POWER SYSTEMS-II (EE3102PC)

UNIT -1

PERFORMANCE OF TRANSMISSION LINES

Types of transmission lines

- A transmission line always has, series resistance, series inductive reactance and shunt capacitive reactance.
- The resistance is dependent upon the material from which the conductor is made.
- The inductance is formed as the conductor is surrounded by the magnetic lines of force.
- The capacitance of the line is formed as the conductor is carrying current acts as a capacitor with the earth which is always at lower potential then the conductor and the air between them forms a dielectric medium.
- Thus, the performance of transmission lines is dependent upon these three line constants. For instance, the voltage drop in the line depends upon the values of the above three line constants.
- Similarly, the resistance of the transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency.

To determine the transmission line performance, it is classified as:
a. Short transmission lines: Up to 50 km – 80 km (<20 kV)
b. Medium transmission lines: Up to 80 km – 200 km (20 kV to 100 kV)
c. Long transmission lines: More than 160 km or 200 km (>100 kV)

Performance of transmission lines:

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency.

Voltage regulation

When a transmission line is carrying current, there is a voltage drop in the line due to resistance and reactance of the line. The result is that receiving end voltage VR is generally less than the sending voltage VS.
The Voltage drop(V_S-V_R) in the line expressed as a percentage of receiving end voltage VR is called voltage regulation.

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

It is desirable that the voltage regulation of transmission line should be low i.e. the increase in load current should make very little difference in the receiving end voltage.

Transmission efficiency

The power obtained at receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.
The ratio of receiving end power to the sending end power of a transmission line is known as the transmission efficiency of line.

$$\%\text{Transmission Efficency, } \eta = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + \text{ losses}} \times 100$$

❖ Performance of short transmission lines

- The capacitance of short lines is negligible and usually not considered. Therefore, only resistance and inductance of the line are considered.
- Here, the line resistance and inductance are shown as lumped or concentrated instead of being distributed.

Let,

$R =$ Resistance per phase

$L =$ Inductance per phase

$C =$ Capacitance per phase

$X_L =$ Inductive reactance per phase

$X_C =$ Capacitive reactance per phase

$V_R =$ Receiving end voltage per phase

$V_S =$ Sending end voltage per phase

$I_R =$ Receiving end current per phase

$I_S =$ Sending end current per phase

$I_C =$ Capacitive current per phase

$\cos\phi_R =$ Receiving end power factor

$\cos\phi_S =$ Sending end power factor

δ=Angle between sending end and receiving end voltage

Solution under Vector Notation:

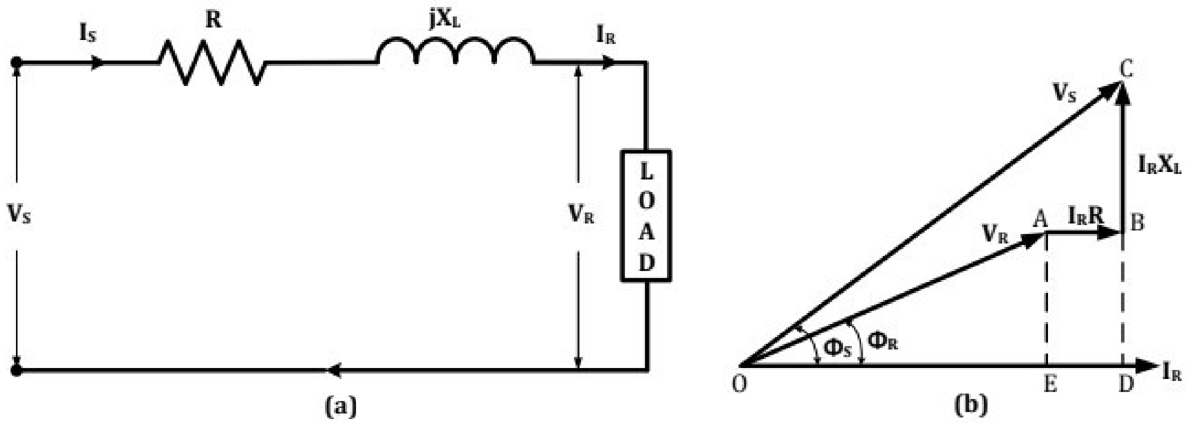


Figure 1. 1 Short Transmission Line (a) Circuit and (b) Vector Diagram (Current as Reference)

$$(OC)^2 = (OD)^2 + (DC)^2$$
$$= (OE + ED)^2 + (DB + BC)^2$$
$$V_S^2 = (V_R \cos\phi_R + I_R R)^2 + (V_R \sin\phi_R + I_R X_L)^2$$
$$V_S = \sqrt{(V_R \cos\phi_R + I_R R)^2 + (V_R \sin\phi_R + I_R X_L)^2}$$

Solution under Complex Notation:

- It is often convenient to make the line calculation in complex notation.

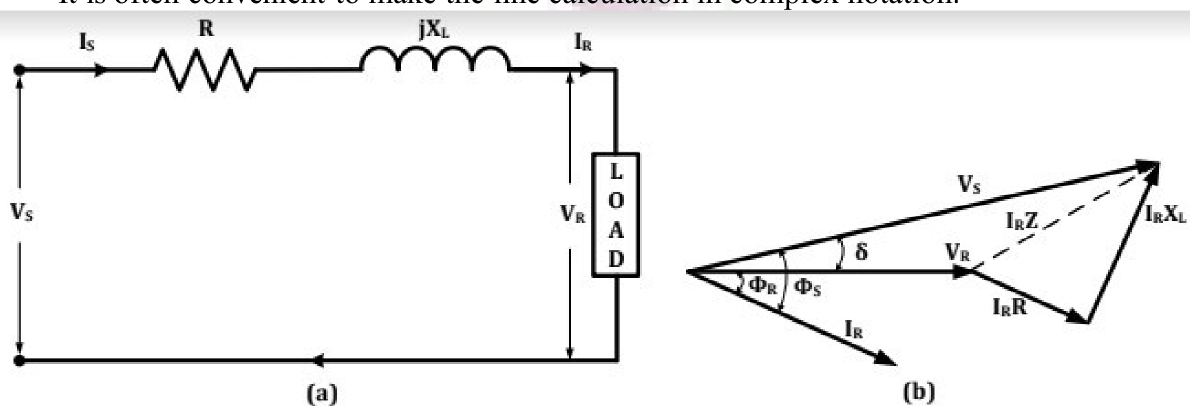


Figure 1. 2 Short Transmission Line (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$Z = R + jX_L$$
$$V_S = V_R + I_R Z$$
$$= (V_R + j0) + I_R(\cos\phi_R - j\sin\phi_R)(R + jX_L)$$
$$V_S = V_S\angle\phi_S$$

- Hence,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\ \text{Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3V_R I_R \cos\phi_R}{3V_R I_R \cos\phi_R + \ losses} \times 100$$

$$= \frac{3V_R I_R \cos\phi_R}{3V_R I_R \cos\phi_R + \ 3I_R^2 R} \times 100$$

Characteristics of medium transmission line

In short transmission line calculations, the effect of the line capacitance is neglected because each line has smaller lengths and transmit power at relatively low voltages (<20kV).

As the length (usually >80 km) and voltage (usually >20 kV) of the line increases, the capacitance gradually becomes of greater importance and cannot be neglected.

The capacitance of the line is uniformly distributed over its entire length. However, to make the calculations simple, the capacitance of the system is assumed to be divided up in lumped or concentrated form of capacitors across the line at one or more points.

- The most common methods of representations of medium transmission lines are
  - (i) End condenser method
  - (ii) Nominal T method
  - (iii) Nominal π method

❖ Performance of medium transmission line using end condenser method

In this method, the capacitance of the line is lumped or concentrated at the receiving end. This method of localizing the line capacitance at the load end overestimates the effect of capacitance.

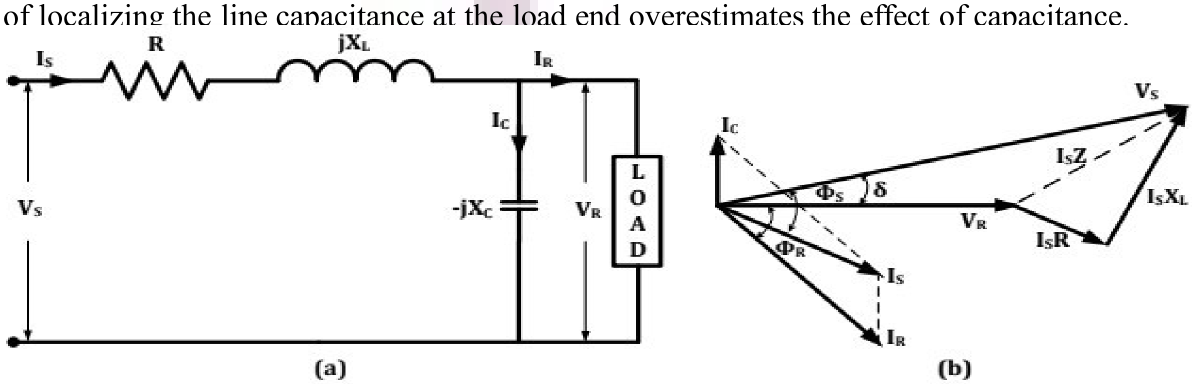


Figure 1. 3 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

$R =$ Resistance per phase
$L =$ Inductance per phase
$C =$ Capacitance per phase
$X_L =$ Inductive reactance per phase
$X_C =$ Capacitive reactance per phase
$V_R =$ Receiving end voltage per phase
$V_S =$ Sending end voltage per phase
$I_R =$ Receiving end current per phase
$I_S =$ Sending end current per phase
$I_C =$ Capacitive current per phase
$\cos\phi_R =$ Receiving end power factor
$\cos\phi_S =$ Sending end power factor
$\delta =$ Angle between sending end and receiving end voltage

Hence,

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$I_C = \frac{V_R}{X_C}\angle 90 = V_R\omega C\angle 90 = jV_R\omega C = jV_R 2\pi fC$$
$$\begin{aligned} I_S &= I_R + I_C \\ &= I_R(\cos\phi_R - j\sin\phi_R) + j2\pi fCV_R \\ &= I_R\cos\phi_R - jI_R\sin\phi_R + j2\pi fCV_R \\ &= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_R) \end{aligned}$$
$$\begin{aligned} V_S &= V_R + I_S Z \\ &= V_R + I_S(R + jX_L) \\ &= V_R + j0 + \left(I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_R)\right)(R + jX_L) \end{aligned}$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$
$$\begin{aligned} \%\text{ Transmission Efficiency} &= \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ losses}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ 3I_S^2 R}\times 100 \end{aligned}$$

- **Performance of medium transmission line using Nominal T method**

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line.

