UNIT-4:COMPLEX VARIABLES

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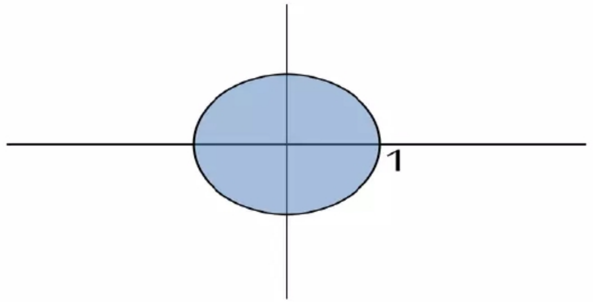
Open Disks or Neighborhoods

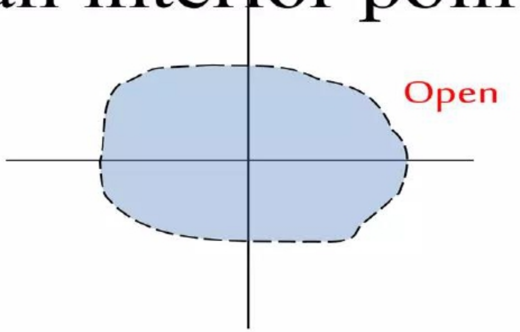
Definition: The set of all points z which satisfy the inequality $|z - z_0| < \rho$, where ρ is a positive real number is called an open disc or neighborhood of z_0.

Remark: The unit disk, i.e., the neighborhood $|z| < 1$, is of particular significance.



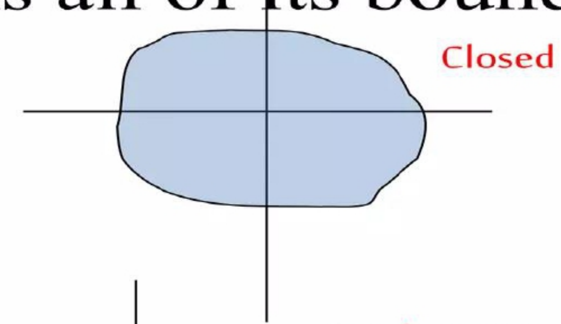
Open Set:

Definition. If every point of a set S is an interior point of S, we say that S is an *open set*.



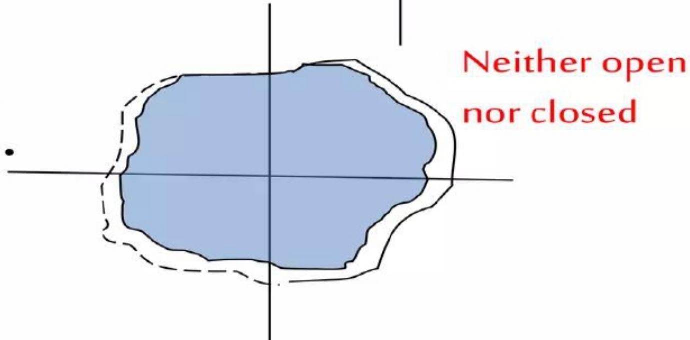
Closed Set:

Definition. If $B(S) \subset S$, i.e., if S contains all of its boundary points, then it is called a *closed set*.



Neither open nor closed:

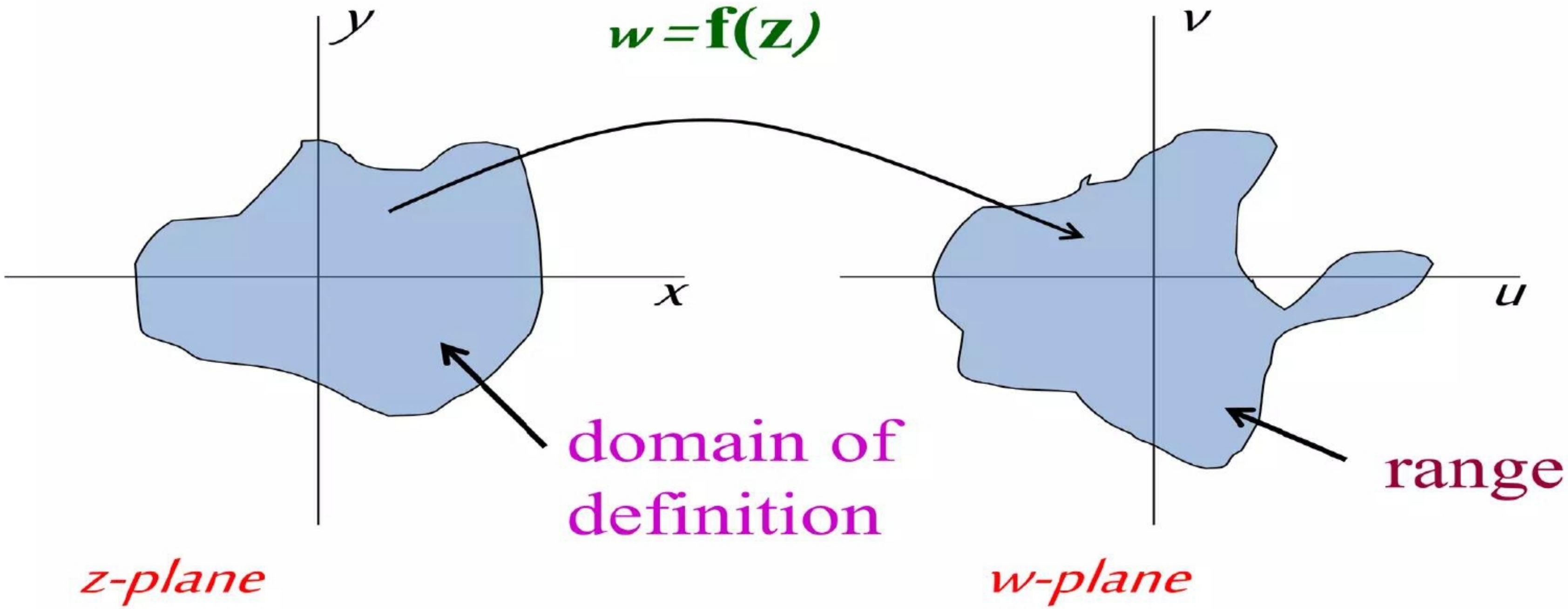
Sets may be neither open nor closed.



