UNIT-II

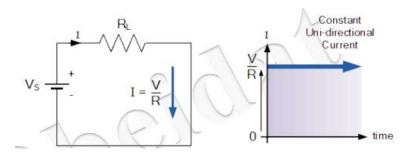
A.C Circuits

Topics

- Introduction to sinusoidal waveforms
- Phasor representation
- The concept of power and power factor
- Analysis of 1-phase RLC series and parallel circuits
- Resonance in series R-L-C circuit
- Three-phase balanced circuits
- voltage and current relations in star and delta connections

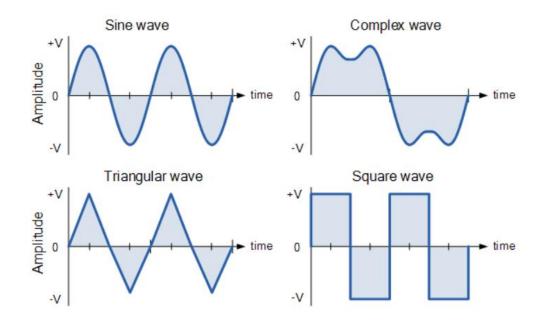
Introduction:

 A DC voltage or current has a fixed magnitude (amplitude) and a definite direction associated with it. And do not change their values with regards to time.



- The term **alternating indicates** only that the **waveform alternates between two prescribed levels** in a set time sequence and periodic in nature.
- A function f(t) is said to be periodic if it satisfies f(t) = f(t + nT) for all t and for all integers n.

Types of ac waveforms



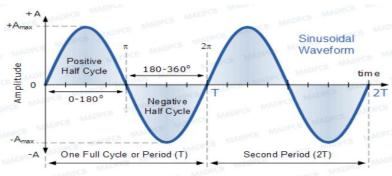
Terminology:

- **Amplitude**: The maximum value of the waveform from its zero position is known as amplitude.
- Time period/periodic time (T): The time taken by the waveform to complete one cycle is known as time period and is measured in seconds.
- Frequency (f): The number of times the waveform repeats itself within a one second time period.
- Cycle: one complete set of positive and negative values of alternating quantity. One cycle corresponds to 360 degrees electrical or 2π radians

Frequency,
$$(f) = \frac{1}{\text{Periodic Time}} = \frac{1}{\text{T}}$$
 Hertz

or

Periodic Time, (T) =
$$\frac{1}{\text{Frequency}} = \frac{1}{f}$$
 seconds



Mathematically a sine wave can be represented as

$$V(t) = V_m \sin \omega t$$

V(t) = instantaneous voltage at a specific time t

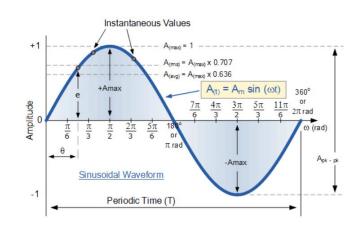
Vm = the maximum or peak voltage of the waveform.

 ω = the angular velocity =2 π f radians per second

Radians =
$$\left(\frac{\pi}{180^{\circ}}\right) \times (\text{degrees})$$

Degrees =
$$\left(\frac{180^{\circ}}{\pi}\right) \times \text{(radians)}$$

 $sin(\omega t)$ = the voltage varies sinusoidally over time



- Instantaneous value: The value of an alternating quantity at any instant of time.
- **Peak value**: The maximum value of the wave during positive half cycle or maximum value of the wave during negative half cycle.
- Peak to peak value: the value of an ac waveform from positive peak to negative peak. (volts or amperes)

Average value: the average of all the instantaneous values of an alternating quantity over one half cycle is known as average value.

• For a symmetrical AC waveform, the average value over a full cycle is zero because the positive and negative

half-cycles cancel each other.

