UNIT-I

DC circuits

Topics

- Basic definitions
- Ohm's law
- KVL and KCL
- superposition Theorems
- Thevenin’s theorem
- Norton’s theorem
- Transient response of RL, RC circuits

Definition of Basic Terms

- Active Element- An element which can generate or produce the energy is called as Active element.

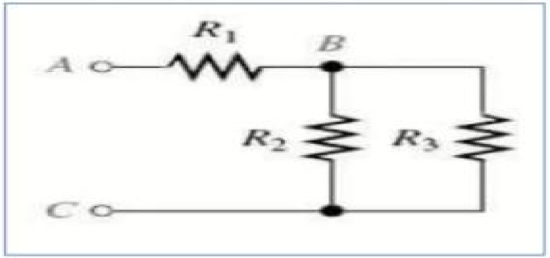
e.g.- Voltage Source, Current source, genearator etc

- Passive element- An element which can not generate energy is called as Passive element.

e.g.- Resistance, Inductance, Capacitance

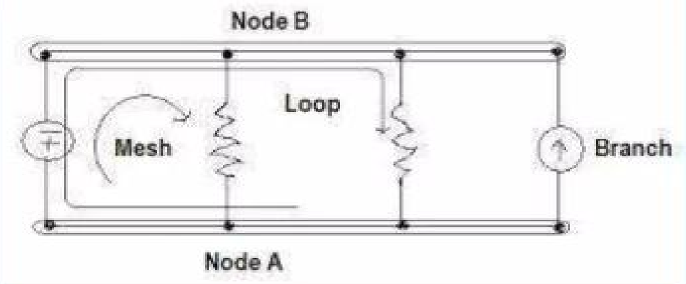
Definition of Basic Terms

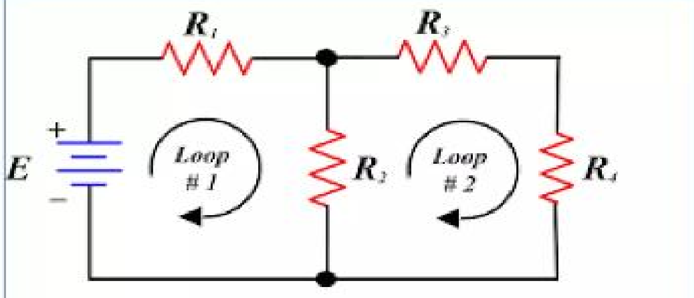
- Active Network- If a network consist of an energy source then it is called as Active network.
- Passive Network- If a network does not have an energy source then it is called as Passive network.



Definition of Basic Terms

- Branch- It is the group of elements connected between two junctions.
- Mesh/ Loop- It's the closed path formed in a network which start & end at same point
- Node/ Junction- It is the common point on the network where two or more branches are connected.





Two loop network

Definition of Basic Terms

Node, Branch, Loop, Mesh

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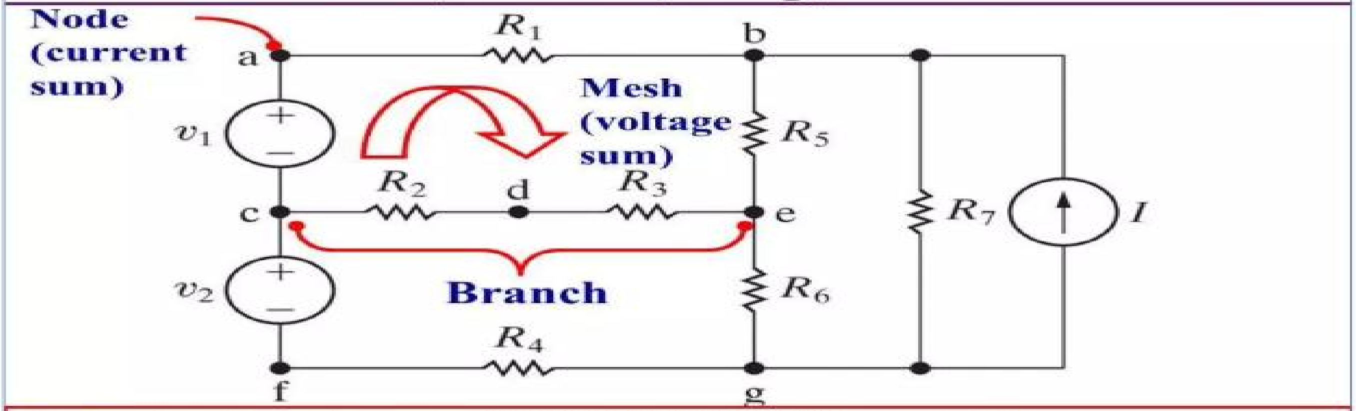


