DS4102PC : Machine Learning

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References

TextBooks:

1. TomM.Mitchell,MachineLearning,IndiaEdition2013,McGraw HillEducation.

Reference Books:

1. TrevorHastie,RobertTibshirani,JeromeFriedman,hTheElements of Statistical Learning, 2nd edition, springer seriesin statistics.
2. EthemAlpaydın,Introductiontomachinelearning,secondedition,MITpress.

Prerequisites

For Machine Learning Course were commend that students meet the following prerequisites:

- Basic programmingskills(in Python)
- Algorithm design
- Basics of probability & statistics

Content

Unit–1	Introduction, Concept Learning, Decision Tree
Unit–2	LearningArtificialNeuralNetworks-1,ArtificialNeural Networks-2, Evaluating Hypothesis,
Unit–3	BayesianLearning,Computationallearningtheory, Instance Based Learning,
Unit–4	GeneticAlgorithms,LearningSetsofRules, Reinforcement Learning
Unit–5	AnalyticalLearning-1,AnalyticalLearning-2,CombiningInductiveand Analytical Learning

UNIT-1

Machine Learning Introduction

Ever since computers were invented, we have wondered whether they might be made to learn.

If we could understand how to program them to learn-to improve automatically with experience-the impact would be dramatic.

Imagine computers learning from medical records which treatments are most effective for new diseases

Houseslearningfromexperiencetooptimizeenergycostsbasedontheparticularusage patterns of their occupants.

Personal software assistants learning the evolving interests of their users in order to highlight especially relevant stories from the online morning newspaper

Example of Successful Learning

- Learning to recognize spoken words
- Learning to drive an autonomous vehicle
- Learning to classify new astronomical structures
- Learning to play world-class backgammon

WhyisMachineLearningImportant?

- Sometaskscannotbedefinedwell,exceptbyexamples(e.g., recognizing people).
- Relationshipsandcorrelationscanbehiddenwithinlargeamountsof data.Machine Learning/DataMining maybe able to find these relationships.
- Humandesignersoftenproducemachinesthatdonotworkaswellas desired in the environments in which they are used.
- Theamountofknowledgeavailableaboutcertaintasksmightbetoolarge for explicit encoding by humans (e.g., medical diagnostic).
- Environmentschangeovertime.
- Newknowledgeabouttasksisconstantlybeingdiscoveredbyhumans.It may be difficult to continuously re-designsystems “by hand”.

AreasofInfluenceforMachineLearning

- ***Statistics:***Howbesttousesamplesdrawnfromunknownprobabilitydistributionsto help decidefrom which distributionsomenewsampleisdrawn?
- ***Brain Models***: Non-linear elements with weighted inputs (Artificial NeuralNetworks)havebeensuggestedassimplemodelsofbiologicalneurons.
- ***AdaptiveControlTheory***:Howtodealwithcontrollingaprocesshavingunknown parameters that must be estimated during operation?
- ***Psychology***:Howtomodelhumanperformanceonvariouslearningtasks?
- ***ArtificialIntelligence***:Howtowritealgorithmstoacquiretheknowledgehumansare able to acquire, atleast, as well as humans?
- ***EvolutionaryModels***:Howtomodelcertainaspectsofbiologicalevolutionto improve the performance of computer programs?

MachineLearning:ADefinition

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured byP,improveswithexperienceE.

Why“Learn”?

Learningissusedwhen:

- Humanexpertise doesnot exist(navigatingonMars)
- Humansare unabletoexplaintheirexpertise(speech recognition)
- Solution changes intime(routingona computernetwork)
- Solutionneedstobe adapted toparticular cases(userbiometrics)

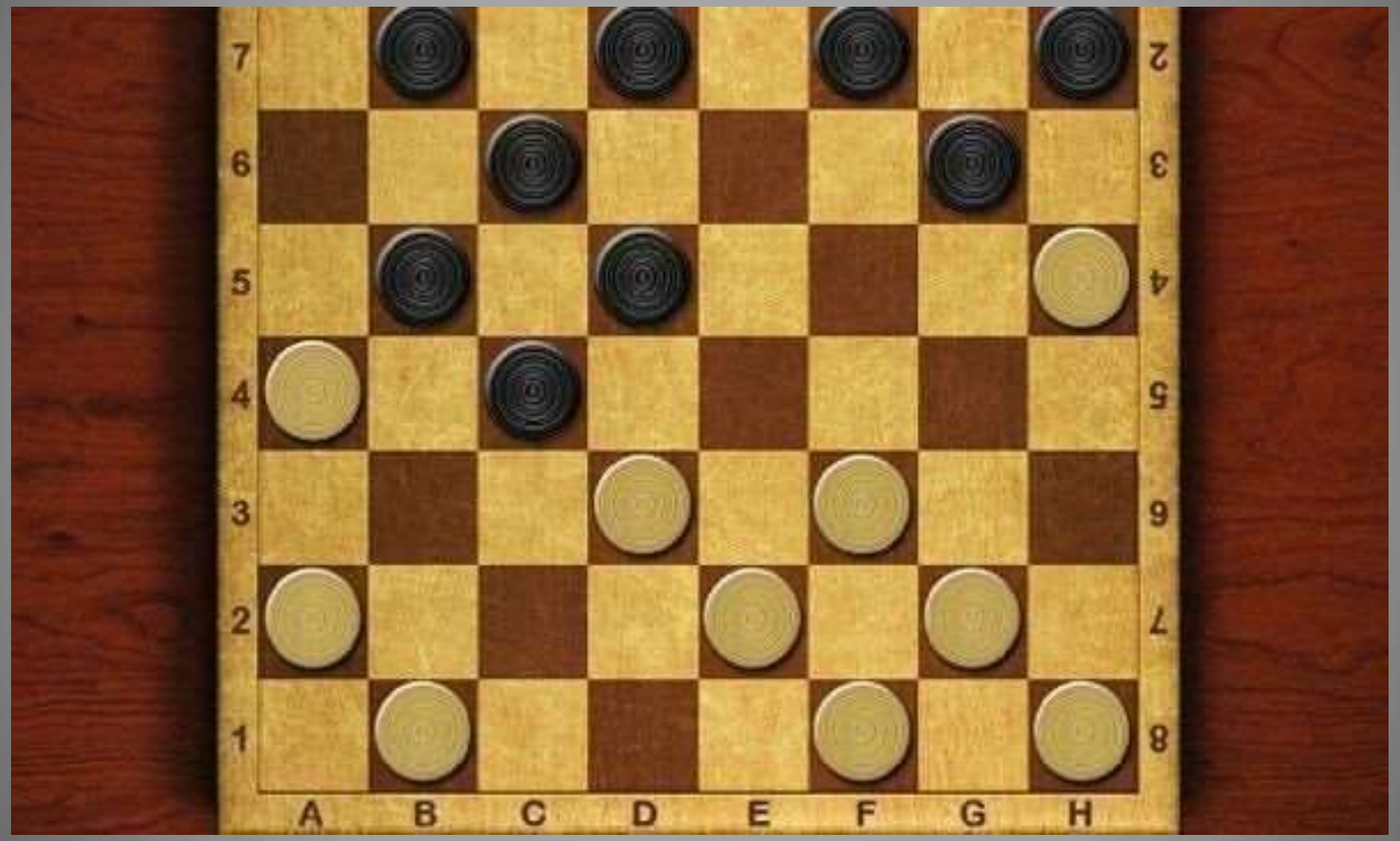
Well-PosedLearningProblem

Definition: A computer program is said to learn from experience E with respect to someclassoftasksT andperformancemeasureP,ifitsperformance attasksinT,as measured by P, improves with experience E.

Tohave awell-definedlearning problem,three featuresneedstobeidentified:

1. The classoftasks
2. Themeasureofperformancetobe improved
3. The sourceof experience

CheckersGame



GameBasics

- Checkers is played by two players. Each player begins the game with 12 coloreddiscs.(Onesetofpiecesisblackandtheotherred.)Eachplayerplaceshis or her pieces on the 12 dark squares closest to him or her. Black moves first. Players then alternate moves.
- Theboard consistsof64squares,alternatingbetween32darkand 32lightsquares.
- Itispositionedsothateachplayerhasalightsquareontherightsidecornerclosest to him or her.
- A player wins the game when the opponent cannot make a move. In most cases, thisisbecausealloftheopponent'spieceshavebeencaptured,butitcouldalsobe because all of his pieces are blocked in.

RulesoftheGame

- Moves are allowed only on the dark squares, so pieces always move diagonally. Single pieces are always limited to forward moves (toward the opponent).
- Apiecemakinganon-capturingmove(notinvolvingajump)maymoveonlyone square.
- Apiecemakingacapturingmove(ajump)leapsoveroneoftheopponent'spieces, landinginastraightdiagonallineontheotherside.Onlyonepiecemaybecaptured in a single jump; however, multiple jumps are allowed during a single turn.
- When apiece iscaptured, itisremoved fromtheboard.
- Ifaplayerisabletomake acapture,thereisnooption;the jumpmustbemade.
- If morethanone captureis available,the playerisfreetochoosewhichever heor she prefers.

RulesoftheGameCont.

- Whenapiecereachesthefurthestrowfromtheplayerwhocontrolsthatpiece,itis crownedandbecomesaking.Oneofthepieceswhichhadbeencapturedisplaced on top of the king so that it is twice as high as a single piece.
- Kingsarelimitedtomovingdiagonallybutmaymovebothforwardandbackward. (Remember that single pieces, i.e. non-kings, are always limited to forward moves.)
- Kings may combine jumps in several directions, forward and backward, on the same turn. Single pieces may shift direction diagonally during a multiple captureturn, but must always jump forward (toward the opponent).

Well-DefinedLearningProblem

Acheckerslearning problem:

- TaskT:playingcheckers
- Performancemeasure P:percent ofgames wonagainstopponents
- Training experienceE:playing practicegames against itself

Ahandwritingrecognition learning problem:

- TaskT:recognizingandclassifyinghandwrittenwordswithinimages
- PerformancemeasureP:percentofwordscorrectlyclassified
- TrainingexperienceE:adatabaseofhandwrittenwordswith given classifications

Arobotdriving learningproblem:

- TaskT:drivingonpublicfour-lanehighwaysusingvisionsensors
- Performance measureP:averagedistancetravelledbeforeanerror(asjudgedby human overseer)
- TrainingexperienceE:asequence ofimages andsteeringcommands recorded While observingahumandriver

DesigningaLearningSystem

1. ChoosingtheTrainingExperience
2. ChoosingtheTargetFunction
3. ChoosingaRepresentationfortheTarget Function
4. ChoosingaFunctionApproximationAlgorithm
    1. Estimating trainingvalues
    2. Adjusting theweights
5. TheFinalDesign

1.ChoosingtheTrainingExperience

- Thefirstdesignchoiceistochoosethetypeoftrainingexperiencefromwhich the system will learn.
- Thetypeoftrainingexperienceavailablecanhaveasignificantimpact on success or failure of the learner.

Therearethreeattributeswhichimpactonsuccessorfailureofthe learner

1. Whetherthetrainingexperienceprovidesdirectorindirectfeedbackregarding the choices made by the performance system.
2. Thedegreetowhichthelearnercontrols thesequence oftrainingexamples
3. Howwellitrepresentsthedistributionofexamplesoverwhichthefinal system performance mustbe measured.

1. Whetherthetrainingexperienceprovidesdirectorindirectfeedbackregarding the choices madeby the performance system ■

Forexample,incheckersgame:

- Inlearningto playcheckers,thesystemmightlearnfromdirecttraining examples consisting ofindividual ***Checkersboardstates andthecorrectmoveforeach***.

- Indirect trainingexamplesconsistingofthe ***movesequencesand final outcomesofvariousgames played***.

- Theinformationaboutthecorrectnessofspecificmovesearlyinthegamemustbeinferredindirectly from the fact that the game was eventually won or lost.

- Herethelearnerfacesanadditionalproblemof ***creditassignment***,ordeterminingthedegreetowhicheach move in the sequence deserves creditor blame for the finaloutcome.

- Creditassignment canbeaparticularlydifficultproblembecausethegamecanbelostevenwhenearly moves are optimal,if the seare followed later by poor moves.

- Hence,learningfromdirect trainingfeedbackistypically easier thanlearningfromindirectfeedback.

2. Asecond important attribute of the training experience *is the degree to which the learner controls the sequence of training examples*

Forexample,incheckersgame:

- The learner might depends on the teacher to select informative board states and to provide the correct move foreach.
- Alternatively,thelearnermightitselfproposeboardstatesthatitfindsparticularlyconfusingandaskthe teacher for the correct move.
- The learner may have complete control over both the board states and (indirect) training classifications, as it does when it learns by playing against itself with no teacher present.
- Noticeinthislastcasethelearnermaychoosebetweenexperimentingwithnovelboardstatesthatithasnotyet considered, or honing its skill by playing minor variations of lines of play it currently finds mostpromising.

3. A third attribute of the training experience is how well it represents *the distributionofexamples*overwhichthefinalsystemperformancemustbe measured ■

Learning ismostreliablewhenthe trainingexamplesfollowadistribution similartothatoffuture test examples.

Forexample,incheckersgame:

- In checkers learning scenario, the performance metric P is the percent of games the system wins in the worldtournament.

- If its training experience E consists only of games played against itself, there is an danger that this training experiencemight not be fully representative of the distribution ofsituations overwhich it will later be tested. Forexample,thelearnermightneverencountercertaincrucialboardstatesthatareverylikelytobeplayedby the human checkers champion.

- It is necessary to learn from a distribution of examples that is somewhat different from those on which the finalsystemwillbeevaluated.Suchsituationsareproblematicbecausemasteryofonedistributionofexamples will not necessary lead to strong performance over some other distribution.

3. Choosing the Target Function

The next design choice is to determine exactly what type of knowledge will be learned and how this will be used by the performance program.

- Let's begin with a checkers-playing program that can generate the legal moves from any boardstate.
- The program needs only to learn how to choose the best move from among these legalmoves.Thislearningtaskisrepresentativeofalargeclassoftasksforwhich the legal moves that define some large search space are known a priori, but for which the best search strategy is not known.

Given this setting where we must learn to choose among the legal moves, the most obviouschoiceforthetypeofinformationtobelearnedisaprogram,orfunction,that chooses the best move for any given board state.

1. Let ***ChooseMove***bethetarget functionand thenotationis

$$ChooseMove: B \longrightarrow M$$

Which indicate that this function accepts as input any board from the set of legal boardstates Band produces as output somemove from the set of legal moves M.

ChooseMove is an choice for the target function in checkers example, but this function will turn out to be very difficult to learn given the kind of indirect training experience available to our system

2. Analternative targetfunction is an***evaluationfunction*** thatassignsa ***numericalscore*** to any given boardstate

LetthetargetfunctionVandthenotation

$$V : B \rightarrow R$$

Whichdenotethat Vmaps any legalboardstate fromthesetBto somerealvalue

We intend for this target function V to assign higher scores to better board states. If thesystemcansuccessfullylearnsuchatargetfunctionV,itcaneasilyuseittoselect the best move from any current board position.

Let usdefinethe targetvalueV(b)foranarbitraryboardstatebinB,asfollows:

1. Ifbisa finalboardstatethat is won,thenV(b)=100
2. Ifbisa finalboard state that islost,thenV(b)=-100
3. Ifb is a finalboard statethat is drawn,thenV(b)=0
4. Ifb is a nota finalstatein the game,thenV(b)=V(b'),

Whereb'isthebestfinalboardstatethatcanbeachievedstartingfromband playing optimally until the end of the game

3. ChoosingaRepresentationforthe TargetFunction

Letuschooseasimplerepresentation-foranygivenboardstate,thefunctionwill be calculated as a linear combination of the following board features:

XL: the number of black pieces on the
boardx2: the number of red pieces on the
boardx3:thenumberofblackkingsonthe
boardx4: the number of red kings on the
board
X5:thenumberofblackpiecesthreatenedbyred(i.e.,whichcan be captured on red's next turn)
x6: the number ofred pieces threatened byblack

$$\hat{V}(b) = w_0 + w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6$$

Where,

- w_0 through w_6 are numerical coefficients, or weights, to be chosen by the learning algorithm.

- Learned values for the weights w_1 through w_6 will determine the relative Importance of the various board features in determining the value of the board
  - The weight w_0 will provide an additive constant to the board value

Partialdesignofacheckerslearningprogram:

- TaskT:playingcheckers
- PerformancemeasureP:percentofgameswonintheworldtournament
- TrainingexperienceE:gamesplayedagainstitself
- Targetfunction:V:Board $\longrightarrow$ R
- Targetfunctionrepresentation

$$\hat{V}(b) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 + w_5 x_5 + w_6 x_6$$

The first three items above correspond to the specification of the learning task, whereasthefinaltwoitemsconstitutedesignchoicesfortheimplementationof the learning program.

4. ChoosingaFunctionApproximation Algorithm

- Inordertolearnthetargetfunctionwerequireasetoftrainingexamples,each describing a specific board state band the training value $V_{train}(b)$ for b.

- Eachtraining exampleis anorderedpairoftheform$(b, V_{train}(b))$.

- For instance, the following training example describes a board state b in whichblackhaswonthegame(note$x_2=0$indicatesthatredhasnoremaining pieces) and for which the target function value$V_{train}(b)$ is therefore +100.

$$((x_1=3, x_2=0, x_3=1, x_4=0, x_5=0, x_6=0), +100)$$

FunctionApproximationProcedure

1. Derivetrainingexamplesfromtheindirecttrainingexperienceavailableto the learner
2. Adjuststheweights$w_i$tobestfitthesetraining examples

Where,
$\hat{V}$ is the learner's current approximation to V
Successor(b) denotes the next board state following for which it is again the program's turn to move

Rule for estimating training values

$$V_{train}(b) \leftarrow \hat{V}(Successor(b))$$

Specifythe learning algorithmforchoosingthe weights $w_i$tobest fit theset of training examples $\{(b, V_{train}(b))\}$

Afirststepistodefinewhat wemeanbythebestfit tothetrainingdata.

- Onecommonapproach istodefinethebest hypothesis,orsetofweights,as that which minimizes the squared error E between the training values and the values predicted by the hypothesis.

$$E \equiv \sum_{\langle b, V_{train}(b)\rangle \in \text{training examples}} (V_{train}(b) - \hat{V}(b))^2$$

- Severalalgorithmsareknownforfindingweightsofalinearfunction that minimizeE.

