NARSIMHA REDDY
ENGINEERING COLLEGE
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UNIT – I

MATRICES

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Matrix: A set of mn real or complex numbers or functions displayed as an array of m horizontal lines (called rows) and n vertical lines (called columns) is called a matrix of order (m, n) or $m \times n$ (read as m by n) .The numbers or functions are called the elements or entries of the matrix and are enclosed within brackets [ ] or ( )
Matrices are denoted with capital letters A , B , C ..& elements are denoted with small letters a ,b , c ….letters i and j are used as suffixes on the a , b , c …to denote the row and columns position respectively of the corresponding entry .
Thus

$$A=[a_{ij}]=\begin{bmatrix} a_{11} & a_{12}\dots & a_{1j}\dots & a_{1n} \\ a_{21} & a_{22}\dots & a_{2j}\dots & a_{2n} \\ a_{i1} & a_{i2}\dots & a_{ij}\dots & a_{in} \\ a_{m1} & a_{m2}\dots & a_{mj}\dots & a_{mn} \end{bmatrix} \quad \text{where } 1\le i\le m$$

$$1\le j \le n$$

is a matrix with m rows and n columns

Types of matrices :
Real matrix: A matrix whose elements are all real numbers or function is called a real matrix

Ex : $\begin{bmatrix} -1 & 0 \\ 2 & -2 \\ 13 & 5 \end{bmatrix}$, $\begin{bmatrix} 0 & -1 \end{bmatrix}$.

Complex matrix: A matrix which contains at least one complex numbers or function as on element is called a complex matrix

Ex : $\begin{bmatrix} 1 & -i \\ 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 7 & 3+i \\ 13 & 8 \end{bmatrix}$

Row matrix: A matrix with only one row is called a row matrix or row vector .It is a matrix of order $1 \times n$ for some positive integer n.

Ex :[-3 7 0 2 11] ; [7 4 8]

Column matrix: A matrix with only one column is called a column matrix or column vector .It is a matrix of order $m \times 1$ for some positive integer m .

Ex: $\begin{bmatrix} 0 \\ 2 \\ 16 \end{bmatrix}$ & $\begin{bmatrix} 5 \\ 12 \\ 6 \end{bmatrix}$

Square matrix: A matrix in which the number of rows and the number of columns are equal is called a square matrix

Ex: $\begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$,$\begin{bmatrix} 0 & 5 & 3 \\ 7 & 6 & 4 \\ -3 & 0 & 2 \end{bmatrix}$

A square matrix of order $n \times n$ is simply described as an n- square matrix.
Diagonal matrix: A square matrix [aij] with ij $= 0$ for i G j is called a diagonal matrix

Ex: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ & $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & -8 \end{bmatrix}$; $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Scalar matrix: A diagonal matrix which consists all the elements are equal in the diagonal is called scalar matrix.

Ex: $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

Zero or null matrix : A matrix in which every entry is zero is called a zero matrix or null matrix and is denoted by o.

EX: $0_{3\times2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $0_{1\times2} = [0\ 0]$

Unit matrix (or) Identity matrix :A diagonal matrix in which all the diagonal elements are equal to unity or 1 is called unit matrix (or) Identity matrix and is denoted by I.

Ex: $I = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$

Rectangular matrix: A matrix in which the numbers of rows and the numbers of columns may not be equal is called a rectangular matrix .

Ex: $\begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 5 \end{bmatrix}$, $\begin{bmatrix} 5 & -3 \\ 0 & 1 \\ 12 & 4 \end{bmatrix}$

Upper triangular matrix : A square matrix A=[aij] in which aij = 0 for i>j is called an upper triangular matrix .

Ex: $\begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 6 & -2 \\ 0 & 5 \end{bmatrix}$

Lower triangular matrix: A square matrix A =$[aij]_{n\times n}$in which = 0 for i<j is called a lower triangular matrix

Ex: $\begin{bmatrix} -1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 7 \end{bmatrix}$, $\begin{bmatrix} 11 & 0 \\ 6 & 8 \end{bmatrix}$

Triangular matrix: A matrix which is called either upper triangular or lower triangular is called as triangular matrix .

Idempotent matrix: A square matrix which remains the same under multiplication by itself is called an idempotent matrix. In other words ,a square matrix A is called idempotent matrix if ^{2} =A .

Ex: $_2$= $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $_3$= $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $_2$= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $_3$= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Involuntary matrix: A matrix which is its own inverse is called on involuntary matrix .In other words ,a square matrix A is involuntary if ^{2} =I.

Nilpotent matrix: A square matrix which vanishes when it is raised to some positive integral power m is called a nilpotent matrix .In other words a square matrix A which is such that =o for some m belongs to N, is called a nilpotent matrix .

Ex: A= $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

$^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,i.e; $^2 = 0$,here m=2

Periodic matrix: If a square matrix and is such that
$^{+1} =$ for some positive integer n then A is called a periodic matrix
The least positive integer p such that $^{+1} =$ holds is called the period of A and is denoted by P(A).
Note : A periodic matrix of period one is on idempotent matrix

Ex: $A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = A$

Hence A is a periodic matrix of period one

Transpose of a matrix: The matrix obtained from any given matrix A, by interchanging its rows and columns is called the transpose of A and it is denoted by ^{1}or

Ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

Properties of transpose of a matrix:

If A^T and B^T be the transposes of A and B respectively , then

1) $(A^T)^T = A$
2) $(A+B)^T = A^T + B^T$, A and B being of the same order
3) $(KA)^T = K.A^T$, K is a scalar
4) $(AB)^T = B^T A^T$, A and B being conformable for multiplication.

Trace of a square matrix : The sum of the elements along the main diagonal of a square matrix ,A is called the trace of A and written as
Trace(A)= $a_{12}+a_{22}+\ldots . a_{nn}$

Tr (A)= $\sum^{n}_{i=1}$ ij

Properties of trace of A

Tr (KA) = K. Tr (A), where K is a scalar
Tr (A+B) = Tr(A) + Tr (B)

Tr (AB) = Tr (BA)

Equal matrix : Two matrices A=[a_{ij}] and B=[b_{ij}] are said to be equal if and only if

(1) A and B are of the same type (or order) and

(2) $a_{ij} = b_{ij}$ for every i and j .

Addition of matrices:
Let A =$[a_{ij}]_{m\times n}$; B[b_{ij}] be two matrices .The matrix C=$[c_{ij}]_{m\times n}$ where $c_{ij}= a_{ij}+b_{ij}$ is called the sum of the matrices A and B. The sum of A and B is denoted by A+B

Difference of two matrices: If A and B are two matrices of the same type (order) then A+(-B) is taken as A-B.