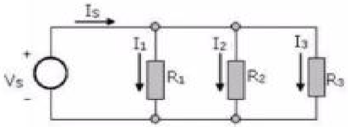
Node: A point where two or more circuit elements join

Mesh: A loop that does not enclose any other loops

Branch: A circuit path that contains two nodes

Loop: A path with starting node=last node; can contain many meshes

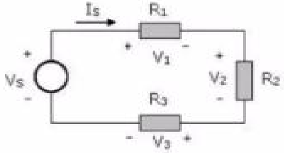
Series- Parallel



Parallel Circuit

$I_S = I_1 + I_2 + I_3$

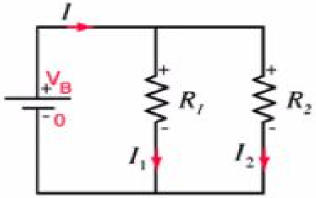
$= V_S/R_1 + V_S/R_2 + V_S/R_3$



Series Circuit

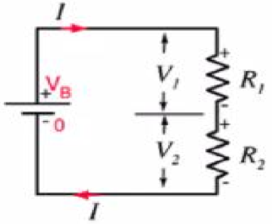
$V_S = V_1 + V_2 + V_3$

$= I_SR_1 + I_SR_2 + I_SR_3$



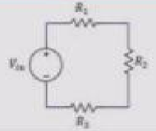
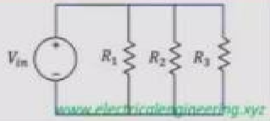
Parallel resistors

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2}$$



Series resistors

$$R_{equivalent} = R_1 + R_2$$

	Series	Parallel
How it looks		
Voltage	$V_{in} = V_1 + V_2 + V_3$	$V_{in} = V_1 = V_2 = V_3$
Current	$I_{series} = I_1 = I_2 = I_3$	$I_{in} = I_1 + I_2 + I_3$
Resistance	$R_{eq} = R_1 + R_2 + R_3$	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
Features	If one components burns current becomes inactive	If one component burns current stops only through that branch rest part works fine

Ohms Law

- **Ohm's law** states that the current through a conductor between two points is directly proportional to the voltage across the two points.

OR

- Ohm's law states that the voltage across a conductor is directly proportional to the current flowing through it, provided all physical conditions and temperature, remain constant.

$$V = IR$$

In the equation, the constant of proportionality, R is Resistance and has units of ohms, with symbol Ω.

The same formula can be rewritten in order to calculate the current and resistance respectively as follows:

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

Kirchhoff's Current Law

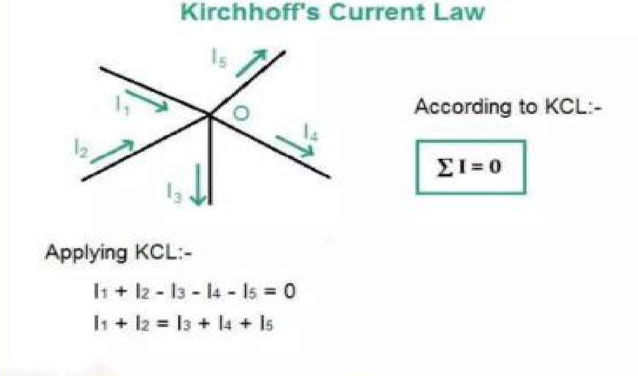
- The algebraic sum of all currents meeting at junction point is equal to zero.

