POWER SYSTEMS-II (EE3102PC)

UNIT -1

PERFORMANCE OF TRANSMISSION LINES

Types of transmission lines

- A transmission line always has, series resistance, series inductive reactance and shunt capacitive reactance.
- The resistance is dependent upon the material from which the conductor is made.
- The inductance is formed as the conductor is surrounded by the magnetic lines of force.
- The capacitance of the line is formed as the conductor is carrying current acts as a capacitor with the earth which is always at lower potential then the conductor and the air between them forms a dielectric medium.
- Thus, the performance of transmission lines is dependent upon these three line constants. For instance, the voltage drop in the line depends upon the values of the above three line constants.
- Similarly, the resistance of the transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency.

To determine the transmission line performance, it is classified as:
a. Short transmission lines: Up to 50 km – 80 km (<20 kV)
b. Medium transmission lines: Up to 80 km – 200 km (20 kV to 100 kV)
c. Long transmission lines: More than 160 km or 200 km (>100 kV)

Performance of transmission lines:

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency.

Voltage regulation

When a transmission line is carrying current, there is a voltage drop in the line due to resistance and reactance of the line. The result is that receiving end voltage VR is generally less than the sending voltage VS.

The Voltage drop(V_S-V_R) in the line expressed as a percentage of receiving end voltage VR is called voltage regulation.

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

It is desirable that the voltage regulation of transmission line should be low i.e. the increase in load current should make very little difference in the receiving end voltage.

Transmission efficiency

The power obtained at receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the transmission efficiency of line.

$$\%\text{Transmission Efficency, } \eta = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_R I_R \cos\phi_R + \text{ losses}} \times 100$$

❖ Performance of short transmission lines

- The capacitance of short lines is negligible and usually not considered. Therefore, only resistance and inductance of the line are considered.
- Here, the line resistance and inductance are shown as lumped or concentrated instead of being distributed.

Let,

R = Resistance per phase

L = Inductance per phase

C = Capacitance per phase

X_L = Inductive reactance per phase

X_C = Capacitive reactance per phase

V_R = Receiving end voltage per phase

V_S = Sending end voltage per phase

I_R = Receiving end current per phase

I_S = Sending end current per phase

I_C = Capacitive current per phase

$\cos\phi_R$ = Receiving end power factor

$\cos\phi_S$ = Sending end power factor

δ=Angle between sending end and receiving end voltage

Solution under Vector Notation:

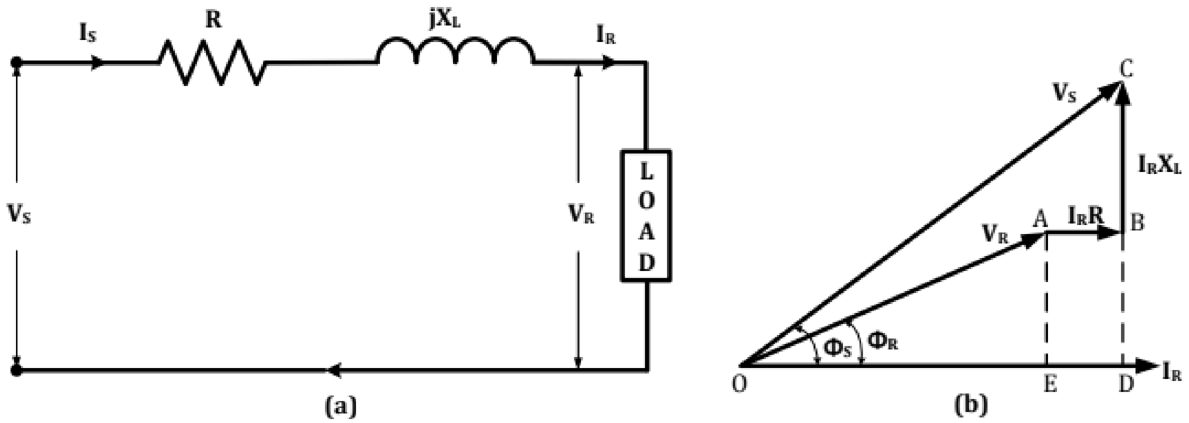


Figure 1. 1 Short Transmission Line (a) Circuit and (b) Vector Diagram (Current as Reference)

$$(OC)^2 = (OD)^2 + (DC)^2$$
$$= (OE + ED)^2 + (DB + BC)^2$$
$$V_S^2 = (V_R \cos\phi_R + I_R R)^2 + (V_R \sin\phi_R + I_R X_L)^2$$
$$V_S = \sqrt{(V_R \cos\phi_R + I_R R)^2 + (V_R \sin\phi_R + I_R X_L)^2}$$

Solution under Complex Notation:

- It is often convenient to make the line calculation in complex notation.

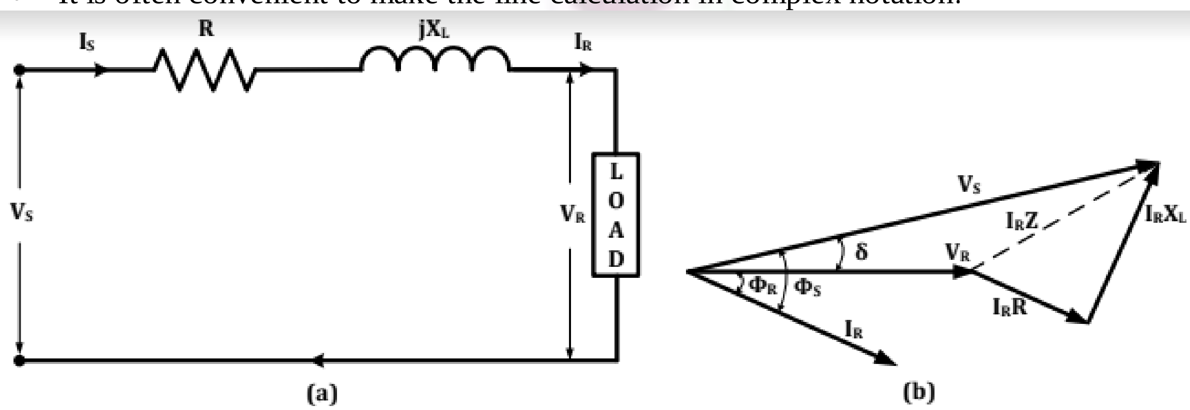


Figure 1. 2 Short Transmission Line (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$Z = R + jX_L$$
$$V_S = V_R + I_R Z$$
$$= (V_R + j0) + I_R(\cos\phi_R - j\sin\phi_R)(R + jX_L)$$
$$V_S = V_S\angle\phi_S$$

- Hence,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3V_R I_R \cos\phi_R}{3V_R I_R \cos\phi_R + \textit{losses}} \times 100$$

$$= \frac{3V_R I_R \cos\phi_R}{3V_R I_R \cos\phi_R + 3I_R^2 R} \times 100$$

Characteristics of medium transmission line

In short transmission line calculations, the effect of the line capacitance is neglected because each line has smaller lengths and transmit power at relatively low voltages (<20kV).

As the length (usually >80 km) and voltage (usually >20 kV) of the line increases, the capacitance gradually becomes of greater importance and cannot be neglected.

The capacitance of the line is uniformly distributed over its entire length. However, to make the calculations simple, the capacitance of the system is assumed to be divided up in lumped or concentrated form of capacitors across the line at one or more points.

- The most common methods of representations of medium transmission lines are
  - (i) End condenser method
  - (ii) Nominal T method
  - (iii) Nominal π method

❖ Performance of medium transmission line using end condenser method

In this method, the capacitance of the line is lumped or concentrated at the receiving end. This method of localizing the line capacitance at the load end overestimates the effect of capacitance.

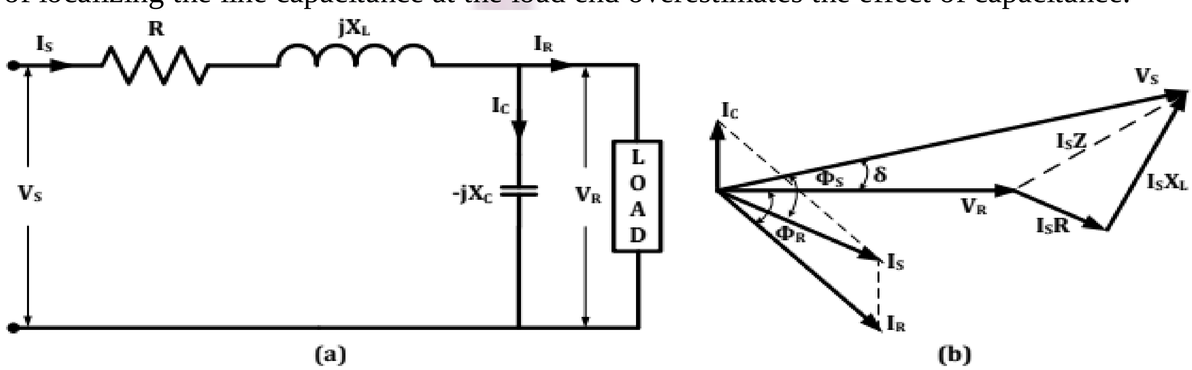


Figure 1. 3 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

$R =$ Resistance per phase
$L =$ Inductance per phase
$C =$ Capacitance per phase
$X_L =$ Inductive reactance per phase
$X_C =$ Capacitive reactance per phase
$V_R =$ Receiving end voltage per phase
$V_S =$ Sending end voltage per phase
$I_R =$ Receiving end current per phase
$I_S =$ Sending end current per phase
$I_C =$ Capacitive current per phase
$\cos\phi_R =$ Receiving end power factor
$\cos\phi_S =$ Sending end power factor
δ=Angle between sending end and receiving end voltage

Hence,

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$I_C = \frac{V_R}{X_C}\angle 90 = V_R\omega C\angle 90 = jV_R\omega C = jV_R 2\pi fC$$
$$\begin{aligned} I_S &= I_R + I_C \\ &= I_R(\cos\phi_R - j\sin\phi_R) + j2\pi fCV_R \\ &= I_R\cos\phi_R - jI_R\sin\phi_R + j2\pi fCV_R \\ &= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_R) \end{aligned}$$
$$\begin{aligned} V_S &= V_R + I_S Z \\ &= V_R + I_S(R + jX_L) \\ &= V_R + j0 + \left(I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_R)\right)(R + jX_L) \end{aligned}$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$
$$\begin{aligned} \%\ \text{Transmission Efficiency} &= \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ losses}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ 3I_S^2 R}\times 100 \end{aligned}$$

- **Performance of medium transmission line using Nominal T method**

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line.

Half of the line resistance and reactance are lumped on the both side and full charging current flows over half the line.

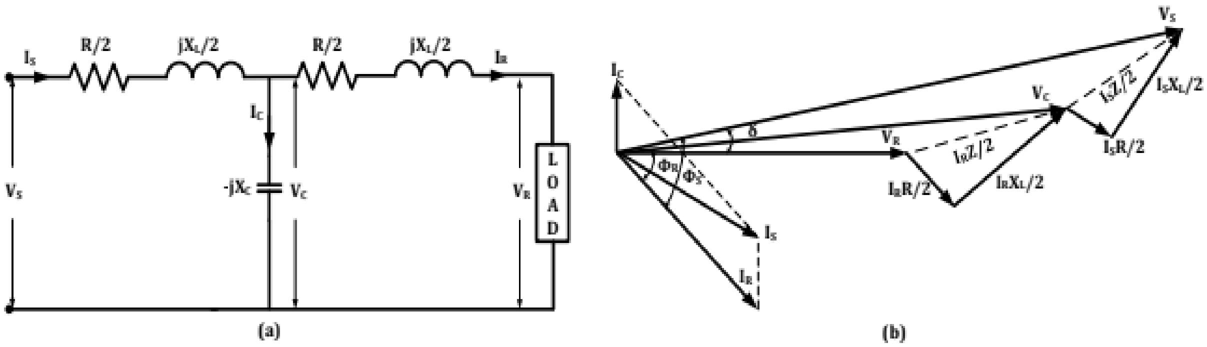


Figure 1. 4 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

R = Resistance per phase

L = Inductance per phase

C = Capacitance per phase

X_L = Inductive reactance per phase

X_C = Capacitive reactance per phase

V_R = Receiving end voltage per phase

V_S = Sending end voltage per phase

I_R = Receiving end current per phase

I_S = Sending end current per phase

I_C = Capacitive current per phase

$\cos\phi_R$ = Receiving end power factor

$\cos\phi_S$ = Sending end power factor

δ = Angle between sending end and receiving end voltage

Hence,

$$V_R = V_R\angle 0 = V_R + j0$$
$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$
$$V_C = V_R + I_R\frac{Z}{2} = V_R + j0 + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$
$$I_C = \frac{V_C}{X_C}\angle 90 = V_C\omega C\angle 90 = jV_C\omega C = jV_C 2\pi fC$$
$$\begin{aligned} I_S &= I_R + I_C \\ &= I_R(\cos\phi_R - j\sin\phi_R) + j2\pi fCV_C \\ &= I_R\cos\phi_R - jI_R\sin\phi_R + j2\pi fCV_C \\ &= I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_C) \end{aligned}$$
$$\begin{aligned} V_S &= V_C + I_S\frac{Z}{2} \\ &= \left(V_R + j0 + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)\right) + \left(I_R\cos\phi_R + j(-I_R\sin\phi_R + 2\pi fCV_C)\right)\left(\frac{R}{2} + j\frac{X_L}{2}\right) \end{aligned}$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$
$$\begin{aligned} \%\ \text{Transmission Efficiency} &= \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ losses}\times 100 \\ &= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \ 3I_S^2\frac{R}{2} + 3I_R^2\frac{R}{2}}\times 100 \end{aligned}$$

❖ Performance of medium transmission line using Nominal π method

In this method, the capacitance of each conductor i.e. line to neutral is divided into two halves; one half being lumped at the sending end and the other half at the receiving end.

It is obvious that capacitance at the sending end has no effect on the line drop. However, it's charging current must be added to the line current to obtain the total sending end current.

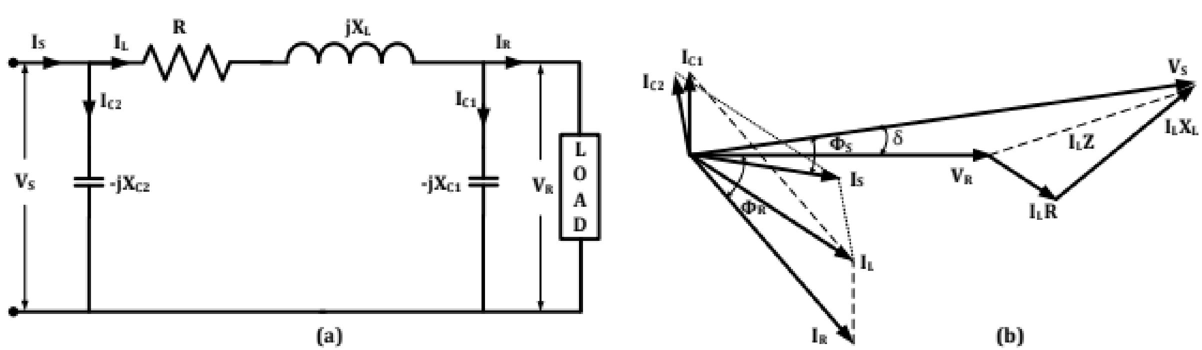


Figure 1. 5 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

Let,

$R =$ Resistance per phase

$L =$ Inductance per phase

$C =$ Capacitance per phase

$X_L =$ Inductive reactance per phase

$X_C =$ Capacitive reactance per phase

$V_R =$ Receiving end voltage per phase

$V_S =$ Sending end voltage per phase

$I_R =$ Receiving end current per phase

$I_S =$ Sending end current per phase

$I_C =$ Capacitive current per phase

$\cos\phi_R =$ Receiving end power factor

$\cos\phi_S =$ Sending end power factor

$\delta =$ Angle between sending end and receiving end voltage

$$V_R = V_R\angle 0 = V_R + j0$$

$$I_R = I_R\angle -\phi_R = I_R(\cos\phi_R - j\sin\phi_R)$$

$$I_{C1} = \frac{V_R}{X_{C1}}\angle 90 = V_R\omega\frac{C}{2}\angle 90 = jV_R\omega\frac{C}{2} = jV_R 2\pi f\frac{C}{2}$$

$$I_{C2} = \frac{V_S}{X_{C2}}\angle 90 = V_S\omega\frac{C}{2}\angle 90 = jV_S\omega\frac{C}{2} = jV_S 2\pi f\frac{C}{2}$$

$$I_L = I_R + I_{C1}$$

$$I_S = I_L + I_{C2}$$

$$V_S = V_R + I_L Z = V_R + I_L(R + jX_L)$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$

$$\%\ \text{Transmission Efficiency} = \frac{V_R I_R\cos\phi_R}{V_S I_S\cos\phi_S}\times 100$$

$$= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + \mathit{losses}}\times 100$$

$$= \frac{3V_R I_R\cos\phi_R}{3V_R I_R\cos\phi_R + 3I_L^2 R}\times 100$$

Characteristics of long transmission lines

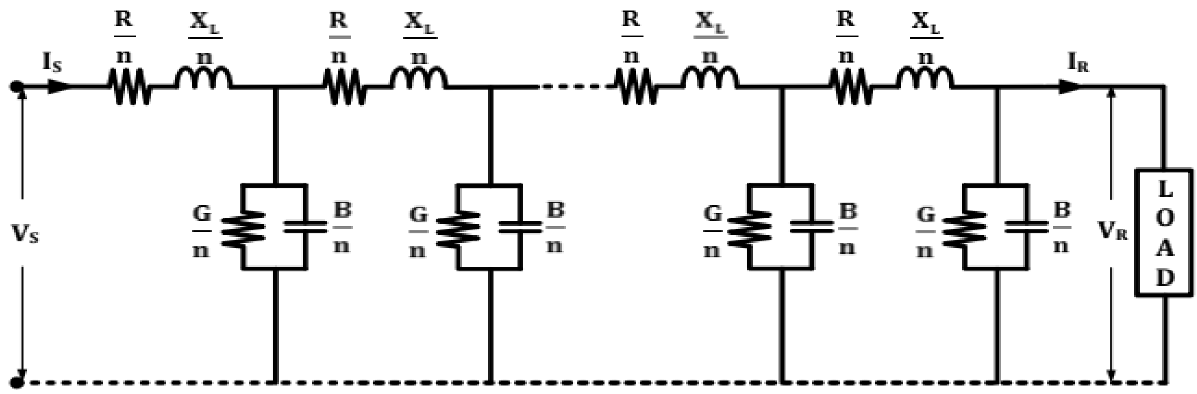


Figure 1. 6 Equivalent Circuit of Long Transmission Line

In equivalent circuit of a 3-phase long transmission line on a phase-neutral basis, the whole line is divided into n sections, each section having line constants 1/n th of those for the whole line.

The line constants are uniformly distributed over the entire length of line.

The resistance and inductive reactance are series elements.

The leakage susceptance (B) i.e. due to capacitance between line and neutral and leakage conductance (G) i.e. due to energy losses through leakages over the insulators or corona loss are shunt elements. Hence, admittance = $\sqrt{G^2 + B^2}$

The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached.

❖ Performance of long transmission line

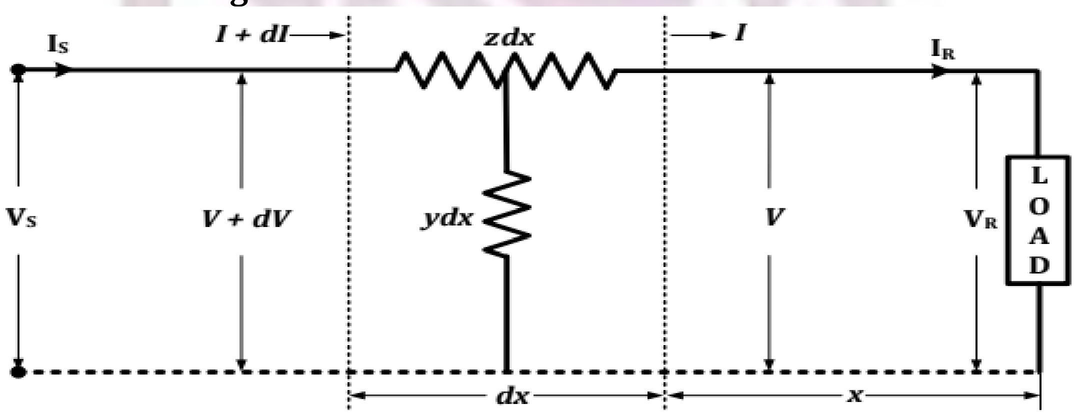


Figure 1. 7 Small Element of a Long Transmission Line

Consider a small element in the line of length dx situated at a distance x from the receiving end.

z=Series impedance of the line per unit length
y=Shunt admittance of the line per unit length
Z=Total series impedance of the line
Y=Total shunt admittance of the line
zdx=Series impedance of the small element dx
ydx=Shunt admittance of the small element dx
V=Voltage at end of the element towards receiving end
$V+dV$=Voltage at the end of element towards sending end
$I+dI$=Current entering the small element dx
I=Current leaving the small element dx

As current entering the element is I + dI and leaving the element is I. Hence voltage drop across small element dv and current through the shunt element is dI.

$$dv = Izdx \qquad \Rightarrow \frac{dv}{dx} = Iz$$

$$dI = Vydx \qquad \Rightarrow \frac{dI}{dx} = Vy$$

Now differentiating above equation

$$\frac{d^2V}{dx} = \frac{dI}{dx}z = Vyz \qquad \Rightarrow \frac{d^2V}{dx} - Vyz = 0$$

The solution of this differential equation is

$$V = K_1 Cosh\left(x\sqrt{yz}\right) + K_2 Sinh\left(x\sqrt{yz}\right)$$

Differentiating this equation, we get

$$\frac{dV}{dx} = \left(\sqrt{yz}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{yz}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore Iz = \left(\sqrt{yz}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{yz}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore I = \left(\sqrt{\frac{y}{z}}\right)K_1 Sinh\left(x\sqrt{yz}\right) + \left(\sqrt{\frac{y}{z}}\right)K_2 Cosh\left(x\sqrt{yz}\right)$$

$$\therefore I = \sqrt{\frac{y}{z}}\left(K_1 Sinh\left(x\sqrt{yz}\right) + K_2 Cosh\left(x\sqrt{yz}\right)\right)$$

The values of K1 and K2 can be found by applying end conditions at x=0, V=VR and I=IR.

$$K_1 = V_R \;\; \& \;\; K_2 = \sqrt{\frac{z}{y}}I_R$$

Substituting the values of K1 and K2 in equations

$$V = V_R Cosh\left(x\sqrt{yz}\right) + \sqrt{\frac{z}{y}}I_R Sinh\left(x\sqrt{yz}\right)$$

$$I = \sqrt{\frac{y}{z}}\left(V_R Sinh\left(x\sqrt{yz}\right) + \sqrt{\frac{z}{y}}I_R Cosh\left(x\sqrt{yz}\right)\right)$$

The sending end voltage VS and sending end current IS can be obtained by putting x = l in the above equations.

$$V_S = V_R Cosh\left(l\sqrt{yz}\right) + \sqrt{\frac{z}{y}} I_R Sinh\left(l\sqrt{yz}\right)$$

$$I_S = \sqrt{\frac{y}{z}}\left(V_R Sinh\left(l\sqrt{yz}\right) + \sqrt{\frac{z}{y}} I_R Cosh\left(l\sqrt{yz}\right)\right)$$

Let,

$$l\sqrt{yz} = \sqrt{l^2 yz} = \sqrt{(ly)(lz)} = \sqrt{YZ}$$

$$\sqrt{\frac{y}{z}} = \sqrt{\frac{ly}{lz}} = \sqrt{\frac{Y}{Z}}$$

So,

$$V_S = V_R Cosh\left(\sqrt{YZ}\right) + I_R \sqrt{\frac{Z}{Y}} Sinh\left(\sqrt{YZ}\right)$$

$$I_S = V_R \sqrt{\frac{Y}{Z}} Sinh\left(\sqrt{YZ}\right) + I_R Cosh\left(\sqrt{YZ}\right)$$

Where,

$$Cosh\left(\sqrt{YZ}\right) = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \cdots\cdots\right)$$

$$Sinh\left(\sqrt{YZ}\right) = \left(\sqrt{YZ} + \frac{(YZ)^{\frac{3}{2}}}{6} + \cdots\cdots\right)$$

Also,

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$\%\ \text{Transmission Efficiency} = \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

Generalized circuit constants of a transmission lines

In any four terminal network, the input voltage and input current can be expressed in terms of output voltage and current.

When voltage VR and current IR are selected as independent variable and voltage VS and current IS are dependent variable, network can be characterized by following set of equation. A, B, C and D are the generalized circuit constants of the transmission line and are complex numbers.

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

The constants A and D are dimensionless whereas the dimensions of B and C are ohms and siemens respectively. For a given transmission line A=D and AD-BC=1.

A. Generalized circuit constants (ABCD parameters) of short transmission lines

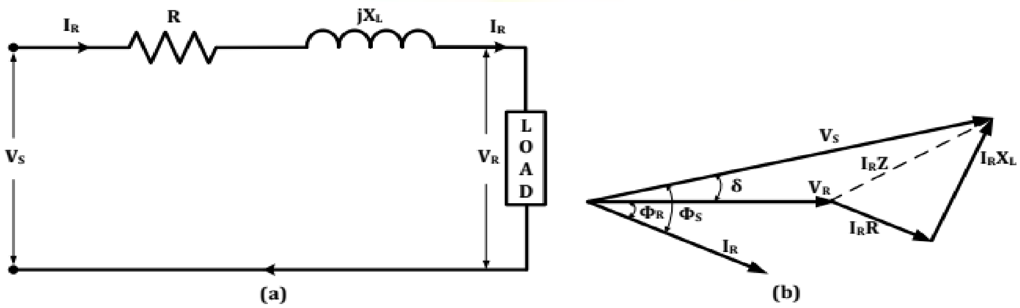


Figure 1. 8 Short Transmission Line (a) Circuit and (b) Vector Diagram (Voltage as Reference)

For short transmission line,

$$V_S = V_R + I_R Z$$
$$I_S = I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$

B. Generalized circuit constants (ABCD parameters) of medium transmission lines - End condenser method

In this method, the capacitance of the line is lumped or concentrated at the receiving end. This method of localizing the line capacitance at the load end overestimates the effect of capacitance.

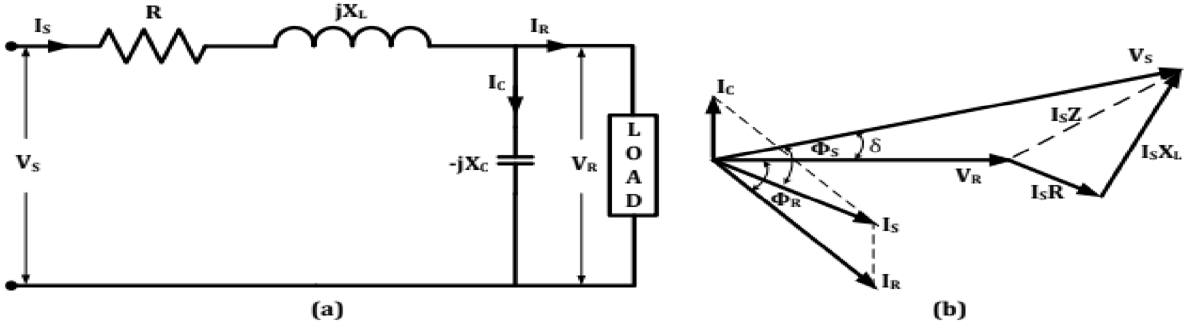


Figure 1. 9 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_S = V_R + I_S Z$$
$$= V_R + (I_C + I_R) Z$$
$$= V_R + ZI_C + ZI_R$$
$$= V_R + Z(YV_R) + ZI_R$$
$$V_S = (1 + YZ)V_R + ZI_R$$

$$I_S = I_C + I_R$$
$$I_S = YV_R + I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 + YZ \qquad B = Z$$
$$C = Y \qquad D = 1$$

C. Generalized circuit constants (ABCD parameters) of medium transmission lines - Nominal T method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line.

Half of the line resistance and reactance are lumped on the both side and full charging current flows over half the line.

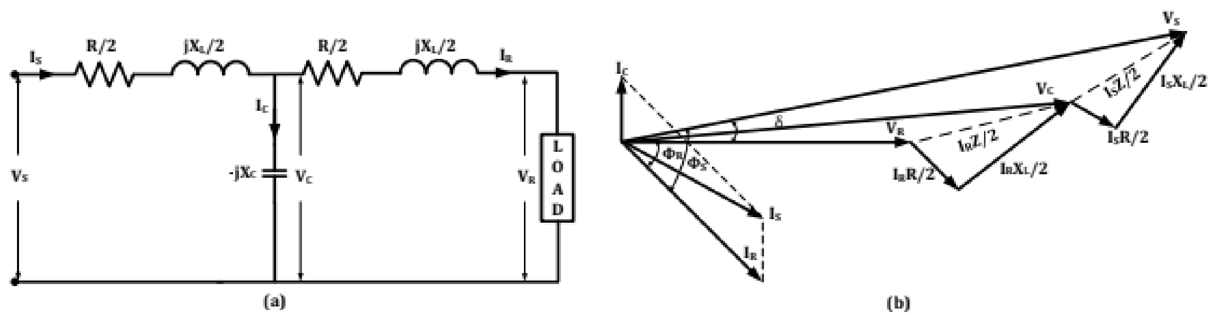


Figure 1. 10 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_S = V_C + I_S \frac{Z}{2}$$
$$= V_C + (I_R + I_C)\frac{Z}{2}$$
$$= V_C + (I_R + V_C Y)\frac{Z}{2}$$
$$= V_C\left(1 + \frac{YZ}{2}\right) + I_R \frac{Z}{2}$$
$$= \left(V_R + I_R \frac{Z}{2}\right)\left(1 + \frac{YZ}{2}\right) + I_R \frac{Z}{2}$$
$$= \left(1 + \frac{YZ}{2}\right)V_R + \left(\frac{Z}{2} + \frac{YZ^2}{4} + \frac{Z}{2}\right)I_R$$
$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + \left(Z + \frac{YZ^2}{4}\right)I_R$$

$$I_S = I_R + I_C$$
$$= I_R + V_C Y$$
$$= I_R + \left(V_R + I_R \frac{Z}{2}\right)Y$$
$$I_S = YV_R + \left(1 + \frac{YZ}{2}\right)I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 + \frac{YZ}{2} \qquad B = Z + \frac{YZ^2}{4}$$
$$C = Y \qquad D = 1 + \frac{YZ}{2}$$

D. Generalized circuit constants (ABCD parameters) of medium transmission lines - Nominal π method

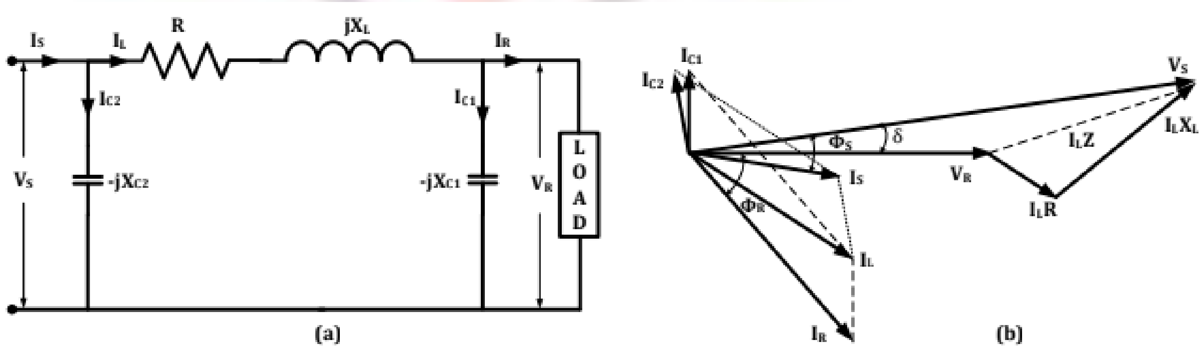


Figure 1. 11 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

In this method, the capacitance of each conductor i.e. line to neutral is divided into two halves; one half being lumped at the sending end and the other half at the receiving end.

It is obvious that capacitance at the sending end has no effect on the line drop. However, it's charging current must be added to the line current to obtain the total sending end current.

$$V_S = V_R + I_L Z$$
$$= V_R + (I_R + I_{C1})Z$$
$$= V_R + \left(I_R + V_R\frac{Y}{2}\right)Z$$
$$V_S = \left(1 + \frac{YZ}{2}\right)V_R + ZI_R$$

$$I_S = I_L + I_{C2}$$
$$= I_R + I_{C1} + I_{C2}$$
$$= I_R + V_R\frac{Y}{2} + V_S\frac{Y}{2}$$
$$= I_R + V_R\frac{Y}{2} + (V_R + I_L Z)\frac{Y}{2}$$
$$= I_R + V_R\frac{Y}{2} + V_R\frac{Y}{2} + I_L\frac{YZ}{2}$$
$$= I_R + V_R\frac{Y}{2} + V_R\frac{Y}{2} + \left(I_R + V_R\frac{Y}{2}\right)\frac{YZ}{2}$$
$$= I_R + \frac{Y}{2}V_R + \frac{Y}{2}V_R + \frac{YZ}{2}I_R + \frac{Y^2Z}{4}V_R$$
$$I_S = \left(Y + \frac{Y^2Z}{4}\right)V_R + \left(1 + \frac{YZ}{2}\right)I_R$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = 1 + \frac{YZ}{2} \qquad B = Z$$
$$C = Y + \frac{Y^2Z}{4} \qquad D = 1 + \frac{YZ}{2}$$

E. Generalized circuit constants (ABCD parameters) of long transmission lines

By rigorous method, the sending end voltage and current of a long transmission line are given by

$$V_S = V_R Cosh\left(\sqrt{YZ}\right) + I_R\sqrt{\frac{Z}{Y}}Sinh\left(\sqrt{YZ}\right)$$
$$I_S = V_R\sqrt{\frac{Y}{Z}}Sinh\left(\sqrt{YZ}\right) + I_R Cosh\left(\sqrt{YZ}\right)$$

Comparing these equation with basic equation of generalized circuit constants

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

Hence,

$$A = Cosh\left(\sqrt{YZ}\right) \qquad B = \sqrt{\frac{Z}{Y}}Sinh\left(\sqrt{YZ}\right)$$

$$C = \sqrt{\frac{Y}{Z}}Sinh\left(\sqrt{YZ}\right) \qquad D = Cosh\left(\sqrt{YZ}\right)$$

❖ Power flow through a transmission line

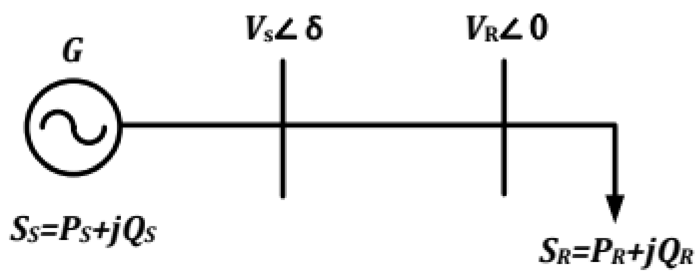


Figure 1. 12 Transmission Line Power Flow

Let,

R = Resistance per phase
L = Inductance per phase
C = Capacitance per phase
X_L = Inductive reactance per phase
X_C = Capacitive reactance per phase
V_R = Receiving end voltage per phase
V_S = Sending end voltage per phase
I_R = Receiving end current per phase
I_S = Sending end current per phase
I_C = Capacitive current per phase
$\cos\phi_R$ = Receiving end power factor
$\cos\phi_S$ = Sending end power factor
δ = Angle between sending end and receiving end voltage

Generalized line constants (ABCD parameter) are

$$A = A\angle\alpha \qquad B = B\angle\beta$$

$$C = C\angle\gamma \qquad D = D\angle\Delta$$

Relation for receiving end power can be derived with the use of relation of sending end voltage in terms of ABCD parameters of transmission line.

$$V_S = AV_R + BI_R$$

$$BI_R = V_S - AV_R$$

$$I_R = \frac{V_S - AV_R}{B}$$

$$= \frac{V_S\angle\delta - (A\angle\alpha)(V_R\angle 0)}{B\angle\beta}$$

$$= \frac{V_S\angle\delta}{B\angle\beta} - \frac{(A\angle\alpha)(V_R\angle 0)}{B\angle\beta}$$

$$= \frac{V_S}{B}\angle(\delta-\beta) - \frac{AV_R}{B}\angle(\alpha-\beta)$$

$$I_R^* = \frac{V_S}{B}\angle(\beta-\delta) - \frac{AV_R}{B}\angle(\beta-\alpha)$$

$$S_R = P_R + jQ_R$$

$$S_R = V_R I_R^*$$

$$= (V_R\angle 0)\left(\frac{V_S}{B}\angle(\beta-\delta) - \frac{AV_R}{B}\angle(\beta-\alpha)\right)$$

$$= \frac{V_R V_S}{B}\angle(\beta-\delta) - \frac{AV_R^2}{B}\angle(\beta-\alpha)$$

$$= \left(\frac{V_R V_S}{B}Cos(\beta-\delta) + j\frac{V_R V_S}{B}Sin(\beta-\delta)\right) - \left(\frac{AV_R^2}{B}Cos(\beta-\alpha) + j\frac{AV_R^2}{B}Sin(\beta-\alpha)\right)$$

$$= \left(\frac{V_R V_S}{B}Cos(\beta-\delta) - \frac{AV_R^2}{B}Cos(\beta-\alpha)\right) + j\left(\frac{V_R V_S}{B}Sin(\beta-\delta) - \frac{AV_R^2}{B}Sin(\beta-\alpha)\right)$$

$$\therefore P_R = \frac{V_R V_S}{B}Cos(\beta-\delta) - \frac{AV_R^2}{B}Cos(\beta-\alpha)$$

$$\therefore Q_R = \frac{V_R V_S}{B}Sin(\beta-\delta) - \frac{AV_R^2}{B}Sin(\beta-\alpha)$$

Relation for sending end power can be derived with the use of relation of sending end current in terms of ABCD parameters of transmission line.

$$I_S = CV_R + DI_R \Rightarrow I_R = \frac{I_S - CV_R}{D}$$

$$V_S = AV_R + BI_R$$

$$= AV_R + \frac{B}{D}\left(I_S - CV_R\right)$$

$$DV_S = ADV_R + B\left(I_S - CV_R\right)$$

$$= ADV_R + BI_S - BCV_R$$

$$= (AD - BC)V_R + BI_S$$

$$DV_S = V_R + BI_S$$

$$I_S = \frac{DV_S - V_R}{B}$$

$$= \frac{(D\angle\Delta)(V_S\angle\delta) - V_R\angle 0}{B\angle\beta}$$

$$= \frac{(D\angle\Delta)(V_S\angle\delta)}{B\angle\beta} - \frac{V_R\angle 0}{B\angle\beta}$$

$$= \frac{DV_S}{B}\angle(\Delta + \delta - \beta) - \frac{V_R}{B}\angle(-\beta)$$

$$I_S^* = \frac{DV_S}{B}\angle(\beta - \Delta - \delta) - \frac{V_R}{B}\angle(\beta)$$

$$S_S = P_S + jQ_S$$

$$S_S = V_S I_S^*$$

$$= (V_S\angle\delta)\left(\frac{DV_S}{B}\angle(\beta - \Delta - \delta) - \frac{V_R}{B}\angle(\beta)\right)$$

$$= \frac{DV_S^2}{B}\angle(\beta - \Delta) - \frac{V_S V_R}{B}\angle(\beta + \delta)$$

$$= \left(\frac{DV_S^2}{B}Cos(\beta - \Delta) + j\frac{DV_S^2}{B}Sin(\beta - \Delta)\right) - \left(\frac{V_S V_R}{B}Cos(\beta + \delta) + j\frac{V_S V_R}{B}Sin(\beta + \delta)\right)$$

$$= \left(\frac{DV_S^2}{B}Cos(\beta - \Delta) - \frac{V_S V_R}{B}Cos(\beta + \delta)\right) + j\left(\frac{DV_S^2}{B}Sin(\beta - \Delta) - \frac{V_S V_R}{B}Sin(\beta + \delta)\right)$$

$$\therefore P_S = \frac{DV_S^2}{B}Cos(\beta - \Delta) - \frac{V_S V_R}{B}Cos(\beta + \delta)$$

$$\therefore Q_S = \frac{DV_S^2}{B}Sin(\beta - \Delta) - \frac{V_S V_R}{B}Sin(\beta + \delta)$$

- **Power circle diagram for transmission line**

For transmission line analysis, it is required to find sending end voltage and sending end current while receiving end voltage and receiving end current is known.

Unique feature of transmission line power circuit is that it operates at fixed frequency, hence it follows all the mathematical function of circle.

A. Receiving end power circle diagram

The circle drawn with receiving end active power component as horizontal coordinate and receiving end reactive power component as vertical coordinate is called the receiving end power circle diagram.

Receiving end active power component and receiving end reactive power component are

$$P_R = \frac{V_R V_S}{B} Cos(\beta - \delta) - \frac{AV_R^2}{B} Cos(\beta - \alpha)$$

$$\therefore P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha) = \frac{V_R V_S}{B} Cos(\beta - \delta)$$

$$Q_R = \frac{V_R V_S}{B} Sin(\beta - \delta) - \frac{AV_R^2}{B} Sin(\beta - \alpha)$$

$$\therefore Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha) = \frac{V_R V_S}{B} Sin(\beta - \delta)$$

Taking square on both the side of equation and adding it.

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \left(\frac{V_R V_S}{B} Cos(\beta - \delta)\right)^2 + \left(\frac{V_R V_S}{B} Sin(\beta - \delta)\right)^2$$

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \frac{V_R^2 V_S^2}{B^2}\left(Cos^2(\beta - \delta) + Sin^2(\beta - \delta)\right)$$

$$\left(P_R + \frac{AV_R^2}{B} Cos(\beta - \alpha)\right)^2 + \left(Q_R + \frac{AV_R^2}{B} Sin(\beta - \alpha)\right)^2 = \frac{V_R^2 V_S^2}{B^2}$$

Equation of circle is

$$(x - h)^2 + (y - g)^2 = r^2$$

The coordinates of the centre of the circle and radius of the circle can be given as

$$\text{x-Coordinate of the centre of circle} = -\frac{AV_R^2}{B} Cos(\beta - \alpha)$$

$$\text{y-Coordinate of the centre of circle} = -\frac{AV_R^2}{B} Sin(\beta - \alpha)$$

$$\text{Radius of circle} = \frac{V_S V_R}{B}$$

$$\text{Radius of circle} = \frac{V_S V_R}{B}$$

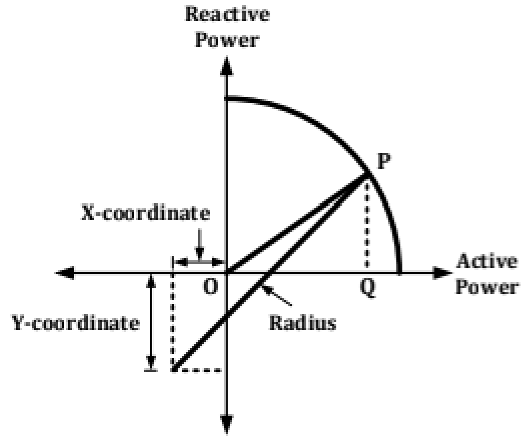


Figure 1. 13 Receiving end Circle Diagram

Construction of circle diagram

O Plot centre of circle with suitable scale.

O From centre draw an arc with calculated radius.

O Draw load line OP from origin at an angle □R with horizontal and let it cut the circle at point P.

O Measure line OQ i.e. receiving end active power

O Measure line PQ i.e. receiving end reactive power

O Draw a horizontal line from centre of circle intersecting vertical axis at point L and circle at the point M.

O Measure line LM i.e. maximum power for receiving end.

When sending end voltage and receiving end voltages are the phase quantity then power indicted on x-axis and y-axis are watts and VAR per phase values.

When sending end voltage and receiving end voltages are the line quantity then power indicted on x-axis and y-axis are watts and VAR for all three phases.

B. Sending end power circle diagram

The circle drawn with sending end active power component as horizontal coordinate and sending end reactive power component as vertical coordinate is called the sending end power circle diagram.

Sending end active power component and sending end reactive power component are

$$P_S = \frac{DV_S^2}{B} Cos(\beta - \Delta) - \frac{V_S V_R}{B} Cos(\beta + \delta)$$

$$\therefore P_S - \frac{DV_S^2}{B} Cos(\beta - \Delta) = -\frac{V_S V_R}{B} Cos(\beta + \delta)$$

$$Q_S = \frac{DV_S^2}{B} Sin(\beta - \Delta) - \frac{V_S V_R}{B} Sin(\beta + \delta)$$

$$\therefore Q_S - \frac{DV_S^2}{B} Sin(\beta - \Delta) = -\frac{V_S V_R}{B} Sin(\beta + \delta)$$

Taking square on both the side of equation and adding it.

$$\left(P_S - \frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2 + \left(Q_S - \frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2 = \left(-\frac{V_SV_R}{B}Cos(\beta+\delta)\right)^2 + \left(-\frac{V_SV_R}{B}Sin(\beta+\delta)\right)^2$$

$$\left(P_S - \frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2 + \left(Q_S - \frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2 = \frac{V_S^2V_R^2}{B^2}\left(Cos^2(\beta+\delta)+Sin^2(\beta+\delta)\right)$$

$$\left(P_S - \frac{DV_S^2}{B}Cos(\beta-\Delta)\right)^2 + \left(Q_S - \frac{DV_S^2}{B}Sin(\beta-\Delta)\right)^2 = \frac{V_S^2V_R^2}{B^2}$$

Equation of circle is

$$(x-h)^2 + (y-g)^2 = r^2$$

The coordinates of the centre of the circle and radius of the circle can be given as

$$\text{x-Coordinate of the centre of circle} = \frac{DV_S^2}{B}Cos(\beta-\Delta)$$

$$\text{y-Coordinate of the centre of circle} = \frac{DV_S^2}{B}Sin(\beta-\Delta)$$

$$\text{Radius of circle} = \frac{V_SV_R}{B}$$

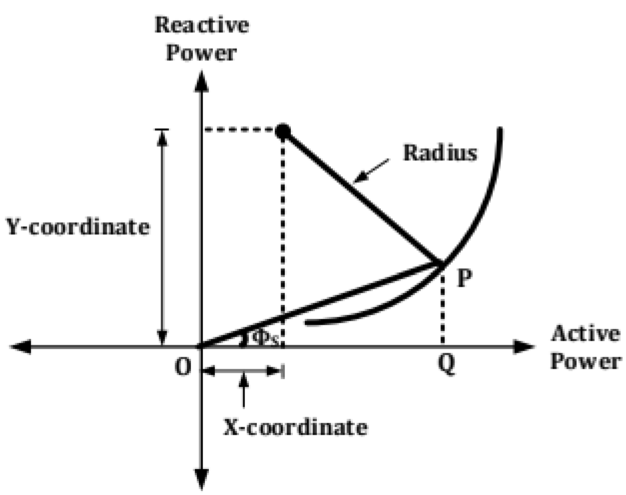


Figure 1. 14 Sending end Circle Diagram

Construction of circle diagram

O Plot centre of circle with suitable scale.

O From centre draw an arc with calculated radius.

O Draw load line OP from origin at an angle □S with horizontal and let it cut the circle at point P.

O Measure line OQ i.e. sending end active power

O Measure line PQ i.e. sending end reactive power

O Draw a horizontal line from centre of circle intersecting vertical axis at point L and circle at the point M.

O Measure line LM i.e. maximum power for sending end.