• Used in Analysis of rectifiers, pulsating DC signals

mathematically,

For non sine wave

$$V_{avg} = \frac{\text{sum of all the mid-ordinates}}{\text{number of mid-ordinates}}$$

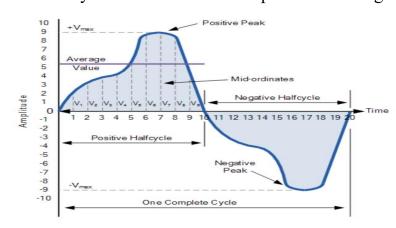
$$V_{avg} = \frac{\bigvee_1 + \bigvee_2 + \bigvee_3 + \bigvee_4 + \ldots + \bigvee_n}{n}$$

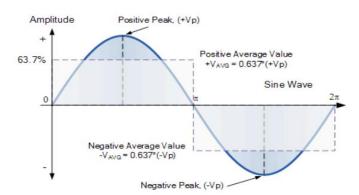
For sinusoidal waveform

Vavg =
$$\frac{1}{T} \int_{0}^{T} Vm \sin \omega t$$

Where $T = \pi$ radians for half cycle

$$V_{avg} = \frac{1}{\pi} \int_0^{\pi} Vm \sin \omega t = 2Vm/\pi = 0.637Vm$$





Root mean square value(RMS) or effective value:

• The RMS value is the effective or DC equivalent value of an AC waveform. It represents the steady DC current or voltage that would dissipate the same amount of power as the AC source over a given time.

• Used in Power calculations, device ratings

Mathematically,

$$V_{RMS} = \sqrt{\frac{\text{sum of mid-ordinate (voltages)}^2}{\text{number of mid-ordinates}}}$$

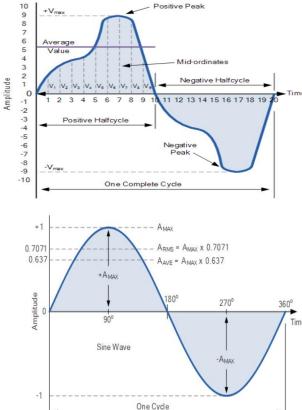
$$V_{RMS} = \sqrt{\frac{V_1^2 + V_2^2 + V_3^2 + V_4^2 + ... + V_n^2}{n}}$$

For sinusoidal waveform

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T [V(t)]^2 dt}$$

Where $V(t) = V_m \sin \omega t$ and $T = \pi$ radians for half cycle

$$V_{RMS} = \sqrt{\frac{1}{\pi} \int_{0}^{\pi} (V_{m} \sin \omega t)^{2} d(\omega t)} = \frac{V_{m}}{\sqrt{2}} = 0.7071 \text{ Vm}$$



Form factor: The ratio of its Root Mean Square (RMS) value to its average value.

Form factor =
$$\frac{V_{RMS}}{V_{avg}}$$

This dimensionless ratio indicates how closely a waveform resembles an ideal direct current (DC) signal.

It is used to identify the shape and distortion of a waveform

For a sinusoidal waveform, the form factor is approximately 1.11

Peak factor or crest factor: The ratio of the maximum (peak) value to the RMS value of an alternating quantity.

Peak factor =
$$\frac{V_{peak}}{V_{RMS}}$$

For a sinusoidal waveform, the peak factor is approximately 1.414

It indicates the potential for voltage or current spikes, which is crucial for circuit design and safety.

A higher peak factor means the waveform has a more pronounced peak relative to its RMS value

Phasor representation

A **phasor** is a rotating vector that represents a sinusoidal quantity like an alternating voltage or current. It is a convenient way to simplify calculations in AC circuits by representing **both the magnitude (length of the vector) and phase angle (direction)** of the sinusoid as a complex number, which eliminates the need to work with time-varying differential equations

The **phasor** as a rotating vector, rotates in counter-clockwise at a constant angular velocity(ω) in radians per second.

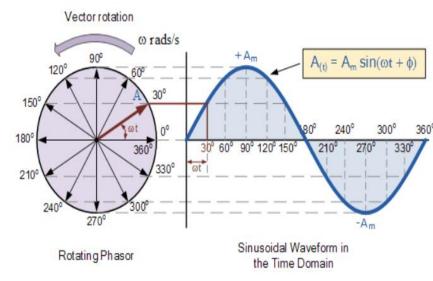
The length of the vector represents the amplitude(Am) of the sinusoidal waveform.

The sinusoidal waveform is the projection of the rotating phasor on to the vertical axis over time.