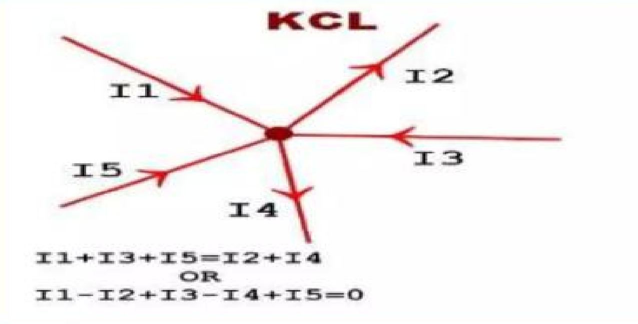
i.e. $\Sigma I = 0$

- At Junction,

Incoming current = Outgoing current

- The total current entering a junction or a node is equal to the current leaving the node.





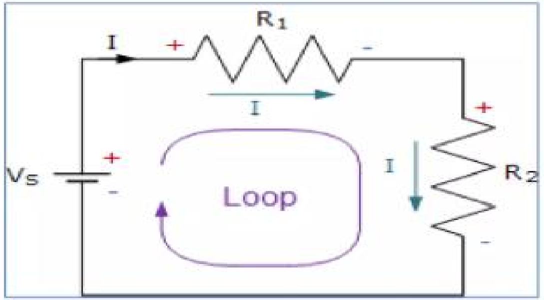
Kirchhoff's Voltage Law

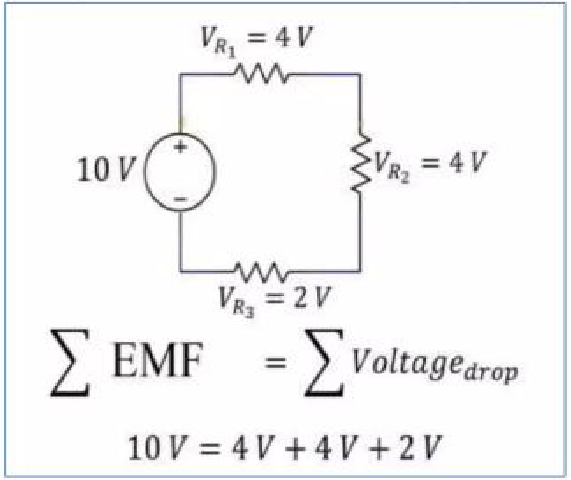
- In any closed loop, the algebraic sum of EMF's & algebraic sum of voltage drops is equals to zero.

i.e. Σ EMF + Σ Voltage Drops = 0

Σ EMF = Σ Voltage Drops

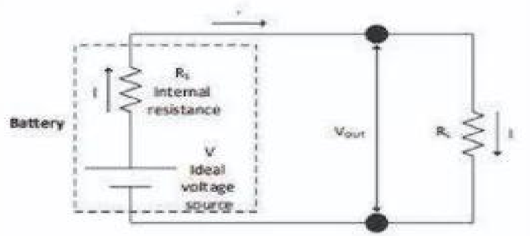
- In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop.

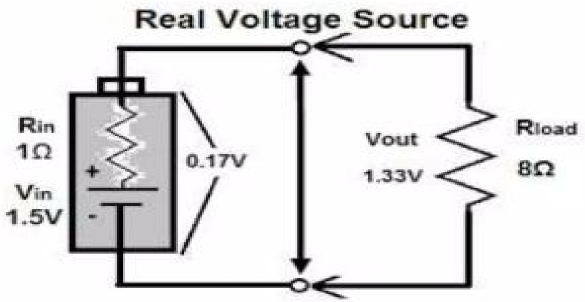




Internal Resistance of Source

- It is the resistance which causes internal voltage drop in the Source.
- All the practical voltage & current sources are always has internal resistance.
- Due to internal resistance the terminal voltage is always less than Source voltage.
- The voltage across terminals of Cell or Battery is called as Terminal Voltage.





