ELECTRONICS DEVICES AND CIRCUITS

UNIT-I DIODE AND DIODE APPLICATIONS

UNIT : I

PN junction diode – I-V characteristics

Diode resistance and capacitance

Diode models (Ideal, Simplified, Piecewise Linear)

Rectifiers – Half-wave, Full-wave (Center-tap and bridge)

Capacitor filter for rectifiers

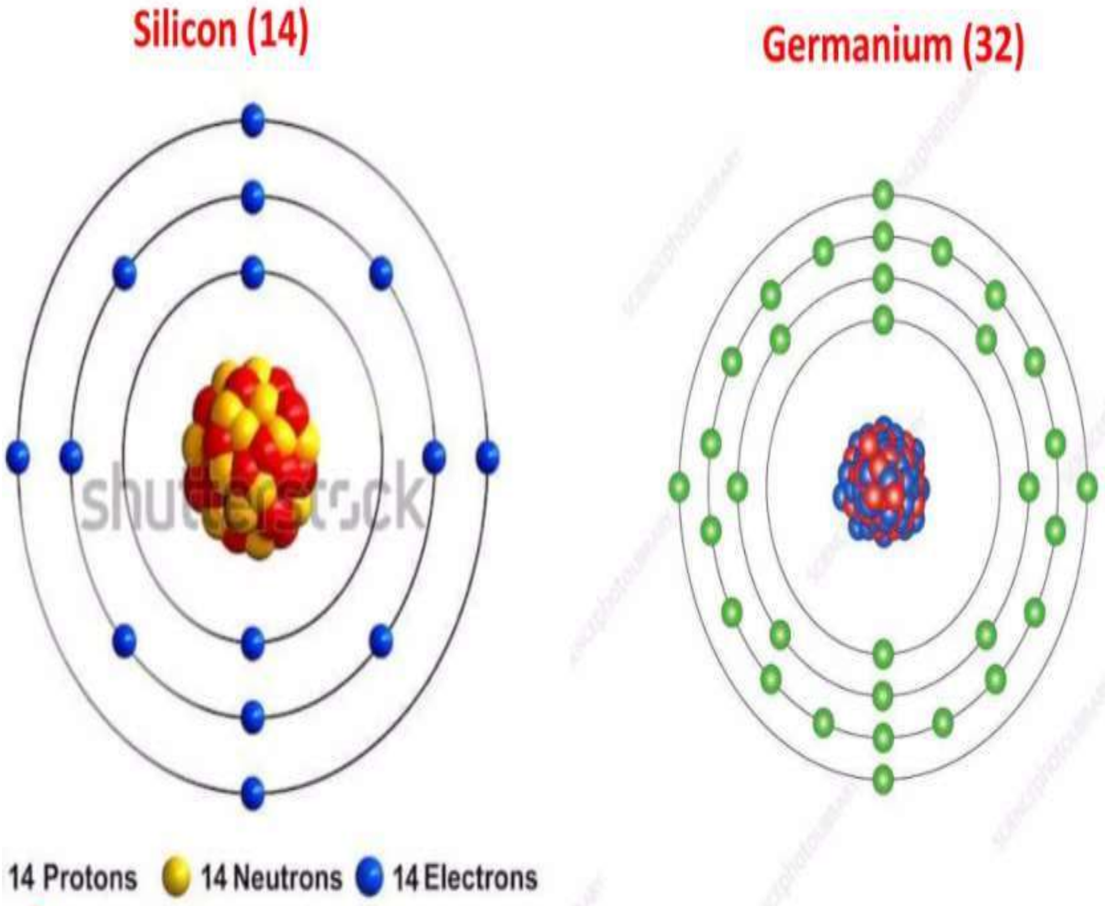
Clippers and clampers

Zener diode – I-V characteristics and voltage regulation

Semiconductor

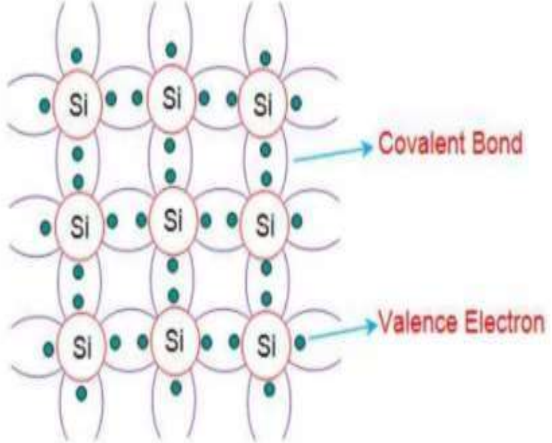
- A semiconductor is a material which has electrical conductivity to a degree between that of a metal and that of an insulator.
- Conductivity of
  - Silicon ➔ 50 x 10^3 Ω–cm
  - germanium ➔ 50 Ω-cm
- Semiconductors are the foundation of modern electronics including
  - transistors,
  - solar cells,
  - light -emitting diodes (LEDs),
  - quantum dots,
  - digital and analog integrated circuits

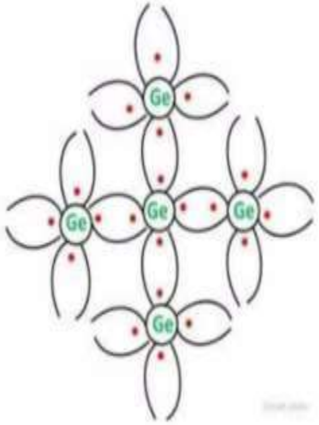
Silicon Vs Germanium



Intrinsic Semiconductor

- A pure form of Semiconductor
- The concentration of electrons (n_i) in the conduction band = concentration of holes (p_i) in the valance band. (**$n_i = p_i$**)
- Conductivity is poor
- Eg. Pure Silicon, Pure Germanium (Tetravalent)





Extrinsic Semiconductor

- A Impure form of Semiconductor
- To increase the conductivity of intrinsic semiconductor, a small amount of impurity (Pentavalent or Trivalent) is added.
- This process of adding impurity is known as **Doping**.
- 1 or 2 atoms of impurity for 10^6 intrinsic atoms.
- Electron concentration ≠ Hole concentration
- One type of carrier will predominate in an extrinsic semiconductor

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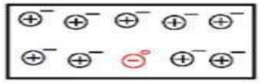
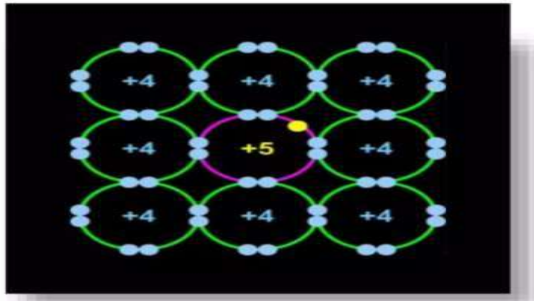
Classification of Extrinsic Semiconductor

- N Type Semiconductor
- P Type Semiconductor

N Type Semiconductor

- A small amount of pentavalent impurities is added
- It is denoting one extra electron for conduction, so it is called donor impurity (Donors)
- $+^{ve}$ charged Ions
- Electron concentration > Hole Concentration
- Most commonly used dopants are
  - Arsenic,
  - Antimony and
  - Phosphorus





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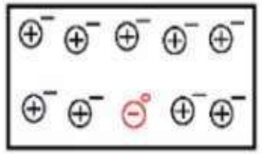
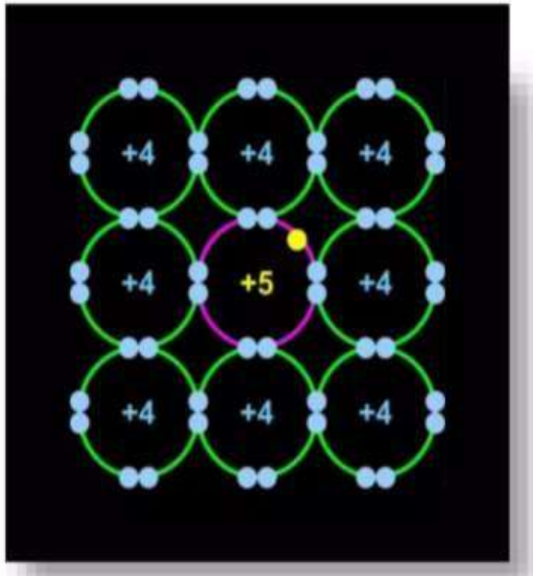
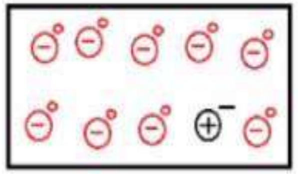
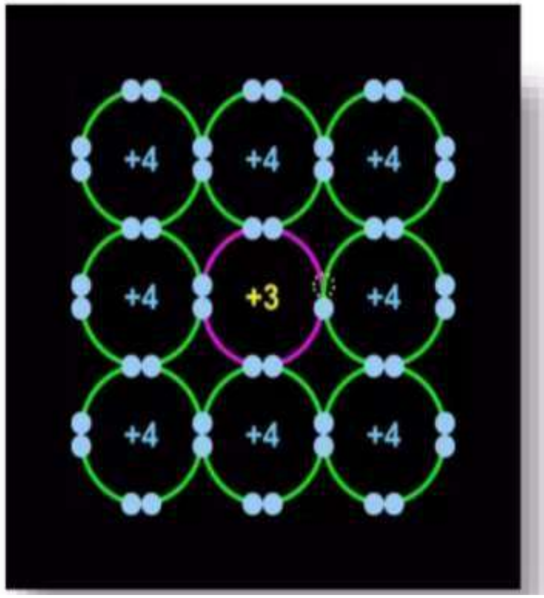




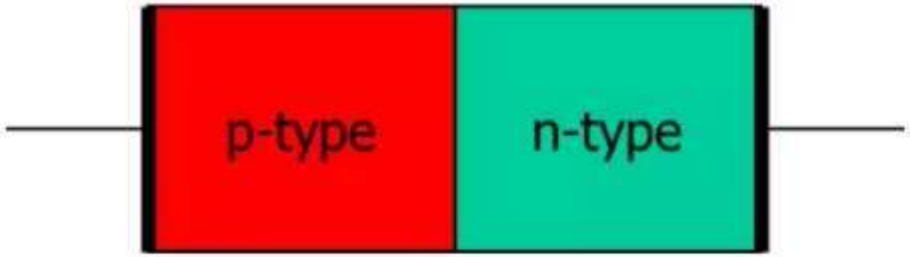
P Type Semiconductor

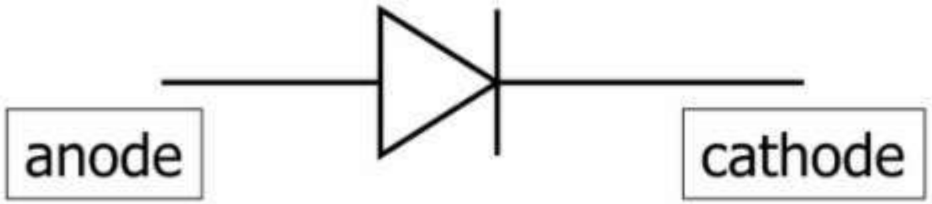
- A small amount of **trivalent impurities** is added
- It accepts free electrons in the place of hole, so it is called Acceptor impurity (Acceptors)
- $-^{ve}$ charged Ions
- Hole concentration > Electron Concentration
- Most commonly used dopants are
  - Aluminum,
  - Boron, and
  - Gallium