Onesuchalgorithmiscalledthe***leastmeansquares***,or***LMStrainingrule***.Foreach observed training example it adjusts the weights asmall amount in the direction that reduces the error on this training example

LMS weightupdate rule:-For each training example

(b,V_{train}(b))Usethecurrentweightstocalculate$\hat{v}$(b)

Foreachweightw_i,updateit as

$$w_i \leftarrow w_i + \eta(V_{train}(b) - \hat{v}(b))\hat{x}_i$$

hereηisasmallconstant(e.g.,0.1)thatmoderatesthesizeoftheweight

update.Working of weight update rule

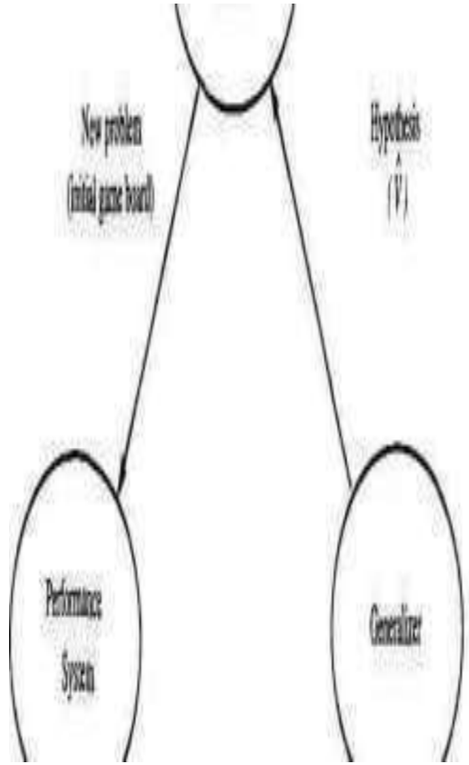
- Whenthe error (Vtrain(b)-V(b))isẑero,no weightsarechanged.
- When (Vtrain(b)-V(b))îspositive(i.e.,whenV(b)istoo[illegible]ofV(b)ˆ, reducing the error.

Ifthevalueofsomefeature$x_i$iszero,thenitsweightisnotalteredregardlessof the

- error, so that the only weights updated are those whose features actually occur on the training example board.

Thefinaldesignofcheckerslearningsystemcanbedescribedbyfourdistinct program modules that represent the central components in many learning systems

5. TheFinalDesign

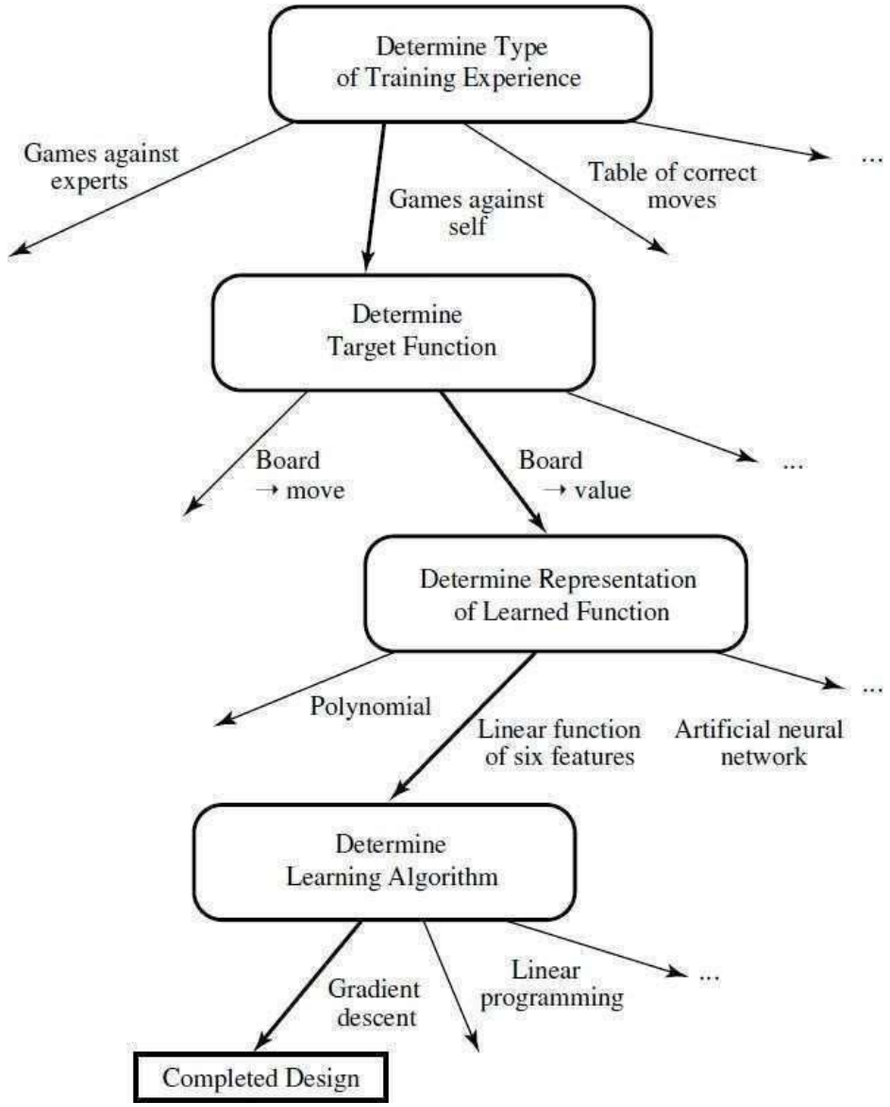

New problem
(initial game board)
Hypothesis
(V)
Performance
System
Generalizer

1. The Performance System is the module that must solve the given performance taskby using the learned target function(s).
Ittakesaninstanceofanewproblem(newgame)asinputandproducesatraceofits solution (game history) as output.
In checkers game, the strategy used by the Performance System to select its next move at each step is determined by the learned $\hat{v}$ evaluation function. Therefore, weexpect its performance to improve as this evaluation function becomes increasingly accurate.

2. The Critic takes as input the history or trace of the game and produces as output a setoftrainingexamplesofthetargetfunction.Asshowninthediagram,eachtraining example in this case corresponds to some game state in the trace, along with an estimateV_{train} of the target function value for this example.

3. TheGeneralizertakesasinputthetrainingexamplesandproducesan output hypothes is that is its estimate of the target function.
It generalizes from the specific training examples, hypothesizing a general functionthatcoverstheseexamplesandothercasesbeyondthetrainingexamples. Inour example, the Generalizer corresponds to the LMS algorithm, and the output hypothesis is the function$\hat{V}$described bythe learned weightsw_0, .. . , W6.

4. TheExperimentGeneratortakesasinputthecurrenthypothesisandoutputsanew problem(i.e.,initial board state) forthe Performance Systemto explore.Its roleisto pick new practice problems that will maximize the learning rate of the over all system.
Inourexample,theExperimentGeneratoralwaysproposesthesameinitialgame board to begin a new game.


Determine Type of Training Experience
Games against experts
Games against self
Table of correct moves
...
Determine Target Function
Board → move
Board → value
...
Determine Representation of Learned Function
Polynomial
Linear function of six features
Artificial neural network
...
Determine Learning Algorithm
Gradient descent
Linear programming
...
Completed Design

PerspectivesofMachineLearning

Perspectiveofmachinelearninginvolvessearchingverylarge space of possible hypothesis to determine one that

Best fits the observed data and any prior knowledge heldby learner.

Issuesin MachineLearning

- What algorithms exist for learning general target functions from specific training examples? In what settings will particular algorithms converge to the desired function, given sufficient training data?Which algorithms perform best for which types of problems and representations?

- Howmuchtrainingdataissufficient?Whatgeneralboundscanbefound torelate theconfidenceinlearnedhypothesestotheamountoftrainingexperienceandthe character of the learner's hypothesis space?

- When and how can prior knowledge held by the learner guide the process of generalizingfromexamples?Canpriorknowledgebehelpfulevenwhenitisonly approximately correct?

- What is thebest strategyforchoosing a useful next training experience, and how does the choice of this strategy alter the complexity of the learning problem?

- What is the best way to reduce the learning task to one or more function approximation problems? Put another way, what specific functions should the system attempt tolearn? Can this process itself be automated?

- Howcanthelearnerautomaticallyalteritsrepresentationtoimproveitsabilityto represent and learn the target function?

ConceptLearning

- Learning involves acquiring general concepts from specific training examples. Example: People continually learn general concepts or categories such as "bird,""car,""situationsinwhichIshouldstudymoreinordertopasstheexam,"etc.
- Each such concept can be viewed as describing some subset of objects or events defined over a larger set
- Alternatively, each concept can be though to fasaBoolean-valued function defined over this larger set. (Example: A function defined over all animals, whose value is true for birds and false for other animals).

Conceptlearning-InferringaBoolean-valuedfunctionfromtrainingexamplesof its input andoutput

AConceptLearningTask

Consider the example task of learning the target concept "DaysonwhichmyfriendAldoenjoyshisfavoritewatersport."

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Table- byasetofattributes
Describesasetof example days, each represented

Whathypothes isrepresentation is provided tothelearner?

Let’sconsiderasimplerepresentationinwhicheachhypothesisconsistsofaconjunction of constraints on the instance attributes.

Leteachhypothesisbeavectorofsixconstraints,specifyingthevaluesofthesix attributes Sky, AirTemp, Humidity,Wind,Water, and Forecast.

Foreachattribute, thehypothesiswill either

- Indicatebya"?'thatanyvalueisacceptableforthis attribute,
- Specifyasingle required value(e.g.,Warm)forthe attribute,or
- Indicatebya"Φ"that novalueis acceptable

The hypothesis that PERSON enjoys his favo rite sport only on cold days with highhumidity (independent of the values of the other attributes) is represented by theexpression

(?, Cold, High,?,?,?)

Themostgeneralhypothesis-thateverydayisapositiveexample-isrepresentedby
(?,?,?,?,?,?)

The most specific possible hypothesis-that day is a positive example-is norepresented by

Notation

Theset ofitemsover whichtheconcept is definediscalledthesetof ***instances***,whichwedenotebyX.

Example:Xisthesetofall possibledays,eachrepresentedbytheattributes:Sky, AirTemp, Humidity,Wind,Water,and Forecast

Theconceptorfunctiontobelearnediscalledthe***targetconcept***,whichwedenote by c.

ccan be anyBooleanvalued function defined overtheinstancesXc:X{O, 1}

Example:The target conceptcorrespondsto thevalueoftheattribute***EnjoySport*** (i.e.,c(x)=1if***EnjoySport***=Yes,andc(x)=0if***EnjoySport***=No).

- Instancesforwhichc(x)=1arecalledpositiveexamples,ormembersofthe target concept.
- Instancesforwhichc(x)=0arecallednegativeexamples,ornon-membersofthe target concept.
- Theorderedpair(x,c(x))todescribethetrainingexampleconsistingofthe instancex and its target conceptvalue c(x).
- ***D***todenotethe set ofavailabletraining examples
- The symbol ***H*** to denote the set of all possible hypotheses that the learner mayconsiderregardingtheidentityofthetargetconcept.Eachhypothesis ***h*** in ***H***represents a Boolean-valued function defined over X

$$h: X \rightarrow \{0,1\}$$

- Thegoal ofthelearneristofindahypothesissuchthath(x)=c(x)forallxin ***X***.

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

- **Given:**
  - Instances X: Possible days, each described by the attributes
    - *Sky* (with possible values *Sunny*, *Cloudy*, and *Rainy*),
    - *AirTemp* (with values *Warm* and *Cold*),
    - *Humidity* (with values *Normal* and *High*),
    - *Wind* (with values *Strong* and *Weak*),
    - *Water* (with values *Warm* and *Cool*), and
    - *Forecast* (with values *Same* and *Change*).
  - Hypotheses H: Each hypothesis is described by a conjunction of constraints on the attributes *Sky*, *AirTemp*, *Humidity*, *Wind*, *Water*, and *Forecast*. The constraints may be "?" (any value is acceptable), "$\emptyset$" (no value is acceptable), or a specific value.
  - Target concept c: $EnjoySport : X \rightarrow \{0, 1\}$
  - Training examples D: Positive and negative examples of the target function (see Table 2.1).
- **Determine:**
  - A hypothesis h in H such that $h(x) = c(x)$ for all x in X.

TABLE The *EnjoySport* concept learning task.

Any hypothesis foundto approximate the target function well over a sufficiently large set of training examples will also approximate the target function well overother unobserved examples.

TheInductiveLearningHypothesis

ConceptlearningasSearch

- Conceptlearningcanbeviewedasthetaskofsearchingthroughalargespace of hypotheses implicitly defined bythe hypothesis representation.
- Thegoalofthissearchistofindthehypothesisthatbestfitsthetraining examples.

Example,theinstances***X***andhypotheses***H***inthe***EnjoySport***learningtask. The attribute ***Sky*** has three possible values, and ***AirTemp,Humidity ,Wind,WaterForecast***eachhavetwopossiblevalues,theinstancespaceX contains exactly

- 3.2.2.2.2.2=96 Distinctinstances
- 5.4.4.4.4.4=5120syntacticallydistinct hypotheses withinH.

Everyhypothesiscontainingoneormore"Φ"symbolsrepresentstheemptysetof instances; that is, it classifies every instance as ***negative.***

- 1+(4.3.3.3.3.3)=973.Semantically distinct hypotheses

General-to-SpecificOrderingofHypotheses

- Considerthetwohypotheses

  h_1= (Sunny,?,?,Strong,?,?)

  h_2= (Sunny,?,?,?,?,?)

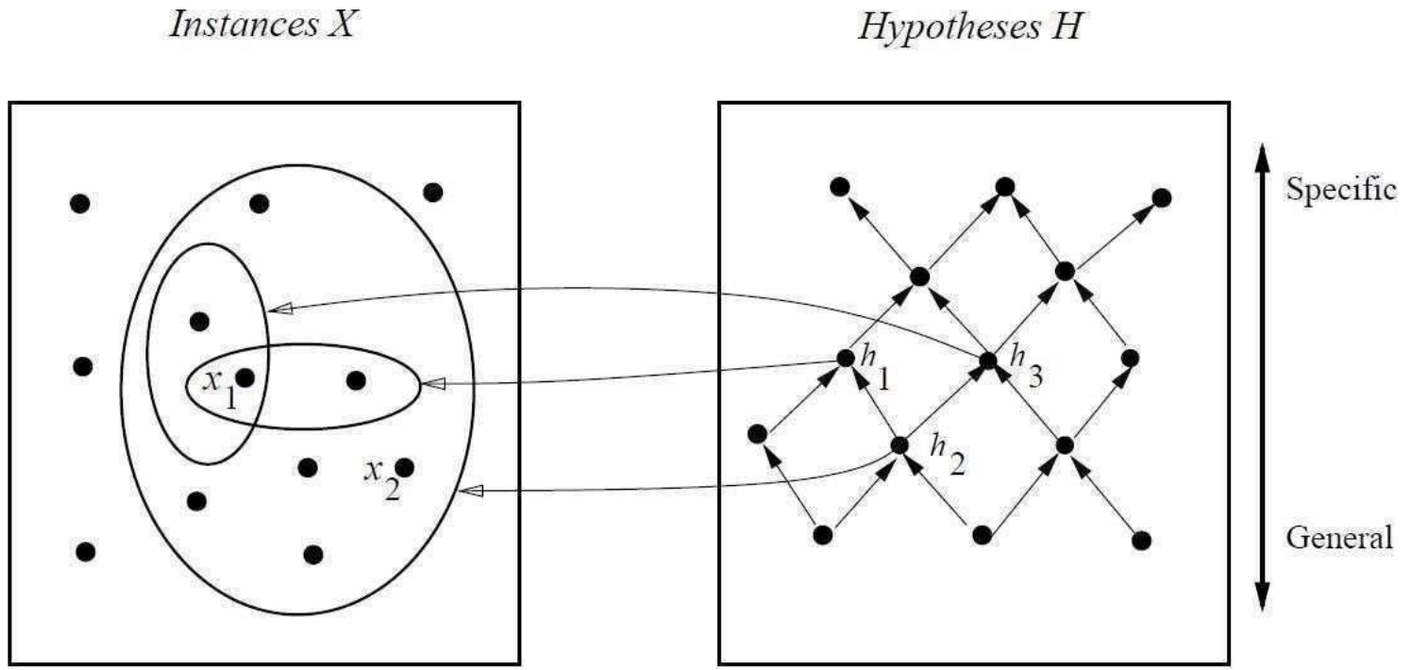
- Considerthesetsofinstancesthatareclassifiedpositivebyh$_1$andbyh$_2$.
- h$_2$imposesfewerconstraintsontheinstance, itclassifies more instancesaspositive. So, any instance classified positive by h$_1$ will also be classified positiveby h$_2$.Therefore, h$_2$is more generalthan h$_1$.