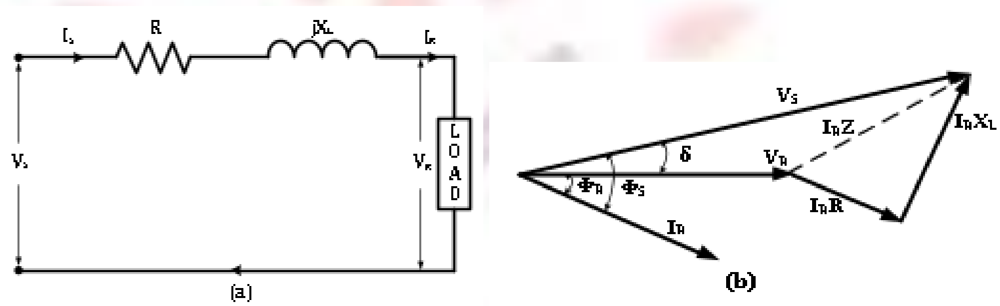
When sending end voltage and receiving end voltages are the phase quantity then power indicted on x-axis and y-axis are watts and VAR per phase values.

When sending end voltage and receiving end voltages are the line quantity then power indicted on x-axis and y-axis are watts and VAR for all three phases.

PROBLEMS:

1. A three phase overhead transmission line delivers 1100 kW at 33 kV at 0.8 pf lagging. The total resistance and inductive reactance per phase of the line are 10 Ω and 15 Ω respectively. Find (a). % voltage regulation and (b). % transmission efficiency.

Solution:



$$V_R = 33\times10^3\angle0 = 33\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = R + jX_L = 10 + j15 = 18\angle56.3$$

$$P_R = \sqrt{3}V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{1100\times10^3}{\sqrt{3}\times33\times10^3\times0.8} = 24\ A$$

$$I_R = I_R\angle-\phi_R$$
$$= 24\angle-36.86$$
$$= 19.20 - j14.39$$

$$I_S = I_R$$
$$= 24\angle-36.86\ A$$

$$
\begin{aligned}
V_S &= V_R + I_R Z \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + (24\angle -36.86)(10 + j15) \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + (24\angle -36.86)(18\angle 56.3) \\
&= \frac{33\times10^3}{\sqrt{3}}\angle 0 + 432\angle 19.44 \\
&= 19052.55 + j0 + 407.37 + j143.77 \\
&= 19459.92 + j143.77 \\
&= 19460.45\angle 0.42 \text{ V/ph} \\
V_S &= 19.46\angle 0.42 \text{ kV/ph}
\end{aligned}
$$

$$
\begin{aligned}
V_S &= \sqrt{3}\times 19.46\angle 0.42 \\
&= 33.70\angle 0.42 \text{ kV}
\end{aligned}
$$

$$
\begin{aligned}
\phi_S &= 36.86 + 0.42 = 37.28 \\
\cos\phi_S &= \cos(37.28) = 0.79(lag)
\end{aligned}
$$

$$
\begin{aligned}
\%\text{Voltage Regulation} &= \frac{V_S - V_R}{V_R}\times 100 \\
&= \frac{\left(\frac{33.70\times10^3}{\sqrt{3}}\right) - \left(\frac{33\times10^3}{\sqrt{3}}\right)}{\left(\frac{33\times10^3}{\sqrt{3}}\right)}\times 100 \\
&= 2.12
\end{aligned}
$$

$$
\begin{aligned}
\%\text{ Transmission Efficiency} &= \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S}\times 100 \\
&= \frac{3\times\left(\frac{33\times10^3}{\sqrt{3}}\right)\times 24\times 0.8}{3\times\left(\frac{33.70\times10^3}{\sqrt{3}}\right)\times 24\times 0.79}\times 100 \\
&= 99.16
\end{aligned}
$$

2. A medium three phase transmission line 100 km long has following constants:

   Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the total capacitance of the line is localized at the receiving end alone. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). % voltage regulation and (b). % transmission efficiency.

Solution:

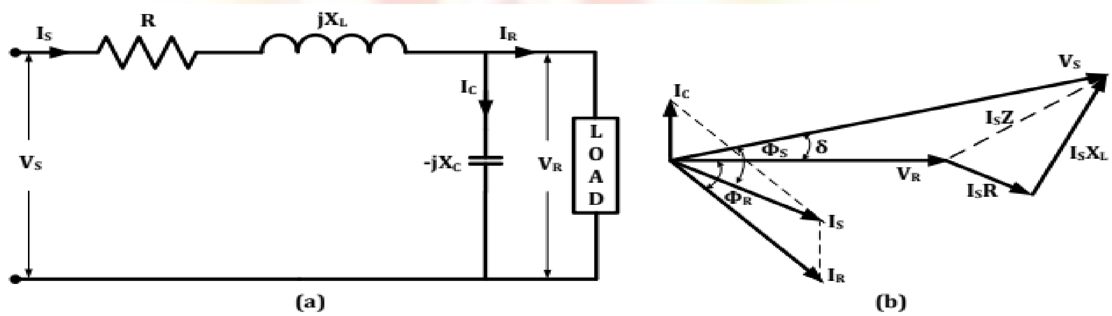


Figure 1. 3 End Condenser Method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = 132\times10^3\angle0 = 132\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$R = 0.15\times100 = 15\ \Omega/ph$$
$$X_L = 0.377\times100 = 37.7\ \Omega/ph$$
$$X_C = 31.87\times100 = 3187\ \Omega/ph$$

$$P_R = \sqrt{3}V_R I_R\cos\phi_R$$
$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{72\times10^6}{\sqrt{3}\times132\times10^3\times0.8} = 393.64\ A$$

$$I_R = I_R\angle-\phi_R = 393.64\angle-36.86 = 314.95 - j236.12$$

$$I_c = \frac{V_R}{-jX_c}$$

$$= \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{-j3187}$$

$$= \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{3187\angle -90}$$

$$= 23.91\angle 90 \text{ A}$$

$$I_S = I_R + I_C$$

$$= 393.64\angle -36.86 + 23.91\angle 90$$

$$= 314.95 - j236.12 + 0 + 23.91j$$

$$= 314.95 - 212.21j$$

$$= 379.77\angle -33.97 \text{ A}$$

$$V_s = V_R + I_s Z$$

$$= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (379.77\angle -33.97)(15 + j37.7)$$

$$= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (379.77\angle -33.97)(40.57\angle 68.3)$$

$$= \frac{132\times10^3}{\sqrt{3}}\angle 0 + 15407.26\angle 34.33$$

$$= 76210 + 0j + 12723.36 + 8689.05j$$

$$= 88933.36 + 8689.05j$$

$$= 89356.82\angle 5.58 \quad \text{V/ph}$$

$$= 89.35\angle 5.58 \quad \text{kV/ph}$$

$$V_s = \sqrt{3}\times 89.35\angle 5.58 \text{ kV}$$

$$= 154.75\angle 5.58 \text{ kV}$$

$$\phi_s = 33.97 + 5.58 = 39.58$$

$$\cos\phi_s = \cos(39.58) = 0.77(lag)$$

$$\%\text{Voltage Regulation} = \frac{V_s - V_R}{V_R}\times 100$$

$$= \frac{\left(\frac{154.75\times10^3}{\sqrt{3}}\right) - \left(\frac{132\times10^3}{\sqrt{3}}\right)}{\left(\frac{132\times10^3}{\sqrt{3}}\right)}\times 100$$

$$= 16.66$$

]

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3\times\left(\dfrac{132\times10^3}{\sqrt{3}}\right)\times 393.64\times0.8}{3\times\left(\dfrac{154.75\times10^3}{\sqrt{3}}\right)\times379.77\times0.77}\times100$$

$$= 91.85$$

3. A medium three phase transmission line 100 km long has following constants:
   Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the total capacitance of the line is localized at middle point of the line. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). % voltage regulation and (b). % transmission efficiency.

Solution:

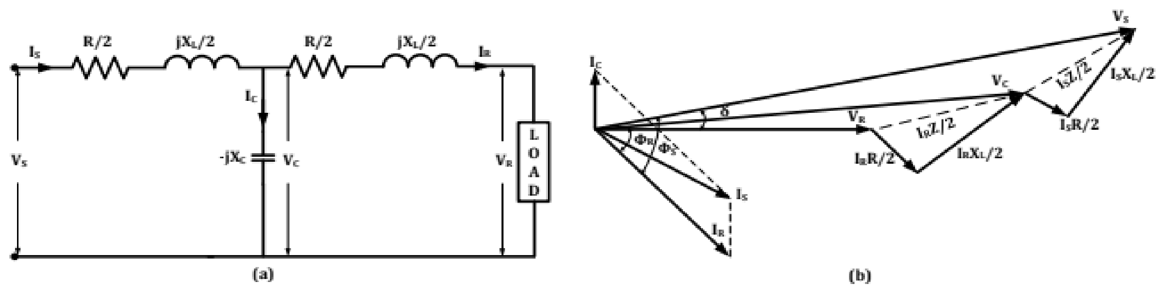


Figure 1. 4 Nominal T method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = 132\times10^3\angle0 = 132\times10^3 + j0$$

$$\cos\phi_R = 0.8\,(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$R = 0.15\times100 = 15\ \Omega/ph$$
$$X_L = 0.377\times100 = 37.7\ \Omega/ph$$
$$X_C = 31.87\times100 = 3187\ \Omega/ph$$

$$P_R = \sqrt{3}V_R I_R\cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{72\times10^6}{\sqrt{3}\times132\times10^3\times0.8} = 393.64\ A$$

$$I_R = I_R\angle-\phi_R = 393.64\angle-36.86 = 314.95 - j236.12$$

$$V_C = V_R\angle 0 + (I_R\angle -\phi_R)\left(\frac{R+jX_L}{2}\right)$$

$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + (393.64\angle -36.86)\left(\frac{15+j37.7}{2}\right)$$

$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + (393.64\angle -36.86)(20.28\angle 68.30)$$

$$= \left(\frac{132\times10^3}{\sqrt{3}}\angle 0\right) + 7983.01\angle 31.44$$

$$= 76210.23 + 0j + 6810.99 + 4163.98j$$

$$= 83021.22 + 4163.98j$$

$$= 83125.57\angle 2.87 \text{ V}$$

$$I_C = \frac{V_C}{-jX_C} = \frac{83125.57\angle 2.87}{-j3187} = \frac{83125.57\angle 2.87}{3187\angle -90} = 26.08\angle 92.87 \text{ A}$$

$$I_S = I_R + I_C$$

$$= 393.64\angle -36.86 + 26.08\angle 92.87$$

$$= 314.95 - 236.12j - 1.30 + 26.04j$$

$$= 313.65 - 210.08j$$

$$= 377.5\angle -33.81 \text{ A}$$

$$V_S = V_C\angle\theta_C + (I_S\angle \pm \phi_S)\left(\frac{R+jX_L}{2}\right)$$

$$= 83125.57\angle 2.87 + (377.5\angle -33.81)\left(\frac{15+j37.7}{2}\right)$$

$$= 83125.57\angle 2.87 + (377.5\angle -33.81)(20.28\angle 68.30)$$

$$= 83125.57\angle 2.87 + 7655.7\angle 34.49$$

$$= 83021.22 + 4163.98j + 6310.01 + 4335.13j$$

$$= 89331.23 + 8499.11j$$

$$= 89734.62\angle 5.43 \text{ V/ph}$$

$$= 89.73\angle 5.43 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times 89.73\angle 5.43 \text{ kV}$$

$$= 155.41\angle 5.43 \text{ kV}$$

$$\phi_S = 33.81 + 5.43 = 39.24$$

$$\cos\phi_S = \cos(39.24) = 0.77(lag)$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times 100$$

$$= \frac{\left(\frac{155.41\times10^3}{\sqrt{3}}\right) - \left(\frac{132\times10^3}{\sqrt{3}}\right)}{\left(\frac{132\times10^3}{\sqrt{3}}\right)}\times 100$$

$$= 17.73$$

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3 \times \left(\frac{132 \times 10^3}{\sqrt{3}}\right) \times 393.64 \times 0.8}{3 \times \left(\frac{155.41 \times 10^3}{\sqrt{3}}\right) \times 377.5 \times 0.77} \times 100$$

$$= 92.01$$

---

4. A medium three phase transmission line 100 km long has following constants: Resistance/km/phase: 0.15 Ω, Reactance/km/phase: 0.377 Ω, Capacitive Reactance/km/phase: 31.87 Ω, Receiving end line Voltage: 132 kV. Assume the half of capacitance of the line is lumped at both the end. The line is delivering load of 72 MW at 0.8 pf lagging. Find (a). % voltage regulation and (b). % transmission efficiency.

Solution:

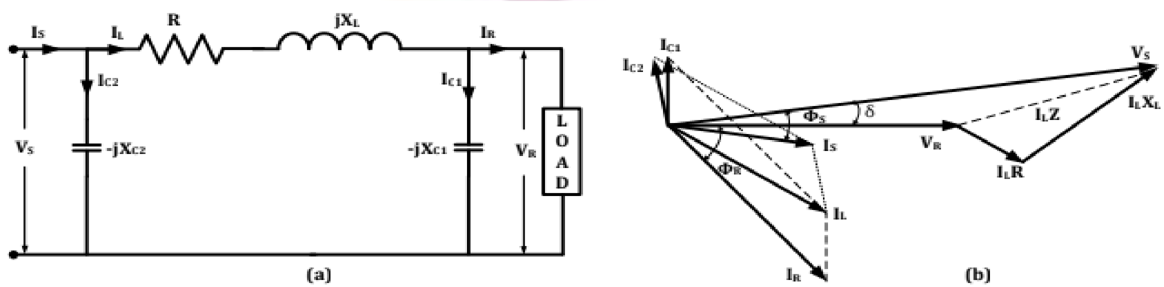


Figure 1. 11 Nominal π method (a) Circuit and (b) Vector Diagram (Voltage as Reference)

$$V_R = 132 \times 10^3 \angle 0 = 132 \times 10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \quad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$R = 0.15 \times 100 = 15 \ \Omega/ph$$

$$X_L = 0.377 \times 100 = 37.7 \ \Omega/ph$$

$$X_C = 31.87 \times 100 = 3187 \ \Omega/ph$$

$$X_{C1} = 2X_C = 6374 \ \Omega/ph$$

$$X_{C2} = 2X_C = 6374 \ \Omega/ph$$

$$P_R = \sqrt{3} V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3} V_R \cos\phi_R} = \frac{72 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0.8} = 393.64 \ A$$

$$I_R = I_R \angle -\phi_R = 393.64 \angle -36.86 = 314.95 - j236.12$$

$$I_{C1} = \frac{V_R}{-jX_{C1}} = \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{-j6374} = \frac{\frac{132\times10^3}{\sqrt{3}}\angle 0}{6374\angle -90} = 11.95\angle 90 \text{ A}$$

$$\begin{aligned}
I_L &= I_R + I_{C1} \\
&= 393.64\angle -36.86 + 11.95\angle 90 \\
&= 314.95 - 236.12j + 0 + 11.95j \\
&= 314.95 - 224.17j \\
&= 386.58\angle -35.44 \text{ A}
\end{aligned}$$

$$\begin{aligned}
V_S &= V_R + I_L Z \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (386.58\angle -35.44)(15 + j37.7) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + (386.58\angle -35.44)(40.57\angle 68.30) \\
&= \frac{132\times10^3}{\sqrt{3}}\angle 0 + 15683.55\angle 32.86 \\
&= 76210.23 + 0j + 13174.16 + 8509.70j \\
&= 89384.39 + 8509.70j \\
&= 89788.55\angle 5.43 \text{ V} \\
&= 89.78\angle 5.43 \text{ kV}
\end{aligned}$$

$$\begin{aligned}
V_S &= \sqrt{3}\times 89.78\angle 5.43 \text{ kV} \\
&= 155.5\angle 5.43 \text{ kV}
\end{aligned}$$

$$\begin{aligned}
I_{C2} &= \frac{V_S}{-jX_{C2}} \\
&= \frac{89788.55\angle 5.43}{-j6374} \\
&= \frac{89788.55\angle 5.43}{6374\angle -90} \\
&= 14.08\angle 95.43 \text{ A}
\end{aligned}$$

$$\begin{aligned}
I_S &= I_L + I_{C2} \\
&= 386.58\angle -35.44 + 14.08\angle 95.43 \\
&= 314.95 - 224.17j - 1.33 + 14.01j \\
&= 313.62 - 210.16j \\
&= 377.52\angle -33.82 \text{ A}
\end{aligned}$$

$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R} \times 100$$

$$= \frac{\left(\frac{155.5 \times 10^3}{\sqrt{3}}\right) - \left(\frac{132 \times 10^3}{\sqrt{3}}\right)}{\left(\frac{132 \times 10^3}{\sqrt{3}}\right)} \times 100$$

$$= 17.80$$

$$\%\text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3 \times \left(\frac{132 \times 10^3}{\sqrt{3}}\right) \times 393.64 \times 0.8}{3 \times \left(\frac{155.5 \times 10^3}{\sqrt{3}}\right) \times 377.52 \times 0.77} \times 100$$

$$= 91.96$$

Comparison of Performance of Medium Transmission Lines

Quantity		End condenser method	Nominal T method	Nominal π method
P_R	MW	72	72	72
V_R	kV	$132\angle 0$	$132\angle 0$	$132\angle 0$
I_R	A	$393.64\angle -36.86$	$393.64\angle -36.86$	$393.64\angle -36.86$
$\cos\Phi_R$	lag	0.8	0.8	0.8
P_S	MW	78.38	78.19	78.28
V_S	kV	$154.75\angle 5.58$	$155.41\angle 5.43$	$155.50\angle 5.43$
I_S	A	$379.77\angle -33.97$	$377.50\angle -33.81$	$377.52\angle -33.82$
$\cos\Phi_S$	lag	0.77	0.77	0.77
%VR	---	16.66	17.73	17.80
%η	---	91.85	92.08	91.96

5. A 3-Φ overhead transmission line has a total series impedance per phase at $200\angle 80°$ Ω/ph and a total shunt admittance at $0.0013\angle 90°$ Siemens/ph. The line delivered a load at 80 MW at 0.8 power factor lagging and 220 kV

between the lines. Determine sending end current and sending end voltage by rigorous method.

Solution:

$$V_R = 220\times10^3\angle0 = 220\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = 200\angle80 = 34.72 + 196.96j\ \Omega/ph$$
$$Y = 0.0013\angle90 = 0 + 0.0013j\ /ph$$

$$P_R = \sqrt{3}V_R I_R\cos\phi_R$$
$$I_R = \frac{P_R}{\sqrt{3}V_R\cos\phi_R} = \frac{80\times10^6}{\sqrt{3}\times220\times10^3\times0.8} = 262.43\ A$$

$$I_R = I_R\angle-\phi_R = 262.43\angle-36.86 = 209.97 - 157.42j$$

$$Cosh(\sqrt{YZ}) = \left(1 + \frac{YZ}{2} + \frac{Y^2Z^2}{24} + \cdots\right)$$
$$= 1 + \frac{(0.0013\angle90)(200\angle80)}{2} + \cdots$$
$$= 1 + 0.13\angle170$$
$$= 1 - 0.128 + 0.0225j$$
$$= 0.872 + 0.0225j$$
$$= 0.872\angle1.47$$

$$Sinh(\sqrt{YZ}) = \left(\sqrt{YZ} + \frac{(YZ)^{\frac{3}{2}}}{6} + \cdots\right)$$
$$= \sqrt{(0.0013\angle90)(200\angle80)} + \cdots$$
$$= \sqrt{0.26\angle170}$$
$$= 0.50\angle85$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0.0013\angle90}{200\angle80}}$$
$$= \sqrt{6.5\times10^{-6}\angle10}$$
$$= 2.54\times10^{-3}\angle5$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{200\angle80}{0.0013\angle90}}$$
$$= \sqrt{153846.15\angle-10}$$
$$= 392.23\angle-5$$

$$V_S = V_R Cosh\left(\sqrt{YZ}\right) + I_R\sqrt{\frac{Z}{Y}}Sinh\left(\sqrt{YZ}\right)$$

$$= \left(\frac{220\times10^3}{\sqrt{3}}\angle0\right)(0.872\angle1.47) + (262.43\angle-36.86)(392.23\angle-5)(0.50\angle85)$$

$$= 110758.87\angle1.47 + 51466.45\angle43.14$$

$$= 110722.41 + 2841.35j + 37554.30 + 35191.90j$$

$$= 148276.71 + 38033.25j$$

$$= 153076.81\angle14.38 \text{ V/ph}$$

$$= 153.07\angle14.38 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times153.07\angle14.38$$

$$= 265.12\angle14.38 \text{ kV}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}}Sinh\left(\sqrt{YZ}\right) + I_R Cosh\left(\sqrt{YZ}\right)$$

$$= \left(\frac{220\times10^3}{\sqrt{3}}\angle0\right)(2.54\times10^{-3}\angle5)(0.50\angle85) + (262.43\angle-36.86)(0.872\angle1.47)$$

$$= 161.31\angle90 + 228.83\angle-35.39$$

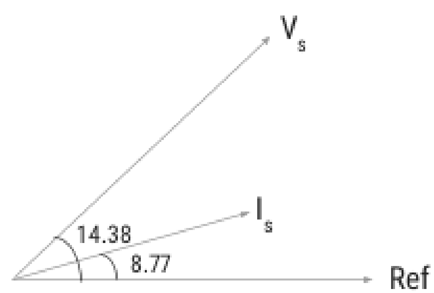
$$= 0 + 161.31j + 186.54 - 132.52j$$

$$= 186.54 + 28.79j$$

$$= 188.74\angle8.77 \text{ A}$$

$$\phi_S = 14.38 - 8.77 = 5.61$$

$$\cos\phi_S = \cos(5.61) = 0.99(lag)$$



$$\%\text{Voltage Regulation} = \frac{V_S - V_R}{V_R}\times100$$

$$= \frac{\left(\frac{265.12\times10^3}{\sqrt{3}}\right) - \left(\frac{220\times10^3}{\sqrt{3}}\right)}{\left(\frac{220\times10^3}{\sqrt{3}}\right)}\times100$$

$$= 20.50$$

$$\% \text{ Transmission Efficiency} = \frac{3V_R I_R \cos\phi_R}{3V_S I_S \cos\phi_S} \times 100$$

$$= \frac{3\times\left(\frac{220\times10^3}{\sqrt{3}}\right)\times 262.43 \times 0.8}{3\times\left(\frac{265.12\times10^3}{\sqrt{3}}\right)\times 188.74\times 0.99}\times 100$$

$$= 93.23$$

---

6. A 3-Φ overhead transmission line has a total series impedance per phase at 200∠80° Ω/ph and a total shunt admittance at 0.0013∠90° Siemens/ph. The line delivered a load at 80 MW at 0.8 power factor lagging and 220 kV between the lines. Determine sending end current and sending end voltage by ABCD parameter method.

Solution:

$$V_R = 220\times10^3\angle 0 = 220\times10^3 + j0$$

$$\cos\phi_R = 0.8(lag) \qquad \Rightarrow \phi_R = \cos^{-1}(0.8) = 36.86$$

$$Z = 200\angle 80 = 34.72 + 196.96j \ \Omega / ph$$

$$Y = 0.0013\angle 90 = 0 + 0.0013j \ ℧ \ / ph$$

$$P_R = \sqrt{3} V_R I_R \cos\phi_R$$

$$I_R = \frac{P_R}{\sqrt{3} V_R \cos\phi_R} = \frac{80\times10^6}{\sqrt{3}\times 220\times10^3\times 0.8} = 262.43\ A$$

$$I_R = I_R\angle -\phi_R = 262.43\angle -36.86 = 209.97 - 157.42j$$

A). Short transmission lines:

$A = 1$

$B = Z = 200\angle 80$

$C = 0$

$D = 1$

$V_S = AV_R + BI_R$

$= (1)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$

$= 127017.05\angle 0 + 52486\angle 43.14$

$= 127017.05 + 0j + 38298.25 + 35889.05j$

$= 165315.30 + 35889.05j$

$= 169166.1\angle 12.24$ V/ph

$= 169.16\angle 12.24$ kV/ph

$V_S = \sqrt{3}\times 169.16\angle 12.24$

$= 292.99\angle 12.24$ kV

$I_S = CV_R + DI_R$

$= (0)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (1)(262.43\angle -36.86)$

$= 262.43\angle -36.86$ A

B). Medium transmission lines (End condenser method):

$A = 1 + YZ$

$= 1 + (0.0013\angle 90)(200\angle 80)$

$= 1 + 0.26\angle 170$

$= 1 - 0.25 + 0.045j$

$= 0.75 + 0.045j$

$= 0.75\angle 3.43$

$B = Z = 200\angle 80$

$C = Y = 0.0013\angle 90$

$D = 1$

$V_S = AV_R + BI_R$

$= (0.75\angle 3.43)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$

$= 952627.79\angle 3.43 + 52486\angle 43.14$

$= 95092.14 + 5699.48j + 38298.25 + 35889.05j$

$= 133390.39 + 41588.53j$

$= 139723.30\angle 17.31$ V/ph

$= 139.72\angle 17.31$ kV/ph

$V_S = \sqrt{3}\times 139.72\angle 17.31$

$= 242.00\angle 17.31$ kV

$I_S = CV_R + DI_R$

$= (0.0013\angle 90)\left(\frac{220\times 10^3 \angle 0}{\sqrt{3}}\right) + (1)(262.43\angle -36.86)$

$= 165.12\angle 90 + 262.43\angle -36.86$

$= 0j + 165.12j + 209.97 - 157.42j$

$= 209.97 + 7.7j$

$= 210.11\angle 2.1$ A

C). Medium transmission lines (Nominal T method):

$$A = 1+\frac{YZ}{2}$$
$$= 1+\frac{(0.0013\angle 90)(200\angle 80)}{2}$$
$$= 1+0.13\angle 170$$
$$= 1-0.128+0.022j$$
$$= 0.875+0.022j$$
$$= 0.875\angle 1.44$$

$$B = Z+\frac{YZ^2}{4}$$
$$= 200\angle 80+\frac{(0.0013\angle 90)(200\angle 80)^2}{4}$$
$$= 200\angle 80+13\angle 250$$
$$= 34.72+196.96j-4.44-12.21j$$
$$= 30.28+184.75j$$
$$= 187.21\angle 80.69$$

$$C = Y = 0.0013\angle 90$$
$$D = 1+\frac{YZ}{2} = 0.875\angle 1.44$$

$$V_s = AV_R + BI_R$$
$$=(0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right)+(187.21\angle 80.69)(262.43\angle -36.86)$$
$$=111139.92\angle 1.44+49129.52\angle 43.83$$
$$=111104.82+2792.95j+35441.92+34023.22j$$
$$=146546.74+36816.17j$$
$$=151100.55\angle 14.10 \text{ V/ph}$$
$$=151.10\angle 14.10 \text{ kV/ph}$$

$$V_s = \sqrt{3}\times 151.10\angle 14.10$$
$$=261.71\angle 14.10 \text{ kV}$$

$$I_s = CV_R + DI_R$$
$$=(0.0013\angle 90)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right)+(0.875\angle 1.44)(262.43\angle -36.86)$$
$$=165.12\angle 90+229.62\angle -35.42$$
$$=0j+165.12j+187.12-133.07j$$
$$=187.12+32.05j$$
$$=189.84\angle 9.71 \text{ A}$$

D). Medium transmission lines (Nominal π method):

$$A = 1 + \frac{YZ}{2}$$
$$= 1 + \frac{(0.0013\angle 90)(200\angle 80)}{2}$$
$$= 1 + 0.13\angle 170$$
$$= 1 - 0.128 + 0.022j$$
$$= 0.875 + 0.022j$$
$$= 0.875\angle 1.44$$
$$B = Z = 200\angle 80$$

$$C = Y + \frac{Y^2 Z}{4}$$
$$= 0.0013\angle 90 + \frac{(0.0013\angle 90)^2(200\angle 80)}{4}$$
$$= 0.0013\angle 90 + 8.45\times 10^{-5}\angle 260$$
$$= 0 + 0.0013j - 1.46\times 10^{-5} - 8.32\times 10^{-5}j$$
$$= -1.46\times 10^{-5} + 1.21\times 10^{-3}j$$
$$= 1.21\times 10^{-3}\angle 90.68$$
$$D = 1 + \frac{YZ}{2} = 0.875\angle 1.44$$

$$V_s = AV_R + BI_R$$
$$= (0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$$
$$= 111139.92\angle 1.44 + 52486\angle 43.14$$
$$= 111104.82 + 2792.95j + 38298.25 + 35889.05j$$
$$= 149403.07 + 38682j$$
$$= 154329.43\angle 14.51 \text{ V/ph}$$
$$= 154.32\angle 14.51 \text{ kV/ph}$$

$$V_s = \sqrt{3}\times 154.32\angle 14.51$$
$$= 267.29\angle 14.51 \text{ kV}$$

$$I_s = CV_R + DI_R$$
$$= (1.21\times 10^{-3}\angle 90.68)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (0.875\angle 1.44)(262.43\angle -36.86)$$
$$= 153.69\angle 90.68 + 229.62\angle -35.42$$
$$= -1.82 + 153.67j + 187.12 - 133.07j$$
$$= 185.3 + 20.6j$$
$$= 186.44\angle 6.34 \text{ A}$$

E). Long transmission lines:

$$A = Cosh(\sqrt{YZ})$$
$$= 1 + \frac{YZ}{2}$$
$$= 1 + \frac{(0.0013\angle 90)(200\angle 80)}{2}$$
$$= 1 + 0.13\angle 170$$
$$= 1 - 0.128 + 0.022j$$
$$= 0.875 + 0.022j$$
$$= 0.875\angle 1.44$$

$$B = \sqrt{\frac{Z}{Y}} Sinh(\sqrt{YZ}) = \sqrt{\frac{Z}{Y}}\sqrt{YZ} = Z$$
$$= 200\angle 80$$
$$C = \sqrt{\frac{Y}{Z}} Sinh(\sqrt{YZ}) = \sqrt{\frac{Y}{Z}}\sqrt{YZ} = Y$$
$$= 0.0013\angle 90$$
$$D = Cosh(\sqrt{YZ}) = 0.875\angle 1.44$$

$$V_S = AV_R + BI_R$$
$$= (0.875\angle 1.44)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (200\angle 80)(262.43\angle -36.86)$$
$$= 111139.92\angle 1.44 + 52486\angle 43.14$$
$$= 111104.82 + 2792.95j + 38298.25 + 35889.05j$$
$$= 149403.07 + 38682j$$
$$= 154329.43\angle 14.51 \text{ V/ph}$$
$$= 154.32\angle 14.51 \text{ kV/ph}$$

$$V_S = \sqrt{3}\times 154.32\angle 14.51$$
$$= 267.29\angle 14.51 \text{ kV}$$

$$I_S = CV_R + DI_R$$
$$= (0.0013\angle 90)\left(\frac{220\times 10^3\angle 0}{\sqrt{3}}\right) + (0.875\angle 1.44)(262.43\angle -36.86)$$
$$= 165.12\angle 90 + 229.62\angle -35.42$$
$$= 0 + 165.12j + 187.12 - 133.07j$$
$$= 187.12 + 20.6j$$
$$= 188.25\angle 6.28 \text{ A}$$

$V = V_{rated} \pm 5\%$

if $V > V_{rated} + 5\%$ (or) $V > 1.05$ pu
if $V < V_{rated} - 5\%$ (or) $V < 0.95$ pu
} then V – control has to be done.
only under steady state

* steady state over voltages are due to –

Practical (i) Ferranti effect (loading < SIL)

(ii) leading p.f loads.

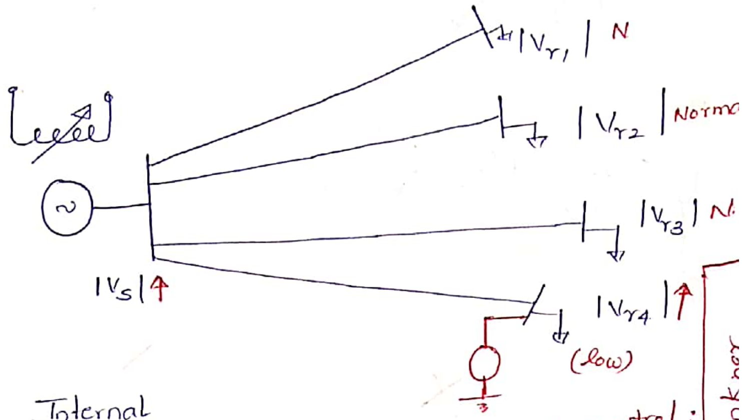
Scanned with CamScanner

(ii) steady state under vtgs are due

More practical

(i) lagging p.f loads : heavy load ①

② p.f is too low



Internal
(or)
Excitation control Method for V-control :

Excitation of alternator changed to control vtg at required buses.

* other bus vtgs will be disturbed.
(disadvantage)

External Method for V-control :

External compensating devices are placed at locations, where ever it is required to improve voltage.

→ P, Q, |V|, δ, Excitation, f.

| P, δ, f |

| Q, |V|, Excitation |

} independent groups

↓

voltage stability (or)

Voltage control Analysis.

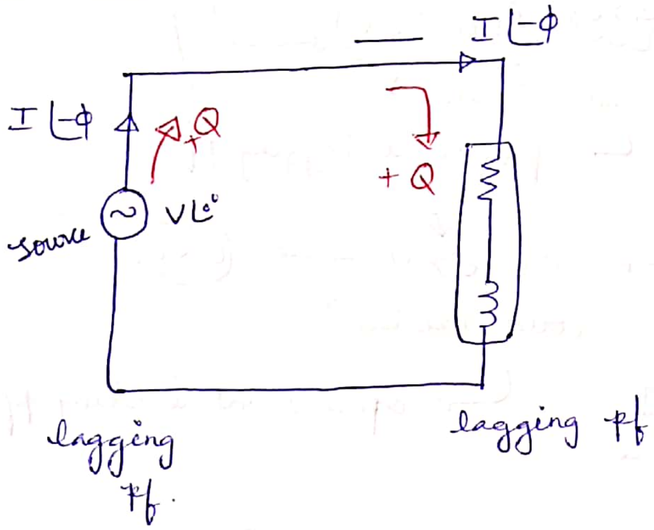
For |V| - control reactive power compensation will be done.

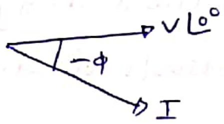
Scanned with CamScanner

Synchronous Generator : comes under "internal control method".

Source of Q (alternator) : Over Excitation (lagging pf)

$Q = \frac{V}{X}[E\cos\delta - V]$ ($E\cos\delta > V$)





load

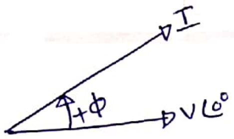
Q>0 → absorbs Q lagging
delivers leading Q

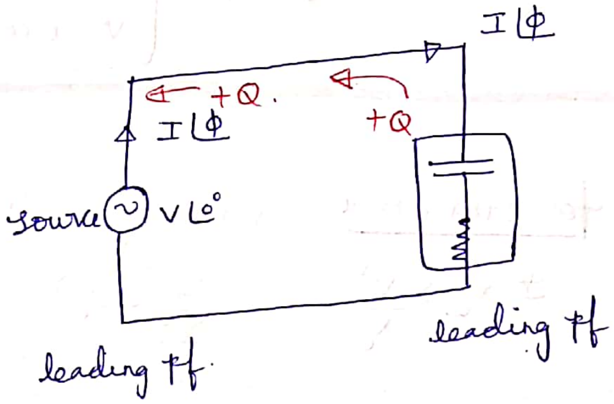
source

Q>0 → delivers lagging Q
absorbs leading Q

Sink of Q : under Excitation (leading pf)

$Q = \frac{V}{X}[E\cos\delta - V]$ ($E\cos\delta < V$)





load

Q<0 → absorbs leading Q
delivers lagging Q

source

Q<0 → delivers leading Q.
absorbs lagging Q

Neither sink (or) source of Q : Normal Excitation (UPF) → Q = 0 ($E\cos\delta = V$)

Induction Generator (→ source) : Excitation = 0 → sink of Q

Induction machine always absorbs lagging Q. (Q>0)

whenever source absorbs lagging Q ; that means it has to receive from external source, which is delivers lagging Q

∴ so it is operated at leading Q.

Induction motor (→ load) : Excitation = 0 → Sink of Q

IM absorbs lagging Q (Q>0)

whenever load absorbs lagging Q, so pf is lagging Q.

Synchronous motor: (load)

case (i): sink of Q (sink = absorbs)

$$Q = \frac{E}{X}\left[V - E\cos\delta\right]$$

Q>0 (E cosδ < V)

acts like inductor
- ✓ ∴ absorbs lagging Q. (under Excited)
- ✓ delivers leading Q ↳ operates at lagging pf

case (ii): source of Q → E cosδ > V → Q<0

(over Excited)

acts like capacitor
- ✓ delivers lagging Q. ↳ operates at leading pf.
- ✓ absorbs leading Q.

case (iii) Normal Excitation → Neither sink nor source Q.
→ unity pf.

$Q = 0$
$V = E\cos\delta$

Transmission line

case i → sink of Q — for over load (loading > SIL)

$$I^2 X_L > \frac{V^2}{X_C}$$

$Z_L < Z_C$

case (ii) → source of Q — for light load (loading < SIL)

$$I^2 X_L < \frac{V^2}{X_C}$$

$Z_L > Z_C$

case iii → Neither sink nor source Q

$$I^2 X_L = \frac{V^2}{X_C}$$

for lossless line at loading = SIL

surge impedance loading

$Z_L = Z_C$

Basic V – control devices :

1. shunt capacitor bank
2. shunt reactor bank
3. series capacitor (→ only series connected.)

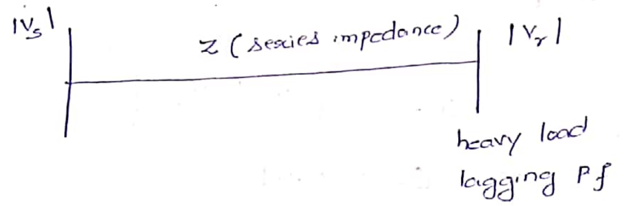
} static Devices

4. synchronous condenser.
5. Synchronous coil

} Dynamic devices.

6. synchronous phase modifier / phase advancer.

Scanned with CamScanner

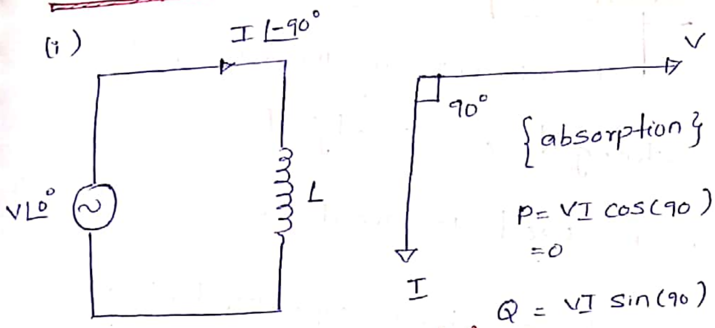


To improve $|V_r|$ vtg drop $= IZ$ has to be chang…

* series impedance of line has to be modified. ⟶ series connected v-control d…
* current in the line has to be modified. ⟶ shunt connected v-control device.

Inductor :

(i)



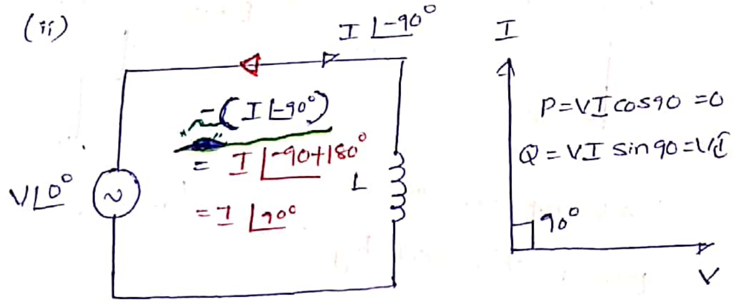
{absorption}

$P = VI\cos(90)$
$= 0$

$Q = VI\sin(90)$
$= VI$

'L' absorbs Lagging 'Q'

(ii)



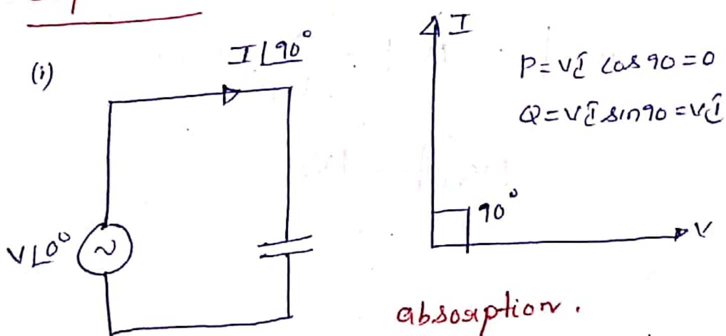
$P = VI\cos 90 = 0$
$Q = VI\sin 90 = VI$

{Delivery}

'L' delievers leading 'Q'

Capacitor :

(i)



$P = VI\cos 90 = 0$
$Q = VI\sin 90 = VI$

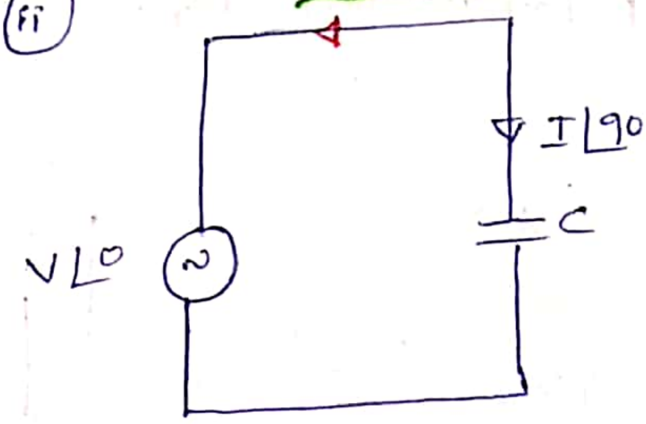
absorption.

'c' absorbs leading 'Q'

Scanned with CamScanner

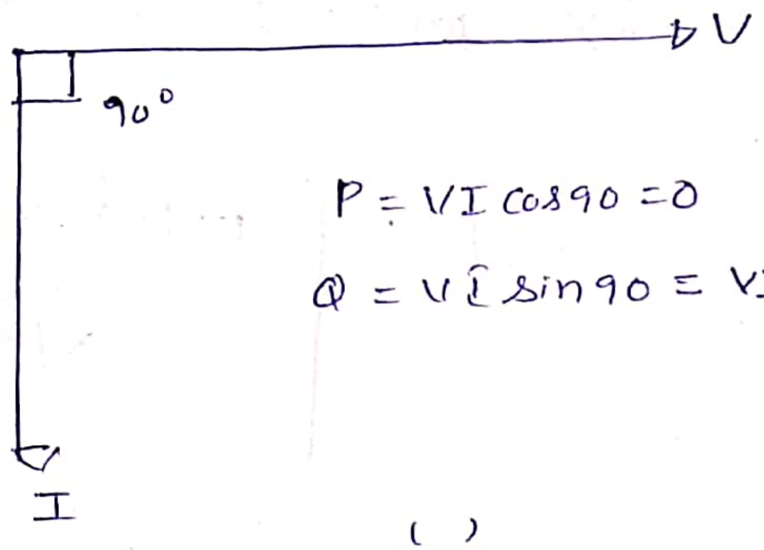
(ii)

$-(I\angle 90^\circ) = I\angle 90^\circ - 180^\circ = I\angle -90^\circ$



{delievering}

ged



device

$P = VI\cos 90 = 0$

$Q = VI\sin 90 = VI$

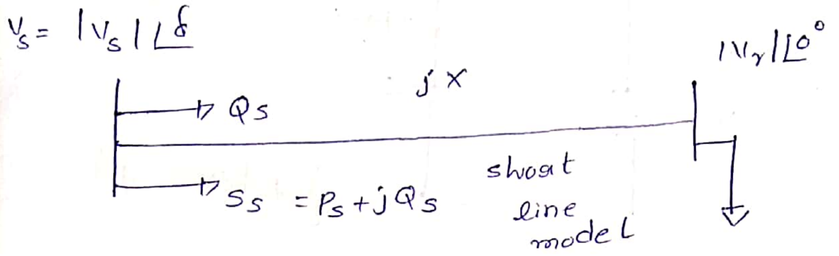
'c' delivers lagging 'Q'.

* A sink of lagging Q acts as source of leading Q

A source of lagging Q acts as sink of leading 'Q'.

Scanned with CamScanner

Mathematical Relation b/n $|V_r|$ and Q_s



Complex power: $S_s = V_s I_s^*$

$$S_s = |V_s| \angle \delta \left[\frac{|V_s| \angle \delta - |V_r| \angle 0°}{X \angle 90°} \right]^*$$

$$= |V_s| \angle \delta \left[\frac{|V_s|}{X} \angle \delta - 90 - \frac{|V_r|}{X} \angle -90 \right]^*$$

$$= |V_s| \angle \delta \left[\frac{|V_s|}{X} \angle 90 - \delta - \frac{|V_r|}{X} \angle 90° \right]$$

$$S_s = \frac{|V_s|^2}{X} \angle 90 - \frac{|V_s||V_r|}{X} \angle 90 + \delta$$

$\downarrow$

$P_s + jQ_s$

Net reactive power (or) Sending end reactive power

$$Q_s = \frac{|V_s|^2}{X} \sin 90 - \frac{|V_s||V_r|}{X} \sin(90 + \delta)$$

$$Q_s = \frac{|V_s|^2}{X} - \frac{|V_s||V_r|}{X} \cos \delta$$

$$Q_s = \frac{|V_s|}{X} \left[|V_s| - |V_r| \cos \delta \right]$$

Scanned with CamScanner

(or) Q_{net} : if system is highly stable

$\Rightarrow \cos\delta \simeq 1$

$$Q_s \simeq \frac{|V_s|}{X}\left[|V_s| - |V_r|\right]$$

$$Q_s \propto \left[|V_s| - |V_r|\right]$$

$$Q_s \propto \Delta |V|$$

$Q_s \propto$ Magnitude of vtg regulation

From ①: $Q_s \frac{X}{|V_s|} = |V_s| - |V_r|$

$$|V_r| = |V_s| - Q_s \frac{X}{|V_s|}$$

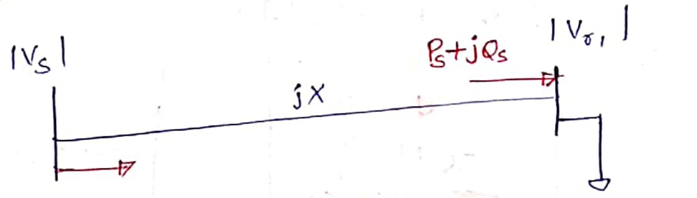
as $Q_s \downarrow$, $|V_r| \uparrow$

shunt capacitor bank:

→ current in the line gets changed (or) modified.

→ used to avoid Low vtg profile

→ without shunt capacitor bank:



Assumption reactive power absorbed by line is zero.

$$P_s = P_{load} \; ; \; Q_s = Q_{load}$$

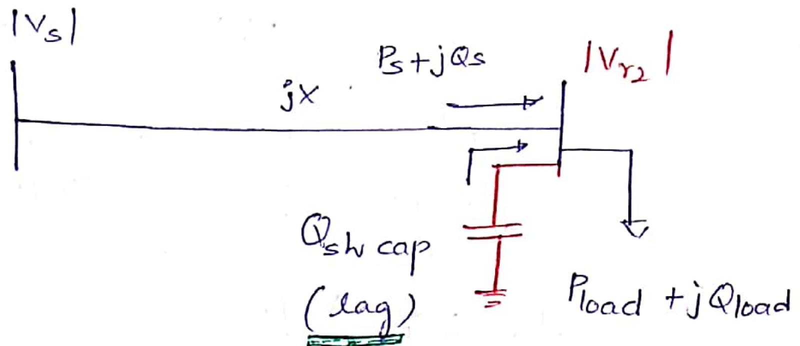
$$|V_{r1}| = |V_s| - \frac{X}{|V_s|} Q_s$$

$$|V_{r1}| = |V_s| - \frac{X}{|V_s|} Q_{load} \rightarrow ①.$$

$$|V_{r1}| < |V_s|.$$

Scanned with CamScanner

With shunt capacitor bank :



→ delivers lagging Q

$P_s = P_{load}$; $Q_s + Q_{shcap} = Q_{load}$.