Superposition Theorem :

- *The superposition theorem extends the use of Ohm's Law to circuits with multiple sources.*
- **Definition** :- *The current through, or voltage across, an element in a linear bilateral network equal to the algebraic sum of the currents or voltages produced independently by each source.*
- *The Superposition theorem is very helpful in determining the voltage across an element or current through a branch when the circuit contains multiple number of voltage or current sources.*

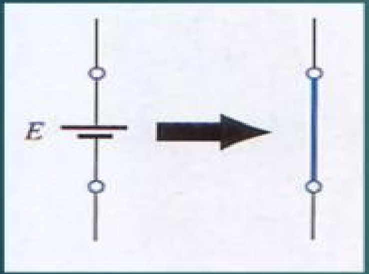
Condition:

- In order to apply the superposition theorem to a network, certain conditions must be met :

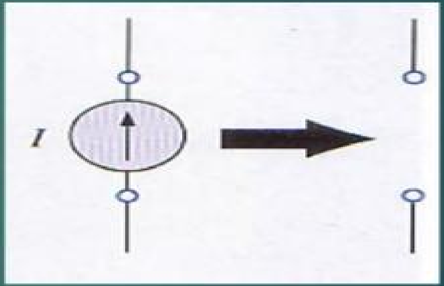
1. All the components must be linear, for e.g.- the current is proportional to the applied voltage (for resistors), flux linkage is proportional to current (in inductors), etc.
2. All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
3. Passive components may be used. These are components such as resistors, capacitors, and inductors, that do not amplify or rectify.
4. Active components may not be used. Active components include transistors, semiconductor diodes, and electron tubes. Such components are never bilateral and seldom linear.

Procedure for applying Superposition Theorem:

- *Circuits Containing* **Only Independent Sources**
- *Consider only one source to be active at a time.*
- *Remove all other* **IDEAL VOLTAGE SOURCES** *by* **SHORT CIRCUIT** *& all other* **IDEAL CURRENT SOURCES** *by* **OPEN CIRCUIT.**



Voltage source is replaced by a Short Circuit



Current source is replaced by a Open Circuit

Steps:

1) Select any one source and short all other voltage sources and open all current sources if internal impedance is not known. If known replace them by their impedance.

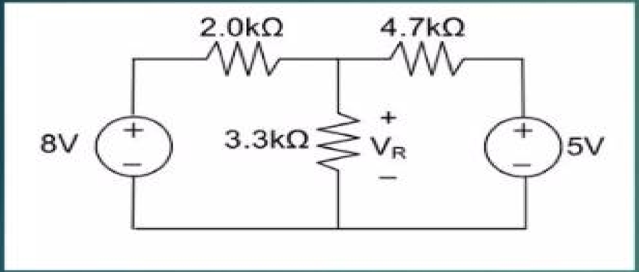
2) Find out the current or voltage across the required element, due to the source under consideration.

3) Repeat the above steps for all other sources.

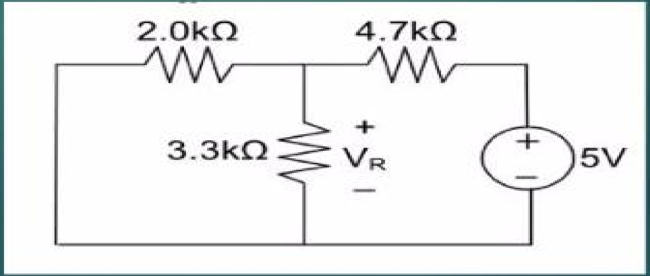
4) Add all the individual effects produced by individual sources to obtain the total current in or across the voltage element.

Example:

Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in figure below.



