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Estd.2007

UNIT-III

SINGLE VARIABLE CALCULUS

Let y=f(x) be a function continuous in the closed internal [a,b]. This means that if

$a < c < b$,

$\lim\limits_{x \to c} f(x) = f(c)$ and $\lim\limits_{x \to a+0} f(x) = f(a)$, $\lim\limits_{x \to b-0} f(x) = f(b)$

Let y = f(x) be differentiable in the closed interval [a,b]. This means that if $a < c < b$, the derivative of f(x) at x = c exists.

i.e., $\lim\limits_{x \to c} \frac{f(x) - f(c)}{x-c}$ exists

Further $\lim\limits_{x \to a+0} \frac{f(x) - f(a)}{x-a}$ and $\lim\limits_{x \to b-0} \frac{f(x) - f(b)}{x-b}$ exists.

Geometrically, if f(x) in a continuous function in the closed interval [a,b], the graph of y=f(x) is a continuous curve for the points x in [a,b]. If f(x) is derived in closed [a,b], there exists a unique tangent to the curve at every point in the interval [a,b]. This is shown in the following figures (1), (2), & (3).

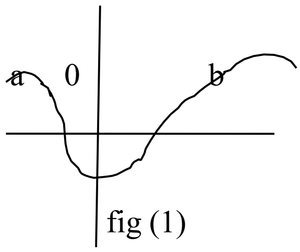


fig (1)

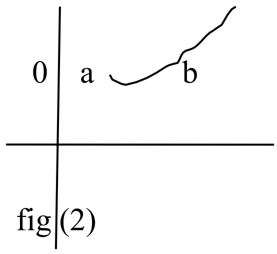
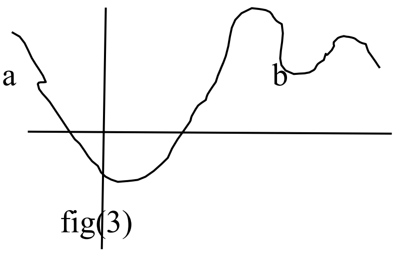


fig (2)



fig(3)

Mean Value Theorems

I) Rolle's Theorem

Statement : Let f(x) be a function such that

i) It is continuous in closed interval [a,b]

ii) It is differentiable in open interval [a,b] and

iii) f(a) = f(b)

Then there exists at least one point c in open interval (a,b) such that $f^1(c)= 0$

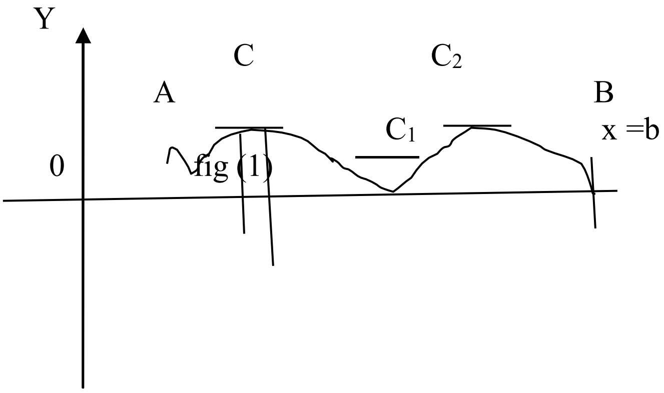
Geometric interpretation of Roll's theorem

Consider the portion AB of the curve y=f(x), lying between x = a and x = b such that

i) It goes continuously from A to B

ii) It has a tangent at every point between A and B, and

iii) Ordinate of A = ordinate of B



From the above fig(1), it is self evident that there is at least one point c (may be more) of the curve at which the tangent is parallel to the x – axis.

i.e. slope of the tangent at c (x = c) = 0. But the slope of the tangent at c is the value of the different co-efficient of f(x) with respect to x, therefore $f^1(c) = 0$.

Hence the theorem is proved.

Eg : 1) Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{\;}$ or $e^{-x} \sin x$ in $[o,\pi]$

Solution : given $f(x) = \frac{\sin x}{\;}$

i) We know that every polynomial is continuous in [a,b] so that sin x & e^{-x} are also continuous function is $[0,\pi]$

$\therefore \frac{\sin x}{\;}$ is also continuous in $[o,\pi]$

ii) Since sin x and e^x are derivable in $[0,\pi]$

$\therefore \frac{\sin x}{\;}$ is also continuous in $[o,\pi]$

iii) $F(o) = \frac{\sin o}{\;} = o$ and $f(\pi) = \frac{\sin}{\;} = 0$

$\therefore f(o) = f(\pi)$

Thus all the three conditions of Roll's theorem are satisfied.

$\therefore$ there exists $c \in (a,b)$ such that $f^1(c) = 0$

$\therefore (c-a)^{m-1} (c - b)^{n-1} [(m+n) c - (mb+na)] = 0$

$\rightarrow (m+n) c - (mb+na) = 0$

$\rightarrow (m+n) c - mb+na$

$\rightarrow c = \frac{mb+na}{+} E (a,b)$

[since the point $c \in (a,b)$ divides a and b internally in the ratio m:n]

$\therefore$ Roll's theorem is verified.

(3) verify Rolle's theorem for the function log[$\frac{x2 + ab}{(\; + \;)}$] in [a,b], a > o , b > o

Solution : let f(x) = log $\frac{x^2-ab}{(x^2-ab)}$,

= log (x^2 +ab) – log x (a+b)

= log (x^2 +ab) – log x – log x(a+b)

i) Since f(x) is a composite function of continuous functions in [a,b], it is continuous in [a,b].

ii) $f^1(x) = \frac{1}{^2+} 2x - \frac{1}{} = \frac{x^2-ab}{(\; ^2+ \;)}$, which exists ∀ x∈ (a,b)

∴ f(x) is derivable in (a,b)

iii) f(a) = log[$\frac{^2+}{a^2-ab)}$] = log1 = 0

f(b) = log [$\frac{^2+}{^2+ \;)}$] = log1 = 0

∴ f(x) = f(b)

Thus f(x) satisfies all the three conditions of Rolle's theorem.

∴there exists c∈ (a,b) such that $f^1(c)$ =0

i.e., $\frac{c^2-ab}{(\; ^2+ \;)}$ = 0

i.e., c^2 – ab = 0

i.e., c^2 =ab

i.e., c = ± $\sqrt{ab}$

∴ c = $\sqrt{ab}$ ∈ (a,b)

Hence Rolle's theorem is verified.

(4) Using Rolle's theorem, show that g(x) = gx^3 – 6x^2 – 2x +1 has a zero between 0 and 1.

Solution:

i) since g(x) being a polynomial.