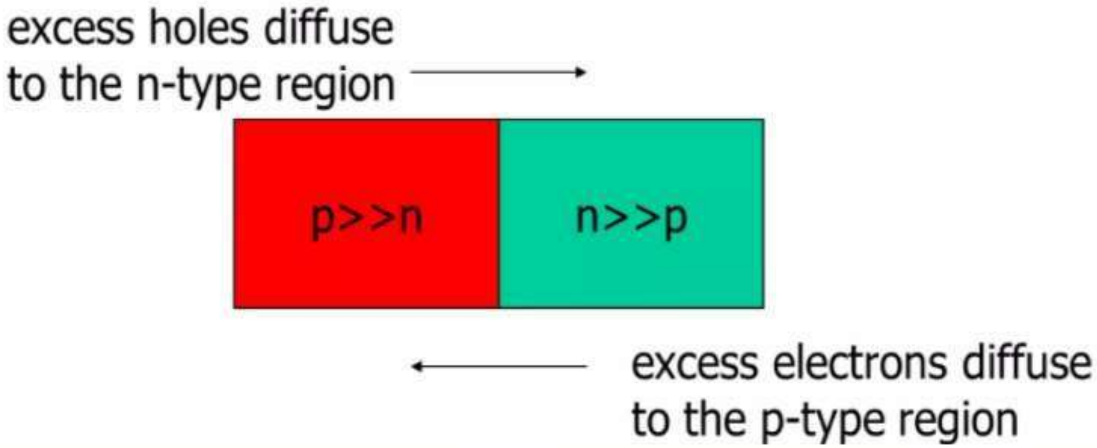


PN Junction Diode





Dopant distribution in PN Junction Diode



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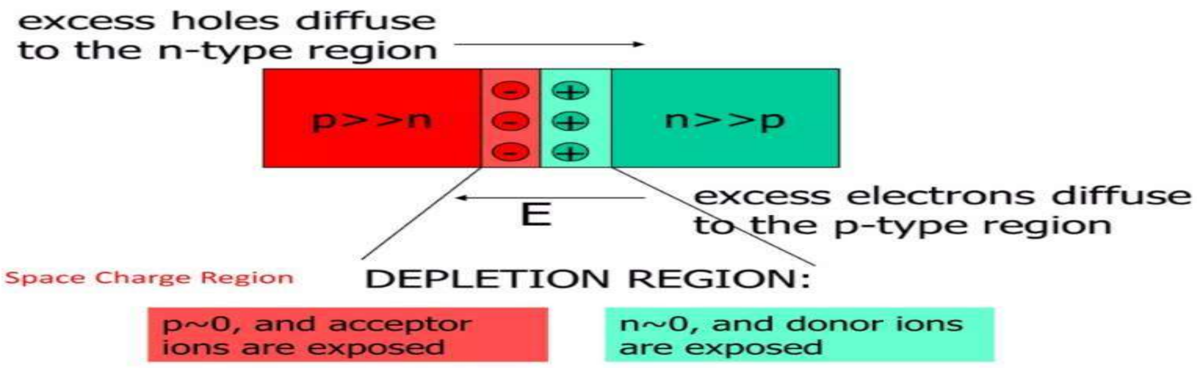

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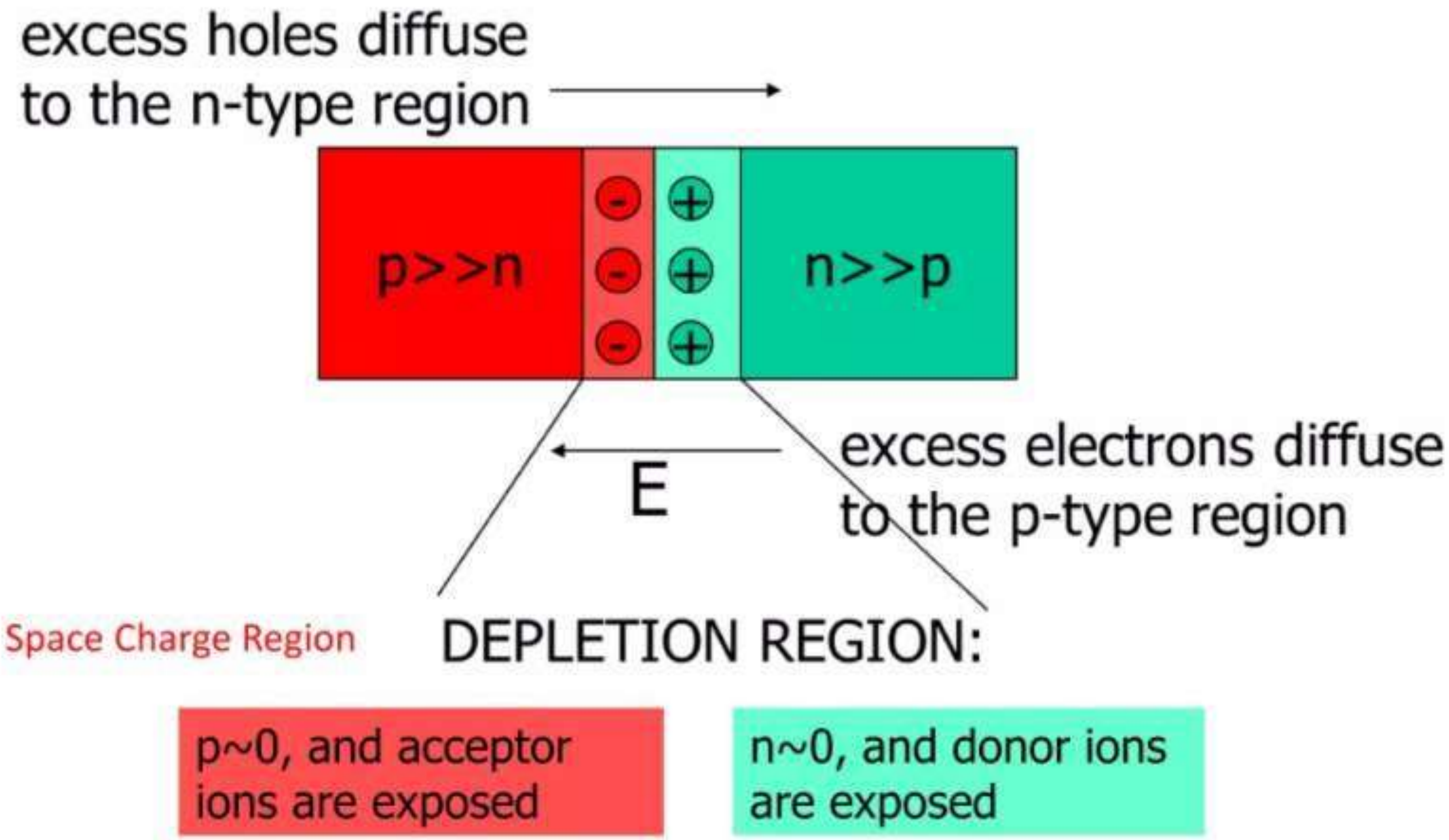
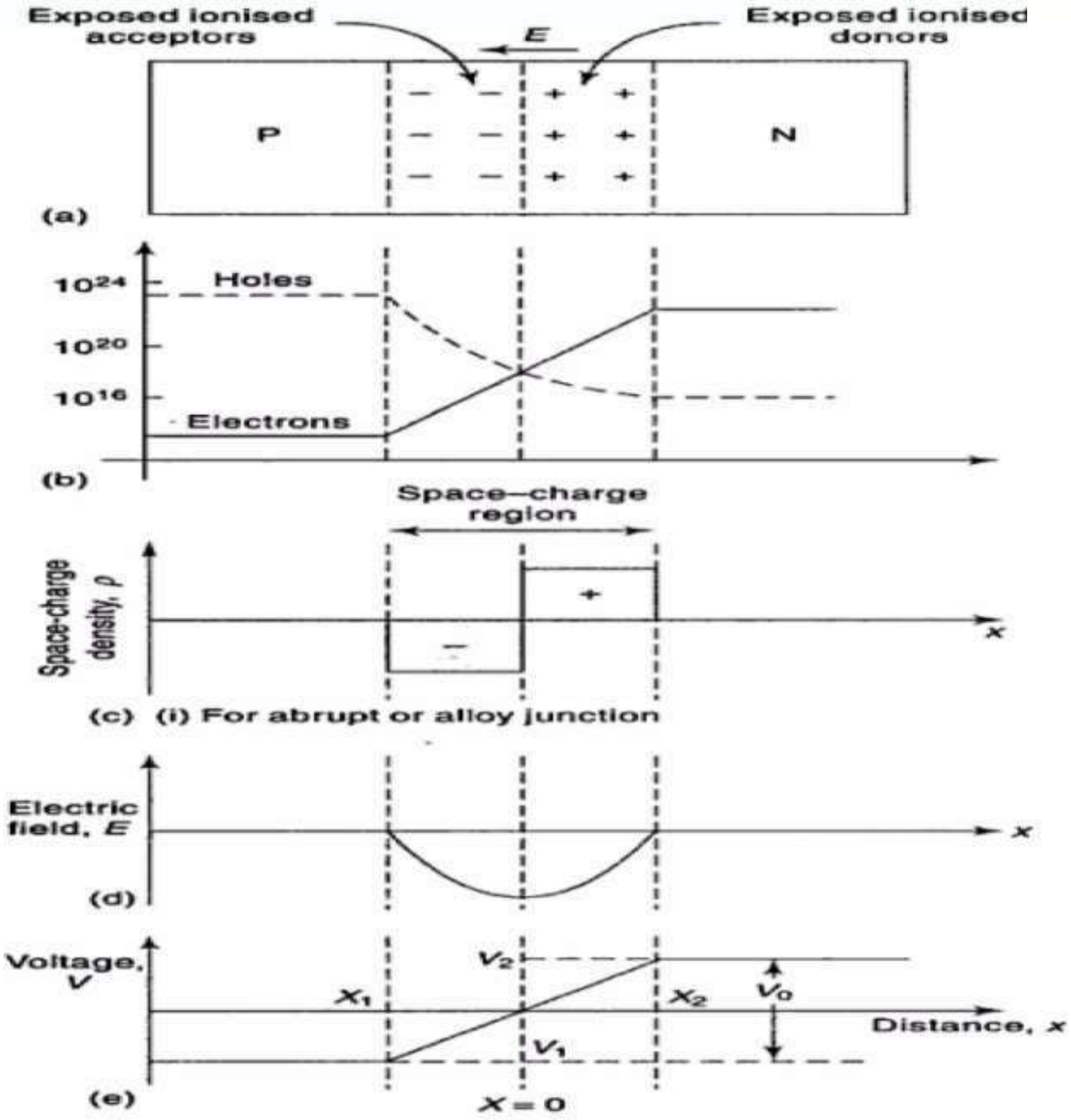
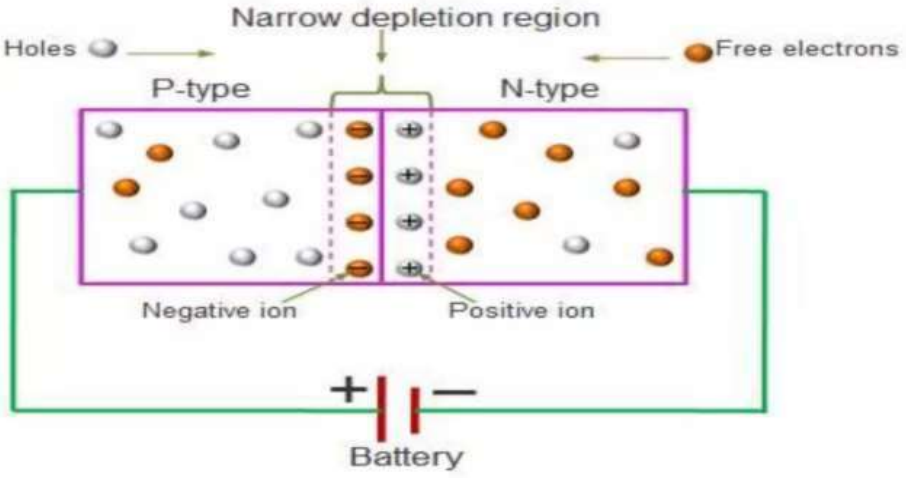


