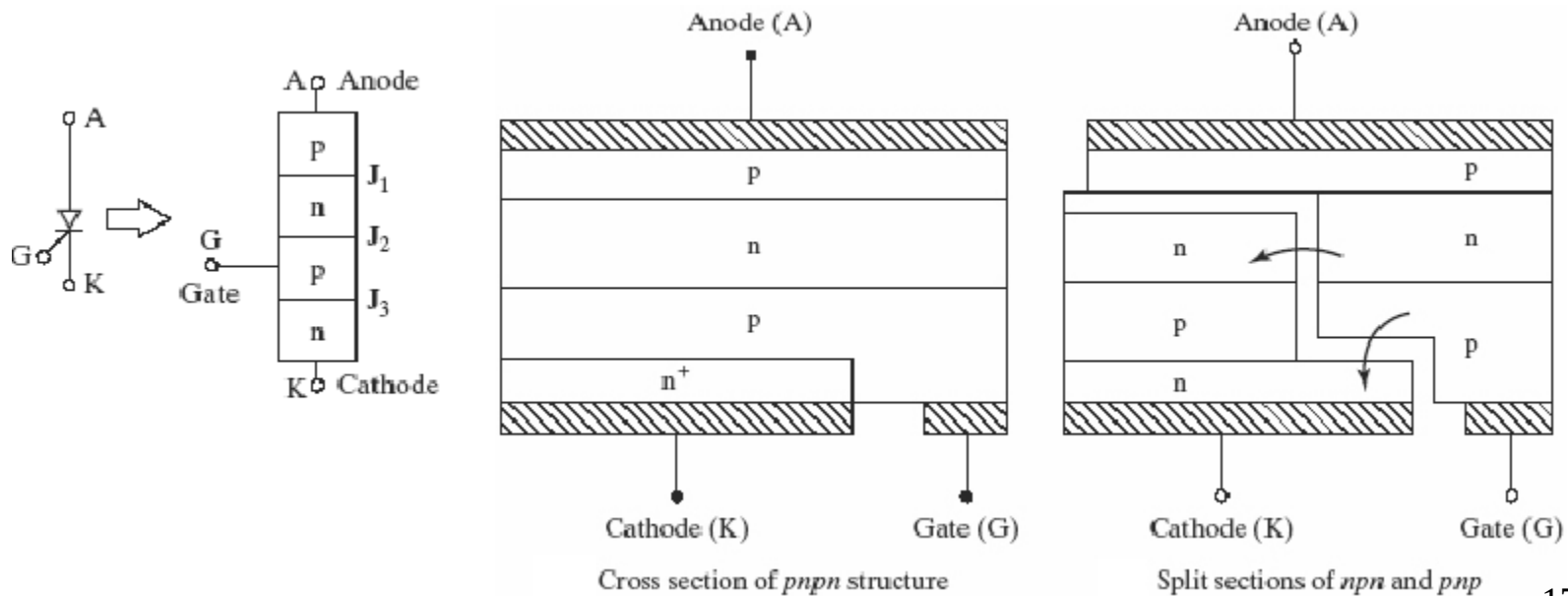
Switching Characteristic (IV)

- Forward breakdown voltage V_{BO}
 - The voltage of avalanche breakdown
- Latching current I_L
 - The minimum anode current required to maintain the thyristor in the on-state immediately after it is turned on and the gate signal has been removed
- Holding current I_H
 - The minimum anode current to maintain the thyristor in the on-state
- $I_L > I_H$



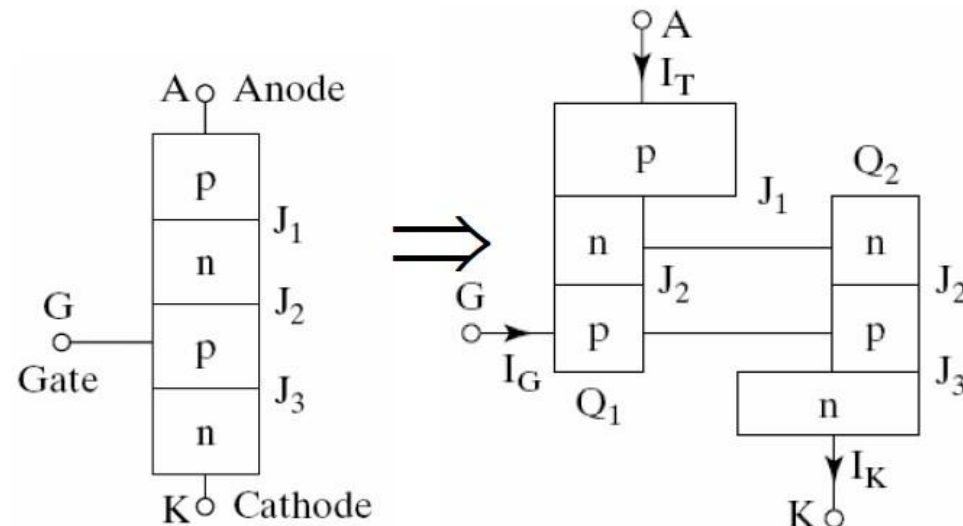
Symbol and construction

The thyristor is a four-layer, three terminal semiconducting device, with each layer consisting of alternately N-type or P-type material, for example P-N-P-N. The main terminals, labeled anode and cathode, are across the full four layers, and the control terminal, called the gate, is attached to p-type material near the cathode.



Different types of Thyristors

- Silicon Controlled Rectifier (SCR).
- TRIAC.
- DIAC.
- Silicon Unilateral
in low voltage av



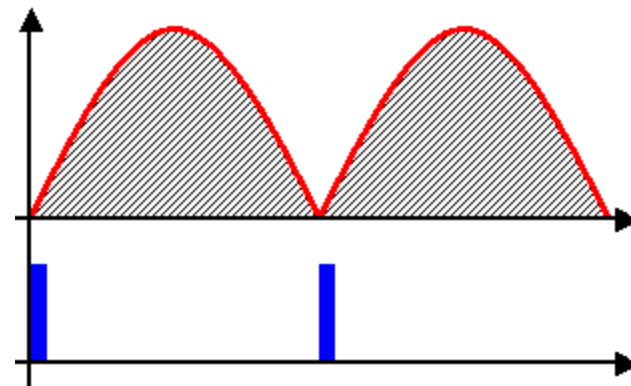
Construction of 18

Application

- Mainly used where high currents and voltages are involved, and are often used to control alternating currents, where the change of polarity of the current causes the device to switch off automatically; referred to as Zero Cross operation.
- Thyristors can be used as the control elements for phase angle triggered controllers, also known as phase fired controllers.

Cntd...

- In power supplies for digital circuits, thyristor can be used as a sort of "circuit breaker" or "crowbar" to prevent a failure in the power supply from damaging downstream components, by shorting the power supply output to ground



Load voltage regulated by thyristor phase control.

Red trace: load voltage

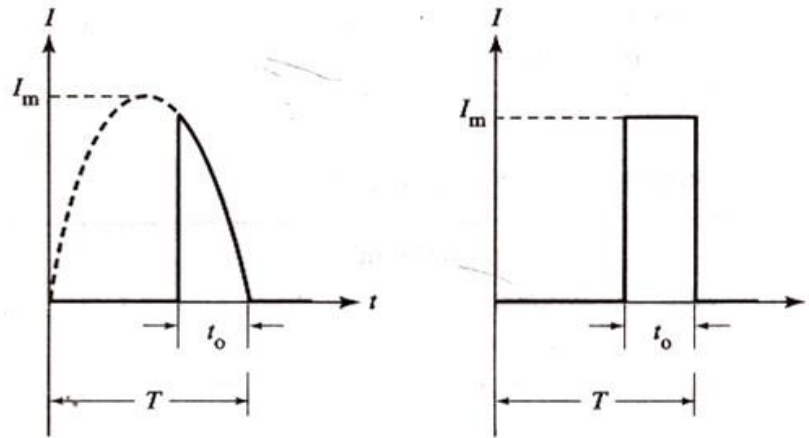
Blue trace: trigger signal.

SCR Ratings

(a) SCR Current Ratings

1- Maximum Repetitive RMS current Rating

- Average on-state current is the maximum average current value that can be carried by the SCR in its on state.
- RMS value of nonsinusoidal waveform is simplified by approximating it by rectangular waveform.
- This approximation give higher RMS value, but leaves slight safety factor.



After approximating, the RMS value of the current can be found from

$$I_{\text{RMS}} = \sqrt{\frac{I_{\text{m}}^2 t_{\text{o}}}{T}}$$

- Average value of pulse is

$$I_{\text{AVE}} = \frac{I_{\text{m}} t_{\text{o}}}{T}$$

- Form factor is

$$f_{\text{o}} = \frac{I_{\text{RMS}}}{I_{\text{AVE}}}$$

- Knowing the form factor for given waveform, RMS current can be obtained from

$$I_{\text{RMS}} = fo(I_{\text{AVE}})$$

- Maximum repetitive RMS current is given by

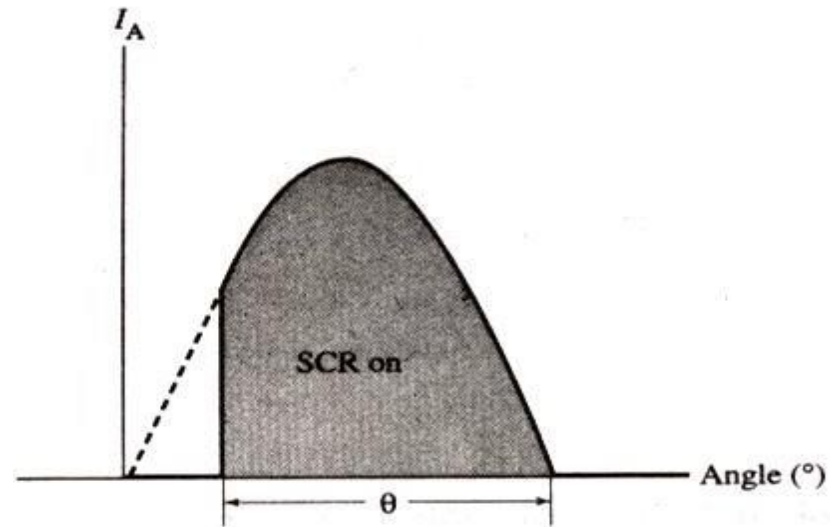
$$I_{\text{T(RMS)}} = fo(I_{\text{T(AVE)}})$$

- Conduction angle verses form factor

Conduction angle (θ)	Form factor (fo)
20°	5.0
40°	3.5
60°	2.7
80°	2.3
100°	2.0
120°	1.8
140°	1.6
160°	1.4
180°	1.3

Conduction Angle

- *Duration for which SCR is on.* It is measured as shown



2- Surge Current Rating

Peak anode current that SCR can handle for brief duration.

3 Latching current

Minimum anode current that must flow through the SCR in order for it to stay on initially after gate signal is removed.

4 Holding Current

Minimum value of anode current, required to maintain SCR in conducting state.

(b) SCR Voltage Ratings

1 Peak repetitive forward blocking voltage

Maximum instantaneous voltage that SCR can block in forward direction.

2 Peak Repetitive Reverse Voltage

Maximum instantaneous voltage that SCR can withstand, without breakdown, in reverse direction.

3 Non-repetitive peak reverse voltage

Maximum transient reverse voltage that SCR can withstand.

(c) SCR Rate-of-Change Ratings

1 (di/dt rating)

Critical rate of rise of on-state current. It is the rate at which anode current increases and must be less than rate at which conduction area increases.

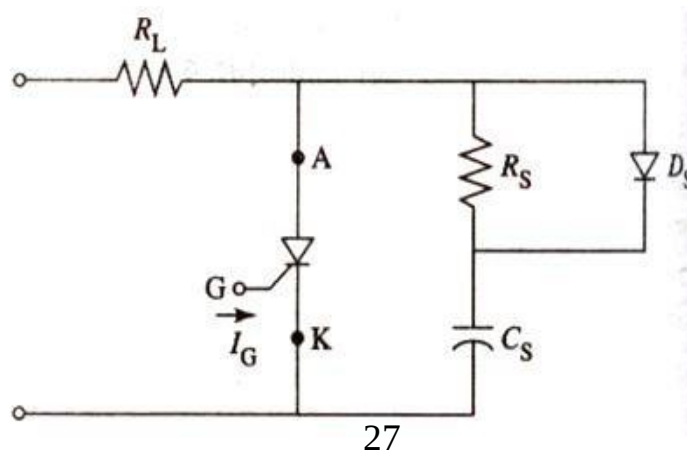
To prevent damage to SCR by high di/dt value, small inductance is added in series with device.

Value of required inductance is

$$L \geq \frac{V_p}{(di/dt)_{\max}}$$

2 dv/dt rating

Maximum rise time of a voltage pulse that can be applied to the SCR in the off state without causing it to fire. Unscheduled firing due to high value of dv/dt can be prevented by using RC snubber circuit.



(d) Gate Parameters

1 Maximum Gate Peak Inverse Voltage

Maximum value of negative DC voltage that can be applied without damaging the gate-cathode junction.

2 Maximum Gate Trigger Current

Maximum DC gate current allowed to turn on the device.

3 Maximum gate trigger voltage

DC voltage necessary to produce maximum gate trigger current.

4 Maximum Gate Power Dissipation

Maximum instantaneous product of gate current and gate voltage that can exist during forward-bias.

5 Minimum gate trigger voltage

Minimum DC gate-to-cathode voltage required to trigger the SCR.

6 Minimum gate trigger current

Minimum DC gate current necessary to turn SCR on.

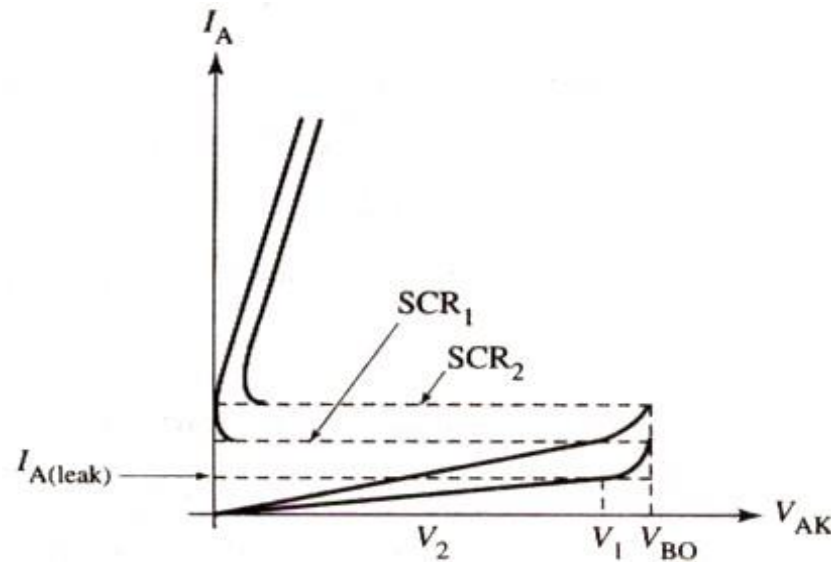
Series and Parallel SCR **Connections**

SCRs are connected in series and parallel to extend voltage and current ratings.

For high-voltage, high-current applications, series-parallel combinations of SCRs are used.

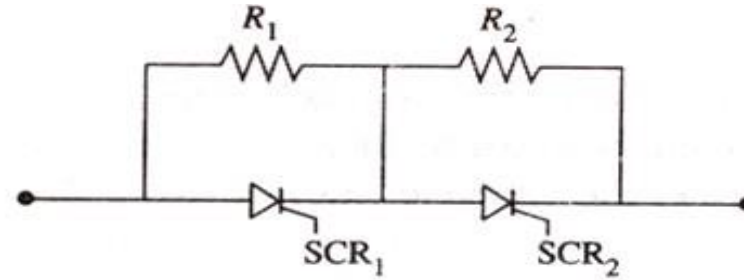
SCRs in Series

- Unequal distribution of voltage across two series SCRs.

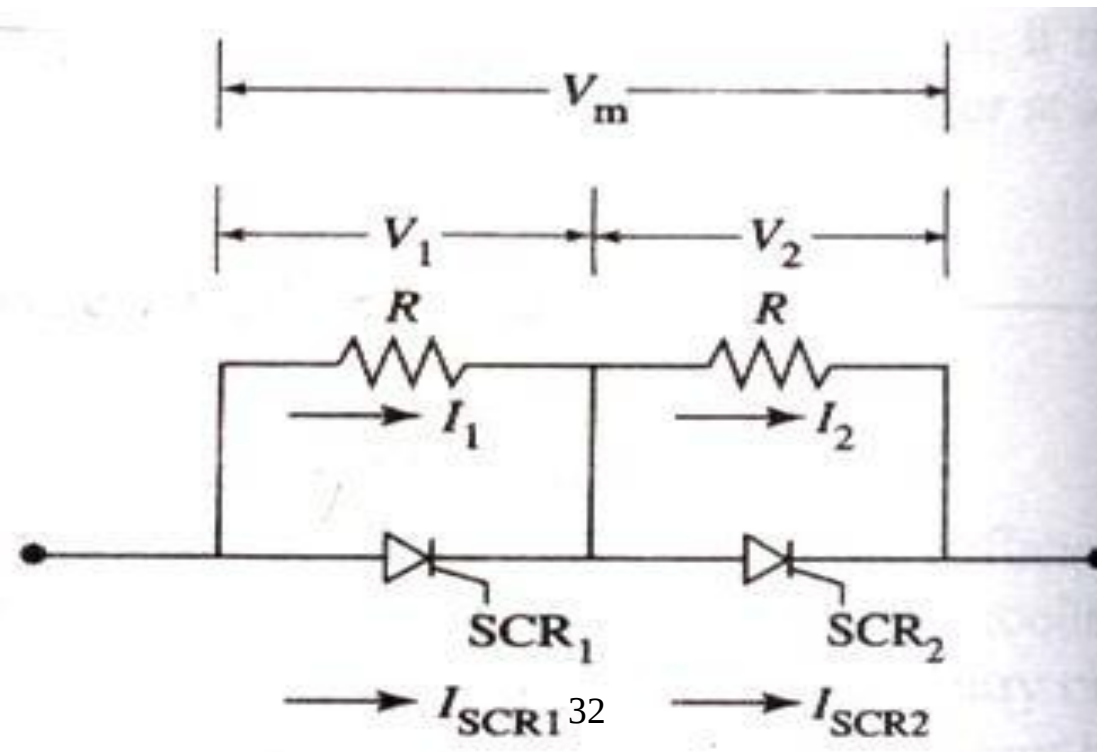


- Two SCRs do not share the same supply voltage. Maximum voltage that SCRs can block is $V_1 + V_2$, not $2V_{BO}$.

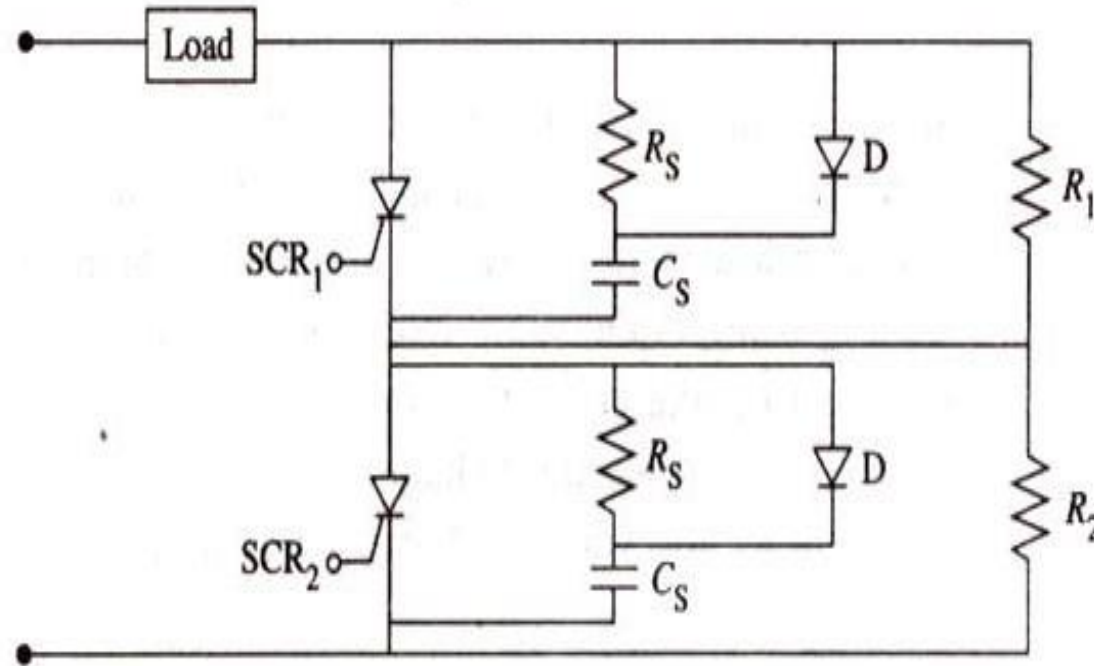
- **Resistance equalization**



- **Voltage equalization**

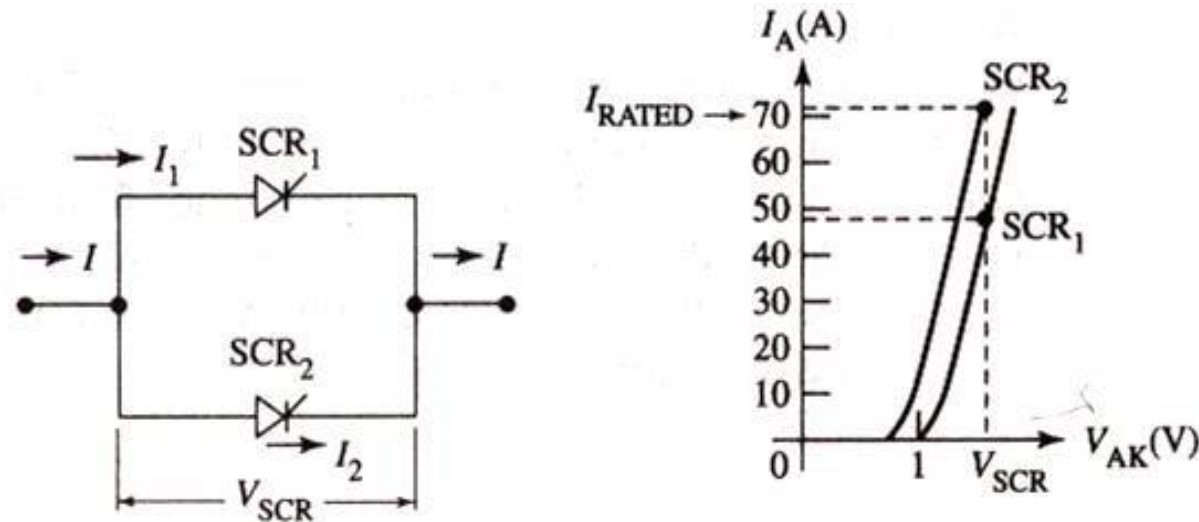


- RC equalization for SCRs connected in series.



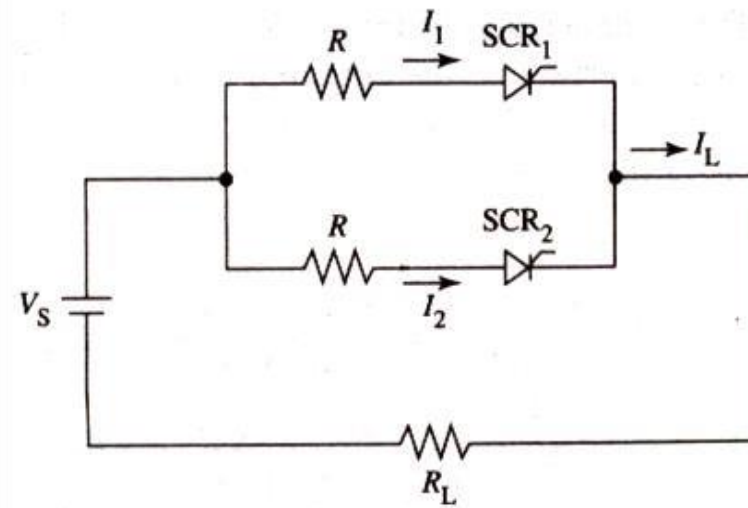
SCRs In Parallel

- Unequal current sharing between two SCRs is shown:



- Total rated current of parallel connection is $I_1 + I_2$, not $2I_2$.

- With unmatched SCRs, equal current sharing is achieved by adding low value resistor or inductor in series with each SCR, as shown below.



- Value of resistance

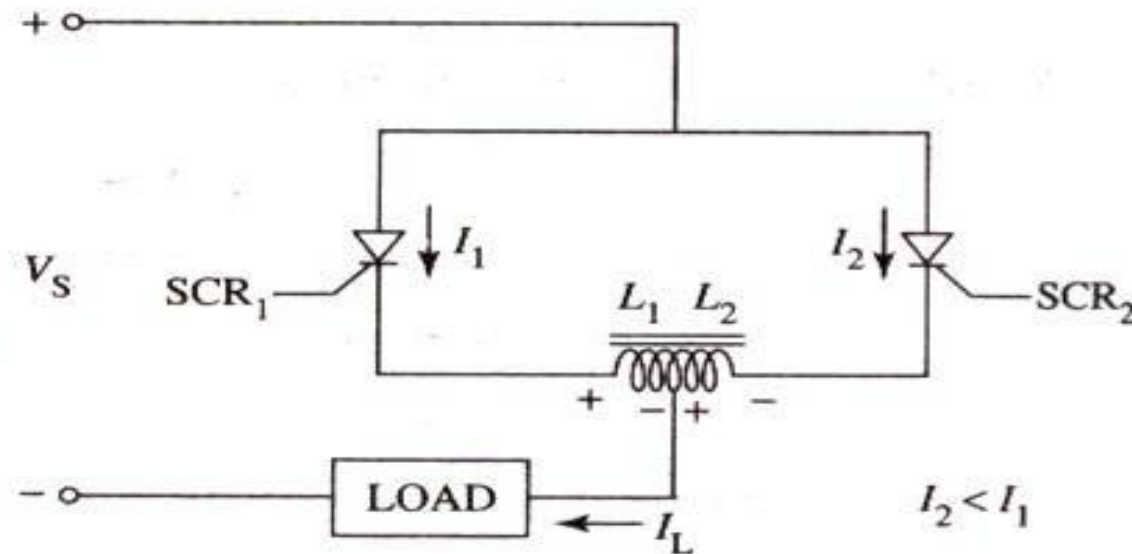
$$R = \frac{V_1 - V_2}{I_2 - I_1}$$

○ Current sharing in SCRs with parallel reactors

Equalization using resistors is inefficient due to

- ❖ *Extra power loss*
- ❖ *Noncompensation for unequal SCR turn-on and turn-off times.*
- ❖ *Damage due to overloading*

SCRs with center-tapped reactors is shown below.



SCR Gate-Triggering Circuits

○ Triggering circuits provide firing signal to turn on the SCR at precisely the correct time.

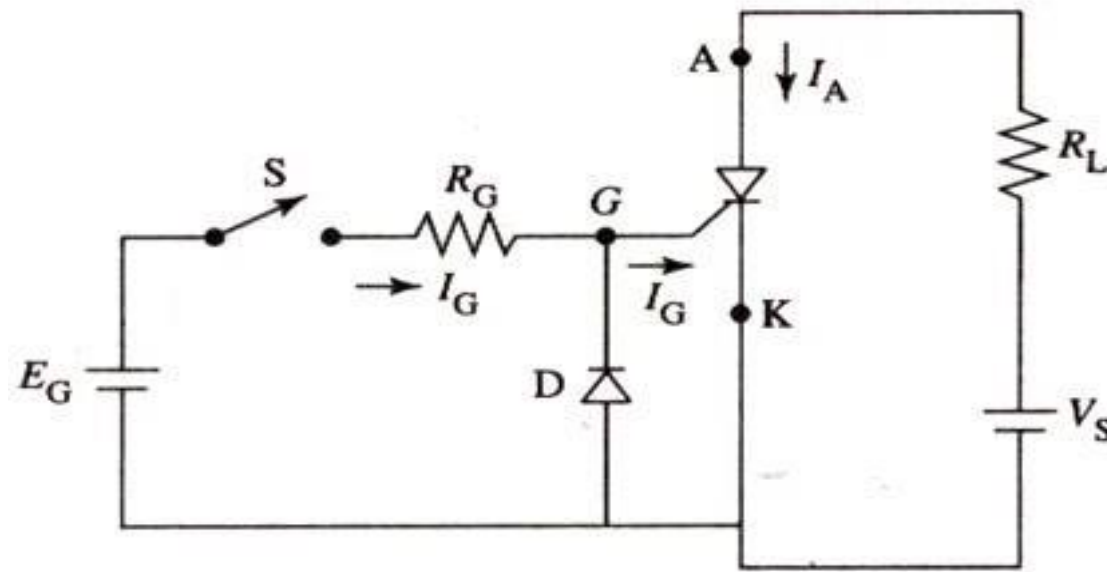
○ Firing circuits must have following properties

1. Produce gate signal of suitable magnitude and sufficiently short rise time.
2. Produce gate signal of adequate duration.
3. Provide accurate firing control over the required range.
4. Ensure that triggering does not occur from false signals or noise
5. In AC applications, ensure that the gate signal is applied when the SCR is forward-biased
6. In three-phase circuits, provide gate pulses that are 120° apart with respect to the reference point
7. Ensure simultaneous triggering of SCRs connected in series or in parallel.

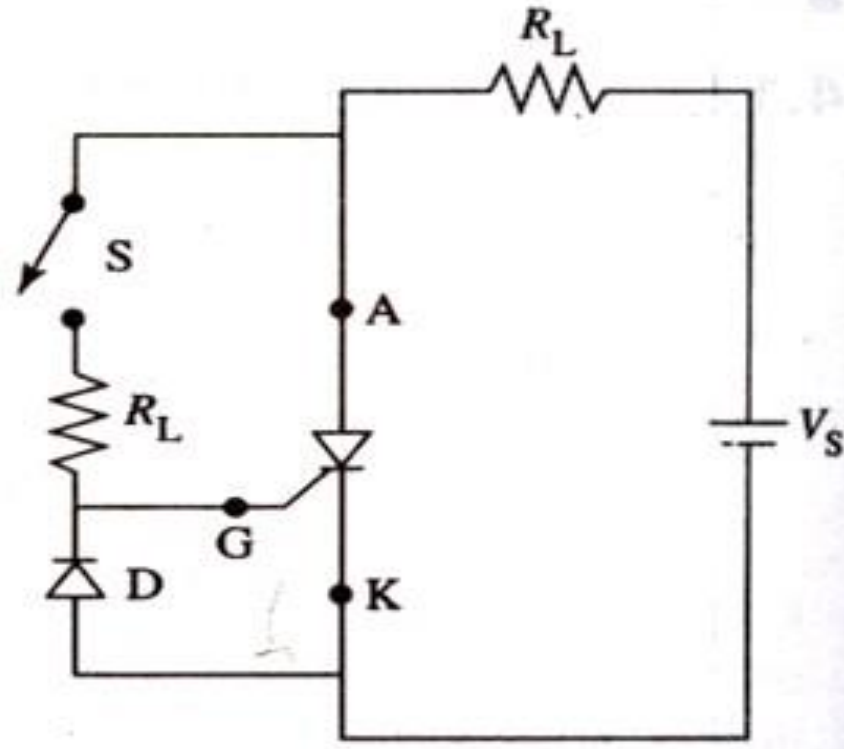
Types Of Gate Firing Signals

1. DC signals
2. Pulse signals
3. AC signals

(a) DC Gating Signal From Separate Source



DC Gating signals from Same Source



Disadvantage of DC gating Signals

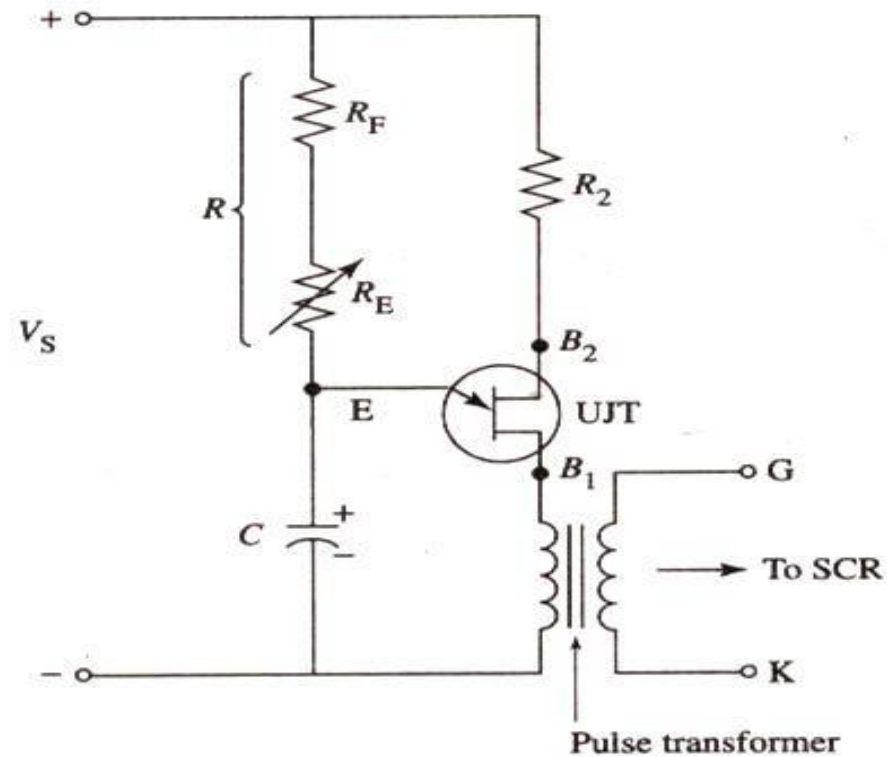
1. Constant DC gate signal causes gate power dissipation
2. DC gate signals are not used for firing SCRs in AC applications, because presence of positive gate signal during negative half cycle would increase the reverse anode current and possibly destroy the device.

(2) Pulse Signals

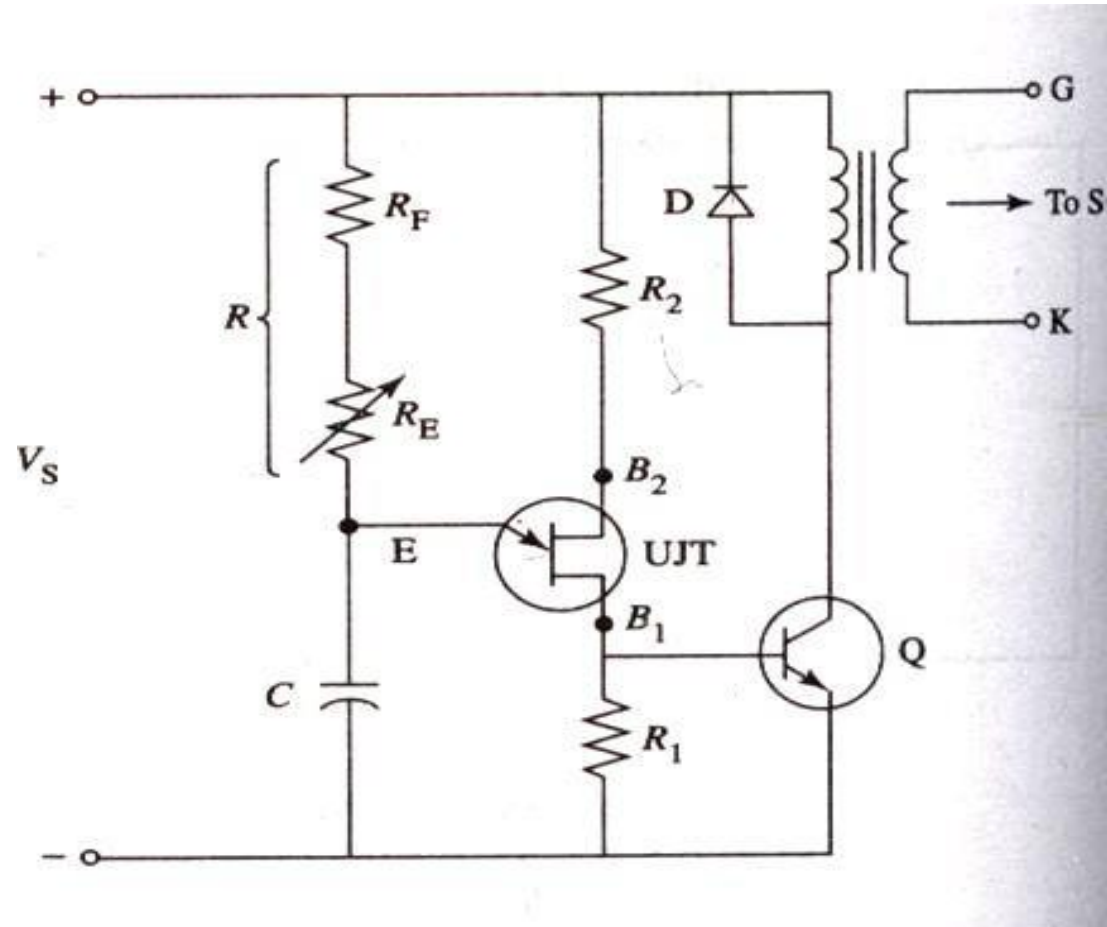
1. Instead of continuous DC signal, single pulse or train of pulses is generated.
2. It provides precise control of point at which SCR is fired.
3. It provides electrical isolation between SCR and gate-trigger circuit.

SCR trigger circuits using UJT oscillator

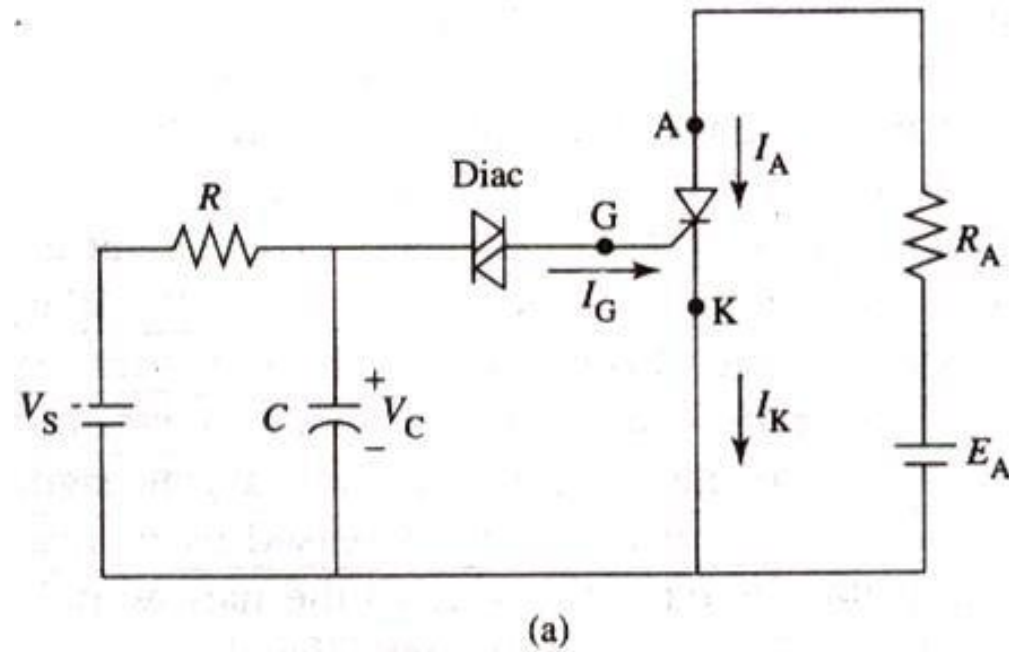
○ Circuit A



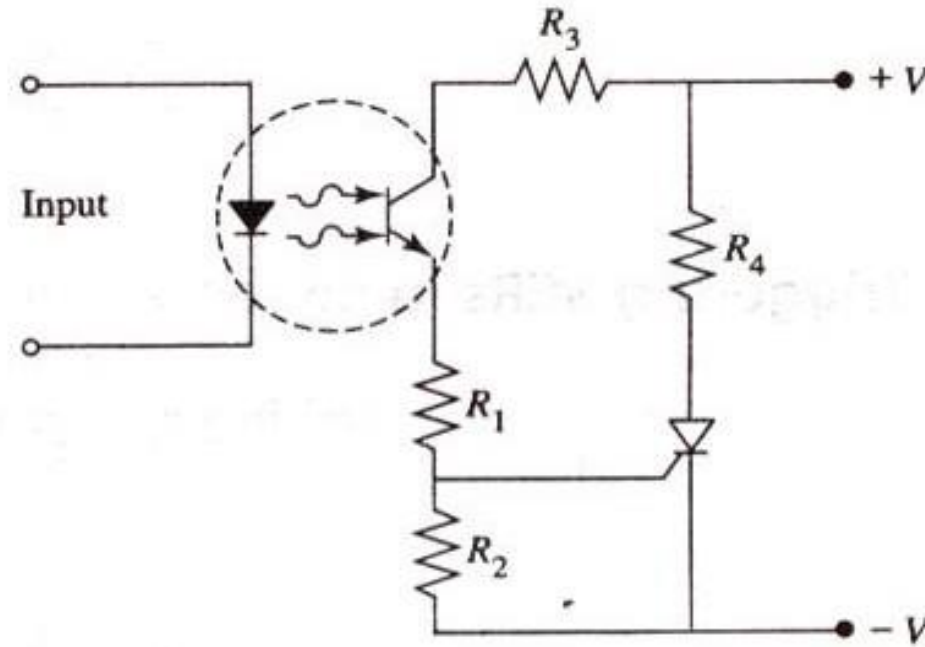
Circuit B



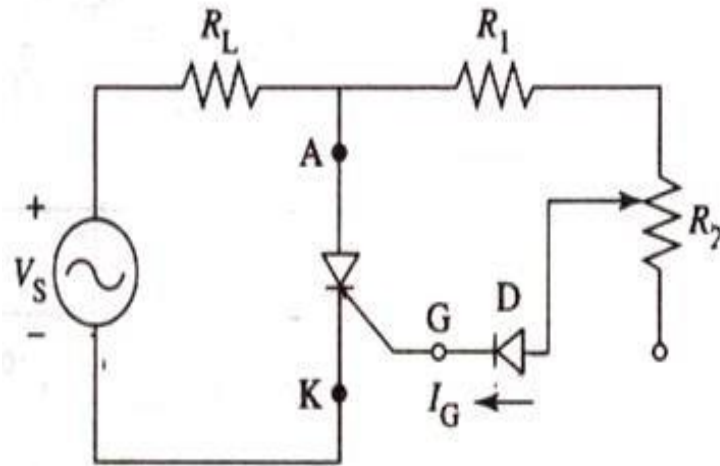
SCR trigger circuit using DIAC



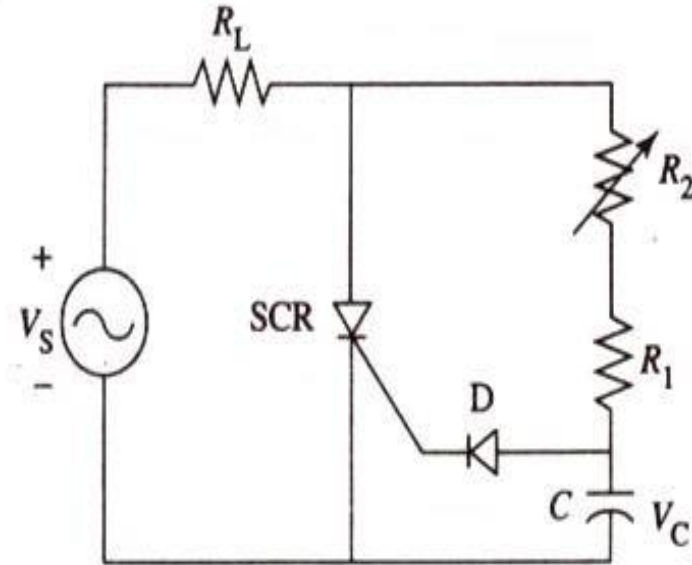
SCR trigger circuit using Optocoupler



(c) AC Signals

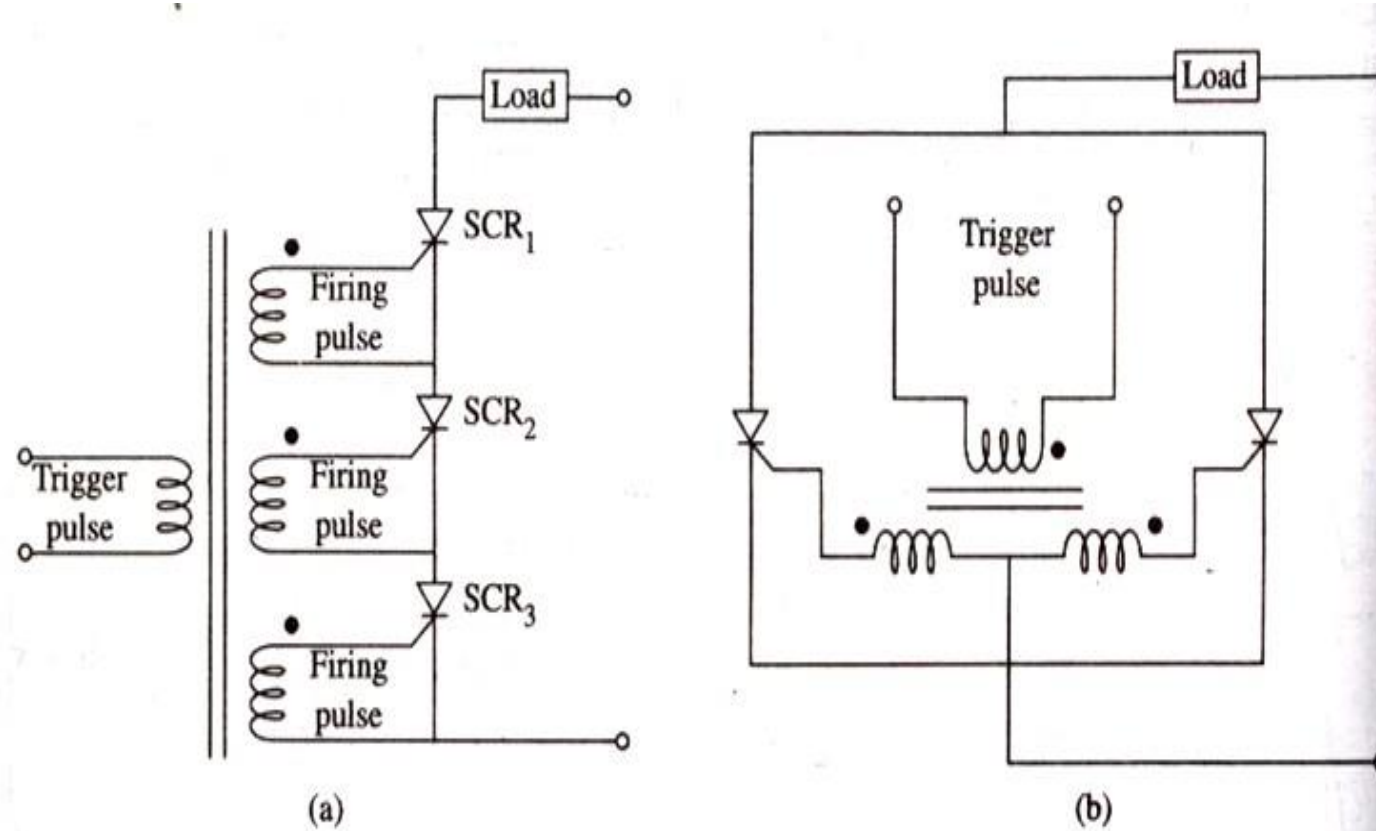


Resistive phase control
phase control



RC

Triggering SCRs in Series and in Parallel



SCR Turnoff (Commutation)

Circuits

○ What is Commutation?

The process of turning off an SCR is called commutation.

It is achieved by

1. Reducing anode current below holding current
2. Make anode negative with respect to cathode

○ Types of commutation are:

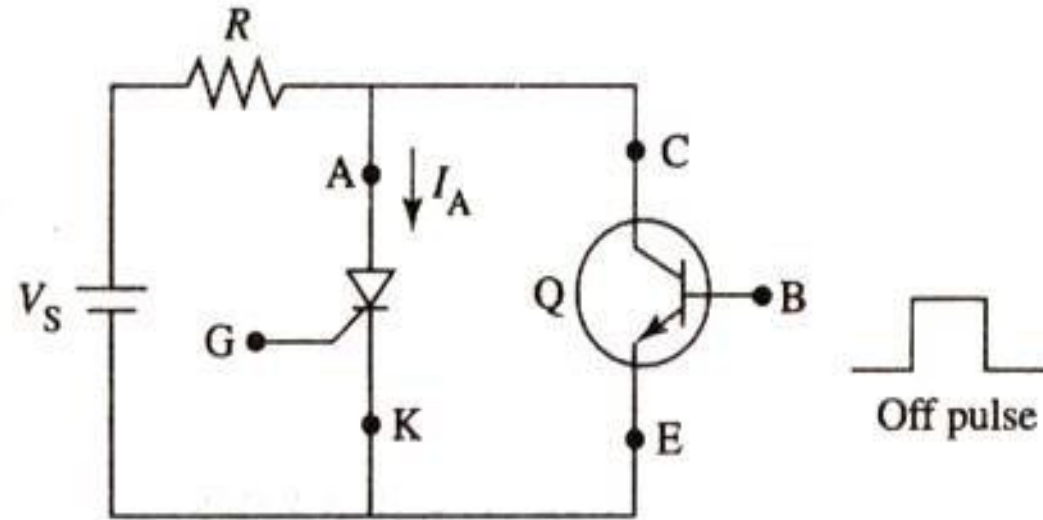
1. Natural or line commutation
2. Forced commutation

SCR Turnoff Methods

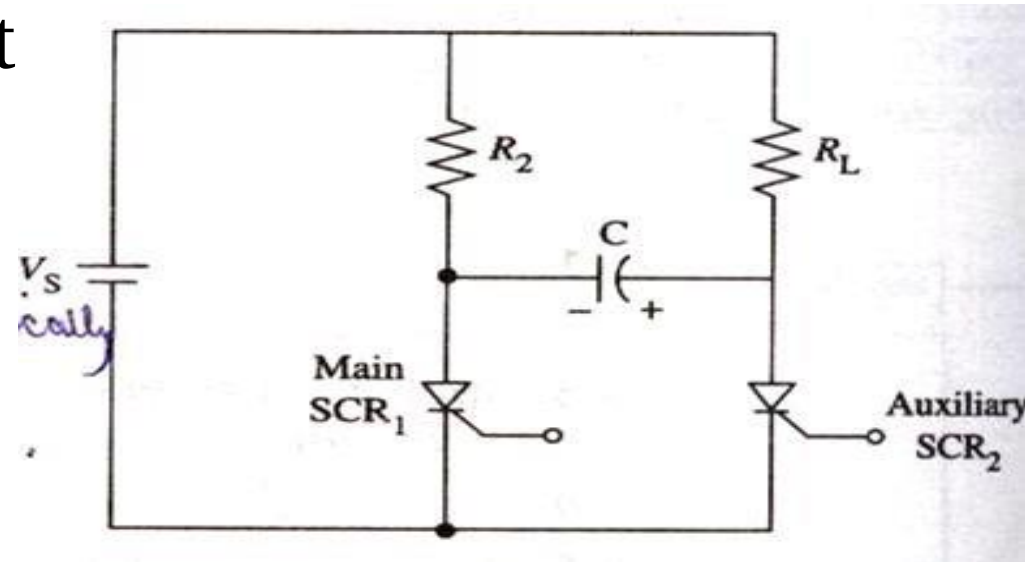
1. Diverting the anode current to an alternate path
2. Shorting the SCR from anode to cathode
3. Applying a reverse voltage (by making the cathode positive with respect to the anode) across the SCR
4. Forcing the anode current to zero for a brief period
5. Opening the external path from its anode supply voltage
6. Momentarily reducing supply voltage to zero

(1) Capacitor Commutation

- SCR turnoff circuit using a transistor switch^h



- SCR turnoff circuit using commutation capacit

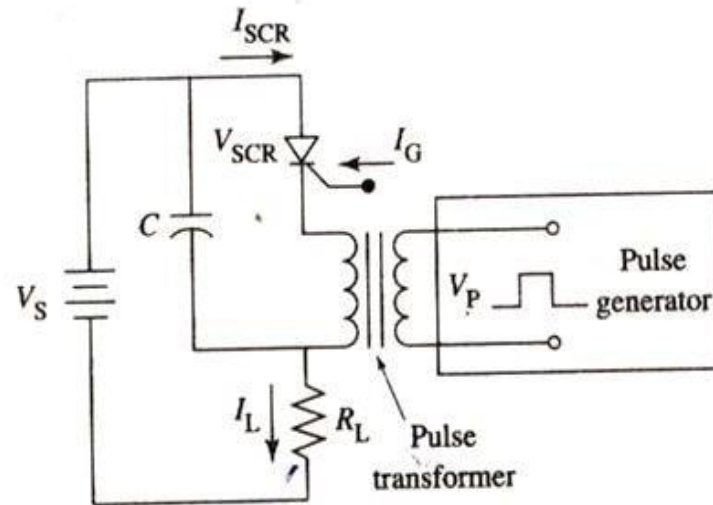


- Value of capacitance is determined by:

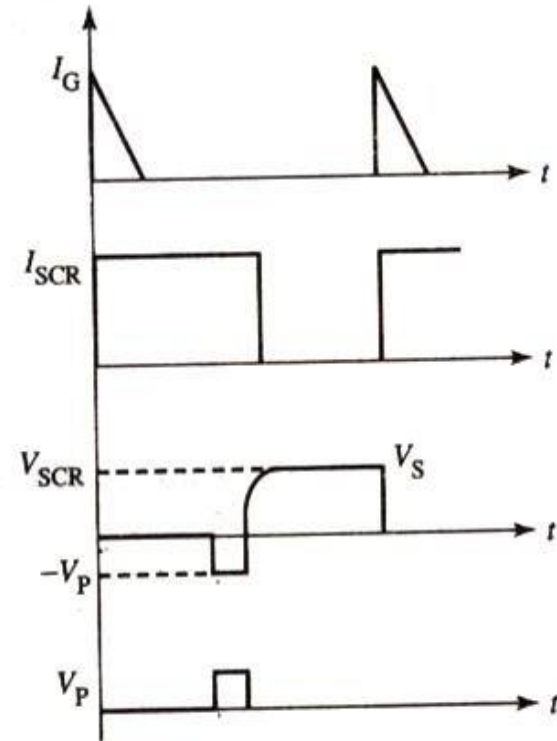
$$C \geq t_{OFF}$$

$$0.693R_L$$

(2) Commutation By External Source



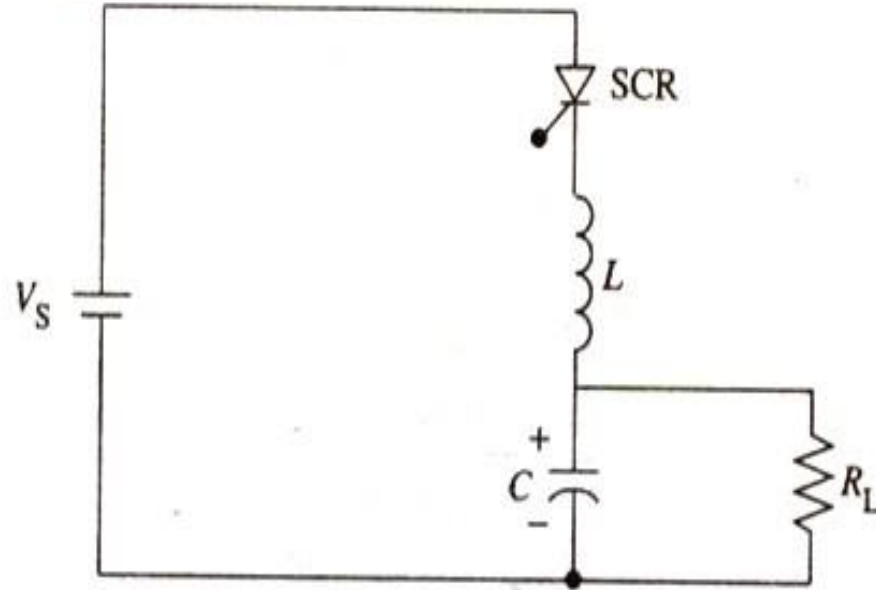
(a)



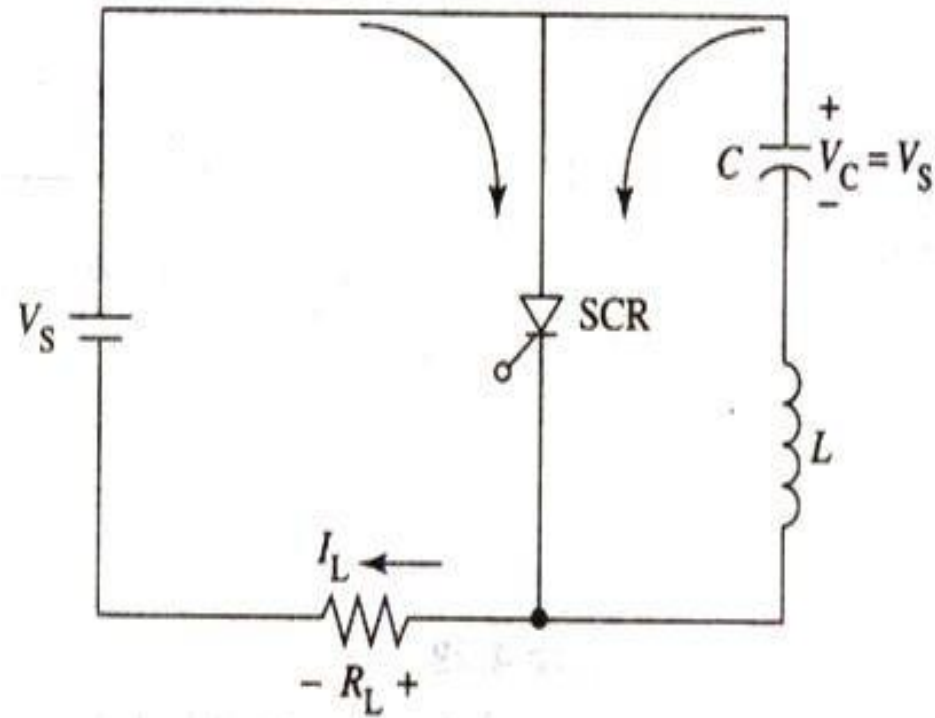
(b)

(3) Commutation by Resonance

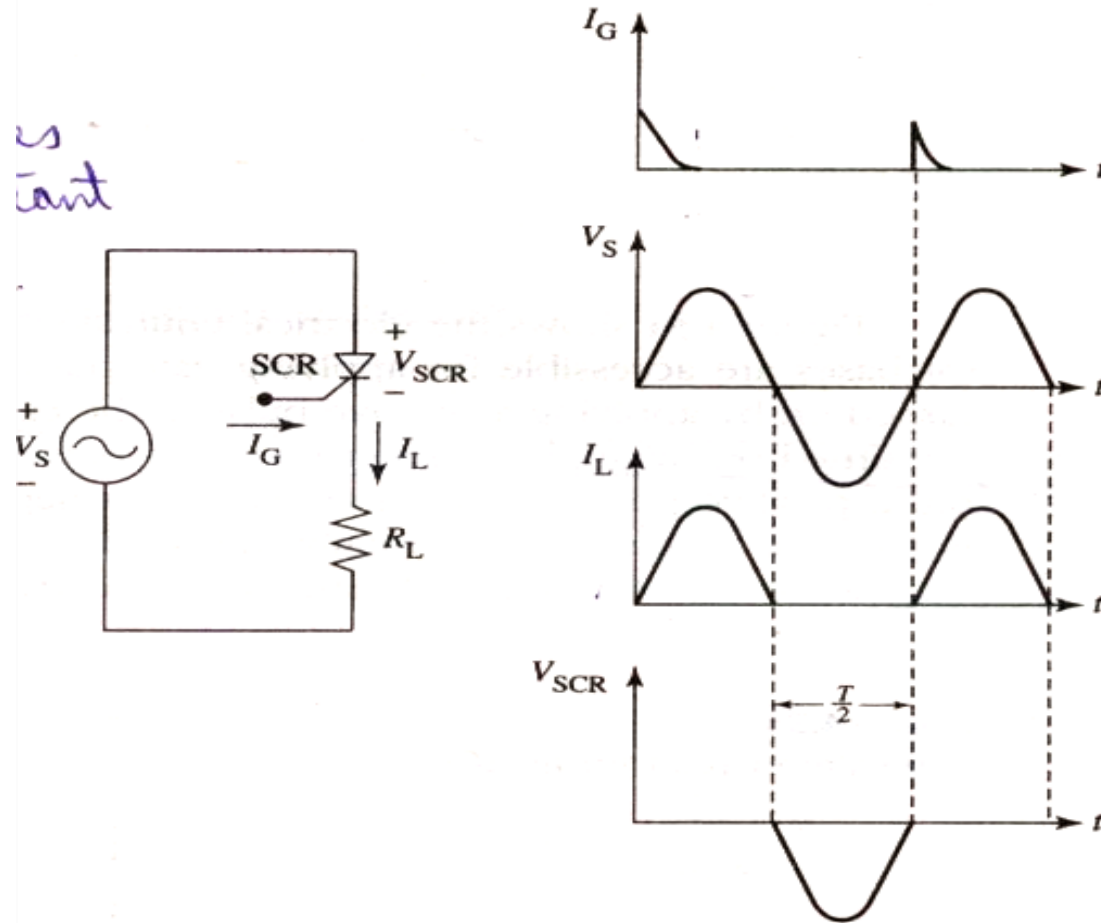
. Series resonant turnoff circuit



- Parallel resonant turnoff circuit

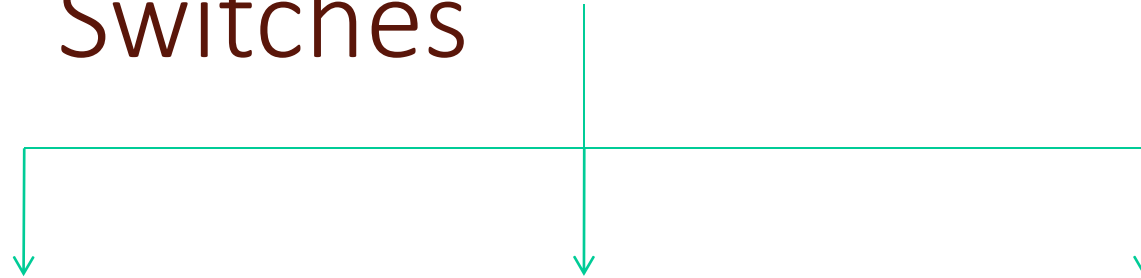


(4) AC line commutation



Other members of Thyristor Family

Power Semiconductor Switches



Power Diodes
2 layer device

Power Transistors
3 layer Device

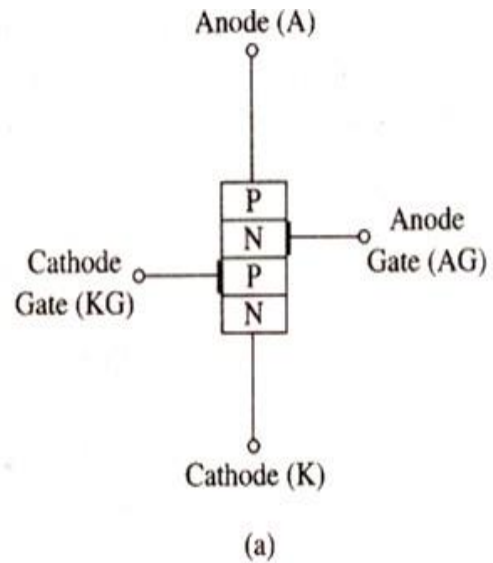
Thyristors
4 layer Device

- **Thyristor devices can convert and control large amounts of power in AC or DC systems while using very low power for control.**
- **Thyristor family includes**
 - 1- Silicon controlled switch (SCR)
 - 2- Gate-turnoff thyristor (GTO)
 - 3 Triac
 - 4 Diac
 - 5 Silicon controlled switch (SCS)
 - 6- Mos-controlled switch (MCT)

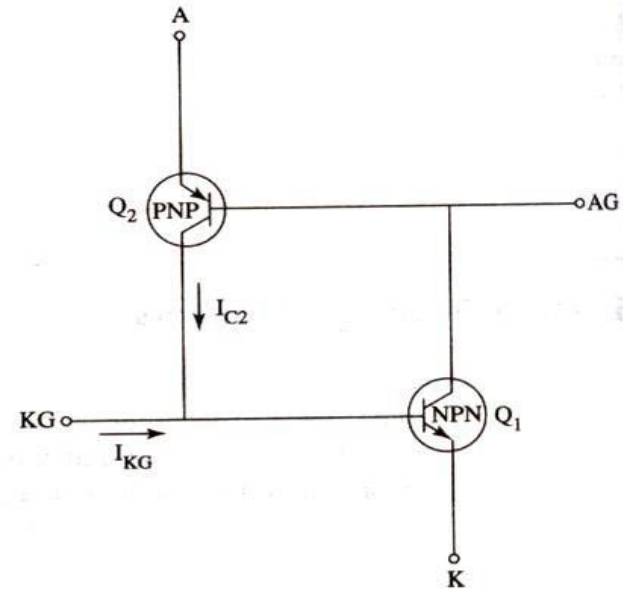
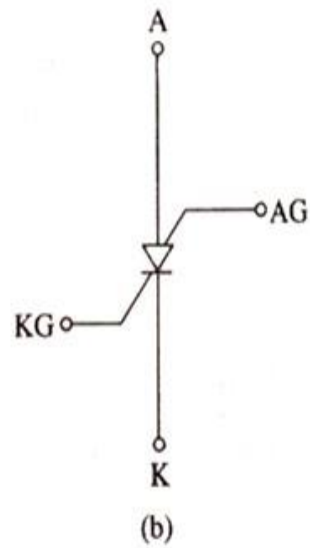
Other Types of Thyristors

1. Silicon Controlled Switch (SCS)
2. Gate Turnoff Thyristor (GTO)
3. DIAC
4. TRIAC
5. MOS-Controlled Thyristor (MCT)

1. SCS

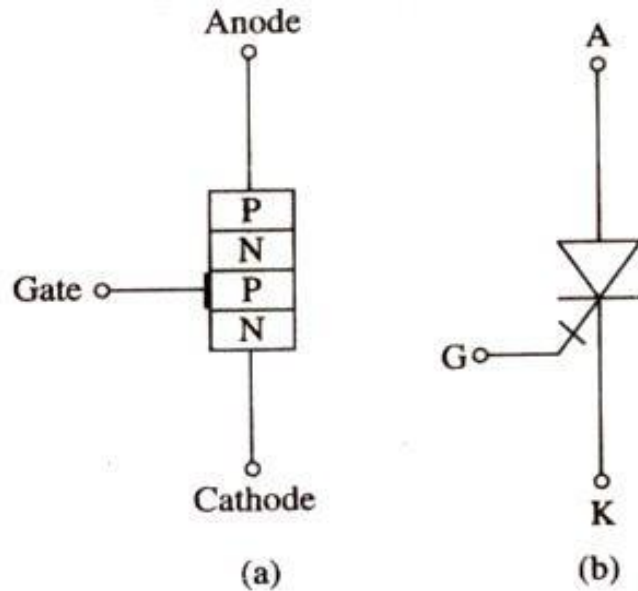


Structure
Symbol

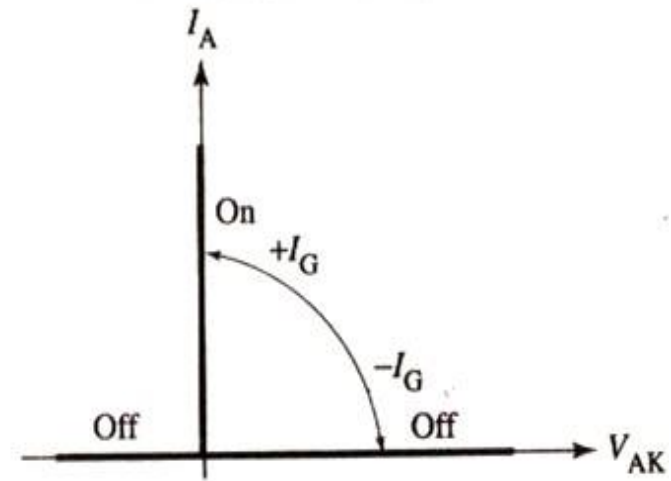


Equivalent circuit
for SCS

(2) GTO

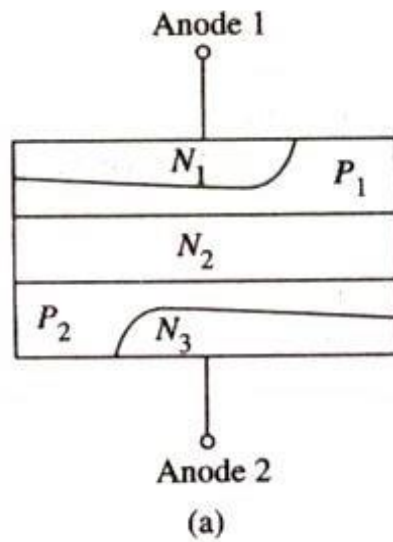


Structure
Symbol

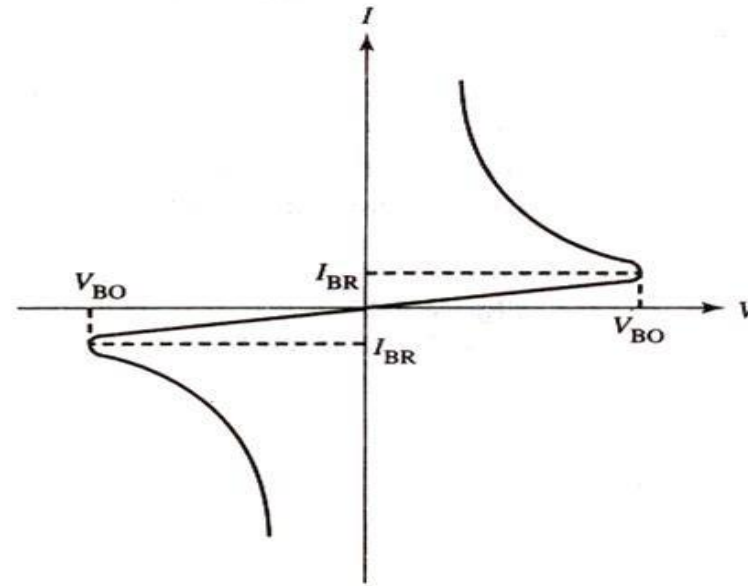


GTO Ideal VI
characteristics

(3) DIAC

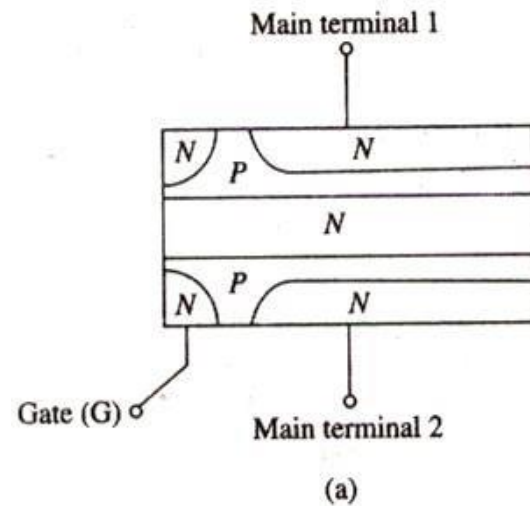


Structure
Symbol

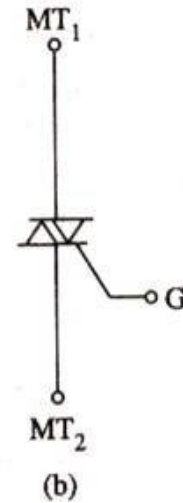


VI characteristics
of diac

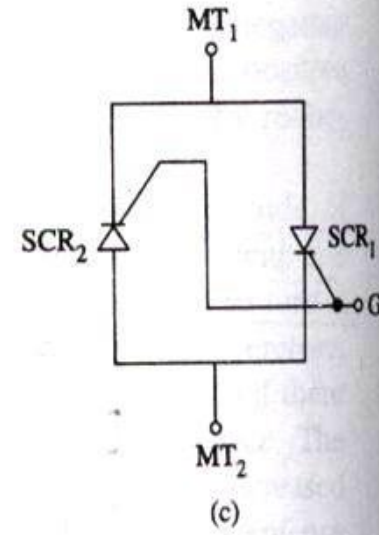
(4) Triac



Structure
equivalent circuit

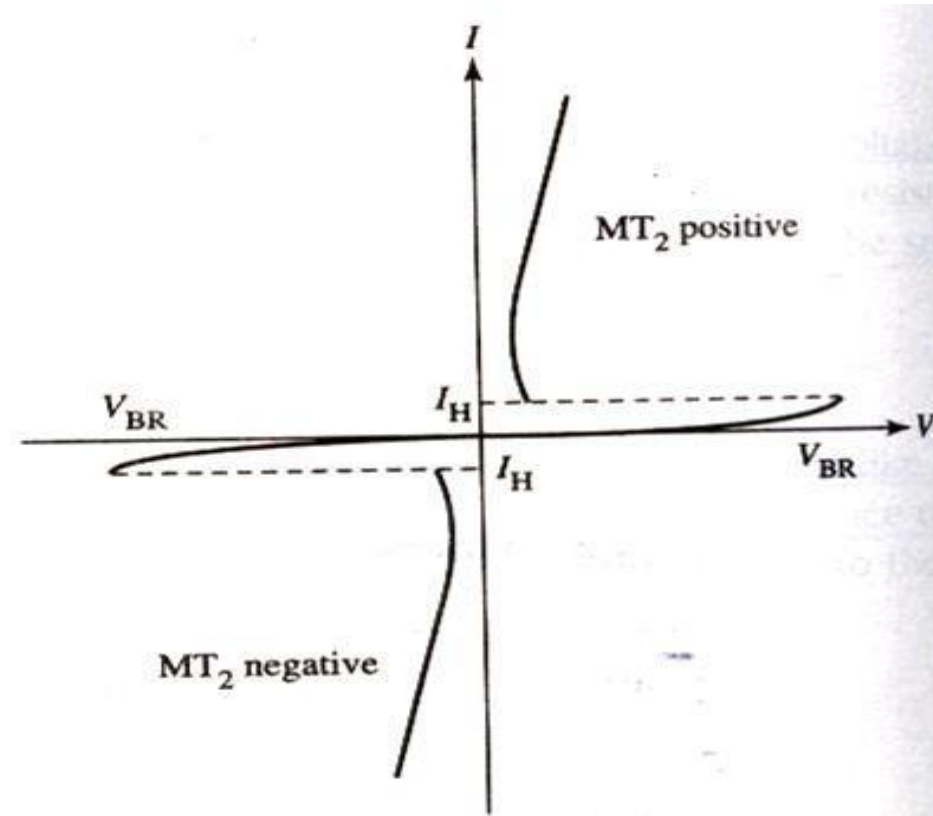


Symbol

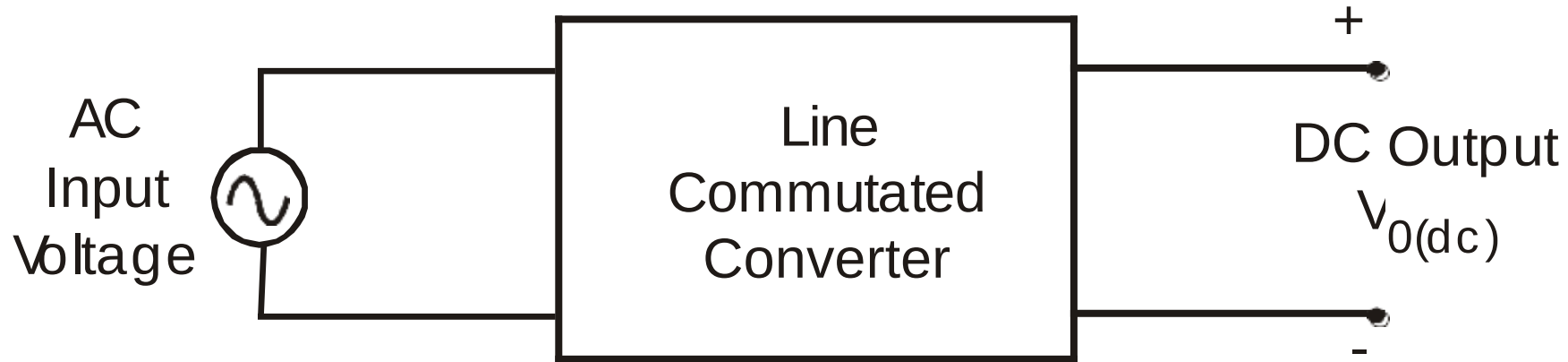


SCR

Triac VI characteristics



**UNIT-II AC-DC
Converters(Phase Controlled
Rectifiers)**



- Type of input: Fixed voltage, fixed frequency ac power supply.
- Type of output: Variable dc output voltage
- Type of commutation: Natural / AC line commutation

Different types of Line Commutated Converters

- AC to DC Converters (Phase controlled rectifiers)
- AC to AC converters (AC voltage controllers)
- AC to AC converters (Cyclo-converters) at low output frequency.

Differences Between Diode Rectifiers & Phase Controlled Rectifiers

Cntd...

- The diode rectifiers are referred to as uncontrolled rectifiers .
- The diode rectifiers give a fixed dc output voltage .
- Each diode conducts for one half cycle.
- Diode conduction angle = 180° or π radians.
- We can not control the dc output voltage or the average dc load current in a diode rectifier circuit.

Cntd...

Single phase half wave diode rectifier gives an

$$\text{Average dc output voltage } V_{O(dc)} = \frac{V_m}{\pi}$$

Single phase full wave diode rectifier gives an

$$\text{Average dc output voltage } V_{O(dc)} = \frac{2V_m}{\pi}$$

Applications of Phase Controlled Rectifiers

- DC motor control in steel mills, paper and textile mills employing dc motor drives.
- AC fed traction system using dc traction motor.
- Electro-chemical and electro-metallurgical processes.
- Magnet power supplies.
- Portable hand tool drives.

Classification of Phase Controlled Rectifiers

- Single Phase Controlled Rectifiers.
- Three Phase Controlled Rectifiers.

Different types of Single Phase Controlled Rectifiers.

- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
 - Using a center tapped transformer.
 - Full wave bridge circuit.
 - Semi converter.
 - Full converter.