∴ it is continuous on [0,1]

ii) since the derivative of g(x) exists for all x ε(0,1)

∴ it is derivable on (0,1)

iii) g(0)=1, and g(1) = 8-6-2+1=1

∴ g(0) =g(1)

Hence all the conditions of Rolle's theorem are satisfied on [0,1]

Therefore , there exists a number c ∈ (0,1) such that

$g^1(c)$ =0

Now $g^1(x) = 24x^2 - 12x - 2$

$\therefore g^1(c) = 0$

i.e., $24c^2 - 12c - 2 = 0$

i.e., $12c^2 - 6c - 1 = 0$

i.e.,$c = \frac{3 \pm \sqrt{21}}{12}$

i.e. c = 0.63 or -0.132

Here clearly c = 0.63 ∈ (0.1)

Thus there exists at least one root between 0 & 1

5) Verify whether Rolle's therorem can be applied to the following functions in the intervals cited :

i) f(x) = tan x in [0,π]

ii) $f(x) = \frac{1}{2}$ in [-1, 1]

ii) $f(x) = x^3$ in [1,3]

solution:

i) F(x) = tan x in [0,π] since f(x) is discontinuous at x = π/2

Thus the condition (1) of Rolle's theorem is not satisfied.

Hence we can't apply Rolle's theorem here.

ii) $f(x) = \frac{1}{2}$ in [-1, 1]

Here f(x) is discontinuous at x = 0

Hence Rolle's theorem can't be applied.

iii) $f(x) = x^3$ in [1,3]

Here clearly f(x) is continuous on [1,3] and derivable on (1,3)

But f(1) G f(3)

i.e., condition (3) of Rolle's theorem fails

Hence we can't apply Rolle's theorem for $f(x) = x^3$ in [1,3]

Exercise : (A)

I) verify Rolle's theorem for the following functions in the intervals indicated.

i) x^2 in [-1,1] ii) $x(x+3)\, e^{-x/2}$ in [-3,0]

iii) $x^{2/3} - 2x^{1/3}$ in (0,8) iv) $\frac{x^{2-x-6}}{x-1}$ in (-2,3)

v) $x^2 - 2x - 3$ in (1,-3) vi) |x| in [-1,1]

answers : i) c =0 ii) c = -2 iii) c = 1 iv) not applicable

v) c =1 vi) not applicable.

II) Langrange's means value theorem :- (LMVT)

Statement: let f(x) be a function such that

i) It is continuous is closed interval [a,b] and

ii) Differentiable in open interval [a,b]

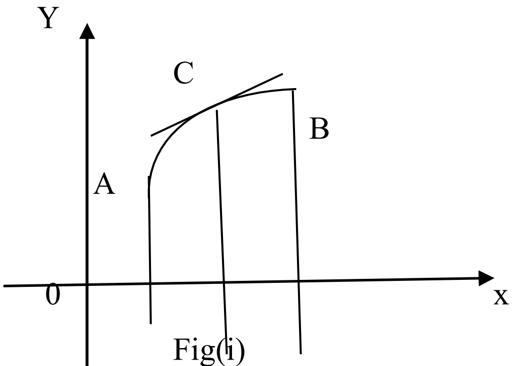
Then there exists at least one point of x say c in open interval (a,b) i.e. a < c < b such that

$$f^1(c) = \frac{f(b) - f(a)}{b - a}$$

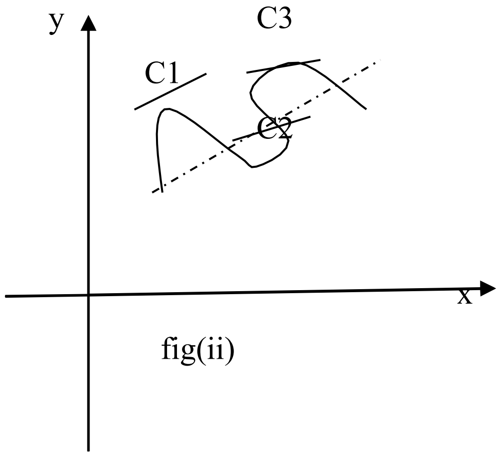
Note : Langrange's mean value theorem is also known as first mean value theorem of differential calculus.

Geometric interpretation of Lagrange's mean value theorem

Let A,B be the points on the curve y = f(x) corresponding to x = a and x=b so that A = [a,f(a)] and B=[b,f(b)], shown in figure (i)&(ii) below.



Fig(i)



fig(ii)

$\therefore$ slope of chord AB $= \frac{f(b) - f(a)}{b - a}$

By lagranges mean value theorem, the slope of the chord AB = $f^1(c)$, the slope of the tangent of the curve at c(x=c)

Hence the lagrange's mean value theorem asserts that if a curve AB has a tangent at each of its points, then there exists at least one point C on this curve, the tangent at which is parallel to the chord AB.

Another form of Lagrange's mean value theorem

Let f(x) be a function such that

i) It is contiunuous in the closed interval [a,a+b],

ii) $f^1(x)$ exists in the open interval (a,a+b)

Then there exists at least one number θ $(0 < \theta < 1)$

such that $f(a+b) = f(a) + hf^{1}(a+\theta b)$

Solved examples

Eg (1) : Verify Lagrange's mean value theorem for

$$f(x) = x^3 - x^2 - 5x + 3 \text{ in } [0,4]$$

solution :

Since f(x) is a polynomial so that it is continuous and derivable for every value of x.

In particular, f(x) is continuous in closed interval [0,4] and derivable in open interval (0,4).

Hence by Lagange's mean value theorem, there exists a point c in open interval (0,4) such that

$$f^{1}(c) = \frac{f(4)-f(0)}{4-0}$$

i.e., $3c^2 - 2c - 5 = \frac{f(4)-f(0)}{4}$ ------- (1) $(\because f^{1}(x) = 3x^2 - 2x - 5)$

Here $f(4) = 4^3 - 4^2 - 5(4) + 3 = 64-16-20+3 = 31$

and $f(0) = 3$

from (1), we have $3c^2 - 2c - 5 = 7$

$$= 3c^2 - 2c - 12 = 0$$

$$\therefore c = \frac{2 \pm \sqrt{4+144}}{6} = \frac{2 \pm \sqrt{148}}{6} = \frac{1 \pm \sqrt{37}}{3}$$

Here clearly $c = \frac{1+\sqrt{37}}{3}$ c (0,4)

2) Verify lagrage's mean value theorem for $f(x) = \log_e x$ in [1,e]

Solution: given $f(x) = \log_e x \Rightarrow f^{1}(x) = \frac{1}{}$

Since f(x) is a polynomial so that it is continuous in [1,e] and derivable in [1,e]

$\therefore$ By lagrage's mean value theorem, there exists a point c c (1,e) such that

$$f^{1}(c) = \frac{f(e)-f(1)}{e-1} = \frac{1-0}{e-1} = \frac{1}{e-1} \text{ --- (1)}$$

but $f^{1}(c) = \frac{1}{}$

$$\frac{1}{C} = \frac{1}{e-1}$$

$\therefore c = e - 1$ c (1,e)

Hence lagrange's mean value theorem is verified.

