UNIT- II

AC CIRCUITS

REPRESENTATION OF SINUSOIDAL WAVEFORMS

PEAK AND RMS VALUES

PHASOR REPRESENTATION

REAL POWER, REACTIVE POWER, APPARENT POWER,

POWER FACTOR,

ANALYSIS OF SINGLE PHASE AC CIRCUITS

CONSISTING OF R, L, C, RL, RC, RLC combinatio
(series and Par

RESONANCE IN SERIES RLC CIRCUIT

THREE PHASE BALANCED CIRCUITS

VOLTAGE AND CURRENT RELATIONS IN STAR A

DELTA CONNECTIONS

AC CIRCUITS

Introduction:

We deal so far with cases in which the currents are steady and in one direction, this is called Direct Current (DC).

- The use of direct currents is limited to a few applications. Ex: Charging of Batteries, Electroplating, Electric Traction etc,

In AC system, the Voltage acting in the circuit changes polarity at regular intervals of time and the resulting current (called alternating current) changes direction accordingly.

- The AC system has offered so many advantages that at present Electrical Energy is universally generated, transmitted and used in the form of alternating current.

- Even when DC Energy is necessary, it is a common practice to convert AC into DC by means of rotary converters (or) Rectifiers.

~~ADVANTAGES~~:

Alternating current: It is the current in which magnitude and direction varies with time

ADVANTAGES:

1. Generation of AC is cheaper and easier than that of DC.
2. AC can be easily step-up (or) step-down using a device called Transformer.
3. AC machines are simple, Robust and do not require much attention for their repairs & maintenance during their use.
4. The Switch gear (Ex. Switches, circuit breakers etc) for AC system is simpler than the DC system.
5. AC can be easily converted into DC with the help of rectifiers.
6. When AC is supplied at higher voltages in long distance transmission, the line losses are small compared to a DC transmission.

REPRESENTATION OF SINUSOIDAL WAVEFORM:

Many times, alternating voltages and currents are represented by a sinusoidal wave (or) simply a sinusoid. It is a very common type of alternating current (AC) and alternating voltage. The sinusoidal wave is generally referred to as a sine wave.

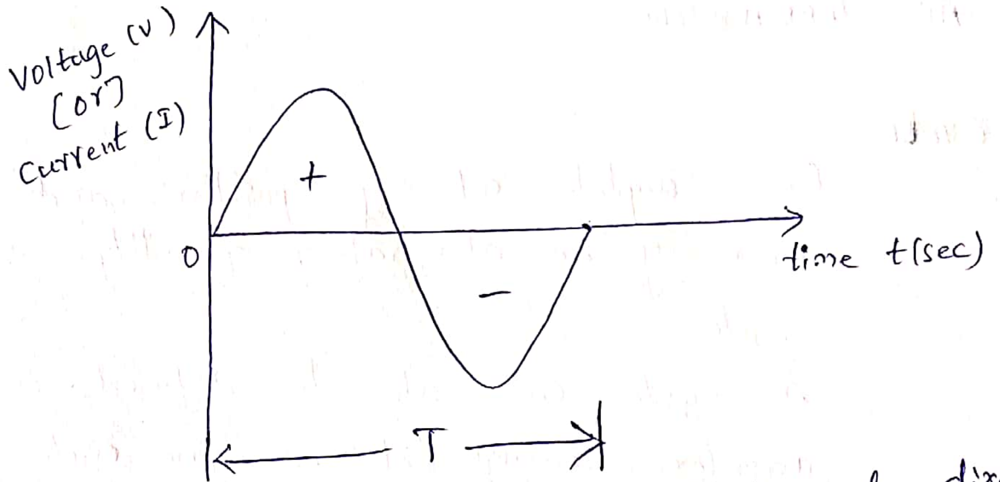
Waveform: defined as the voltage (or) current that fluctuates with time periodically with change in polarity and direction.

In general, the sine wave is more useful than other waveforms like pulse, sawtooth, square etc.,

A sinusoidal function is easy to analyse. The sinusoid function is easy to generate and it is more useful in the power industry.

The shape of a sinusoidal waveform is shown in fig...

The waveform may be either a current waveform (or) a voltage waveform



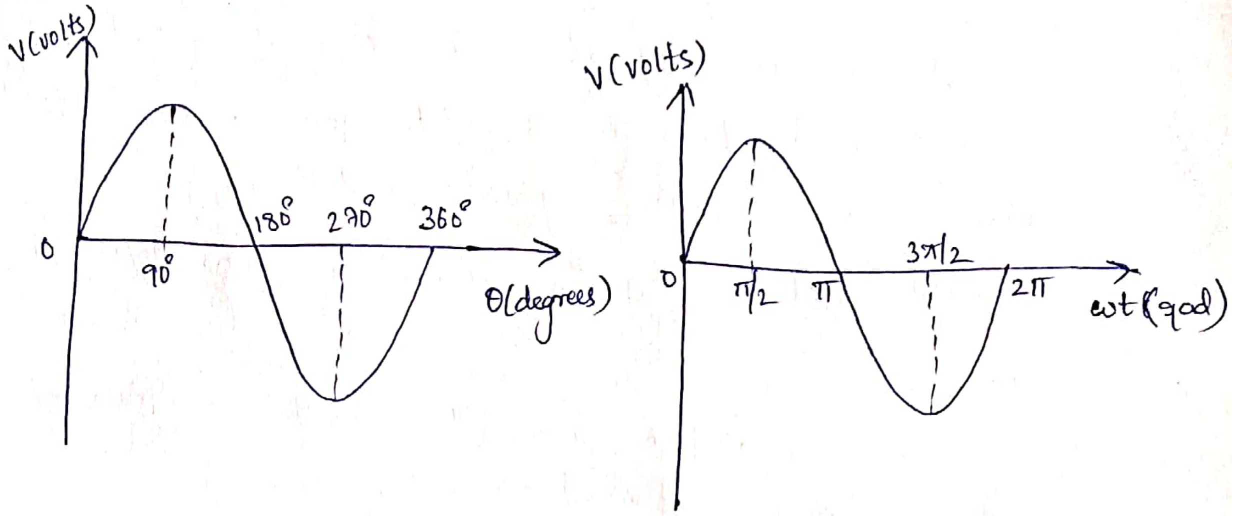
Here the wave changes in magnitude and direction with time. If we start at time $t=0$ then the wave goes to a maximum value and returns to zero, & then decrease to a negative maximum value before returning to zero.

ANGULAR RELATION OF A SINE WAVE:

A sine wave can be measured along x-axis on a time base which is frequency dependent.

A sine wave can be expressed in terms of an angular measurement.

This angular measurement is expressed in degrees (or) radians.



AC TERMINOLOGIES:

CYCLES

One complete set of positive and negative values of an alternating quantity is known as cycle.

- A cycle can also be defined in terms of angular measurement as one cycle corresponds to 360° electrical (or) 2π radians.

TIME PERIOD:

The time taken by an alternating quantity to complete one full cycle is called its Time Period.

- Represented by the letter 'T'.

FREQUENCY:

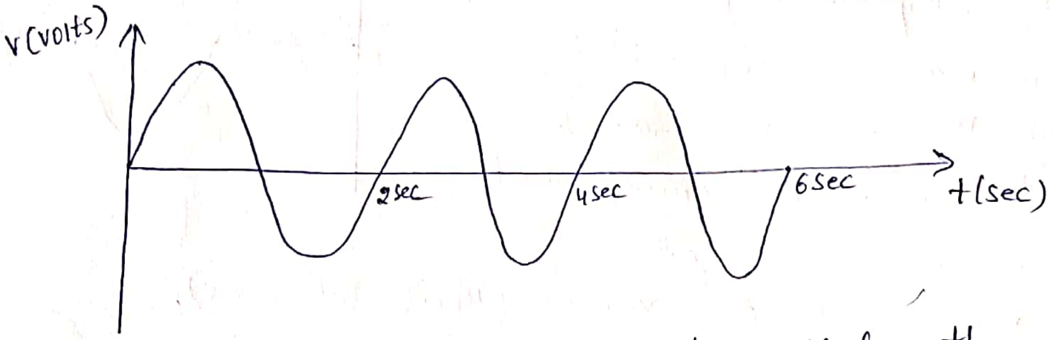
Number of cycles that occur in one second is called frequency. (F).

- It is measured in cycles/sec (or) Hertz (i.e., c/s (or) HZ)

- One Hertz is Equal to 1 c/s.
- 50HZ is Equal to 50 c/s.

* Relation between time period (T) and frequency (F) is given by $\boxed{f = \frac{1}{T}}$ (or) $\boxed{T = \frac{1}{F}}$

(P) What is the Time period of sine wave shown in fig..



Sol: From sine wave, it is clear that the sine wave takes two seconds to complete one period in Each cycle.

$\boxed{\therefore T = 2 \text{ sec}}$

(P) The time period of a sine wave is 20 milli seconds, what is the frequency.

Sol: $T = 20\,ms$

$f = ?$

$$f = \frac{1}{T} = \frac{1}{20ms} = \frac{1}{20 \times 10^{-3}} = \frac{1000}{20} = 50 \text{ c/s}$$

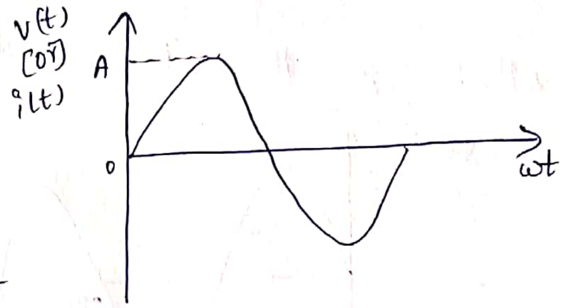
$\boxed{F = 50 \text{ c/s (or) } 50 \text{ HZ}}$

AMPLITUDE: The maximum value (either for positive Half cycle (or) for Negative Half cycle) attained by an alternating quantity is called its Amplitude.

- It is also called as "peak value."
- The amplitude of an alternating voltage (or) alternating current is designated by V_m (or) I_m.

SINE WAVE EQUATION:

A Sine wave is represented graphically as shown in the figure.