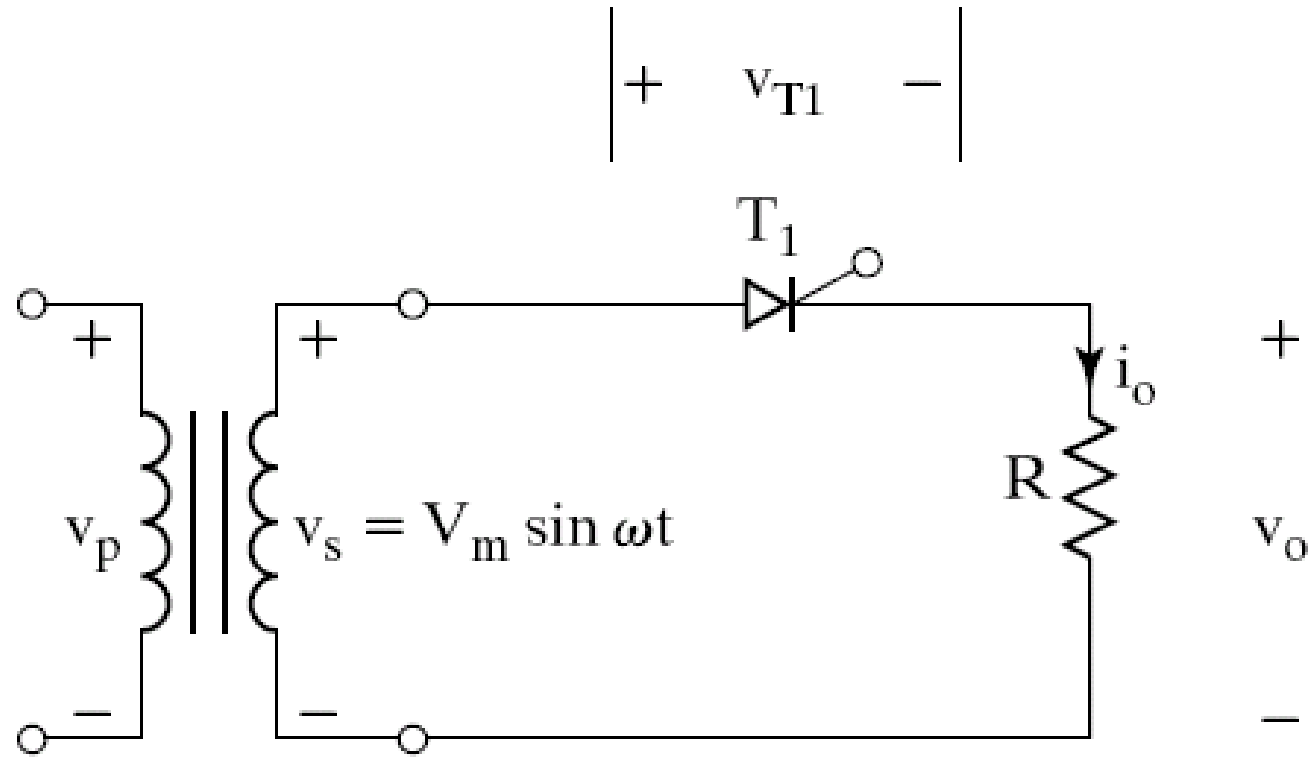
Different Types of Three Phase Controlled Rectifiers

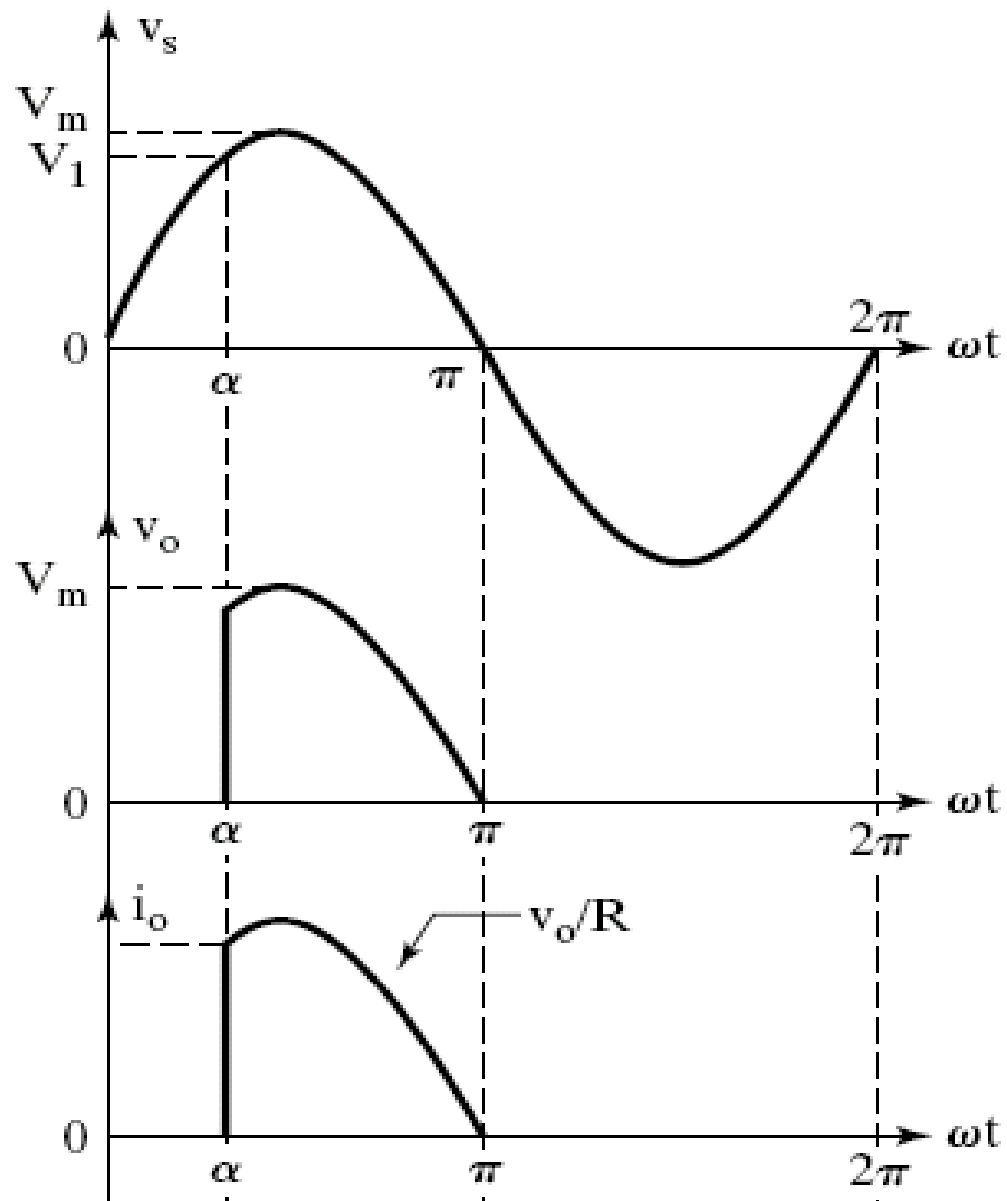
- Half wave controlled rectifiers.
- Full wave controlled rectifiers.
 - Semi converter (half controlled bridge converter).
 - Full converter (fully controlled bridge converter).

Principle of Phase Controlled Rectifier Operation

Principle of Phase Controlled Rectifier Operation

Single Phase Half-Wave Thyristor Converter with Resistive Load

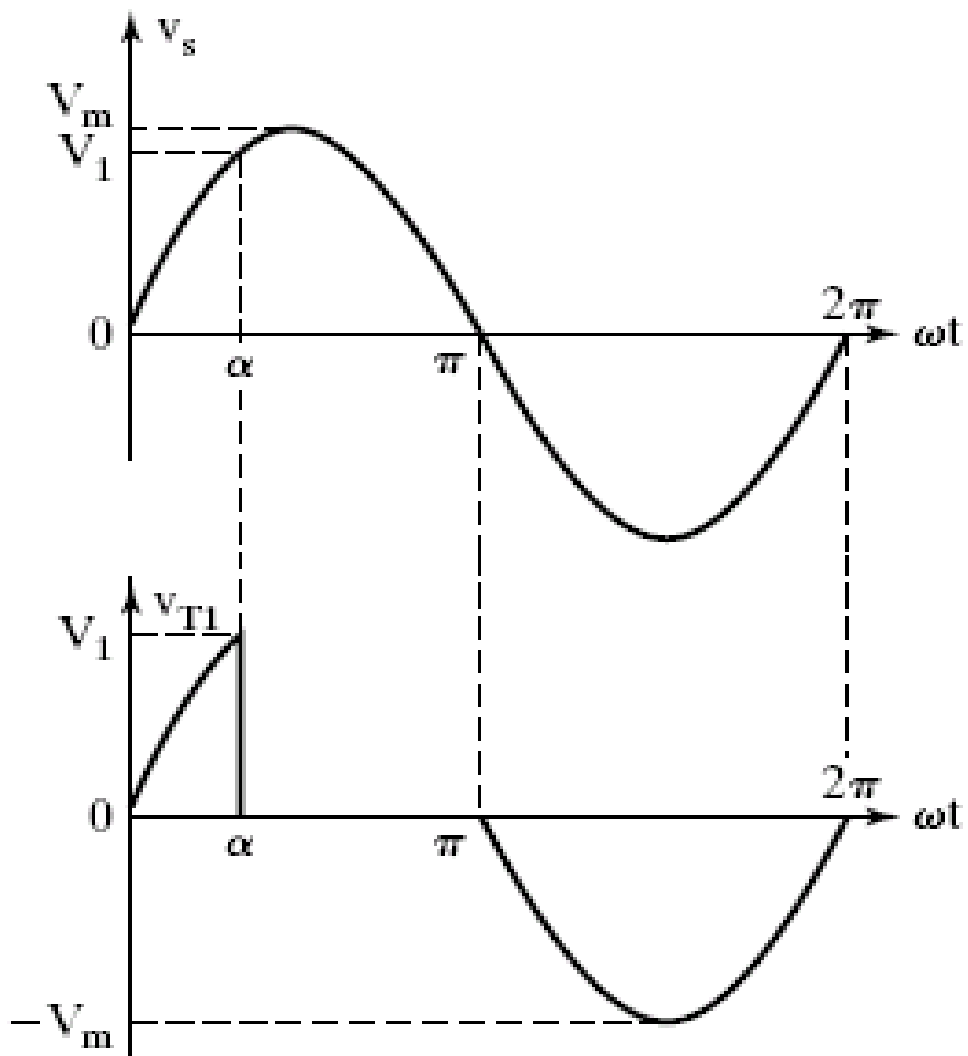




Supply Voltage

Output Voltage

Output (load)
Current



Supply Voltage

Thyristor Voltage

Equations

$$v_s = V_m \sin \omega t = \text{ i/p ac supply voltage}$$

$$V_m = \text{max. value of i/p ac supply voltage}$$

$$V_s = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply voltage}$$

$$v_o = v_L = \text{output voltage across the load}$$

When the thyristor is triggered at $\omega t = \alpha$

$$v_O = v_L = V_m \sin \omega t; \quad \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{v_O}{R} = \text{Load current}; \quad \omega t = \alpha \text{ to } \pi$$

$$i_O = i_L = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t; \quad \omega t = \alpha \text{ to } \pi$$

$$\text{Where } I_m = \frac{V_m}{R} = \text{max. value of load current}$$

To Derive an Expression for the
Average (DC)
Output Voltage Across The Load

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_0^{2\pi} v_O \cdot d(\omega t);$$

$$v_O = V_m \sin \omega t \text{ for } \omega t = \alpha \text{ to } \pi$$

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cdot d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin \omega t \cdot d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_{\alpha}^{\pi} \sin \omega t \cdot d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [-\cos \pi + \cos \alpha]; \quad \cos \pi = -1$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [1 + \cos \alpha]; \quad V_m = \sqrt{2}V_s$$

Maximum average (dc) o/p
voltage is obtained when $\alpha = 0$
and the maximum dc output voltage

$$V_{dc(\max)} = V_{dm} = \frac{V_m}{2\pi} (1 + \cos 0); \cos(0) = 1$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{V_m}{\pi}$$

Cntd...

$$V_{O(dc)} = \frac{V_m}{2\pi} [1 + \cos \alpha] ; V_m = \sqrt{2}V_s$$

The average dc output voltage can be varied by varying the trigger angle α from 0 to a maximum of 180° (π radians)

We can plot the control characteristic

$(V_{O(dc)} \text{ vs } \alpha)$ by using the equation for $V_{O(dc)}$

Control Characteristic
of
Single Phase Half Wave Phase
Controlled Rectifier
with
Resistive Load

Cntd...

The average dc output voltage is given by the expression

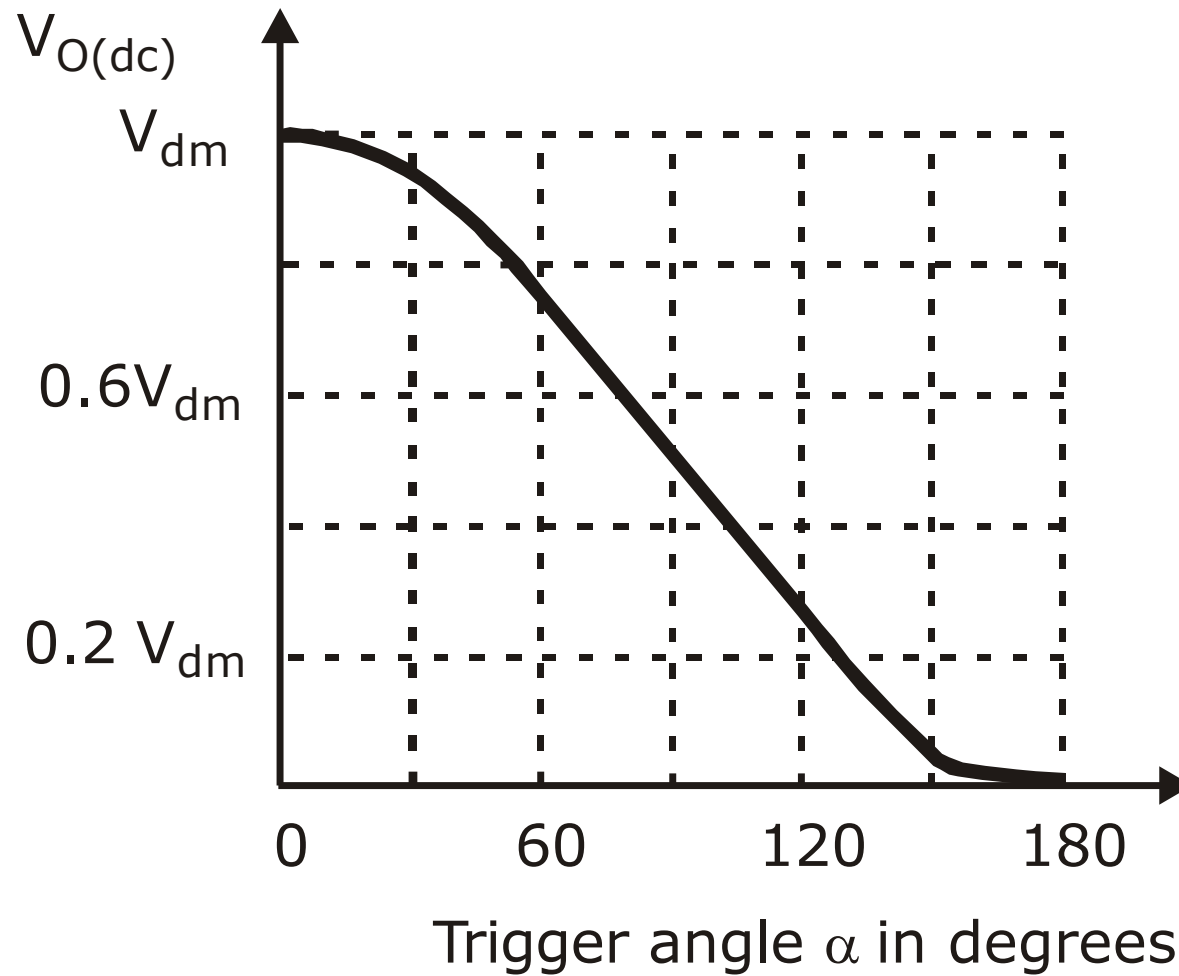
$$V_{O(dc)} = \frac{V_m}{2\pi} [1 + \cos \alpha]$$

We can obtain the control characteristic by plotting the expression for the dc output voltage as a function of trigger angle α

Trigger angle α in degrees	$V_{O(dc)}$	%
0	$V_{dm} = \frac{V_m}{\pi}$	100% V_{dm}
30°	$0.933 V_{dm}$	93.3 % V_{dm}
60°	$0.75 V_{dm}$	75 % V_{dm}
90°	$0.5 V_{dm}$	50 % V_{dm}
120°	$0.25 V_{dm}$	25 % V_{dm}
150°	$0.06698 V_{dm}$	6.69 % V_{dm}
180°	0	0

$$V_{dm} = \frac{V_m}{\pi} = V_{dc(max)}$$

Control Characteristic



Normalizing the dc output
voltage with respect to V_{dm} , the
Normalized output voltage

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{\frac{V_m}{2\pi} (1 + \cos \alpha)}{\frac{V_m}{\pi}}$$

$$V_n = \frac{V_{dc}}{V_{dm}} = \frac{1}{2} (1 + \cos \alpha) = V_{dcn}$$

To Derive An Expression for the
RMS Value of Output Voltage of a
Single Phase Half Wave Controlled
Rectifier With Resistive Load

Cntd...

The RMS output voltage is given by

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_0^{2\pi} v_o^2 . d(\omega t) \right]$$

Output voltage $v_o = V_m \sin \omega t$; for $\omega t = \alpha$ to π

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}}$$

Cntd...

By substituting $\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$, we get

$$V_{O(RMS)} = \left[\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \frac{(1 - \cos 2\omega t)}{2} d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{4\pi} \left\{ \int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t d(\omega t) \right\} \right]^{\frac{1}{2}}$$

Cntd...

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left\{ (\omega t) \Big|_{\alpha}^{\pi} - \left(\frac{\sin 2\omega t}{2} \right) \Big|_{\alpha}^{\pi} \right\} \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left((\pi - \alpha) - \frac{(\sin 2\pi - \sin 2\alpha)}{2} \right) \right]^{\frac{1}{2}} ; \sin 2\pi = 0$$

$$V_{O(RMS)} = \frac{V_m}{2} \left[\frac{1}{\pi} \left((\pi - \alpha) + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \left((\pi - \alpha) + \frac{\sin 2\alpha}{2} \right)^{\frac{1}{2}}$$

Performance Parameters Of Phase Controlled Rectifiers

Cntd...

Output dc power (avg. or dc o/p
power delivered to the load)

$$P_{O(dc)} = V_{O(dc)} \times I_{O(dc)} ; \text{ i.e., } P_{dc} = V_{dc} \times I_{dc}$$

Where

$$V_{O(dc)} = V_{dc} = \text{avg./ dc value of o/p voltage.}$$

$$I_{O(dc)} = I_{dc} = \text{avg./dc value of o/p current}$$

Cntd...
Output ac
power

$$P_{O(ac)} = V_{O(RMS)} \times I_{O(RMS)}$$

Efficiency of Rectification (Rectification Ratio)

$$\text{Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}}; \quad \% \text{ Efficiency } \eta = \frac{P_{O(dc)}}{P_{O(ac)}} \times 100$$

The o/p voltage consists of two components

The dc component $V_{O(dc)}$

The ac /ripple component $V_{ac} = V_{r(rms)}$

Cntd...

The total RMS value of output voltage is given by

$$V_{O(RMS)} = \sqrt{V_{O(dc)}^2 + V_{r(rms)}^2}$$

$$\therefore V_{ac} = V_{r(rms)} = \sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}$$

Form Factor (FF) which is a measure of the shape of the output voltage is given by

$$FF = \frac{V_{O(RMS)}}{V_{O(dc)}} = \frac{\text{RMS output (load) voltage}}{\text{DC load output (load) voltage}}$$

Cntd...

The Ripple Factor (RF) w.r.t. o/p voltage w/f

$$r_v = RF = \frac{V_{r(rms)}}{V_{O(dc)}} = \frac{V_{ac}}{V_{dc}}$$

$$r_v = \frac{\sqrt{V_{O(RMS)}^2 - V_{O(dc)}^2}}{V_{O(dc)}} = \sqrt{\left[\frac{V_{O(RMS)}}{V_{O(dc)}} \right]^2 - 1}$$

$$\therefore r_v = \sqrt{FF^2 - 1}$$

Cntd...

$$\text{Current Ripple Factor } r_i = \frac{I_{r(rms)}}{I_{O(dc)}} = \frac{I_{ac}}{I_{dc}}$$

$$\text{Where } I_{r(rms)} = I_{ac} = \sqrt{I_{O(RMS)}^2 - I_{O(dc)}^2}$$

$V_{r(pp)}$ = peak to peak ac ripple output voltage

$$V_{r(pp)} = V_{O(max)} - V_{O(min)}$$

$I_{r(pp)}$ = peak to peak ac ripple load current

$$I_{r(pp)} = I_{O(max)} - I_{O(min)}$$

Cntd...

Transformer Utilization Factor (TUF)

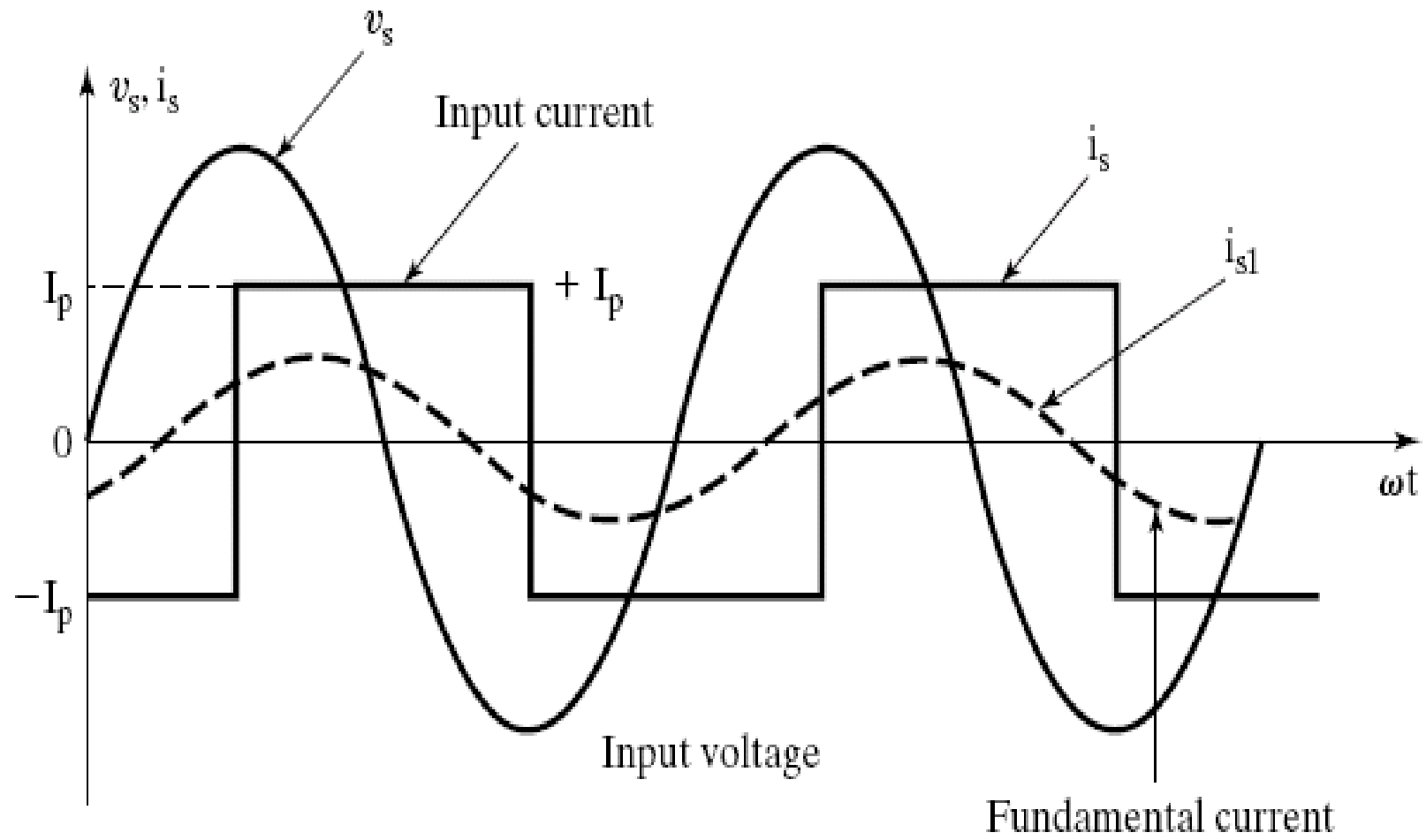
$$TUF = \frac{P_{O(dc)}}{V_S \times I_S}$$

Where

V_S = RMS supply (secondary) voltage

I_S = RMS supply (secondary) current

Cntd...



Cntd...

Wher

e

$v_s =$ Supply voltage at the transformer secondary side

$i_s =$ i/p supply current

(transformer secondary winding current)

$i_{s1} =$ Fundamental component of the i/p supply current

$I_P =$ Peak value of the input supply current

$\phi =$ Phase angle difference between (sine wave components) the fundamental components of i/p supply current & the input supply voltage.

Cntd...

ϕ = Displacement angle (phase angle) For an RL load
 ϕ = Displacement angle = Load impedance angle

$$\therefore \phi = \tan^{-1} \left(\frac{\omega L}{R} \right) \text{ for an RL load}$$

Displacement Factor (DF) or
Fundamental Power Factor

$$DF = \cos \phi$$

Cntd...

Harmonic Factor (HF) or

Total Harmonic Distortion Factor ; THD

$$HF = \left[\frac{I_S^2 - I_{S1}^2}{I_{S1}^2} \right]^{\frac{1}{2}} = \left[\left(\frac{I_S}{I_{S1}} \right)^2 - 1 \right]^{\frac{1}{2}}$$

Where

I_S = RMS value of input supply current.

I_{S1} = RMS value of fundamental component of the i/p supply current.

Cntd...

Input Power Factor (PF)

$$PF = \frac{V_s I_{s1}}{V_s I_s} \cos\phi = \frac{I_{s1}}{I_s} \cos\phi$$

The Crest Factor (CF)

$$CF = \frac{I_{s(peak)}}{I_s} = \frac{\text{Peak input supply current}}{\text{RMS input supply current}}$$

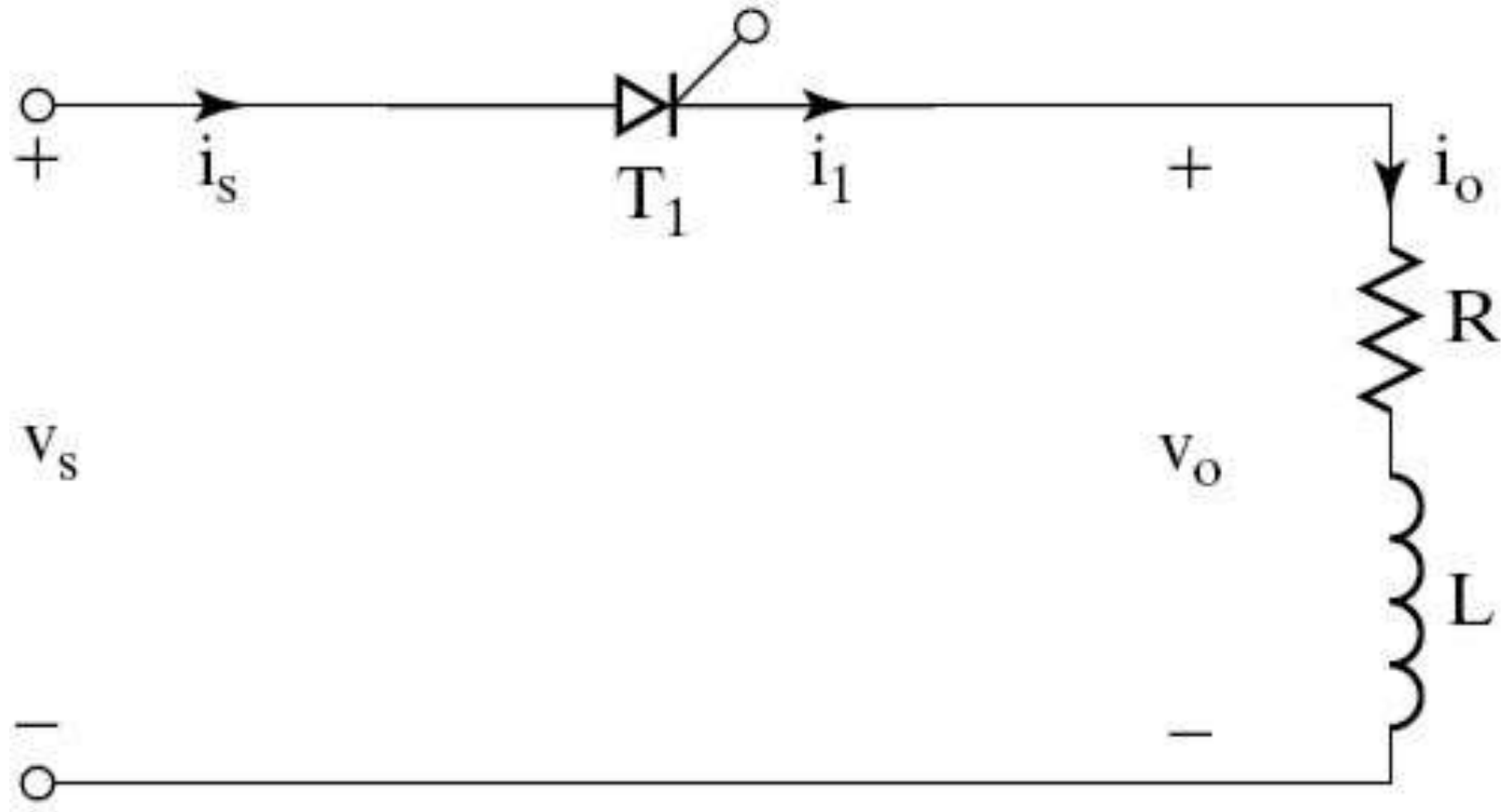
For an Ideal Controlled Rectifier

$$FF = 1; \eta = 100\% ; V_{ac} = V_{r(rms)} = 0 ; TUF = 1;$$

$$RF = r_v = 0 ; HF = THD = 0; PF = DPF = 1$$

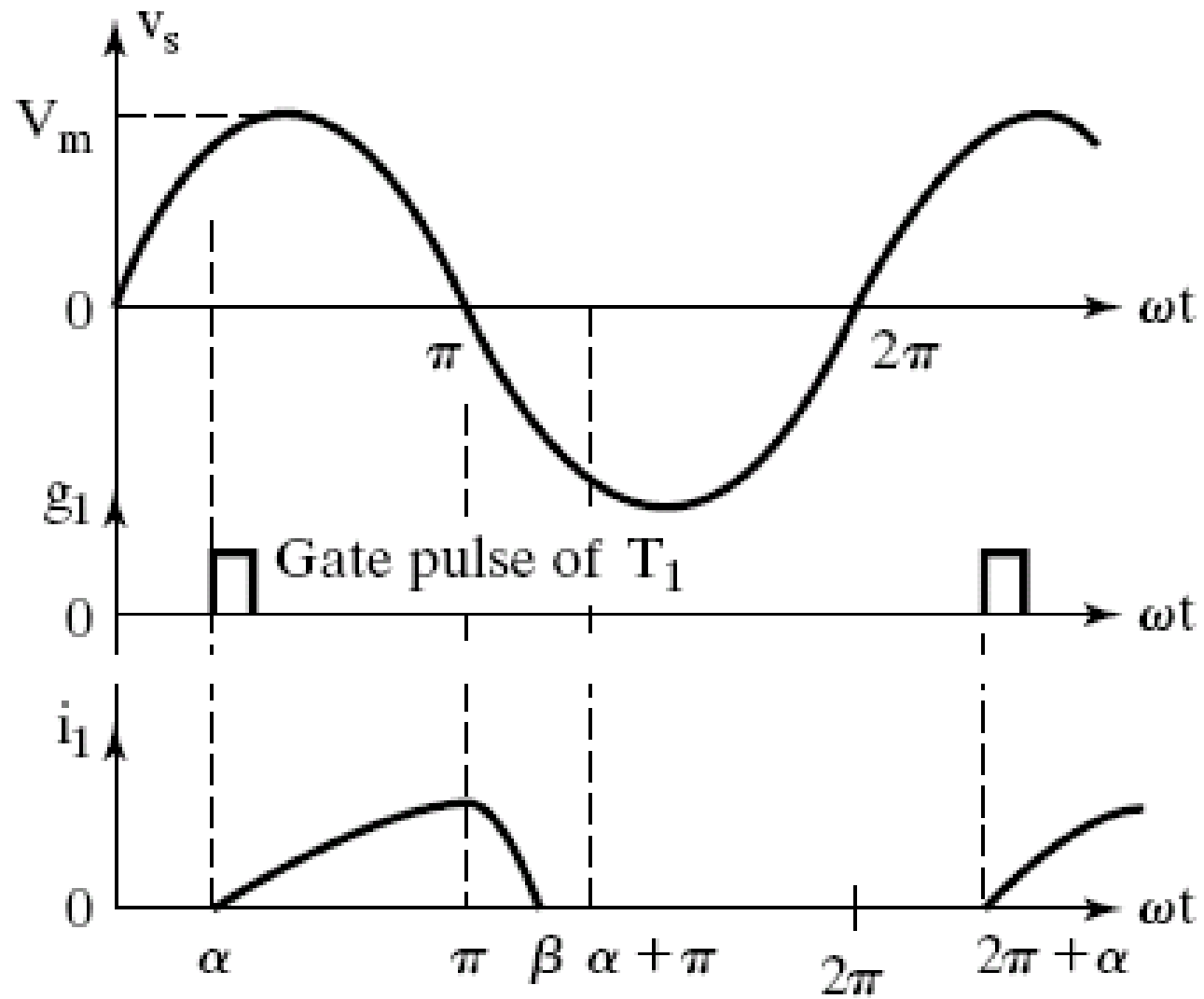
Single Phase Half Wave Controlled Rectifier With An RL Load

Cntd...

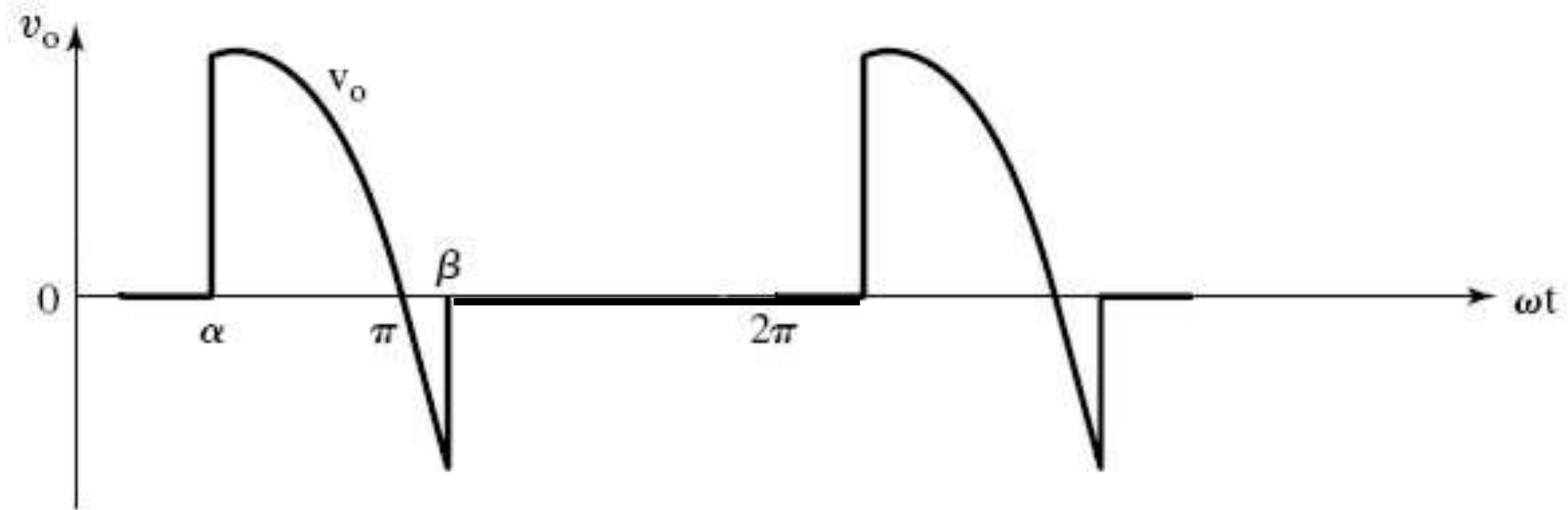


Input Supply Voltage (V_s)
&
Thyristor (Output) Current
Waveforms

Cntd...



Output (Load) Voltage Waveform



Cntd...

To Derive An Expression For
The Output
(Load) Current, During $\omega t = \alpha$ to β
When Thyristor T_1 Conducts

Cntd...

Assuming T_1 is triggered $\omega t = \alpha$,
we can write the equation,

$$L \left(\frac{di_o}{dt} \right) + Ri_o = V_m \sin \omega t ; \alpha \leq \omega t \leq \beta$$

General expression for the output current,

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) + A_1 e^{\frac{-t}{\tau}}$$

Cntd...

$$V_m = \sqrt{2}V_s = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A_1 e^{\frac{-R}{L} t}$$

Cntd...

Constant A_1 is calculated from

initial condition $i_o = 0$ at $\omega t = \alpha$; $t = \left(\frac{\alpha}{\omega} \right)$

$$i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A_1 e^{\frac{-R}{L} t}$$

$$\therefore A_1 e^{\frac{-R}{L} t} = \frac{-V_m}{Z} \sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Cntd...

Substituting the value of constant A_1 in the general expression for i_o

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right];$$

Where $\alpha \leq \omega t \leq \beta$

Cntd...

Extinction angle β can be calculated by using the condition that $i_o = 0$ at $\omega t = \beta$

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression
For
Average (DC) Load Voltage of a
Single Half Wave Controlled
Rectifier with
RL Load

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \int_0^{2\pi} v_O \cdot d(\omega t)$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_0^{\alpha} v_O \cdot d(\omega t) + \int_{\alpha}^{\beta} v_O \cdot d(\omega t) + \int_{\beta}^{2\pi} v_O \cdot d(\omega t) \right]$$

$$v_O = 0 \text{ for } \omega t = 0 \text{ to } \alpha \text{ \& for } \omega t = \beta \text{ to } 2\pi$$

$$\therefore V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} v_O \cdot d(\omega t) \right];$$

$$v_O = V_m \sin \omega t \text{ for } \omega t = \alpha \text{ to } \beta$$

$$V_{O(dc)} = V_L = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\beta}$$

$$V_{O(dc)} = V_L = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

$$\therefore V_{O(dc)} = V_L = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta)$$

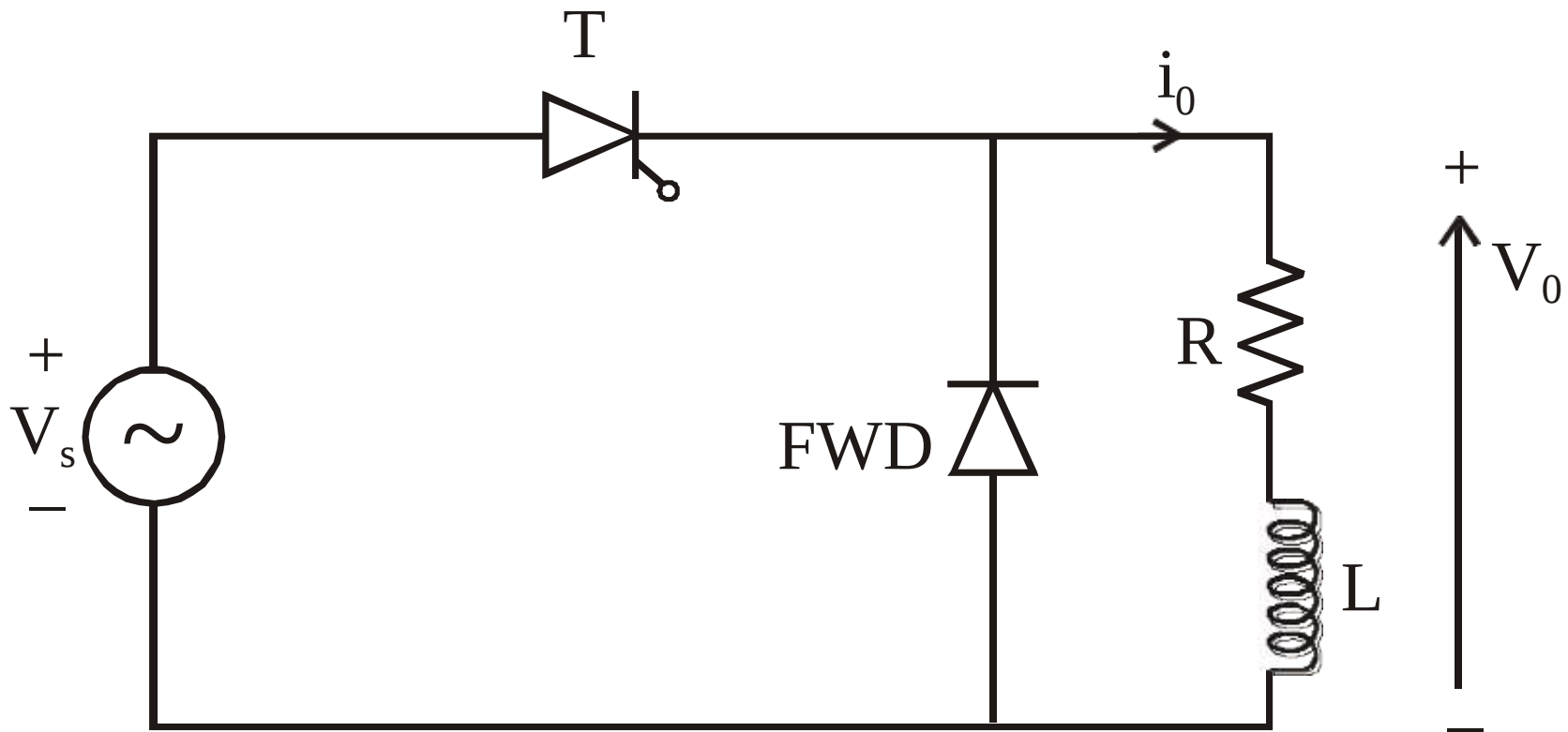
Effect of Load Inductance on the Output

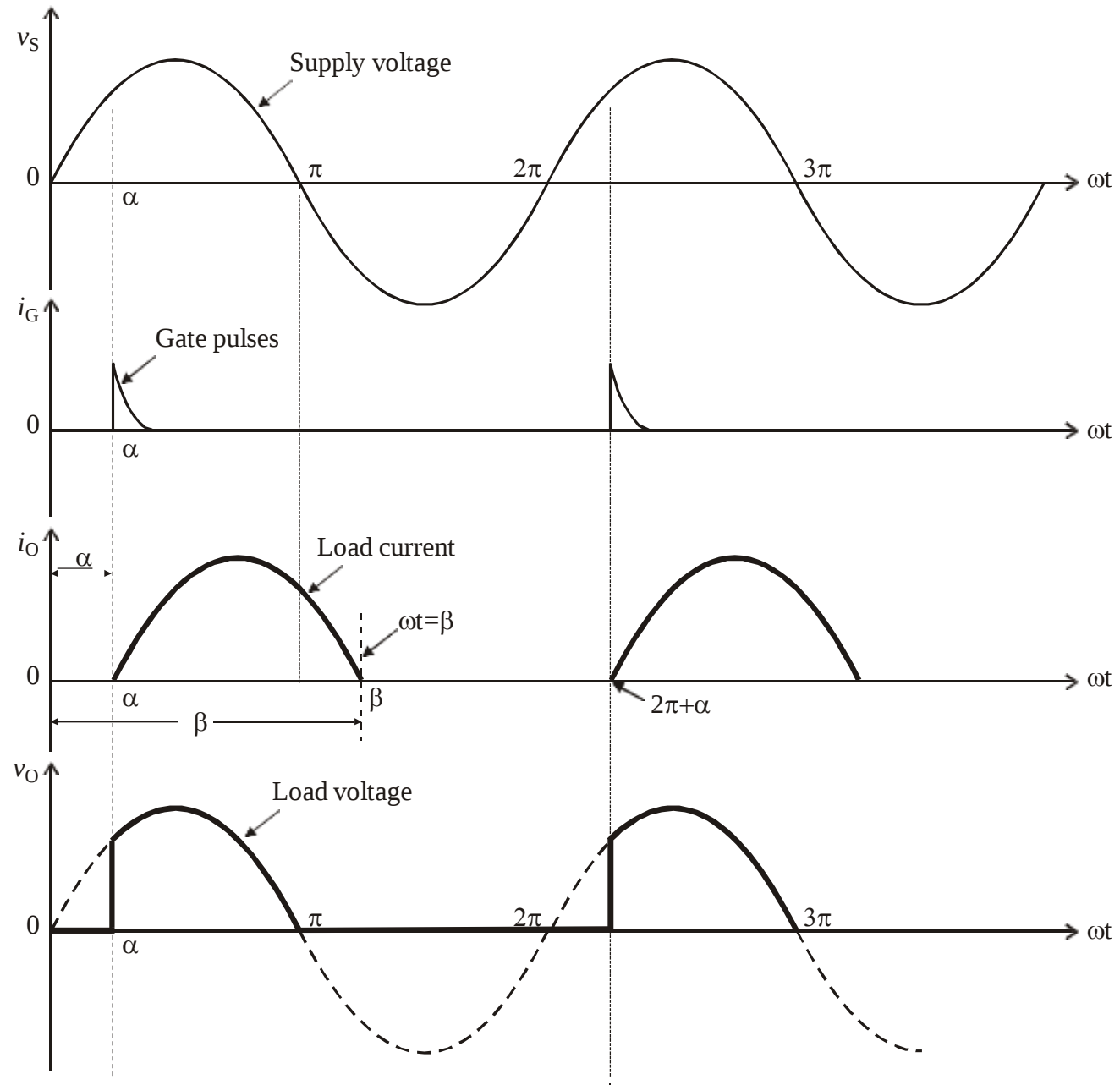
During the period $\omega t = \pi$ to β the instantaneous o/p voltage is negative and this reduces the average or the dc output voltage when compared to a purely resistive load.

Average DC Load Current

$$I_{O(dc)} = I_{L(Avg)} = \frac{V_{O(dc)}}{R_L} = \frac{V_m}{2\pi R_L} (\cos \alpha - \cos \beta)$$

Single Phase Half Wave Controlled Rectifier With RL Load & Free Wheeling Diode





The average output voltage

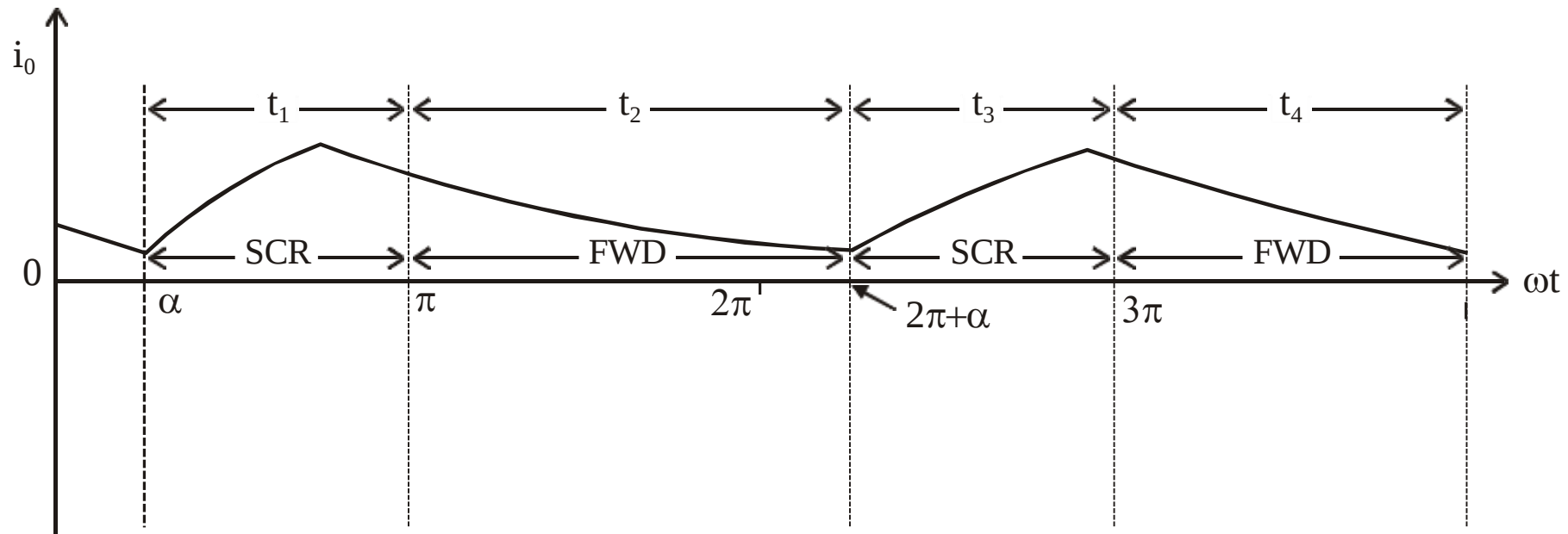
$V_{dc} = \frac{V_m}{2\pi} [1 + \cos \alpha]$ which is the same as that of a purely resistive load.

The following points are to be noted

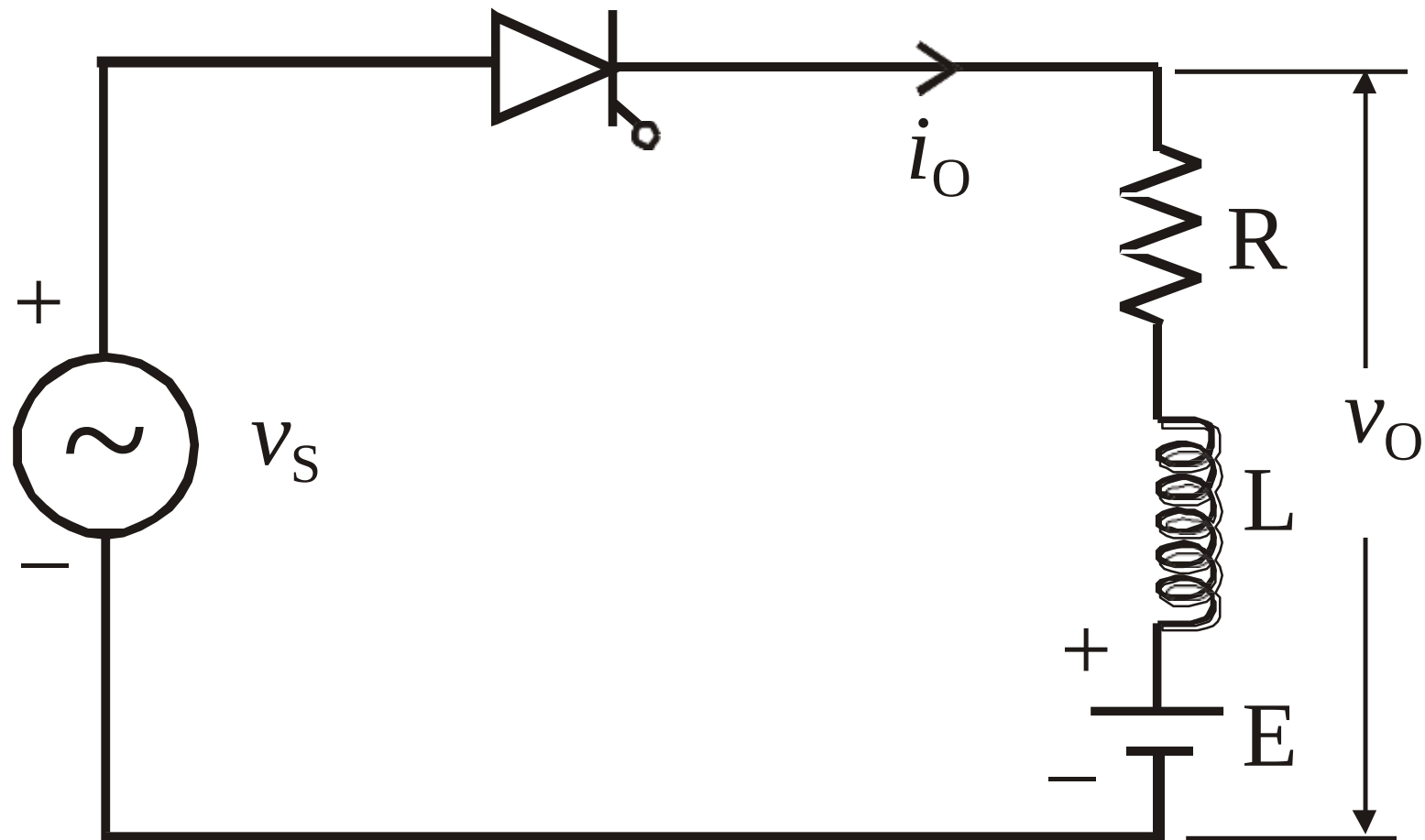
For low value of inductance, the load current tends to become discontinuous.

During the period α to π
the load current is carried by the SCR.
During the period π to β load current is
carried by the free wheeling diode.
The value of β depends on the value of
R and L and the forward resistance
of the FWD.

For Large Load Inductance
the load current does not reach zero,
& we obtain continuous load current



Single Phase Half Wave Controlled Rectifier With A General Load



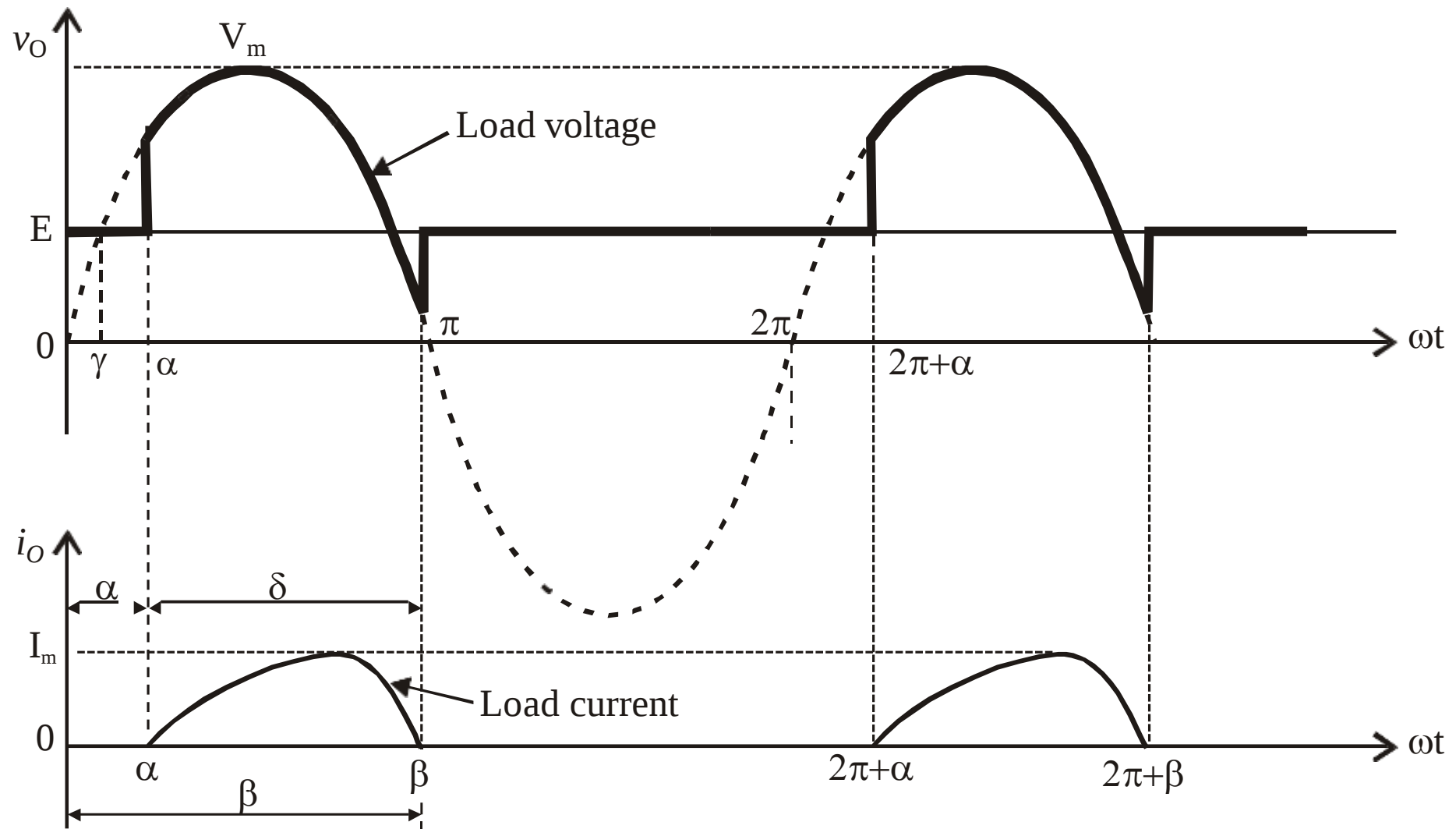
$$\gamma = \sin^{-1} \left(\frac{E}{V_m} \right)$$

For trigger angle $\alpha < \gamma$,

the Thyristor conducts from $\omega t = \gamma$ to β

For trigger angle $\alpha > \gamma$,

the Thyristor conducts from $\omega t = \alpha$ to β



Equations

$$v_S = V_m \sin \omega t = \text{Input supply voltage.}$$

$$v_O = V_m \sin \omega t = \text{o/p (load) voltage}$$

$$\text{for } \omega t = \alpha \text{ to } \beta.$$

$$v_O = E \text{ for } \omega t = 0 \text{ to } \alpha \text{ \&}$$

$$\text{for } \omega t = \beta \text{ to } 2\pi.$$

Expression for the Load Current

When the thyristor is triggered at a delay angle of $\alpha > \gamma$, the eqn. for the circuit can be written as

$$V_m \sin \omega t = i_o \times R + L \left(\frac{di_o}{dt} \right) + E ; \alpha \leq \omega t \leq \beta$$

The general expression for the output load current can be written as

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) - \frac{E}{R} + A e^{\frac{-t}{\tau}}$$

Where

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load Impedance.}$$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

The general expression for the o/p current can

be written as

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{E}{R} + A e^{\frac{-R}{L}t}$$

To find the value of the constant 'A' apply the initial conditions at $\omega t = \alpha$, load current $i_o = 0$, Equating the general expression for the load current to zero at $\omega t = \alpha$, we get

$$i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) - \frac{E}{R} + Ae^{\frac{-R}{L} \times \frac{\alpha}{\omega}}$$

We obtain the value of constant 'A' as

$$A = \left[\frac{E}{R} - \frac{V_m}{Z} \sin(\alpha - \phi) \right] e^{\frac{R}{\omega L} \alpha}$$

Substituting the value of the constant 'A' in the expression for the load current; we get the complete expression for the output load current as

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) - \frac{E}{R} + \left[\frac{E}{R} - \frac{V_m}{Z} \sin(\alpha - \phi) \right] e^{\frac{-R}{\omega L}(\omega t - \alpha)}$$

To Derive
An
Expression For The Average
Or
DC Load Voltage

$$V_{O(dc)} = \frac{1}{2\pi} \int_0^{2\pi} v_O \cdot d(\omega t)$$

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} v_O \cdot d(\omega t) + \int_{\alpha}^{\beta} v_O \cdot d(\omega t) + \int_{\beta}^{2\pi} v_O \cdot d(\omega t) \right]$$

$v_O = V_m \sin \omega t =$ Output load voltage for $\omega t = \alpha$ to β

$v_O = E$ for $\omega t = 0$ to α & for $\omega t = \beta$ to 2π

$$V_{O(dc)} = \frac{1}{2\pi} \left[\int_0^{\alpha} E \cdot d(\omega t) + \int_{\alpha}^{\beta} V_m \sin \omega t + \int_{\beta}^{2\pi} E \cdot d(\omega t) \right]$$

$$V_{O(dc)} = \frac{1}{2\pi} \left[E(\omega t) \Big|_0^\alpha + V_m (-\cos \omega t) \Big|_\alpha^\beta + E(\omega t) \Big|_\beta^{2\pi} \right]$$

$$V_{O(dc)} = \frac{1}{2\pi} \left[E(\alpha - 0) - V_m (\cos \beta - \cos \alpha) + E(2\pi - \beta) \right]$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [\cos \alpha - \cos \beta] + \frac{E}{2\pi} [2\pi - \beta + \alpha]$$

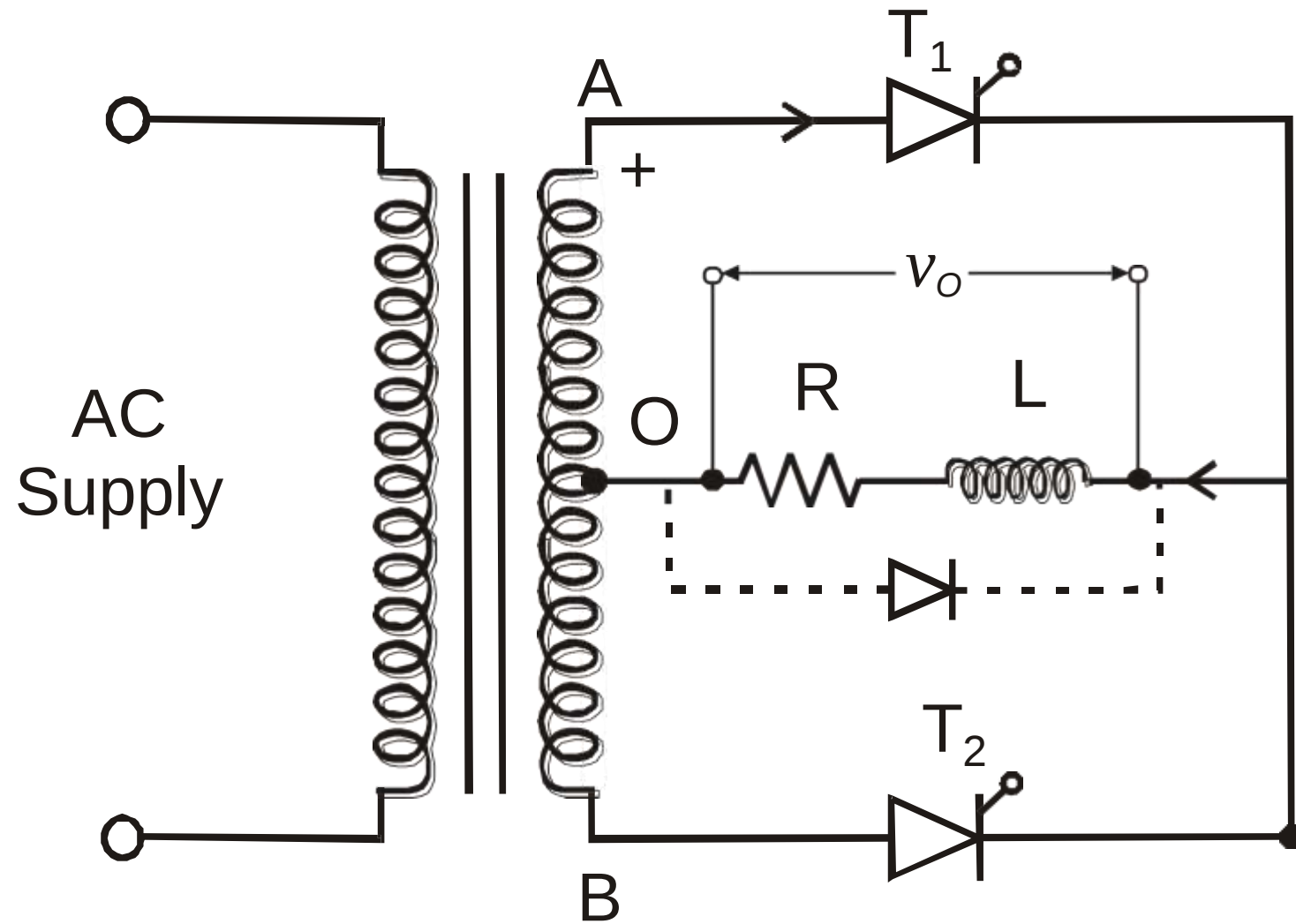
$$V_{O(dc)} = \frac{V_m}{2\pi} (\cos \alpha - \cos \beta) + \left[\frac{2\pi - (\beta - \alpha)}{2\pi} \right] E$$

Conduction angle of thyristor $\delta = (\beta - \alpha)$

RMS Output Voltage can be calculated
by using the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi} \left[\int_0^{2\pi} v_o^2 \cdot d(\omega t) \right]}$$

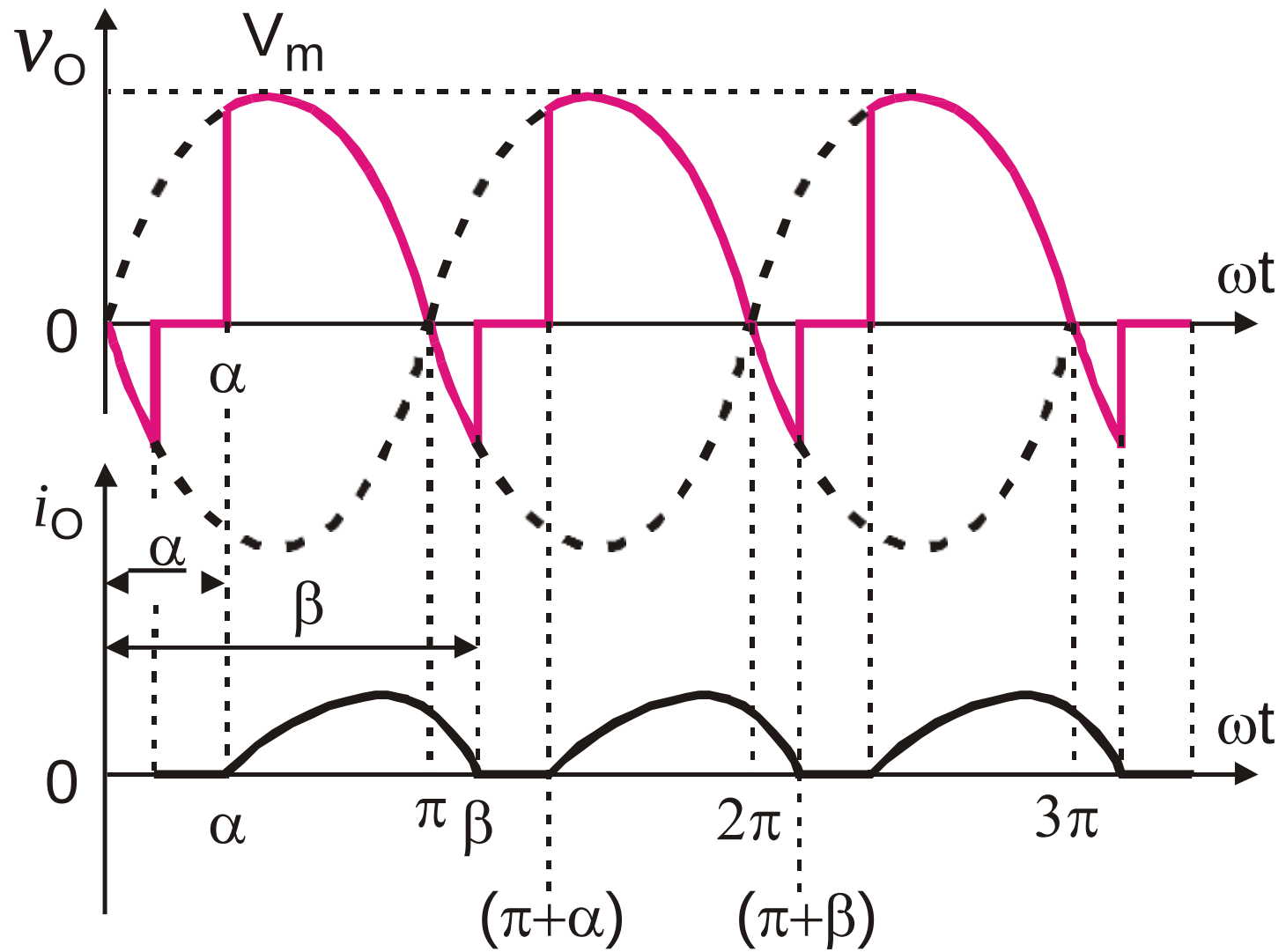
Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer



Discontinuous Load Current Operation without FWD

for

- $\pi < \beta < (\pi + \alpha)$



To Derive An Expression For
The Output
(Load) Current, During $\omega t = \alpha$ to β
When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,
we can write the equation,

$$L \left(\frac{di_o}{dt} \right) + Ri_o = V_m \sin \omega t ; \alpha \leq \omega t \leq \beta$$

General expression for the output current,

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) + A_1 e^{\frac{-t}{\tau}}$$

$$V_m = \sqrt{2}V_s = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A_1 e^{\frac{-R}{L} t}$$

Constant A_1 is calculated from

initial condition $i_o = 0$ at $\omega t = \alpha$; $t = \left(\frac{\alpha}{\omega} \right)$

$$i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

$$\therefore A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z} \sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Substituting the value of constant A_1 in the general expression for i_o

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right];$$

Where $\alpha \leq \omega t \leq \beta$

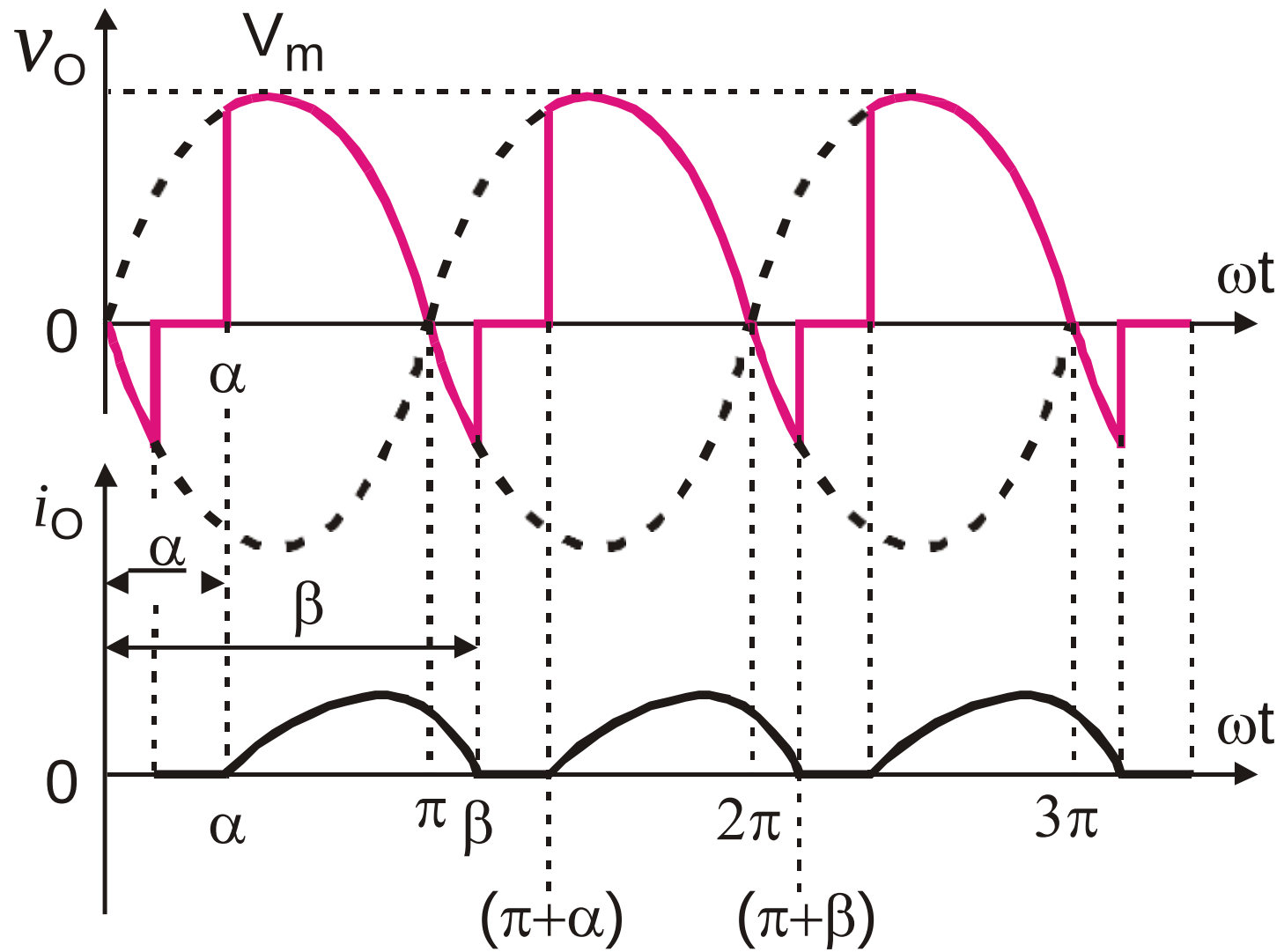
Extinction angle β can be calculated by using the condition that $i_o = 0$ at $\omega t = \beta$

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The
DC Output Voltage Of
A Single Phase Full Wave
Controlled Rectifier With RL Load
(Without FWD)



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{\beta} v_o . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible (i.e., $L \approx 0$)

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \pi)$$

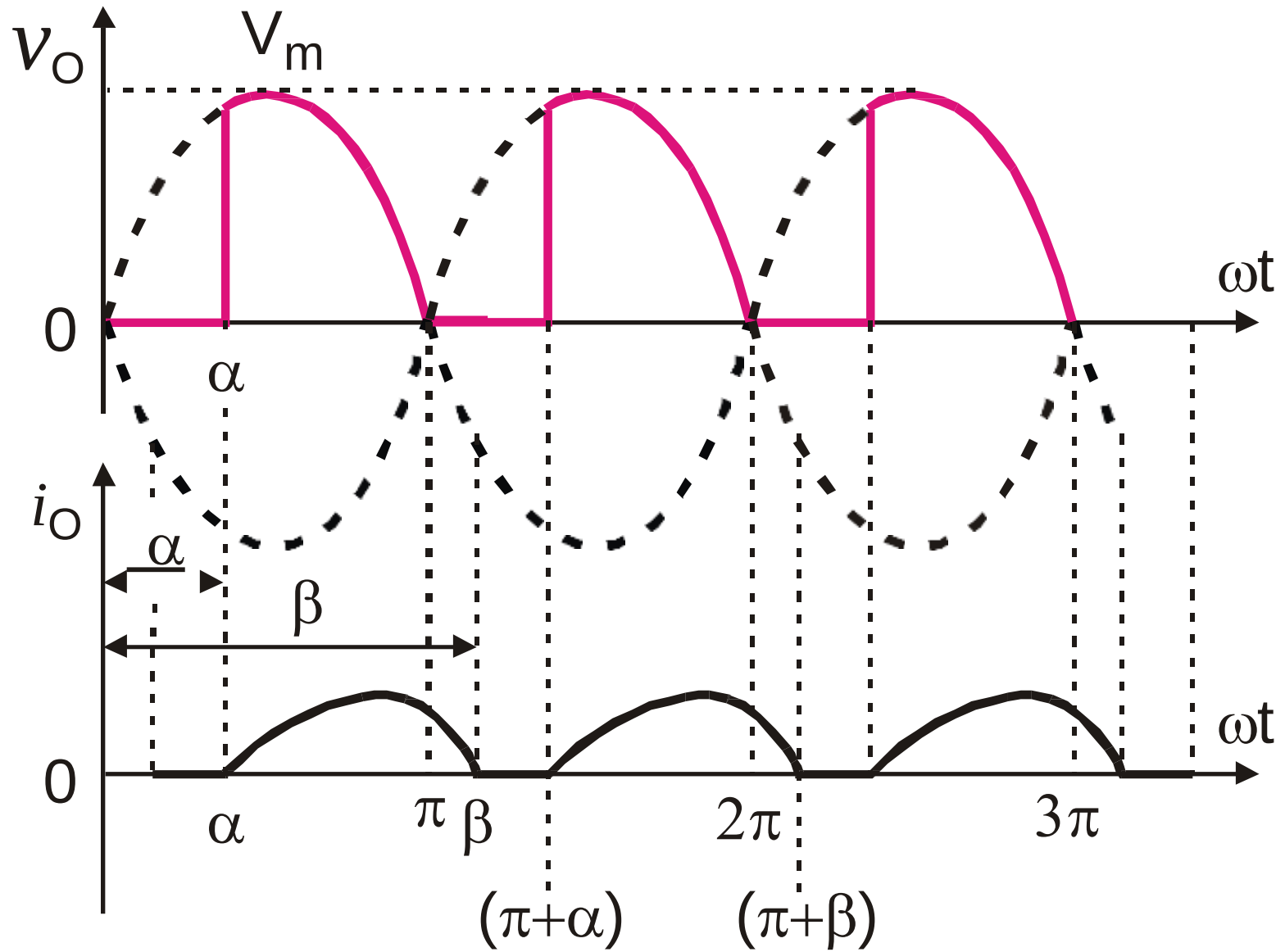
$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - (-1))$$

$$V_{O(dc)} = \frac{V_m}{\pi} (1 + \cos \alpha); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d (\omega t) \right]}$$

Discontinuous Load Current Operation with FWD



Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_o \approx 0$ during discontinuous load current.