Half of the line resistance and reactance are lumped on the both side and full charging current flows over half the line.

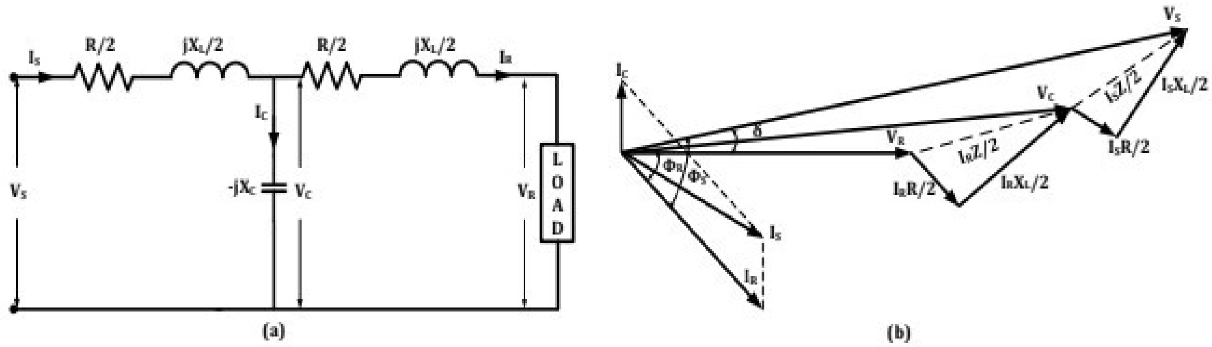


Figure 1. 4 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

$R =$ Resistance per phase

$L =$ Inductance per phase

$C =$ Capacitance per phase

$X_L =$ Inductive reactance per phase

$X_C =$ Capacitive reactance per phase

$V_R =$ Receiving end voltage per phase

$V_S =$ Sending end voltage per phase

$I_R =$ Receiving end current per phase

$I_S =$ Sending end current per phase

$I_C =$ Capacitive current per phase

$\cos\phi_R =$ Receiving end power factor

$\cos\phi_S =$ Sending end power factor

$\delta =$ Angle between sending end and receiving end voltage

Hence,

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$V_C = V_R + I_R\frac{Z}{2} = V_R + j0 + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$
$$I_C = \frac{V_C}{X_C}\angle 90 = V_C\omega C\angle 90 = jV_C\omega C = jV_C 2\pi fC$$
$$\begin{aligned} I_S &= I_R + I_C \\ &= I_R(\cos\phi_R - j\sin\phi_R) + j2\pi fCV_C \\ &= I_R\cos\phi_R - jI_R\sin\phi_R + j2\pi fCV_C \\ &= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_C) \end{aligned}$$
$$\begin{aligned} V_S &= V_C + I_S\frac{Z}{2} \\ &= \left(V_R + j0 + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)\right) + \left(I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_C)\right)\left(\frac{R}{2} + j\frac{X_L}{2}\right) \end{aligned}$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$
$$\begin{aligned} \%\text{ Transmission Efficiency} &= \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ losses}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ 3I_S^2\frac{R}{2} + 3I_R^2\frac{R}{2}}\times 100 \end{aligned}$$

❖ Performance of medium transmission line using Nominal π method

In this method, the capacitance of each conductor i.e. line to neutral is divided into two halves; one half being lumped at the sending end and the other half at the receiving end.

It is obvious that capacitance at the sending end has no effect on the line drop. However, it's charging current must be added to the line current to obtain the total sending end current.

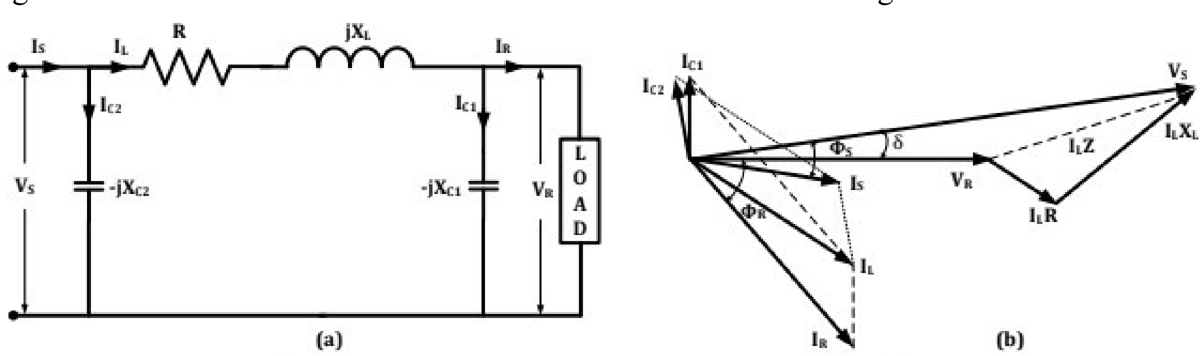


Figure 1. 5 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

$R =$ Resistance per phase

$L =$ Inductance per phase

$C =$ Capacitance per phase

$X_L =$ Inductive reactance per phase

$X_C =$ Capacitive reactance per phase

$V_R =$ Receiving end voltage per phase

$V_S =$ Sending end voltage per phase

$I_R =$ Receiving end current per phase

$I_S =$ Sending end current per phase

$I_C =$ Capacitive current per phase

$\cos\phi_R =$ Receiving end power factor

$\cos\phi_S =$ Sending end power factor

δ=Angle between sending end and receiving end voltage

$$V_R = V_R\angle 0 = V_R + j0$$

$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$

$$I_{C1} = \frac{V_R}{X_{C1}}\angle 90 = V_R\omega\frac{C}{2}\angle 90 = jV_R\omega\frac{C}{2} = jV_R 2\pi f\frac{C}{2}$$

$$I_{C2} = \frac{V_S}{X_{C2}}\angle 90 = V_S\omega\frac{C}{2}\angle 90 = jV_S\omega\frac{C}{2} = jV_S 2\pi f\frac{C}{2}$$

$$I_L = I_R + I_{C1}$$

$$I_S = I_L + I_{C2}$$

$$V_S = V_R + I_L Z = V_R + I_L\left(R + jX_L\right)$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$

$$\%\ \text{Transmission Efficiency} = \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100$$

$$= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ losses}\times 100$$

$$= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ 3I_L^2 R}\times 100$$

Characteristics of long transmission lines

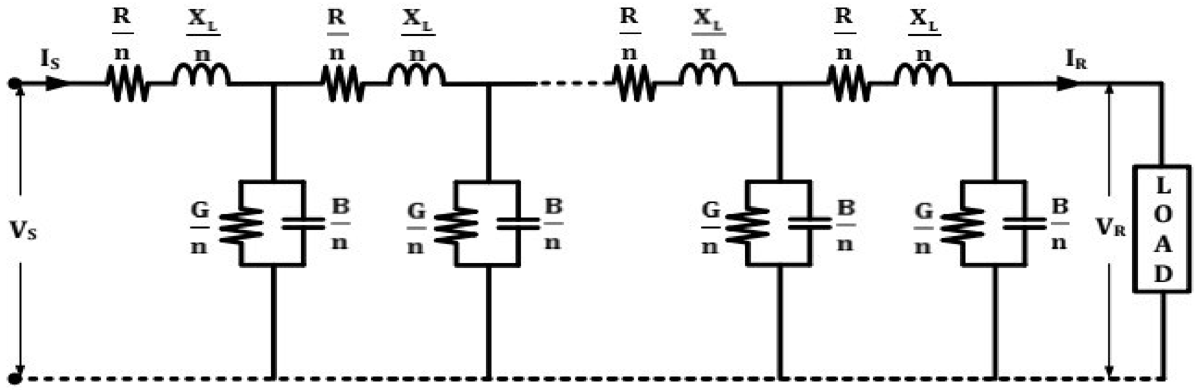


Figure 1. 6 Equivalent Circuit of Long Transmission Line

In equivalent circuit of a 3-phase long transmission line on a phase-neutral basis, the whole line is divided into n sections, each section having line constants 1/n th of those for the whole line.

The line constants are uniformly distributed over the entire length of line.

The resistance and inductive reactance are series elements.

The leakage susceptance (B) i.e. due to capacitance between line and neutral and leakage conductance (G) i.e. due to energy losses through leakages over the insulators or corona loss are shunt elements. Hence, admittance = $\sqrt{G^2 + B^2}$

The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached.