Matrix multiplication :
Let A =$[a_{ik}]_{m\times n}$ and B=$[b_{kj}]_{n\times p}$ then the matrix C=$[c_{ij}]_{m\times p}$

where $c_{ij} = \sum_{=1} a_{ik}*b_{kj}$ is called the product of The matrices A and B in that order and we write C=AB

In the product AB, the matrix A is called the pre-factor and B the post-factor.

If the number of columns of A is equal to the number of rows in B then the

matrices are said to be conformable for multiplication in that order.

Properties of matrix multiplication:

1) Matrix multiplication is associate

   i.e if A,B,C are matrices ,then (AB)C=A(BC)

2) Multiplication of matrices is distributive with respect to addition of matrices

   i.e; A(B+C)=AB+AC and (B+C)A=BA+CA

Note: A(B-C)=AB-AC and (B-C)A= BC-CA

3)If is A is matrix of order $m \times n$ then A I_n= I_n. A=A.

Sub matrix of a matrix : A sub matrix of a matrix A is a matrix obtained from A by deleting some rows and / or some columns of A.

Ex: A= $\begin{bmatrix} 1 & -1 & 0 & 7 \\ 4 & 3 & 2 & 8 \\ -6 & 11 & 0 & 5 \end{bmatrix}_{3\times4}$

The sub matrices of A are

$\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}$ & $\begin{bmatrix} 1 & 0 & 7 \\ 4 & 2 & 8 \\ -6 & 0 & 5 \end{bmatrix}$

Determinant of a square matrix :

With each n-square matrix A=[a_{ij}] , we associate a unique expression called

The determinant of matrix A of order 'n' denoted by det A or $|A|$ or Δ as

defined below If A=[a_{11}], a single element matrix ,then det A=$|A|$=a_{11}

If A= $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a 2- square matrix than

Det. A=$|A|$ =$a_{11}\, a_{22} - a_{21}\, a_{12}$

The expansion of determinants of higher order is through minors, cofactors of an element of the matrix.

Minor and cofactor:

Let A=[aij]$_{n\times n}$ be a square matrix when from A the elements of i^{th} row and j^{th}

column are deleted the determinant of (n-1) rowed matrix M_{ij} is

called the minor of a_{ij} of A and is denoted by $|M_{ij}|$, the signed minor $(-1)^{i+j}$ | M_{ij} |is

called the cofactor of a_{ij} and is denoted by A_{ij}

Ex:

let A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

Minor of 1 is $=|(5 \times 9) - (6 \times 8)|$

$=|45 - 48|$

$=|-3| = 3$

Cofactor of 1 is (-1)

Adjoint of a square matrix: let A be a square matrix of order n .The transpose of the matrix got from A by replacing the elements of A by the corresponding cofactors is called adjoint of A and is denoted by ad j A .

Singular & non singular matrices:

A square matrix 'A' is said to be singular if $|A| = 0$

If $|A|$ G 0 then A is said to be non-singular .

Invertible matrix : A square matrix A is said to be invertible if there exists a matrix B such that AB=BA=I is called an inverse of A.

Note:

1) A matrix is said to be invertible ,if it posses inverse

2) Every invertible matrix possesses a unique inverse
(or)
The inverse of a matrix if it exists is unique.

3) The inverse of A is denote by A^{-1} thus $AA^{-1}=A^{-1}A=I$

4) If A is an invertible matrix and if A=B then $A^{-1}=B^{-1}$

5) If | | G 0 then $A^{-1}=\frac{1}{|\ |}$.(adj A).

Symmetric matrix : A square matrix $A=[a_{ij}]$ is said to be symmetric if $a_{ij}=a_{ji}$ for every ; iand j thus A is symmetric matrix if $A=A^T$ (or) $A^T=A$

Ex: $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

$A^T=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

$\therefore A=A^T$, hence A is symmetric.

Skew –symmetric : A square matrix A=[a_{ij}] is raid to be skew-symmetric if $a_{ij}=a_{ji}$ for every i and j
Thus A is skew symmetric $\Leftrightarrow$ A=-A^T

Ex : Let $A=\begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$

$$A^T=\begin{bmatrix} 0 & -a & b \\ a & 0 & -c \\ -b & c & 0 \end{bmatrix}=-\begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}=-A$$

$\therefore A^T=-A \Rightarrow A=-A^T$

$\therefore$A is skew –symmetric

Orthogonal matrix: A square matrix A is raid to be orthogonal if $AA^T=A^TA=I$.
That is $A^T=A^{-1}$.

Conjugate of a matrix: The matrix obtained form any given matrix A, on replacing its element by the corresponding conjugate complex numbers is called the conjugate of A and is denoted by $\bar{A}$

Ex :$A=\begin{bmatrix} 2 & 3i & 2-5i \\ -i & 0 & 4i+3 \end{bmatrix}$, then $\bar{A}=\begin{bmatrix} 2 & -3i & 2+5i \\ i & 0 & -4i+3 \end{bmatrix}$

Note :

If $\bar{A}$ and $\bar{B}$ be the conjugates of A and B respectively

Then 1) $\overline{(\bar{A})}$=A

2) $\overline{(A+B)}=\bar{A}+\bar{B}$

3) $\overline{(KA)}=\bar{K}.\bar{A}$, K being any complex number

4) $\overline{(AB)}=\bar{A}.\bar{B}$, A and B being conformable for multiplication

The transport of the conjugate of a square matrix

If A is a square matrix and its conjugate is $\bar{A}$, then the transpose of $\bar{A}$ is $(\bar{A})^T$. It can be easily seen that $(\bar{A})^T=\overline{(A^T)}$. The transposed conjugate of A is denoted by A^θ

$A^\theta=(\bar{A})^T=\overline{(A^T)}$.

Note :

1) $(A^\theta)^\theta= A$
2) $(A\pm B)^\theta = A^\theta \pm B^\theta$
3) $(KA)^\theta = \bar{K}.A^\theta$ where k is a complex number
4) $(AB)^\theta = A^\theta B^\theta$

Hermitian matrix: A square matrix A such that $A^T = \bar{A}$ or $(\bar{A})^T = A$ is called a Hermitian matrix .