3) State whether langrange's mean value theorem can be applied to the following function in the interval indicated justify your answer.

$$F(x) = x^{3/4} \text{ in } [-1,1]$$

Solution :

Given $f(x) = x^{1/3}$

Clearly f(x) is continuous in closed interval[-1,1]

But $f^1(x) = \frac{1}{3}\, x\, \frac{1}{3}\text{-}1 = \frac{1}{3\ \ ^{2/3}}$ is not derivable at x = 0.

Hence it is not derivable in open interval (-1,1)

Hence we can't apply lagrange's mean value theorem.

Exercise : (B)

1) Verify lagrange's mean value theorem for the following functions in the intervals indicated.

i) Cos x in [0,π/2] ii) |x| in [-1,1]

iii) x^3-$2x^2$ in [2,5] v) $2x^2 - 7x+10$; a-2, b=5

2) Find C of the lagrange's theorem for

F(x) = (x-1) (x-2) (x-3) on [0,4] ans: $C=\frac{16\pm3}{3}$

3) State whether LMVT can be applicable for the function

$F(x) = \frac{1}{\ }$ in [-1,1] ans: not applicable

Eg:

1) If a < b , prove that $\frac{b-a}{1+\ \ ^2} < \tan^{-1} a < \frac{b-a}{1+\ \ ^2}$ using lagrange's mean value theorem reduce the following

i) $\frac{\ }{4}+\frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\ }{4}+\frac{1}{6}$

ii) $\frac{5\ \ +4}{20} < \tan^{-1}2 < \frac{\ +2}{4}$

Solution :

Consider $f(x) = \tan^{-1}x$ in [a,b] for 0 < a < b < 1

Since f(x) is continuous in closed interval [a,b] and derivable in open interval[a,b] we can apply lagrange's mean value theorem.

Hence exists a point c c (a,b) such that

$f^1(x) = \frac{f(b)-f(a)}{b-a}$

Hence $f^1(x) = \frac{1}{1+\ \ ^2}$

$f^1(c) = \frac{1}{1+\ \ ^2}$

Thus there exists a point c, a<c<b such that

$\frac{1}{1+\ \ ^2} = \frac{\tan^{-1}b-\tan^{-1}a}{b-a}$ ------- (1)

We have a < c < b

$1+a^2 < 1+c^2 < 1+b^2$ ------------(2)

$\frac{1}{1+\ ^2} > \frac{1}{1+\ ^2} > \frac{1}{1+\ ^2}$

Using 1) and 2), we have

$\frac{1}{1+\ ^2} > \frac{tan^{-1}b - tan^{-1}a}{b-a} > \frac{1}{1+\ ^2}$

or $\frac{b-a}{1+\ ^2} < \tan^{-1}b - \tan^{-1}a < \frac{ba}{1+\ ^2}$ ------- (3)

Hence the result.

Deduction:

i) We have $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+\ ^2}$ ---- (4)

Put $b = \frac{4}{3}$, a=1, we get

$= \frac{\frac{4}{3}-1}{1+\frac{16}{9}} < \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}(1) < \frac{\frac{4}{3}-1}{1+1^2}$

$\rightarrow \frac{\frac{4-3}{3}}{\frac{25}{9}} < \tan^{-1}\left(\frac{4}{3}\right) - \frac{\pi}{4} < \frac{\frac{4-3}{3}}{2}$

$\rightarrow \frac{3}{25} + \frac{\pi}{4} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$

ii) Put b=2 and a=1 in (4), we get

$\frac{2-1}{1+2^2} < \tan^{-1}(2) - \tan^{-1}(1) < \frac{2-1}{1+2^2}$

$\rightarrow \frac{1}{5} < \tan^{-1}(2) - \pi/4 > \frac{1}{2}$

$\rightarrow \frac{1}{5} + \frac{\pi}{4} < \tan^{-1}(2) < \frac{\pi}{4} + \frac{1}{2}$

or $\frac{4+5}{20} < \tan^{-1}(2) < \frac{2+}{4}$

2) Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\frac{3}{4} \le \frac{\pi}{6} \pm \frac{1}{8}$ using langrange's mean value theorem.

Solution : let $f(x) = \sin^{-1}(x)$, which is continuous and differentiable .

Now $f^1(x) = \frac{1}{\sqrt{1-x^2}}$ -- $f^1(c) = \frac{1}{\sqrt{1-c^2}}$

By Langrange's mean value theorem, there exist c c (a,b) such that a < c<b and

$f^1(c) = \frac{f(b)-f(a)}{b-a}$

i.e, $\frac{1}{\sqrt{1-c^2}} = \frac{\sin^{-1}b - \sin^{-1}a}{b-a}$ ----- (1)

We have a < c < b

Then $a^2 < c^2 < b^2$

$\rightarrow 1-a^2 > 1-c^2 > 1-b^2$

$\rightarrow \sqrt{1-a^2} > \sqrt{1-c^2} > \sqrt{1-b^2}$

$\rightarrow \frac{1}{\sqrt{1-a^2}} > \frac{1}{\sqrt{1-c^2}} > \frac{1}{\sqrt{1-b^2}}$

$\rightarrow \frac{1}{\sqrt{1-a^2}} < \frac{\sin^{-1}b - \sin^{-1}a}{b-a} < \frac{1}{\sqrt{1-b^2}}$

$\rightarrow \frac{b-a}{1+a^2} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{1+\ ^2}$

Put a =1/2 and b =3/5

$\rightarrow \frac{\frac{3}{5}-\frac{1}{2}}{\sqrt{1-\frac{}{4}}} < \sin^{-1}\frac{3}{4} - \sin^{-1}\frac{1}{2} < \frac{\frac{3}{5}-\frac{1}{2}}{\sqrt{1-(\frac{1}{4})^2}}$

$\rightarrow \frac{2}{10\sqrt{3}} < \sin^{-1}\frac{3}{4} - \frac{\ }{6} < \frac{1}{8}$

$\frac{\ }{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\frac{3}{5} < \frac{\ }{6} + \frac{1}{8}$

3) Prove using mean value theorem | sin u- sinv | ≤ | u – v|

Solution : if u = v , there , is nothing to prove.