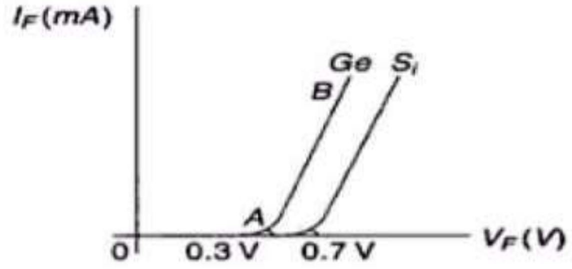


PN Junction Forward bias



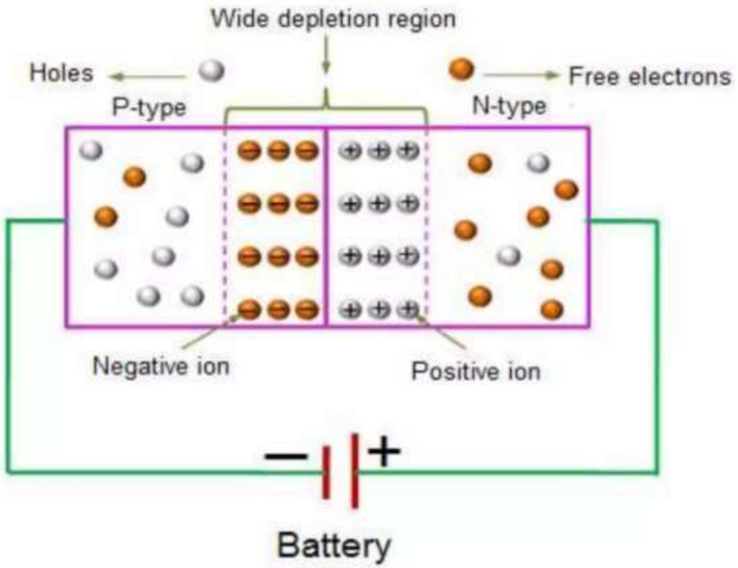
Forward bias

Forward bias Characteristics



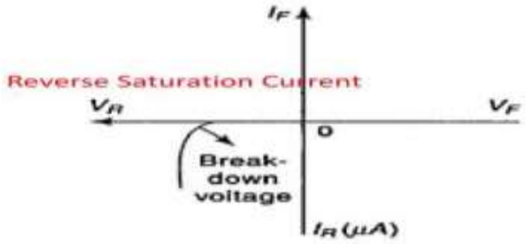
$$W = \left[\frac{2\varepsilon_o\,\varepsilon_r (V_o - V_F)}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) \right]^{1/2}$$

PN Junction Reverse bias



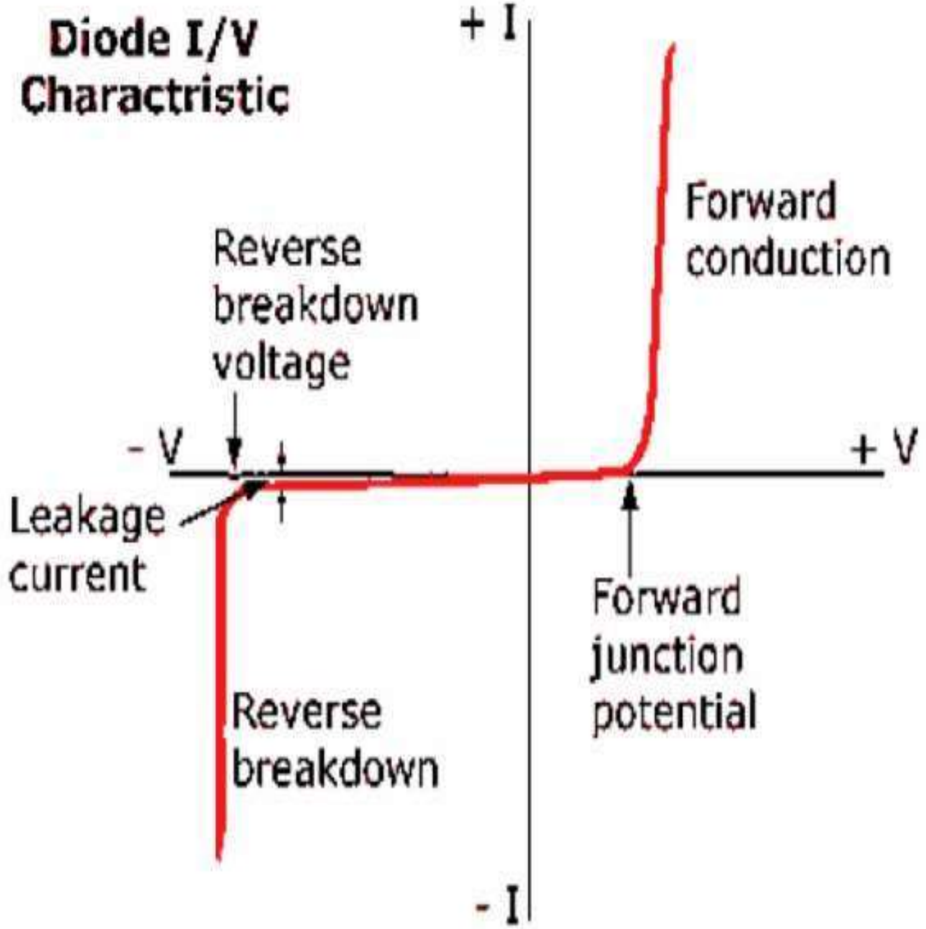
Reverse bias

Reverse bias Characteristics



$$W = \left[\frac{2\varepsilon_o\, \varepsilon_r (V_o + V_R)}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)\right]^{1/2}$$

Characteristics of PN Junction



Transition Capacitance

- The parallel layers of oppositely charged immobile ions on the two side of the junction form the capacitance C_T
- C_T is Transition or Space charge or depletion region capacitance.

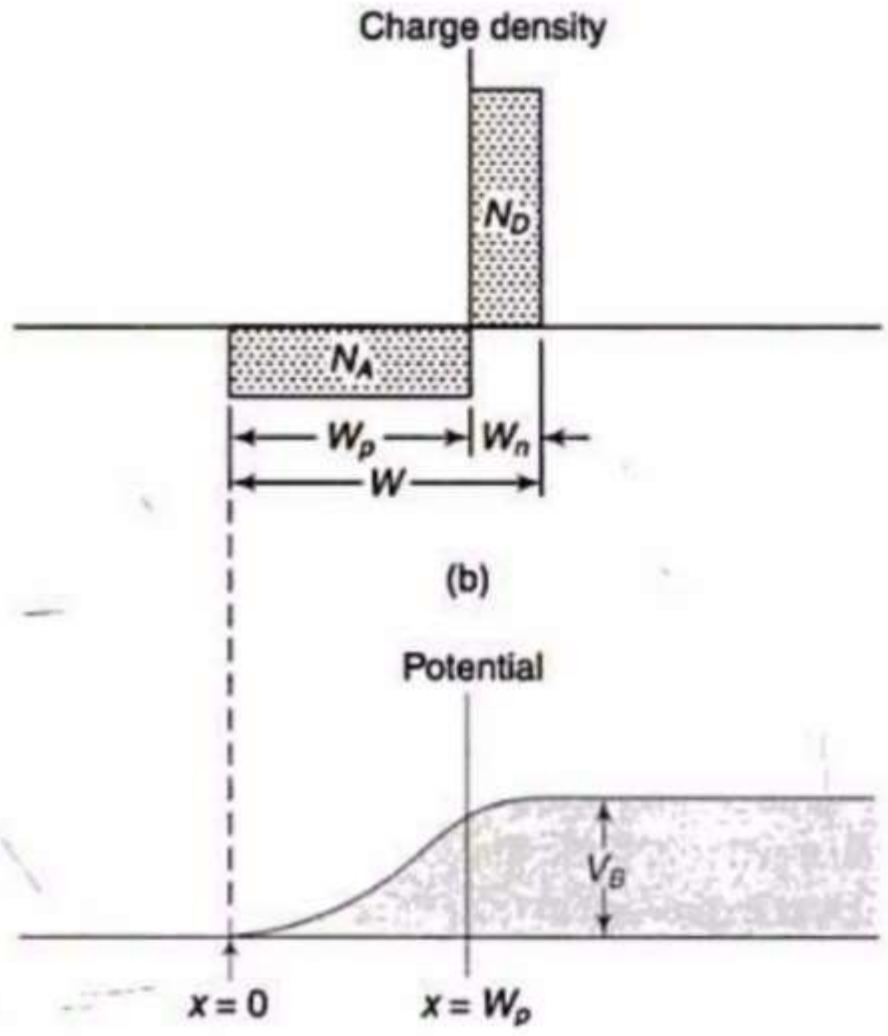
$$C_T = \left|\frac{dQ}{dV}\right|$$

Where dQ is the increase in charge and dV is the change in voltage.

- The Total Charge density of a p type material with area of the junction A is given by,

$$Q = q\, N_A\, W\, A$$

Transition Capacitance



Transition Capacitance

- The relation between potential and charge density is given by the Poisson's equation.

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\varepsilon_o \varepsilon_r}$$

$$\frac{d^2V}{dx^2} = \frac{qN_A}{\varepsilon_o \varepsilon_r}$$

$$V = \frac{qN_A x^2}{2\varepsilon_o \varepsilon r}$$

- At $x = W_p$, $V = V_B$,

$$V_B = \frac{qN_A W^2}{2\varepsilon}$$

- Differentiating w.r.to V, we get .

$$1 = \frac{qN_A\, 2W}{2\varepsilon}\left|\frac{dW}{dV}\right|$$

$$\left|\frac{dW}{dV}\right| = \frac{\varepsilon}{qN_A W}$$

$$C_T = \left|\frac{dQ}{dV}\right| = A\, q\, N_A \left|\frac{dW}{dV}\right|$$

$$= A\, q\, N_A \frac{\varepsilon}{qN_A W}$$

$$C_T = \frac{\varepsilon A}{W}.$$

Diffusion Capacitance

- The capacitance that exists in a forward biased junction is called a diffusion or storage capacitance (C_D)
- $C_D >> C_T$

$$C_D = \frac{dQ}{dV},$$

Where dQ represents change in the number of minority carrier when change in voltage.

$$Q = \int_0^\infty AeP_n(0)e^{-x/L_p}\,dx = \left[\frac{AeP_n(0)e^{-x/L_p}}{1/L_p}\right]_0^\infty$$

$$= L_p\, AeP_n(0)$$

$$C_D = \frac{dQ}{dV} = AeL_p \frac{d[P_n(0)]}{dV}$$

Diffusion hole current in the N side $I_{pn}(x)$

$$I_{pn}(x) = \frac{AeD_p P_n(0)}{L_p} e^{-x/L_p}$$

At, x = 0

$$I_{pn}(0) = \frac{AeD_p P_n(0)}{L_p}$$

$I_{pn}(0) = I$

$$I = \frac{AeD_p P_n(0)}{L_p}$$

$$P_n(0) = \frac{IL_p}{AeD_p}$$

Diffusion Capacitance

Diff. w.r.to V

$$\frac{d[P_n(0)]}{dV} = \frac{dI}{dV}\frac{L_p}{AeD_p}$$

$$C_D = \frac{dQ}{dV} = \frac{dI}{dV}\frac{L_p^2}{D_p}$$

$$C_D = g\tau$$

$$g = \frac{dI}{dV}$$

$$\tau = \frac{L_p^2}{D_p}$$

$$C_D = \frac{\tau I}{\eta V_T}$$

From diode current equation, $g = \frac{I}{\eta V_T}$

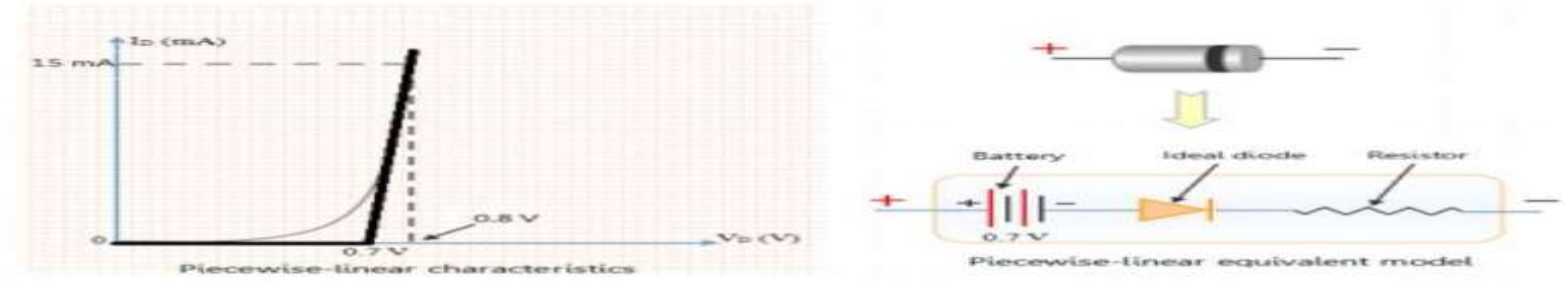
Diode Equivalent Circuit

- An equivalent circuit is nothing but a combination of elements (R,L,C) that best represents the actual terminal characteristics of the device.
- it simply means the diode in the circuit can be replaced by other elements without severely affecting the behavior of circuit.