Ex : $A = \begin{bmatrix} 4 & 1+3i \\ 1-3i & 7 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$ & $A^T = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$

$\bar{A} = A^T$

∴ A is hermitian

Skew –Her mitian matrix:

A square matrix A such that $A^T = -\bar{A}$ or $(\bar{A})^T = -A$ is called a skew-Hermitian matrix

Ex : $A = \begin{bmatrix} -3i & 2+i \\ 2+i & -i \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 3i & 2-i \\ -2-i & i \end{bmatrix}$, $A^T = \begin{bmatrix} -3i & -2+i \\ 2+i & -i \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 3i & -2-i \\ 2-i & i \end{bmatrix}$

$-A = \begin{bmatrix} 3i & -2-i \\ 2-i & i \end{bmatrix}$

∴ $(\bar{A})^T = -A$

∴ A is skew –Hermition

Unitary matrix : A square matrix A is said to be unitary if . = . = .

Theorem:Every square matrix can be expressed as the sum of a symmetric and skew- symmetric matrices.

Proof : Let A be any square matrix

If can be written as $A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T) = P+Q$

Here

$P = \frac{1}{2}(A+A^T)$, $Q = \frac{1}{2}(A-A^T)$

$P^T = ((\frac{1}{2}(A+A^T))^T$, $Q^T = (\frac{1}{2}(A-A^T))^T$

$=\frac{1}{2}(A^T+(A^T)^T)$

$=\frac{1}{2}(A^T+A)$

$=\frac{1}{2}(A+A^T)$

$=P$

$=\frac{1}{2}(A^T-(A^T)^T)$

$=\frac{1}{2}(A^T-A)$

$=\frac{1}{2}(A-A^T)$

$=-Q$

∴P is symmetric ∴Q is skew-symmetric

A=P+Q

Thus square matrix=symmetric + skew –symmetric.
Hence every square matrix can be expressed as sum of symmetric & skew symmetric matrices

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Ex:Show that every square matrix is uniquely expressible as the sum of Hermitian and skew-Hermitian matrix.

Proof:

Let A be any square matrix:

It can be expressed as A= $\frac{1}{2}$(A+) + $\frac{1}{2}$(A-) = P+Q

Here P = $\frac{1}{2}$(A+) , Q = $\frac{1}{2}$(A-)

Now (A+) = +( )

= +A

= A+

∴A+A^θ is a hermitian matrix

$\frac{1}{2}$(A+) is also hermitian matrix

∴P is hermitian matrix

Now $(A-A^\theta) = A^\theta - (A^\theta)$

$= A^\theta - A$

$= -(A-A^\theta)$

Hence A$-A^\theta$ is skew – hermitian

$\frac{1}{2}$(A-) is also skew hermitian matrix

∴ Q is skew hermitian

Thus p is hermitian & Q is skew hermitian matrix

∴A = P+Q

Hence every square matrix is expressible as sum of hermitian &skew-hermitian

Prove that the following matrix is orthogonal

$$\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix},$$

Sol:

$$A=\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$AA^T=\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{4}{9}+\frac{1}{9}+\frac{4}{9} & -\frac{4}{9}+\frac{2}{9}+\frac{2}{9} & -\frac{2}{9}-\frac{2}{9}+\frac{4}{9} \\ -\frac{4}{9}+\frac{2}{9}+\frac{2}{9} & \frac{4}{9}+\frac{4}{9}+\frac{1}{9} & \frac{2}{9}-\frac{4}{9}+\frac{2}{9} \\ -\frac{2}{9}-\frac{2}{9}+\frac{4}{9} & \frac{2}{9}-\frac{4}{9}+\frac{2}{9} & \frac{1}{9}+\frac{4}{9}+\frac{4}{9} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

Hence the given matrix A is orthogonal

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Determine the values a, b, c when $\begin{bmatrix} 0 & 2 & \\ & & -c \\ & -b & \end{bmatrix}$ is orthogonal

Sol: Given $A=\begin{bmatrix} 0 & 2 & \\ & & -c \\ a & -b & \end{bmatrix}$

$$AA^T=\begin{bmatrix} 0 & 2 & \\ a & b & -c \\ a & -b & c \end{bmatrix}\begin{bmatrix} & 0 & \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

$$=\begin{bmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-a^2 \\ -2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+a^2 \end{bmatrix}=I$$

(∴ since A is orthogonal i.e; $AA^T=I$)

Sol:

$2b^2-c^2=0$, $\quad a^2-b^2-c^2=0$

$C=\pm\sqrt{2b^2}$ $\quad a^2=b^2+c^2$

$=\pm\sqrt{2}.\,b$ $\quad =b^2+2b^2$

$= 3b^2$

$a = \pm\sqrt{3}.\ b$

$4b^2+c^2 = 1$ $\quad c = \pm\ \sqrt{2}\ .b$

$4b^2+2b^2=1$ $\quad = \sqrt{2}\ .\ 1/\sqrt{6}$

$6b^2 = 1$ $\quad = \pm\ 1/\sqrt{3}$

$b = \pm\ 1/\sqrt{6}$

$\therefore\ a = \pm\sqrt{3}\cdot 1/\sqrt{6}$

$= \pm\ \sqrt{3}\cdot 1/\sqrt{3}.\ \sqrt{2}$ $\quad = \pm\ 1/\sqrt{2}$

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Find ad joint of inverse of a matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

SOL: Given $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$|\ | = 1(-12-12)-1(-4-6)+3(-4+6)$

$= -8 \neq 0$

$\therefore$ A is non singular $\Rightarrow A^{-1}$ exists

Cofactor of first row :

Cofactor of $1 = (-1)^{1+1}\begin{vmatrix} 3 & -3 \\ -4 & -4 \end{vmatrix} = -12-12 = --24$

" " $1 = (-1)^{1+2}\begin{vmatrix} 1 & -3 \\ -2 & -4 \end{vmatrix} = -(-4-6) = 10$

$3 = (-1)^{1+3}\begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix} = +(-4+6) = +2$

Cofactors of second row:

Cofactor of $1 = (-1)^{2+1}\begin{vmatrix} 1 & 3 \\ -4 & -4 \end{vmatrix} = -(-4+12) = -8$

" " $3 = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix} = -(-4+6) = 2$

$-3 = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ -2 & -4 \end{vmatrix} = -1(-4+2) = 2$

Cofactor of 3 row:

Cofactor of $-2 = (-1)^{3+1}\begin{vmatrix} 1 & 3 \\ 3 & -3 \end{vmatrix} = (-3-9) = -12$

$-4 = = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix} = -(-3-3) = -6$

$-4 = = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = (3-1) = 2$

The matrix formed by cofactors elements of A is

$$B = \begin{bmatrix} -24 & 10 & 2 \\ -8 & 2 & 2 \\ -12 & 6 & 2 \end{bmatrix}$$

$$Adj = B^T = \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{adjA}{|\ |} = \frac{-1}{8}\begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ \frac{-5}{4} & \frac{-1}{4} & \frac{-3}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{-1}{4} \end{bmatrix}$$