To Derive an Expression
For The
DC Output Voltage For
A
Single Phase Full Wave Controlled
Rectifier
With RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d (\omega t)$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d (\omega t)$$

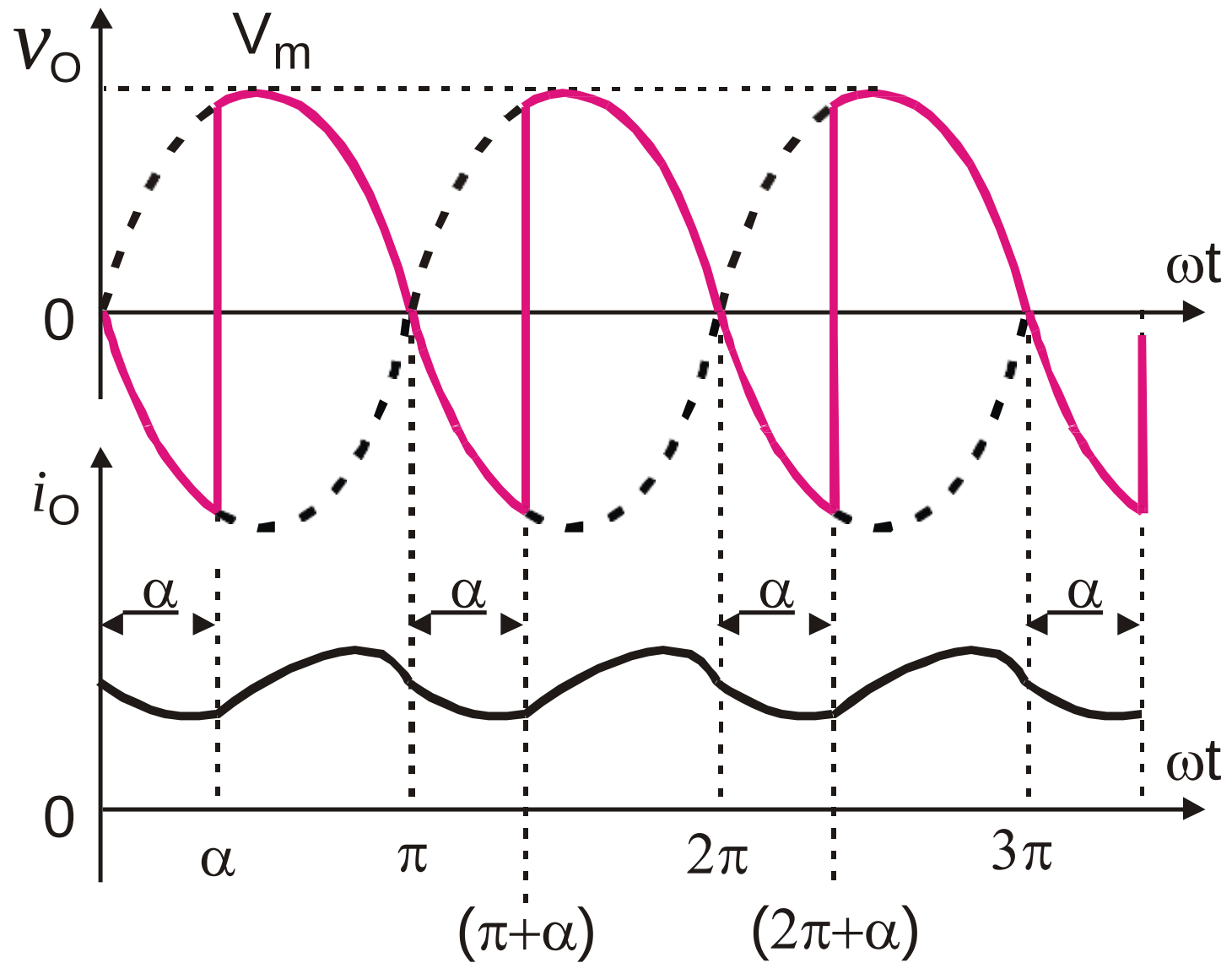
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] ; \cos \pi = -1$$

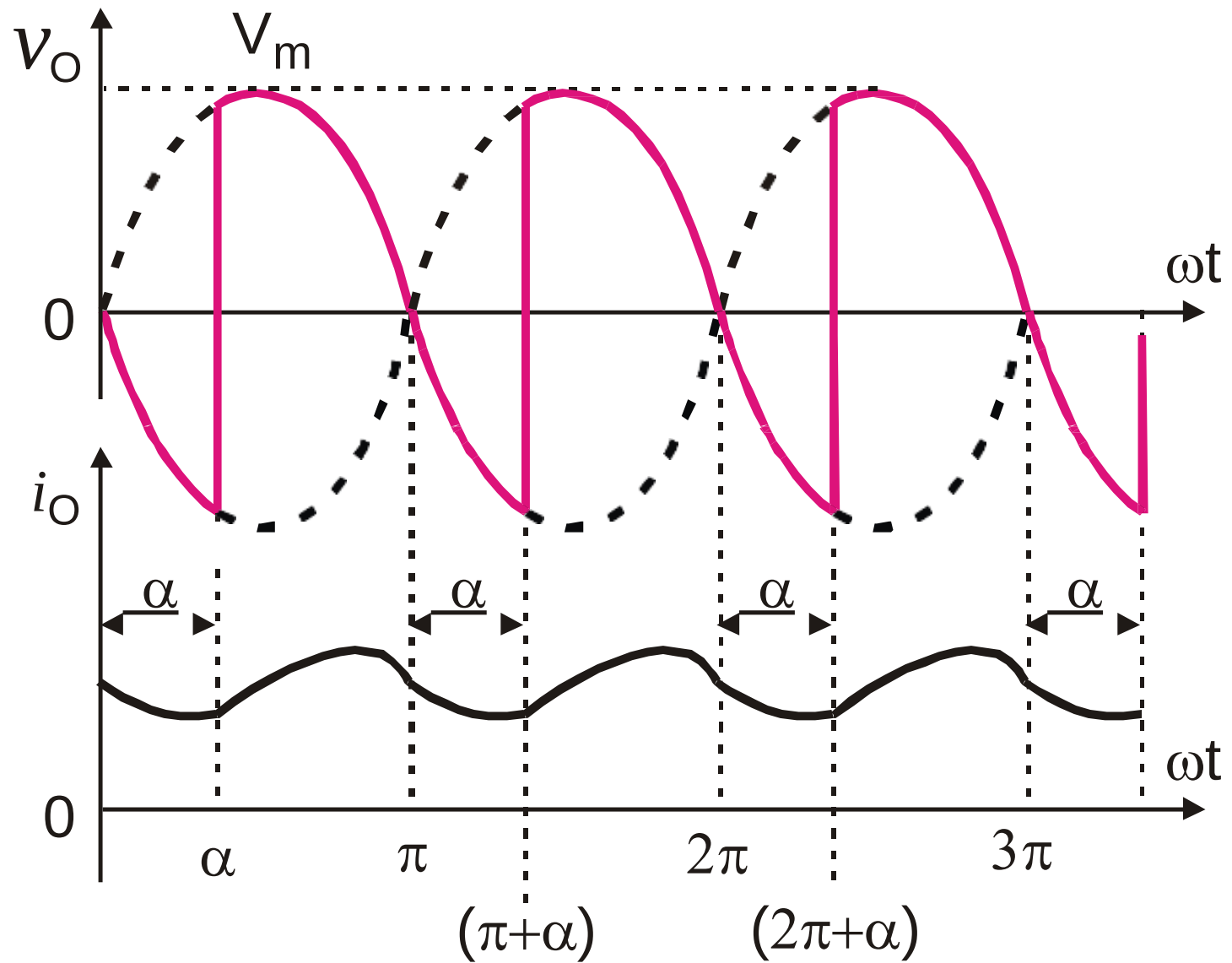
$$\therefore V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)



To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled
Rectifier For Continuous Current
Operation without FWD



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{(\pi + \alpha)} v_O . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi + \alpha)} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi + \alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi} \left[\cos \alpha - \cos (\pi + \alpha) \right] ;$$

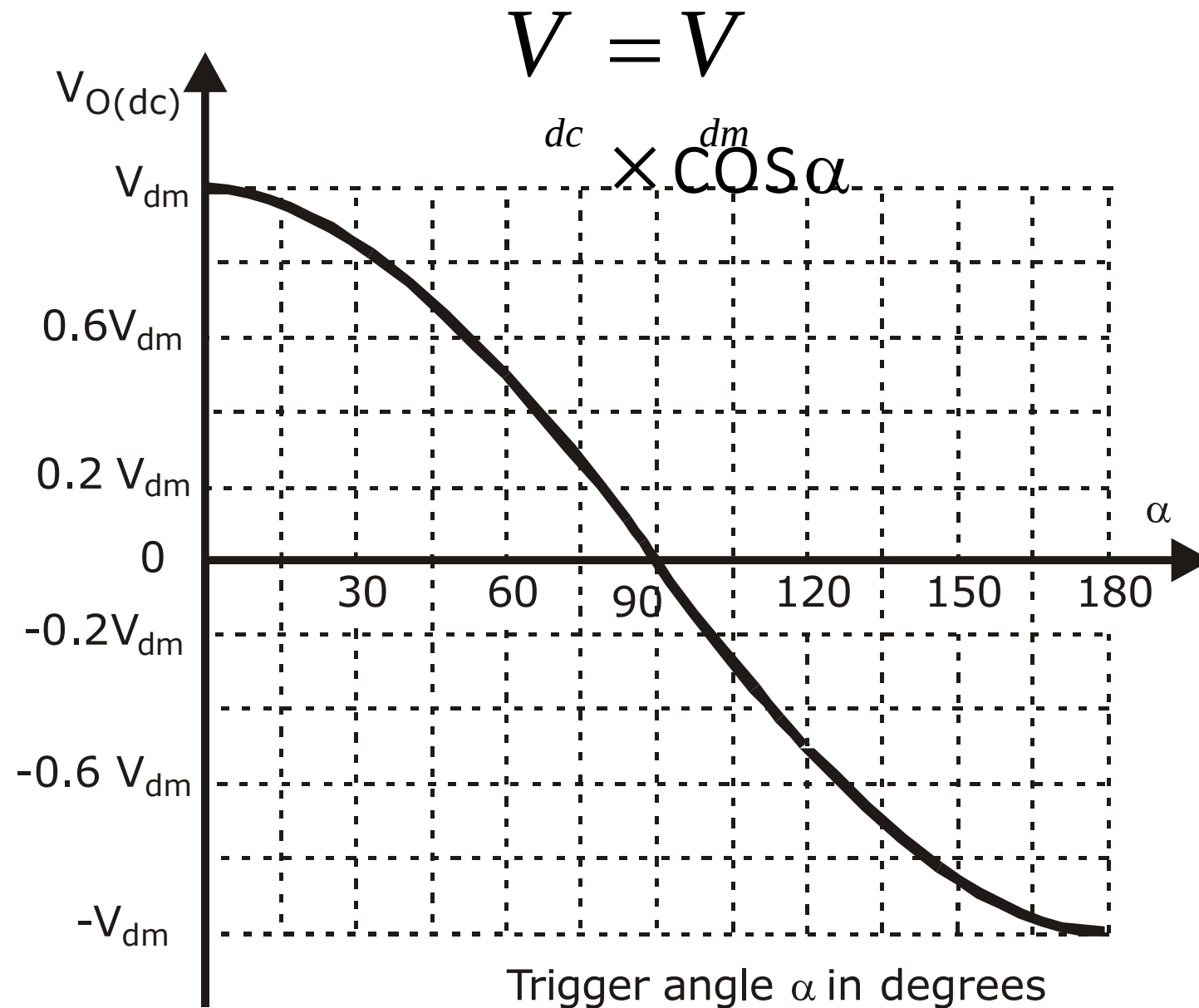
$$\cos (\pi + \alpha) = -\cos \alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} [\cos \alpha + \cos \alpha]$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

- By plotting $V_{O(dc)}$ *versus* α ,
we obtain the control characteristic of a
single phase full wave controlled rectifier
with RL load for continuous load current
operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi} \right)$	Maximum dc output voltage $V_{dc(max)} = V_{dm} = \left(\frac{2V_m}{\pi} \right)$
30^0	$0.866 V_{dm}$	$V_{dc} = V_{dm} \times \cos \alpha$
60^0	$0.5 V_{dm}$	
90^0	$0 V_{dm}$	
120^0	$-0.5 V_{dm}$	
150^0	$-0.866 V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi} \right)$	



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α , 0 to 90° (*i.e.*, $0 \leq \alpha \leq 90^\circ$);

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = (V_{dc} \times I_{dc})$ is positive

Converter operates as a **Controlled Rectifier**.

Power flow is from the ac source to the load.

For trigger angle α , 90° to 180°

$$(i.e., 90^\circ \leq \alpha \leq 180^\circ),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = (V_{dc} \times I_{dc}) \text{ is negative.}$$

In this case the converter operates
as a **Line Commutated Inverter**.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the
i/p source.

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase Full Wave Bridge Controlled Rectifier

Single Phase

Full Wave Bridge Controlled

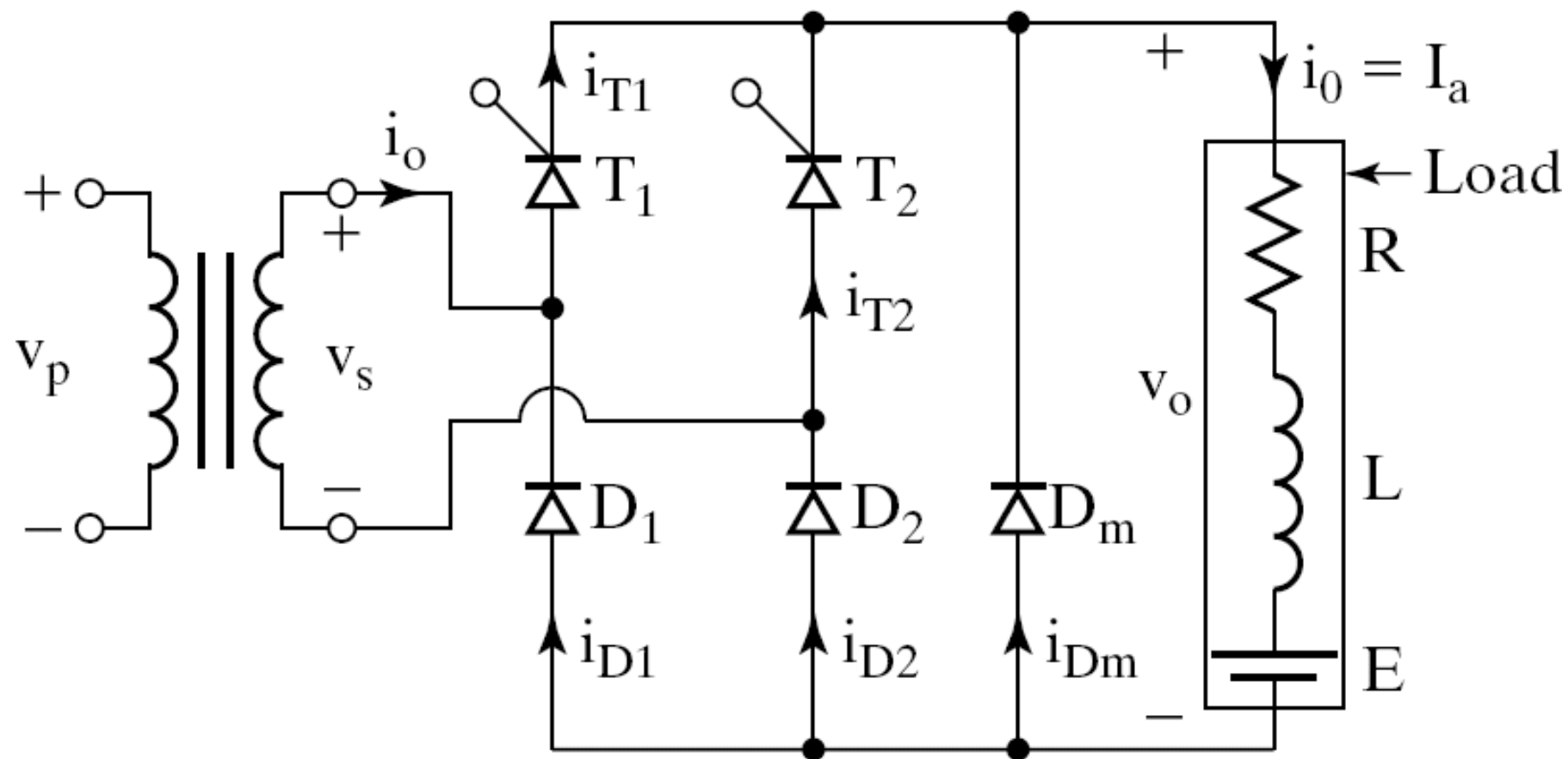
Rectifier

2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter
(Semi-Converter)
- Fully Controlled Bridge Converter
(Full Converter)

*The bridge full wave controlled rectifier does not
require a centre tapped transformer*

Single Phase
Full Wave Half Controlled Bridge
Converter
(Single Phase Semi Converter)



Trigger Pattern of Thyristors

Thyristor T_1 is triggered at

$$\omega t = \alpha, \text{ at } \omega t = (2\pi + \alpha), \dots$$

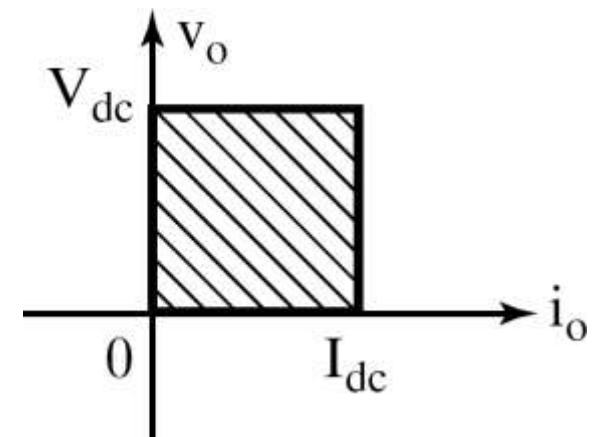
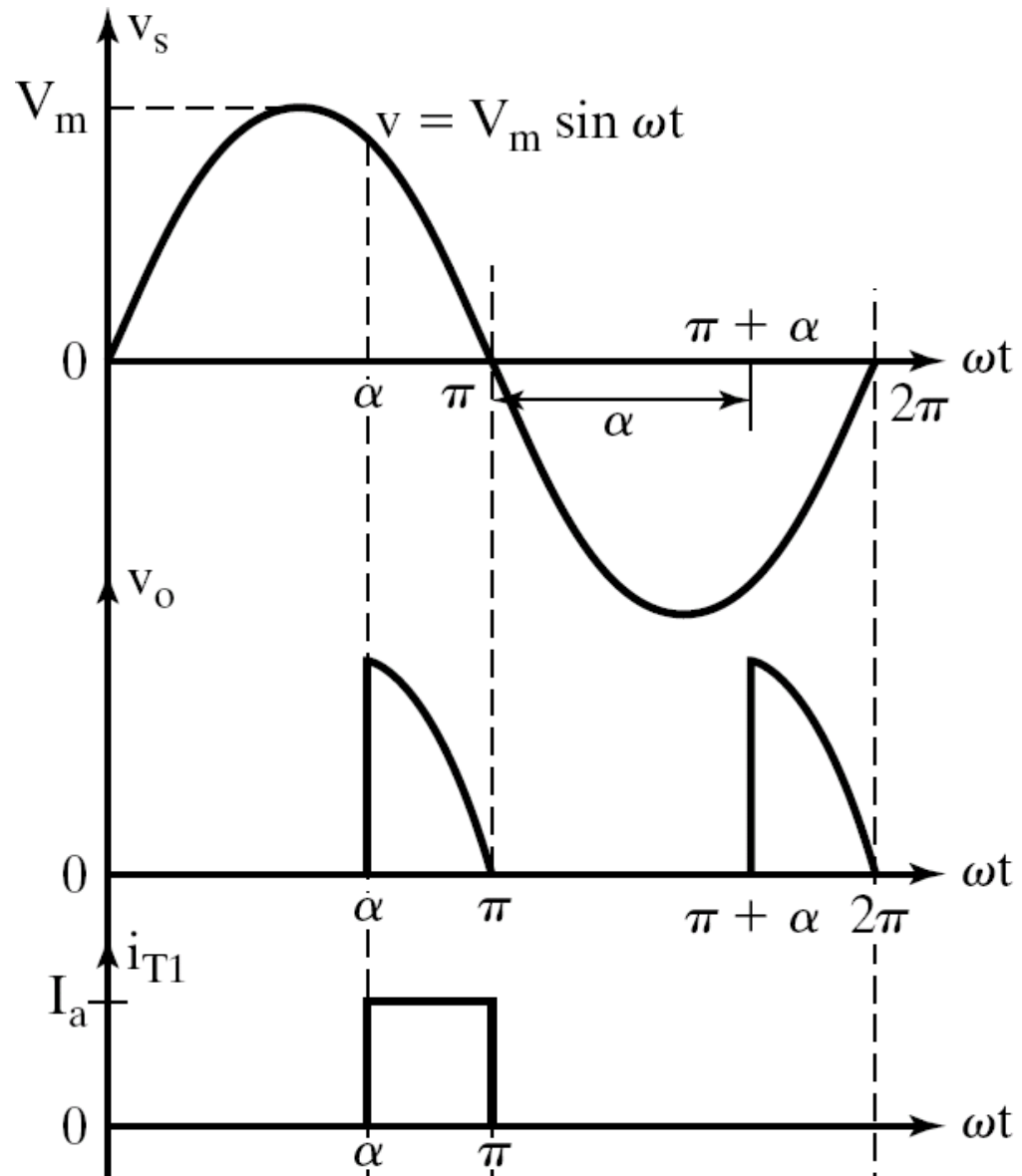
Thyristor T_2 is triggered at

$$\omega t = (\pi + \alpha), \text{ at } \omega t = (3\pi + \alpha), \dots$$

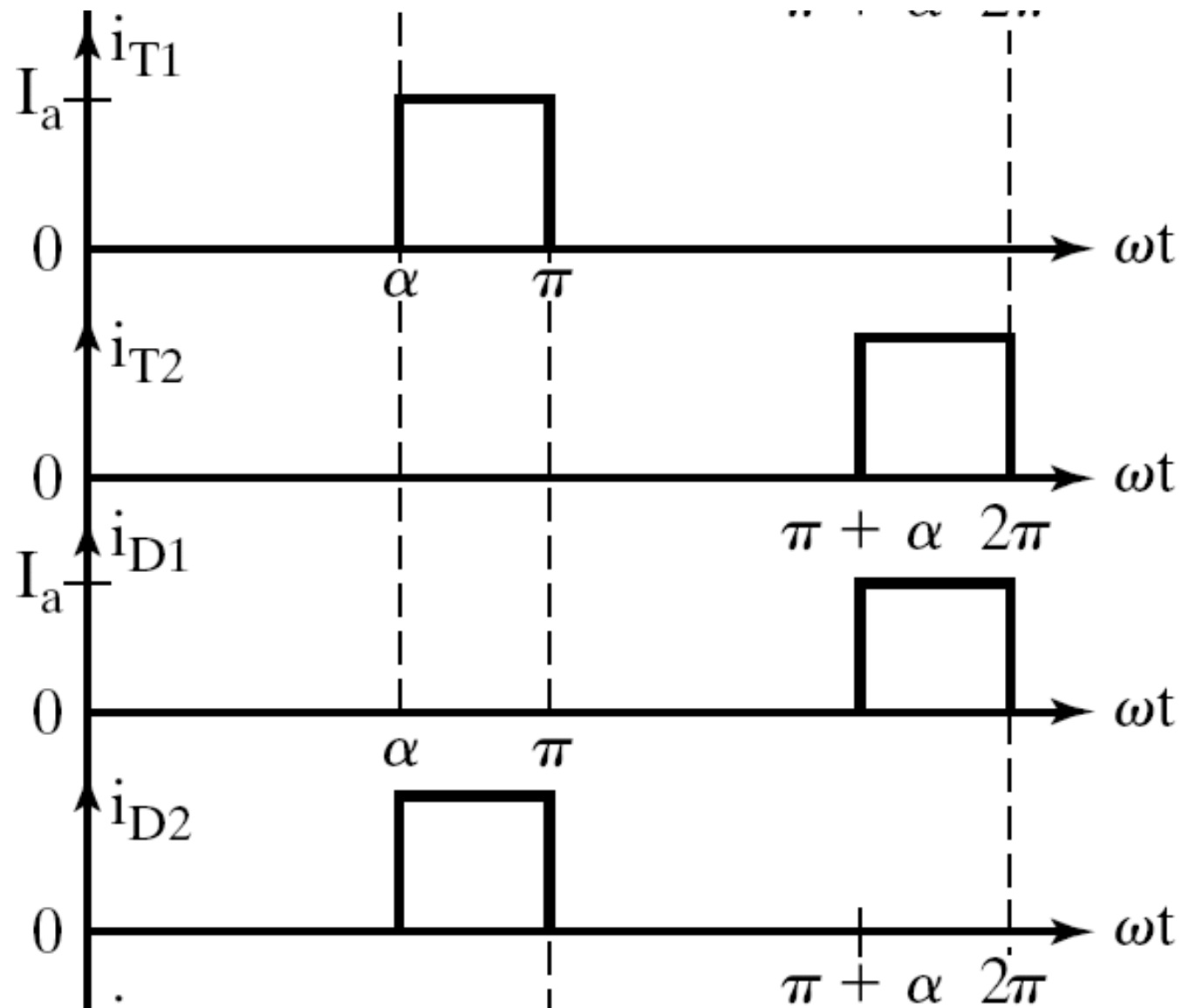
The time delay between the gating

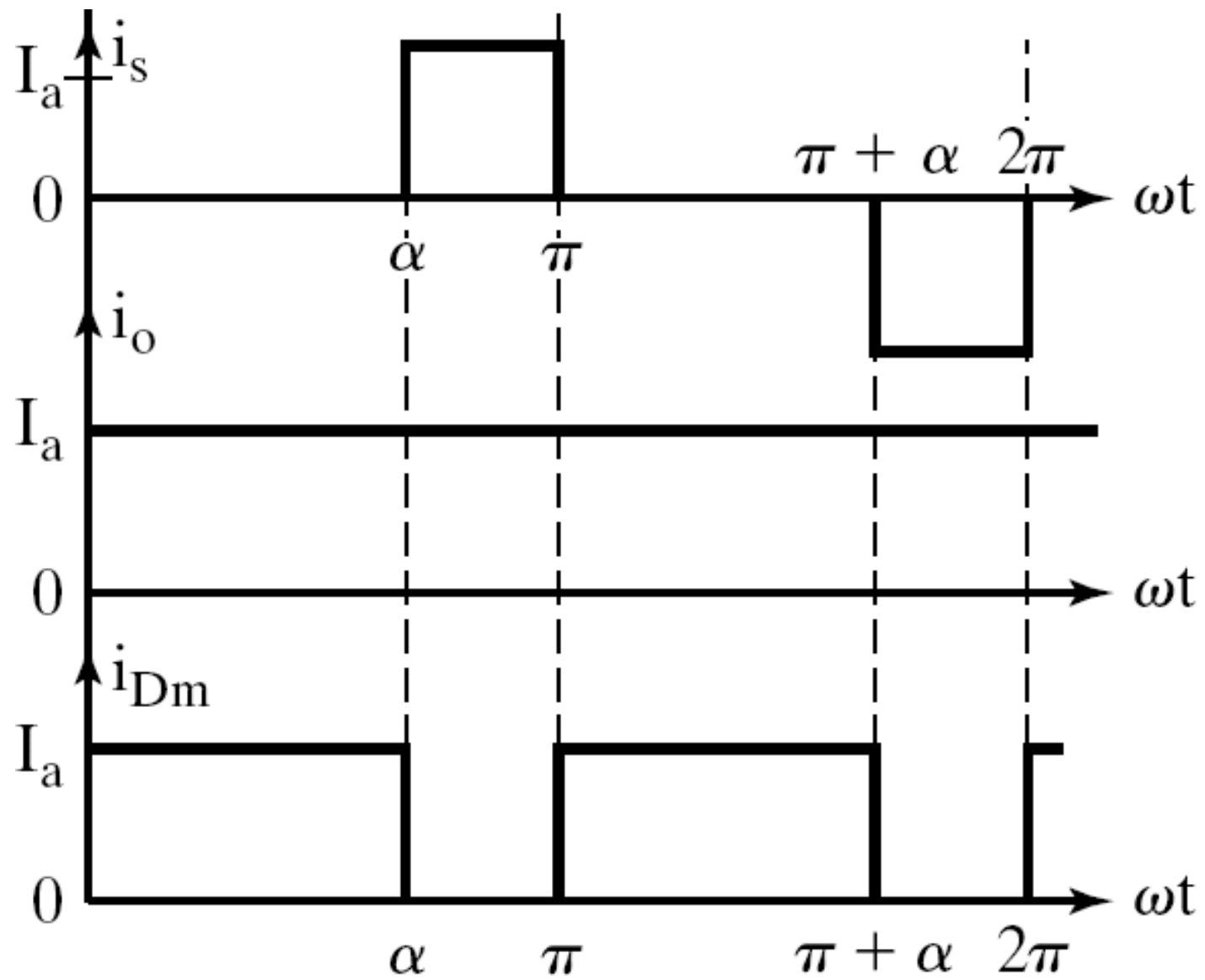
signals of T_1 & $T_2 = \pi$ radians or 180°

Waveforms of
single phase semi-converter
with general load & FWD
for $\alpha > 90^\circ$



Single Quadrant
Operation





Thyristor T_1 & D_1

conduct

from $\omega t = \alpha$ to

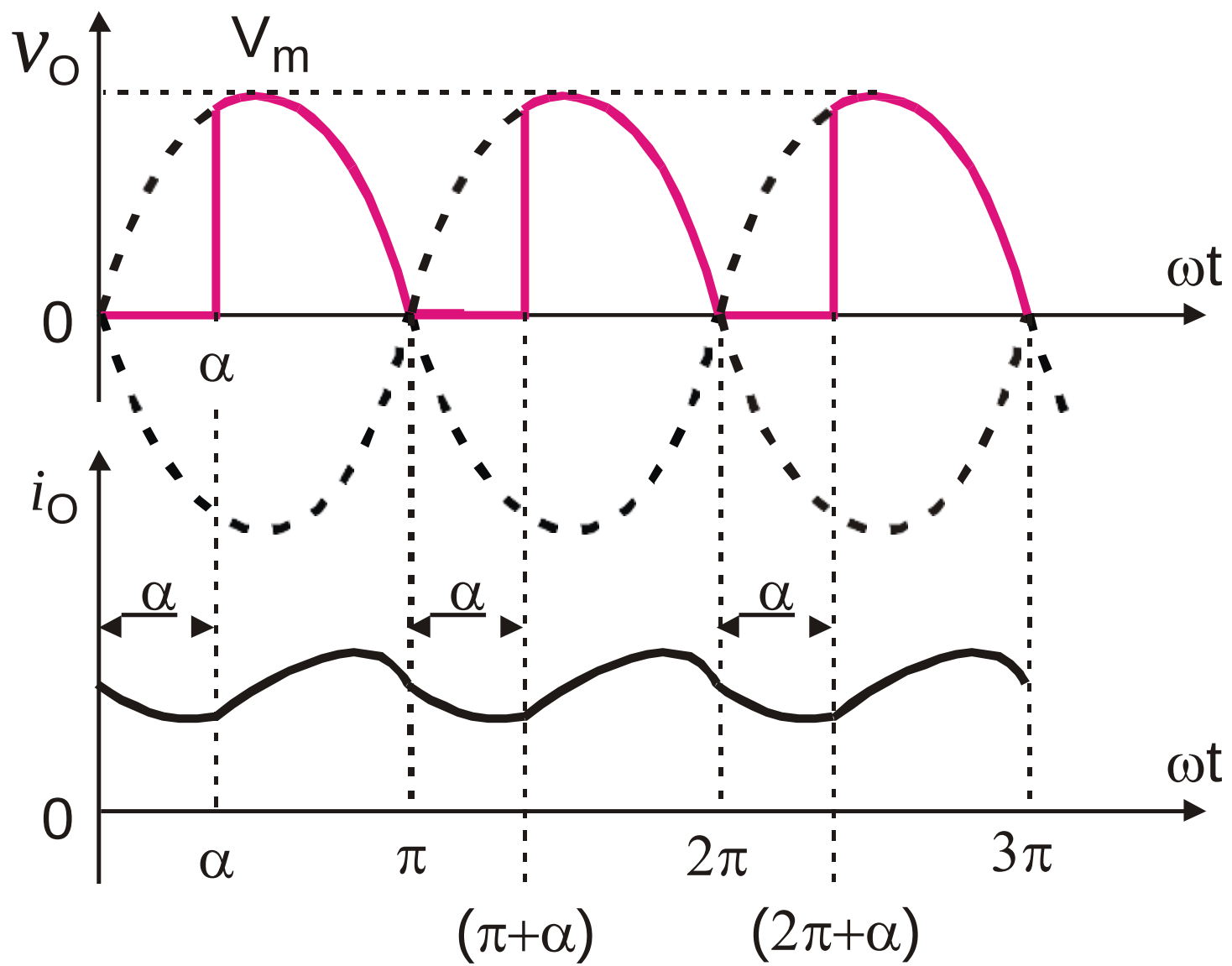
Thyristor T_2 & D_2 conduct

from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts during

$\omega t = 0$ to α , π to $(\pi + \alpha)$, ...

Load Voltage & Load Current
Waveform of Single Phase Semi
Converter for
 $\alpha < 90^\circ$
& Continuous load current operation



To Derive an Expression
For The
DC Output Voltage of
A
Single Phase Semi-Converter With
R,L, & E Load & FWD
For Continuous, Ripple Free Load
Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d (\omega t)$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] ; \cos \pi = -1$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π .

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi} (1 + \cos \alpha)}{\left(\frac{2V_m}{\pi} \right)} = \frac{1}{2} (1 + \cos \alpha)$$

RMS O/P Voltage $V_{O(RMS)}$

$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}}$$

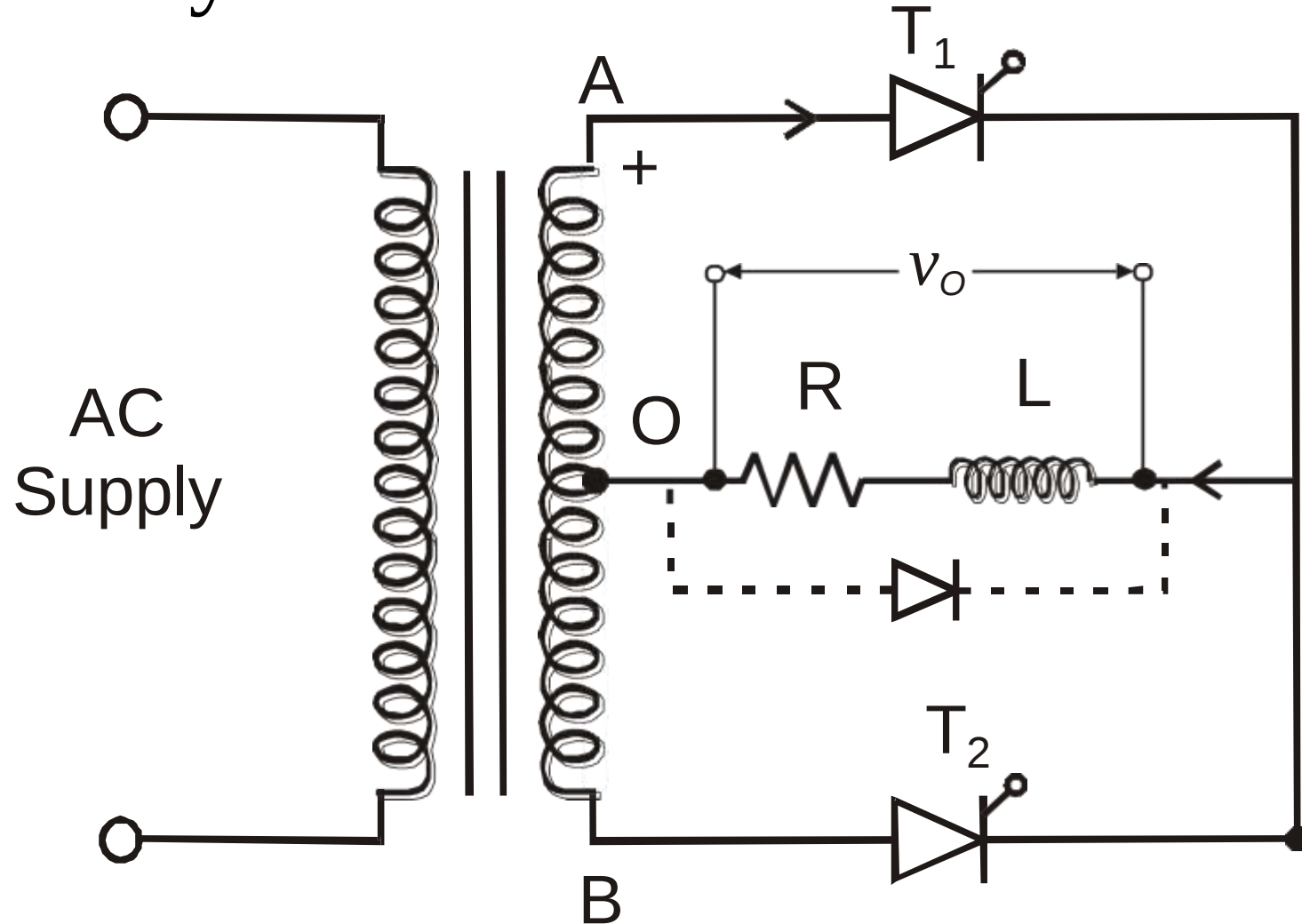
$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Single Phase Full Wave Controlled Rectifier

Single Phase Full Wave Controlled Rectifier Using A Center Tapped Transformer

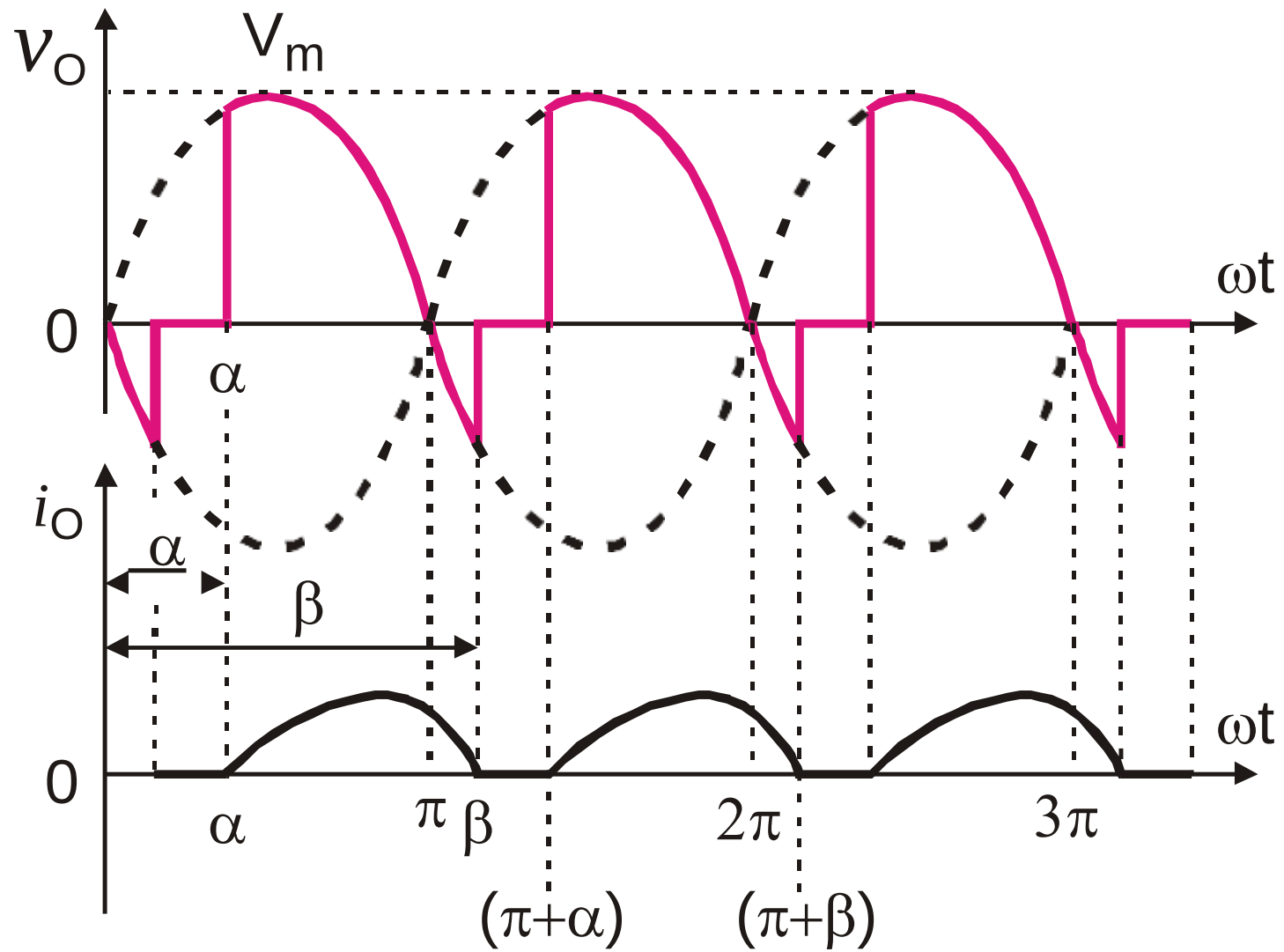
Single Phase Midpoint type Fully controlled Rectifier



Discontinuous Load
Current Operation
without FWD

for

- $\pi < \beta < (\pi + \alpha)$



To Derive An Expression For
The Output
(Load) Current, During $\omega t = \alpha$ to β
When Thyristor T_1 Conducts

Assuming T_1 is triggered $\omega t = \alpha$,
we can write the equation,

$$L \left(\frac{di_o}{dt} \right) + Ri_o = V_m \sin \omega t ; \alpha \leq \omega t \leq \beta$$

General expression for the output current,

$$i_o = \frac{V_m}{Z} \sin (\omega t - \phi) + A_1 e^{\frac{-t}{\tau}}$$

$$V_m = \sqrt{2}V_s = \text{maximum supply voltage.}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right) = \text{Load impedance angle.}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

$\therefore$ general expression for the output load current

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + A_1 e^{\frac{-R}{L} t}$$

Constant A_1 is calculated from

initial condition $i_o = 0$ at $\omega t = \alpha$; $t = \left(\frac{\alpha}{\omega} \right)$

$$i_o = 0 = \frac{V_m}{Z} \sin(\alpha - \phi) + A_1 e^{\frac{-R}{L} t}$$

$$\therefore A_1 e^{\frac{-R}{L} t} = \frac{-V_m}{Z} \sin(\alpha - \phi)$$

We get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

Substituting the value of constant A_1 in the general expression for i_o

$$i_o = \frac{V_m}{Z} \sin(\omega t - \phi) + e^{\frac{-R}{\omega L}(\omega t - \alpha)} \left[\frac{-V_m}{Z} \sin(\alpha - \phi) \right]$$

$\therefore$ we obtain the final expression for the inductive load current

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right];$$

Where $\alpha \leq \omega t \leq \beta$

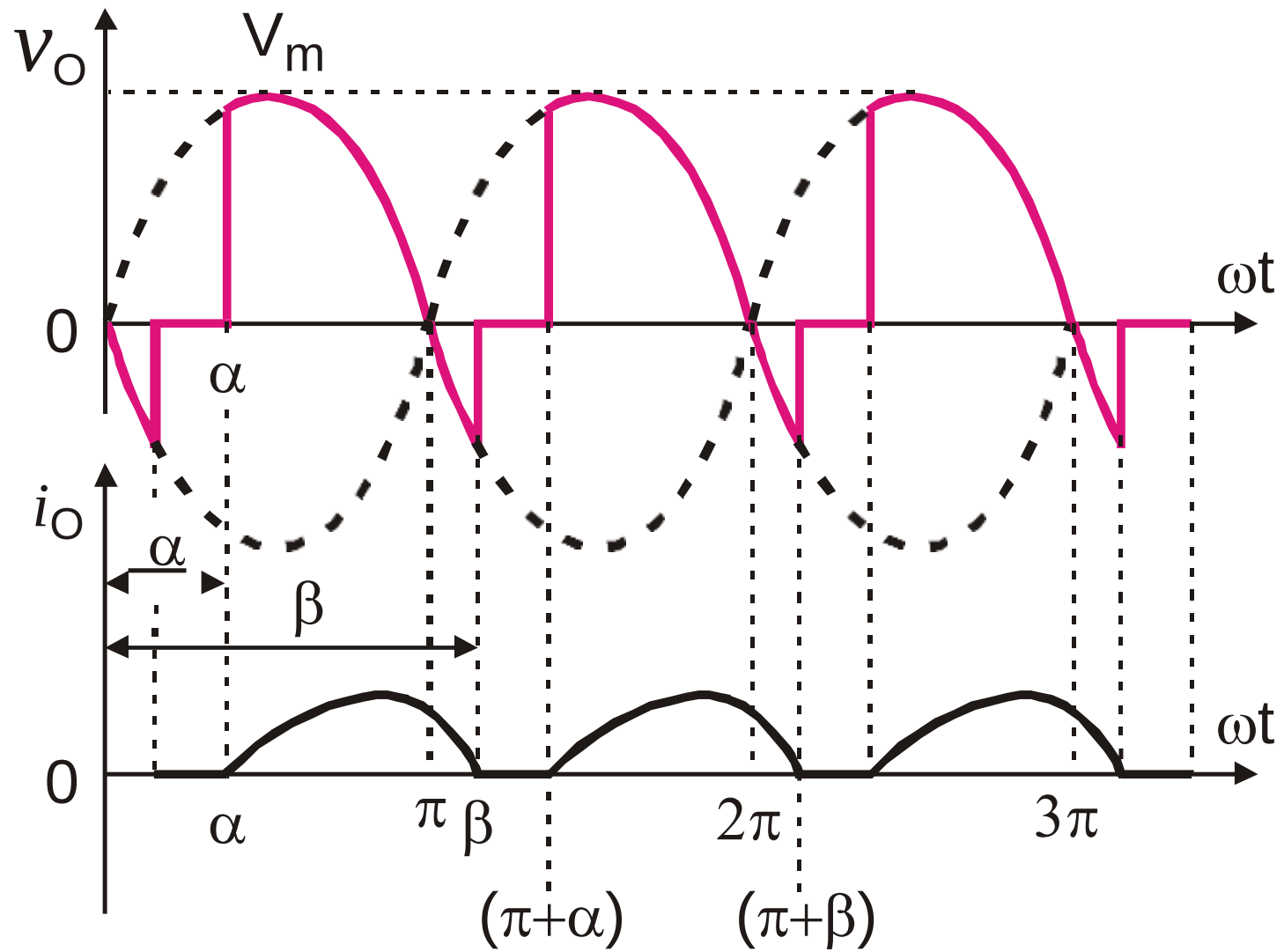
Extinction angle β can be calculated by using the condition that $i_o = 0$ at $\omega t = \beta$

$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] = 0$$

$$\therefore \sin(\beta - \phi) = e^{\frac{-R}{\omega L}(\beta - \alpha)} \times \sin(\alpha - \phi)$$

β can be calculated by solving the above eqn.

To Derive An Expression For The DC Output
Voltage Of
A Single Phase Full Wave Controlled
Rectifier With RL Load
(Without FWD)



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{\beta} v_o . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{\beta} \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (\cos \alpha - \cos \beta)$$

When the load inductance is negligible (i.e., $L \approx 0$)

Extinction angle $\beta = \pi$ radians

Hence the average or dc output voltage for R load

$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - \cos \pi)$$

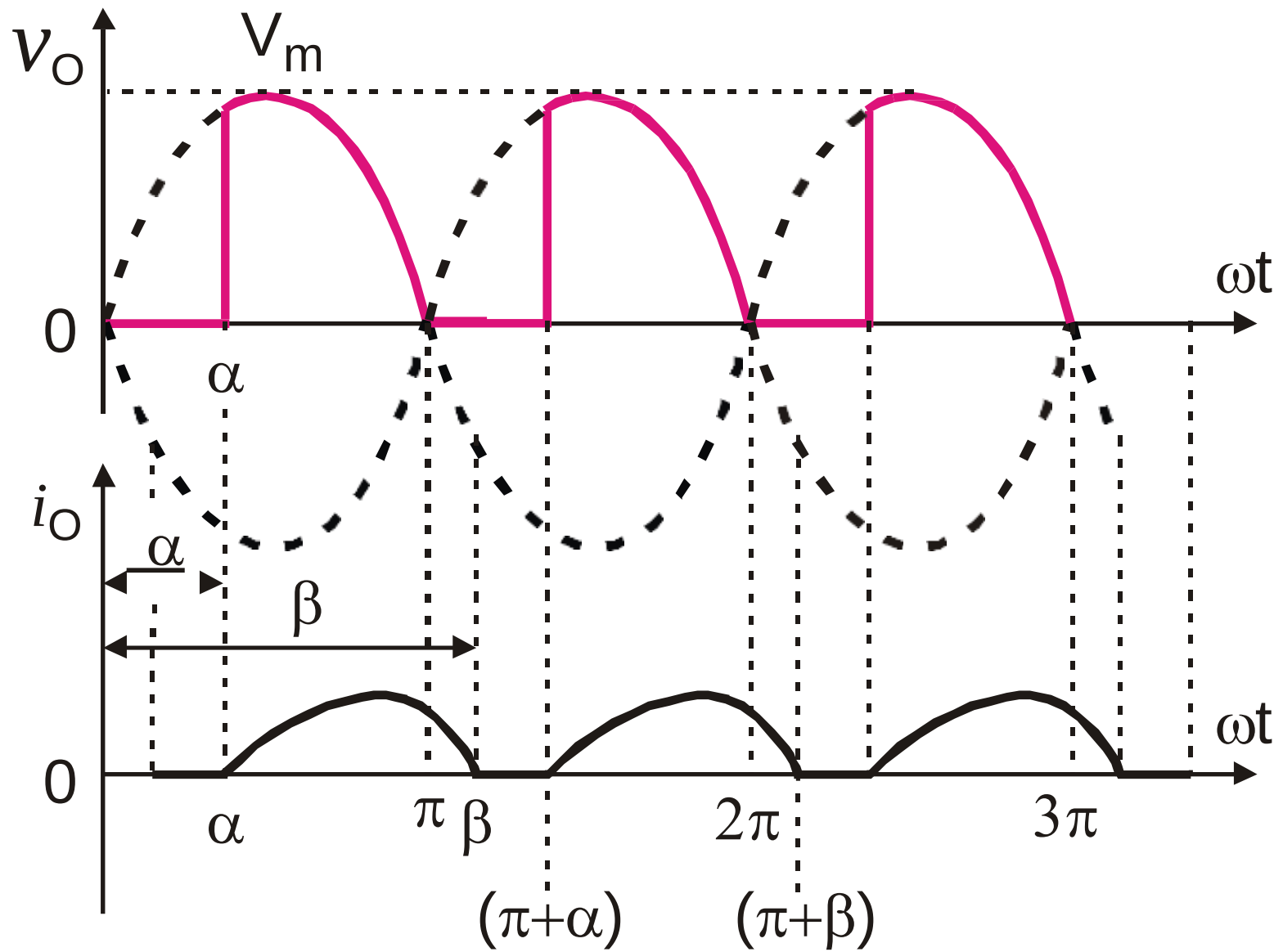
$$V_{O(dc)} = \frac{V_m}{\pi} (\cos \alpha - (-1))$$

$$V_{O(dc)} = \frac{V_m}{\pi} (1 + \cos \alpha); \text{ for R load, when } \beta = \pi$$

To calculate the RMS output voltage we use the expression

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi} \left[\int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]}$$

Discontinuous Load Current Operation with FWD



Thyristor T_1 is triggered at $\omega t = \alpha$;

T_1 conducts from $\omega t = \alpha$ to π

Thyristor T_2 is triggered at $\omega t = (\pi + \alpha)$;

T_2 conducts from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts from $\omega t = \pi$ to β &

$v_o \approx 0$ during discontinuous load current.

To Derive an Expression For The DC Output
Voltage For A Single Phase Full Wave
Controlled Rectifier With
RL Load & FWD

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d (\omega t)$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d (\omega t)$$

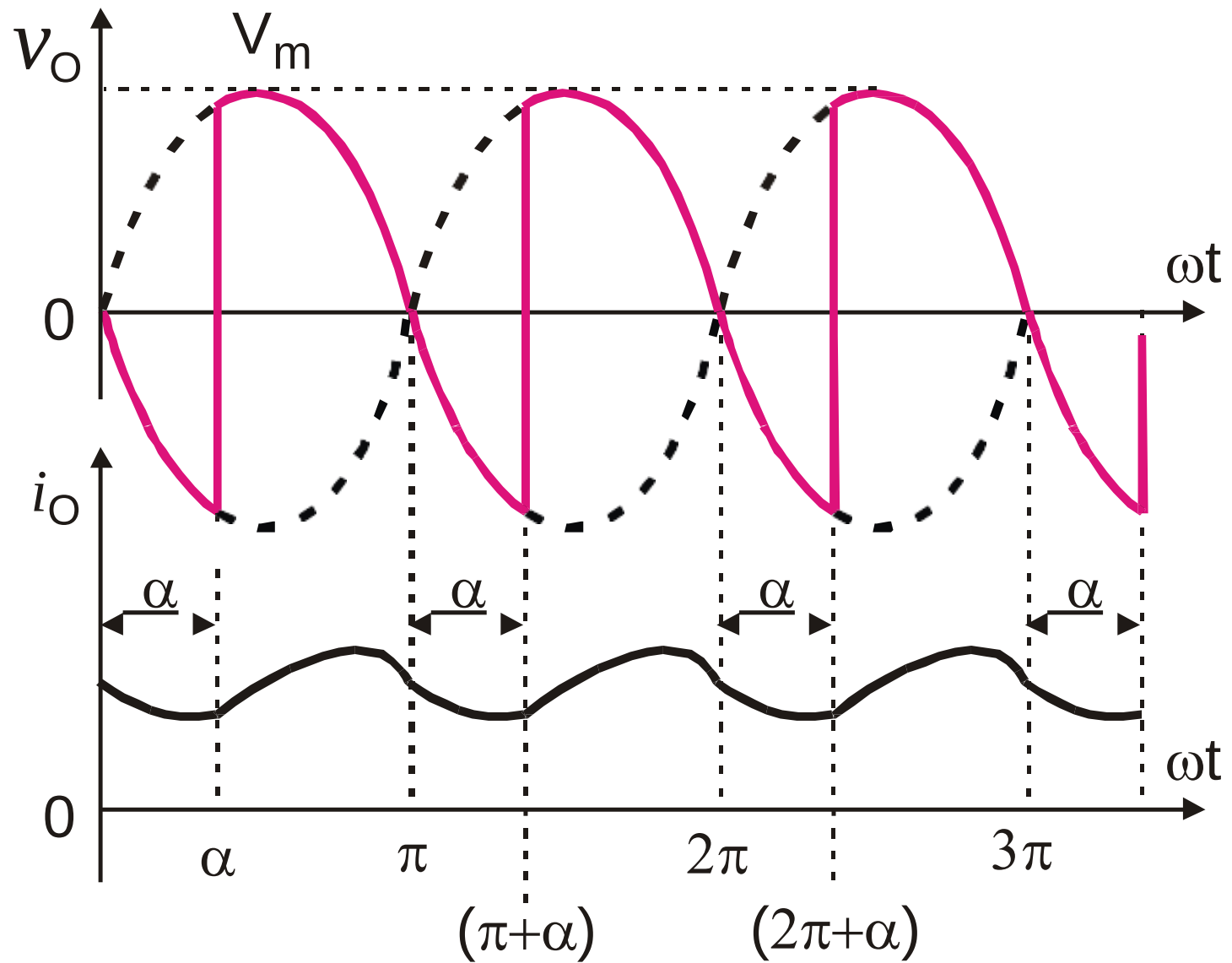
$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] ; \cos \pi = -1$$

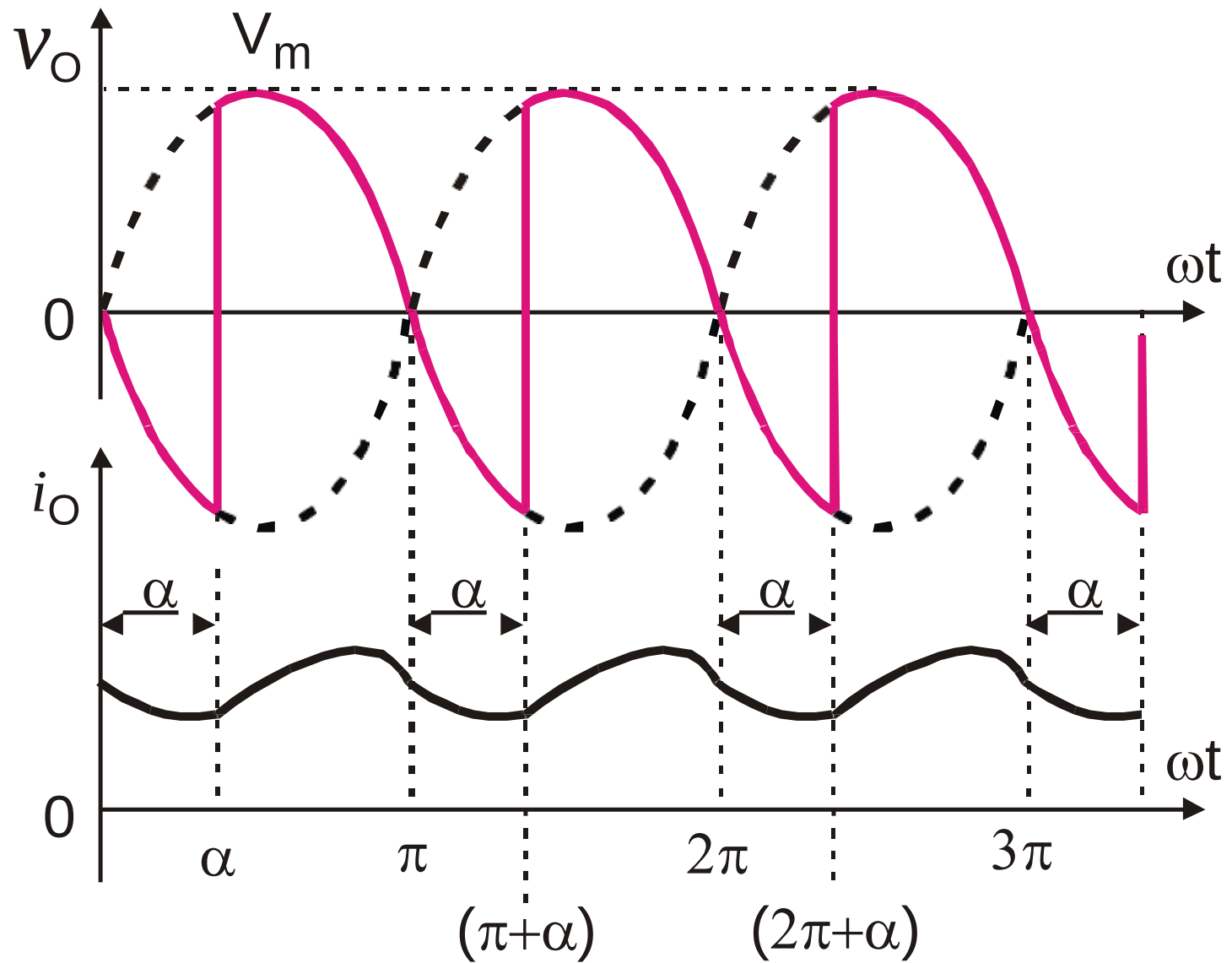
$$\therefore V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

- The load current is discontinuous for low values of load inductance and for large values of trigger angles.
- For large values of load inductance the load current flows continuously without falling to zero.
- Generally the load current is continuous for large load inductance and for low trigger angles.

Continuous Load Current Operation (Without FWD)



To Derive
An Expression For
Average / DC Output Voltage
Of
Single Phase Full Wave Controlled
Rectifier For Continuous Current
Operation without FWD



$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t = \alpha}^{(\pi + \alpha)} v_O . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \left[\int_{\alpha}^{(\pi + \alpha)} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \Big/_{\alpha}^{(\pi + \alpha)} \right]$$

$$V_{O(dc)} = V_{dc}$$

$$= \frac{V_m}{\pi} \left[\cos \alpha - \cos (\pi + \alpha) \right] ;$$

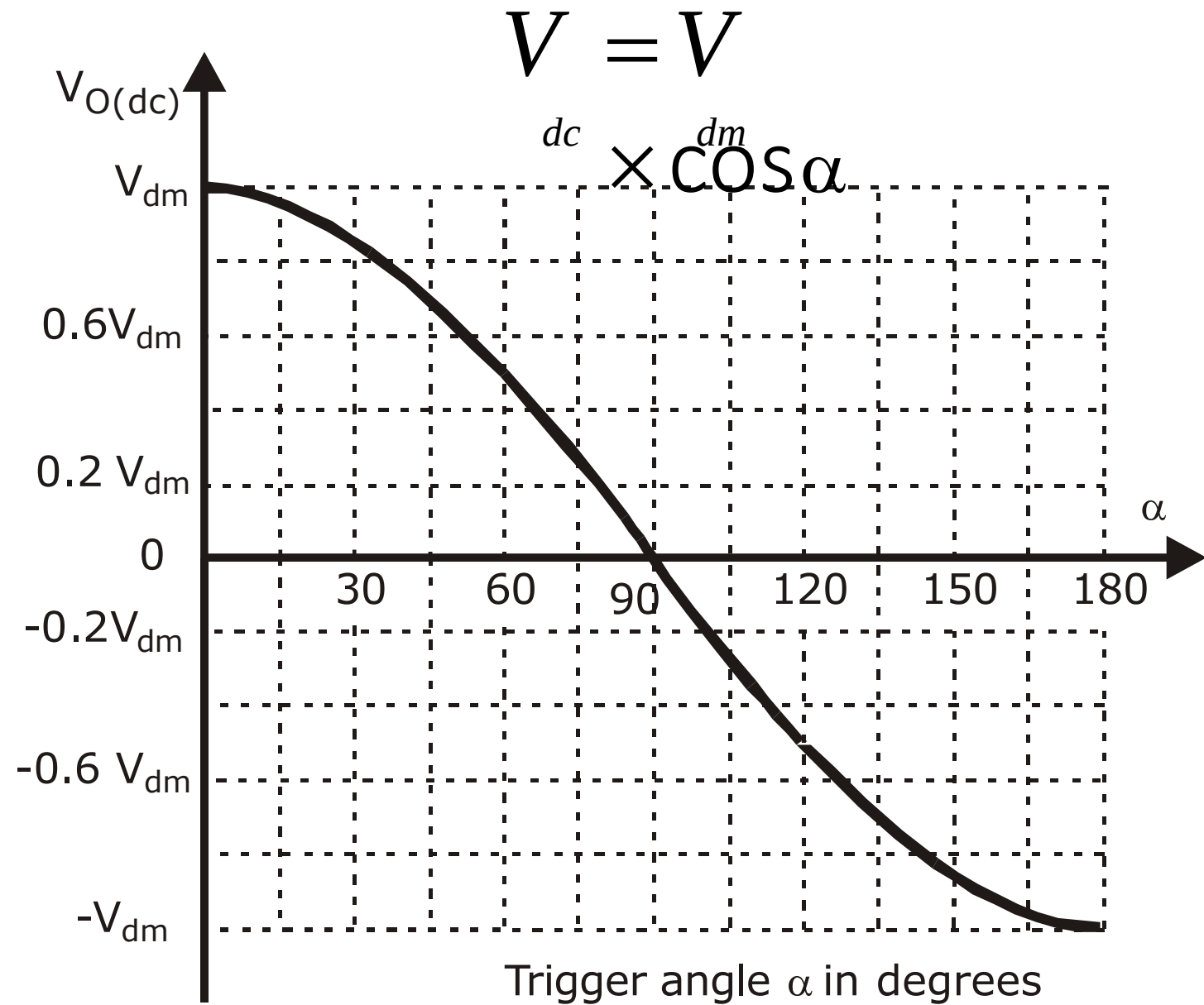
$$\cos (\pi + \alpha) = -\cos \alpha$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} [\cos \alpha + \cos \alpha]$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

- By plotting $V_{O(dc)}$ *versus* α ,
we obtain the control characteristic of a
single phase full wave controlled rectifier
with RL load for continuous load current
operation without FWD

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi} \right)$	Maximum dc output voltage $V_{dc(max)} = V_{dm} = \left(\frac{2V_m}{\pi} \right)$
30^0	$0.866 V_{dm}$	$V_{dc} = V_{dm} \times \cos \alpha$
60^0	$0.5 V_{dm}$	
90^0	$0 V_{dm}$	
120^0	$-0.5 V_{dm}$	
150^0	$-0.866 V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi} \right)$	



By varying the trigger angle we can vary the output dc voltage across the load. Hence we can control the dc output power flow to the load.

For trigger angle α , 0 to 90° (*i.e., $0 \leq \alpha \leq 90^\circ$*);

$\cos\alpha$ is positive and hence V_{dc} is positive

V_{dc} & I_{dc} are positive ; $P_{dc} = (V_{dc} \times I_{dc})$ is positive

Converter operates as a **Controlled Rectifier.**

Power flow is from the ac source to the load.

For trigger angle α , 90° to 180°

$$(i.e., 90^\circ \leq \alpha \leq 180^\circ),$$

$\cos\alpha$ is negative and hence

V_{dc} is negative; I_{dc} is positive ;

$$P_{dc} = (V_{dc} \times I_{dc}) \text{ is negative.}$$

In this case the converter operates
as a **Line Commutated Inverter**.

Power flows from the load ckt. to the i/p ac source.

The inductive load energy is fed back to the
i/p source.

Single Phase Full Wave Bridge Controlled Rectifier

Drawbacks Of Full Wave Controlled Rectifier With Centre Tapped Transformer

- We require a centre tapped transformer which is quite heavier and bulky.
- Cost of the transformer is higher for the required dc output voltage & output power.
- Hence full wave bridge converters are preferred.

Single Phase Full Wave Bridge Controlled Rectifier

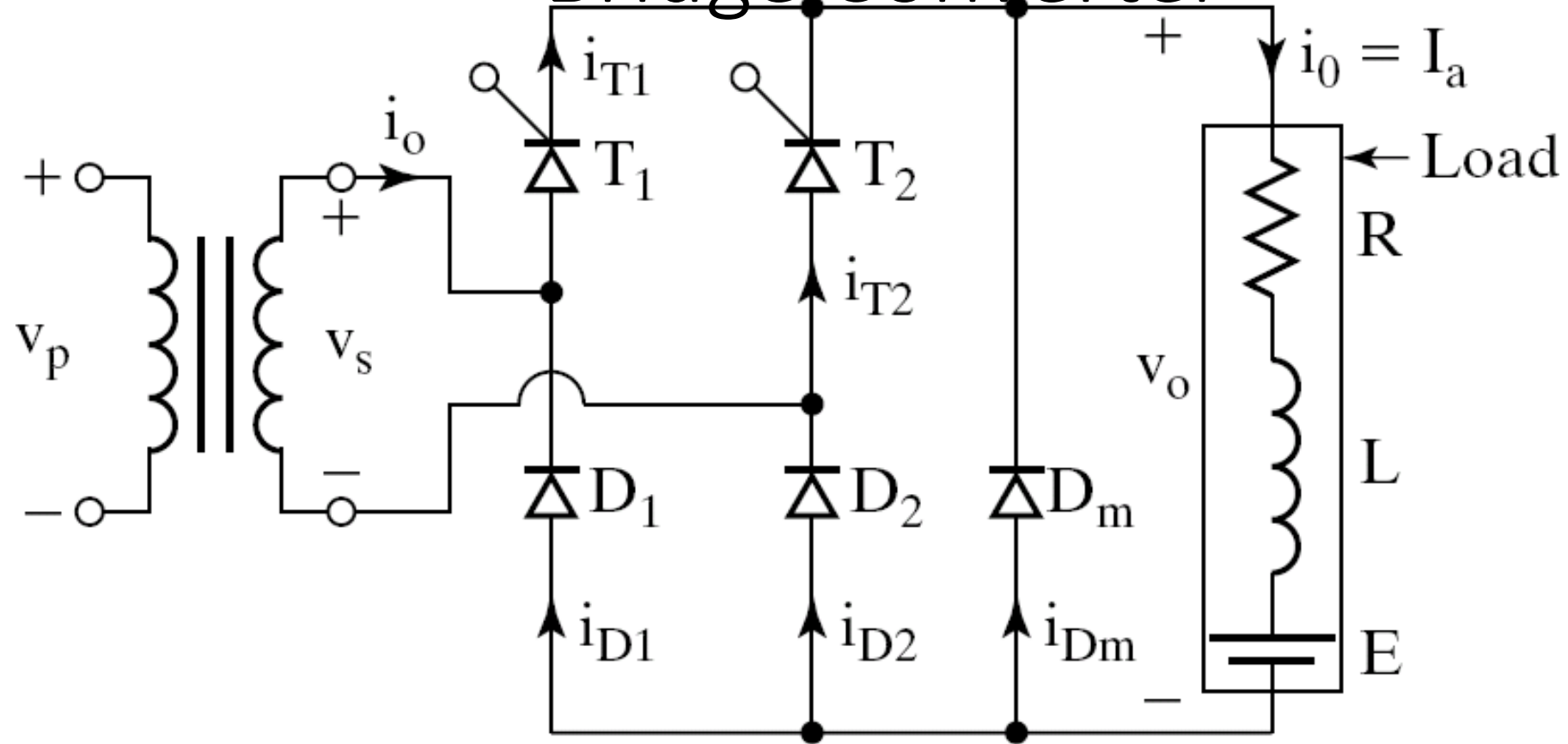
2 types of FW Bridge Controlled Rectifiers are

- Half Controlled Bridge Converter
(Semi-Converter)
- Fully Controlled Bridge Converter
(Full Converter)

The bridge full wave controlled rectifier does not require a centre tapped transformer

Single Phase
Full Wave Half Controlled Bridge
Converter
(Single Phase Semi Converter)

Single Phase Full Wave Half Controlled Bridge Converter



Trigger Pattern of Thyristors

Thyristor T_1 is triggered at

$$\omega t = \alpha, \text{ at } \omega t = (2\pi + \alpha), \dots$$

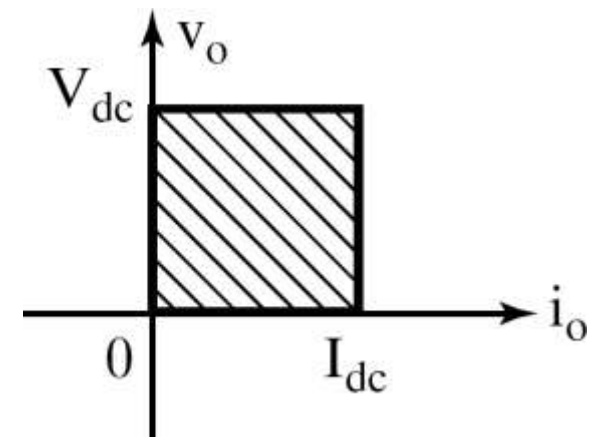
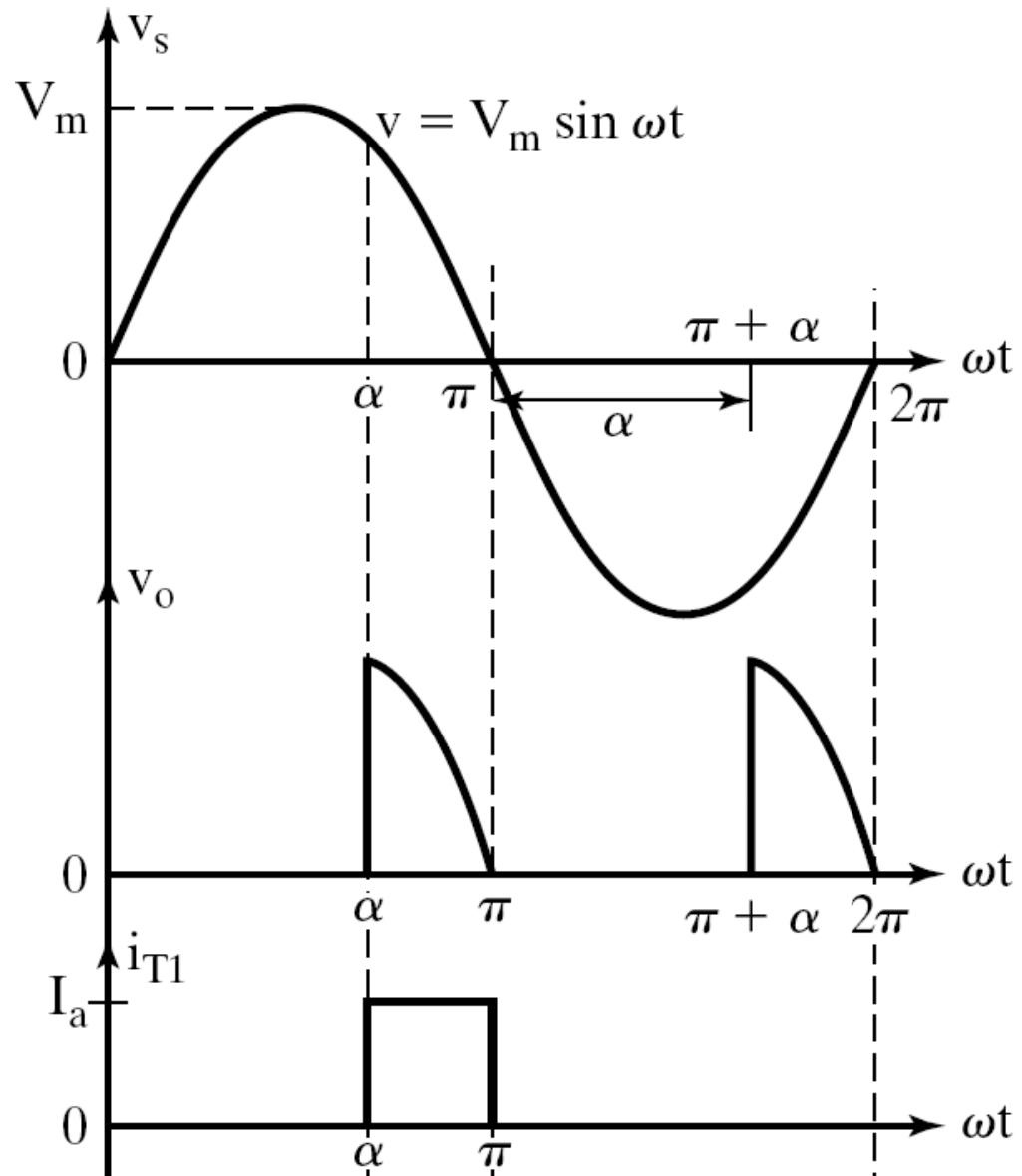
Thyristor T_2 is triggered at

$$\omega t = (\pi + \alpha), \text{ at } \omega t = (3\pi + \alpha), \dots$$

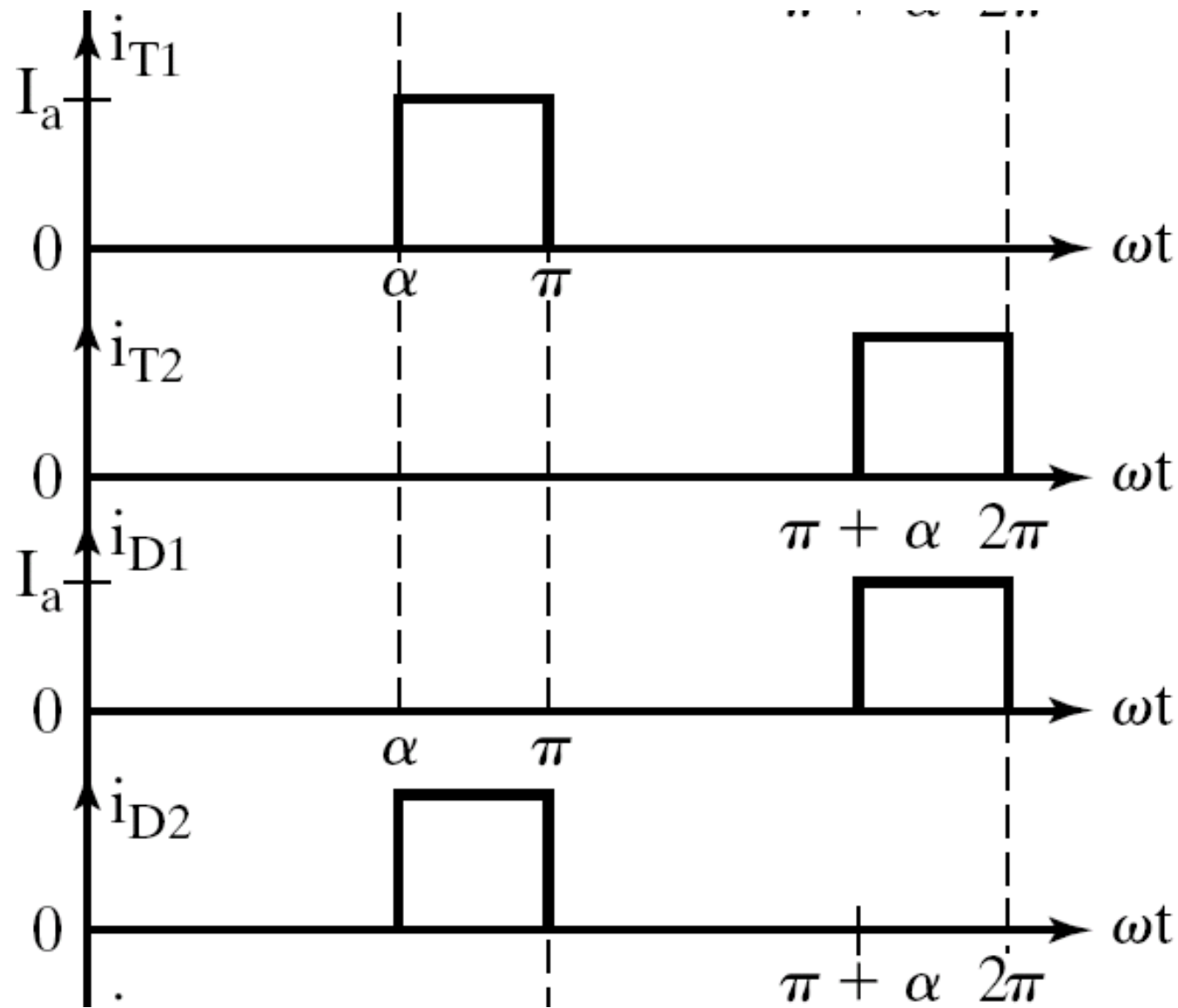
The time delay between the gating

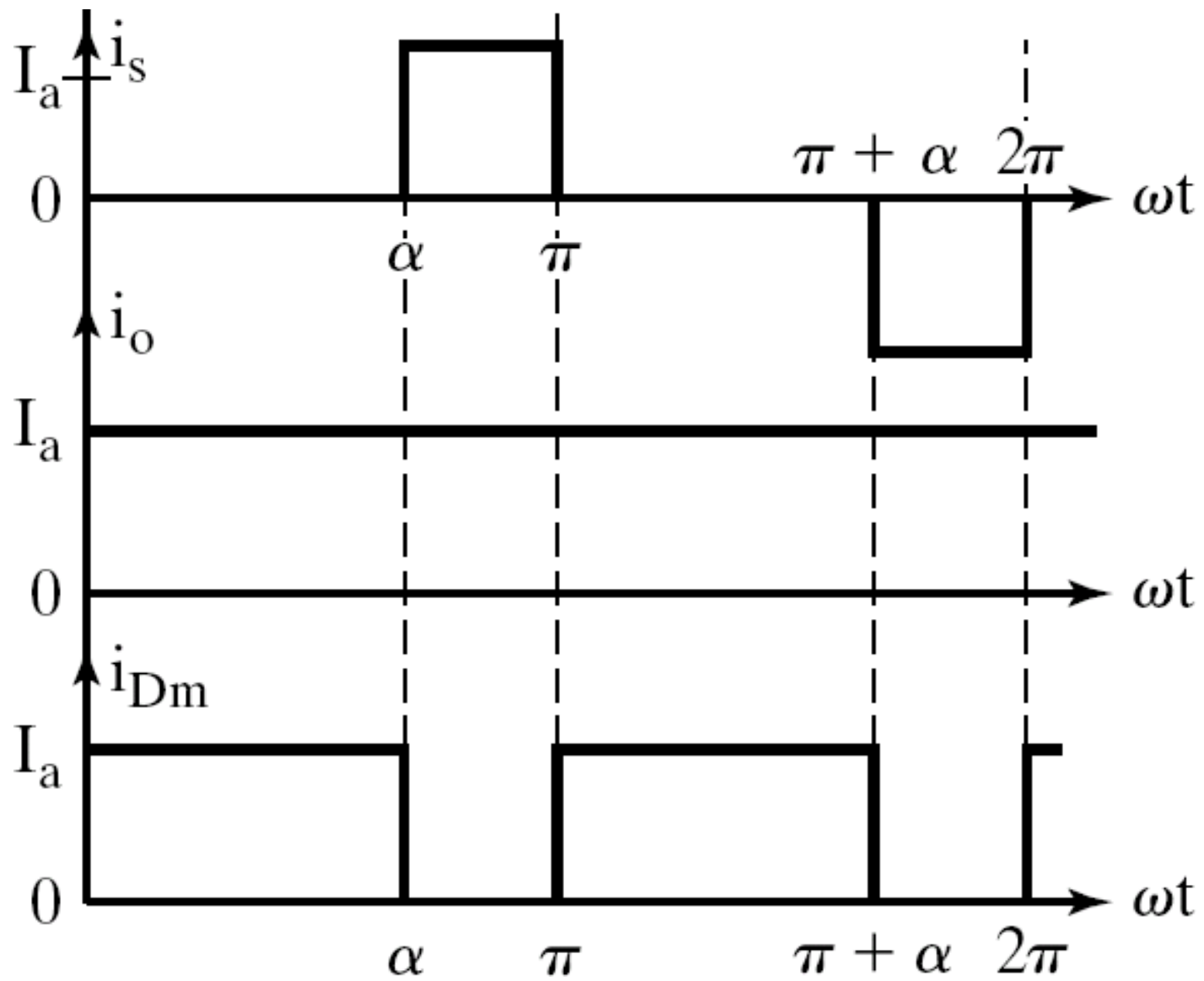
signals of T_1 & $T_2 = \pi$ radians or 180°

Waveforms of
single phase semi-converter
with general load & FWD
for $\alpha > 90^\circ$



Single Quadrant
Operation





Thyristor T_1 & D_1

conduct

from $\omega t = \alpha$ to

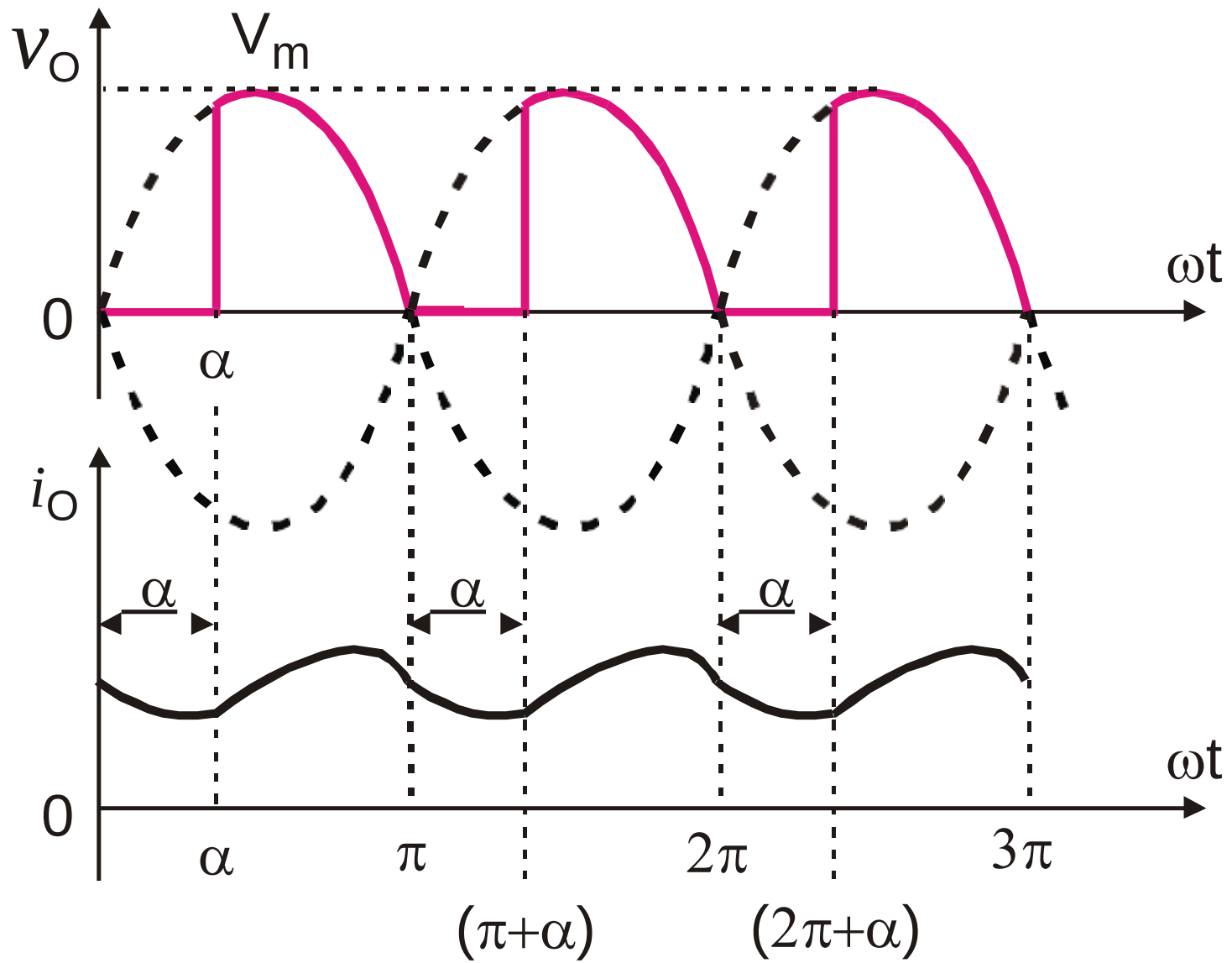
Thyristor T_2 & D_2 conduct

from $\omega t = (\pi + \alpha)$ to 2π

FWD conducts during

$\omega t = 0$ to α , π to $(\pi + \alpha)$, ...