$\Rightarrow Q_s = Q_{load} - Q_{shcap}$.

$$|V_{r_2}| = |V_s| - \frac{X}{|V_s|} Q_s$$

$$|V_{r_2}| = |V_s| - \frac{X}{|V_s|} (Q_{load} - Q_{shcap}) \longrightarrow (2).$$

from (1) & (2) : $|V_{r_2}| > |V_{r_1}|$

Magnitude vtg increment at receiving end side ;

$$\Delta V_c = |V_{r_2}| - |V_{r_1}| \Rightarrow eq^n(2) - eq^n(1)$$

$$\Delta V_c = \frac{X}{|V_s|} Q_{shunt\ capacitor}$$

Rating of capacitor bank for vtg rise ;

$$Q_{sh\ cap} = \frac{\Delta V_c \cdot |V_s|}{X}$$

$X \rightarrow$ reactance of trans. line.

F.

(i)

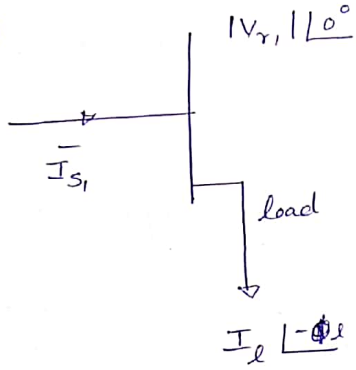
(ii)

1

Scanned with CamScanner

phasor diagram :

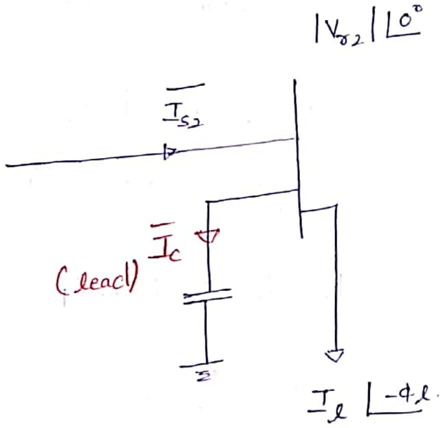
(i)



$\cos\phi_l$ lag

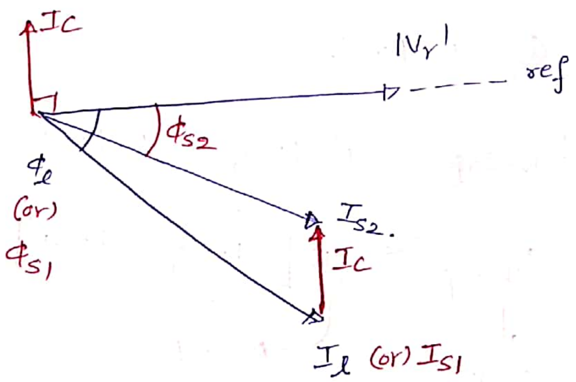
$$\bar{I}_{s1} = I_l \angle -\phi_l$$

(ii)



(1)

$$\bar{I}_{s2} = I_l \angle -\phi_l + \bar{I}_c$$



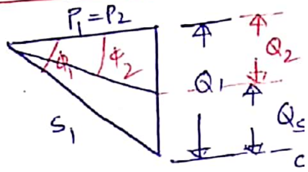
$$|I_{s2}| < |I_{s1}|$$

$$\phi_{s2} < \phi_{s1}$$

net pf ; $\cos\phi_{s2} > \cos\phi_{s1}$ net pf got improved.

Practically shunt capacitor known as p.f correction device. Because p.f improvement is more appreciable as compared to voltage improvement.

* To improve p.f from $\cos\phi_1$ lag to $\cos\phi_2$ lag:



$P_1, Q_1 \rightarrow$ supplied by source without shunt capacitor bank

$P_2, Q_2 \rightarrow$ supplied by source with shunt capacitor bank

$P_2 = P_1$; $Q_2 < Q_1$

$\Rightarrow Q_{\text{shunt capacitor}} = Q_1 - Q_2$.

$P = S\cos\phi \Rightarrow S = P/\cos\phi$

$Q = S\sin\phi \Rightarrow Q = \frac{P}{\cos\phi} \cdot \sin\phi$

$\Rightarrow \boxed{Q = P\tan\phi}$

$Q_1 = P_1 \tan\phi_1$

$Q_2 = P_2 \tan\phi_2$

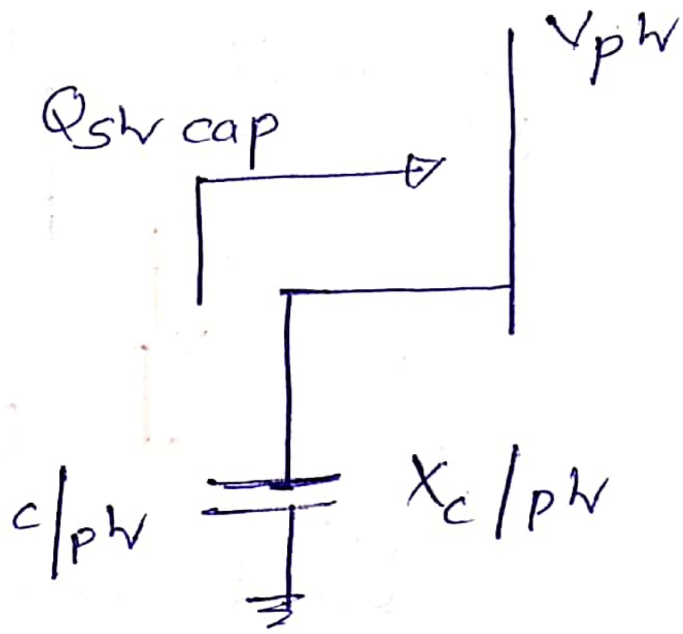
$Q_{sh\ cap} = P_1\tan\phi_1 - P_2\tan\phi_2$

$Q_{sh\ cap} = P_1[\tan\phi_1 - \tan\phi_2]$

$Q_{sh\ cap} \rightarrow 3\text{-}\phi$; $P_1 \rightarrow 3\text{-}\phi$

$\rightarrow$ For less less.

* During operation:



$$Q_{sh\,cap}/ph = \frac{V_{ph}^2}{X_C/ph}$$

shunt capacitor

$$Q_{sh\,cap}\ (3-\phi) = 3 \cdot \frac{V_{ph}^2}{X_C/ph}$$

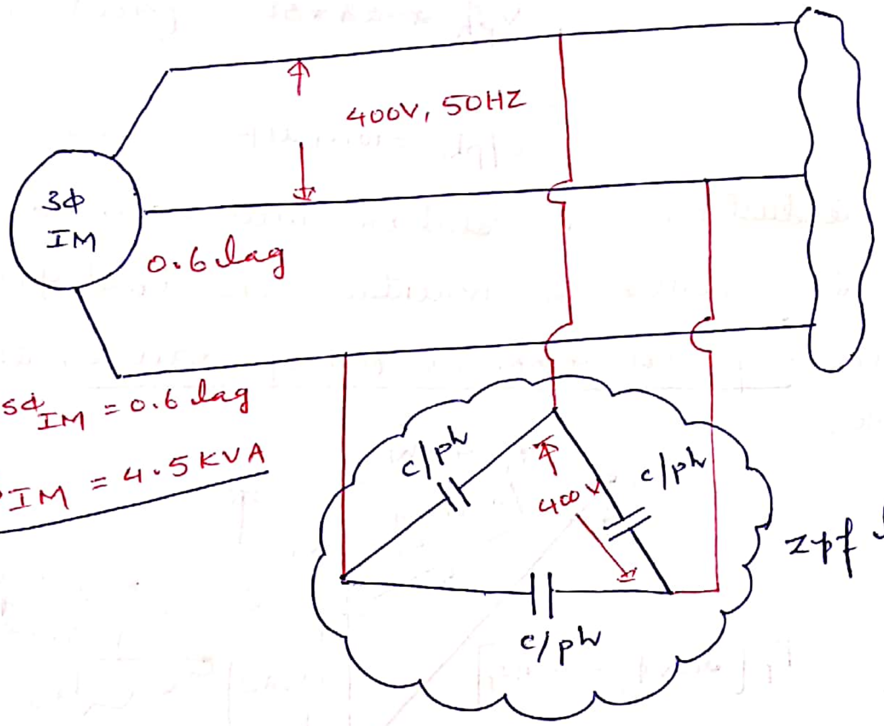
$$= 3 \cdot V_{ph}^2 \cdot 2\pi f\, C/ph.$$

$$\boxed{Q_{sh\,cap} \propto V^2 \cdot f}$$

Scanned with CamScanner

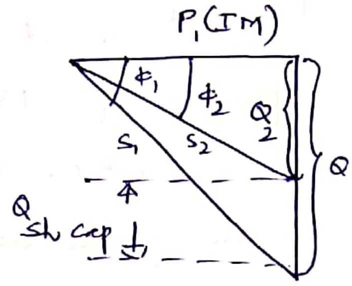
Example : A 3ϕ induction motor rated at 400V, 50HZ coupled to a pump is running at a lower power factor of 0.6. The input is 4.5 KVA. It is proposed to improve the power factor to 0.8 by connecting a delta connected capacitor bank. Find the value of capacitance per phase.

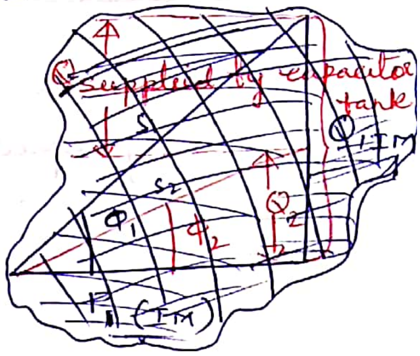
Solution :



$\cos\phi_{IM} = 0.6$ lag

$S_{IM} = 4.5$ KVA





$$Q_{1\,IM} = Q_c + Q_{source}.$$

$\cos\phi_1 = 0.6$ lag ; $\cos\phi_2 = 0.8$ lag.

$$Q_{shunt\ capacitor} = P_1\left[\tan\phi_1 - \tan\phi_2\right]$$
$$= Q_1 - Q_2$$
$$= P_1\left[\tan\phi_1 - \tan\phi_2\right]$$

$Q_1 = S_1 \sin\phi_1$
$P_1 = S_1 \cos\phi_1$
$S_1 = \dfrac{P_1}{\cos\phi_1}$
$Q_1 = \dfrac{P_1}{\cos\phi_1} \cdot \sin\phi_1$
$Q_1 = P_1 \tan\phi_1$
$Q_2 = P_1 \tan\phi_2$

$$P_1 = S_1\cos\phi_1 = P_{IM} = S_{IM}\cos\phi_{IM}$$
$$P_1 = 4.5 * 0.6 = 2.7\ \text{KW}\ (3\phi)$$

$$Q_{shunt\ capacitor\ (3\phi)} = 2.7\left(\tan(\cos^{-1}(0.6)) - \tan(\cos^{-1}(0.8))\right)$$
$$= 2.7(1.33 - 0.75) = 1.575\ \text{KVAR}.$$

Scanned with CamScanner

$Q_{shunt\ capacitor} / ph = \frac{1.575}{3} = 0.525\ KVAR$

$$Q_{sh\ cap}/ph = \frac{V_{ph}^2}{X_c/ph} = V_{ph}^2 * \omega(c/ph)$$

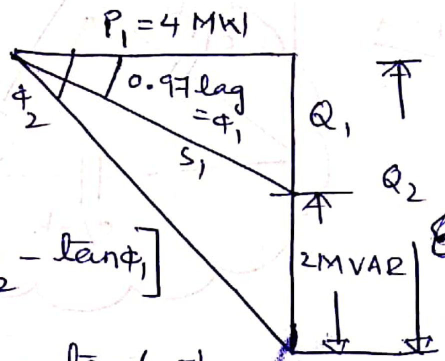
$$= V_{ph}^2 * 2\pi f * c/ph$$

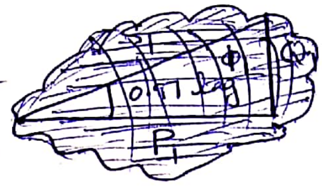
$$c/ph = \frac{Q_{sh\ cap}/ph}{V_{ph}^2 * 2\pi * 50} = \frac{0.525 * 10^3}{(400)^2 * 2\pi * 50}$$

$$c/ph = 10.4\ \mu F.$$

Example ② At an industrial sub-station with a 4 MW load, a capacitor of 2 MVAR is installed to maintain the load power factor at 0.97 lag. If the capacitor tank is out of service, the load power factor will be ---

<u>Example solution :-</u>





Reactive power supplied by capacitor $= P_1 [\tan\phi_2 - \tan\phi_1]$

$$2 = 4 [\tan\phi_2 - \tan(\cos^{-1}(0.97))]$$

$$\frac{1}{2} = \tan\phi_2 - 0.25 \Rightarrow 0.75 = \tan\phi_2$$

$$\phi_2 = 36.87^\circ$$

$$\text{power factor} = \cos\phi_2 = 0.8\ \text{lagging}$$

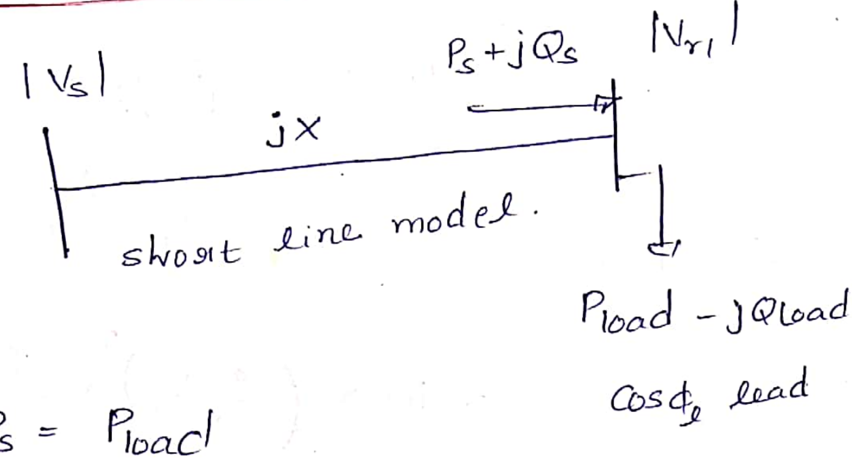
Scanned with CamScanner

Shunt reactor or Shunt inductor:

↳ current in the line gets modified.

→ used to avoid over vtgs under steady state conditions

Without shunt reactor:



$P_s = P_{load}$

$Q_s = -Q_{load}$

$$|V_{r1}| = |V_s| - \frac{X}{|V_s|} Q_s$$

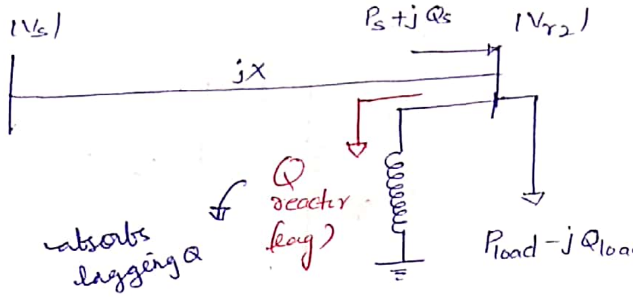
$$= |V_s| - \frac{X}{|V_s|}(-Q_{load})$$

$$|V_{r1}| = |V_s| + \frac{X}{|V_s|} Q_{load} \longrightarrow (1)$$

$$|V_{r1}| > |V_s|.$$

Scanned with CamScanner

With shunt reactor :



$P_s = P_{Load}$; $Q_s = Q_{reactor} - Q_{load}$.

$$|V_{r2}| = |V_s| - \frac{X}{|V_s|} Q_s$$

$$|V_{r2}| = |V_s| - \frac{X}{|V_s|} (Q_{reactor} - Q_{load})$$

$\Longrightarrow$ ②

$$|V_{r2}| = |V_s| + \frac{X}{|V_s|} (Q_{load} - Q_{reactor})$$

from ① & ② $\Rightarrow$ $|V_{r2}| < |V_{r1}|$

* decrement in vtg magnitude

$$\Delta V_r = |V_{r1}| - |V_{r2}| \Rightarrow eq^n ① - eq^n ②$$

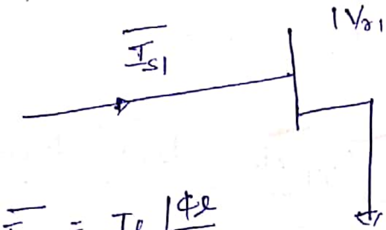
$$\Delta V_r = \frac{X}{|V_s|} Q_{reactor}.$$

* Rating of $Q_{reactor} = \dfrac{\Delta V_r |V_s|}{X_{th}}$

X_{th} ↓ From Z-bus elements.

phasor diagram :

(i)

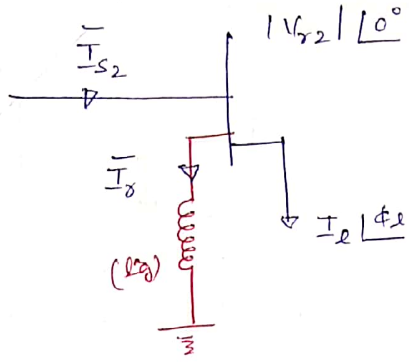


$\bar{I}_{s1} = I_l \angle \phi_l$ $\qquad$ $I_l \angle +\phi_l$.

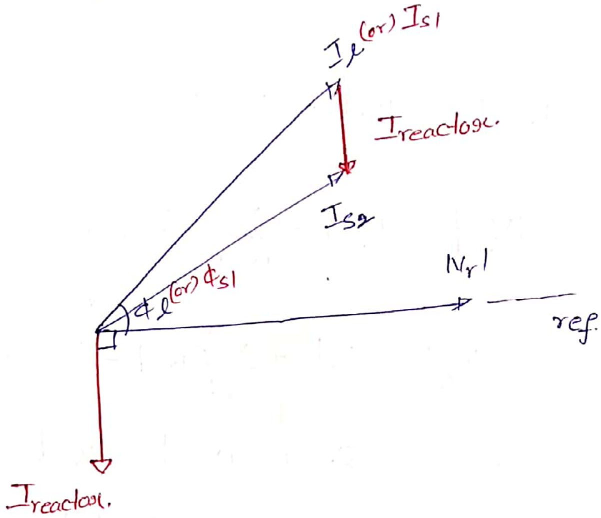
$\cos\phi_l$ leading

Scanned with CamScanner

(ii)



$\bar{I}_{s_2} = I_l \angle \phi_l + \bar{I}_{reactor}$



$|I_{s_2}| < |I_{s_1}|$

$\phi_{s_2} < \phi_{s_1}$

$\cos\phi_{s_2} > \cos\phi_{s_1}$ (leading p.f)

Net p.f of system got improved.

Provided that load is leading in nature.

Practically shunt reactor → Not a p.f correction device.

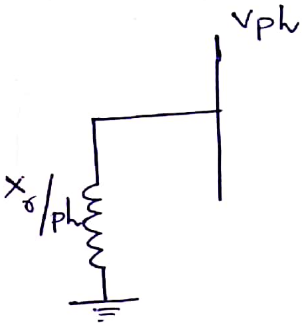
→ Ferranti Effect control device.

* To change p.f from $\cos\phi_1$ lead to $\cos\phi_2$ lead

$Q_{reactor}$ (abs) $= P_1 [\tan\phi_1 - \tan\phi_2]$

↓ 3-ϕ   ↓ 3-ϕ

Example: A shunt reactor of 100 MVAR is operated at 98% of its rated voltage and at 96% of its rated frequency. The reactive power absorbed by the reactor is ---



$$Q_{reactor\,(3\text{-}\phi)} \propto \frac{V^2}{f}$$

$$Q_{reactor} = \frac{V_{ph}^2}{X_r|ph}$$

$$Q_{reactor\,(3\phi)} = \frac{3V_{ph}^2}{X_r|ph}$$

$$Q_{reactor\,(3\text{-}\phi)} = \frac{3V_{ph}^2}{(2\pi f)(L|p}$$

$Q_{reactor\,1} = 100\,MVAR \longrightarrow V_1\ f$

$Q_{reactor\,2} = ? \longrightarrow 0.98V,\ 0.96f$

$$Q_{reactor} \propto \frac{V^2}{f}$$

$$\frac{Q_{reactor\,2}}{Q_{reactor\,1}} = \frac{V_2^2/f_2}{V_1^2/f_1}$$

$$Q_{reactor\,2} = Q_{reactor\,1} * \left(\frac{V_2}{V_1}\right)^2 * \left(\frac{f_1}{f_2}\right)$$

$$Q_{reactor\,2} = 100 * \left(\frac{0.98V}{V}\right)^2 * \left(\frac{f}{0.96f}\right)$$

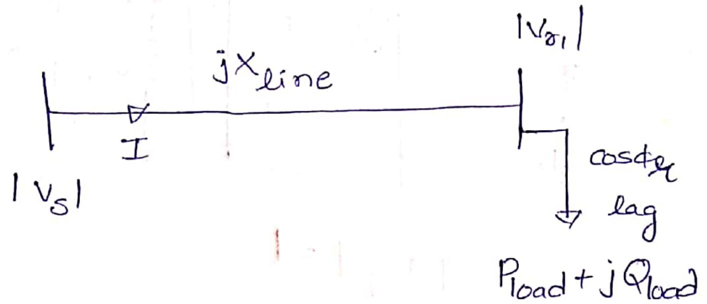
$$= 100.04\ MVAR.$$

Scanned with CamScanner

Series capacitor:

- To reduce net series reactance of tr. line.
- Used to avoid under vtgs

* **Without series capacitor:**



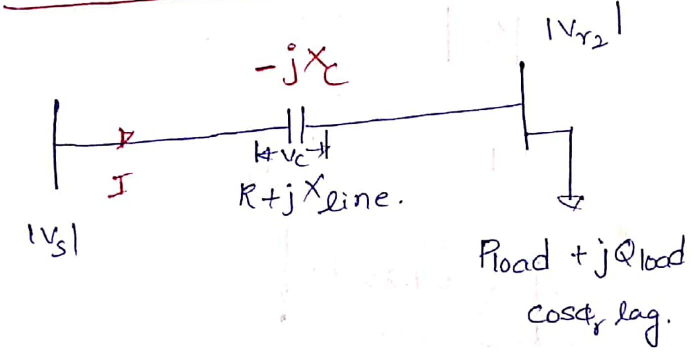
$$|V_s| - |V_{r_1}| =$$

$$\left\{\begin{array}{l} \bar{V_s} - \bar{V}_{r_1} = I(jX_{line}) \\ |\bar{V_s} - \bar{V}_{r_1}| = IX_{line} \neq |V_s| - |V_{r_1}| \end{array}\right\}$$

$$\underbrace{|V_s| - |V_{r_1}|}_{\Delta V_1} = IR\cos\phi_r + IX_{line}\sin\phi_r \quad \text{(1)}$$

$$|V_{r_1}| < |V_s|$$

* **With series capacitor:**



* Due to tr. line impedance changes, change in (I) magnitude of line is very less.

* A little improvement in pf occurs which is not appreciable in practical system

$$\underbrace{|V_s| - |V_r|}_{\Delta V_2} = IR\cos\phi_r + I(X_{line} - X_c)\sin\phi_r \longrightarrow ②$$

From ① × ② eq^{n}s.

$$\boxed{\Delta V_2 < \Delta V_1}$$

i.e $|V_s| - |V_{r_2}| < |V_s| - |V_{r_1}|$

$$-|V_{r_2}| < -|V_{r_1}|$$

$$\boxed{|V_{r_2}| > |V_{r_1}|}$$

* Magnitude increment in vtg:

$$\Delta V_c = \Delta V_1 - \Delta V_2 \Longrightarrow \text{eq}^n① - \text{eq}^n②$$

$$= IX_c \sin\phi_r$$

* Magnitude vtg across series capacitor

$$|V_c| = IX_c.$$

* 'Q' delivered by series capacitor;

$$Q_{series\ capacitor}/ph = I^2 \times X_c/ph$$

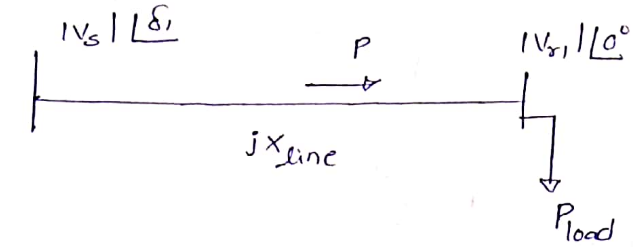
$$Q_{series\ capacitor} (3-\phi) = 3I^2 X_c/ph$$

$$= 3 \cdot \frac{I^2}{2\pi f C/ph}$$

$$Q_{series\ capacitor} \propto \frac{I^2}{f}.$$

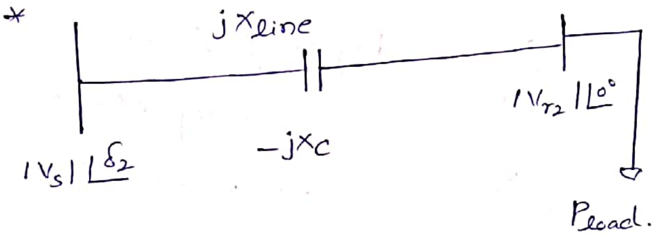
Scanned with CamScanner

Consequence ① :



$$P_1 = \frac{|V_s||V_{r_1}|}{X_{line}} \sin\delta_1$$

$$P_{max_1} = \frac{|V_s||V_{r_1}|}{X_{line}}$$



$$P_2 = \frac{|V_s||V_{r_2}|}{(X_{line} - X_C)} \sin\delta_2$$

$$P_{max_2}\uparrow = \frac{|V_s||V_{r_2}|}{(X_{line} - X_C)\downarrow}$$

$$\boxed{P_{max_2} > P_{max_1}}$$

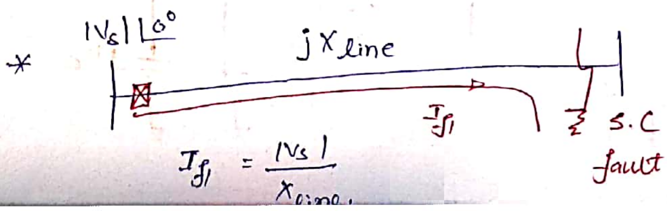
$$P_3 = P_{max_2}\uparrow \sin\delta_2\downarrow$$

$\downarrow$ Const

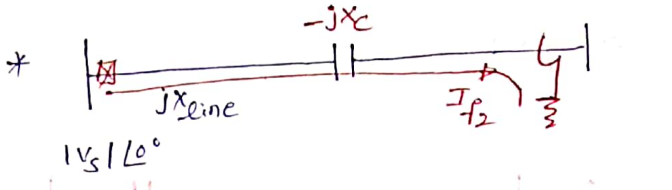
$\boxed{\delta_2 < \delta_1}$ → angular stability gets improved

Series capacitor is known as stability improvement device (practically)

Consequence ② :



$$I_{f_1} = \frac{|V_s|}{X_{line}}$$



$$I_{f_2}\uparrow = \frac{|V_s|}{(X_{line} - X_C)\downarrow}$$

Fault level in the system will be ↑sed.

* Breaking capacity of CB ↑, size & cost of CB ↑

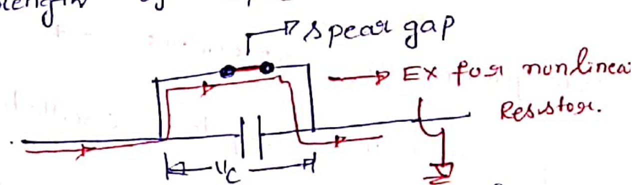
* $|V_C| = |I_{f_2}| X_C$

* Series connected device → high current rating, low vtg rating
* shunt connected device → high v rating, low I-rating.

During s.c fault, as $I_{f_2}\uparrow$, $|V_C|\uparrow$

⇒ Capacitor may be damaged.

Due to high vtgs dielectric strength of capacitor destroyed.



To avoid damage occuring for the series capacitor a sphere gap / spark gap will be connected in ||er to series capacitor.

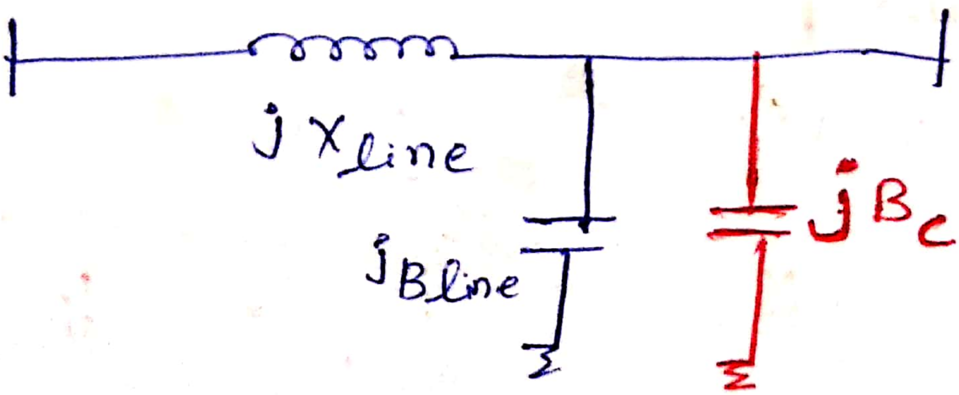
* With series capacitor : placement

$P_{max} \uparrow$; SIL $\uparrow$ (P_{max} (or) angular stability improved.)

$\downarrow$

ideal power transfer.

* With shunt capacitor : P.f



Scanned with CamScanner

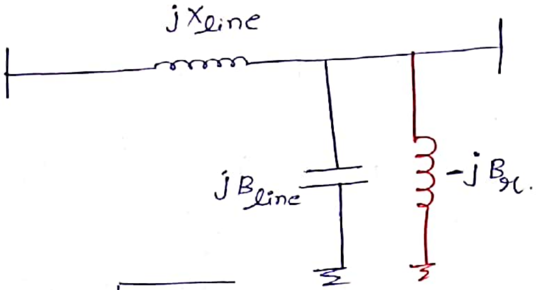
$$Z_{C_2}\downarrow = \sqrt{\frac{X_{line}}{(B_{line}+B_c)\uparrow}}$$

SIL ↑ ;

$$P_{max_2} = \frac{|V_s||V_{r_2}|}{X_{line}}$$

* A little increment (or) improvement in the stability occurs. ?

* With shunt reactor : (Ferranti Effect)

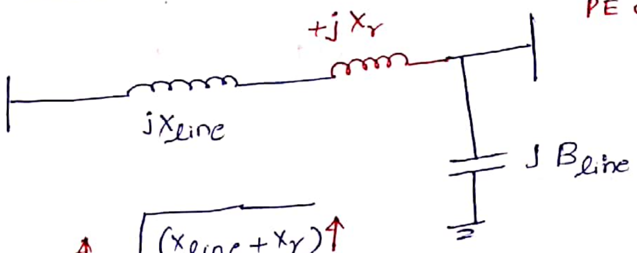


$$Z_{C_2}\uparrow = \sqrt{\frac{X_{line}}{(B_{line}-B_r)\downarrow}}$$

SIL ↓

* A little decrement in Pmax occurs. ?

* With series reactor : Fault current limiter. Smoothing reactor in PE ckts & HVDC



$$Z_{C_2}\uparrow = \sqrt{\frac{(X_{line}+X_r)\uparrow}{B_{line}}}$$

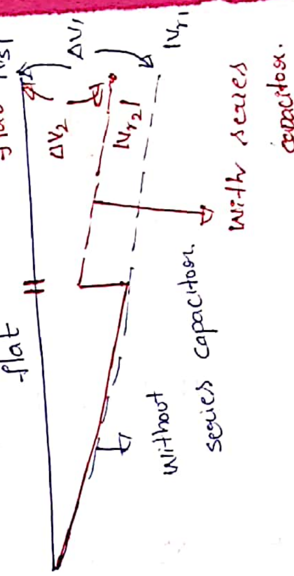
SIL ↓,

$$P_{max2}\downarrow = \frac{|V_s||V_{r_2}|}{(X_{line}+X_r)\uparrow}$$

Series capacitor

1. Reactance of line got reduced

2.

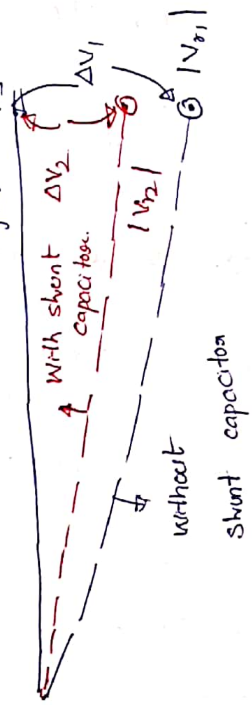


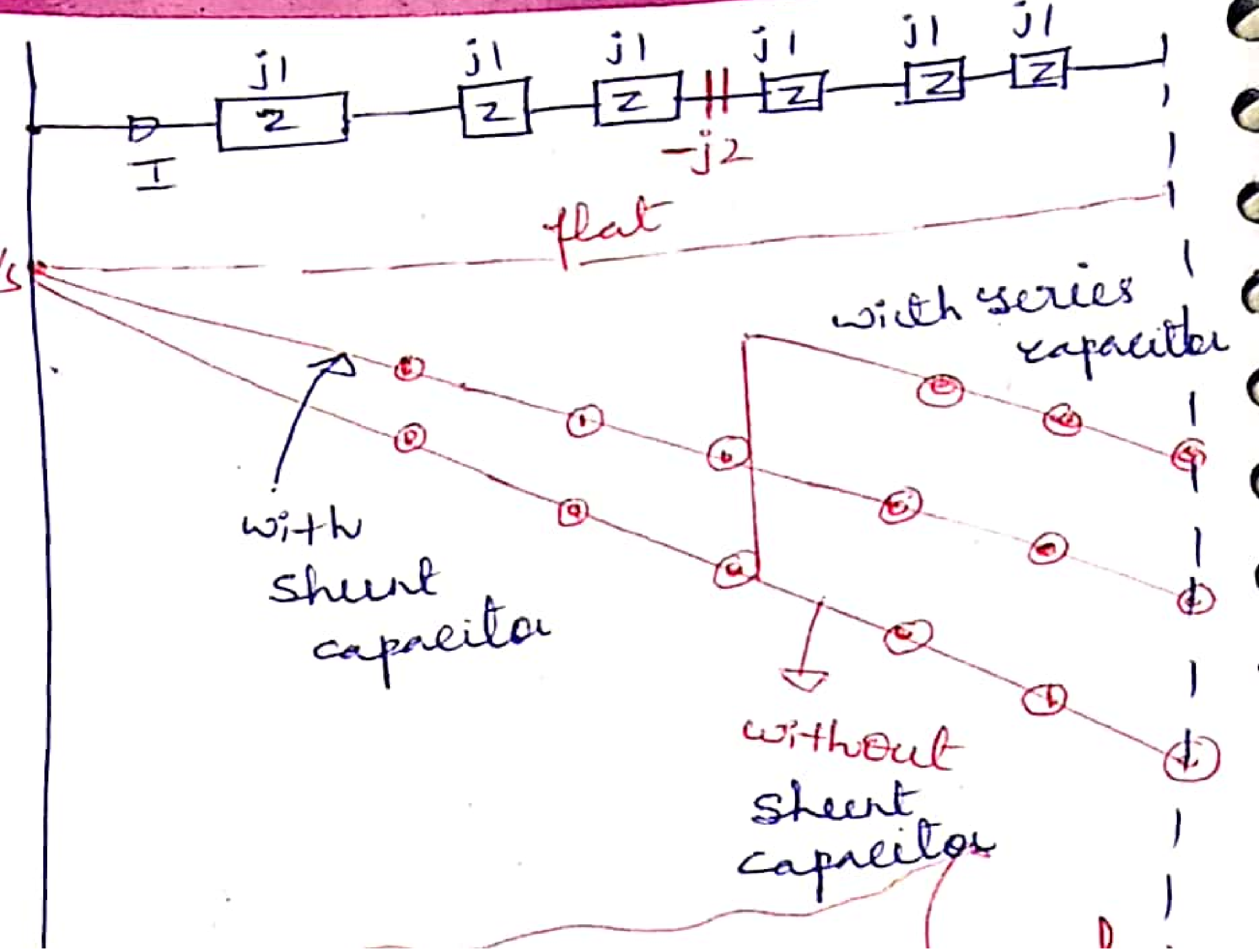
Differences b/n shunt & series capacitor

Shunt capacitor

1. current in the line gets changed

2.

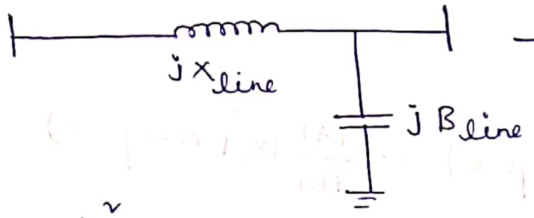




Scanned with CamScanner

Problem: A lossless transmission line having surge impedance loading (SIL) of 2280 MW. A series capacitive compensation of 30% is placed. Then SIL of the compensated transmission line will be

Solution:



→ approximate model

$$SIL = \frac{V^2}{Z_c}$$

$$Z_c = \sqrt{\frac{A}{C}\cdot\frac{B}{D}} = \sqrt{\frac{B}{D}} = \sqrt{\frac{Z}{Y}}$$

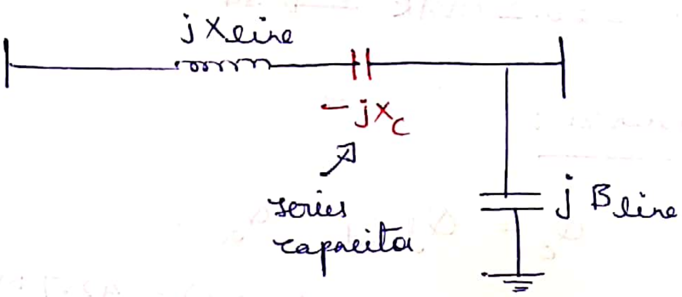
$$Z_c = \sqrt{\frac{X_{line}}{B_{line}}}$$

Before compensation:

(without series capacitor)

$$SIL_1 = \frac{V^2}{Z_{c_1}} = 2280\text{ MW}$$

with series capacitor (After compensation):



30% compensation:

$$\frac{X_c}{X_{line}} = \text{degree of series compensation}$$

$$= 0.3 \qquad 30\% = \frac{30}{100} = 0.3$$

$$\boxed{X_c = 0.3\,X_{line}}$$

$$Z_{c_2} = \sqrt{\frac{X_{line} - X_c}{B_{line}}} = \sqrt{\frac{X_{line} - 0.3X_{line}}{B_{line}}}$$

$$Z_{c_2} = \sqrt{\frac{0.7X_{line}}{B_{line}}} = \sqrt{0.7}\,Z_{c_1}$$

$$\boxed{Z_{c_2} = \sqrt{0.7}\,Z_{c_1}}$$

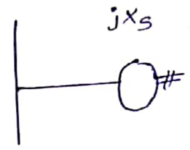
$$SIL_2 = \frac{V^2}{Z_{c_2}} = \frac{V^2}{\sqrt{0.7}\,Z_{c_1}} = \frac{2280}{\sqrt{0.7}}$$

($SIL_2 = 2725$ MW)

Scanned with CamScanner

Dynamic Devices :

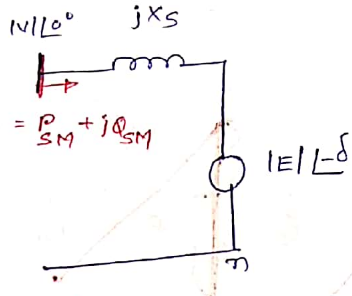
Synchronous motor ;



(Assume lossless)

$V = |V|\angle 0^\circ$

⇓



real power absorbed ;

$$P_{SM} = \frac{|E||V|}{X_s}\sin\delta$$

reactive power absorbed

$$Q_{SM\,(abs)} = \frac{|V|}{X_s}\left[|V| - |E|\cos\delta\right]$$

'Q' delivered by SM into the bus.

$$Q_{SM\,(supply)} = \frac{|V|}{X_s}\left[|E|\cos\delta - |V|\right]$$

Synchronous Condenser : (capacitor)

over excited synchronous motor, at No load condition.

For lossless motor ⇒ $P_{SM} = 0$

⇒ $\sin\delta = 0$

$\boxed{\delta = 0^\circ}$

$$\boxed{Q_{SM\,(supply)}\Big|_{\delta=0^\circ} = \frac{|V|}{X_s}\left[|E| - |V|\right]}$$

Scanned with CamScanner

Due to over Excitation ⇒ $I_f \uparrow, \phi \uparrow, |E| \uparrow$

such that $|E| > |V|$

$$Q_{SM\,(supply)}\Big|_{\delta=0^\circ} = +Ve.$$

means ↓

synchronous condenser → injects

lagging 'Q' into the system.

→ similar to shunt capacitor bank.

→ p.f correction device practically.

$$Q_{\text{synchronous condensor}} = P_1\left[\tan\phi_1 - \tan\phi_2\right]$$

→ p.f is ZPF Lead [$P_{SM} = 0$]

↳ ideal.

→ p.f is 0.1 lead – 0.2 lead (practical)

Scanned with CamScanner

Synchronous coil : (inductor)

under excited S.M. under No Load Condition.

→ Under Excitation, $I_f \downarrow$, $\phi \downarrow$, $|E| \downarrow$ such that $|E| < |V|$

→ Q_{SM} (supply) $\Big|_{\delta = 0°}$ = −ve

→ Synchronous coil absorbs Q from the system.

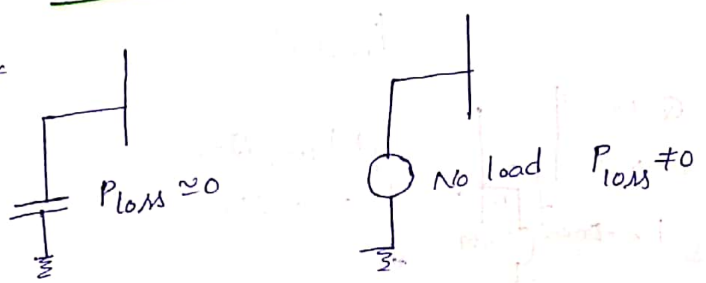
→ participation similar to shunt reactor.

→ Ferranti Effect control device.

→ Zpf lag (ideal)
0.1 to 0.2 lag (practical)

Technical disadvantages of dynamic devices compared to static devices :

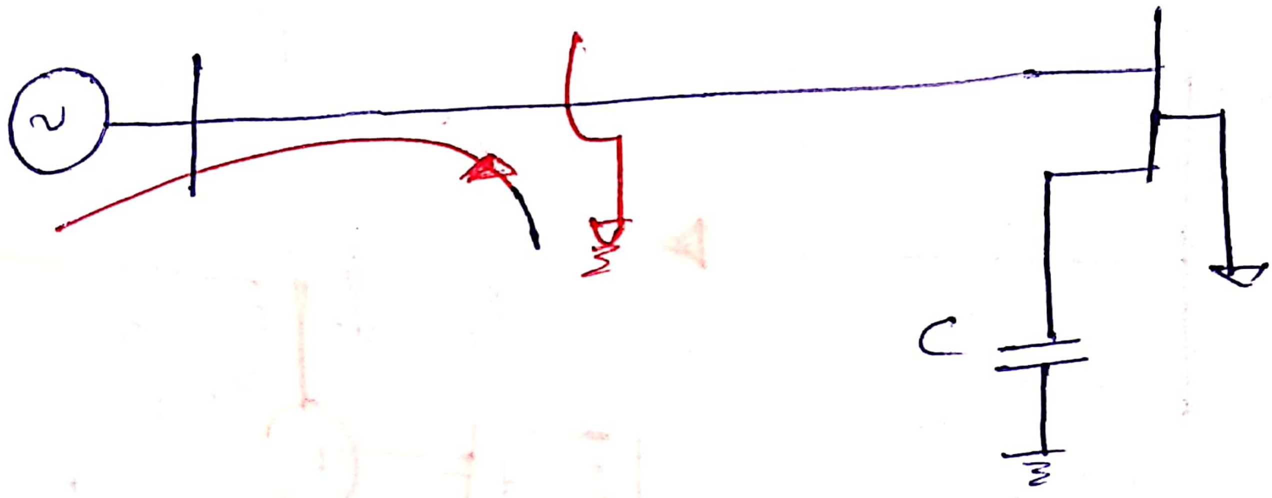
* seperate dc battery is required.
* stability problem is more.
*

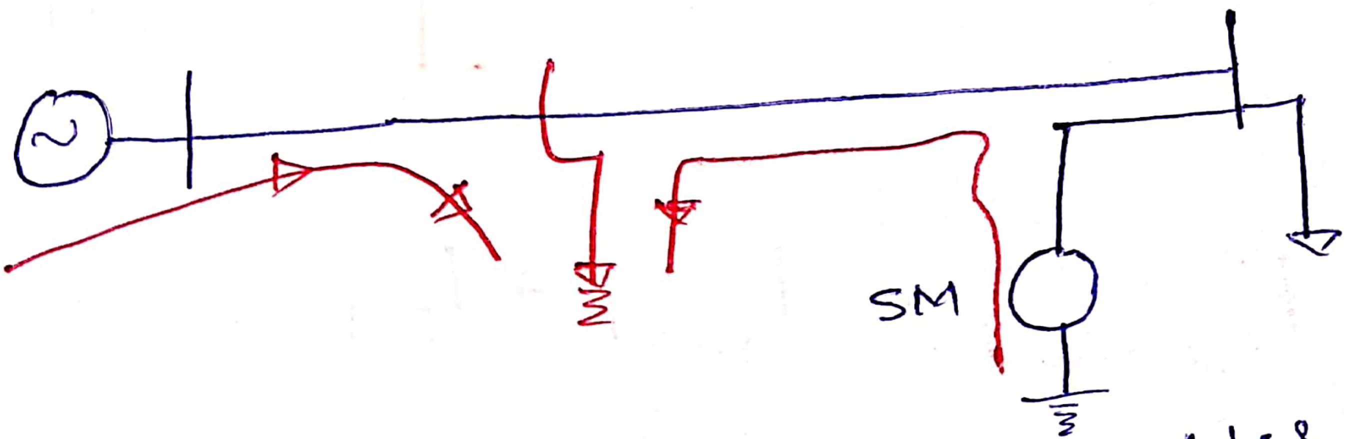


* synchronous motor → $T_{st} = 0$
starting methods are required.

Scanned with CamScanner

*



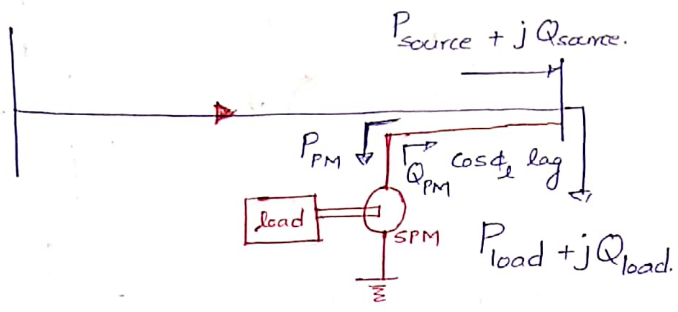


Fault level in the system ↑ses due to S.M.

Synchronous phase modifier (or) phase advancer:

over excited synchronous motor having mechanical load on the shaft.