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Express $A = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

Sol: Let $P = \frac{1}{2}(A+A^T)$ & $Q = \frac{1}{2}(A-A^T)$

$$A+A^T = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix} + \begin{bmatrix} 2 & 14 & 3 \\ -4 & 7 & 5 \\ 9 & 13 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 & 12 \\ 10 & 14 & 18 \\ 12 & 18 & 22 \end{bmatrix}$$

$$P = \frac{1}{2}(A+A^T) = \frac{1}{2}\begin{bmatrix} 4 & 10 & 12 \\ 10 & 14 & 18 \\ 12 & 18 & 22 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 6 \\ 5 & 7 & 9 \\ 6 & 9 & 11 \end{bmatrix}$$

$$P^T = \begin{bmatrix} 2 & 5 & 6 \\ 5 & 7 & 9 \\ 6 & 9 & 11 \end{bmatrix}$$

$\therefore p^T = P$; Hence p is symmetric

$$A-A^T = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 14 & 3 \\ -4 & 7 & 5 \\ 9 & 13 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -18 & 6 \\ 18 & 0 & 8 \\ -6 & -8 & 0 \end{bmatrix}$$

$$Q=\frac{1}{2}\begin{bmatrix}0 & -18 & 6\\ 18 & 0 & 8\\ -6 & -8 & 0\end{bmatrix}=\begin{bmatrix}0 & -9 & 3\\ 9 & 0 & 4\\ -3 & -4 & 0\end{bmatrix}$$

$$Q^T=\begin{bmatrix}0 & 9 & -3\\ -9 & 0 & -4\\ 3 & 4 & 0\end{bmatrix} \;\&\; -Q=\begin{bmatrix}0 & -9 & 3\\ 9 & 0 & 4\\ -3 & -4 & 0\end{bmatrix}$$

∴ $Q^T = -Q$, hence Q is skew –symmetric

$$\text{Now } P+Q=\begin{bmatrix}2 & 5 & 6\\ 5 & 7 & 9\\ 6 & 9 & 11\end{bmatrix}+\begin{bmatrix}0 & -9 & 3\\ 9 & 0 & 4\\ -3 & -4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2 & -4 & 9\\ 14 & 7 & 13\\ 3 & 5 & 11\end{bmatrix}=A=\text{ Given matrix}$$

∴ A=P+Q

∴ Every square matrix can be expressed as sum of symmetric & skew symmetric matrices .

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Ex: Express the matrix $\begin{bmatrix}1+i & 2 & 5-5i\\ 2i & 2+i & 4+2i\\ -1+i & -4 & 7\end{bmatrix}$ as the sum of hermitian and skew hermitian matrices.

Sol: Given $A=\begin{bmatrix}1+i & 2 & 5-5i\\ 2i & 2+i & 4+2i\\ -1+i & -4 & 7\end{bmatrix}$

$$\bar{A}=\begin{bmatrix}1-i & 2 & 5+5i\\ -2i & 2-i & 4-2i\\ -1-i & -4 & 7\end{bmatrix}$$

$$A^{\theta}=(\bar{A})^T=\begin{bmatrix}1-i & -2i & -1-i\\ 2 & 2-i & -4\\ 5+5i & 4-2i & 7\end{bmatrix}$$

$$\text{Now } A+\;=\begin{bmatrix}1+i & 2 & 5-5i\\ 2i & 2+i & 4+2i\\ -1+i & -4 & 7\end{bmatrix}+\begin{bmatrix}1-i & -2i & -1-i\\ 2 & 2-i & -4\\ 5+5i & 4-2i & 7\end{bmatrix}$$

$$=\begin{bmatrix} 2 & 2-2i & 4-6i \\ 2+2i & 4 & 2i \\ 4+6i & 2i & 14 \end{bmatrix}$$

$$P=\frac{1}{2}(A+A^{\theta})=\begin{bmatrix} 1 & 1-i & 2-3i \\ 1-i & 2 & -i \\ 2-3i & -i & 7 \end{bmatrix}$$

$$\bar{P}=\begin{bmatrix} 1 & 1-i & 2-3i \\ 1-i & 2 & -i \\ 2-3i & -i & 7 \end{bmatrix}$$

$$P^{\theta}=(\bar{P})^{T}=\begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & i & 7 \end{bmatrix}$$

$\therefore P^{\theta}=$ P, P is Hermition

Now A- $=\begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}-\begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$

$$=\begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

$Q=\frac{1}{2}$ (A- $)=\frac{1}{2}\begin{bmatrix} 2i & 2+2i & 6-4i \\ -2+2i & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$

$$=\begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

$$\bar{Q}=\begin{bmatrix} -i & 1-i & 3+2i \\ -1-i & -i & 4-i \\ -3+2i & -4-i & 0 \end{bmatrix}$$

$$Q^{\theta}=(\bar{Q})^{T}=\begin{bmatrix} -i & -1-i & -3+2i \\ 1-i & -i & -4-i \\ 3+2i & 4-i & 0 \end{bmatrix}$$

$$-Q=\begin{bmatrix} -i & -1-i & -3+2i \\ 1-i & -i & -4-i \\ 3+2i & 4-i & 0 \end{bmatrix}$$

$\therefore Q^{\theta} = -Q$, hence Q is skew hermition

Now $P+Q = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & i & 7 \end{bmatrix} + \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$

$=A$

Hence every square matrix can be expressible as sum of Hermition & Skew-Hermition

Exercise

1) Find the ad joint and inverse of $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

2) Compute the ad joint and inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

3) Prove that $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$ is orthogonal

4) Show that $\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal

5) Express the matrix $\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as a sum of symmetric and skew symmetric matrices

6) Express the matrix $\begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$ as a sum of hermit ion and skew hermit ion matrices

7) Show that the matrix $A = \begin{bmatrix} 3i & 2+i \\ -2+i & -i \end{bmatrix}$ is skew hermition matrix.

Elementary transformations (or operations) on a matrix

1) Interchange of two rows: If i^{th} row and j^{th} row are interchanged , it is denoted by $R_i \Leftarrow\Rightarrow R_j$

2) Multiplication of each element of a row with a non zero scalar .If i^{th} row is multiplication with k then it is denoted by $R_i \Rightarrow kR_i$

3) Multiplication every element of a row with a non zero scalar and adding to the corresponding elements of another row
If all the elements of i^{th} row are multiplied with k and added to the corresponding elements of j^{th} row then it is denoted by $R_i \Rightarrow R_j + kR_i$
By column transformations will be denoted by c instead of R.