Diode Equivalent circuit

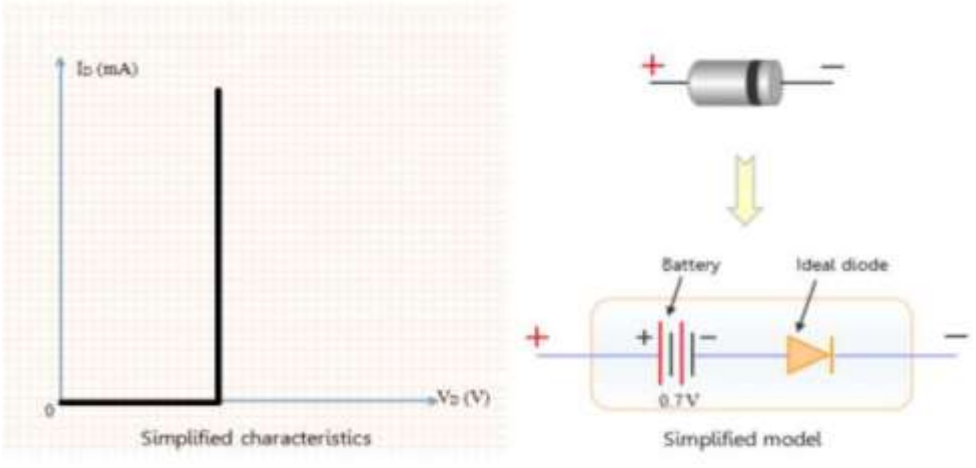
Three models with increasing accuracy are listed below:

1. **Piecewise-Linear Equivalent Circuit :** A technique for obtaining an equivalent circuit for a diode is to approximate the characteristics of the device by straight-line segments



Piecewise-Linear Equivalent Circuit

2.SIMPLIFIED EQUIVALENT CIRCUIT

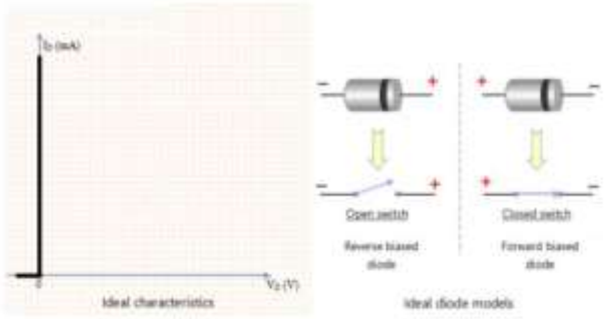


Simplified Equivalent Circuit

The battery simply indicates that it opposes the flow of current in forward direction until 0.7 V. As the voltage becomes larger than 0.7 V, the current starts flowing in forward direction.

The horizontal line indicates that the current flowing through diode is zero for voltages between 0 and 0.7 V.

3. Ideal Diode Model

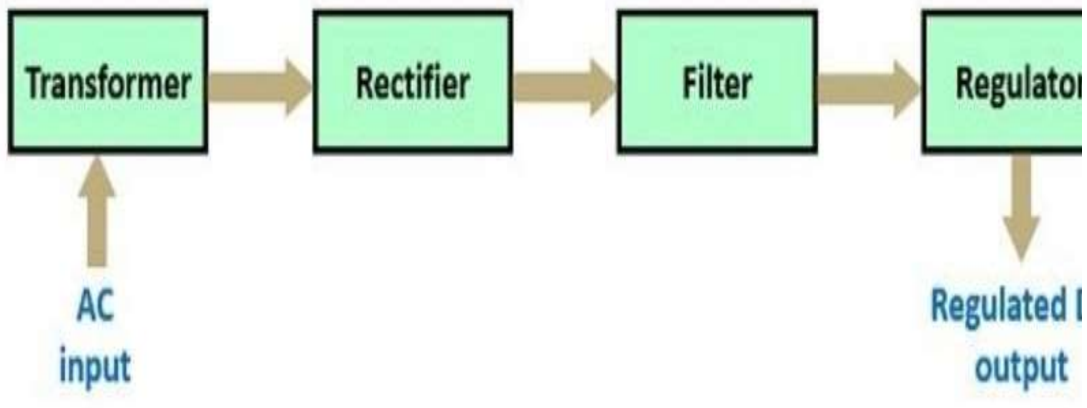


Ideal Diode Model

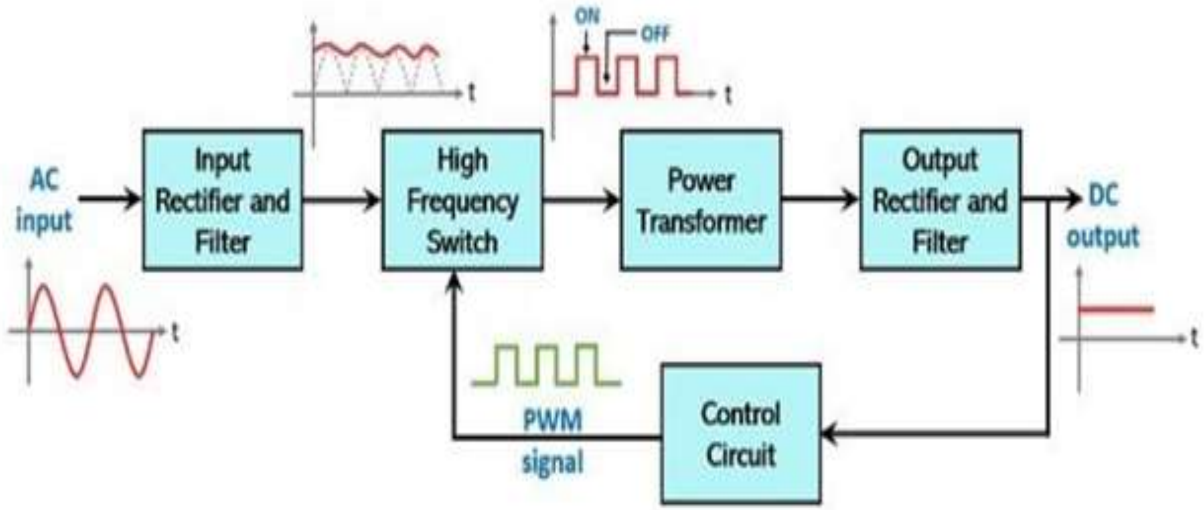
1. Ideal diode allows the flow of forward current for any value of forward bias voltage. Hence, Ideal diode can be modeled as closed switch under forward bias condition. This is shown in the figure.
2. Ideal diode allows zero current to flow under reverse biased condition. Hence it can be modeled as open switch. This is indicated in the figure.

POWER SUPPLY:

- Linear power supply
- Switched mode power supply



Block diagram of Linear Power Supply



Block diagram of Switch Mode Power Supply

RECTIFIER

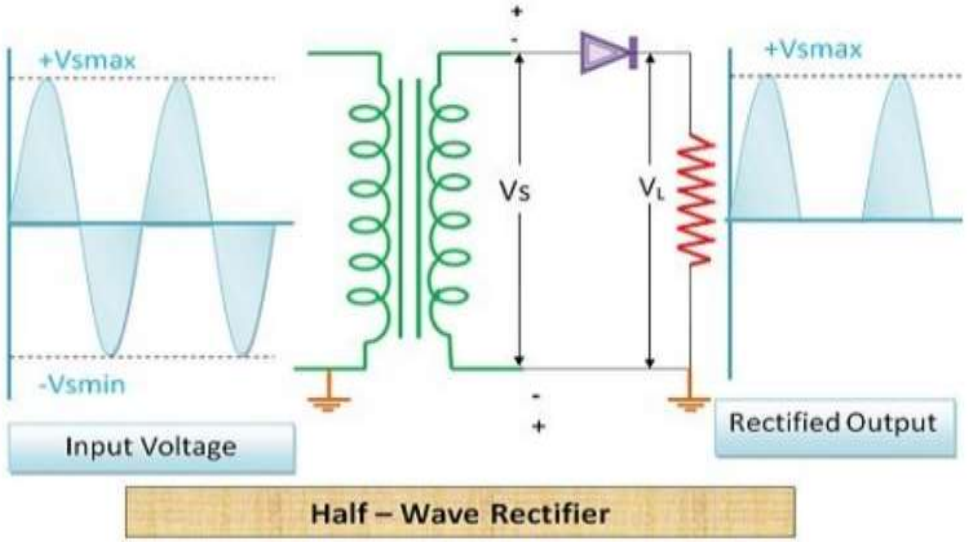
A rectifier is a device which converts a.c. voltage (bi-directional) to pulsating d.c. voltage (Uni-directional).

Any electrical device which offers a low resistance to the current in one direction but a high resistance to the current in the opposite direction is called rectifier.

Normal household power is AC while batteries provide DC, and converting from AC to DC is called rectification. Diodes are used so commonly for this purpose that they are sometimes called rectifiers, although there are other types of rectifying devices.

Half Wave Rectifiers

Definition: **Half wave rectifier** is that in which the half cycle of AC voltage gets converted into **pulsating DC** voltage. The remaining half cycle of AC is suppressed by rectifier circuit or the output DC current for remaining half cycle is zero.



Let V_i be the voltage to the primary of the transformer and given by the equation

$$V_i = V_m \sin \omega t;\ V_m >> V_\gamma$$

Ripple factor (Γ) The ratio of rms value of a.c. component to the d.c. component in the output is known as *ripple factor* (Γ).

$$\Gamma = \frac{\text{rms value of a.c. component}}{\text{d.c. value of component}} = \frac{V_{r,\text{rms}}}{V_{\text{d.c.}}}$$

where $$V_{r,\text{rms}} = \sqrt{V_{\text{rms}}^2 - V_{\text{d.c.}}^2}$$

$$\Gamma = \sqrt{\left(\frac{V_{\text{rms}}}{V_{\text{d.c.}}}\right)^2 - 1}$$

If the values of diode forward resistance (r_f) and the transformer secondary winding resistance (r_s) are also taken into account, then

$$V_{\text{d.c.}} = \frac{V_m}{\pi} - I_{\text{d.c.}}(r_s + r_f)$$

$$I_{\text{d.c.}} = \frac{V_{\text{d.c.}}}{(r_s + r_f) + R_L} = \frac{V_m}{\pi(r_s + r_f + R_L)}$$

The rms voltage at the load resistance can be calculated as

$$V_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)\right]^{\frac{1}{2}}$$

$$= V_m\left[\frac{1}{4\pi}\int_0^{\pi} (1 - \cos 2\,\omega t)\, d(\omega t)\right]^{\frac{1}{2}} = \frac{V_m}{2}$$

Therefore, $$\Gamma = \sqrt{\left[\frac{V_m/2}{V_m/\pi}\right]^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

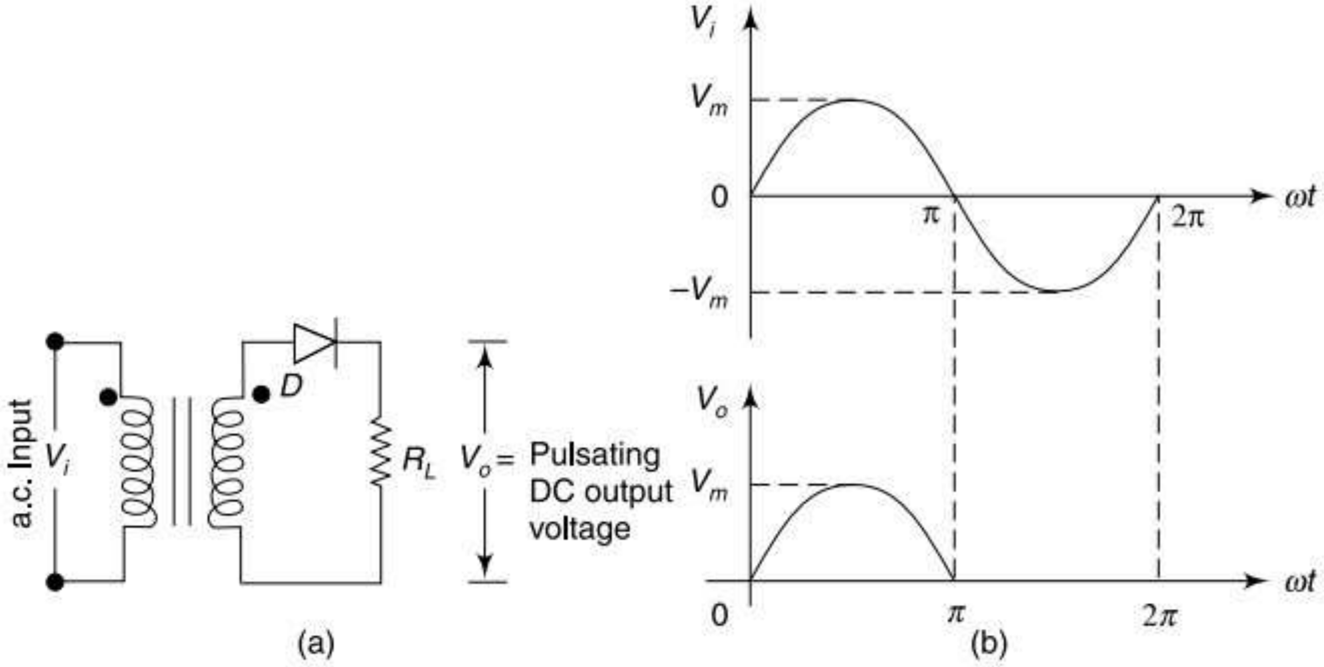


Fig. 3.3 *(a) Basic structure of a half-wave rectifier, (b) Input output waveforms of half wave rectifier*