If u > v , then consider the function

F(u) = sin u on [v,u]

Clearly, f is continuous on [v,u] and derivable on(v,u)

∴ By Lagrange's mean valve theorem, there exists c c (v,u)

Such that $\frac{f(u)-f(v)}{u-v} = f^1(c)$

$\frac{\sin u - sing\ v}{u-v} = \cos c$

But | cos c| ≤ 1

∴ $\left|\frac{\sin u - sing\ v}{u-v}\right| \leq 1$

If v > u, then in similar manner, we have

|sin v – sin u| ≤ | v – u|

| sin u – sun v| ≤ | u – v | [∴|x| = |x|]

Hence for all u, v E R

| sin u – sun v| ≤ | u – v |

4) show that for any x> 0 , $1+x < e^x < 1+e^x$

Solution:

Let f(x) = e^x defined on [0,x] and derivable on (0,x)

∴ By Lagrange's mean value theorem

There exists a number c c (0,x) such that

$$\frac{f(x)-f(o)}{x-o} = f^1(c)$$

$$\frac{e^x - e^o}{} = e^c$$

$$\frac{e^x - 1}{} = e^c \text{--------- (1)}$$

Now c ε(o,x) i.e., 0 < c < x

$$e^o < e^c < e^x$$

$$1 < \frac{e^x - 1}{} < e^x < \text{from (1)}>$$

$$x < e^x - 1 < xe^x$$

$$1+x < e^x < 1+xe^x$$

Exercise : (C)

1) Find c of cauchys mean value theorem for f(x) = $\sqrt{3}$ and g(x) = $\frac{1}{\sqrt{x}}$ in [a,b]

Solutions :

Clearly f, g are continuous on [a,b]

We have f(x) = $\sqrt{x}$

$$F^1(x) = \frac{1}{2\sqrt{x}}$$

And g(x) = $\frac{1}{\sqrt{x}}$

$g^1(x) = -\frac{1}{2x\sqrt{x}}$, which exists on (a,b)

∴ f,g are differentiable on (a,b)

Also $g^1(x) \neq 0 \ \forall \ x \ \varepsilon(a,b) \ CR^+$

∴ conditions of cauchys mean value theorem are satisfied on (a,b)

∴ there exists c c (a,b) such that

$$\frac{f(b)-f(a)}{g(b)-f(a)} = \frac{f^1(c)}{g^1(c)}$$

$$\frac{\sqrt{b}-\sqrt{a}}{\frac{1}{\sqrt{b}}-\frac{1}{\sqrt{a}}} = \frac{\frac{1}{2\sqrt{c}}}{\frac{1}{2c\sqrt{c}}}$$

$$\frac{\sqrt{b}-\sqrt{}}{\frac{\sqrt{b}-\sqrt{a}}{\sqrt{ab}}} = \frac{2c\sqrt{c}}{\sqrt{c}}$$

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$\frac{\sqrt{ab}\,(\sqrt{b}-\sqrt{a})}{\sqrt{b}-\sqrt{a}} = c$

$\sqrt{ab} = c$

Clearly $c = \sqrt{ab}$ c (a,b)

Hence Cauchy mean value theorem is verified.

2) Find c of Cauchy mean value theorem on [a,b] for

$f(x) = e^x$ and $g(x) = e^x$ (a,b > o)

solution :

given $(x) = e^x$ and $g(x) = e^{-x}$

clearly f, g are continuous on[a,b] and f,g are differentiable on (a,b)

also $g^1(x) = -e^{-x} \neq 0\ \forall\ x\ \varepsilon\ (a,b)$ such that

$\frac{f(b)-f(a)}{g(b)-f(a)} = \frac{f^1(c)}{g^1(c)}$

$\frac{e^b - e^a}{e^{-b} - e^{-a}} = \frac{}{-e^{-c}}$

$\frac{e^b - e^a}{\ } = -e^{2c}$

$\frac{e^b - e^a}{\frac{e^a - e^b}{e^a - e^b}} = -e^{2c}$

$\frac{e^b - e^a}{\frac{e^a - e^b}{+}} = -e^{2c}$

$\frac{e^{a+b}(e^b - e^a)}{-(e^b - e^a)} = -e^{2c}$

$e^{a+b} = e^{2c}$

a+b = 2c

$C = \frac{+}{2}$ c (a,b)

Hence LMVT is verified

Exercise :(D)

1) Verify cauchy mean value theorem for the following

i) $f(x) = \frac{1}{2}$, $g(x) = \frac{1}{}$ on [a,b] ans: $c = \frac{2}{+}$

ii) f(x) = sin x , g(x) = cos x on [o, $\frac{\pi}{2}$ ans : c = π/4

iii) f(x) = log x and g(x) = x^2 in [a,b], b>a>1 show that $\frac{logb-loga}{b-a} = \frac{+}{2\ 2}$

iv) $f(x) = x^2$, $g(x) = x^3$ in [1,2] ans : $c = \frac{14}{9}$

Taylor's theorem

Statement: If f : [a,b] →R is such that

i) f^{n-1} is continuous on [a,b]

ii) f^{n-1} is derivable on (a,b) or $f^{(n)}$ exists on (a,b) then there exists a point c c (a,b) such that

$$f(b) = f(a) + \frac{b-a}{1!} f^1(a) + \frac{(b-a)^2}{2!} f^1(a) + \text{---} + \frac{(b-a)^{n-1}}{n-1} f^{n-1}(a) + R_n$$

i) Scholmitch – Roche's form of remainder:

$$R_n = \frac{(b-a)^p (b-c)^{n-p} f^n(c)}{(n-1)!} \quad \text{------ (1)}$$

ii) Lagrange's form of remainder : put p=1, in (1) we get

$$R_n = \frac{(b-a)^n f^n(c)}{!}$$

iii) Cauchy's form remainder : put p=1 in (1), we get

$$R_n = \frac{(b-a)\ \ (b-c)^{n-1} f^n(c)}{(n-1)!}$$

Note : (x) = f(a) +(x-a) $f^1(a) + \frac{(x-a)^2}{2!} f^{11}(a)$+--------- is called Taylor's series for f(x) about

x =a

Machlaurin's theorem

Statement: If f:[0,x] →R is such that

i) f^{n-1} is continuous on [0,x]

ii) f^{n-1} is derivable on(0,x) then there exists a real number θ c (0,1) such that

$$f(x) = f(0) + x f^1(0) + \frac{x^2}{2!} f''(0) + \text{---------} + x^{n-1} f^{(n-1)}(0) + R_n$$

i) **Roche's form of remainder**:

$$R_n = \frac{x^n (1-\theta)^{n-p} f^n(\theta x)}{(n-1)!} \quad \text{--------- (1)}$$

ii) **Langrange's form remainder** : put p=n in (1)

We get $R_n = \frac{}{!} f^n(\theta x)$

iii) **Cauchys form of remainder** : put p=1 in (1)

We get $R_n = \frac{x(1-\theta)^{n-p} f^n(\theta x)}{(n-1)!}$

Note : $f(x) = f(0) + xf^1(0) + \frac{2}{2!} f'''(0)$+-----+$\frac{x^n}{!} f^n(0)$ +--- is called maclaurin's series expansion of f(x).

Solved examples

1) Obtain Taylor's series expansion of $f(x) = e^x$ in powers of x+1

Obtain the talylor's series expansion of e^x about x = -1.