General-to-SpecificOrderingofHypotheses

- Givenhypotheses $h_j$andh_k,$h_j$ismore-general-thanor-equaldo$h_k$ifandonlyifanyinstancethatsatisfies$h_k$alsosatisfies$\boldsymbol{h}_i$

Definition:Let $h_j$and$h_k$beBoolean-valuedfunctionsdefinedoverX.Then$h_j$ismoregeneral-than-or-equal-toh_k(written$h_j \geq h_k$)ifand only if

$$(\forall x \in X)[(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$



x_1 = <Sunny, Warm, High, Strong, Cool, Same>
x_2 = <Sunny, Warm, High, Light, Warm, Same>

h_1 = <Sunny, ?, ?, Strong, ?, ?>
h_2 = <Sunny, ?, ?, ?, ?, ?>
h_3 = <Sunny, ?, ?, ?, Cool, ?>

- Inthe figure, thebox on the leftrepresentsthesetXofallinstances, the box on the right theset H ofall hypotheses.
- Eachhypothesiscorrespondstosomes ubsetofX-thesubsetofinstancesthatitclassifies positive.
- The arrows connecting hypothesesrepresent the ***more - general - than***relation,withthearrowpointingt owardthelessgeneralhypothesis.
- Notethesubsetofinstancescharacteri zedby$h_2$subsumesthesubset characterizedbyh_1,hence$h_2$is more-general–thanh_1

FIND-S:FindingaMaximallySpecific Hypothesis

FIND-SAlgorithm

1. Initialize*h*tothemostspecifichypothesisin*H*
2. Foreachpositivetraininginstance*x*

   Foreachattributeconstraint$a_i$in*h*

   Iftheconstraint$a_i$issatisfiedby*x*

   Thendonothing

   Elsereplace$a_i$in*h*bythenextmoregeneralconstraintthatissatisfiedby *x*
3. Outputhypothesis*h*

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

ThefirststepofFIND-Sistoinitialize***h***tothemostspecifichypothesisin***H***

h-(Ø,Ø,Ø,Ø,Ø,Ø)

x_1=<*SunnyWarmNormalStrongWarmSame*>,+

Observing the first training example, it is clear that our hypothesis is too specific. Inparticular, none of the "Ø" constraints in h are satisfied by this example, so each isreplacedbythe nextmore general constraintthat fitsthe example

h_1=<SunnyWarmNormalStrongWarmSame>

Thishisstillveryspecific;itassertsthatallinstancesarenegativeexceptforthesinglepositive training example

x_2=<*Sunny,Warm,High,Strong,Warm,Same*>,+

The second training example forces the algorithm to further generalize h, this timesubstituting a "?' in place of any attribute value in h that is not satisfied by the newexample

h_2=<SunnyWarm?StrongWarmSame>

x3=<Rainy,Cold,High,Strong,Warm,Change>,-

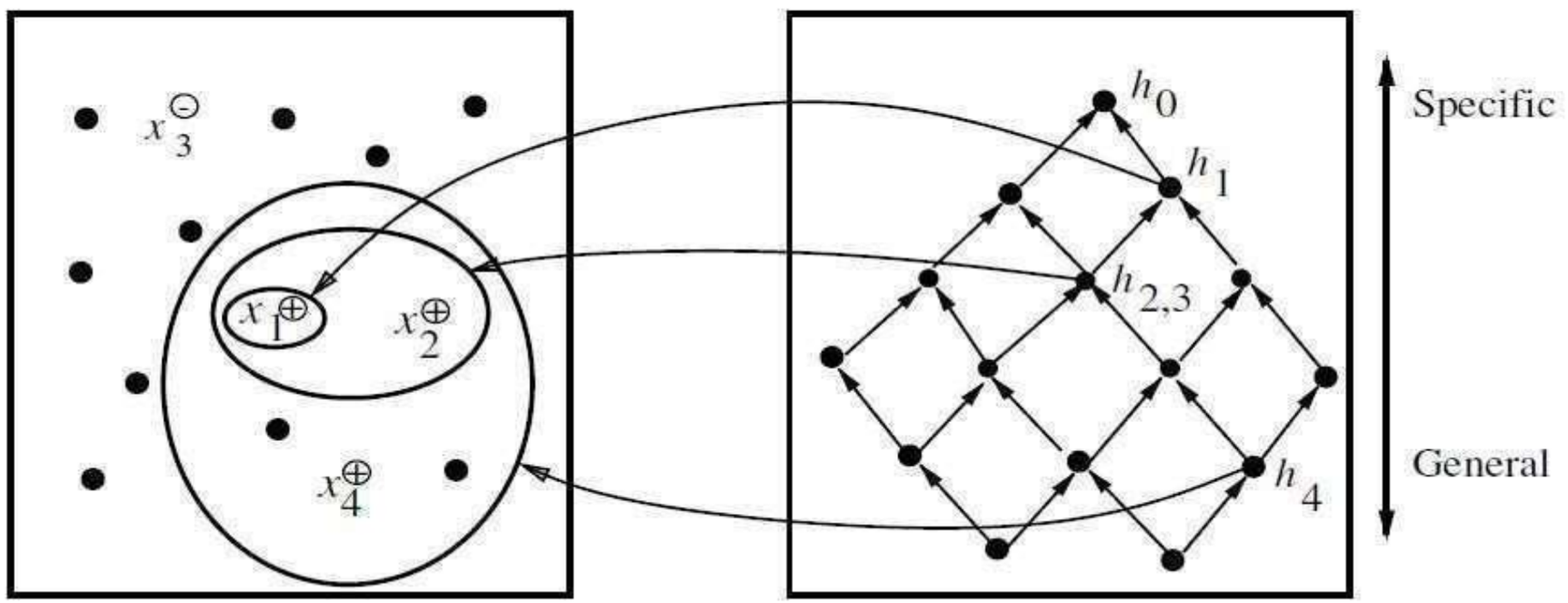
Uponencounteringthethirdtrainingthealgorithmmakesnochangetoh.TheFIND-Salgorithm simply ignores everynegative example.

h3=<SunnyWarm?StrongWarmSame>

x4=<SunnyWarmHighStrongCoolChange>,+

Thefourthexampleleadstoafurthergeneralizationofh

h4=<SunnyWarm?Strong??>



x_1 = <*Sunny Warm Normal Strong Warm Same*>, +

x_2 = <*Sunny Warm High Strong Warm Same*>, +

x_3 = <*Rainy Cold High Strong Warm Change*>, -

x_4 = <*Sunny Warm High Strong Cool Change*>, +

h_0 = <∅, ∅, ∅, ∅, ∅, ∅>

h_1 = <*Sunny Warm Normal Strong Warm San*

h_2 = <*Sunny Warm ? Strong Warm Same*>

h_3 = <*Sunny Warm ? Strong Warm Same*>

h_4 = <*Sunny Warm ? Strong ? ?* >

ThekeypropertyoftheFIND-Salgorithmis

- FIND-S is guaranteed to output the most specific hypothesis within H that isconsistentwith thepositive training examples
- FIND-S algorithm’s final hypothesis will also be consistent with the negativeexamples provided the correct target concept is contained in H, and provided thetrainingexamples are correct.

UnansweredbyFIND-S

1. Hasthe learnerconvergedtothecorrecttargetconcept?
2. Whyprefertheмostspecifichypothesis?
3. Arethetrainingexamplesconsistent?
4. Whatiftherеareseveralmaximallyspecificconsistenthypotheses?

VersionSpaceandCANDIDATEE LIMINATIONAlgorithm

ThekeyideaintheCANDIDATE-ELIMINATIONalgorithmistooutputadescriptionofthesetofall***hypothesesconsistentwiththetrainingexamples***

Representation

- ***Definition:***Ahypothesishis**consistent**with**a**setoftrainingexamples***D***if**and**only if***h(x)*=***c(x)***foreachexample(x,***c(x))***in***D.***

$$Consistent(h,D) \equiv (\forall \langle x, c(x) \rangle \in D) h(x) = c(x))$$

Notedifferencebetweendefinitionsof*consistent*and*satisfies*

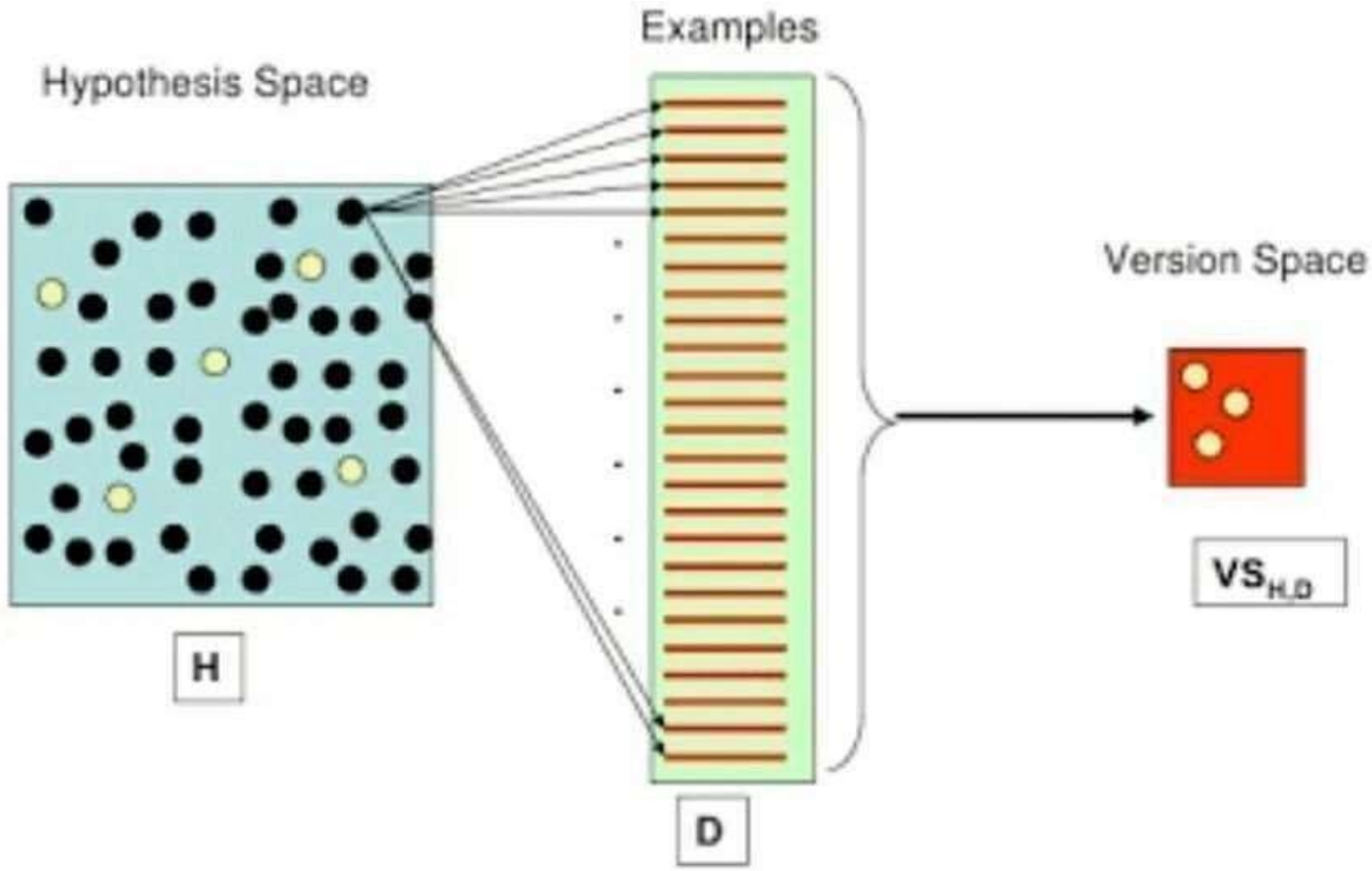
- Anexample*x*issaidto***satisfy***hypothesis***h***when***h(x)***=1,regardless ofwhether*x*isapositiveornegativeexampleofthetargetconcept.
- anexample*x*issaidto***consistent***withhypothesis***h***iff***h(x)=c(x)***

VersionSpace

ArepresentationofthesetofallhypotheseswhichareconsistentwithD

Definition: The **version space,** denoted $VS_{H,D}$with respect to hypothesisspace ***H***and training examples D, is the subset of hypotheses from ***H*** consistent with thetraining examplesin D

$$VS_{H,D} \equiv \{h \in H | Consistent(h,D)\}$$


Examples
Hypothesis Space
Version Space
H
D
VS H,D

TheLIST-THEN-ELIMINATE Algorithm

TheLIST-THEN-ELIMINATEalgorithmfirstinitializestheversionspacetocontainall hypotheses in H and then eliminates any hypothesis found inconsistent with anytraining example.

TheLIST–THEN–ELIMINATE

Algorithm

1. ***VersionSpace****c*alistcontainingeveryhypothesisinH
2. Foreachtrainingexample,***(x,c(x))***

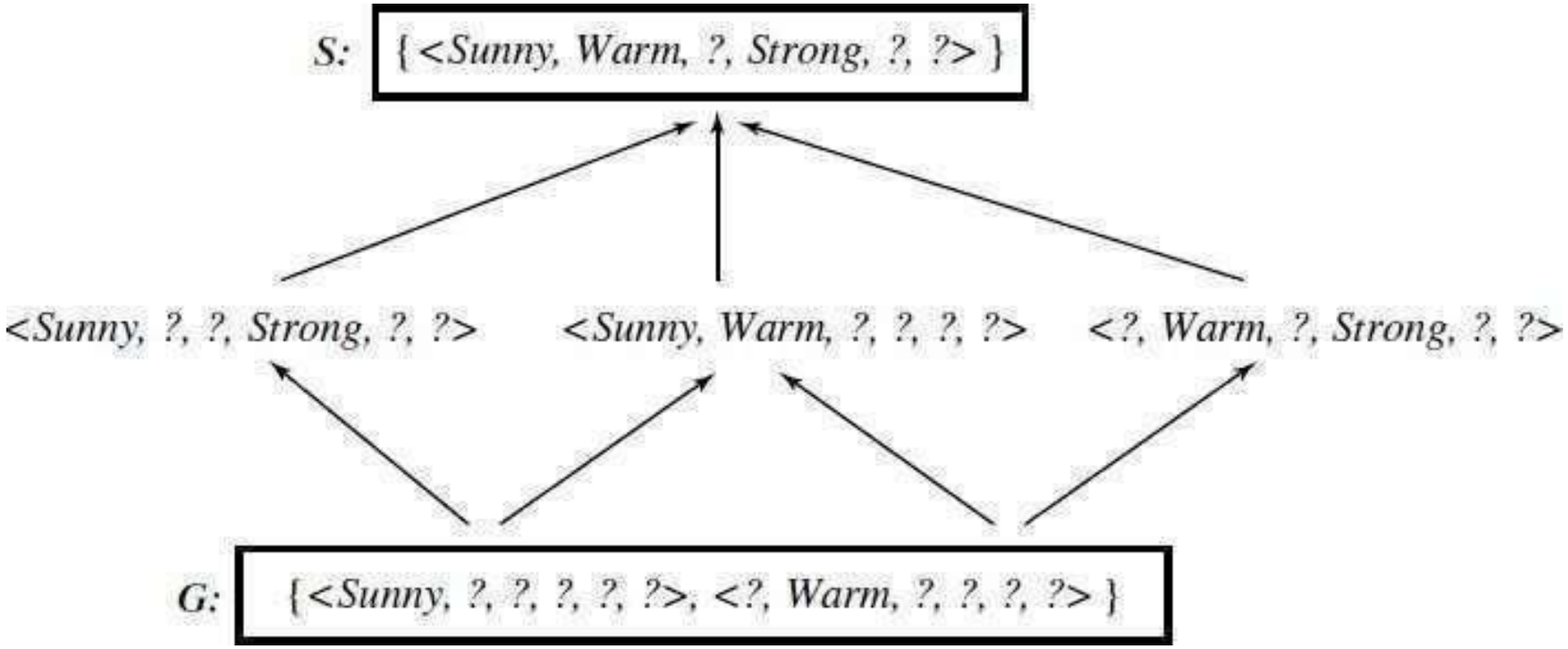
   removefrom***VersionSpace***anyhypothesis***h***forwhich***h(x)≠c(x)***
3. Outputthelistofhypothesesin***VersionSpace***

TheLIST-THEN-ELIMINATEAlgorithm

- **List-Then-Eliminate**worksinprinciple,solongasversionspaceisfinite.
- However,sinceitrequiresexhaustiveenumerationofallhypothesesinpracticeitisnot feasible.

AMoreCompactRepresentationforVersionSpaces

- Theversionspaceisrepresentedbyitsmostgeneralandleastgeneralmembers.
- Thesemembersformgeneralandspecificboundarysetsthatdelimittheversionspace withinthe partiallyordered hypothesis space.



- Aversionspacewithitsgeneral and specific boundarysets.
- The version space includes allsix hypotheses shown here, butcanberepresentedmoresimply by ***S***and***G.***
- Arrowsindicateinstanceof the*more-general-than* relation.Thisistheversion spaceforthe *Enjoysport*conceptlearning
- problemandtrainingexamples describedinbelowtable

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

$$G \equiv \{g \in H \mid Consistent(g,D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g',D)]\}$$

Definition: The **specific boundary** S, with respect to hypothesis space *H* and training data *D*, is the set of minimally general (i.e., maximally specific) members of *H* consistent with *D*.

$$S \equiv \{s \in H \mid Consistent(s,D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s',D)]\}$$

VersionSpacerepresentationtheorem

Theorem: Let X be an arbitrary set of instances and Let H be a set ofBoolean-valued hypotheses defined over X. Let c : X →{O, 1} be an arbitrary target conceptdefined over X, and let D be an arbitrary set of training examples {(x, c(x))). For allX, H,c, and D such that Sand G are well defined,

$$VS_{H,D}=\{h\in H|(\exists s\in S)(\exists g\in G)(g\geq_g h\geq_g s)\}$$

$$VS_{H,D}=\{h\in H\mid(\exists s\in S)(\exists g\in G)(g\geq_g h \geq_g s)\}$$

ToProve:

1. Everyhsatisfyingtherighthandsideoftheaboveexpressionisin $VS_{H,D}$
2. Everymemberof $VS_{H,D}$ satisfiestheright-handsideoftheexpression

Sketchofproof:

1. letg,h,sbearbitrarymembersofG,H,Srespectivelywith $g\geq_g h\geq_g s$

By the definition of ***S,*** **s** must be satisfied by all positive examples in D. Because $h \geq_g s$,h must alsobe satisfied byall positive examples inD.

Bythedefinitionof***G,***gcannotbesatisfiedbyanynegativeexampleinD,andbecause $g\geq_g h$ hcannot be satisfied by any negative example in D. Because h is satisfied by all positive examples in Dandbynonegativeexamples inD,his consistent with D,and thereforehisa memberof $VS_{H,D}$

2. Itcanbeprovenbyassumingsomehin $VS_{H,D}$,thatdoesnotsatisfytheright-handsideoftheexpression,thenshowingthatthisleadstoaninconsistency

TheCANDIDATE-ELIMINATIONLearningAlgorithm

The CANDIDATE-ELIMINTION algorithm computes the ***version space*** containingall hypotheses from H that are consistent with an observed sequence of trainingexamples.

