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**Year& Semester : II B.Tech., I SEM**

**Faculty Name : Dr. K. PAVANI**

**Program Name : B.Tech- ECE, EEE**

**Name of the Course : NUMERICAL METHODS AND COMPLEX VARIABLES PPT**

**Course Code : NMCV** (23MA301)

# UNIT-1 Fourier Series and Fourier Transforms

your roots to success...

# * CONTENTS *

- FOURIER SERIES.
- APPLICATION OF FOURIER SERIES :-
  - FORCED OSCILLATION.
  - APPROXIMATION BY TRIGNOMETRIC POLYNOMIALS.

**FOURIER SERIES can be generally written as,**

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx),$$

**Where,**

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \quad \text{......... (1.1)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \quad \text{......... (1.2)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx \quad \text{......... (1.3)}$$

**Fourier series make use of the orthogonality relationships of the sine and cosine functions.**

# BASIS FORMULAE OF FOURIER SERIES

**The Fourier series of a periodic function $f(x)$ with period 2π is defined as the trigonometric series with the coefficient a0, an and bn, known as** *FOURIER COEFFICIENTS*, **determined by formulae (1.1), (1.2) and (1.3).**

**The individual terms in Fourier Series are known as** *HARMONICS.*

**Every function $f(x)$ of period 2π satisfying following conditions known as** *DIRICHLET'S CONDITIONS*, **can be expressed in the form of Fourier series.**

CONDITIONS :-

1. $f$(x) is bounded and single value.

   ( A function $f$(x) is called single valued if each point in the domain, it has unique value in the range.)

2. $f$(x) has at most, a finite no. of maxima and minima in the interval.

3. $f$(x) has at most, a finite no. of discontinuities in the interval.

- **EXAMPLE:**

  $\sin^{-1}x$, we can say that the function $\sin^{-1}x$ cant be expressed as Fourier series as it is not a single valued function.

  tanx, also in the interval (0,2п) cannot be expressed as a Fourier Series because it is infinite at x= п/2.

# FOURIER SERIES FOR EVEN AND ODD FUNCTIONS

## EVEN FUNCTIONS

**If function *f*(x) is an even periodic function with the period 2L (–L ≤ x ≤ L), then *f*(x)cos(n□x/L) is even while *f*(x)sin(n□x/L) is odd.**

**Thus the Fourier series expansion of an even periodic function *f*(x) with period 2L (–L ≤ x ≤ L) is given by,**

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{\pi n x}{L}$$

**Where,** $$a_0 = \frac{2}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad n = 1, 2, \cdots$$

$$b_n = 0$$

# ODD FUNCTIONS

**If function *f* (x) is an even periodic function with the period 2L (–L ≤ x ≤ L), then *f* (x)cos(n⊓x/L) is even while *f* (x)sin(n⊓x/L) is odd.**

**Thus the Fourier series expansion of an odd periodic function *f* (x) with period 2L (–L ≤ x ≤ L) is given by,**

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

**Where,**

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx \qquad n = 1,2,\cdots$$

# EXAMPLES..

Question.: Find the fourier series of f(x) = $x^2$+x , - ≤ x ≤.

Solution.: The fourier series of $f$(x) is given by,

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx),$$

Using above,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x^2 + x)dx$$

$$= \frac{1}{\pi}\left(\frac{x^3}{3} + \frac{x^2}{2}\right)_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left(\frac{\pi^3}{3} + \frac{\pi^2}{2} + \frac{\pi^3}{3} - \frac{\pi^2}{2}\right) = \frac{2\pi^3}{3} = a_0$$

Now,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x^2 + x)\cos nx dx$$

$$= \frac{1}{\pi}\left[(x^2 + x)\left(\frac{\sin nx}{n}\right) - (2x+1)\left(\frac{-\cos nx}{n^2}\right) + (2)\left(\frac{-\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(2\pi+1)\frac{\cos n\pi}{n^2} - (-2\pi+1)\frac{\cos n\pi}{n^2}\right]$$

$$= \frac{1}{\pi}\left[(2\pi+1)\frac{(-1)^n}{n^2} - (-2\pi+1)\frac{(-1)^n}{n^2}\right]$$

$$= \frac{4(-1)^n}{n^2}$$

Now,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x^2 + x)\sin nx dx$$

$$= \frac{1}{\pi}\left[(x^2 + x)\left(-\frac{\cos nx}{n}\right) - (2x+1)\left(-\frac{\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{(\pi^2 + \pi)}{n}(-1)^n + \frac{(\pi^2 + \pi)}{n}(-1)^n\right]$$

$$= \frac{(-1)^n}{\pi n}\left[-\pi^2 - \pi + \pi^2 - \pi\right]$$

$$= -\frac{2(-1)^n}{n}$$

Hence fourier series of, f(x) = $x^2$+x,

$$x^2 + x = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4(-1)^n}{n^2}\cos nx - \frac{2(-1)^n}{n}\sin nx\right]$$

# Fourier Transforms

# What is Fourier Transform

- ***Fourier Transform***, named after ***Joseph Fourier***, is a mathematical transformation employed to transform signals between time(or spatial) domain and frequency domain.
- It is a tool that breaks a waveform (a function or signal) into an alternate representation, characterized by *sine and cosines*.
- It shows that any waveform can be re-written as *the weighted sum of sinusoidal* functions.

# Forward Fourier and Inverse Fourier transforms

- Given an image a and its Fourier transform A
  - Then *the forward transform* goes from the spatial domain (either continuous or discrete) to the frequency domain which is always continuous.

$$Forward - \qquad A = \mathfrak{F}\{a\}$$

  - The *inverse* goes from the frequency domain to the spatial domain.

$$Inverse - \qquad a = \mathfrak{F}^{-1}\{A\}$$

# Fourier Transform

- The Fourier transform of F(u) of a *single variable, continuous function*, f(x)

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

- The Fourier transform of F(u,v) of a *double variable, continuous function*, f(x,y)

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

# Fourier Transform

- The Fourier Transform of a *discrete function of one variable*, f(x), x=0,1,2,. . . ,M-1,

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-2j\pi ux/M}$$

u=0,1,2,. . . ,M-1.

- The concept of Frequency domain follows *Euler's formula*

$$e^{j\theta} = \cos\theta + j\sin\theta$$

# Fourier Transform

- Fourier Transform (in one dimension)

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\left[\cos 2\pi ux / M - j\sin 2\pi ux / M\right]$$

- Each term of the Fourier transform is composed of the sum of all values of the function *f(x)*.
- The values of *f(x)* are multiplied by sine and cosines of various frequencies.
- Each of the M term of *F*(u) is called the *frequency component* of the transform.
- The domain (values of u) over which the values of F(u) range is appropriately called the *frequency domain*.