Solution : let $f(x) = e^x$ about x = -1

Here a = -1

$\therefore f(x) = e^x \; f^1(x) = e^x \; f^1(a) = e^{-1}$

$f''(x) = e^x$------ $f''(a) = e^{-1}$

We know that the Talylor's series expansion of f(x) about x =a is

$f(x) = f(a) + (x-a)\, f^1(a) + \frac{(x-a)^2}{2!} f^{11}(a) +$---------(1)

put $f(x) = e^x$ & a=-1 in (1) , we get

$e^x = f(-1) + (x+1)\, f^1(-1) + \frac{(\ +1)^2}{2!} f^{11}(-1) +$ ------

$e^x = e^{-1} + (x+1)\, e^{-1} + \frac{(\ +2)^2}{2!} +$ ------

$e^x = e^{-1}\,[\, 1+ (x+1) + \frac{(\ +2)^2}{2!} +$ ------] is the required Taylor's series expansion about x =-1

2) Show that $\frac{\sin^{-1}x}{\sqrt{1-x^2}} = x+4\frac{x^3}{3!}+$ ------

Let $f(x) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ then f(0) = 0

$\sqrt{1-x^2}\, f(x) = \sin^{-1}x$-------- (1)

Differentiating (1) w.r.t. x, we get

$\sqrt{1-x^2}\, f^1(x) + f(x)\left(\frac{-2x}{2\sqrt{1-x^2}}\right) = \frac{1}{\sqrt{1-x^2}}$

$(1-x^2)\, f^1(x) - xf(x) = 1$--------------(2)

Now $f^1(0) = 1$

Differentiate (2) w.r.t. x, we get

$(1-x^2)\, f^{11}(x) + f^1(x)\,(-2x) - xf^1(x) - f(x) = 0$-------(3)

$(1-x^2)\, f^{11}(x) - 3xf^1(x) - f(x) = 0$

Then $f^{11}(0) = 0$

Differentiate (3) w.r.t. x, we get

$(1-x^2)\; f^{111}(x) - 2x\, f^{11}(x) - 3f^1(x) - 3xf^{11}(x) - f^1(x) = 0$

$(1-x^2)\, f^{111}(x) - 5xf^{11}(x) - 4f^1(x) = 0$

$f^{111}(o) - 4f^1(o) = 0$

$f^{111}(o) = 4$ $(\therefore f^1(0) = 1)$

Similarly $f^{IV}(O) = 0$

We have by Taylor's theorem,
$F(x) = f(o)+1.x+\frac{x^2}{2!} f^{11}(0)+\frac{x^3}{3!} f^{111}(0)+\frac{x^4}{4!} f^{iv}(0)+$----

$\frac{sin^{-1}x}{\sqrt{1-x^2}}$ = 0 +1.x + $\frac{}{2!}$ f^{11}(0) + $\frac{3}{3!}$ f^{111}(0) + $\frac{4}{4!}$ f^{iv}(0)+----

= x + 4$\frac{3}{3!}$ + -----

3) Show that log(1+e^x) = log2 + $\frac{}{2}$ + $\frac{2}{8}$ - $\frac{24}{192}$ +-------and hence reduce that

$\frac{2}{+1}$ = $\frac{1}{2}$ + $\frac{}{4}$ - $\frac{}{48}$ + ------

Solution : let f(x) = log(1+e^x) then f(0) = log2

Differentiate successively w.r.t. x, w get

f^1(x) = $\frac{x}{1+}$ ∴ f^1(0) = $\frac{1}{1+1}$ = $\frac{1}{2}$

f^{11}(x) = $\frac{(1+e^x)^x - e^x e^x}{(1+\ )^2}$ = $\frac{}{(1+\ )^2}$ ∴ f^1(o) = $\frac{1}{(1+1)^2}$ = $\frac{1}{4}$

f^{111}(X) = $\frac{(1+e^x)^2 e^x - 2e^x(1-e^x)e^x}{(1+\ )^4}$ = $\frac{(1-e^x)e^x[e^x + e^{2x} - 2e^{2x}]}{(1+\ )^4}$

= $\frac{x - e^{2x}}{(1+\ )^3}$

∴ f^{111}(o) = 0

$\frac{(1+e^x)^3(e^x - 2e^{2x}) - (e^x - e^{2x})\ 3(1+e^x)^2 e^x}{(1+\ )^6}$

= $\frac{(1+e^x)(e^x - 2e^{2x}) - 3e^x(1-1)}{(1-1)^4}$ = $\frac{2}{16}$ = $\frac{1}{8}$

Substituting the values of f(o), f^{II}(o),----------------in the maclaurin's series

f(x) = f(0) + xf^1(0) + $\frac{2}{2!}$ f^{11}(o) + $\frac{3}{3!}$$f^{111}$ (o) + ------

We get log(1+e^x) = log2 +x($\frac{1}{2}$) + $\frac{2}{2!}$ ($\frac{1}{4}$) + $\frac{3}{3!}$ (0) + $\frac{}{4!}$ (-$\frac{1}{8}$) + ----

log(1+e^x) = log2 + $\frac{}{2}$ + $\frac{2}{8}$ - $\frac{4}{192}$ + ----------------(1)

Deduction :

Differentiating the result given by (1) w.r.t x,

We get $\frac{1}{1+\ 2}$ e^x = $\frac{1}{2}$ + $\frac{2}{8}$ - $\frac{3}{48}$ + -------------

4) Verify Taylor's theorem for f(x) = (1-x)$^{5/2}$ with lagrange's form of remainder upto 2 terms in the interval [0,1].

Solution: consider f(x) = (1-x)$^{5/2}$ in [0,1]

i) f(x), f^1(x) are continuous in [0,1]

ii) f^{II}(x) is differentiable in (0,1)

Thus f(x) satisfies the conditions of Taylor's theorem.

We consider Taylor's theorem with Lagrange's form of remainder

$f(x) = f(o) + xf^1(o) + \frac{2}{2!}\ f^{11}(o)$ with $0<\theta < 1$------ (1)

Here n =p=2, a=0, and x =1

$f(x) = (1-x)^{5/2}$ then $f(0) = 1$

$f^1(x)= \frac{5}{2}(1-x)^{3/2}$ then $f^1(0)= - 5/2$

$f^{II}(x) = \frac{15}{4}(1-x)^{1/2}$ then $f^{II}(\theta x) = \frac{15}{4}(1-\theta x)^{1/2}$

i.e., $f^{II}(\theta) = \frac{15}{4}\ (1-\theta)^{1/2}$

and $f(1) = 0$

From (1), we have $f(x) = f(0)+xf^1(0) + \frac{2}{2!}\ f^{11}(\theta x)$

Substituting the above values, we get

$\theta = \frac{9}{25} = 0.36$

$\therefore$ θ lies between 0 and 1.

Thus Taylor's theorem is verified.