Scanned with CamScanner



$$P_{source} = P_{load} + P_{PM}$$

$$Q_{source} + Q_{PM} = Q_{load}$$

$$Q_{source} = Q_{load} - Q_{PM}$$

$$P_{PM}\text{ (absorbed)} = \frac{|E||V|}{X_s}\sin\delta, \quad \delta \neq 0$$

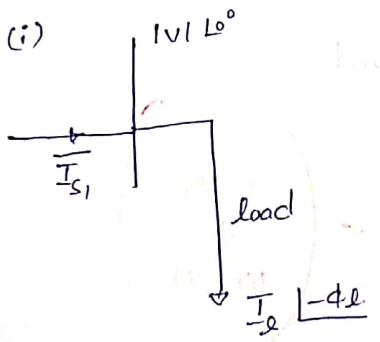
$$Q_{PM}\text{ (supply)} = \frac{|V|}{X_s}\left[|E|\cos\delta - |V|\right]$$

$$\text{P.f}, \; \cos\phi_{PM} = \frac{P_{PM}}{\sqrt{P_{PM}^2 + Q_{PM}^2}} \text{ leading}$$

$$Q_{PM}\text{ (supply)} = +ve \Rightarrow |E|\cos\delta > |V|$$

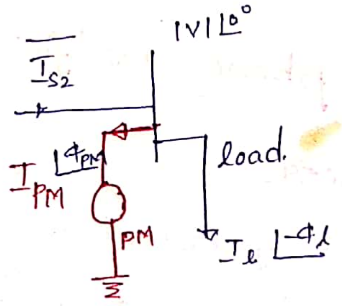
phasor diagram:

(i)



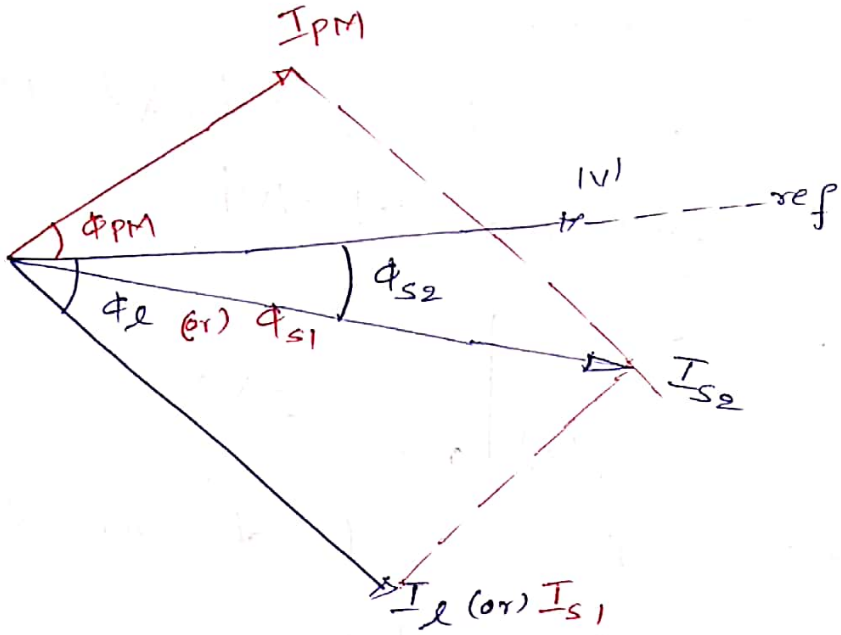
$$\bar{I}_{S1} = I_l \angle -\phi_l$$

(ii)



$$\bar{I}_{S2} = I_l \angle -\phi_l + I_{PM} \angle \phi_{PM}$$

Scanned with CamScanner



$\phi_{s2} < \phi_{s1}$

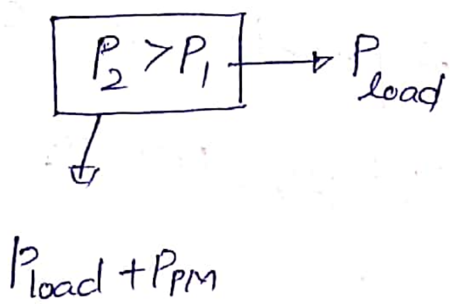
So net pf : $\cos\phi_{s2} > \cos\phi_{s1}$.

Net pf of system was improved.

∴ Practical pf correction device.

* To improve p.f from $\cos\phi_1$ lagging to $\cos\phi_2$ lagging

$$\phi_{PM\ (supply)} = P_1 \tan\phi_1 - P_2 \tan\phi_2$$



problem ①: A 3ϕ induction motor delivers (output) 500HP at an efficiency of 90% when the operating power factor is 0.8 lag. A loaded synchronous motor with a power consumption of 120 KW is connected in parallel with the induction motor. If overall power factor is unity then find

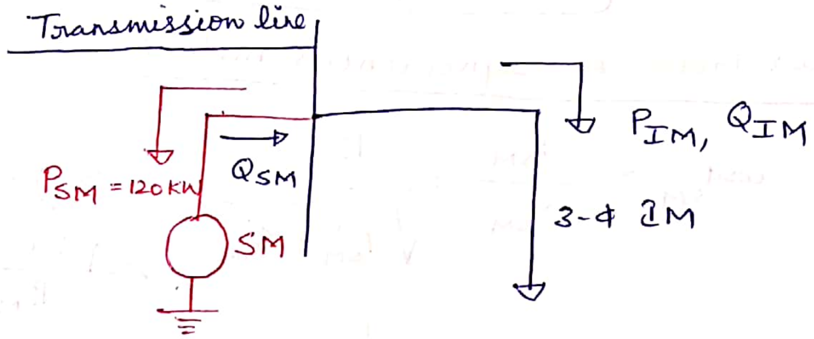
i) Reactive power to be injected by synchronous motor.

ii) power factor of synchronous motor

iii) KVA rating of synchronous motor

iv) KVA supplied by source without and with synchronous motor connection.

solution:



3ϕ Induction Motor:

motor delivers (output) → $P_{out} = 500HP$ 1HP = 746W

$= 500 * 746$

$= 373 KW$

Efficiency $(\eta_{IM}) = 90\%$ Pf of IM $(\cos\phi_{IM}) = 0.8$ lag

3ϕ synchronous motor:

P_{SM} (power consumption) = 120KW

Then overall power factor is $(\cos\phi) = 1$

from 3ϕ IM data:

$$P_{IM} = \frac{P_{out}}{\eta} = \frac{373\ KW}{0.9} = 414.4\ KW$$

$$Q_{IM}\ (\text{absorbed}) = P_{IM} \cdot \tan\phi_{IM}$$

$$= 414.4 \times 10^3 * \tan(\cos^{-1}(0.8))$$

$$= 310.83\ KVAR$$

Scanned with CamScanner

As source power factor is unity ; $Q_{source} = 0$

i) Reactive power to be injected by synchronous motor :

power balance equations

$$P_{source} = P_{IM} + P_{SM}$$

$$P_{source} = 414.4 + 120 = 534.4 \text{ KW}$$

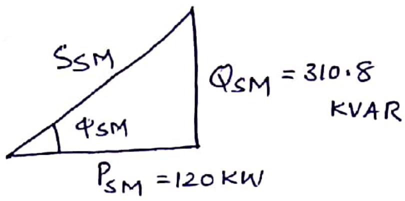
$$Q_{source} + Q_{SM} = Q_{IM}$$

$$0 + Q_{SM} = 310.8 \text{ KVAR}$$

$$\boxed{Q_{SM}(\text{supply}) = 310.8 \text{ KVAR}}$$

ii) power factor of synchronous motor :

$$\cos\phi_{SM} = \frac{P_{SM}}{S_{SM}} = \frac{P_{SM}}{\sqrt{P_{SM}^2 + Q_{SM}^2}}$$



$$= \frac{120}{\sqrt{(120)^2 + (310.8)^2}} = 0.36 \text{ leading}$$

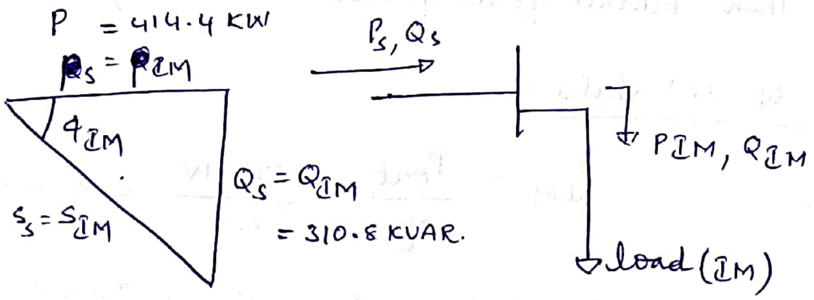
iii) KVA rating of synchronous motor :

from above power Δ^{le} $(S_{SM}) = \sqrt{P_{SM}^2 + Q_{SM}^2}$

$$= 333.16 \text{ KVA}.$$

iv) KVA supplied by source without and with synchronous motor connection :

case i) : without SM : then we have only IM



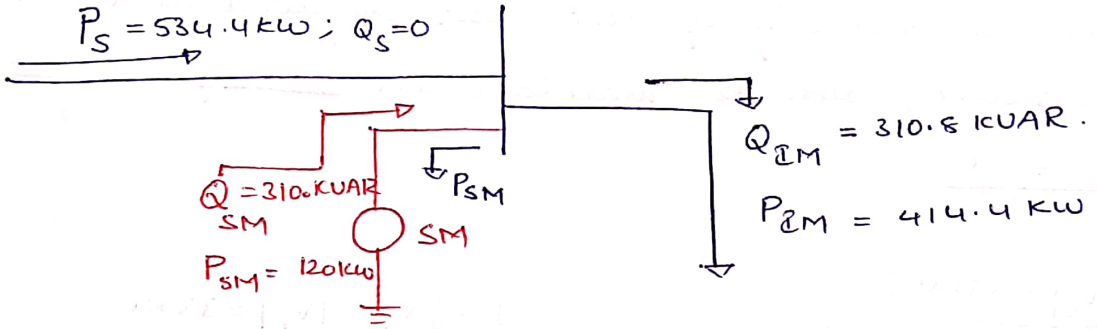
$$S_{IM} = S_{source}(S_s) = \sqrt{P_{IM}^2 + Q_{IM}^2}$$

$$= \sqrt{(414.4)^2 + (310.8)^2} = 518 \text{ KVA}$$

Scanned with CamScanner

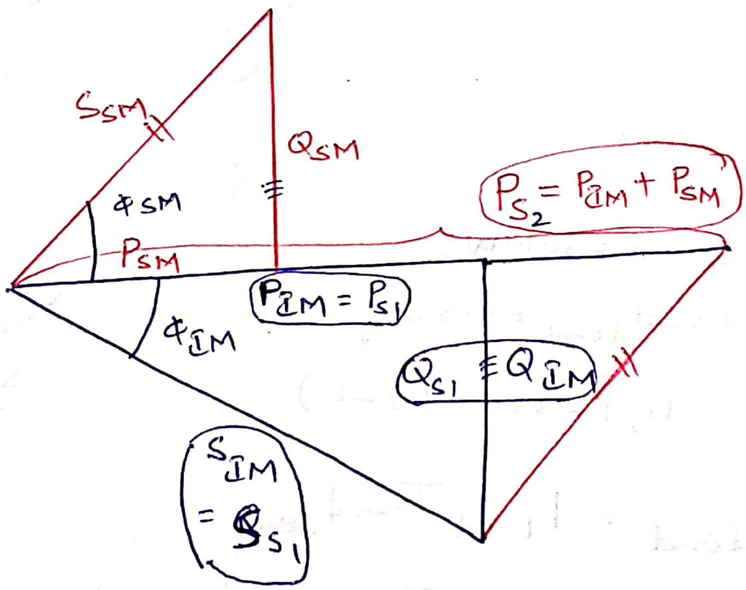
case ii) : with synchronous motor :

$P_{source} = 534.4\ KW$; $Q_{source} = 0$ (because overall pf is unity)



from above diagram $Q_{SM} = Q_{IM} = 310.8\ KVAR.$

$P_s = P_{SM} + P_{IM} = 534.4\ KW$



, $Q_{S_2} = 0$; so $S_{S_2} = P_{S_2}$

$$S_{source} = \sqrt{P_{source}^2 + Q_{source}^2} = \sqrt{(534.4)^2 + 0^2}$$

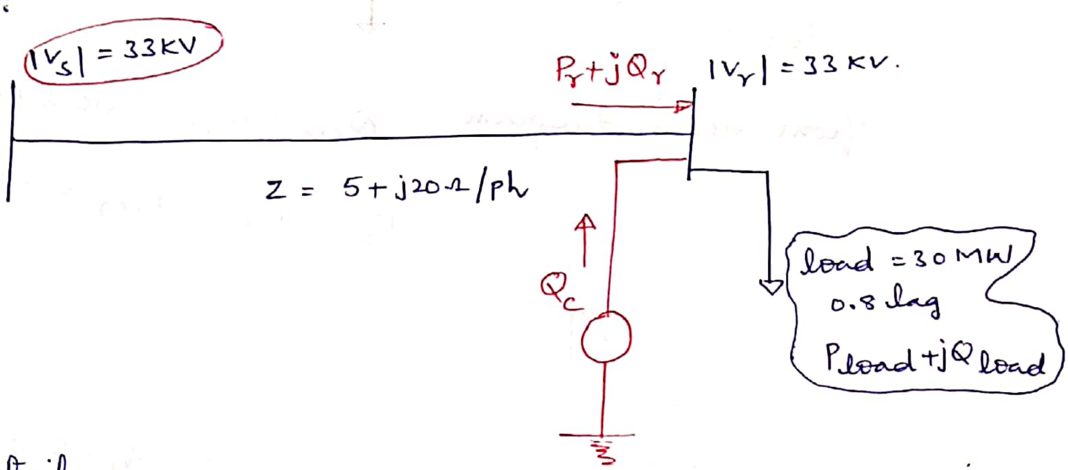
$$= 534.4\ KW.$$

Scanned with CamScanner

Problem (2): A 3-ϕ line having an impedance of 5+j20 ohms per phase delivers a load of 30MW at a power factor of 0.8 lag and voltage of 33KV, 50HZ.

i) Determine the capacity of condenser required to be installed at the receiving side to maintain the sending end voltage also at 33KV.

Solution:

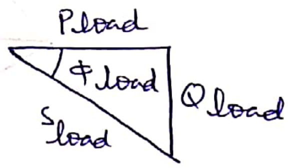


load details:

$P_{load} = 30MW$

$\cos\phi_{load} = 0.8$ lag

$|V_r| = 33$ KV (L-L)



$$Q_{load} = P_{load} \tan\phi_{load}$$

$$= 30 * \tan(\cos^{-1}(0.8)) = 22.5 \text{ MVAR}$$

Power balance equations:

$$P_r = P_{load} = 30MW$$

$$Q_r + Q_c = Q_{load}$$

$$Q_c = Q_{load} - Q_r$$

$$Q_r = \frac{|V_s||V_r|}{|B|} \sin(\beta-\delta) - \frac{|A|}{|B|}|V_r|^2 \sin(\beta-\alpha)$$

For transmission line → short transmission line case

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ & 20.6\angle 75.96^\circ \\ 0 & 1\angle 0^\circ \end{bmatrix}$$

Scanned with CamScanner

$|A| = 1$; $\alpha = 0^\circ$; $|B| = 20.6$; $\beta = 76^\circ$

$A = |A| \angle \alpha$

$B = |B| \angle \beta = Z = R + jX = 5 + j20\,\Omega/ph$

To get δ :

$$P_r = \frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \cos(\beta - \alpha)$$

$$30 = \frac{33 * 33}{20.6} \cos(76 - \delta) - \frac{1}{20.6} (33)^2 \cos(76 - 0^\circ)$$

$$\boxed{\delta = 40.04^\circ}$$

Now;

$$Q_r = \frac{|V_s||V_r|}{|B|} \sin(\beta - \delta) - \frac{|A|}{|B|} |V_r|^2 \sin(\beta - \alpha)$$

$$= \frac{33 \times 33}{20.6} \sin(76^\circ - 40.04^\circ) - \frac{1}{20.6} * (33)^2 \sin(76^\circ - 0^\circ)$$

$$Q_r = -20.2 \text{ MVAR} \longrightarrow 3\phi$$

Rating of condensor :

$$Q_c = Q_{load} - Q_r$$

$$= 22.5 - (-20.2) = 42.7 \text{ MVAR } (3\phi)$$

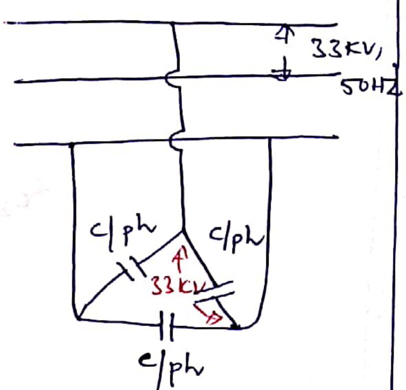
ii) If the condenser was interpreted as a Δ- connected capacitor bank then what is capacitance per phase in capacitor bank.

$$Q_c/ph = \frac{42.7}{3} = 14.23 \text{ MVAR}$$

$$Q_c/ph = \frac{V_{ph}^2}{X_c/ph} = V_{ph}^2 (2\pi f\, c/ph)$$

$$c/ph = \frac{Q_c/ph}{V_{ph}^2 (2\pi f)} = \frac{14.23}{(33)^2 * 2\pi * 50}$$

$$\boxed{c/ph = 41.58\,\mu F}$$



Tap Changing Transformers

The main job of a transformer is to transform electric energy from one voltage level to another. Almost all power transformers on transmission lines are provided with taps for ratio control *i.e.*, control of secondary voltage. There are two types of tap changing transformers:

(*i*) Off-load tap changing transformers.

(*ii*) On-load (under-load) tap changing transformers.

The tap changing transformers do not control the voltage by regulating the flow of reactive vars but by changing the transformation ratio, the voltage in the secondary circuit is varied and voltage control is obtained. This method is the most popular as it can be used for controlling voltages at all levels.

Scanned with CamScanner

Figure 10.7 refers to the off-load tap changing transformer which requires the disconnection of the transformer when the tap setting is to be changed. The modern practice is to use on-load tap changing transformer which is shown in Fig. 10.8. In the position shown the voltage is a maximum and since the currents divide equally and flow in opposition through the coil between Q_1 and Q_2, the resultant flux is zero and hence minimum impedance. To reduce the voltage, the following operations are required in sequence : (*i*) open Q_1; (*ii*) move selector switch S_1 to the next contact; (*iii*) close Q_1; (*iv*) open Q_2; (*v*) move selector switch S_2 to the next contact; and (*vi*) close Q_2.

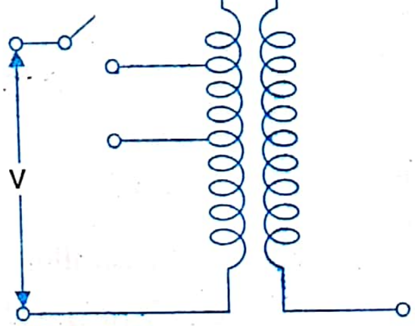


Fig. 10.7 Off-load tap changing transformer.

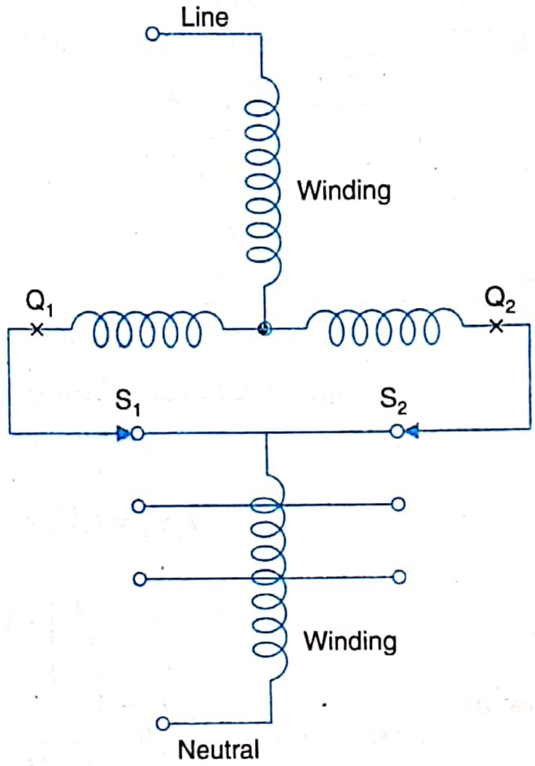


Fig. 10.8 On-load tap changing transformer.

Thus six operations are required for one change in tap position. The voltage change between taps is often 1.25 per cent of the nominal voltage where nominal voltages are the voltages at the ends of the transmission line and the actual voltages are t_sV_1 and t_rV_2 where t_s and t_r are the fractions of the nominal transformation ratios, *i.e.*, the tap ratio/nominal ratio.

Consider the operation of a radial transmission line with tap changing transformers at both the ends as shown in Fig. 10.9. It is desired to find out the tap changing ratios required to completely compensate for the voltage drop in the line. We assume here that the product of t_s and t_r is unity as this ensures that the overall voltage level remains of the same order and that the minimum range of taps on both transformers is used.

From Fig. 10.9, we have

$$t_sV_1 = t_rV_2 + IZ \tag{10.12}$$

We know that the approximate line drop is given as

$$IZ = \Delta V = v_r \cos\phi + v_x \sin\phi \tag{10.13}$$
$$= IR\cos\phi + IX\sin\phi$$

Scanned with CamScanner

$$= R \cdot I \cos\varphi + X \cdot I \sin\varphi$$

$$= \frac{R \cdot P}{V_r} + \frac{X \cdot Q}{V_r}$$

$$= \frac{RP + XQ}{t_r V_2} \tag{10.14}$$

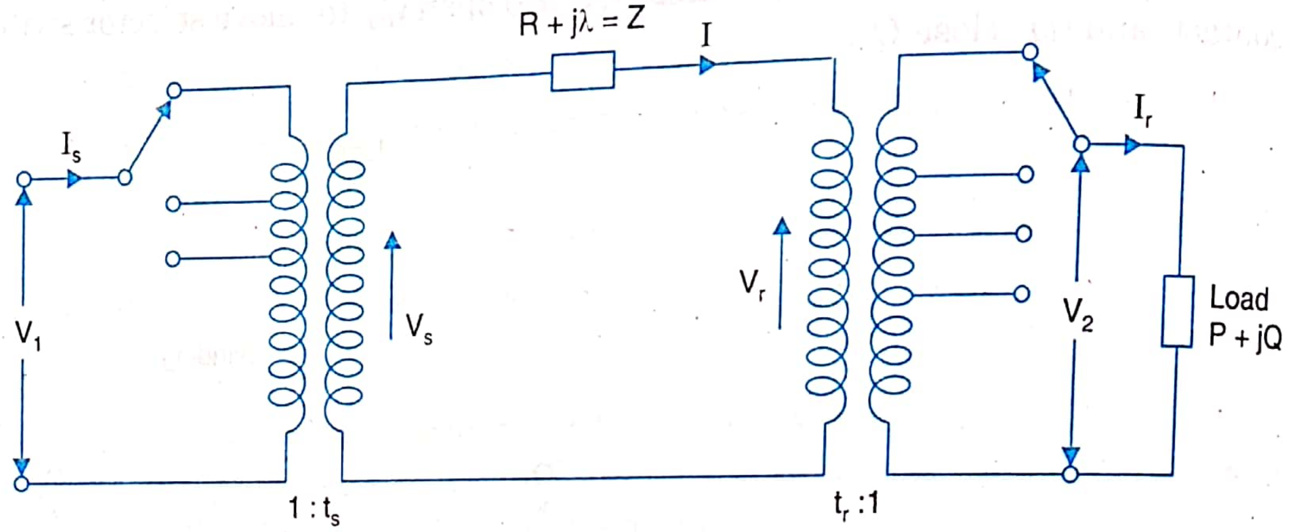


Fig. 10.9 Radial transmission line with on-load tap changing transformer at both the ends.

$$\therefore \qquad t_s V_1 = t_r V_2 + \frac{RP + XQ}{t_r V_2} \tag{10.15}$$

$$t_s = \frac{1}{V_1}\left[t_r V_2 + \frac{RP + XQ}{t_r V_2}\right] \tag{10.16}$$

Now as

$$t_s t_r = 1 \tag{10.17}$$

$$t_s = \frac{1}{V_1}\left[\frac{V_2}{t_s} + \frac{RP + XQ}{V_2 / t_s}\right]$$

or

$$t_s^2 = \frac{V_2}{V_1} + \left(\frac{RP + XQ}{V_2 V_1}\right) t_s^2$$

or

$$t_s^2\left[1 - \frac{RP + XQ}{V_1 V_2}\right] = \frac{V_2}{V_1} \tag{10.18}$$

From the equation (10.18), for particular values of V_2 and V_1 and the load requirements P and Q, the value of t_s can be obtained.

The tap changing operation is normally motor operated. A closed loop control of the secondary voltage level is possible.

Scanned with CamScanner

UNIT-3

PER UNIT SYSTEM REPRESENTATION OF POWER SYSTEMS

INTRODUCTION

The various components of a power system like alternators, transformers, induction motors etc. have their voltage, power, current and impedance ratings in kV, kVA, kA and kΩ respectively. The components or various sections of power system may operate at different voltage and power levels. It will be convenient for analysis of power system if the voltage, power, current and impedance ratings of components of power system are expressed with reference to a common value called base value. Hence for analysis purpose, a base value is chosen for voltage, power, current and impedance. Then all the voltage, power, current and impedance ratings of the components are expressed as a percentage or per unit of the base value.

Voltage, kilo volt-amperes, current and impedance are so related that, the selection of base values for any two of them determines the base values of the remaining two. Usually base power rating in Mega volt-amperes and base voltage in Kilo volts are the quantities selected for the base.

PER UNIT QUANTITIES

In a large interconnected power system with various voltage levels and various capacity equipments, it has been found quite convenient to work with per unit (p.u) system of quantities for analysis purpose, rather than in absolute values of quantities.

Per unit value of any quantity is defined as the ratio of actual value to the chosen base value in the same unit. In a power system the basic quantities of importance are voltage, power, current and impedance.

$$i.e\ Perunit\ value = \frac{actual\ value\ in\ any\ unit}{base\ value\ in\ the\ same\ unit}$$

Percentage value = Per unit value $\times 100$

The per unit value is particularly convenient in power systems as the various sections of power systems are connected through transformer and have different voltage levels.

For single phase systems

Let Base voltage amperes= $(VA)_b$

Base voltage = V_b in volts

Base current= I_b in amps

Base impedance = Z_b in ohms

Then the base current and impedance are given by

Base current, $I_b = \frac{(VA)_b}{V_b}$

Base impedance, $Z_b = \frac{V_b}{I_b} = \frac{V_b^2}{(VA)_b}$ in ohms

If the actual impedance is Z(ohms), its per unit value is given by

Z(p.u)= $\frac{Z}{Z_b} = Z \times \frac{(VA)_b}{V_b^2}$

For a power system, practical choices of base values are

Base mega volt amperes= $(MVA)_b$

or Base kilo volt amperes = $(KVA)_b$

Base kilo volts = $(kV)_b$

Base current, $I_b = \frac{(KVA)_b}{(kV)_b} = \frac{(KVA)_b/1000}{(kV)_b/1000} = \frac{(MVA)_b}{(kV)_b} \times 1000$

Base impedance, $Z_b = \frac{V_b}{I_b} = \frac{V_b/1000}{I_b/1000}$

$$= 1000 \frac{(kV)_b}{I_b}$$

$$= 1000 \times \frac{(kV)_b}{(KVA)_b/(kV)_b}$$

$$= 1000 \times \frac{(kV)_b^2}{(KVA)_b}$$

$$= \frac{(kV)_b^2}{(KVA)_b/1000} = \frac{(kV)_b^2}{(MVA)_b}$$

$$\therefore Z_b = 1000 \times \frac{(kV)_b^2}{(KVA)_b} = \frac{(kV)_b^2}{(MVA)_b}$$

Per unit impedance, $Z_{p.u} = \frac{Z}{Z_b} = Z \times \frac{(MVA)_b}{(kV)_b^2} = Z \times \frac{(KVA)_b}{(kV)_b^2} . \frac{1}{1000}$

For three phase system

Three phase base mega volt amps = $(MVA)_b$

Line to line base kilo volts = $(kV)_b$

Assume star connection

Base current $I_b = \dfrac{(KVA)_b}{\sqrt{3}\times(kV)_b} = \dfrac{(KVA)_b/1000}{\sqrt{3}\times(kV)_b/1000} = 1000\dfrac{(MVA)_b}{\sqrt{3}\times(kV)_b}$

Base impedance, $Z_b = \dfrac{V_b}{\sqrt{3}I_b} = \dfrac{V_b/1000}{\sqrt{3}I_b/1000}$

$$= 1000\times\frac{(kV)_b}{\sqrt{3}I_b}$$

$$= 1000\times\frac{(kV)_b}{\sqrt{3}\times1000\dfrac{(MVA)_b}{\sqrt{3}\times(kV)_b}}$$

$$= \frac{(kV)_b^2}{(MVA)_b} = \frac{1000(kV)_b^2}{(KVA)_b}$$

Per input impedance $Z_{p.u} = \dfrac{Z}{Z_b} = \dfrac{Z}{1000\dfrac{(kV)_b^2}{(KVA)_b}} = Z\times\dfrac{(KVA)_b}{(kV)_b^2}\times\dfrac{1}{1000} = Z\times\dfrac{(MVA)_b}{(kV)_b^2}$

Change of Base:

Normally, the impedances are specified on the rating of the equipment. Hence, there is a need to change the p.u values from the base of the equipment rating (old value) to that of the chosen system base (new value).

When MVA base is changed from $(MVA)_{b,old}$ to $(MVA)_{b,new}$ and KV base is changed from $(KV)_{b,old}$ to $(KV)_{b,new}$ the per unit impedance from the above equation is

$$Z(p.u)_{new} = Z(p.u)_{old}\times\frac{(MVA)_{b,new}}{(MVA)_{b,old}}\times\frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

SELECTION OF BASE VALUES

The selection of base values of kVA and kV is made in order to reduce the work required for the calculations as much as possible. In a power system network the same kVA base should be used in all parts of the system. It may be total kVA or MVA of the system, the largest kVA of a section, or any round figure such as 10, 100, 1000 kVA etc. After the selection of base kVA, the base voltage for a particular selection should be chosen. The rated voltages of the largest section may be taken as the base voltage for the section. The base voltages for other sections will be fixed depending upon transformer ratios. When once the selection of base kVA and base voltages for different sections are made, the per unit impedances of various sections can be made for drawing single line diagram of the system

PER UNIT IMPEDANCE DIAGRAM OF A POWER SYSTEM

From a single line diagram of a power system we can directly draw the impedance diagram by using the following steps

1. Choose an appropriate common MVA or KVA base for the system.
2. Consider the system to be divided into a number of sections based on the location of transformers.
3. Choose an appropriate kV base in one of the sections. Calculate kV bases of the other sections using the voltage ratio of transformers.
4. Calculate per unit values of voltages and impedances in each section and connect them as per the topology of the single line diagram. The result is the single phase per unit impedance diagram.

ADVANTAGES OF PER UNIT SYSTEM

1. Manufactures usually specify the impedance values of device or machine in percent or per-unit on the base of the name plate rating. If any data is not available, it is easier to assume its per unit value then its numerical value
2. The per unit impedance of circuit element connected by transformers expressed in a proper base will be same if it is referred to either side of transformer
3. The per unit values of impedances lie in a narrow range even though the ohmic values may differ widely for machines of different ratings. Therefore any erroneous date can be easily identified.
4. Power system contain a large number of transformers, the ohmic value of impedance as referred to secondary is different from the values as referred to the primary. However if the base values are selected properly , the per unit impedance is the same on the two sides of a transformer.
5. The per unit impedance of transformers in 3-phase circuits is not effected by the connection of transformer.

Problem 1

A 3-Ø generator with rating 1000KVA, 66 KV has its armature resistance and synchronous reactance as 60Ω/phase and 90Ω/phase. Calculate p.u impedance of the generator?

Solution:

Take generator ratings are chosen as base kV and base KVA

Base kilo volt, $(kV)_b$ =66 kV

Base kilo volt amp,$(KVA)_b$ =1000 KVA

$$Z_{p.u} = \frac{Z_{ohm}}{1000} \times \frac{(KVA)_b}{(kV)_b}$$

$$= \frac{(60 + j90)}{1000} \times \frac{1000}{66^2} = (0.01377 + j0.02066)\, p.u$$

Problem 2

a) A generator is rated 600MVA, 35kV. Its star-connected winding has a reactance of 1.4p.u. Find the ohmic value of the reactance of winding.

b) If the generator is working in a circuit for which the specified values are 400MVA, 30KV, then find the p.u value of reactance of generator winding on the specified base.

Solution:

a) Base kilo volt $(kV)_b$ =35kV

Base mega volt amps ,$(MVA)_b$ =600MVA

$X_{p.u}$ =1.4 p.u

$$X_{p.u} = X_{ohm} \times \frac{(MVA)_b}{(kV)_b^2}$$

$$\Rightarrow X_{ohm} = X_{p.u} \times \frac{(kV)_b^2}{(MVA)_b}$$

$$= 1.4 \times \frac{35^2}{600} = 2.858\Omega / phase$$

b) $$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

$$= 1.4 \times \frac{35^2}{30^2} \times \frac{400}{600}$$

$$= 1.2702\, p.u$$

Problem 3

A 120MVA, 19.5 KV generator has a synchronous reactance of 0.15 p.u and it is connected to T/L through a T/F rated 150MVA , 18/230 KV (Y/Δ) with X=0.1 p.u

a) Calculate the p.u reactance's by taking generator rating as the base values
b) Calculate the p.u reactance by taking T/F rating as the base values
c) Calculate the p.u reactance for a base value 100MVA and 220KV on H.T side of T/F

Solution

a) Base mega volt amps ,$(MVA)_{b,new}$ =120MVA

Base kilo volt , $(kV)_b$ =19.5kV

Since the generator rating chosen as the base values a, its p.u reactance does not change

The p.u reactance of generator =0.15p.u

The new p.u reactance of T/F , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.1 \times \frac{18^2}{19.5^2} \times \frac{120}{150}$$
$$= 0.0682\, p.u$$

b) Base mega volt amps ,$(MVA)_{b,new}$ =150MVA

Base kilo volt, $(kV)_b$ =18kV

Since the T/F rating chosen as the base values, its p.u reactance does not change

The p.u reactance of T/F =0.1p.u

The new p.u reactance of T/F , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.15 \times \frac{19.5^2}{18^2} \times \frac{150}{120}$$
$$= 0.22\, p.u$$

c) Base mega volt amps ,$(MVA)_{b,new}$ =100MVA

Base kilo volt, $(kV)_b$ =220 kV

The new p.u reactance of T/F , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.1\times\frac{230^2}{220^2}\times\frac{100}{150}$$
$$=0.0729\,p.u$$

The generator connected to L.T side of transformer

Base KV referred to L.T side of T/F = base KV on H.T side $\times\frac{L.T\ voltage\quad rating}{H.T\ voltage\quad rating}$

$$= 220\times\frac{18}{230} = 17.22\,p.u$$

There fore $(KV)_{b,new}$=17.22p.u

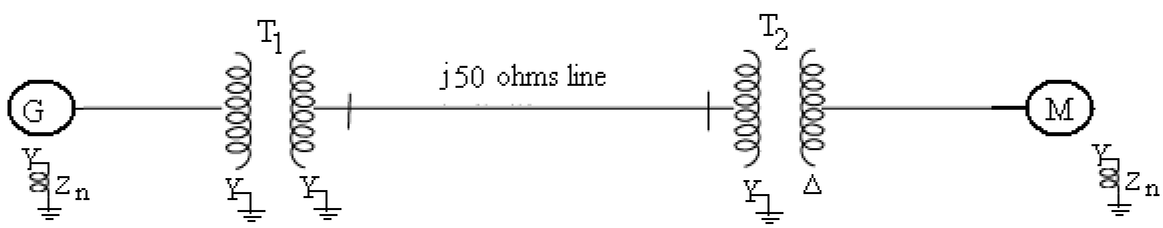
The new p.u reactance of generator , $X(p.u)_{new} = X(p.u)_{old}\times\frac{(MVA)_{b,new}}{(MVA)_{b,old}}\times\frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.15\times\frac{19.5^2}{17.22^2}\times\frac{100}{120}$$
$$=0.1603\,p.u$$

ADDITIONAL SOLVED PROBLEMS

Problem 1

Draw the reactance diagram for the power system shown in figure . Neglect resistance and use a base of 100MVA, 220KV in 50Ω line .The ratings of the generator ,motor and T/F are given below



Generator : 40MVA,25kV, X''=20%

Synchronous motor : 50MVA, 11kV, X''=30%

Y-Y transformer : 40MVA,33/220 kV, X=15%

Y-Δ transformer: 30MVA, 11/220 kV(Δ/Y), X=15%

Solution:

Base mega volt amps ,$(MVA)_{b,new}$ =100MVA

Base kilo volt, $(kV)_b$ =220 kV

Reactance of T/L:

$$X_{p.u} = X_{ohm} \times \frac{(MVA)_b}{(kV)_b^2} = 50 \times \frac{100}{220^2} = 0.1033 p.u$$

Reactance of T/F , T_1:

Base KV referred to L.T side of T/F, T_1 = base KV on H.T side $\times \frac{L.T\ voltage\ \ rating}{H.T\ voltage\ \ rating}$

$$= 220 \times \frac{33}{220} = 33kV$$

There fore $(kV)_{b,new}$=33 kV

The new p.u reactance of transformer T_1 is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

$$= 0.15 \times \frac{33^2}{33^2} \times \frac{100}{40} = 0.375 p.u$$

Reactance of generator, G:

The new p.u reactance of generator (G) is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

$$= 0.2 \times \frac{25^2}{33^2} \times \frac{100}{40} = 0.287 p.u$$

Reactance of T/F, T_2:

Base KV referred to L.T side of T/F, T_2 = base kV on H.T side $\times \frac{L.T\ voltage\ \ rating}{H.T\ voltage\ \ rating}$

$$= 220 \times \frac{11}{220} = 11kV$$

There fore $(KV)_{b,new}$=11 kV

The new p.u reactance of transformer ,T_2 is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

$$= 0.15 \times \frac{11^2}{11^2} \times \frac{100}{30} = 0.5\,p.u$$

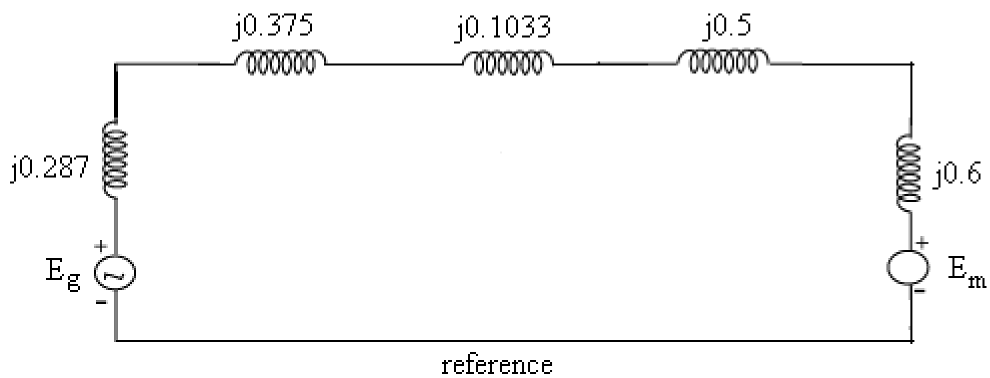
Reactance of synchronous motor :

The new p.u reactance of synch. motor (G) is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$$

$$= 0.15 \times \frac{11^2}{11^2} \times \frac{100}{50} = 0.6\,p.u$$

The reactance diagram is shown in the following figure



Problem-2: Show that p.u impedance of a transformer referred to either HV or LV side is same? **[JNTU, Regular, Nov-2009]**

Solution: Consider a 1-phase transformer having V_1 and V_2 as the primary and secondary voltages and I_1 and I_2 as the primary and secondary currents. Then, $V_1/V_2 = I_2/I_1 = K$

Let impedance as referred to primary is Z_1.

Base volt amperes= $V_1 I_1 = V_2 I_2$

Base voltage = V_1 for primary and

= V_2 for secondary

Base impedance for primary= V_1 / I_1

Per unit impedance when referred to primary is given by

$$Z_1(p.u) = \frac{Z_1}{V_1 / I_1} = \frac{I_1 V_1}{Z_1} \text{ ----------- (1)}$$

Actual impedance when referred transformer secondary= $Z_1(V_2/V_1)^2$

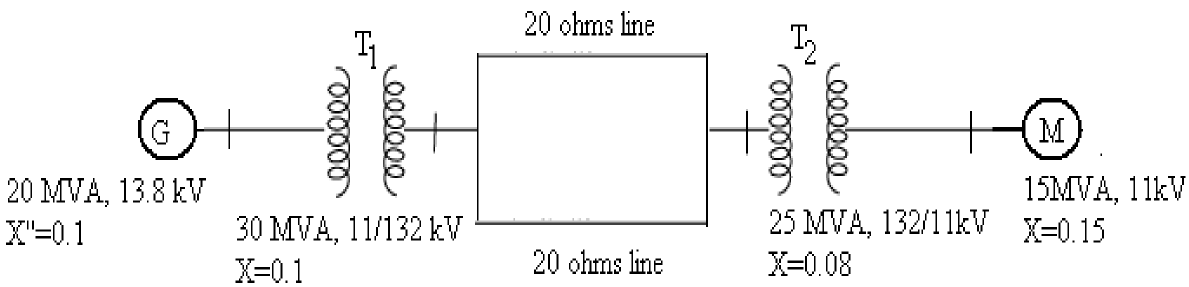
Base impedance for secondary= V_2 / I_2

Per unit impedance when referred to secondary

$$Z_2(p.u) = \frac{Z_1(V_2/V_1)^2}{V_2/I_2} = \frac{Z_1V_2I_2}{(V_1)^2} = \frac{Z_1V_1I_1}{(V_1)^2} = \frac{I_1V_1}{Z_1} \text{-------- (2)}$$

From eqns.(1) and (2), it is clear that p.u impedance of a transformer referred to either HV or LV side is same.

Problem-3: Draw the per unit impedance diagram of the network shown in the figure. Choose base quantities as the generator values? **[JNTU, Reguylar,Nov-2009]**



Solution: Let us choose the base values as 20MVA, 13.8 kV

Reactance of generator, G:

Per unit reactance of the generator = 0.1 p.u, since the generator values are taken as base values.

Reactance of T/F, T_1:

The new p.u reactance of transformer T_1 is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$$

$$= 0.1 \times \frac{11^2}{13.8^2} \times \frac{20}{30} = 0.04235\,p.u$$

Reactance of transmission line:

Reactance of transmission line = 20Ω

Total reactance of transmission line =10Ω

Base KV referred to H.T side of T/F, T_1 = base KV on L.T side $\times \frac{H.T\ voltage\quad rating}{L.T\ voltage\quad rating}$

$$= 20 \times \frac{132}{20} = 132kV$$

There fore $(kV)_{b,new}$=132 kV

$$Z_{p.u} = Z_{ohms} \times \frac{(MVA)_b}{(kV)_b^2} = 10 \times \frac{20}{132^2} = 0.011478\, p.u$$

Reactance of T/F, T_2:

Base KV referred to H.T side of T/F, T_2 = base KV on L.T side $\times \frac{H.T\ voltage\quad rating}{L.T\ voltage\quad rating}$

$$= 132 \times \frac{11}{132} = 11kV$$

There fore $(KV)_{b,new}$=11 kV

The new p.u reactance of transformer ,T_2 is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

$$= 0.08 \times \frac{11^2}{11^2} \times \frac{20}{25} = 0.064\, p.u$$

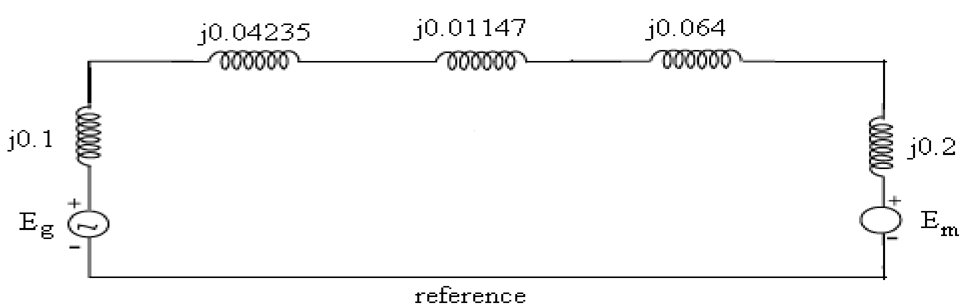
Reactance of motor, M:

The new p.u reactance of M_1 on new base is

$$X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)_{b,old}^2}{(kV)_{b,new}^2}$$

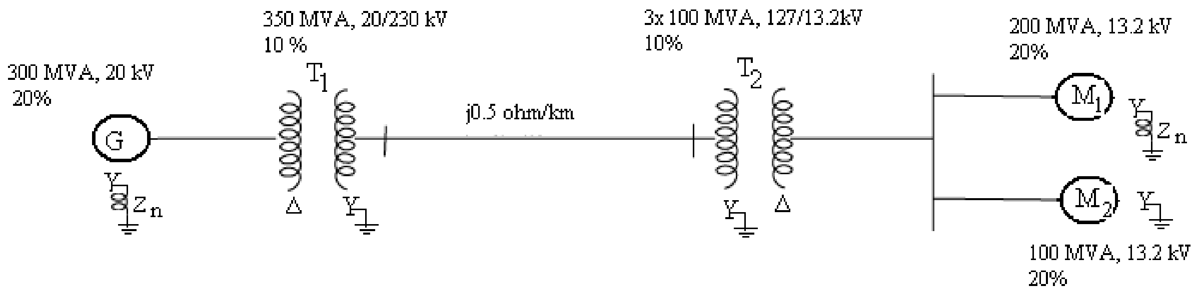
$$= 0.15 \times \frac{11^2}{11^2} \times \frac{20}{15} = 0.2\, p.u$$

The reactance diagram is shown in the following figure



Problem-3: A 300MVA, 20KV,3-Ø generator has a sub transient reactance of 20% .The generator supplies two synchronous motors through a 64 km T/L having T/Fs at both the ends as shown figure .In this T_1 is a 3-Ø transformer and T_2 is made of made of 3 single phase transformers of rating 100MVA, 127/13.2 KV ,10% reactance .Series reactance of the T/L is

0.5Ω/km . Draw the reactance diagram with all the reactance marked in p.u . Select the generator rating as base values.



Solution:

Let Base mega volt amps ,$(MVA)_{b,new}$ =300MVA

Base kilo volt, $(kV)_b$ =20 kV

Reactance of generators

Since the generator rating and the base values are same, the generator p.u reactance does not change. There fore, the p.u reactance of generator = 20%= 0.2p.u

Reactance of T/F, T_1

The new p.u reactance of transformer (T_1) , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.1 \times \frac{20^2}{20^2} \times \frac{300}{350} = 0.0857\, p.u$$

Reactance of transmission line:

Reactance of T/L = 0.5Ω/km

Total reactance of T/L=0.5×64=32Ω

Base KV referred to H.T side of T/F, T_1 = base KV on L.T side $\times \frac{H.T\ voltage\quad rating}{L.T\ voltage\quad rating}$

$$= 20 \times \frac{230}{20} = 230kV$$

There fore $(kV)_{b,new}$=230 kV

$$Z_{p.u} = Z_{ohms} \times \frac{(MVA)_b}{(KV)^2_b} = 32 \times \frac{300}{230^2} = 0.1815\, p.u$$

Reactance of T/F, T_2

The T/F, T_2 a 3-Ø T/F bank formed using three numbers of single phase transformers with voltage rating 127/13.2KV.In this the high voltage side is star connected and low voltage side is delta connected.