The equation for the sinusoidal waveform is given as

$$A(t)=Am \sin(\omega t+\Phi)$$

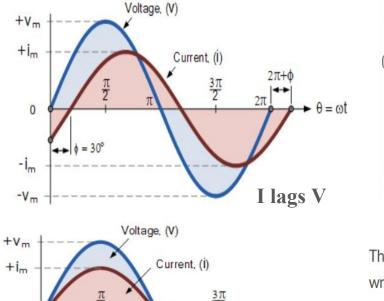
where Am= amplitude, ω = angular frequency, t= time,

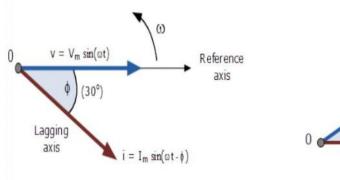


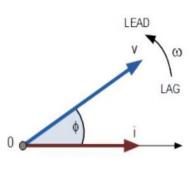
The phase angle represents the initial position of the phasor at t=0. In the diagram, the phasor starts at an angle of 30° from the horizontal axis.

Phase: the phase of sine wave is an angular measurement that specifies the position of the sine wave relative to a reference. It is measured in degrees or radians from a reference point.

Phase difference: the difference in phase between two waveforms is called phase difference.







Voltage, (V) $+i_{m}$ 0 $\frac{\pi}{2}$ π $\frac{3\pi}{2}$ 2π $\theta = \omega t$ $-i_{m}$ $-v_{m}$ I and V are in phase

The generalised mathematical expression to define these two sinusoidal quantities will be written as:

$$v_{(t)} = V_{m} \sin(\omega t)$$

$$i_{(t)} = I_{m} \sin(\omega t - \phi t)$$

Significance of operator j:

j Operator is a mathematical operator, when it is multiplied with any vector, rotates that vector by 90 degree in counter clockwise direction.

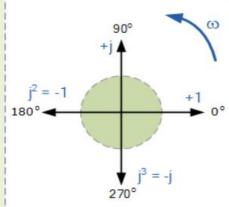
Vector Rotation of the j-operator



180° rotation:
$$j^2 = (\sqrt{-1})^2 = -1$$

180° rotation:
$$j^2 = (\sqrt{-1})^2 = -1$$
270° rotation: $j^3 = (\sqrt{-1})^3 = -j$

$$360^{\circ}$$
 rotation: $j^4 = \left(\sqrt{-1}\right)^4 = \pm 1$



Phasors are expressed in any of the following forms:

1. Rectangular or complex or cartesian form

Where:

Z - is the Complex Number representing the Vector

X - is the Real part or the Active component

y - is the Imaginary part or the Reactive component

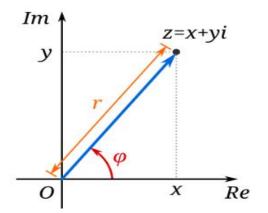
 $j - is defined by \sqrt{-1}$

1. Polar form:

Phasor $z=r\angle \phi$

Where
$$r = |z| = \sqrt{x^2 + y^2}$$
 and $\phi = \tan^{-1} \frac{y}{x}$

- 1. Trigonometric form
- 2. Exponential form



Phasor Diagrams Summary

phasor diagrams are a projection of a rotating vector onto a horizontal axis which represents the instantaneous value. As phasor diagrams can be drawn to represent any instant of time and therefore any angle, the reference phasor of an alternating quantity is always drawn along the positive x-axis direction.

- Vectors, Phasors and **Phasor Diagrams** ONLY apply to sinusoidal AC alternating quantities.
- Phasor Diagrams can be used to represent two or more stationary sinusoidal quantities at any instant in time.
- Generally the reference phasor is drawn along the horizontal axis and at that instant in time the other phasors are drawn.