Initialize G to the set of maximally general hypotheses in HInitialize S to the set of maximally specific hypotheses in HForeachtraining example d, do

- Ifdisapositiveexample
  - RemovefromGanyhypothesisinconsistentwithd
  - ForeachhypothesissinSthatisnotconsistentwithd
    - RemovesfromS
    - AddtoSallminimalgeneralizationshofssuchthat
      - hisconsistentwithd,andsomememberofGismoregeneralthanh
    - RemovefromSanyhypothesisthatismoregeneralthananotherhypothesisinS

- Ifdisanegativeexample
  - RemovefromSanyhypothesisinconsistentwithd
  - ForeachhypothesisginGthatisnotconsistentwithd
    - RemovegfromG
    - AddtoGallminimalspecializationshofgsuchthat
      - hisconsistentwithd,andsomememberofSismorespecificthanh
    - RemovefromGanyhypothesisthatislessgeneralthananotherhypothesisinG

AnIllustrativeExample

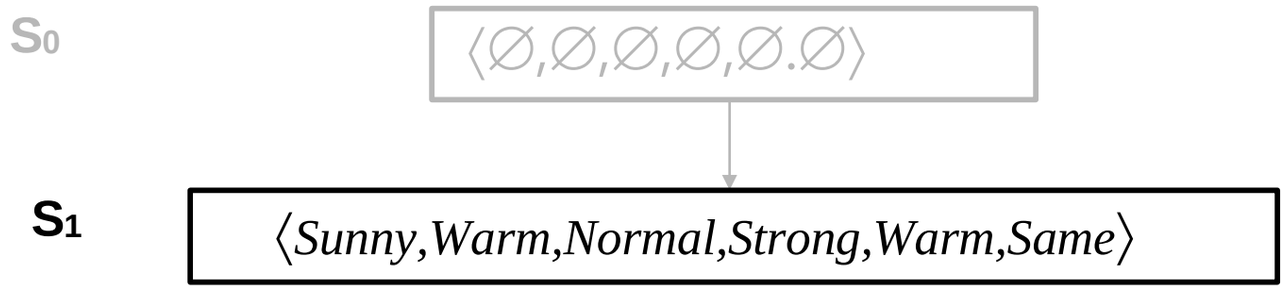
TheboundarysetsarefirstinitializedtoG_0andS_0,themostgeneralandmostspecifichypotheses in H.

S_0 $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$

G_0 $\langle ?, ?, ?, ?, ?, ? \rangle$

Fortrainingexampled,

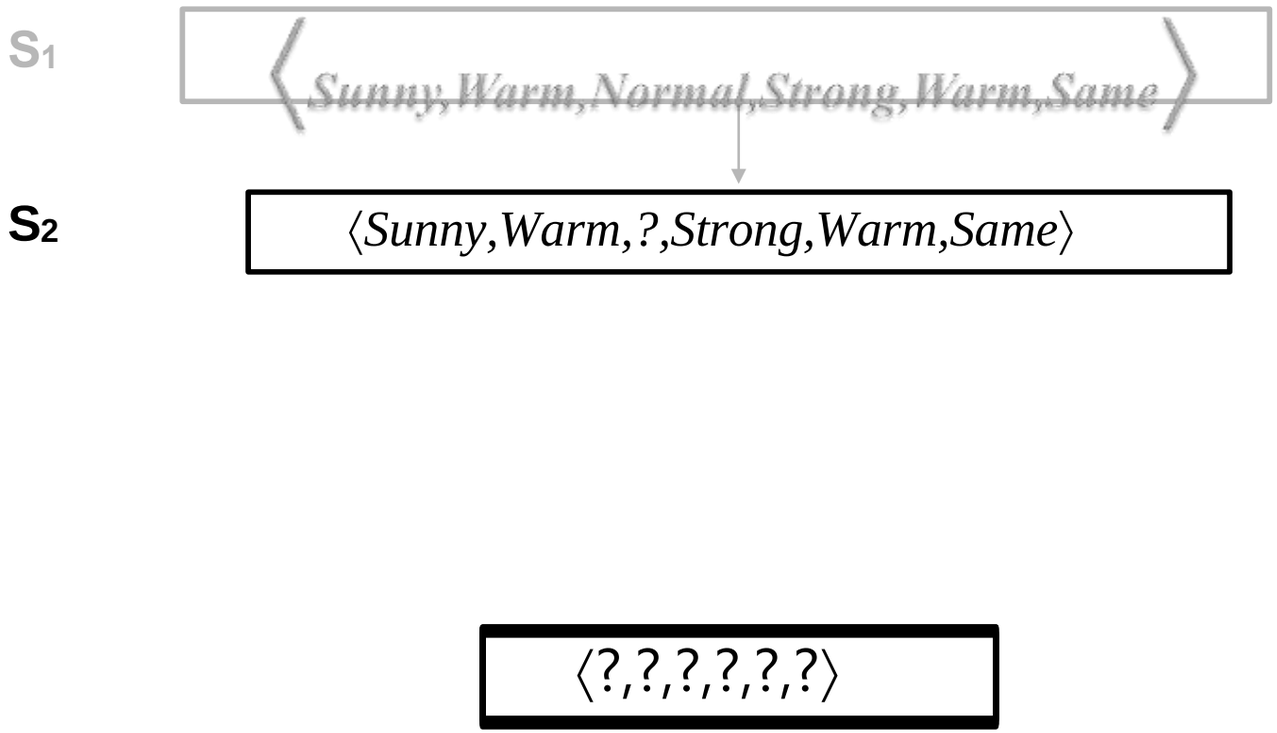
⟨Sunny,Warm,Normal,Strong,Warm,Same⟩+



⟨?,?,?,?,?,?⟩

Fortrainingexampled,

⟨Sunny,Warm,High,Strong,Warm,Same⟩+



Fortrainingexampled,

⟨Rainy,Cold,High,Strong,Warm,Change⟩–

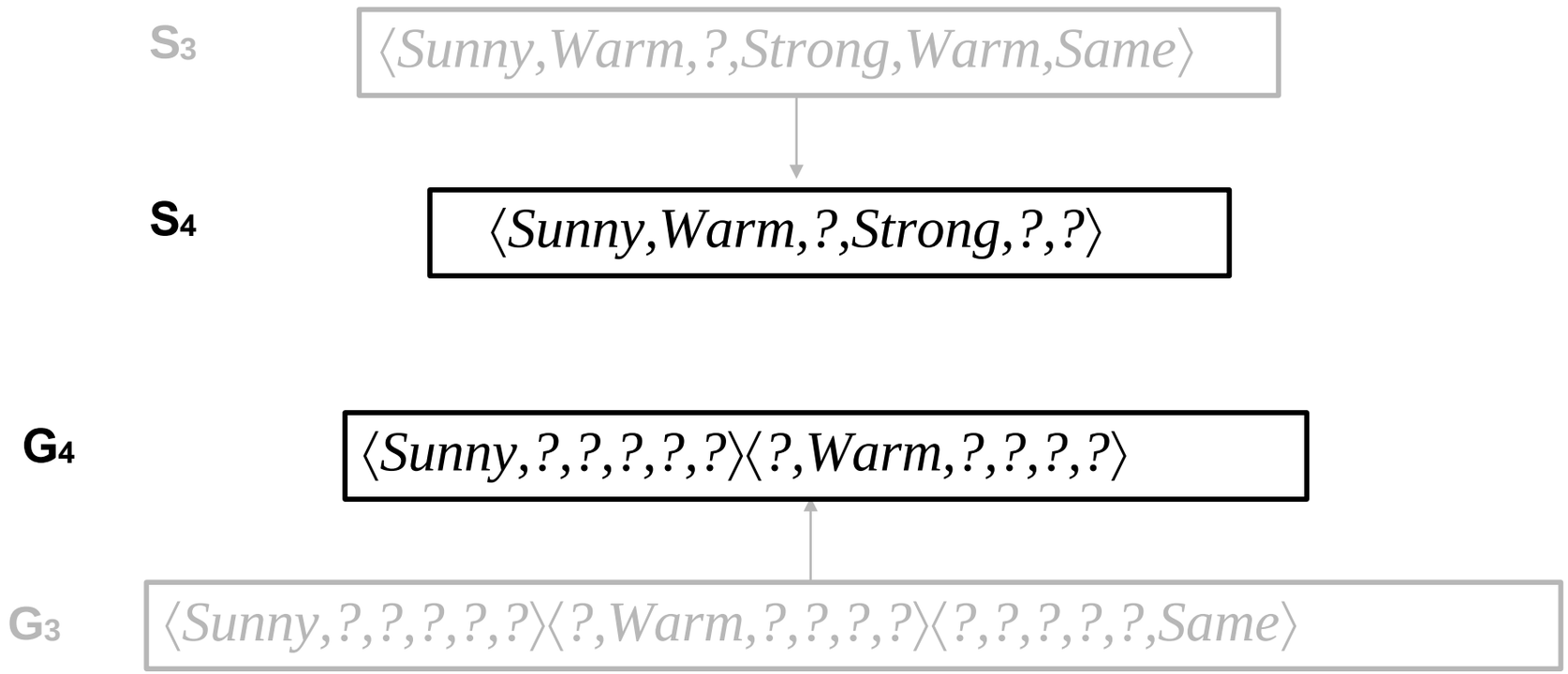
S_2, S_3 ⟨Sunny,Warm,?,Strong,Warm,Same⟩

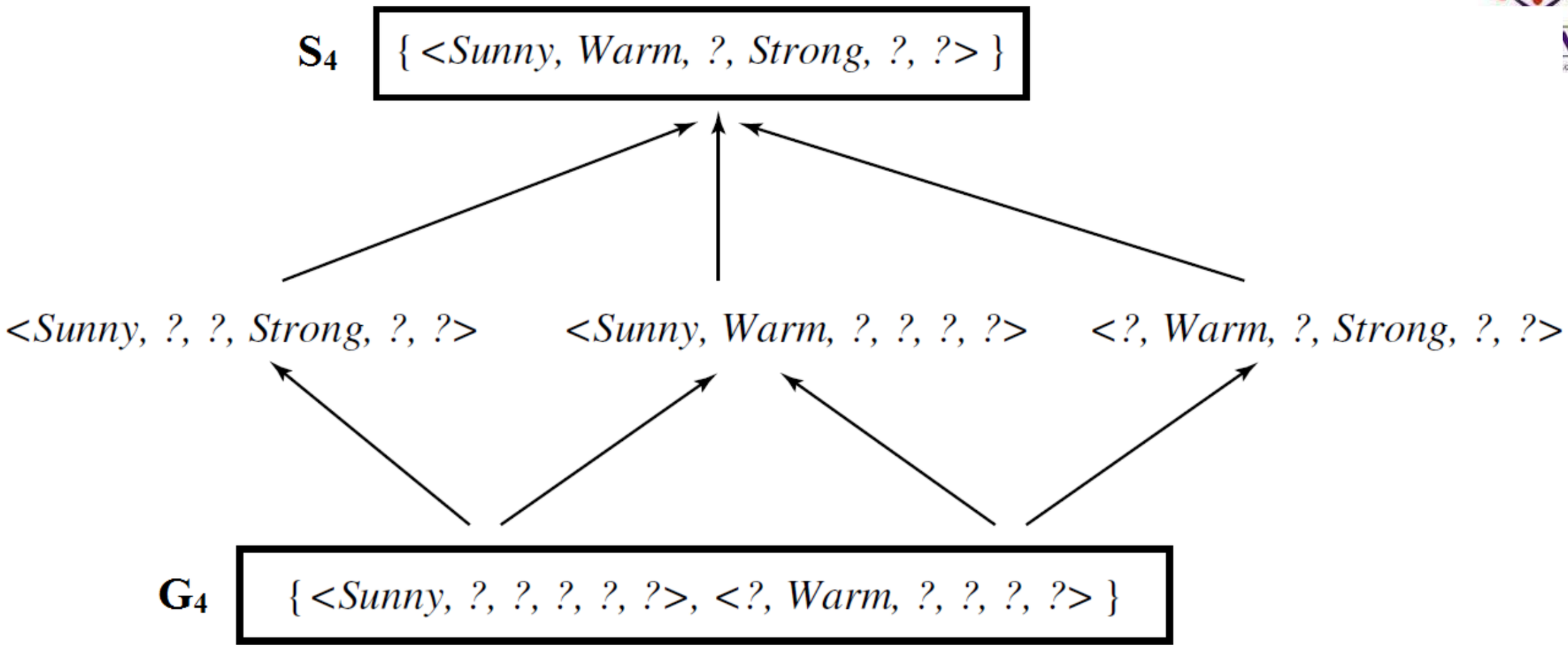
G_3 ⟨Sunny,?,?,?,?,?⟩⟨?,Warm,?,?,?,?⟩⟨?,?,?,?,?,Same⟩

G_2 ⟨?,?,?,?,?,?⟩

Fortrainingexampled,

⟨Sunny,Warm,High,Strong,CoolChange⟩+





The final version space for the ***EnjoySport***concept learning problem and

InductiveBias

Thefundamentalquestionsforinductiveinference

- Whatifthetargetconceptisnotcontainedinthehypothesisspace?
- Canweavoidthisdifficultybyusing ahypothesisspacethatincludeseverypossiblehypothesis?
- Howdoesthesizeofthishypothesisspaceinfluencetheabilityofthealgorithmtogeneralizet ounobservedinstances?
- Howdoesthesizeofthehypothesisspaceinfluencethenumberoftrainingexamples thatmustbeobserved?

Effectofincompletehypothesisspace

Precedingalgorithmsworkiftargetfunctionis inH

Willgenerallynotworkiftargetfunctionnotin H

Considerfollowingexampleswhichrepresenttargetfunction "sky=sunnyorsky=cloudy":

⟨SunnyWarmNormalStrongCoolChange⟩	Y
⟨CloudyWarmNormalStrongCoolChange⟩	Y
⟨RainyWarmNormalStrongCoolChange⟩	N

IfapplyCandidateEliminationalgorithmasbefore,endupwithemptyVersionSpaceAfterf

irsttwotrainingexample

S=⟨?WarmNormalStrongCoolChange⟩

Newhypothesisisoverlygeneralanditcoversthethirdnegativetrainingexample!Our

AnUnbiasedLearner

Incompletehypothesisspace

- IfcnotinH,thenconsidergeneralizingrepresentationofHtocontainc
- ThesizeoftheinstancespaceXofdaysdescribedbythesixavailableattributesis 96.Thenumberofdistinctsubsetsthatcanbedefinedoverasetxcontaining|X| elements(i.e., thesizeofthepowersetofX)is$2^{|X|}$
- Recallthatthereare96instancesin***EnjoySport***;hencethereare2^{96}possiblehypothesesinfullspaceH
- CandothisbyusingfullpropositionalcalculuswithAND,OR,NOT
- HenceHdefinedonlybyconjunctionsofattributesisbiased(containingonly973h’s)

- Let us reformulate the ***Enjoysport***learning task in an unbiased way by defining a newhypothesisspace***H'***thatcanrepresenteverysubsetofinstances;thatis,letH'correspondtothepowersetofX.
- Onewayto definesuchan H'istoallowarbitrarydisjunctions,conjunctions,andnegationsofourearlierhypotheses.

Forinstance,thetargetconcept***"Sky=Sunny***or***Sky=Cloudy"***couldthenbedescribedas

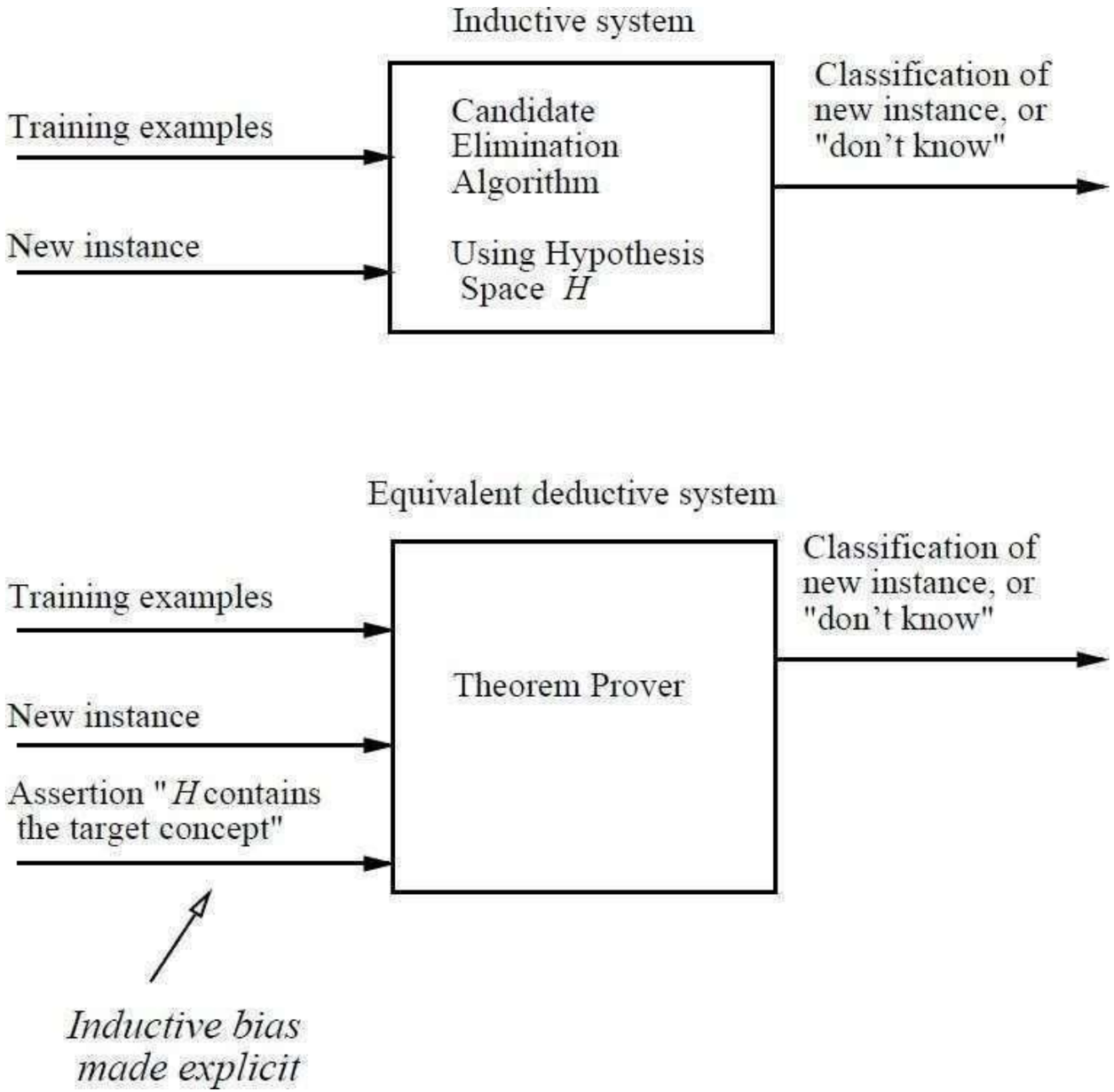
(Sunny,?,?,?,?,?)V***(Cloudy,***?,?,?,?,?)

Definition:

ConsideraconceptlearningalgorithmLforthesetofinstancesX.