Step 1: *Remove the 8V power supply from the original circuit, such that the new circuit becomes as the following and then measure voltage across resistor.*

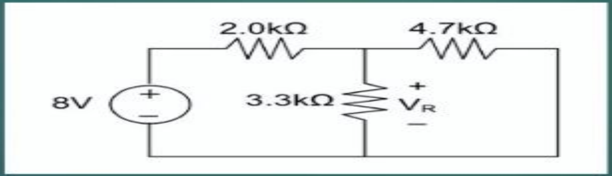


Here 3.3K and 2K are in parallel, therefore resultant resistance will be 1.245K.

Using voltage divider rule voltage across 1.245K will be

V1= [1.245/(1.245+4.7)]*5 = 1.047V

Step 2: Remove the 5V power supply from the original circuit such that the new circuit becomes as the following and then measure voltage across resistor.



Here 3.3K and 4.7K are in parallel, therefore resultant resistance will be 1.938K.

Using voltage divider rule voltage across 1.938K will be

V2= [1.938/(1.938+2)]*8 = 3.9377V

Therefore voltage drop across 3.3K resistor is V1+V2 = 1.047+3.9377=4.9847 v

Thevenin's theorem:

Thevenin's theorem states that: **Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor as in Fig. 4.6 (a).**

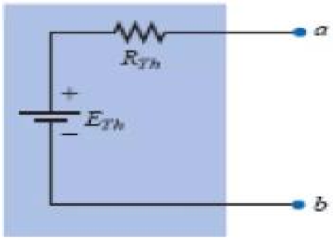


Fig. 4.6(a)

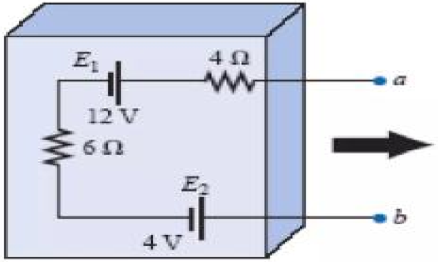


Fig. 4.6(b)

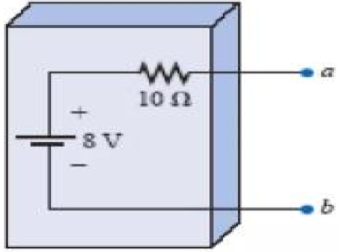
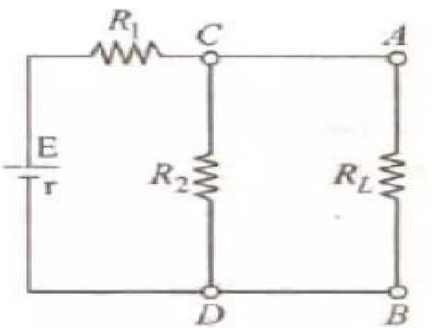
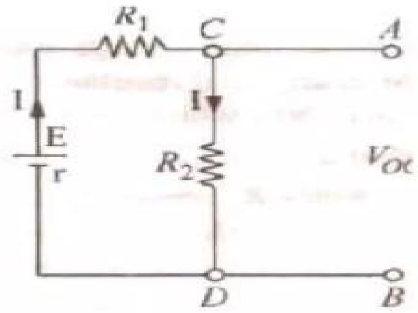


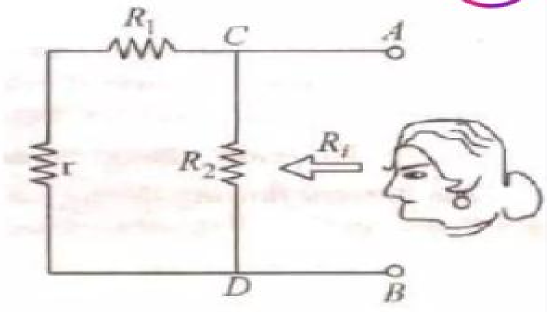
Fig. 4.6(c)

- a mathematical technique for replacing a given network, as viewed from two output terminals,
- by *a single voltage source with a series resistance*
- It makes the solution of complicated networks (particularly, electronic networks) quite quick and easy.
- Application of this extremely useful

Let, it is required to find current flowing through load resistance R_L as in Fig.