# Properties of Fourier Transforms

- ***Linearity***
  - Scaling a function scales it's transform pair. Adding two functions corresponds to adding the two frequency spectrum.

If $h(x) \leftrightarrow H(f)$ then $ah(x) \leftrightarrow aH(f)$

If $h(x) \leftrightarrow H(f)$, $g(x) \leftrightarrow G(f)$ then $h(x)+g(x) \leftrightarrow H(f)+G(f)$

- ***Scaling Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(at) \Leftrightarrow \frac{1}{|a|} F(\omega/a)$$

# Properties of Fourier Transforms

- ***Time Differentiation***
  - If
    $$f(t) \Leftrightarrow F(\omega)$$
  - Then
    $$\frac{df}{dt} \Leftrightarrow j\omega F(\omega)$$
- ***Convolution Property***
  - If
    $$f_1(t) \Leftrightarrow F_1(\omega) \quad \text{and} \quad f_2(t) \Leftrightarrow F_2(\omega)$$
  - Then
    $$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega)F_2(\omega)$$
  - (where * is convolution) and
    $$f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}F_1(\omega) * F_2(\omega)$$

# Properties of Fourier Transforms

- ***Frequency-shift Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(t)e^{j\omega_0 t} \Leftrightarrow F(\omega - \omega_0)$$

- ***Time-Shift Property***
  - If

$$f(t) \Leftrightarrow F(\omega)$$

  - Then

$$f(t - t_0) \Leftrightarrow F(\omega)e^{-j\omega t_0}$$

  - In other words, a shift in time corresponds to a change in phase in the Fourier transform.

# Fourier Transformation in Image processing

- Used to access the *geometric characteristics* of a spatial domain image.
  - Fourier domain decompose the image into its *sinusoidal components.*

- In most implementations
  - Fourier image is shifted in such a way that the $F(0,0)$ represent the center of the image.
  - The further away from the center an image point is, the higher is its corresponding frequency.

# Fourier Transformation in Image processing

- The Fourier Transform is used in a wide range in image processing
  - Image filtering,
  - Image applications
    - Image analysis,
    - Image filtering,
    - Image reconstruction, and
    - Image compression.

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# UNIT-2 Numerical Methods-I

# Numerical Methods

**Numerical Methods:**

Algorithms that are used to obtain numerical solutions of a mathematical problem.

**Why do we need them?**

1. No analytical solution exists,
2. An analytical solution is difficult to obtain or not practical.

# What do we need?

**Basic Needs in the Numerical Methods:**

- Practical:

  Can be computed in a reasonable amount of time.

- Accurate:
  - Good approximate to the true value,
  - Information about the approximation error (Bounds, error order,… ).

# Algebraic functions

The general form of an Algebraic function:

$$f_n y^n + f_{n-1} y^{n-1} + \cdots + f_1 y + f_0 = 0$$

$f_i$ = an $i$-th order polynomial.

Example : $\underbrace{(2x^3 + 6x + 3)}_{f_3} y^3 + \underbrace{(x - 3)}_{f_2} y^2 + \underbrace{7}_{f_0} = 0$

Polynomials are a simple class of algebraic function

$$f_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

$a_i$'s are constants.

# Transcendental functions

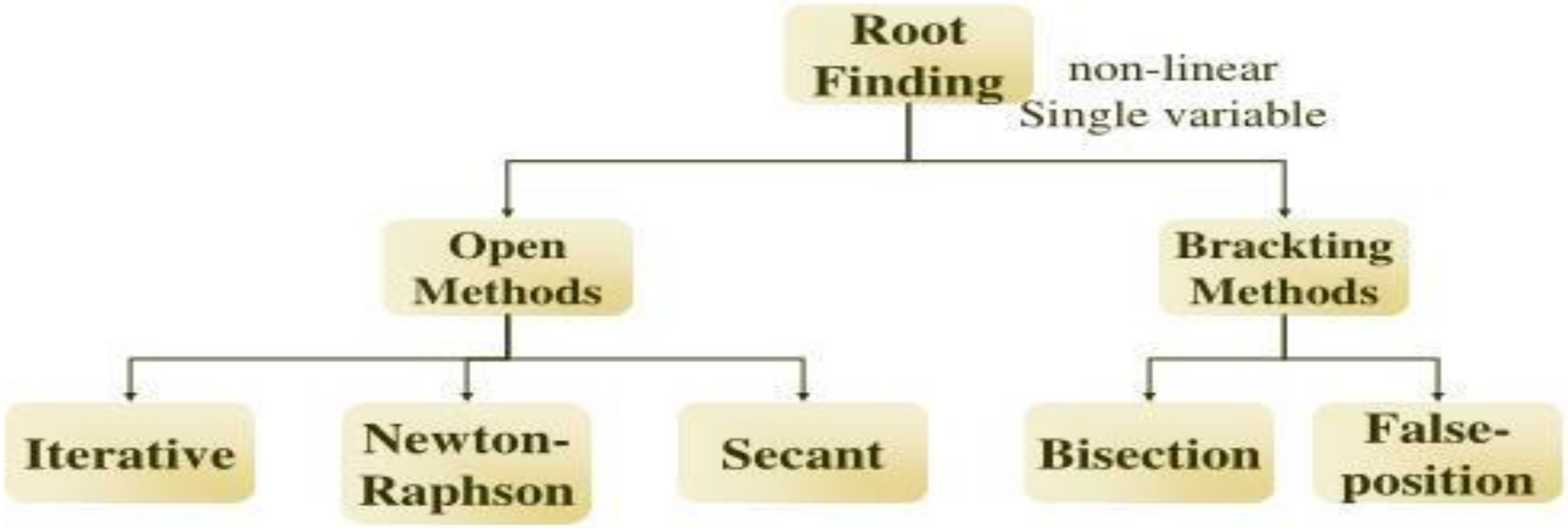
- A transcendental function is non-algebraic.
- May include trigonometric, exponential, logarithmic functions
- Examples:

$$f(x) = \ln x^2 - 1$$

$$f(x) = e^{-0.2x} \sin(3x - 0.5)$$

# Equation Solving

- Given an approximate location (initial value)
- find a single real root



# DEFINITION

The **bisection method** in mathematics is a root finding method which repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing.