❖ Performance of long transmission line

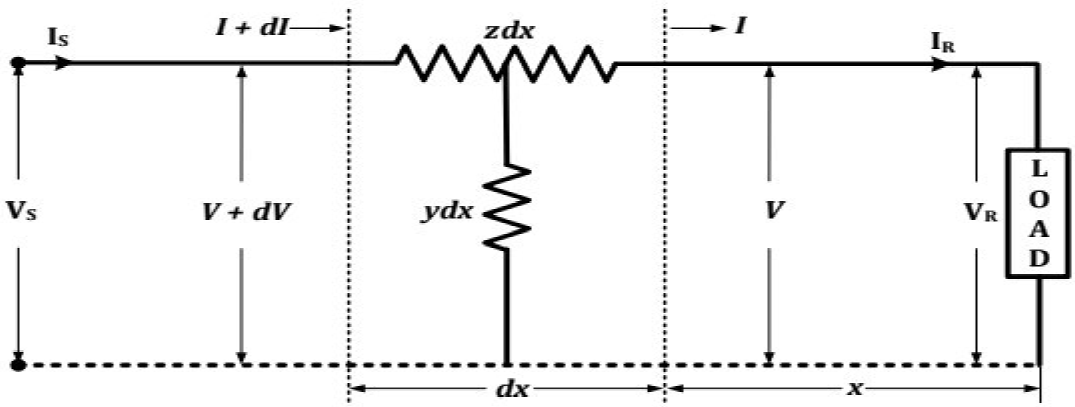


Figure 1. 7 Small Element of a Long Transmission Line

Consider a small element in the line of length dx situated at a distance x from the receiving end.

z=Series impedance of the line per unit length
y=Shunt admittance of the line per unit length
Z=Total series impedance of the line
Y=Total shunt admittance of the line
zdx=Series impedance of the small element dx
ydx=Shunt admittance of the small element dx
V=Voltage at end of the element towards receiving end
$V+dV$=Voltage at the end of element towards sending end
$I+dI$=Current entering the small element dx
I=Current leaving the small element dx

As current entering the element is I + dI and leaving the element is I. Hence voltage drop across small element dv and current through the shunt element is dI.

$$dv = Izdx \qquad \Rightarrow \frac{dv}{dx} = Iz$$

$$dI = Vydx \qquad \Rightarrow \frac{dI}{dx} = Vy$$

Now differentiating above equation

$$\frac{d^2V}{dx} = \frac{dI}{dx}z = Vyz \qquad \Rightarrow \frac{d^2V}{dx} - Vyz = 0$$

The solution of this differential equation is

$$V = K_1 Cosh\left(x\sqrt{yz}\right) + K_2 Sinh\left(x\sqrt{yz}\right)$$

Differentiating this equation, we get

$$\frac{dV}{dx} = \left(\sqrt{yz}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{yz}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore Iz = \left(\sqrt{yz}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{yz}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore I = \left(\sqrt{\frac{y}{z}}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{\frac{y}{z}}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore I = \sqrt{\frac{y}{z}}\left(K_1 Sinh\left(x\sqrt{yz}\right) + K_2 Cosh\left(x\sqrt{yz}\right)\right)$$

The values of K1 and K2 can be found by applying end conditions at x=0, V=VR and I=IR.

$$K_1 = V_R \ \ \& \ \ K_2 = \sqrt{\frac{z}{y}}I_R$$

Substituting the values of K1 and K2 in equations

$$V = V_R Cosh\left(x\sqrt{yz}\right) + \sqrt{\frac{z}{y}}I_R Sinh\left(x\sqrt{yz}\right)$$

$$I = \sqrt{\frac{y}{z}}\left(V_R Sinh\left(x\sqrt{yz}\right) + \sqrt{\frac{z}{y}}I_R Cosh\left(x\sqrt{yz}\right)\right)$$

The sending end voltage VS and sending end current IS can be obtained by putting x = l in the above equations.

$$V_S = V_R Cosh\left(l\sqrt{yz}\right) + \sqrt{\frac{z}{y}} I_R Sinh\left(l\sqrt{yz}\right)$$

$$I_S = \sqrt{\frac{y}{z}}\left(V_R Sinh\left(l\sqrt{yz}\right) + \sqrt{\frac{z}{y}} I_R Cosh\left(l\sqrt{yz}\right)\right)$$

Let,

$$l\sqrt{yz} = \sqrt{l^2 yz} = \sqrt{(ly)(lz)} = \sqrt{YZ}$$

$$\sqrt{\frac{y}{z}} = \sqrt{\frac{ly}{lz}} = \sqrt{\frac{Y}{Z}}$$

So,

$$V_S = V_R Cosh\left(\sqrt{YZ}\right) + I_R \sqrt{\frac{Z}{Y}} Sinh\left(\sqrt{YZ}\right)$$

$$I_S = V_R \sqrt{\frac{Y}{Z}} Sinh\left(\sqrt{YZ}\right) + I_R Cosh\left(\sqrt{YZ}\right)$$

Where,

$$Cosh\left(\sqrt{YZ}\right) = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \cdots\cdots\right)$$

$$Sinh\left(\sqrt{YZ}\right) = \left(\sqrt{YZ} + \frac{(YZ)^{\frac{3}{2}}}{6} + \cdots\cdots\right)$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\ \text{Transmission Efficiency} = \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

Generalized circuit constants of a transmission lines

In any four terminal network, the input voltage and input current can be expressed in terms of output voltage and current.

When voltage VR and current IR are selected as independent variable and voltage VS and current IS are dependent variable, network can be characterized by following set of equation. A, B, C and D are the generalized circuit constants of the transmission line and are complex numbers.

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The constants A and D are dimensionless whereas the dimensions of B and C are ohms and siemens respectively. For a given transmission line A=D and AD-BC=1.

A. Generalized circuit constants (ABCD parameters) of short transmission lines

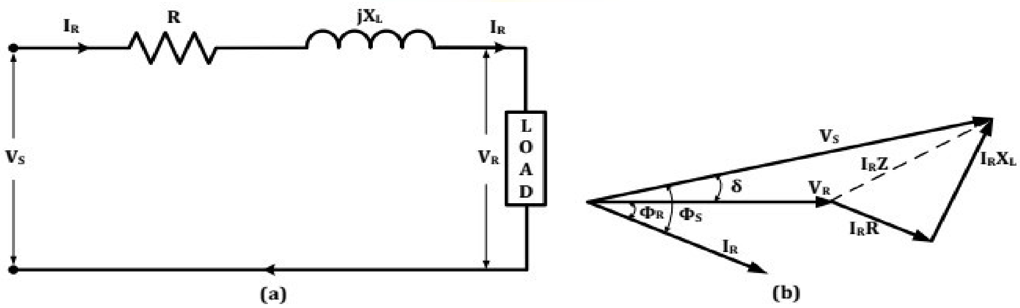


Figure 1. 8 Short Transmission Line (a) Circuit and (b) Vector Diagram (Voltage as Reference)

For short transmission line,

$$V_S = V_R + I_R Z$$
$$I_S = I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$

B. Generalized circuit constants (ABCD parameters) of medium transmission lines - End condenser method

In this method, the capacitance of the line is lumped or concentrated at the receiving end. This method of localizing the line capacitance at the load end overestimates the effect of capacitance.

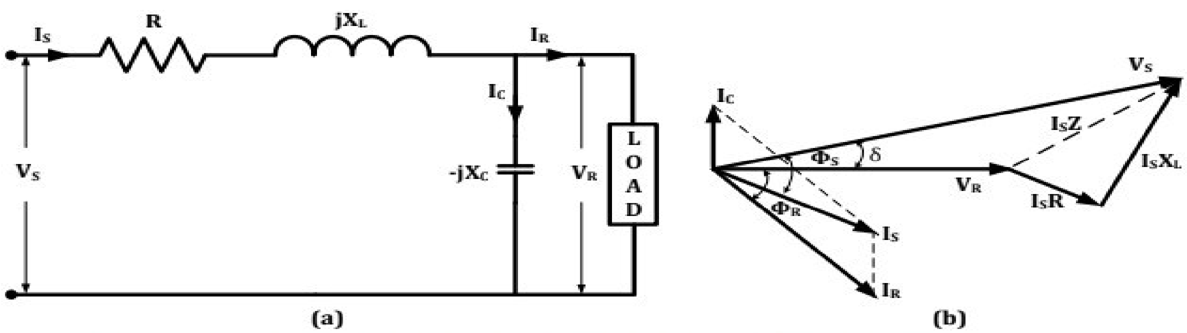


Figure 1. 9 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_S = V_R + I_S Z$$
$$= V_R + (I_C + I_R)Z$$
$$= V_R + ZI_C + ZI_R$$
$$= V_R + Z(YV_R) + ZI_R$$
$$V_S = (1+YZ)V_R + ZI_R$$

$$I_S = I_C + I_R$$
$$I_S = YV_R + I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1+YZ \qquad B = Z$$
$$C = Y \qquad D = 1$$

C. Generalized circuit constants (ABCD parameters) of medium transmission lines - Nominal T method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line.

Half of the line resistance and reactance are lumped on the both side and full charging current flows over half the line.