We will proceed as under:

1. Remove *RL*from the circuit terminals *A* and *B* and redraw the circuit as shown in Fig. 4.7. Obviously, the terminals have become open-circuited. ·

2. Calculate the open-circuit voltage *Voc* which appears across terminals *A* and *B* when they are open *i.e.* when R_L is removed.

Voc = drop across $R_2 = IR_2$, where *I* is the circuit current when *A* and *B* are open

$$I = \frac{E}{R_1 + R_2 + r} \quad \therefore \quad V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r}$$

It is also called 'Thevenin voltage' V_{th}

If the internal resistance is not given by the circuit, then consider r = 0.

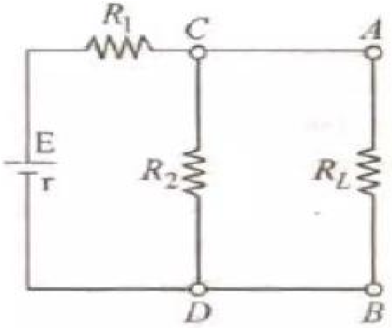


Fig. 4.7(a)

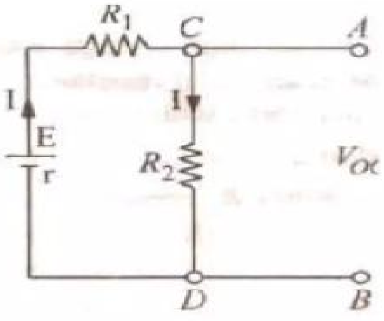


Fig. 4.7(b)

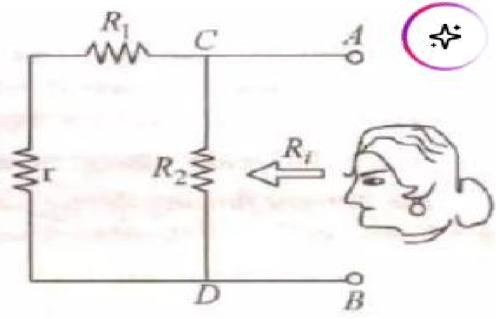


Fig. 4.7(c)

- Now, imagine the battery to be removed from the circuit, leaving its internal resistance *r* behind and redraw the circuit, as shown in Fig. 4.7
- When viewed *inwards* from terminals *A* and *B*, the circuit consists of two parallel paths: one containing *R2* and the other containing *(Rt + r)*
- Equivalent resistance of-the network, as viewed from these terminals

$$R = R_2 \parallel (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is also called, *Thevenin resistance R_{Th}.

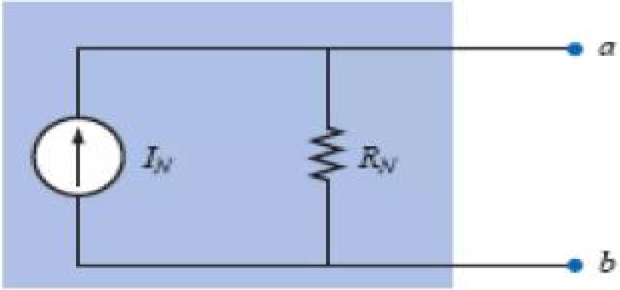
Steps to follow to apply thevenin's theorem

1. Remove that portion of the network across which the Thevenin's equivalent circuit is to be found.
2. Mark the terminals of the remaining two terminal network.
3. calculate R_{TH} by first setting all sources to zero (Voltage sources are replaced by SC and current sources by OC) and then finding the resultant resistance between the two marked terminals (internal resistances of the sources must remain when the sources are set to zero).
4. Calculate V_{TH} by first returning all sources to their original position and finding the open circuit voltage (V_{OC}) between the terminals. It is the same voltage that would be measured by a voltmeter placed between the marked terminals.
5. Draw the Thevenin equivalent circuit with the portion of the circuit previously removed between the terminals of the equivalent circuit.