It is a very simple and robust method, but it is also relatively slow. Because of this, it is often used to obtain a rough approximation to a solution which is then used as a starting point for more rapidly converging methods .The method is also called the binary search method or the dichotomy method.

# ALGORITHM

Step 1: Choose two approximations A and B (B>A) such that

$f(A)*f(B)<0$

Step 2: Evaluate the midpoint C of [A,B] given by

$C=(A+B)/2$

- Step 3: If f(C)*f(B)<0 then rename B & C as A & B. If not rename of C as B . Then apply the formula of Step 2.
- Step 4:Stop evolution when the different of two successive values of C obtained from Step 2 is numerically less than E, the prescribed accuracy .

**Newton-Raphson method**, also known as the **Newton's Method**, is the simplest and fastest approach to find the root of a function.

It is an open bracket method and requires only one initial guess.

Newton's method is often used to improve the result or value of the root obtained from other methods.

This method is more useful when the first derivative of f(x) is a large value.

Newton's Method: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This is <u>Newton's Method</u> of finding roots. It is an example of an <u>Algorithm</u> (a specific set of computational steps.)

It is sometimes called the <u>Newton-Raphson</u> method

This is a <u>Recursive</u> algorithm because a set of steps are repeated with the previous answer put in the next repetition. Each repetition is called an <u>Iteration</u>.

→

- # Direct Methods

Direct methods give the roots of non-linear equations in a finite number of steps. In addition, these methods are capable of giving all the roots at the same time.

For e.g., the roots of the quadratic equation

$ax^2+bx+c=0$ where a#0

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- # Iterative Methods

Iterative methods also known as trial and error methods, are based on the idea of successive approximations. They start with one or more initial approximations to the root and obtain a sequence of approximations by repeating a fixed sequence of steps till the solution with reasonable accuracy is obtained. Iterative methods, generally, give one root at a time.

# Interpolation

Let the function $y=f(x)$ take the values $y_0, y_1, y_2, \dots, y_n$ corresponding to the values $x_0, x_1, x_2, \dots, x_n$ of x. The process of finding the value of y corresponding to any value of $x=x_i$ between $x_0$ and $x_n$ is called interpolation.

# Newton's Forward Interpolation

- Let the function y=f(x) take the values y0,y1,y2,....,yn corresponding to the values x0,x1,x2,....,xn of x. Suppose it is required to evaluate f(x) for x=x0 + rh , where r is any real number.

- Formula :

$$Yn(x) = yo + r\Delta yo + \frac{r(r-1)}{2!}\Delta^2 yo + \frac{r(r-1)(r-2)}{3!}\Delta^3 yo + \cdots$$

$$\text{where } r = \frac{x - x0}{h}$$

# Newton's Backword Interpolation

- Let the function y=f(x) take the values y0,y1,y2,...,yn corresponding to the values x0,x1,x2,...,xn of x. Suppose it is required to evaluate f(x) for x=x0 + r*h , where r is any real number.

- Formula :

- $y_n(x) = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \cdots$

where $r = \frac{x - x_n}{h}$

# Gauss Forward Central Difference Formula

$$Y = f(x) = P_n(x) = Y_0 + P\Delta Y_0 + \frac{P(P-1)\,\Delta^2 Y_{-1}}{2!} + \frac{P(P-1)(P-2)\,\Delta^3 Y_{-1}}{3!} + \frac{P(P-1)(P-2)(P-3)\,\Delta^4 Y_{-2}}{4!}$$

Where $P = X - X_0/h$

# Gauss Backward Central Difference Formula

$$Y = f(x) = P_n(x) = Y_0 + P\Delta Y_0 + \frac{P(P+1)}{2!}\Delta^2 Y_{-1} + \frac{P(P+1)(P+2)}{3!}\Delta^3 Y_{-1} + \frac{P(P+1)(P+2)(P+3)}{4!}\Delta^4 Y_{-2}$$

Where $P = X - X_0/h$

# Lagrange interpolation

Lagrange polynomials are used for polynomial interpolation.

For a given set of distinct points $x_j$ and numbers $y_j$.
Lagrange's interpolation is also an *Nth* degree polynomial approximation to $f(x)$.

# Formula

$$y = \frac{(x-x_1)(x-x_2).....(x-x_n)}{(x_0-x_1)(x_0-x_2).....(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2).....(x-x_n)}{(x_1-x_0)(x_1-x_2).....(x_1-x_n)}y_1 + .. + \frac{(x-x_1)(x-x_1).....(x-x_{n-1})}{(x_n-x_0)(x_0-x_1).....(x_n-x_{n-1})}y_n$$

- This is called Lagrange's interpolation formula and can be used and unequal intervals.

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# UNIT-3 Numerical Methods-II