- LetcbeanarbitraryconceptdefinedoverX
- LetD_c={(x,c(x))}beanarbitrarysetoftrainingexamplesofc.
- Let$L(x_i,D_c)$denotetheclassificationassignedtotheinstance$x_i$byLaftertrainingonthedataD_c.
- TheinductivebiasofLisany minimalsetofassertionsBsuch thatforanytargetconceptcandcorrespondingtrainingexamplesD_c

$$(\forall\langle x_i \in X)[(B \wedge D_c \wedge x_i) \vdash L(x_i,D_c)]$$



Modelling inductive systems byequivalentdeductivesystems.

Theinput-outputbehavioroftheCANDIDATE-ELIMINATION algorithm using a hypothesis space ***H*** is identical to that of a deductivetheoremproverutilizingthe assertion ***"H*** contains the target concept." Thisassertion is therefore called the***inductivebias***ofthe CANDIDATE-ELIMINATION algorithm.

.

characterizinginductivesystems by theirinductive biasallowsmodellingthembytheirequivalentdeductivesystems.Thisprovidesawayto compareinductive systemsaccording to theirpolicies forgeneralizing beyond the observedtrainingdata

DECISIONTREELEARNING

DECISIONTREEREPRESENTATION

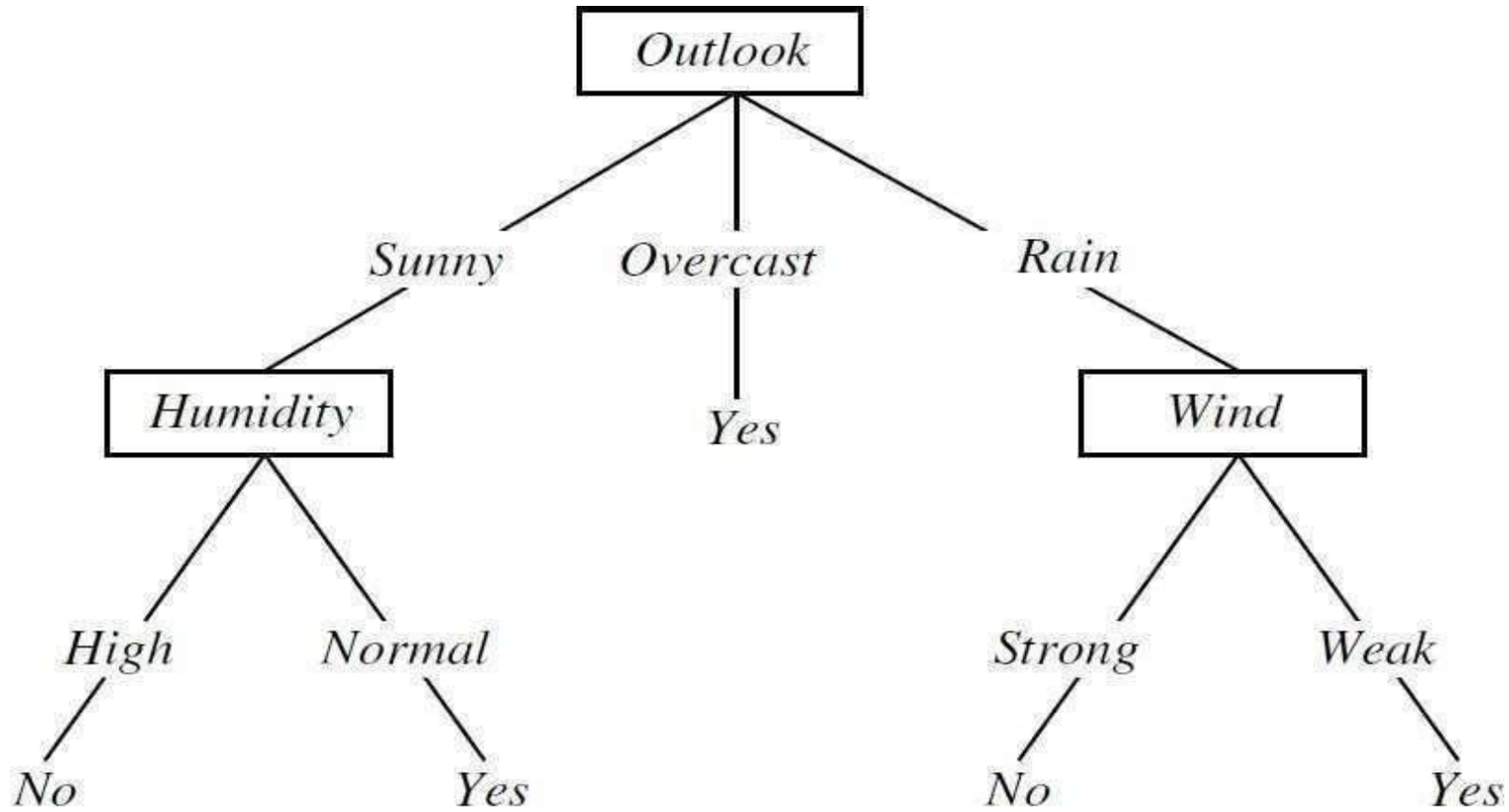


FIGURE: A decision tree for theconcept ***PlayTennis***.Anexam pleisclassified by sortingit through the tree totheappropriateleaf node, then returningtheclassific ationassociated with thisleaf

- Decision trees classify instances by sorting them down the tree from the root tosome leaf node, which provides the classificationof the instance.

- Each node in the tree specifies a test of some attribute of the instance, and eachbranch descending from that node corresponds to one of the possible values forthisattribute.

- Aninstanceisclassifiedbystartingattherootnodeofthetree,testingtheattribute specified by this node, then moving down the tree branch correspondingtothevalueoftheattributeinthegivenexample.Thisprocessisthenrepea tedforthe subtree rootedat the new node.

- Decisiontreesrepresentadisjunctionofconjunctionsofconstraintsontheattribute values of instances.
- Eachpathfromthetreeroottoaleafcorrespondstoaconjunctionofattributetests,and the treeitself toa disjunction of these conjunctions

Forexample,
Thedecisiontreeshowninabovefigurecorrespondstothe expression

(Outlook =Sunny AHumidity = Normal)
(Outlook=Overcast)
(Outlook=RainAWind=Weak)

APPROPRIATEPROBLEMSFOR DECISIONTREELEARNING

Decisiontreelearningisgenerallybestsuitedtoproblemswiththefollowingcharacteristics:

1. ***Instancesarerepresentedbyattribute-valuepairs***–Instancesaredescribedbyafixed set of attributes and their values
2. ***The target function has discrete output values*** – The decision tree assigns aBoolean classification (e.g., yes or no) to each example. Decision tree methodseasilyextendtolearningfunctions withmorethantwo possibleoutputvalues.
3. ***Disjunctivedescriptionsmayberequired***

4. ***Thetrainingdatamaycontainerrors***–Decisiontreelearningmethodsarerobust to errors, both errors in classifications of the training examples and errorsinthe attribute values that describe these examples.

5. ***Thetrainingdatamaycontainmissingattributevalues–***Decisiontreemethodscanbe used evenwhen some training exampleshaveunknown values

- Decision tree learning has been applied to problems such as learning to classify*medical patients by their disease*, *equipment malfunctions by their cause*, and*loan applicants by their likelihood ofdefaulting on payments*.

- Suchproblems,inwhichthetaskistoclassifyexamplesintooneofadiscrete setofpossible categories, are often referred toas***classification problems.***

THEBASICDECISIONTREE ALGORITHM

- Mostalgorithmsthathavebeendevelopedforlearningdecisiontreesarevariations on a core algorithm that employs a top-down, greedy search through thespaceofpossibledecisiontrees.ThisapproachisexemplifiedbytheID3algorithmand itssuccessor C4.5

WhatistheID3algorithm?

- ID3standsforIterativeDichotomiser3
- ID3isaprecursortotheC4.5Algorithm.
- TheID3algorithmwasinventedbyRossQuinlanin1975
- Usedtogenerateadecisiontreefromagivendatasetbyemployingatop-down,greedysearch, to test each attribute at every nodeof thetree.
- Theresultingtreeisusedtoclassifyfuturesamples.

ID3algorithm

ID3(Examples,Target_attribute,Attributes)

Examples are the training examples. Target_attribute is the attribute whose value is tobe predictedby the tree. Attributes is a list of other attributes that may be tested by the learned decision tree.Returnsadecisiontree that correctlyclassifies thegiven Examples.

- CreateaRootnodeforthetree
- IfallExamplesarepositive,Returnthesingle-nodetreeRoot,withlabel=+
- IfallExamplesarenegative,Returnthesingle-nodetreeRoot,withlabel=-
- IfAttributesisempty,Returnthesingle-nodetreeRoot,withlabel=mostcommonvalueofTarget_attributeinExamples

- OtherwiseBegin
  - A←theattributefromAttributesthatbest*classifiesExamples
  - ThedecisionattributeforRoot←A
  - Foreachpossiblevalue,v_i,ofA,
    - Addanewtreebranchbelow*Root*,correspondingtothetestA=v_i
    - Let$Examples_{vi}$,bethesubsetofExamplesthathavevalue$v_i$for*A*
    - If$Examples_{vi}$,isempty
      - Thenbelowthisnewbranchaddaleafnodewithlabel=mostcommonvalueofTarget_attribuleinExamples
      - Elsebelowthisnewbranchaddthesubtree ID3(*Examples*$_{vi}$,Targe_tattribute,Attributes–{A}))
- End
- ReturnRoot

*Thebestattributeistheonewithhighestinformationgain

WhichAttributeIstheBestClassifier?

- ThecentralchoiceintheID3algorithmisselectingwhichattributetotestat eachnode in thetree.
- Astatisticalpropertycalled***informationgain***thatmeasureshowwellagivenattributeseparatesthetraining examples accordingto theirtargetclassification.
- ID3uses***informationgain***measuretoselectamongthecandidateattributesateach step while growingthe tree.

ENTROPYMEASURESHOMOGENEITYOFEXAMPLES

- Todefineinformationgain,webeginbydefiningameasurecalledentropy. *Entropymeasurestheimpurityofacollectionofexamples.*
- GivenacollectionS,containingpositiveandnegativeexamplesofsometargetconcept,theentropy of Srelative to this Booleanclassificationis

$$Entropy\,(S) \equiv -p_{\oplus} log_2 p_{\oplus} - p_{\ominus} log_2 p_{\ominus}$$

Where,

p_+isthepropportionofpositiveexamplesinS

p-istheproportionofnegativeexamplesinS.

Example:Entropy

- Suppose S is a collection of 14 examples of some boolean concept, including 9positive and 5 negative examples. Then the entropy of S relative to this booleanclassificationis

$$Entropy([9+, 5-]) = -(9/14)\log_2(9/14) - (5/14)\log_2(5/14)$$
$$= 0.940$$

- Theentropyis0ifallmembersofSbelongtothesameclass
- Theentropyis1whenthecollectioncontainsanequalnumberofpositiveandnegative examples
- Ifthecollectioncontainsunequalnumbersofpositiveandnegativeexamples,theentropyis between 0 and 1

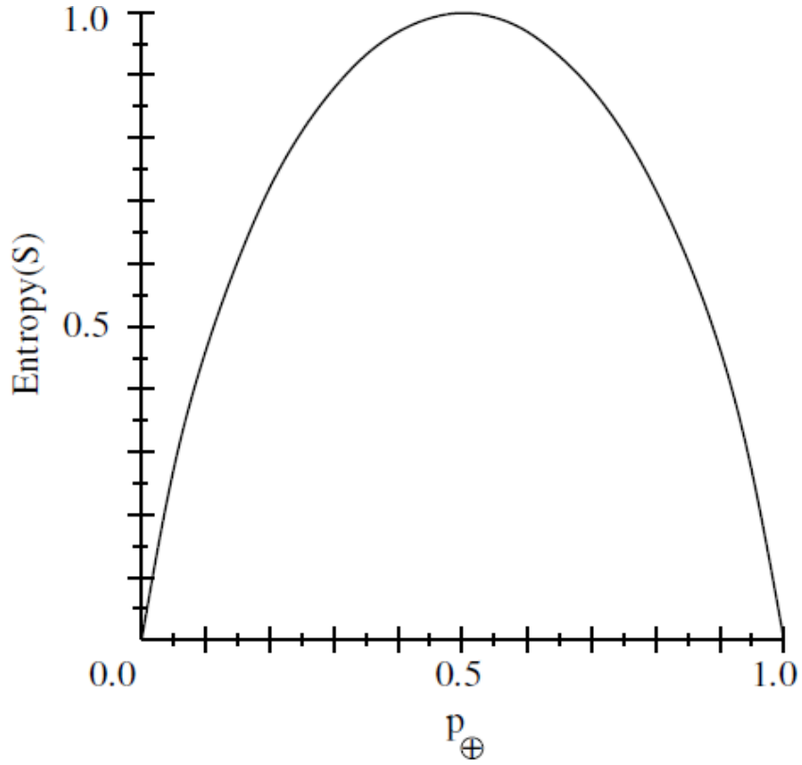


FIGURE The entropy function relative to a boolean classification, as the proportion, $p_{\oplus}$, of positive examples varies between 0 and 1.

INFORMATIONGAINMEASURESTHEEXPECTEDREDUCTIO NINENTROPY

- ***Informationgain,*** istheexpectedreductioninentropycausedbypartitioning theexamplesaccording tothis attribute.
- Theinformationgain,Gain(S,A)ofanattributeA,relativetoacollectionofexamplesS, is defined as

$$Gain(S, A) = Entropy(S) - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} Entropy(S_v)$$

Example:Informationgain

Let, $Values(Wind)=\{Weak, Strong\}$

S $=[9+,5-]$

S_{Weak} $=[6+,2-]$

S_{Strong} $=[3+,3-]$

Informationgainofattribute*Wind*:

$$Gain(S,Wind)=Entropy(S)-8/14\,Entropy(S_{Weak})-6/14\,Entropy(S_{Strong})$$

$$=0.94-(8/14)*0.811-(6/14)*1.00$$

$$=0.048$$

AnIllustrativeExample

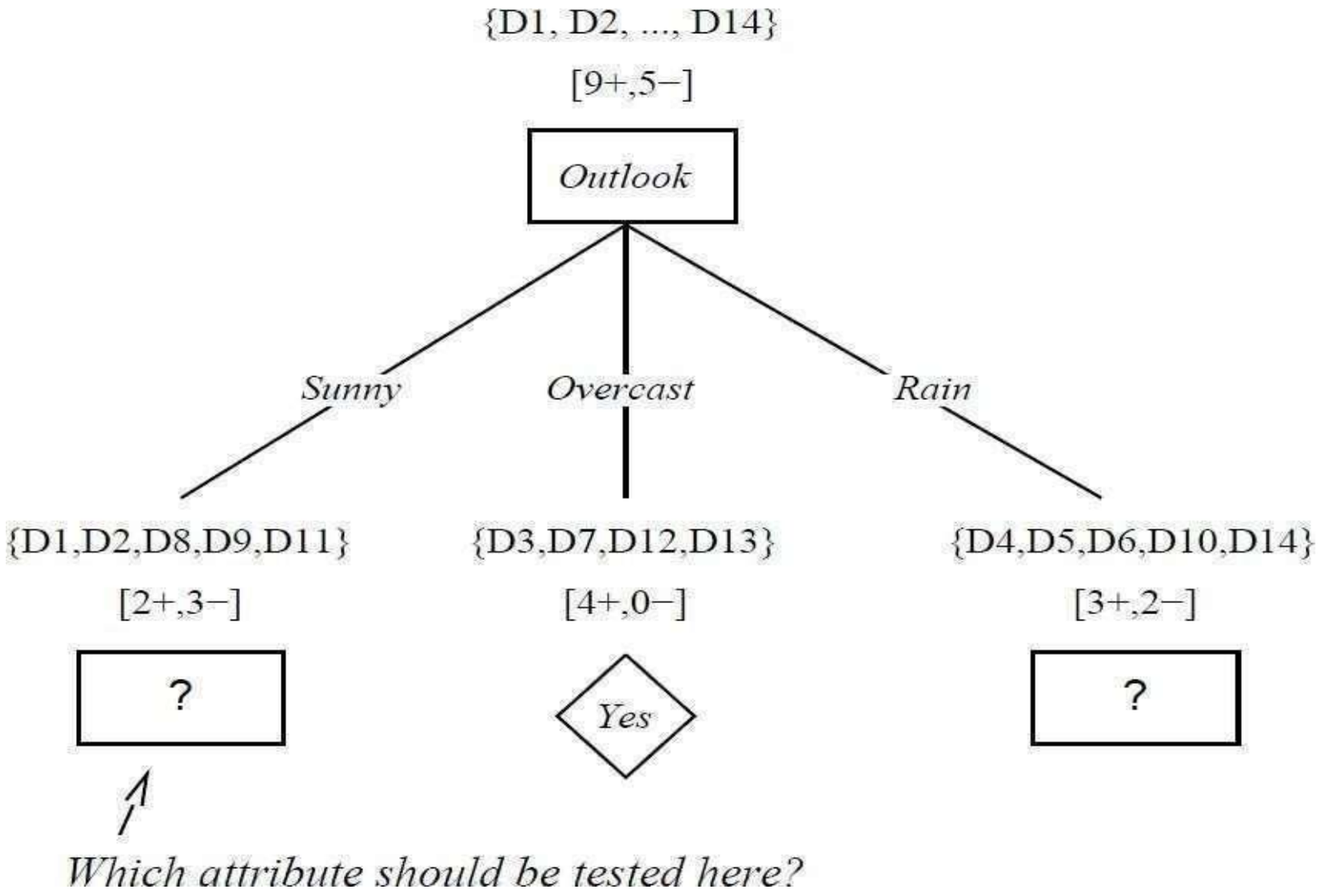
- ToillustratetheoperationofID3,considerthelearningtask representedbythetrainingexamples of below table.
- Herethetargetattribute***PlayTennis***,which canhave values***yes*** or***no***fordifferentdays.
- Considerthefirststepthroughthealgorithm,inwhichthetopmostnodeofthedecision treeis created.