 All phasors are drawn referenced to the horizontal zero axis.
- Phasor diagrams can be drawn to represent more than two sinusoids. They can be either voltage, current or some other alternating quantity but the frequency of all of them **must be the same**.
- All phasors are drawn rotating in an anticlockwise direction. All the phasors ahead of the reference phasor are said to be "leading" while all the phasors behind the reference phasor are said to be "lagging".
- Generally, the length of a phasor represents the r.m.s. value of the sinusoidal quantity rather than its maximum value.
- Sinusoids of different frequencies cannot be represented on the same phasor diagram due to the different speed of the vectors. At any instant in time the phase angle between them will be different.
- Two or more vectors can be added or subtracted together and become a single vector, called a **Resultant Vector**.

Single Phase AC circuit:

1.AC circuit with a pure resistance:

Consider an electrical circuit consists of a pure resistance R with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 19. The current i flowing in the circuit is expressed as

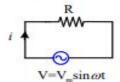


Figure 1

$$i = \frac{V_m sin\omega t}{R} = I_m sin\omega t$$

where $I_m = \frac{V_m}{B}$

The instantaneous power consumed by the resistance R in the above circuit is

$$P = vi = (V_m sin\omega t)(I_m sin\omega t)$$

$$= V_m I_m sin^2(\omega t)$$

$$= V_m I_m \frac{1 - cos2\omega t}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t$$

The equation consists of two terms. The first term is called as the constant power term. The second term is consists of $\frac{V_m I_m}{2} cos2\omega t$ which is periodically varying with frequency 2ω , twice the input frequency. The average power over a period of time is zero.

$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t$$

$$P_{av} = \int_0^{2\pi} \left(\frac{V_m I_m}{2} - \frac{V_m I_m}{2} cos2\omega t\right)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \int_0^{2\pi} \frac{1}{2\pi} (cos2\omega t) d\omega t$$

$$= \frac{V_m I_m}{2} - 0$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$= VI$$

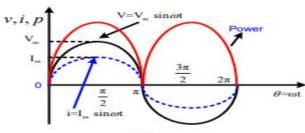


Figure 2

2.AC circuit with a pure Inductance:

Consider an electrical circuit consists of a inductor L with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 3. The current i flowing in the circuit is expressed as



Figure 3

$$V_L = -L \frac{dl}{di}$$

$$\begin{split} dl &= \frac{V_L}{L}dt = \frac{1}{L}V_m sin\omega t \ dt \\ i &= \frac{V_m}{L}\int sin\omega t \ dt \\ &= \frac{V_m}{\omega L}(-cos\omega t) \\ &= \frac{V_m}{X_L} sin(\omega t - \frac{\pi}{2}) \\ &= I_m sin\left(\omega t - \frac{\pi}{2}\right) \end{split}$$

The instantaneous power consumed by the inductance L in the above circuit is

$$P = vi = (V_m sin\omega t)I_m sin\left(\omega t - \frac{\pi}{2}\right)$$

 $= V_m I_m sin(\omega t)(-cos\omega t)$
 $= -\frac{1}{2}V_m I_m sin2\omega t$

The power consumed by the inductance consists of $\frac{V_m I_m}{2} cos 2\omega t$ which is periodically varying with frequency 2ω , twice the input frequency.

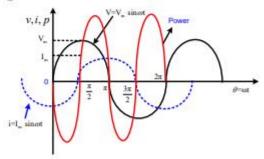


Figure 4

3. AC circuit with a pure capacitance:

Consider an electrical circuit consists of a capacitor C with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 3. The current i flowing in the circuit is expressed as



Figure 5

$$i = \frac{dq}{dt} = \frac{dCv}{dt}$$

$$= C\frac{d}{dt}V_m sin\omega t$$

$$= \omega CV_m cos\omega t$$

$$= \frac{V_m}{1/\omega C} sin(\omega t + \pi/2)$$

$$= \frac{V_m}{XC} sin(\omega t + \pi/2)$$

$$= I_m sin(\omega t + \pi/2)$$

where
$$I_m = \frac{V_m}{X_C}$$
, $X_C = \frac{V_m}{\omega C}$

The instantaneous power consumed by the inductance L in the above circuit is

$$P = vi = (V_m sin\omega t)I_m sin\left(\omega t - \frac{\pi}{2}\right)$$

 $= V_m I_m sin(\omega t)(-cos\omega t)$
 $= -\frac{1}{2}V_m I_m sin2\omega t$

The instantaneous power is.

$$P = vi = (V_m sin\omega t)I_m sin\left(\omega t + \frac{\pi}{2}\right)$$

 $= V_m I_m sin\omega t cos\omega$
 $= \frac{1}{2}V_m I_m sin2\omega t$

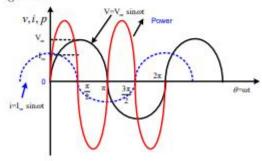


Figure 6

Series R-L circuit:

Consider an electrical circuit consists of a resistor R and inductor L connected in series with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 7. The voltage across the resistor is V_R and across the inductor is V_L .