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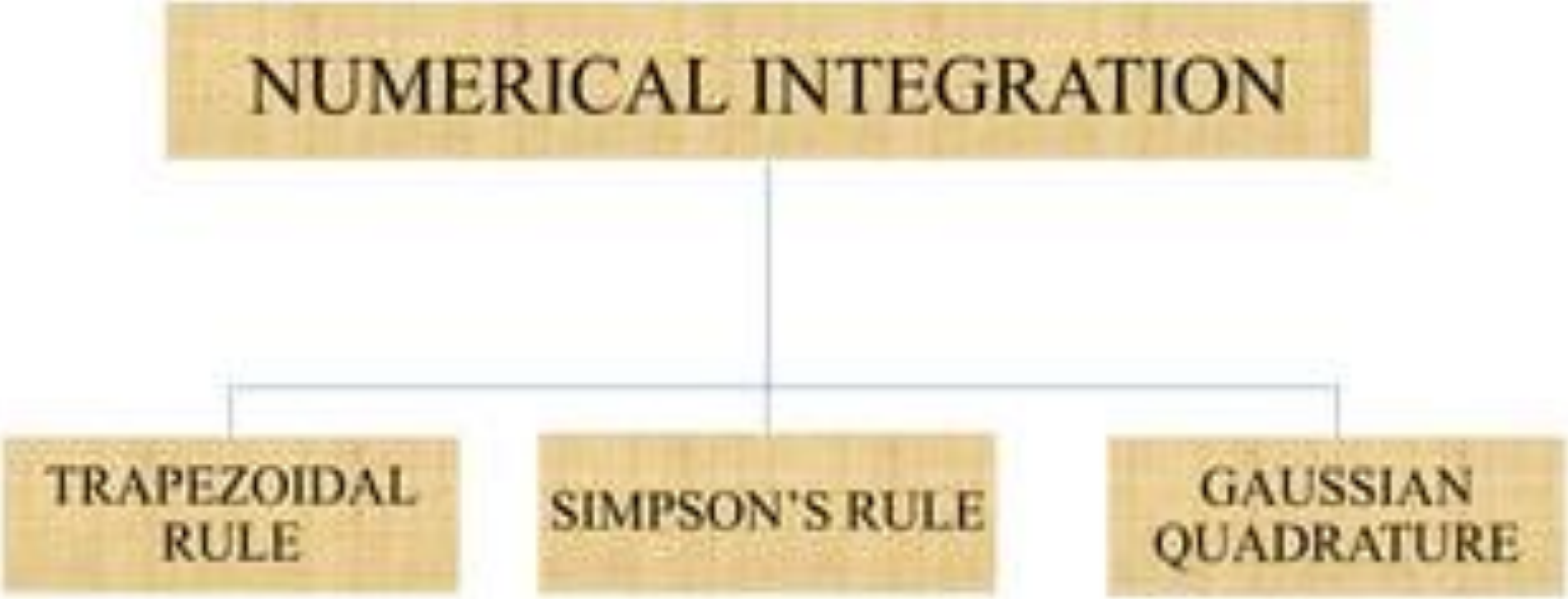
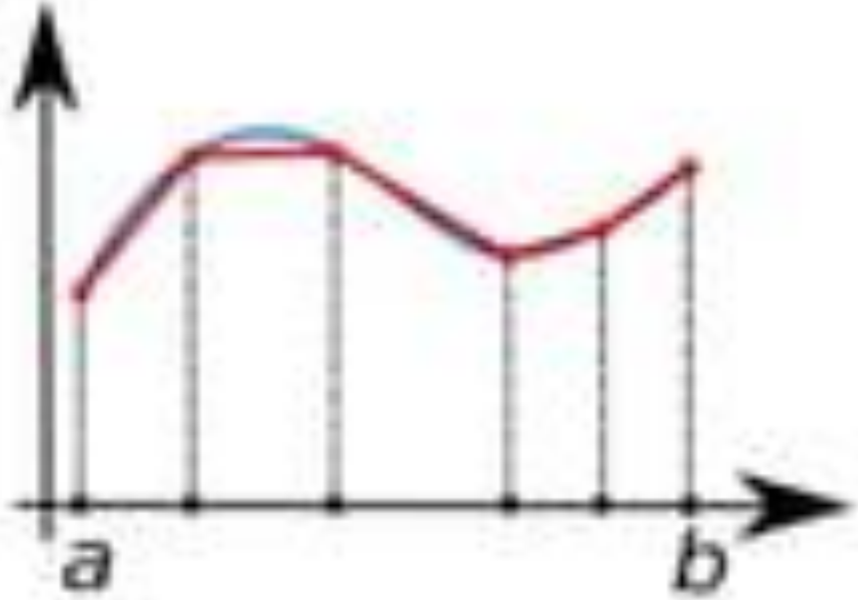
# WHAT IS NUMERICAL INTEGRATION?

- Approximate computation of a definite integral using numerical techniques
- Tabulated at regularly spaced intervals
- Gives the approximate calculation
- Different from analytical integration


NUMERICAL INTEGRATION
TRAPEZOIDAL RULE
SIMPSON'S RULE
GAUSSIAN QUADRATURE

# TRAPEZOIDAL RULE

- Fundamental method of Numerical Integration
- Trapezoids are used for finding the area under curve
- Trapezoids better fits the curve, less error



- Evaluating, $I = \int_a^b f(x)dx$ by Trapezoidal Rule
- Consider an expanded view of a general region
- Area of each trapezoid be,

$$\int_{x_{i-1}}^{x_i} f(x)dx \approx \frac{f_{i-1} + f_i}{2}(\Delta x)$$

And,

$$\int_{x_i}^{x_{i+1}} f(x)dx \approx \frac{f_i + f_{i+1}}{2}(\Delta x)$$

- The integral over two panel is given by,

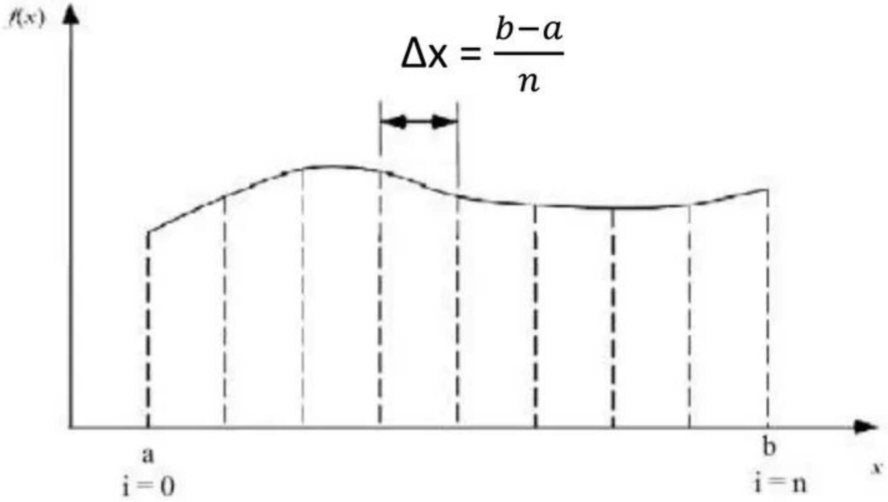
$$\int_{x_{i-1}}^{x_{i+1}} f(x)dx \approx \frac{f_i + f_{i+1}}{2}(\Delta x) + \frac{f_{i-1} + f_i}{2}(\Delta x) = \frac{\Delta x}{2}(f_{i-1} + 2f_i + f_{i+1})$$