Load Voltage & Load Current Waveform of
Single Phase Semi Converter for
 $\alpha < 90^\circ$ & Continuous load current
operation



To Derive an Expression
For The
DC Output Voltage of
A
Single Phase Semi-Converter With
R,L, & E Load & FWD
For Continuous, Ripple Free Load
Current Operation

$$V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\omega t=0}^{\pi} v_O . d (\omega t)$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{1}{\pi} \int_{\alpha}^{\pi} V_m \sin \omega t . d (\omega t)$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi}$$

$$V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} \left[-\cos \pi + \cos \alpha \right] ; \cos \pi = -1$$

$$\therefore V_{O(dc)} = V_{dc} = \frac{V_m}{\pi} (1 + \cos \alpha)$$

V_{dc} can be varied from a max.

value of $\frac{2V_m}{\pi}$ to 0 by varying α from 0 to π .

For $\alpha = 0$, The max. dc o/p voltage obtained is

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

Normalized dc o/p voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dn}} = \frac{\frac{V_m}{\pi} (1 + \cos \alpha)}{\left(\frac{2V_m}{\pi} \right)} = \frac{1}{2} (1 + \cos \alpha)$$

RMS O/P Voltage $V_{O(RMS)}$

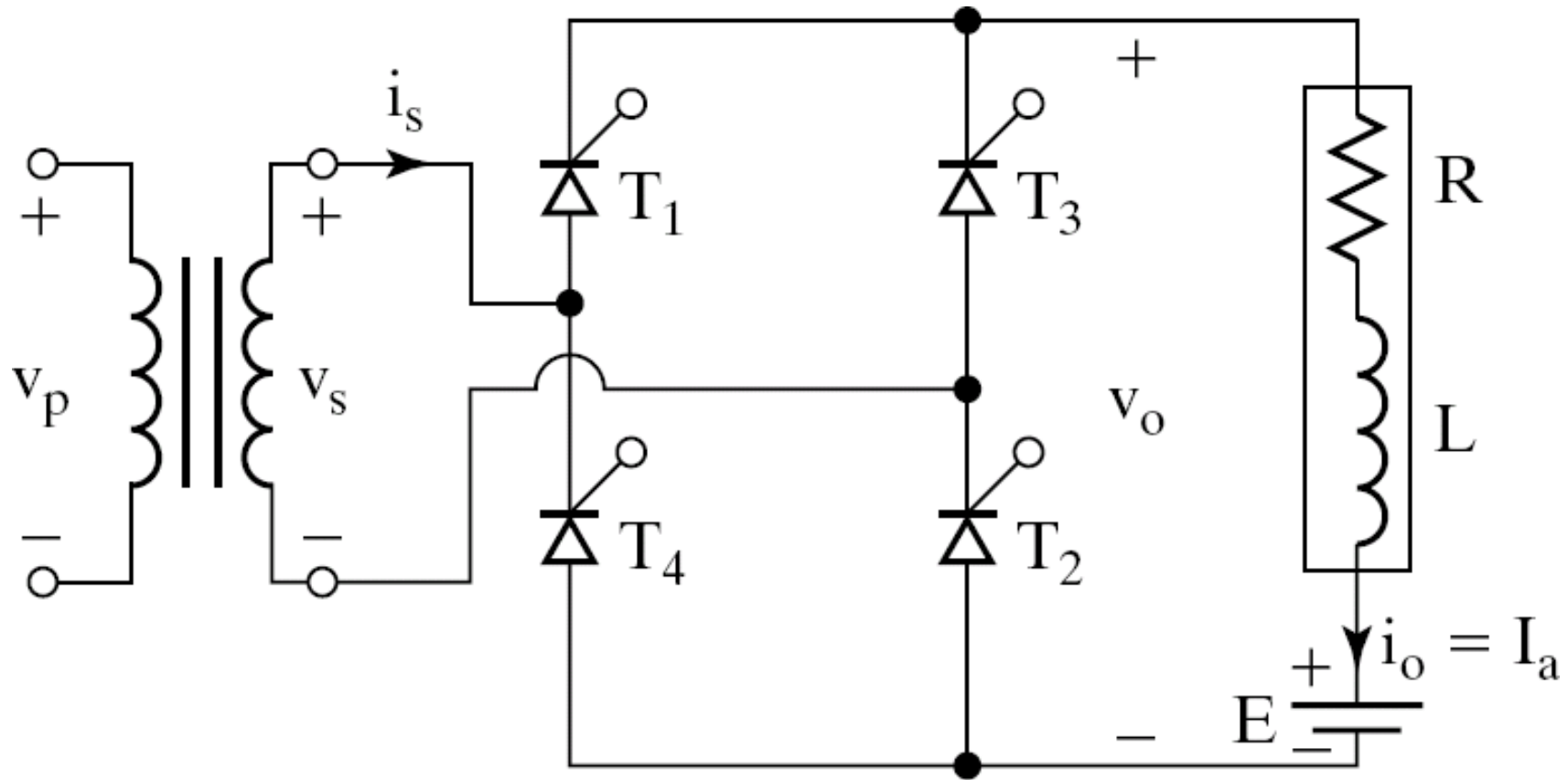
$$V_{O(RMS)} = \left[\frac{2}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \cdot d(\omega t) \right]^{\frac{1}{2}}$$

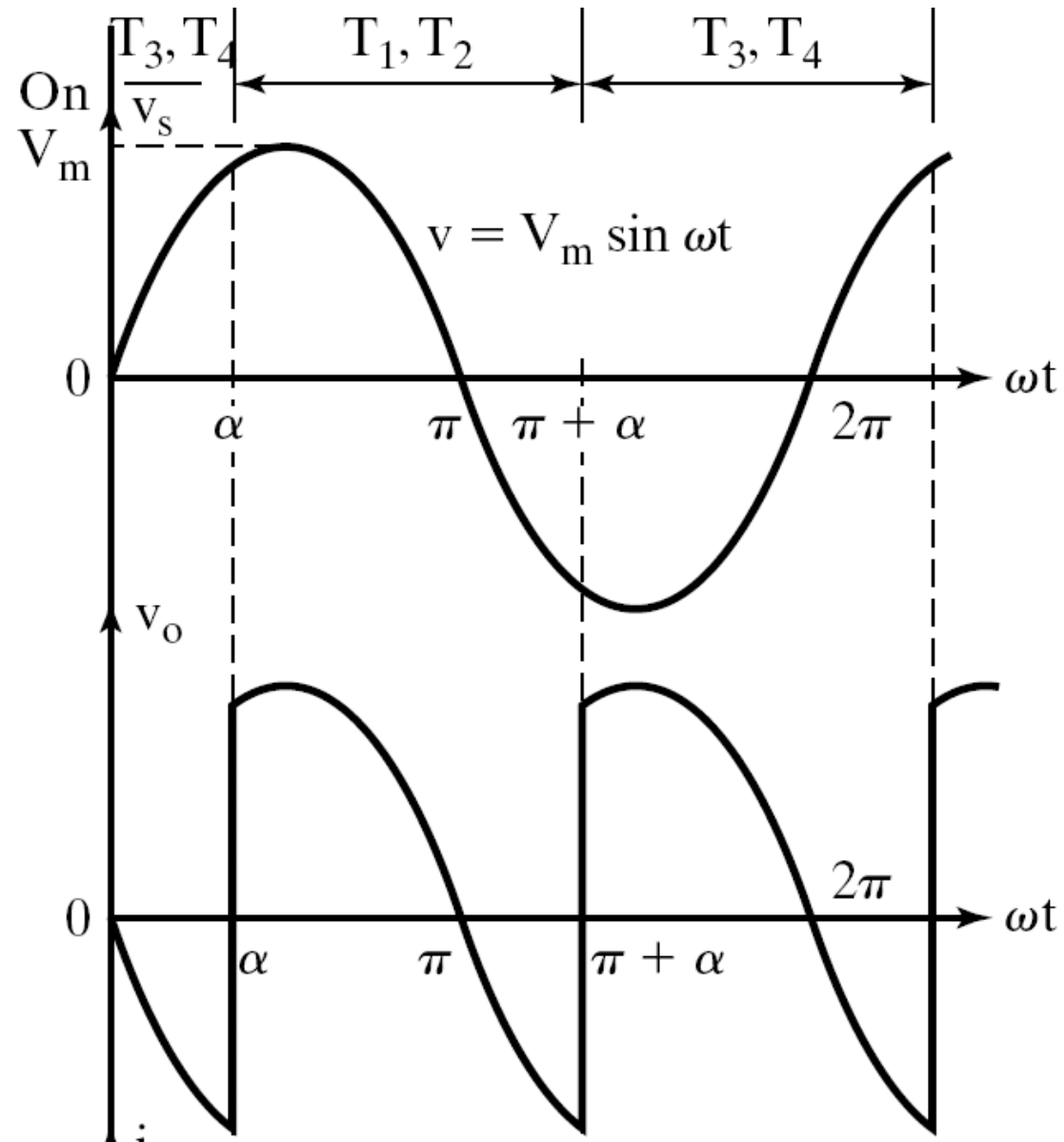
$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

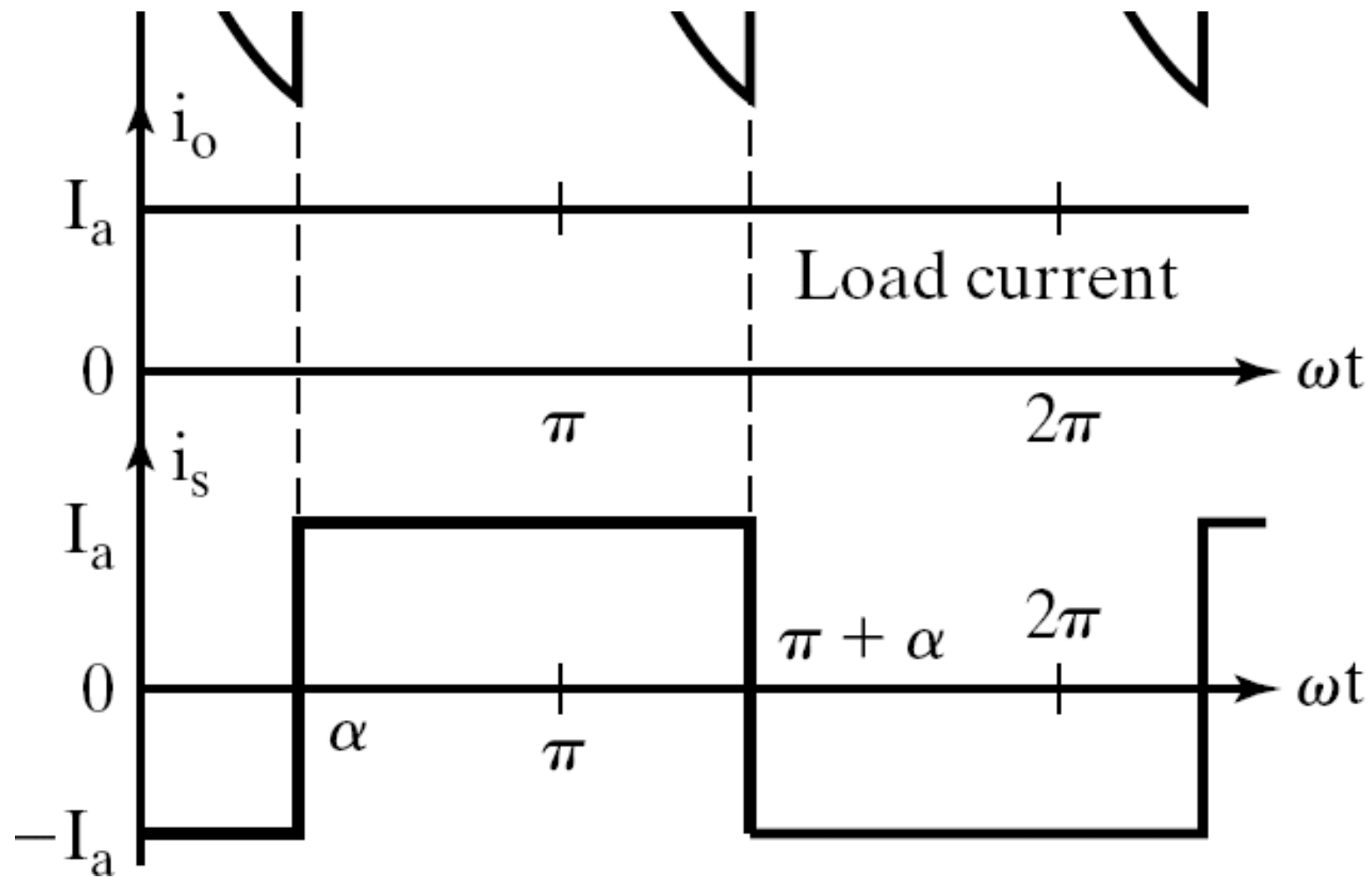
Single Phase Full Converter

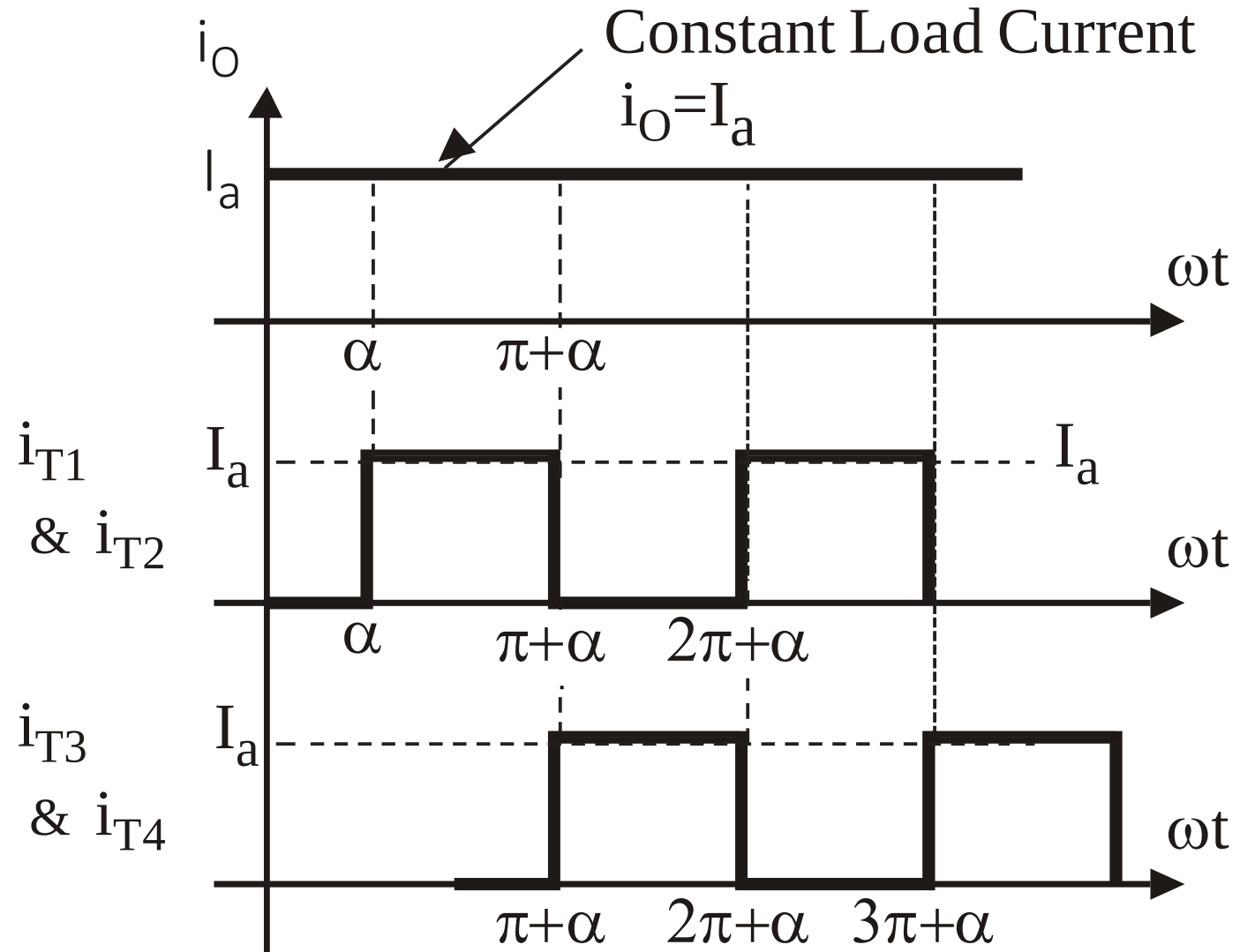
Single Phase Full Converter



Waveforms of
Single Phase Full Converter
Assuming Continuous (Constant
Load Current)
&
Ripple Free Load Current







To Derive An Expression For The
Average DC Output Voltage of a
Single Phase Full Converter
assuming Continuous & Constant
Load Current

The average dc output voltage can be determined by using the expression

$$V_{O(dc)} = V_{dc} = \frac{1}{2\pi} \left[\int_0^{2\pi} v_o \cdot d(\omega t) \right];$$

The o/p voltage waveform consists of two o/p pulses during the input supply time period of 0 to 2π radians. Hence the Average or dc o/p voltage can be calculated as

$$V_{O(dc)} = V_{dc} = \frac{2}{2\pi} \left[\int_{\alpha}^{\pi + \alpha} V_m \sin \omega t . d (\omega t) \right]$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{\pi + \alpha}$$

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos \alpha$$

Maximum average dc output voltage is calculated for a trigger angle $\alpha = 0^\circ$ and is obtained as

$$V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi} \times \cos(0) = \frac{2V_m}{\pi}$$

$$\therefore V_{dc(\max)} = V_{dm} = \frac{2V_m}{\pi}$$

The normalized average output voltage is given by

$$V_{dcn} = V_n = \frac{V_{O(dc)}}{V_{dc(max)}} = \frac{V_{dc}}{V_{dm}}$$
$$\therefore V_{dcn} = V_n = \frac{\frac{2V_m \cos \alpha}{\pi}}{\frac{2V_m}{\pi}} = \cos \alpha$$

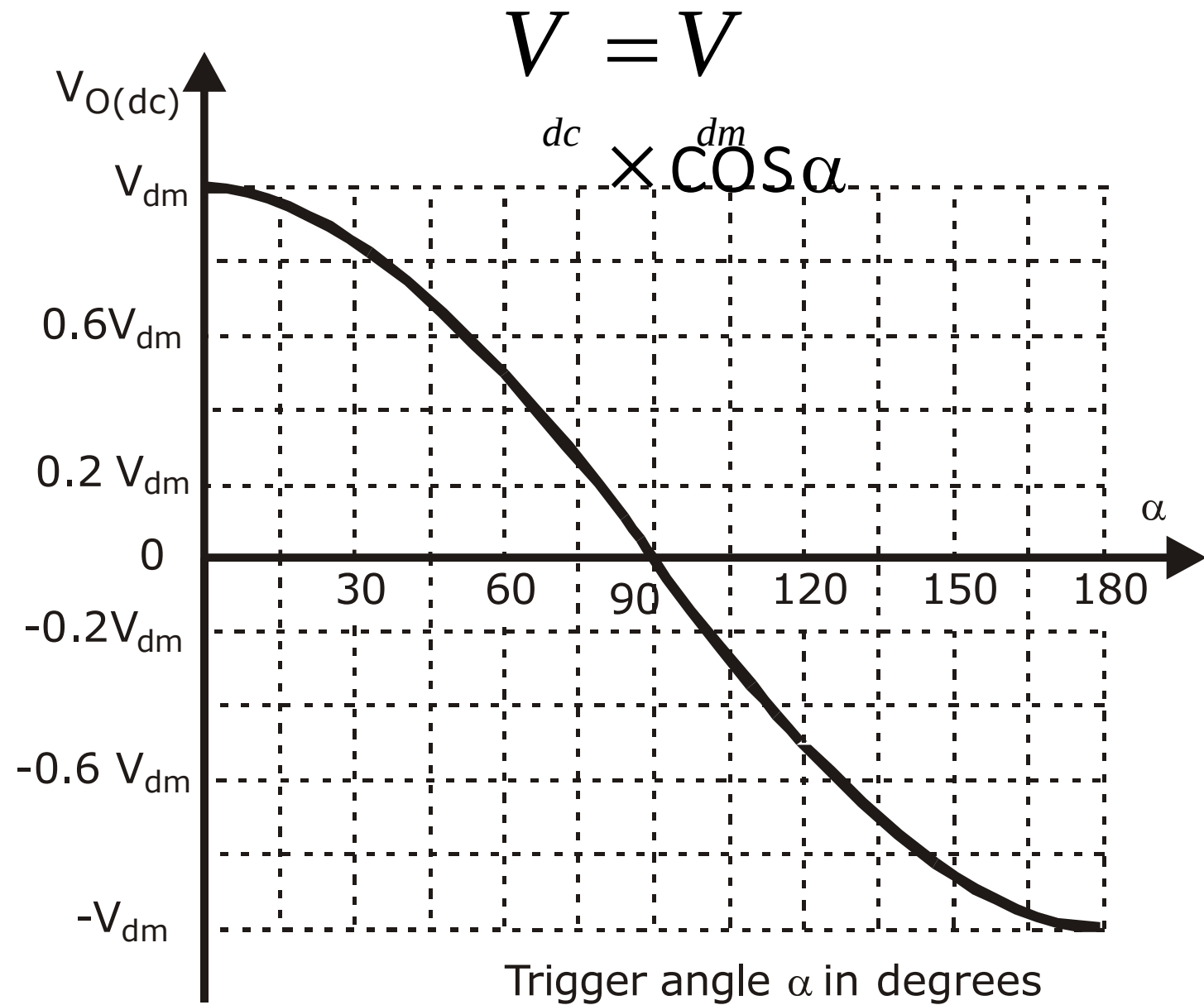
By plotting $V_{O(dc)}$ *versus* α ,
we obtain the control characteristic of a
single phase full wave fully controlled
bridge converter
(single phase full converter)
for constant & continuous
load current operation.

To plot the control characteristic of a Single Phase Full Converter for constant & continuous load current operation.

- We use the equation for the average/ dc output voltage

$$V_{O(dc)} = V_{dc} = \frac{2V_m}{\pi} \cos\alpha$$

Trigger angle α in degrees	$V_{O(dc)}$	Remarks
0	$V_{dm} = \left(\frac{2V_m}{\pi} \right)$	Maximum dc output voltage $V_{dc(max)} = V_{dm} = \left(\frac{2V_m}{\pi} \right)$
30^0	$0.866 V_{dm}$	
60^0	$0.5 V_{dm}$	
90^0	$0 V_{dm}$	
120^0	$-0.5 V_{dm}$	
150^0	$-0.866 V_{dm}$	
180^0	$-V_{dm} = -\left(\frac{2V_m}{\pi} \right)$	



- During the period from $\omega t = \alpha$ to π the input voltage v_s and the input current i_s are both positive and the power flows from the supply to the load.
- The converter is said to be operated in the rectification mode

Controlled Rectifier Operation

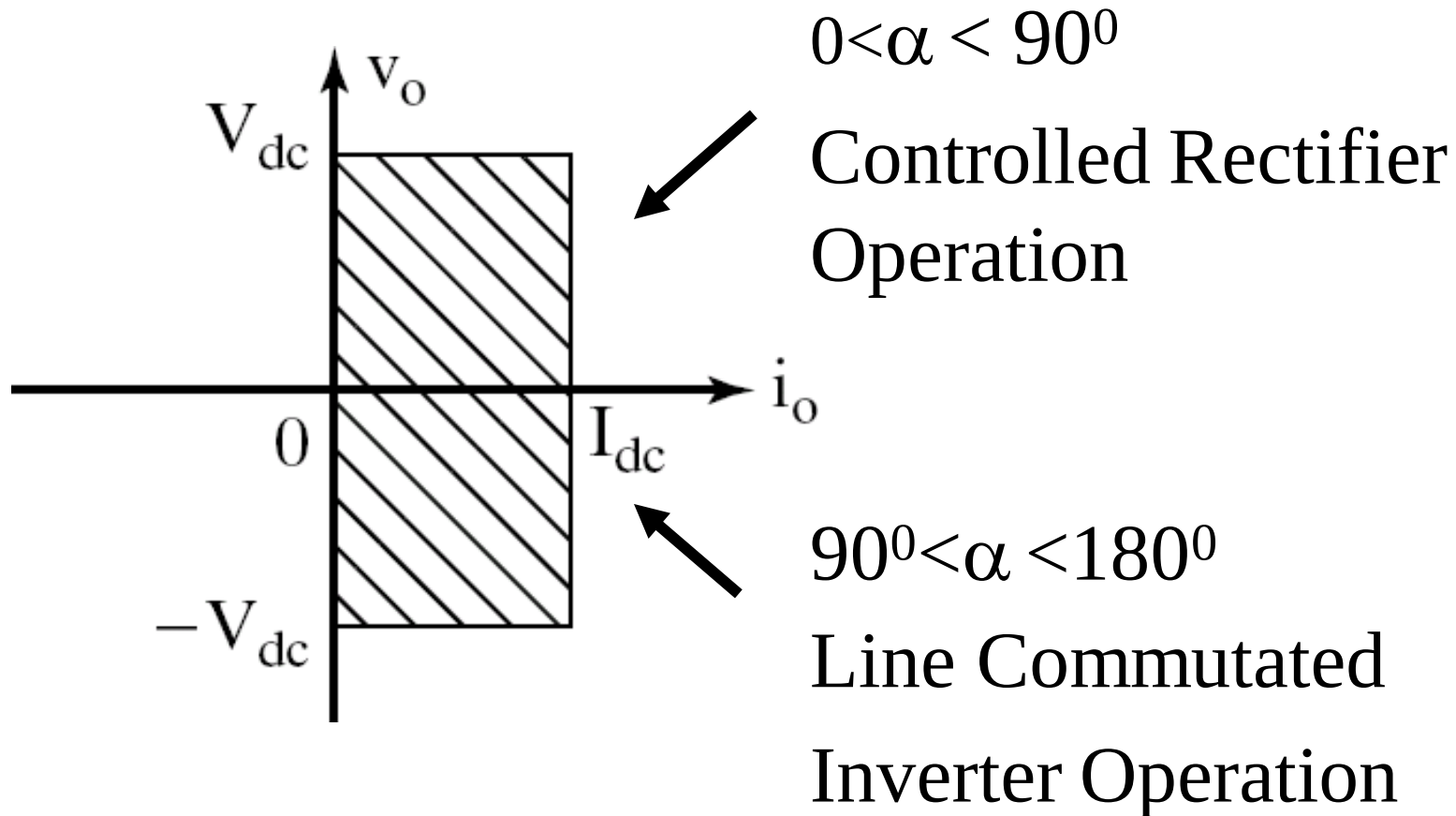
for $0 < \alpha < 90^\circ$

- During the period from $\omega t = \pi$ to $(\pi + \alpha)$, the input voltage v_s is negative and the input current i_s is positive and the output power becomes negative and there will be reverse power flow from the load circuit to the supply.
- The converter is said to be operated in the inversion mode.

Line Commutated Inverter Operation
for $90^\circ < \alpha < 180^\circ$

Two Quadrant Operation of a Single Phase Full Converter

Two Quadrant Operation of a Single Phase Full Converter



To Derive An
Expression For The
RMS Value Of The Output Voltage

The rms value of the output voltage
is calculated as

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi} \left[\int_0^{2\pi} v_o^2 \cdot d(\omega t) \right]}$$

The single phase full converter gives two output voltage pulses during the input supply time period and hence the single phase full converter is referred to as a two pulse converter. The rms output voltage can be calculated as

$$V_{O(RMS)} = \sqrt{\frac{2}{2\pi} \left[\int_{\alpha}^{\pi + \alpha} v_o^2 \cdot d(\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{1}{\pi} \left[\int_{\alpha}^{\pi + \alpha} V_m^2 \sin^2 \omega t . d (\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi} \left[\int_{\alpha}^{\pi + \alpha} \sin^2 \omega t . d (\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\pi} \left[\int_{\alpha}^{\pi + \alpha} \frac{(1 - \cos 2\omega t)}{2} . d (\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi + \alpha} d (\omega t) - \int_{\alpha}^{\pi + \alpha} \cos 2\omega t . d (\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[\left(\omega t \right) \Big/_{\alpha}^{\pi + \alpha} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{\pi + \alpha} \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[\left(\pi + \alpha - \alpha \right) - \left(\frac{\sin 2(\pi + \alpha) - \sin 2\alpha}{2} \right) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[\left(\pi \right) - \left(\frac{\sin (2\pi + 2\alpha) - \sin 2\alpha}{2} \right) \right]};$$

$$\sin (2\pi + 2\alpha) = \sin 2\alpha$$

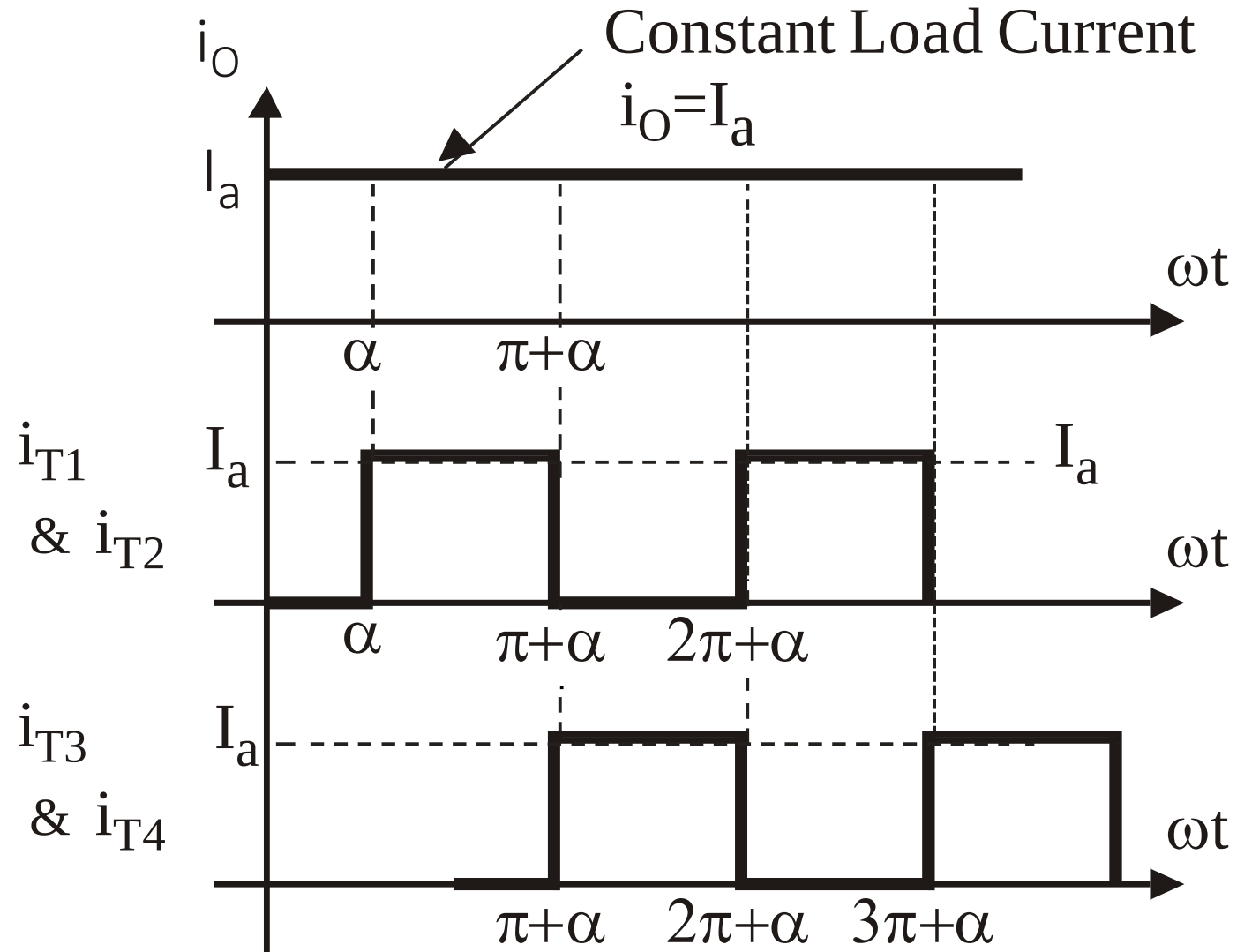
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[(\pi) - \left(\frac{\sin 2\alpha - \sin 2\alpha}{2} \right) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} (\pi) - 0} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_s$$

Hence the rms output voltage is same as the rms input supply voltage

Thyristor Current Waveforms



The rms thyristor current can be calculated as

$$I_{T(RMS)} = \frac{I_{O(RMS)}}{\sqrt{2}}$$

The average thyristor current can be calculated as

$$I_{T(Avg)} = \frac{I_{O(dc)}}{2}$$

THREE PHASE LINE COMMUTATED CONVERTERS

Introduction to Three phase converters

3 Phase Controlled Rectifiers

- Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load

Features of 3-phase controlled rectifiers

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage.
- Higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current.

- Extensively used in high power variable speed industrial dc drives.
- Three single phase half-wave converters can be connected together to form a three phase half-wave converter.

Classification of 3-phase converters

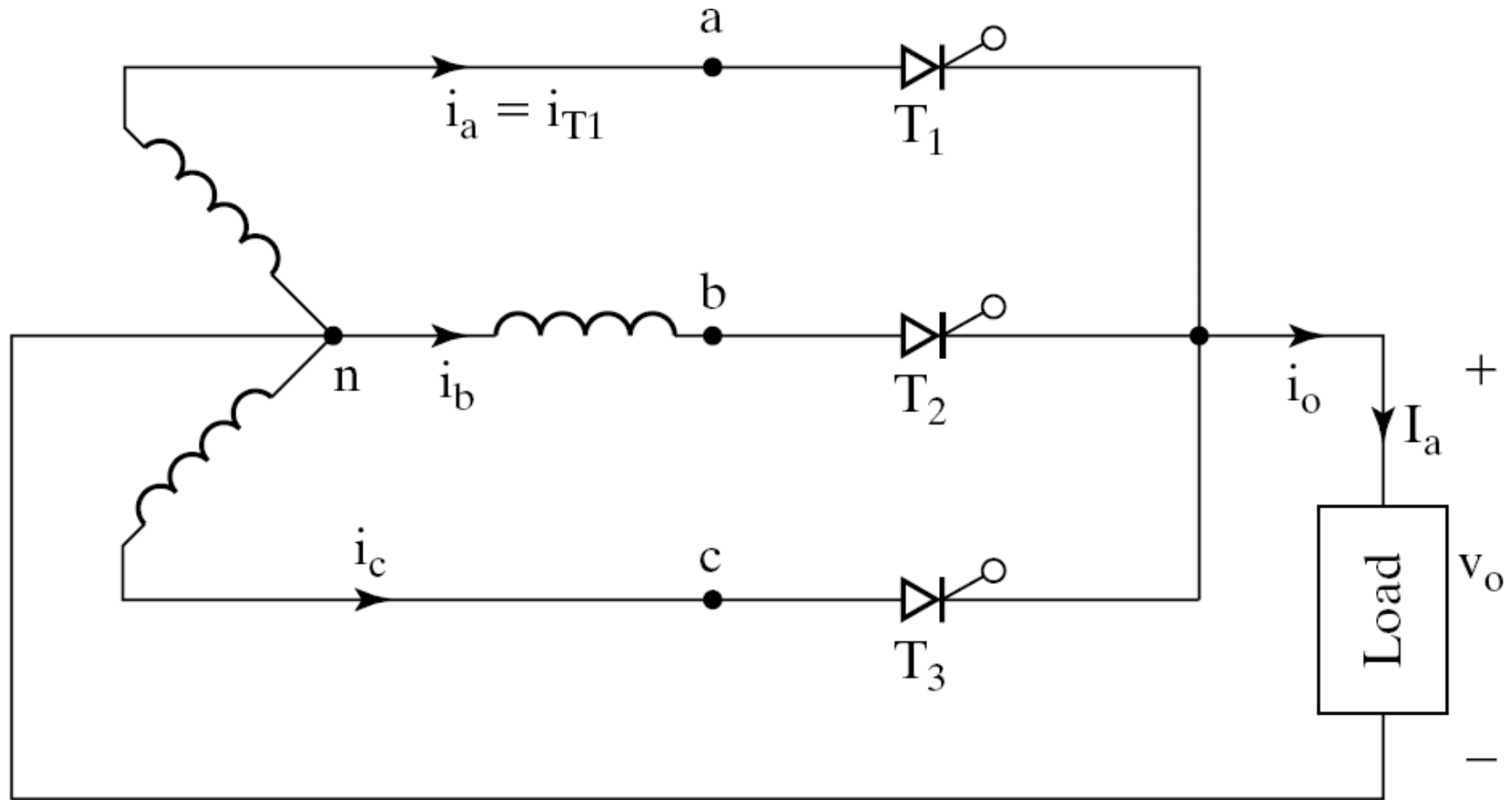
- 3-phase half wave converter
- 3-phase semi converter
- 3-phase full converter
- 3- phase dual converter

Classification according to no of pulses in the output wave

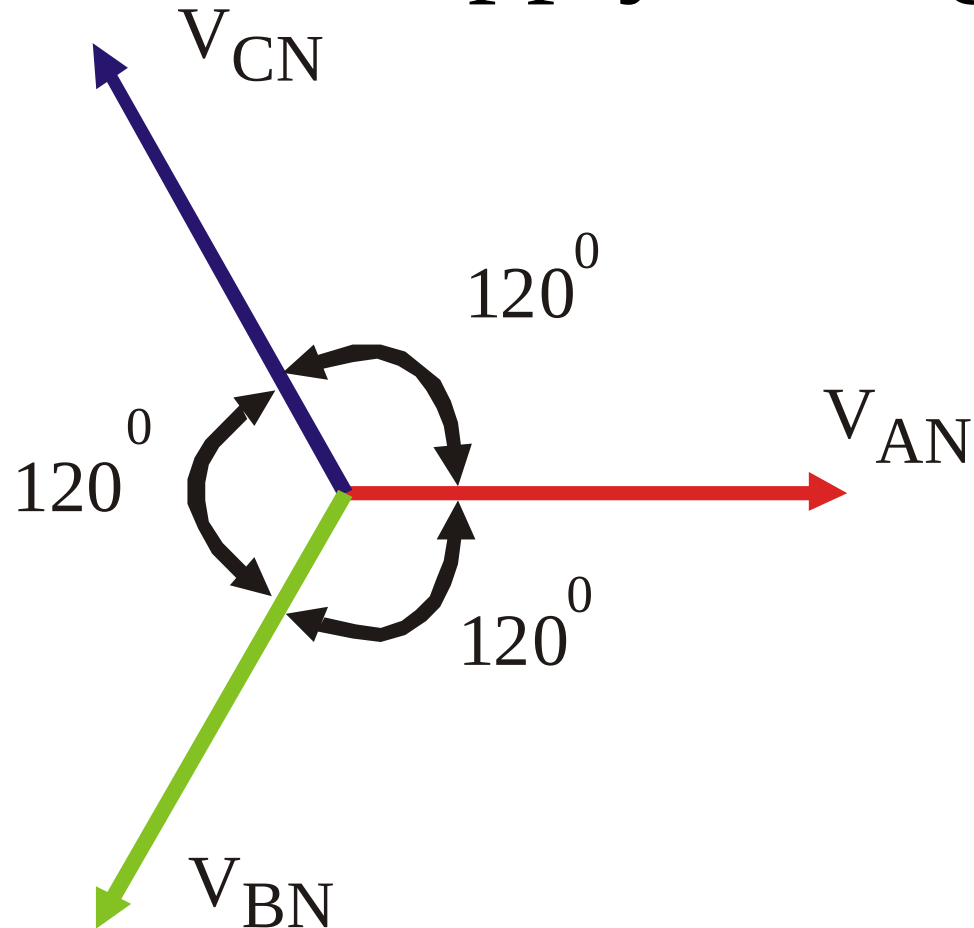
- 3- pulse converter
- 6-pulse converter
- 12- pulse converter

3-Phase
Half Wave Converter
(3-Pulse Converter)
with
R-L Load
Continuous & Constant
Load Current Operation

Circuit Diagram of 3- pulse converter



Vector Diagram of 3 Phase Supply Voltages



$$V_{RN} = V_{AN}$$

$$V_{YN} = V_{BN}$$

$$V_{BN} = V_{CN}$$

3 Phase Supply Voltage Equations

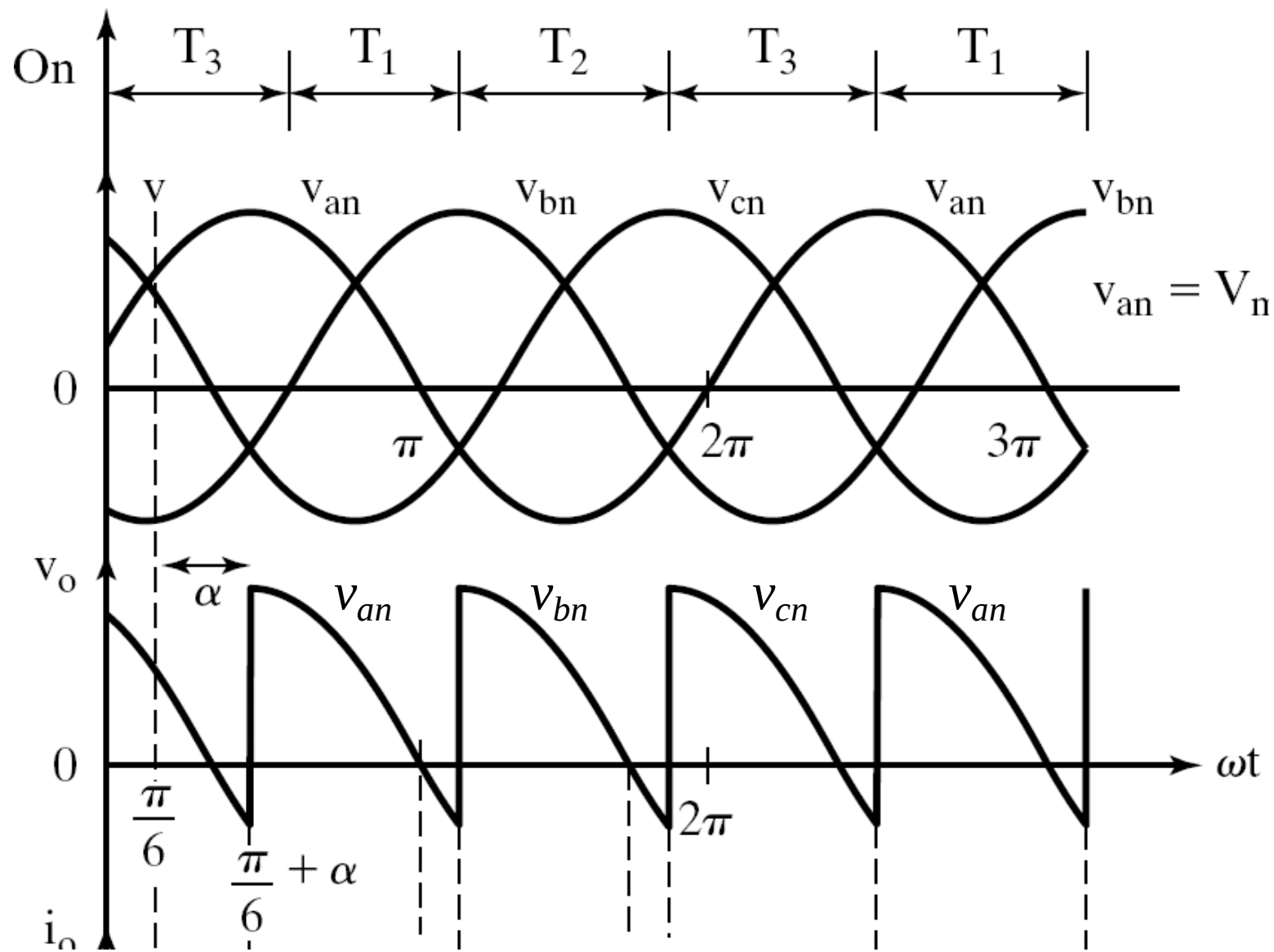
We define three line to neutral voltages
(3 phase voltages) as follows

$$V_{RN} = V_{an} = V_m \sin \omega t;$$

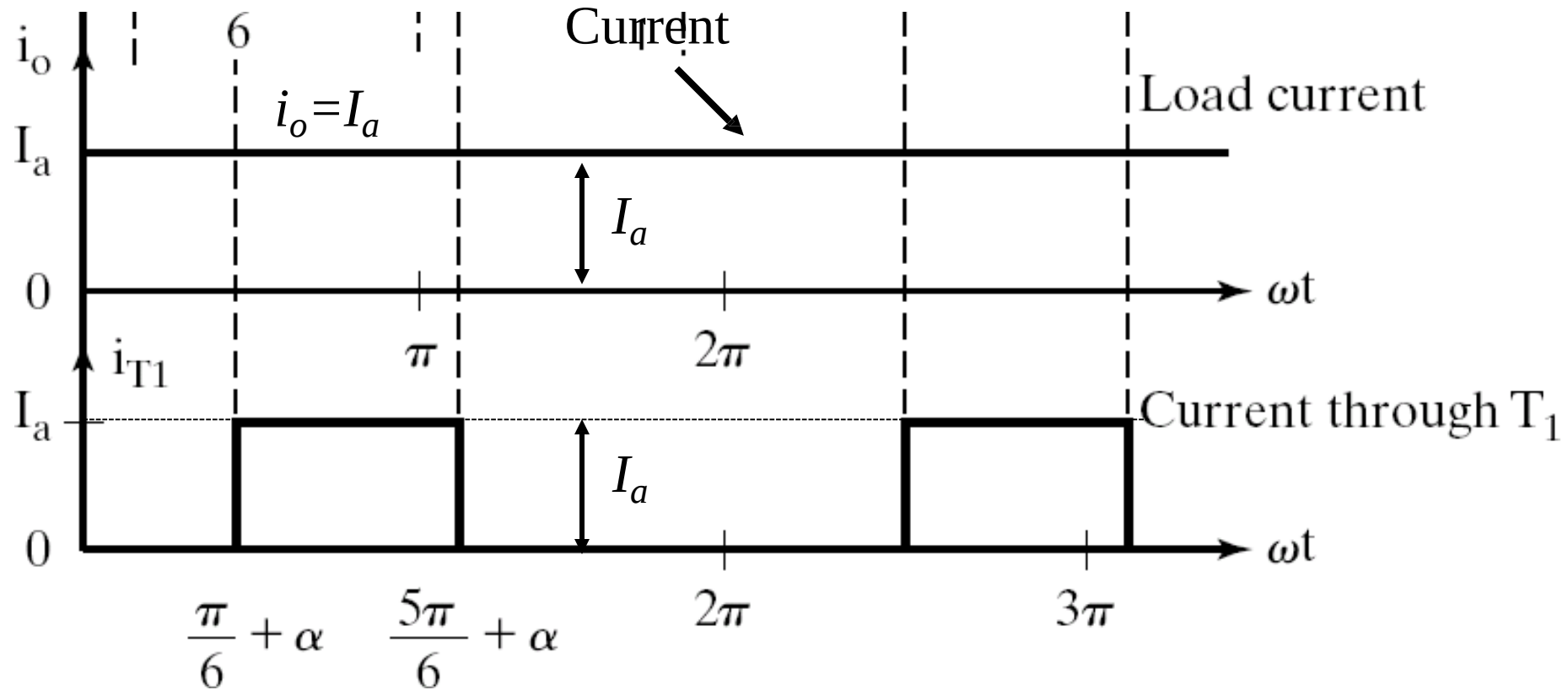
$V_m =$ Max. Phase Voltage

$$\begin{aligned} V_{YN} = V_{bn} &= V_m \sin \left(\omega t - \frac{2\pi}{3} \right) \\ &= V_m \sin (\omega t - 120^\circ) \end{aligned}$$

$$\begin{aligned} V_{BN} = V_{cn} &= V_m \sin \left(\omega t + \frac{2\pi}{3} \right) \\ &= V_m \sin (\omega t + 120^\circ) \\ &= V_m \sin (\omega t - 240^\circ) \end{aligned}$$



Each thyristor conducts for
 $2\pi/3$ (120°) Constant Load



To Derive an
Expression for the
Average Output Voltage of a
3-Phase Half Wave Converter
with RL Load
for Continuous Load Current

T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha \right) = (30^\circ + \alpha)$

T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha \right) = (150^\circ + \alpha)$

T_3 is triggered at $\omega t = \left(\frac{7\pi}{6} + \alpha \right) = (270^\circ + \alpha)$

Each thyristor conducts for 120° or $\frac{2\pi}{3}$ radians

If the reference phase voltage is

$v_{RN} = v_{an} = V_m \sin \omega t$, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m \sin \omega t \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} \sin \omega t . d (\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\left(-\cos \omega t \right) / \right]_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha}$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos \left(\frac{5\pi}{6} + \alpha \right) + \cos \left(\frac{\pi}{6} + \alpha \right) \right]$$

Note from the trigonometric relationship

$$\cos(A+B) = (\cos A \cos B - \sin A \sin B)$$
$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos\left(\frac{5\pi}{6}\right) \cos(\alpha) + \sin\left(\frac{5\pi}{6}\right) \sin(\alpha) \right. \\ \left. + \cos\left(\frac{\pi}{6}\right) \cos(\alpha) - \sin\left(\frac{\pi}{6}\right) \sin(\alpha) \right]$$
$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos(150^\circ) \cos(\alpha) + \sin(150^\circ) \sin(\alpha) \right. \\ \left. + \cos(30^\circ) \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\begin{aligned} & -\cos(180^\circ - 30^\circ) \cos(\alpha) + \sin(180^\circ - 30^\circ) \sin(\alpha) \\ & + \cos(30^\circ) \cos(\alpha) - \sin(30^\circ) \sin(\alpha) \end{aligned} \right]$$

Note: $\cos(180^\circ - 30^\circ) = -\cos(30^\circ)$

$\sin(180^\circ - 30^\circ) = \sin(30^\circ)$

$$\therefore V_{dc} = \frac{3V_m}{2\pi} \left[\begin{aligned} & +\cos(30^\circ) \cos(\alpha) + \cancel{\sin(30^\circ) \sin(\alpha)} \\ & + \cos(30^\circ) \cos(\alpha) - \cancel{\sin(30^\circ) \sin(\alpha)} \end{aligned} \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[2 \cos(30^\circ) \cos(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[2 \times \frac{\sqrt{3}}{2} \cos(\alpha) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\sqrt{3} \cos(\alpha) \right] = \frac{3\sqrt{3}V_m}{2\pi} \cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi} \cos(\alpha)$$

Where $V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is

given $V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3} V_m}{2\pi}$

Where V_m is the peak phase voltage.

And the normalized average output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos \alpha$$

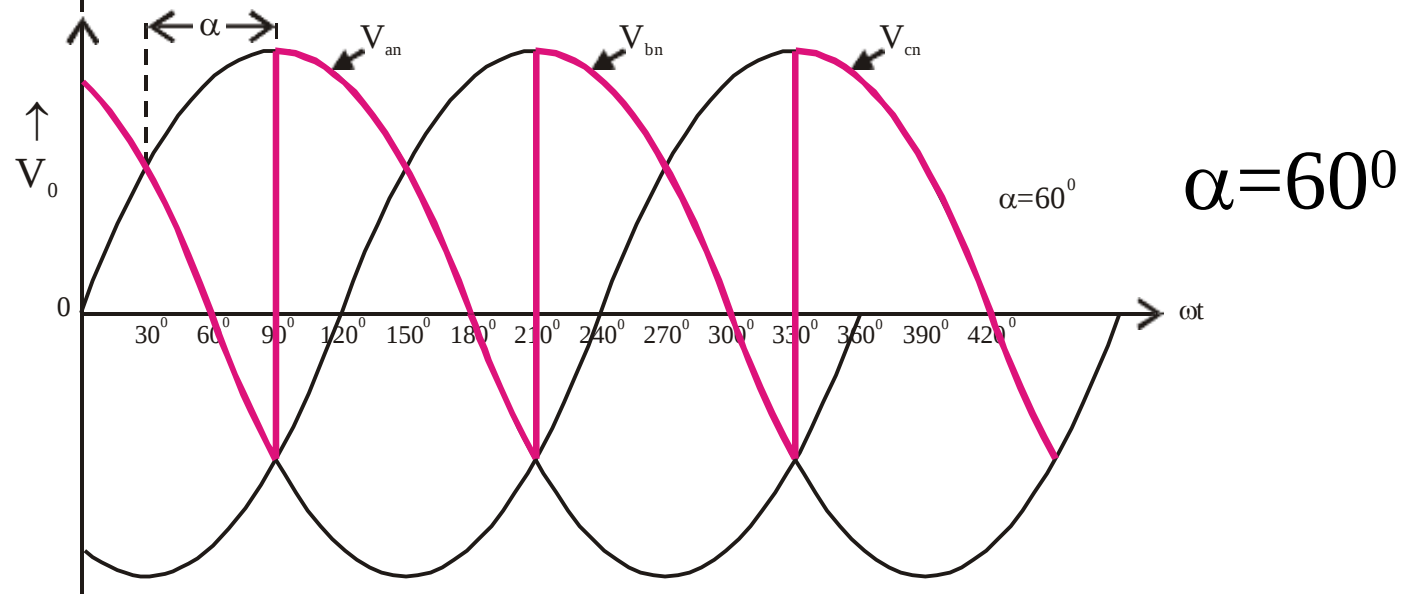
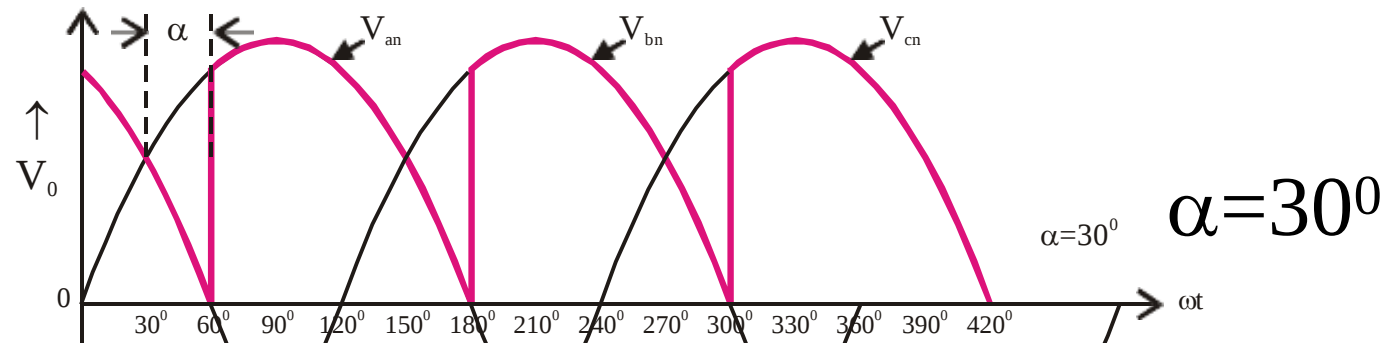
The rms value of output voltage is found by using the equation

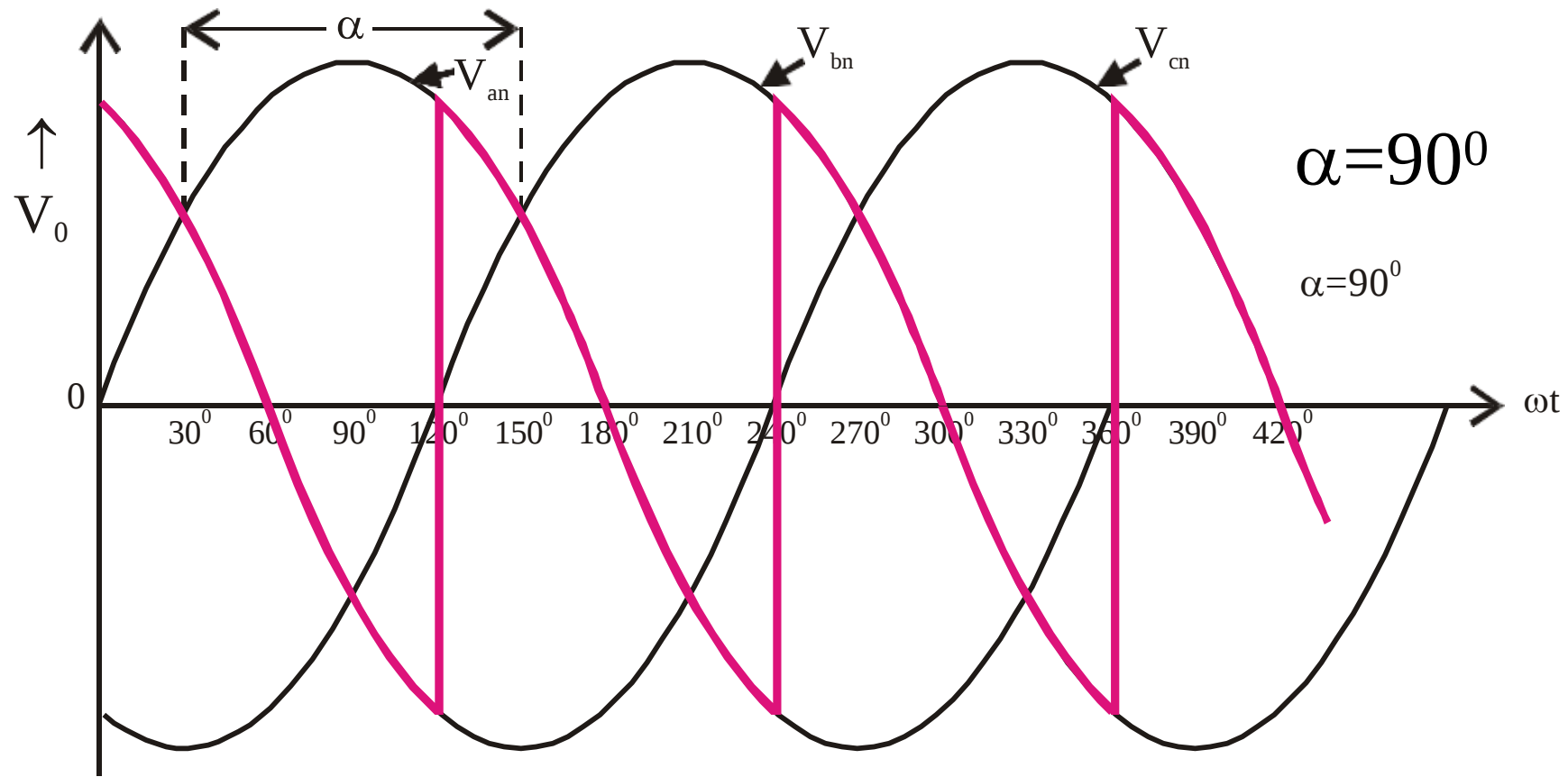
$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{5\pi}{6} + \alpha} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{\frac{1}{2}}$$

and we obtain

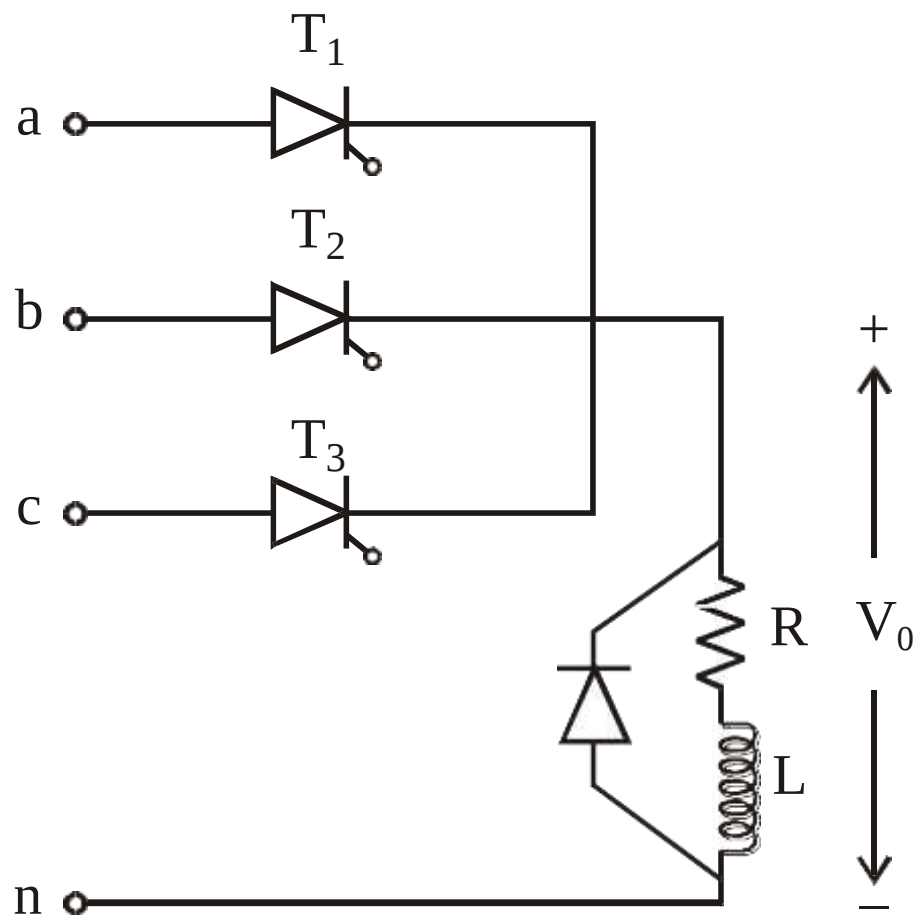
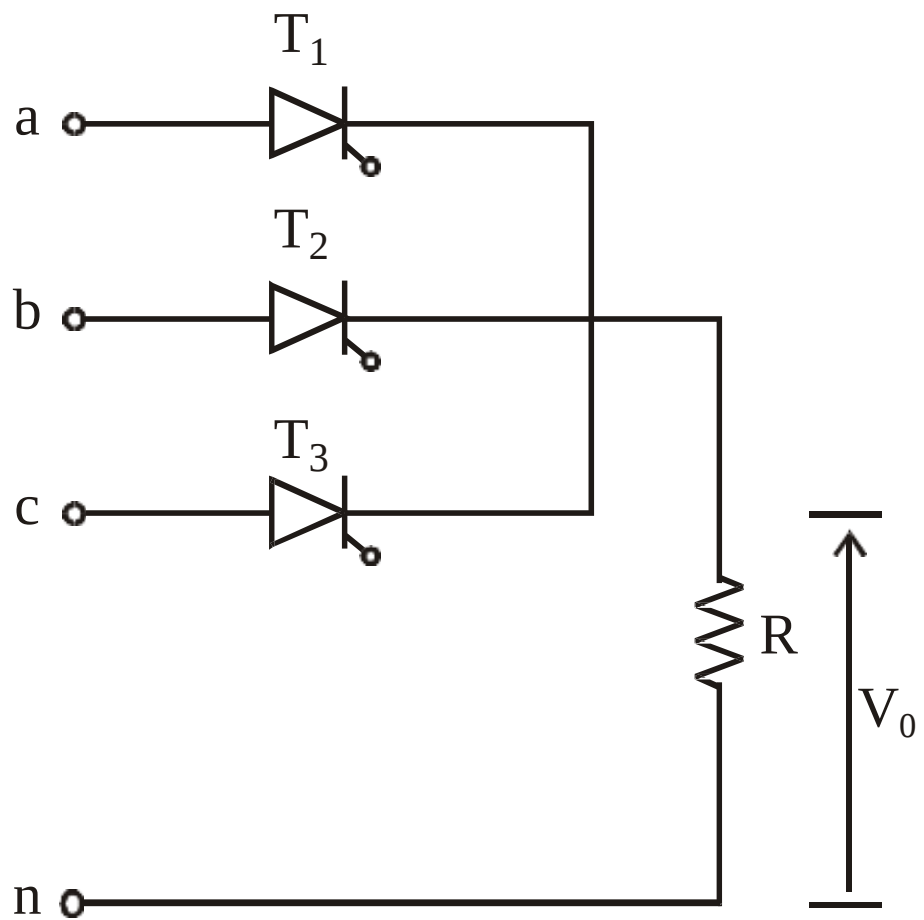
$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{1}{6} + \frac{\sqrt{3}}{8\pi} \cos 2\alpha \right]^{\frac{1}{2}}$$

3 Phase Half Wave
Controlled Rectifier Output
Voltage Waveforms For RL Load
at
Different Trigger Angles

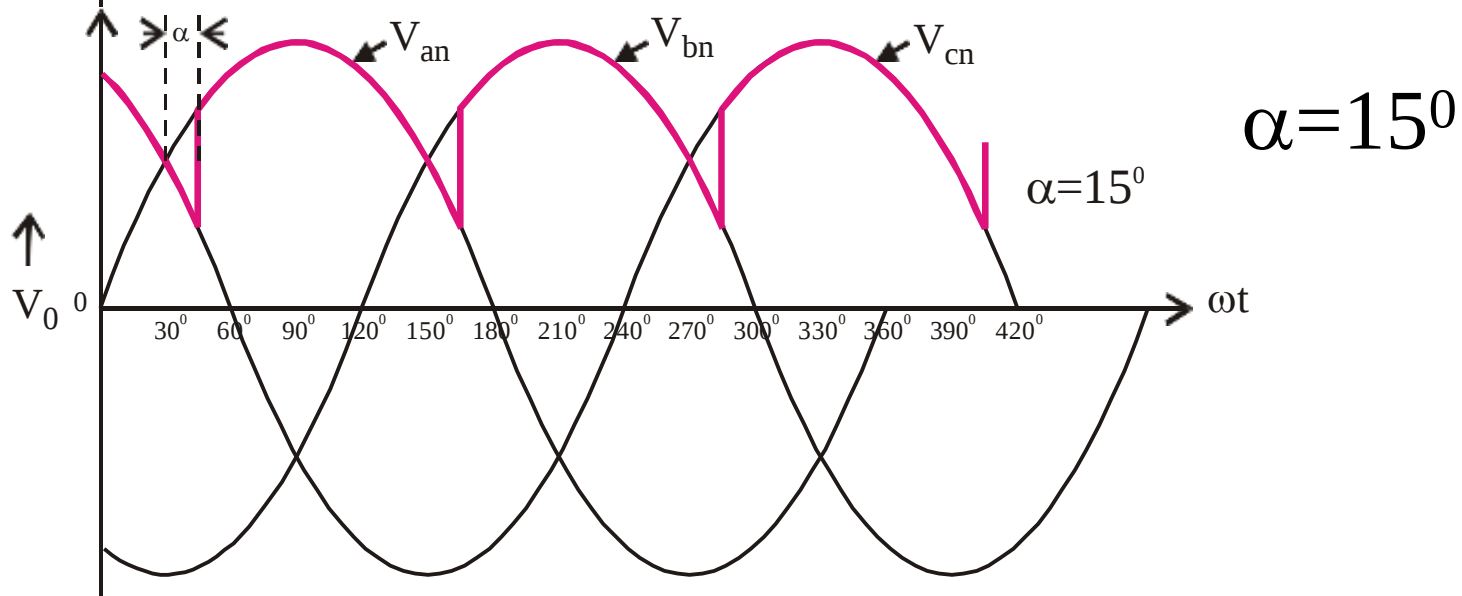
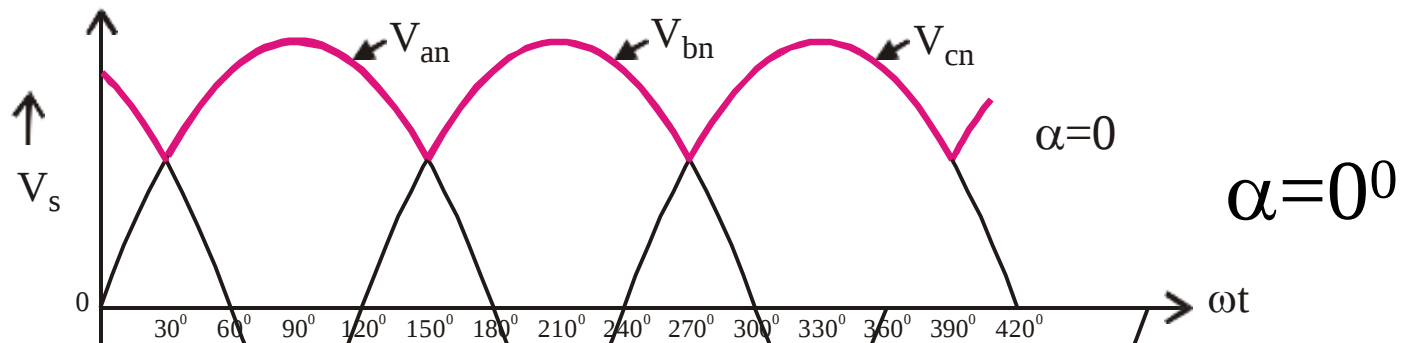


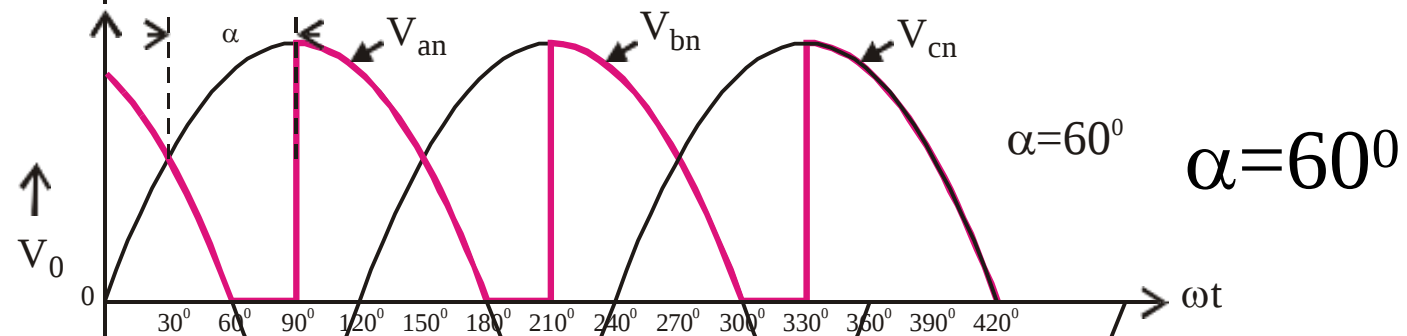
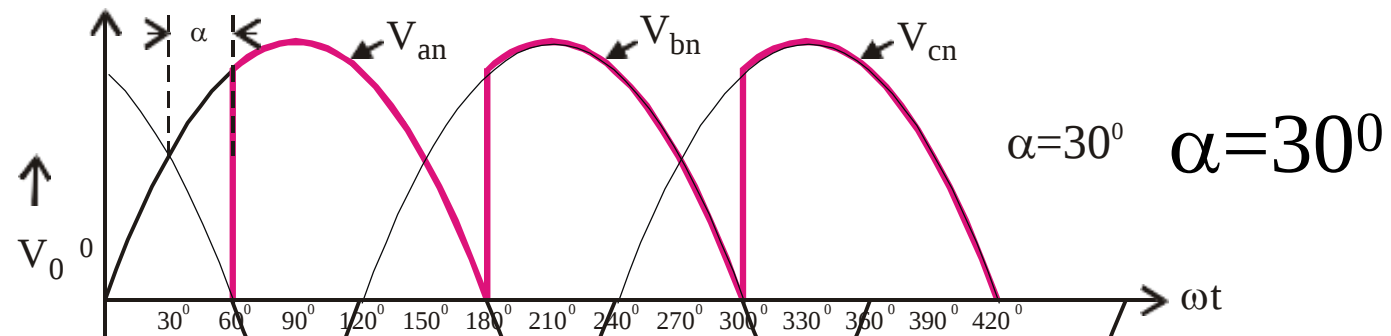