- The amplitude of a sine wave is represented on vertical axis (Y-axis).
- The angular measurement in degrees (or) radians is represented on X-axis (Horizontal axis).
- Amplitude (A) is the maximum value of the voltage (or) current on the Y-axis.
- In general, the sine wave is represented by the equation.

$$V(t) = V_m \sin \omega t \quad — ①$$

Equ. ① states that any point on the sine wave represented by instantaneous value v(t) is equal to the maximum value times the sine of the angular frequency at that point.

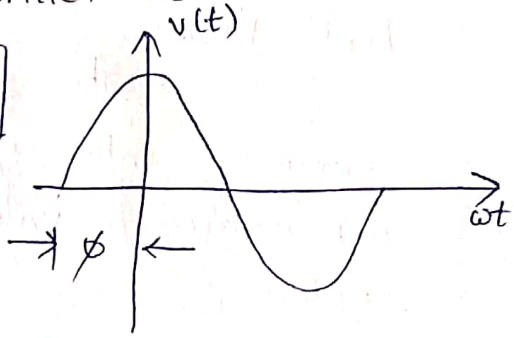
Ex: If a certain sine wave voltage has peak value of 20V, the instantaneous voltage at a point $\pi/4$ radians along the horizontal axis can be calculated by

$$V(t) = V_m \sin \omega t$$

$$= 20 \sin(\pi/4) = 20 \times 0.707 = 14.14V$$

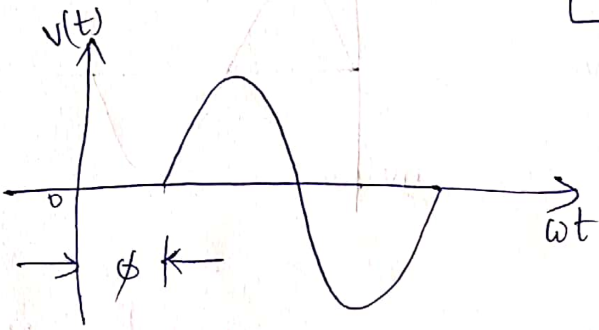
→ When a sine wave is shifted to the left of the reference wave by a certain angle ϕ as shown in fig. The expression can be written as

$$\boxed{v(t) = V_m \sin(\omega t + \phi)}$$



→ When the sine wave is shifted to the right of the reference wave by a certain angle ϕ as shown in fig... The general expression is

$$\boxed{v(t) = V_m (\sin \omega t - \phi)}$$

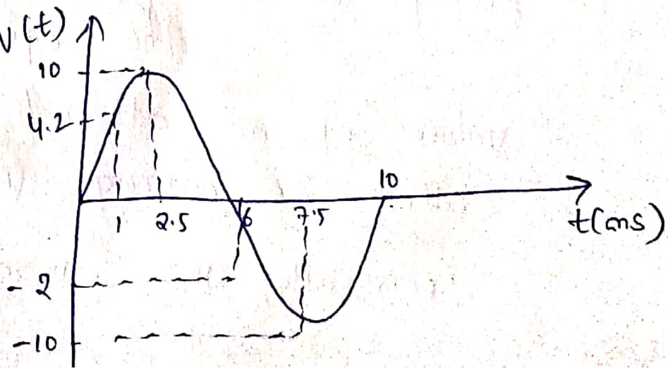


Voltage and current values of a sine wave:

As the magnitude of the waveform is not constant, the waveform can be measured in different ways. They are:

Instantaneous Value: The value of an alternating quantity at any instant is called instantaneous value.

- At 1ms, the value is 4.2V
- At 2.5ms, the value is 10V

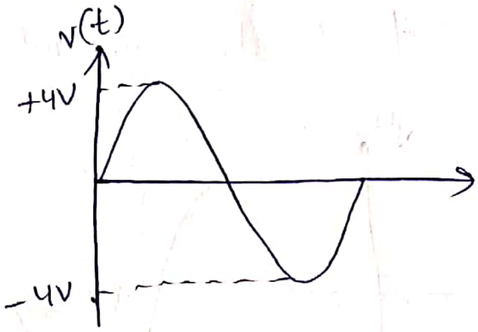


- At 6ms, the value is $-2V$.
- At 7.5ms, the value is $-10V$.

Peak Value:

The peak value of the sine wave is the maximum value of the wave during positive half cycle (or) maximum value of the wave during negative half cycle.

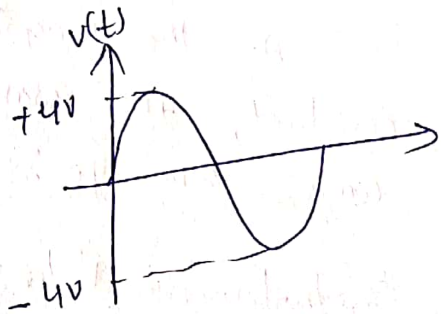
- Since the value of these two cycles are equal in magnitude, then the sine wave is characterised by a single peak value.



- Here, the peak value of the sine wave is 4V.

Peak to Peak Value:

The peak to peak value of a sine wave is the value from the positive to the negative peak.



- Here, the peak to peak value is 8V.

Average Value:

The arithmetical average of all the values of an alternating quantity over one cycle is called its average value.

- Generally, the average value of any function

$v(t)$, with time period 'T' is given by

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt.$$

i.e., Average value = $\frac{\text{Area Under the curve}}{\text{Base}}$

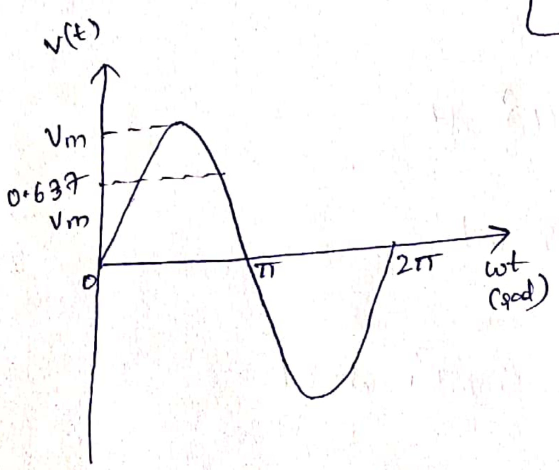
- The average value of a sine wave over a complete cycle is always zero.
- Because, positive half cycle is exactly equal to the negative half cycle so the net area is zero.
- So, the average value of a sine wave is defined over a Half cycle and not for a full cycle.

(i.e.,) The average value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

So, the average value of the sine wave is given by

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \; d\omega t$$

$[\text{As } v(t) = V_m \sin\omega t]$



$$= \frac{1}{\pi} V_m \int_0^{\pi} \sin\omega t \; d\omega t$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{-V_m}{\pi}\left[\cos\pi - \cos 0\right]$$

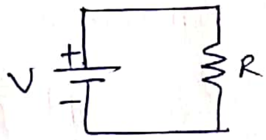
$$= \frac{-V_m}{\pi}\left[(-1)-1\right] = \frac{2V_m}{\pi} = 0.367\,V_m$$

$$\boxed{\therefore\ V_{avg} = 0.637 V_m}$$

Root Mean Square Value (or) Effective Value:

The root mean square (Rms) value of a sine wave is a measure of the heating Effect of the wave.

When a resistor is connected across a DC Voltage source, a certain amount of heat is generated in the resistor in a given time.



Similarly, a resistor is connected across a AC Voltage source a certain amount of heat is produced for the same time.

The heat produced in the resistor is same as DC source, that term is the Effective value. This Effective Value is called as the Rms value.

Let, the Rms value of any function with time period 'T' is given by

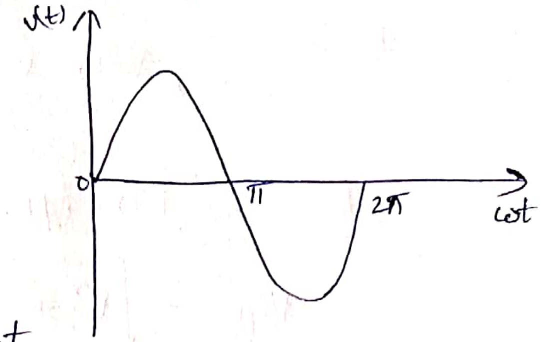
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt}$$

As $V(t) = V_m . \sin \omega t$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin\omega t)^2\, d\,\omega t}$$

Rms value for a sinusoidal voltage is as follows:

for complete one cycle.



$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [V(t)]^2\, dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\omega t\, d\omega t} \qquad \left[\sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$\cancel{= \sqrt{\frac{V_m^2}{2\pi}\times\frac{1}{2}\int_0^{2\pi}\frac{\cos 2\omega t}{2}\, d\omega t}}$$

$$\cancel{= \sqrt{\frac{V_m^2}{4\pi}\left[\sin 2\omega t\right]_0^{2\pi} \cdot \left[d\omega t\right]_0^{2\pi}}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1}{2} - \frac{\cos 2\omega t}{2}\right] d\omega t}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\int_0^{2\pi}\frac{1}{2}\, d\omega t - \int_0^{2\pi}\frac{\cos 2\omega t}{2}\, d\omega t\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\, \frac{1}{2}\int_0^{2\pi} d\omega t - \int_0^{2\pi}\frac{\cos 2\omega t}{2}\, d\omega t}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[(\omega t)_0^{2\pi} - \left(\frac{\sin 2\omega t}{2}\right)_0^{2\pi}\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[(2\pi - 0) - \left(\frac{\sin 2(2\pi)}{2} - \frac{\sin 2(0)}{2}\right)\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[2\pi - \frac{\sin 4\pi}{2} - \frac{\sin 0}{2}\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[2\pi - 0 - 0\right]}$$

$$= \sqrt{\frac{V_m^2}{\cancel{4\pi}_{2}} \times \cancel{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

$$\boxed{V_{rms} = 0.707\, V_m}$$

If the function consists of a number of Sinusoidal terms. i.e., $V(t) = V_0 + (V_{c_1} \cos\omega t + V_{c_2} \cos 2\omega t + \dots) + (V_{s_1} \sin\omega t + V_{s_2} \sin 2\omega t + \dots)$

The rms (or) Effective Value is given by

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_{c_1}^2 + V_{c_2}^2 + \dots) + \frac{1}{2}(V_{s_1}^2 + V_{s_2}^2 + \dots)}$$

(P) Find the RMS value of the voltage wave whose Equation $V(t) = 10 + 200 \sin(\omega t - 30^\circ) + 100 \cos 2\omega t - 50 \sin(5\omega t + 60^\circ)$

Sol:

$V_0 = 10$; $V_{S_1} = 200$; $V_{C_1} = 100$; $V_{S_2} = 50$

$$V_{rms} = \sqrt{10^2 + \frac{(200)^2}{2} + \frac{(100)^2}{2} + \frac{(50)^2}{2}}$$

$$= \sqrt{100 + 20000 + 5000 + 1250}$$

$$V_{rms} = 162.327\,V$$

(P) A Sinusoidal current wave is given by $i = 50 \sin 100\pi t$

Determine a) The greatest rate of change of current.

b) Derive Average and Rms values of current

c) The time interval between a maximum value and the next zero value of current.