Complex Function:

- **Definition:** Complex function of a complex variable. Let $\Phi \subset C$. A *function f* defined on Φ is a rule which assigns to each $z \in \Phi$ a complex number w. The number w is called a *value of f at z* and is denoted by $f(z)$, i.e.,
  $w = f(z)$.
  The set Φ is called the *domain of definition of f*. Although the domain of definition is often a domain, it need not be.

Graph of Complex Function:

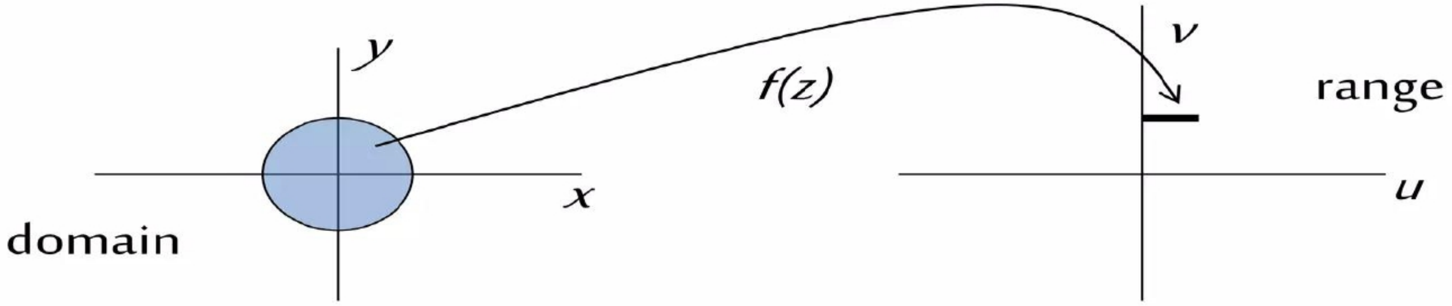


Example :

➢ Describe the range of the function $f(z) = x^2 + 2i$, defined on (the domain is) the unit disk $|z| \leq 1$.

Solution : We have $u(x\ ,\ y) = x^2$ and $v(x\ ,\ y) = 2$. Thus as z varies over the closed unit disk, u varies between 0 and 1, and v is constant (=2).

Therefore $w = f(z) = u(x\ ,\ y) + iv(x\ ,\ y) = x^2 + 2i$ is a line segment from $w = 2i$ to $w = 1 + 2i$.



Analytic function

Definition : A complex function is said to be analytic on a region if it is complex differentiable at every point.

- The terms holomorphic function , differentiable function, and complex differentiable function are sometimes used interchangeably with "analytic function“
- If a complex function is analytic on a region, it is infinitely differentiable.
- A complex function may fail to be analytic at one or more points through the presence of singularities, or along lines or line segments through

- A necessary condition for a complex function to be analytic is

  Let f(x, y) = u(x, y) + i v(x, y) be a complex function.

  Since x = $(\frac{z+\bar{z}}{2})$ and y = $(\frac{z-\bar{z}}{2})$

  substituting for x and y gives ,

  $f(z,\bar{z}) = u(x, y) + iv(x, y)$.

A necessary condition for $f(z, \bar{z})$ to be analytic is

$\partial f / \partial z = 0$.

Therefore a necessary condition for f = u + iv to be analytic is that f depends only on z.

In terms of the of the real and imaginary parts u, v of f

$\partial u / \partial x = \partial v / \partial y$

$\partial u / \partial y = - \partial v / \partial x$

The above equations are known as the **Cauchy-Riemann** equations.

Necessary and sufficient conditions for a function to be analytic

- The necessary and sufficient conditions for a function $f = u + iv$ to be analytic are that:
- 1. The four partial derivatives of its real and imaginary parts

$$\partial u/\partial x,\ \partial v/\partial x,$$
$$\partial u/\partial y,\ \partial v/\partial y$$

satisfy the Cauchy-Riemann equations

2. The four partial derivatives of its real and imaginary parts $\partial u/\partial x$, $\partial v/\partial y$, $\partial u/\partial y$, $\partial v/\partial x$ are continuous.