3 Phase Half Wave
Controlled Rectifier With
R Load
and
RL Load with FWD



3 Phase Half Wave
Controlled Rectifier Output
Voltage Waveforms For R Load
or RL Load with FWD
at
Different Trigger Angles





To Derive An Expression For The Average
Or Dc Output Voltage Of A
3 Phase Half Wave Converter With
Resistive Load Or RL Load With FWD

T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha \right) = (30^\circ + \alpha)$

T_1 conducts from $(30^\circ + \alpha)$ to 180° ;

$$v_O = v_{an} = V_m \sin \omega t$$

T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha \right) = (150^\circ + \alpha)$

T_2 conducts from $(150^\circ + \alpha)$ to 300° ;

$$v_O = v_{bn} = V_m \sin (\omega t - 120^\circ)$$

T_3 is triggered at $\omega t = \left(\frac{7\pi}{6} + \alpha \right) = (270^\circ + \alpha)$

T_3 conducts from $(270^\circ + \alpha)$ to 420° ;

$$\begin{aligned} v_O = v_{cn} &= V_m \sin(\omega t - 240^\circ) \\ &= V_m \sin(\omega t + 120^\circ) \end{aligned}$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^0}^{180^0} v_o .d (\omega t) \right]$$

$$v_o = v_{an} = V_m \sin \omega t; \text{ for } \omega t = (\alpha + 30^0) \text{ to } (180^0)$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\alpha+30^0}^{180^0} V_m \sin \omega t .d (\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\int_{\alpha+30^0}^{180^0} \sin \omega t .d (\omega t) \right]$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[\frac{-\cos \omega t}{\alpha + 30^\circ} \right]_{180^\circ}$$

$$V_{dc} = \frac{3V_m}{2\pi} \left[-\cos 180^\circ + \cos (\alpha + 30^\circ) \right]$$

$\therefore \cos 180^\circ = -1$, we get

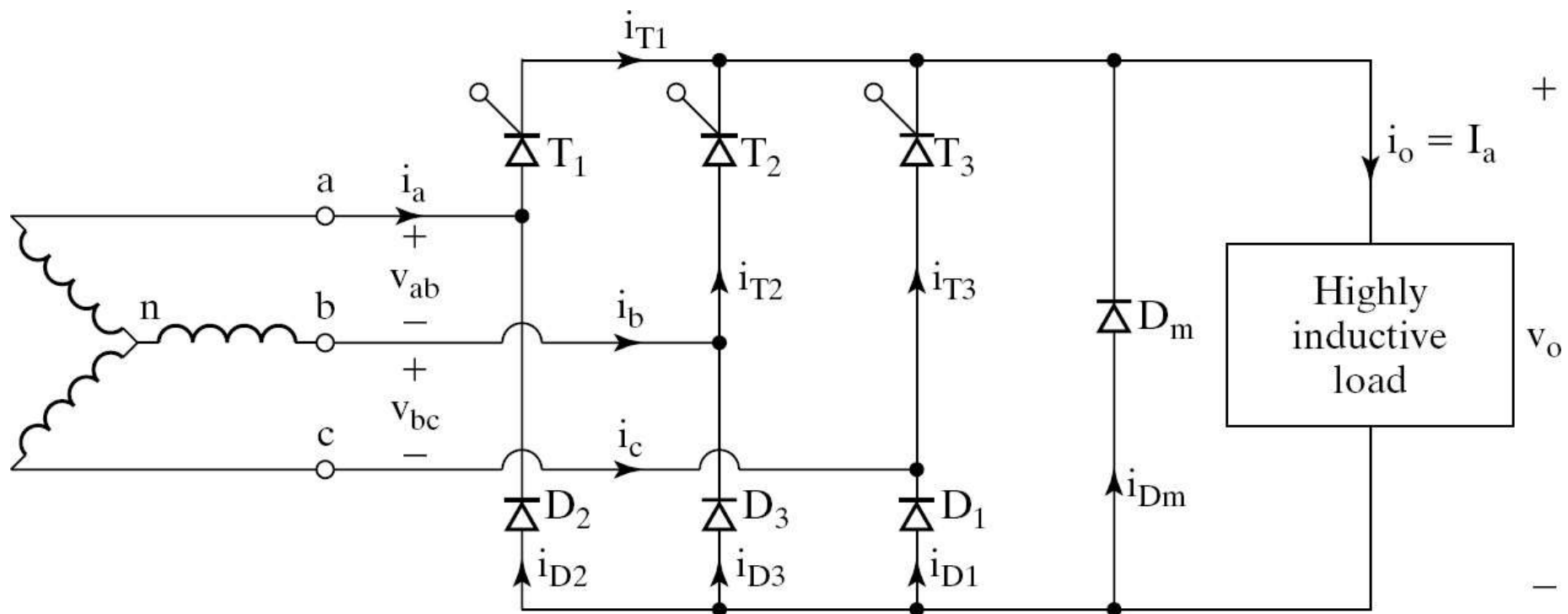
$$V_{dc} = \frac{3V_m}{2\pi} \left[1 + \cos (\alpha + 30^\circ) \right]$$

Three Phase Semi-converters

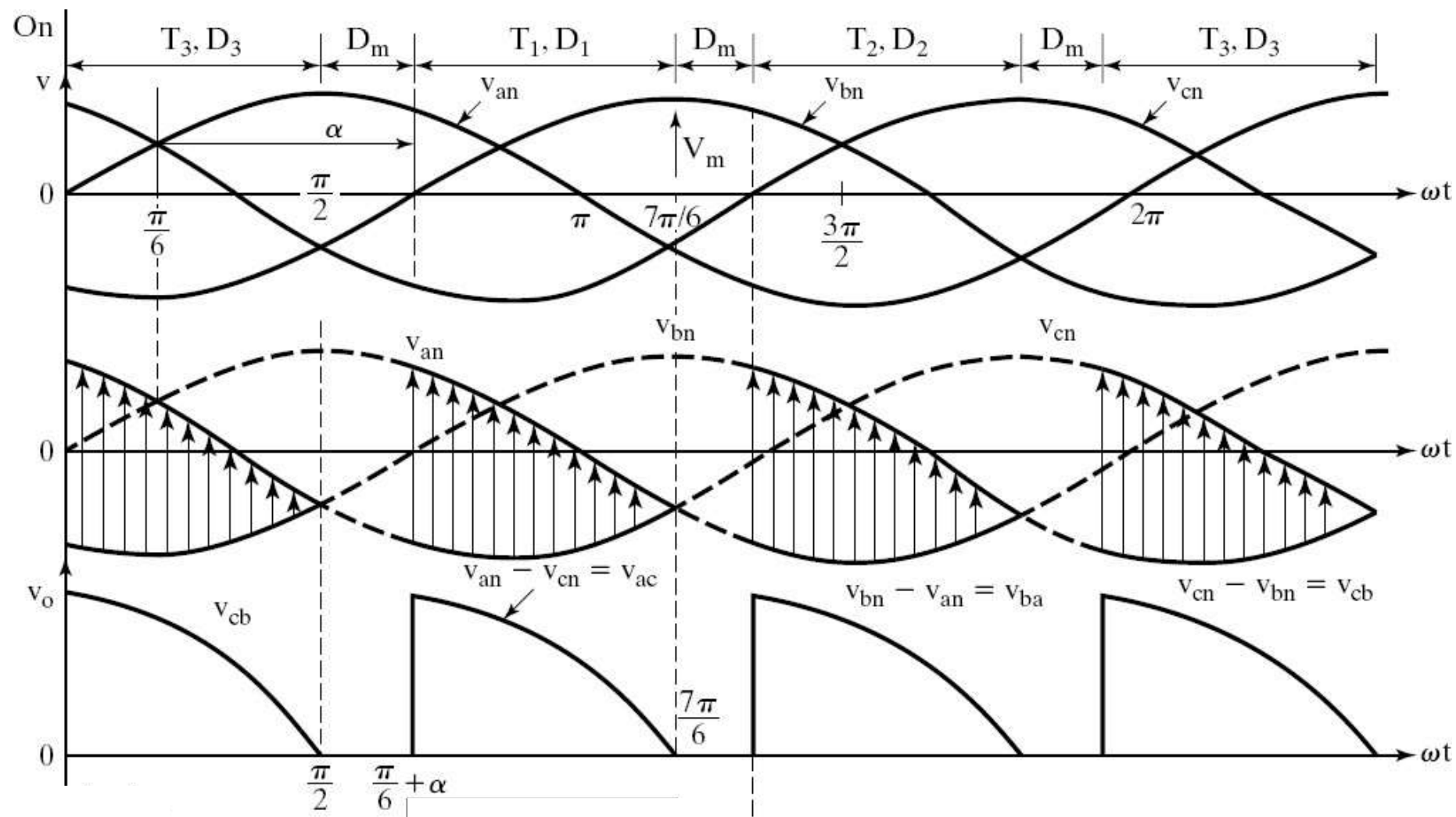
Three Phase Semi-converters

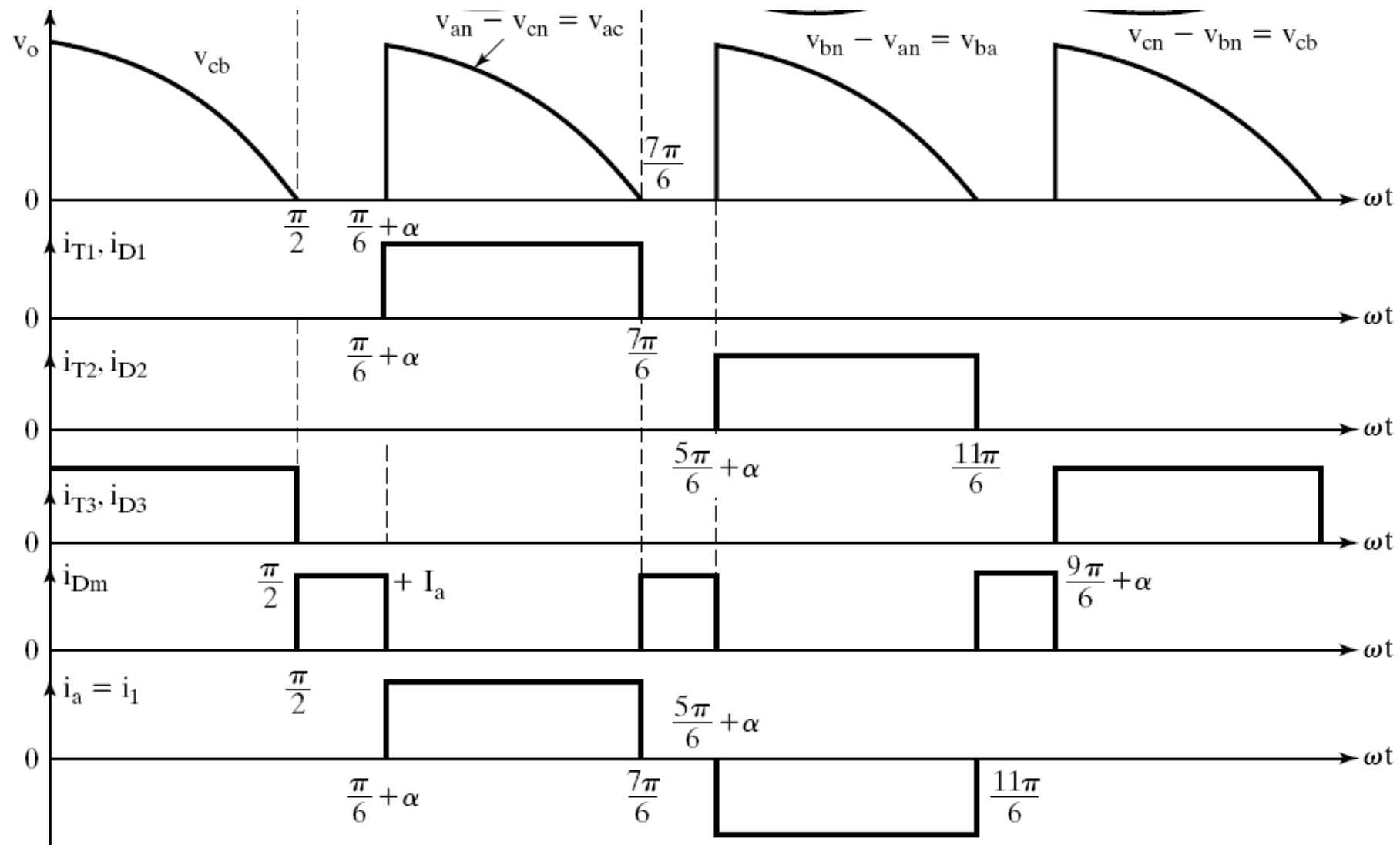
- 3 Phase semi-converters are used in Industrial dc drive applications upto 120kW power output.
- Single quadrant operation is possible.
- Power factor decreases as the delay angle increases.
- Power factor is better than that of 3 phase half wave converter.

3 Phase
Half Controlled Bridge Converter
(Semi Converter)
with Highly Inductive Load &
Continuous Ripple free Load Current



Wave forms of 3 Phase Semiconverter
for
 $\alpha > 60^\circ$





3 phase semiconverter output ripple frequency
of output voltage is $3f_s$

The delay angle α can be varied from 0 to π

During the period

$$30^\circ \leq \omega t < 210^\circ$$

$$\frac{\pi}{6} \leq \omega t < \frac{7\pi}{6}, \text{ thyristor } T_1 \text{ is forward biased}$$

If thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$,

T_1 & D_1 conduct together and the line to line voltage v_{ac} appears across the load.

At $\omega t = \frac{7\pi}{6}$, v_{ac} becomes negative & FWD D_m conducts.

The load current continues to flow through FWD D_m ;

T_1 and D_1 are turned off.

If FWD D_m is not used the T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha \right)$, and Free wheeling action would be accomplished through T_1 & D_2 .

If the delay angle $\alpha \leq \frac{\pi}{3}$, each thyristor conducts for $\frac{2\pi}{3}$ and the FWD D_m does not conduct.

We define three line neutral voltages

(3 phase voltages) as follows

$$V_{RN} = V_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{Max. Phase Voltage}$$

$$V_{YN} = V_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ)$$

$$V_{BN} = V_{cn} = V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ) \\ = V_m \sin (\omega t - 240^\circ)$$

V_m is the peak phase voltage of a wye-connected source

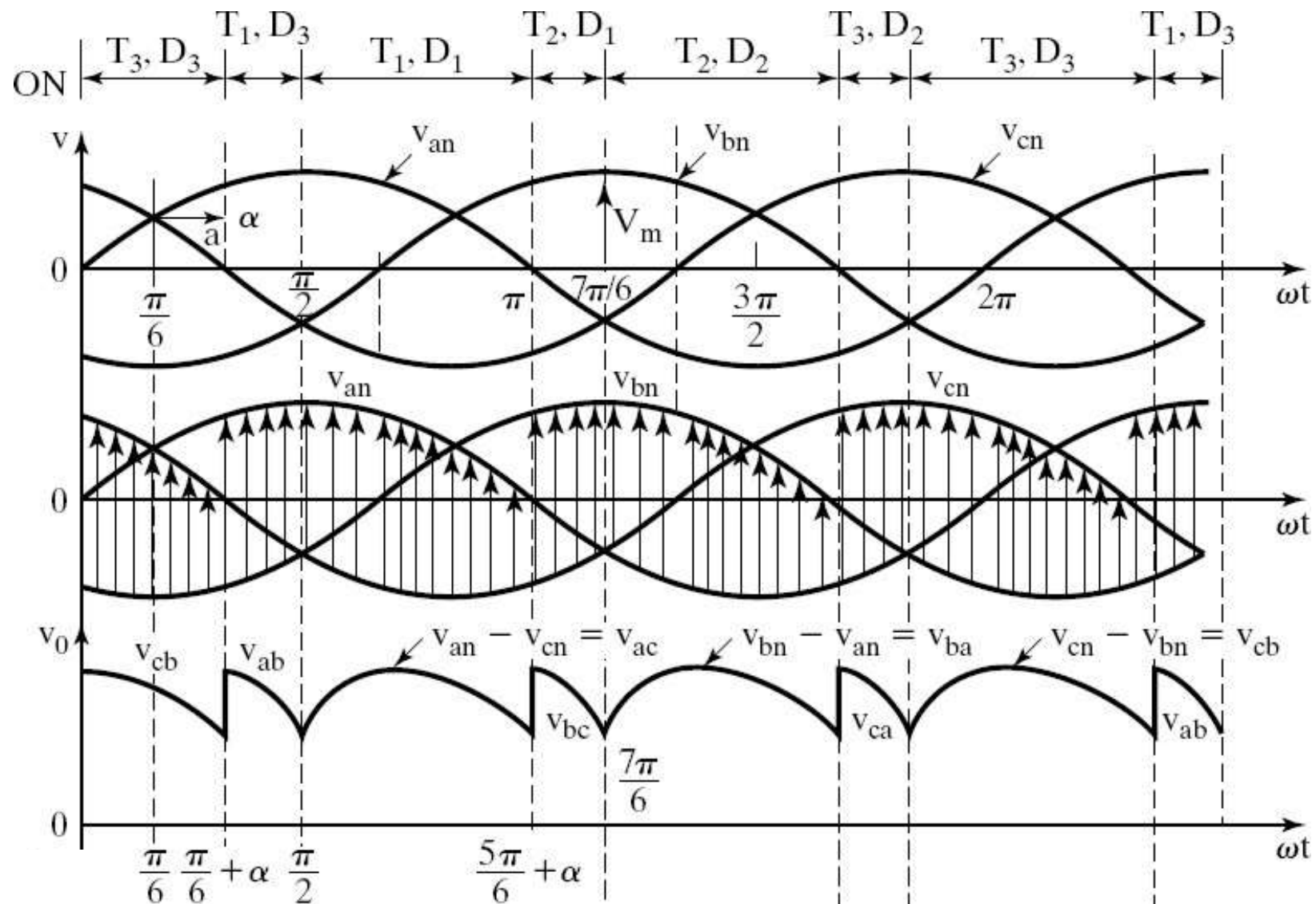
$$V_{RB} = V_{ac} = (v_{an} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$$

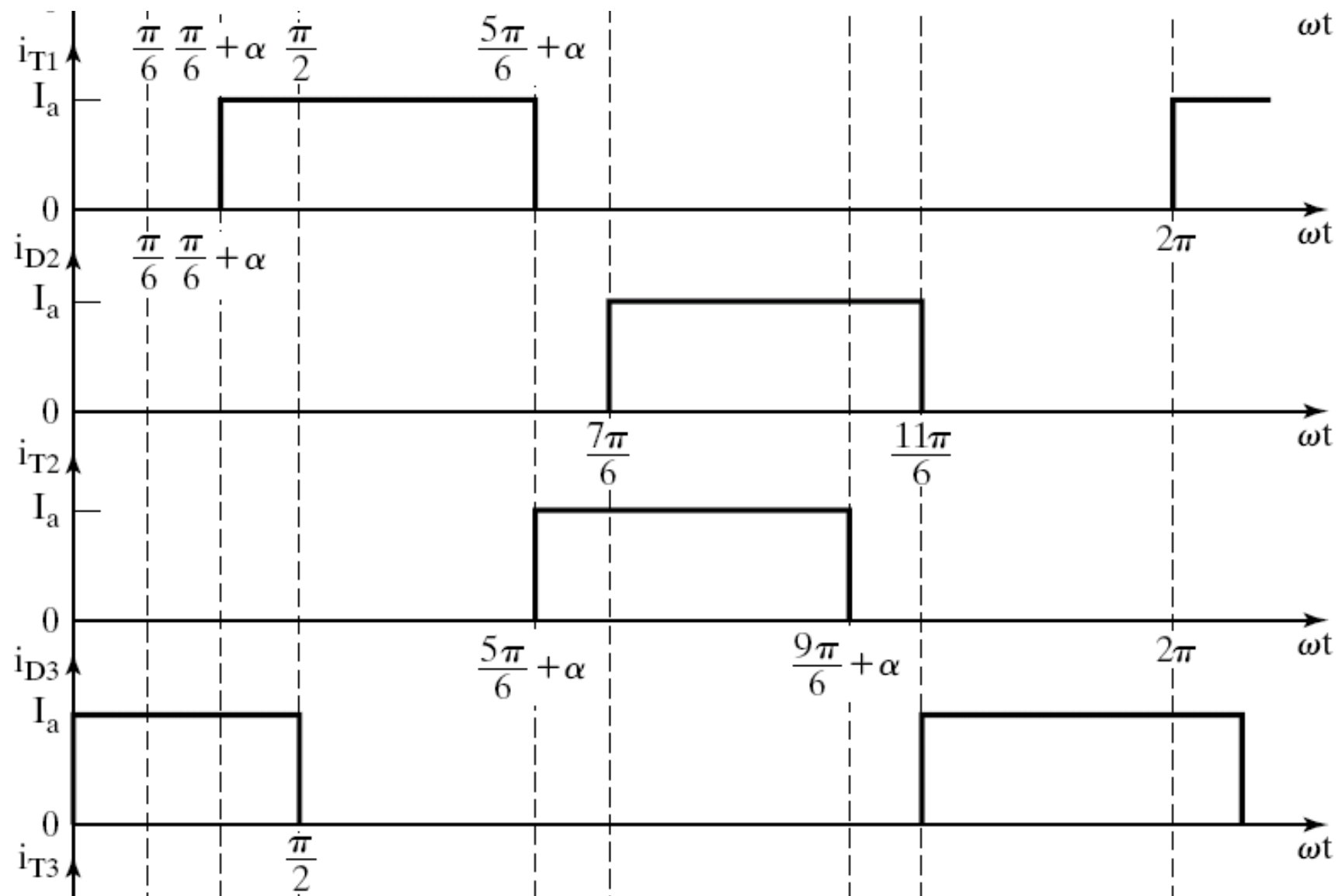
$$V_{YR} = V_{ba} = (v_{bn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

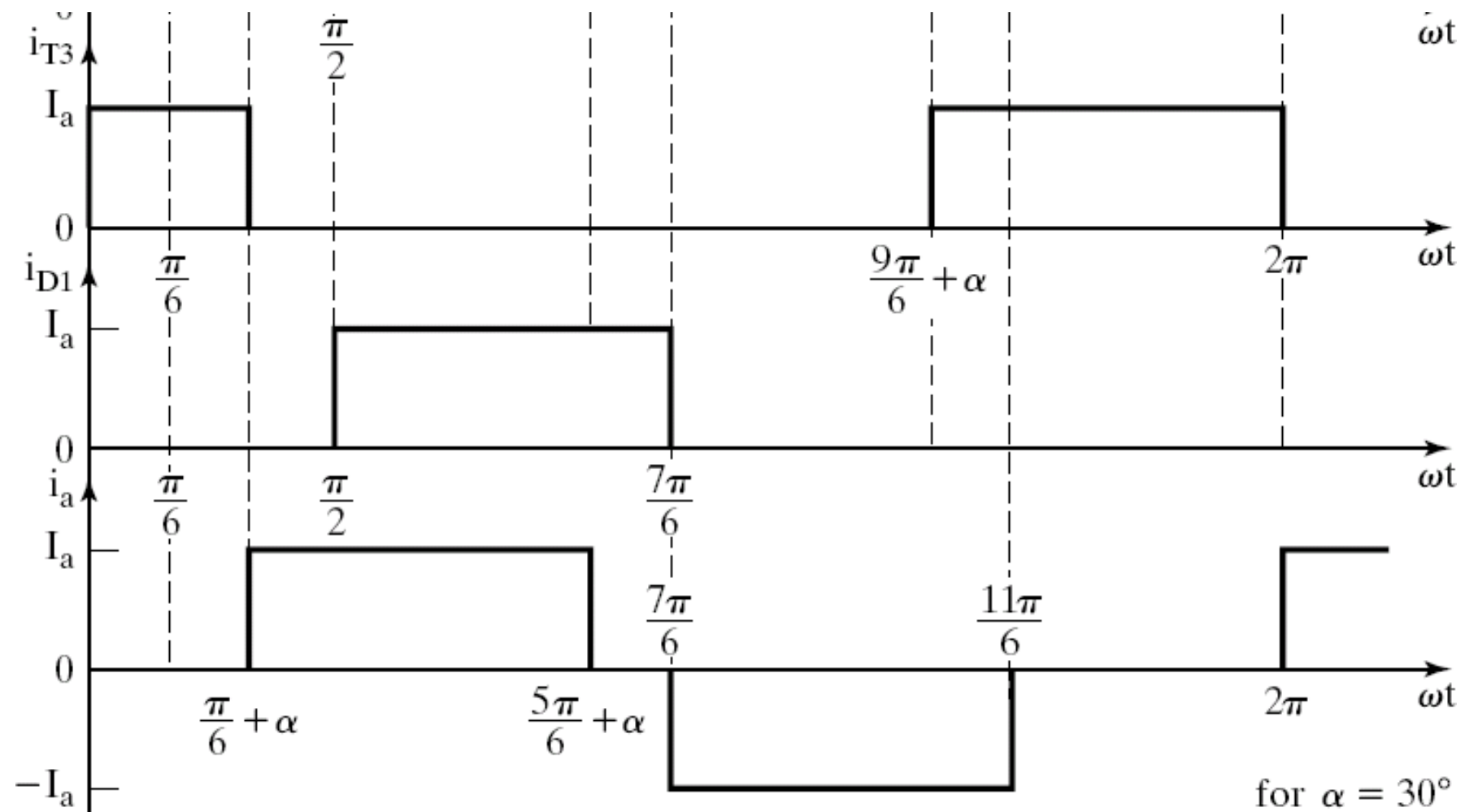
$$V_{BY} = V_{cb} = (v_{cn} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$V_{RY} = V_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

Wave forms of 3 Phase Semiconverter
for
 $\alpha \leq 60^\circ$







To derive an Expression for the
Average Output Voltage of 3 Phase
Semi-converter for $\alpha > \pi / 3$
and Discontinuous Output Voltage

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage:

the Average output voltage is found from

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{7\pi/6} v_{ac} \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3} V_m \sin\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos\alpha)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi} (1 + \cos\alpha)$$

$V_{mL} = \sqrt{3}V_m =$ Max. value of line-to-line supply voltage

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos \alpha)$$

The rms output voltage is found from

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} v_{ac}^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2 \left(\omega t - \frac{\pi}{6} \right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m \left[\frac{3}{4\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2} \right) \right]^{\frac{1}{2}}$$

Average or DC Output Voltage
of a
3-Phase Semi-converter
for $\alpha \leq \pi / 3$,
and Continuous Output Voltage

For $\alpha \leq \frac{\pi}{3}$, and continuous output voltage

$$V_{dc} = \frac{3}{2\pi} \left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab} \cdot d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac} \cdot d(\omega t) \right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi} (1 + \cos\alpha)$$

$$V_n = \frac{V_{dc}}{V_{dm} \cos \alpha} = 0.5(1 +$$

RMS value of o/p voltage is calculated by using the equation

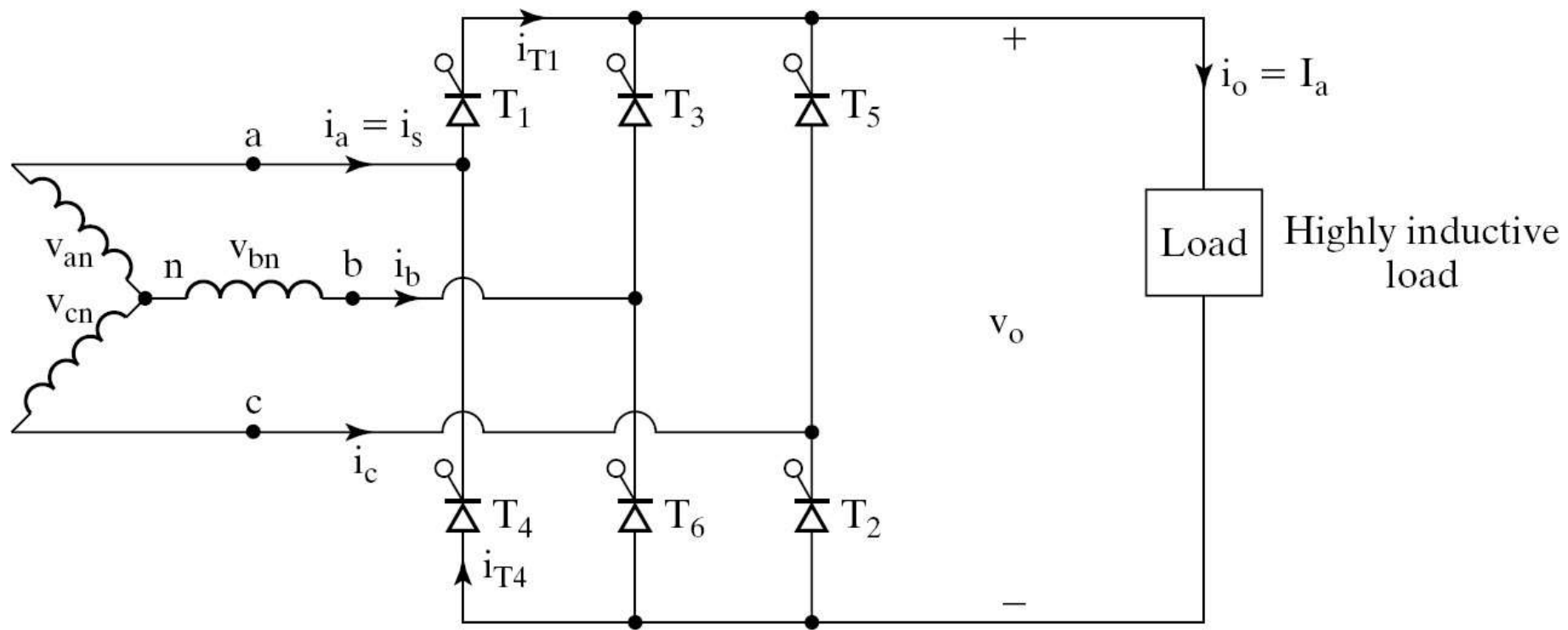
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2 \cdot d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

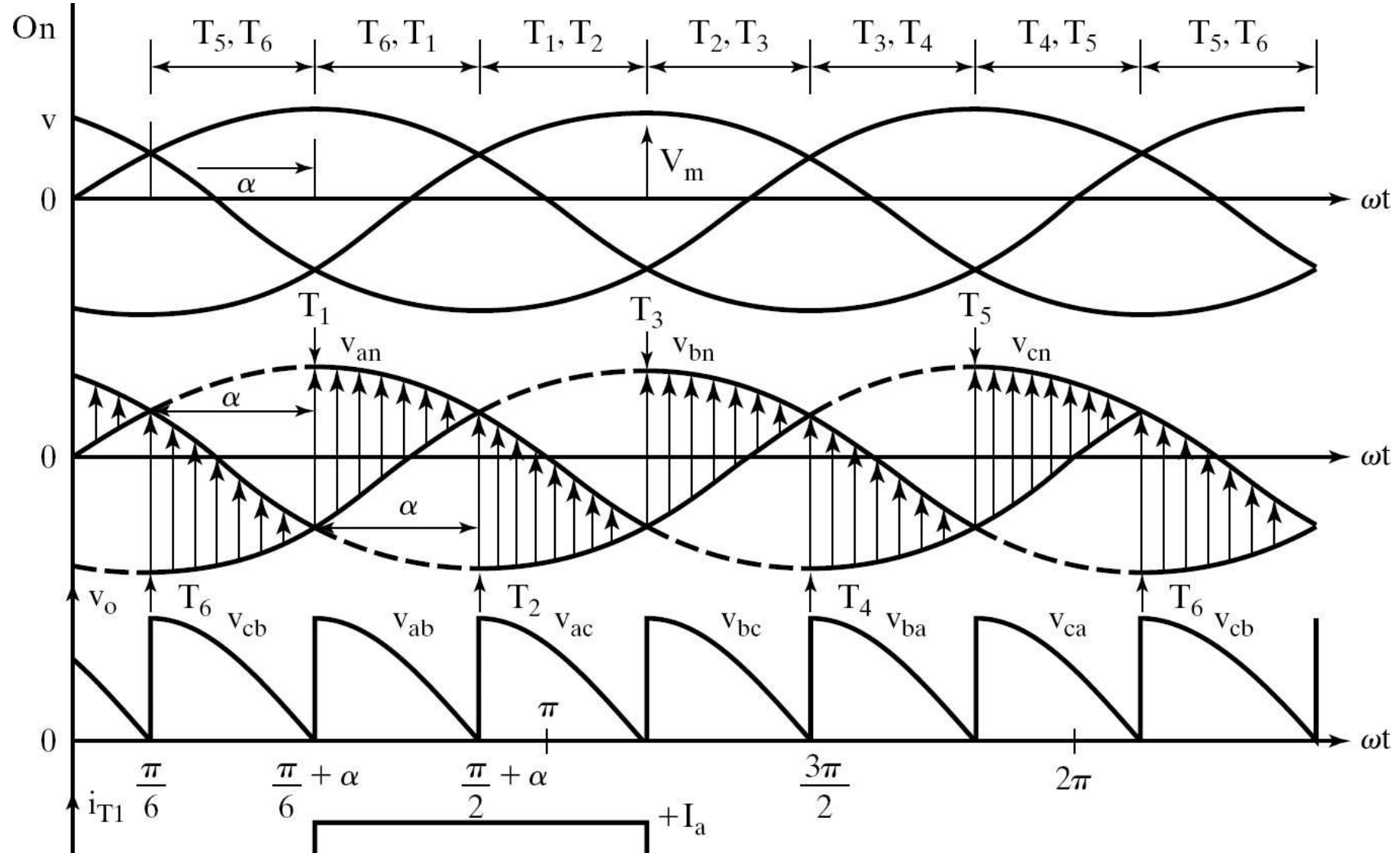
$$V_{O(rms)} = \sqrt{3} V_m \left[\frac{3}{4\pi} \left(\frac{2\pi}{3} + \sqrt{3} \cos^2 \alpha \right) \right]^{\frac{1}{2}}$$

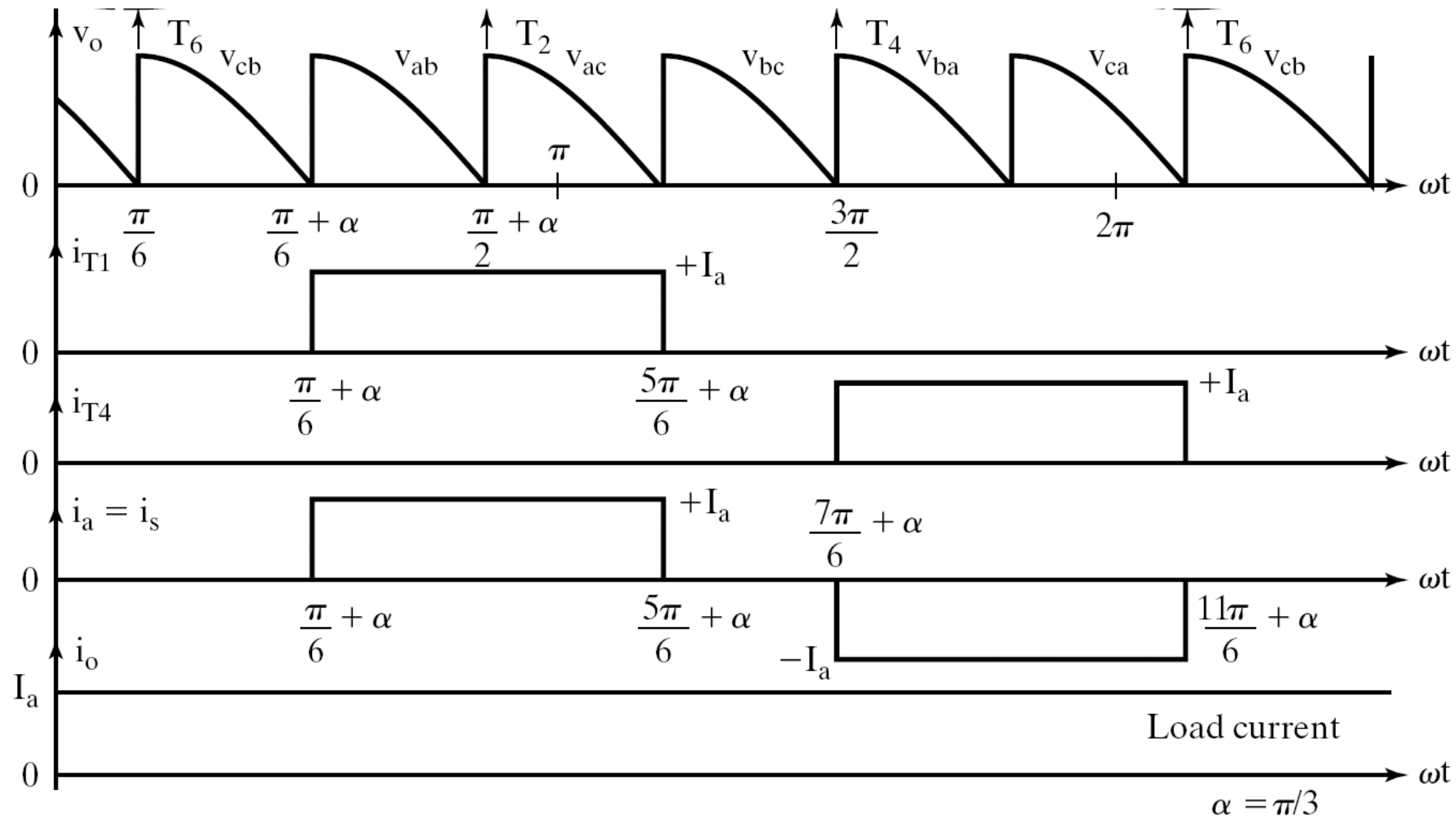
Three Phase Full Converter

Three Phase Full Converter

- 3 Phase Fully Controlled Full Wave Bridge Converter.
- Known as a 6-pulse converter.
- Used in industrial applications up to 120kW output power.
- Two quadrant operation is possible.







- The thyristors are triggered at an interval of $\frac{\pi}{3}$
- The frequency of output ripple voltage is $6f_s$.
- T_1 is triggered at $\omega t = (\pi/6 + \alpha)$, T_6 is already conducting when T_1 is turned ON.
- During the interval $(\pi/6 + \alpha)$ to $(\pi/2 + \alpha)$, T_1 and T_6 conduct together & the output load voltage is equal to $v_{ab} = (v_{an} - v_{bn})$

- T_2 is triggered at $\omega t = (\pi/2 + \alpha)$, T_6 turns off naturally as it is reverse biased as soon as T_2 is triggered.
- During the interval $(\pi/2 + \alpha)$ to $(5\pi/6 + \alpha)$, T_1 and T_2 conduct together & the output load voltage $v_O = v_{ac} = (v_{an} - v_{cn})$
- Thyristors are numbered in the order in which they are triggered.
- The thyristor triggering sequence is 12, 23, 34, 45, 56, 61, 12, 23, 34,

We define three line neutral voltages
(3 phase voltages) as follows

$$V_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ)$$

$$\begin{aligned} v_{BN} = v_{cn} &= V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ) \\ &= V_m \sin (\omega t - 240^\circ) \end{aligned}$$

V_m is the peak phase voltage of a wye-connected source.

The corresponding line-to-line supply voltages are

$$V_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{YB} = v_{bc} = (v_{bn} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{BR} = v_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To Derive An Expression For
The Average Output Voltage Of
3-phase Full Converter
With Highly Inductive Load
Assuming Continuous And
Constant Load Current

The output load voltage consists of 6 voltage pulses over a period of 2π radians, Hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_o \cdot d\omega t \ ;$$

$$v_o = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{\pi} \cos\alpha = \frac{3V_{mL}}{\pi} \cos\alpha$$

Where $V_{mL} = \sqrt{3}V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle $\alpha = 0$,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_o^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

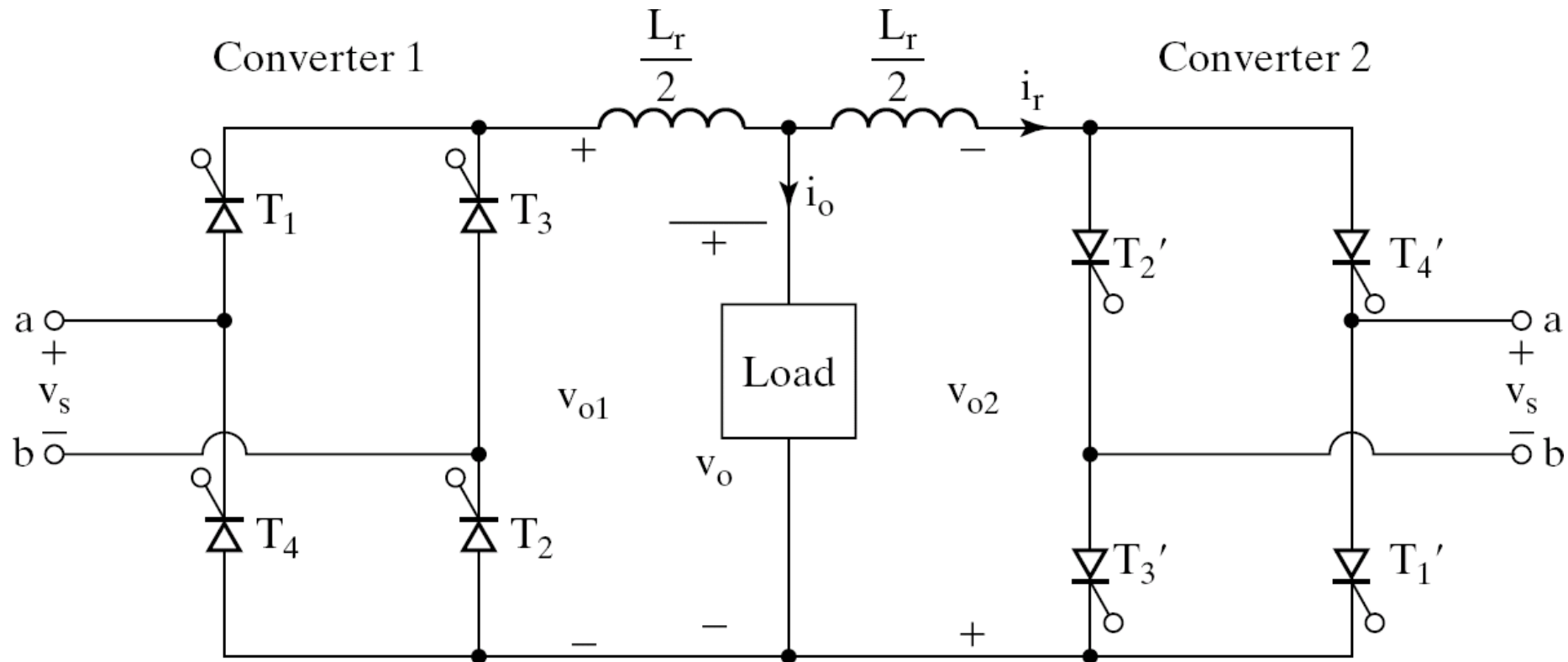
$$V_{O(rms)} = \left[\frac{6}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} v_{ab}^2 \cdot d(\omega t) \right]^{\frac{1}{2}}$$

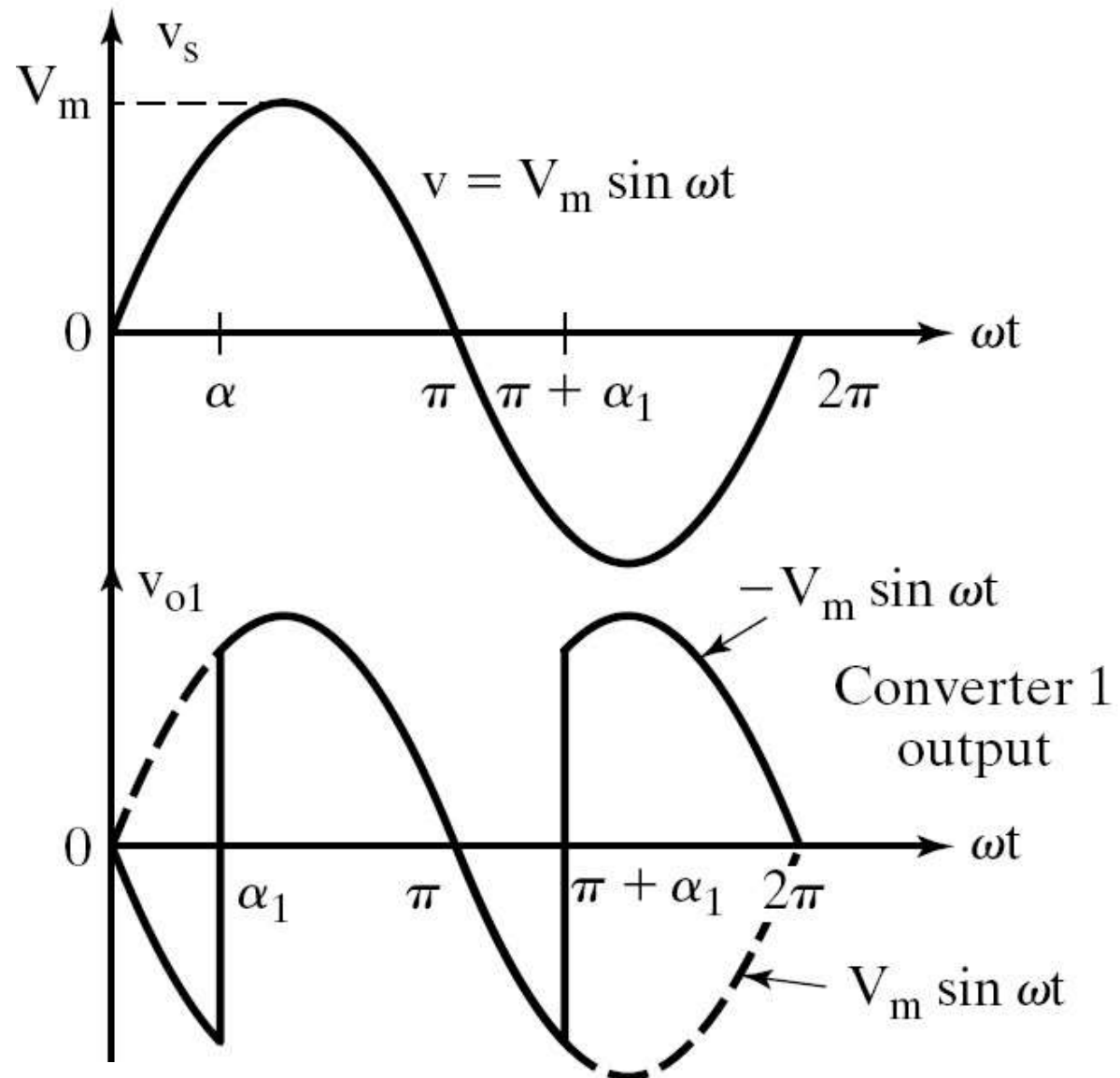
$$V_{O(rms)} = \left[\frac{3}{2\pi} \int_{\frac{\pi}{6} + \alpha}^{\frac{\pi}{2} + \alpha} 3V_m^2 \sin^2 \left(\omega t + \frac{\pi}{6} \right) \cdot d(\omega t) \right]^{\frac{1}{2}}$$

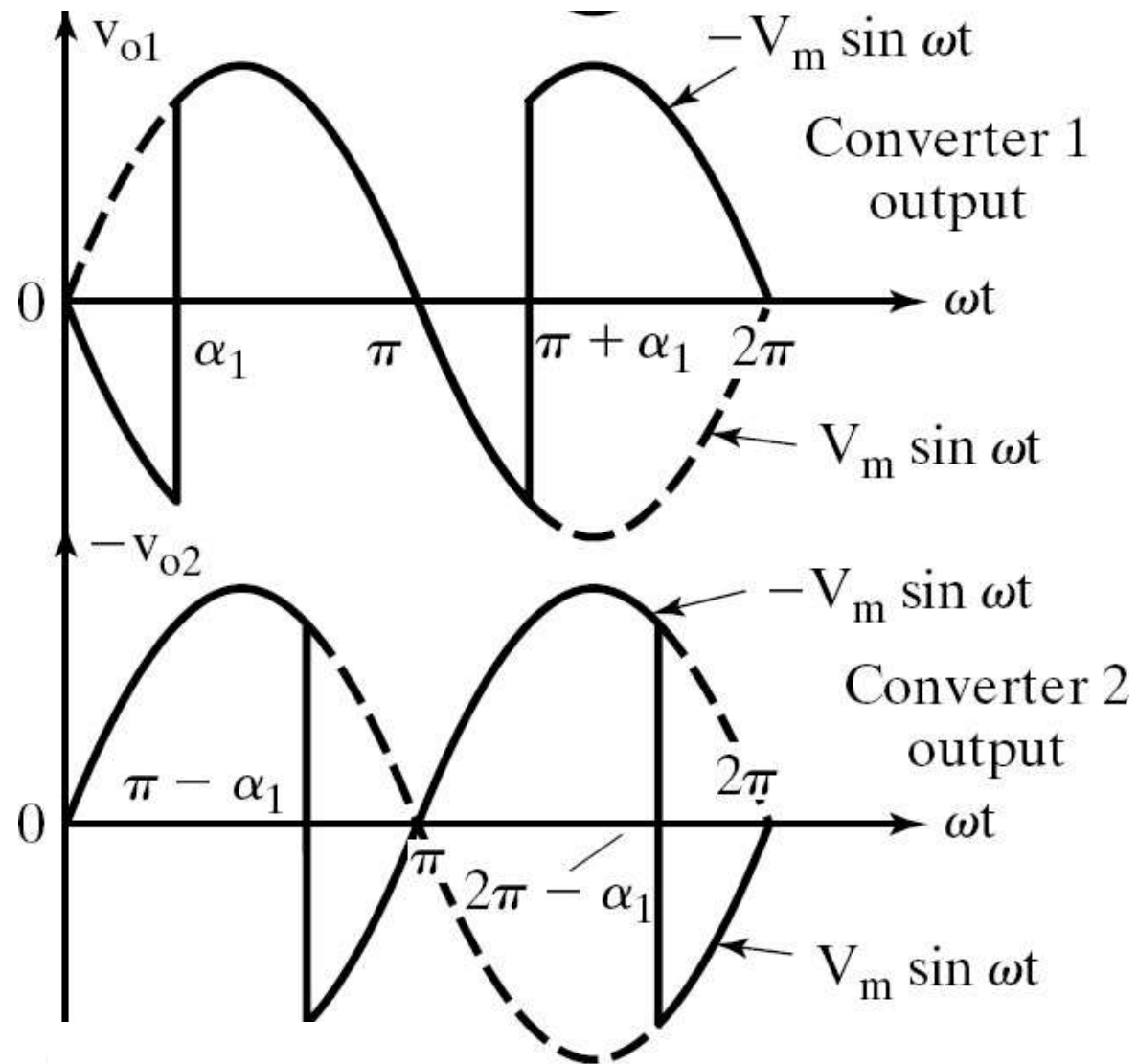
$$V_{O(rms)} = \sqrt{3}V_m \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right)^{\frac{1}{2}}$$

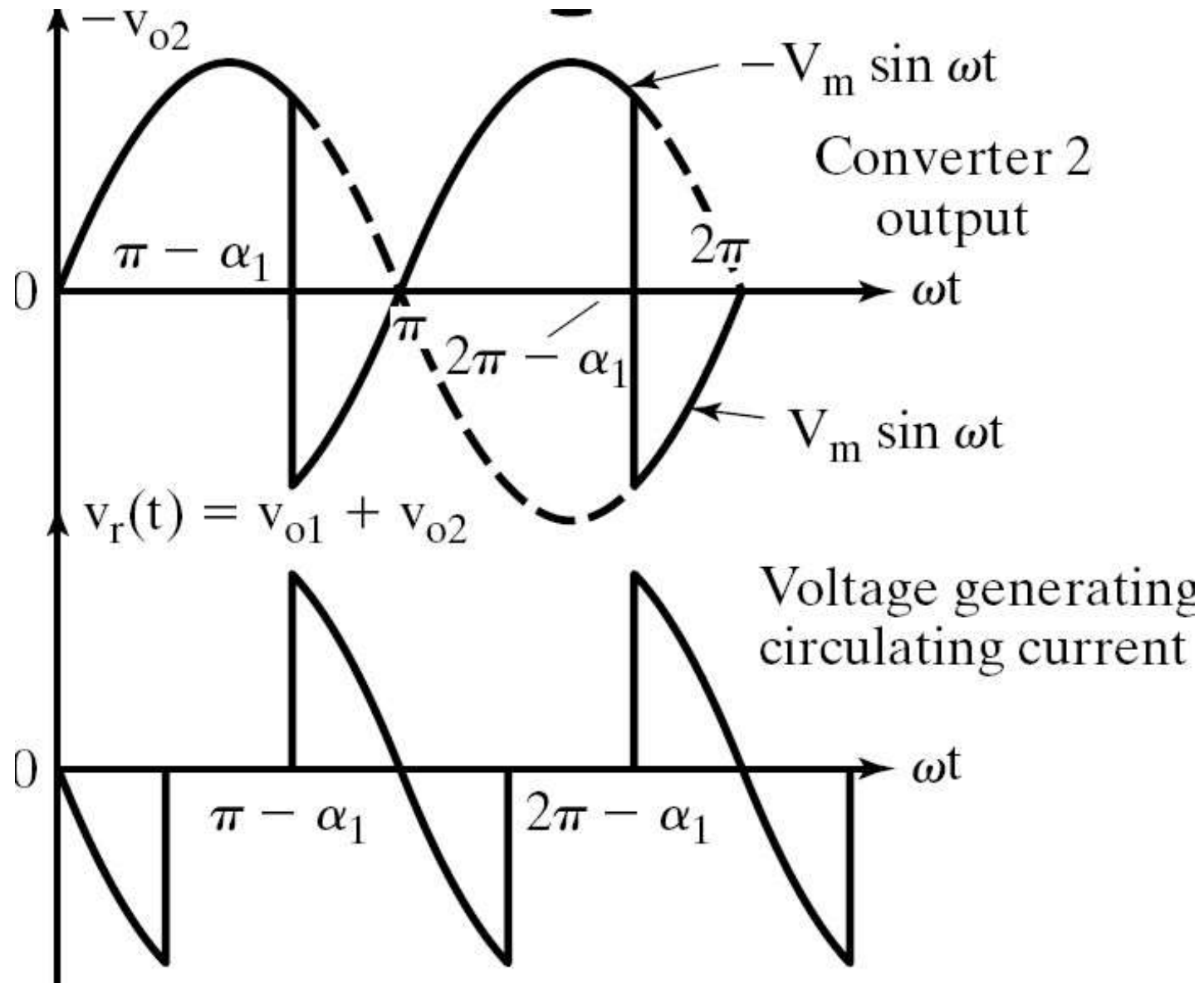
Single Phase Dual Converter

Single Phase Dual Converter









The average dc output voltage of converter 1 is

$$V_{dc1} = \frac{2V_m}{\pi} \cos\alpha_1$$

The average dc output voltage of converter 2 is

$$V_{dc2} = \frac{2V_m}{\pi} \cos\alpha_2$$

In the dual converter operation one converter is operated as a controlled rectifier with $\alpha < 90^\circ$ & the second converter is operated as a line commutated inverter in the inversion mode with $\alpha > 90^\circ$

$$\therefore V_{dc1} = -V_{dc2}$$

$$\frac{2V_m}{\pi} \cos \alpha_1 = \frac{-2V_m}{\pi} \cos \alpha_2 = \frac{2V_m}{\pi} (-\cos \alpha_2)$$

$$\therefore \cos \alpha_1 = -\cos \alpha_2$$

or

$$\cos \alpha_2 = -\cos \alpha_1 = \cos(\pi - \alpha_1)$$

$$\therefore \alpha_2 = (\pi - \alpha_1) \text{ or}$$

$$(\alpha_1 + \alpha_2) = \pi \text{ radians}$$

Which gives

$$\alpha_2 = (\pi - \alpha_1)$$

To Obtain an Expression
for the
Instantaneous Circulating Current

- v_{O1} = Instantaneous o/p voltage of converter 1.
- v_{O2} = Instantaneous o/p voltage of converter 2.
- The circulating current i_r can be determined by integrating the instantaneous voltage difference (which is the voltage drop across the circulating current reactor L_r), starting from $\omega t = (2\pi - \alpha_1)$.
- As the two average output voltages during the interval $\omega t = (\pi + \alpha_1)$ to $(2\pi - \alpha_1)$ are equal and opposite their contribution to the instantaneous circulating current i_r is zero.

$$i_r = \frac{1}{\omega L_r} \left[\int_{(2\pi - \alpha_1)}^{\omega t} v_r \cdot d(\omega t) \right] ; \quad v_r = (v_{O1} - v_{O2})$$

As the o/p voltage v_{O2} is negative

$$v_r = (v_{O1} + v_{O2})$$

$$\therefore i_r = \frac{1}{\omega L_r} \left[\int_{(2\pi - \alpha_1)}^{\omega t} (v_{O1} + v_{O2}) \cdot d(\omega t) \right] ;$$

$$v_{O1} = -V_m \sin \omega t \text{ for } (2\pi - \alpha_1) \text{ to } \omega t$$

$$i_r = \frac{V_m}{\omega L_r} \left[\int_{(2\pi - \alpha_1)}^{\omega t} -\sin \omega t . d(\omega t) - \int_{(2\pi - \alpha_1)}^{\omega t} \sin \omega t . d(\omega t) \right]$$

$$i_r = \frac{2V_m}{\omega L_r} (\cos \omega t - \cos \alpha_1)$$

The instantaneous value of the circulating current depends on the delay angle.

For trigger angle (delay angle) $\alpha_1 = 0$,
the magnitude of circulating current becomes min.
when $\omega t = n\pi$, $n = 0, 2, 4, \dots$ & magnitude becomes
max. when $\omega t = n\pi$, $n = 1, 3, 5, \dots$

If the peak load current is I_p , one of the
converters that controls the power flow
may carry a peak current of

$$\left(I_p + \frac{4V_m}{\omega L_r} \right),$$

wher

e

$$I_p = I_{L(\max)} = \frac{V_m}{R_L},$$

&

$$i_{r(\max)} = \frac{4V_m}{\omega L_r} = \text{max. circulating current}$$

Different Modes Of Operation of Dual converter

- Non-circulating current (circulating current free) mode of operation.
- Circulating current mode of operation.

Non-Circulating Current Mode of Operation

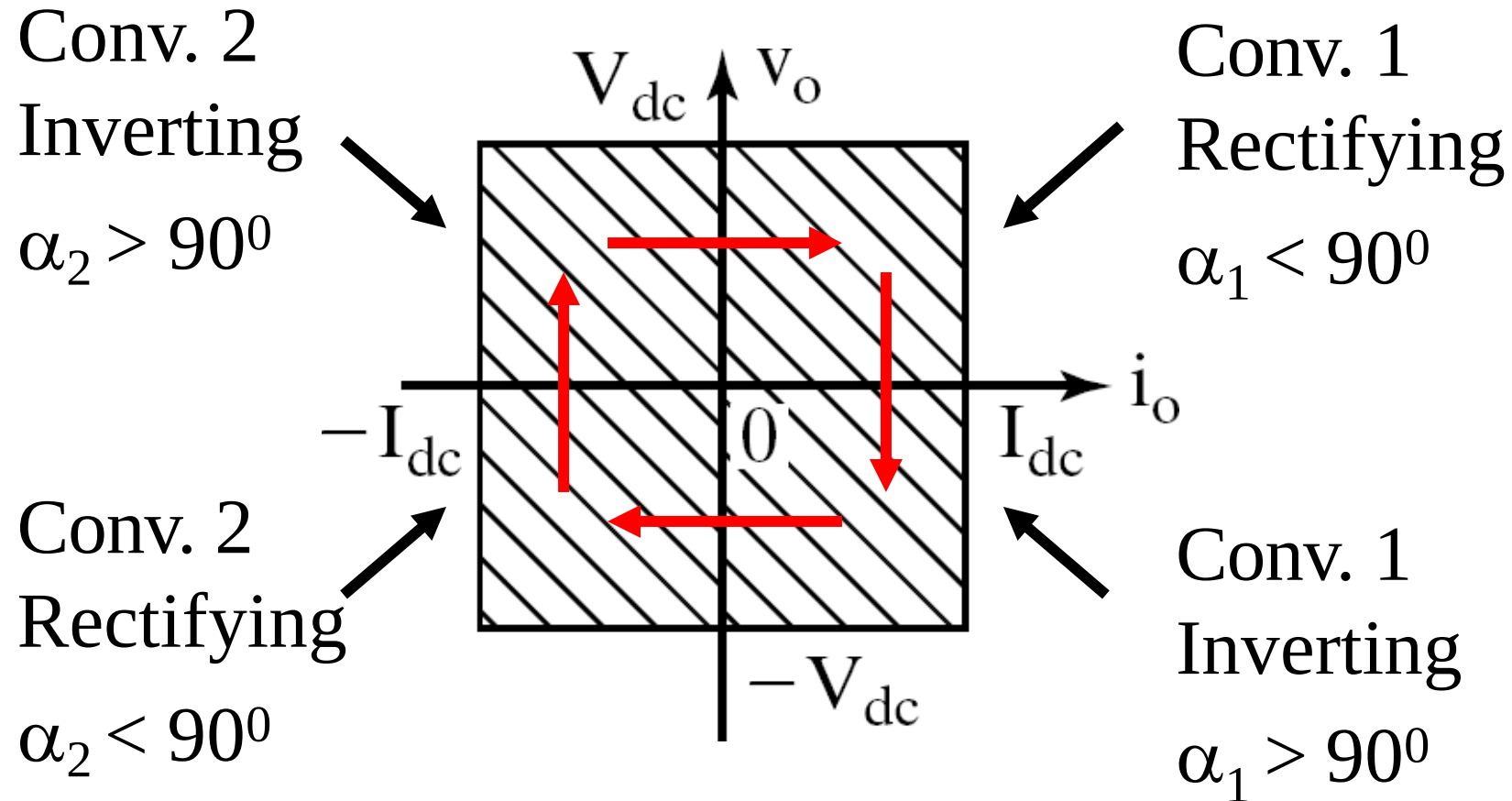
- In this mode only one converter is operated at a time.
- When converter 1 is ON, $0 < \alpha_1 < 90^\circ$
- V_{dc} is positive and I_{dc} is positive.
- When converter 2 is ON, $0 < \alpha_2 < 90^\circ$
- V_{dc} is negative and I_{dc} is negative.

Circulating Current Mode Of Operation

- In this mode, both the converters are switched ON and operated at the same time.
- The trigger angles α_1 and α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^\circ$; $\alpha_2 = (180^\circ - \alpha_1)$.

- When $0 < \alpha_1 < 90^\circ$, converter 1 operates as a controlled rectifier and converter 2 operates as an inverter with $90^\circ < \alpha_2 < 180^\circ$.
- In this case V_{dc} and I_{dc} , both are positive.
- When $90^\circ < \alpha_1 < 180^\circ$, converter 1 operates as an Inverter and converter 2 operated as a controlled rectifier by adjusting its trigger angle α_2 such that $0 < \alpha_2 < 90^\circ$.
- In this case V_{dc} and I_{dc} , both are negative.

Four Quadrant Operation



Advantages of Circulating Current Mode Of Operation

- The circulating current maintains continuous conduction of both the converters over the complete control range, independent of the load.
- One converter always operates as a rectifier and the other converter operates as an inverter, the power flow in either direction at any time is possible.

- As both the converters are in continuous conduction we obtain faster dynamic response. i.e., the time response for changing from one quadrant operation to another is faster.

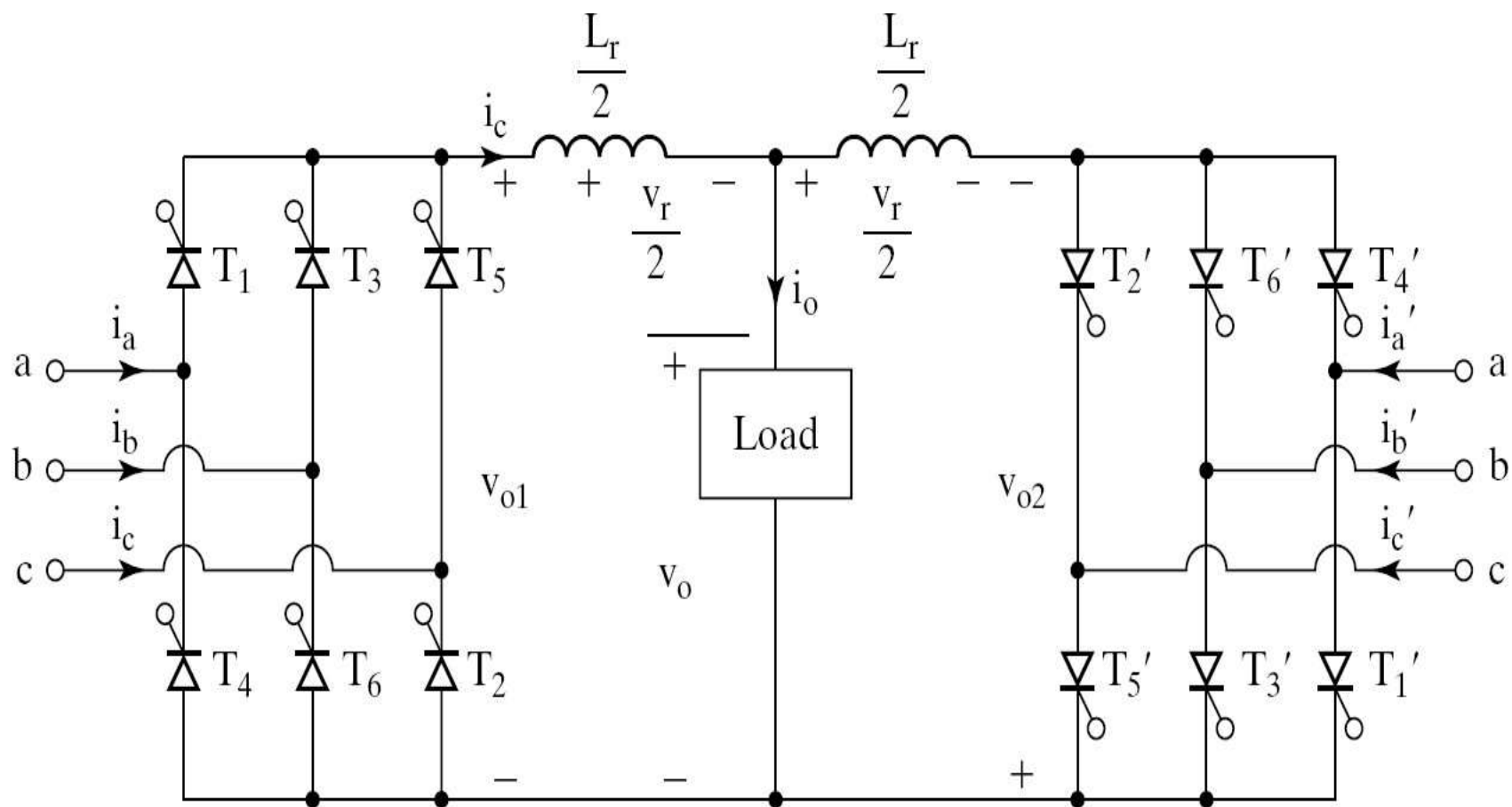
Disadvantages of Circulating Current Mode Of Operation

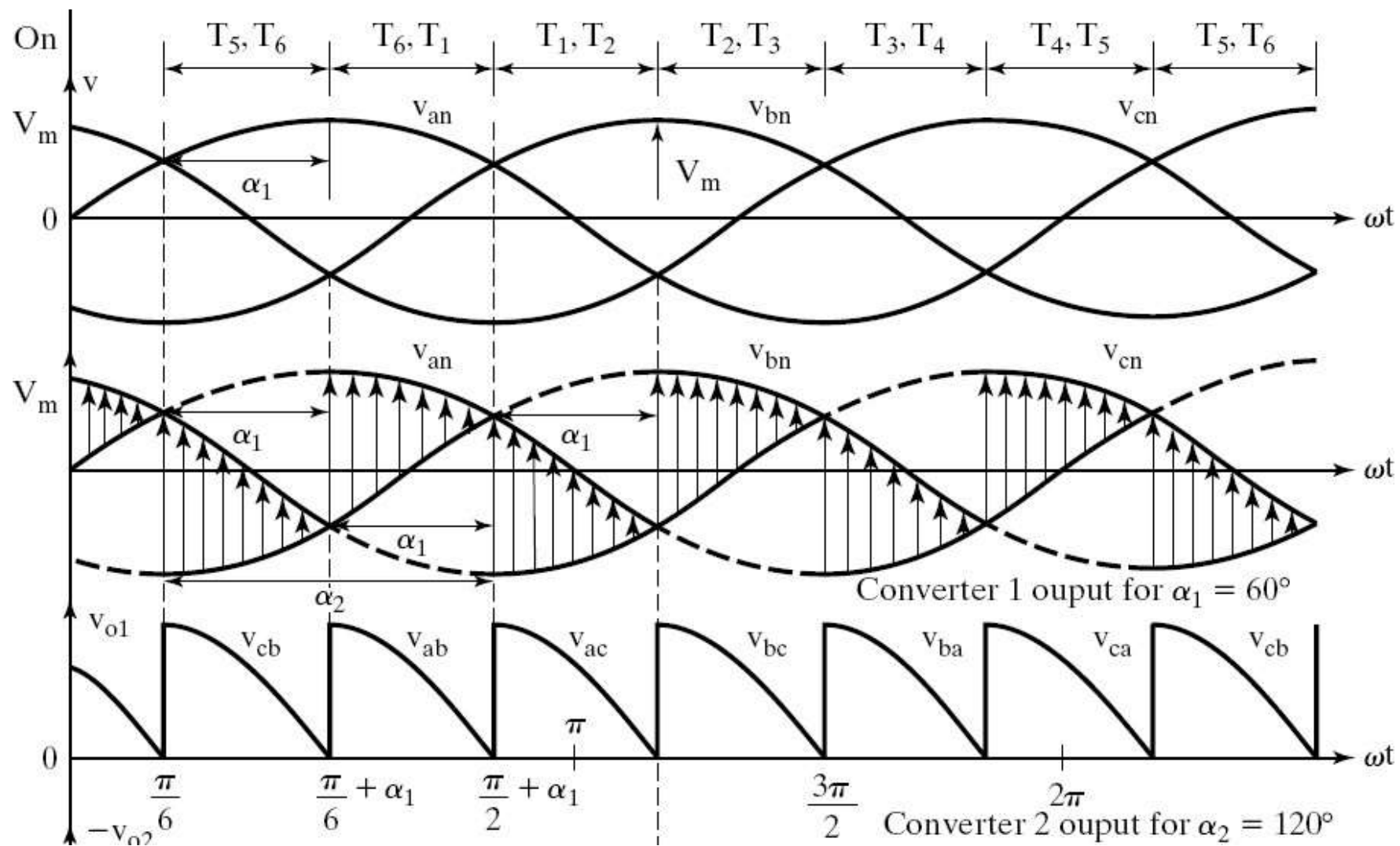
- There is always a circulating current flowing between the converters.
- When the load current falls to zero, there will be a circulating current flowing between the converters so we need to connect circulating current reactors in order to limit the peak circulating current to safe level.
- The converter thyristors should be rated to carry a peak current much greater than the peak load current.

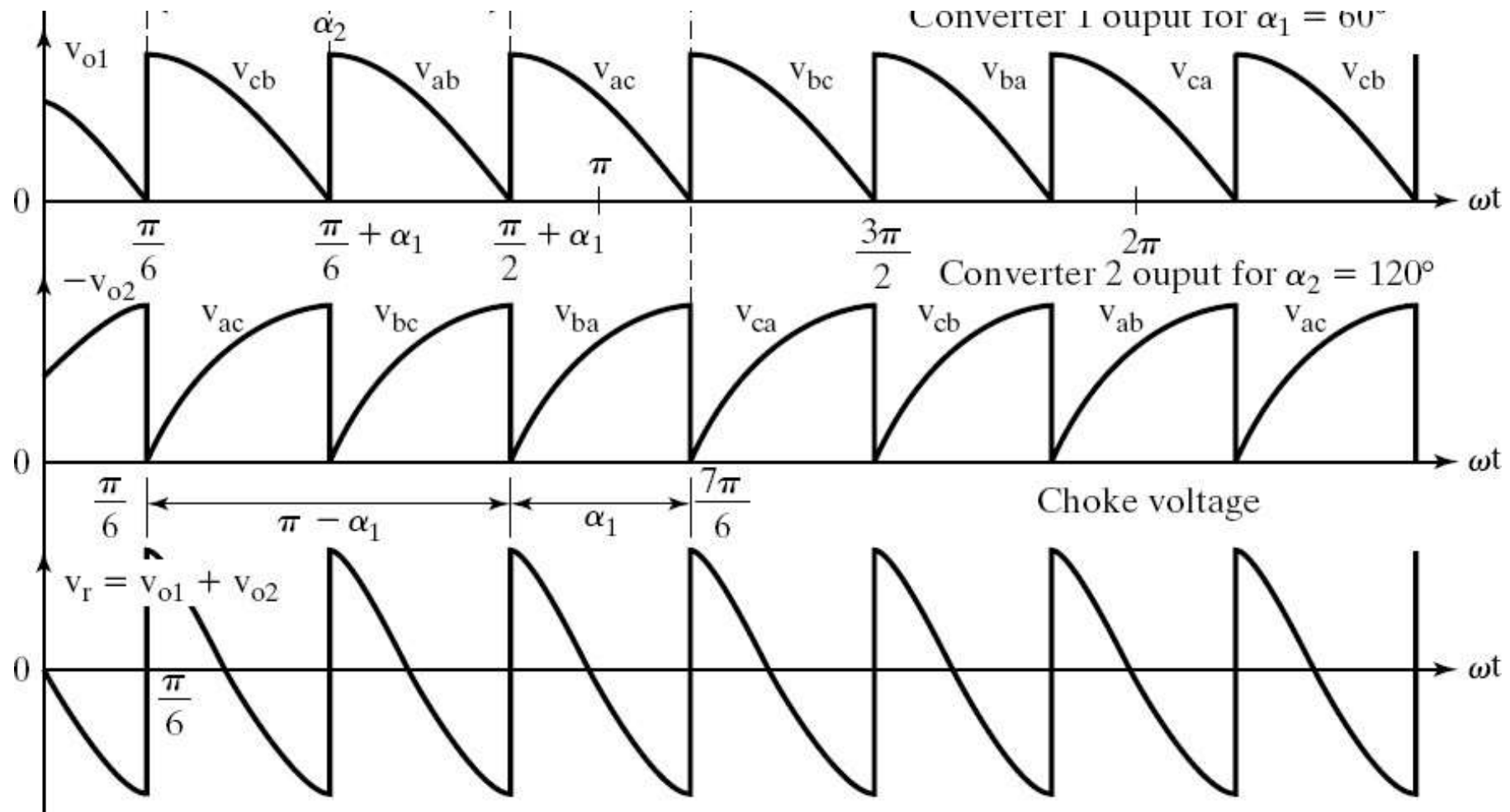
Three Phase Dual Converters

Three Phase Dual Converters

- For four quadrant operation in many industrial variable speed dc drives , 3 phase dual converters are used.
- Used for applications up to 2 mega watt output power level.
- Dual converter consists of two 3 phase full converters which are connected in parallel & in opposite directions across a common load.







Outputs of Converters 1 & 2

- During the interval $(\pi/6 + \alpha_1)$ to $(\pi/2 + \alpha_1)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We define three line neutral voltages
(3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ;$$

$$V_m = \text{Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin \left(\omega t - \frac{2\pi}{3} \right) = V_m \sin (\omega t - 120^\circ)$$

$$\begin{aligned} v_{BN} = v_{cn} &= V_m \sin \left(\omega t + \frac{2\pi}{3} \right) = V_m \sin (\omega t + 120^\circ) \\ &= V_m \sin (\omega t - 240^\circ) \end{aligned}$$

The corresponding line-to-line supply voltages are

$$V_{RY} = V_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{YB} = V_{bc} = (v_{bn} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{BR} = V_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

To obtain an Expression for the Circulating Current

- If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $(\pi/6 + \alpha_1) \leq \omega t \leq (\pi/2 + \alpha_1)$ is given by

$$V_r = V_{O1} + V_{O2} = V_{ab} - V_{bc}$$

$$v_r = \sqrt{3}V_m \left[\sin\left(\omega t + \frac{\pi}{6}\right) - \sin\left(\omega t - \frac{\pi}{2}\right) \right]$$

$$v_r = 3V_m \cos\left(\omega t - \frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

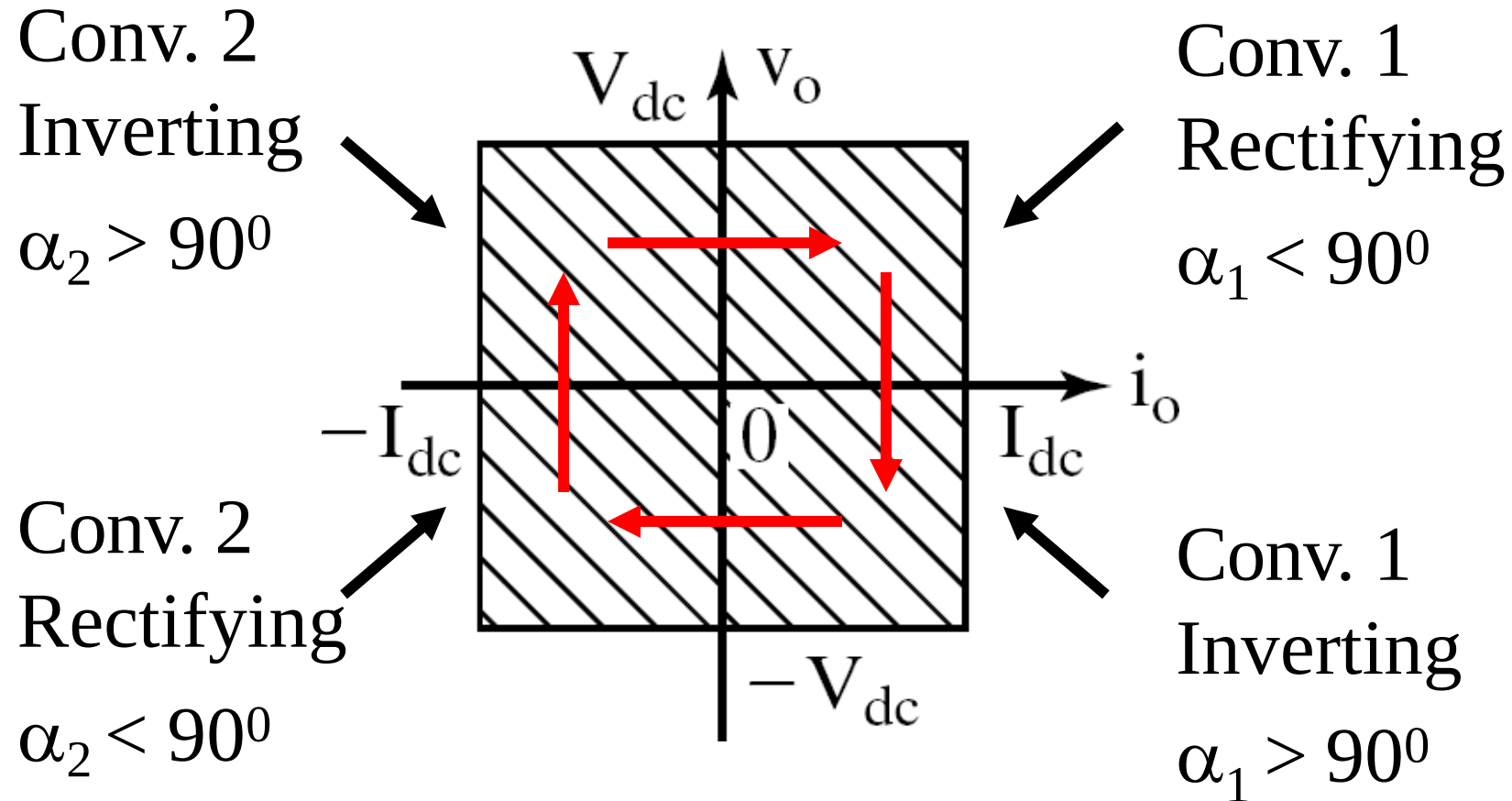
$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} v_r \cdot d(\omega t)$$

$$i_r(t) = \frac{1}{\omega L_r} \int_{\frac{\pi}{6} + \alpha_1}^{\omega t} 3V_m \cos\left(\omega t - \frac{\pi}{6}\right) \cdot d(\omega t)$$

$$i_r(t) = \frac{3V_m}{\omega L_r} \left[\sin\left(\omega t - \frac{\pi}{6}\right) - \sin\alpha_1 \right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r}$$

Four Quadrant Operation



Contd...