Zero row & non –zero row: If all the elements in a row of a matrix are zeros , then it is called zero row and if there is at least one non zero element in a row then it is called a non –zero row.

Rank of a matrix: Let A is be an $m \times n$ matrix .If A is null matrix , we define its rank to be 0 (zero).

If A is non zero matrix ,we say that 'r' is the rank of A if

(i) every $(r+1)^{th}$ order minor of A is 0(zero) and
(ii) there exists at least one r^{th} order minor of A which is not zero

Rank of A is denoted by $\rho(A)$

Note:

1) Every matrix will have rank

2) Rank of a matrix is unique

3) ρ (A) 1 when A is a non-zero matrix

4) If A is a matrix of order $m \times n$ rank of A= ρ (A) $\leq$min(m,n)

5)If ρ (A) = r then every minor of A of order r+1 or more is zero

6)Rank of the identity matrix I_n is n

7)If A is a matrix of order 'n' and A is non-singular (i.e; det AG0) then ρ (A)=n.

8)The rank of the transpose of a matrix is the some as that of the original matrix(i.e; ρ (A)= ρ (A^T))

9) If A and B are two equivalent matrices then rank A= rank B

10)if A and B are two equivalent matrixes then rank A = rank B

Problems:-

1) Find the rank of the matrix $A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}_{3x3}$

Sol: Det A of given matrix (A) = -1(18-1) – 0(9+5)+(3+30) = -17-0+198

= 181$\neq$0

$\therefore$ A is non – singular third order matrix

$\therefore$ rank of A = ρ (A) = 3 = order of given matrix.

2) Find rank of the matrix $\begin{bmatrix} 1 & -2 & -1 \\ -3 & 3 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

Sol:- det A = (A) = 1(12-0) – (-2) (-12-0) -1(-6-6)

= 12-24+12=0

$\therefore$ A is singular

Let us take a submatrix of given matrix

$B=\begin{bmatrix} 1 & -2 \\ -3 & 3 \end{bmatrix} \Rightarrow \{B\} = 3-6 = -3\neq 0$

Rank of given matrix = submatrix rank = P(A) = 2

3) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 8 & 7 & 8 & 5 \end{bmatrix}$

Sol:- Here the matrix is of order 3x4. Its rank $\leq$ min (3,4) = 3

Let us consider the submatrix of given matrix

$B = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 8 & 7 & 0 \end{bmatrix}$

[B] = 1(0-49)-2(0-56)+3(35-48) = - 49+112-39

= 24 $\neq$ 0

$\therefore$ Rank of the matrix ρ (A) = 3 = order of submatrix

Echelon form:-

The Echelon form of a matrix A is an equivalent matrix, obtained by finite number of elementary operations on A by the following way.

1) The zero rows, if any, are below a nonzero row

2) The first nonzero entry in each nonzero row is one (1)

3) The number of zeros before the first nonzero entry in a row is lessthan the number of such zeros in the next row immediately below it.

Note:- (i) Condition (2) is optinal

(ii) The rank of A is equal to the number of nonzero rows in its echelon form.

Solved Problems:

1) Find the rank of the matrix by echelon form $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

Sol:- Given A $= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_1$; $R_3 \rightarrow R_3 - 2R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$

$R_3 \rightarrow R_3 - 2R_3$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ ρ (A) = Rank of A = number of non zero rows = 2

2) Find the rank of the matrix $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -15 \end{bmatrix}$

Sol :- Given A = $\begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -15 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$; $R_3 \to 2R_3 + R_1$

$\sim \begin{bmatrix} 4 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\therefore$ Rank of A = ρ (A) = Number of non zero rows = 1

3) Find the value of K such that the rank of A = $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2

Sol:- Given A = $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & +1 & -1 & 1 \\ 0 & -2 & +1 & -2 \\ 0 & -2 & +3 & -2 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$\sim \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & +1 & -2 \\ 0 & 0 & -k+2 & 0 \end{bmatrix}$

Give rank of A is 2, there will be only two non zero rows

$\therefore$ Third row must be zero row $\Rightarrow$ 2-K=0

$\Rightarrow$ K = 2

Exercise:-

Find the rank of the following matrixs by using echelon form

(1) $\begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$ (ans) 3

(2) $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ (ans) 2

(3) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (ans) 2

(4) $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$ (ans) 2

(5) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ (ans) 2

(6) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ (ans) 3

(7) find the value of K it the rank of the matrix A is e where

$A= \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ -3 & 1 & 0 & 2 \\ 1 & 1 & 5 & 0 \end{bmatrix}$

(8) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ (ans) 3

Normal form:

Every m x n matrix of rank r can be reduced to the for [Ir 0] or Ir or (3) $\begin{bmatrix} & 0 \\ 0 & 0 \end{bmatrix}$ by a finale number of elementary row or column transformations. Here 'r' indicates rank of the matrix.

Solved Problems:

1) Find the rank of the matrix by using normal form where $A= \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$

Sol:- Given $A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$

$R_1 \leftrightarrow R_3$

$\sim \begin{bmatrix} 1 & -3 & -1 \\ 3 & -2 & 4 \\ 2 & 3 & 7 \end{bmatrix}$

$R_2 \rightarrow R_2 - 3R_1$; $R_3 \rightarrow R_3 - 2R_1$

$\sim \begin{bmatrix} 1 & -3 & -1 \\ 0 & 7 & 7 \\ 0 & 9 & 9 \end{bmatrix}$

$C_2 \rightarrow C_2 + 3C_1$; $C_3 \rightarrow C_3 + C_1$

$A \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 7 \\ 0 & 9 & 9 \end{bmatrix}$

$R_2 \rightarrow R_2 \frac{1}{7}$, $R_3 \rightarrow R_3 . \frac{1}{9}$

$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_2$

$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$C_3 \rightarrow C_3 - C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

Rank of A = ρ (A) = r = 2 = unit matrix order

2) Find the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ by using normal form.

Sol: Given A = $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

$C_1 \leftrightarrow C_2$

$A \sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 1 & 2 & 3 & 1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_1$

$\sim \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$

$C_3 \rightarrow C_3 - 2C, C_4 \rightarrow C_4 + 2C_1$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -6 & 6 \\ 0 & 2 & -3 & 3 \end{bmatrix}$

$R_{3+} \rightarrow 2R_{3\,-}R_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -6 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2 \rightarrow C_2 \cdot \frac{1}{4}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 \rightarrow C_3 + 6C_2, C_4 \rightarrow C_4 - 6C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} I2 & 0 \\ 0 & 0 \end{bmatrix}$

Rank of A = ρ (A) = r = 2

Exercise :

Find the rank of the following matrix by using normal form

1) $\begin{bmatrix} -8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ ans (3)

2) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ ans (3)

3) $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ ans (2)

4) $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ ans (4)

5) $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$ ans (4)

Inverse of Non-singular matrix by Gauss – Jordan method:-

We can find the inverse of a non-singular square matrix using elementary row operations only.