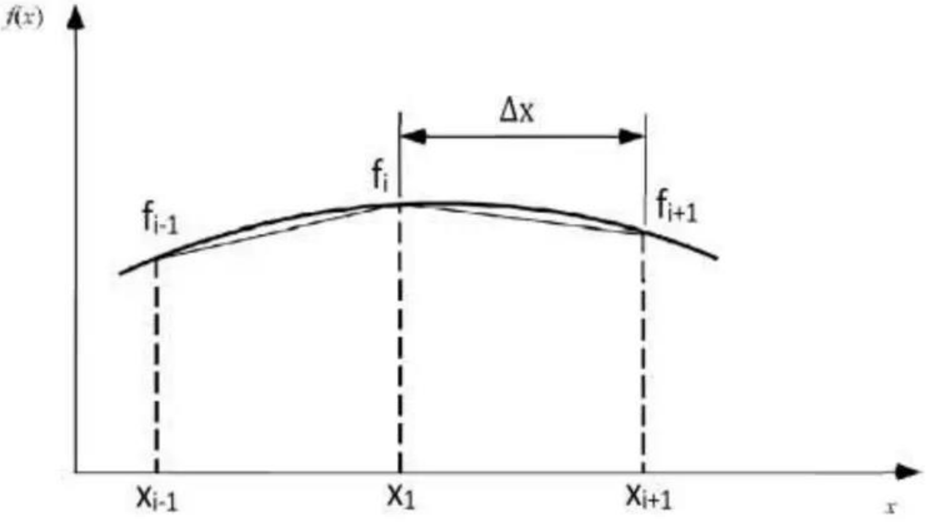
- The Trapezoidal Rule approximation to a integral over the entire

Interval is ,

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\{(f_0 + f_n) + 2(f_1 + f_2 + \cdots + f_{n-1})\}$$

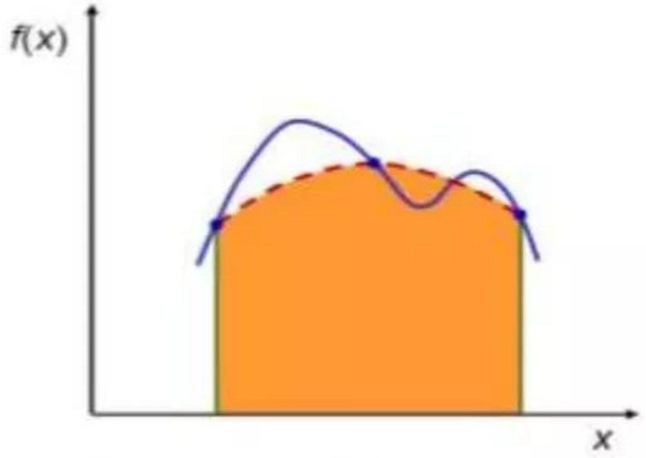
Where $f_o = f(a)\ and\ f_n = f(b)$



# SIMPSON'S 1/3 RULE

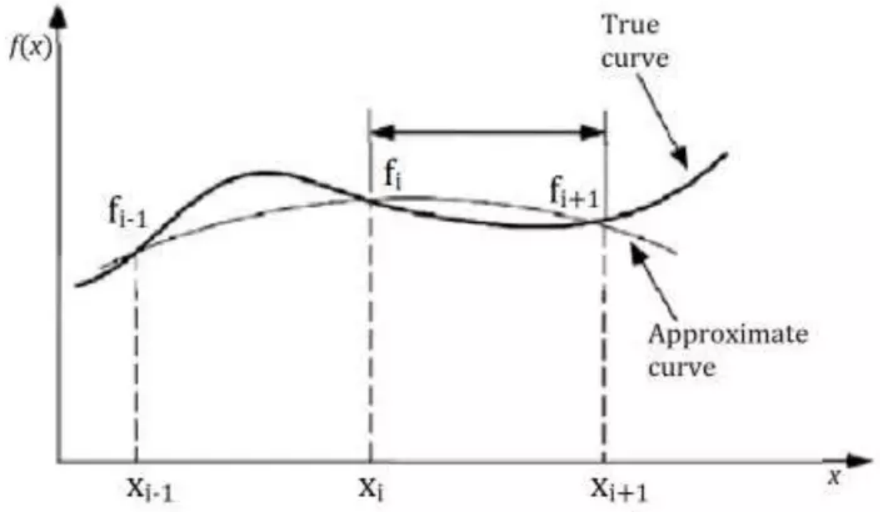
- A further improvement over Trapezoidal rule is Simpson's Rule
- Based on use of parabolic arcs (quadratic function) to approximate the curve instead of the straight lines employed in the trapezoid rule
- Connecting 3 points, the number of interval has to be even




Simpson's 1/3 rule
Quadratic connecting 3 points

- Approximating the area of the panel by the area under the parabola is given by,

$$\int_{x_{i-1}}^{x_{i+1}} f_2(x)dx = \frac{\Delta x}{3}[f(x_{i-1}) + 4f(x_i) + f(x_{i+1})]$$




- The Simpson's rule approximation to the integral over the entire interval is

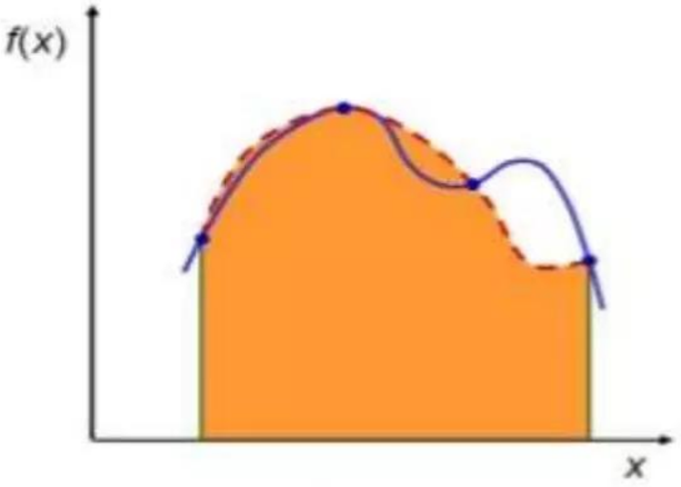
$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 2f_{n-2} + 4f_{n-1} + f_n)$$

Where $f_o = f(a)$ $and$ $f_n = f(b)$

# SIMPSON'S 3/8 RULE

- Consists of taking the area under a cubic equation connecting four points.
- Number of interval must be multiple of three
- The formula for Simpson's 3/8 rule is

$$\int_a^b f(x)dx = \frac{3\Delta x}{8}[(f_0 + f_n) + 2(f_3 + f_6 + \cdots + f_{n-3}) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-1})]$$




Simpson's 3/8 rule
Cubic connecting 4 points

# GAUSSIAN QUADRATURE

- Sampling points may not be equally spaced in practical problems
- Both positions of the sampling points and the weights have been optimized
- Gaussian Quadrature formula can be expressed as,