5) Obtain the Maclaurins series expression of the following functions.

i) e^x ii) sin x iii) $\log_e(1+x)$

solutions:

i) let $f(x) = e^x$ then

$f^1(x) = f^{11}(x) = f^{111}(x)$ =-------- $=e^x$

$\therefore\ f(0) = f^1(0) = f^{11}(0) = f^{111}(0)$-------- $=e^0 = 1$

The Maclaurins series expression of f(x) is given by

$f(x) = f(0) + xf^1(0) + \frac{2}{2!}\ f^{11}(0) +$ ------ $+ \frac{}{!} f^n(0) +$-------

i.e., $e^x = 1 + \frac{}{1!} + \frac{2}{2!} + \frac{3}{3!} +$ ------ $+ \frac{}{!} +$ -----

ii) let $f(x) = \sin x$ then $f(0) = \sin 0 = 0$

Then $f^1(x) = \cos x \rightarrow f^1(0) = \cos 0 = 1$

$f^{11}(x) = -\sin x \rightarrow f^{11}(0) = -\sin 0 = 0$

$f^{111}(x) = -\cos x \rightarrow f^{111}(0) = -\cos 0 = -1$

$f^{IV}(x) = \sin x \rightarrow f^{1v}(0) = \sin 0 = 0$

substituting all these values in maclarins series of f(x) given by ,

$$f(x) = f(0) + xf^{1}(0) + \frac{x^2}{2!} f^{11}(0) + \frac{x^3}{3!} f^{111}(0) + \frac{x^4}{4!} f^{iv}(0) + -----$$

$$\sin x = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(0) + -----$$

$$\sin x = x - \frac{x^3}{3!} + --------$$

iii) let $f(x) = \log_e (1+x)$

$f^{I}(x) = \frac{1}{1+x} \rightarrow f^{1}(0) = \frac{1}{1+0} = 1$

$f^{II}(x) = \frac{1}{(1+x)^2} \rightarrow f^{11}(0) = \frac{1}{(1+0)^2} = 1$

$f^{III}(x) = \frac{2}{(1+x)^3} \rightarrow f^{111}(0) = \frac{2}{(1+0)^3} = 2$

$f^{IV}(x) = \frac{-6}{(1+x)^4} \rightarrow f^{IV}(0) = \frac{-6}{(1+0)^4} = -6$

substituting all these values in maclurins series expansion of f(x) given by,

$$f(x) = f(0) + xf^{1}(0) + \frac{x^2}{2!} f^{11}(0) + \frac{x^3}{3!} f^{111}(0) + \frac{x^4}{4!} f^{iv}(0) + -----$$

we get , $\log(1-x) = 0 + x(1) + \frac{x^2}{2!} -1 + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) + ------$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ------$$

Exercise: (E)

1) Obtain the maclaurins series for the following functions.

   i) Cos x ii) sin x iii) $(1-x)^n$

2) Obtain the Taylor's series expansion of sinx in powers of $x - \frac{\pi}{4}$

3) Write Taylor's series for $f(x) = (1-x)^{5/2}$ with lagrange's form of remainder upto 3 terms in the interval [0,1].

Applications of definite integral's

Definite integral:

Definition

Given a function f(x) that is continuous on the interval [a,b] we divide the interval into n sub intervals of equal width Δx and from each interval choose a point , x_i^*. Then the definite integral of f(x) a to b is

$\int^{b} f(x)dx = \lim_{n\to\infty} \sum_{i=1} f(x_i^*)\,\Delta x$

The integration procedure helps us in evaluating length of plane curves, volume of solids of revolutions, surface area of solids of revolution, determination of centre of mass of a plane mass distribution etc.,

Surface areas of Revolution:

Equation of curve	Axis of revolution	Surface area
Cartesion form:		$S = 2\pi\int^{b} y\sqrt{1+(\frac{dy}{dx})^2}\,dx$
i) Y = f(x)	X – axis	
ii) X= f(y)	Y – axis	$S= 2\pi\int^{d} y\sqrt{1+(\frac{dy}{dx})^2}\,dy$

Solved examples

1) Find the area of the surface of the revolution generated by revolving about the x – axis of the arc of the parabola $y^2=12x$ from x =0 to x=3

Solution: given $y^2 = 12x$

$y = 2\sqrt{3}\sqrt{x}$

$\frac{dy}{dx} = 2\sqrt{3}\,\frac{1}{2\sqrt{x}} = \sqrt{\frac{3}{x}}$

$\therefore$ Surface area $= 2\pi\int^{b} y\sqrt{1+(\frac{dy}{dx})^2}\,dx$

$= 2\pi\int_0^3 2\sqrt{3}\sqrt{x}\sqrt{1+\frac{3}{x}}\,dx$

$= 4\pi\sqrt{3}\int_0^3 \sqrt{x}\sqrt{1+\frac{x+3}{x}}\,dx$

$= 4\pi\sqrt{3}\int_0^3 (1+x)^{\frac{1}{2}}$

$= 4\pi\sqrt{3}\,\left[\frac{x+3^{3/2}}{3/2}\right]$

$= \frac{8\sqrt{3}}{3}\,[(6)^{3/2} - (3)^{3/2}]$

$= \frac{8}{\sqrt{3}}\,(3)^{3/2}\,[(2)^{3/2} - 1]$

$= 24\pi\,[2\sqrt{2} - 1]$

2) Find the area of the surface of revolution generates by revolving one area of the curve y=sinx about the x – axis .

Solution: given curve is y = sin x

Here x varies from 0 to π/2

$\therefore \frac{dy}{dx} = \cos x$

Hence required surface area

$= 2\pi \int_0^{\pi/2} y \sqrt{1 + (\frac{dy}{dx})^2}\, dx$

$= 2\pi \int_0^{\pi/2} \sin x \sqrt{1 + \cos^2 x}\, dx$

$= 2\pi \int_0^1 \sqrt{1 + t^2}\, dt$ (putting cos x = t)

$= 2\pi \left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\sinh^{-1} t\right]_0^1$

$= 2\pi \left[\frac{1}{2}\sqrt{2} + \frac{1}{2}\sinh^{-1}(1) - 0 - 0\right]$

$= \pi [\sqrt{2} + \sinh^{-1}(1)]$

3) The area of the curve $x = y^3$ between y =0 and y=2 is revolved about y-axis. Find the area of surface so generated.

Solution : given curve is $x = y^3$

Then $\frac{dx}{dy} = 3y^2$

$\therefore$ required surface area $= 2\pi \int_0^2 x \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

$= 2\pi \int_0^2 y^3 \sqrt{1 + (3y^2)^2}\, dy$

$= 2\pi \int_0^2 y^3 \sqrt{1 + 9y^4}\, dy$

$= 2\pi \int_1^{145} \frac{\sqrt{t}}{36} dt$ (putting $1+9y^4=t$)

$= \frac{\pi}{18}\left[\frac{2}{3} t^{3/2}\right]_1^{145}$

$= \frac{\pi}{27}[(145)^{3/2} - 1]$

Exercise: (F)

1) Find the surface area generated by the revolution of an arc of the catenary $y = C \cosh \frac{x}{c}$ about x –axis ans : $\pi c^2[1+\frac{\sinh 2}{2}$

2) Find the area of the surface of revolution generated by revolving the arc of the curve $a^2 y = x^3$ from x =0 to x =a about the x –axis ans: $\frac{\pi}{27}[10\sqrt{10} - 1]$

3) Find the surface area of s phere of radius 'a' ans: $4\pi a^2$

Volumes of solids of revolution:

Region		Volume of solid generated
Castesion form		
i)	y=f(x) the x – axis and the lines x =a , x=b	$V = \pi \int^{b} y^2\ dx$
ii)	x=g(y) the y – axis and the lines y=c, y=d	$V = \pi \int^{a} x^2\ dy$
iii)	y =y_1(x), y=y_2 (x) the x – axis and ordinates x=a, x=b	$V = \pi \int^{b} (x_2^2 - x_1^2)\ dy$

Solved examples:

1) Find the volume of a sphere of radius 'a'.