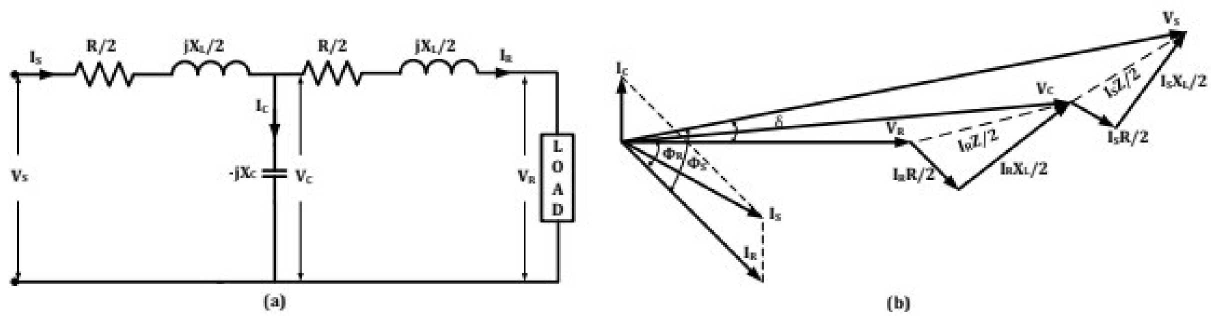


Figure 1. 10 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_S = V_C + I_S \frac{Z}{2}$$
$$= V_C + (I_R + I_C)\frac{Z}{2}$$
$$= V_C + (I_R + V_C Y)\frac{Z}{2}$$
$$= V_C\left(1 + \frac{YZ}{2}\right) + I_R \frac{Z}{2}$$
$$= \left(V_R + I_R \frac{Z}{2}\right)\left(1 + \frac{YZ}{2}\right) + I_R \frac{Z}{2}$$
$$= \left(1 + \frac{YZ}{2}\right)V_R + \left(\frac{Z}{2} + \frac{YZ^2}{4} + \frac{Z}{2}\right)I_R$$
$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + \left(Z + \frac{YZ^2}{4}\right)I_R$$

$$I_S = I_R + I_C$$
$$= I_R + V_C Y$$
$$= I_R + \left(V_R + I_R \frac{Z}{2}\right)Y$$
$$I_S = YV_R + \left(1 + \frac{YZ}{2}\right)I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 + \frac{YZ}{2} \qquad B = Z + \frac{YZ^2}{4}$$
$$C = Y \qquad D = 1 + \frac{YZ}{2}$$

D. Generalized circuit constants (ABCD parameters) of medium transmission lines - Nominal π method

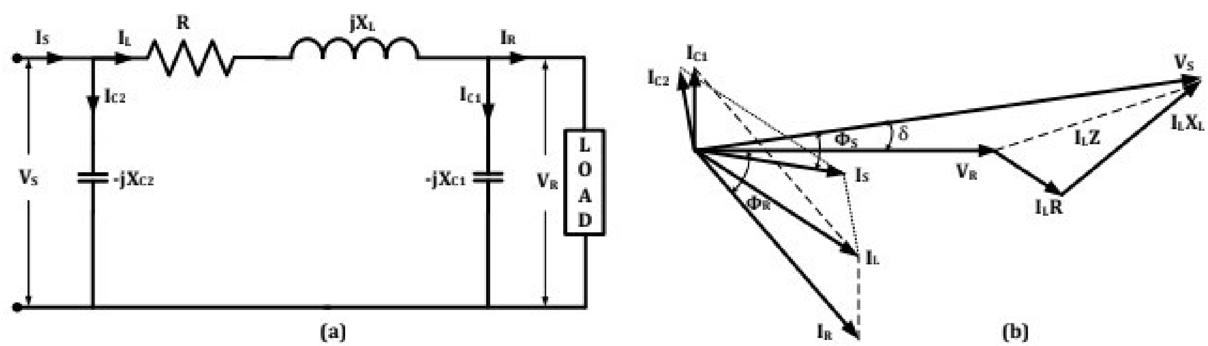


Figure 1. 11 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

In this method, the capacitance of each conductor i.e. line to neutral is divided into two halves; one half being lumped at the sending end and the other half at the receiving end.

It is obvious that capacitance at the sending end has no effect on the line drop. However, it's charging current must be added to the line current to obtain the total sending end current.

$$V_S = V_R + I_L Z$$
$$= V_R + (I_R + I_{C1})Z$$
$$= V_R + \left(I_R + V_R \frac{Y}{2}\right)Z$$
$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + ZI_R$$

$$I_S = I_L + I_{C2}$$
$$= I_R + I_{C1} + I_{C2}$$
$$= I_R + V_R \frac{Y}{2} + V_S \frac{Y}{2}$$
$$= I_R + V_R \frac{Y}{2} + (V_R + I_L Z)\frac{Y}{2}$$
$$= I_R + V_R \frac{Y}{2} + V_R \frac{Y}{2} + I_L \frac{YZ}{2}$$
$$= I_R + V_R \frac{Y}{2} + V_R \frac{Y}{2} + \left(I_R + V_R \frac{Y}{2}\right)\frac{YZ}{2}$$
$$= I_R + \frac{Y}{2}V_R + \frac{Y}{2}V_R + \frac{YZ}{2}I_R + \frac{Y^2 Z}{4}V_R$$
$$I_S = \left(Y + \frac{Y^2 Z}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 + \frac{YZ}{2} \qquad B = Z$$
$$C = Y + \frac{Y^2 Z}{4} \qquad D = 1 + \frac{YZ}{2}$$

E. Generalized circuit constants (ABCD parameters) of long transmission lines

By rigorous method, the sending end voltage and current of a long transmission line are given by

$$V_S = V_R Cosh(\sqrt{YZ}) + I_R \sqrt{\frac{Z}{Y}} Sinh(\sqrt{YZ})$$
$$I_S = V_R \sqrt{\frac{Y}{Z}} Sinh(\sqrt{YZ}) + I_R Cosh(\sqrt{YZ})$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = Cosh(\sqrt{YZ}) \qquad B = \sqrt{\frac{Z}{Y}} Sinh(\sqrt{YZ})$$

$$C = \sqrt{\frac{Y}{Z}} Sinh(\sqrt{YZ}) \qquad D = Cosh(\sqrt{YZ})$$

- **Power flow through a transmission line**

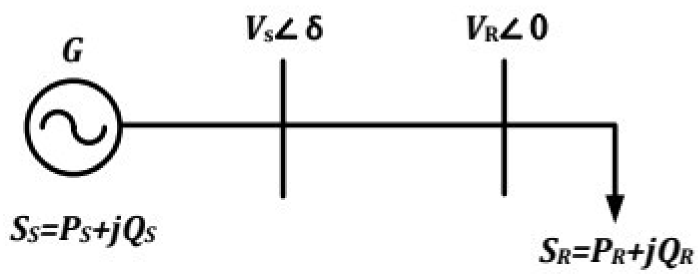


Figure 1. 12 Transmission Line Power Flow

Let,

R = Resistance per phase
L = Inductance per phase
C = Capacitance per phase
X_L = Inductive reactance per phase
X_C = Capacitive reactance per phase
V_R = Receiving end voltage per phase
V_S = Sending end voltage per phase
I_R = Receiving end current per phase
I_S = Sending end current per phase
I_C = Capacitive current per phase
$\cos\phi_R$ = Receiving end power factor
$\cos\phi_S$ = Sending end power factor
δ=Angle between sending end and receiving end voltage

Generalized line constants (ABCD parameter) are

$$A = A\angle\alpha \qquad B = B\angle\beta$$

$$C = C\angle\gamma \qquad D = D\angle\Delta$$

Relation for receiving end power can be derived with the use of relation of sending end voltage in terms of ABCD parameters of transmission line.

$$V_S = AV_R + BI_R$$
$$BI_R = V_S - AV_R$$
$$I_R = \frac{V_S - AV_R}{B}$$
$$= \frac{V_S\angle\delta - (A\angle\alpha)(V_R\angle 0)}{B\angle\beta}$$
$$= \frac{V_S\angle\delta}{B\angle\beta} - \frac{(A\angle\alpha)(V_R\angle 0)}{B\angle\beta}$$
$$= \frac{V_S}{B}\angle(\delta-\beta) - \frac{AV_R}{B}\angle(\alpha-\beta)$$
$$I_R^* = \frac{V_S}{B}\angle(\beta-\delta) - \frac{AV_R}{B}\angle(\beta-\alpha)$$

$$S_R = P_R + jQ_R$$
$$S_R = V_R I_R^*$$
$$= (V_R\angle 0)\left(\frac{V_S}{B}\angle(\beta-\delta) - \frac{AV_R}{B}\angle(\beta-\alpha)\right)$$
$$= \frac{V_R V_S}{B}\angle(\beta-\delta) - \frac{AV_R^2}{B}\angle(\beta-\alpha)$$
$$= \left(\frac{V_R V_S}{B}Cos(\beta-\delta) + j\frac{V_R V_S}{B}Sin(\beta-\delta)\right) - \left(\frac{AV_R^2}{B}Cos(\beta-\alpha) + j\frac{AV_R^2}{B}Sin(\beta-\alpha)\right)$$
$$= \left(\frac{V_R V_S}{B}Cos(\beta-\delta) - \frac{AV_R^2}{B}Cos(\beta-\alpha)\right) + j\left(\frac{V_R V_S}{B}Sin(\beta-\delta) - \frac{AV_R^2}{B}Sin(\beta-\alpha)\right)$$

$$\therefore P_R = \frac{V_R V_S}{B}Cos(\beta-\delta) - \frac{AV_R^2}{B}Cos(\beta-\alpha)$$
$$\therefore Q_R = \frac{V_R V_S}{B}Sin(\beta-\delta) - \frac{AV_R^2}{B}Sin(\beta-\alpha)$$

Relation for sending end power can be derived with the use of relation of sending end current in terms of ABCD parameters of transmission line.

$$I_S = CV_R + DI_R \;\Rightarrow I_R = \frac{I_S - CV_R}{D}$$

$$V_S = AV_R + BI_R$$

$$= AV_R + \frac{B}{D}\left(I_S - CV_R\right)$$

$$DV_S = ADV_R + B\left(I_S - CV_R\right)$$

$$= ADV_R + BI_S - BCV_R$$

$$= (AD - BC)V_R + BI_S$$

$$DV_S = V_R + BI_S$$

$$I_S = \frac{DV_S - V_R}{B}$$

$$= \frac{(D\angle\Delta)(V_S\angle\delta) - V_R\angle 0}{B\angle\beta}$$

$$= \frac{(D\angle\Delta)(V_S\angle\delta)}{B\angle\beta} - \frac{V_R\angle 0}{B\angle\beta}$$

$$= \frac{DV_S}{B}\angle(\Delta + \delta - \beta) - \frac{V_R}{B}\angle(-\beta)$$

$$I_S^* = \frac{DV_S}{B}\angle(\beta - \Delta - \delta) - \frac{V_R}{B}\angle(\beta)$$

$$S_S = P_S + jQ_S$$

$$S_S = V_S I_S^*$$

$$= (V_S\angle\delta)\left(\frac{DV_S}{B}\angle(\beta - \Delta - \delta) - \frac{V_R}{B}\angle(\beta)\right)$$

$$= \frac{DV_S^2}{B}\angle(\beta - \Delta) - \frac{V_S V_R}{B}\angle(\beta + \delta)$$

$$= \left(\frac{DV_S^2}{B}Cos(\beta - \Delta) + j\frac{DV_S^2}{B}Sin(\beta - \Delta)\right) - \left(\frac{V_S V_R}{B}Cos(\beta + \delta) + j\frac{V_S V_R}{B}Sin(\beta + \delta)\right)$$

$$= \left(\frac{DV_S^2}{B}Cos(\beta - \Delta) - \frac{V_S V_R}{B}Cos(\beta + \delta)\right) + j\left(\frac{DV_S^2}{B}Sin(\beta - \Delta) - \frac{V_S V_R}{B}Sin(\beta + \delta)\right)$$

$$\therefore P_S = \frac{DV_S^2}{B}Cos(\beta - \Delta) - \frac{V_S V_R}{B}Cos(\beta + \delta)$$

$$\therefore Q_S = \frac{DV_S^2}{B}Sin(\beta - \Delta) - \frac{V_S V_R}{B}Sin(\beta + \delta)$$

- **Power circle diagram for transmission line**

For transmission line analysis, it is required to find sending end voltage and sending end current while receiving end voltage and receiving end current is known.