Suppose A is a nonsingular square matrix of order n we write $A= I_nA$

Now we apply elementary row operations only to the matrix A and the prefactor I_n of the R.H.S. We will do this till we get an equation of the form $I_n = BA$. Then abviously B is the inverse of A.

1) Find the inverse of the Matrix $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ by using Gaus – Jordan Method

Sol:- Given A = $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

Write $A = I_n.A$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix} . A$$

$R_2 \to R_2.(\frac{-1}{3})$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & -1 & 1 \end{bmatrix} . A$$

$R_1 \rightarrow R_1 - R_2$; $R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & -2/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ -1/3 & 2/3 & 0 \\ -2/3 & 1/3 & 1 \end{bmatrix}$$

$R_3 \rightarrow R3(-3/2)$

$$\begin{bmatrix} 1 & 0 & 4/3 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 1 & -1/2 & -3/2 \end{bmatrix} . A$$

$R_1 \rightarrow R_1 - 4/3.R_3$; $R_2 \rightarrow R_2 + 1/3.R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & -3/2 \end{bmatrix} . A$$

$I_{3x3} = B.A$ where $B = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & -3/2 \end{bmatrix}$ is the inverse of given matrix.

Exercise:

Find the inverse of the following matrixes by using Gaugs – Jordan method.

1) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

2) $\begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

3) $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

Solution of linear System of equations:

An equation of the form $a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots + a_nx_n = b$……..............(1)

Where $x_1, x_2, \ldots, x_n$ are unknowns and $a_1, a_2, \ldots, a_n$, b are constants is called a linear equations in n unknowns consider the system of m linear equations in n unknowns .

$x_1, x_2, \ldots, x_n$ as given below

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = b1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = b2$

.. . .

.. . .

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = bm$......................(2)

where aij's and b_1, b_2 ……… b_m are constants. An ordered n- tuple $(x_1, x_2 \ldots, x_n)$ satisfying all equations in (2) is called a solution of the system (2).

The System of equations in (2) can be written in matrix from A X= B...........(3)

Where A = [aij], $x = (x_1, x_2 \ldots, x_n)^T$, $B = (b_1.b_2 \ldots b_m)^T$

The Matrix [A/B] is called the augmented matrix of the system(2)

If B=0 in (3), the system is said to be Homogeneous otherwise the system is said to be non – homogeneous.

* The system AX = 0 is always consistent since X = 0 (i.e., $x_1=0, x_2=0, \ldots X_n=0$) is always a solution of AX = 0 This solution is called Trival solution of the system.

* Given AX = 0, we try to decide whether it has a solution $X \neq 0$. Such a solution, if exists, is called a non-Trival solution

* If there is a least one solution for the given system is said to consistent, if the system does not have any solution, the system is said to be inconsistent.

Solution of Non-homogeneous system of equations:

The system AX=B is consistent i.e., it has a solution (unique or infinite) if and only if rank A = rank [A/B]

i) If rank of A = rank of [A/B] = r<n then the system is consistent and it has infinitely many solutions. There r = rank, n = number of unknowns in the system.

ii) If rant of A = rank of [A/B] = r = n then the system has unique solution.

iii) If rank of A≠ rank [A/B] then the system is inconsistent i.e., It has no solution.

Solved Problems:

1) Solve the system of equations x+2y+3z=1; 2x+37+8z=2 ; x+y+z = 3

Sol: Given system can be written in matrix form

as
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix} [\;] = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
$$A \quad X = B$$

Augmented matrix of the given system

$$[A/B] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 8 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & -2 & 2 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & 2 \end{bmatrix}$$

∴ rank of A = rank [A/B] = r = 3 = number of unknowns = n

∴ n = r = 3

∴ The given system is consistent and it has unique solution. The solution is as follows from the last augmented matrix we can write as

$-4z = 2$

$z = \frac{-1}{2}$

$-y+2z = 0$

$2z = y$

$2(\frac{-1}{2}) = y$

$Y=-1$

$x+2y+3z = 1$

$x = 1-2y-3z$

$= 1-2(-1)-3(\frac{-1}{2})$

$= 1+2+\frac{3}{2}$

$X= 9/2$

∴ The solution of given system : $x=9/2$; $y= -1$, $z=-1/2$

2) Solve the system of equations $x+2y+z =14$

$3x+4y+z = 11$

$2x+3y+z = 11$

Sol:- Given system can be written in matrix form as

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} [\ ] = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$$

A X = B

The augmented matrix of the given system as

$$[A/B] = \begin{bmatrix} 1 & 2 & 1 & 14 \\ 3 & 4 & 1 & 11 \\ 2 & 3 & 1 & 11 \end{bmatrix}$$

$R_2 \to R_2-3R_1$; $R_3 \to R_3-2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 14 \\ 0 & -2 & -2 & -31 \\ 0 & -1 & -1 & -17 \end{bmatrix}$$

$R_3 \to 2R_3-R_2$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 14 \\ 0 & -2 & -2 & -31 \\ 0 & 0 & -0 & -3 \end{bmatrix}$$

Rank of A = 2≠3 = rank of AB

∴ The given system has no solution, i.e., the system is inconsistent

3) Show that the system $x+y+z=6$; $x+2y+3z = 14$; $x+4y+7z = 30$ are consistent and solve them.

Sol:- Given system can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} [\ ] = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

Augmented matrix

$$[A/B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of A = rank of AB = r = 2 < 3 = n = number of unknowns

∴ The system has consistent and it has infinitely many solutions.