Solution :

Sphere is formed by the revolution of the area enclosed by a semi circle its diameter

Equation to circle of radius 'a' is $x^2+y^2 = a^2$---------(1)

Then $y^2 = a^2-x^2$

In semi circle 'x' varies from –a to a.

∴ Required volume = $\pi \int_{-a}^{a} {}^{2}\ dx$

$= \pi \int_{-a}^{a} (a^2-x^2)\ dx$

$= \pi \left[a^2x - \frac{x^3}{3}\right]_{a}^{a}$

$= \pi \left[a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right]$

$= \pi \left[2a^2 - \frac{2a^3}{3}\right]$

$= \frac{4\ \ {}^{3}}{3}$ cubic units

2) Find the volume of the solid that result when the region enclosed by the curve $y=x^3$, y=0, y=1 is revolved about y – axis .

Solution :

Given curve is $y = x^3$

Then $x=y^{1/3}$

∴ Required volume = $\pi \int_0^1 x^2\ dy$

$= \pi \int_0^1 (y^{1/3})^2\ dy$

$= [\frac{5/3}{5/3}]_0^1$

$= \frac{3}{5}[\ (1)^{5/3}\ -0]$

$= \frac{3}{5}$ cu. units

3) Find the area of the solid generated by revolving the arc of the parabola $x^2 = 12y$, bounded by its latusrectum about y – axis.

Solution:

Given parabola is

$x^2 = 12y = 4(3)y$ (i.e $x^2 = 4ay$)

let 'O' be the vertex and LL^1 be the latusrectum as shown in fig.

for the arc OL, y varies from 0 to 3.

∴ Required volume = 2(volume generated by the revolution about the y – axis of the area OLC)

$= 2\pi \int_0^3 x^2\ dy$

$=2\pi \int_0^3 (12)\ dy$

$= 24\pi[\frac{y^2}{2}]_0^3 = 108\pi$ cubic units

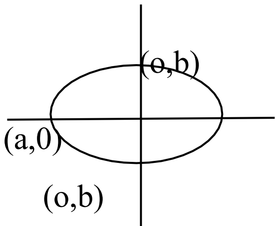
4) Find the volume of the solid generated by revolving the ellipse $\frac{\ ^2}{\ ^2} + \frac{\ ^2}{\ ^2} = 1$ (0 < b < a) about the major axis.

Solution :

Given equation of the ellipse is

$\frac{\ ^2}{\ ^2} + \frac{\ ^2}{\ ^2} = 1$

When y =0, x = ± a



∴ major axis of the ellipse is x = -a to +a

∴ The volume of the solid generated by the given ellipse revolving about the major axis

$= \int_{-a}\ \ ^2\ dx$

$=2\pi \int_0^a y^2\ dx$

$= 2\pi \int_0 (b^2 - \frac{b^2}{\ ^2}\ ^2)\ dy$

$= 2\pi\ [\ b^2x - \frac{b^2}{2}\frac{x^3}{3}]_0^a$

$= 2\pi\ [b^2a - \frac{b^2}{2}\frac{a^3}{3}\ - (0)]$

$= 2\pi\, [\, ab^2 - \frac{ab^2}{3}] = \frac{4}{3}\ \pi ab^2$

Exercise :(G)

1) Find the volume got by the revolution of the area bounded by x – axis, the catenary

$y = a \cosh (\frac{x}{})$ about the x-axis between the ordinates $x = \pm a$

Ans : $\pi a^3 (1+\frac{1}{2}\sinh 2)$

2) Find the volume of the solid when ellipse $\frac{2}{2}\frac{2}{2}$ = 1, (o< b< a) rotates about minor axis

Ans: $\frac{4\ \ 2}{3}$

Objective type Questions

1. The value of c of Rolle's theorem for $f(x) = \frac{sinx}{}$ in ((0,Π) is

   a) Π b) $\frac{\pi}{4}$ c) $\frac{}{3}$ d) $\frac{}{2}$

2. Using which mean value theorem, we can calculate approximately the value of $(65)^{1/6}$ in the easier way

   a) Cauch's b) Lagrange's c) Taylor's II order d) Rolle's

3. The value of Cauchy's mean value theorem for $(x) = e^x$ and $g(x) = e^{-x}$ defined on [a,b], o<a<b is

   a) $\sqrt{ab}$ b) $\frac{a-b}{2}$ c) $\frac{+}{2}$ d) $\frac{2}{+}$

4. If f(x) is continuous in [a,b], $f^1(x)$ exists for every value of x in (a,b), f(a)=f(b), there exists at least one value c of x in (a,b) such that $f^1(c)$ = ___________

a) 0 b) a+b c) c d) b

5. Lagrange's mean value theorem for f(x) = sec x in (0.2Π) is

a) Applicable b) not applicable due to non-differentiability

c) applicable and c= $\frac{}{2}$ d) not applicable due to discontinuity

6. F(a+h) = f(a) + $hf^{1}(a)$ + $\frac{h^2}{2}$ $f^{II}(a)$ +......................+$\frac{h^2}{2}$ $f^{n}(a+\theta h)$ is called