Unique feature of transmission line power circuit is that it operates at fixed frequency, hence it follows all the mathematical function of circle.

A. Receiving end power circle diagram

The circle drawn with receiving end active power component as horizontal coordinate and receiving end reactive power component as vertical coordinate is called the receiving end power circle diagram.

Receiving end active power component and receiving end reactive power component are

$$P_R = \frac{V_R V_S}{B} Cos(\beta - \delta) - \frac{AV_R^2}{B} Cos(\beta - \alpha)$$

$$\therefore P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha) = \frac{V_R V_S}{B} Cos(\beta - \delta)$$

$$Q_R = \frac{V_R V_S}{B} Sin(\beta - \delta) - \frac{AV_R^2}{B} Sin(\beta - \alpha)$$

$$\therefore Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha) = \frac{V_R V_S}{B} Sin(\beta - \delta)$$

Taking square on both the side of equation and adding it.

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \left(\frac{V_R V_S}{B} Cos(\beta - \delta)\right)^2 + \left(\frac{V_R V_S}{B} Sin(\beta - \delta)\right)^2$$

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \frac{V_R^2 V_S^2}{B^2}\left(Cos^2(\beta - \delta) + Sin^2(\beta - \delta)\right)$$

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \frac{V_R^2 V_S^2}{B^2}$$

Equation of circle is

$$(x - h)^2 + (y - g)^2 = r^2$$

The coordinates of the centre of the circle and radius of the circle can be given as

$$\text{x-Coordinate of the centre of circle} = -\frac{AV_R^2}{B} Cos(\beta - \alpha)$$

$$\text{y-Coordinate of the centre of circle} = -\frac{AV_R^2}{B} Sin(\beta - \alpha)$$

$$\text{Radius of circle} = \frac{V_S V_R}{B}$$

$$\text{Radius of circle} = \frac{V_S V_R}{B}$$

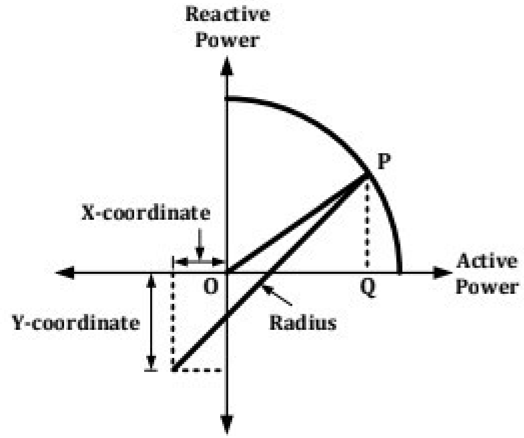


Figure 1. 13 Receiving end Circle Diagram

Construction of circle diagram

O Plot centre of circle with suitable scale.

O From centre draw an arc with calculated radius.

O Draw load line OP from origin at an angle [redacted] with horizontal and let it cut the circle at point P.

O Measure line OQ i.e. receiving end active power

O Measure line PQ i.e. receiving end reactive power

O Draw a horizontal line from centre of circle intersecting vertical axis at point L and circle at the point M.

O Measure line LM i.e. maximum power for receiving end.

When sending end voltage and receiving end voltages are the phase quantity then power indicted on x-axis and y-axis are watts and VAR per phase values.

When sending end voltage and receiving end voltages are the line quantity then power indicted on x-axis and y-axis are watts and VAR for all three phases.

B. Sending end power circle diagram

The circle drawn with sending end active power component as horizontal coordinate and sending end reactive power component as vertical coordinate is called the sending end power circle diagram.

Sending end active power component and sending end reactive power component are

$$P_S = \frac{DV_S^2}{B} Cos(\beta - \Delta) - \frac{V_S V_R}{B} Cos(\beta + \delta)$$

$$\therefore P_S - \frac{DV_S^2}{B} Cos(\beta - \Delta) = -\frac{V_S V_R}{B} Cos(\beta + \delta)$$

$$Q_S = \frac{DV_S^2}{B} Sin(\beta - \Delta) - \frac{V_S V_R}{B} Sin(\beta + \delta)$$

$$\therefore Q_S - \frac{DV_S^2}{B} Sin(\beta - \Delta) = -\frac{V_S V_R}{B} Sin(\beta + \delta)$$

Taking square on both the side of equation and adding it.

$$\left(P_S-\frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2+\left(Q_S-\frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2=\left(-\frac{V_SV_R}{B}Cos(\beta+\delta)\right)^2+\left(-\frac{V_SV_R}{B}Sin(\beta+\delta)\right)^2$$

$$\left(P_S-\frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2+\left(Q_S-\frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2=\frac{V_S^2V_R^2}{B^2}\left(Cos^2(\beta+\delta)+Sin^2(\beta+\delta)\right)$$

$$\left(P_S-\frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2+\left(Q_S-\frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2=\frac{V_S^2V_R^2}{B^2}$$

Equation of circle is

$$(x-h)^2+(y-g)^2=r^2$$

The coordinates of the centre of the circle and radius of the circle can be given as

$$\text{x-Coordinate of the centre of circle}=\frac{DV_S^2}{B}Cos(\beta-\Delta)$$

$$\text{y-Coordinate of the centre of circle}=\frac{DV_S^2}{B}Sin(\beta-\Delta)$$

$$\text{Radius of circle}=\frac{V_SV_R}{B}$$

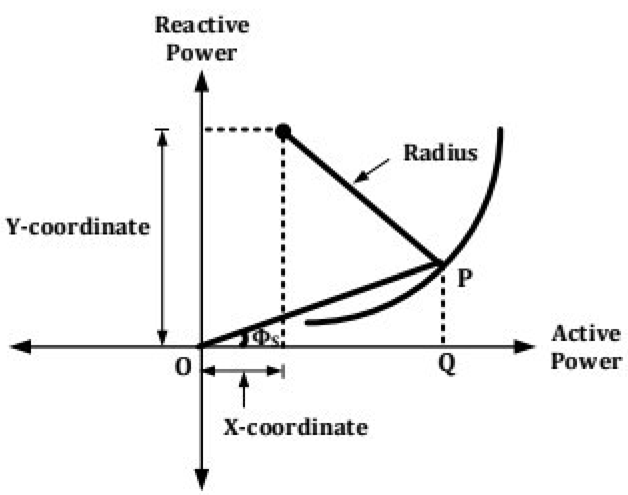


Figure 1. 14 Sending end Circle Diagram

Construction of circle diagram

- O Plot centre of circle with suitable scale.
- O From centre draw an arc with calculated radius.
- O Draw load line OP from origin at an angle [redacted] with horizontal and let it cut the circle at point P.
- O Measure line OQ i.e. sending end active power
- O Measure line PQ i.e. sending end reactive power
- O Draw a horizontal line from centre of circle intersecting vertical axis at point L and circle at the point M.
- O Measure line LM i.e. maximum power for sending end.

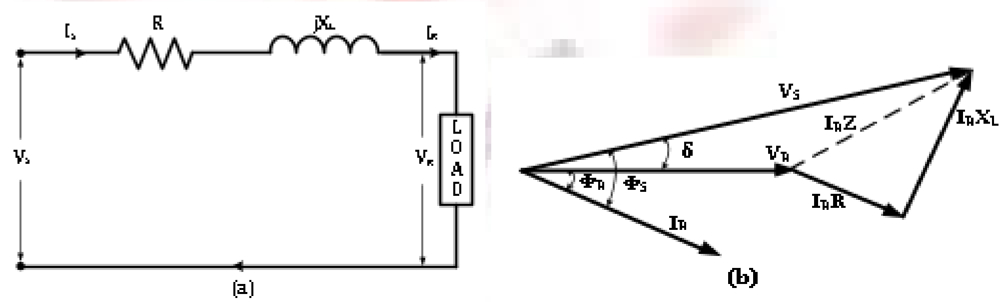
When sending end voltage and receiving end voltages are the phase quantity then power indicted on x-axis and y-axis are watts and VAR per phase values.

When sending end voltage and receiving end voltages are the line quantity then power indicted on x-axis and y-axis are watts and VAR for all three phases.

PROBLEMS:

1. A three phase overhead transmission line delivers 1100 kW at 33 kV at 0.8 pf lagging. The total resistance and inductive reactance per phase of the line are 10 Ω and 15 Ω respectively. Find (a). %voltage regulation and (b). % transmission efficiency.