Sol:

$i = 50 \sin \underline{100\pi t}$ $\{As\ i = I_m \sin \omega t\}$

$\sin\omega t = \sin 2\pi f t$

$\omega = \underline{2\pi f}$

$2\pi f = 100\pi$

$f = \frac{100\pi}{2\pi} = 50Hz$

$\frac{di}{dt} = 50 \times 100\pi \cos 100\pi t = 5000\pi \cos 100\pi t$

$$\left(\frac{di}{dt}\right)_{max} = 5000\pi$$

(b) Average value:
$$I_{avg} = \frac{2I_m}{\pi} = \frac{2\times 50}{\pi} = \frac{2\times 50}{3.14}$$

$$I_{avg} = 31.82A$$

$$\left[\text{As } V_{avg} = \frac{2V_m}{\pi}\right]$$

$(I_m = 50)$

Rms Value: $I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35A$

$$\left(V_{rms} = \frac{V_{max}}{\sqrt{2}}\right)$$

(c) Time Interval, $T = \frac{1}{f} = \frac{1}{50} = 0.020 = 20\,ms$

$\therefore$ Time Interval $= \frac{20}{4} = 5\,ms$

(P) A Sine wave has a peak value of 25V. Determine the following value a) Rms b) Peak to Peak c) Average

Sol: a) Rms value of the sine wave

$V_{rms} = 0.707V_m = 0.707\times 25 = 17.68V$

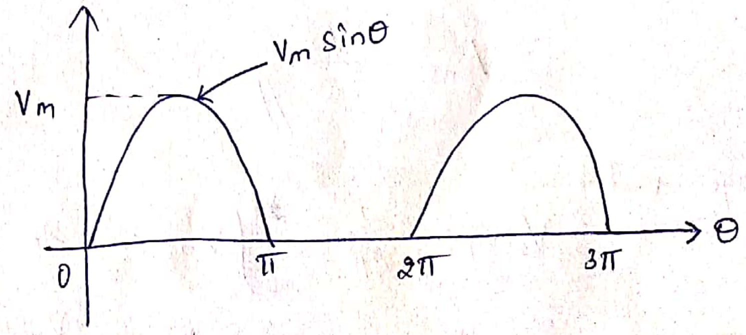
b) Peak to Peak value of the sine wave: $V_{pp} = 2V_m$

$V_{pp} = 2\times 25 = 50V$

c) Average value of the sine wave

$V_{avg} = 0.637V_m = 0.637\times 25 = 15.93V$

(P) Find Rms and Average value of the following waveform.



Sol: Rms value $V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta \, d\theta}$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right] d\theta} \qquad \left\{\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right\}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\int_0^{\pi}\frac{1}{2}\,d\theta - \int_0^{\pi}\frac{\cos 2\theta}{2}\,d\theta\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\frac{1}{2}\int_0^{\pi} d\theta - \frac{1}{2}\int_0^{\pi}\cos 2\theta \, d\theta\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\,\frac{1}{2}\left[\int_0^{\pi} d\theta - \int_0^{\pi}\cos 2\theta \, d\theta\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[(\theta)_0^{\pi} - (\sin 2\theta)_0^{\pi}\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[(\pi - 0) - (\sin 2(\pi) - \sin 2(0))\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\pi - (0-0)\right]}$$

$$= \sqrt{\frac{V_m^2}{4\pi}(\pi - 0)}$$

$$= \sqrt{\frac{V_m^2}{4\cancel{\pi}} \times \cancel{\pi}}$$

$$= \sqrt{\frac{V_m^2}{4}}$$

$$V_{rms} = \frac{V_m}{2}$$

Average value

$$V_{avg} : \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta \, d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{2\pi}$$

$$V_{avg} = \frac{V_m}{\pi}$$

FORM FACTOR:

It is defined as the ratio of RMS value to the Average value of the wave

— denoted by K_f.

$$\text{form factor} = \frac{\text{Rms Value}}{\text{Average Value}}$$

For a Sinusoidal alternating Voltage:

$$\text{Form factor} = \frac{V_{Rms}}{V_{avg}} = \frac{V_m/\sqrt{2}}{2V_m/\pi}$$

$$\boxed{K_f = 1.11}$$

PEAK FACTOR (OR) CREST FACTOR:

It is defined as the ratio of the peak value to the Rms value of the wave

— denoted by K_p.

$$\text{Peak Value } (K_p) = \frac{\text{Peak Value}}{\text{Rms Value}}$$

For a Sinusoidal alternating Voltage:

$$\text{Peak Value} = \frac{V_m}{V_{Rms}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}}$$

$$\boxed{K_p = 1.414}$$

(P) The Equation of an AC is $i(t) = 42.42 \sin(628t)$ Determine i, Rms Value ii, Average Value (iii) form factor iv, Peak factor

Sol:

$i(t) = 42.42 \sin 628t$

Comparing with $i(t) = I_m \sin \omega t$

$I_m = 42.42$

$\omega = 628$

i, Rms Value : $I_{Rms} = \frac{I_m}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} = 30 A$

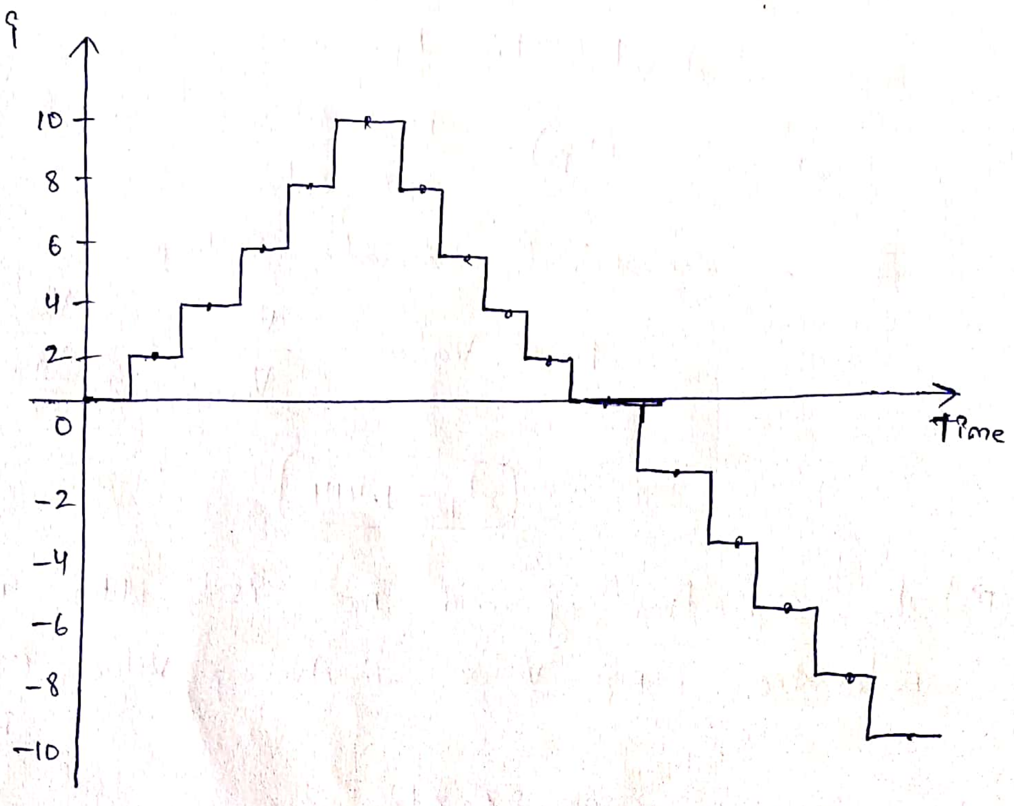
ii, Average Value : $I_{avg} = \frac{2I_m}{\pi} = \frac{2 \times 42.42}{\pi} = 27 A$

iii, form factor : $K_f = \frac{\text{Rms Value}}{\text{Average Value}} = \frac{30}{27} = 1.11$

iv, Peak factor : $K_p = \frac{\text{Peak Value}}{\text{Rms Value}} = \frac{42.42}{30} = 1.414$

(P) Calculate the Rms value, Average Value, form factor, Peak factor of a periodic current having following values for equal time intervals changing suddenly from one value to next as 0, 2, 4, 6, 8, 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10, −8 ......