V_{av} is the average or the d.c. content of the voltage across the load and is given by

$$V_{av} = V_{\text{d.c.}} = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t d(\omega t) + \int_{\pi}^{2\pi} 0.\text{d}(\omega t)\right]$$

$$= \frac{V_m}{2\pi}[-\cos \omega t]_0^{\pi} = \frac{V_m}{\pi}$$

Therefore, $$I_{\text{d.c}} = \frac{V_{\text{d.c.}}}{R_L} = \frac{V_m}{\pi R_L} = \frac{I_m}{\pi}$$

Efficiency (η) The ratio of d.c. output power to a.c. input power is known as *rectifier efficiency* (η).

$$\eta = \frac{\text{d.c. output power}}{\text{a.c. input power}} = \frac{P_{\text{d.c.}}}{P_{\text{a.c.}}}$$

$$= \frac{\dfrac{(V_{\text{d.c.}})^2}{R_L}}{\dfrac{(V_{\text{rms}})^2}{R_L}} = \frac{\left(\dfrac{V_m}{\pi}\right)^2}{\left(\dfrac{V_m}{2}\right)^2} = \frac{4}{\pi^2} = 0.406 = 40.6\%$$

The maximum efficiency of a half-wave rectifier is 40.6%.

Peak Inverse Voltage (PIV) It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half cycle. For half-wave rectifier, PIV is V_m.

Transformer Utilisation Factor (TUF) In the design of any power supply, the rating of the transformer should be determined. This can be done with a knowledge of the d.c. power delivered to the load and the type of rectifying circuit used.

$$\text{TUF} = \frac{\text{d.c. power delivered to the load}}{\text{a.c. rating of the transformer secondary}}$$

$$= \frac{P_{\text{d.c.}}}{P_{\text{a.c.}}\text{ rated}}$$

In the half-wave rectifying circuit, the rated voltage of the transformer secondary is $V_m/\sqrt{2}$, but the actual rms current flowing through the winding is only $\frac{I_m}{2}$, not $I_m/\sqrt{2}$.

$$\text{TUF} = \frac{\frac{I_m^2}{\pi^2}R_L}{\frac{V_m}{\sqrt{2}} \times \frac{I_m}{2}} = \frac{\frac{V_m^2}{\pi^2}\frac{1}{R_L}}{\frac{V_m}{\sqrt{2}}\frac{V_m}{2R_L}} = \frac{2\sqrt{2}}{\pi^2} = 0.287$$

The TUF for a half-wave rectifier is 0.287.

(a) *Form factor*

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}}$$

$$= \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} = 1.57$$

(b) *Peak factor*

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}}$$

$$= \frac{V_m}{V_m/2} = 2$$

Full Wave Rectifier

Definition: Full wave rectifier is the semiconductor devices which convert complete cycle of AC into pulsating DC.

Unlike half wave rectifiers which uses only half wave of the input AC cycle, full wave rectifiers utilize full wave.

The lower efficiency drawback of half wave rectifier can be overcome by using full wave rectifier.

Ripple factor (Γ)

$$\Gamma = \sqrt{\left(\frac{V_{rms}}{V_{d.c.}}\right)^2 - 1}$$

The average voltage or d.c. voltage available across the load resistance

$$V_{d.c.} = \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$

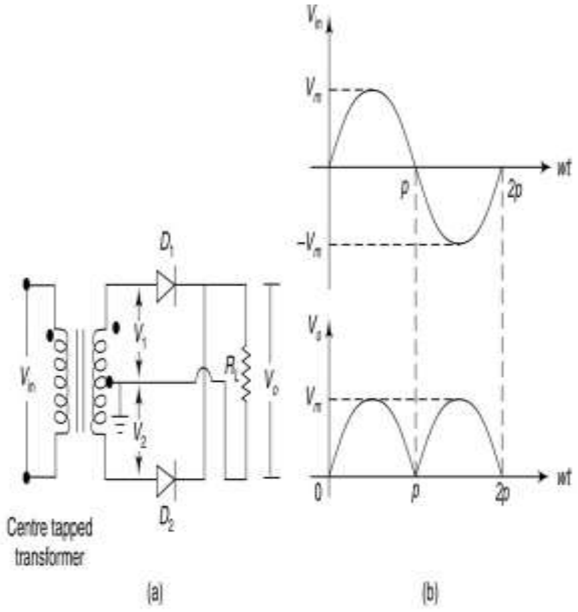
$$= \frac{V_m}{\pi}[-\cos \omega t]_0^{\pi} = \frac{2V_m}{\pi}$$

$$I_{d.c.} = \frac{V_{d.c.}}{R_L} = \frac{2V_m}{\pi R_L} = \frac{2I_m}{\pi} \text{ and } I_{rms} = \frac{I_m}{\sqrt{2}}$$

If the diode forward resistance (r_f) and the transformer secondary winding resistance (r_s) are included in the analysis, then

$$V_{d.c} = \frac{2V_m}{\pi} - I_{d.c.}(r_s + r_f)$$

$$I_{d.c.} = \frac{V_{d.c.}}{(r_s + r_f) + R_L} = \frac{2V_m}{\pi(r_s + r_f + R_L)}$$



RMS value of the voltage at the load resistance is

$$V_{rms} = \sqrt{\left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \, d(\omega t)\right]} = \frac{V_m}{\sqrt{2}}$$

Therefore, $$\Gamma = \sqrt{\left(\frac{V_m/\sqrt{2}}{2V_m/\pi}\right)^2 - 1} = \sqrt{\frac{\pi^2}{8} - 1} = 0.482$$

Efficiency (η) The ratio of d.c. output power to a.c. input power is known as rectifier efficiency (η).

$$\eta = \frac{\text{d.c. output power}}{\text{a.c. input power}} = \frac{P_{d.c.}}{P_{a.c.}}$$

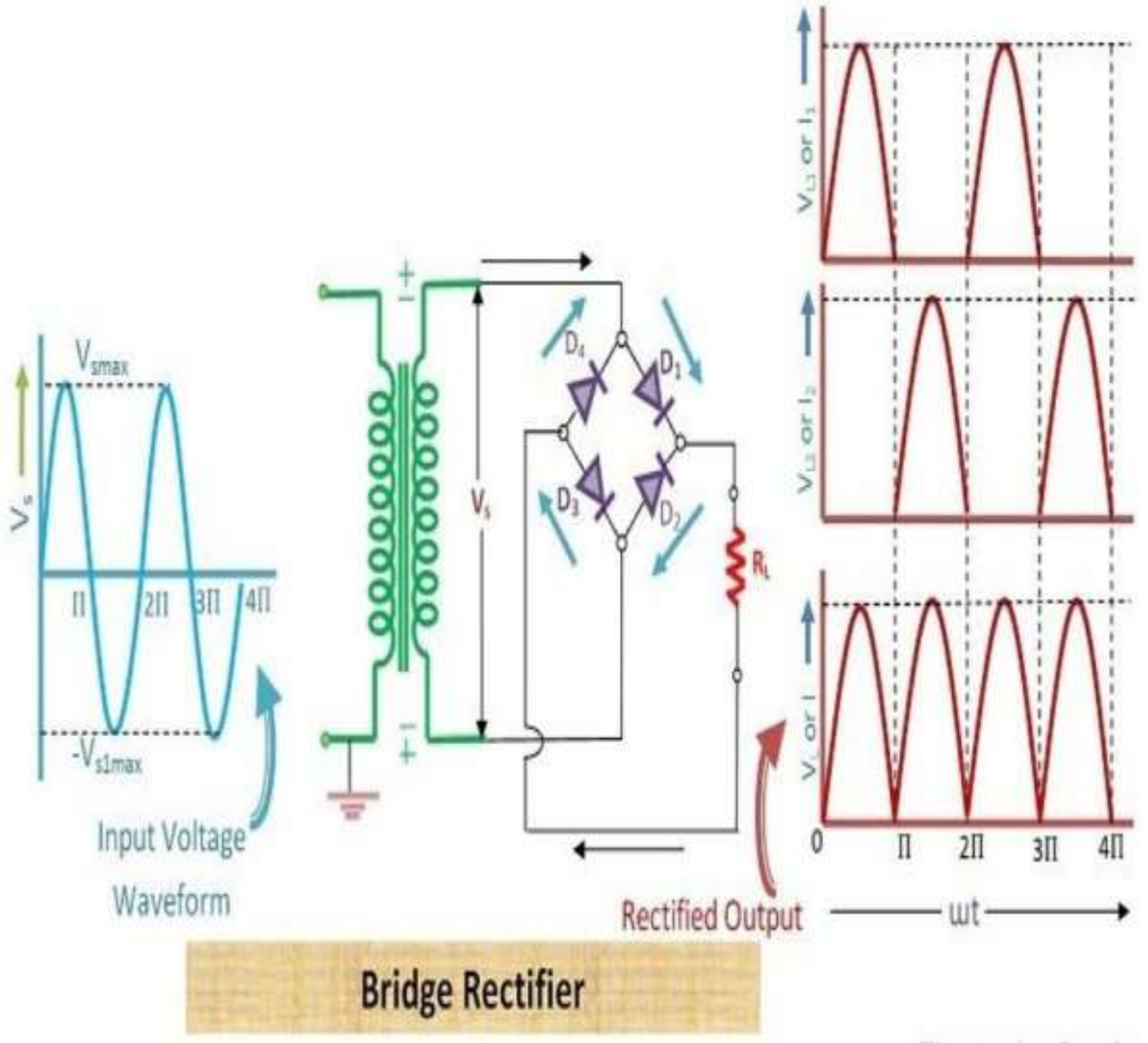
$$= \frac{(V_{d.c.})^2/R_L}{(V_{rms})^2/R_L} = \frac{\left[\frac{2V_m}{\pi}\right]^2}{\left[\frac{V_m}{\sqrt{2}}\right]^2} = \frac{8}{\pi^2} = 0.812 = 81.2\%$$

The maximum efficiency of a full-wave rectifier is 81.2%.

Full WAVE Bridge Rectifier

Definition: Bridge rectifier is formed by connecting **four diodes** in the form of a **Wheatstone bridge**. It also provides full wave rectification.

During the first half of AC cycle, two diodes are forward biased and during the second half of AC cycle, the other two diodes become forward biased.