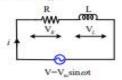
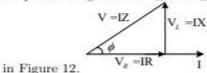


Figure 7

The current flowing in the network I, the voltage across resistor V_R is in phase with I and the voltage across inductor V_L leads the current by 90°. The phasor diagram of I V_R are V_L as shown



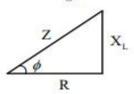


Figure 8

Figure 9

From the phasor diagram, the the resultant voltage V can be expressed as

$$V = \sqrt{V_R^2 + V_L^2}$$

 $= \sqrt{(IR)^2 + (IX_L)^2}$
 $= I\sqrt{R^2 + X_L^2}$
 $= I\sqrt{R^2 + X_L^2}$
 $= IZ$

where Z is

$$Z = \sqrt{R^2 + X_L^2}$$

The phase angle Φ between the voltage and and the current is

$$\Phi = tan^{-1}\frac{X_L}{R}$$

The instantaneous power consumed by the circuit is

$$\begin{split} P &= vi = (V_m sin\omega t)I_m sin \left(\omega t - \phi\right) \\ &= \frac{1}{2}V_m I_m [cos\phi - cos(2\omega t - \phi) \\ &= \frac{1}{2}V_m I_m cos\phi - \frac{1}{2}V_m I_m cos(2\omega t - \phi) \end{split}$$

The second term is periodically varying quantity and its frequency is twice the applied frequency. Average of the power is zero. The first term is called as the constant power term which represents the power consumed in the circuit.

$$P = \frac{1}{2}V_mI_mcos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}cos\phi$$
$$= VIcos\phi$$

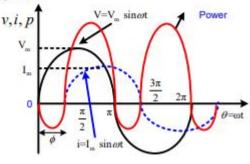


Figure 10

Series R-C circuit:

Consider an electrical circuit consists of a resistor R and capacitor C are connected in series with an alternating voltage $V = V_m sin\omega t$ as shown in the Figure 7. The voltage across the resistor is V_R and across the capacitor is V_C .

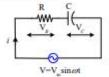


Figure 11

The current flowing in the network I, the voltage across resistor V_R is in phase with I and the voltage across capacitor V_C lags the current by 90°. The phasor diagram of I, V_R and V_L are as shown in Figure ?? (a).

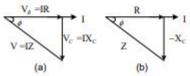


Figure 12

From the phasor diagram, the the resultant voltage V can be expressed as

$$V = \sqrt{V_R^2 + V_C^2}$$

 $= \sqrt{(IR)^2 + (-IX_C)^2}$
 $= I\sqrt{R^2 + X_C^2}$
 $= I\sqrt{R^2 + X_C^2}$
 $= IZ$

where Z is

$$Z = \sqrt{R^2 + X_C^2}$$

The phase angle Φ between the voltage and and the current is

$$\Phi = tan^{-1}\frac{X_C}{R}$$

The instantaneous power consumed by the circuit is

$$\begin{split} P &= vi = (V_m sin\omega t)I_m sin\left(\omega t + \phi\right) \\ &= \frac{1}{2}V_m I_m [cos\phi - cos(2\omega t - \phi) \\ &= \frac{1}{2}V_m I_m cos\phi - \frac{1}{2}V_m I_m cos(2\omega t - \phi) \end{split}$$

The second term is periodically varying quantity and its frequency is twice the applied frequency. Average of the power is zero. The first term is called as the constant power term which represents the power consumed in the circuit.

$$P = \frac{1}{2}V_mI_mcos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}cos\phi$$

= $VIcos\phi$

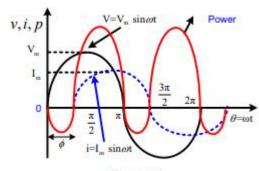


Figure 13

Introduction to three phase system



Why three phase system?