Sol:



Average Value of the current is given by

$$\text{Average Value} = \frac{0+2+4+6+8+10+8+6+4+2}{10} = 5A$$

Rms value of current:

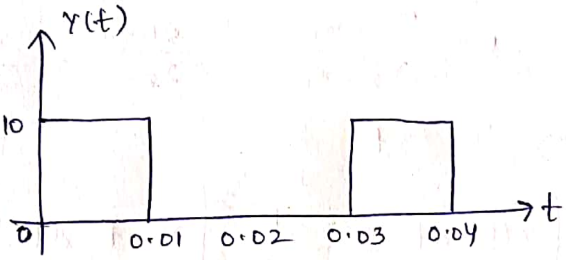
$$\sqrt{\frac{0^2+2^2+4^2+6^2+8^2+10^2+8^2+6^2+4^2+2^2}{10}}$$

$$= 5.8309 \text{ A}$$

$$\text{Form factor: } k_f = \frac{\text{Rms Value}}{\text{Average Value}} = \frac{5.8309}{5} = 1.16 \text{ A}$$

$$\text{Peak factor: } k_p = \frac{\text{Maximum Value}}{\text{Rms Value}} = \frac{10}{5.8309} = 1.715$$

(P) Find the Average and Effective Values, form factor and Peak factor for the given waveform as shown below:



Sol: For $0 < t < 0.01$, $y(t) = 10$

$0.01 < t < 0.03$, $y(t) = 0$

$$\frac{1}{0.03}\left[\int_0^{0.01} 10\, dt + \int_{0.01}^{0.03} 0\, dt\right]$$

$$Y_{avg} = \frac{1}{T}\int_0^T y(t)\, dt = \frac{1}{0.03}\int_0^{0.01} 10\, dt = 10 \times \frac{0.01}{0.03}$$

$$\boxed{Y_{avg} = 3.33}$$

$$Y_{Rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2\, dt}$$

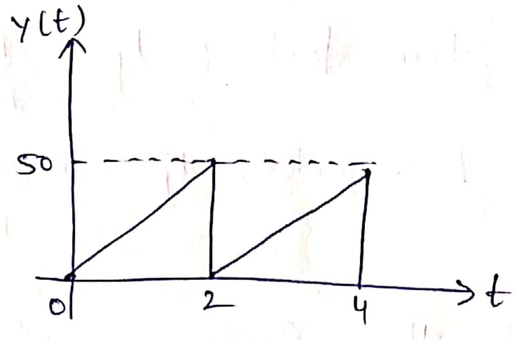
$$= \sqrt{\frac{1}{0.03}\int_0^{0.01} 10^2\, dt} = \sqrt{100 \times \frac{0.01}{0.03}} = 5.773$$

$$\boxed{Y_{Rms} = 5.773}$$

Form Factor : $\dfrac{Y_{RMS}}{Y_{AVG}} = \dfrac{5.773}{3.33} = 1.733$

Peak factor : $\dfrac{Y_{MAX}}{Y_{Rms}} = \dfrac{10}{5.773} = 1.732$

(P) Find the average and Effective values, form factor and peak factor of the Saw-tooth waveform as shown in figure.



Sol: for $0 < t < 2$;

$y_2 = 50 \quad y_1 = 0 \qquad x_2 = 0 \quad x_1 = 2$

$$y = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - 0}{2 - 0} = \frac{50}{2} = 25t$$

$$\Rightarrow y = 25t$$

$$Y_{Avg} = \frac{1}{T}\int_0^T y(t)\,dt = \frac{1}{2}\int_0^2 25t\,dt$$

$$= \frac{25}{2}\int_0^2 t\,dt$$

$$= 25$$

$$Y_{Rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2\,dt} \cancel{= 16.63} = \sqrt{\frac{1}{2}\int_0^2 25^2 t^2\,dt}$$

$$= \sqrt{\frac{625}{2}\int_0^2 t^2\,dt}$$

$$= 28.86$$

Form factor $= \dfrac{Y_{Rms}}{Y_{Avg}} = \dfrac{28.86}{25} = 1.15$

Peak factor $= \dfrac{Y_m}{Y_{Rms}} = \dfrac{50}{28.86} = 1.732$

PHASE OF AN ALTERNATING QUANTITY

Phase:
The phase of an sine wave is an angular measurement that specifies the position of the sine wave relative to a reference.

Phase Difference:
The difference in phase between two waves is called phase difference.

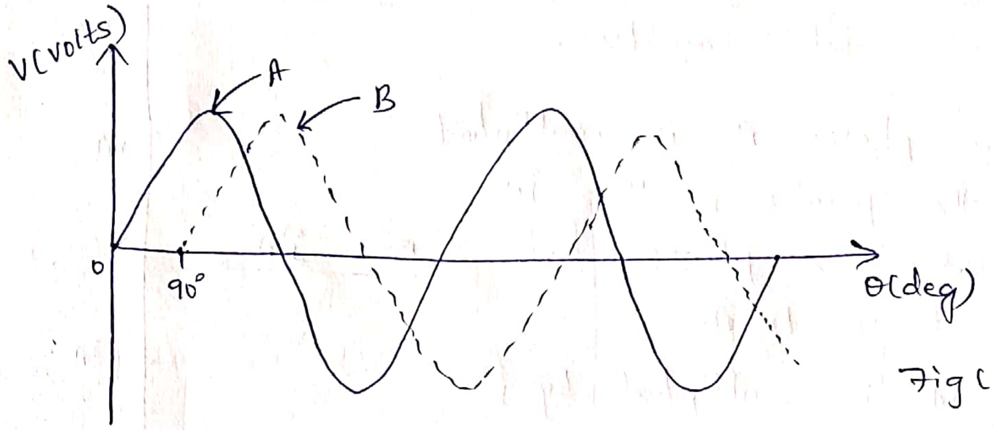
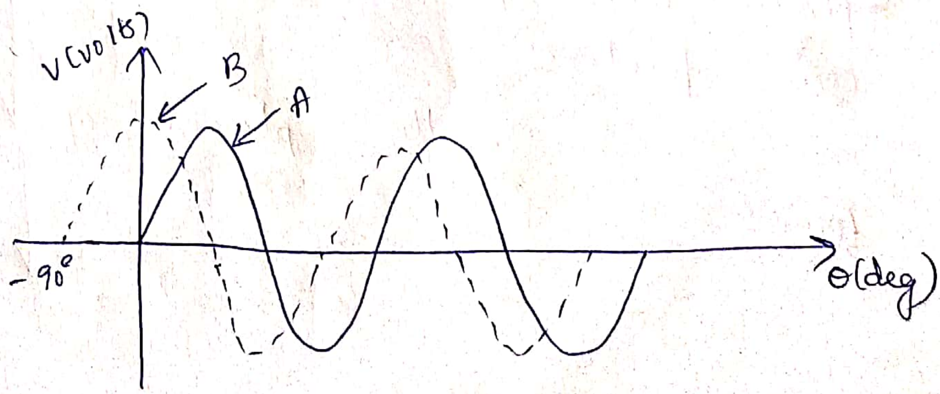


Fig (1)

From the graph,
the sine wave is shifted to the Right by 90° by dotted lines

- So, there is a phase angle of 90° between A and B.
- The waveform 'B' is lagging behind the waveform 'A' by 90°.
(or)
- The sine wave 'A' is leading the waveform 'B' by 90°.

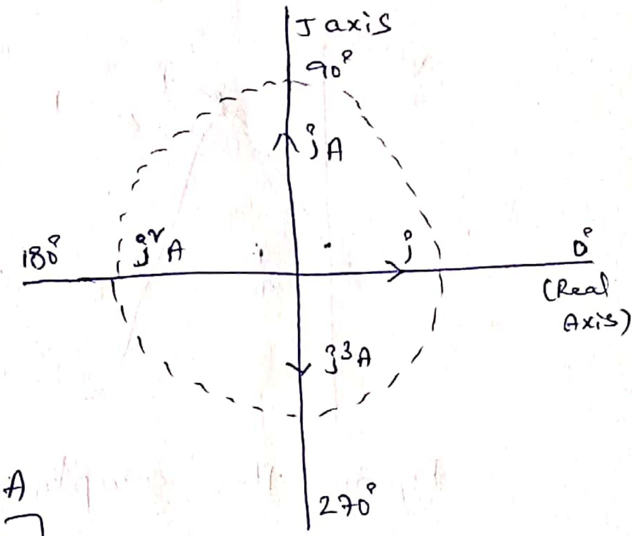


In fig (2),

The sine wave A is lagging behind the waveform B by 90°.

- In both the ~~cla~~ cases, the phase difference is 90°.

J-operator: The multiplication of any phasor by the operator j causes the phasor to rotate through 90° in anti clock wise direction without affecting its magnitude.



- If a phasor A is multiplied by the operator 'j' then jA represents a phasor which is at 90° to A.
- If jA is multiplied by j

then, $(jA)\,j = j^2 A = -1 \cdot A = -A$

$$\boxed{j^2 = -1 \quad (\text{or}) \quad j = \sqrt{-1}}$$

||ly $(j^2 A) \cdot j = j^3 A = j^2 \cdot j \cdot A = -1 \cdot j \cdot A = -jA$ //

$(j^3 A) \cdot j = j^4 A = j^2 \cdot j^2 \cdot A = -1 \times -1 \times A = A$ //

Phasor Representation:

Sinusoids are expressed in terms of phasors which are more convenient to work with sine & cosine functions.

So, the phasors are expressed in following forms:

i, Rectangular (or) complex (or) Cartesian form:

$z = a + jb$

a - real part of z

b - Imaginary part of z

ii, Polar form: $z = r \angle \theta$

iii, Trigonometrical form: $z = r(\cos\theta + j\sin\theta)$

iv, Exponential form: $z = re^{j\theta}$

where, $r = \sqrt{a^2 + b^2}$ is the magnitude of z

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$ is the phase of z

Now, conversion of Rectangular to polar form

$z = a + jb$ to $z = r \angle \theta$

Conversion of polar form to Rectangular form

$z = r \angle \theta$ to $z = a + jb$

where, $a = r\cos\theta$

$b = r\sin\theta$

Addition of phasors:

let $z_1 = a_1 + jb_1$ & $z_2 = a_2 + jb_2$

$z = z_1 + z_2$

$z = a_1 + jb_1 + a_2 + jb_2$

$z = a_1 + a_2 + jb_1 + jb_2$

$z = a_1 + a_2 + j(b_1 + b_2)$

Magnitude: $r = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2}$

phase: $\theta = \tan^{-1}\left(\frac{b_1 + b_2}{a_1 + a_2}\right)$

Subtraction of phasors:

let $z_1 = a_1 - jb_1$ & $z_2 = a_2 - jb_2$

$$z = z_1 - z_2$$

$$z = (a_1 - jb_1) - (a_2 - jb_2)$$

$$z = a_1 - a_2 - jb_1 + jb_2$$

$$z = (a_1 - a_2) - j(b_1 - b_2)$$

magnitude : $r = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

phase : $\theta = \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right)$

Multiplication of phasors:

$$z = z_1 \times z_2$$

$z_1 = r_1 \angle \theta_1$ & $z_2 = r_2 \angle \theta_2$

$$z = z_1 \times z_2 = r_1 \angle \theta_1 \times r_2 \angle \theta_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

Division of phasors:

$z_1 = r_1 \angle \theta_1$ & $z_2 = r_2 \angle \theta_2$

$$z = \frac{z_1}{z_2} = \frac{r_1 \angle \theta_1}{r_2 \angle \theta_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

$$z = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$$

(P) Compute the following operations:

i) $a + b$

ii) $a - b$

iii) $a \times b$

iv) $a \div b$ where $a = 6 + j8$; $b = 3 - j4$

Sol:

i, $a + b = 6 + j8 + 3 - j4 = 9 + j4$

magnitude : $r = \sqrt{a^2 + b^2} = \sqrt{9^2 + 4^2} = 9.85$

phase : $\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{4}{9}\right) = 23.94°$

$\therefore a + b = 9.85 \angle 23.94°$

ii, $a - b = (6 + j8) - (3 - j4) = 6 + j8 - 3 + j4 = 3 + j12$

magnitude : $r = \sqrt{a^2 + b^2} = \sqrt{3^2 + 12^2} = 12.37$

phase : $\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{12}{3}\right) = 75.96°$

$\therefore a - b = 12.37 \angle 75.96°$

iii, $a \times b = (6 + j8)(3 - j4)$

$= 18 - j24 + j24 + 32$

$= 50 + j0$

$\therefore a \times b = 50 \angle 0°$

iv, ~~$a \div b$~~ $a \div b = \frac{a}{b} = \frac{6 + j8}{3 - j4} = \frac{\sqrt{6^2 + 8^2} \angle \tan^{-1}\left(\frac{8}{6}\right)}{\sqrt{3^2 + 4^2} \angle \tan^{-1}\left(-\frac{4}{3}\right)}$

$= \frac{10 \angle 53.13°}{5 \angle -53.13°}$

$\therefore a \div b = 2 \angle 106.26°$

P Hw

i, $\sqrt{40 \angle 50° + 20 \angle -30°}$

ii, $\frac{10 \angle -30° + (3 - j4)}{(2 + j4)(3 - j5)}$

→ A sinusoidal Voltage $V = 100\angle 60°$ volts is applied across a circuit having an impedance of $Z = (3+j4)\,\Omega$. Find the magnitude of current and its phase angle with respect to Voltage.

Sol: $V = 100\angle 60°$

$Z = 3 + j4\,\Omega$

$$I = \frac{V}{Z} = \frac{100\angle 60°}{3+j4} = \frac{100\angle 60°}{5\angle 53.13°} = 20\angle 6.87°$$

Magnitude of current : $I = 20A$

phase Angle : $60° - 6.87°$

$= 53.13°$ //

H/w

i) Evaluate $\sqrt{40\angle 50° + 20\angle -30°}$

Convert the given polar form to Rectangular form.

$40\angle 50° = 40(\cos 50° + j\sin 50°)$

$= 40\,(0.6427 + j\,0.766)$

$= 25.7 + j\,30.64$

$20\angle -30° = 20\left[\cos(-30°) + j\sin(-30°)\right]$

$= 20\left[+0.866 + j(-0.5)\right]$

$= 17.36 - j10$

Adding two values : $25.7 + j30.64 + 17.36 - j10$

$= 43.06 + j20.64$

$\theta = \tan^{-1}\left(\frac{b}{a}\right) \Rightarrow \tan^{-1}\left(\frac{20.64}{43.06}\right) = 25.6°$

$r = \sqrt{a^2+b^2} = \sqrt{(43.06)^2 + (20.64)^2} = 47.75$ $\quad \therefore 47.75\angle 25.6°$

Taking the square root

$$\sqrt{47.72 \angle 25.63^\circ}$$

ii, $\dfrac{10\angle -30^\circ + (3-j4)}{(2+j4)(3-j5)^*}$

Sol: $10\angle -30^\circ = 10\left[\cos(-30^\circ) + j\sin(-30^\circ)\right]$

$= 10\left[0.866 + j(-0.5)\right]$

$= 8.66 - j5$

As, $10\angle -30^\circ + (3-j4) = 8.66 - j5 + 3 - j4$

$= 8.66 + 3 - j5 - j4$

$= 11.66 - j9$

$(2+j4)(3-j5)^* = (2+j4)(3+j5) = -14 + j22$

Now, $\dfrac{11.66 - j9}{-14 + j22} = \dfrac{14.73 \angle -37.66^\circ}{26.07 \angle 57.52^\circ} = \dfrac{14.73}{26.07} \angle -37.66 - 57.52$

$= 0.565 \angle \cancel{160.31}\ 95.18^\circ$

$\Rightarrow 11.66 - j9 = \sqrt{(11.66)^2 + (9)^2} = 14.73$

$\theta = \tan^{-1}\left(\dfrac{-9}{11.66}\right) = -37.66^\circ$

$\Big\}\ 14.73 \angle -37.66^\circ$

$|||^{ly}$ $-14 + j22 = \sqrt{(-14)^2 + (22)^2} = 26.07$

$\theta = \tan^{-1}\left(\dfrac{22}{-14}\right) = -57.52$

$\Big\}\ 26.07 \angle -57.5$

SINGLE PHASE AC CIRCUITS:

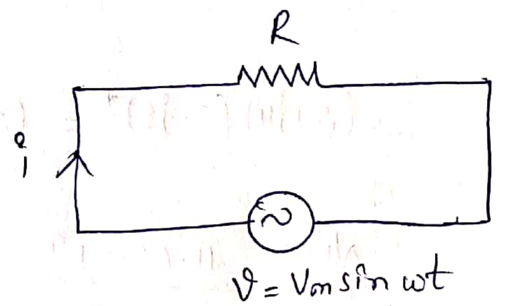
The Resistance (R), Inductance (L) and Capacitance (C) are 3 basic elements of any electrical network (or) circuit.

So, to analyse any Electric circuit it is necessary to understand the following three cases:

- AC through Pure Resistor
- AC through Pure Inductor
- AC through Pure Capacitor

→ AC through Pure Resistor:

Consider an AC circuit with pure resistance 'R' (ohms) connected across an alternating voltage (V), and current (i) is flowing in the circuit as shown in figure:



Let, $v = V_m \sin \omega t$

According to ohm's law

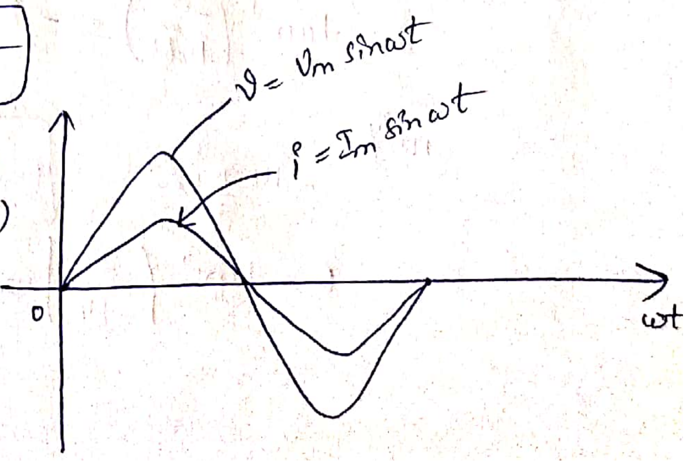
$V = IR$

$$I = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R} \sin \omega t$$

We can write $\left\{ \frac{V_m}{R} = I_m \right\}$

$$\boxed{i = I_m \sin \omega t}$$

Waveform of voltage (V) and current (i)



- The phase difference between voltage and current in case of pure resistor is zero (i.e., $\phi = 0^\circ$)
- So, power factor = $\cos\phi = \cos 0 = 1$ (unity)
- In case of pure resistor, the power factor is unity.
- The voltage and current both are inphase, as

$$O \longrightarrow I \longrightarrow V$$ (phasor diagram)

<u>Power (P)</u>: The instantaneous power in AC circuits can be obtained by taking product of the instantaneous value of current and voltage

$$P = V \times i$$

$$= V_m \sin\omega t \times I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t \qquad \left\{ \because \sin^2\theta = \frac{1-\cos 2\theta}{2} \right\}$$

$$= V_m I_m \left(\frac{1-\cos 2\omega t}{2} \right)$$

$$= \frac{V_m I_m}{2} (1 - \cos 2\omega t)$$

$$\boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t}$$

<u>Average Power (P)</u>: $P_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

$$P = V_{rms} \times I_{rms}$$

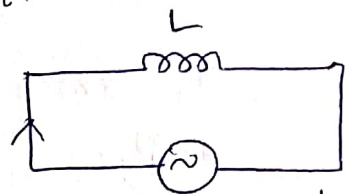
$$P = V \times I$$

(or)

$$P = I^2 R \text{ (or) } \frac{V^2}{R} \text{ Watts}$$

→ **AC through Pure Inductor:**

Consider an AC circuit with pure inductance 'L' (Henry) connected across an alternating voltage 'V' and current 'i' is flowing in the circuit.



$v = V_m \sin \omega t$

let, $v = V_m \sin \omega t$

Voltage across inductor:

$$V_L = L\frac{di}{dt}$$

$$V = L\frac{di}{dt}$$

$$\frac{V}{L} = \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V}{L}$$

$$\frac{di}{dt} = \frac{V_m \sin\omega t}{L}$$

$$di = \frac{V_m \sin\omega t}{L} dt$$

Integrating on both sides

$$\int di = \int \frac{V_m \sin\omega t}{L} dt$$

$$i = \frac{V_m}{L}\int \sin\omega t \, dt \qquad \left[\because \int \sin\theta = -\cos\theta\right]$$

$$i = \frac{V_m}{L}(-\cos\omega t)$$

$$i = \frac{-V_m \cos\omega t}{\omega L}$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t) \qquad \left[\because \sin(\omega t - 90^\circ) = -\cos\omega t\right]$$

$$i = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

As $\omega = 2\pi f$

$$i = \frac{V_m}{2\pi f L} \sin(\omega t - 90^\circ)$$

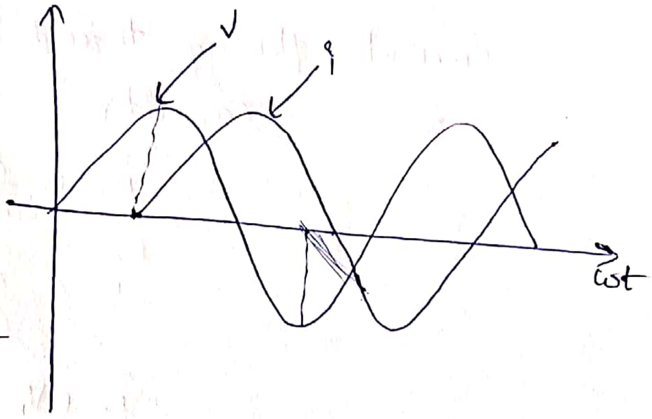
$$\{\because X_L = 2\pi f L\}$$

$$i = \frac{V_m}{X_L} \sin(\omega t - 90^\circ)$$

$$\left[\because \frac{V_m}{X_L} = I_m\right]$$

$$\boxed{i = I_m \sin \omega t - 90^\circ}$$

Wave form for Voltage (v) and Current (I)



- The phase difference between Voltage (v) & Current (I) in case of pure inductor is 90°.
- Power factor: $\cos\phi = \cos 90^\circ = 0$.
- So, the power factor for pure inductor is zero.
- Voltage leads the current by 90°.