Voltage ratio of line voltage of 3-Ω T/F bank = $\frac{\sqrt{3}\times 127}{13.2} = \frac{220}{13.2}$

Base KV referred to H.T side of T/F, T_2 = base KV on L.T side $\times \frac{H.T\ voltage\quad rating}{L.T\ voltage\quad rating}$

$$= 230 \times \frac{13.2}{220} = 13.8kV$$

There fore $(kV)_{b,new}$=13.8 kV

The new p.u reactance of transformer (T_2) , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.1 \times \frac{13.2^2}{13.8^2} \times \frac{300}{3\times 100}$$
$$= 0.0915\,p.u$$

Reactance of motor M_1

The new p.u reactance of M_1 on new base, $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

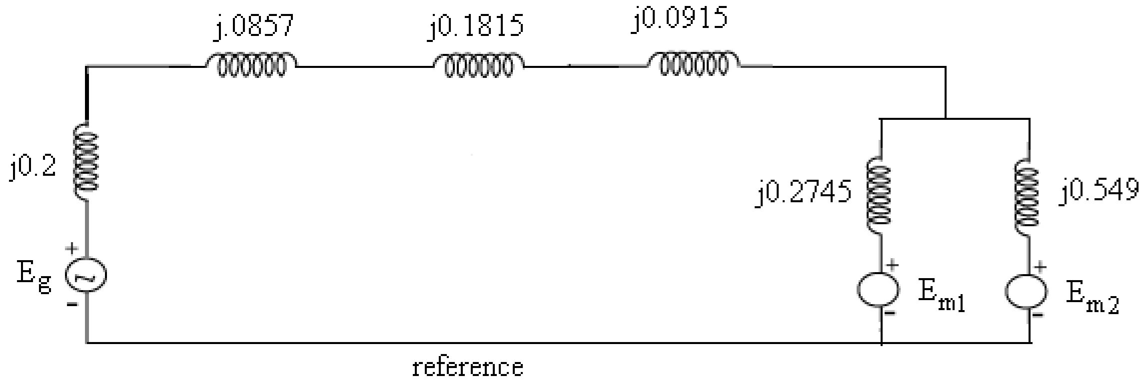
$$= 0.1 \times \frac{13.2^2}{13.8^2} \times \frac{300}{100}$$
$$= 0.2745\,p.u$$

Reactance of motor M_2

The new p.u reactance of M_2 on new base , $X(p.u)_{new} = X(p.u)_{old} \times \frac{(MVA)_{b,new}}{(MVA)_{b,old}} \times \frac{(kV)^2_{b,old}}{(kV)^2_{b,new}}$

$$= 0.2 \times \frac{13.2^2}{13.8^2} \times \frac{300}{100}$$
$$= 0.549\,p.u$$

The reactance diagram is shown in the following figure



TRAVELING WAVES ON TRANSMISSION LINES

INTRODUCTION TO TRANSIENTS

Introduction to Transients:

- Transient phenomenon is an a periodic function of time and does not last longer. The duration for which they last is very insignificant as compared with the operating time of the system. Yet they are very important because depending upon the severity of these transients, the system may result into black out in a city, shut down of a plant, fires in some buildings, etc.
- The power system can be considered as made up of linear impedance elements of resistance, inductance and capacitance. The circuit is normally energized and carries load until a fault suddenly occurs. The fault, then, corresponds to the closing of a switch (or switches, depending upon the type of fault) in the electrical circuit. The closing of this switch changes the circuit so that a new distribution of currents and voltages is brought about. This redistribution is accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high.
- It is very important to realize that this redistribution of currents and voltages cannot take place instantaneously for the following reasons:

1. The electromagnetic energy stored by an inductance L is ½ LI^2 where I is the instantaneous value of current. Assuming inductance to be constant the change in magnetic energy requires change in current which an inductor is opposed by an emf of magnitude L dI/dt. . In order to change the current instantaneously dt = 0 and therefore L dI/0 is infinity,

i.e., to bring about instantaneous change in current the emf in the inductor should become infinity which is practically not possible and, therefore, it can be said that the change of energy in an inductor is gradual.

2. The electrostatic energy stored by a capacitor C is given by ½ CV^2 where V is the instantaneous value of voltage. Assuming capacitance to be constant, the change in energy requires change in voltage across the capacitor. Since, for a capacitor, $dV/dt = I/C$, to bring instantaneous change in voltage, i.e., for $dt = 0$ the change in current required is infinite which again cannot be achieved in practice and, therefore, it can be said that change in energy in a capacitor is also gradual.

- There are only two components L and C in an electrical circuit which store energy and we have seen that the change in energy through these components is gradual and, therefore, the redistribution of energy following a circuit change takes a finite time. The third component, the resistance R, consumes energy. At any time, the principle of conservation of energy in an electrical circuit applies, i.e. the rate of generation of energy is equal to the rate of storage of energy plus the rate of energy consumption.
- It is clear that the three simple facts, namely,

1. The current cannot change instantaneously through an inductor,

2. The voltage across a capacitor cannot change instantaneously,

3. The law of conservation of energy must hold good, are fundamental to the phenomenon of transients in electric power systems.

- From the above it can be said that in order to have transients in an electrical system the following requirements should be met:

1. Either inductor or capacitor or both should be present.

2. A sudden change in the form of a fault or any switching operation should take place.

- There are two components of voltages in a power system during transient period:

1. Fundamental frequency voltages, and

2. Natural frequency voltages usually of short duration which are superimposed upon the fundamental frequency voltages. There is third component also known as harmonic voltages resulting from unbalanced currents flowing in rotating machines in which the reactances in the direct and quadrature axes are unequal.

- Natural frequency voltages appear immediately after the sudden occurrence of a fault. They simply add to the fundamental frequency voltages. Since resultant voltages are of greater importance from a practical viewpoint it will be preferable to speak of the fundamental frequency and natural frequency components simply as a transient voltage.
- The transient voltages are affected by the number of connections and the arrangements of the circuits.
- Transients, in which only one form of energy-storage, magnetic or electric is concerned, are called single energy transients, where both magnetic and electric energies are contained in or accepted by the circuit, double energy transients are involved.

Transients with D.C. Source

Transients with DC source:

(a) Resistance only:

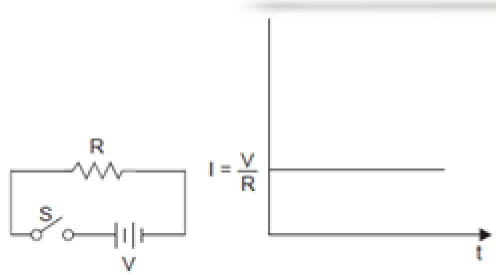


Fig: Resistive Circuit

As soon as the switch S is closed, the current in the circuit will be determined according to Ohm's law.

$$I = \frac{V}{R}$$

Now, transients will be there in the circuit.

(b) Inductance only:

When switch S is closed, the current in the circuit will be given by

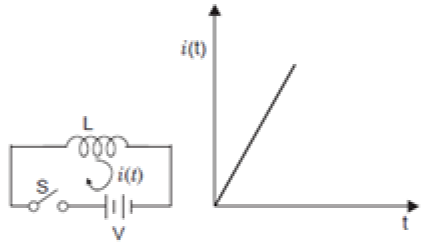


Fig: Inductive Circuit

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{Ls} = \frac{V}{L} \cdot \frac{1}{s^2}$$

$$i(t) = \frac{V}{L} t$$

This shows that when a pure inductance is switched on to a d.c. source, the current at t = 0 is zero and this increases linearly with time till for infinite time it becomes infinity. In practice, of course, a choke coil will have some finite resistance, however small; the current will settle down to the value V/R, where R is the resistance of the coil.

(c) Capacitance only:

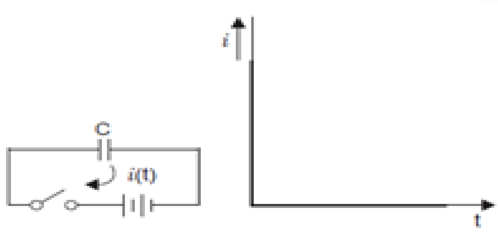


Fig: Capacitance circuit

When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} Cs = VC$$

which is an impulse of strength (magnitude) VC.

(d) R-L Circuit:

When switch S is closed, the current in the circuit is given by

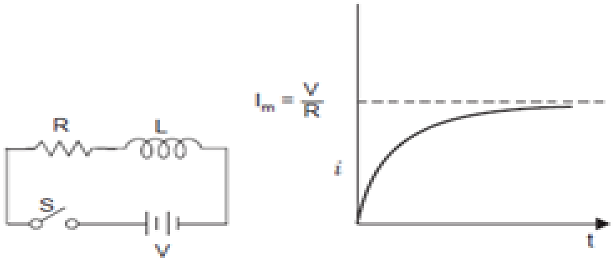


Fig: R-L circuit

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s}\frac{1}{R+Ls} = \frac{V}{s}\frac{1/L}{s+R/L}$$

$$= \frac{V}{L}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]\frac{L}{R}$$

$$= \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

$$i(t) = \frac{V}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right]$$

It can be seen from the expression that the current will reach V/R value after infinite time. Also it can be seen that the inductor just after closing of the switch behaves as an open circuit and that is why the current at $t = 0$ is zero. When $t = L/R$,

$$i(t) = \frac{V}{R}\left(1 - \frac{1}{e}\right) = I_m\left(1 - \frac{1}{e}\right) = 0.632\ I_m$$

At time t = L/R the current in the circuit is 63.2% of the maximum value reached in the circuit. This time in seconds is called the time-constant of the circuit. The larger the value of inductance in the circuit as compared with resistance the slower will be the buildup of current in the circuit. The energy stored in the inductor under steady state condition will be ½ LI^2 m , where $I_m = V/R$.

Transients with A.C. Source

Transients with AC source:

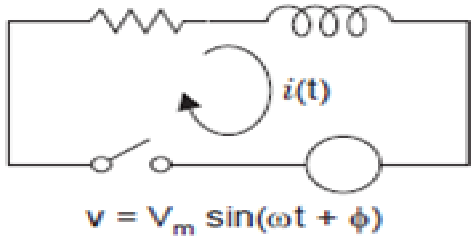


Fig: R-L circuit

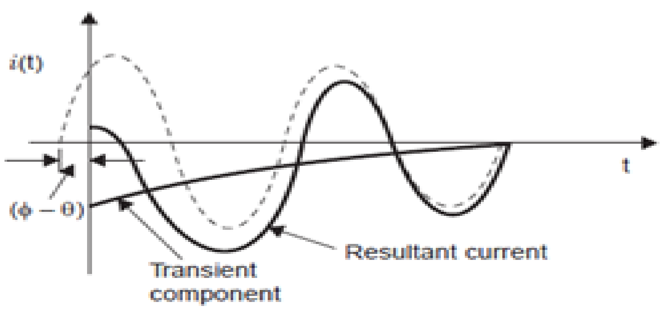


Fig: Asymmetrical alternating current

R-L Circuit:

When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = V_m \left\{ \frac{\omega \cos\phi}{s^2+\omega^2} + \frac{s \sin\phi}{s^2+\omega^2} \right\} \frac{1}{R+Ls}$$

$$= \frac{V_m}{L} \left\{ \frac{\omega \cos\phi}{s^2+\omega^2} + \frac{s \sin\phi}{s^2+\omega^2} \right\} \frac{1}{s+R/L}$$

Let $\frac{R}{L} = a$; then

$$I(s) = \frac{V_m}{L}\left\{\frac{\omega\cos\phi}{(s+a)(s^2+\omega^2)} + \frac{s\sin\phi}{(s+a)(s^2+\omega^2)}\right\}$$

Now $$\frac{1}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{1}{(s+a)} + \frac{a}{(s^2+\omega^2)} - \frac{s}{(s^2+\omega^2)}\right\}$$

and $$\frac{s}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{as}{s^2+\omega^2} + \frac{\omega^2}{s^2+\omega^2} - \frac{a}{s+a}\right\}$$

Therefore $$\mathrm{L}^{-1}I(s) = \frac{V_m}{(a^2+\omega^2)L}\left[\omega\cos\phi\left\{e^{-at} + \frac{a}{\omega}\sin\omega t - \cos\omega t\right\}\right.$$
$$\left. + \sin\phi\{a\cos\omega t + \omega\sin\omega t - ae^{-at}\}\right]$$

The equation can be further simplified to

$$i(t) = \frac{V_m}{L\sqrt{a^2+\omega^2}}\{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)e^{-at}\}$$
$$= \frac{V_m}{(R^2+\omega^2L^2)^{1/2}}\{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)\,e^{-at}\}$$

where $$\theta = \tan^{-1}\frac{\omega L}{R}.$$

- The first term in the expression above is the steady state sinusoidal variation and the second term is the transient part of it which vanishes theoretically after infinite time. But practically, it vanishes very quickly after two or three cycles. The transient decay as is seen depends upon the time constant 1/a = L/R of the circuit.
- Also at t = 0 it can be seen that the transient component equals the steady state component and since the transient component is negative the net current is zero at t = 0. It can be seen that the transient component will be zero in case the switching on of the voltage wave is done when θ = φ, i.e., when the wave is passing through an angle $\varphi = \tan^{-1}(\omega L)/R$.

- This is the situation when we have no transient even though the circuit contains inductance and there is switching operation also. On the other hand if $\varphi - \theta = \pm \pi/2$, the transient term will have its maximum value and the first peak of the resulting current will be twice the peak value of the sinusoidal steady state component.

TRAVELING WAVES ON TRANSMISSION LINES

Traveling waves on transmission lines:

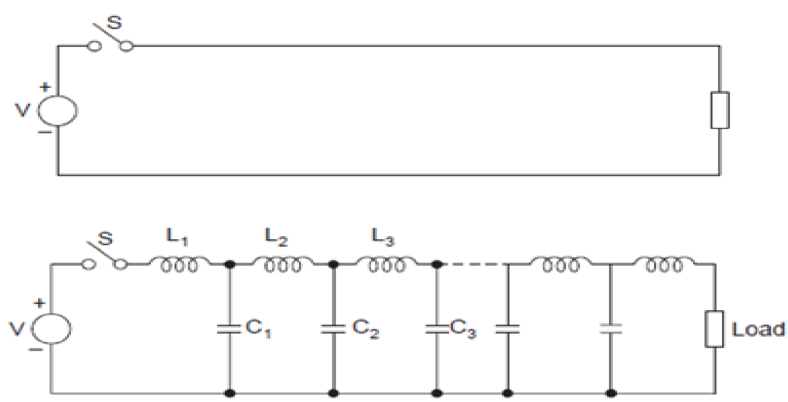


Fig: (a) Long Transmission line (b) Equivalent section of a long transmission line

- Some parts of a power system where the approach of transients is inadequate. The most obvious example is the transmission line. Here the parameters L, C and R are uniformly distributed over the length of the line.
- For steady state operation of the line the transmission lines could be represented by lumped parameters but for the transient behavior of the lines they must be represented by their actual circuits i.e., distributed parameters.
- We say that for a 50 Hz supply and short transmission line the sending end current equals the receiving end current and the change in voltage from sending end to receiving end is smooth. This is not so when transmission line is subjected to a transient. To understand the traveling wave phenomenon over transmission lines consider Fig.(a). The line is assumed to be lossless. Let L and C be the inductance and capacitance respectively per unit length of the line.
- The line has been represented in Fig. (b) by a large number of L and C sections.
- When switch S is closed, the inductance L_1 acts as an open circuit and C_1 as short circuit instantaneously. The same instant the next section cannot be

charged because the voltage across the capacitor C_1 is zero. So unless the capacitor C_1 is charged to some value whatsoever, charging of the capacitor C_2 through L_2 is not possible which, of course, will take some finite time.

- The same argument applies to the third section, fourth section and so on. So we see that the voltage at the successive sections builds up gradually. This gradual buildup of voltage over the transmission line conductors can be regarded as though a voltage wave is traveling from one end to the other end and the gradual charging of the capacitances is due to the associated current wave.
- Now it is desired to find out expressions for the relation between the voltage and current waves traveling over the transmission lines and their velocity of propagation.

Open End Line

Open End Line:

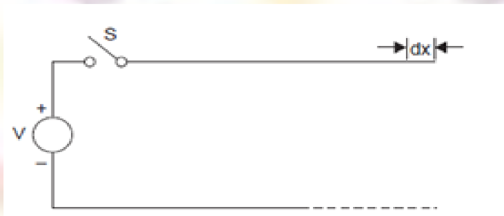


Fig: Case of an open-ended line

- When switch S is closed, a voltage and current wave of magnitudes V and I respectively travel towards the open end. These waves are related by equation:

$$\frac{V}{I} = Z$$

where Z is the characteristic impedance of the line. Consider the last element dx of the line, because, it is here where the wave is going to see a change in impedance, an impedance different from Z (infinite impedance as the line is open-ended).

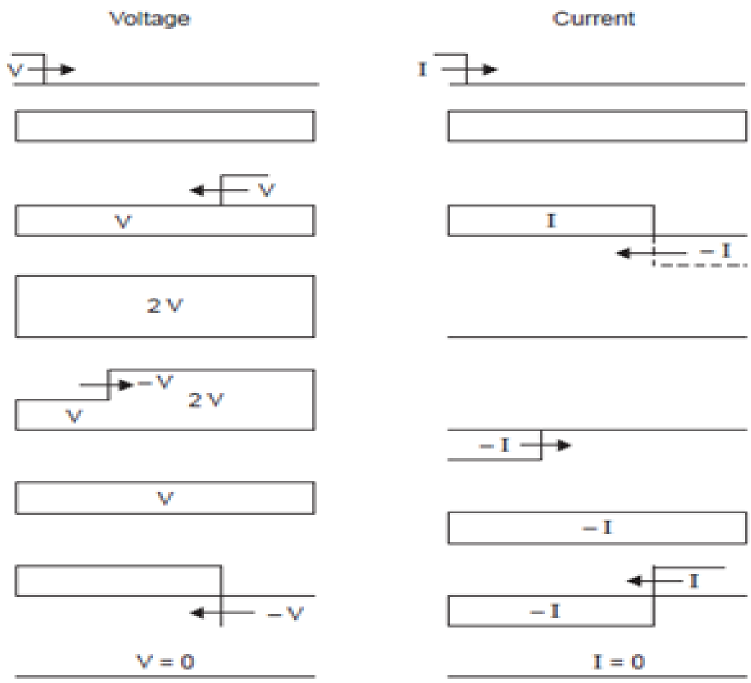


Fig: Variation of voltage and current in an open-ended line

- The electromagnetic energy stored by the element dx is given by ½ L*dx*I^2 and electrostatic energy in the element dx, ½ C*dx*V^2. Since the current at the open end is zero, the electromagnetic energy vanishes and is transformed into electrostatic energy. As a result, let the change in voltage be e; then

$$\frac{1}{2} L dx I^2 = \frac{1}{2} C dx e^2$$

or $$\left(\frac{e}{I}\right)^2 = \frac{L}{C}$$

or $$e = IZ = V$$

- This means the potential of the open end is raised by V volts; therefore, the total potential of the open end when the wave reaches this end is

V. V = 2V

- The wave that starts traveling over the line when the switch S is closed, could be considered as the incident wave and after the wave reaches the open end, the rise in potential V could be considered due to a wave which is reflected at the open end and actual voltage at the open end could be considered as the refracted or transmitted wave and is thus

Refracted wave = Incident wave Reflected wave

- We have seen that for an open end line a traveling wave is reflected back with positive sign and coefficient of reflection as unity.
- Let us see now about the current wave. As soon as the incident current wave I reaches the open end, the current at the open end is zero, this could be explained by saying that a current wave of I magnitude travels back over the transmission line. This means for an open end line, a current wave is reflected with negative sign and coefficient of reflection unity.
- After the voltage and current waves are reflected back from the open end, they reach the source end, the voltage over the line becomes 2V and the current is zero. The voltage at source end cannot be more than the source voltage V, therefore, a voltage wave of –V and current wave of –I is reflected back into the line. It can be seen that after the waves have travelled through a distance of 4l where l is the length of the line, they would have wiped out both the current and voltage waves, leaving the line momentarily in its original state. The above cycle repeats itself.

Short Circuited Line:

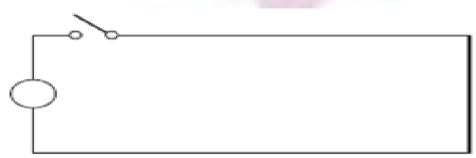


Fig: Case of a short-circuited line

- When switch S is closed, a voltage wave of magnitude V and current wave of magnitude I start traveling towards the shorted end. Consider again the last element dx where the electrostatic energy stored by the element is ½ $C dx V^2$ and electromagnetic energy ½ $L dx I^2$.
- Since the voltage at the shorted end is zero, the electrostatic energy vanishes and is transformed into electromagnetic energy. As a result, let the change in the current be i; then

$$\frac{1}{2} CdxV^2 = \frac{1}{2} Ldxi^2$$

or $$V = iZ$$

or $$i = \frac{V}{Z} = I$$

- This means the increase in current is I amperes. As a result the total current at the shorted end, when the current wave reaches the end is $I + I = 2I$ amperes. This could be considered due to a reflected current wave of magnitude I amperes. Therefore, for a short-circuited end the current wave is reflected back with positive sign and coefficient of reflection as unity.

- Since the voltage at the shorted end is zero, a voltage wave of – V could be considered to have been reflected back into the line, i.e., the current wave in case of short-circuited end is reflected back with positive sign and with coefficient of reflection as unity, whereas the voltage wave is reflected back with negative sign and coefficient of reflection as in the variation of voltage and current over the line is explained in figure of the variation of voltage and current.
- It is seen from above that the voltage wave periodically reduces to zero after it has travelled through a distance of twice the length of the line whereas after each reflection at either end the current is built up by an amount $V/Z_n = I$.

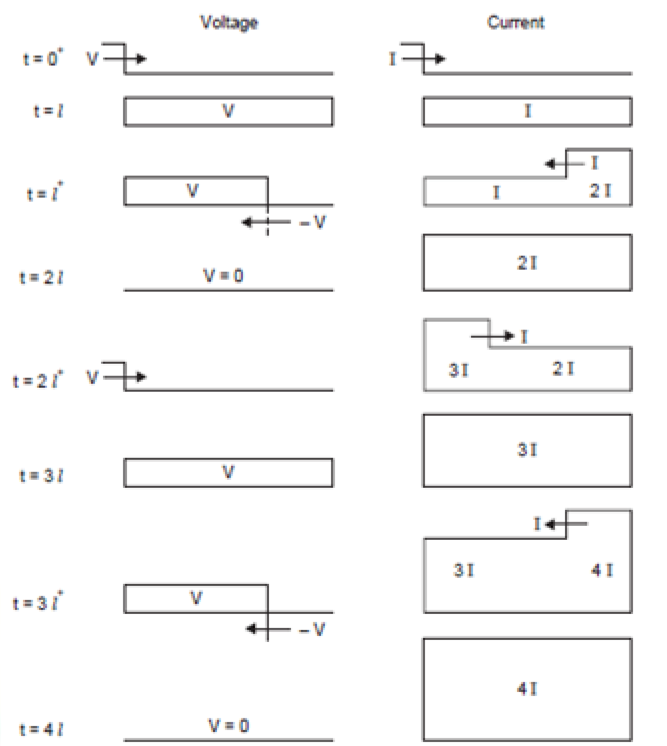


Fig: Variation of voltage and current in a short ended line

- Theoretically, the reflections will be infinite and therefore, the current will reach infinite value. But practically in an actual system the current will be limited by the resistance of the line and the final value of the current will be I′ = V/R, where R is the resistance of transmission line.

Line Terminated Through a Resistance

Line terminated through a resistance:

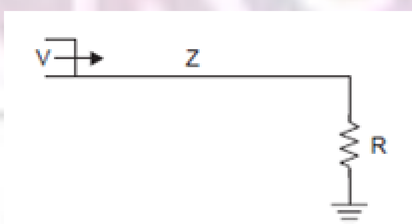


Fig: Line terminated through a resistance

- Let Z be the surge impedance of the line terminated through a resistance R. It is seen that whatever be the value of the terminating impedance whether it is open or short circuited, one of the two voltage or current waves is reflected back with negative sign.

- Also, since the reflected wave travels along the overhead line or over the line along which the incident wave travelled, therefore, the following relation holds good for reflected voltage and current waves.

$$I' = \frac{V'}{Z}$$

where V′ and I′ are the reflected voltage and current waves.

- Also,

Refracted or transmitted wave = Incident wave Reflected wave

- Let V″ and I″ be the refracted voltage and current waves into the resistor R when the incident waves V and I reach the resistance R. The following relations hold good:

$$I = \frac{V}{Z}$$

$$I' = -\frac{V'}{Z}$$

$$I'' = \frac{V''}{Z}$$

Since $I'' = I + I'$ and $V'' = V + V'$, using these relations, we have

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} = \frac{V}{Z} - \frac{V'' - V}{Z} = \frac{2V}{Z} - \frac{V''}{Z}$$

or $$V'' = \frac{2VR}{Z + R}$$

and current $$I'' = \frac{2V}{R+Z} = \frac{V}{Z} \cdot \frac{2Z}{R+Z} = I \cdot \frac{2Z}{R+Z}$$

- Similarly substituting for V″ in terms of (V V′), we have

$$\frac{V+V'}{R}=\frac{V}{Z}-\frac{V'}{Z}$$

or
$$V'=V\frac{R-Z}{R+Z}$$

and
$$I'=-\frac{V'}{Z}=-\frac{V}{Z}\frac{(R-Z)}{R+Z}$$

From the relations above, the coefficient of refraction for current waves

$$=\frac{2Z}{R+Z}$$

and for voltage waves
$$=\frac{2R}{R+Z}$$

Similarly, the coefficient of reflection for current waves

$$=-\frac{R-Z}{R+Z}$$

and for voltage waves
$$=+\frac{R-Z}{R+Z}$$

Line Connected to a Cable

Line connected to a cable:

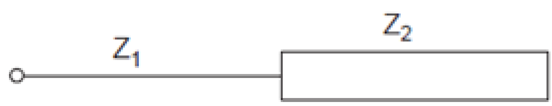


Fig: Line connected to a cable

- A wave travels over the line and enters the cable. Since the wave looks into a different impedance, it suffers reflection and refraction at the junction and the refracted voltage wave is given by

$$V''=V\frac{2Z_2}{Z_1+Z_2}$$

- The other waves can be obtained. The impedance of the overhead line and cable are approximately 400 ohms and 40 ohms respectively. With these values it can be seen that the voltage entering the cable will be

$$V'' = V \cdot \frac{2 \times 40}{40 + 400} = \frac{2}{11} V$$

i.e., it is about 20% of the incident voltage V.

- It is for this reason that an overhead line is terminated near a station by connecting the station equipment to the overhead line through a short length of underground cable. Besides the reduction in the magnitude of the voltage wave, the steepness is also reduced because of the capacitance of the cable.

- The reduction in steepness is very important because this is one of the factors for reducing the voltage distribution along the windings of the equipment. While connecting the overhead line to a station equipment through a cable.

- It is important to note that the length of the cable should not be very short (should not be shorter than the expected length of the wave) otherwise successive reflections at the junction may result in piling up of voltage and the voltage at the junction may reach the incident voltage.

Reflection and Refraction at a T-junction

Reflection and Refraction at a T-Junction:

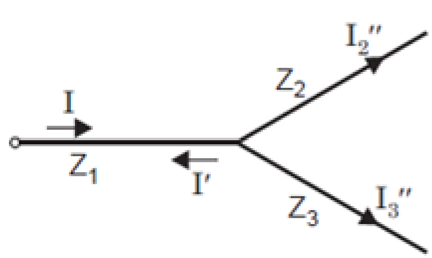


Fig: A Bi-furcated line

- A voltage wave V is traveling over the line with surge impedance Z1 as shown in figure. When it reaches the junction, it looks a change in impedance and, therefore, suffers reflection and refraction.

- Let V_2'', I_2'' and V_3'', I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively. Since Z_2 and Z_3 form a parallel path as far as the surge wave is concerned,

$$\mathbf{V_2'' = V_3'' = V''.}$$

- Therefore, the following relations hold good.

$$V + V' = V''$$

$$I = \frac{V}{Z_1},\ I' = -\frac{V'}{Z_1}$$

$$I_2'' = \frac{V''}{Z_2} \quad \text{and} \quad I_3'' = \frac{V''}{Z_3}$$

$$I + I' = I_2'' + I_3''$$

- Substituting the values of Current in the above equation we get,

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

Substituting for $V' = V'' - V$,

$$\frac{V}{Z_1} - \frac{V'' - V}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right)$$

or

$$V'' = \frac{2V/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

Similarly other quantities can be derived.

Line Terminated through a Capacitance

Line terminated through a capacitance:

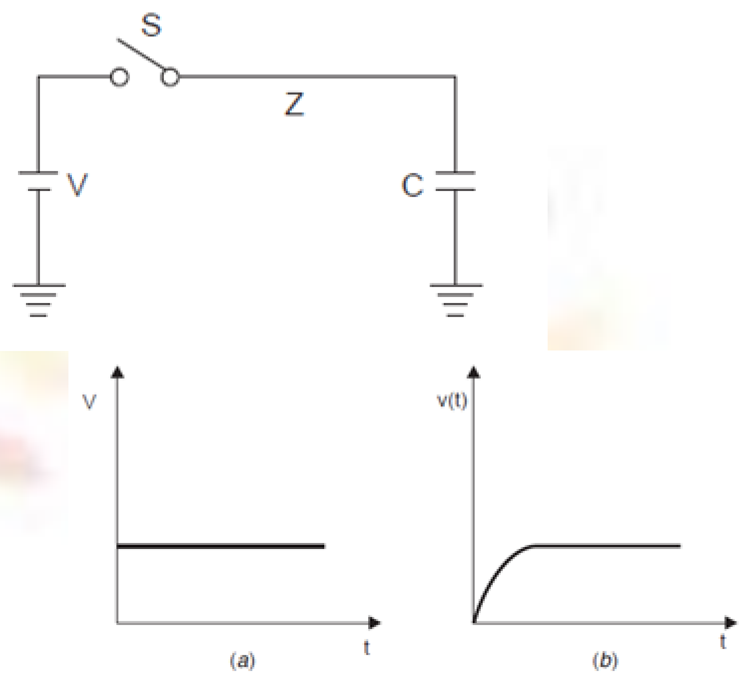


Fig: Line terminated through a capacitance *Fig: (a) Incident voltage and (b) Voltage across capacitor*

- Here a d.c. surge of infinite length travels over the line of surge impedance Z and is incident on the capacitor as shown in figure of line terminated through a capacitance.
- The voltage across the capacitor i.e., the refracted voltage needs to be found out. The refracted voltage, is given by
-

$$V''(s) = \frac{2 \times 1/Cs}{Z + 1/Cs} \cdot \frac{V}{s} = \frac{2V}{s} \frac{1}{ZCs+1}$$

$$= \frac{2V}{s} \cdot \frac{1/ZC}{s+1/ZC} = 2V\left[\frac{1}{s} - \frac{1}{s+1/ZC}\right]$$

$$v''(t) = 2V[1 - e^{-t/ZC}]$$

- The variation of voltage is shown in the figure of graphs showing voltage across capacitor.
- Since terminating impedance is not a transmission line, V″(s) is not a traveling wave but it is the voltage across the capacitor C.

Capacitor Connection at a T

Capacitor Connection at a T:

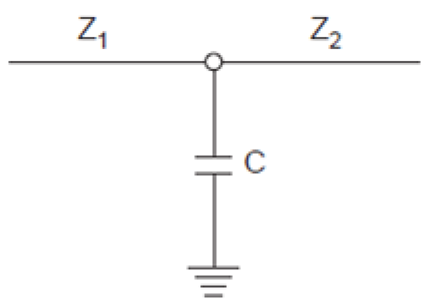


Fig: Capacitor connector at T

The voltage across capacitor is given by the equation

$$V''(s) = \frac{2V/Z_1 s}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs} = \frac{2VZ_2}{s} \cdot \frac{(1/Z_1Z_2C)}{\frac{Z_1+Z_2}{Z_1Z_2C} + s} = \frac{2V}{sZ_1C} \cdot \frac{1}{s + \frac{Z_1Z_2}{Z_1Z_2C}}$$

$$\text{Let } \frac{Z_1+Z_2}{Z_1Z_2C} = \alpha; \text{ then}$$

$$V''(s) = \frac{2V}{s} \cdot \frac{1/Z_1C}{s+\alpha}$$

or
$$V''(s) = \frac{2V}{s} \cdot \frac{Z_2}{Z_1+Z_2} \cdot \frac{(Z_1+Z_2)/Z_1Z_2C}{(s+\alpha)}$$

$$= \frac{2V}{s} \cdot \frac{Z_2}{Z_1+Z_2} \cdot \frac{\alpha}{s+\alpha} = \frac{2VZ_2}{Z_1+Z_2}\left[\frac{1}{s} - \frac{1}{s+\alpha}\right]$$

or
$$v''(t) = \frac{2V.Z_2}{Z_1+Z_2}\left[1 - \exp\left(-\frac{Z_1+Z_2}{Z_1Z_2C}t\right)\right]$$

The variation of the wave is shown as:

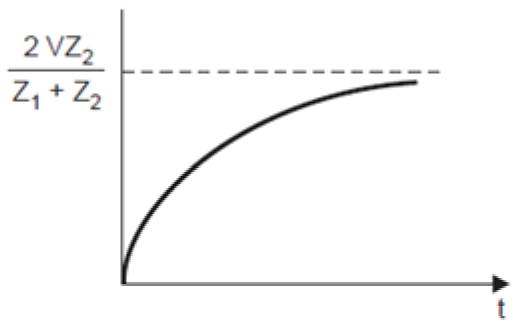


Fig: Variation of voltage across the capacitor

Attenuation of Traveling Waves

Attenuation of Traveling Waves:

Let R, L, C and G be the resistance, inductance, capacitance and conductance respectively per unit length of a line. Let the value of voltage and current waves at x = 0 be V_0 and I_0. Our objective is to find the values of voltage and current waves when they have travelled through a distance of x units over the overhead line. Let the time taken be t units when voltage and current waves are V and I respectively. To travel a distance of dx, let the time taken be dt. The equivalent circuit for the differential length dx of the line is shown.

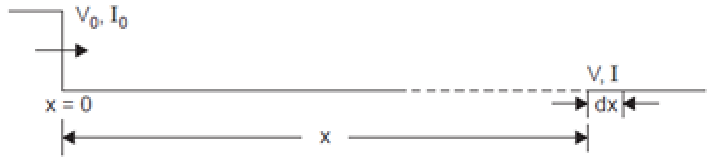


Fig: Travelling wave on a lossy line

The power loss in the differential element is

$$dp = I^2 R\,dx + V^2 G\,dx$$

Also power at a distance x. $VI = p = I^2 Z_n$

Differential power, $dp = -2IZ_n dI$

where Z_n is the natural impedance of the line. Here negative sign has been assigned as there is reduction in power as the wave travels with time.

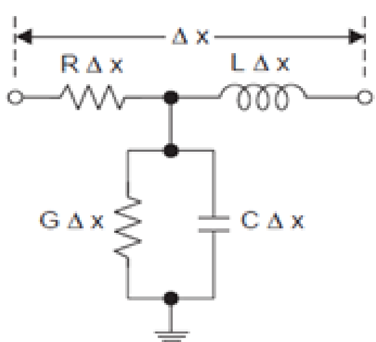


Fig: Differential element of transmission line

Equating the values of *dp*, we get,

$$-2IZ_n dI = I^2Rdx + V^2Gdx$$
$$= I^2Rdx + I^2Z_n^2Gdx$$

or $$dI = -\frac{I(R+GZ_n^2)}{2Z_n}dx$$

or $$\frac{dI}{I} = -\frac{(R+GZ_n^2)}{2Z_n}dx$$

or $$\ln I = -\frac{(R+GZ_n^2)}{2Z_n}x + A$$

At $x = 0$, $I = I_0$, $\therefore\ A = \ln I_0$

or $$\ln\frac{I}{I_0} = -\frac{R+GZ_n^2}{2Z_n}x = -ax \text{ (say)}$$

where $$a = \frac{R+GZ_n^2}{2Z_n}.$$

$$\therefore\quad I = I_0e^{-ax}.$$

Similarly it can be proved that $V = V_0e^{-ax}$. This shows that the current and voltage waves get attenuated exponentially as they travel over the line and the magnitude of attenuation depends upon the parameters of the line. Since the value of resistance depends not only on the size of the conductors but also on the shape and length of the waves.

CAPACITANCE SWITCHING

Capacitance Switching:

- The switching of a capacitance such as disconnecting a line or a cable or a bank of capacitor poses serious problems in power systems in terms of abnormally high voltages across the circuit breaker contacts. Under this situation the current leads the voltage by about 90°.

- Assuming that the current interruption takes place when it is passing through zero value the capacitor will be charged to maximum voltage. Since the capacitor is now isolated from the source, it retains its charge as shown in figure and because of trapping of this charge, half a cycle after the current zero the voltage across the circuit breaker contact is 2V which may prove to be dangerous and may result in the circuit breaker restrike. This is equivalent to closing the switch suddenly which will result into oscillations in the circuit at the natural frequency $f = \frac{1}{2\pi\sqrt{LC}}$

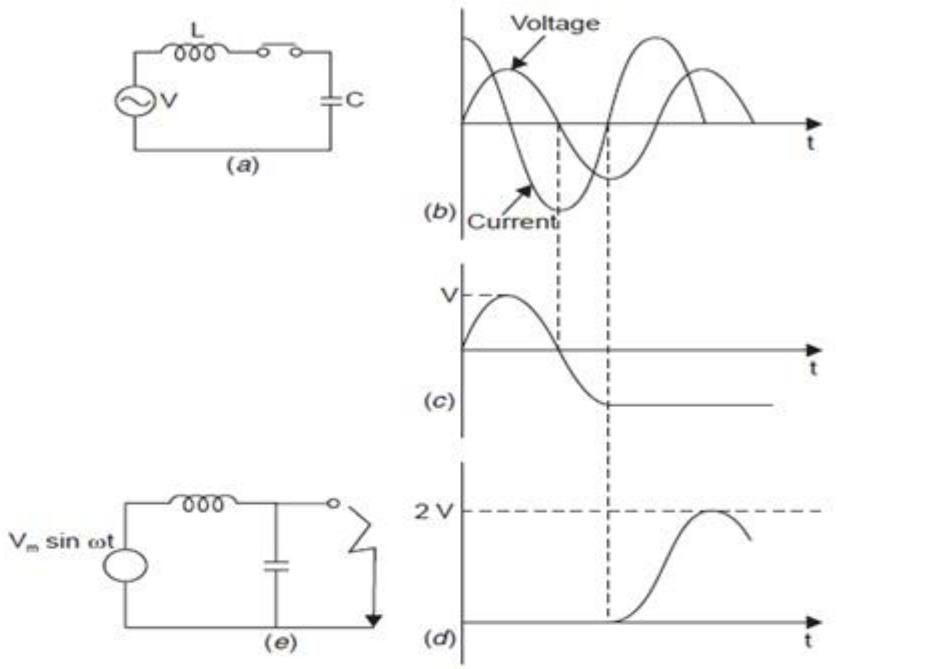


Fig: (a) Equivalent circuit for capacitor switching; (b) System voltage and current; (c) Capacitor voltage; (d) Voltage across the switch; (e) Equivalent circuit for 3-φ fault.

- The circuit condition corresponds to Fig. (e). The only difference between the two circuits is that whereas in Fig. the capacitor is charged to a voltage V, in Fig.(e) it is assumed to be without charge. Therefore, the voltage across the capacitor reaches 3V. Since the source voltage is V, the voltage across the breaker contacts after another half cycle will be 4V which may cause another restrike.

- This phenomenon may theoretically continue indefinitely, increasing the voltage by successive increments of 2V. This may result into an external flashover or the failure of the capacitor. This is due to the inability of the circuit breaker to provide sufficient dielectric strength to the contacts to avoid restrikes after they are opened first.

UNIT 4

OVERVOLTAGE PROTECTION

Overvoltage Protection:

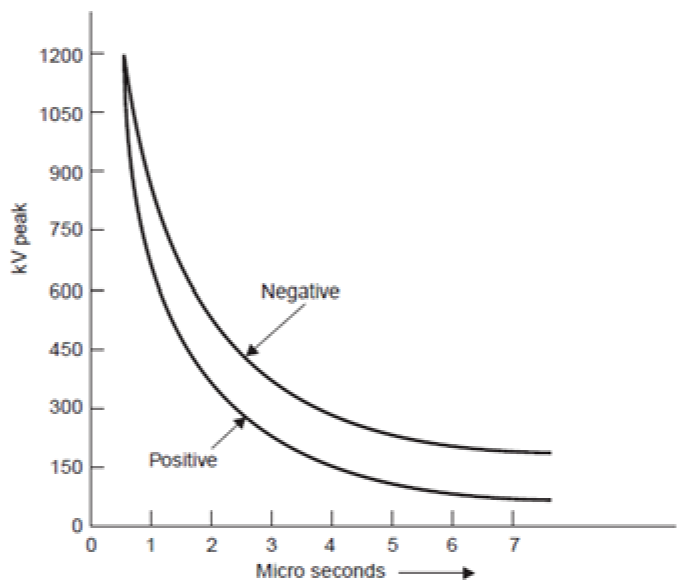


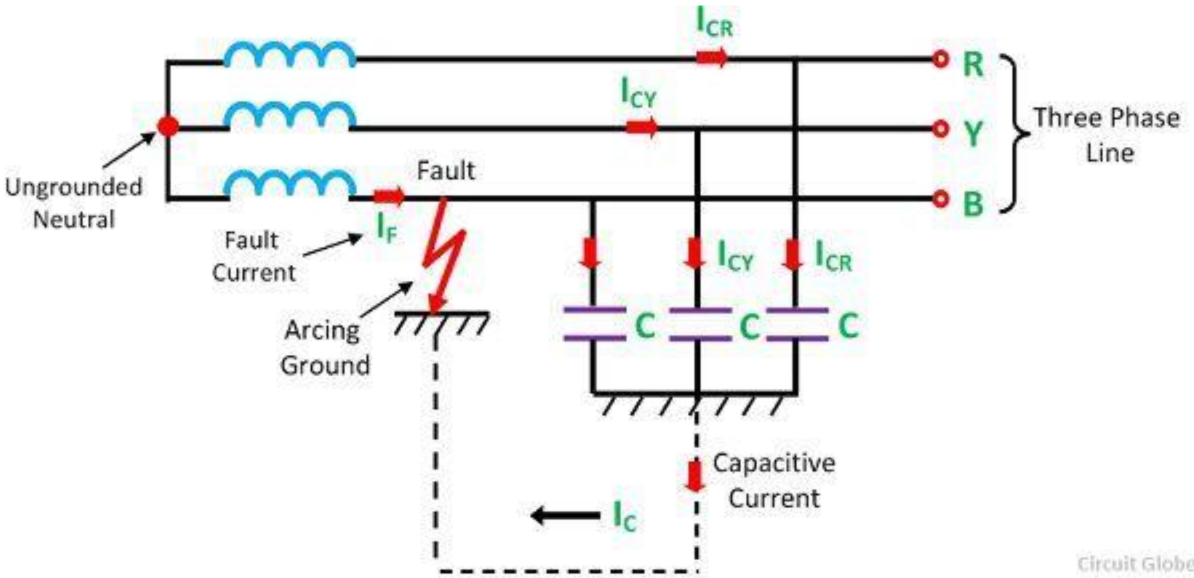
Fig: Volt-time curves of gaps for positive and negative polarity

- The causes of over-voltages in the system have been studied extensively in previous sections. Basically, there are two sources: (i) external over-voltages due to mainly lightning, and (ii) internal over-voltages mainly due to switching operation. The system can be protected against external over-voltages using what are known as shielding methods which do not allow an arc path to form between the line conductors and ground, thereby giving inherent protection in the line design.
- For protection against internal voltages normally non-shielding methods are used which allow an arc path between the ground structure and the line conductor but means are provided to quench the arc.
- The use of ground wire is a shielding method whereas the use of spark gaps, and lightning arresters are the non-shielding methods. We will study first the non-shielding methods and then the shielding methods.

- However, the non-shielding methods can also be used for external over voltages. The non-shielding methods are based upon the principle of insulation breakdown as the overvoltage is incident on the protective device; thereby a part of the energy content in the overvoltage is discharged to the ground through the protective device.
- The insulation breakdown is not only a function of voltage but it depends upon the time for which it is applied and also it depends upon the shape and size of the electrodes used. The steeper the shape of the voltage wave, the larger will be the magnitude of voltage required for breakdown; this is because an expenditure of energy is required for the rupture of any dielectric, whether gaseous, liquid or solid, and energy involves time.
- The energy criterion for various insulations can be compared in terms of a common term known as Impulse Ratio which is defined as the ratio of breakdown voltage due to an impulse of specified shape to the breakdown voltage at power frequency.
- The impulse ratio for sphere gap is unity because this gap has a fairly uniform field and the breakdown takes place on the field ionization phenomenon mainly whereas for a needle gap it varies between 1.5 to 2.3 depending upon the frequency and gap length. This ratio is higher than unity because of the non-uniform field between the electrodes. The impulse ratio of a gap of given geometry and dimension is greater with solid than with air dielectric.
- The insulators should have a high impulse ratio for an economic design whereas the lightning arresters should have a low impulse ratio so that a surge incident on the lightning arrester may be-by passed to the ground instead of passing it on to the apparatus.
- The volt-time characteristics of gaps having one electrode grounded depend upon the polarity of the voltage wave. From Fig. it is seen that the volt-time characteristic for positive polarity is lower than the negative polarity, i.e. the breakdown voltage for a negative impulse is greater than for a positive because of the nearness of earthed metal or of current carrying conductors. For post insulators the negative polarity wave has a high breakdown value whereas for suspension insulators the reverse is true.

1). Arcing Ground Phenomena:

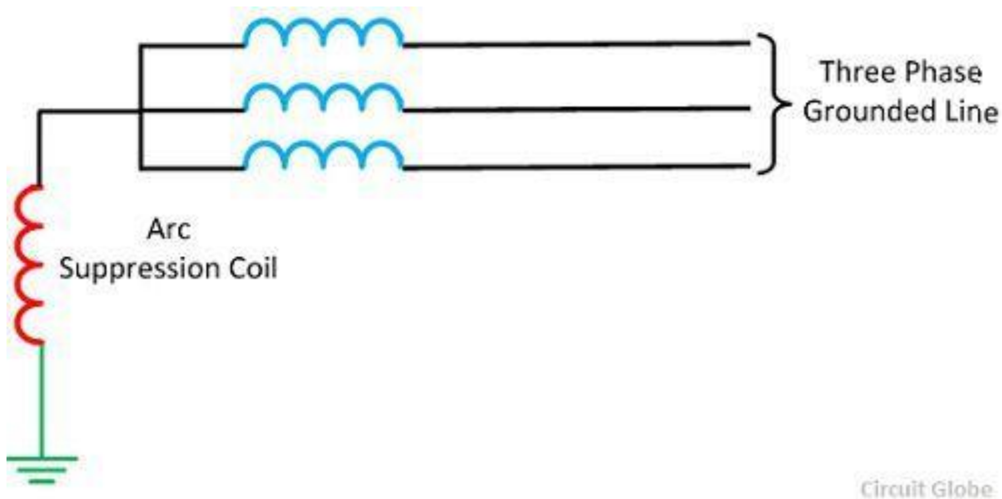
In a three phase line, each phase has a capacitance on earth. When the fault occurs on any of the phases, then the capacitive fault current flows into the ground. If the fault current exceeds 4 – 5 amperes, then it is sufficient to maintain the arc in the ionised path of the fault, even though the fault has cleared itself.



The capacitive current over 4 to 5 ampere flows through the fault give rise to an arc in the ionised path of the fault. With the formation of the arc, the voltage across it becomes zero, and therefore the arc is extinguished. The potential of the fault current restored due to which the formation of a second arc takes places. The phenomenon of intermitting arcing is called the arcing grounding.

The alternating extinction and reignition of the charging current flowing in the arc build up the potential of the other two healthy conductors due to the setting of the high-frequency oscillations. The high-frequency oscillations are superimposed on the network and produce the surge voltage as high as six times the normal value. The overvoltage damages the healthy conductor at some other points of the system.