❖Any system utilizing more than one winding is called **Poly phase system**

❖Phase angle (Φ)between adjacent phases

- \bullet In 3 phase system Φ = 360/3 = 120
- **❖**In 4 phase system Φ = 360/4 = 90
- **❖In N phase system \Phi= 360/N (Where N = no of phases)**
- ❖ In three phase system, 3 independent winding are displaced by 120°

- Designing of high rated machines are complicated in single phase system.
- Three phase motors has good power factor and efficiency
- For power generation and power transmission three phase system is more convenient

Generation of 3- φ EMF and phasor diagram

$$\begin{split} V_{RN} &= V_M \sin(\theta) = V_M \sin(\omega t) \\ V_{YN} &= V_M \sin(\theta - 120) = V_M \sin(\omega t - 120) - \text{emf eqns} \\ V_{RN} &= V_M \sin(\theta - 240) = V_M \sin(\omega t - 240) \end{split}$$

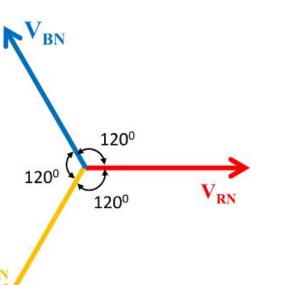
For a balanced supply RMS Value of all the

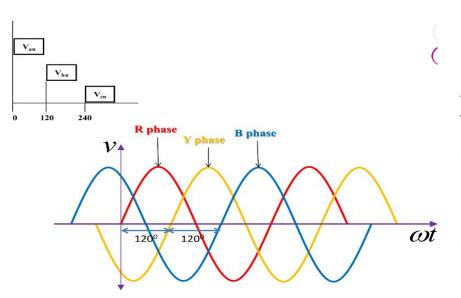
 $\left|V_{RN}\right| = \left|V_{YN}\right| = \left|V_{BN}\right| = \frac{V_M}{\sqrt{2}}$

Voltages in polar form

$$V_{\scriptscriptstyle RN}\langle 0 \ V_{\scriptscriptstyle YN}\langle -120 \$$

 $V_{\scriptscriptstyle RM}\langle -240$

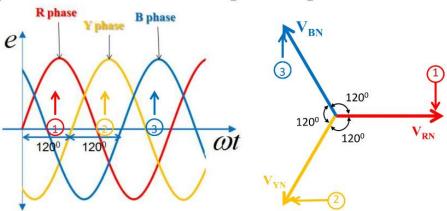




3 phase voltage

Phase sequence

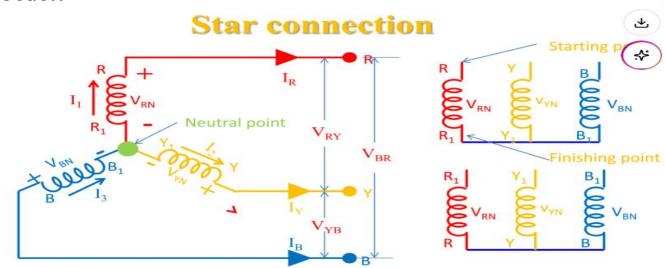
The order in which the phase voltages of a three phase system attains their peak or maximum positive values is called the phase sequence



The phase sequence is RYB

Two possible connections are

- i. Star connection
- ii. Delta connection



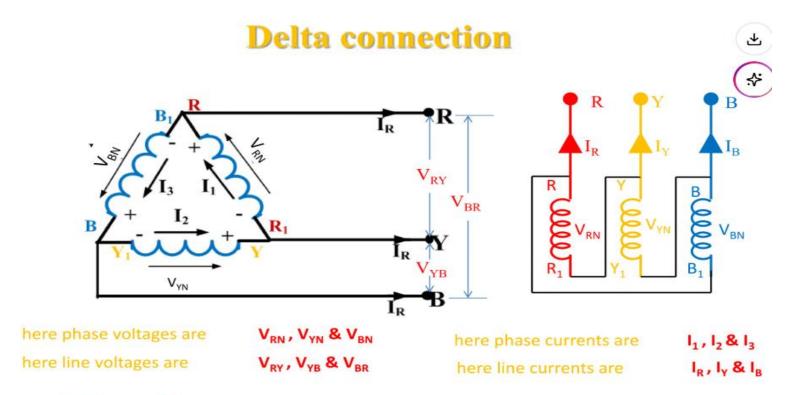
- Reduced total no of conductors
- Neutral point or star point