Power (P):

$$P = V \times i$$

$$= V_m \sin \omega t \times I_m \sin(\omega t - 90^\circ)$$

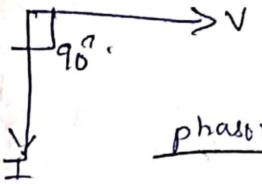
$$= V_m \sin \omega t \times I_m (-\cos \omega t)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$[\because \sin 2\theta = 2 \sin\theta \cos\theta]$$

$$\boxed{P = -\frac{V_m I_m}{2} \sin 2\omega t}$$

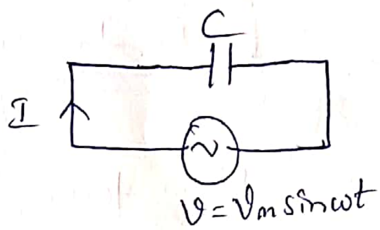
Average Power: $P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t = 0.$



phasor diagram.

→ **AC through Pure Capacitor:**

Consider an AC circuit with pure capacitance 'C' (Farads) connected across an alternating voltage 'v' and current 'i' is flowing in the circuit.



let, $v = V_m \sin\omega t$

Current flowing through the capacitor:

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d}{dt}(V_m \sin\omega t)$$

$$i = CV_m \frac{d}{dt}\sin\omega t \qquad \left[\because \frac{d}{dx}\sin x = \cos x \cdot (1)\right.$$

$$i = CV_m \cos\omega t \frac{d}{dt}\cancel{(\omega t)}(\omega t)$$

$$i = CV_m \cos\omega t \cdot \omega$$

$$i = CV_m\omega \cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}}\cos\omega t \qquad \left[\because \frac{V_m}{1}\times\frac{\omega c}{1} = V_m\omega c\right]$$

$$i = \frac{V_m}{X_C}\cos\omega t \qquad X_C = \frac{1}{2\pi f C}$$

$$\omega = 2\pi f$$

$$\left\{\frac{V_m}{X_C} = I_m\right\} \qquad \boxed{X_C = \frac{1}{\omega C}}$$

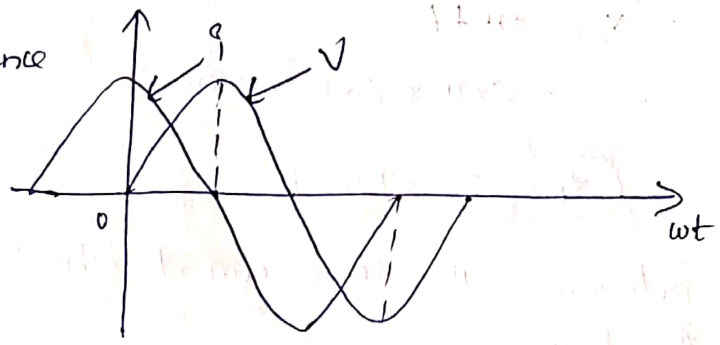
$$\boxed{i = I_m \cos\omega t}$$

$\{\sin(\omega t + 90°) = \cos \omega t\}$

$$\boxed{i = I_m \sin(\omega t + 90°)}$$

Wave form for voltage and current.

- The phase difference between voltage and current in phase of pure capacitor is 90°.



- So, power factor : $\cos\phi = \cos 90° = 0$.
- The power factor for pure capacitor is zero.
- The power factor for pure capacitor by 90°.
- Voltage lags the current by 90°.

Power (P) $= V \times i$

$= V_m \sin\omega t \times I_m \sin(\omega t + 90°)$

$= V_m \sin\omega t \times I_m \cos\omega t$

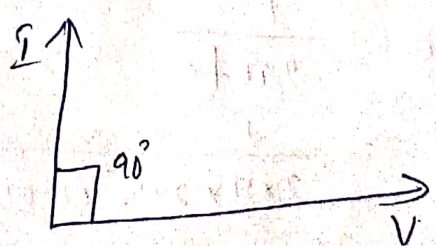
$= V_m I_m \sin\omega t \cos\omega t$

$$\boxed{P = \frac{V_m I_m}{2} \sin 2\omega t}$$

Average power:

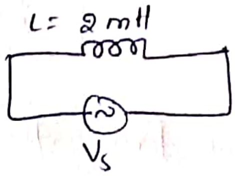
$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p\, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

phasor diagram:



Problems:

→ A sinusoidal voltage is applied to the circuit shown in figure. The frequency is 3kHz. Determine the inductive reactance.



Sol: $L = 2mH = 2\times10^{-3} H$

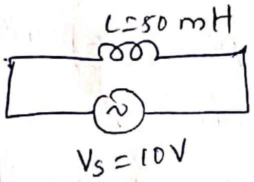
$f = 3kHz$

$X_L = 2\pi f L$

$= 2\times\pi\times 3\times10^{3}\times 2\times10^{-3}$

$\boxed{X_L = 37.69\,\Omega}$

→ Determine the rms current in the circuit as shown in the figure.



Sol: Rms current $(I_{rms}) = ?$

$L = 50mH$

$V = 10V$

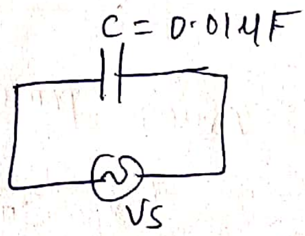
$I = \frac{V}{X_L} = \frac{V_{rms}}{X_L}$

$X_L = 2\pi f L = 2\times\pi\times50\times50\times10^{-3} = 3.14\times10^{3}$

$I = \frac{10}{3.14\times10^{3}} = 3.18\,mA$

$\boxed{I_{rms} = 3.18\,mA}$

→ A sinusoidal voltage is applied to a capacitor as shown in figure. The frequency of the sine wave is 2kHz. Determine the capacitive reactance.



Sol: $C = 0.01\mu F$

$f = 2kHz$

$X_C = \frac{1}{2\pi f C}$

$= \frac{1}{2\times\pi\times2\times10^{3}\times0.01\times10^{-6}} = 7.96\,k\Omega$

$\boxed{X_C = 7.96\,k\Omega}$

→ Determine the rms current in the circuit.

Sol: $C = 0.01\,\mu F$

$V = 5V = V_{rms}$

$f = 5 KHZ = 5\times10^3$ HZ

$$X_C = \frac{1}{2\pi f C}$$

$$= 1/2\pi \times 5\times10^3 \times 0.01\times10^{-6} = 3.18\,K\Omega$$

$$I_{rms} = \frac{V_{rms}}{X_C} = 5/3.18\times10^3 = 1.57\,mA$$

$$\boxed{I_{rms} = 1.57\ mA}$$

→ An AC circuit consists of pure resistance of 20Ω and is connected to an AC supply of 250V, 50HZ. Determine

i, current flowing through the circuit.

ii, Power absorbed by the circuit.

iii, Equations for voltage & current.

Sol: $V = 250V$, $50 HZ = f$, $R = 20\Omega$

i, $I = V/R = 250/20 = 12.5A$

ii, $P = VI = 250\times12.5 = 3125W$

iii, Equations for voltage & current

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_m = V_{max} = \sqrt{2}\cdot V_{rms} = \sqrt{2}\times 250 = 353.6V$$

$$I_m = I_{max} = \sqrt{2}\cdot I_{rms} = \sqrt{2}\times12.5 = 17.68A$$

$$v = V_m \sin\omega t = 353.6 \sin 2\pi f t$$
$$= 353.6 \sin(314)t$$

$$i = I_m \sin\omega t = 17.68 \sin 2\pi f t$$
$$= 17.68 \sin(314)t$$

→ A pure inductive coil allows a current of 10A to flow from 230V, 50HZ supply. Determine:

i, Inductance of the coil
(ii) Power absorbed by the circuit
iii, Equations for Voltage & Current.

Sol: $I = 10A$, $V = 230V$, $f = 50HZ$

i, $L = ?$

$X_L = 2\pi f L$

$L = \frac{X_L}{2\pi f}$

$X_L = \frac{V_{rms}}{I} = \frac{230}{10} = 23\Omega$

$L = \frac{23}{2\pi \times 50} = 0.073H$

$\boxed{L = 0.073H}$

ii, Power absorbed by pure Inductor is zero. (i.e., $P = 0$)

iii, Equations for Voltage & Current

$V_{max} = \sqrt{2} V_{rms} = \sqrt{2} \times 230 = 325.3V$

$I_{max} = \sqrt{2} I_{rms} = \sqrt{2} \times 10 = 14.14V$

$v = V_{max} \sin \omega t$

$= 325.3 \sin 2\pi f t$

$= 325.3 \sin (314)t$

$i = I_{max} \sin(\omega t - 90°)$

$= 14.14 \sin(2\pi f t - 90°)$

$= 14.14 \sin(314t - 90°)$

→ A capacitor has a capacitance of 318 μF connected across 200V, 50HZ supply. Determine

i, capacitive Reactance
ii, Rms value of current
iii, Equations for voltage & current

Sol: $C = 318\mu F = 318 \times 10^{-6} F$

$V = 200V$

$f = 50HZ$

i, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}} = 10\Omega$

ii, Rms value of current: $I_{rms} = ?$

$I_{rms} = \frac{V}{X_C} = \frac{20}{10} = 20A$

iii) Equations of voltage & current

$$V = V_m \sin \omega t$$
$$I = I_m \sin(\omega t + 90^\circ)$$

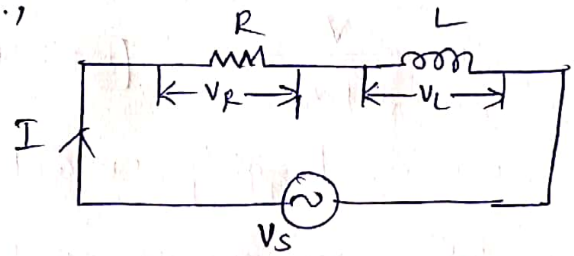
$$V_m = V_{max} = \sqrt{2}\, V_{rms} = \sqrt{2} \times 200 = 282.84V$$
$$I_m = I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2} \times 20 = 28.28A$$

$$V = 282.84 \sin \omega t = 282.84 \sin(2\pi f t) = 282.84 \sin(314)t$$
$$i = 28.28 \sin(\omega t + 90^\circ) = 28.28 \sin(314t + 90^\circ)$$

Note: In pure capacitor, current leads the voltage by 90°.