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Theinformationgainvaluesforallfourattributesare

- Gain(S,Outlook) =0.246
- Gain(S,Humidity) =0.151
- Gain(S,Wind) =0.048
- Gain(S,Temperature) =0.029

- Accordingtotheinformationgainmeasure,the***Outlook***attributeprovidesthebest prediction of the target attribute, ***PlayTennis***, over the training examples.Therefore,***Outlook***isselectedasthedecisionattributefortherootnode, andbranchesarecreatedbelowtherootforeachofitspossiblevaluesi.e.,


{D1, D2, ..., D14}
[9+,5−]
Outlook
Sunny
Overcast
Rain
{D1,D2,D8,D9,D11}
[2+,3−]
{D3,D7,D12,D13}
[4+,0−]
{D4,D5,D6,D10,D14}
[3+,2−]
?
Yes
?
Which attribute should be tested here?

$S_{sunny} = \{D1, D2, D8, D9, D11\}$

$Gain(S_{sunny}, Humidity) = .970 - (3/5)\,0.0 - (2/5)\,0.0 = .970$

$Gain(S_{sunny}, Temperature) = .970 - (2/5)\,0.0 - (2/5)\,1.0 - (1/5)\,0.0 = .570$

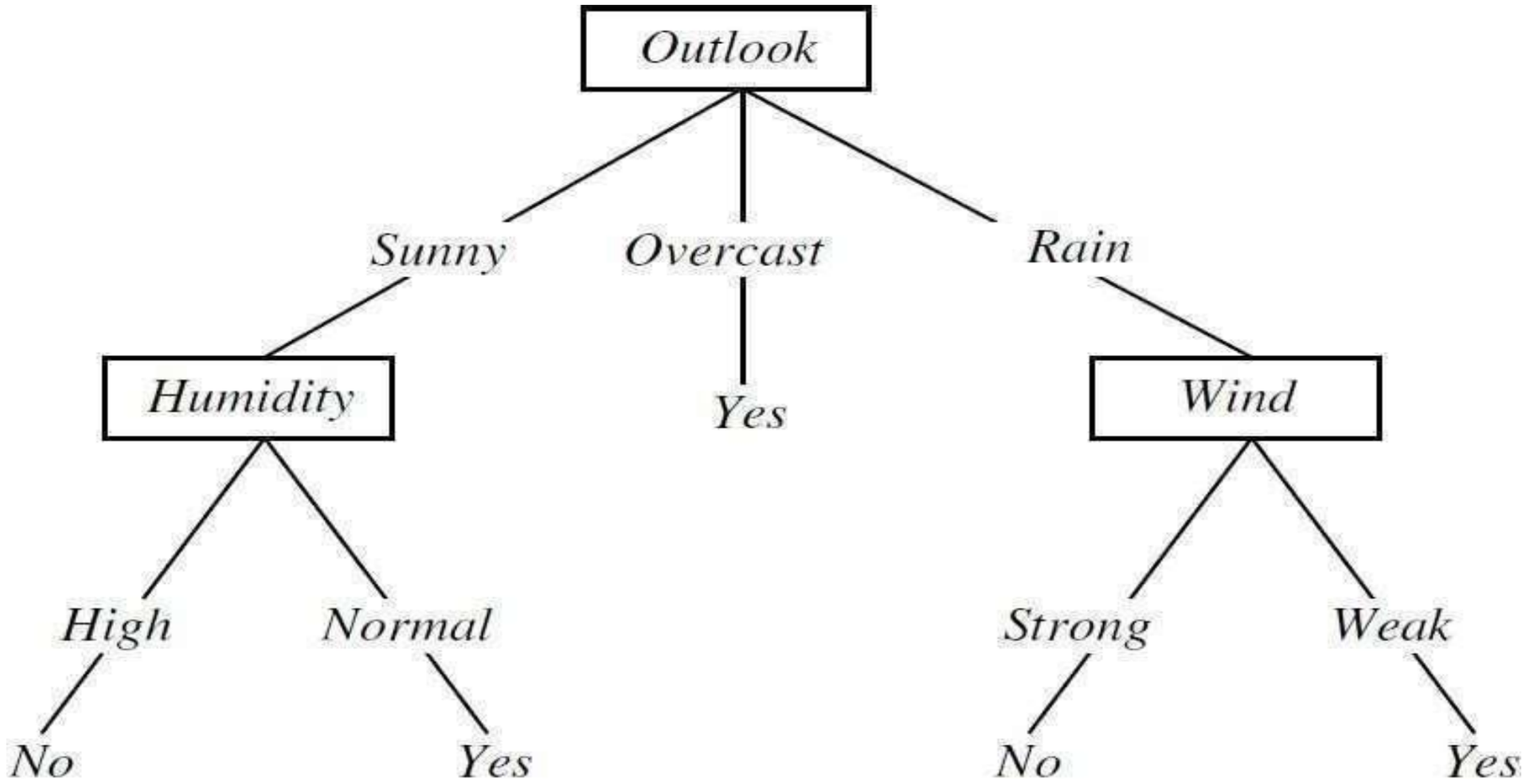
$Gain(S_{sunny}, Wind) = .970 - (2/5)\,1.0 - (3/5)\,.918 = .019$

$S_{Rain} = \{D4, D5, D6, D10, D14\}$

$$Gain(S_{Rain}, Humidity) = 0.970 - (2/5)1.0 - (3/5)0.917 = 0.019$$

$$Gain(S_{Rain}, Temperature) = 0.970 - (0/5)0.0 - (3/5)0.918 - (2/5)1.0 = 0.019$$

$$Gain(S_{Rain}, Wind) = 0.970 - (3/5)0.0 - (2/5)0.0 = 0.970$$


Outlook
Sunny
Overcast
Rain
Humidity
Yes
Wind
High
Normal
Strong
Weak
No
Yes
No
Yes

HYPOTHESISSPACESEARCHINDECISIONTREEL EARNING

- ID3 can be characterized as searching a space of hypotheses for one that fits thetraining examples.
- ThehypothesisspacesearchedbyID3isthesetofpossibledecisiontrees.
- ID3 performs a ***simple-to complex, hill-climbing search*** through this hypothesisspace,beginningwiththeemptytree,thenconsideringprogressivelymoreela boratehypothesesinsearchofadecisiontreethatcorrectlyclassifiesthetraining data

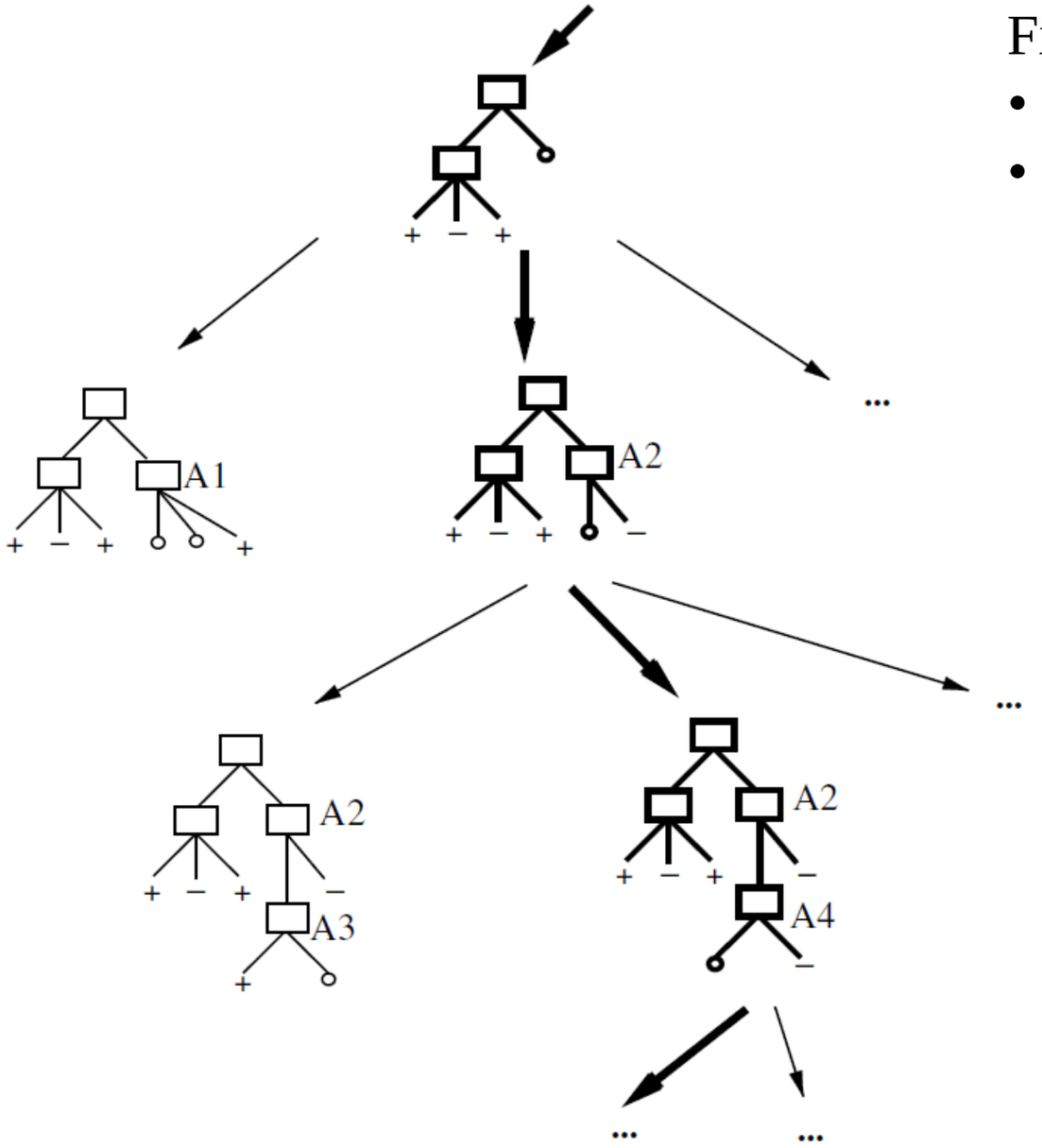


Figure:

- HypothesisspacesearchbyID3.
- ID3searchesthroughthespaceofpossible decision trees from simplest toincreasinglycomplex,guidedbytheinformationgain heuristic

1. ID3's hypothesis space of all deision trees is a ***complete*** space of finite discrete-valued functions, relative to the available attributes. Because every finite discrete-valuedfunction can be represented by some decision tree

- ID3avoidsoneofthemajorrisksofmethodsthat***searchincompletehypothesisspaces*** :thatthehypothesisspacemightnotcontainthetargetfunction.

2. ID3maintains***onlyasinglecurrenthypothesis***asitsearchesthroughthespaceof decision trees.

Forexample,withtheearlierversionspacecandidateeliminationmethod,whichmaint ains the set of ***all*** hypotheses consistent with the available trainingexamples.

Bydeterminingonlyasinglehypothesis,ID3losesthecapabilitiesthatfollowfromexplici tlyrepresenting allconsistenthypotheses.

For example, it does not have the ability to determine how many alternativedecisiontreesareconsistentwiththeavailabletrainingdata,orto pose newinstancequeriesthat optimallyresolveamongthesecompetinghypotheses

3. ID3 in its pure form performs ***no backtracking in its search***. Once it selects anattribute to test at a particular level in the tree, it never backtracks to reconsider thischoice.

- In the case of **ID3,** a locally optimal solution corresponds to the decision tree itselects along the single search path it explores. However, this locally optimalsolution may be less desirable than trees that would have been encountered along adifferentbranch of the search.

4. ID3*usesalltrainingexamplesateachstep*inthesearchtomakestatisticallybaseddecisions regarding how to refineits current hypothesis.

- Oneadvantageofusingstatisticalpropertiesofalltheexamplesisthattheresultingsearch ismuch ***less sensitiveto errors***inindividualtraining examples.
- **ID3**canbeeasilyextendedtohandlenoisytrainingdataby modifyingitsterminationcriteriontoaccepthypotheses thatimperfectlyfitthetrainingdata.

INDUCTIVEBIASINDECISIONTREELEARNING

Inductive bias is the set of assumptions that, together with the training data,deductivelyjustifytheclassificationsassignedbythelearnertofutureinstances

Givenacollectionoftrainingexamples,therearetypicallymanydecisiontreesconsistent with theseexamples.Which ofthese decisiontreesdoes ID3choose?

ID3searchstrategy

(a) selectsinfavourofshortertreesoverlongerones

(b) selectstreesthatplacetheattributeswith highestinformationgainclosesttotheroot.

ApproximateinductivebiasofID3: Shortertreesarepreferredoverlargertrees

- Consideranalgorithmthatbeginswiththeemptytreeandsearches***breadthfirst*** throughprogressivelymorecomplextrees.
- Firstconsideringalltreesofdepth1,thenalltreesofdepth2,etc.
- Once itfinds a decision tree consistent with thetraining data, it returns thesmallestconsistenttreeatthatsearchdepth(e.g.,thetreewiththefewestnodes).
- LetuscallthisbreadthfirstsearchalgorithmBFS-ID3.
- BFS-ID3findsashortestdecisiontreeandthusexhibitsthebias"shortertrees arepreferredover longer trees.

- ID3 can be viewed as an efficient approximation to BFS-ID3, using a greedyheuristic search to attempt to find the shortest tree without conducting the entirebreadth-firstsearchthrough thehypothesis space.
- Because ID3 uses the information gain heuristic and a hill climbing strategy, itexhibitsa more complexbiasthan BFS-ID3.
- In particular, it does not always find the shortest consistent tree, and it is biased tofavourtreesthat place attributeswithhigh informationgain closest tothe root.

RestrictionBiasesandPreferenceBiases

DifferencebetweenthetypesofinductivebiasexhibitedbyID3andbytheCANDIDATE-ELIMINATIONAlgorithm.

ID3

- ID3searchesacompletehypothesisspace
- Itsearchesincompletelythroughthisspace,fromsimpletocomplexhypotheses,untilitstermination condition ismet
- Itsinductivebiasissolelyaconsequenceoftheorderingofhypothesesbyitssearchstrategy. Itshypothesisspace introducesno additional bias

CANDIDATE-ELIMINATIONAlgorithm

- TheversionspaceCANDIDATE-ELIMINATIONAlgorithmsearchesanincompletehypothesisspace
- Itsearchesthisspacecompletely,findingeveryhypothesisconsistentwiththetrainingdata.
- Its inductive bias is solely a consequence of the expressive power of its hypothesisrepresentation.Its search strategy introducesno additionalbias

RestrictionBiasesandPreferenceBiases

- The inductive bias of ID3 is a *preference* for certain hypotheses over others (e.g.,preference for shorter hypotheses over larger hypotheses), with no hard restrictiononthehypothesesthatcanbeeventuallyenumerated.Thisformofbiasis called a***preferencebias***or ***a search bias***.

- The bias of the CANDIDATE ELIMINATION algorithm is in the form of a*categorical* restriction on the set of hypotheses considered. This form of bias istypicallycalled a***restriction bias***or ***alanguage bias***.

Whichtypeofinductivebiasispreferredinordertogeneralizebeyondthetrainingdata,apreferenc e bias or restriction bias?

- A preference bias is more desirable than a restriction bias, because it allows thelearner to work within a complete hypothesis space that is assured to contain theunknown target function.
- In contrast, a restriction bias that strictly limits the set of potential hypotheses isgenerally less desirable, because it introduces thepossibilityofexcluding theunknown target function altogether.

Occam'srazor

Occam's razor: is the problem-solving principle that the simplest solution tends to bethe right one. When presented with competing hypotheses to solve a problem, oneshouldselect the solution with the fewest assumptions.

Occam'srazor:"***Preferthesimplesthypothesisthatfitsthedata***".

WhyPreferShortHypotheses?

<u>Argumentinfavour:</u>

Fewershorthypothesesthanlongones:

- Shorthypothesesfitsthetrainingdatawhichare*lesslikely*tobe*coincident*
- Longerhypothesesfitsthetrainingdatamightbe*coincident.*

Manycomplexhypothesesthatfitthecurrenttrainingdatabutfailtogeneralizecorrectly tosubsequent data.

Argumentopposed:

- There are few small trees, and our priori chance of finding one consistent with anarbitrary set of data is therefore small. The difficulty here is that there are verymanysmallsetsofhypothesesthatonecandefine*butunderstoodbyfewerlearner*.
- The size of a hypothesis is determined by the representation used ***internally*** by thelearner.Occam'srazorwillproduce*twodifferenthypothesesfromthesametrainingexampleswhenitisappliedbytwolearners*,bothjustifyingtheircontradictory conclusions by Occam's razor. On this basis we might be tempted torejectOccam's razor altogether.

ISSUESINDECISIONTREELEARNING

1. AvoidingOverfittingtheData

   Reduced error

   pruningRulepost-pruning

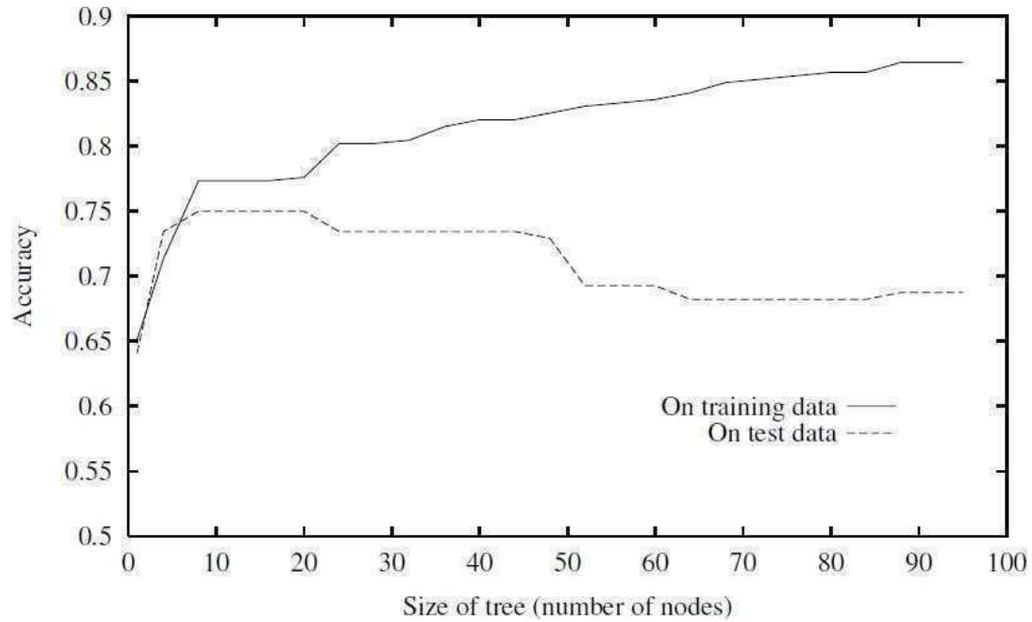
2. IncorporatingContinuous-ValuedAttributes
3. AlternativeMeasuresforSelectingAttributes
4. HandlingTrainingExampleswithMissingAttributeValues
5. HandlingAttributeswithDifferingCosts

1. AvoidingOverfittingtheData

- The ID3 algorithm grows each branch of the tree just deeply enough to perfectlyclassify the training examples but it can lead to difficulties when there is noise inthe data, or when the number of training examples is too small to produce arepresentative sample of the true target function. This algorithm can produce treesthat***overfit***the training examples.