Solution:



$$V_R = 33\times10^3\angle 0 = 33\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = R + jX_L = 10 + j15 = 18\angle 56.3$$

$$P_R = \sqrt{3}V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{1100\times10^3}{\sqrt{3}\times33\times10^3\times0.8} = 24\ A$$

$$\begin{aligned} I_R &= I_R\angle-\phi_R \\ &= 24\angle-36.86 \\ &= 19.20 - j14.39 \end{aligned}$$

$$\begin{aligned} I_S &= I_R \\ &= 24\angle-36.86\ A \end{aligned}$$

$$
\begin{aligned}
V_S &= V_R + I_R Z \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + (24\angle -36.86)(10 + j15) \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + (24\angle -36.86)(18\angle 56.3) \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + 432\angle 19.44 \\
&= 19052.55 + j0 + 407.37 + j143.77 \\
&= 19459.92 + j143.77 \\
&= 19460.45\angle 0.42 \text{ V/ph} \\
V_S &= 19.46\angle 0.42 \text{ kV/ph}
\end{aligned}
$$

$$
\begin{aligned}
V_S &= \sqrt{3}\times 19.46\angle 0.42 \\
&= 33.70\angle 0.42 \text{ kV}
\end{aligned}
$$

$$
\begin{aligned}
\phi_S &= 36.86 + 0.42 = 37.28 \\
\cos\phi_S &= \cos(37.28) = 0.79(lag)
\end{aligned}
$$

$$
\begin{aligned}
\%\text{Voltage Regulation} &= \frac{V_S - V_R}{V_R}\times 100 \\
&= \frac{\left(\frac{33.70\times10^3}{\sqrt{3}}\right) - \left(\frac{33\times10^3}{\sqrt{3}}\right)}{\left(\frac{33\times10^3}{\sqrt{3}}\right)}\times 100 \\
&= 2.12
\end{aligned}
$$

$$
\begin{aligned}
\%\text{ Transmission Efficiency} &= \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S}\times 100 \\
&= \frac{3\times\left(\frac{33\times10^3}{\sqrt{3}}\right)\times 24\times 0.8}{3\times\left(\frac{33.70\times10^3}{\sqrt{3}}\right)\times 24\times 0.79}\times 100 \\
&= 99.16
\end{aligned}
$$

2. A medium three phase transmission line 100 km long has following constants:

   Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the total capacitance of the line is localized at the receiving end alone. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). %voltage regulation and (b). %transmission efficiency.

Solution:

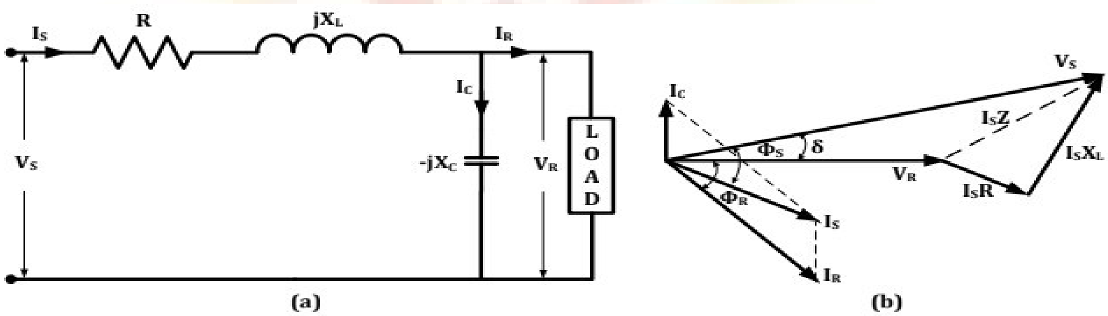


Figure 1. 3 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = 132\times10^3\angle 0 = 132\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$R = 0.15\times100 = 15\ \Omega/ph$$
$$X_L = 0.377\times100 = 37.7\ \Omega/ph$$
$$X_C = 31.87\times100 = 3187\ \Omega/ph$$

$$P_R = \sqrt{3}V_R I_R\cos\phi_R$$
$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R}$$
$$= \frac{72\times10^6}{\sqrt{3}\times132\times10^3\times0.8}$$
$$= 393.64\ A$$

$$I_R = I_R\angle-\phi_R$$
$$= 393.64\angle-36.86$$
$$= 314.95 - j236.12$$

$$
\begin{aligned}
I_C &= \frac{V_R}{-jX_C} \\
&= \frac{\dfrac{132\times10^3}{\sqrt{3}}\angle 0}{-j3187} \\
&= \frac{\dfrac{132\times10^3}{\sqrt{3}}\angle 0}{3187\angle -90} \\
&= 23.91\angle 90 \text{ A}
\end{aligned}
$$

$$
\begin{aligned}
I_S &= I_R + I_C \\
&= 393.64\angle -36.86 + 23.91\angle 90 \\
&= 314.95 - j236.12 + 0 + 23.91j \\
&= 314.95 - 212.21j \\
&= 379.77\angle -33.97 \text{ A}
\end{aligned}
$$

$$
\begin{aligned}
V_S &= V_R + I_S Z \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (379.77\angle -33.97)(15 + j37.7) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (379.77\angle -33.97)(40.57\angle 68.3) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + 15407.26\angle 34.33 \\
&= 76210 + 0j + 12723.36 + 8689.05j \\
&= 88933.36 + 8689.05j \\
&= 89356.82\angle 5.58 \quad \text{V/ph} \\
&= 89.35\angle 5.58 \quad \text{kV/ph}
\end{aligned}
$$

$$
\begin{aligned}
V_S &= \sqrt{3}\times 89.35\angle 5.58 \text{ kV} \\
&= 154.75\angle 5.58 \text{ kV}
\end{aligned}
$$

$$
\begin{aligned}
\phi_S &= 33.97 + 5.58 = 39.58 \\
\cos\phi_S &= \cos(39.58) = 0.77(lag)
\end{aligned}
$$

$$
\begin{aligned}
\%\text{Voltage Regulation} &= \frac{V_S - V_R}{V_R}\times 100 \\
&= \frac{\left(\dfrac{154.75\times10^3}{\sqrt{3}}\right) - \left(\dfrac{132\times10^3}{\sqrt{3}}\right)}{\left(\dfrac{132\times10^3}{\sqrt{3}}\right)}\times 100 \\
&= 16.66
\end{aligned}
$$

]

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3\times\left(\frac{132\times10^3}{\sqrt{3}}\right)\times 393.64\times0.8}{3\times\left(\frac{154.75\times10^3}{\sqrt{3}}\right)\times379.77\times0.77}\times100$$

$$= 91.85$$

3. A medium three phase transmission line 100 km long has following constants:

Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the total capacitance of the line is localized at middle point of the line. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). %voltage regulation and (b). %transmission efficiency.

Solution:

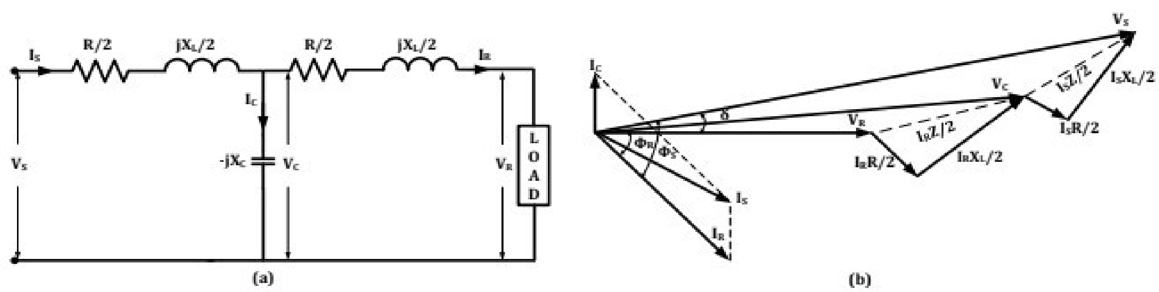


Figure 1. 4 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = 132\times10^3\angle0 = 132\times10^3 + j0$$

$$\cos\phi_R = 0.8\,(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$R = 0.15\times100 = 15\ \Omega/ph$$
$$X_L = 0.377\times100 = 37.7\ \Omega/ph$$
$$X_C = 31.87\times100 = 3187\ \Omega/ph$$

$$P_R = \sqrt{3}V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{72\times10^6}{\sqrt{3}\times132\times10^3\times0.8} = 393.64\ A$$

$$I_R = I_R\angle-\phi_R = 393.64\angle-36.86 = 314.95 - j236.12$$

$$V_C = V_R\angle 0 + (I_R\angle -\phi_R)\left(\frac{R+jX_L}{2}\right)$$
$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + (393.64\angle -36.86)\left(\frac{15+j37.7}{2}\right)$$
$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + (393.64\angle -36.86)(20.28\angle 68.30)$$
$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + 7983.01\angle 31.44$$
$$= 76210.23 + 0j + 6810.99 + 4163.98j$$
$$= 83021.22 + 4163.98j$$
$$= 83125.57\angle 2.87 \text{ V}$$

$$I_C = \frac{V_C}{-jX_C}$$
$$= \frac{83125.57\angle 2.87}{-j3187}$$
$$= \frac{83125.57\angle 2.87}{3187\angle -90}$$
$$= 26.08\angle 92.87 \text{ A}$$

$$I_S = I_R + I_C$$
$$= 393.64\angle -36.86 + 26.08\angle 92.87$$
$$= 314.95 - 236.12j - 1.30 + 26.04j$$
$$= 313.65 - 210.08j$$
$$= 377.5\angle -33.81 \text{ A}$$

$$V_S = V_C\angle\theta_C + (I_S\angle \pm \phi_S)\left(\frac{R+jX_L}{2}\right)$$
$$= 83125.57\angle 2.87 + (377.5\angle -33.81)\left(\frac{15+j37.7}{2}\right)$$
$$= 83125.57\angle 2.87 + (377.5\angle -33.81)(20.28\angle 68.30)$$
$$= 83125.57\angle 2.87 + 7655.7\angle 34.49$$
$$= 83021.22 + 4163.98j + 6310.01 + 4335.13j$$
$$= 89331.23 + 8499.11j$$
$$= 89734.62\angle 5.43 \text{ V/ph}$$
$$= 89.73\angle 5.43 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times 89.73\angle 5.43 \text{ kV}$$
$$= 155.41\angle 5.43 \text{ kV}$$

$$\phi_S = 33.81 + 5.43 = 39.24$$
$$\cos\phi_S = \cos(39.24) = 0.77(lag)$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$
$$= \frac{\left(\frac{155.41\times10^3}{\sqrt{3}}\right) - \left(\frac{132\times10^3}{\sqrt{3}}\right)}{\left(\frac{132\times10^3}{\sqrt{3}}\right)}\times 100$$
$$= 17.73$$

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3\times\left(\frac{132\times10^3}{\sqrt{3}}\right)\times393.64\times0.8}{3\times\left(\frac{155.41\times10^3}{\sqrt{3}}\right)\times377.5\times0.77}\times100$$

$$= 92.01$$

---

4. A medium three phase transmission line 100 km long has following constants: Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the half of capacitance of the line is lumped at both the end. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). %voltage regulation and (b). %transmission efficiency.