SINUSOIDAL RESPONSE OF SERIES RL CIRCUIT:

Consider an AC circuit consisting of Resistance (R) and Inductance (L) in series combination as shown in figure..,



I – Total current of the circuit
V_R – Voltage across the Resistor
V_L – Voltage across the Inductor
V_s – Voltage Source

By KVL:

$$V = V_R + V_L$$
$$= IR + IX_L$$
$$= IR + Ij X_L$$
$$= I(R + jX_L)$$

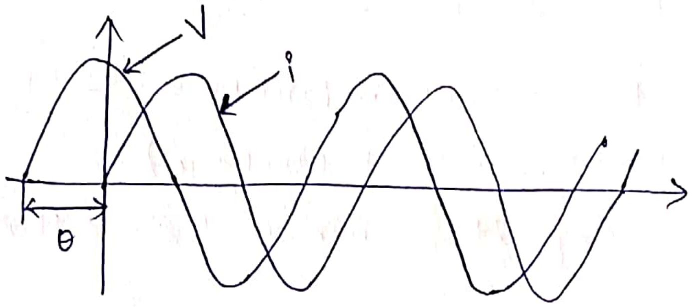
As $V = IZ$

$$Z = R + jX_L$$
$$|Z| = \sqrt{R^2 + (X_L)^2}$$

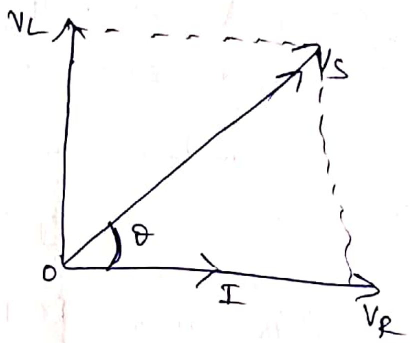
phase angle $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$

Total Impedance (Z) : $\sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$

Wave form of Voltage & current :



Phasor diagram



In graph the voltage response leads ~~by~~ phase angle 'θ'.

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Instantaneous Current : $i = I_m \sin(\omega t - \theta)$

$$i = \frac{V}{\sqrt{R^2 + X_L^2}} \sin\left[\omega t - \tan^{-1}\left(\frac{X_L}{R}\right)\right]$$

Note : This circuit is a lagging current circuit.

(P) A coil having a resistance of 7Ω & inductance of 31.8 mH is connected to 230V, 50Hz supply. Calculate (i) the current (ii) Phase Angle (iii) Power factor (iv) Power Consumed.

Sol: $R = 7\Omega$; $L = 31.8\,mH$; $V = 230V$; $f = 50Hz$

(i) $I = \frac{V}{Z}$

$Z = \sqrt{R^2 + X_L^2}$

$X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\Omega$

$Z = \sqrt{7^2 + 10^2} = 12.2\Omega$

$I = \frac{V}{Z} = \frac{230}{12.2} = 18.85A$

$\boxed{I = 18.85A}$

ii, phase angle $(\theta) = \tan^{-1}\left(\frac{10}{7}\right) = 55^\circ$ lag

iii, Power factor : $\cos\phi = \cos 55^\circ = 0.573$ lag

iv, Power Consumed : $P = VI\cos\phi = 230 \times 18.85 \times 0.573$

$P = 2484.2$ W

S.N ⇒ P A coil having a resistance of 6Ω and inductance of 0.03H is connected across 230V, 50Hz supply. Determine
i, current
ii, phase angle
iii, Power factor
iv, Power consumed

Sol: i, $I = 20.53$ A

ii, phase angle $\phi = 57.5^\circ$

iii, P.f : $\cos\phi = \cos 57.5^\circ = 0.54$

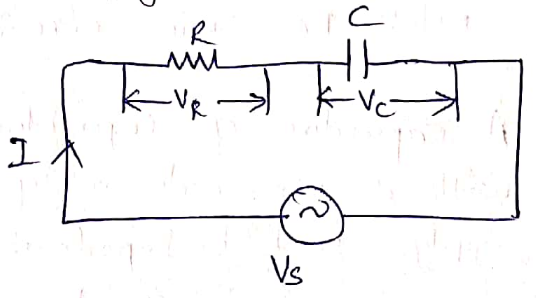
iv, Power Consumed $(P) = VI\cos\phi$

$= 230 \times 20.53 \times 0.54$

$= 2.537$ kW

SINUSOIDAL RESPONSE OF SERIES RC CIRCUIT:

Consider the RC Series circuit excited by sinusoidal voltage as shown in figure:



By KVL:

$$V_s = V_R + V_C$$

$$= IR + I \cdot \frac{1}{j\omega C}$$

$$= I\left(R + \frac{1}{j\omega C}\right)$$

$$= I\left(R - \frac{j}{\omega C}\right)$$

$\left[\because \text{As } X_C = \frac{1}{\omega C}\right]$

$$X_C = \frac{1}{2\pi f C}$$

$$\omega = 2\pi f$$

$\left[\because X_C = 1/\omega C\right]$

$$= I\left(R - j \cdot \frac{1}{\omega C}\right)$$

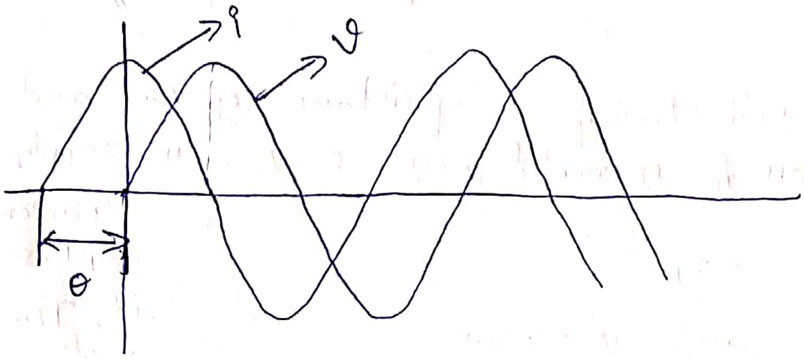
$$= I\,(R - jX_C)$$

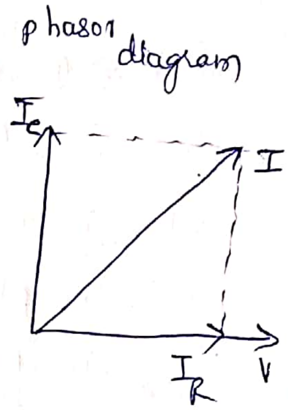
$$Z = R - jX_C$$

$$|Z| = \sqrt{R^2 - (jX_C)^2} = \sqrt{R^2 + X_C^2}$$

Phase angle $(\theta) = \tan^{-1}\left(\frac{-X_C}{R}\right)$

Wave form of voltage and current;





In graph the voltage response lags the current by phase angle 'θ'

$$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

Instantaneous Current : $i = I_m \sin(\omega t + 90°)$

$$= \frac{V}{\sqrt{R^2 + X_C^2}} \sin\left[\omega t - \tan^{-1}\left(\frac{-X_C}{R}\right)\right]$$

<u>Note:</u> RC series circuit is a leading current circuit.

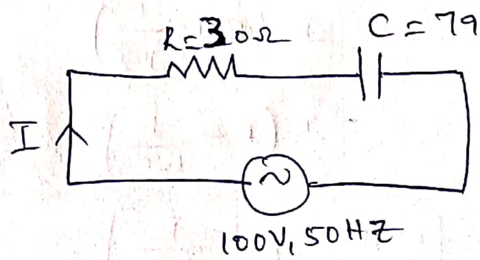
(P) A capacitor of capacitance $79.5\,\mu F$ is connected in series with a Non-inductive resistance of $30\,\Omega$, across a 100V, 50HZ supply. Find i, Impedance (ii) Current (iii) Phase Angle (iv) Equation for instantaneous value of current.

<u>Sol:</u> $C = 79.5\,\mu F = 79.5 \times 10^{-6}\,F$

$R = 30\,\Omega$

$V = 100V$

$f = 50HZ$



i, Impedance $(Z) = \sqrt{R^2 + X_C^2}$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 79.5 \times 10^{-6}} = 40\,\Omega$$

$Z = \sqrt{(30)^2 + (40)^2}$

$Z = 50\Omega$

ii, Current $(I) = \frac{V}{Z} = \frac{100}{50} = 2A$

iii, Phase Angle $(\theta) = \tan^{-1}\left(-\frac{40}{30}\right) = +53^\circ$ lead.

iv, Instantaneous Value of current $(I_{max}) =$

$$I_{max} = \sqrt{2} \times I_{rms}$$

$$I = I_{rms} = 2A$$

$$I_m = \sqrt{2} \times 2 = 2.828\ A$$

$i = I_m \sin(\omega t + \theta)$

$= 2.828 \sin(314t + 53^\circ)$ //

(P) A capacitor has a capacitance of $6\mu F$ connected in series with a resistance of 15Ω across a 230V, 50HZ supply. Calculate i, Current ii, Impedance iii, Phase Angle iv, Equations for voltage & current.