- There are two different modes of operation.
 - Circulating current free
(non circulating) mode of operation
 - Circulating current mode of operation

Non Circulating Current Mode Of Operation

- In this mode of operation only one converter is switched on at a time
- When the converter 1 is switched on,
For $\alpha_1 < 90^\circ$ the converter 1 operates in the Rectification mode
 V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.
- Power flows from ac source to the load

- When the converter 1 is on,
For $\alpha_1 > 90^\circ$ the converter 1 operates in the
Inversion mode
 V_{dc} is negative, I_{dc} is positive and the average
load power P_{dc} is negative.
- Power flows from load circuit to ac source.

- When the converter 2 is switched on,
For $\alpha_2 < 90^\circ$ the converter 2 operates in the Rectification mode
 V_{dc} is negative, I_{dc} is negative and the average load power P_{dc} is positive.
- The output load voltage & load current reverse when converter 2 is on.
- Power flows from ac source to the load

- When the converter 2 is switched on,
For $\alpha_2 > 90^\circ$ the converter 2 operates in the
Inversion mode
 V_{dc} is positive, I_{dc} is negative and the average
load power P_{dc} is negative.
- Power flows from load to the ac source.
- Energy is supplied from the load circuit to the
ac supply.

Circulating Current Mode Of Operation

- Both the converters are switched on at the same time.
- One converter operates in the rectification mode while the other operates in the inversion mode.
- Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^\circ$

- When $\alpha_1 < 90^\circ$, converter 1 operates as a controlled rectifier. α_2 is made greater than 90° and converter 2 operates as an Inverter.
- V_{dc} is positive & I_{dc} is positive and P_{dc} is positive.

UNIT-III

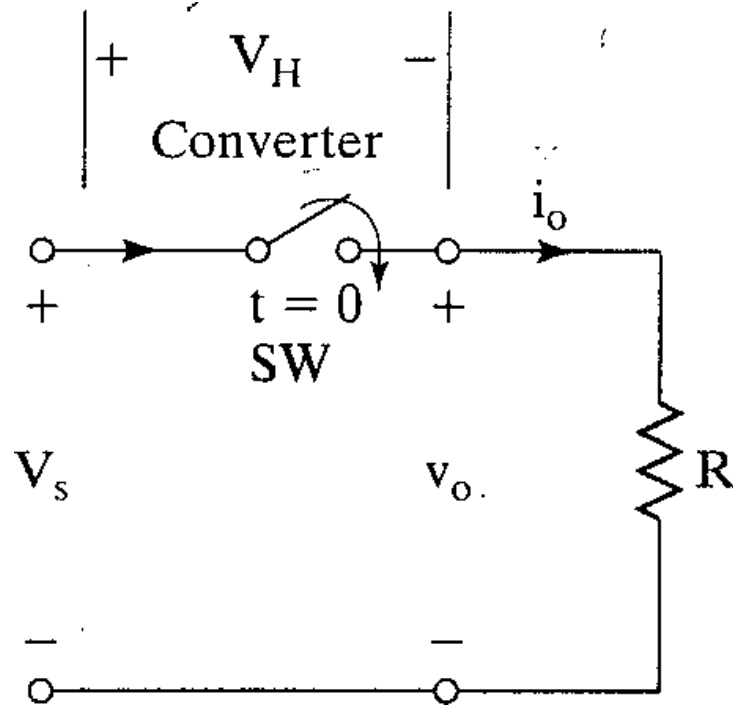
DC-DC Converters

(Chopper/SMPS)

DC-DC Converters

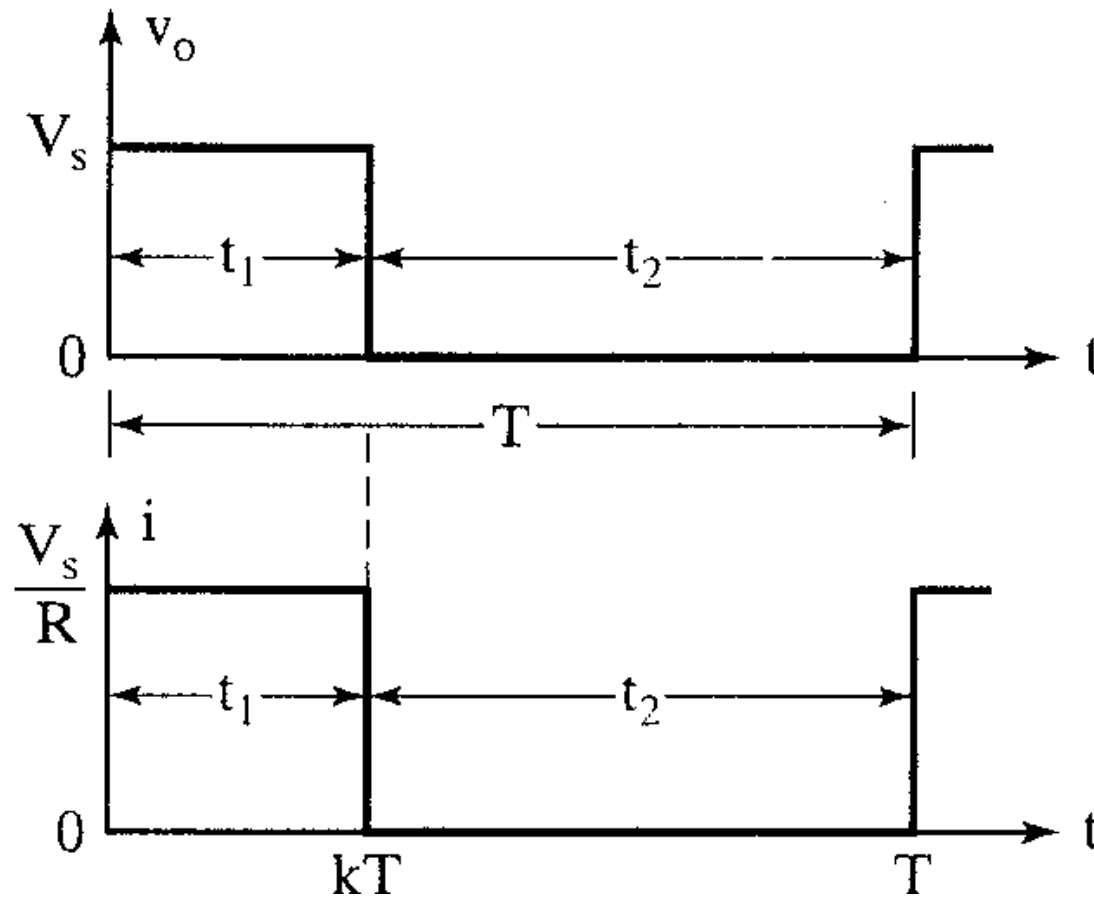
- Convert a fixed DC Source into a Variable DC Source
- DC equivalent to an AC transformer with variable turns ratio
- Step-up and Step-down versions
- Applications
 - Motor Control
 - Voltage Regulators

Step-down Operation

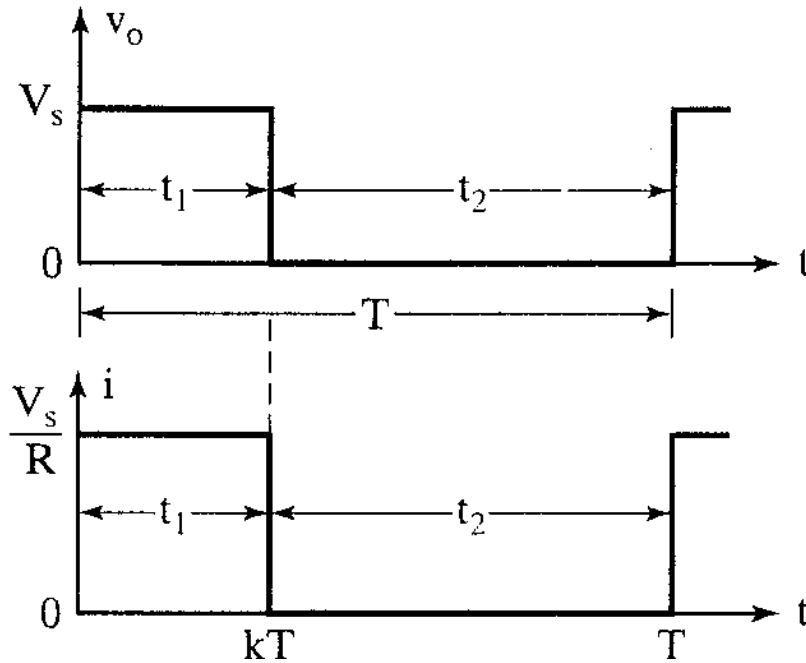


- Switch SW is known as a “Chopper”
- Use BJT, MOSFET, or IGBT
- Close for time t_1
 - V_s appears across R
- Open for time t_2
 - Voltage across $R = 0$
- Repeat
- Period $T = t_1 + t_2$

Waveforms for the Step-Down Converter



Average Value of the Output Voltage



$$V_a = \frac{1}{T} \int_0^{t_1} v_o dt$$

$$V_a = \frac{1}{T} \int_0^{t_1} V_s dt$$

$$V_a = \frac{t_1}{T} V_s = f t_1 V_s$$

$$V_a = k V_s$$

Average Value of the Load Current

$$I_a = \frac{V_a}{R} = \frac{kV_s}{R}$$

$T = \text{period}$

$k = \frac{t_1}{T} = \text{duty cycle}$

$f = \text{frequency}$

rms Value of the output voltage

$$V_O = \left(\frac{1}{T} \int_0^{kT} v_O^2 dt \right)^{\frac{1}{2}}$$

$$V_O = \left(\frac{1}{T} \int_0^{kT} V_S^2 dt \right)^{\frac{1}{2}}$$

$$V_O = \sqrt{k} V_S$$

If the converter is “lossless”, $P_{in} = P_{out}$

$$P_{in} = \frac{1}{T} \int_0^{kT} v_O i dt$$

$$P_{in} = \frac{1}{T} \int_0^{kT} \frac{v_O^2}{R} dt$$

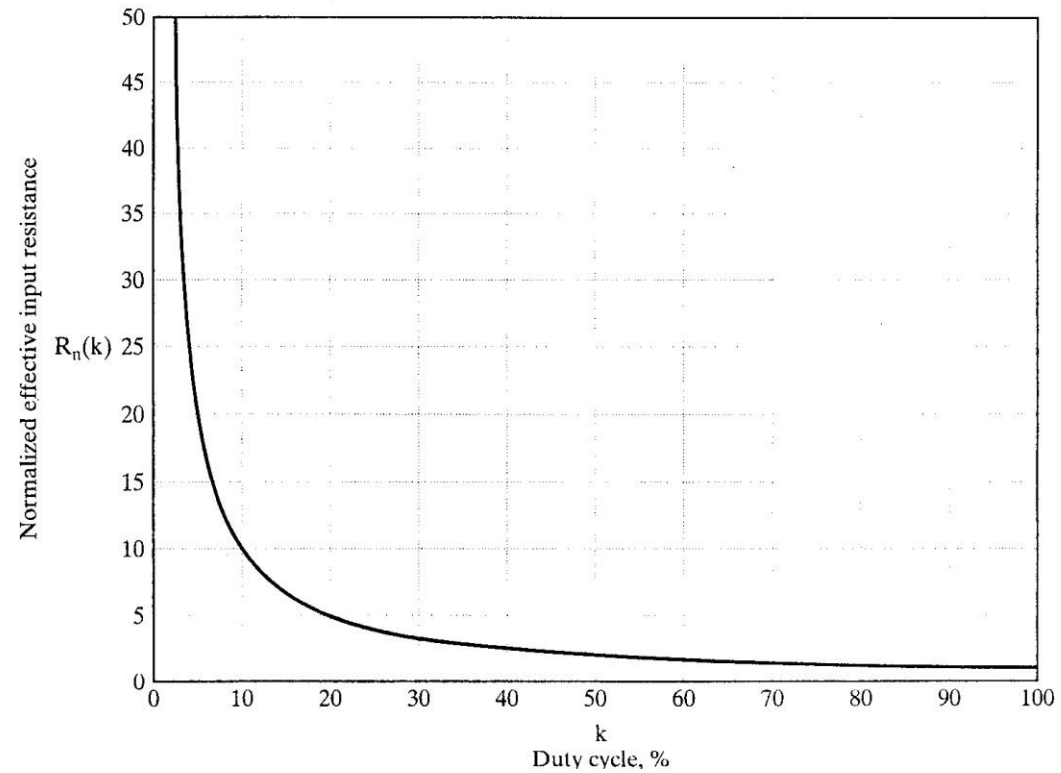
$$P_{in} = \frac{1}{T} \frac{V_S^2}{R} kT$$

$$P_{in} = k \frac{V_S^2}{R}$$

Effective Input Resistance seen by V_S

$$R_i = \frac{V_S}{I_a} = \frac{V_S}{k \frac{V_S}{R}}$$

$$R_i = \frac{R}{k}$$

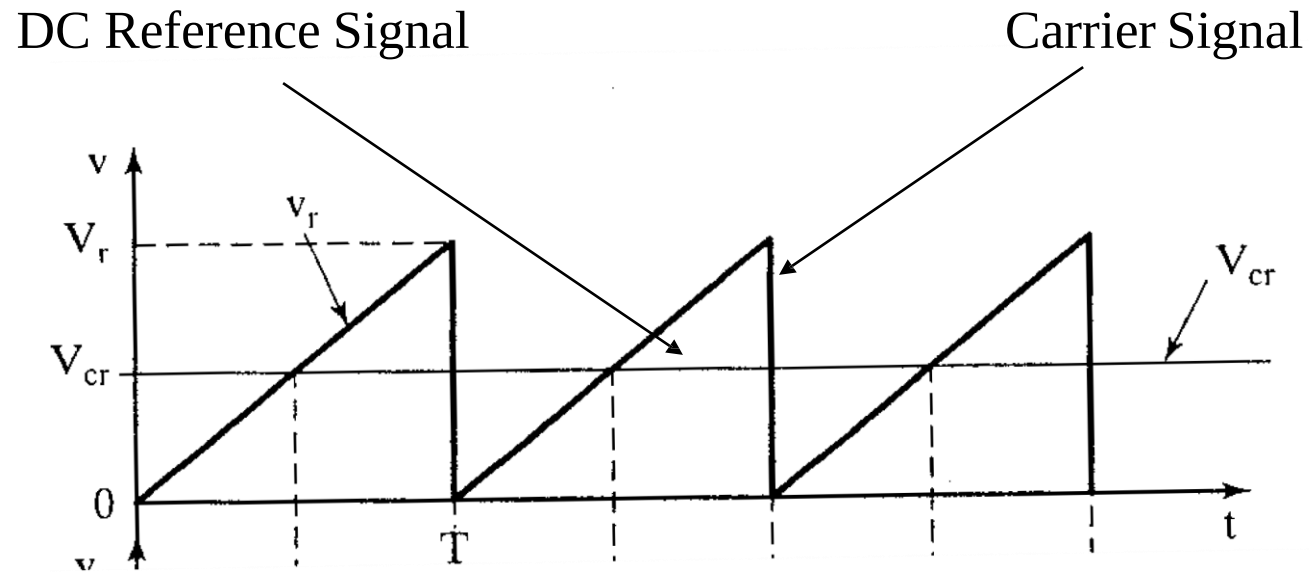


Modes of Operation

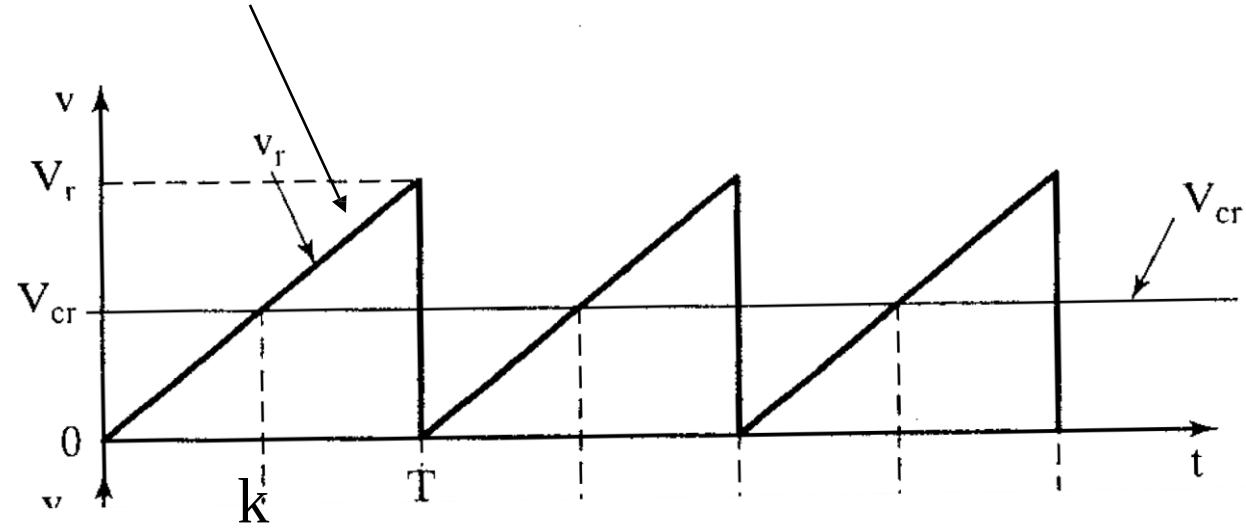
- Constant – frequency operation
 - Period T held constant, t_1 varied
 - Width of the pulse changes
 - “Pulse-width modulation”, PWM
- Variable -- frequency operation
 - Change the chopping frequency (period T)
 - Either t_1 or t_2 is kept constant
 - “Frequency modulation”

Generation of Duty Cycle

- Compare a dc reference signal with a saw-tooth carrier signal



$$v_r = \frac{V_r}{T} k$$

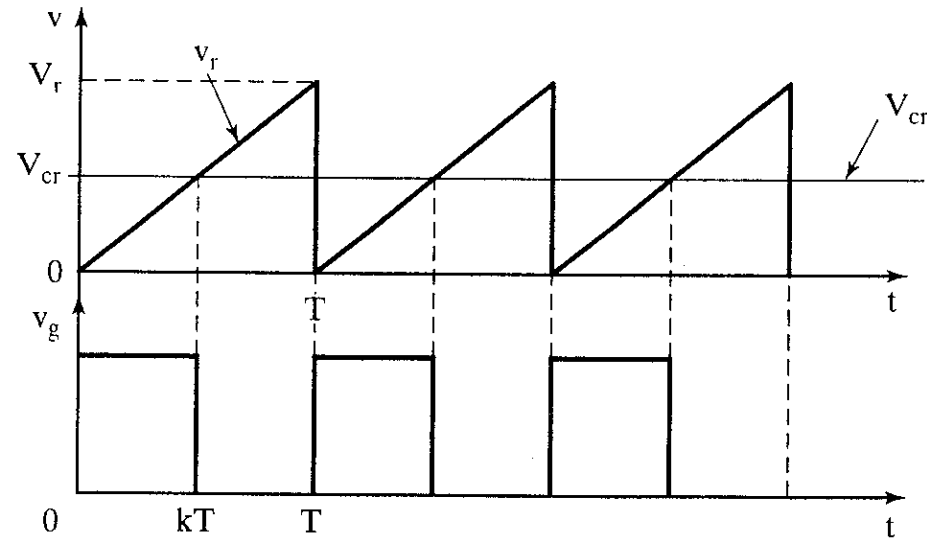


$$v_r = V_{cr} @ t = kT$$

$$V_{cr} = \frac{V_r}{T} kT$$

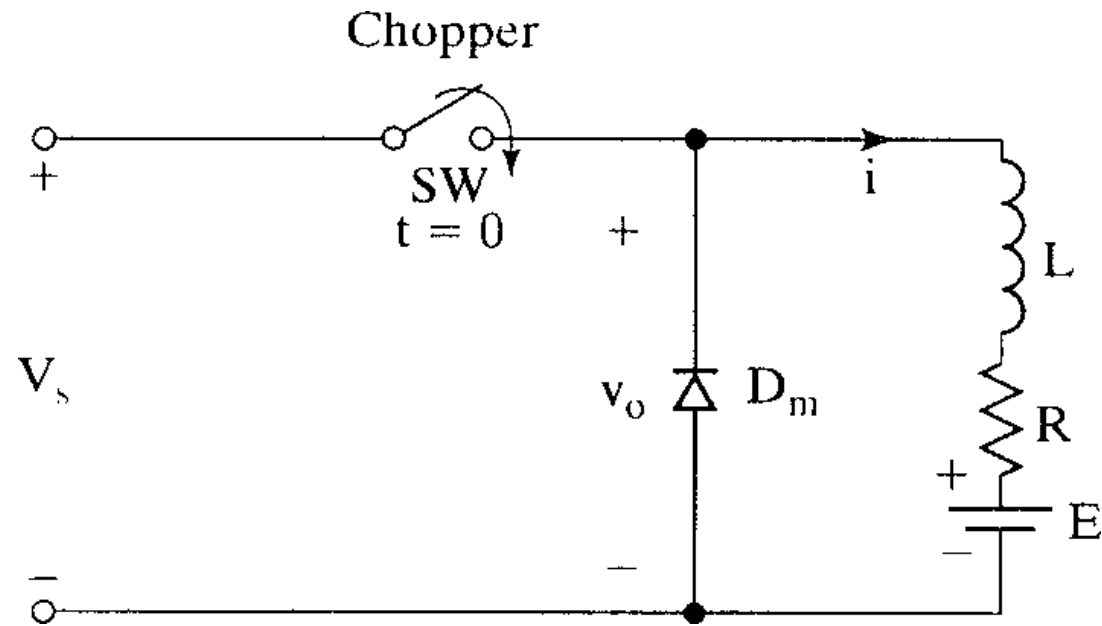
$$k = \frac{V_{cr}}{V_r} = M$$

To generate the gating signal

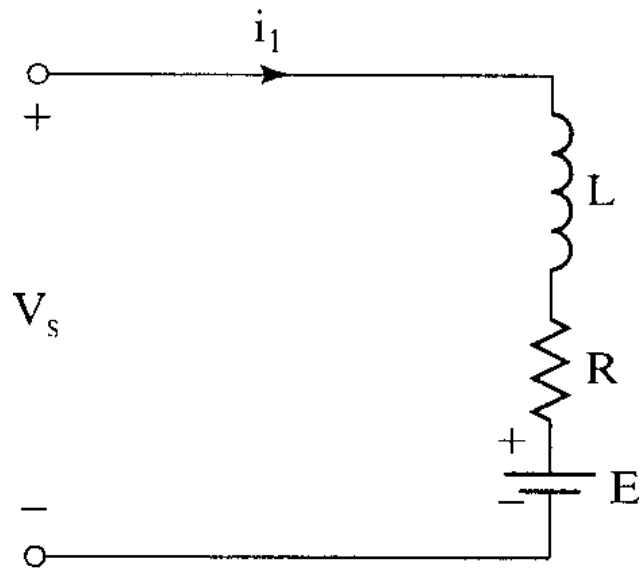


- Generate the triangular waveform of period T , v_r , and the dc carrier signal, v_{cr}
- Compare to generate the difference $v_c - v_{cr}$
- Apply to a “hard limiter” to “square off”

Step-Down Converter with RL Load



Mode 1: Switch Closed



$$V_s = Ri_1 + L \frac{di_1}{dt} + E$$

$$i_1(t)_{(t=0)} = I_1$$

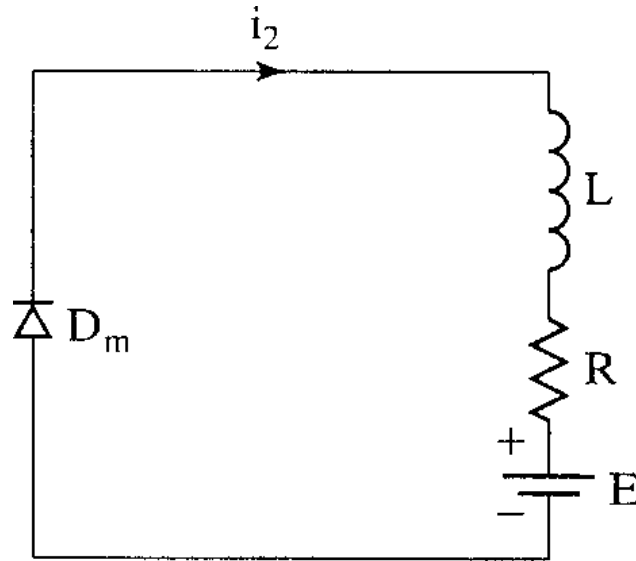
Mode 1

$$i_1(t) = I_1 e^{-\frac{R}{L}t} + \frac{V_s - E}{R} \left(1 - e^{-\frac{R}{L}t}\right) \quad t \leq t_1(kT)$$

$$t = t_1 = kT$$

$$i_1(kT) = I_2$$

Mode 2: Switch Open



Mode 2

$$0 = Ri_2 + L \frac{di_2}{dt} + E$$

$$i_2(t = 0) = I_2$$

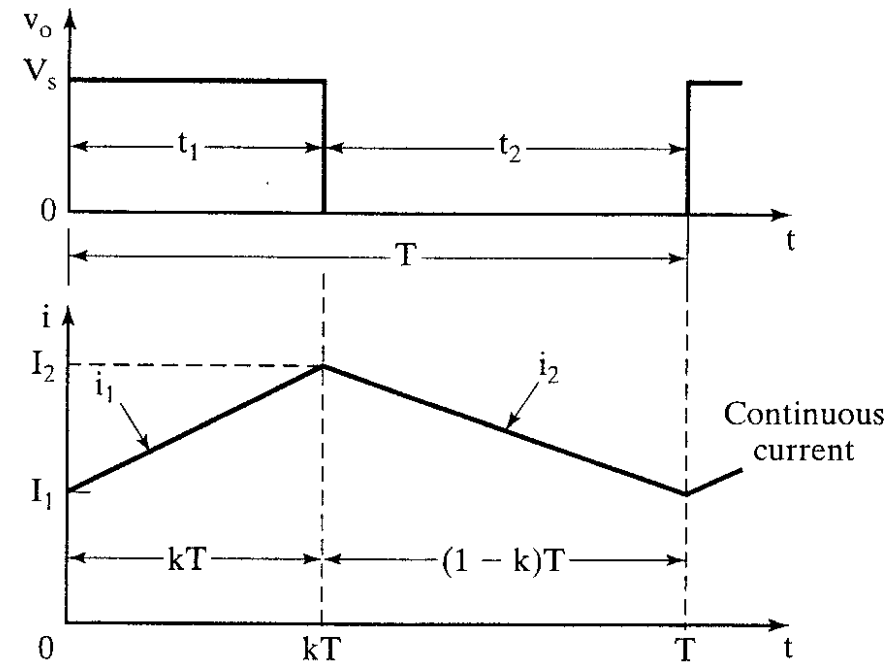
$$i_2(t) = I_2 e^{-\frac{R}{L}t} - \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$

$$0 \leq t \leq t_2 = (1 - k)T$$

$$@t = t_2 = (1 - K)T$$

$$i_2(t_2) = I_3 = I_1$$

Current for “Continuous” Mode



$$I_1 = \frac{V_s}{R} \left(\frac{e^{kz} - 1}{e^z - 1} \right) - \frac{E}{R}$$

$$I_2 = \frac{V_s}{R} \left(\frac{e^{-kz} - 1}{e^z - 1} \right) - \frac{E}{R}$$

$$Z = \frac{TR}{L}$$

$$\Delta I = \frac{V_s}{R} \left[\frac{1 - e^{-kz} + e^{-z} - e^{-(1-k)z}}{1 - e^{-z}} \right]$$

$$\Delta I_{\max} \cong \frac{V_s}{4fL}$$

For Continuous Current

$$I_1 \geq 0$$

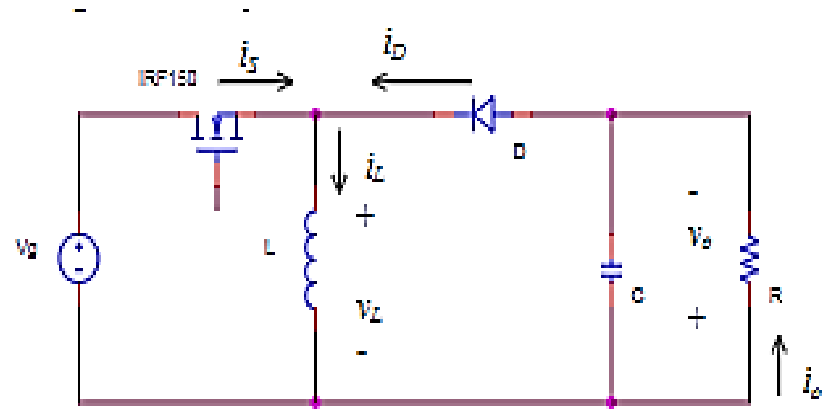
$$\left[\frac{e^{kz} - 1}{e^z - 1} - \frac{E}{V_s} \right] \geq 0$$

Define the load emf ratio

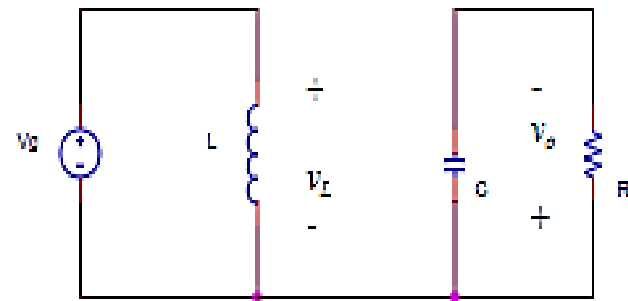
$$\chi = \frac{E}{V_s}$$

$$\chi = \frac{E}{V_s} \leq \frac{e^{kz} - 1}{e^z - 1}$$

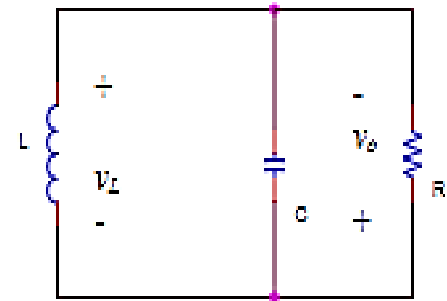
Boost converter operation



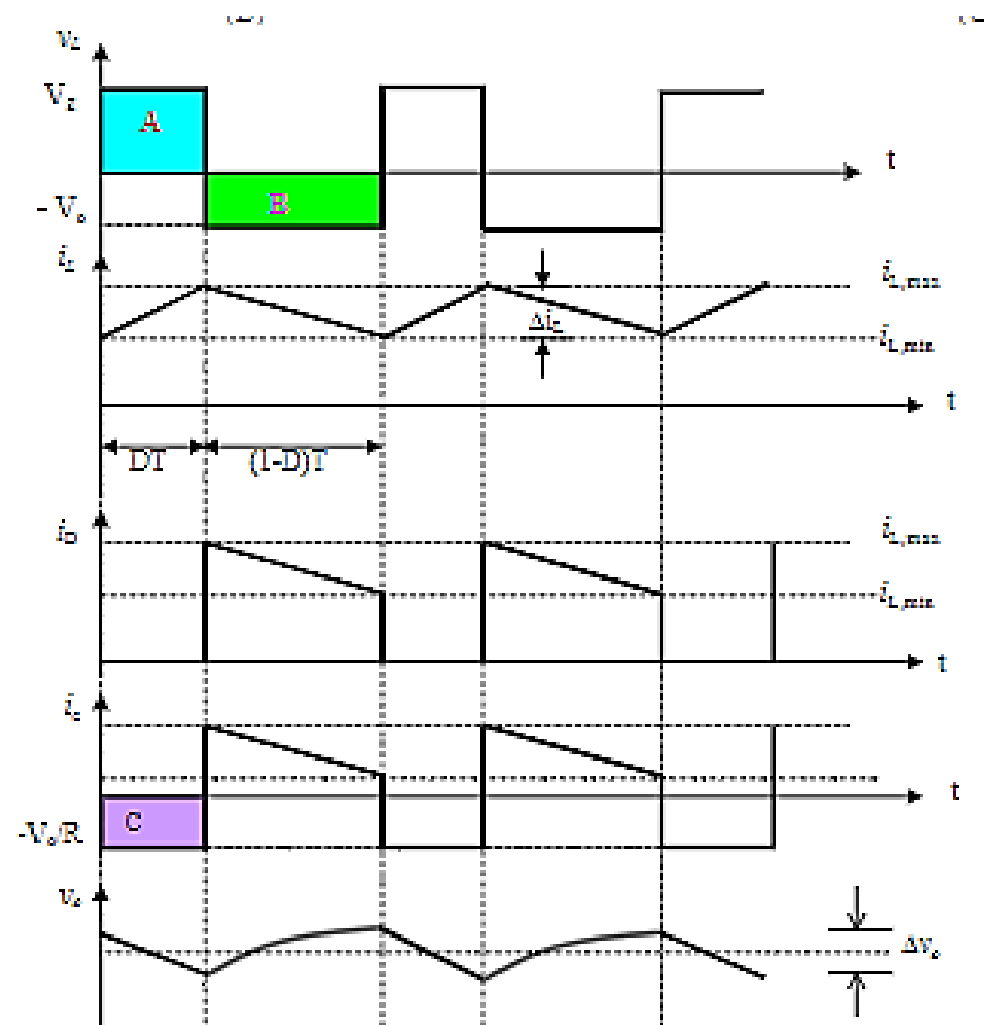
(a)



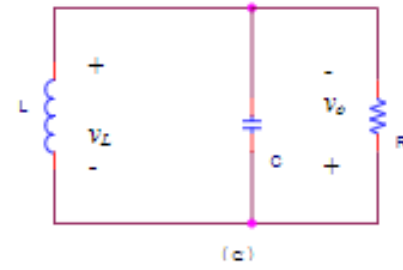
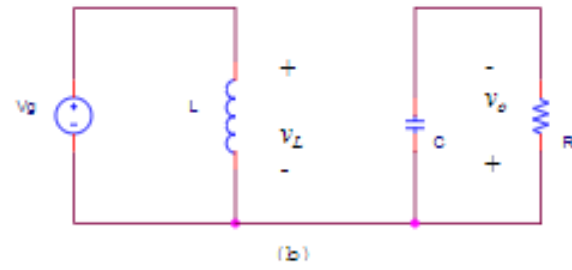
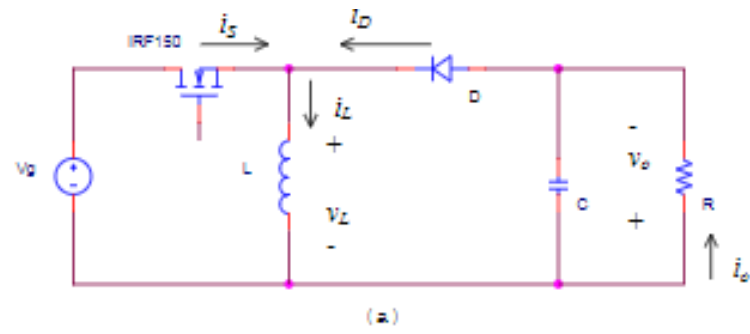
(b)



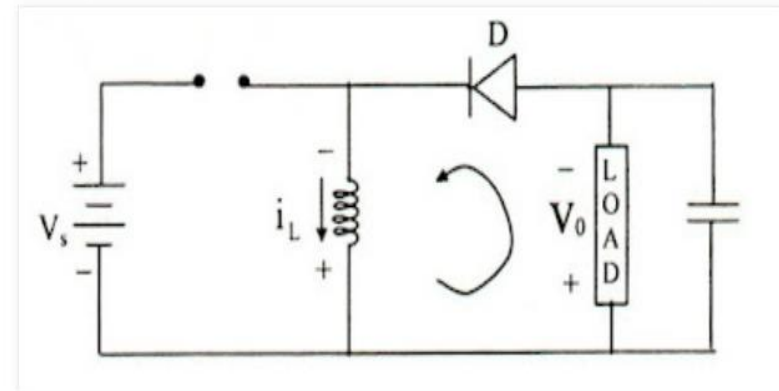
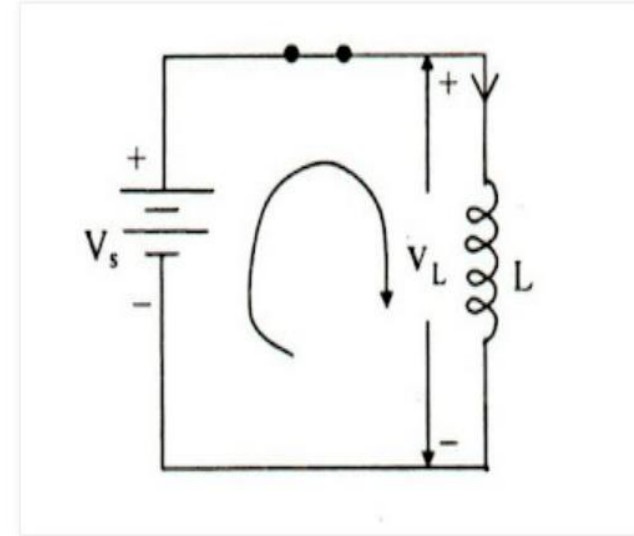
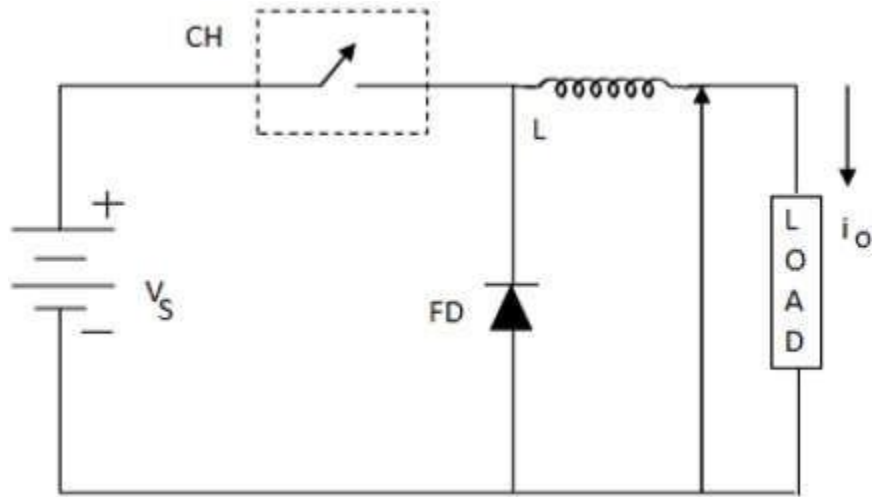
(c)

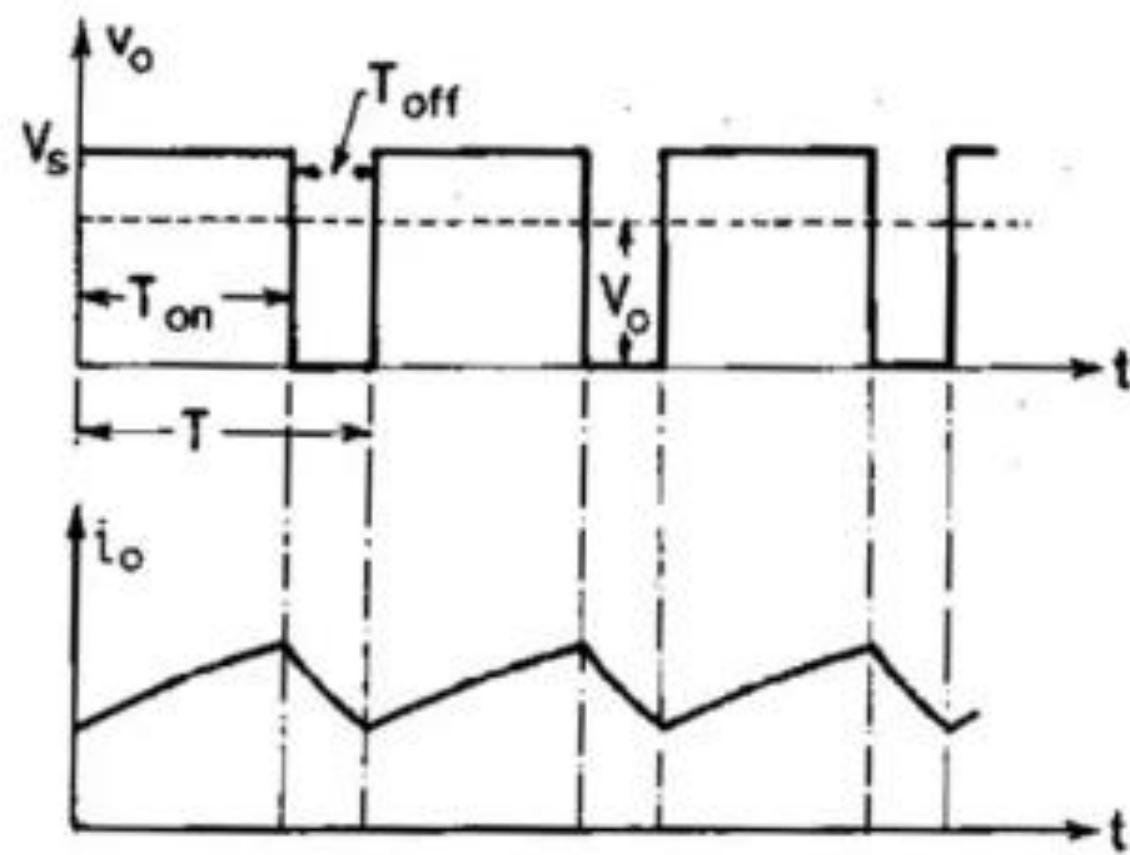


Step Up/ Step Down Chopper



Step Down Chopper





UNIT-IV

AC-DC Inverters

Inverter

- Single phase half bridge
- Single phase full bridge
- Inverter Control
 - Frequency Control
 - Voltage Control
 - Harmonic Control (minimisation)

DC-to-AC converters are known as *inverters*. The function of an inverter is to change a dc input voltage to a symmetrical ac output voltage of desired magnitude and frequency.

- The output voltage could be fixed or variable at a fixed or variable frequency.
- A variable output voltage can be obtained by varying the input dc voltage and maintaining the gain of the inverter constant.
- For low- and medium-power applications, square-wave or quasi- square-wave voltages may be acceptable; and for high-power applications, low distorted sinusoidal waveforms are required.
- With the availability of high-speed power semiconductor devices, the harmonic contents of output voltage can be minimized or reduced significantly by switching techniques.

Inverters can be broadly classified into two types:

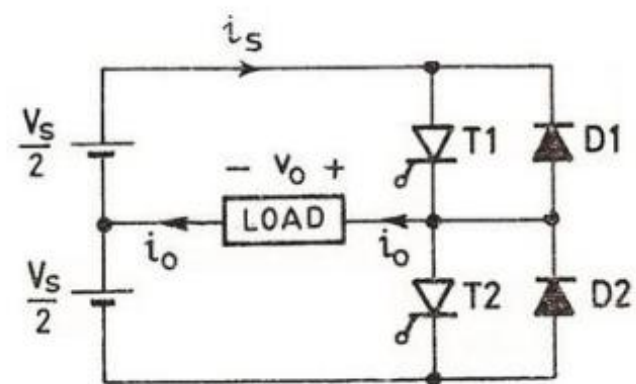
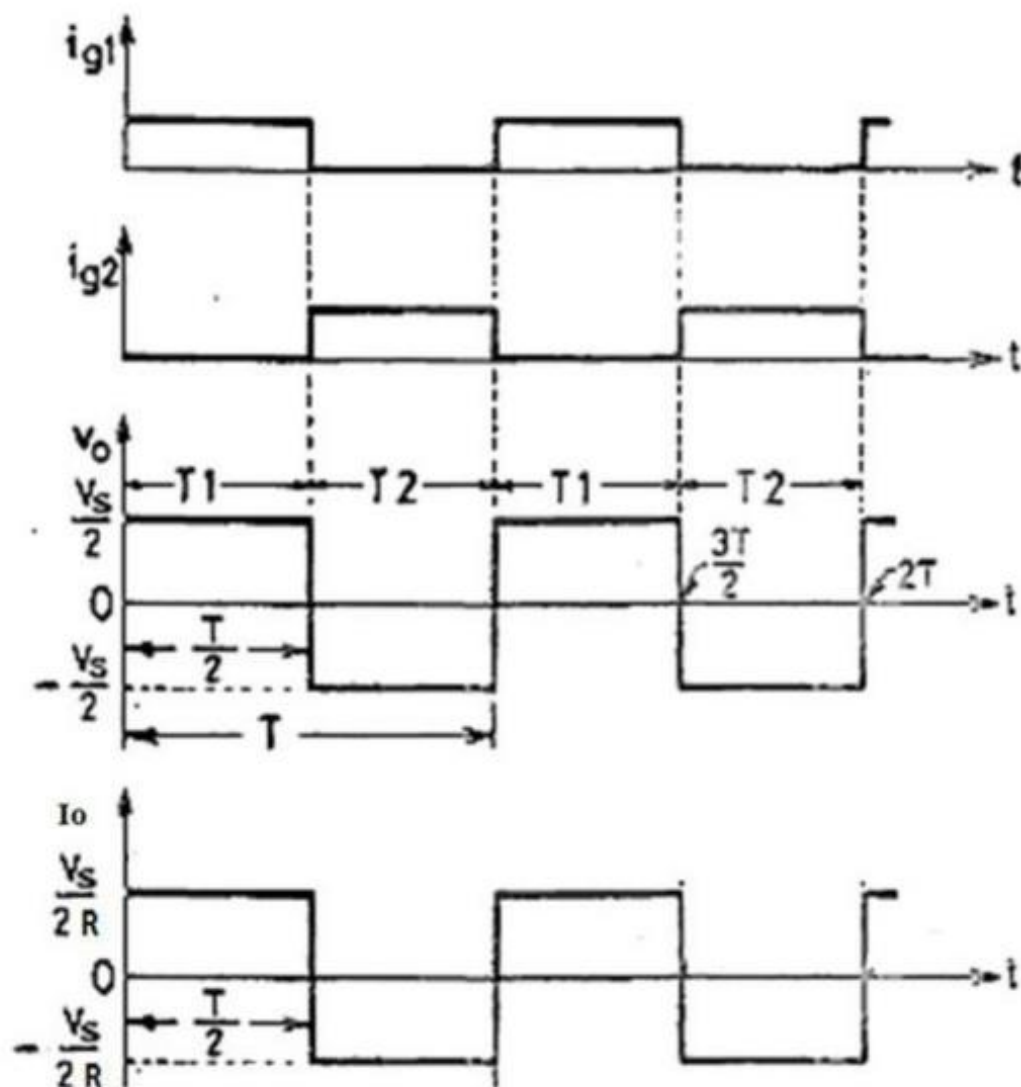
(I) Single-phase Inverters

(2) Three-phase Inverters

- Each type can use controlled turn-on and turn-off devices (e.g., BJTs, MOSFETs, IGBTs, GTOs) or forced-commutated thyristors depending on applications.

- These inverters generally use PWM control signals for producing an ac output voltage. An inverter is called a **voltage- Source inverter (VSI)** if the input voltage remains constant, a **current-Source inverter (CSI)** if the input current is maintained constant, and a **variable dc linked inverter** if the input voltage is controllable.

Single Phase Half-Bridge Inverter with R-load-Waveforms



Single Phase Half-Bridge Inverter with R-load-

Modes of operation

T1 is turned on upto $t = T/2$

Load voltage $V_o = V_s/2$

T2 is turned on upto $t = T$

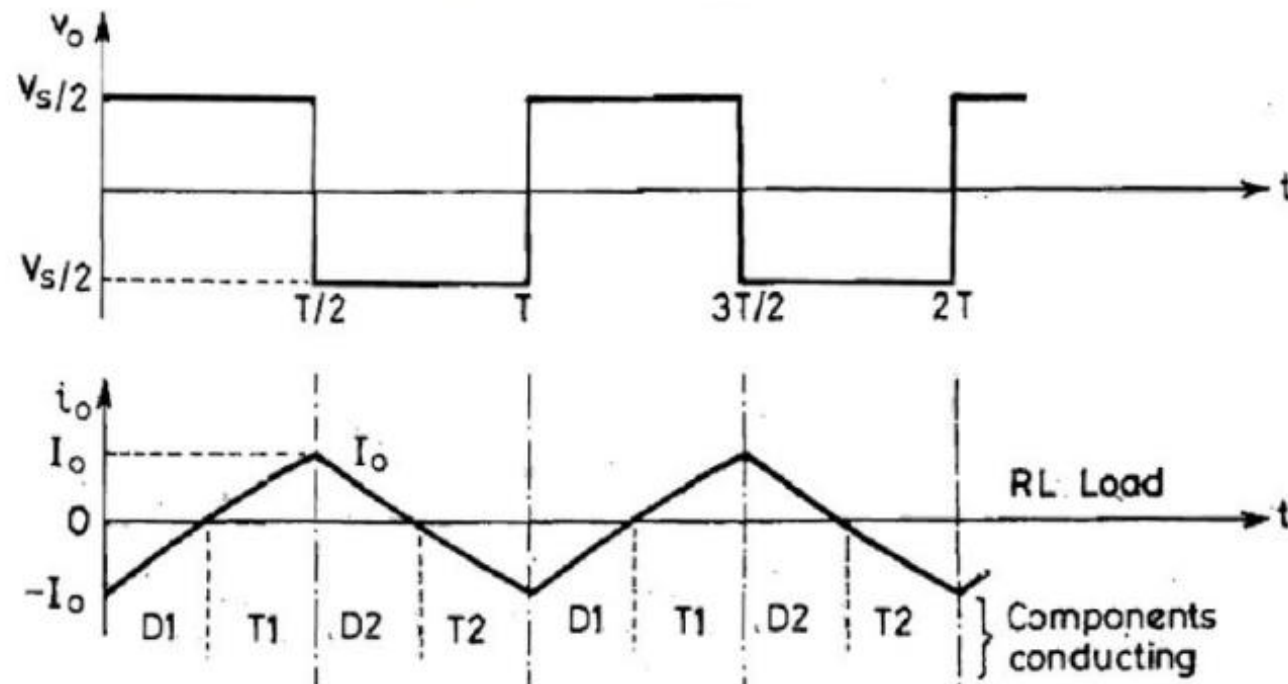
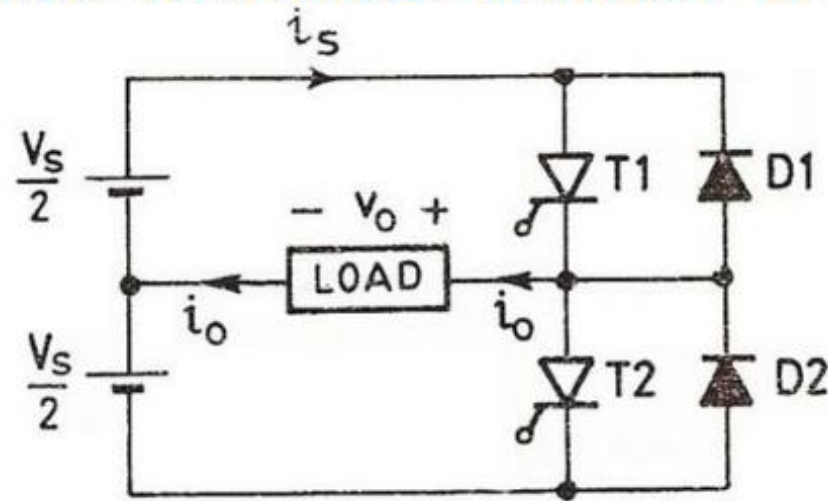
Load Voltage $V_o = -V_s/2$

T1 and **T2** are switched complementary

The rms output voltage

$$V_o = \left(\frac{2}{T} \int_0^{T/2} \frac{V_s^2}{4} dt \right)^{1/2} = \frac{V_s}{2}$$

Single-phase half-bridge inverter –RL load

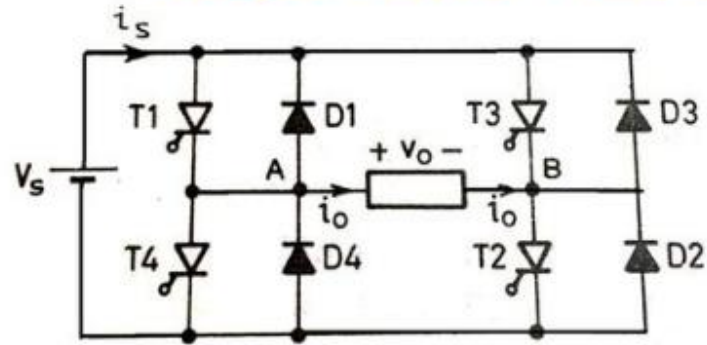


Single Phase Half-Bridge Inverter with RL load-

Modes of operation

- **For an inductive load**, the load current cannot change immediately with the output voltage.
- When T_1 is turned off at $t = T/2$, the load current would continue to flow through D_2 , load, and the lower half of the dc source until the current falls to zero.
- when T_2 is turned off at $t = T$, the load current flows through D_1 , load, and the upper half of the dc source.
- When diode D_1 or D_2 conducts, energy is fed back to the dc source and these diodes are known as *feedback diodes*.

Single Phase Full-Bridge Inverter with R-load



At $t=T/2$

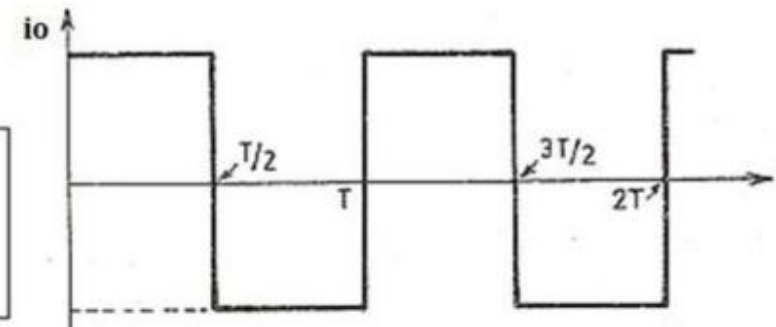
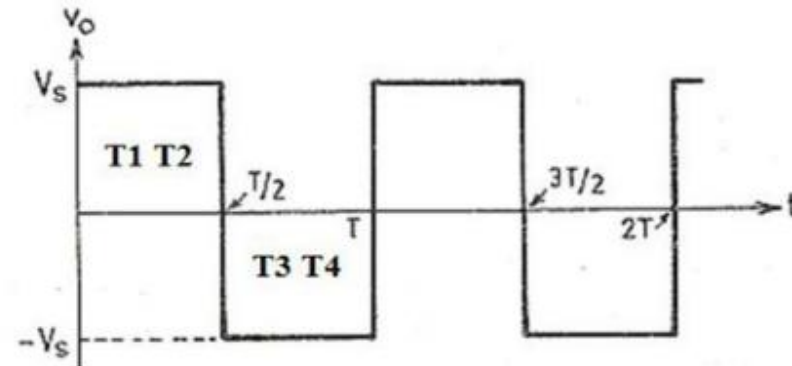
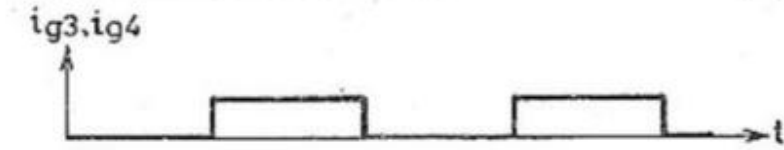
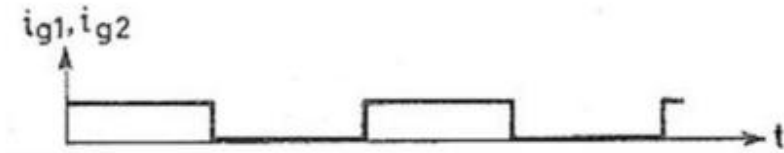
T1 and T2 are ON then $V_o = V_s$

At $t=T$

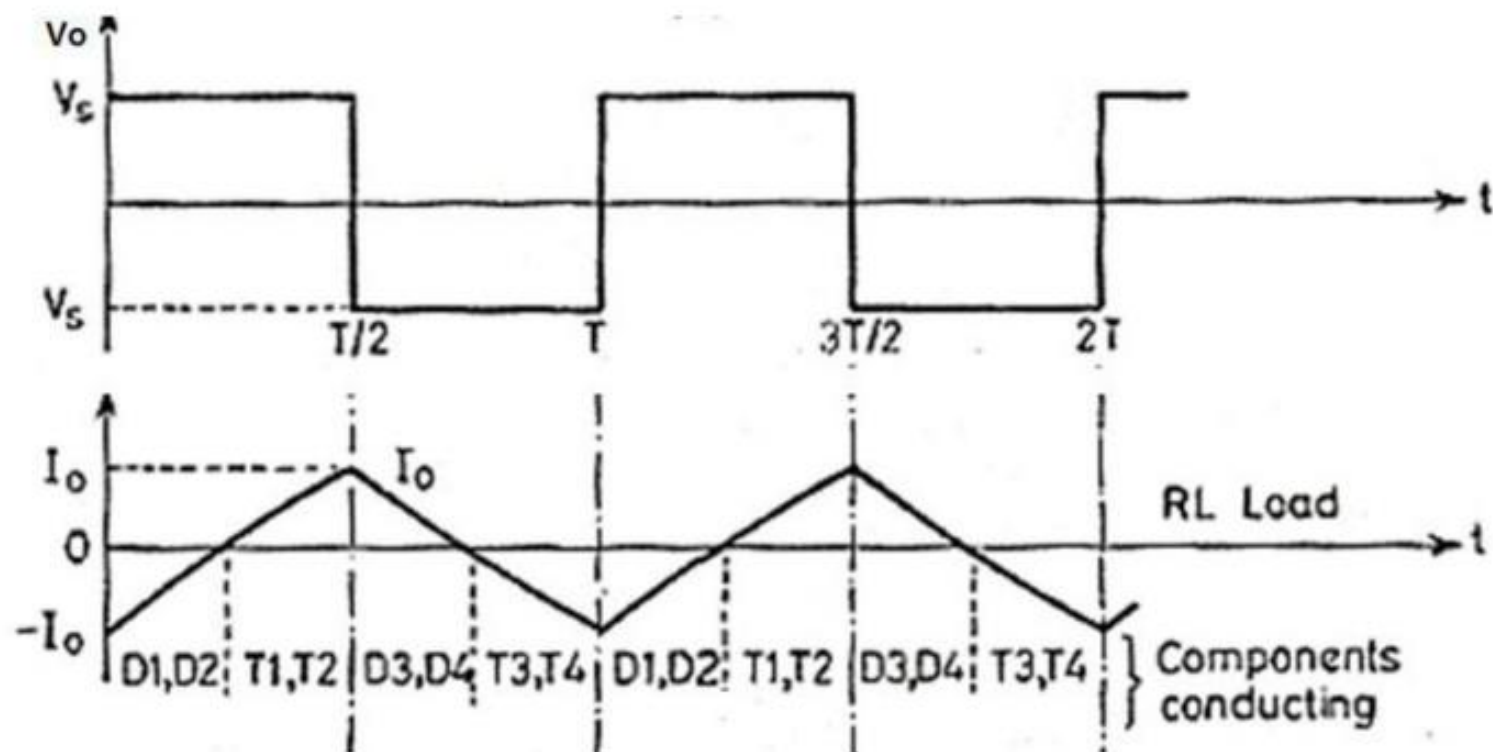
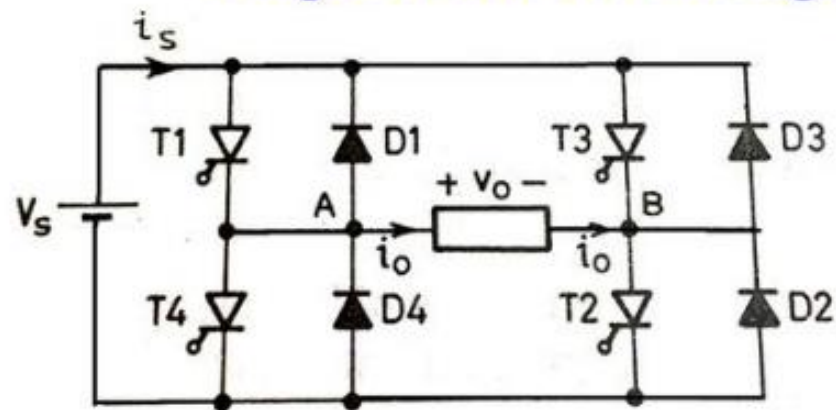
T3 and T4 are ON then $V_o = -V_s$

The rms output voltage

$$V_o = \left(\frac{2}{T} \int_0^{T/2} V_s^2 dt \right)^{1/2} = V_s$$



Single Phase Full-Bridge Inverter with RL load



Single Phase Full-Bridge Inverter with RL load

- In the waveform of i_o , Before $t = 0$, thyristors T3, T4 are conducting and load current i_o is flowing from B to A, i.e. in the reversed direction, This current is shown as $-I_o$ at $t = 0$.
- After T3, T4 are turned off at $t = 0$, current i_o cannot change its direction immediately because of the nature of load. As a result, diodes D1, D2 start conducting after $t = 0$ and allow i_o to flow against the supply voltage V_s . As soon as D1, D2 begin to conduct, load is subjected to V_s as shown.
- Though T1, T2 are gated at $t = 0$, these SCRs will not turn on as these are reverse biased by voltage drops across diodes D1 and D2.
- When load current through D1, D2 falls to zero, T1 and T2 become forward biased by source voltage V_s , T1 and T2 therefore get turned on as these are gated for a period $T/2$ sec.
- Now load current i_o flows in the positive direction from A to B.
- At $t = T/2$; T1, T2 are turned off by forced commutation and as load current cannot reverse immediately, diodes D3, D4 come into conduction to allow the flow of current i_o after $T/2$.
- Thyristors T3, T4, though gated, will not turn on as these are reverse biased by the voltage drop in diodes D3, D4. When current in diodes D3, D4 drops to zero; T3, T4 are turned on as these are already gated.

Fourier Analysis Of Single-phase Inverter Output Voltage

Output Voltage waves can be resolved into Fourier series equation as follows:

$$v_0 = \sum_{n=1, 3, 5, \dots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t \text{ volts}$$

Single-phase half-bridge inverter

$$v_0 = \sum_{n=1, 3, 5, \dots}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t \text{ volts}$$

Single-phase Full-bridge inverter

n is the order of the harmonic and $\omega = 2\pi f$ is the frequency of the output voltage

- The instantaneous output voltage can be expressed in Fourier series as

$$\begin{aligned} v_0 &= \sum_{n=1,3,5,\dots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t \\ &= 0 \quad \text{for } n = 2, 4, \dots \end{aligned} \quad (2)$$

is frequency of output voltage.

For $n=1$ the eq(2) gives the rms of fundamental component.

$$V_1 = \frac{2V_s}{\sqrt{2} \pi} = 0.45V_s \quad (3)$$

PERFORMANCE PARAMETERS

The output of practical inverters contain harmonics and the quality of an inverter is normally evaluated in terms of the following performance parameters.

Harmonic factor of nth harmonic- HF_n

The harmonic factor (of the nth harmonic), which is a measure of individual harmonic contribution, is defined as

$$HF_n = \frac{V_n}{V_1} \quad (4)$$

where V_1 is the rms value of the fundamental component and V_n is the rms value of the nth harmonic component.

Total harmonic distortion THD

The total harmonic distortion, which is a measure of closeness in shape between a waveform and its fundamental component, is defined as

$$\text{THD} = \frac{\sqrt{V_0^2 - V_1^2}}{V_1} \quad (5)$$

THD gives the total harmonic content, but it does not indicate the level of each harmonic component. If a filter is used at the output of inverters, the higher-order harmonics would be attenuated more effectively. Therefore, a knowledge of both the frequency and magnitude of each harmonic is important.

Distortion factor DF

The distortion factor indicates the amount of harmonic distortion that remains in a particular waveform after the harmonics that waveform have been subjected to a second-order attenuation (i.e., divided by n^2).

DF is a measure of effectiveness in reducing unwanted harmonics.

$$\mathbf{DF} = \frac{1}{V_1} \sqrt{\sum_{n=3,5,7}^{\infty} \left(\frac{V_n}{n^2}\right)^2} \quad (6)$$

DF of nth harmonics component is defined as

$$\boxed{\mathbf{DF}_n = \frac{V_n}{V_1 n^2}} \quad (7)$$

Inverter Control Techniques

- Control Techniques (frequency, voltage and harmonics)
 - Frequency Control
 - Determined by frequency of fundamental switching pattern
 - Voltage Control (consequential harmonics)
 - Vary d.c. input voltage
 - Quasi-square
 - Notching
 - Pulse Width Modulation (PWM) – variable width notching

4.4 Three Phase DC-AC Converters

Three phase inverters are normally used for high power applications. The advantages of a three phase inverter are:

The frequency of the output voltage waveform depends on the switching rate of the switches and hence

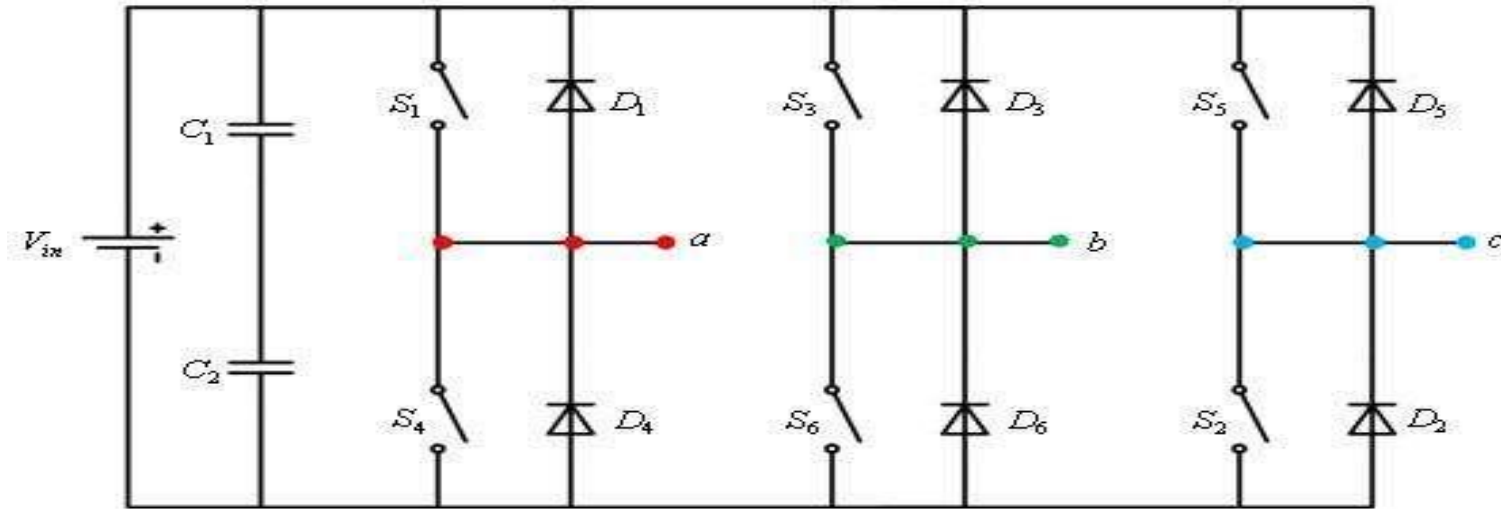
can be varied over a wide range.

- The direction of rotation of the motor can be reversed by changing the output phase sequence of the inverter.

The ac output voltage can be controlled by varying the dc link voltage.