The surge voltage due to arcing ground can remove by using the arc suppression coil or Peterson coil. The arc suppression coil has an iron cored tapped reactor connected in neutral to ground connection.



The reactor of the arc suppression coil extinguishes the arcing ground by neutralising the capacitive current. The Peterson coil isolates the system, in which the healthy phases continue supplies power and avoid the complete shut down on the system till the fault was located and isolated**.**

OVER VOLTAGE DUE TO ARCING GROUND

Over voltage due to Arching Ground:

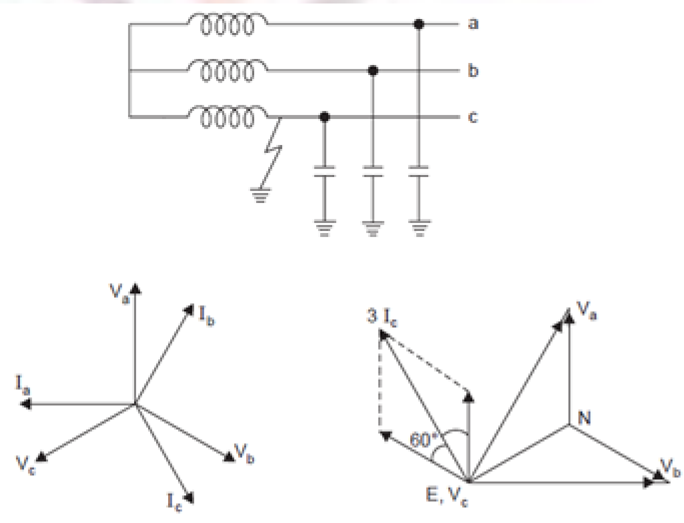


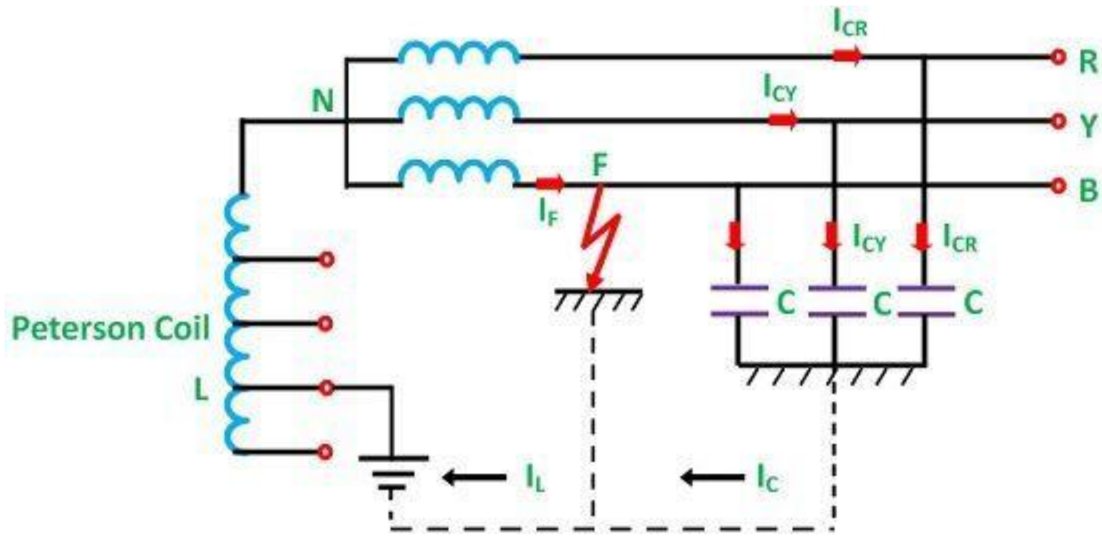
Fig: (a) 3-phase system with isolated neutral; (b) Phasor diagram under healthy condition; (c) Phasor diagram under faulted condition.

- The Figure shows a 3-phase system with isolated neutral. The shunt capacitances are also shown. Under balanced conditions and complete transposed transmission lines, the potential of the neutral is near the ground potential and the currents in various phases through the shunt capacitors are leading their corresponding voltages by 90°.
- They are displaced from each other by 120° so that the net sum of the three currents is zero. Say there is line-to-ground fault on one of the three phases (say phase 'c').
- The voltage across the shunt capacitor of that phase reduces to zero whereas those of the healthy phases become line-to-line voltages and now they are displaced by 60° rather than 120°. The net charging current now is three times the phase current under balanced conditions.
- These currents flow through the fault and the windings of the alternator. The magnitude of this current is often sufficient to sustain an arc and, therefore, we have an arcing ground. This could be due to a flashover of a support insulator. Here this flashover acts as a switch.
- If the arc extinguishes when the current is passing through zero value, the capacitors in phases a and b are charged to line voltages. The voltage across the line and the grounded points of the post insulator will be the super-position of the capacitor voltage and the generator voltage and this voltage may be good enough to cause flashover which is equivalent to restrike in a circuit breaker.
- Because of the presence of the inductance of the generator winding, the capacitances will form an oscillatory circuit and these oscillations may build up to still higher voltages and the arc may reignite causing further transient disturbances which may finally lead to complete rupture of the post insulators.

PETERSON COIL GROUNDING

Peterson coil is an iron core reactor connected between transformer neutral and ground. It is used for limiting the capacitance earth fault current which is flowing when the line ground fault occurs in the line. The coil is provided with the tapping so that it can be adjusted with the capacitance of the system. The reactance is selected so that the current through the reactor is equal to the small line charging current which would flow into the line-to-ground fault.

Consider an LG fault in phase **B** at a point F as shown in the figure below. The line-to-ground voltage of phase **B** becomes zero. The voltage of the phases R and Y increase from phase values to line values.



Line-Ground fault on phase B

Circuit Globe

The resultant of I_{CR} and I_{CY} is I_C.

$$I_c = I_{CR} + I_{CY}$$

$$I_{CR} = \frac{V_{CR}}{X_{CR}} = \frac{\sqrt{3}V_P}{X_C}$$

$$I_{CY} = \frac{V_{CY}}{X_{CY}} = \frac{\sqrt{3}V_P}{X_C}$$

$$I_{CR} = I_{CY}$$

From the phasor diagram

$$I_C = \sqrt{3}I_{CR} = \sqrt{3}I_{CY}$$

$$I_C = \frac{\sqrt{3} \times \sqrt{3}V_P}{X_C} = \frac{3V_P}{X_C}$$

For balanced conditions

$$I_C = I_L$$

If IC is equal to I_L there will be no current through the ground, and there will be no tendency of the arcing grounds to occur. With the help of Peterson coil neutral grounding, arc resistance is reduced to such a small value that it is usually self-extinguishing. Therefore, Peterson coil is also known as a ground fault neutralizer or arc suppression coil.

Peterson coil is rated for a short time of about 5 minutes, or it is designed to carry its rated current continuously. It reduces the transient fault which occurs due to lightning and also minimized the single line-to-ground voltage drops.

LIGHTNING PHENOMENON

Lightning Phenomenon:

Lightning has been a source of wonder to mankind for thousands of years. Scotland points out that any real scientific search for the first time was made into the phenomenon of lightning by Franklin in 18th century.

Before going into the various theories explaining the charge formation in a thunder cloud and the mechanism of lightning, it is desirable to review some of the accepted facts concerning the thunder cloud and the associated phenomenon.

1. The height of the cloud base above the surrounding ground level may vary from 160 to 9,500 m. The charged centers which are responsible for lightning are in the range of 300 to 1500 m.

2. The maximum charge on a cloud is of the order of 10 coulombs which is built up exponentially over a period of perhaps many seconds or even minutes.

3. The maximum potential of a cloud lies approximately within the range of 10 MV to 100 MV.

4. The energy in a lightning stroke may be of the order of 250 kWhr.

5. Raindrops:

(a) Raindrops elongate and become unstable under an electric field, the limiting diameter being 0.3 cm in a field of 100 kV/cm.

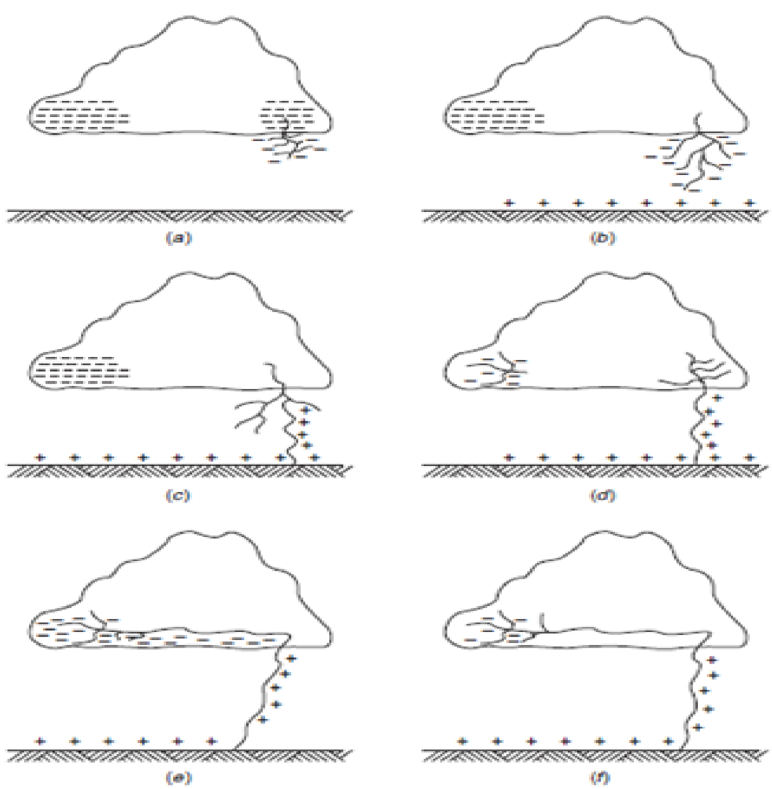
(b) A free falling raindrop attains a constant velocity with respect to the air depending upon its size. This velocity is 800 cms/sec. for drops of the size 0.25 cm dia. and is zero for spray. This means that in case the air currents are moving upwards with a velocity greater than 800 cm/sec, no rain drop can fall.

(c) Falling raindrops greater than 0.5 cm in diameter become unstable and break up into smaller drops.

(d) When a drop is broken up by air currents, the water particles become positively charged and the air negatively charged.

(e) When an ice crystal strikes with air currents, the ice crystal is negatively charged and the air positively charged.

Mechanism of Lightning Stroke:



- Lightning phenomenon is the discharge of the cloud to the ground. The cloud and the ground form two plates of a gigantic capacitor and the dielectric medium is air. Since the lower part of the cloud is negatively charged, the earth is positively charged by induction.
- Lightning discharge will require the puncture of the air between the cloud and the earth. For breakdown of air at STP condition the electric field required is 30 kV/cm peak. But in a cloud where the moisture content in the air is large and also because of the high altitude (lower pressure) it is seen that for breakdown of air the electric field required is only 10 kV/cm.
- The mechanism of lightning discharge is best explained with the help of the figure. After a gradient of approximately 10 kV/cm is set up in the cloud, the air surrounding gets ionized.
- At this a streamer (Fig. (a)) starts from the cloud towards the earth which cannot be detected with the naked eye; only a spot travelling is detected. The current in the streamer is of the order of 100 amperes and the speed of the streamer is 0.16 m/μ sec. This streamer is known as pilot streamer because this leads to the lightning phenomenon.
- Depending upon the state of ionization of the air surrounding the streamer, it is branched to several paths and this is known as stepped leader (Fig. (b)). The leader steps are of the order of 50 m in length and are accomplished in about a microsecond. The charge is brought from the cloud through the already ionized path to these pauses.
- The air surrounding these pauses is again ionized and the leader in this way reaches the earth (Fig. (c)). Once the stepped leader has made contact with the earth it is believed that a power return stroke (Fig.(c)) moves very fast up towards the cloud through the already ionized path by the leader. This streamer is very intense where the current varies between 1000 amps and 200,000 amps and the speed is about 10% that of light.
- It is here where the –ve charge of the cloud is being neutralized by the positive induced charge on the earth (Fig. (d)). It is this instant which gives rise to lightning flash which we observe with our naked eye. There may be another cell of charges in the cloud near the neutralized charged cell.
- This charged cell will try to neutralize through this ionized path. This streamer is known as dart leader (Fig. (e)). The velocity of the dart leader is about 3% of the velocity of light. The effect of the dart leader is much more severe than that of the return stroke.
- The discharge current in the return streamer is relatively very large but as it lasts only for a few microseconds the energy contained in the streamer is small and

hence this streamer is known as cold lightning stroke whereas the dart leader is known as hot lightning stroke because even though the current in this leader is relatively smaller but it lasts for some milliseconds and therefore the energy contained in this leader is relatively larger. It is found that each thunder cloud may contain as many as 40 charged cells and a heavy lightning stroke may occur. This is known as multiple strokes.

LINE DESIGN BASED ON LIGHTNING:

The severity of switching surges for voltage 400 kV and above is much more than that due to lightning voltages. All the same it is desired to protect the transmission lines against direct lightning strokes. The object of good line design is to reduce the number of outages caused by lightning.

- To achieve this, the following actions are required.

  (i) The incidence of stroke on to power conductor should be minimized.

  (ii) The effect of those strokes which are incident on the system should be minimized.

- To achieve (i) we know that lightning normally falls on tall objects; thus tall towers are more vulnerable to lightning than the smaller towers. In order to keep smaller tower height for a particular ground clearance, the span lengths will decrease which requires more number of towers and hence the associated accessories like insulators etc. The cost will go up very high. Therefore, a compromise has to be made so that adequate clearance is provided, at the same time keeping longer span and hence lesser number of towers.
- With a particular number of towers the chances of incidence of lightning on power conductors can be minimized by placing a ground wire at the top of the tower structure.
- Once the stroke is incident on the ground wire, the lightning current propagates in both the directions along the ground wire. The tower presents a discontinuity to the travelling waves; therefore they suffer reflections and refraction. The system is, then, equivalent to a line bifurcated at the tower point.
- The voltage and current transmitted into the tower will depend upon the surge impedance of the tower and the ground impedance (tower footing resistance) of the tower. If it is low, the wave reflected back up the tower will largely remove the potential existing due to the incident wave.

- In this way the chance of flashover is eliminated. If, on the other hand, the incident wave encounters a high ground impedance, positive reflection will take place and the potential on the top of the tower structure will be raised rather than lowered.
- It is, therefore, desired that for good line design high surge impedances in the ground wire circuits, the tower structures and the tower footing should be avoided

Switching Surge Test Voltage Characteristics:

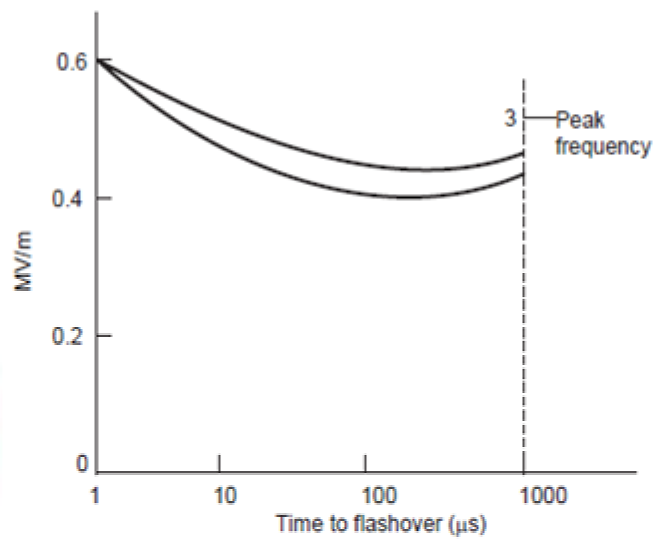


Fig: Variation of F.O. V/m as a function of time to flashover

- Switching surges assume great importance for designing insulation of overhead lines operating at voltages more than 345 kV. It has been observed that the flashover voltage for various geometrical arrangements under unidirectional switching surge voltages decreases with increasing the front duration of the surge and the minimum switching surge corresponds to the range between 100 and 500 μ sec.
- However, time to half the value has no effect as flashover takes place either at the crest or before the crest of the switching surge. The figure gives the relationship between the critical flashover voltage per metre as a function of time to flashover for on a 3 m rod-rod gap and a conductor-plane gap. It can be seen that the standard impulse voltage (1/50 μ sec) gives highest flashover

voltage and switching surge voltage with front time varying between 100 to 500 μ sec has lower flashover voltage as compared to power frequency voltage.

- The flashover voltage not only depends upon the crest time but upon the gap spacing and humidity for the same crest time surges. It has been observed that the switching surge voltage per meter gap length decreases drastically with increase in gap length and, therefore, for ultra-high voltage system, costly design clearances are required.
- Therefore, it is important to know the behavior of external insulation with different configuration under positive switching surges as it has been found that for nearly all gap configurations which are of practical interest positive switching impulse is lower than the negative polarity switching impulse. It has also been observed that if the humidity varies between 3 to 16 gm/m3, the breakdown voltage of positive and gaps increases approximately 1.7% for 1 gm/m3 increase in absolute humidity.
- For testing purposes the switching surge has been standardized with wave front time 250 μ sec ± 20% and wave tail time 2500 to ± 60% μ sec. It is known that the shape of the electrode has a decided effect on the flashover voltage of the insulation.
- Lot of experimental work has been carried on the switching surge flash over voltage for long gaps using rod-plane gap and it has been attempted to correlate these voltages with switching surge flash over voltage of other configuration electrodes. Several investigators have shown that if the gap length varies between 2 to 8 m, the 50% positive switching surge flash over for any configuration is given by the expression

$$\mathbf{V_{50} = 500\, \mathit{kd}^{0.6}\ kV}$$

where d is the gap length in metres, k is the gap factor which is a function of electrode geometry. For rod-plane gaps K = 1.0. Thus K represents a proportionality constant and is equal to 50% flash over voltage of any gap geometry to that of a rod-plane gap for the same gap spacing i.e.,

$$k = \frac{V_{50}}{V_{50}\text{rod-plane gap}}$$

- The expression for V_{50} applies to switching impulse of constant crest time. A more general expression which applies to longer times to crest has been proposed as follows:

$$V_{50} = \frac{3450\,K}{1 + \frac{8}{d}}\,kv$$

where K and d have the same meaning as in the equation above. The gap factor K depends mainly on the gap geometry and hence on the field distribution in the gap.

Horn Gaps:

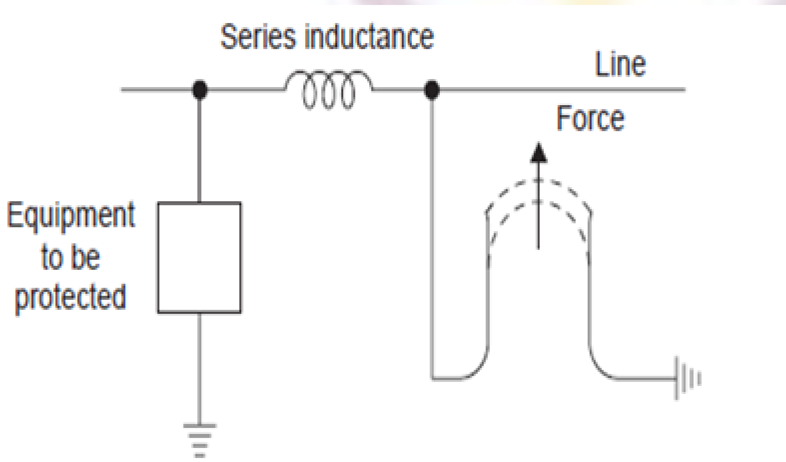


Fig: Horn gap connected in the system for protection

- The horn gap consists of two horn-shaped rods separated by a small distance. One end of this is connected to be line and the other to the earth as shown in Fig, with or without a series resistance.
- The choke connected between the equipment to be protected and the horn gap serves two purposes:

  i. The steepness of the wave incident on the equipment to be protected is reduced.
  ii. It reflects the voltage surge back on to the horn.

- Whenever a surge voltage exceeds the breakdown value of the gap a discharge takes place and the energy content in the rest part of the wave is by-passed to the ground.
- An arc is set up between the gap, which acts like a flexible conductor and rises upwards under the influence of the electro-magnetic forces, thus increasing the length of the arc which eventually blows out.
- There are two major drawbacks of the horn gap:

  i. The time of operation of the gap is quite large as compared to the modern protective gear.
  ii. If used on isolated neutral the horn gap may constitute a vicious kind of arcing ground. For these reasons, the horn gap has almost vanished from important power lines.

Surge Diverters - Rod Gap:

- The following are the basic requirements of a surge diverter:

  (i) It should not pass any current at normal or abnormal (normally 5% more than the normal voltage) power frequency voltage.

  (ii) It should breakdown as quickly as possible after the abnormal high frequency voltage arrives.

  (iii) It should not only protect the equipment for which it is used but should discharge the surge current without damaging itself.

  (iv) It should interrupt the power frequency follow current after the surge is discharged to ground.

- There are mainly three types of surge diverters:

  (i) Rod gap,

  (ii) Protector tube or expulsion type of lightning arrester &

  (iii) Valve type of lightning arrester.

ROD GAP:

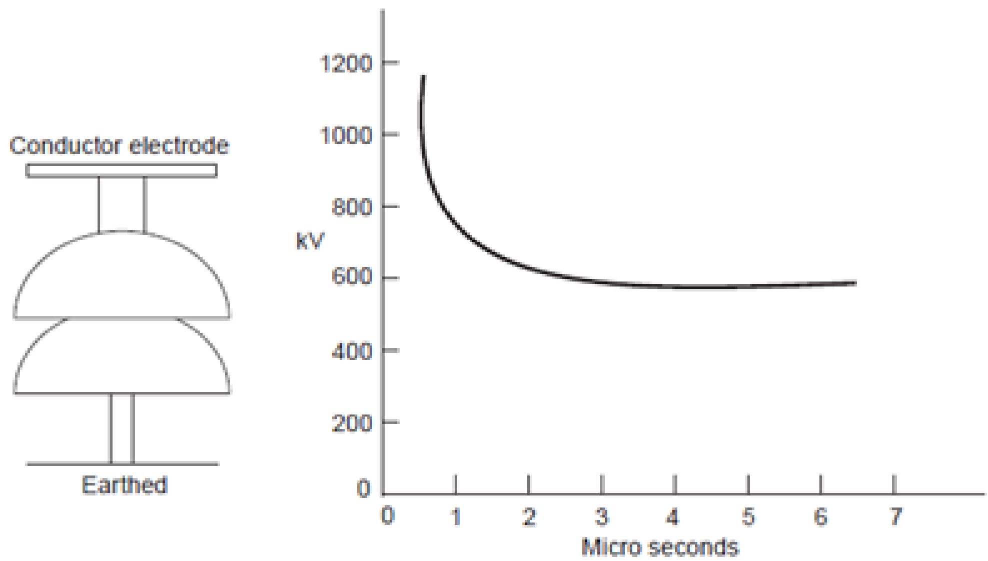


Fig: A rod gap & Volt-time characteristics of a rod gap

- This type of surge diverter is perhaps the simplest, cheapest and most rugged one. The figure shows one such gap for a breaker bushing. This may take the form of arcing ring. It also shows the breakdown characteristics (volt-time) of a rod gap.
- For a given gap and wave shape of the voltage, the time for breakdown varies approximately inversely with the applied voltage. The time to flashover for positive polarity are lower than for negative polarities.
- Also it is found that the flashover voltage depends to some extent on the length of the lower (grounded) rod.
- For low values of this length there is a reasonable difference between positive (lower value) and negative flashover voltages. Usually a length of 1.5 to 2.0 times the gap spacing is good enough to reduce this difference to a reasonable amount.
- The gap setting normally chosen is such that its breakdown voltage is not less than 30% below the voltage withstand level of the equipment to be protected.

Even though rod gap is the cheapest form of protection, it suffers from the major disadvantage that it does not satisfy one of the basic requirements of a lightning arrester listed at no. (iv) i.e., it does not interrupt the power frequency follow current. This means that every operation of the rod gap results in a L-G fault and the breakers must operate to de-energize the circuit to clear the flashover. The rod gap, therefore, is generally used as back up protection

Surge Diverters - Expulsion type of lightning arrester:

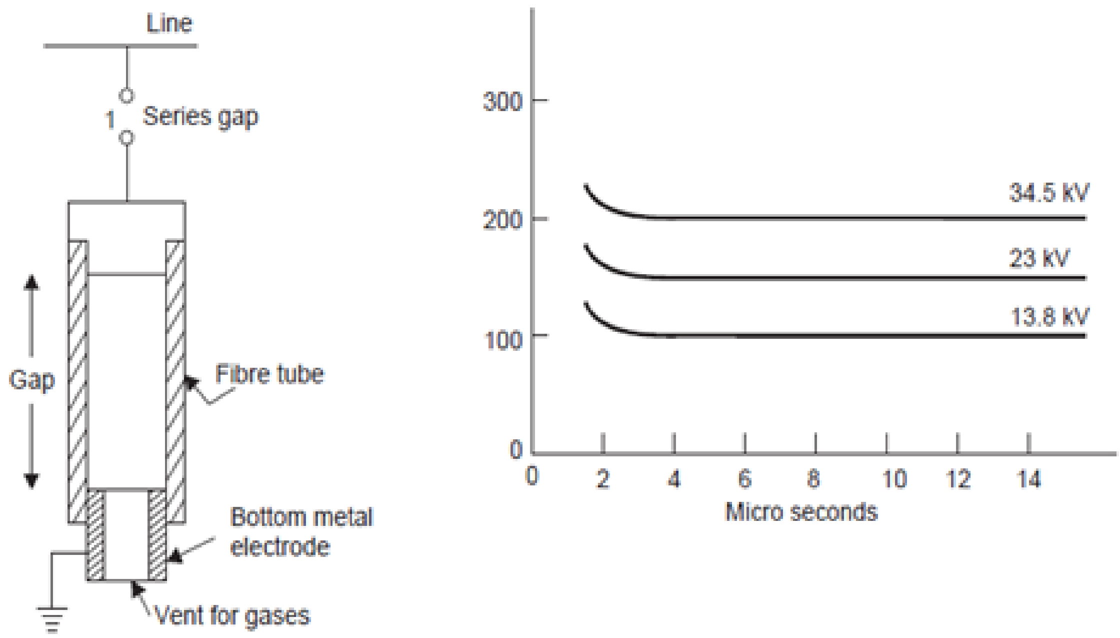


Fig: Expulsion type lightning arrester & Volt-time characteristics of expulsion gaps

- An improvement of the rod gap is the expulsion tube which consists of

  i. a series gap (1) external to the tube which is good enough to withstand normal system voltage, thereby there is no possibility of corona or leakage current across the tube
  ii. a tube which has a fibre lining on the inner side which is a highly gas evolving material
  iii. a spark gap (2) in the tube; and
  iv. an open vent at the lower end for the gases to be expelled.

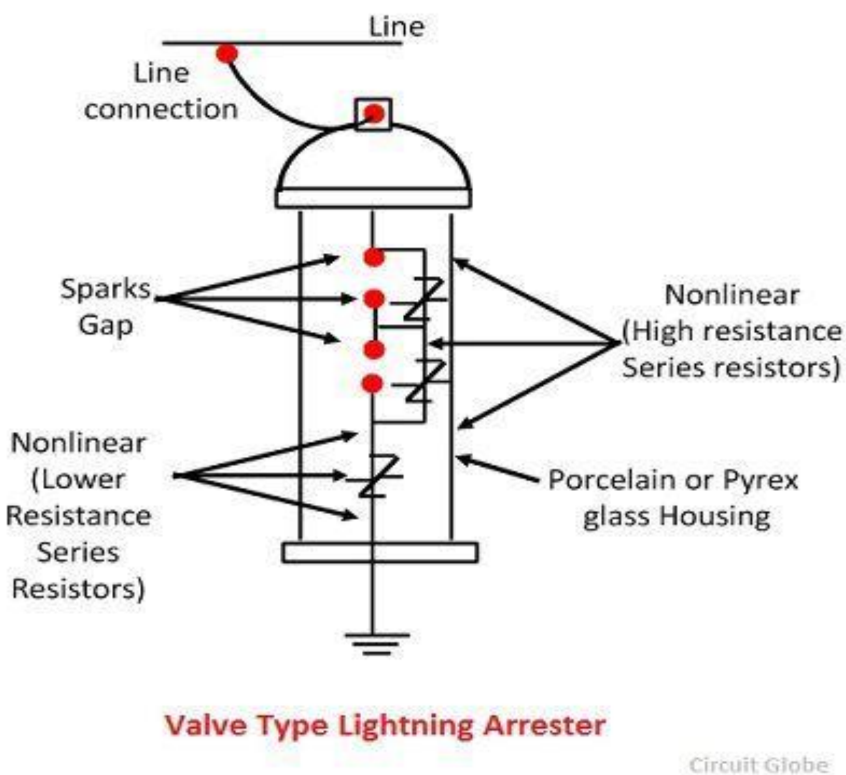
- It is desired that the breakdown voltage of a tube must be lower than that of the insulation for which it is used. When a surge voltage is incident on the expulsion tube the series gap is spanned and an arc is formed between the electrodes within the tube.
- The heat of the arc vaporizes some of the organic material of the tube wall causing a high gas pressure to build up in the tube. The resulting neutral gas creates lot of turbulence within the tube and is expelled out from the open bottom vent of the tube and it extinguishes the arc at the first current zero.

- At this instant the rate of buildup of insulation strength is greater than the RRRV. Very high currents have been interrupted using these tubes. The breakdown voltage of expulsion tubes is slightly lower than for plain rod gaps for the same spacing.
- With each operation of the tube the diameter of the tube (fibre lining) increases; thereby the insulation characteristics of the tube will lower down even though not materially. The volt-time characteristics of the expulsion tube are somewhat better than the rod gap and have the ability to interrupt power voltage after flashover.

1).Surge Diverters - Valve type lighning arresters:

Definition: The lightning arrester which consists the single or multi-gaps connected in series with the current controlling element, such type of arrester is known as the lightning arrester. The gap between the electrodes intercepts the flow of current through the arrester except when the voltage across the gap raises beyond the critical gap flashover. The valve type arrester is also known as gap surge diverter or silicon carbide surge diverter with a series gap.

Construction of Valve Type Lightning Arrester

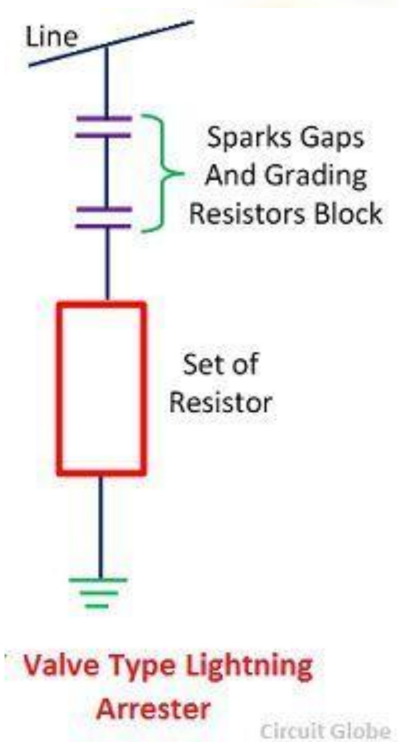


The valve type arrester consists of a multiple spark gap assembly in series with the resistor of nonlinear element. The each spark gap has two elements. For non-uniform distribution between the gap, the non-linear resistors are connected in parallel across the each gap.

The resistor elements are made up of silicon carbide with inorganic binders. The whole arrangement is enclosed in a sealed porcelain housing filled with nitrogen gas or SF6 gas.

Working of Valve Type Lightning Arrester

For low voltage, there is no spark-over across the gaps due to the effect of parallel resistor. The slow changes in applied voltage are not injurious to the system. But when the rapid changes in voltage occur across the terminal of the arrester the air gap spark of the current is discharged to ground through the non-linear resistor which offers very small resistance.

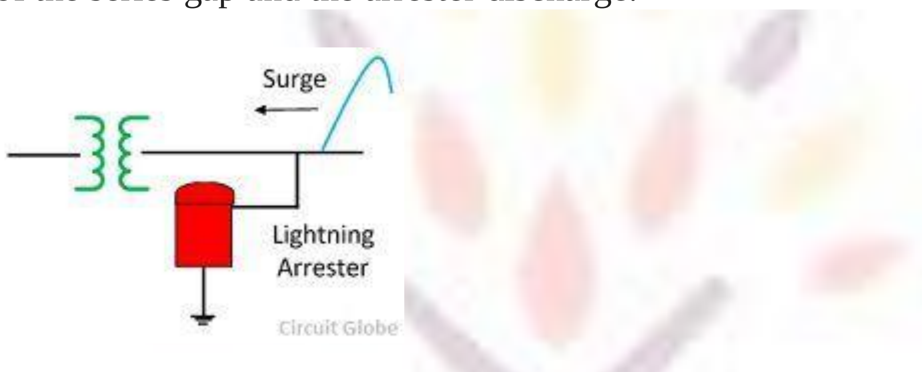


After the passage of the surge, the impressed voltage across the arrester falls, and the arrester resistance increases until the normal voltage restores. When the surge diverter disappears, a small current at low power frequency flow in the path produced by the flash over. This current is known as the power follow current.

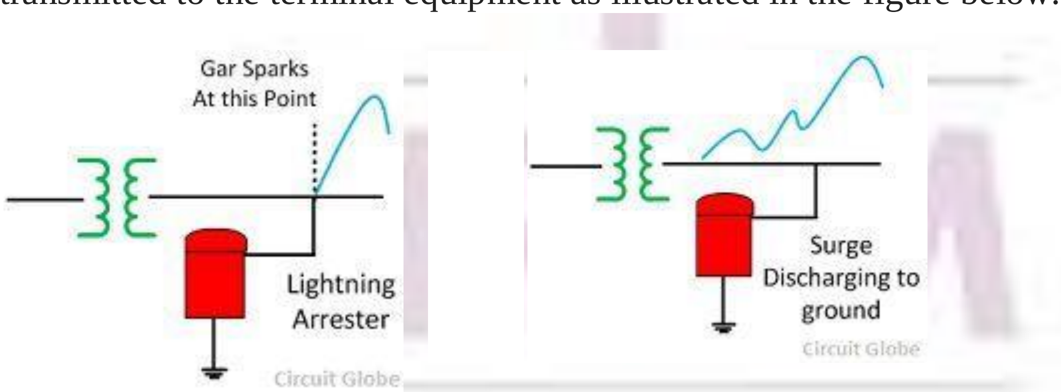
The magnitude of the power follows current decreases to the value which can be interrupted by the spark gap as they recover their dielectric strength. The power follow current is extinguished at the first current and the supply remains uninterrupted. The arrester is ready for the normal operation.This is called resealing of the lightning arrester.

Stage of Valve Type Lightning Arrester

When the surge reaches the transformer, it meets the lightning arrester as shown in the figure below. For approximately 0.25μs the voltage attained the breakdown value of the series gap and the arrester discharge.

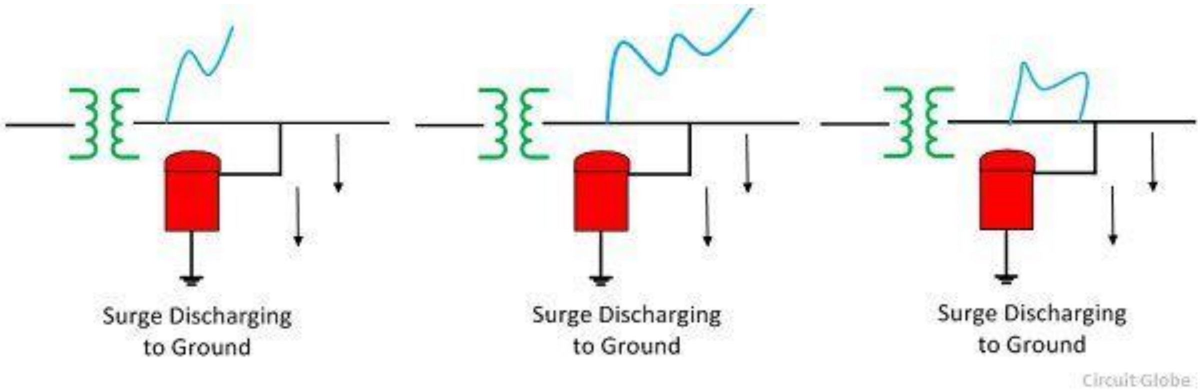


When the surge voltage increases, the resistance of non-linear element drops, thus allowing the further surge energy to discharge. So restricted the voltage transmitted to the terminal equipment as illustrated in the figure below.



When the voltage reduces, the current passes to the ground also decreases while the resistance increases. The lightning arrester attaining a stage when the current flow is interrupted by the spark gap and the arrester is sealed again.

The maximum voltage developed across the arrester terminal and transmitted to the terminal equipment is known as the discharge value of the arrester.



The valve type lightning arrester may be station types, line types, arresters for the protection of the rotating machine distribution type or secondary type.

Types of Valve Type Lightning Arrester

Station Type Valve Lightning Arrester – This type of valve is mainly employed for the protection of the critical power equipment in the circuit of 2.2kV to 400kV and higher. They have the high capacity of energy dissipation.

Line Type Lightning Arrester – The line type arresters are used for the protection of substation equipment. Their cross-sectional area is smaller, lighter in weight and cheaper in cost. They permit higher surge voltage across their terminal in comparison to station type and have lower surge carrying capacity.

Distribution arrester – Such type of arrester is usually mounted on the pole and are employed for the protection of the generators and motors.

Secondary arrester is meant for the protection of low voltage apparatus. The arrester for the protection of rotating machine is designed for the protection of generators and motors.

2).Surge Diverters - Valve type lighning arresters:

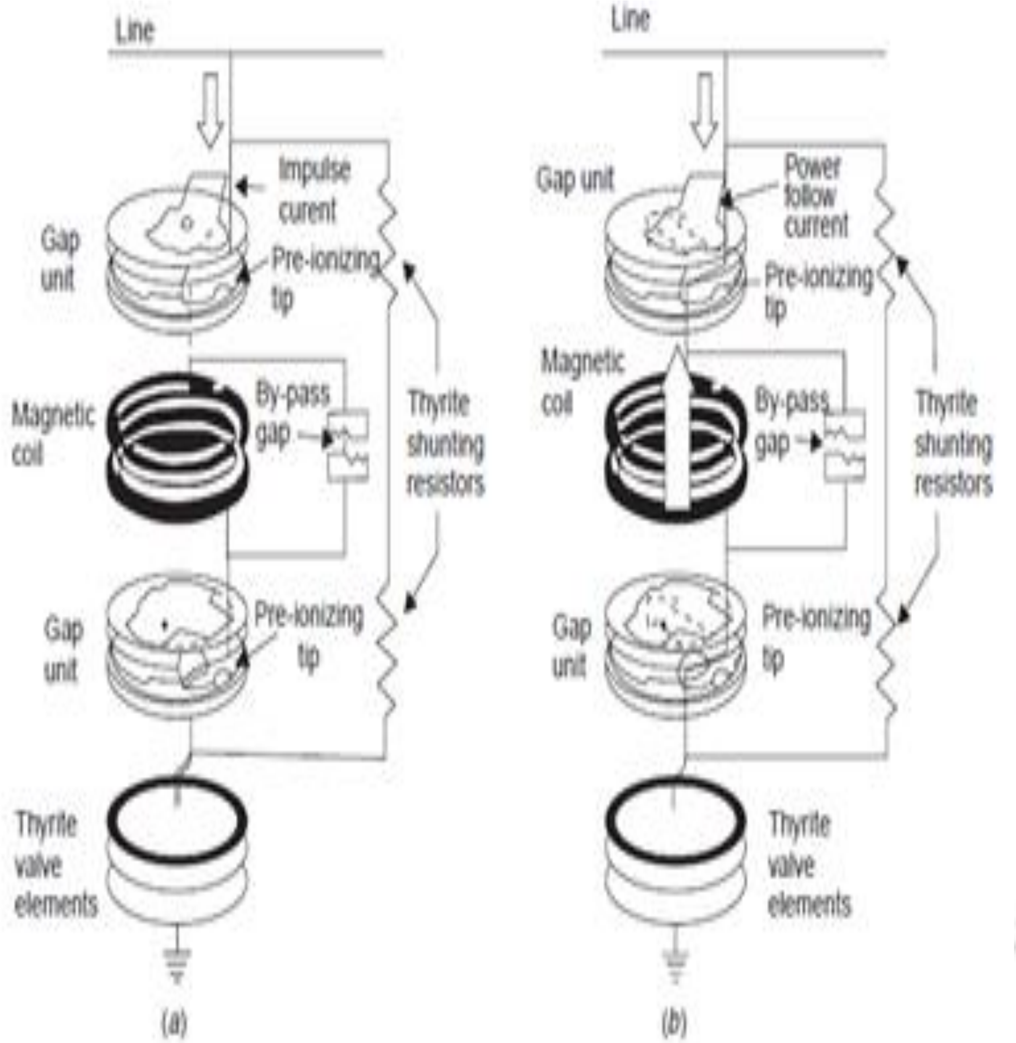


Fig: 1 Schematic diagram of valve-type arrester indicating path of (a) Surge current, (b) Follow current.

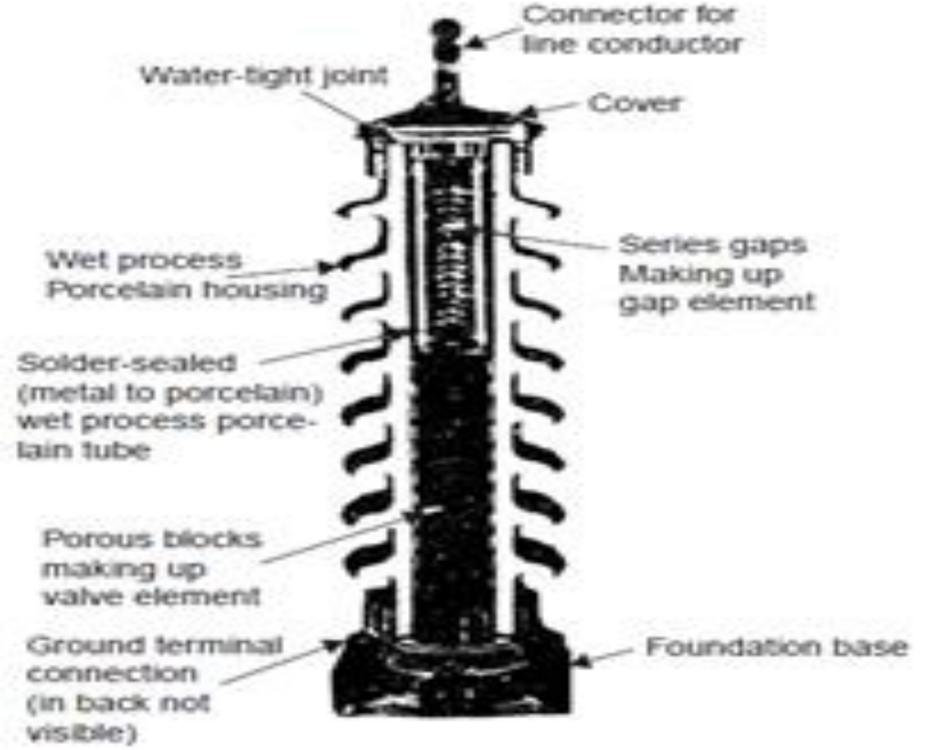


Fig: 2 Valve-type lightning arrester

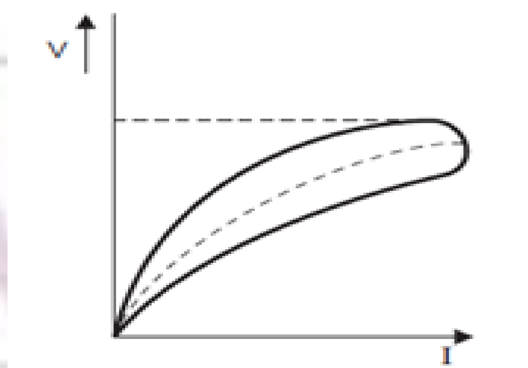


Fig: 3 Volt-ampere characteristic of valve-type LA

- An improved but more expensive surge diverter is the valve type of lightning arrester or a non-linear surge diverter. A porcelain bushing contains a number of series gaps, coil units and the valve elements of the non-linear resistance material usually made of silicon carbide disc, the latter possessing low resistance to high currents and high resistance to low currents.

- The characteristic is usually expressed as $I = KV_n$ where n lies between 2 and 6 and K is constant, a function of the geometry and dimension of the resistor. The non-linear characteristic is attributed to the properties of the electrical contacts between the grains of silicon carbide.
- The discs are 90 mm in diameter and 25 mm thick. A grading ring or a high resistance is connected across the disc so that the system voltage is evenly distributed over the discs. The high resistance keeps the inner assembly dry due to some heat generated. Figure 3 shows the volt-ampere characteristics of a non-linear resistance of the required type. The closed curve represents the dynamic characteristic corresponding to the application of a voltage surge whereas the dotted line represents the static characteristic.
- The voltage corresponding to the horizontal tangent to the dynamic characteristic is known as the residual voltage (IR drop) and is the peak value of the voltage during the discharge of the surge current. This voltage varies from 3 kV to 6 kV depending upon the type of arrester i.e., whether station or line type, the magnitude and wave shape of the discharge current.
- The spark gaps are so designed that they give an impulse ratio of unity to the surge diverter and as a result they are unable to interrupt high values of current and the follow up currents are limited to 20 to 30 A. The impulse breakdown strength of a diverter is smaller than the residual voltage, and therefore, from the point of view of insulation coordination residual voltage decides the protection level. The operation of the arrester can be easily understood with the help of Fig. 1 (a) and (b).
- When a surge voltage is incident at the terminal of the arrester it causes the two gap units to flashover, thereby a path is provided to the surge to the ground through the coil element and the non-linear resistor element. Because of the high frequency of the surge, the coil develops sufficient voltage across its terminals to cause the by-pass gap to flashover.
- With this the coil is removed from the circuit and the voltage across the LA is the IR drop due to the non-linear element. This condition continues till power frequency currents follow the pre-ionized path. For power frequency the impedance of the coil is very low and, therefore, the arc will become unstable and the current will be transferred to the coil (Fig. 1 (b)).
- The magnetic field developed by the follow current in the coil reacts with this current in the arcs of the gap assemblies, causing them to be driven into arc quenching chambers which are an integral part of the gap unit. The arc is extinguished at the first current zero by cooling and lengthening the arc and also

because the current and voltage are almost in phase. Thus the diverter comes back to normal state after discharging the surge to the ground successfully.

LOCATION AND RATING OF LIGHTNING ARRESTERS

Location of lightning arresters:

The normal practice is to locate the lightning arrester as close as possible to the equipment to be protected. The following are the reasons for the practice:

(i) This reduces the chances of surges entering the circuit between the protective equipment and the equipment to be protected.

(ii) If there is a distance between the two, a steep fronted wave after being incident on the lightning arrester, which sparks over corresponding to its spark over voltage, enters the transformer after travelling over the lead between the two. The wave suffers reflection at the terminal and, therefore, the total voltage at the terminal of the transformer is the sum of reflected and the incident voltage which approaches nearly twice the incident voltage i.e., the transformer may experience a surge twice as high as that of the lightning arrester. If the lightning arrester is right at the terminals this could not occur.

(iii) If L is the inductance of the lead between the two, and IR the residual voltage of the lightning arrester, the voltage incident at the transformer terminal will be $V = IR + L\frac{di}{dt}$, where di/dt is the rate of change of the surge current. It is possible to provide some separation between the two, where a capacitor is connected at the terminals of the equipment to be protected. This reduces the steepness of the wave and hence the rate di/dt and this also reduces the stress distribution over the winding of the equipment.