Solution:

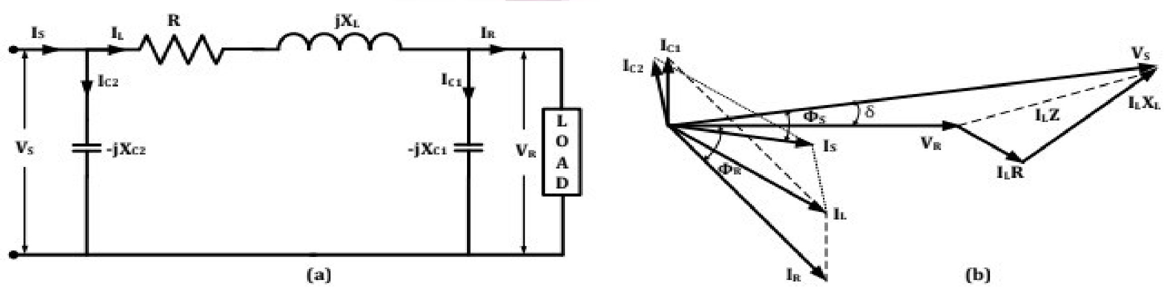


Figure 1. 11 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$V_R = 132\times10^3\angle0 = 132\times10^3 + j0$

$\cos\phi_R = 0.8(lag) \quad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$

$R = 0.15\times100 = 15\ \Omega/ph$

$X_L = 0.377\times100 = 37.7\ \Omega/ph$

$X_C = 31.87\times100 = 3187\ \Omega/ph$

$X_{C1} = 2X_C = 6374\ \Omega/ph$

$X_{C2} = 2X_C = 6374\ \Omega/ph$

$$P_R = \sqrt{3}V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{72\times10^6}{\sqrt{3}\times132\times10^3\times0.8} = 393.64\ A$$

$$I_R = I_R\angle-\phi_R = 393.64\angle-36.86 = 314.95 - j236.12$$

$$I_{C1} = \frac{V_R}{-jX_{C1}} = \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{-j6374} = \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{6374\angle -90} = 11.95\angle 90 \text{ A}$$

$$\begin{aligned}
I_L &= I_R + I_{C1} \\
&= 393.64\angle -36.86 + 11.95\angle 90 \\
&= 314.95 - 236.12j + 0 + 11.95j \\
&= 314.95 - 224.17j \\
&= 386.58\angle -35.44 \text{ A}
\end{aligned}$$

$$\begin{aligned}
V_S &= V_R + I_L Z \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (386.58\angle -35.44)(15 + j37.7) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (386.58\angle -35.44)(40.57\angle 68.30) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + 15683.55\angle 32.86 \\
&= 76210.23 + 0j + 13174.16 + 8509.70j \\
&= 89384.39 + 8509.70j \\
&= 89788.55\angle 5.43 \text{ V} \\
&= 89.78\angle 5.43 \text{ kV}
\end{aligned}$$

$$\begin{aligned}
V_S &= \sqrt{3}\times 89.78\angle 5.43 \text{ kV} \\
&= 155.5\angle 5.43 \text{ kV}
\end{aligned}$$

$$\begin{aligned}
I_{C2} &= \frac{V_S}{-jX_{C2}} \\
&= \frac{89788.55\angle 5.43}{-j6374} \\
&= \frac{89788.55\angle 5.43}{6374\angle -90} \\
&= 14.08\angle 95.43 \text{ A}
\end{aligned}$$

$$\begin{aligned}
I_S &= I_L + I_{C2} \\
&= 386.58\angle -35.44 + 14.08\angle 95.43 \\
&= 314.95 - 224.17j - 1.33 + 14.01j \\
&= 313.62 - 210.16j \\
&= 377.52\angle -33.82 \text{ A}
\end{aligned}$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$= \frac{\left(\frac{155.5\times10^3}{\sqrt{3}}\right) - \left(\frac{132\times10^3}{\sqrt{3}}\right)}{\left(\frac{132\times10^3}{\sqrt{3}}\right)} \times 100$$

$$= 17.80$$

$$\%\text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3\times\left(\frac{132\times10^3}{\sqrt{3}}\right)\times 393.64\times0.8}{3\times\left(\frac{155.5\times10^3}{\sqrt{3}}\right)\times377.52\times0.77} \times 100$$

$$= 91.96$$

Comparison of Performance of Medium Transmission Lines

Quantity		End condenser method	Nominal T method	Nominal π method
P_R	MW	72	72	72
V_R	kV	$132\angle0$	$132\angle0$	$132\angle0$
I_R	A	$393.64\angle-36.86$	$393.64\angle-36.86$	$393.64\angle-36.86$
$\text{Cos}\Phi_R$	lag	0.8	0.8	0.8
P_S	MW	78.38	78.19	78.28
V_S	kV	$154.75\angle5.58$	$155.41\angle5.43$	$155.50\angle5.43$
I_S	A	$379.77\angle-33.97$	$377.50\angle-33.81$	$377.52\angle-33.82$
$\text{Cos}\Phi_S$	lag	0.77	0.77	0.77
%VR	---	16.66	17.73	17.80
%η	---	91.85	92.08	91.96

5. A 3-Φ overhead transmission line has a total series impedance per phase at $200\angle80°$ Ω/ph and a total shunt admittance at $0.0013\angle90°$ Siemens/ph. The line delivered a load at 80 MW at 0.8 power factor lagging and 220 kV

between the lines. Determine sending end current and sending end voltage by rigorous method.

Solution:

$$V_R = 220\times10^3\angle 0 = 220\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = 200\angle 80 = 34.72 + 196.96j\ \Omega/ph$$
$$Y = 0.0013\angle 90 = 0 + 0.0013j\ /ph$$

$$P_R = \sqrt{3}V_R I_R\cos\phi_R$$
$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R}$$
$$= \frac{80\times10^6}{\sqrt{3}\times220\times10^3\times0.8}$$
$$= 262.43\ A$$

$$I_R = I_R\angle-\phi_R$$
$$= 262.43\angle-36.86$$
$$= 209.97 - 157.42j$$

$$Cosh\left(\sqrt{YZ}\right) = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \cdots\right)$$
$$= 1 + \frac{(0.0013\angle 90)(200\angle 80)}{2} + \cdots$$
$$= 1 + 0.13\angle 170$$
$$= 1 - 0.128 + 0.0225j$$
$$= 0.872 + 0.0225j$$
$$= 0.872\angle 1.47$$

$$Sinh\left(\sqrt{YZ}\right) = \left(\sqrt{YZ} + \frac{(YZ)^{\frac{3}{2}}}{6} + \cdots\right)$$
$$= \sqrt{(0.0013\angle 90)(200\angle 80)} + \cdots$$
$$= \sqrt{0.26\angle 170}$$
$$= 0.50\angle 85$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0.0013\angle 90}{200\angle 80}}$$
$$= \sqrt{6.5\times10^{-6}\angle 10}$$
$$= 2.54\times10^{-3}\angle 5$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{200\angle 80}{0.0013\angle 90}}$$
$$= \sqrt{153846.15\angle-10}$$
$$= 392.23\angle-5$$

$$V_S = V_R Cosh\left(\sqrt{YZ}\right) + I_R\sqrt{\frac{Z}{Y}} Sinh\left(\sqrt{YZ}\right)$$

$$= \left(\frac{220\times10^3}{\sqrt{3}}\angle 0\right)(0.872\angle 1.47) + (262.43\angle -36.86)(392.23\angle -5)(0.50\angle 85)$$

$$= 110758.87\angle 1.47 + 51466.45\angle 43.14$$

$$= 110722.41 + 2841.35j + 37554.30 + 35191.90j$$

$$= 148276.71 + 38033.25j$$

$$= 153076.81\angle 14.38 \text{ V/ph}$$

$$= 153.07\angle 14.38 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times 153.07\angle 14.38$$

$$= 265.12\angle 14.38 \text{ kV}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}} Sinh\left(\sqrt{YZ}\right) + I_R Cosh\left(\sqrt{YZ}\right)$$

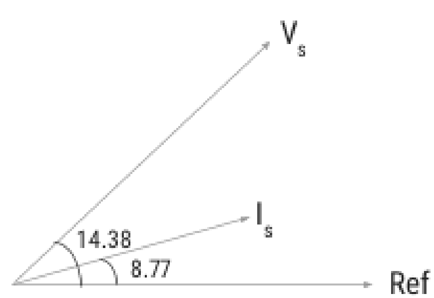
$$= \left(\frac{220\times10^3}{\sqrt{3}}\angle 0\right)(2.54\times10^{-3}\angle 5)(0.50\angle 85) + (262.43\angle -36.86)(0.872\angle 1.47)$$

$$= 161.31\angle 90 + 228.83\angle -35.39$$

$$= 0 + 161.31j + 186.54 - 132.52j$$

$$= 186.54 + 28.79j$$

$$= 188.74\angle 8.77 \text{ A}$$



$$\phi_S = 14.38 - 8.77 = 5.61$$

$$\cos\phi_S = \cos(5.61) = 0.99(lag)$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$

$$= \frac{\left(\frac{265.12\times10^3}{\sqrt{3}}\right) - \left(\frac{220\times10^3}{\sqrt{3}}\right)}{\left(\frac{220\times10^3}{\sqrt{3}}\right)}\times 100$$

$$= 20.50$$

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3 \times \left(\frac{220\times10^3}{\sqrt{3}}\right) \times 262.43\ \times 0.8}{3 \times \left(\frac{265.12\times10^3}{\sqrt{3}}\right) \times 188.74 \times 0.99} \times 100$$

$$= 93.23$$

---

6. A 3-Φ overhead transmission line has a total series impedance per phase at 200∠80° Ω/ph and a total shunt admittance at 0.0013∠90° Siemens/ph. The line delivered a load at 80 MW at 0.8 power factor lagging and 220 kV between the lines. Determine sending end current and sending end voltage by ABCD parameter method.