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} w_i f(x_i)$$

- It produces most accurate approximation

# One point Gaussian Quadrature

- There is only sampling point $(n = 1)$
- $\int_{-1}^{1} f(x)dx = w_1 f(x_1)$
- For one point Gaussian Quadrature the sampling point $x_1 = 0$ and weight $w_1 = 2$
- For two point Gaussian Quadrature the sampling points $x_1 = -1/\sqrt{3}$, $x_2 = 1/\sqrt{3}$ and weight $w_1 = w_2 = 1$

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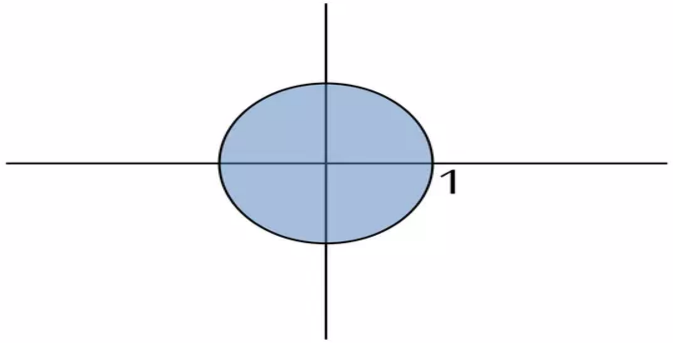
# UNIT-4:COMPLEX VARIABLES

# Open Disks or Neighborhoods

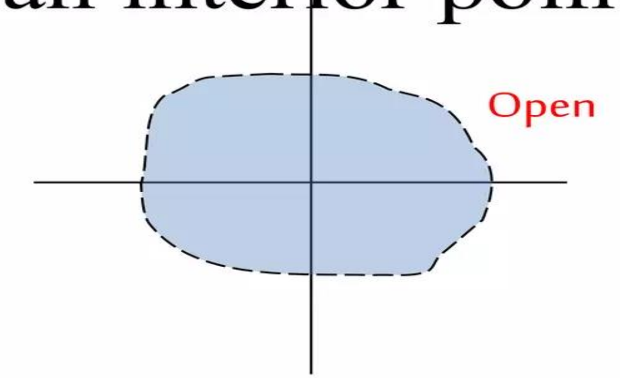
**Definition:** The set of all points z which satisfy the inequality $|z - z_0| < \rho$, where ρ is a positive real number is called an open disc or neighborhood of $z_0$.

**Remark:** The unit disk, i.e., the neighborhood $|z| < 1$, is of particular significance.



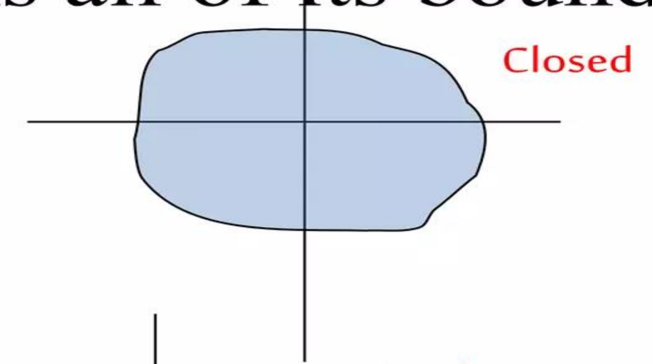
# Open Set:

**Definition**. If every point of a set $S$ is an interior point of $S$, we say that $S$ is an *open set*.



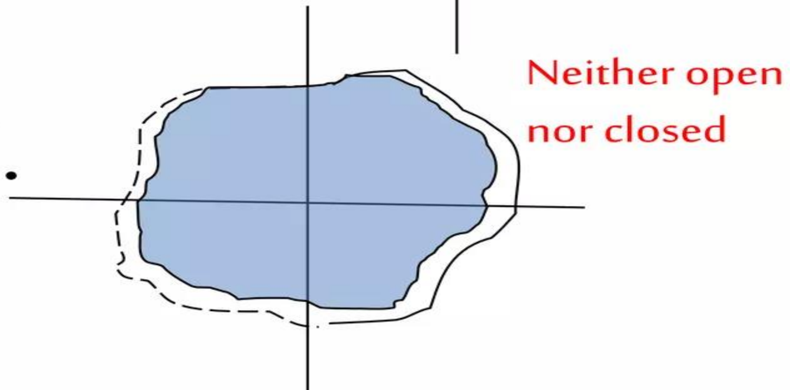

# Closed Set:

**Definition**. If $B(S) \subset S$, i.e., if $S$ contains all of its boundary points, then it is called a *closed set*.




# Neither open nor closed:

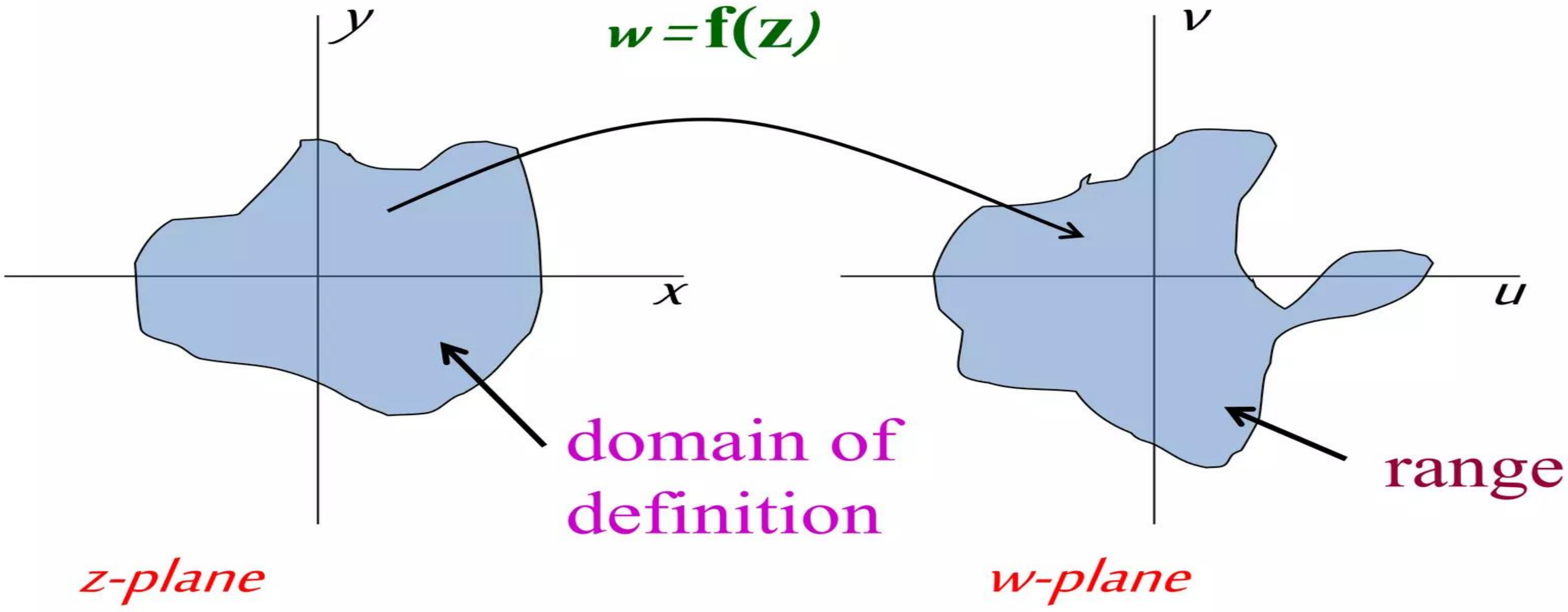
Sets may be neither open nor closed.