COMPARISON OF RECTIFIERS

Particulars	*Type of rectifier*		
	Half-wave	*Full-wave*	*Bridge*
No. of diodes	1	2	4
Maximum efficiency	40.6%	81.2%	81.2%
$V_{d.c.}$ (no load)	V_m/π	$2V_m/\pi$	$2V_m/\pi$
Average current/diode	$I_{d.c.}$	$I_{d.c.}/2$	$I_{d.c.}/2$
Ripple factor	1.21	0.48	0.48
Peak inverse voltage	V_m	$2V_m$	V_m
Output frequency	f	$2f$	$2f$
Transformer utilisation factor	0.287	0.693	0.812
Form factor	1.57	1.11	1.11
Peak factor	2	$\sqrt{2}$	$\sqrt{2}$

Capacitor filter

An inexpensive filter for light load found in the capacitor filter which connected directly across the load

The charge it has acquired $= V_{r,\,p\text{-}p} \times C$

The charge it has lost $= I_{\text{d.c.}} \times T_2$

Therefore, $= I_{\text{d.c.}} \times T_2$

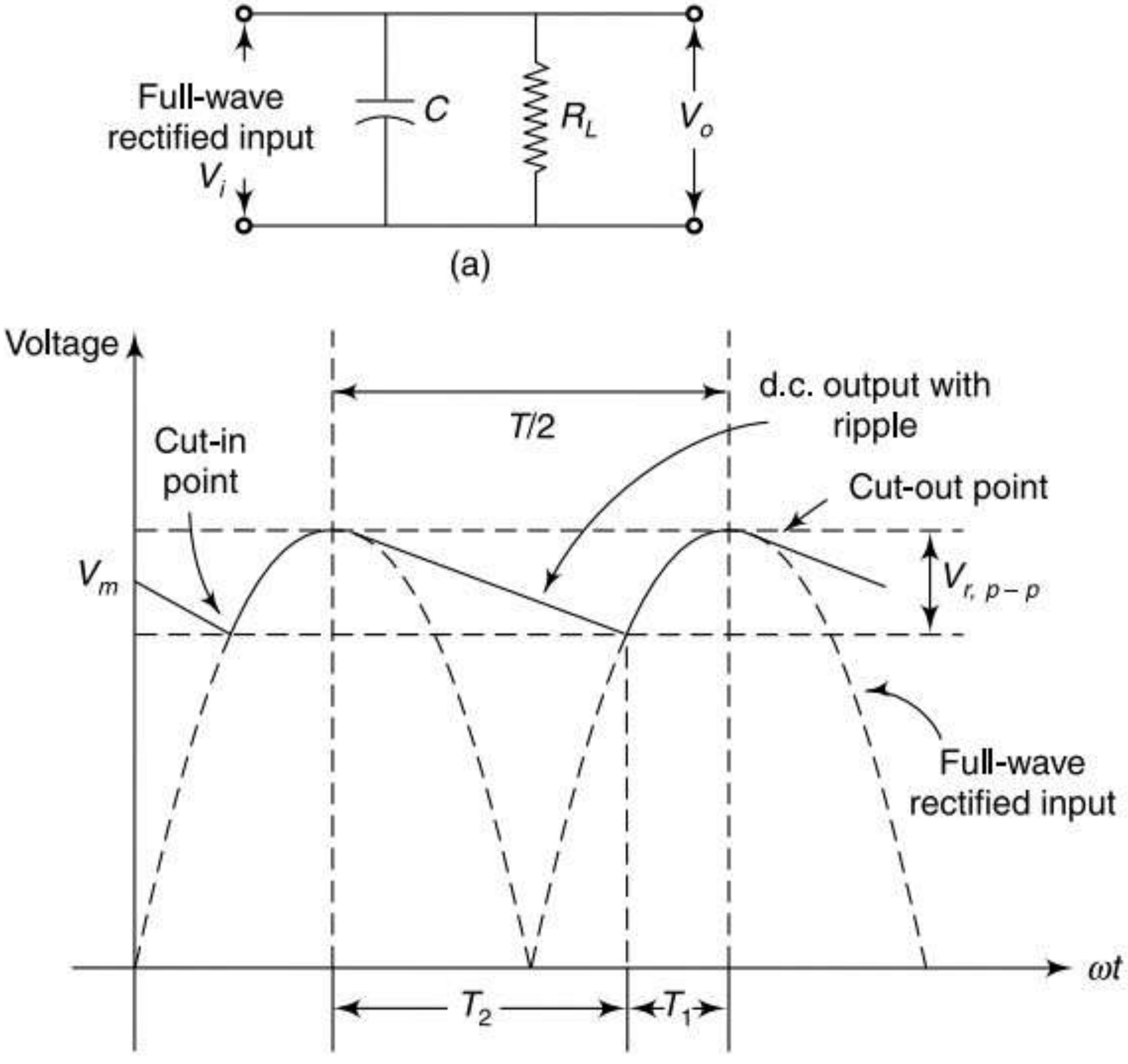


Fig. 3.10 *(a) Capacitor filter (b) Ripple voltage triangular waveform*

i.e. $$T_2 = \frac{T}{2} = \frac{1}{2f}, \quad \text{then} \quad V_{r,p-p} = \frac{I_{\text{d.c.}}}{2fC}$$

With the assumptions made above, the ripple waveform will be triangular in nature and the rms value of the ripple is given by

$$V_{r,\text{rms}} = \frac{V_{r,p-p}}{2\sqrt{3}}$$

Therefore from the above equation, we have

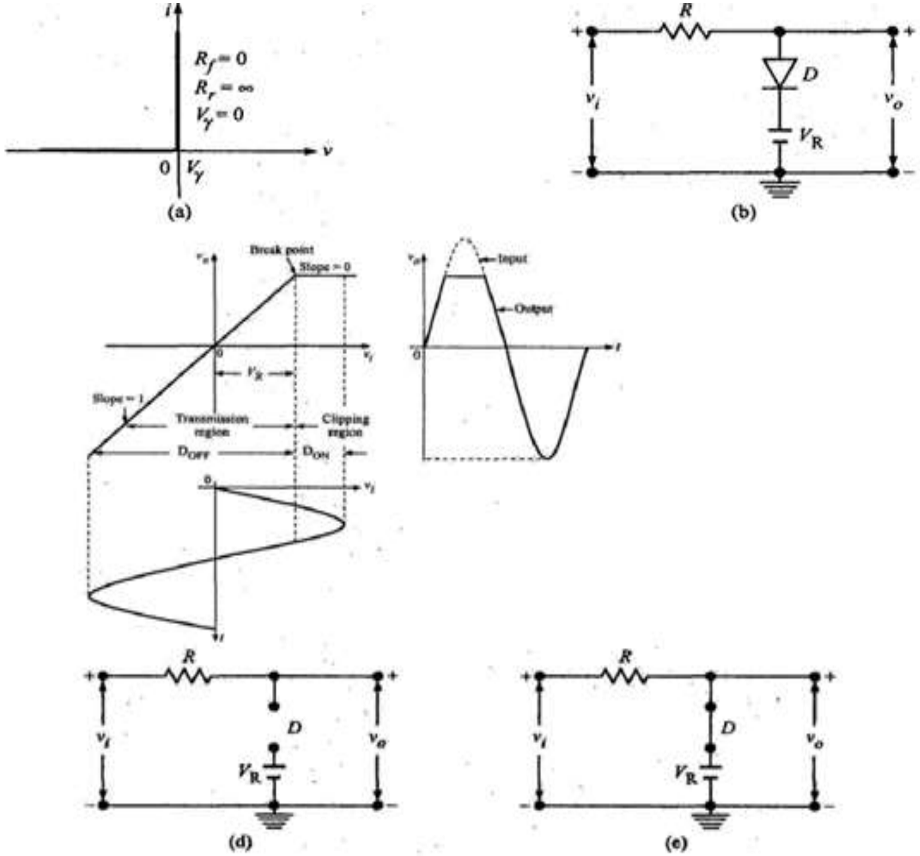
$$V_{r,\text{rms}} = \frac{I_{\text{d.c.}}}{4\sqrt{3}\,fC}$$

$$= \frac{V_{\text{d.c.}}}{4\sqrt{3}\,fCR_L}, \text{ since } I_{\text{d.c.}} = \frac{V_{\text{d.c.}}}{R_L}$$

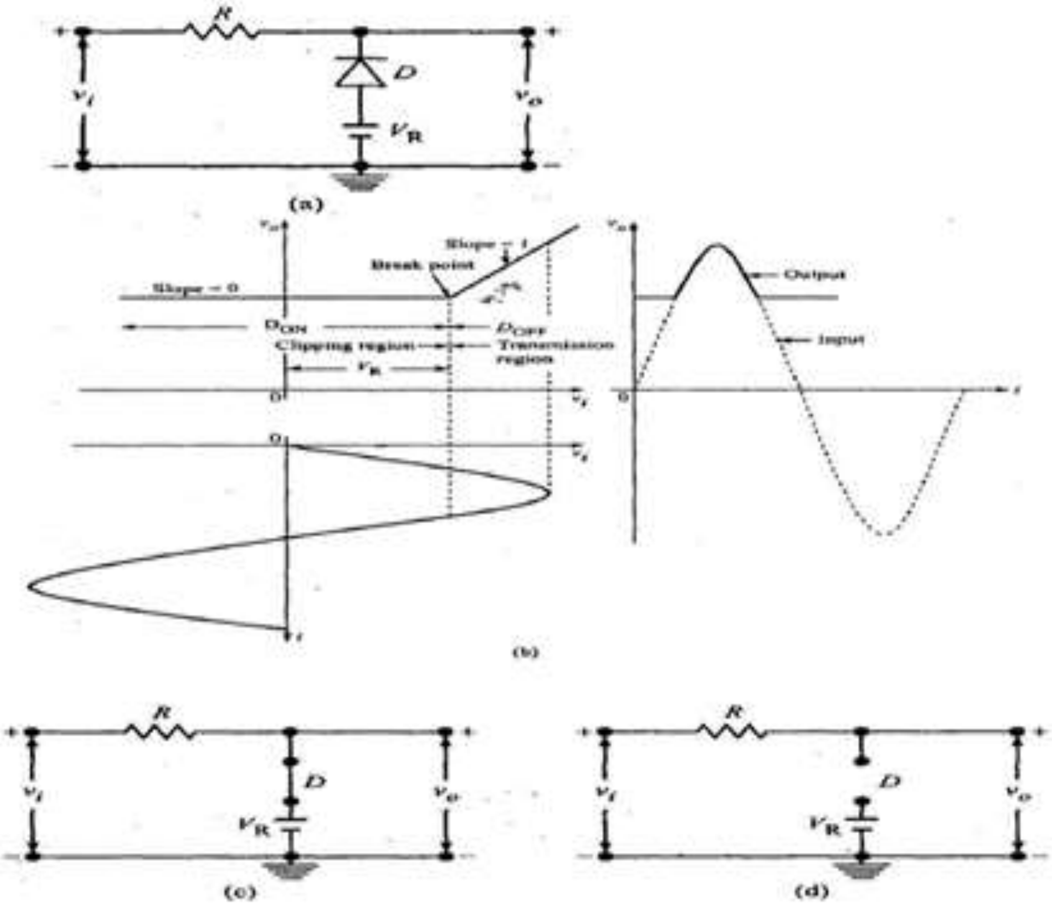
Therefore, ripple factor $$\Gamma = \frac{V_{r,\text{rms}}}{V_{\text{d.c.}}} = \frac{1}{4\sqrt{3}\,fCR_L}$$