Solution:

$$V_R = 220\times10^3\angle 0 = 220\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = 200\angle 80 = 34.72 + 196.96j\ \Omega/ph$$

$$Y = 0.0013\angle 90 = 0 + 0.0013j\ \square\ /ph$$

$$P_R = \sqrt{3}V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R \cos\phi_R}$$

$$= \frac{80\times10^6}{\sqrt{3}\times220\times10^3\times0.8}$$

$$= 262.43\ A$$

$$I_R = I_R\angle -\phi_R$$

$$= 262.43\ \angle -36.86$$

$$= 209.97 - 157.42j$$

A). Short transmission lines:

$$A = 1$$
$$B = Z = 200\angle 80$$
$$C = 0$$
$$D = 1$$

$$\begin{aligned} V_S &= AV_R + BI_R \\ &= (1)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86) \\ &= 127017.05\angle 0 + 52486\angle 43.14 \\ &= 127017.05 + 0j + 38298.25 + 35889.05j \\ &= 165315.30 + 35889.05j \\ &= 169166.1\angle 12.24 \text{ V/ph} \\ &= 169.16\angle 12.24 \text{ kV/ph} \end{aligned}$$

$$\begin{aligned} V_S &= \sqrt{3}\times 169.16\angle 12.24 \\ &= 292.99\angle 12.24 \text{ kV} \end{aligned}$$

$$\begin{aligned} I_S &= CV_R + DI_R \\ &= (0)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (1)(262.43\angle -36.86) \\ &= 262.43\angle -36.86 \text{ A} \end{aligned}$$

B). Medium transmission lines (End condenser method):

$$\begin{aligned} A &= 1 + YZ \\ &= 1 + (0.0013\angle 90)(200\angle 80) \\ &= 1 + 0.26\angle 170 \\ &= 1 - 0.25 + 0.045j \\ &= 0.75 + 0.045j \\ &= 0.75\angle 3.43 \end{aligned}$$
$$B = Z = 200\angle 80$$
$$C = Y = 0.0013\angle 90$$
$$D = 1$$

$$\begin{aligned} V_S &= AV_R + BI_R \\ &= (0.75\angle 3.43)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86) \\ &= 952627.79\angle 3.43 + 52486\angle 43.14 \\ &= 95092.14 + 5699.48j + 38298.25 + 35889.05j \\ &= 133390.39 + 41588.53j \\ &= 139723.30\angle 17.31 \text{V/ph} \\ &= 139.72\angle 17.31 \text{kV/ph} \end{aligned}$$

$$\begin{aligned} V_S &= \sqrt{3}\times 139.72\angle 17.31 \\ &= 242.00\angle 17.31 \text{kV} \end{aligned}$$

$$\begin{aligned} I_S &= CV_R + DI_R \\ &= (0.0013\angle 90)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (1)(262.43\angle -36.86) \\ &= 165.12\angle 90 + 262.43\angle -36.86 \\ &= 0j + 165.12j + 209.97 - 157.42j \\ &= 209.97 + 7.7j \\ &= 210.11\angle 2.1 \text{ A} \end{aligned}$$

C). Medium transmission lines (Nominal T method):

$$\begin{aligned}A &= 1+\frac{YZ}{2}\\ &= 1+\frac{(0.0013\angle 90)(200\angle 80)}{2}\\ &= 1+0.13\angle 170\\ &= 1-0.128+0.022j\\ &= 0.875+0.022j\\ &= 0.875\angle 1.44\end{aligned}$$

$$\begin{aligned}B &= Z+\frac{YZ^2}{4}\\ &= 200\angle 80+\frac{(0.0013\angle 90)(200\angle 80)^2}{4}\\ &= 200\angle 80+13\angle 250\\ &= 34.72+196.96j-4.44-12.21j\\ &= 30.28+184.75j\\ &= 187.21\angle 80.69\end{aligned}$$

$$C = Y = 0.0013\angle 90$$

$$D = 1+\frac{YZ}{2} = 0.875\angle 1.44$$

$$\begin{aligned}V_S &= AV_R+BI_R\\ &= (0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right)+(187.21\angle 80.69)(262.43\angle -36.86)\\ &= 111139.92\angle 1.44+49129.52\angle 43.83\\ &= 111104.82+2792.95j+35441.92+34023.22j\\ &= 146546.74+36816.17j\\ &= 151100.55\angle 14.10\ \text{V/ph}\\ &= 151.10\angle 14.10\ \text{kV/ph}\end{aligned}$$

$$\begin{aligned}V_S &= \sqrt{3}\times 151.10\angle 14.10\\ &= 261.71\angle 14.10\ \text{kV}\end{aligned}$$

$$\begin{aligned}I_S &= CV_R+DI_R\\ &= (0.0013\angle 90)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right)+(0.875\angle 1.44)(262.43\angle -36.86)\\ &= 165.12\angle 90+229.62\angle -35.42\\ &= 0j+165.12j+187.12-133.07j\\ &= 187.12+32.05j\\ &= 189.84\angle 9.71\ \text{A}\end{aligned}$$

D). Medium transmission lines (Nominal π method):

$$A = 1 + \frac{YZ}{2}$$
$$= 1 + \frac{(0.0013\angle 90)(200\angle 80)}{2}$$
$$= 1 + 0.13\angle 170$$
$$= 1 - 0.128 + 0.022j$$
$$= 0.875 + 0.022j$$
$$= 0.875\angle 1.44$$
$$B = Z = 200\angle 80$$

$$C = Y + \frac{Y^2 Z}{4}$$
$$= 0.0013\angle 90 + \frac{(0.0013\angle 90)^2 (200\angle 80)}{4}$$
$$= 0.0013\angle 90 + 8.45\times 10^{-5}\angle 260$$
$$= 0 + 0.0013j - 1.46\times 10^{-5} - 8.32\times 10^{-5}j$$
$$= -1.46\times 10^{-5} + 1.21\times 10^{-3}j$$
$$= 1.21\times 10^{-3}\angle 90.68$$
$$D = 1 + \frac{YZ}{2} = 0.875\angle 1.44$$

$$V_s = AV_R + BI_R$$
$$= (0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$$
$$= 111139.92\angle 1.44 + 52486\angle 43.14$$
$$= 111104.82 + 2792.95j + 38298.25 + 35889.05j$$
$$= 149403.07 + 38682j$$
$$= 154329.43\angle 14.51 \text{ V/ph}$$
$$= 154.32\angle 14.51 \text{ kV/ph}$$

$$V_s = \sqrt{3}\times 154.32\angle 14.51$$
$$= 267.29\angle 14.51 \text{ kV}$$

$$I_s = CV_R + DI_R$$
$$= (1.21\times 10^{-3}\angle 90.68)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (0.875\angle 1.44)(262.43\angle -36.86)$$
$$= 153.69\angle 90.68 + 229.62\angle -35.42$$
$$= -1.82 + 153.67j + 187.12 - 133.07j$$
$$= 185.3 + 20.6j$$
$$= 186.44\angle 6.34 \text{ A}$$

E). Long transmission lines:

$$A = Cosh\left(\sqrt{YZ}\right)$$
$$= 1 + \frac{YZ}{2}$$
$$= 1 + \frac{(0.0013\angle 90)(200\angle 80)}{2}$$
$$= 1 + 0.13\angle 170$$
$$= 1 - 0.128 + 0.022j$$
$$= 0.875 + 0.022j$$
$$= 0.875\angle 1.44$$

$$B = \sqrt{\frac{Z}{Y}} Sinh\left(\sqrt{YZ}\right) = \sqrt{\frac{Z}{Y}}\sqrt{YZ} = Z$$
$$= 200\angle 80$$
$$C = \sqrt{\frac{Y}{Z}} Sinh\left(\sqrt{YZ}\right) = \sqrt{\frac{Y}{Z}}\sqrt{YZ} = Y$$
$$= 0.0013\angle 90$$
$$D = Cosh\left(\sqrt{YZ}\right) = 0.875\angle 1.44$$

$$V_S = AV_R + BI_R$$
$$= (0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$$
$$= 111139.92\angle 1.44 + 52486\angle 43.14$$
$$= 111104.82 + 2792.95j + 38298.25 + 35889.05j$$
$$= 149403.07 + 38682j$$
$$= 154329.43\angle 14.51 \text{ V/ph}$$
$$= 154.32\angle 14.51 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times 154.32\angle 14.51$$
$$= 267.29\angle 14.51 \text{ kV}$$

$$I_S = CV_R + DI_R$$
$$= (0.0013\angle 90)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (0.875\angle 1.44)(262.43\angle -36.86)$$
$$= 165.12\angle 90 + 229.62\angle -35.42$$
$$= 0 + 165.12j + 187.12 - 133.07j$$
$$= 187.12 + 20.6j$$
$$= 188.25\angle 6.28 \text{ A}$$