a) Taylor's theorem with lagrange form of remainder

b) Caughy's theorem with lagranges form of remainder

c) Raiman's theorem with lagrange form of remainder

d) Lagrange's theorem with lagrange form of remainder

7. If f(x) =f(o) +..............$F^{II}(o)$, then the series is called

a) Maclaurin's Series b) Taylor's Series

c) Cauchy's Series d) lagrange's series

8. The value of Rolle's theorem in (-1,1) for f(x) = x^3-x is

a) 0 b) $\pm\frac{1}{\sqrt{3}}$ c) $\frac{1}{2}$ d) $\pm\frac{1}{\sqrt{2}}$

9. The value of x so that $\frac{f(b)-f(a)}{b-a}$ $f^{1}(x)$ whne a< x<b given f(x)= $\frac{1}{2}$, a=1, b=4

a) $\frac{3}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{9}{4}$

10. The value of c of Cauchy's mean value theorem for the function f(x) = x^2, g(x) = x^3 in the interval [1,2] is

a) $\frac{14}{9}$ b) $\frac{3}{14}$ c) $\frac{17}{9}$ d) $\frac{5}{14}$

11. If f(o)=0, $f^{1}(0)$=1, $f^{II}(0)$=1, $f^{III}(0)$=1, then the machlaurin's expansion of f(x) is given by

a) $x+\frac{2}{2}+\frac{3}{3}+\ldots$ b) $x+\frac{2}{2}+\frac{3}{6}+\ldots$

c) $-x-\frac{2}{2}+\frac{3}{3}+\ldots$ d) $x-\frac{2}{2}+\frac{3}{3}+\ldots$

12. The value c of Rolle's theorem in [$\frac{1}{2}$,2] for f(x) = x^2 + $\frac{1}{2}$is

a) $\frac{3}{4}$ b) $\frac{5}{4}$ c) 1 d) $\frac{3}{2}$

13. Lagrange's mean value theorem for f(x)=secc in (0,2π) is

a) Not applicable due to discontinuity b) applicable & c= $\frac{}{2}$

c) not applicable due to non differentiable d) applicable

14. In the Taylor's theorem, the cauchy's form of remainder is

a) $\frac{h^{n-1} f^{n-1}(a-\theta h)}{}$ b) $h^n f^n (a+\theta h)$

c) $\frac{h(1-\theta)^{n-1} f^{n}(a-\theta h)}{L\, n-1}$ d) $\frac{h^{n+1} f(a-\theta h)}{}$

15. The value of c in Rolle's theorem for f(x) = sinx in(0,πa) is

a) $\frac{1}{}$ b) $\frac{}{4}$ c) $\frac{}{7}$ d) $\frac{\pi}{hn}$

16. The value of c in Rolle's theorem for $f(x)=x^2-x$ in (-1,1)

a) 0 b) 0.5 c) 0.25 d) -0.5

17. The value of c in Rolle's theorem for $f(x) = x^2-x(0,1)$

a) 0 b) 0.5 c) 0.25 d) -0.5

18. The value of c in lagrange's mean value theorem for $f(x) = e^x$ in (0,1) is

a) $Log(e-e^{-1})$ b) loge(c) c) log(e+1) d) log (e-1)

19. The value of c in Cauchy's MVT for $f(x)=e^x$ and $g(x) = e^{-x}$ in (3,7) is

a) 4 b) 5 c) 4.5 d) 6

20. The value of θ if $f(x)=x^2$ & $f(x+h)=f(x)+hf^1(x+\theta h)$

a) -0.5 b) 0.25 c) 0 d) 0.5

21. The value of c in Cauchy's mean value theorem for $f(x) =\sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in (1,4) is

a) 1.5 b) 2 c) 2.5 d) 3

22. The value of c in lagrange's mean value theorem for f(x) = logx in [1,e] is

a) $(e-1)^{-1}$ b) e+1 c) e-1 d) e

23. Lagrange's mean value theorem is not applicable to the function $f(x) = x^{\frac{1}{3}}$ in [-1,1] because

a) F(-1)≠f(1) b) f is not continuous in [-1,1]

c) f is not derivable in (-1,1) d) f is not a objective function

24. Lagrange's MVT is not applicable to the function defined on [-1,1] by $f(x) =x\sin\frac{1}{}$ (x≠0) and f(0)=0 because

a) F(-1) =f(1) b) f is not continuous in [-1,1]

c) f is not deriable in (-1,1) d) f is not a one to one function

25. The value of c for lagrange's MVT for the function f(x) =cosx in $[0,\frac{\pi}{2}]$ is

a) $Cos^{-1} (\frac{2}{})$ b) $\sin^{-1} (\frac{2}{})$ c) $\sin^{-1} (\frac{1}{})$ d) $Cos^{-1} (\frac{1}{})$

26. The value of c for Rolle's theorem for f(x)=(x-a)(x-b) in [a,b] is

a) $-\frac{a+b}{2}$ b) $\sqrt{ab}$ c) a+b d) $\frac{a+b}{2}$

27. The value of c for lagrange's mean value theorem for f(x)=(x-2)(x-3) in [0,1] is

a) 0.5 b) 1 c) 2.5 d) 2

28. The value of c of Rolle's theorem for f(x)=(x-1)(x-2) in [0,3] is

a) 1.5 b) 2.5 c) 3 d) 2

29. The value of c of Cauchy's mean value theorem for f(x)=sinx and g(x)=cosx in $[0, \frac{\pi}{2}]$

a) $\frac{\pi}{ð}$ b) $\frac{}{6}$ c) $\frac{}{4}$ d) $\frac{}{3}$

30. Maclaurin's expansion for log(1+x) is

a) $x - \frac{^2}{2} + \frac{^3}{3} - \frac{^4}{4} + \ldots$ b) $x + \frac{^2}{2} + \frac{^3}{3} + \frac{^4}{4} + \ldots$

b) c) $x + \frac{^2}{2!} + \frac{^3}{3!} + \frac{^4}{4!} + \ldots$ d) $x - \frac{^2}{2!} + \frac{^3}{3!} - \frac{^4}{4!} + \ldots$

31. Maclaurin's expansion of cosx is

a) $\sum_{=0}^{\infty} \frac{K^{2r}}{(2\ )!}$ b) $\sum_{=0}^{\infty} \frac{(-1)^r K^{2r}}{(2\ )!}$

c) $\sum_{=0}^{\infty} \frac{(-1)^r (K^{2r+1}}{(2\ +1)!}$ d) $\sum_{=0}^{\infty} \frac{K^{2r+1}}{(2\ +1)!}$

32. The expansion of e^x in powers of (x-1)

a) $E(\sum_{=0}^{\infty} \frac{(1-K)^r}{!})$ b) $e^{-1} \sum_{=0}^{\infty} \frac{(1-K)^r}{!}$

c) $e(\sum_{=0}^{\infty} \frac{(-1)^r (K-1)^r}{!})$ d) $\sum_{=0}^{\infty} \frac{(-1)^r (K-1)^r}{!}$

33. The expansion for sinx in powers of $(x-\frac{}{2})$ is

a) $1- \frac{1}{2}(x- \frac{\pi}{2})^2 + \frac{1}{4}(x - \frac{\pi}{2})^4 - \ldots\ldots$

b) $x+(x-\frac{\pi}{2}) + \frac{1}{3!}(x- \frac{\pi}{2})^3 + \ldots\ldots$

c) $1+\frac{1}{2}(x-\frac{\pi}{2})^2 + \frac{1}{4!}(x-\frac{\pi}{2})^4 + \ldots\ldots$

d) $x- (x-\frac{\pi}{2})^2 + \frac{1}{3!}(x- \frac{\pi}{2})^3 + \ldots\ldots$

34. Volume of the solid generated by revolving y=f(x), the x-axis and the lines x=a, x=b is

a) $\int^b \pi x^2\, dx$ b) $\int^b (y^2 - x^2)\, dx$ c) $\int^b \pi y^2\, dx$ d) none

35. Volume of the solid generated by revolving the area bounded by the curve x=f(x), the y-axis and the lines y=a, y=b is

a) $\int^b \pi x^2\, dx$ b) $\int^b \pi x^2\, dy$ c) $\int^b \pi x^2\, dx$ d) $\int^b \pi y^2\, dy$