- ***Definition-Overfit:***Givenahypothesisspace$H$,ahypothesis$h \in H$issaidtooverfit the training data if there exists some alternative hypothesis $h' \in H$, suchthat$h$hassmallererrorthanh'overthetrainingexamples,buth'hasasmallererrorthan $h$over the entire distribution of instances.

- Thebelowfigureillustratestheimpactofoverfittinginatypicalapplicationofdecisiontreelearning.



- The**horizontalaxis**ofthis plotindicatesthetotalnumberofnodesinthedecisiontree,asthetreeisbeingconstructed.The**verticalaxis**indicates theaccuracy ofpredictionsmadebythe tree.
- The**solidline**showstheaccuracyofthedecisiontreeoverthetrainingexamples.The**brokenline**showsaccuracymeasuredoveranindependentsetoftestexample
- The**accuracy**ofthetreeoverthe**trainingexamplesincreases**monotonicallyasthetreeisgrown.The **accuracy**measuredovertheindependenttestexamples**firstincreases**,**thendecreases**.

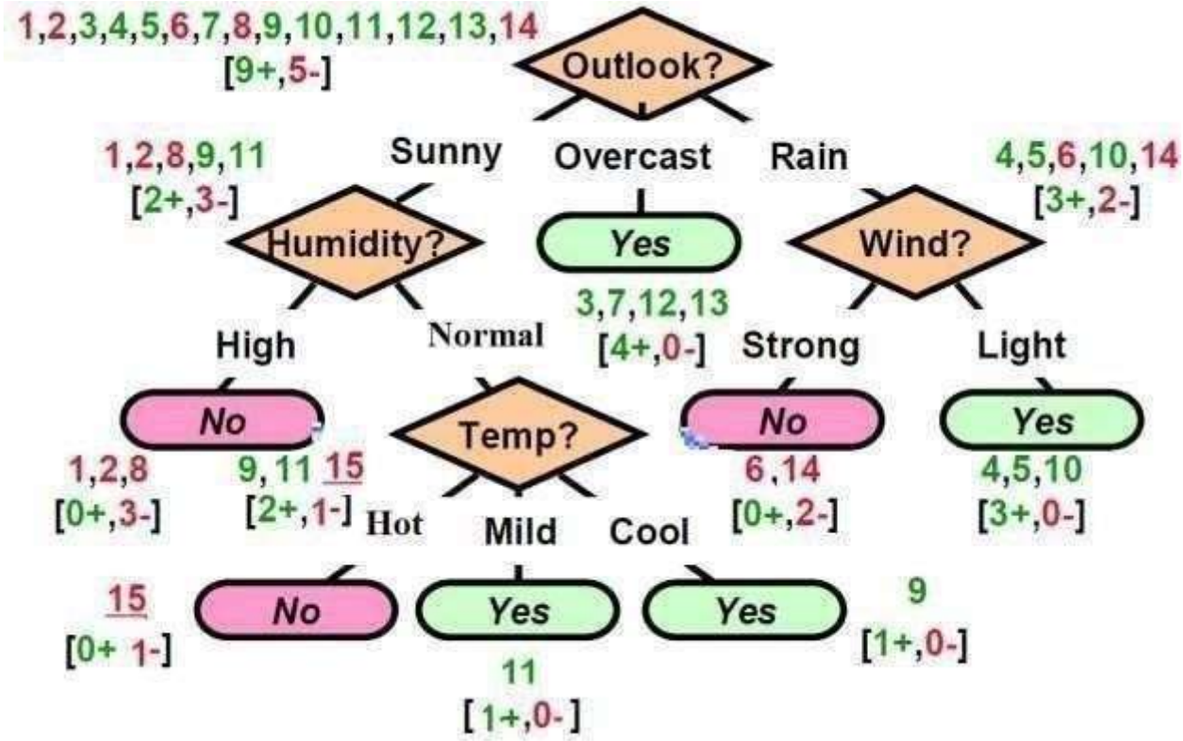
Howcanitbepossiblefortreehtofitthetrainingexamplesbetterthanh',butforittoperformmorepoorly over subsequentexamples?

1. Overfittingcanoccurwhenthetrainingexamplescontainrandomerrorsornoise
2. Whensmallnumbersofexamplesareassociatedwithleafnodes.

NoisyTrainingExample

Example15:<*Sunny,Hot,Normal,Strong,->*

- Exampleisnoisybecausethecorrectlabelis+
- Previouslyconstructedtreemisclassifiesit



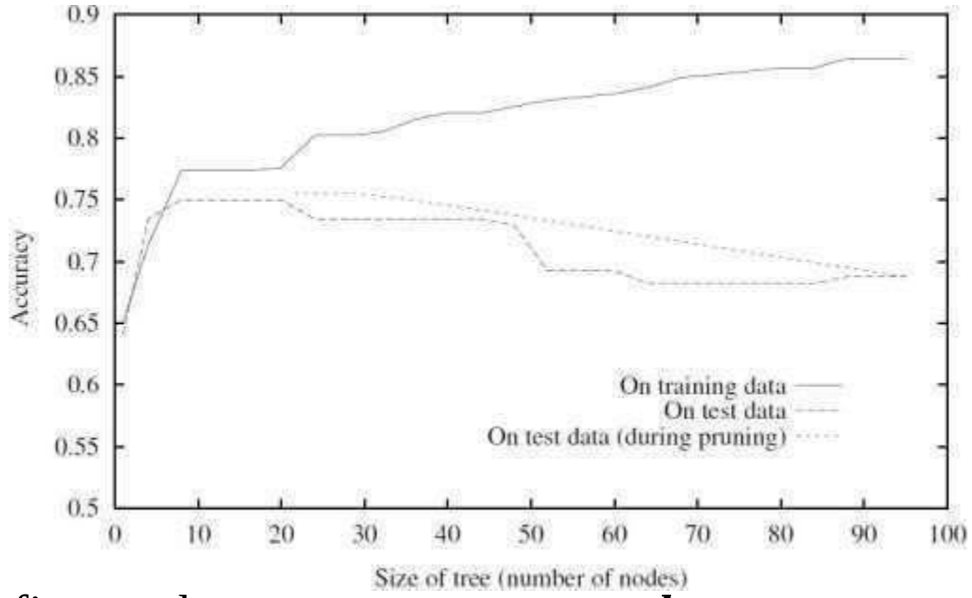
Criterionusedtodeterminethecorrectfinaltreesize

- Useaseparatesetofexamples,distinctfromthetrainingexamples,toevaluatetheutilityof post- pruningnodes from the tree
- Usealltheavailabledatafortraining,butapply*astatisticaltest*toestimatewhetherexpandi ng pruning) a particular node is likely to produce an improvement beyond thetrainingset
- Usemeasureofthecomplexityforencodingthetrainingexamplesandthedecisiontree,h alting growth of the tree when this encoding size is minimized. This approach is theMinimumDescription Length (or called
  - *MDL– Minimize:size(tree)+size(misclassifications(tree))*

Reduced-ErrorPruning

- ***Reduced-error pruning***, is to consider each of the decision nodes in the tree to becandidates for pruning
- ***Pruning*** a decision node consists of removing the subtree rooted at that node,making it a leaf node, and assigning it the most common classification of thetrainingexamples affiliated with that node
- Nodes are removed only if the resulting pruned tree performs no worse than-theoriginalover the validation set.
- Reduced error pruning has the effect that any leaf node added due to coincidentalregularitiesinthetrainingsetislikelytobeprunedbecausethesesamecoincidencesare unlikelyto occur inthe validation set

Theimpactofreduced-errorpruningontheaccuracyofthedecisiontreeisillustratedinbelowfigure



- The additional line in figure shows ***accuracy over the test examples*** as the tree is pruned. Whenpruning begins, the tree is at its maximum size and lowest accuracy over the test set. As pruningproceeds,the numberofnodes isreduced andaccuracy overthetestset increases.
- The available data has been split into three subsets: the training examples, the validation examplesused for pruning the tree, and a set of test examples used to provide an unbiased estimate ofaccuracyoverfutureunseen examples.Theplotshowsaccuracyoverthetrainingandtestsets.

Summary

- Statisticaltheoryprovidesabasisforestimatingthetrueerror ($error_D h$)ofhypothesish,basedonitsobservederror($error_S h$) overasampleSofdata.
- In general, the problem of estimating confidence intervals isapproachedbyidentifyingtheparametertobeestimated($error_D h$)and anestimator($error_S h$)forthisquantity.
- Becausetheestimatorisarandomvariableitcanbecharacterisedbyt heprobability distributionthat governs its value.

unbiasedestima$_2$tor,theobservedvalueoftheestimatorislikelyto vary fromone experimentto another.

The varianceofthedistributiongoverning theestimat
estimateislikely to orcharacteriseshowwidelythis

Confidenceintervalscanthenbecalculatedbydeterminingthein
terval that containsthe desired probability mass under
thisdistribution.

Acause ofestimationerroristhevarianceintheestimate.Evenwithan

Prodigy and Explanation Based Learning

Prodigy defines a set of target concepts to learn, e.g., which operator given the current state takes you to the goal state?

An example of a rule learned by Prodigy in the block-stacking problem is:

IF One subgoal to be solved is On(x,y) AND
One subgoal to be solved is On(y,z)
THEN Solve the subgoal On(y,z) before On(x,y)

Prodigy and Explanation Based Learning

The rationale behind the rule is that it would avoid a conflict when stacking blocks.

Prodigy learns by first encountering a conflict, then explaining the reason for the conflict and creating a rule like the one above.

Experiments show an improvement in efficiency by a factor of two to four.

Problems with EBL

- ✓ The number of control rules that must be learned is very large.
- ✓ If the control rules are many, much time will be spent looking for the best rule.

Utility analysis is used to determine what rules to keep and what rules to forget.

Prodigy:

328 possible rules ⟶ 69 pass test ⟶ 19 were retained

Problems with EBL

- ✓ Another problem with EBL is that it is sometimes intractable to create an explanation for the target concept.

  For example, in chess, learning a concept like:
  "states for which operator A leads to a solution"
  The search here grows exponentially.

Summary

- Different from inductive learning, analytical learning looks for a hypothesis that fit the background knowledge and covers the training examples.

- Explanation based learning is one kind of analytical learning that divides into three steps:
  a. Explain the target value for the current example
  b. Analyze the explanation (generalize)
  c. Refine the hypothesis

Summary

- Prolog-EBG constructs intermediate features after analyzing examples.
- Explanation based learning can be used to find search control rules.
- In all cases we depend on a perfect domain theory.

Machine Learning

Chapter 12. Combining Inductive and Analytical Learning

Tom M. Mitchell

Inductive and Analytical Learning

Inductive learning

- Hypothesis fits data
- Statistical inference
- Requires little prior knowledge
- Syntactic inductive bias

Analytical learning

- Hypothesis fits domain the
- Deductive inference
- Learns from scarce data
- Bias is domain theory

What We Would Like

Inductive learning ⟵ Analytical learning

Plentiful data
No prior knowledge

Perfect prior knowledge
Scarce data

General purpose learning method:

- No domain theory → learn as well as inductive methods
- Perfect domain theory → learn as well as Prolog-EBG
- Accomodate arbitrary and unknown errors in domain theory
- Accomodate arbitrary and unknown errors in training data

Domain theory:

Cup ← Stable, Liftable, Open Vessel

Stable ← BottomIsFlat

Liftable ← Graspable, Light

Graspable ← HasHandle

Open Vessel ← HasConcavity, ConcavityPointsUp

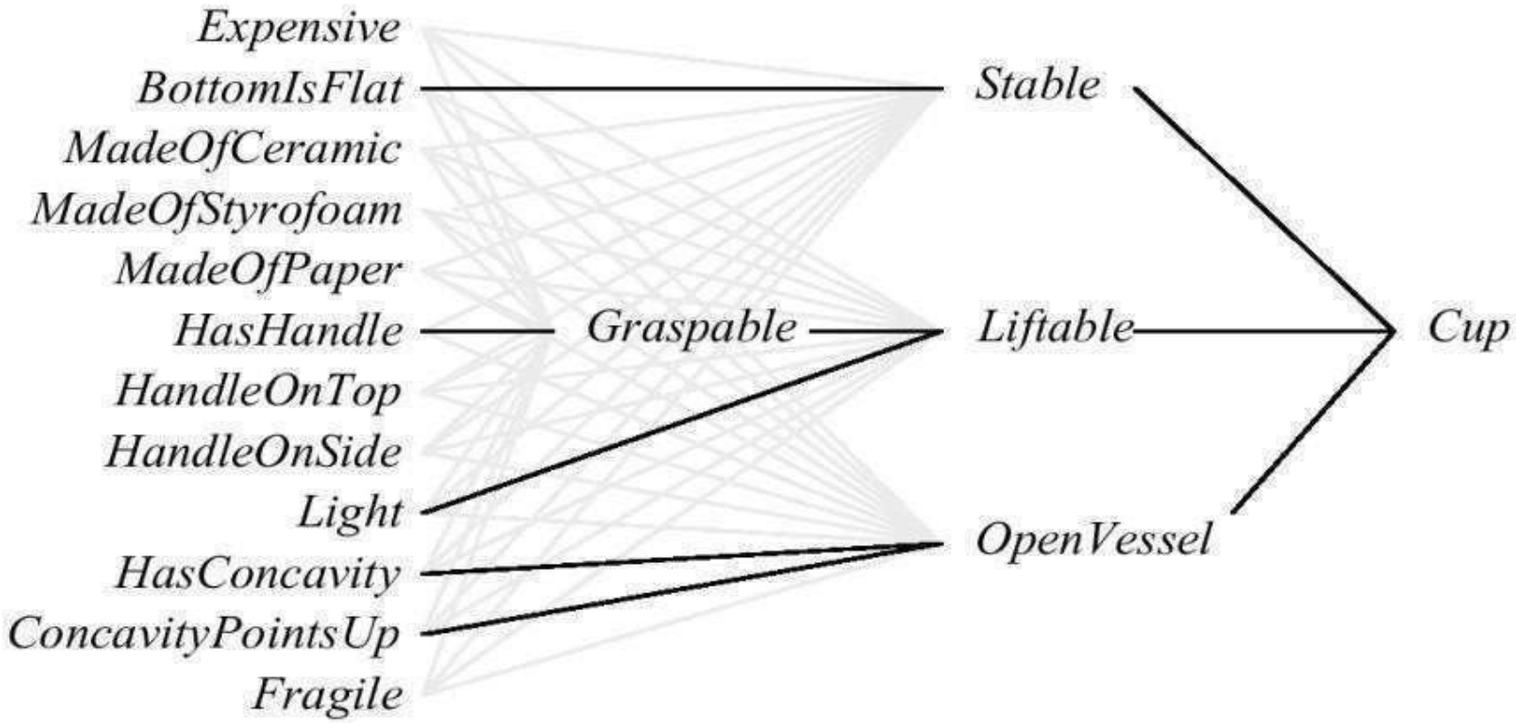
Training examples:

	Cups				Non-Cups					
BottomIsFlat	✓	✓	✓	✓	✓	✓	✓			✓
ConcavityPoints Up	✓	✓	✓	✓	✓		✓	✓		
Expensive	✓		✓				✓		✓	
Fragile	✓	✓			✓	✓		✓		✓
HandleOnTop					✓		✓			
HandleOnSide	✓			✓					✓	
HasConcavity	✓	✓	✓	✓	✓		✓	✓	✓	✓
HasHandle	✓			✓	✓		✓		✓	
Light	✓	✓	✓	✓	✓	✓	✓		✓	
MadeOfCeramic	✓				✓		✓	✓		
MadeOfPaper				✓					✓	
MadeOfStyrofoam		✓	✓			✓				✓

KBANN

- KBANN (data D, domain theory B)
  1. Create a feedforward network h equivalent to B
  2. Use BACKPROP to tune h to t D

Neural Net Equivalent to Domain Theory



Creating Network Equivalent to Domain Theory

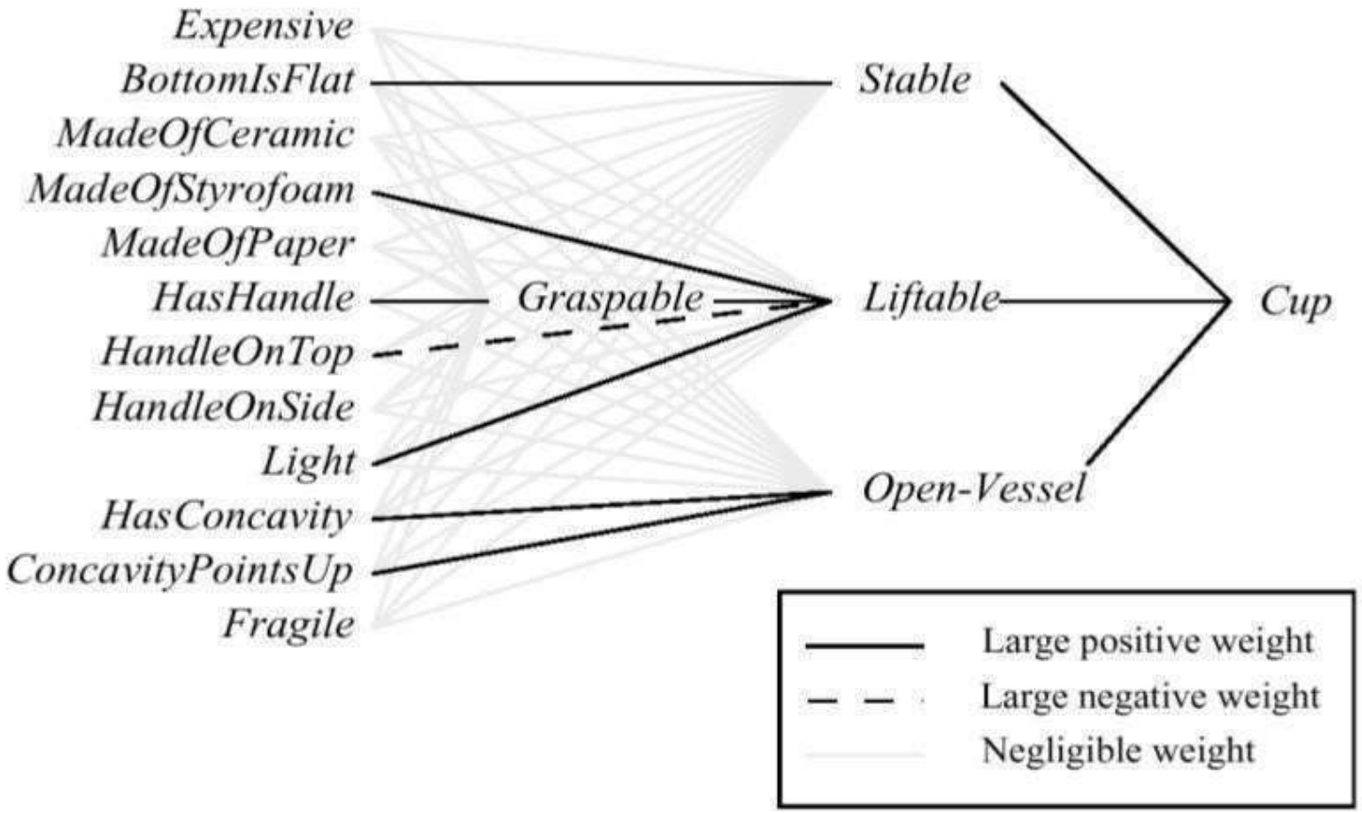
Create one unit per horn clause rule (i.e., an AND unit)