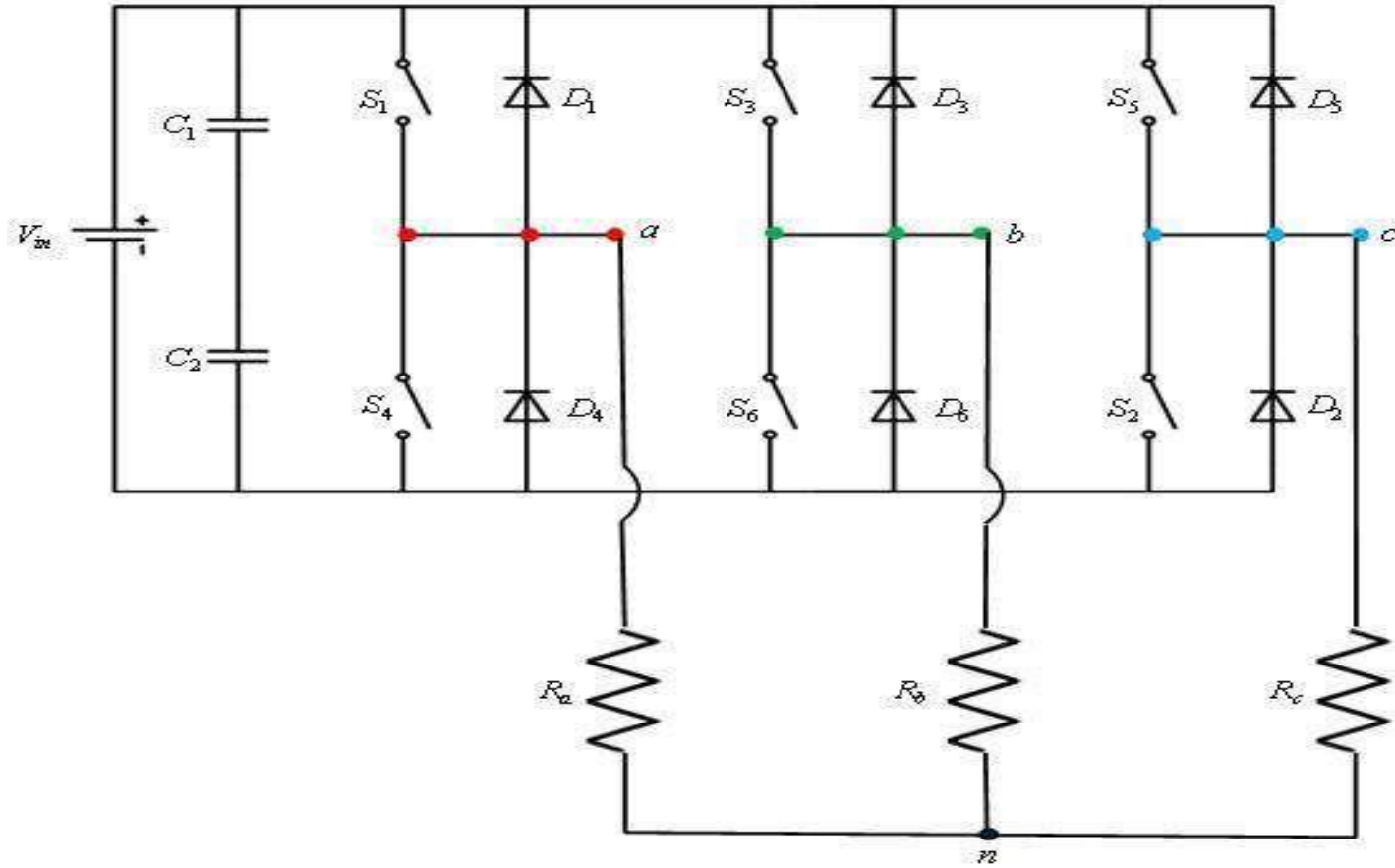
The general configuration of a three phase DC-AC inverter is shown in **Figure Two** types of control signals can be applied to the switches:

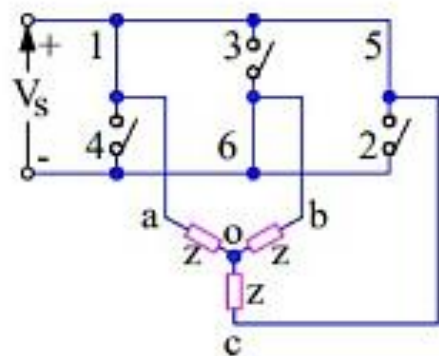
- 180° conduction
- 120° conduction



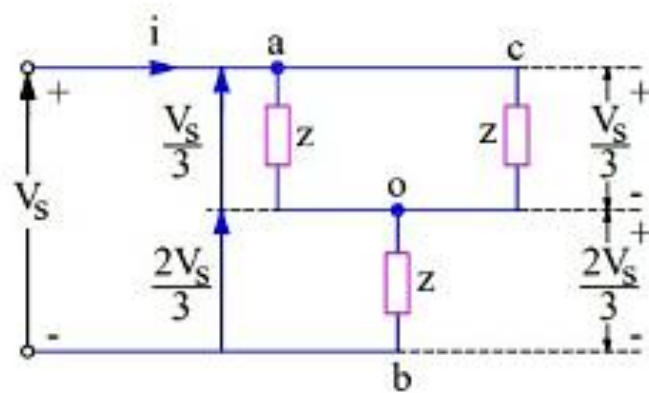
Circuit diagram of three phase bridge inverter

180-Degree Conduction with Star Connected Resistive Load

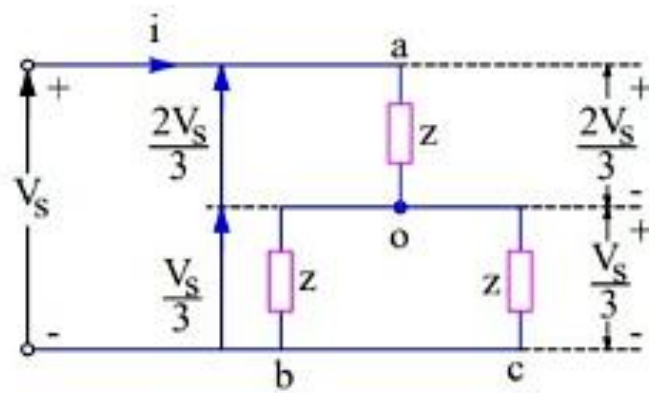
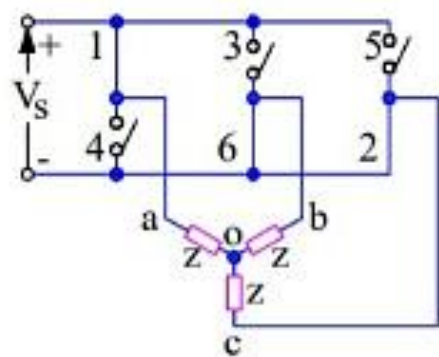




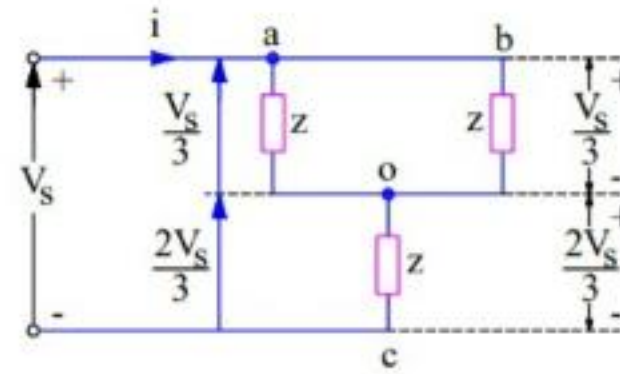
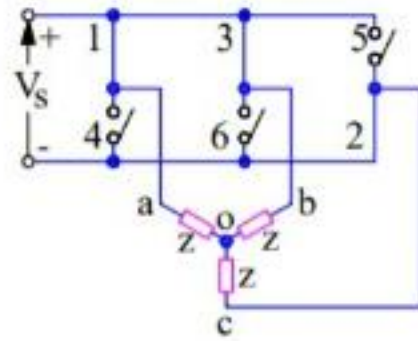
Step-I



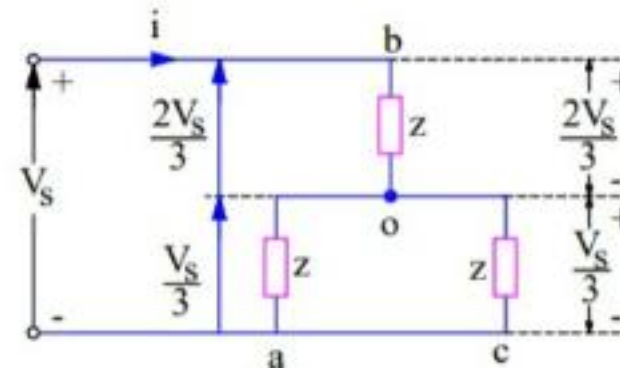
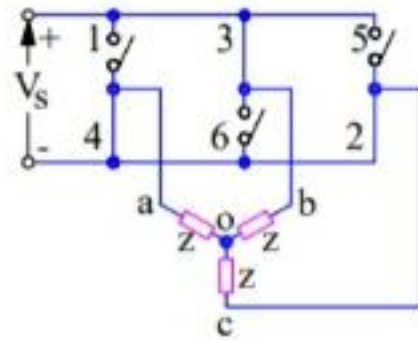
Step-II



Step-III

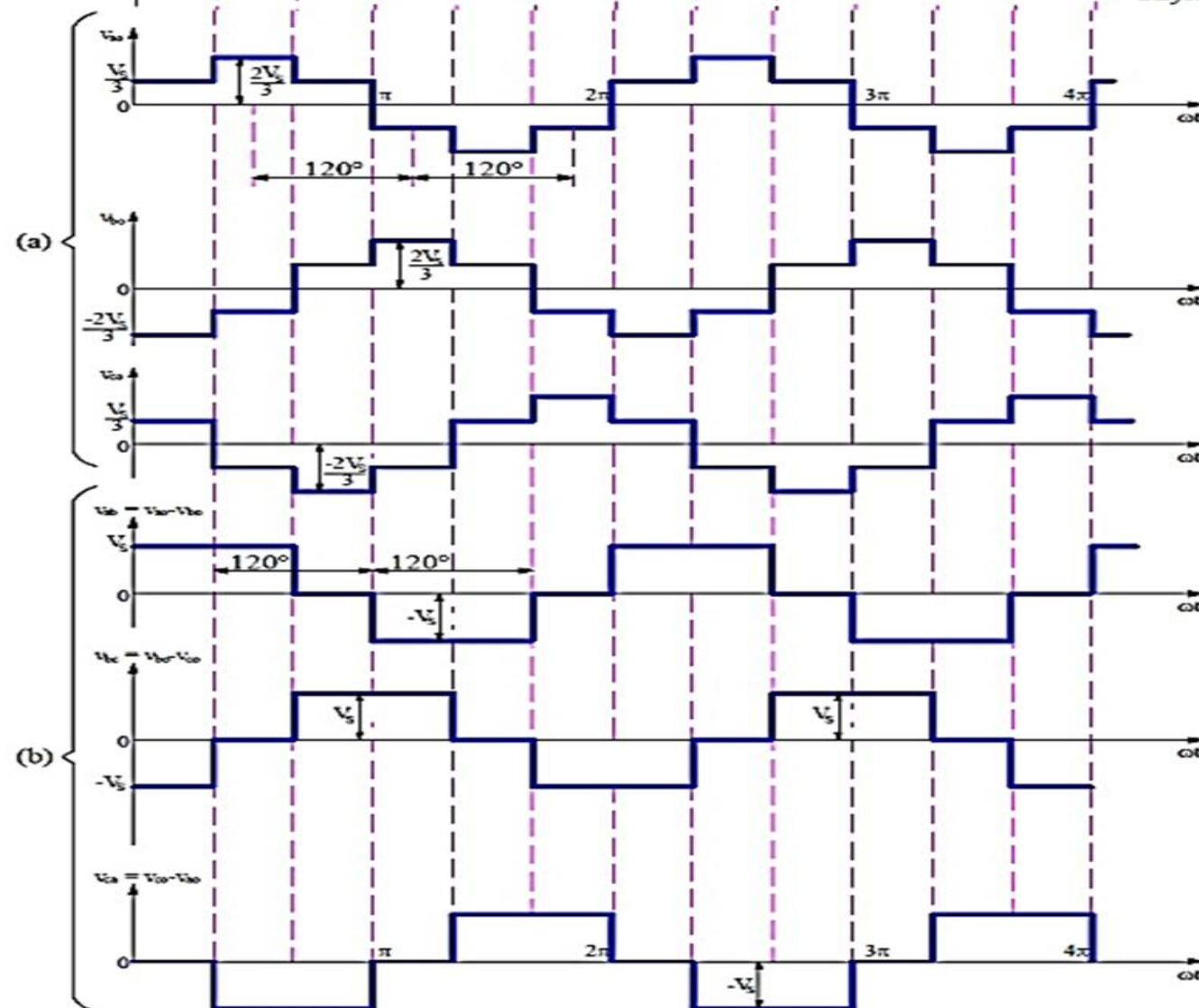


Step-IV

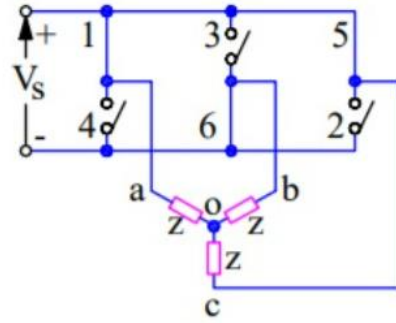


180°		180°	
T1		T4	
T6	T3	T6	T3
T5	T2	T5	T2

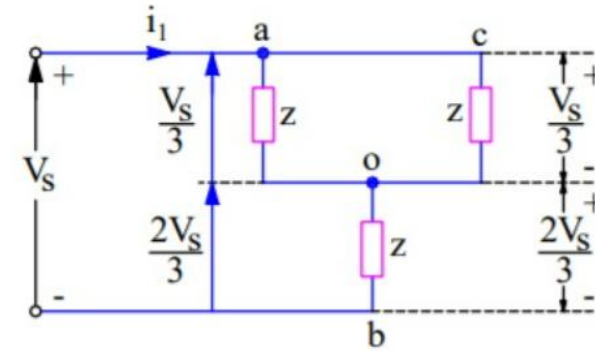
Steps { 0° 60° 120° 180° 240° 300° 360° 60° 120° 180° 240° 300° 360°
 I II III IV V VI I II III IV V VI } Conductor
 5,6,1 6,1,2 1,2,3 2,3,4 3,4,5 4,5,6 5,6,1 6,1,2 1,2,3 2,3,4 3,4,5 4,5,6 } Thyristors



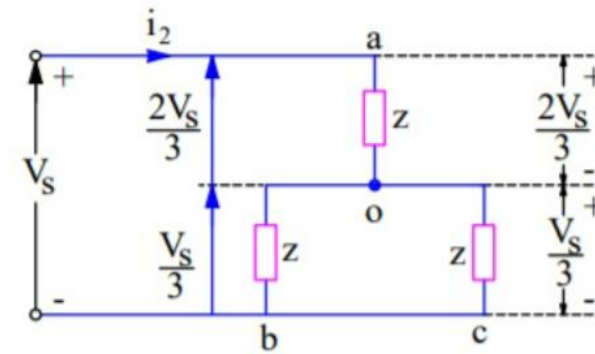
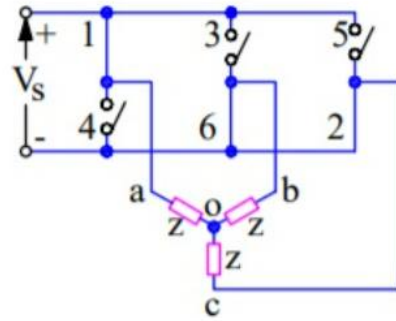
Three Phase DC-AC Converters with 120 degree conduction mode



Step-I



Step-II

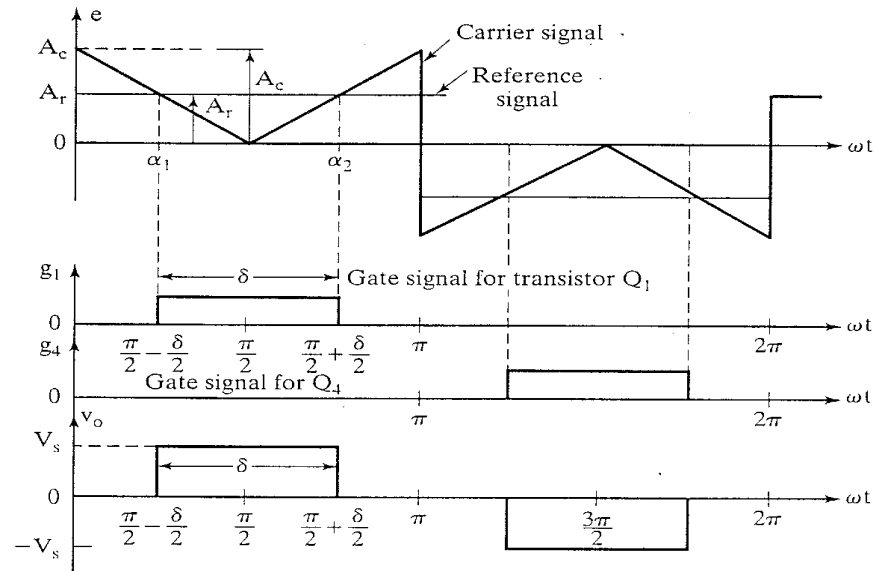


120-degree conduction

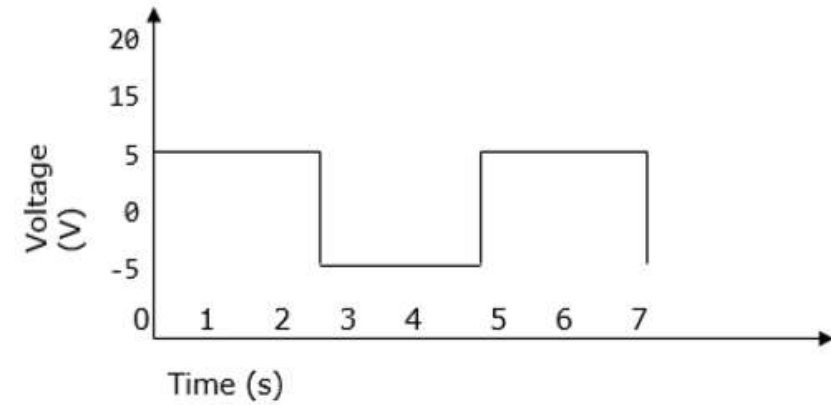
Step	Thyristor conducting	V_{Rn}	V_{Yn}	V_{Bn}	$\vec{v}$
1	6,1	$V_s/2$	$-V_s/2$	0	$\frac{\sqrt{3}V_s}{2}(-30^\circ)$
2	1,2	$V_s/2$	0	$-V_s/2$	$\frac{\sqrt{3}V_s}{2}(30^\circ)$
3	2,3	0	$V_s/2$	$-V_s/2$	$\frac{\sqrt{3}V_s}{2}(90^\circ)$
4	3,4	$-V_s/2$	$V_s/2$	0	$\frac{\sqrt{3}V_s}{2}(150^\circ)$
5	4,5	$-V_s/2$	0	$V_s/2$	$\frac{\sqrt{3}V}{2}(210^\circ)$
6	5,6	0	$-V_s/2$	$V_s/2$	$\frac{\sqrt{3}V}{2}(-30^\circ)$

Voltage control techniques for inverters

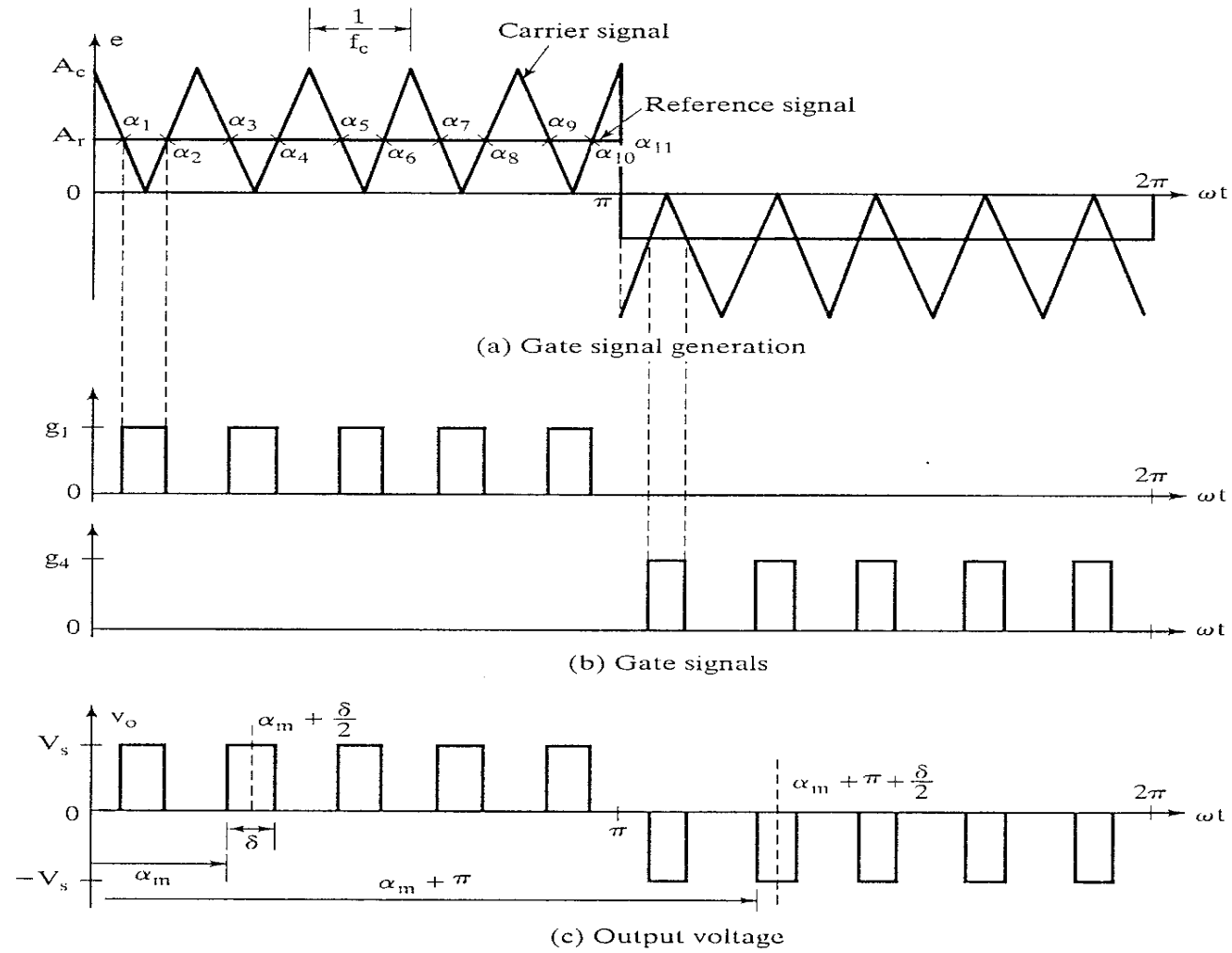
Single Pulse width modulation techniques

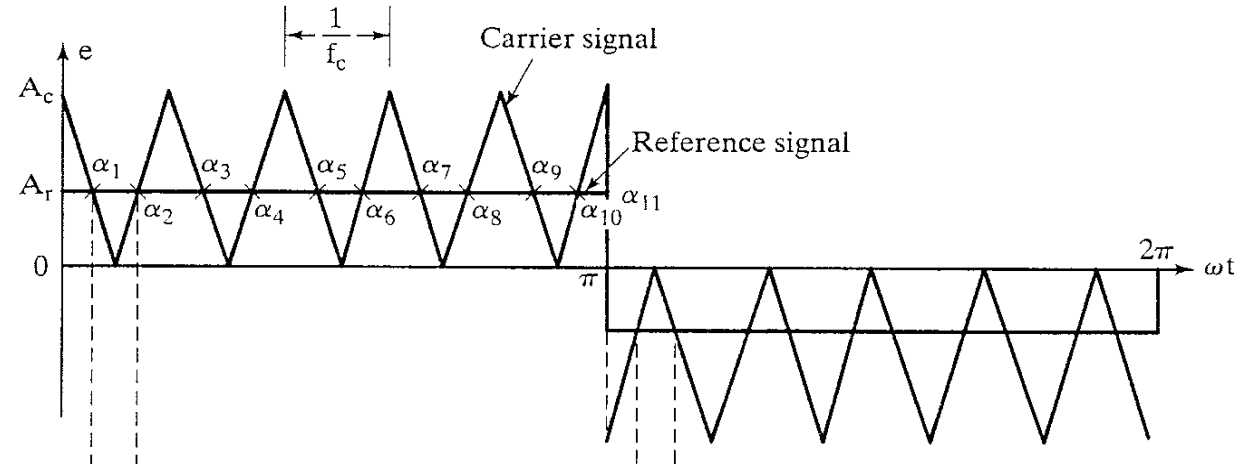


$$v_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_s}{n\pi} \sin \frac{n\delta}{2} \sin n\omega t$$



Multiple Pulse Width Modulation



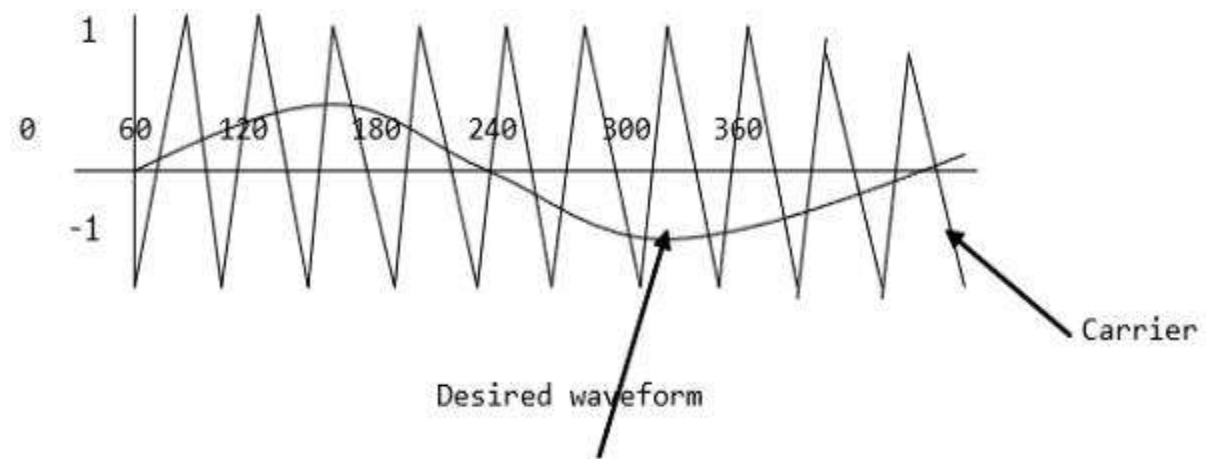


- Compare the Reference Signal with the Carrier
- Frequency of the Reference Signal determines the Output Voltage Frequency
- Frequency of the Carrier determines the number of pulses per half-cycle
- Modulation Index controls the Output Voltage

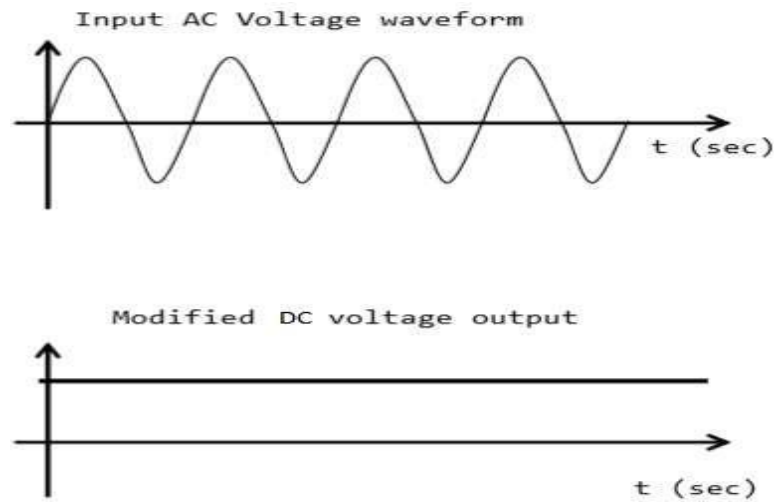
Sinusoidal Pulse Width Modulation

In a simple source voltage inverter, the switches can be turned ON and OFF as needed. During each cycle, the switch is turned on or off once. This results in a square waveform. However, if the switch is turned on for a number of times, a harmonic profile that is improved waveform is obtained.

The sinusoidal PWM waveform is obtained by comparing the desired modulated waveform with a triangular waveform of high frequency. Regardless of whether the voltage of the signal is smaller or larger than that of the carrier waveform, the resulting output voltage of the DC bus is either negative or positive.



Modified Sinusoidal Waveform PWM



Voltage and Harmonic Control

A periodic waveform that has frequency, which is a multiple integral of the fundamental power with frequency of 60Hz is known as a harmonic. Total harmonic distortion (THD) on the other hand refers to the total contribution of all the harmonic current frequencies.

Harmonics are characterized by the pulse that represents the number of rectifiers used in a given circuit.

It is calculated as follows

$$h = (n \times P) + 1 \text{ or } -1$$

Where **n** – is an integer 1, 2, 3, 4....n

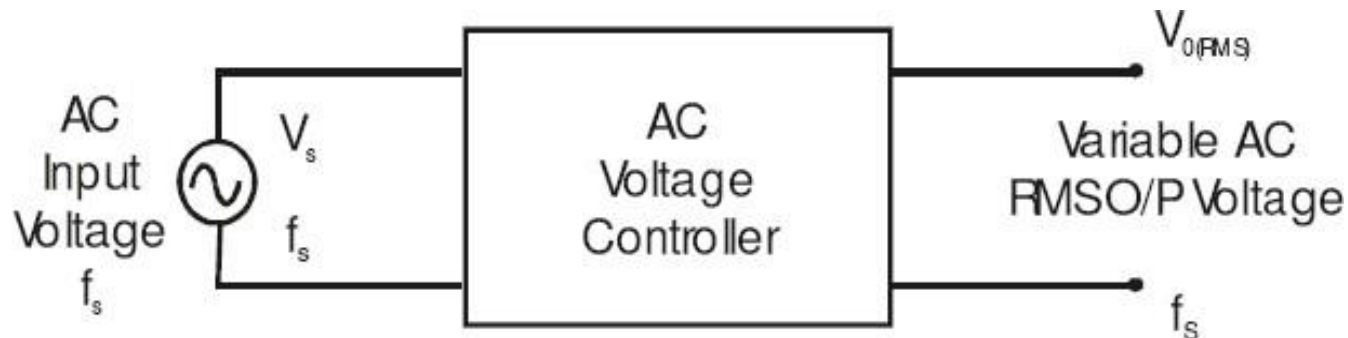
P – Number of rectifiers

UNIT-V

AC-AC Converters

Ac Voltage controller circuits (RMS voltage controllers)

An ac voltage controller is a type of thyristor power converter which is used to convert a fixed voltage, fixed frequency ac input supply to obtain a variable voltage ac output



Applications Of Ac Voltage Controllers

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Industrial heating & Domestic heating.
- Transformer tap changing (on load transformer tap changing).
- Speed control of induction motors C magnet controls.

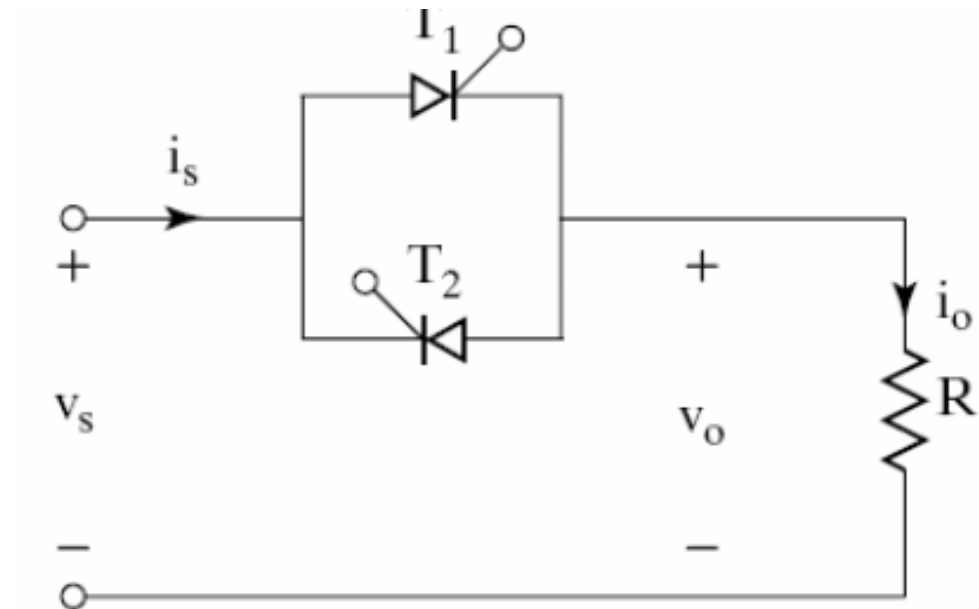
Type Of Ac Voltage Controllers

- Single phase half wave ac voltage controller (Uni-directional controller).
- Single phase full wave ac voltage controller (Bi-directional controller).
- Three phase half wave ac voltage controller (Uni-directional controller).
- Three phase full wave ac voltage controller (Bi-directional Controller)

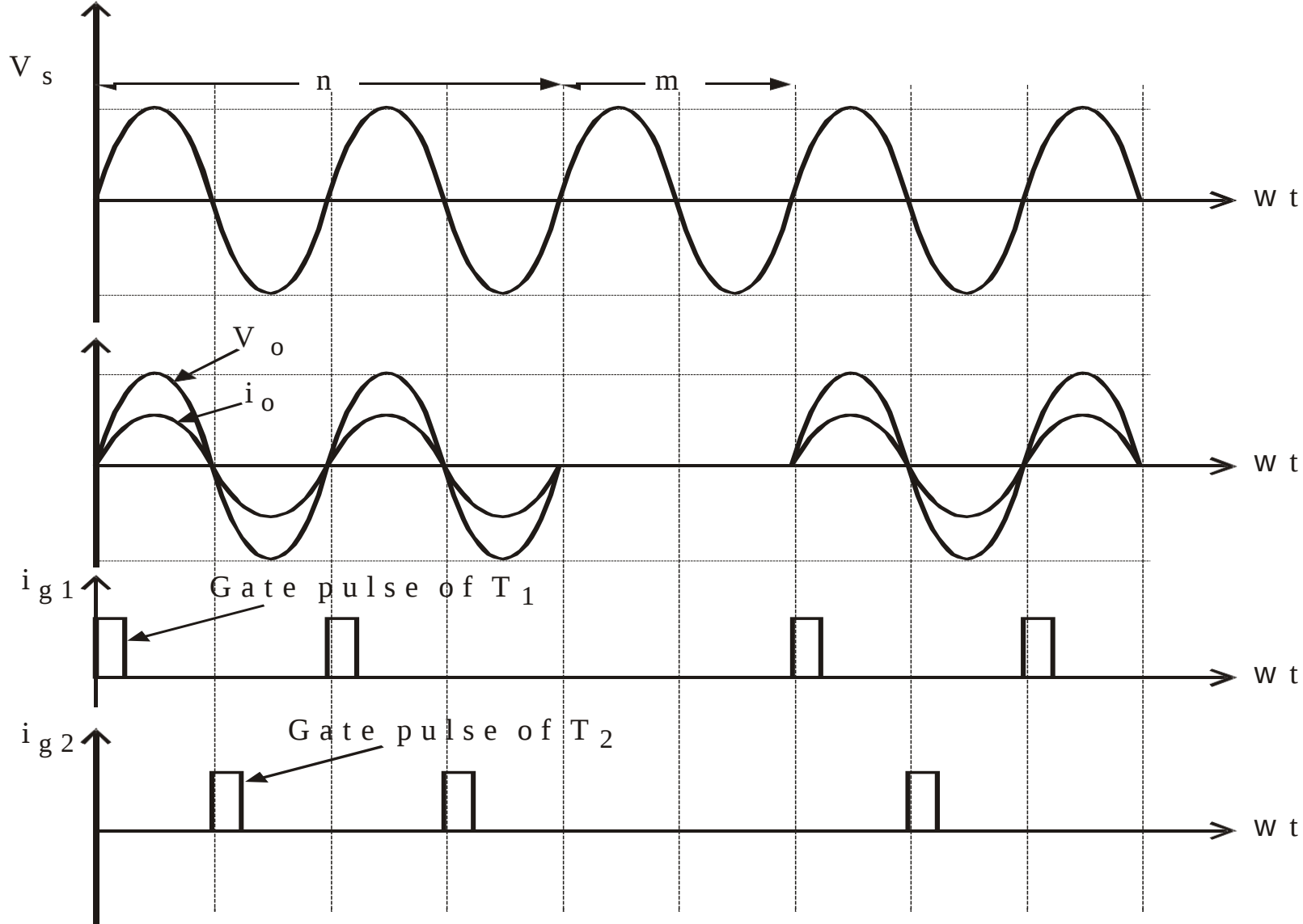
A.C voltage control technique

- On-Off control.
- Phase control.

Principle of ON-OFF Control Technique



Single phase full wave AC voltage controller circuit



For a sine wave i/p ac supply

$$v_s = V_m \sin \omega t = \sqrt{2}V_s \sin \omega t$$

$$V_s = \frac{V_m}{\sqrt{2}} = \text{RMS value of i/p ac supply}$$

$$t_{ON} = n \times T, \quad t_{OFF} = m \times T$$

n = no. of i/p cycles during on time t_{ON}

m = no. of i/p cycles during off time t_{OFF}

$$T = \frac{1}{f} = \text{input cycle time (time period)}$$

f = input supply frequency.

$$t_{ON} = \text{controller on time} = n \times T$$

$$t_{OFF} = \text{controller off time} = m \times T$$

$$T_o = \text{Output time period}$$

$$= (t_{ON} + t_{OFF}) = (nT + mT)$$

For on-off control method

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{t_{ON}}{T_o}} = V_s \sqrt{\frac{t_{ON}}{T_o}}$$

Expression For The RMS Value Of Output Voltage, For ON-OFF Control Method

$$V_{O(RMS)} = \sqrt{\frac{1}{\omega T_o} \int_{\omega t=0}^{\omega t_{ON}} V_m^2 \sin^2 \omega t \cdot d(\omega t)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\omega T_o} \int_0^{\omega t_{ON}} \sin^2 \omega t \cdot d(\omega t)}$$

Substituting for $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{\omega T_o} \int_0^{\omega t_{ON}} \left[\frac{1 - \cos 2\omega t}{2} \right] d(\omega t)}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_o} \left[\int_0^{\omega t_{ON}} d(\omega t) - \int_0^{\omega t_{ON}} \cos 2\omega t \cdot d(\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_o} \left[\left(\omega t \right) \Bigg|_0^{\omega t_{ON}} - \frac{\sin 2\omega t}{2} \Bigg|_0^{\omega t_{ON}} \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\omega T_o} \left[\left(\omega t_{ON} - 0 \right) - \frac{\sin 2\omega t_{ON} - \sin 0}{2} \right]}$$

Here in this case,

t_{ON} = An integral number of
input cycles; Hence

$$t_{ON} = T, 2T, 3T, 4T, 5T, \dots \quad \&$$

$$\omega t_{ON} = 2\pi, 4\pi, 6\pi, 8\pi, 10\pi, \dots$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2 \cancel{\omega} t_{ON}}{2 \cancel{\omega} T_o}} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{t_{ON}}{T_o}}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{t_{ON}}{T_o}} = V_s \sqrt{\frac{t_{ON}}{T_o}}$$

Where $V_{i(RMS)} = \frac{V_m}{\sqrt{2}} = V_s$

$$\begin{aligned}
\frac{t_{ON}}{T_O} &= \frac{t_{ON}}{t_{ON} + t_{OFF}} \\
&= \frac{nT}{nT + mT} = \frac{n}{(n + m)} = k \\
V_{O(RMS)} &= V_S \sqrt{\frac{n}{(m + n)}} = V_S \sqrt{k}
\end{aligned}$$

RMS Out put voltage

$$V_{O(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^{2\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{n}{(m+n)}} = V_{i(RMS)} \sqrt{k} = V_s \sqrt{k}$$

$$V_{O(RMS)} = V_{i(RMS)} \sqrt{k} = V_s \sqrt{k}$$

Duty cycle

$$k = \frac{t_{ON}}{T_o} = \frac{t_{ON}}{(t_{ON} + t_{OFF})} = \frac{nT}{(m+n)T}$$

$$k = \frac{n}{(m+n)}$$

RMS Load Current

$$I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{V_{O(RMS)}}{R_L}$$

Output AC (Load) Power

$$P_O = I_{O(RMS)}^2 \times R_L$$

Input Power factor

$$PF = \frac{P_o}{VA} = \frac{\text{output load power}}{\text{input supply volt amperes}} = \frac{P_o}{V_s I_s}$$

$$PF = \frac{I_{O(RMS)}^2 \times R_L}{V_{i(RMS)} \times I_{in(RMS)}}$$

$$I_s = I_{in(RMS)} = \text{RMS input supply current.}$$

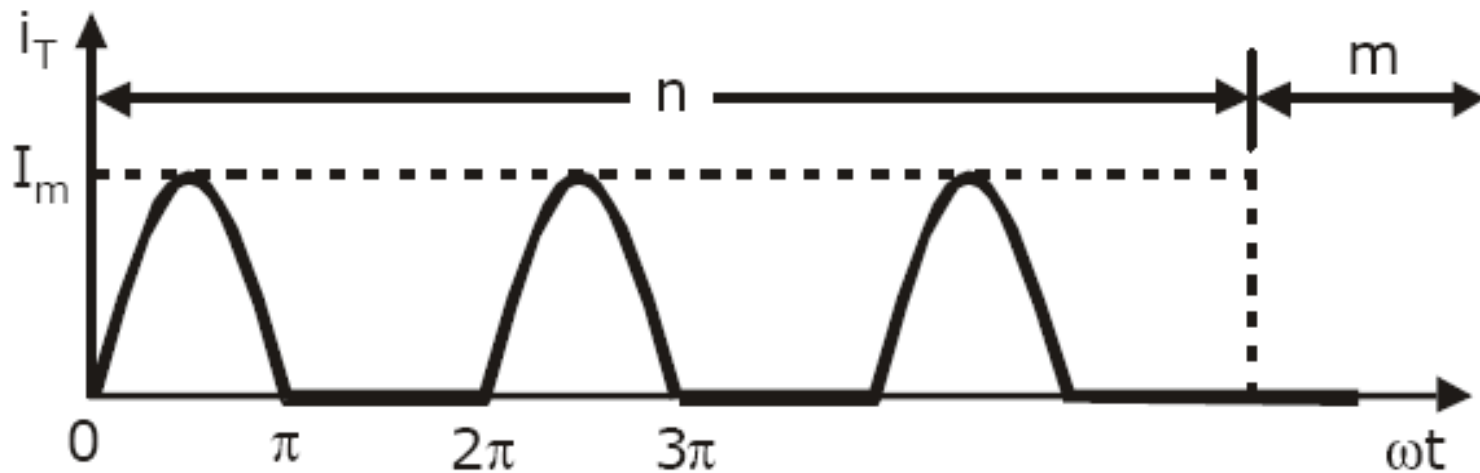
$$= \text{same as RMS load current } I_{O(RMS)}$$

$$\begin{aligned}
 PF &= \frac{I_{O(RMS)}^2 \times R_L}{V_{i(RMS)} \times I_{in(RMS)}} = \frac{V_{O(RMS)}}{V_{i(RMS)}} \\
 &= \frac{V_{i(RMS)} \sqrt{k}}{V_{i(RMS)}} = \sqrt{k}
 \end{aligned}$$

$$PF = \sqrt{k} = \sqrt{\frac{n}{m+n}}$$

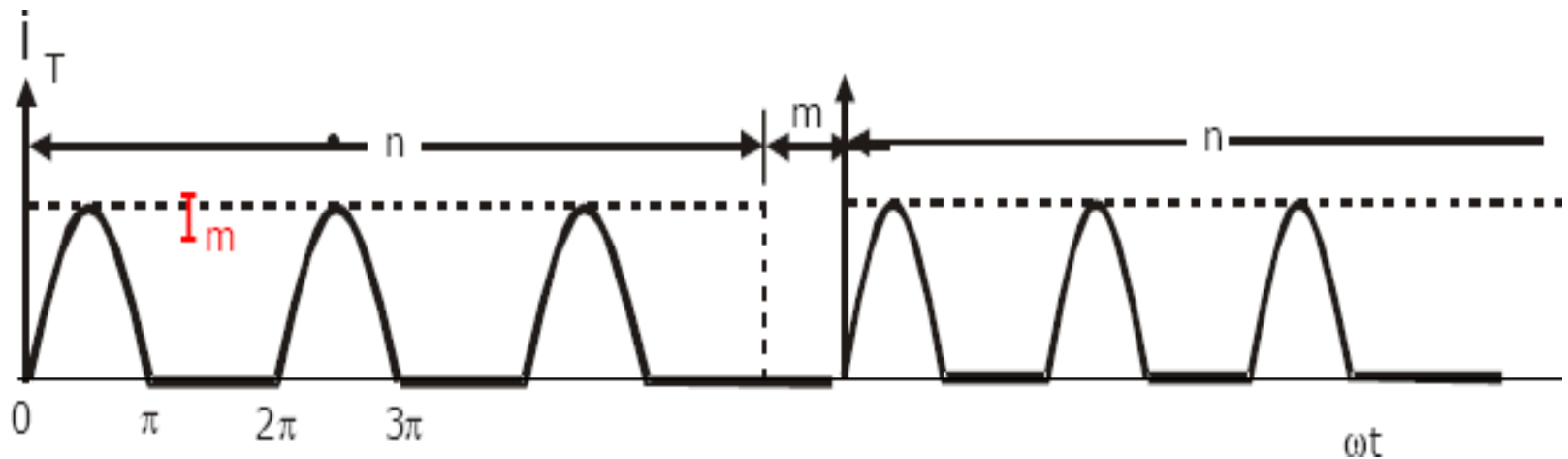
The Average Current Of Thyristor

Waveform of Thyristor Current



$$I_m = \frac{V_m}{R_L} = \text{max. or peak thyristor current}$$

Waveform of Thyristor current



$$I_{T(Avg)} = \frac{n}{2\pi(m+n)} \int_0^\pi I_m \sin \omega t . d(\omega t)$$

$$I_{T(Avg)} = \left(\frac{I_m}{\pi} \right) \frac{n}{(n+m)}$$

$$I_{T(Avg)} = \left(\frac{I_m}{\pi} \right) k$$

RMS Thyristor Current

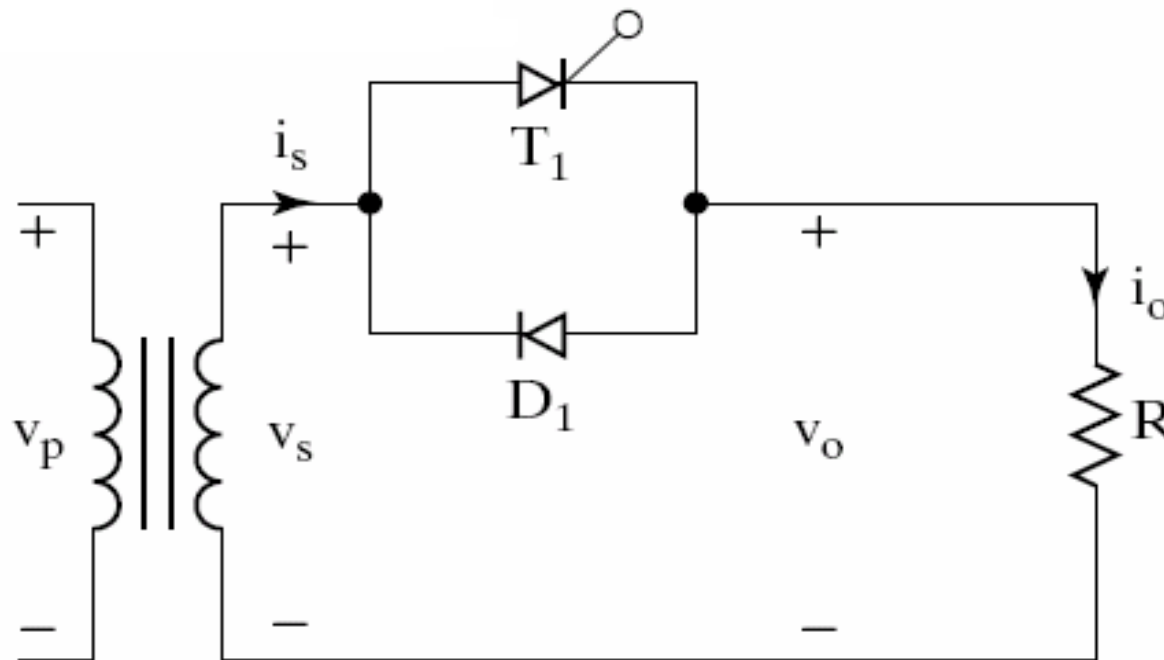
$$I_{T(RMS)} = \left[\frac{n}{2\pi(n+m)} \int_0^\pi I_m^2 \sin^2 \omega t \cdot d(\omega t) \right]^{1/2}$$

$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{\frac{n}{(m+n)}} = \frac{I_m}{2} \sqrt{k}$$

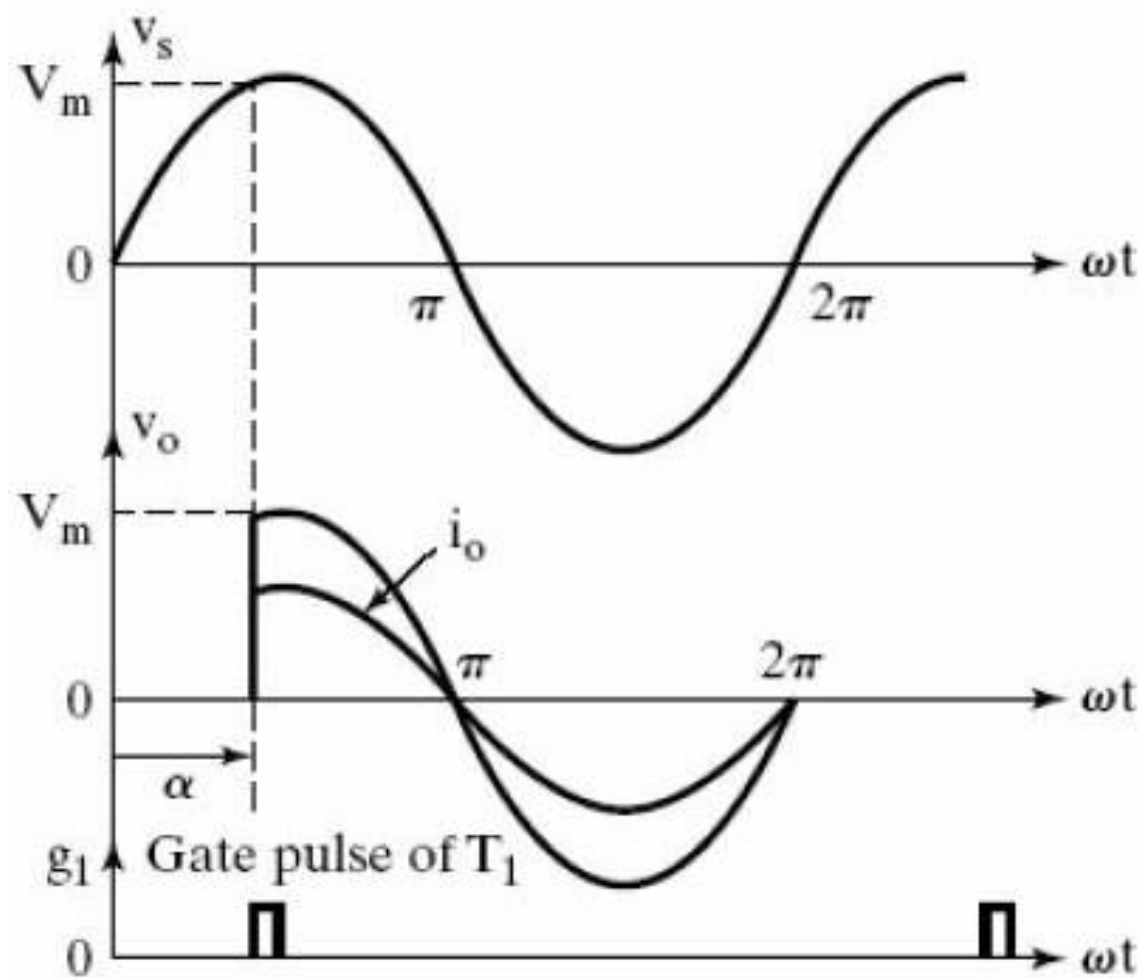
$$I_{T(RMS)} = \frac{I_m}{2} \sqrt{k}$$

Principle Of AC Phase Control
And
Operation of single Phase half-Wave
A.C Phase controller

Principle Of AC Phase Control



**Single phase Half-wave AC phase controller
(Unidirectional Controller)**



Equations

- **Input AC Supply Voltage**

$$v_s = V_m \sin \omega t$$

$$V_S = V_{in(RMS)} = \frac{V_m}{\sqrt{2}} = \text{RMS value of}$$

input supply voltage

Output Load Voltage

$$v_o = v_L = 0;$$

for $\omega t = 0$ to α

$$v_o = v_L = V_m \sin \omega t;$$

for $\omega t = \alpha$ to 2π

Out Put Load Current

$$i_o = i_L = \frac{v_o}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t ;$$

for $\omega t = \alpha$ to 2π

$$i_o = i_L = 0;$$

for $\omega t = 0$ to α

Expression For RMS Out put Load Voltage

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi} \left[\int_{\alpha}^{2\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t) \right]}$$
$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi} \left[\int_{\alpha}^{2\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) \cdot d(\omega t) \right]}$$

$$V_{O(RMS)} = \sqrt{\frac{V_m^2}{4\pi} \left[\int_{\alpha}^{2\pi} (1 - \cos 2\omega t) . d(\omega t) \right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{\left[\int_{\alpha}^{2\pi} d(\omega t) - \int_{\alpha}^{2\pi} \cos 2\omega t . d\omega t \right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{\left[\left(\omega t \right) \Big/_{\alpha}^{2\pi} - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{2\pi} \right]}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) - \left(\frac{\sin 2\omega t}{2} \right) \Big/_{\alpha}^{2\pi}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) - \left\{ \frac{\sin 4\pi}{2} - \frac{\sin 2\alpha}{2} \right\}}$$

$$V_{O(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}} \sqrt{(2\pi - \alpha) + \frac{\sin 2\alpha}{2}}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

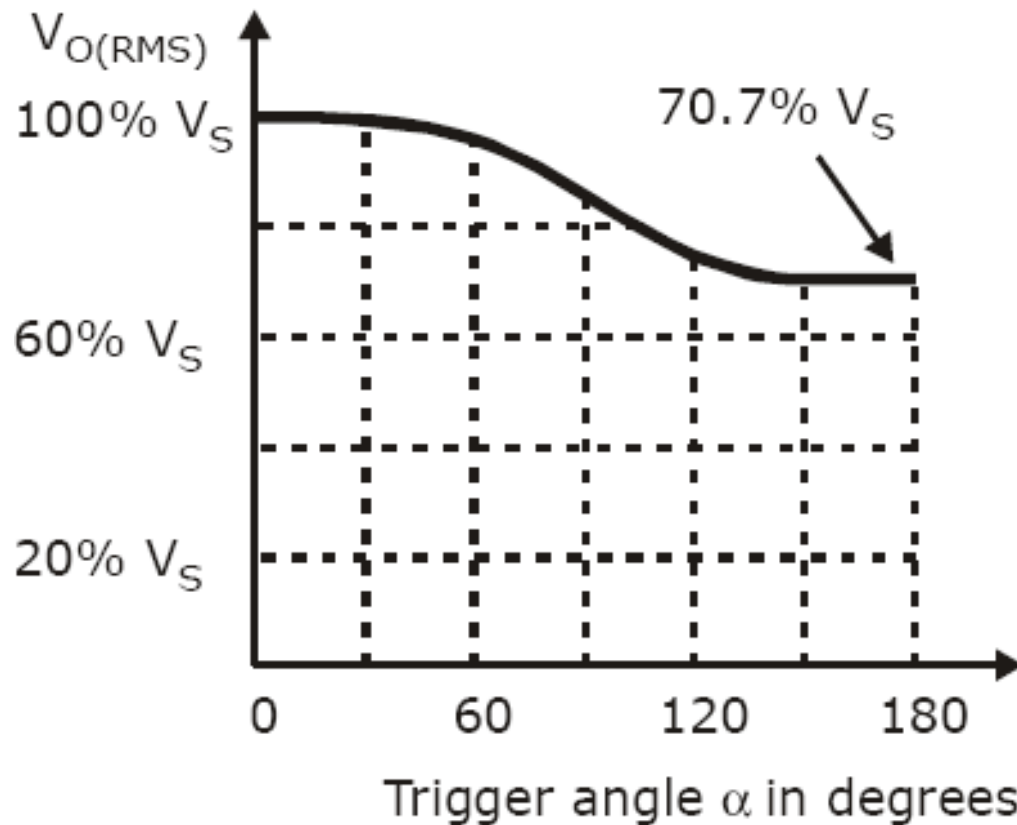
$$V_{O(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{2\pi} \left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi} \left[(2\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

where $V_{i(RMS)} = V_S = \frac{V_m}{\sqrt{2}} =$

RMS value of input supply voltage

Control Characteristics



Average Value of Out put Voltage

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \int_{\alpha}^{2\pi} \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos \omega t \right]_{\alpha}^{2\pi}$$

$$V_{O(dc)} = \frac{V_m}{2\pi} [-\cos 2\pi + \cos \alpha] ; \cos 2\pi = 1$$

$$V_{dc} = \frac{V_m}{2\pi} [\cos \alpha - 1] ; V_m = \sqrt{2}V_s$$

Hence,

$$V_{dc} = \frac{\sqrt{2}V_s}{2\pi}(\cos \alpha - 1)$$

When α is varied from 0 to π

$$V_{dc} \text{ varies from 0 to } \frac{-V_m}{\pi}$$

Disadvantages

- Output load voltage has a DC component as the two halves are not symmetrical with respect to '0' level.
- Limited range of RMS output voltage control from 100% of V_s to 70.7% of V_s , when we vary the trigger angle from zero to 180 degrees.

Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-Load

Single Phase Full Wave Ac Voltage Controller With R-Load

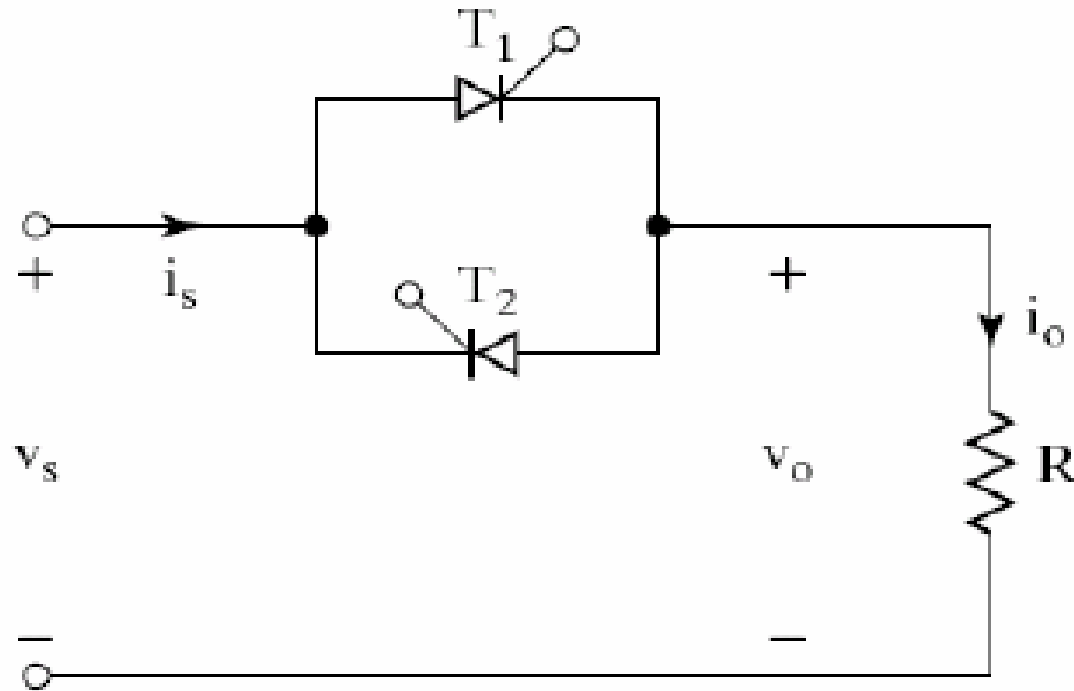
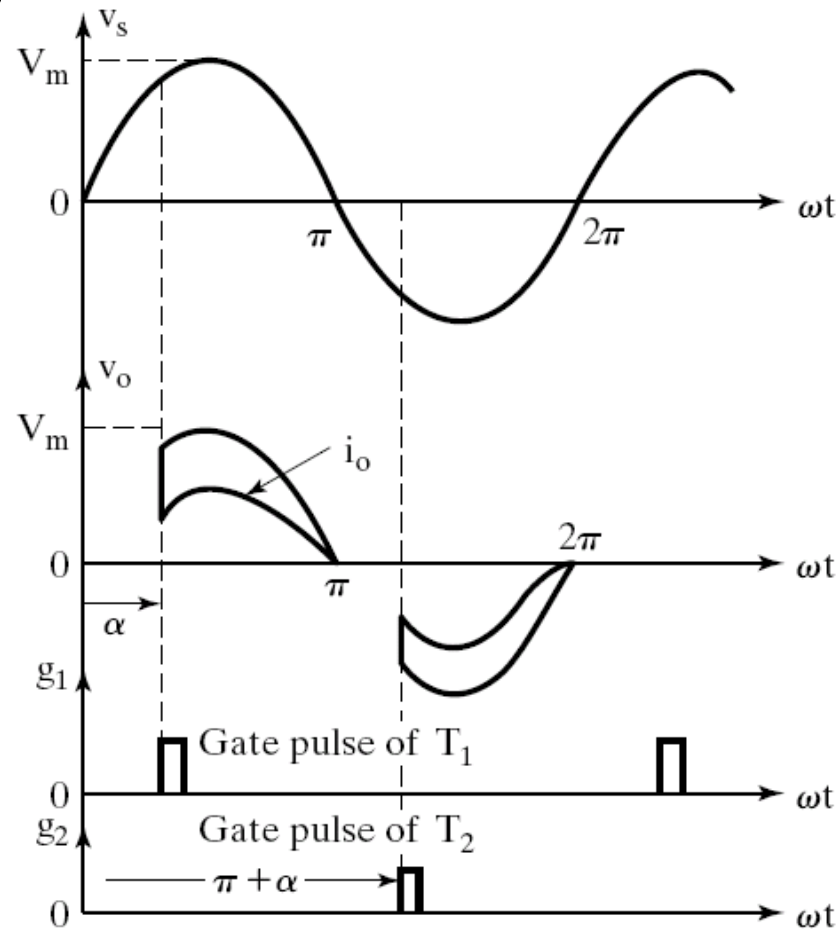


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using SCR

Waveforms of single phase full wave ac voltage controller



Expression for RMS output voltage

$$V_{L(RMS)}^2 = \frac{1}{\pi} \int_0^{\pi} V_m^2 \sin^2 \omega t \cdot d\omega t$$

$$V_{L(RMS)}^2 = \frac{1}{2\pi} \int_0^{2\pi} v_L^2 \cdot d(\omega t)$$

$$V_{L(RMS)}^2 = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} (V_m \sin \omega t)^2 d(\omega t) + \int_{\pi + \alpha}^{2\pi} (V_m \sin \omega t)^2 d(\omega t) \right]$$

Contd...

$$= \frac{1}{2\pi} \left[V_m^2 \int_{\alpha}^{\pi} \sin^2 \omega t \cdot d(\omega t) + V_m^2 \int_{\pi+\alpha}^{2\pi} \sin^2 \omega t \cdot d(\omega t) \right]$$

$$= \frac{1}{2\pi} \left[V_m^2 \int_{\alpha}^{\pi} \sin^2 \omega t \cdot d(\omega t) + V_m^2 \int_{\pi+\alpha}^{2\pi} \sin^2 \omega t \cdot d(\omega t) \right]$$

$$= \frac{V_m^2}{2\pi} \left[\int_{\alpha}^{\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) + \int_{\pi+\alpha}^{2\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) \right]$$

$$= \frac{V_m^2}{2\pi \times 2} \left[\int_{\alpha}^{\pi} d(\omega t) - \int_{\alpha}^{\pi} \cos 2\omega t \cdot d(\omega t) + \int_{\pi+\alpha}^{2\pi} d(\omega t) - \int_{\pi+\alpha}^{2\pi} \cos 2\omega t \cdot d(\omega t) \right]$$

$$= \frac{V_m^2}{4\pi} \left[\left(\omega t \right) \Big/_{\alpha}^{\pi} + \left(\omega t \right) \Big/_{\pi+\alpha}^{2\pi} - \left[\frac{\sin 2\omega t}{2} \right]_{\alpha}^{\pi} - \left[\frac{\sin 2\omega t}{2} \right]_{\pi+\alpha}^{2\pi} \right]$$

$$= \frac{V_m^2}{4\pi} \left[(\pi - \alpha) + (\pi - \alpha) - \frac{1}{2}(\sin 2\pi - \sin 2\alpha) - \frac{1}{2}(\sin 4\pi - \sin 2(\pi + \alpha)) \right]$$

$$= \frac{V_m^2}{4\pi} \left[2(\pi - \alpha) - \frac{1}{2}(0 - \sin 2\alpha) - \frac{1}{2}(0 - \sin 2(\pi + \alpha)) \right]$$

$$= \frac{V_m^2}{4\pi} \left[2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2(\pi + \alpha)}{2} \right]$$

$$= \frac{V_m^2}{4\pi} \left[2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin(2\pi + 2\alpha)}{2} \right]$$

$$= \frac{V_m^2}{4\pi} \left[2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{1}{2}(\sin 2\pi \cdot \cos 2\alpha + \cos 2\pi \cdot \sin 2\alpha) \right]$$

$$\sin 2\pi = 0 \quad \& \quad \cos 2\pi = 1$$

Therefore,

$$V_{L(RMS)}^2 = \frac{V_m^2}{4\pi} \left[2(\pi - \alpha) + \frac{\sin 2\alpha}{2} + \frac{\sin 2\alpha}{2} \right]$$

$$V_{L(RMS)}^2 = \frac{V_m^2}{4\pi} \left[(2\pi - 2\alpha) + \sin 2\alpha \right]$$

- Taking the square root, we get

$$V_{L(RMS)} = \frac{V_m}{2\sqrt{\pi}} \sqrt{[(2\pi - 2\alpha) + \sin 2\alpha]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}\sqrt{2\pi}} \sqrt{[(2\pi - 2\alpha) + \sin 2\alpha]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} [(2\pi - 2\alpha) + \sin 2\alpha]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi} \left[2 \left\{ (\pi - \alpha) + \frac{\sin 2\alpha}{2} \right\} \right]}$$

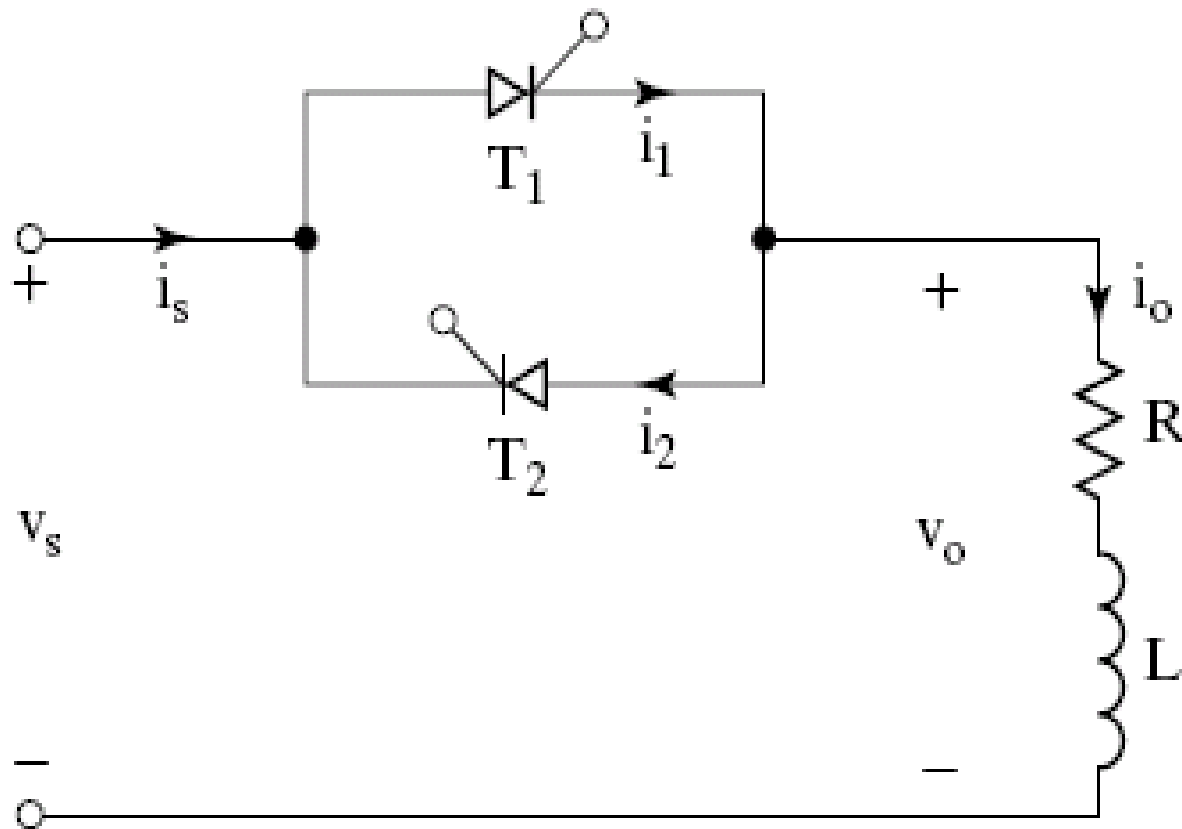
$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

$$V_{L(RMS)} = V_{i(RMS)} \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

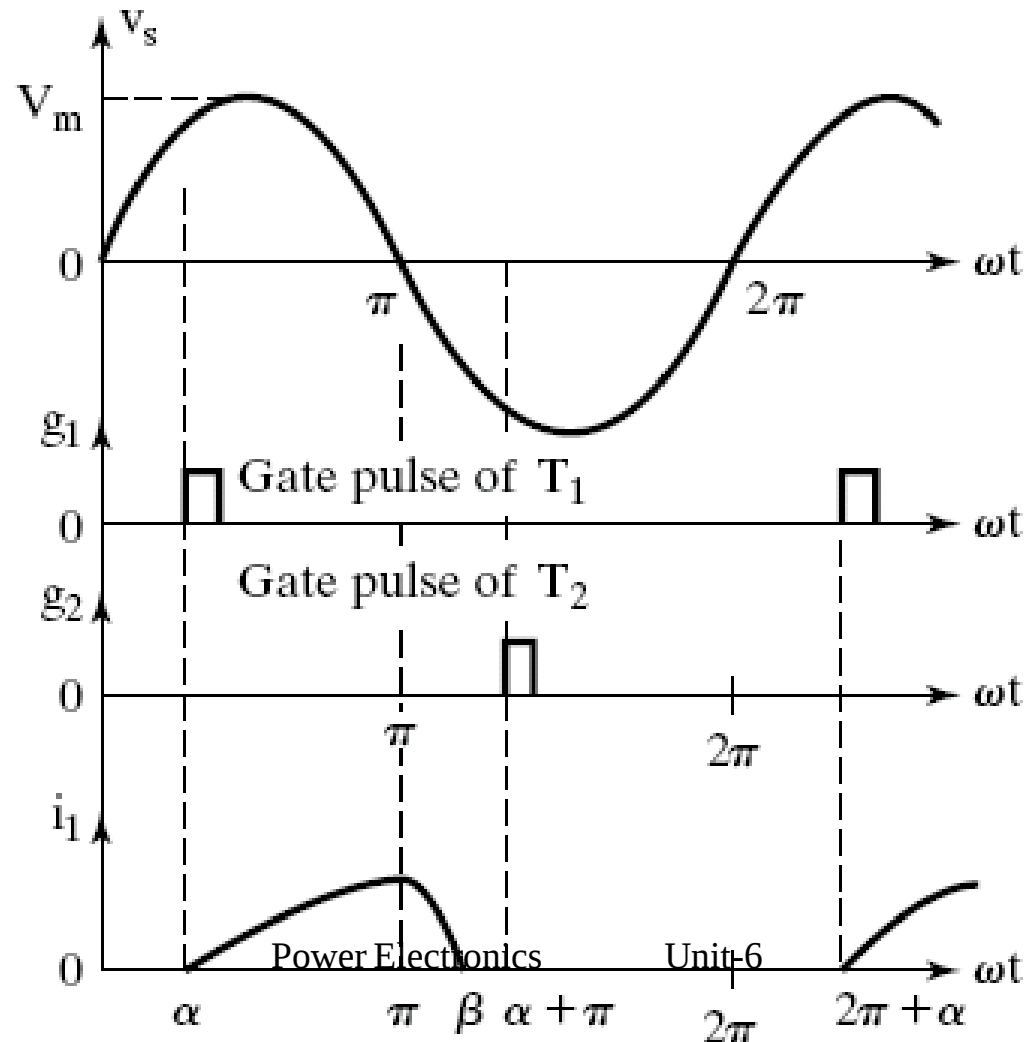
$$V_{L(RMS)} = V_s \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}$$

Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-L Load

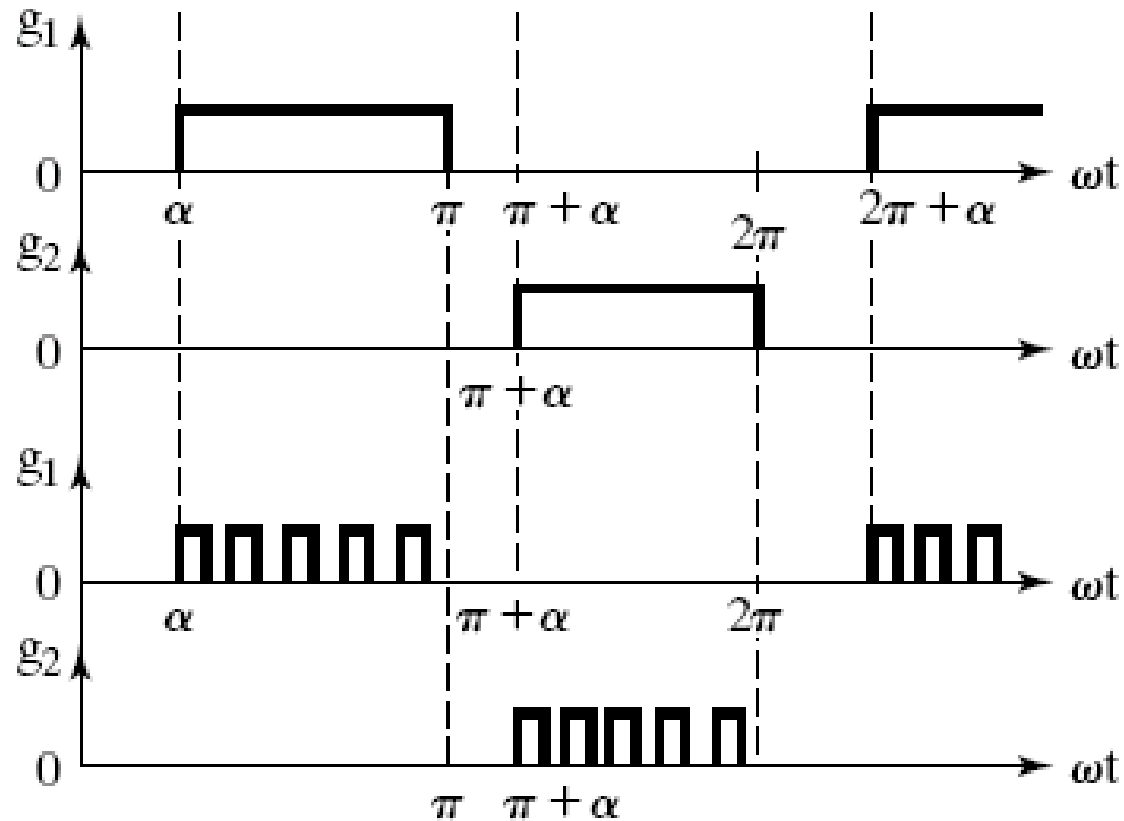
Single Phase Full Wave Ac Voltage Controller (Bidirectional Controller) With R-L Load



Input supply voltage & Thyristor current waveforms

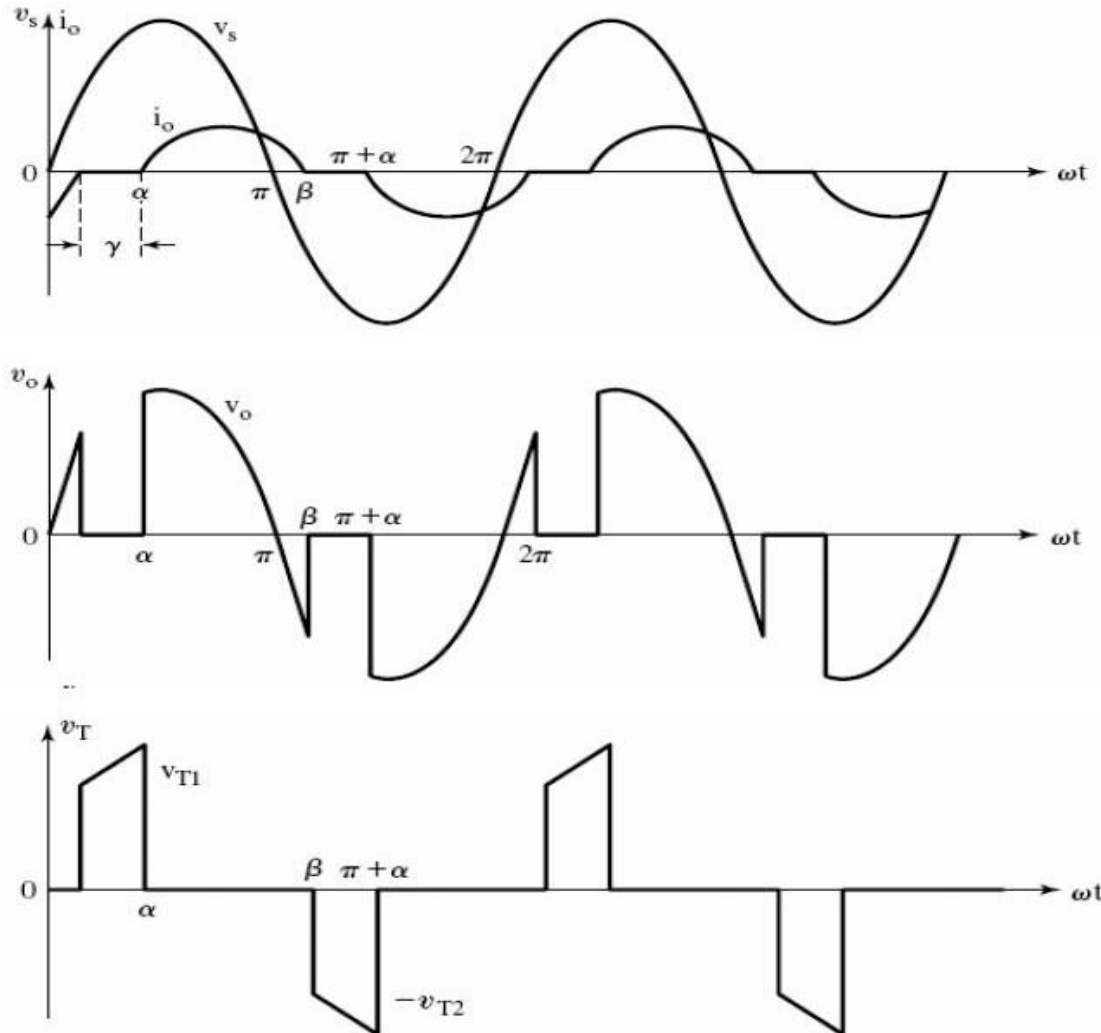


Gating Signals



Waveforms For RL load for for Discontinuous Conduction

$\alpha > \phi$
and



Expression for the inductive load current of a single phase full wave ac voltage controller with RL load

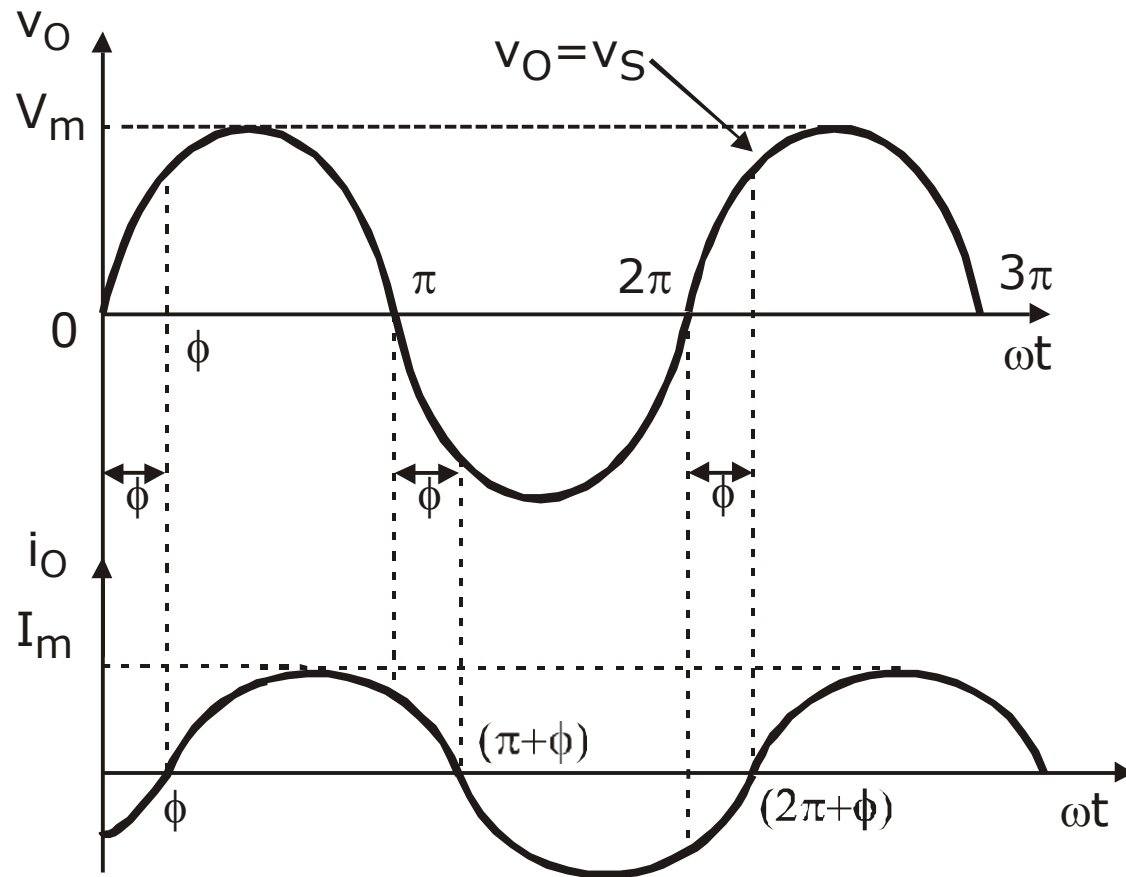
$$i_o = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right]$$

Where $Z = \sqrt{R^2 + (\omega L)^2}$

$$\phi = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

= Load impedance angle (power factor angle of load).

Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \leq \phi$



TRIAC and Its Modes of Operation

TRIAC

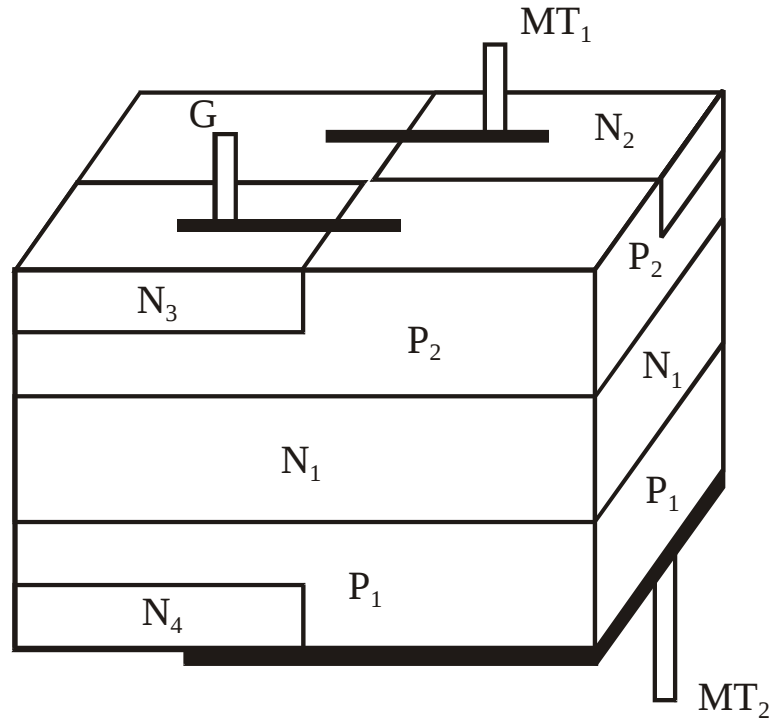


Fig.1 : Triac Structure
Symbol

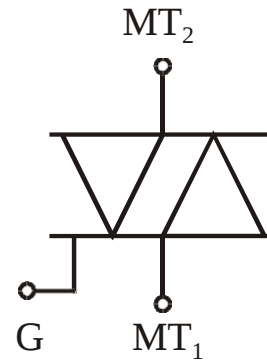
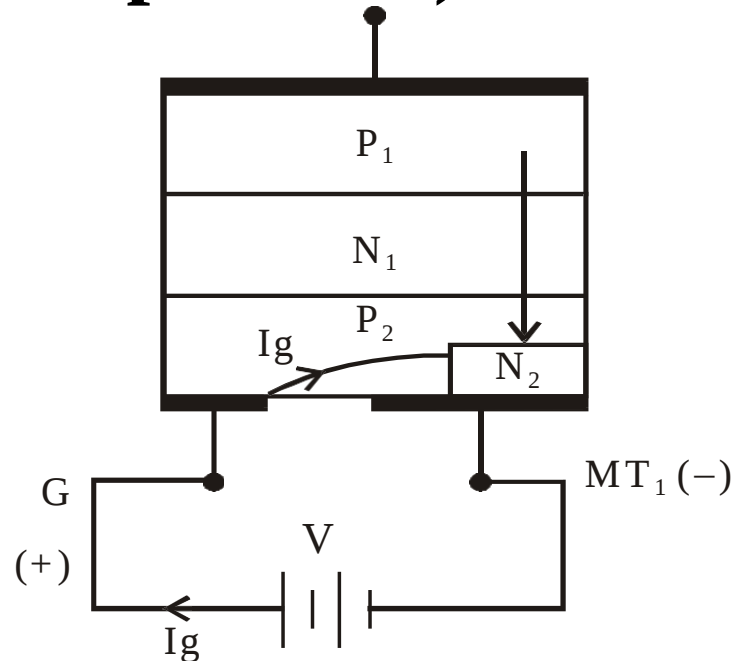


Fig. 2 : Triac

TRIGGERING MODES OF TRIAC

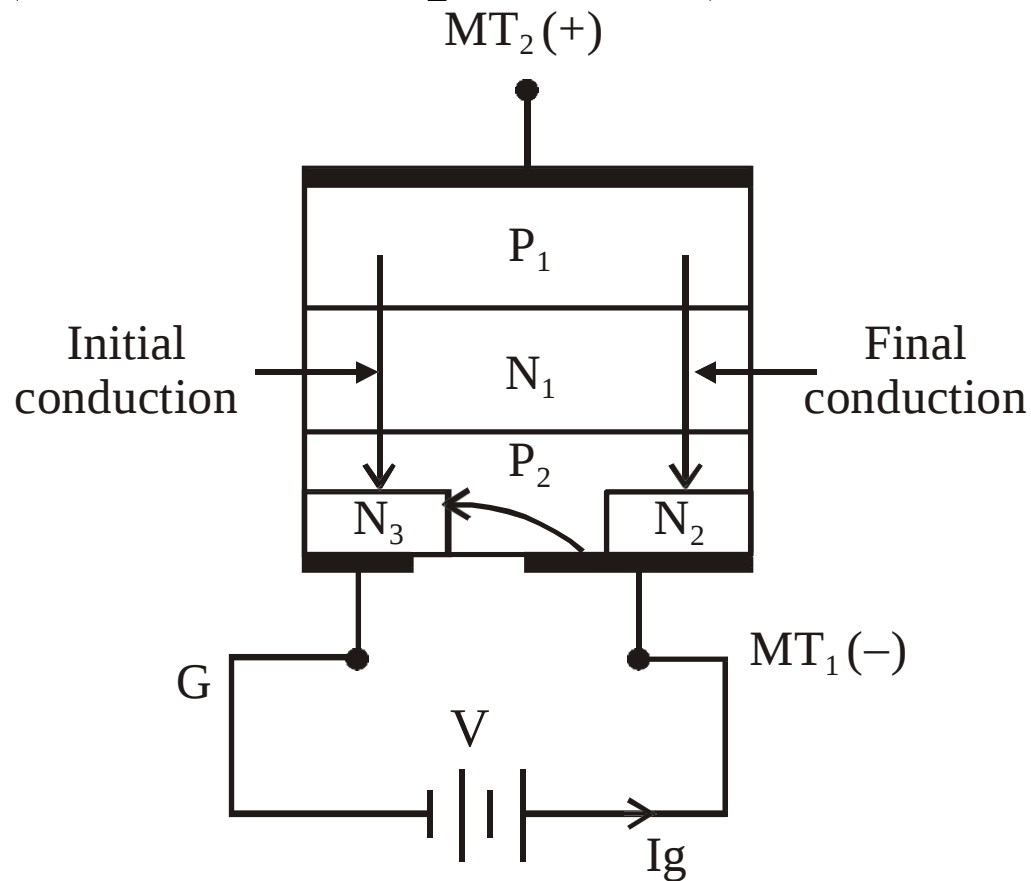
- **MODE 1 : MT1 positive, Positive gate current (I^+ mode of operation)**



- When V_{GT} and gate current are positive with respect to MT1, the gate current flows through P2-N2 junction
- The junction P1-N1 and P2-N2 are forward biased but junction N1-P2 is reverse biased.
- When sufficient number of charge carriers are injected in P2 layer by the gate current the junction N1-P2 breakdown and triac starts conducting through P1N1P2N2 layers.
- Once triac starts conducting the current increases and its V-I characteristics is similar to that of thyristor. Triac in this mode operates in the first-quadrant.

MODE 2

- **MT2 positive, Negative gate current**
(I- mode of operation)

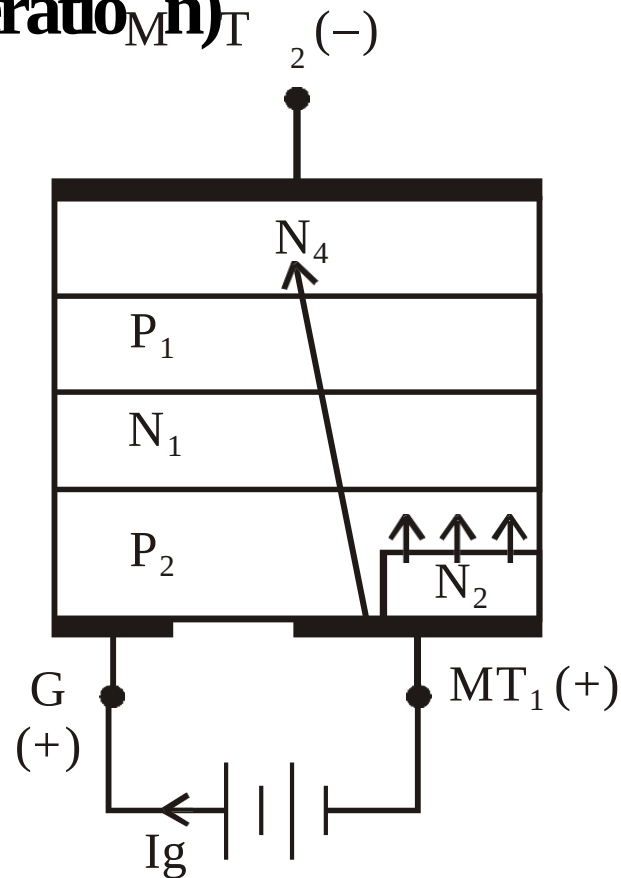


- When MT2 is positive and gate G is negative with respect to MT1 the gate current flows through P2-N3 junction
- The junction P1-N1 and P2-N3 are forward biased but junction N1-P2 is reverse biased. Hence, the triac initially starts conducting through P1N1P2N3 layers.
- As a result the potential of layer between P2-N3 rises towards the potential of MT2.
- Thus, a potential gradient exists across the layer P2 with left hand region at a higher potential than the right hand region.

- This results in a current flow in P2 layer from left to right, forward biasing the P2N2 junction. Now the right hand portion P1-N1 - P2-N2 starts conducting.
- The device operates in first quadrant. When compared to Mode 1, triac with MT2 positive and negative gate current is less sensitive and therefore requires higher gate current for triggering.

MODE 3

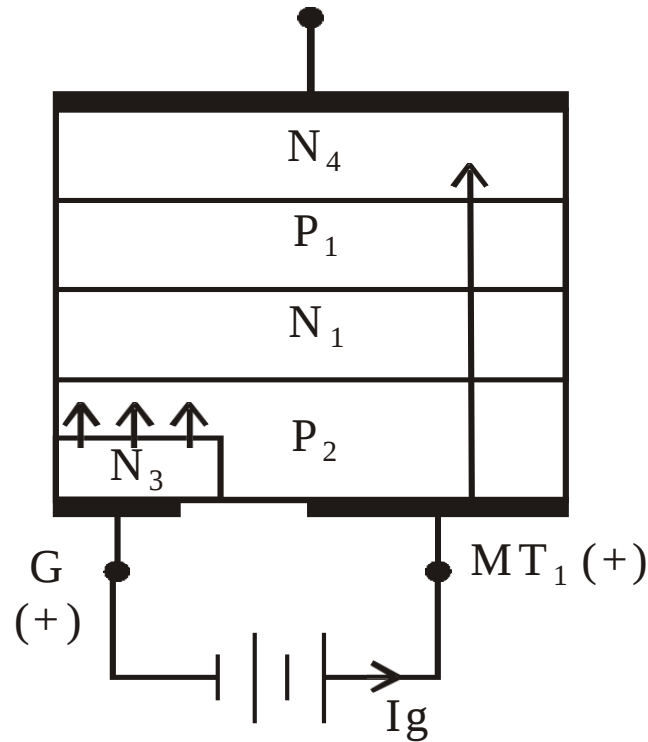
- **MT2 negative, Positive gate current (I_{II}^+ mode of operation)**
 $(I_{M2})_T = (-)$



- When MT2 is negative and gate is positive with respect to MT1 junction P2N2 is forward biased and junction P1-N1 is reverse biased.
- N2 layer injects electrons into P2 layer as shown by arrows in figure below.
- This causes an increase in current flow through junction P2-N1. Resulting in breakdown of reverse biased junction N1-P1.
- Now the device conducts through layers P2N1P1N4 and the current starts increasing, which is limited by an external load.
- The device operates in third quadrant in this mode. Triac in this mode is less sensitive and requires higher gate current for triggering.

MODE 4

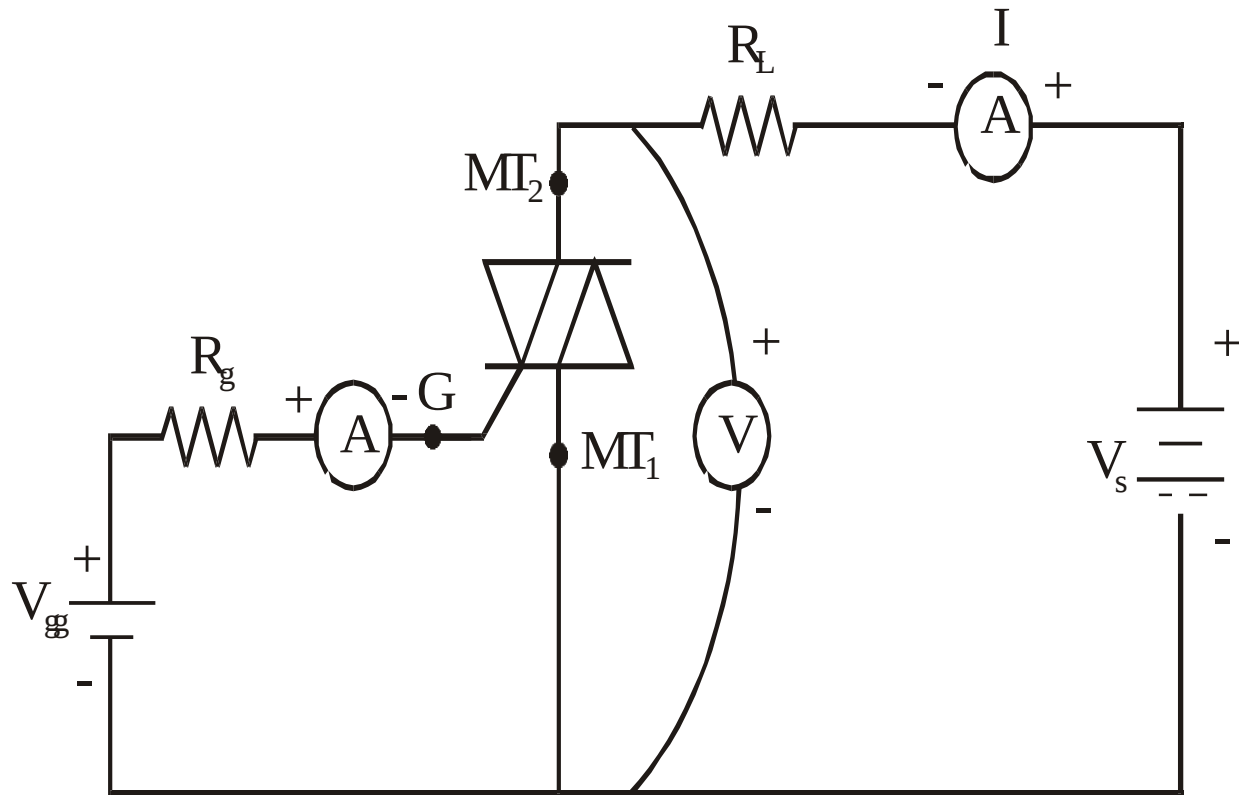
- **MT2 negative, Negative gate current**
(III⁺ mode of operation)
 $MT_2(-)$



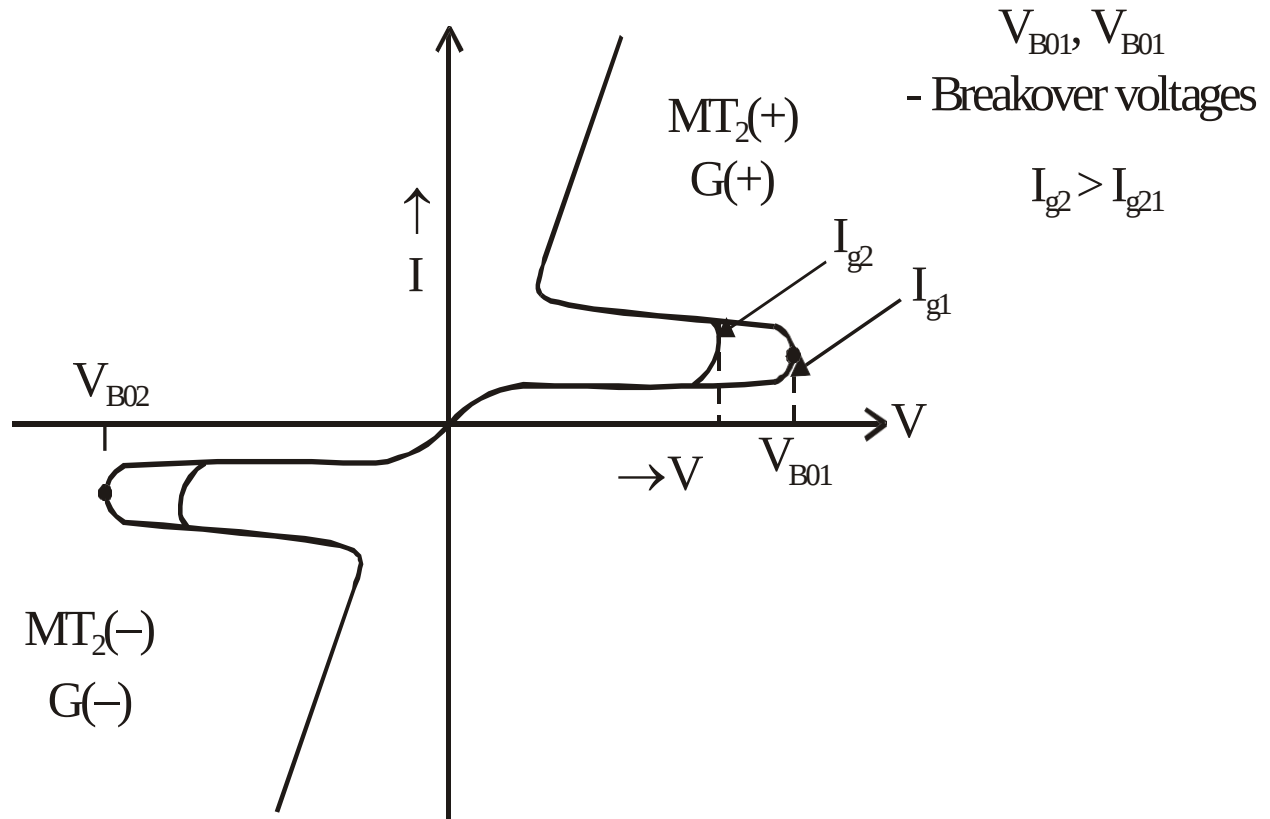
- In this mode both MT2 and gate G are negative with respect to MT1, the gate current flows through P2N3 junction as shown.
- Layer N3 injects electrons as shown by arrows into P2 layer. This results in increase in current flow across P1N1 and the device will turn ON due to increased current in layer N1.
- The current flows through layers P2N1P1N4. Triac is more sensitive in this mode compared to turn ON with positive gate current. (Mode 3).

- Triac sensitivity is greatest in the first quadrant when turned ON with positive gate current and also in third quadrant when turned ON with negative gate current. when V_{AK} is positive with respect to A it is recommended to turn on the triac by a positive gate current.
- When V_{AK} is negative with respect to A it is recommended to turn on the triac by negative gate current. Therefore Mode 1 and Mode 4 are the preferred modes of operation of a triac (mode 1 and mode 4 of operation are normally used).

Triac characteristics

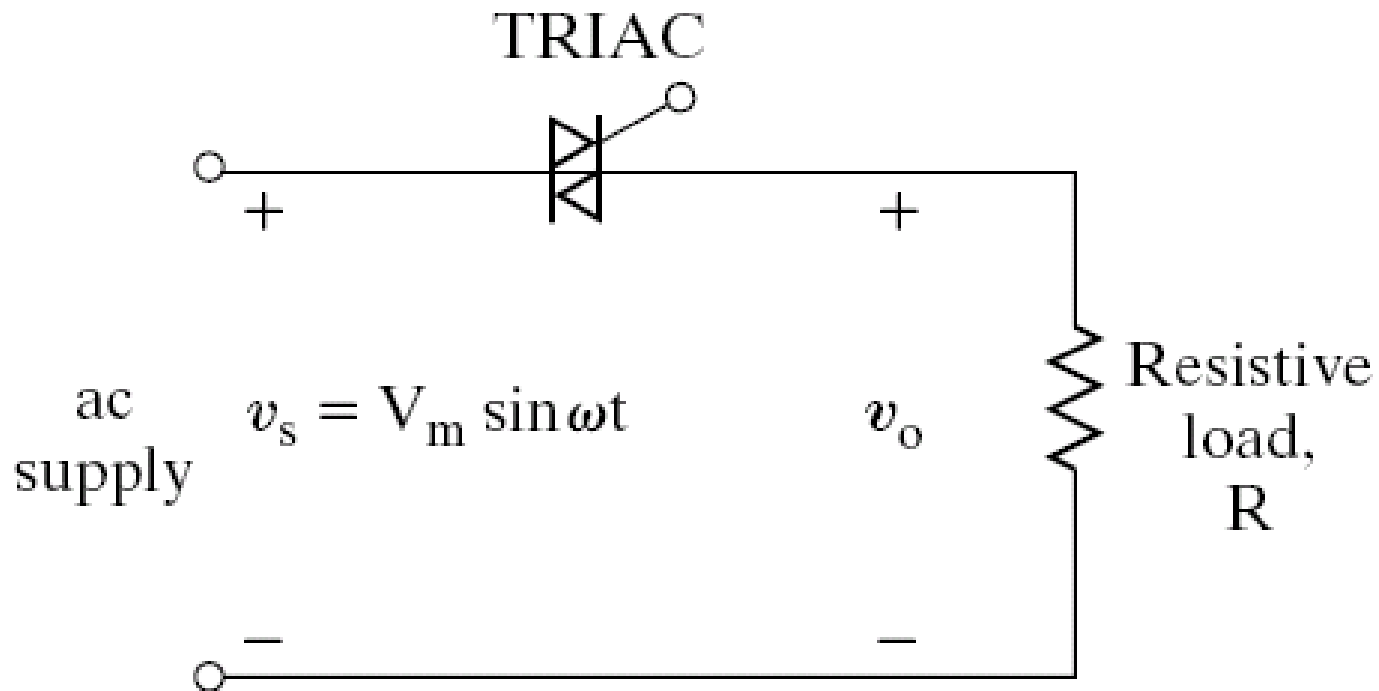


V-I Characteristics of a triac

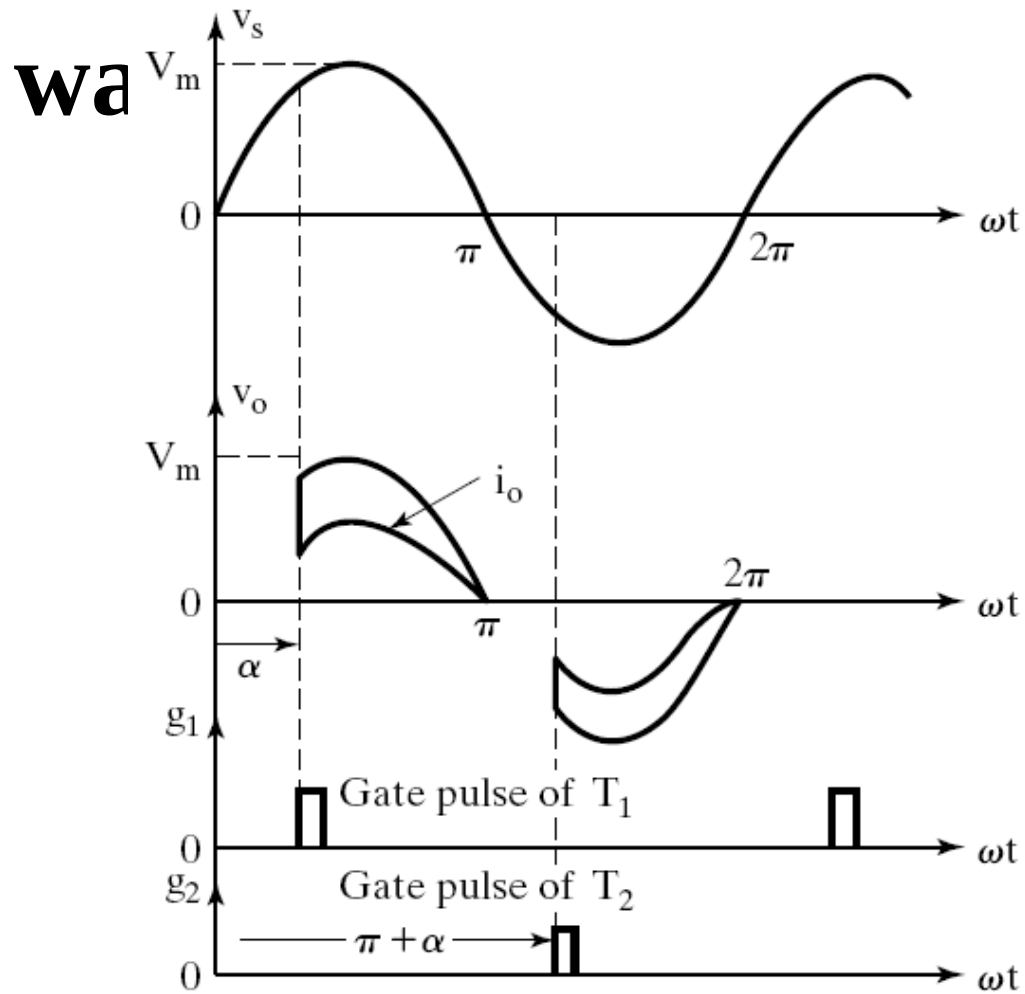


**Single phase full wave ac
voltage controller
(Bi-directional Controller) using
TRIAC**

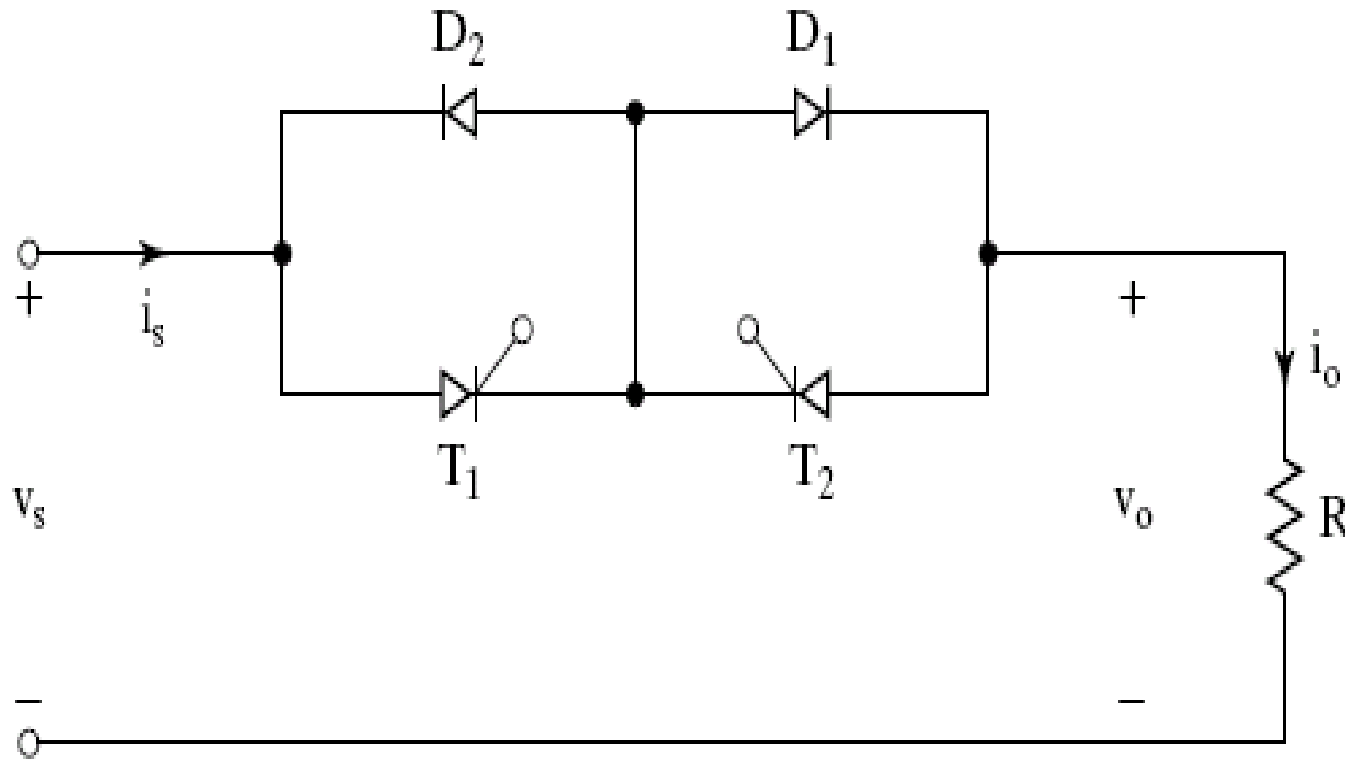
Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC



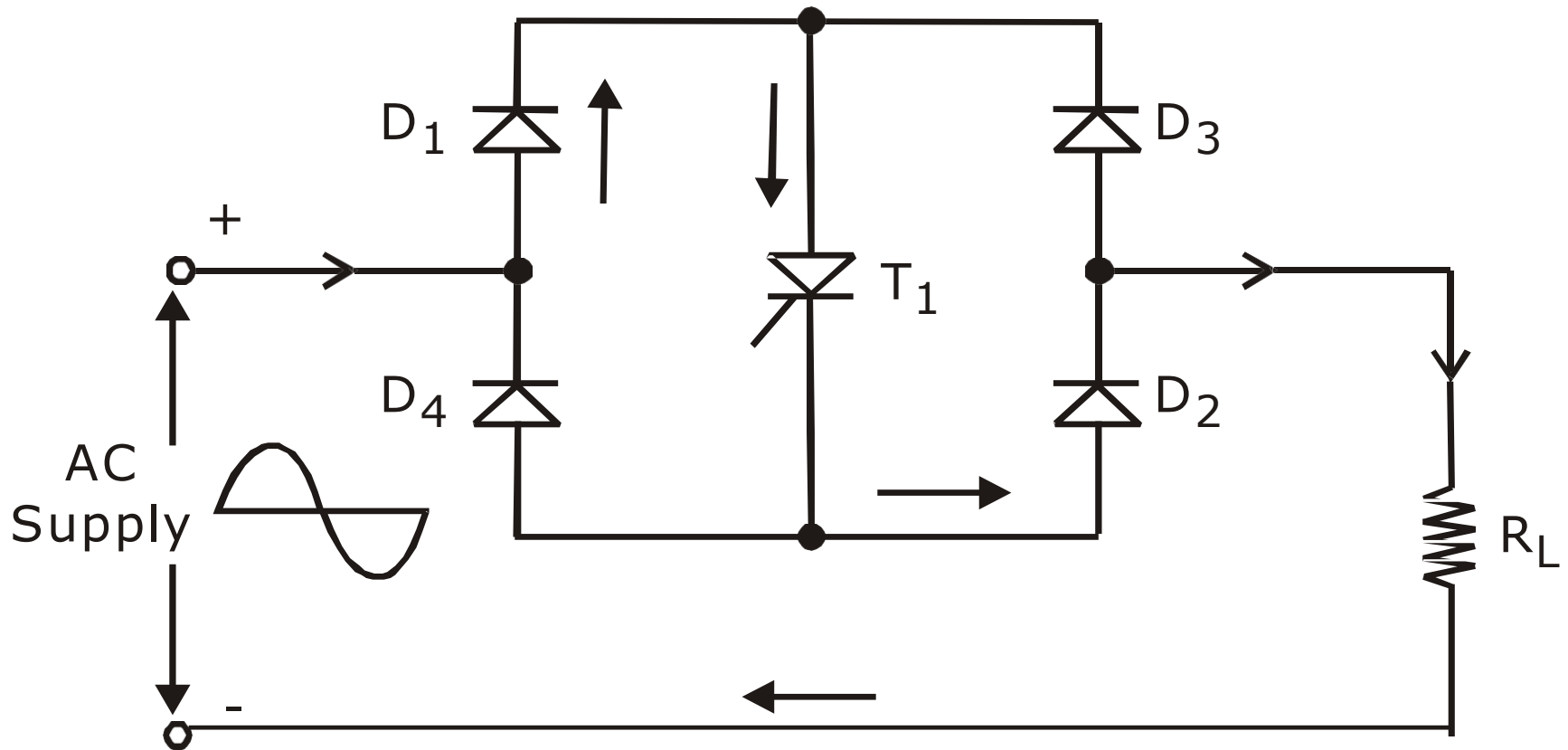
Waveforms of single phase full



Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)



Single Phase Full Wave Ac Voltage Controller Using A Single Thyristor

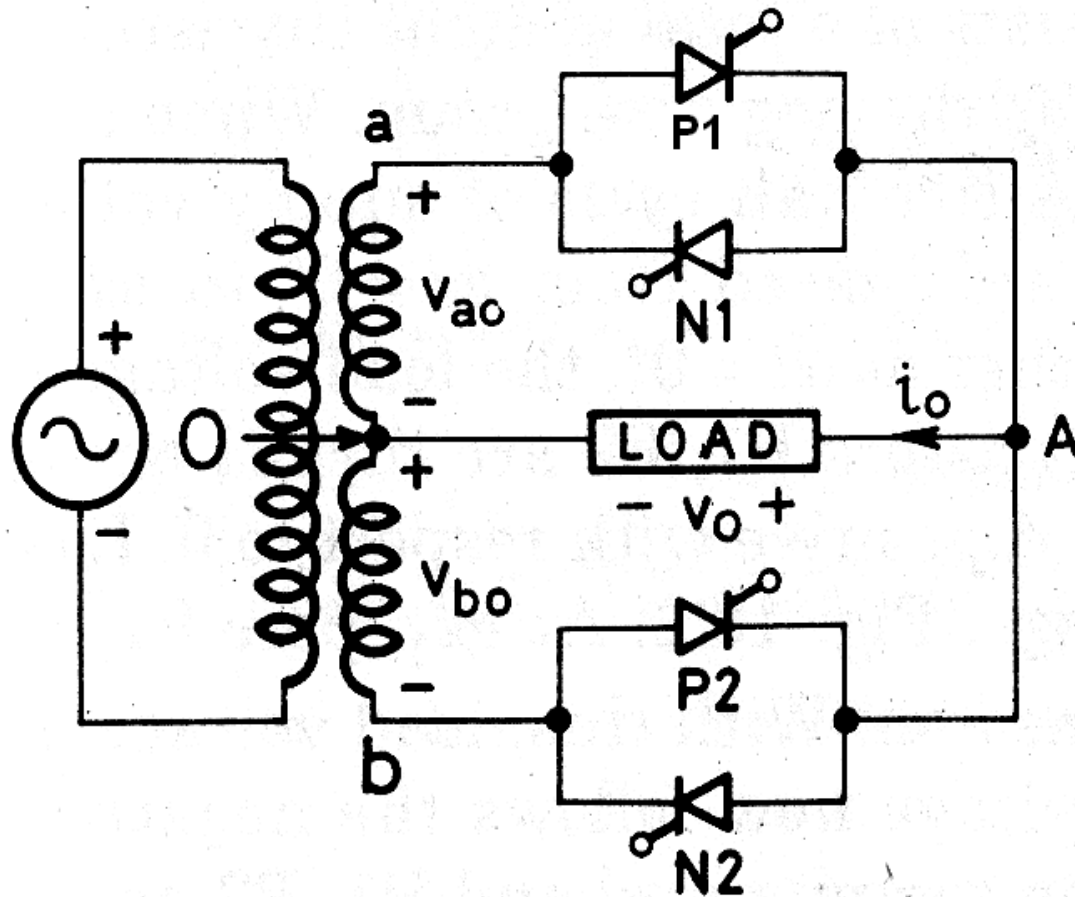


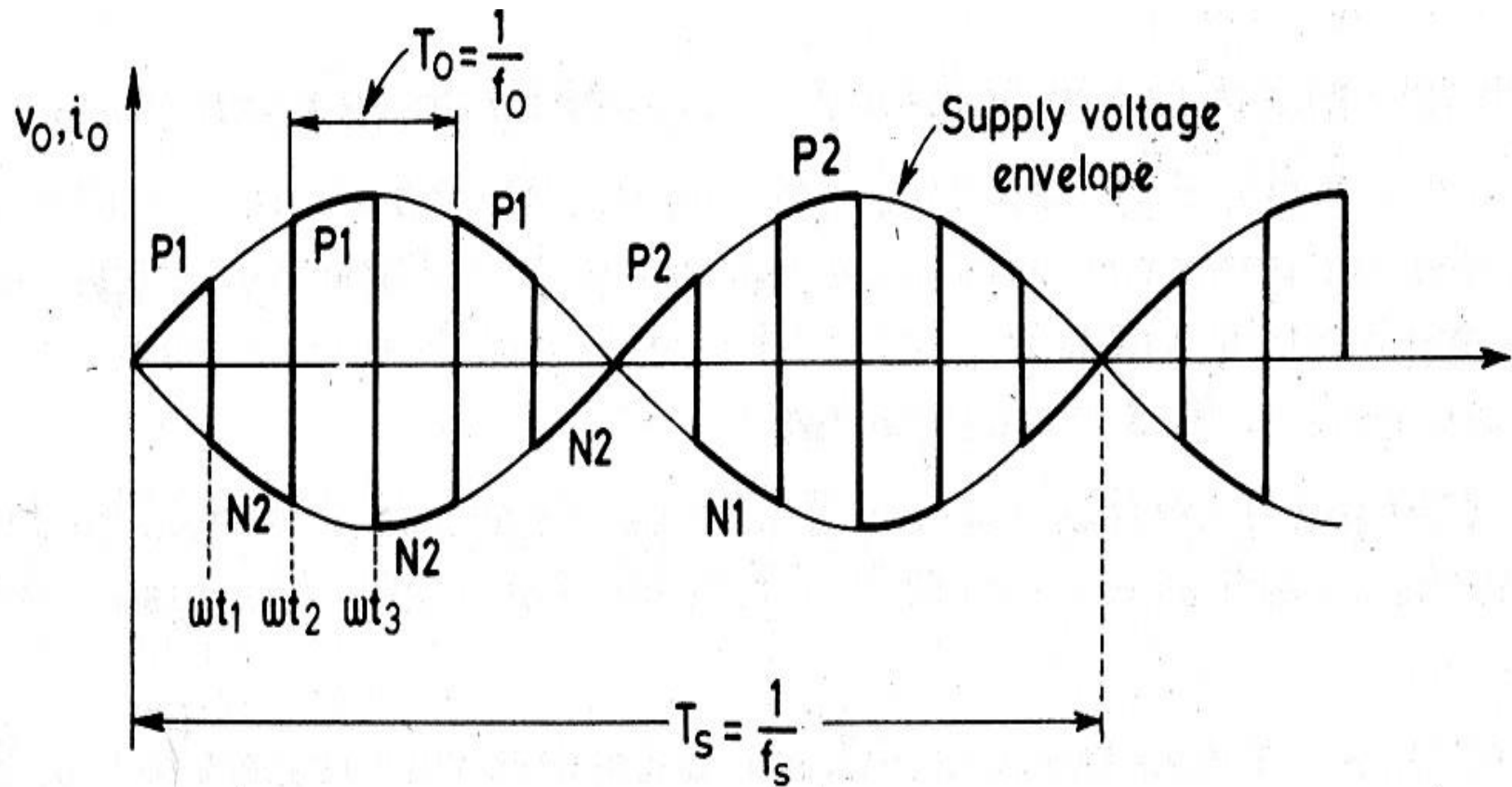
CYCLOCONVERTER

CYCLOCONVERTER

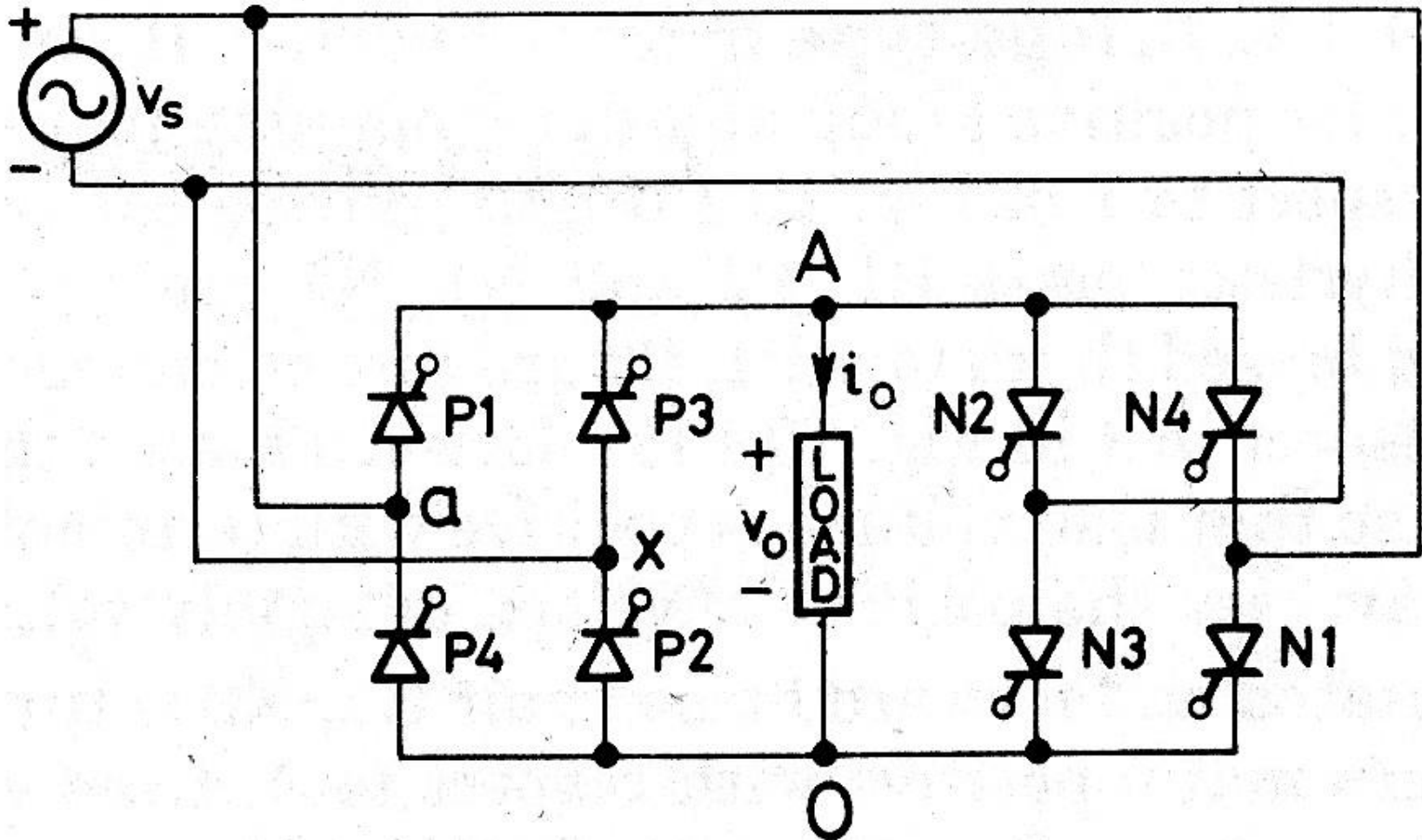
- A device which converts input power at one frequency to the out put power at different frequency with one stage conversion is called a cycloconverter.
- A cycloconverter requires one stage frequency conversion.
- Cycloconverter of two types
 - (i) Step-Up Cycloconverter ($f_o > f_s$)
 - (ii) step-Down Cycloconverter ($f_o < f_s$)

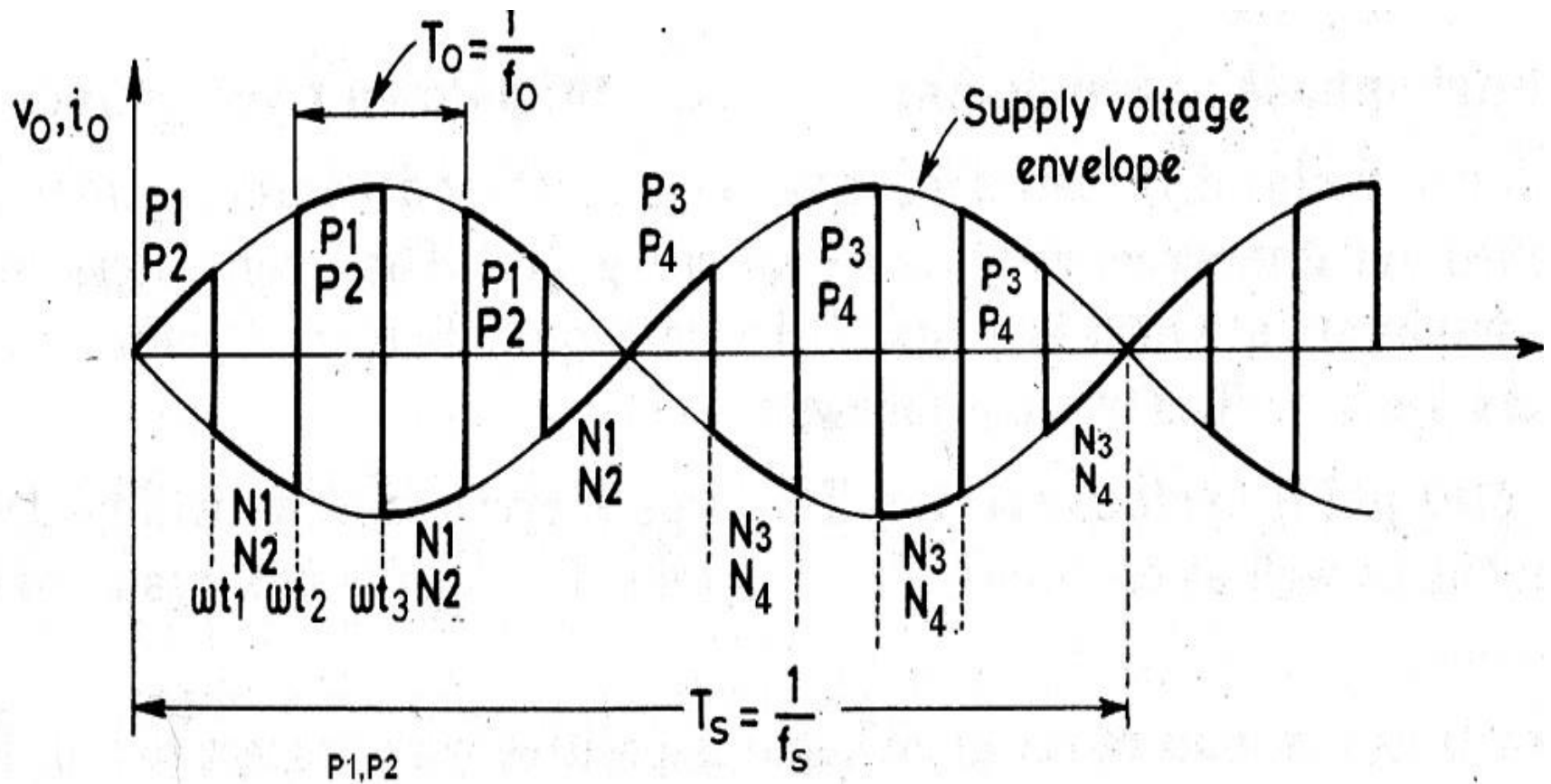
Single phase to single phase Mid point type step-up Cycloconverter with R load



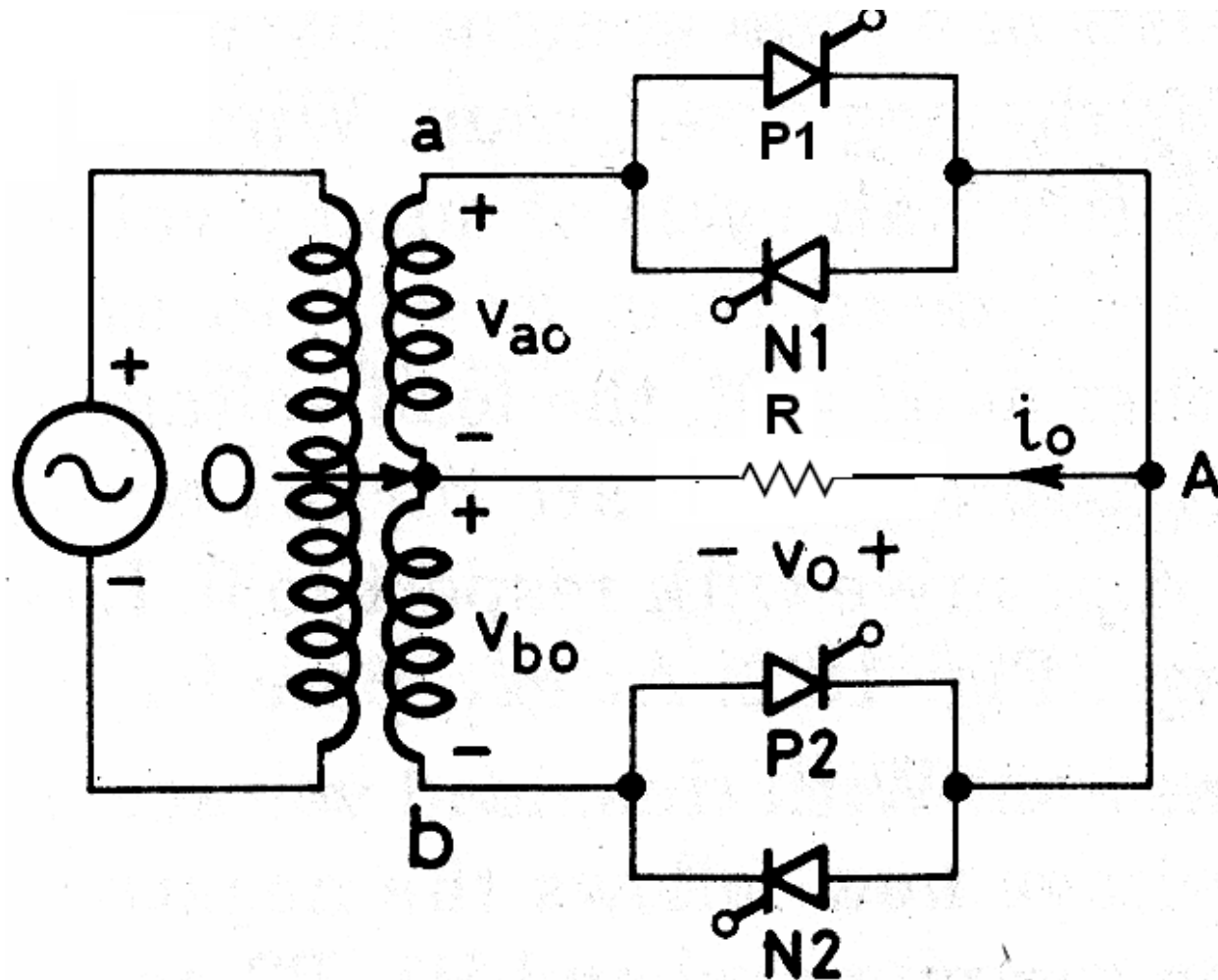


Single phase to single phase Bridge type step-up Cycloconverter with R load





1- ϕ to 1- ϕ Mid point type step-Down Cycloconverter with R load



Output voltage (V_o) and current (I_o) waveform

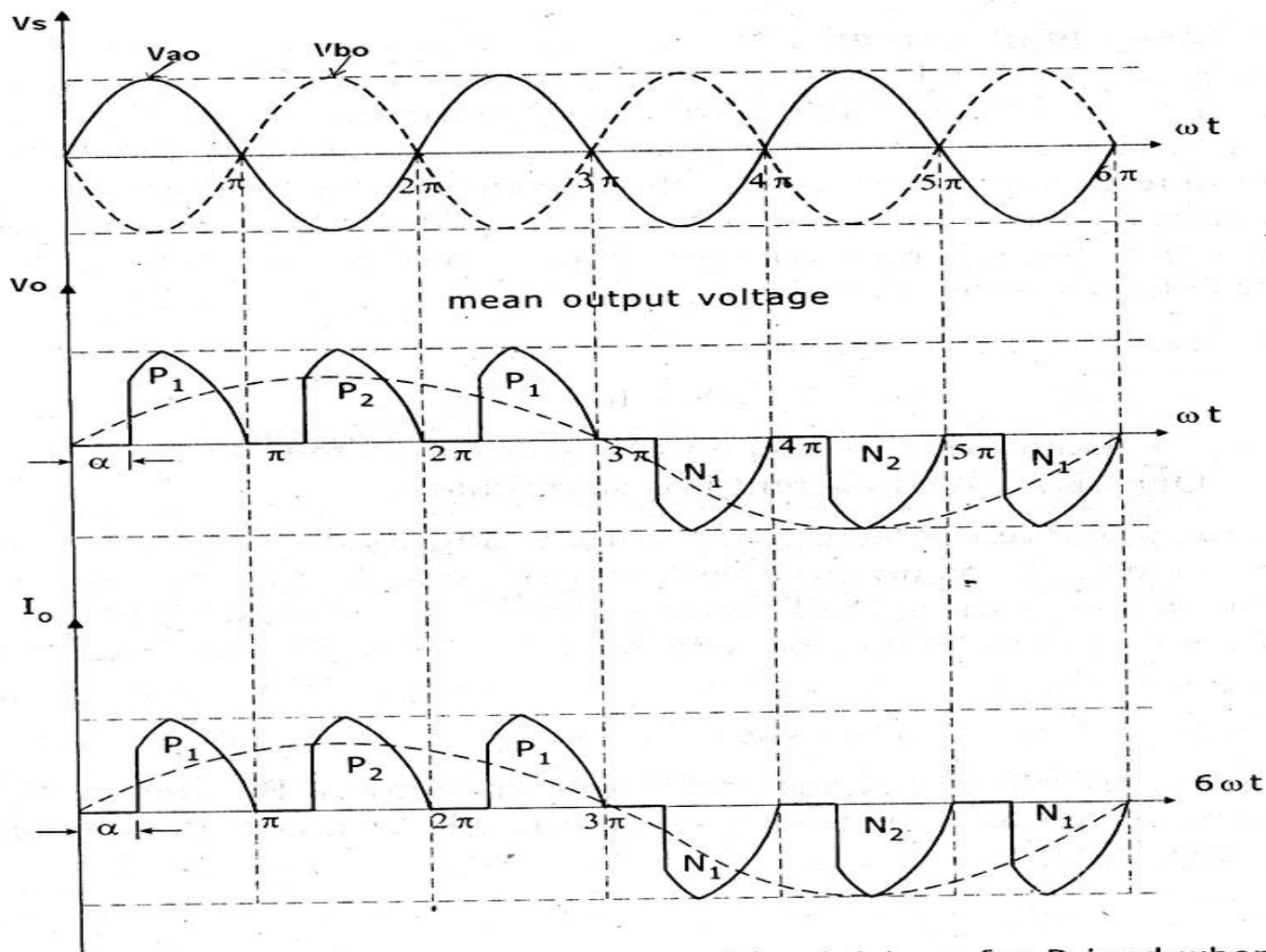
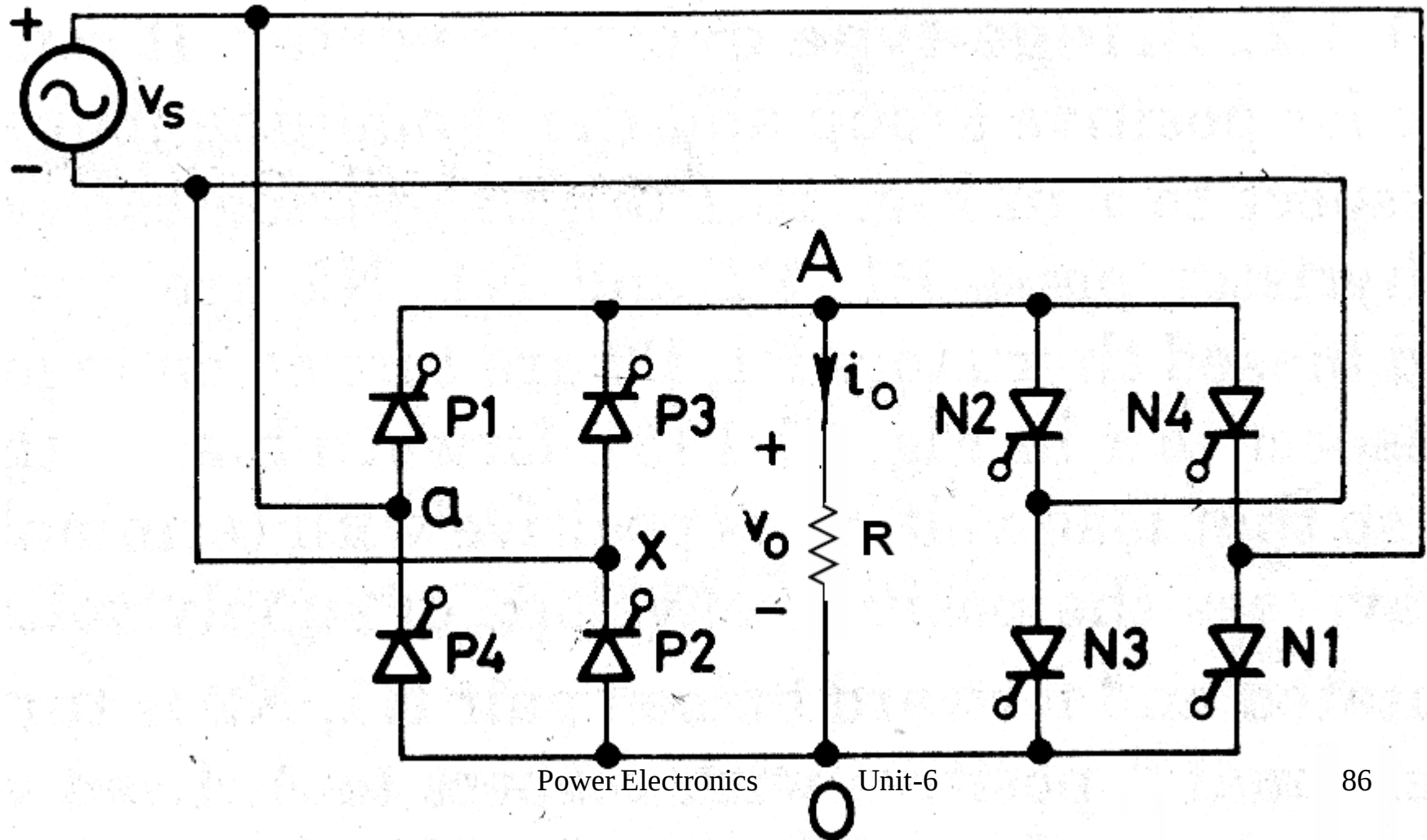


Fig. Step down cyclo converter-mid point type for R load where α is firing angle

1- ϕ to 1- ϕ Bridge type Cyclo-converter with R and R-L load

1- ϕ to 1- ϕ Bridge type step-Down Cycloconverter with R load



Output voltage (V_o) and current (I_o) waveform

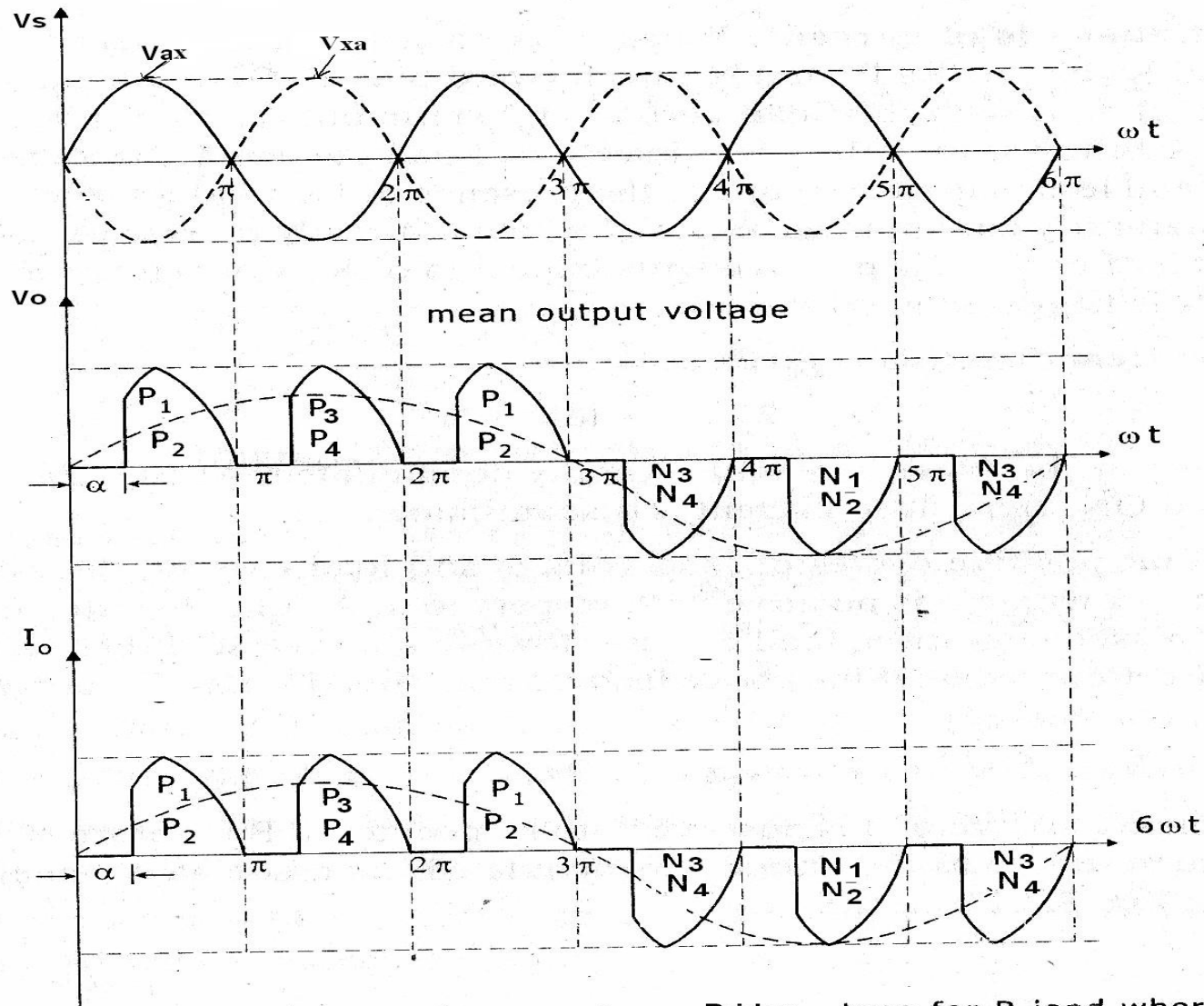
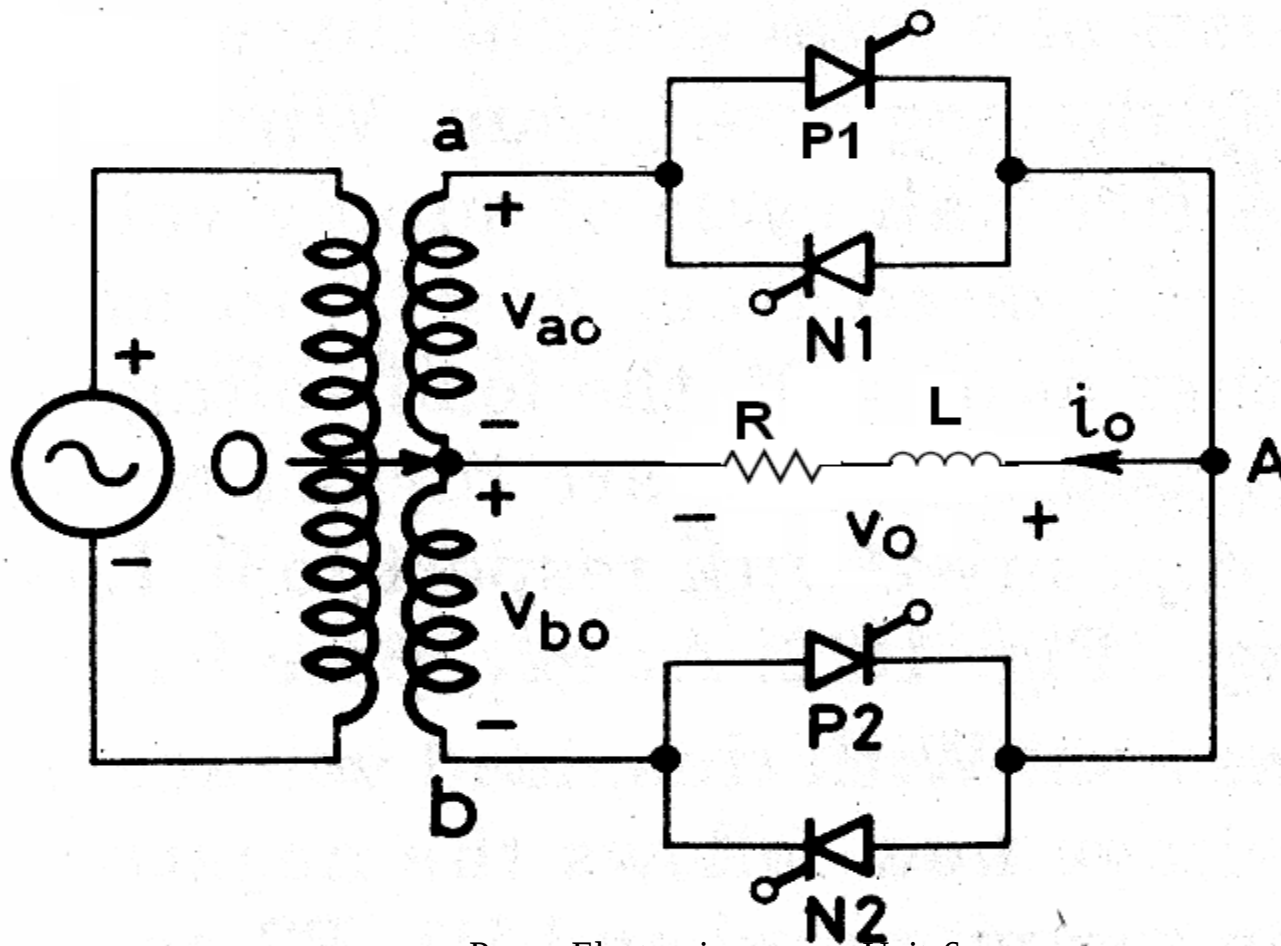
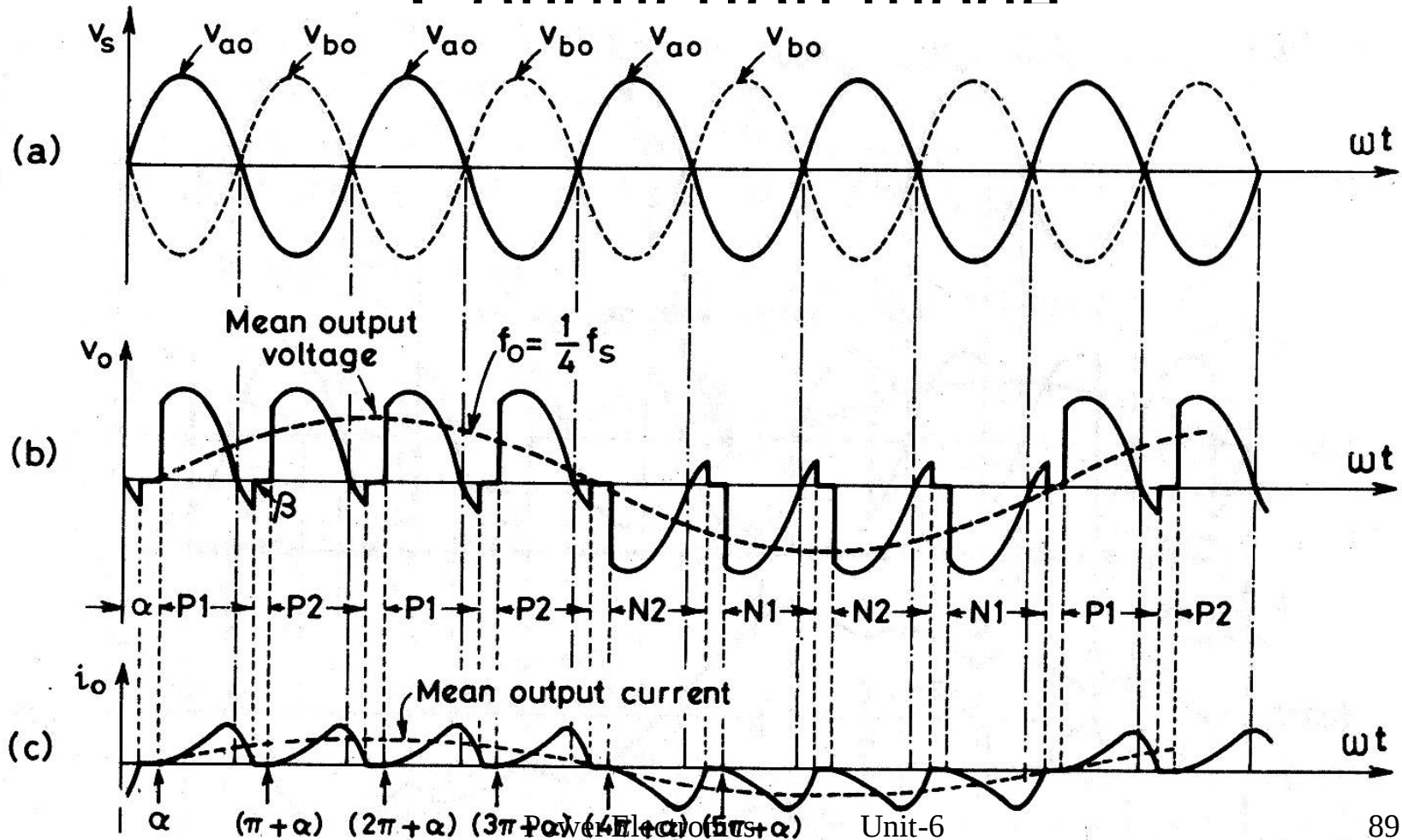


Fig. Step down cyclo converter- Bridge type for R load where α is firing angle

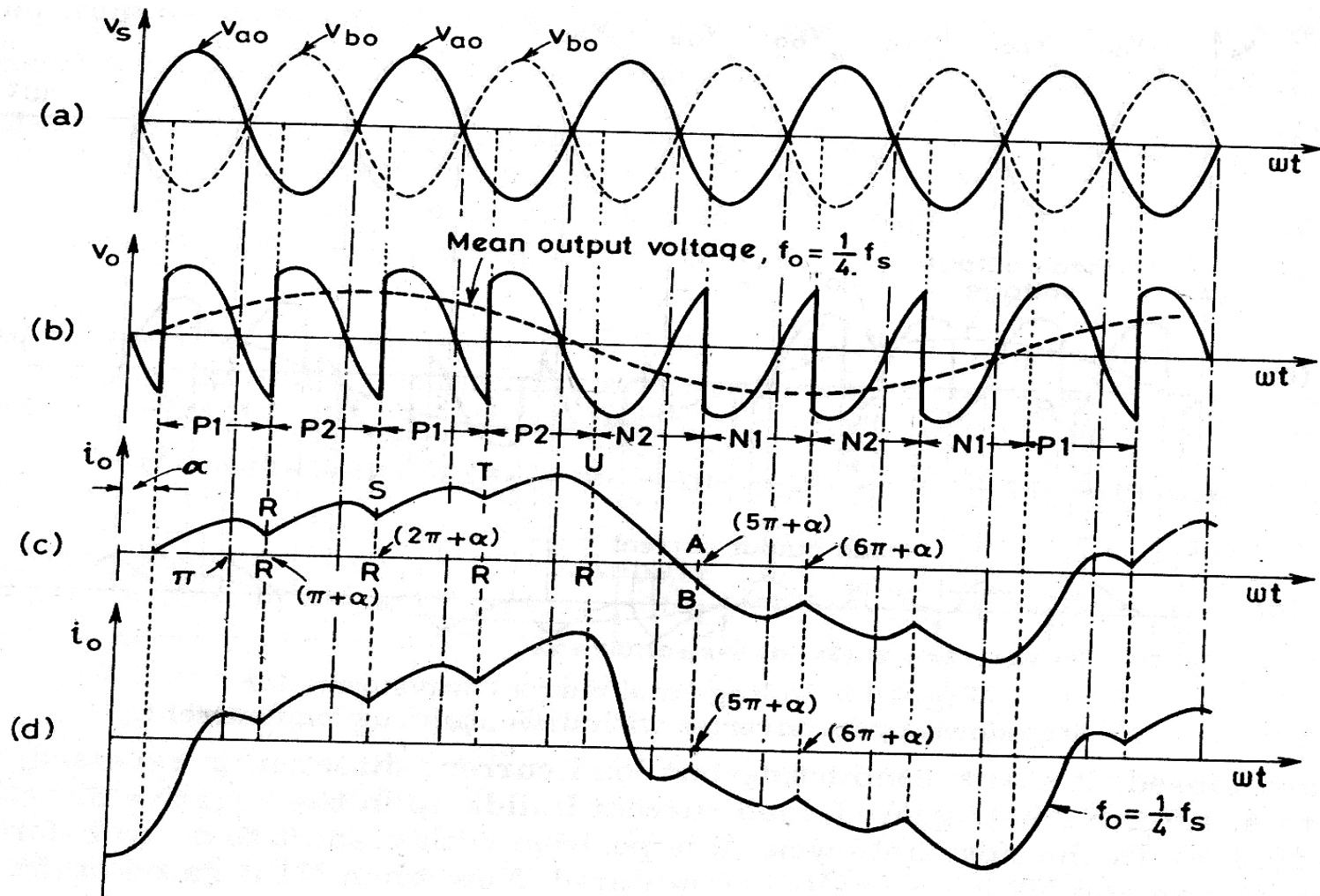
1- ϕ to 1- ϕ Midpoint type step-Down Cycloconverter with R-L load



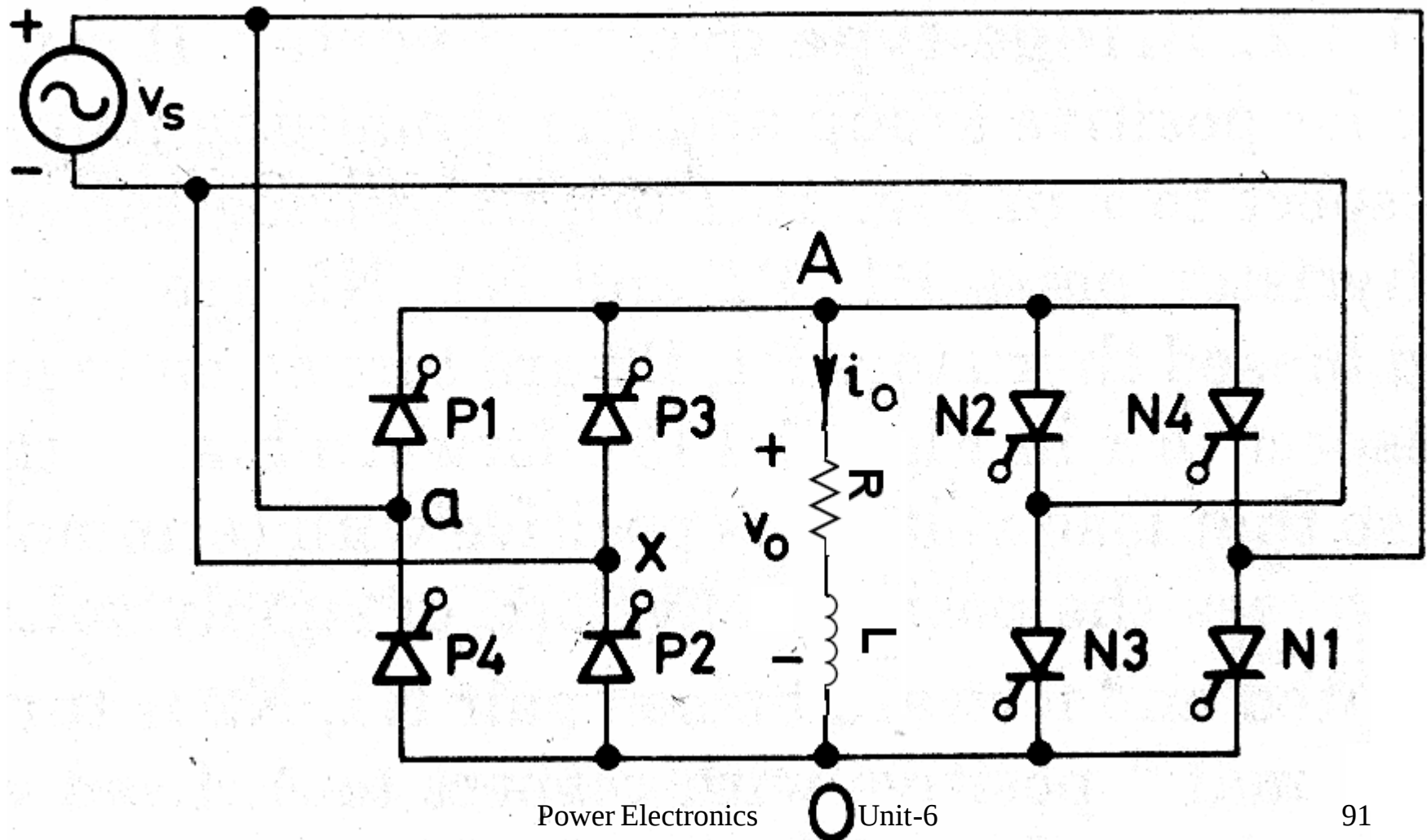
Output voltage (V_o) and current (I_o) waveform for Discontinuous Conduction mode



Output voltage (V_o) and current (I_o) waveform for Continuous Conduction mode

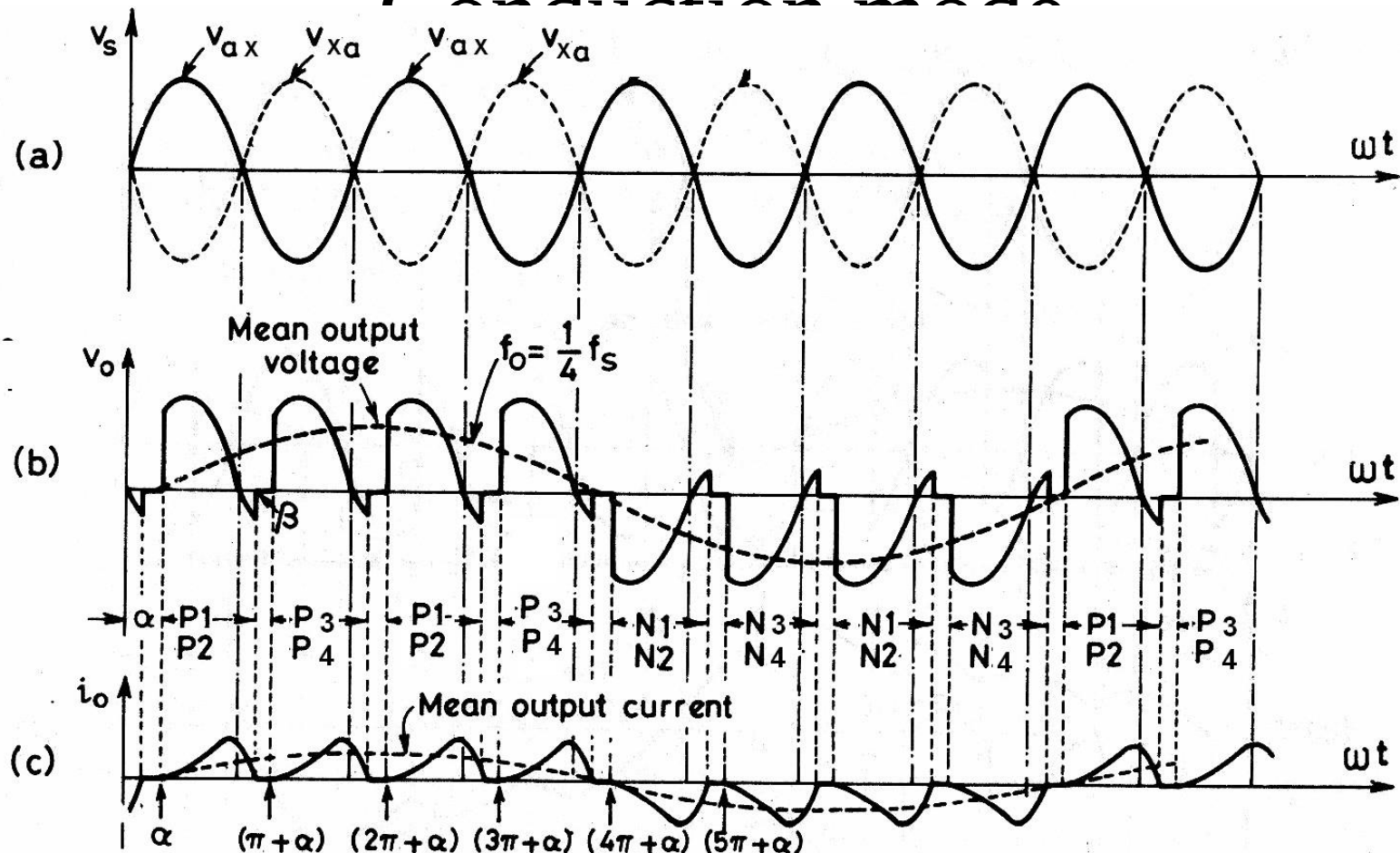


1- ϕ to 1- ϕ Bridge-type step-Down Cycloconverter with R-I load



Output voltage (V_o) and current (I_o) waveform for Discontinuous

Conduction mode



Output voltage (V_o) and current (I_o) waveform for Continuous Conduction mode