phase currents are

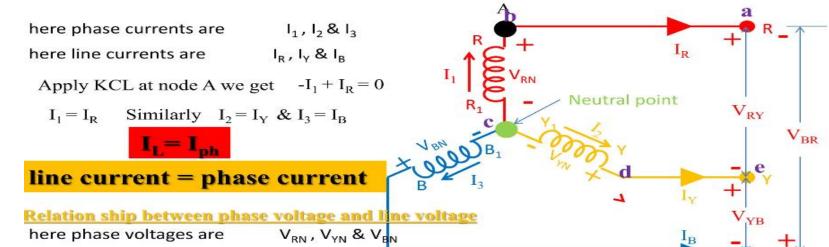




Starting end of one phase being joined to the finishing end of another phase

Line and phase quantities in star connected circut

Relation ship between phase current and line current



Vpv, VvR & VRR

Apply KVL in loop abcdea, we get

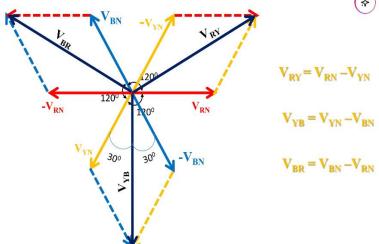
here line voltages are

$$-\mathbf{V}_{RY} + \mathbf{V}_{RN} - \mathbf{V}_{YN} = 0$$
$$\mathbf{V}_{RY} = \mathbf{V}_{RN} - \mathbf{V}_{YN}$$

Similarly
$$V_{YB} = V_{YN} - V_{BN}$$

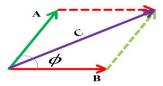
 $V_{BR} = V_{BN} - V_{RN}$

Line and phase quantities in star connected circuit



$$\vec{C} = \vec{A} + \vec{B}$$

$$C = \sqrt{A^2 + B^2 + 2 \times A \times B \times COS(\phi)}$$



If
$$A = B$$
 & $\phi = 60^{\circ}$

$$C = \sqrt{(A)^{2} + (A)^{2} + 2 \times A \times A \times COS(60)}$$

$$C = \sqrt{A^{2} + A^{2} + 2A^{2} \frac{1}{2}}$$

$$C = \sqrt{3 \times A^{2}} \qquad C = \sqrt{3} \times A$$

From phasor diagram

$$V_{YB} = \sqrt{(V_{YN})^2 + (V_{BN})^2 + 2 \times V_{YN} \times V_{BN} \times COS(60)}$$

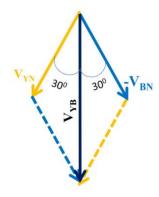
$$COS(60) = \frac{1}{2}, V_{RN} = V_{YN} = V_{BN} = V_{ph}$$

$$V_{RY} = \sqrt{(V_{ph})^2 + (V_{ph})^2 + 2 \times V_{ph} \times V_{ph} \times \frac{1}{2}}$$