- Connect unit inputs to corresponding clause antecedents
- For each non-negated antecedent, corresponding input weight $w \leftarrow W$, where W is some constant
- For each negated antecedent, input weight $w \leftarrow -W$
- Threshold weight $w_0 \leftarrow -(n-.5)W$, where n is number of non-negated antecedents

Finally, add many additional connections with near-zero weights

$$Liftable \leftarrow Graspable, \neg Heavy$$

Result of refining the network



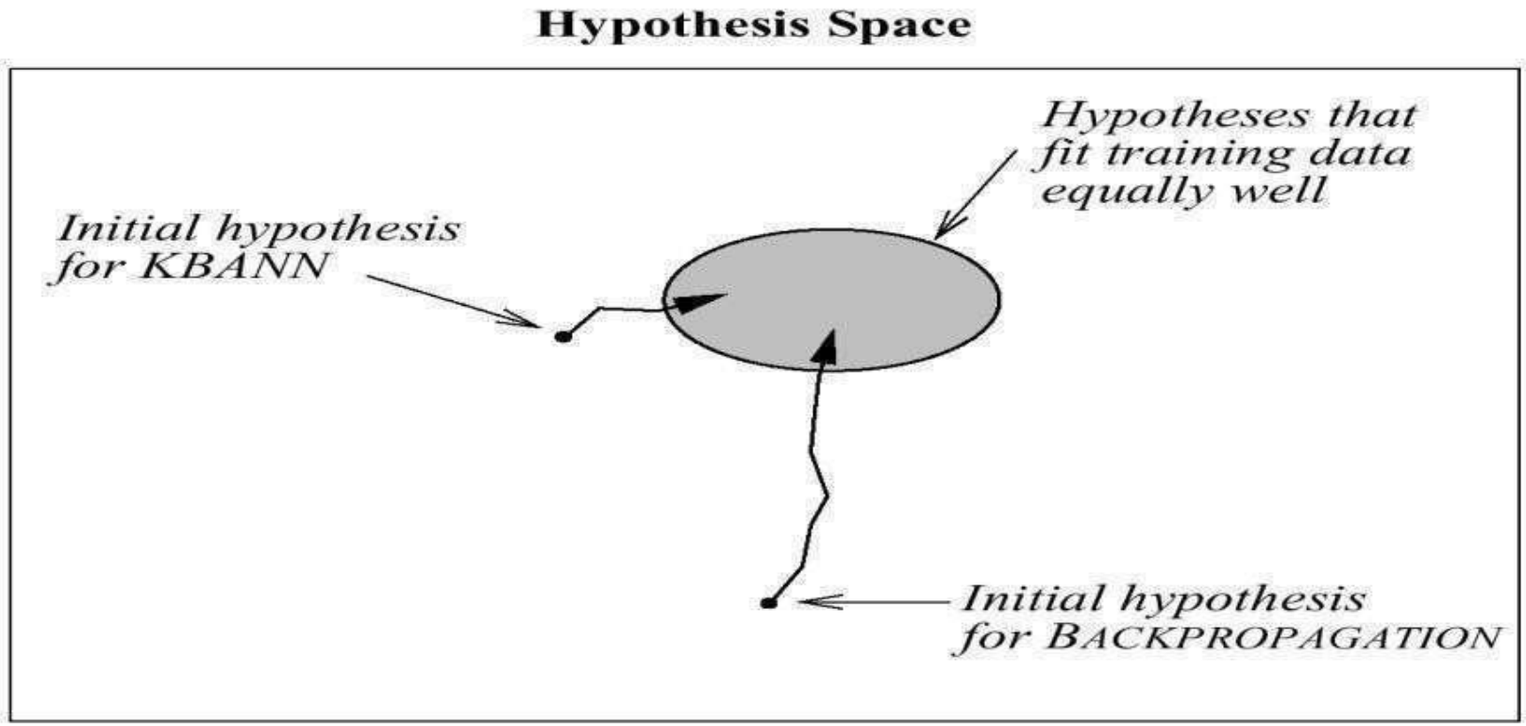
KBANN Results

Classifying promoter regions in DNA leave one out testing:

- Backpropagation : error rate 8/106
- KBANN: 4/106

Similar improvements on other classification, control tasks.

Hypothesis space search in KBANN

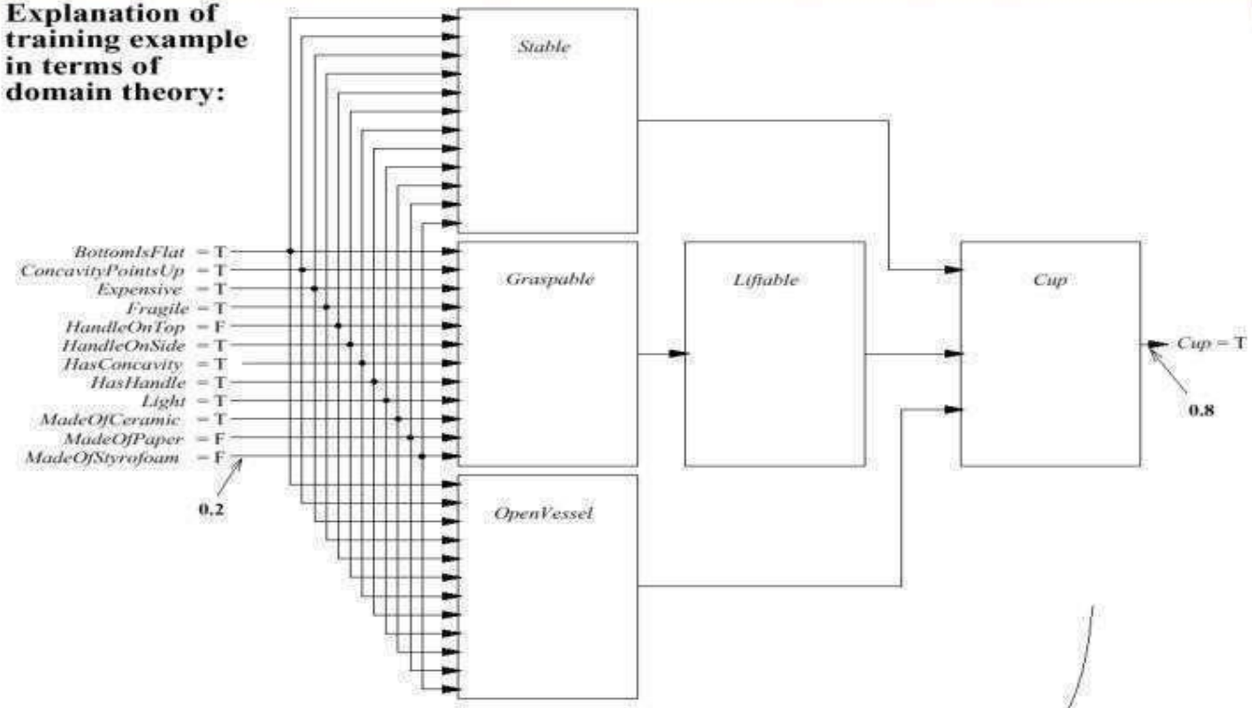


EBNN

Key idea:

- Previously learned approximate domain theory
- Domain theory represented by collection of neural networks
- Learn target function as another neural network

Explanation of training example in terms of domain theory:



Training derivatives

Target network:

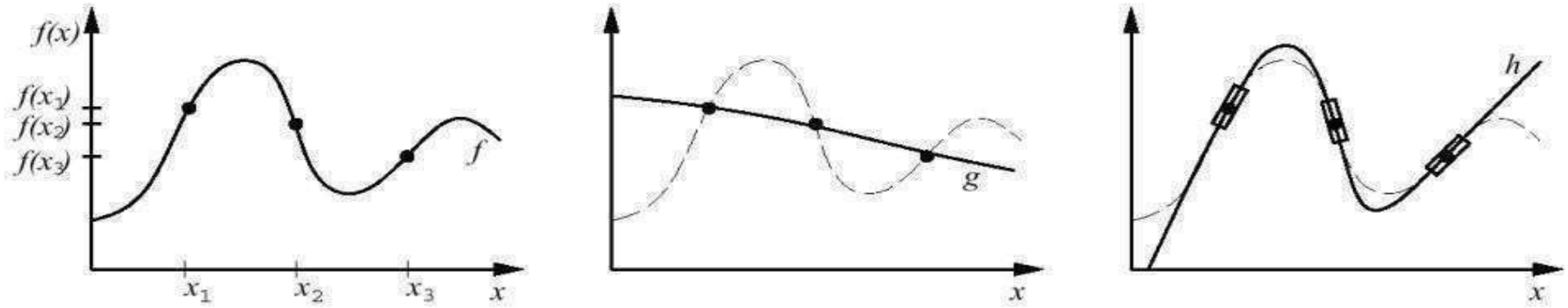
Modified Objective for Gradient Descent

$$E = \sum_i \left[(f(x_i) - \hat{f}(x_i))^2 + \mu_i \sum_j \left(\frac{\partial A(x)}{\partial x^j} - \frac{\partial \hat{f}(x)}{\partial x^j} \right)^2_{(x=x_i)} \right]$$

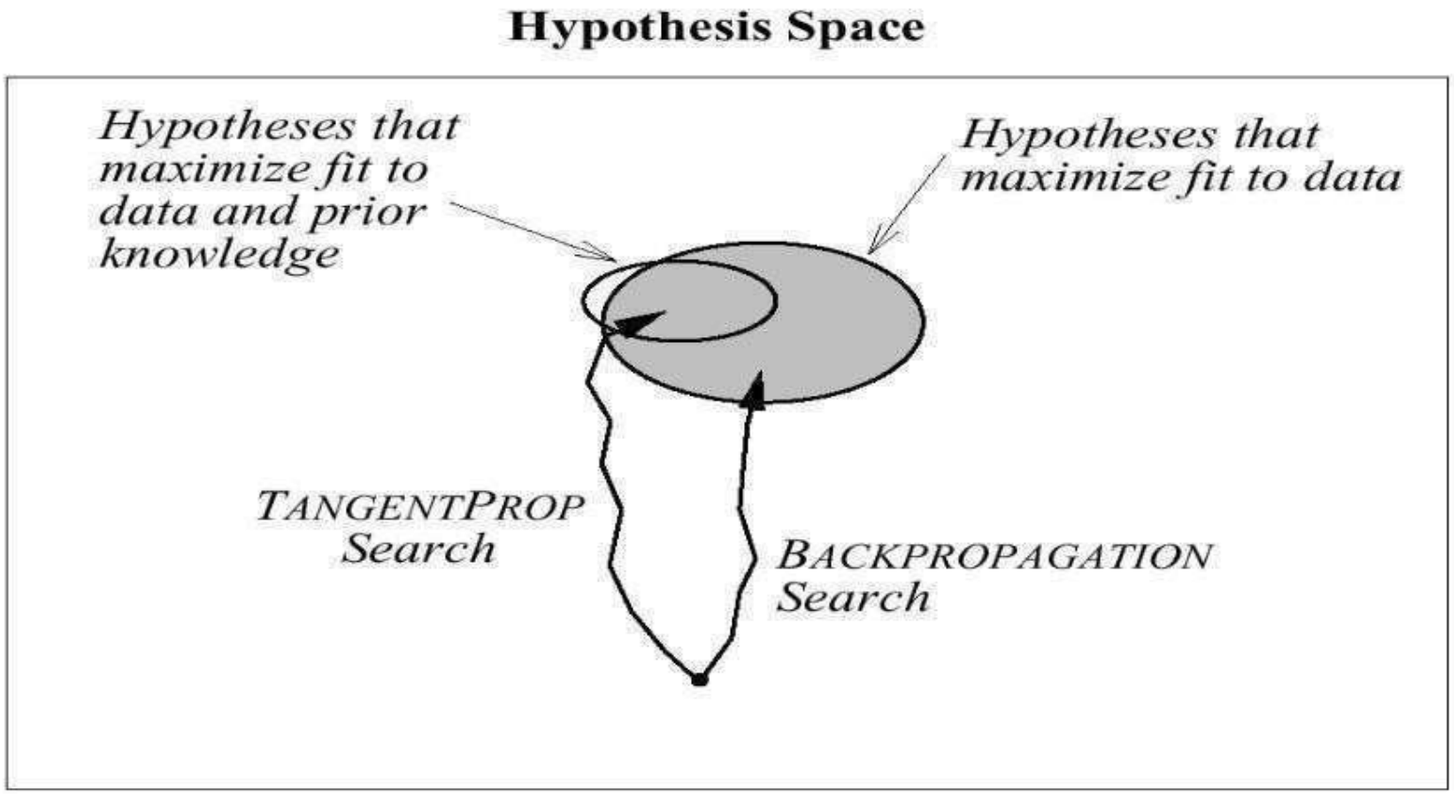
where

$$\mu_i \equiv 1 - \frac{|A(x_i) - f(x_i)|}{c}$$

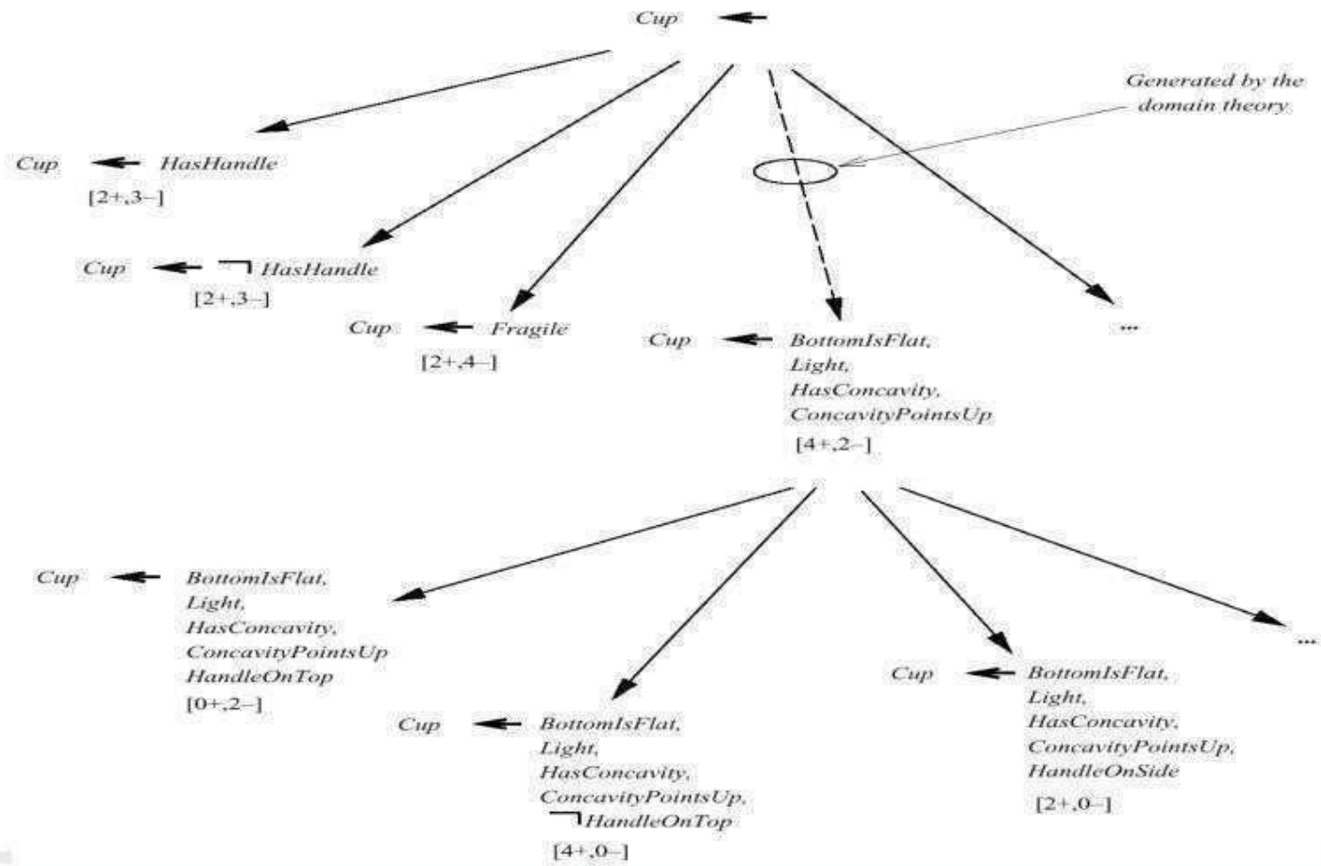
- $f(x)$ is target function
- $\hat{f}(x)$ is neural net approximation to $f(x)$
- $A(x)$ is domain theory approximation to $f(x)$


f(x)
f(x1)
f(x2)
f(x3)
f
x1
x2
x3
x
g
x
h
x

Hypothesis Space Search in EBNN



Search in FOCL



THANK YOU