There are three classes of lightning arresters available:

(i) **Station type**: The voltage ratings of such arresters vary from 3 kV to 312 kV and are designed to discharge currents not less than 100,000 amps. They are used for the protection of substation and power transformers.

(ii) **Line type**: The voltage ratings vary from 20 kV to 73 kV and can discharge currents between 65,000 amps and 100,000 amps. They are used for the protection of distribution transformers, small power transformers and sometimes small substations.

(iii) **Distribution type**: The voltage ratings vary from 8 kV to 15 kV and can discharge currents upto 65,000 amperes. They are used mainly for pole mounted substation for the protection of distribution transformers upto and including the 15 kV classification.

- **Rating of Lightning arresters:**

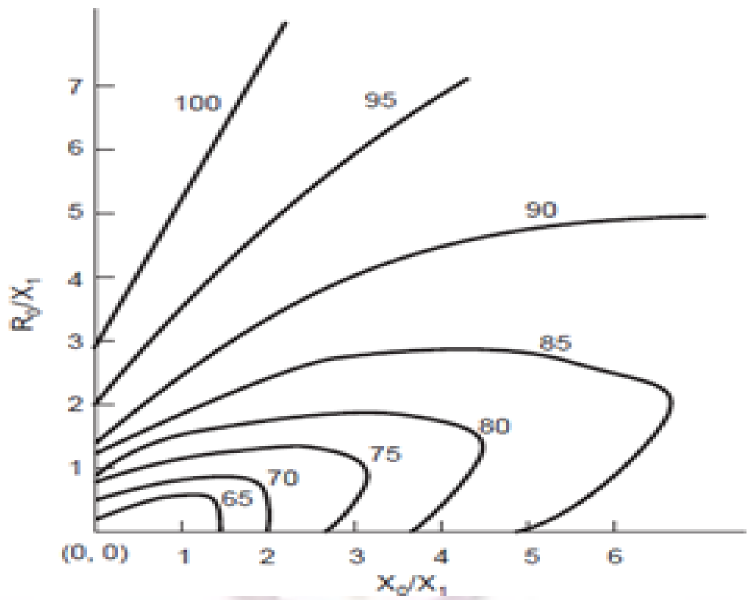


Fig: Maximum line-to-ground voltage at fault location for grounded neutral system under any fault condition(Voltage condition for $R_1 = R_2 = 0.2\ X_1$)

A lightning arrester is expected to discharge surge currents of very large magnitude, thousands of amperes, but since the time is very short in terms of microseconds, the energy that is dissipated through the lightning arrester is small compared with what it would have been if a few amperes of power frequency current had been flown for a few cycles. Therefore, the main considerations in selecting the rating of a lightning arrester is the line-to-ground dynamic voltage to which the arrester may be subjected for any condition of system operation. An allowance of 5% is normally assumed, to take into account the light operating condition under no load at the far end of the line due to Ferranti effect and the sudden loss of load on water wheel generators. This means an arrester of 105% is used on a system where the line to ground voltage may reach line-to-line value during line-to-ground fault condition. The over voltages on a system as is discussed earlier depend upon the neutral grounding condition which is determined by the parameters of the system. We recall that a system is said to be solidly grounded only if

- and $\frac{R_0}{X_1} \leq 1$ $\frac{X_0}{X_1} \leq 3$

and under this condition the line to ground voltage during a L-G fault does not exceed 80% of the L-L voltage and, therefore, an arrester of (80% 0.05 × 80%) = 1.05 × 80% = 84% is required. This is the extreme situation in case of solidly grounded system. In the same system the voltage may be less than 80%; say it may be 75%. In that case the rating of the lightning arrester will be 1.05 × 75% = 78.75%. The over voltages can actually be obtained with the help of pre-calculated curves. One set of curves corresponding to a particular system is given in the figure. For system grounded through Peterson coil, the over voltages may be 100% if it is properly tuned and, therefore, it is customary to apply an arrester of 105% for such systems. Even though there is a risk of overvoltage becoming more than 100% if it is not properly tuned, but it is generally not feasible to select arresters of sufficiently high rating to eliminate all risks of arrester damage due to these reasons. The voltage rating of the arrester, therefore, ranges between 75% to 105% depending upon the neutral grounding condition.

GROUND WIRES

Ground Wires:

- The ground wire is a conductor running parallel to the power conductors of the transmission line and is placed at the top of the tower structure supporting the power conductors (Fig 1 (a)).
- For horizontal configuration of the power line conductors, there are two ground wires to provide effective shielding to power conductors from direct lightning stroke whereas in vertical configuration there is one ground wire.

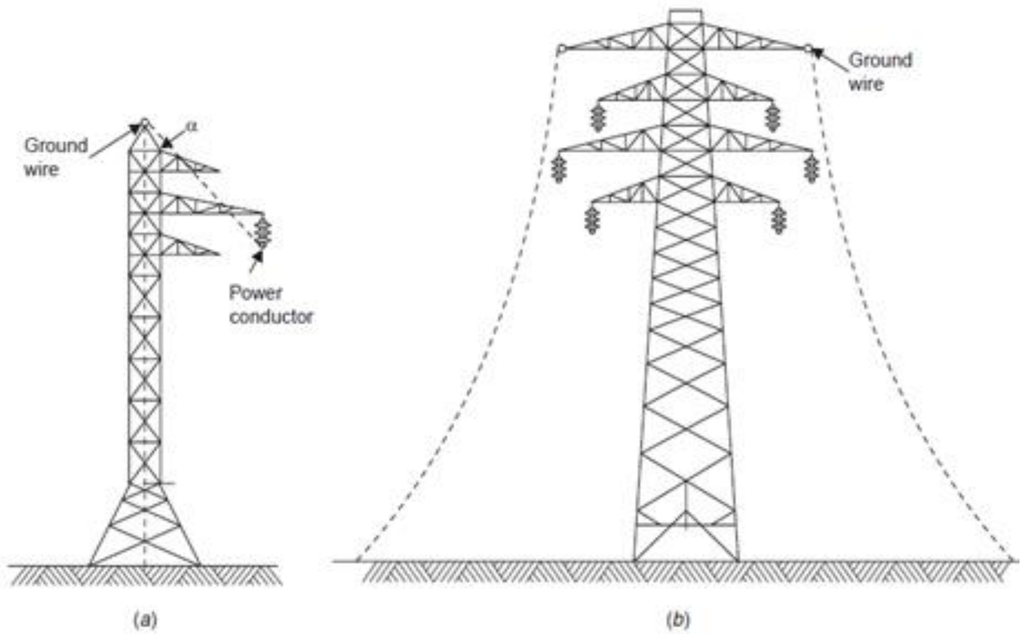


Fig: 1 (a) Protective angle; (b) Protection afforded by two ground wires

- The ground wire is made of galvanized steel wire or in the modern high voltage transmission lines ACSR conductor of the same size as the power conductor is used. The material and size of the conductor are more from mechanical consideration rather than electrical.
- A reduction in the effective ground resistance can be achieved by other relatively simpler and cheaper means. The ground wire serves the following purposes:

  i. It shields the power conductors from direct lightning strokes.
  ii. Whenever a lightning stroke falls on the tower, the ground wires on both sides of the tower provide parallel paths for the stroke, thereby the effective impedance (surge impedance) is reduced and the tower top potential is relatively less.
  iii. There is electric and magnetic coupling between the ground wire and the power conductors, thereby the changes of insulation failure are reduced.

- Protective angle of the ground wire is defined as the angle between the vertical line passing through the ground wire and the line passing through the outermost power conductor (Fig. 1 (a)) and the protective zone is the zone which is a cone with apex at the location of the ground wire and surface generated by line passing through the outermost conductor.

- According to Lacey, a ground wire provides adequate shielding to any power conductor that lies below a quarter circle drawn with its centre at the height of ground wire and with its radius equal to the height of the ground wire above the ground. If two or more ground wires are used, the protective zone between the two adjacent wires can be taken as a semi-circle having as its diameter a line connecting the two ground wires (Fig. 1 (b)).

Ground Rods:

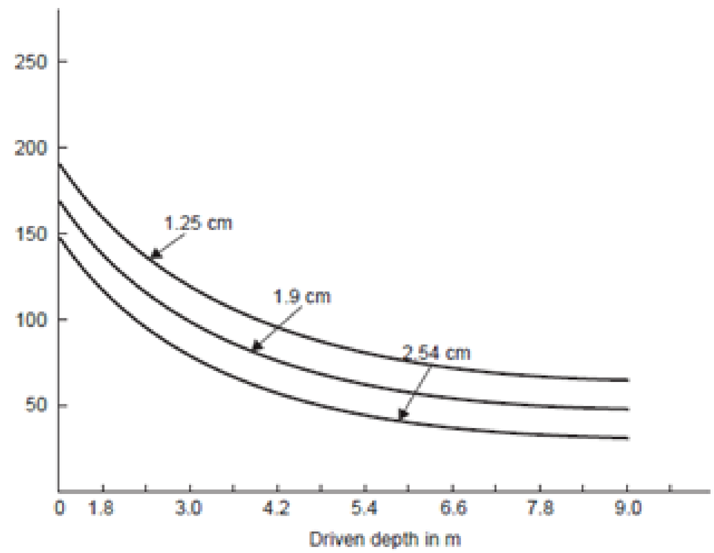


Fig: 1 Ground rod resistance as a function of rod length

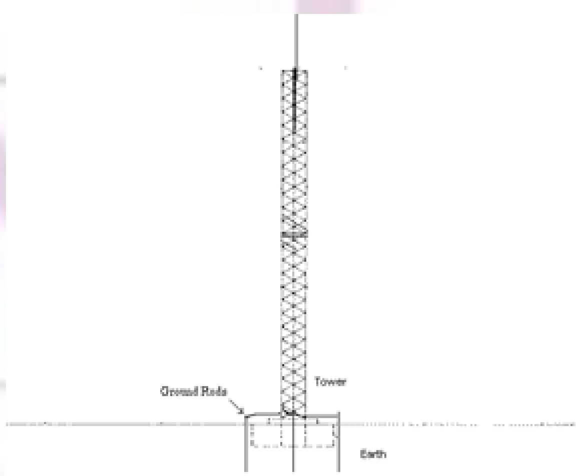


Fig: 2 Ground Rods

- Ground rods are used to reduce the tower footing resistance. These are put into the ground surrounding the tower structure.
- Fig.1 shows the variation of ground resistance with the length and thickness of the ground rods used.
- It is seen that the size (thickness) of the rod does not play a major role in reducing the ground resistance as does the length of the rod. Therefore, it is better to use thin but long rods or many small rods.

Counterpoise:

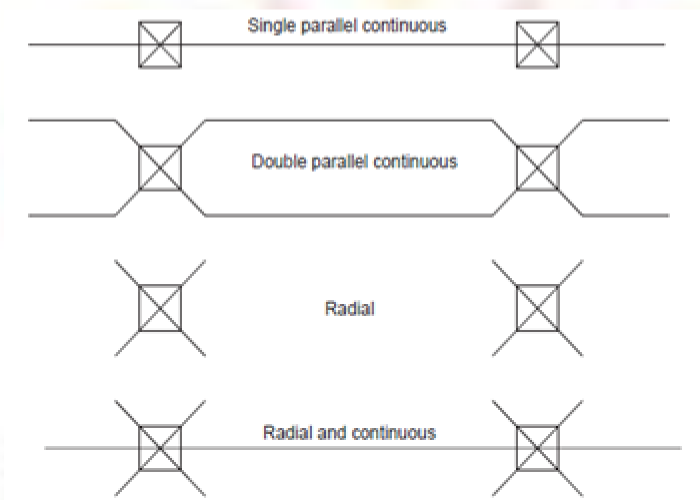


Fig: 1 Arrangement of Counterpoise

- A counterpoise is galvanized steel wire run in parallel or radial or a combination of the two, with respect to the overhead line. The various configurations used are shown in the figure 1. The corners of the squares indicate the location of the tower legs. The lightning stroke as is incident on the tower, discharges to the ground through the tower and then through the counterpoises.
- It is the surge impedance of the counterpoises which is important initially and once the surge has travelled over the counterpoise it is the leakage resistance of the counterpoise that is effective.

- While selecting a suitable counterpoise it is necessary to see that the leakage resistance of the counterpoise should always be smaller than the surge impedance; otherwise, positive reflections of the surge will take place and hence instead of lowering the potential of the tower (by the use of counterpoise) is will be raised.
- The leakage resistance of the counterpoise depends upon the surface area, i.e., whether we have one long continuous counterpoise say 1000 m or four smaller counterpoises of 250 m each, as far as the leakage resistance is concerned it is same, whereas the surge impedance of say 1000 m if it is 200 ohms, then it will be 200/4, if there are four counterpoises of 250 m. each, as these four wires will now be connected in parallel.
- Also if the surge takes say 6 micro-seconds to travel a distance of 1000 m to reduce the surge impedance to leakage impedance, with four of 250 m it will take 1.5. μ sec, that is, the surge will be discharged to ground faster, the shorter the length of the ground wire. It is, therefore, desirable to have many short counterpoises instead of one long counterpoise. But we should not have too many short counterpoises, otherwise the surge impedance will become smaller than the leakage resistance (which is fixed for a counterpoise) and positive reflections will occur.
- The question arises as to why we should have a low value of tower footing resistance. It is clear that, whenever a lightning strikes a power line, a current is injected into the power system. The voltage to which the system will be raised depends upon what impedances the current encounters.
- Say if the lightning stroke strikes a tower, the potential of the tower will depend upon the impedance of the tower. If it is high, the potential of the tower will also be high which will result in flashover of the insulator discs and result in a line-to-ground fault. The flashover will take place from the tower structure to the power conductor and, therefore, it is known as back flashover.

Surge Absorbers:

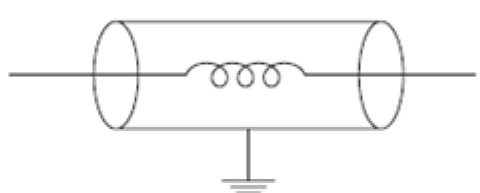


Fig: Ferranti surge absorber

- A surge absorber is a device which absorbs energy contained in a travelling wave. Corona is a means of absorbing energy in the form of corona loss. A short length of cable between the equipment and the overhead line absorbs energy in the travelling wave because of its high capacitance and low inductance.
- Another method of absorbing energy is the use of Ferranti surge absorber which consists of an air core inductor connected in series with the line and surrounded by an earthed metallic sheet called a dissipator. The dissipator is insulated from the inductor by the air as shown in Fig. 2.
- The surge absorber acts like an air cored transformer whose primary is the low inductance inductor and the dissipator acts as the single turn short circuit secondary. Whenever the travelling wave is incident on the surge absorber a part of the energy contained in the wave is dissipated as heat due to transformer action and by eddy currents. Because of the series inductance, the steepness of the wave also is reduced. It is claimed that the stress in the end turns is reduced by 15% with the help of surge absorber.

SURGE PROTECTION OF ROTATING MACHINE

Surge Protection of Rotating machines:

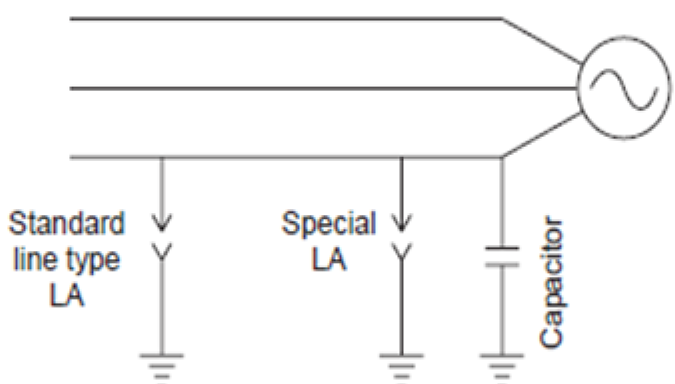


Fig: Surge protection of rotating machine

- A rotating machine is less exposed to lightning surge as compared to transformers. Because of the Limited space available, the insulation on the windings of rotating machines is kept to a minimum.

- The Main difference between the winding of rotating machine and transformer is that in case of rotating Machines the turns are fewer but longer and are deeply buried in the stator slots.
- Surge impedance of Rotating machines in approx. 1000 ω and since the inductance and capacitance of the windings are Large as compared to the overhead lines the velocity of propagation is lower than on the lines. For a Typical machine it is 15 to 20 metres/ μ sec.
- This means that in case of surges with steep fronts, the Voltage will be distributed or concentrated at the first few turns. Since the insulation is not immersed in Oil, its impulse ratio is approx. Unity whereas that of the transformer is more than 2.0.
- The rotating machine should be protected against major and minor insulations. By major insulation Is meant the insulation between winding and the frame and minor insulation means inter-turn Insulation. The rotating machine should be protected against major and minor insulations.
- By major insulation is meant the insulation between winding and the frame and minor insulation means inter-turn insulation.
- The major insulation is normally determined by the expected line-to-ground voltage across the terminal of the machine whereas the minor insulation is determined by the rate of rise of the voltage. Therefore, in order to protect the rotating machine against surges requires limiting the surge voltage magnitude at the machine terminals and sloping the wave front of the incoming surge.
- To protect the major insulation a special lightning arrester is connected at the terminal of the machine and to protect the minor insulation a condenser of suitable rating is connected at the terminals of the machine as shown in the figure.

INSULATION COORDINATION AND OVERVOLTAGE PROTECTION

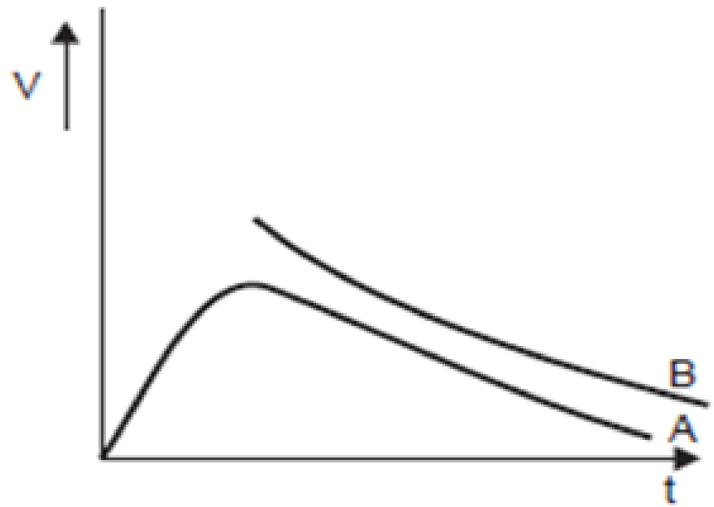


Fig: Volt-time curve A (protecting device and) volt-time curve B (device to be protected)

- Insulation coordination means the correlation of the insulation of the various equipments in a power system to the insulation of the protective devices used for the protection of those equipments against over-voltages.

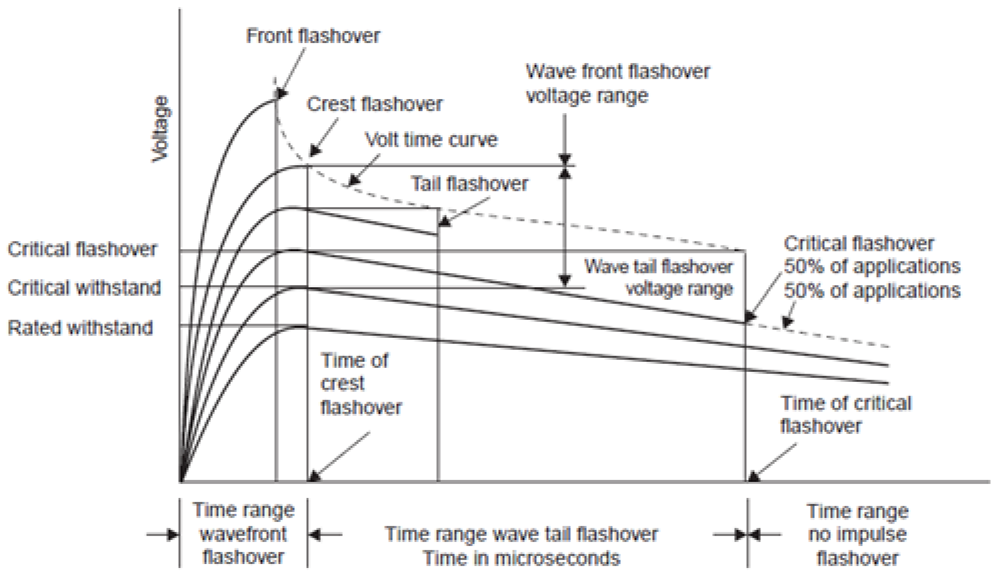
- In a power system various equipments like transformers, circuit breakers, bus supports etc. have different breakdown voltages and hence the volt-time characteristics.

- In order that all the equipments should be properly protected it is desired that the insulation of the various protective devices must be properly coordinated.

- The basic concept of insulation coordination is illustrated in the figure above. Curve A is the volt-time curve of the protective device and B the volt-time curve of the equipment to be protected.

The figure shows the desired positions of the volt-time curves of the protecting device and the equipment to be protected. Thus, any insulation having a withstand voltage strength in excess of the insulation strength of curve B is protected by the protective device of curve A.

Volt-Time Curve:



- The breakdown voltage for a particular insulation of flashover voltage for a gap is a function of both the magnitude of voltage and the time of application of the voltage. The volt-time curve is a graph showing the relation between the crest flashover voltages and the time to flashover for a series of impulse applications of a given wave shape.
- For the construction of volt-time curve the following procedure is adopted. Waves of the same shape but of different peak values are applied to the insulation whose volt-time curve is required. If flashover occurs on the front of the wave, the flashover point gives one point on the volt-time curve.
- The other possibility is that the flashover occurs just at the peak value of the wave; this gives another point on the V-T curve. The third possibility is that the flashover occurs on the tail side of the wave. In this case to find the point on the V-T curve, draw a horizontal line from the peak value of this wave and also draw a vertical line passing through the point where the flashover takes place. The intersection of the horizontal and vertical lines gives the point on the V-T curve. This procedure is nicely shown in the figure.

- The over-voltages against which coordination is required could be caused on the system due to system faults, switching operation or lightning surges. For lower voltages, normally upto about 345 kV, over voltages caused by system faults or switching operations does not cause damage to equipment insulation although they may be detrimental to protective devices.
- Over-voltages caused by lightning are of sufficient magnitude to affect the equipment insulation whereas for voltages above 345 kV it is these switching surges which are more dangerous for the equipments than the lightning surges.
- The problem of coordinating the insulation of the protective equipment involves not only guarding the equipment insulation but also it is desired that the protecting equipment should not be damaged. To assist in the process of insulation coordination, standard insulation levels have been recommended. These insulation levels are defined as follows.
- *Basic impulse insulation levels (BIL)* are reference levels expressed in impulse crest voltage with a standard wave not longer than 1.2/50 μ sec wave. Apparatus insulation as demonstrated by suitable tests shall be equal to or greater than the basic insulation level.
- The problem of insulation coordination can be studied under three steps:

1. Selection of a suitable insulation which is a function of reference class voltage (i.e., 1.05 × operating voltage of the system).
2. The design of the various equipments such that the breakdown or flashover strength of all insulation in the station equals or exceeds the selected level as in (1).
3. Selection of protective devices that will give the apparatus as good protection as can be justified economically.

Statistical Methods for Insulation Coordination:

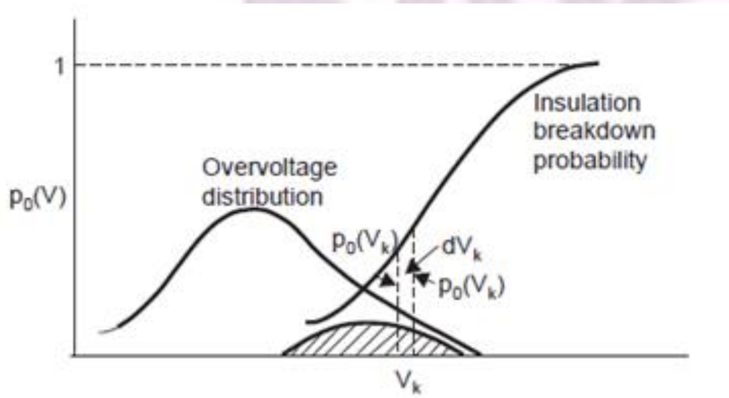


Fig: Overvoltage distribution and Insulation breakdown probability

- Both the over voltages due to lightning or switching and the breakdown strength of the insulating media are of statistical nature. Not all lightning or switching surges are dangerous to the insulation and particular specimen needs not necessarily flashover or punctures at a particular voltage.
- Therefore, it is important to design the insulation of the various equipments to be protected and the devices used for protection not for worst possible condition but for worst probable condition as the cost of insulation for system of the voltage more than 380 kV are proportional to square of the voltage and, therefore any small saving in insulation will result in a large sums when considered for such large modern power system.
- This, however, would involve some level of risk failure. It is desired to accept some level of risk of failure than to design a risk-free but a very costly system. The statistical methods, however, call for a very rigorous experimentation and analysis work so as to find probability of occurrence of over-voltages and probability of failure of insulation.
- It is found that the distribution of breakdown for a given gap follows with some exceptions approximately normal or Gaussian distribution. Similarly the distribution of over voltages on the system also follows the Gaussian distribution.
- In order to coordinate electrical stresses due to over-voltages with the electrical strengths of the dielectric media, it has been found convenient to represent overvoltage distribution in the form of probability density function and the insulation breakdown probability by the cumulative distribution function as shown in the figure.

Unit – 5

Symmetrical Components and Fault Calculations

Any three co-planar vectors V_a, V_b, and V_c can be expressed in terms of three new vectors V_1, V_2 and V_3 by three simultaneous linear equations with constant co-efficients.

$$V_a = a_{11}V_1 + a_{12}V_2 + a_{13}V_3.$$

$$V_b = a_{21}V_1 + a_{22}V_2 + a_{23}V_3.$$

$$V_c = a_{31}V_1 + a_{32}V_2 + a_{33}V_3.$$

Each of the original vector replaced by a set of three vectors making a total of nine vectors.

two conditions should be ~~simplified by the use of the chosen~~ satisfied in selecting systems of components to replace 3-ph current and voltage vectors.

① calculations should be simplified by the use of the chosen system of components. this is possible only if the impedances (or admittances) associated with the components of current (or voltage) can be obtained by calculation or test.

② The system of components chosen should have physical significance and be an aid in determining power system performance.

The three symmetrical component vectors replacing

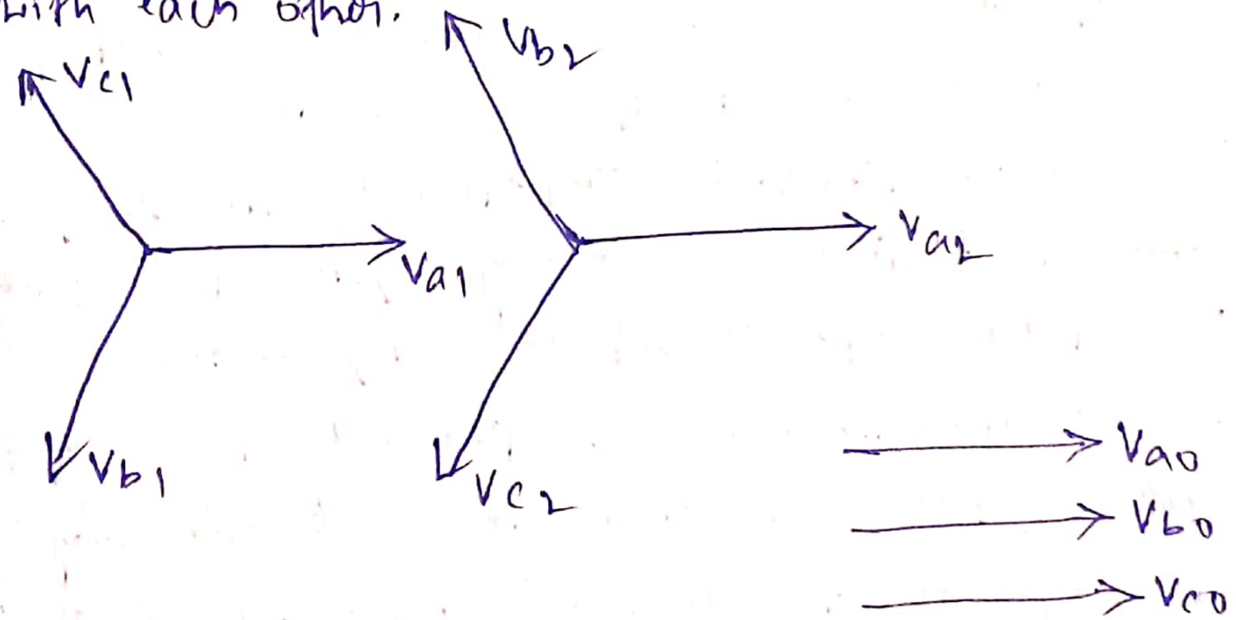
Scanned with CamScanner

V_a, V_B and V_c are

1. Positive Sequence Component :- Three vectors of equal magnitude but displaced in phase from each other by 120° and has the same phase sequence as the original vectors.

2. Negative Sequence Component :- Negative Sequence Component which has phase sequence opposite to original vectors.

3. Zero Sequence Component :- Which has the three vectors of equal magnitude and also are in phase with each other.



Significance of Positive, Negative and Zero Sequence Components :-

A set of positive sequence voltages is applied to the stator winding of the alternator, the direction of rotation of the stator field is same as the rotor, the set of voltages are positive sequence voltages. the direction of rotation of

Scanned with CamScanner

the stator field is opposite to the rotor, the set of voltages are negative sequence voltages.

the zero sequence voltages are single phase voltages gives rise to an alternating field in space.

The following relations b/w the original unbalanced vectors and their corresponding symmetrical components

$$V_a = V_{a1} + V_{a2} + V_{a0} \rightarrow (1)$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \rightarrow (2)$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \rightarrow (3)$$

Here Assume a as a reference phasor, phases b & c are in terms of phase a. Here use is made of operator λ which has a magnitude of unity and rotation through $120°$. When any vector is multiplied by λ, the vector magnitude same but is rotated anticlock wise through $120°$.

$$\lambda = 1\angle 120°$$

In Complex form $\lambda = \cos 120° + j \sin 120°$

$$= -0.5 + j0.866$$

11ly

$$\lambda^2 = -0.5 - j0.866$$

$$\lambda^3 = 1.0 = 1\angle 360°$$

$$\lambda^3 - 1 = 0$$

$$\lambda^2 + \lambda + 1 = 0$$

Scanned with CamScanner

$\lambda^4 = \lambda$

So, the important relations that will be frequently required in power system analysis are

$$\lambda = -0.5 + j0.866 = 1\angle 120^\circ$$

$$\lambda^2 = -0.5 - j0.866 = 1\angle -120^\circ$$

$$\lambda^3 = 1\angle 0$$

$$\lambda^4 = \lambda$$

$$\boxed{\lambda^2 + \lambda + 1 = 0}$$

To determine the relations b/w symmetrical components of phases b and c in terms of symmetrical components of phase a.

$$V_{b1} = \lambda^2 V_{a1}$$

This means that in order to express V_{b1} in terms of V_{a1}. V_{a1} should be rotated anti clock wise through 240°.

$$V_{c1} = \lambda V_{a1}$$

For negative sequence vectors,

$$V_{b2} = \lambda V_{a2}$$

$$V_{c2} = \lambda^2 V_{a2}.$$

For zero sequence vectors, $V_{b0} = V_{a0} = V_{c0}$.

By substituting these relations in eq's ① - ③

$$V_a = V_{a1} + V_{a2} + V_{a0}. \rightarrow ④$$

$$V_b = \lambda^2 V_{a1} + \lambda V_{a2} + V_{a0} \rightarrow ⑤$$

$$V_c = \lambda V_{a1} + \lambda^2 V_{a2} + V_{a0} \rightarrow ⑥$$

Scanned with CamScanner

By comparing the above equations, with V_a, V_b, V_c

$$a_{11} = a_{12} = a_{13} = 1$$

$$a_{21} = \lambda^2, \ a_{22} = \lambda, \ a_{23} = 1$$

$$a_{31} = \lambda, \ a_{32} = \lambda^2, \ a_{33} = 1$$

Phase voltages V_a, V_b and V_c in terms of the symmetrical components of phase a.

If we know the values of V_{a1}, V_{a2} & V_{a0}, the phase voltages V_a, V_b & V_c can be calculated.

Ily Relation b/w phase currents in terms of symmetrical components of currents taking phase a as reference

$$I_a = I_{a1} + I_{a2} + I_{a0} \rightarrow (7)$$

$$I_b = \lambda^2 I_{a1} + \lambda I_{a2} + I_{a0} \rightarrow (8)$$

$$I_c = \lambda I_{a1} + \lambda^2 I_{a2} + I_{a0} \rightarrow (9)$$

If unbalanced phase currents and phase voltages are known in a system. It is required to find out the symmetrical components.

Given values, V_a, V_b, V_c.

Find out V_{a1}, V_{a2}, & V_{a0}. to find out positive sequence component V_{a1}, multiply eqns (4), (5), (6) with $1, \lambda, \lambda^2$ respectively and adding them.

$$V_a + \lambda V_b + \lambda^2 V_c = V_{a1}(1+\lambda^3+\lambda^3) + V_{a2}(1+\lambda^2+\lambda^4)$$

$$+ V_{a0}(1+\lambda+\lambda^2)$$

$$= 3V_{a1} + V_{a2}(1+\lambda^2+\lambda) + 0$$

Scanned with CamScanner

$= 3V_{a1}$ $\quad [\because 1+\lambda+\lambda^2=0]$

$$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_c]$$

For negative sequence component, V_{a2}; multiplying equations (4), (5) & (6) by $1, \lambda^2$ & λ

$$V_a + \lambda^2 V_b + \lambda V_c = V_{a1}(1+\lambda^4+\lambda^2) + V_{a2}(1+\lambda^3+\lambda^3) + V_{a0}(1+\lambda^2+\lambda)$$

$$= 3V_{a2}$$

$$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c]$$

For zero sequence component, V_{a0}, add eqn (4), (5), (6)

$$V_a + V_b + V_c = V_{a1}[1+\lambda^2+\lambda] + V_{a2}[1+\lambda+\lambda^2] + 3V_{a0}$$

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_c] \rightarrow (10)$$

$$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c] \rightarrow (11)$$

$$V_{a0} = \frac{1}{2}[V_a + V_b + V_c] \rightarrow (12)$$

Illy current relations $I_{a1} = \frac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$

$$I_{a2} = \frac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

V_a, V_b & V_c → line to ground, line to neutral, line to line voltages at a point in the n/w (or) they may be generated (or) induced voltages.

Scanned with CamScanner

similarly, currents would be phase currents, line currents, the currents flowing into a fault from the line conductors.

---

(1) The line to ground voltage on the high voltage side of a step-up transformer are 100 kV, 33 kV and 38 kV in phases a, b and c respectively. the voltage of phase a leads that of phase b by 100° and lags that of phase c by 176.5°. Determine analytically the symmetrical components of voltage ?

$$V_a = 100\angle 0°$$

$$V_b = 33\angle -100°$$

$$V_c = 38\angle 176.5°$$

$$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_c]$$

$$= \frac{1}{3}[100\angle 0° + 33\angle -100° \angle 120° + 38\angle 176.5° \angle -120°]$$

$$= \frac{1}{3}[100 + j0 + 33\angle 20° + 38\angle 56.5°]$$

$$= \frac{1}{3}[151.97 + j42.97]$$

$$\boxed{V_{a1} = 50.65 + j\,14.32}$$

$$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c]$$

$$= \frac{1}{3}[100 + j0 + 33\angle -220 + 38\angle 296.5°]$$

$$\boxed{V_{a2} = 30.55 - j\,4.26}$$

Scanned with CamScanner

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$= \frac{1}{3}[100 + j0 + 33\angle{-100} + 38\angle{176.5}]$$

$$= \frac{1}{3}[56.37 - j30.18]$$

$$\boxed{V_{a0} = 18.79 - j10.06}$$

② The line currents in amperes in phases a, b, and c respectively are 500 + j150, 100 − j600 and −300 + j600 referred to the same reference vector. Find the symmetrical component of currents.

Sol:

The line currents are

$$I_a = 500 + j150$$

$$I_b = 100 - j600$$

$$I_c = -300 + j600$$

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

$$= \frac{1}{3}[500 + j150 + 100 - j600 - 300 + j600]$$

$$\boxed{I_{a0} = 100 + j50} \text{ Amps.}$$

$$I_{a1} = \frac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$$

$$= \frac{1}{3}[500 + j150 + (-0.5 + j0.866)(100 - j600) + (-0.5 - j0.866)(-300 + j600)]$$

$$= \frac{1}{3}[1639 + j496.4] = 546.3 + j165.46 \text{ Amp}$$

$$I_{a2} = \frac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$

$$= \frac{1}{3}[500 + j150 + (-0.5 - j0.866)(100 - j600) + (-0.5 + j0.866)(-300 + j600)]$$

$$= \frac{1}{3}[146.3 - j65.46] = 48.8 - j21.82 \text{ Amps.}$$

Scanned with CamScanner

Average Power (3-ph) in terms of Symmetrical Components:-

The Avg power $P = V_a I_a \cos\phi_a + V_b I_b \cos\phi_b + V_c I_c \cos\phi_c \rightarrow (1)$

$= V_a I_a + V_b I_b + V_c I_c$

$= (V_{a1} + V_{a2} + V_{a0})(I_{a1} + I_{a2} + I_{a0}) + (\lambda^2 V_{a1} + \lambda V_{a2} + V_{a0})$

$(\lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}) + (\lambda V_{a1} + \lambda^2 V_{a2} + V_{a0})$

$(\lambda I_{a1} + \lambda^2 I_{a2} + I_{a0})$

Taking first term on R.H.S

$[V_{a1} + V_{a2} + V_{a0}] \cdot [I_{a1} + I_{a2} + I_{a0}] = V_{a1} I_{a1} + V_{a1} I_{a2} + V_{a1} I_{a0}$

$+ V_{a2} I_{a1} + V_{a2} I_{a2} + V_{a2} I_{a0}$

$+ V_{a0} I_{a1} + V_{a0} I_{a2} + V_{a0} I_{a0}$

Expanding second term on R.H.S

$(\lambda^2 V_{a1} + \lambda V_{a2} + V_{a0}) \cdot (\lambda^2 I_{a1} + \lambda I_{a2} + I_{a0})$

$= \lambda^2 V_{a1} \lambda^2 I_{a1} + \lambda^2 V_{a1} \cdot \lambda I_{a2}$

$+ \lambda^2 V_{a1} I_{a0} + \lambda V_{a2} \cdot \lambda^2 I_{a1}$

$+ \lambda V_{a2} \cdot \lambda I_{a2} + \lambda V_{a2} \cdot I_{a0} + V_{a0} \lambda^2 I_{a1}$

$+ V_{a0} \lambda I_{a2} + V_{a0} \cdot I_{a0}$

Now, the dot product of two vectors does not change when both are rotated through the same angle.

$\lambda^2 V_{a1} \cdot \lambda^2 I_{a1} = V_{a1} I_{a1}$

$\lambda^2 V_{a1} \lambda I_{a2} = \lambda V_{a1} \cdot I_{a2}$

Scanned with CamScanner

The addition of the terms after expanding and re arranging

$$P = 3V_{a0}I_{a0} + 3V_{a2}I_{a2} + 3V_{a1}I_{a1} + V_{01}I_{a2}(1+\lambda+\lambda^2)$$
$$+ V_{a1}I_{a0}(1+\lambda+\lambda^2) + V_{a2}I_{a1}(1+\lambda+\lambda^2)$$
$$+ V_{a2}I_{a0}(1+\lambda+\lambda^2) + V_{a0}I_{a1}(1+\lambda+\lambda^2)$$
$$+ V_{a0}I_{a2}(1+\lambda+\lambda^2)$$
$$= 3(V_{a1}\cdot I_{a1} + V_{a2}I_{a2} + V_{a0}\cdot I_{a0})$$
$$= 3\left[|V_{a1}||I_{a1}|\cos\phi_1 + |V_{a2}||I_{a2}|\cos\phi_2 + |V_{a3}||I_{a3}|\cos\phi_0\right] \rightarrow (2)$$

The same power expression can be very easily derived using matrix manipulating.

$$P + jQ = V_a I_a^* + V_b I_b^* + V_c I_c^*$$
$$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$
$$= \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

From equations (4) (5) (6) in the last four

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = AV.$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T = [AV]^T = V^T \cdot A^T$$

Scanned with CamScanner

$$P+jQ = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix}^* \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$P+jQ = \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= 3 \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= 3 \left[V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^* \right]$$

$$\therefore P = 3 \left[|V_{a0}||I_{a0}| \cos\phi_0 + |V_{a1}||I_{a1}| \cos\phi_1 + |V_{a2}||I_{a2}| \cos\phi_2 \right]$$

Scanned with CamScanner

Fault Calculation :- Faults can be classified as

① Shunt Faults (Short ckts).

② Series Faults (open ckts).

→ Shunt type of faults involve power conductor (or) conductors-to-ground (or) short ckt b/w conductors. When ckts are controlled by fuses (or) devices (or) any device which does not open all three phases. one (or) two phases of the ckt may be opened. these are called series type of faults.

→ Shunt faults are characterised by increase in current and fall in voltage and frequency.

→ Series faults are characterised by increase in voltage and frequency and fall in current in the faulted phase.

Shunt faults :-

(i) Line-to-Ground fault

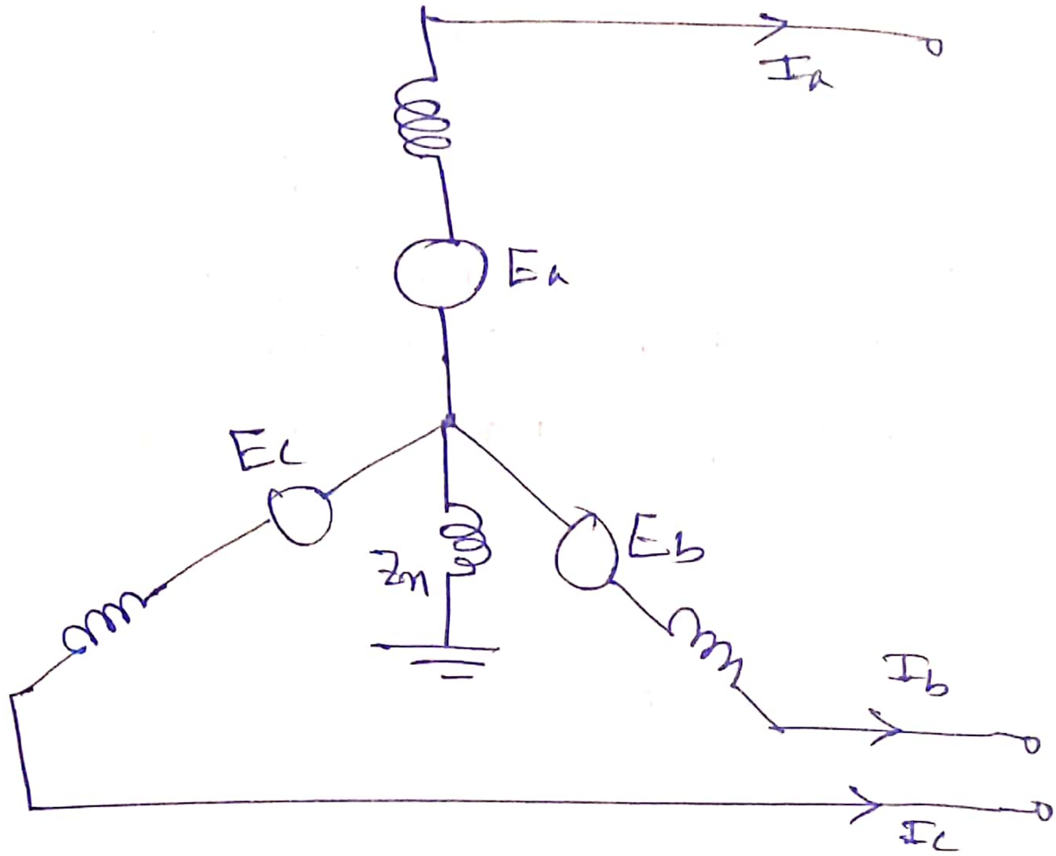
(ii) Line-to-Line fault

(iii) Double Line-to-ground fault

(iv) 3-phase fault

Sequence Network Equations :- These equations will be derived for an unloaded alternator with neutral solidly grounded. Assuming that the system is balanced. i.e. the generated ~~emf~~ voltages are of equal magnitude and displaced by 120°.

Scanned with CamScanner



→ Sequence Impedance per phase are same for all three 3Φ phs. and we are considering initially a balanced system.

→ The positive sequence component of voltage at the fault point is the positive sequence generated voltage minus the drop due to positive sequence current in positive sequence Impedance.

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = E_{a2} - I_{a2} Z_2$$

Since the negative sequence voltage is generated is zero.

$$E_{a2} = 0$$

$$V_{a2} = - I_{a2} Z_2$$

similarly for zero sequence voltage, $E_{a0} = 0$

$$V_{a0} = V_n - I_{a0} Z_{g0}$$

Scanned with CamScanner

$= -3 I_{a0} Z_n - I_{a0} Z_{g0}$

$= -I_{a0} (Z_{g0} + 3 Z_n)$

$Z_{g0} \rightarrow$ Zero Sequence Impedance

$Z_n \rightarrow$ Neutral Impedance.