# Complex Function:

- **Definition:** Complex function of a complex variable. Let $\Phi \subset C$. A *function f* defined on $\Phi$ is a rule which assigns to each $z \in \Phi$ a complex number *w*. The number *w* is called a *value of f at z* and is denoted by *f(z)*, i.e.,
  $w = f(z)$.
  The set $\Phi$ is called the *domain of definition of f*. Although the domain of definition is often a domain, it need not be.

# Graph of Complex Function:

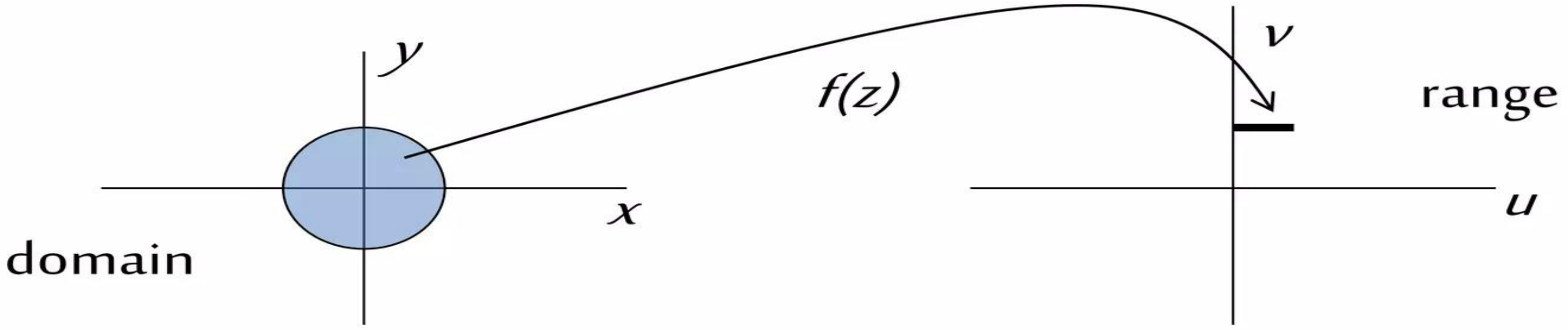


# Example :

➢ Describe the range of the function $f(z) = x^2 + 2i$, defined on (the domain is) the unit disk $|z| \leq 1$.

**Solution :** We have $u(x, y) = x^2$ and $v(x, y) = 2$. Thus as $z$ varies over the closed unit disk, $u$ varies between 0 and 1, and $v$ is constant (=2).

Therefore $w = f(z) = u(x, y) + iv(x, y) = x^2 + 2i$ is a line segment from $w = 2i$ to $w = 1 + 2i$.



# Analytic function

**Definition :** A complex function is said to be analytic on a region if it is complex differentiable at every point.

- The terms holomorphic function , differentiable function, and complex differentiable function are sometimes used interchangeably with "analytic function“
- If a complex function is analytic on a region, it is infinitely differentiable.
- A complex function may fail to be analytic at one or more points through the presence of singularities, or along lines or line segments through

- A necessary condition for a complex function to be analytic is
  Let f(x, y) = u(x, y) + i v(x, y) be a complex function.

  Since $x = (\frac{z+\bar{z}}{2})$ and $y = (\frac{z-\bar{z}}{2})$

  substituting for x and y gives ,

  $$f(z,\bar{z}) = u(x, y) + iv(x, y) \,.$$

A necessary condition for $f(z, \bar{z})$ to be analytic is

$$\partial f / \partial \bar{z} = 0.$$

Therefore a necessary condition for f = u + iv to be analytic is that f depends only on z.

In terms of the of the real and imaginary parts u, v of f

$$\partial u / \partial x = \partial v / \partial y$$

$$\partial u / \partial y = - \partial v / \partial x$$

The above equations are known as the **Cauchy-Riemann** equations.

# Necessary and sufficient conditions for a function to be analytic

- The necessary and sufficient conditions for a function $f = u + iv$ to be analytic are that:
- 1. The four partial derivatives of its real and imaginary parts

$$\partial u / \partial x\ ,\ \partial v / \partial x\ ,$$
$$\partial u / \partial y,\ \partial v / \partial y$$

satisfy the Cauchy-Riemann equations

2. The four partial derivatives of its real and imaginary parts $\partial u / \partial x$ , $\partial v / \partial y$ , $\partial u / \partial y$ , $\partial v / \partial x$ are continuous.