Shunt Clippers

Clipping above reference level

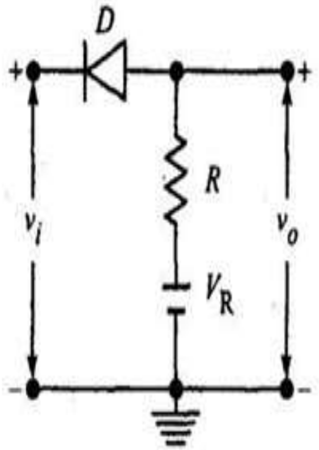


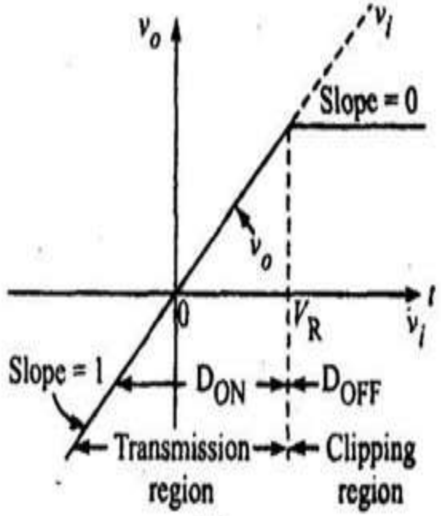
Clipping below reference level



Series Clippers

Clipping above the reference voltage





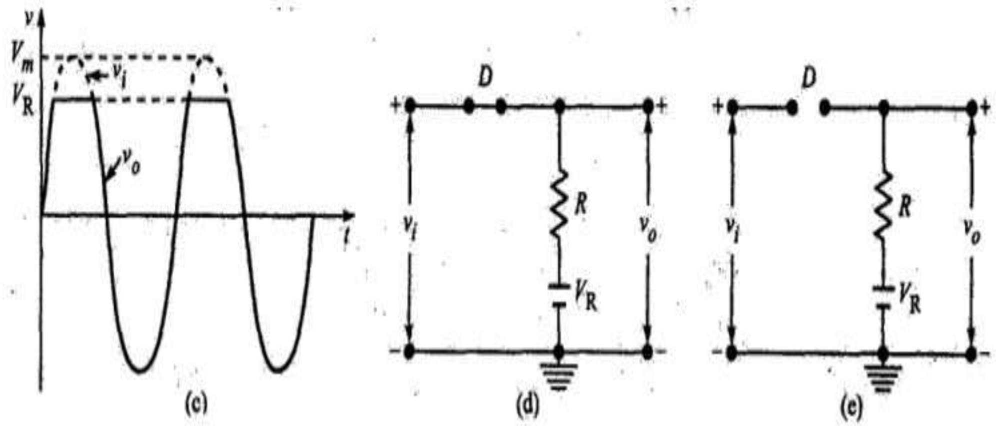
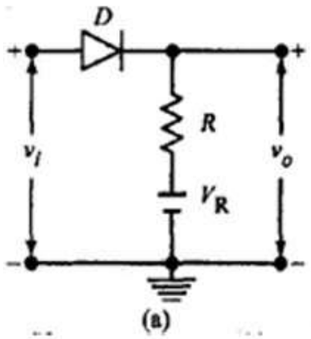
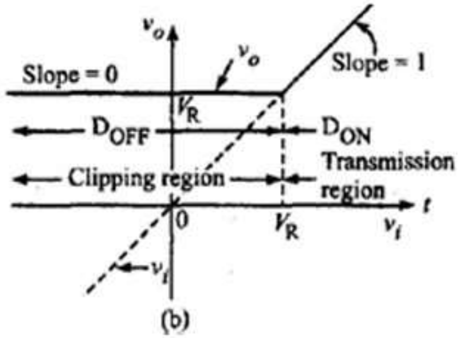


Figure 2.4 (a) Diode series clipper circuit diagram, (b) transfer characteristic, (c) output waveform for a sinusoidal input, (d) equivalent circuit for $v_i < V_R$, and (e) equivalent circuit for $v_i > V_R$.

Clipping below the reference voltage





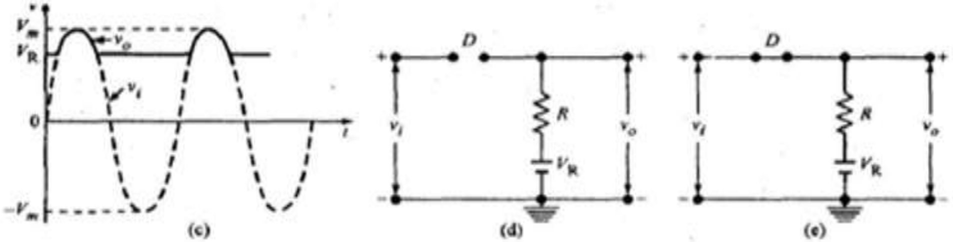


Figure 2.5 (a) Diode series clipper circuit diagram, (b). transfer characteristics, (c) output for a sinusoidal input, (d) equivalent circuit for $v_i < V_R$, and (e) equivalent circuit for $v_i > V_R$.

The circuit works as follows:

CLIPPING AT TWO INDEPENDENT LEVELS

Input v_i	*Output* v_o	*Diode status*
$v_i > V_{R1}$	$v_o = V_{R1}$	D_1 ON, D_2 OFF
$-V_{R2} < v_i < V_{R1}$	$v_o = v_i$	D_1 OFF, D_2 OFF
$v_i < -V_{R2}$	$v_o = -V_{R2}$	D_1 OFF, D_2 ON

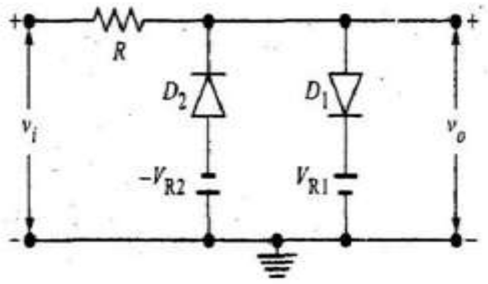


Figure 2.7 A diode clipper which limits at two independent levels.

Figure 2.7 A diode clipper which limits at two independent levels.

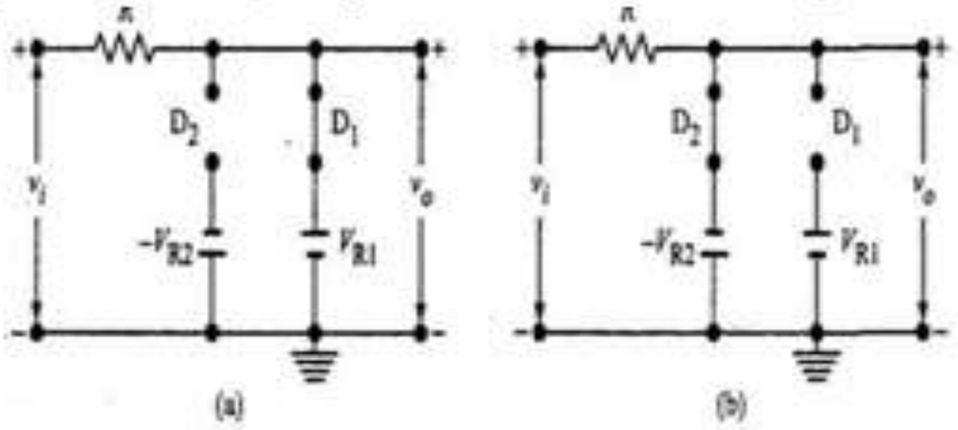


Figure 2.9 (a) Equivalent circuit for $v_i > V_{R1}$ and (b) equivalent circuit for $v_i < -V_{R2}$

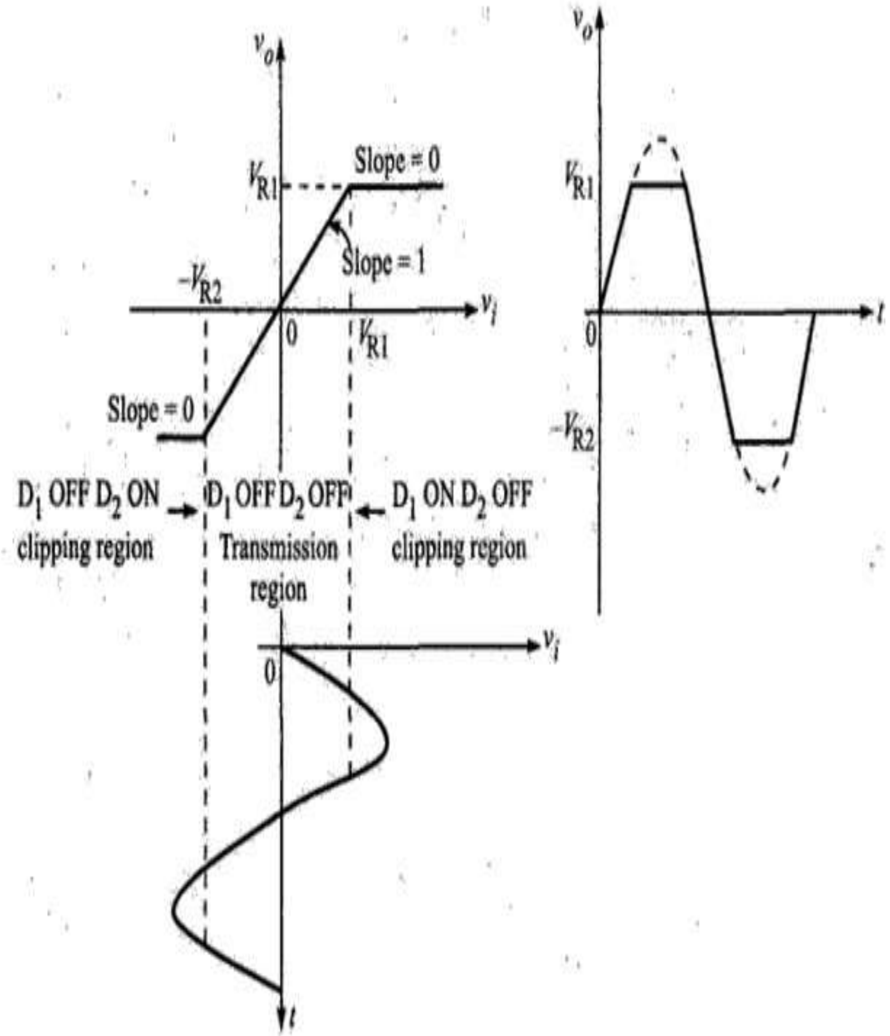


Figure 2.8 The piece-wise linear transfer curve, the input sinusoidal waveform and the corresponding output for the clipper of Figure 2.7.

CLAMPING CIRCUITS

The clamping circuit is often referred to as dc restorer or dc reinserter. In fact, it should be called a dc inserter, because the dc component introduced may be different from the dc component lost during transmission

Classification of clamping circuits

Basically clamping circuits are of two types:

(1) positive-voltage clamping circuits

(2) negative-voltage clamping circuit

Negative Clamper(positive peak clamper)

In negative clamping, the positive extremity of the waveform is fixed at the reference level and the entire waveform appears below the reference, i.e. the output waveform is negatively clamped with respect to the reference level

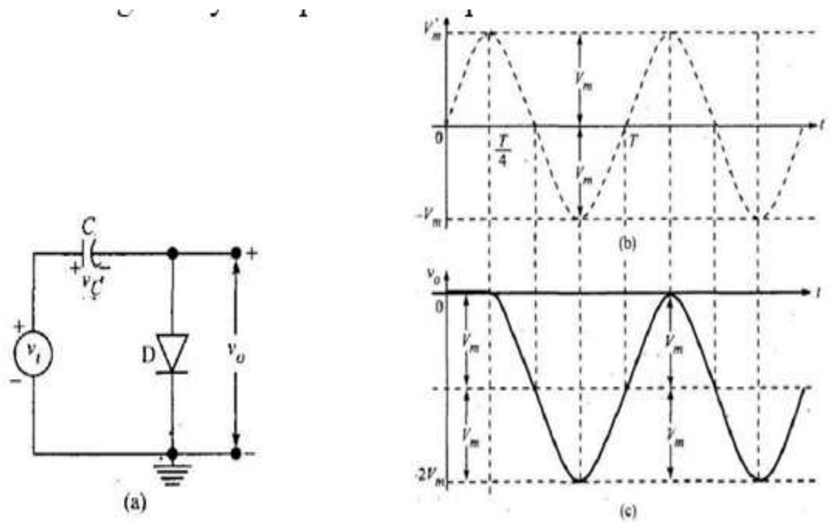


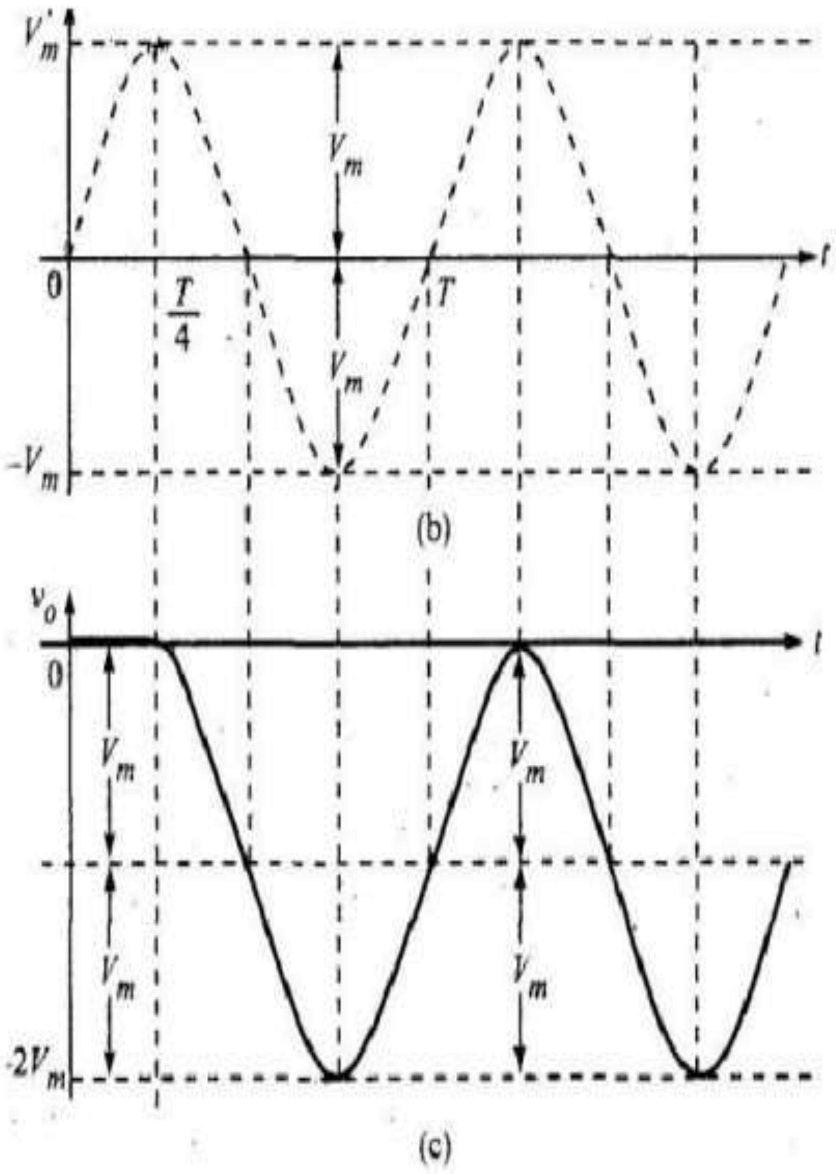
Figure 2.71 (a) A negative clamping circuit, (b) a sinusoidal input, and (c) a steady-state clamped output.