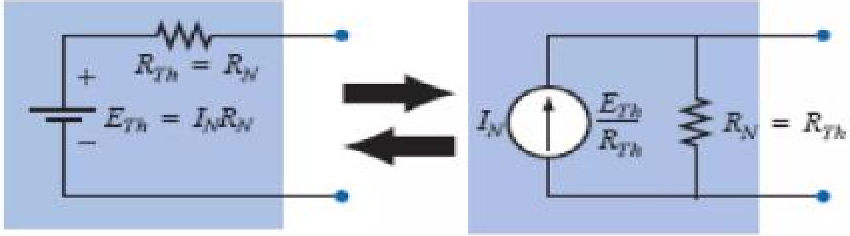
Norton's theorem:

Norton's theorem states that

Any two terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor



- Every voltage source with a series internal resistance has a current source equvalent.
- Current source equivalent of Thevenin's network can be determined by Norton's theorem.



- Steps leading to proper values of I_N and R_N are:
- Remove that portion of the network across which the Thevenin's equivalent circuit is to be found.
- Mark the terminals of the remaining two terminal network.
- calculate R_N by first setting all sources to zero (Voltage sources are replaced by SC and current sources by OC) and then finding the resultant resistance R_N between the two marked terminals (internal resistances of the sources must remain when the sources are set to zero). Since R_N=R_{TH}, the procedure and value obtained using the approach described for Thevenin's theorem will determine the proper value of R_N.
- Calculate I_N by first returning all sources to their original position and finding the short circuit current (I_{SC}) between the terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.
- Draw the Norton equivalent circuit with the portion of the circuit previously removed between the terminals of the equivalent circuit.

Time-domain analysis of first-order RL and RC circuits

- Equations for these circuits, formed using KVL & KCL, consisting of basic elements contain derivatives & integrals of Currents / Voltages .

- Due to above facts equations are not algebraic but are differential in nature.

- Solutions of differential equations are functions of time & not constant as in case of purely resistive circuits.

Series RL Circuit

Fig. 1 shows a series RL circuit connected across a DC source through a switch S. When switch 'S' is close at t>0 the as per KVL network equation will be …

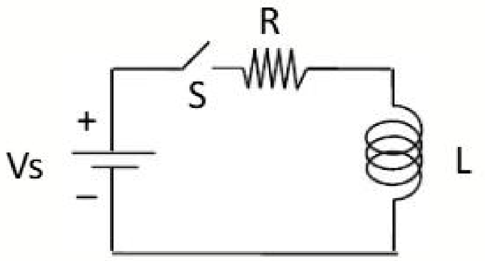


Fig. 1 Series RL Circuit

$$Ri(t) + L\frac{di(t)}{dt} = Vs \quad \text{...................... Eq. 1}$$

Above equation is non homogenous equation linear differential equation of first order. The solution of Eq. 1 will give i(t) which consists of two components i.e.

i) Complimentary function ($i_n(t)$)

Which will satisfy $\frac{di(t)}{dt} + \frac{R}{L} i(t) = 0$

ii) Particular integral ($i_f(t)$)

Which will satisfy $Ri(t) + L\frac{di(t)}{dt} = Vs$

Thus complete solution may be written as …

$$i(t) = i_n(t) + i_f(t) \quad \text{...................... Eq. 2}$$

$$i(t) = I_0 e^{-(\frac{R}{L})t} = I_0 e^{-(\frac{t}{\tau})}$$ .... Eq. 3

Where ζ = L/R time constant of RL circuit

Eq. 3 provides the natural reproduced and is reproduced below....

$$i_n(t) = Ke^{-(\frac{R}{L})t} = Ke^{-(t/\tau)}$$ .... Eq. 4

Eq. 1 can be written with (i(t) = I = constant)

$$RI + L\frac{dI}{dt} = Vs$$ .... Eq. 5

Since I = Constant

$$L\frac{dI}{dt} = 0$$

$$i_f(t) = I = \frac{Vs}{R}$$ .... Eq. 6

Substitute Eq. 4 & Eq.6 in Eq.2 yields the solution of Eq. 1

We get..