The three sequence N/W equations are

$V_{a1} = E_a - I_{a1} Z_1$

$V_{a2} = -I_{a2} Z_2$

$V_{a0} = -I_{a0} Z_0$

$Z_0 = Z_{g0} + 3 Z_n$

Corresponding Sequence N/W for the unloaded alternator

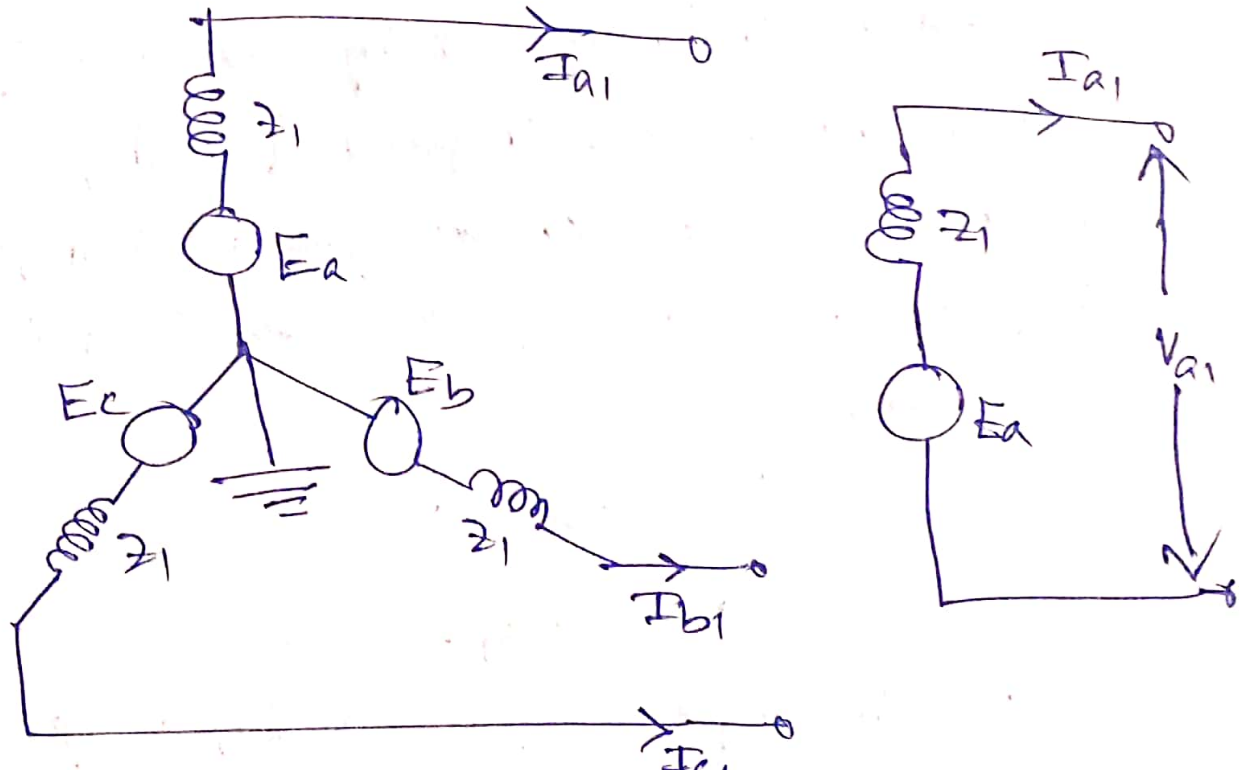
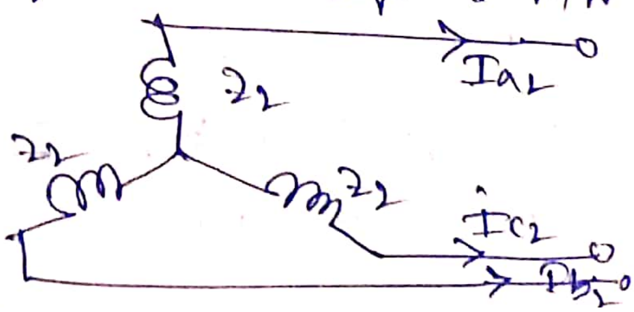


Fig: Positive Sequence N/W



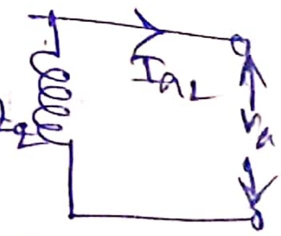


fig: -ve seq. N/W

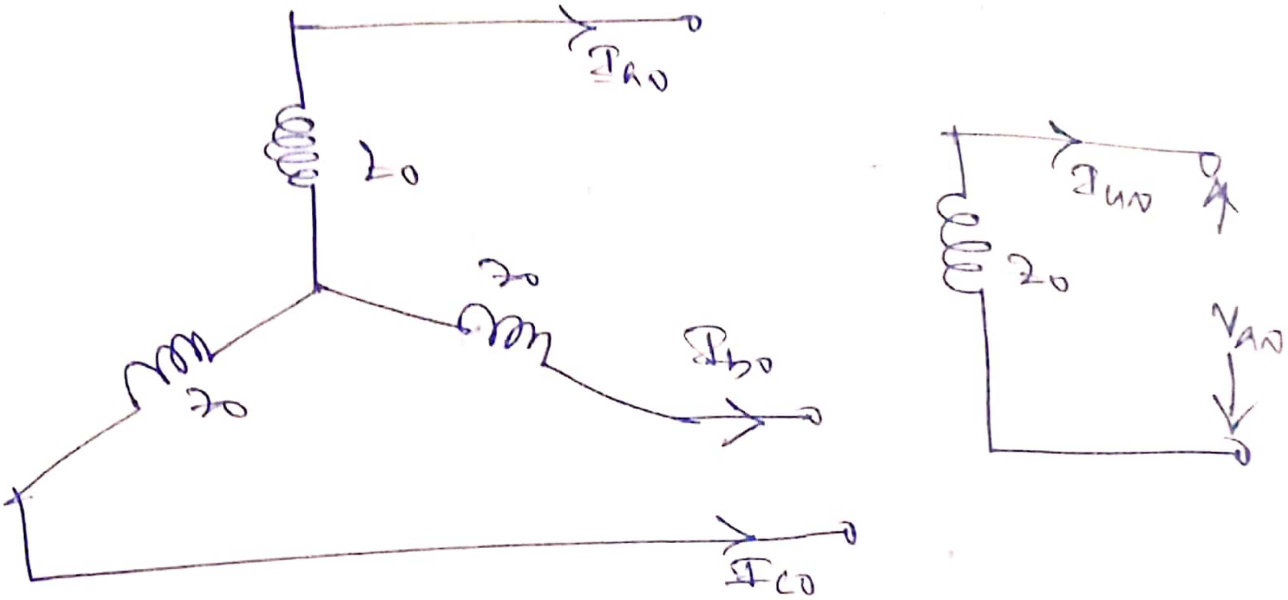


Fig: Zero Sequence N/W.

Solution of three sequence equations and three boundary condition equations in which the phase quantities have been replaced by their symmetrical components of currents and voltages will give the six unknown symmetrical components of currents and voltages.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

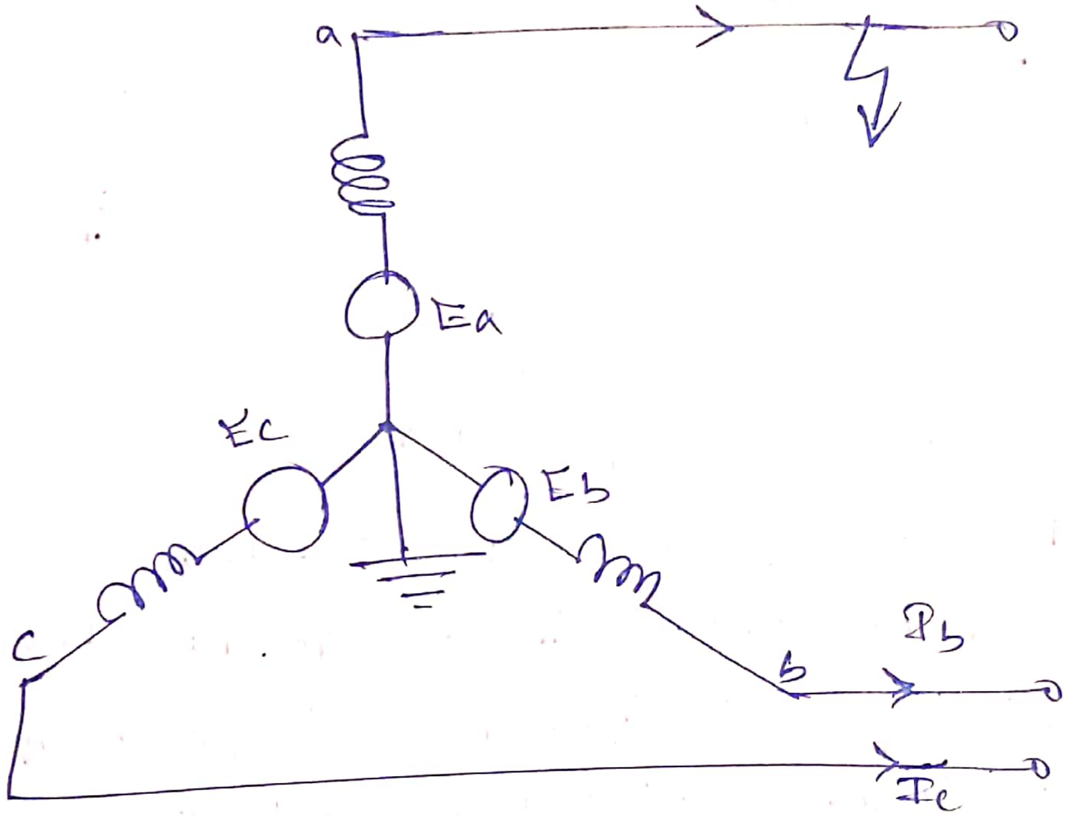
We will analyse a first of all a system where faults take place on an unloaded alternator with neutral solidly grounded.

Single Line-to-Ground fault:- The system to be analysed as shown in the below fig. Let the fault takes place on phase a. The boundary conditions are

$V_a = 0 \rightarrow (1)$

$I_b = 0 \rightarrow (2)$

$I_c = 0 \rightarrow (3)$



The sequence N/W equations are

$$V_{a0} = -I_{a0} Z_0 \rightarrow (4)$$

$$V_{a1} = E_a - I_{a1} \cdot Z_1 \rightarrow (5)$$

$$V_{a2} = -I_{a2} Z_2 \rightarrow (6)$$

The solution of six equations will give six unknowns V_{a0}, V_{a1}, V_{a2} and I_{a0}, I_{a1} & I_{a2}.

$$I_{a1} = \frac{1}{3}\left(I_a + \lambda I_b + \lambda^2 I_c\right) \rightarrow (7)$$

$$I_{a2} = \frac{1}{3}\left(I_a + \lambda^2 I_b + \lambda I_c\right) \rightarrow (8)$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right) \rightarrow (9)$$

By substituting the values of I_b & I_c value in the above eqs

$$\boxed{I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3}} \rightarrow (10)$$

Eq (1) can be written in symmetrical components

$$V_a = 0 = V_{a1} + V_{a2} + V_{a0} \rightarrow (11)$$

now, substituting the values of V_{a0}, V_{a1} and V_{a2} from the sequence N/W equation.

$$E_a - I_{a1} Z_1 - I_{a2} Z_2 - I_{a0} Z_0 = 0 \rightarrow (12)$$

$$I_{a1} = I_{a2} = I_{a0}.$$

then the eq (12) becomes

$$E_a - I_{a1} Z_1 - I_{a1} Z_2 - I_{a1} Z_0 = 0$$

$$E_a = I_{a1} Z_1 + I_{a1} Z_2 + I_{a1} Z_0$$

$$= I_{a1} (Z_1 + Z_2 + Z_0)$$

$$\boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}} \rightarrow (13)$$

From the above eq., to simulate the L-G fault all three sequence N/W's are required and since the currents are all equal in magnitude and phase angle.

∴ the three sequence N/W's must be connected in series.

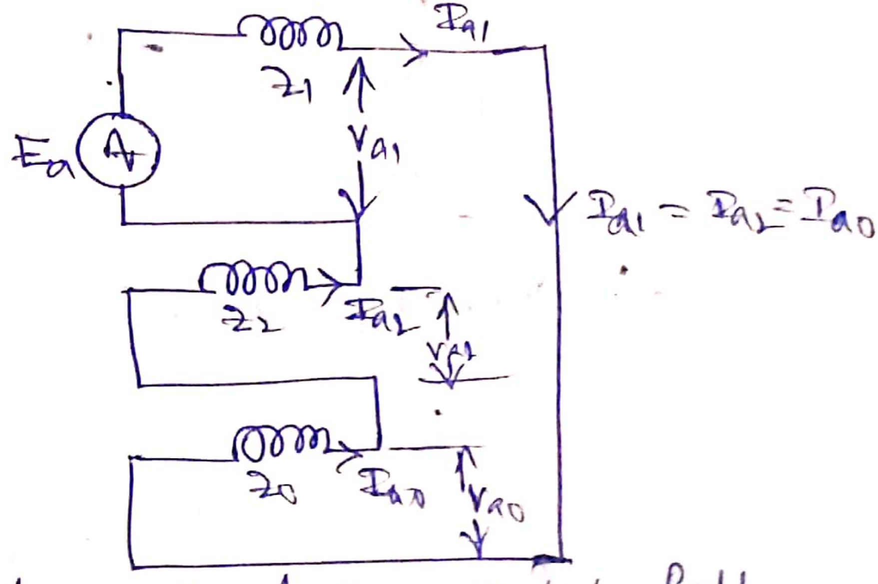


Fig: Inter connection of seq. N/W L-G fault

To calculate the remaining unknown constants V_{a0}, V_{a1}, V_{a2} by using matrix manipulations.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Substituting for $I_b = I_c = 0$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a_0} = I_{a1} = I_{a2} = I_a/3 \rightarrow (14)$$

By substituting Eq (14) in voltage Matrix

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_{a1} Z_0 \\ I_{a1} Z_1 \\ I_{a1} Z_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} -I_{a1} Z_0 \\ E_a - I_{a1} Z_1 \\ -I_{a1} Z_2 \end{bmatrix}$$

$\therefore V_{a0} + V_{a1} + V_{a2} = 0$

$$= -I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2 = 0$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

Scanned with CamScanner

Now, in case of line-to-ground fault, the neutral current $I_n = I_a = I_{a1} + I_{a2} + I_{a0}$

$\therefore$ For the same case $I_{a1} = I_{a2} = I_{a0}$

$\therefore I_n = 3 . I_{a0}$

In case, the neutral is not grounded, the zero sequence impedance becomes infinite

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2 + \infty} = 0$$

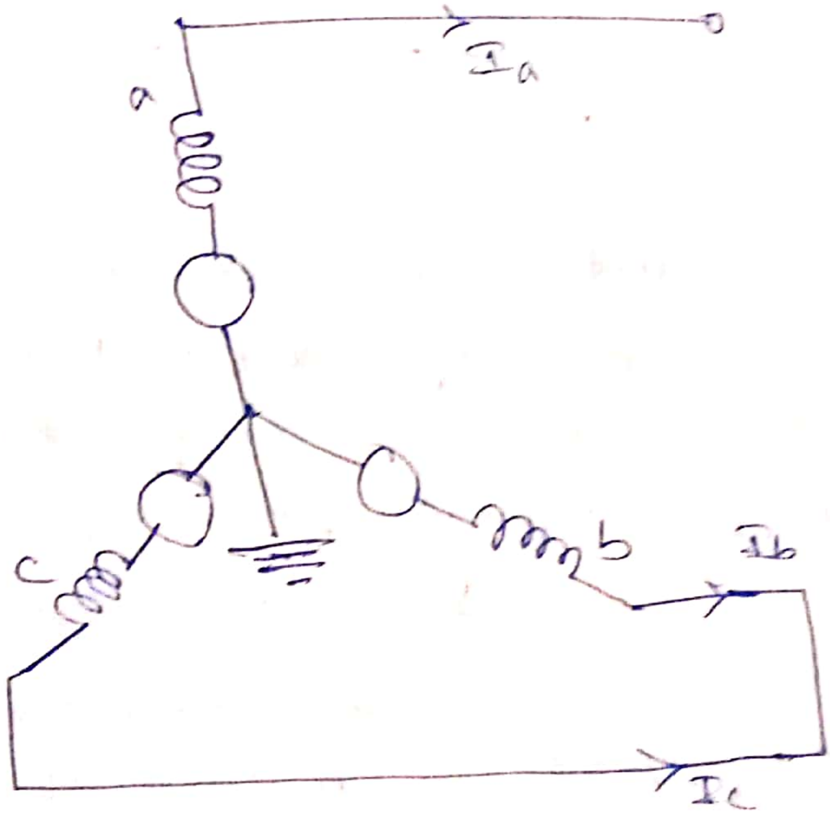
Line-to-Line fault:-

The line-to-line fault takes place on phases b and c. The boundary conditions are

$I_a = 0 \rightarrow (1)$

$I_b + I_c = 0 \rightarrow (2)$

$V_b = V_c \rightarrow (3)$



Scanned with CamScanner

The network sequence equations are

$$V_{a0} = -I_{a0} Z_0 \rightarrow (4)$$

$$V_{a1} = E_a - I_{a1} Z_1 \rightarrow (5)$$

$$V_{a2} = -I_{a2} Z_2 \rightarrow (6)$$

Solution of these six equations will give six unknowns.

using the relations, $I_{a1} = \frac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$

$$I_{a2} = \frac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c].$$

and substituting for I_a, I_b and I_c

$$I_{a1} = \frac{1}{3}[0 + \lambda I_b - \lambda^2 I_b)$$

$$= \frac{1}{3}[\lambda - \lambda^2] I_b$$

$$I_{a2} = \frac{1}{3}[0 + \lambda^2 I_b - \lambda I_b]$$

$$= \frac{I_b}{3}[\lambda^2 - \lambda]$$

& $I_{a0} = \frac{1}{3}[0+0] = 0$

→ It means that for a line-to-line fault, the zero-sequence component of current is absent ~~but~~

→ positive sequence component of current is equal and opposite to negative sequence component of current

$$I_{a1} = -I_{a2} \rightarrow (7)$$

To simulate L-L fault condition zero-sequence N/W is not required and the positive and negative sequence

Scanned with CamScanner

networks are to be connected in opposition

$$I_{a1} = -I_{a2}$$

$$V_b = V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2}$$

$$V_c = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$$

Substituting these relations in Eq (3)

$$V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2} = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$$

$$(\lambda^2 - \lambda) V_{a1} = (\lambda^2 - \lambda) V_{a2}$$

$$\therefore \boxed{V_{a1} = V_{a2}} \rightarrow (8)$$

∴ positive sequence component of voltage equals the -ve sequence component of voltage.

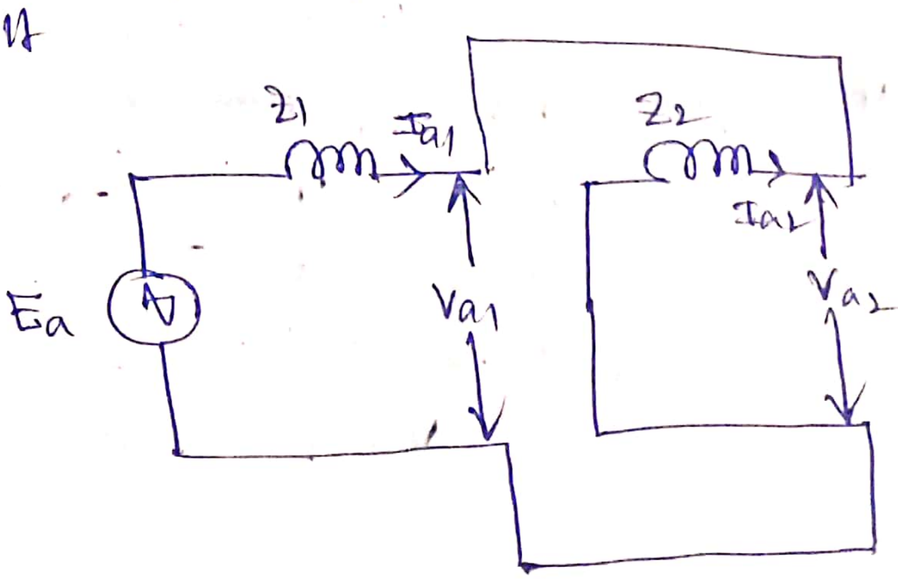
The two sequence N/W's are connected in opposition.

$$V_{a1} = V_{a2}$$

$$E_a - I_{a1} Z_1 = -I_{a2} Z_2 = I_{a1} Z_2$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}$$

The Interconnection of the sequence network for simulation of L-L fault



we have calculated, I_{a1}, I_{a2} and I_{a0}, we can calculate

Scanned with CamScanner

the three symmetrical components of voltages V_{a1}, V_{a2} and V_{a0}.

$I_{a0} = 0 \quad \therefore V_{a0} = 0$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$\therefore I_{a0} = 0$

$I_{a1} = (\lambda - \lambda^2) I_b$

$I_{a2} = (\lambda^2 - \lambda) I_b$

$\therefore I_{a1} = -I_{a2}$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$V_{a0} = \frac{1}{3}[V_a + V_b + V_c] = 0$

$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_b]$

$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_b]$

$\therefore V_{a1} = V_{a2}$

The sequence N/W equations are

$$\begin{bmatrix} 0 \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a2} \end{bmatrix}$$

$V_{a1} = E_a - I_{a1} Z_1 = I_{a1} Z_2$

$$\therefore \boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2}}$$

Scanned with CamScanner

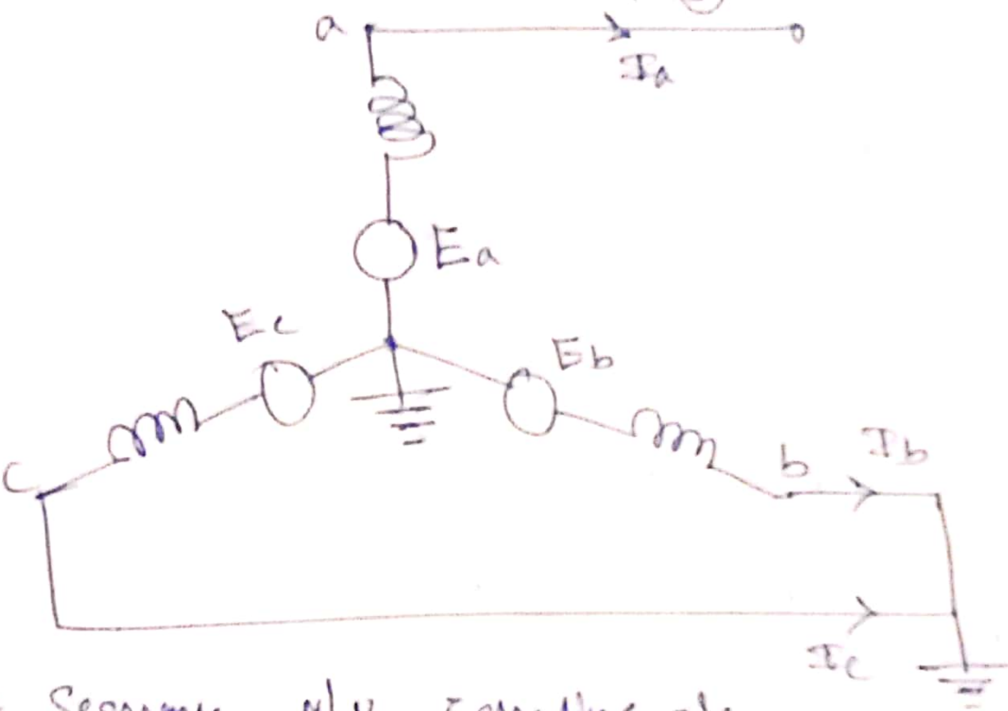
Double line to ground fault :-

Double line to Ground fault takes place in phases b and c, the boundary conditions are

$I_a = 0 \rightarrow (1)$

$V_b = 0 \rightarrow (2)$

$V_c = 0 \rightarrow (3)$



the sequence N/W equations are

$$V_{a0} = -I_{a0} Z_0 \rightarrow (4)$$

$$V_{a1} = E_a - I_{a1} Z_1 \rightarrow (5)$$

$$V_{a2} = -I_{a2} Z_2 \rightarrow (6)$$

The solution of these six equations will give for unknown constants.

$$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$$

$$= \frac{V_a}{3}$$

$$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_c]$$

$$= \frac{V_a}{3}$$

$$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c] = \frac{V_a}{3}$$

Scanned with CamScanner

i.e $V_{a0} = V_{a1} = V_{a2}$ ... — (7)

using this relation of voltages and substituting in sequence n/w equations

$$V_{a0} = V_{a1}$$

$$-I_{a0} Z_0 = E_a - I_{a1} Z_1$$

$$I_{a0} = -\frac{E_a - I_{a1} Z_1}{Z_0} \rightarrow (8)$$

similarly,

$$V_{a2} = V_{a1}$$

$$-I_{a2} Z_2 = E_a - I_{a1} Z_1$$

$$I_{a2} = -\frac{E_a - I_{a1} Z_1}{Z_2} \rightarrow (9)$$

Now from eq (1), $I_a = I_{a1} + I_{a2} + I_{a0} = 0$

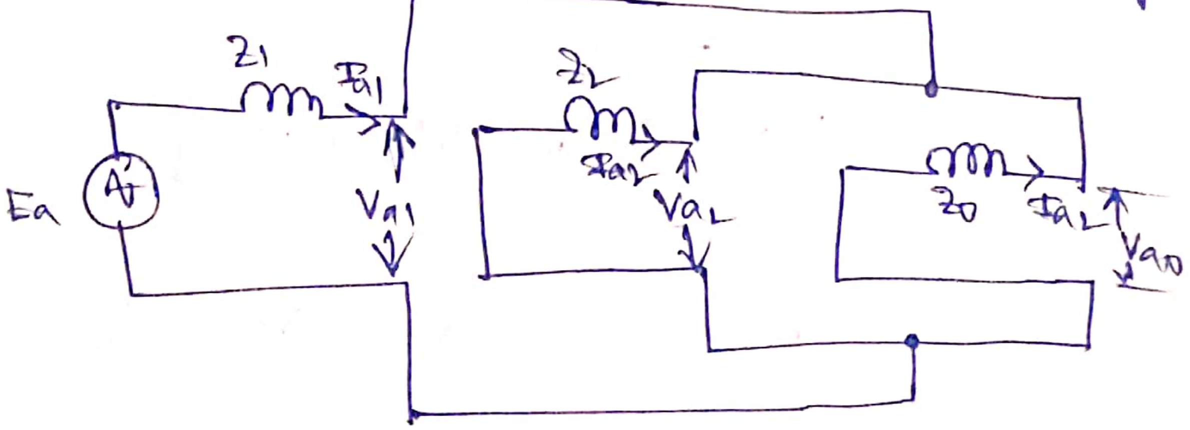
By substituting the values of I_{a2} & I_{a0}

$$I_{a1} - \frac{E_a - I_{a1} Z_1}{Z_2} - \frac{E_a - I_{a1} Z_1}{Z_0} = 0$$

Rearranging the terms gives

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \rightarrow (10)$$

From the above eq, it is all the three sequence n/w's are required to simulate L-L-G and also that the negative and zero sequence n/w's are connected in parallel.



Scanned with CamScanner

NWs are first connected in parallel and then in opposition with the positive sequence network.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2} = V_a/3$$

using these relations in the sequence n/w equations

$$\begin{bmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

these equations are to be solved for I_{a0}, I_{a1} and I_{a2}

Re arranging the terms,

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} -V_{a1} \\ E_1 - V_{a1} \\ -V_{a1} \end{bmatrix}$$

$$AX = B$$

where $X \rightarrow$ current vector.

To find X, pre multiply this equation by A^{-1}

$$X = A^{-1} \cdot B$$

Now,

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

As it is a diagonal Matrix

Scanned with CamScanner

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} -V_{a1} \\ E_a - V_{a1} \\ -V_{a1} \end{bmatrix}$$

$$I_{a0} = \frac{-V_{a1}}{Z_0} = \frac{E_a - I_{a1} Z_1}{Z_0}$$

$$I_{a2} = \frac{-V_{a1}}{Z_2} = -\frac{E_a - I_{a1} Z_1}{Z_2}$$

use the relation $I_{a1} + I_{a2} + I_{a0} = 0$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$$

∴ the neutral current, $I_n = I_b + I_c$

$$= \lambda^2 I_{a1} + \lambda I_{a2} + I_{a0} + \lambda I_{a1} + \lambda^2 I_{a2} + I_{a0}$$

$$= (\lambda^2 + \lambda) I_{a1} + (\lambda + \lambda^2) I_{a2} + 2 I_{a0}$$

$$= -I_{a1} - I_{a2} + 2 I_{a0}$$

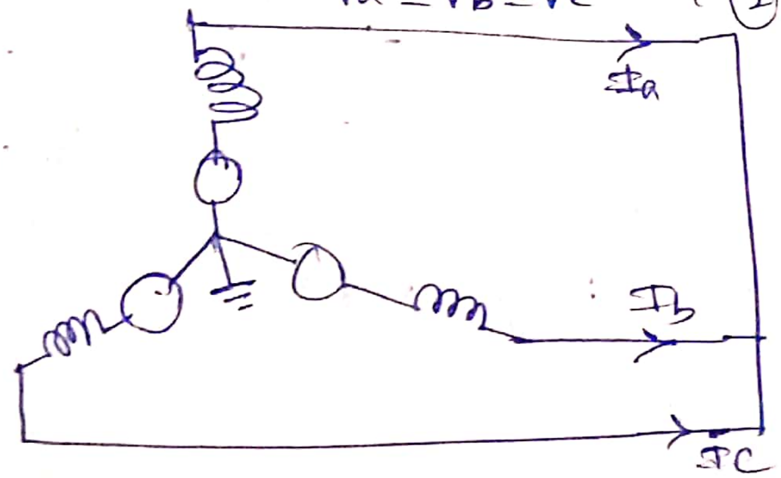
$$= I_{a0} + 2 I_{a0}$$

$$= 3 I_{a0}$$

3-Phase fault:- The boundary conditions are

$I_a + I_b + I_c = 0 \rightarrow (1)$

$V_a = V_b = V_c \rightarrow (2)$



Scanned with CamScanner

Since, $|I_a| = |I_b| = |I_c|$ and if I_a is taken as reference

$$I_b = \lambda^2 I_a, \quad I_c = \lambda I_a$$

using the relation, $I_{a1} = \frac{1}{3}[I_a + \lambda I_b + \lambda^2 I_c]$

and substituting the values of I_b and I_c

$$I_{a1} = \frac{1}{3}[I_a + \lambda^3 I_a + \lambda^3 I_a]$$

$$I_{a1} = I_a \rightarrow (13)$$

$$I_{a2} = \frac{1}{3}[I_a + \lambda^2 I_b + \lambda I_c]$$

substituting for I_b and I_c in terms of I_a

$$I_{a2} = \frac{1}{3}[I_a + \lambda^4 I_a + \lambda^2 I_a]$$

$$= \frac{I_a}{3}[1 + \lambda + \lambda^2]$$

$$I_{a2} = 0 \rightarrow (14)$$

similarly,

$$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$$

$$= 0 \rightarrow (15)$$

It means that for a 3-ph fault zero- as well as negative sequence components of current are absent and the positive sequence component of current is equal to the phase current.

$$V_{a1} = \frac{1}{3}[V_a + \lambda V_b + \lambda^2 V_c]$$

$$= \frac{1}{3}[V_a + \lambda V_a + \lambda^2 V_a]$$

$$= \frac{V_a}{3}[1 + \lambda + \lambda^2] = 0 \rightarrow (16)$$

$$V_{a2} = \frac{1}{3}[V_a + \lambda^2 V_b + \lambda V_c] = 0 \rightarrow (17)$$

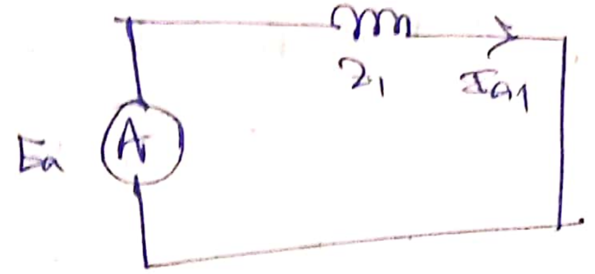
Scanned with CamScanner

$V_{a0} = 0 \rightarrow (P)$

Since $V_{a1} = 0 = E_a - I_{a1} Z_1$

$\therefore I_{a1} = \frac{E_a}{Z_1} \rightarrow (Q)$

The Sequence N/W is shown in fig. below



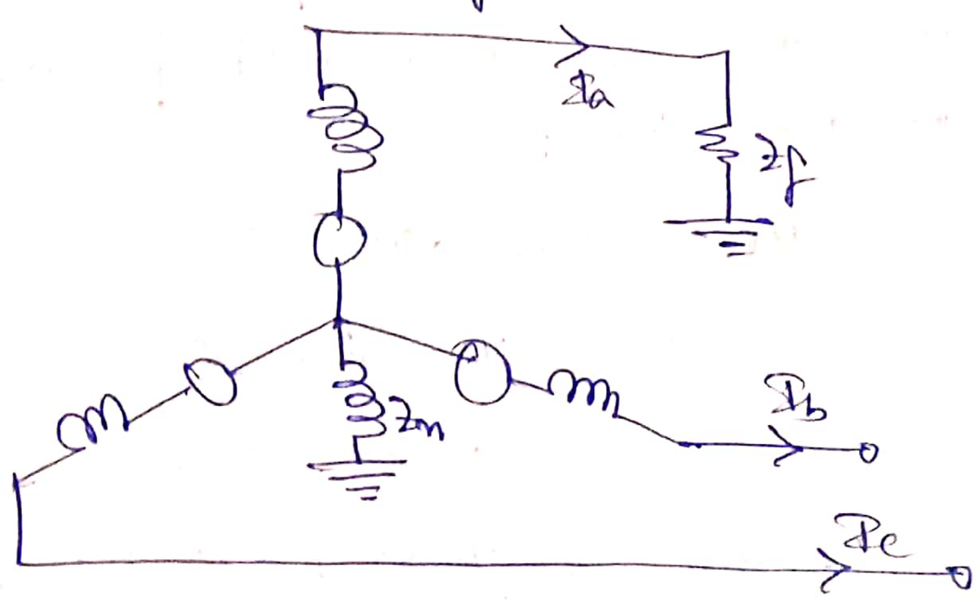
→ Positive Sequence currents are present in all types of faults.

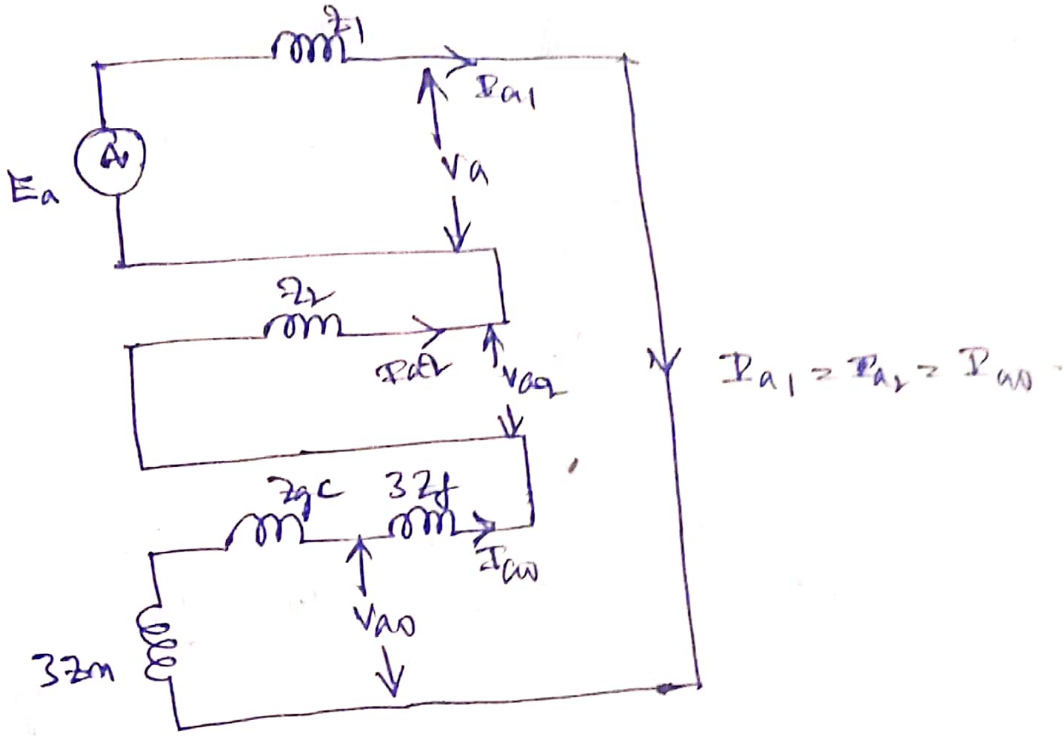
→ Negative Sequence currents are present in all unsymmetrical faults.

→ Zero Sequence currents are present when the neutral of the system is grounded and the fault also involves the ground, and the magnitude of the neutral current is equal to $3 I_{a0}$.

Line to Ground fault with Z_f:-

The fault impedance is Z_f and neutral impedance Z_n





The boundary conditions are $V_a = I_a Z_f$.

$$I_b = 0, \quad I_c = 0$$

$$V_{a0} = -I_{a0}(Z_{g0} + 3Z_n)$$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a2} Z_2$$

The solution of these equations gives the unknown quantities.

$$I_{a1} = I_{a2} = I_{a0} = I_a/3$$

$$V_{a1} + V_{a2} + V_{a0} = V_a = 3 I_{a1}(Z_f)$$

~~$E_a = I_{a1}[Z_1 + Z_2 +$~~

$$E_a - I_{a1}Z_1 - I_{a1}Z_2 - I_{a1}(Z_0 + 3Z_n) = 3 I_{a1} Z_f.$$

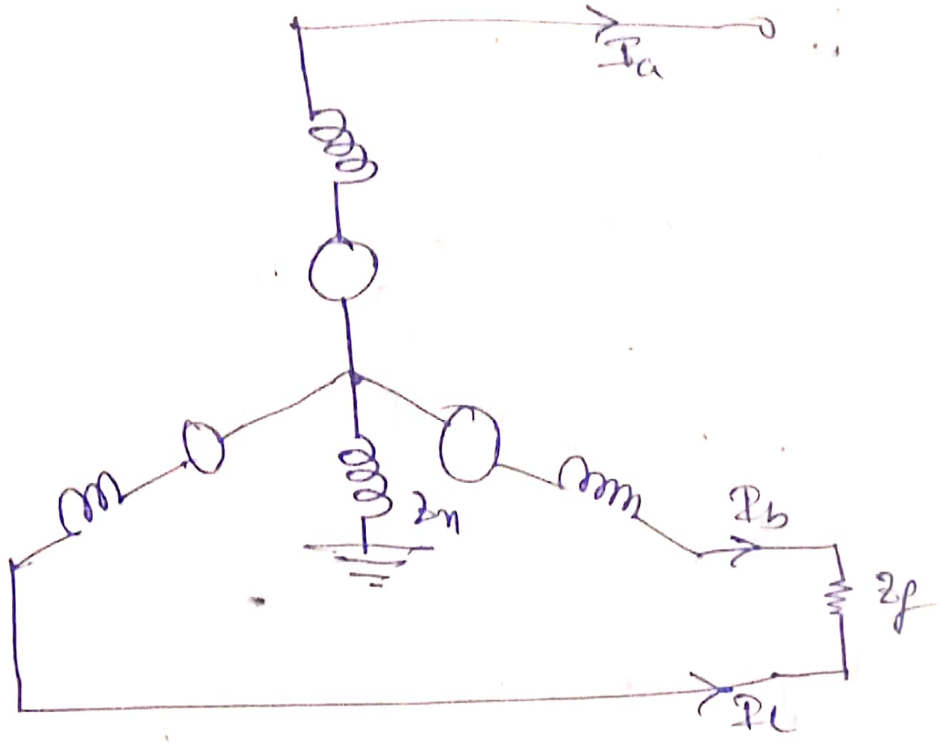
$$E_a = I_{a1}[Z_1 + Z_2 + \{(Z_0 + 3Z_n) + 3Z_f\}]$$

$$I_{a1} = \frac{E_a}{Z_1 + Z_2 + (Z_0 + 3Z_n) + 3Z_f}$$

Since I_{a0}, I_{a1} and I_{a2} are known V_{a0}, V_{a1}, V_{a2} can be calculated from the sequence n/w equations..

Scanned with CamScanner

The line-to-line fault with Z_f :-



The boundary conditions are $I_a = 0$

$I_b + I_c = 0$

$V_b = V_c + I_b \cdot Z_f$

and the sequence n/w equations are

$V_{a1} = E_a - I_{a1} Z_1$

$V_{a2} = -I_{a2} Z_2$

$V_{a0} = -I_{a0} Z_0$

We know, $I_{a1} = -I_{a2}$ & $I_{a0} = 0$

$V_b = V_c + I_b \cdot Z_f$

$V_{a0} + \lambda^2 V_{a1} + \lambda V_{a2} = V_{a0} + \lambda V_{a1} + \lambda^2 V_{a2}$
$+ (\lambda^2 I_{a1} + \lambda I_{a2}) Z_f$.

$\lambda^2 V_{a1} - \lambda V_{a1} = (\lambda^2 - \lambda) V_{a2} + (\lambda^2 I_{a1} - \lambda I_{a1}) Z_f$

$V_{a1} = V_{a2} + I_{a1} Z_f$.

Now substituting for V_{a1} and V_{a2} from the sequence

Scanned with CamScanner

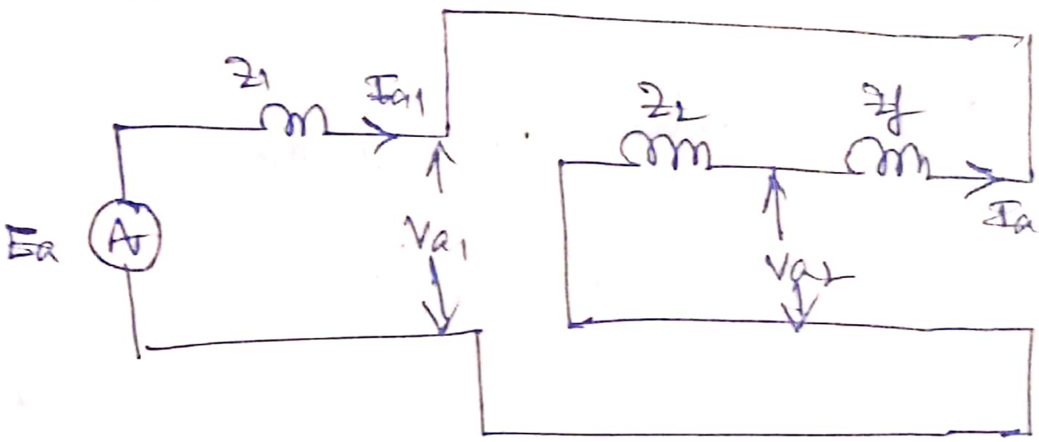
network equations

$$E_a - I_{a1} Z_1 = -I_{a2} Z_2 + I_{a1} Z_f$$

$$E_a - I_{a1} Z_1 = I_{a1} (Z_2 + Z_f)$$

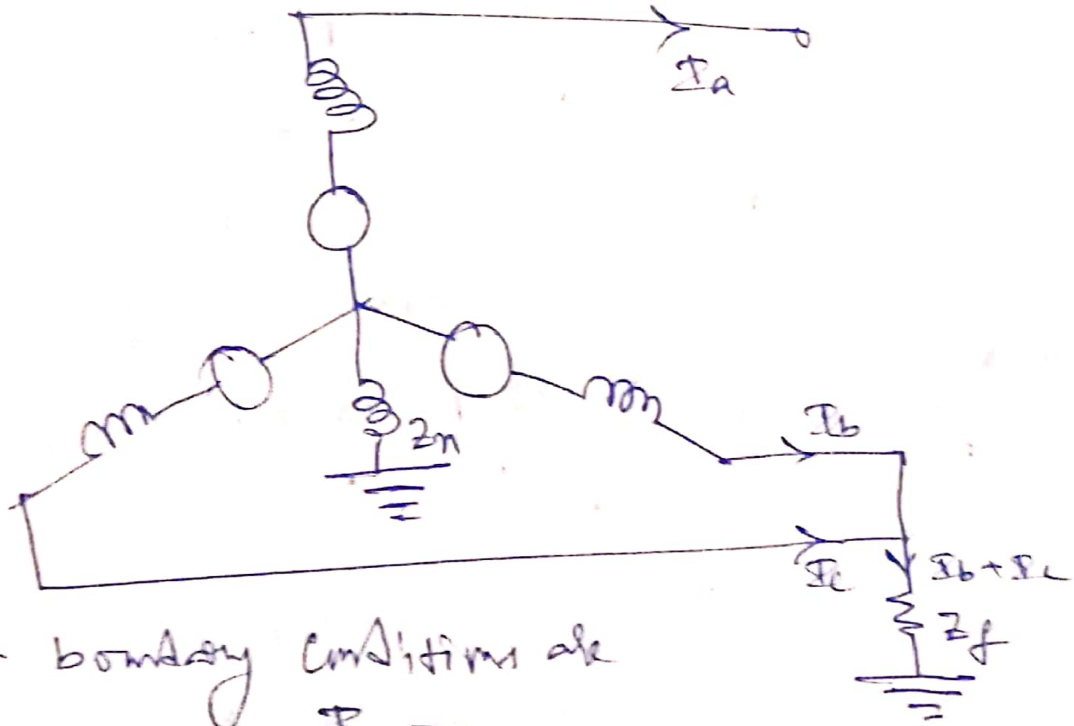
$$\therefore \quad I_{a1} = \frac{E_a}{Z_1 + (Z_2 + Z_f)}$$

The interconnection of sequence n/w is shown in the below fig



Double line to Ground fault :-

Fault Impedance is Z_f and neutral Impedance Z_n.



The boundary conditions are

$$I_a = 0$$

$$V_b = V_c = (I_b + I_c) Z_f \rightarrow (1)$$

and the sequence n/w equations are

Scanned with CamScanner

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = - I_{a2} Z_2$$

$$V_{a0} = - I_{a0} (Z_0 + 3 Z_n)$$

we know that $(I_b + I_c) = 3 I_{a0}$.

From Eq ① $V_b = V_c = 3 I_{a0} Z_f$.

$$\lambda^2 V_{a1} + \lambda V_{a2} + V_{a0} = \lambda V_{a1} + \lambda^2 V_{a2} + V_{a0}$$

$$V_{a1} = V_{a2}.$$

using this relation in Equation $V_b = 3 I_{a0} Z_f$.

$$\lambda^2 V_{a1} + \lambda V_{a1} + V_{a0} = 3 I_{a0} Z_f$$

$$-V_{a1} + V_{a0} = 3 I_{a0} Z_f$$

$$V_{a1} = V_{a0} - 3 I_{a0} Z_f.$$

Substituting for V_{a1} and V_{a0} from the sequence equation and expressing I_{a0} in terms of I_{a1}.

$$E_a - I_{a1} Z_1 = - I_{a0}(Z_0 + 3 Z_n) - 3 I_{a0} Z_f.$$

$$I_{a0} = - \frac{E_a - I_{a1} Z_1}{Z_0 + 3 Z_n + 3 Z_f}$$

similarly making the use of relation $V_{a1} = V_{a2}$, Expressing I_{a2} in terms of I_{a1}

$$E_a - I_{a1} Z_1 = - I_{a2} Z_2$$

$$I_{a2} = \frac{E_a - I_{a1} Z_1}{Z_2}$$

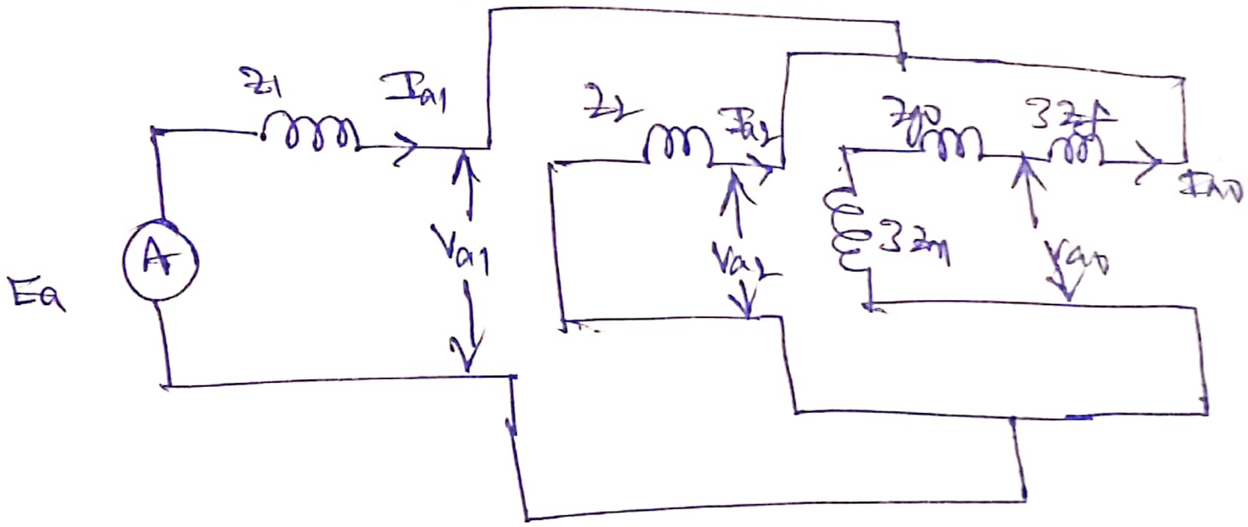
Scanned with CamScanner

Substituting the values of I_{a2} and I_{a0} in the equation $I_a = I_{a1} + I_{a2} + I_{a0} = 0$

$$I_{a1} - \frac{E_a - I_{a1} Z_1}{Z_2} - \frac{E_a - I_{a1} Z_1}{Z_0 + 3Z_n + 3Z_f} = 0$$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2 (Z_0 + 3Z_n + 3Z_f)}{Z_2 + Z_0 + 3Z_n + 3Z_f}}$$

The interconnection of sequence n/w



Scanned with CamScanner