At -the end of the first quarter cycle, the voltage across the capacitor, vc = Vm

after the first quarter cycle, here is no path for the capacitor to discharge. Hence, the voltage across the capacitor remains constant at
vc = Vm

after the first quarter cycle, the output is given by v0 = vi - Vm. During the succeeding cycles, the positive extremity of the signal will be clamped or restored to zero and the output

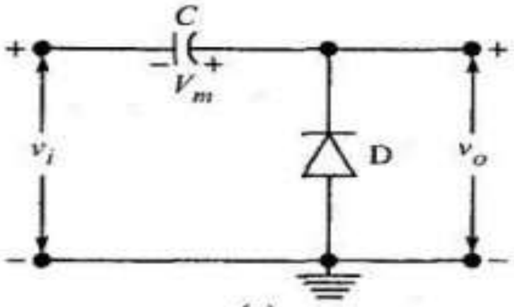
for $v_i = 0$, $v_o = - V_m$.
for $v_i = V_m$, $v_o = 0$,
for $v_i = - V_m$, $v_o = - 2V_m$.

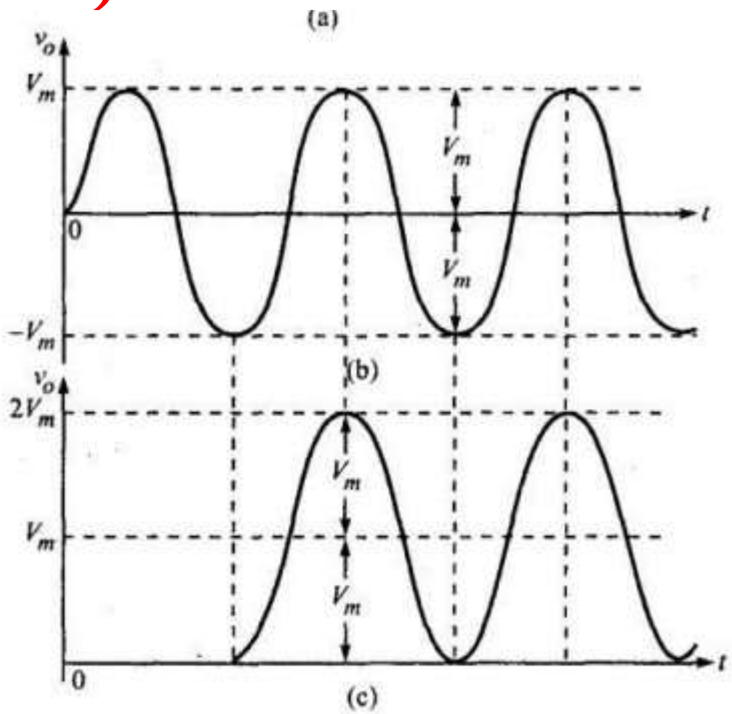


(b)
(c)

Positive Clamper(negative peak clamper)

In positive clamping, the negative extremity of the waveform is fixed at the reference level and the entire waveform appears above the reference level, i.e. the output waveform is positively clamped with reference to the reference level.





(a) A positive clamping circuit, (b) a sinusoidal input, and (c) a steady-state clamped output.

Clamping Circuit Theorem

The clamping circuit theorem states that, for any input waveform under steady-state conditions, the ratio of the area Af under the output voltage curve in the forward direction to that in the reverse direction Ar is equal to the ratio Rf/R

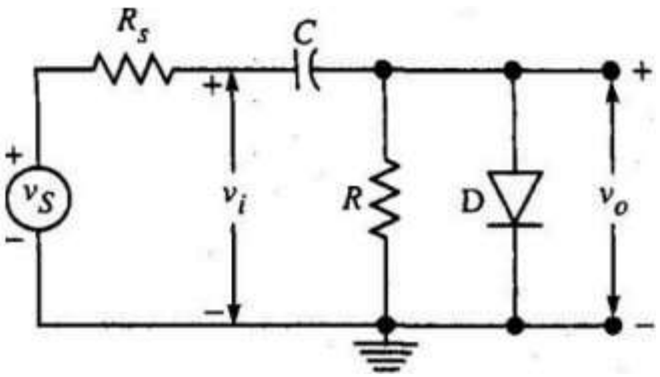
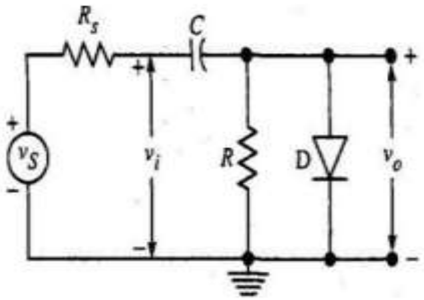


Fig :Clamping circuit considering the source resistance & the diode forward resistance



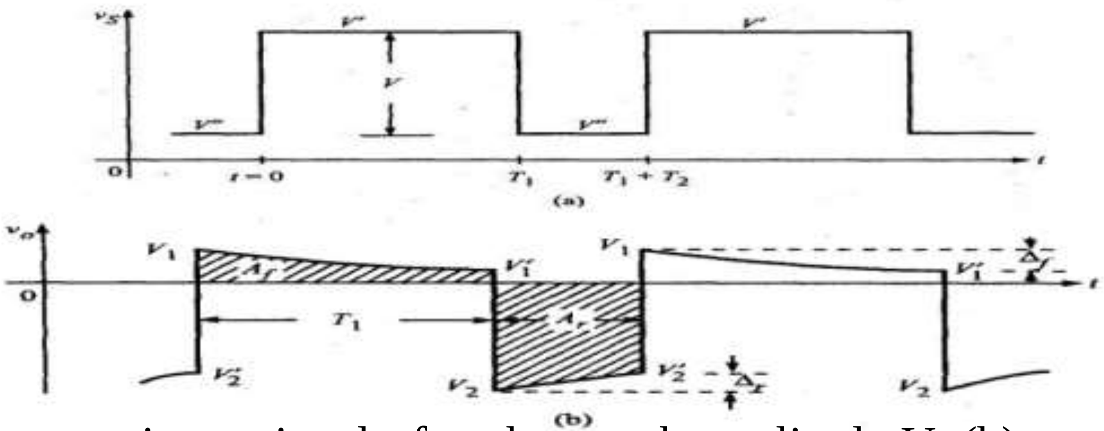
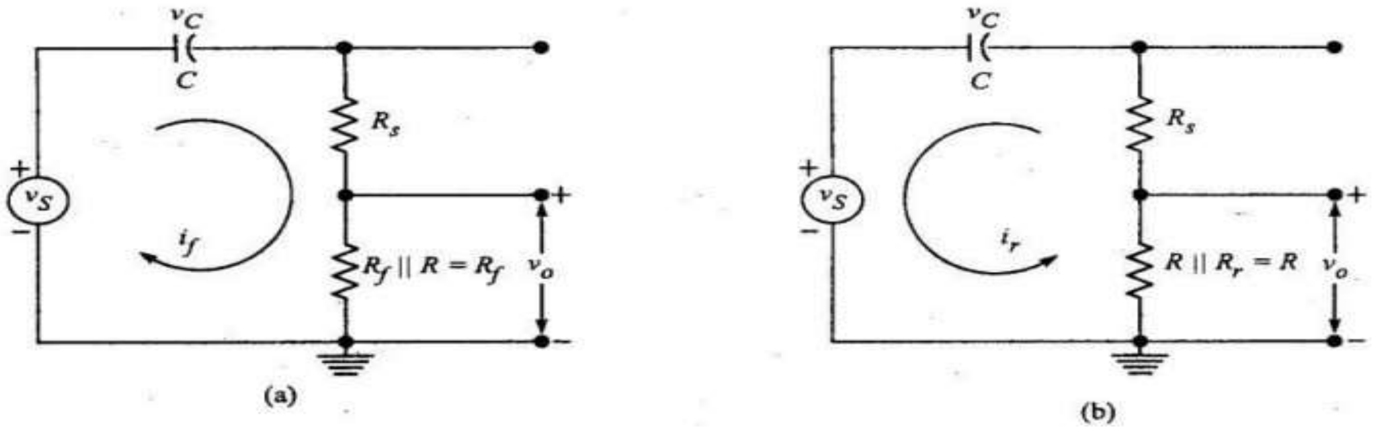


Fig : (a) A square wave input signal of peak-to-peak amplitude V, (b) the general form of the steady-state output of a clamping circuit with;' the input as in (a).

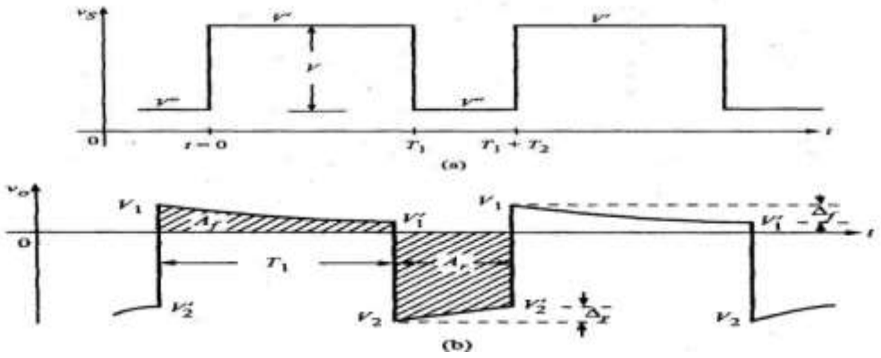


In the interval 0 < t < T, the input is at its upper level, the diode is ON,

If v/(f) is the output waveform in the forward direction, then the capacitor charging current is

$$i_f(t) = \frac{v_f(t)}{R_f}$$

$$Q_g = \int_0^{T_1} i_f(t)dt = \frac{1}{R_f}\int_0^{T_1} v_f(t)dt = \frac{A_f}{R_f}$$



Under steady-state the charge acquired by the Capacitor over one cycle must be equal to zero Therefore, the charge gained in the interval 0 < t < T1, will be equal to the charge lost in the interval T1 < t < T1 + T2, i.e. Qg = Ql

$$Q_g = \int_0^{T_1} i_f(t)dt = \frac{1}{R_f}\int_0^{T_1} v_f(t)dt = \frac{A_f}{R_f}$$

$$Q_l = \int_{T_1}^{T_1+T_2} i_r(t)\,dt = \frac{1}{R}\int_{T_1}^{T_1+T_2} v_r(t)dt = \frac{A_r}{R}$$

$$\frac{A_f}{R_f} = \frac{A_r}{R} \quad \text{i.e.} \quad \frac{A_f}{A_r} = \frac{R_f}{R}$$