$$i(t) = Ke^{-t/\tau} + \frac{Vs}{R} = Ke^{-t/\tau} + I$$ .... Eq. 7

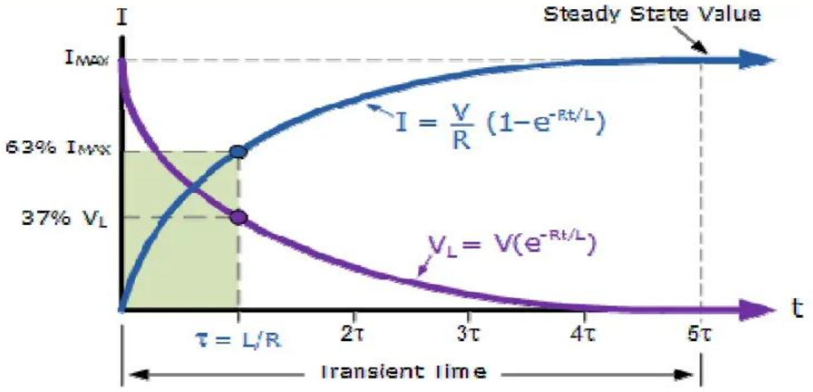
K is determined from initial condition i.e. t=0 Eq. 7 will be ...

$$K = -\frac{Vs}{R} = -I$$ .... Eq. 8

Hence complete solution of Eq. 1 is given by

$$i(t) = \frac{Vs}{R}\left(1 - e^{-(\frac{R}{L}t)}\right)$$

$$i(t) = I(1 - e^{-t/\tau})$$ For t >0



Series RC Circuit

Fig. 2 shows a series RC circuit connected across a DC source through a switch S. It is assumed that capacitor voltage id V_0 When switch 'S' is close at t>0 the as per KVL network equation will be …

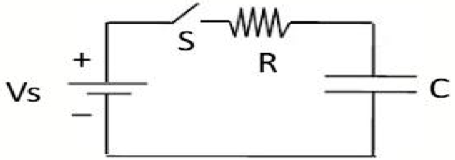


Fig. 2 Series RC Circuit

$$V_s - Ri(t) - v_C(t) = 0 \qquad \text{.................... Eq. 9}$$

For analysis of circuit of Fig. 2 the capacitor voltage Vc(t) is chosen as variable.

Substituting $i(t) = c\frac{dv_c(t)}{dt}$ in Eq. 9 We get.

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$ For t >0) .... Eq. 10

Above Eq.10 is like Eq. 1 it is also non homogenous equation linear differential equation of first order. Therefore solution, solution is also similar to Eq. 1. i.e.

$$v_c(t) = ke^{-t/\tau} + V_s \qquad \text{.................... Eq. 11}$$

In Eq. 11 the time constant is $\tau = RC$

By substituting initial condition Eq. 11 i.e. Vc = V_0 it leads to

$K = V_0 - V_s$

By substituting value of K in Eq. 11 and after simplification we get …

$$v_C(t) = V_0 e^{-(\frac{t}{\tau})} + V_S\left(1 - e^{-\frac{t}{\tau}}\right)V \qquad \text{For } t > 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 12}$$

The expression for the current in the circuit is given by….

$$i(t) = C\frac{dv_C(t)}{dt}$$

$$i(t) = C\left[\left(-\frac{1}{\tau}V_0 e^{-\frac{t}{\tau}}\right) + \left(\frac{1}{\tau}V_S e^{-\frac{1}{\tau}}\right)\right]$$

$$i(t) = \frac{C}{RC}(V_S - V_0)e^{-\frac{t}{\tau}}$$

$$i(t) = \frac{(V_S - V_0)}{R}e^{-\frac{t}{\tau}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 13}$$