Here $x+y+z = 6$

$Y+2z = 8$

Let $z = k$

Now $y = 8-2z=8-2k$

Now $x = 6-y-z$

$= 6-(8-2k)-k$

$x= 6-8+2k-k$

$x= k-2$

∴ The system has infinitely many solutions x=k-2; y=8-2k; z=k

4) Solve the following systems of equations by rank method

$x-3y-8z = -10$

$3x+y-4z = 0$

$2x+5y+6z = 13$

Sol:- The matrix form of given system of equations

$$\begin{bmatrix} 1 & -3 & -8 \\ 3 & 1 & -4 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 13 \end{bmatrix}$$

Augmented matrix of the given system

$$[A/B] = \begin{bmatrix} 1 & -3 & -8 & -10 \\ 3 & 1 & -4 & 0 \\ 2 & 5 & 6 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$; $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 10 & 20 & 30 \\ 0 & 11 & 22 & 33 \end{bmatrix}$$

$R_2 \to R_2(\frac{1}{10})$; $R_3 \to R_3(\frac{1}{11})$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & -3 & -8 & -10 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

∴ Rank of A = rank of AB = r = 2<3=n=number of unknowns

∴ The system has infinitely many solutions

x-3y-8z = -10 & y+2z = 3

let z = k

y = 3-2z

y = 3-2k

& x = -10+3y+8z

= -10+3(3-2k)+8k

= -10+9-6k+8k

X = 2k-1

∴ Sol is x = 2k-1; y=3-2k, =k

For different value of k, system have different solutions i.e., infinitely many solutions

5) For what values of λ and μ the system of equations

2x+3y+5z = 9 have (i) no solution

7x+3y-2x = 8 (ii) unique solution

2x+3y+lz = μ (iii) infinitely many solutions

The matrix form of given system of equations

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

The augumented matrix of given system

$$[A/B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \lambda \end{bmatrix}$$

$R_2 \to 2R_2 - 7R1$; $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{bmatrix}$$

$R_1 \to R1(\frac{1}{2})$

$$\sim \begin{bmatrix} 1 & 3/2 & 5/2 & 9/2 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{bmatrix}$$

Case 1 : λ=5, μ≠9

Then ρ (A) = 2, ρ (AB) = 3

ρ (A) = 2≠3= ρ (AB)

The system has no solution

Case 2:- $\lambda \neq 5, \mu \neq 9$

Then $\rho(A) = \rho(A/B) = r = n = 3$

$\therefore$ The system has unique solution

Case 3: $\lambda = 5, \mu = 9$

Then $\rho(A) = \rho(A/B) = r = 2 < 3 = n$ = number of unknowns

$\therefore$ The system has infinitely many solutions.

Exercise:

1) Find the values of a and b for which the system of equations

$$x+y+z=3$$
$$x+2y+2z=6$$
$$x+9y+az=b$$

have (i)no solution (ii) unique solutions (iii) infinitely many solutions.

2) Find the values of p and qso that the equations

$2x+3y+5z=9$ have (i) no solutions

$7x+3y+2z=8$ (ii) unique solution

$2x+3y+pz=q$ (iii) infinitely many solutions

3) Show that the system of equations $x-4y+7z=14$

$3x+8y-2z=13$

$7x-8y+26z=5$ are not consistent

4) Solve the system of equations $x+y+z=4$; $2x+5y-2z=3$, $x+7y-7z=5$ by rank method.

5) Test for consistency and hence solve the system $x+y+z=6$, $x-y+2z = 5$,$3x+y+z=8$, $2x-2y+3z=7$

6) Test for the consistency of $x+y+z=1$, $x-y+2z=1$, $x-y+2z=5$, $2x-2y+3z=1$

7) Solve the system of equations $x+y+z=6$, $x-y-2z=5$, $3x-y+y+z=-8$

8) Solve the system $2x-y+3z=0$, $3x+2y+z=0$, and $x-4y+5z=0$

9) Solve completely the system of equations

$X+y-2z=3w=0$, $x-2y+w=0$, $4x+y-5z+8w=0$, $5x-7y+2z-w=0$

Consistency of system of homogeneous linear equations:

Consider of system of homogeneous linear equations in n unknowns namely

$$a_{11}x_1+a_{12}x_2+\ldots\ldots+a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2+ \ldots\ldots +a_{2n}x_n = 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2+ \ldots\ldots +a_{mn}x_n = 0$$

This system can be written in matrix form

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{m1} & a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ \cdot \\ \cdot \\ \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

A X = 0

1. If rank of A = n (number of variables)

⇒ The system of equations have only trivial solution (i.e., zero solution)

2. If r<n then the system have an infinitive number of non trivial solutions.

Solved Problems:

1) Find all the solutions of the system of equations

X+2y-z=0, 2x+y+z=0, x-4y+5z=0

Sol. Given system can be written in matrix form

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -4 & 5 \end{bmatrix} [\ ] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix

$$[A/B] = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & -4 & 5 & 0 \end{bmatrix}$$

$R_2 \to R_2-2R_1$; $R_3 \to R_3-R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -6 & 6 & 0 \end{bmatrix}$$

$R_3 \to R_3-2R_2$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of A = rank of AB = r = number of non zero rows = 2<3=n= number of variables

∴ The system has infinitely many solutions from the above matrix

-3y+3z=0 x+2y-z=0

⇒ y=z

Let us consider n-r=3-2=1 arbitrary constants

Let z=k, then y = k

Since x+2y-z=0

⇒ x=z-2y

= z-2y

=k-2k

x=-k

∴ x=-k, y=z=k

2) Solve the system of equations x+y+w=0; y+z=0, x+y+z+w=0, x+y+2z=0

Sol: Given system can be written in matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented matrix

$$[A/B] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$; $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_4$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Rank of A = Rank of AB = r =4=n= number of unknowns

∴Therefore there is no non=zero solution

∴ x=y = z=w=0 is only the trivial solution.

Gauss elimination method:-

This method of solving a system of n liner equations in n unknowns consists of eliminating the coefficients in such a way that the system reduces to upper triangular system which may be solved by back substitution.

Problems:

Solve the equations x+y+z=6, 3x+3y+4z=20, 2x+y+3z=13 by using Gauss elimination method.

Sol matrix from of the given system

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 13 \end{bmatrix}$$

Augmented matrix of the given system

$$[A/B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$; $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Clearly it is an upper triangular matrix from this by back substitution.

z= 2	-y+z=1	x+y+z=6
	z-1=y	x=6-y-z
	2-1=y	=6-1-2
	Y=1	=3
∴ x=3	y=1	z=2

Exercise:

Solve the following system of equations by using Gauss elimination method

1) 3x+y+2z=3, 2x-3y-z=-3, x+2y+z=4

2) 2x+y+z=10, 3x=2y+3z=18, x+4y+9z=16

3) x+y+2z=4, 2x-y+3z=9, 3x-y-z=2 4)

3x+y-z=3, 2x-8y+z=-5, x-2y+9z=8

Gauss Seidel iteration method:

We will consider the system of equations

$a_{11}x_1+a_{12}x_2+a_{13}x_3 = b_1$..........................(1)

$a_{21}x_1 + a_{22}x_2+ a_{23}x_3 = b_2$......................(2)

$a_{31}x_1 + a_{32}x_2+ a_{33}x_3 = b_3$......................(3)

Where the diagonal coefficients are not zero and are large compared to other coefficients such a system is called a "diagonally dominant system".