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**Faculty Name : Dr. K. PAVANI**

**Program Name : B.Tech- ECE, EEE**

**Name of the Course : NUMERICAL METHODS AND COMPLEX VARIABLES PPT**

**Course Code : NMCV** (23MA301)

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# UNIT-5: COMPLEX INTEGRATION

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# Cauchy's theorem

Let f(z) be an analytic function every where within and or a closed curve c then

$$\int_{C} f\,(z) = 0\,,$$

where, c is traversed in positive (anti clock wise) direction

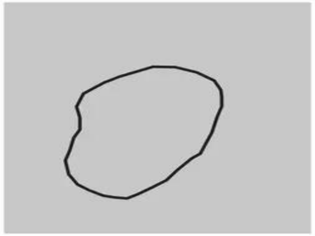


Fig (a)

The shaded grey area is the region and a typical closed curve is shown inside the region

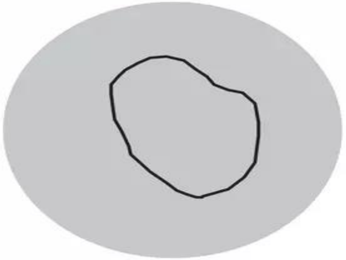


Fig (b)

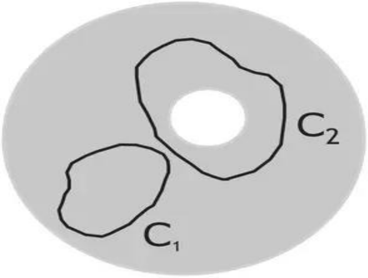



Fig (c)

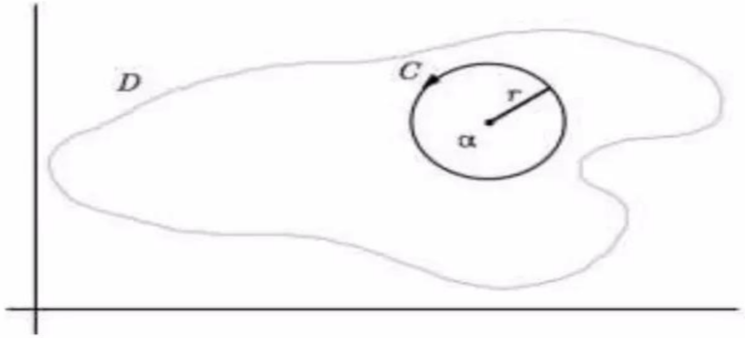
The region contains a hole (the white area inside). The shaded region between the two circles is **not** simply-connected; curve $C_1$ can shrink to a point but curve $C_2$ cannot shrink to a point without leaving the region, due to the hole inside it.

# Cauchy’s Integral Formula

Let f(z) be an analytic function everywhere within and closed curve c

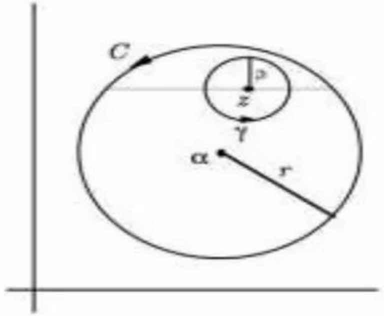
If z=a lies within c then,

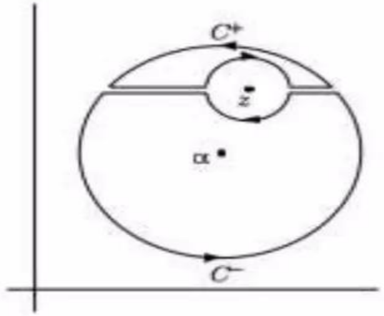
$$f(z_0) = \frac{1}{2\pi i} \oint_c \frac{f(z)}{z - z_0} dz$$



# Proof:

Suppose that γ is a circle of radius ρ and centered at Zo, followed in the counterclockwise direction. Suppose further that ρ is sufficiently small so that γ lies inside C. Note that a horizontal line through the point Zo intersects C at two points and interests γ at two points and gives rise to two lines inside C and outside γ




$$\oint_{C+} \frac{f(z)}{z - z_0} dz = 0.$$

Similarly for $C^-$ the closed path composited by the lower loop, we have

$$\oint_{C-} \frac{f(z)}{z - z_0} dz = 0.$$

It is easily seen that

$$\oint_C \frac{f(z)}{z - z_0} dz - \oint_\gamma \frac{f(z)}{z - z_0} dz = \oint_{C+} \frac{f(z)}{z - z_0} dz + \oint_{C-} \frac{f(z)}{z - z_0} dz = 0.$$

So that

$$\oint_C \frac{f(z)}{z - z_0} dz = \oint_\gamma \frac{f(z)}{z - z_0} dz.$$

We can write

$$\oint_C \frac{f(z)}{z - z_0} dz = f(z_0) \oint_\gamma \frac{1}{z - z_0} dz + \oint_\gamma \frac{f(z) - f(z_0)}{z - z_0} dz. \quad (1)$$

The first integral on the right hand side of (1) is studied in a similar way as in Example C3, and

The first integral on the right hand side of (1) is studied in a similar way as in Example C3, and we have

$$\oint_\gamma \frac{1}{z-z_0} dz = 2\pi i. \tag{2}$$

Recall that $f$ is a continuous function on $z_0$ so for each $\epsilon > 0$ there is some $\delta > 0$ so that for each $|z - z_0| < \delta$, we get $|f(z) - f(z_0)| < \epsilon$. If we choose $\rho$ so that $\rho < \delta$ we get

$$\left| \frac{f(z) - f(z_0)}{z - z_0} \right| < \frac{\epsilon}{\rho}$$

for every $z$ inside the circle $\gamma$. Therefore by Proposition C7, we have

$$\left| \int_\gamma \frac{f(z) - f(z_0)}{z - z_0} dz \right| \leq \oint_\gamma \left| \frac{f(z) - f(z_0)}{z - z_0} \right| dz < \frac{\epsilon}{\rho} 2\pi\rho = 2\pi\epsilon.$$

So by $\epsilon \to 0$ for (1) and by (2), we have

$$\left| \oint_C \frac{f(z)}{z - z_0} dz - f(z_0) 2\pi i \right| \leq 2\pi\epsilon \to 0.$$

□

# Example :

$$\int_C \frac{\cos hz\, dz}{z^2 + z}$$

(b) By Cauchy Integral Theorem,

$$\int_C \frac{\cosh z}{z^2+z} dz = \int_{|z|=r} \frac{\cosh z}{z^2+z} dz + \int_{|z+1|=r} \frac{\cosh z}{z^2+z} dz$$

for $r = 1/2$. By Cauchy Integral Formula,

$$\int_{|z|=r} \frac{\cosh z}{z^2+z} dz = 2\pi i \left. \frac{\cosh(z)}{z+1} \right|_{z=0} = 2\pi i$$

and

$$\int_{|z+1|=r} \frac{\cosh z}{z^2+z} dz = 2\pi i \left. \frac{\cosh z}{z} \right|_{z=-1} = -2\pi i \cosh(-1).$$

Hence

$$\int_C \frac{\cosh z}{z^2+z} dz = 2\pi i(1 - \cosh(-1)).$$