$$V_{RY} = \sqrt{3} \times V_{ph}$$

Similarly

$$V_{RY} = \sqrt{3} \times V_{ph}$$
 and $V_{BR} = \sqrt{3} \times V_{ph}$



Line Voltage = √3 times of phase voltage

$$V_L = \sqrt{3} \times V_{pi}$$

Line and phase quantities in delta connected circuit



 V_{BR}

Relation ship between phase voltage and line voltage

here phase voltages are

$$V_{RN}$$
, $V_{YN} \& V_{BN}$

here line voltages are

$$V_{RY}$$
, V_{YB} & V_{BR}

Apply KVL in loop abcda, we get

$$-V_{RY}+V_{RN}=0$$

$$V_{RY} = V_{RN}$$

Similarly
$$V_{YB} = V_{YN} & V_{BR} = V_{BN}$$

Line voltage = phase voltage



Relation ship between phase current and line current

here phase currents are

$$I_1, I_2 \& I_3$$

here line currents are

$$I_R, I_Y \& I_B$$

Apply KCL at node 'b', we get

$$I_R - I_1 + I_3 = 0$$

$$I_R = I_1 - I_3$$

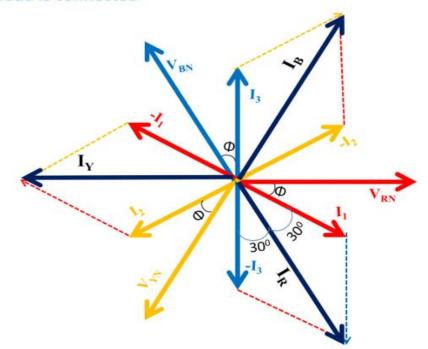
Similarly
$$I_Y = I_2 - I_1 & I_B = I_3 - I_2$$

Line and phase quantities in delta connected circuit

Assume balanced RL load is connected

$$\mathbf{I}_{\mathbf{R}} = \mathbf{I}_1 - \mathbf{I}_3$$

$$\mathbf{I}_{\mathbf{Y}} = \mathbf{I}_2 - \mathbf{I}_1$$



 $\mathbf{I}_{\mathbf{B}} = \mathbf{I}_{\mathbf{3}} - \mathbf{I}_{\mathbf{2}}$

Line and phase quantities in delta connected circuit



For balanced load phase current values are equal in magnitude,

$$I_1 = I_2 = I_3 = I_{ph}$$

$$I_R = \sqrt{(I_1)^2 + (I_3)^2 + 2 \times I_1 \times I_3 \times COS(60)}$$

$$COS(60) = \frac{1}{2}, I_1 = I_2 = I_3 = I_{ph}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 \times I_{ph}^2 \times \frac{1}{2}}$$

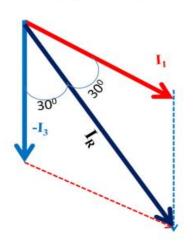
$$I_R = \sqrt{3} \times I_{ph}$$

Similarly

$$I_{Y} = \sqrt{3} \times I_{ph}$$
 and $I_{B} = \sqrt{3} \times I_{ph}$

In delta connection

Line current = $\sqrt{3}$ times of phase current





Power in three phase balanced system

Power in single phase circuit =
$$V_{ph} \times I_{ph} \times \cos \phi$$

Three phase power = $3 \times$ power per phase

Total power =
$$3 \times V_{ph} \times I_{ph} \times \cos \phi$$

In star connected system

$$V_L = \sqrt{3} \times V_{ph}$$
 and $I_L = I_{ph}$

Total power =
$$3 \times V_{ph} \times I_{ph} \times \cos \phi$$

Total power =
$$\sqrt{3} \times (\sqrt{3} \times V_{ph}) \times I_{ph} \times \cos \phi$$

Total power =
$$\sqrt{3} \times V_L \times I_L \times \cos \phi$$

Total power in star circuit

=
$$\sqrt{3} \times \text{Line voltage} \times \text{Line current} \times \text{powerfactor}$$

or
= $3 \times \text{phase voltage} \times \text{phase current} \times \text{powerfactor}$

In delta connected system

$$V_L = V_{ph}$$
 and $I_L = \sqrt{3} \times I_{ph}$

Total power = $3 \times V_{ph} \times I_{ph} \times \cos \phi$

Total power =
$$\sqrt{3} \times V_{ph} \times (\sqrt{3} \times I_{ph}) \times \cos \phi$$

Total power =
$$\sqrt{3} \times V_L \times I_L \times \cos \phi$$

Total power in delta circuit

$$= \sqrt{3} \times \text{Line voltage} \times \text{Line current} \times \text{powerfactor}$$
or
$$= 3 \times \text{phase voltage} \times \text{phase current} \times \text{powerfactor}$$