The system of equations (1) can be written as

$x_1=\frac{1}{11}$ $[b_1-a_{12}x_2-a_{13}x_3]$................(4)

$x_2=\frac{1}{22}$ $[b_2-a_{21}x_1-a_{23}x_3]$...............(5)

$x_3=\frac{1}{33}$ $[b_3-a_{31}x_1-a_{32}x_2]$.............(6)

Let the initial approximate solution be $x_1^{(0)}$, $x_2^{(0)}$, $x_3^{(0)}$ are zero Substitute x_2^0, x_3^0 in (4) we get

$x_1^1 = 1/a_{11}\,[b_1-a_{12}x_2^0-a_{13}x_3^0]$ this is taken as first approximation of x_1

Substitute x_1^1, x_3^0 in (5) we get $x_2^1 = 1/a_{22}[b_2-a_{21}x_1^1-a_{23}x_3^0]$

This is taken as first approximation of x_2 now substitute x_1^1,x_2^1 in (6), we get

$x_3^1 = 1/a_{33}\, [b_3 - a_{31} x_1^1 - a_{32} x_2^1]$

This is taken as first approximation of x_3 continue the same procedure until the desired order of approximation is reached or two successive iterations are nearly same. The final values of x_1, x_2, x_3 obtained an approximate solution of the given system.

1) Use Gauss-Seidel iteration method to solve

$10x+y+z=12;\ 2x+10y+z=13;\ 2x+2y+10z=14$

Sol: Clearly the given system is diagonal by dominant and we write it as

$x = \frac{1}{10}(12-y-z)$ (1)

$y = \frac{1}{10}(13-2x-z)$ (2)

$z = \frac{1}{10}(14-2x-2y)$ (3)

First iteration: We start iteration by taking $y=z=0$ in (1) we get $x_1^1=1.2$

Put $x^1 = 1.2$, $z = 0$ in (2) we get $y^1 = 1.06$

Put $x^1 = 1.2$; $y^1 = 1.06$ (3) we get $z^1 = 0.95$

Second iteration now substitute $y^1 = 1.06$, $z^1 = 0.95$ in (1)

$x^2 = \frac{1}{10}(12-1.06-0.95) = 0.999$

put x^2, z^1 in (2) $y^2 = \frac{1}{10}(13-1.998-0.95) = 1.005$

now substitute x^2, y^2 in (3) $z^2 = \frac{1}{10}(14-1.998-2.010) = 0.999$

Third approximation: now substitute y^2, z^2 in (1)

$x^3 = \frac{1}{10}(12-1.005-0.999) = 1.00$

Put x^3, z^3 in (2) $y^3 = \frac{1}{10}(13-2.0-0.999) = 1.000$

Put y^3, x^3 in (3) $x^3 = \frac{1}{10}(14-2.0-2.0) = 1.00$

Similarly we find fourth approximation of x,y,z and got them as $x^4=1.00$, $y^4=1.00$, $z^4=1.00$

Exercise:

Solve the following system of equations by Gauss – seided method

1) $8x-3y+2z = 20;\ 4x+11y-z=33,\ 6x+3y+12z = 36$

2) $x+10y+z = 6;\ 10x+y+z=6;\ x+y+10z=6$

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Objective Questions
(Theory of Matrix)

Multiple Choice Questions

1. The trace of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 4 & 2 & -7 \end{bmatrix}$ is [ ]

   a) 0 b) -7 c) 7 d) None

2. Which of the following is a scalar matrix [ ]

   a) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

3. A matrix is said to be upper triangular if [ ]

   a) a_{ij} = o, forallij b) aij = 0, forallo<j c) aij = 0, forall i>j d) None

4. The value of 'a' such that the matix $A = \begin{bmatrix} 3-a & 2 & 2 \\ 2 & 4-\square & 1 \\ -2 & -4 & -(1+\square) \end{bmatrix}$ is singular [ ]

   a) 3 b) 2 c) 4 d) 1

5. The rank of non-singular matrix of order 'n' is always [ ]

   a) = n b) < n c) > n d) = 0

6. The rank of singular matrix of order 'n' is [ ]

   a) = n b) < n c) ≥ n d) > n

7. The rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is [ ]

   a) 3 b) 2 c) 1 d) none

8. The rank of the zero matrix of 0 [ ]

   a) 1 b) 2 c) 0 d) cant say

9. If the rank of the matrix $A \begin{bmatrix} 1 & 1 & \\ 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ is < 3 then the value of 'x' is [ ]

   a) 0 b) -1 c) 1 d) none

10. Which of the matrix is in Echelon form [ ]

    a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 6 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

11. Which of the following matrix is in normal form [ ]

    a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. If A is an m x n matrix then rank of A is [ ]

    a) = min[m,n] b) ≥ min[m,n] c) ≤ min [m,n] d) None

$1 \quad 1 \quad -1$

13. The rank if the matrix $A = \begin{bmatrix} 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ is [ ]

a) 0 b) 1 c) 2 d) 3

14. The System AX=B has no solution if [ ]

a) ρ(A)= ρ ([A/B]) b ρ (A) < ρ ([A/B]) c) ρ (A)> ρ ([A/B]) d) None

15. The system of equations x+y+z = 6x+2y+3z=10 and x+2y+3z=5 [ ]

a) Unique sol b) Infinite sol c) No solution d) None

16. If A is a non-singular 3x3 matrix then the system AX=B has [ ]

a) Unique sol b) Infinite sol c) No solution d) None

17. If ρ (A) = r and 'n' is the number of unknowns then the number of linearly Independent solutions of AX = 0 is [ ]

a) n – (r-1) b) n – r c) n – (r+1) d) None

18. The values of a, b for which the system x-3y+4z=5, x+2y+3z=4, x+3y+z=b has a unique solutions are [ ]

a) a=4, b=4, b) a=4, b=5 c) a=4, b≠5 d)a≠4, b≠5

Fill in the Blanks

1. The normal form of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ is ………………

2. The Echelon form of the matrix $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 3 \\ 0 & 1 & -3 \end{bmatrix}$

3. The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is …………….

4. The matrix obtained by applying an elementary transformation is called …………

5. The solution of the system of equations x+3y+2z=0, x+4y+3z=0, x-15y+4z=0 is ….

6. If A is non singular matrix, then the system AX = O has ……………………………..