Sol: i, Current $(I) =$

ii, Impedance $(Z) =$

iii, Phase angle $(\theta) =$

iv, $V_{max} =$

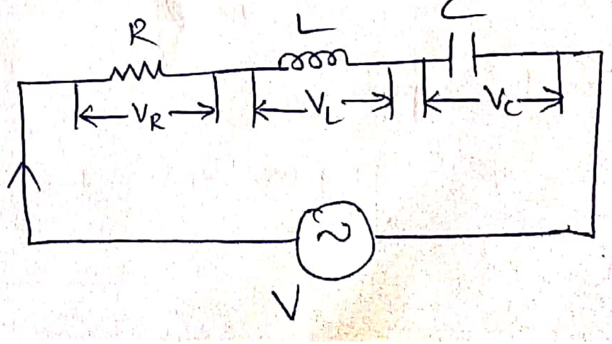
$I_{max} =$

$V =$

$i =$

SINUSOIDAL RESPONSE OF SERIES RLC CIRCUIT:

Consider the RLC series circuit excited by sinusoidal voltage source as shown in figure:



By KVL:

$V = V_R + V_L + V_C$

$V_R = IR$

$V_L = IX_L = IjX_L$

$V_C = IjX_C = IjX_C$

Now,

$$V_S = V_R + V_L + V_C$$

$$= IR + IjX_L - IjX_C$$

$$= I(R + jX_L - jX_C)$$

$$= I[R + j(X_L - X_C)]$$

As : $V = IZ$

$$Z = (R + j(X_L - X_C)]$$

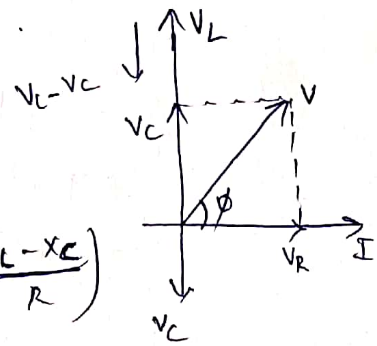
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

case (i): If $X_L > X_C$ (Inductive Nature)

So, the current lags the voltage by $90°$.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$



Total Impedance : $\bar{Z} = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Case (ii): If $X_L < X_C$ (or) $X_C > X_L$ (capacitive Nature)

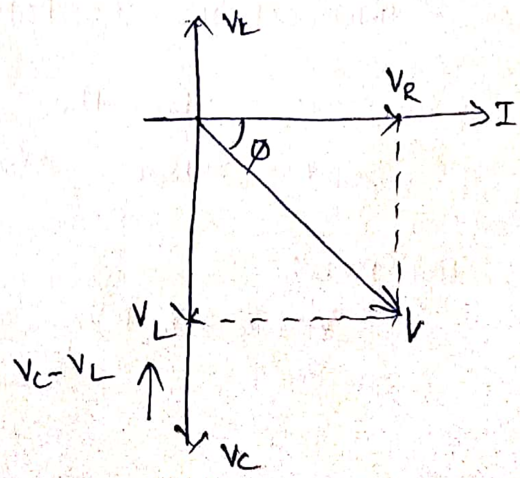
Here the current leads the voltage by $90°$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Total Impedance :

$$\bar{Z} = \sqrt{R^2 + (X_C - X_L)^2} \angle \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

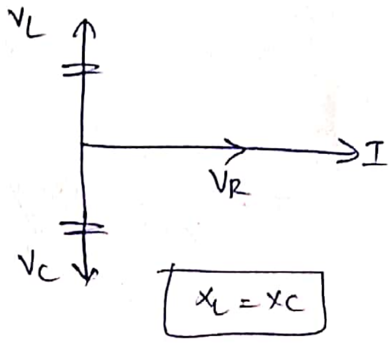


Case (iii) If $X_L = X_C$ (Resistive Nature)

Here, both are inphase

RLC ckt behaves like an Resistive circuit. This condition is known as Resonance Condition.

$(\theta = 0)$.



(P) An inductive coil, having resistance of 8Ω and inductance of 80 mH is connected in series with a Capacitance of $100\mu F$ across a 150V, 50HZ Supply.

(a) The Current
(b) The power factor
(c) The Voltage drops in the Coil and Capacitance.

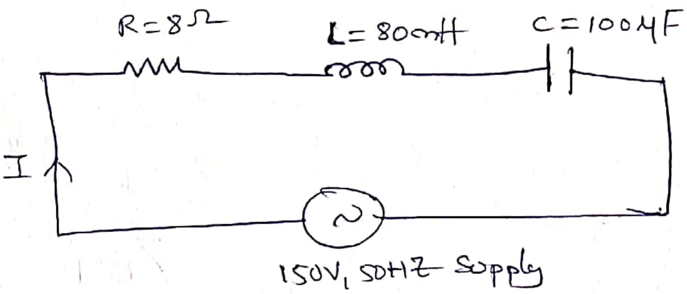
Sol: $R = 8\Omega$

$L = 80 mH$

$C = 100 \mu F$

$V = 150V$

$f = 50HZ$



i, The current $(I) = \frac{V}{Z}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

$X_L = 2\pi f \times 80 \times 10^{-3} = 25.13\Omega$

$X_C = 1 / 2\pi \times 50 \times 100 \times 10^{-6} = 31.83\Omega$

$Z = \sqrt{(8)^2 + (25.13 - 31.83)^2} = 10.435\Omega$

phase angle $(\theta) = \tan^{-1}\left(\frac{25.13 - 31.83}{8}\right)$

$= \tan^{-1}\left(\frac{-6.7}{8}\right)$

$= -39.95°$

$I = \frac{V}{Z} = \frac{150}{10.435\angle -39.95°}$

$I = 14.375 \angle 39.95° A$

ii, Power factor: $\cos\phi = \cos(39.95) = 0.767$ lead

iii, Voltage in the coil and Capacitance:

Voltage drop in the coil (Z_{coil}): $R + jX_L$

$= 8 + j25.13$

$= \sqrt{(8)^2 + (25.13)^2}$

$= 26.375$

$\theta = \tan^{-1}\left(\frac{25.13}{8}\right) = 72.34$

$Z_{coil} = 26.375 \angle 72.34°$

$\therefore V_{coil} = I Z_{coil} = 14.375 \angle 39.95 \times 26.375 \angle 72.34°$

$V_{coil} = 379.14 \angle 112.2$ V.

Voltage drop in capacitor: $(Z_{capacitance})$:

$Z_{cap} = R - jX_C = \sqrt{(8)^2 + (31.83)^2}$

$= 32.82\,\Omega$

$\theta = \tan^{-1}\left(\frac{-31.83}{8}\right)$

$\theta = -75.89°$

$Z_{cap} = 32.82 \angle -75.89°$

$$V_{capacitance} = I \cdot Z_{capacitance}$$

$$= 14.375 \angle 39.95 \times 32.82 \angle -75.89$$

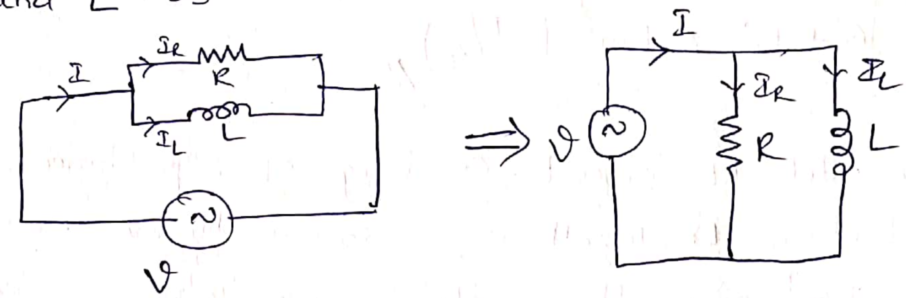
$$= 14.375 \times 32.82 \angle 39.95 + (-75.89)$$

$$= 471.78 \angle 39.95 - 75.89$$

$$= 471.78 \angle -35.94$$

SINUSOIDAL RESPONSE OF PARALLEL RL CIRCUIT:

Consider an AC circuit with parallel combination of R and L as shown in the figure:



KCL:

$$I = I_R + I_L$$

$$= \frac{V}{R} + \frac{V}{jX_L}$$

$$= V\left(\frac{1}{R} + \frac{1}{jX_L}\right)$$

$\because Z = R + jX_L$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{jX_L}$$

$\cancel{Y = G + jB}$

$Y = \frac{1}{Z}$ (Reciprocal of Impedance)

$G = \frac{1}{R}$ (Reciprocal of Resistance)

$B = \frac{1}{X}$ (Reciprocal of Reactance)

$$\boxed{Y = G - jB_L}$$

$$Y = \sqrt{G^2 + B_L^2}$$

$$\text{phase angle } (\theta) = \tan^{-1}\left(\frac{-B_L}{G}\right)$$

$$\theta = \tan^{-1}\left(\frac{-1/X_L}{\frac{1}{R}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-1}{X_L} \cdot \frac{R}{1}\right)$$

$$\theta = \tan^{-1}\left(-R/X_L\right)$$

$$\therefore \bar{Y} = \sqrt{G^2 + B_L^2} \angle \tan^{-1}\left(-R/X_L\right)$$

(P) A parallel RL circuit consisting of resistance $(R) = 10\Omega$ & inductance $(L) = 10\,mH$ is energised by $V = 100 \sin(1000t + 36°)$ volts. Determine the instantaneous value of currents through R and L, hence obtain the total current interms of RMS values.

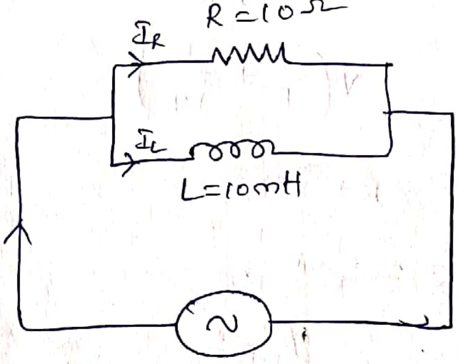
Sol: $R = 10\Omega$ $\quad L = 10\,mH$

$V = 100 \sin(1000t + 36°)$

$i = ?$

$i_R = ?$

$i_L = ?$



$V = 100 \sin(1000t + 36°)$

As $V = V_m \sin \omega t$

$V_m = 100$

$\omega = 1000 = 2\pi f$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71 \angle 36°$$

$$I_R = \frac{V_{rms}}{R} = \frac{70.71}{10} = 7.071 \angle 36°$$

$$I_L = \frac{V}{jX_L}$$

$$= \frac{70.71\angle 36^\circ}{10\angle 90^\circ}$$

$$I_L = 7.071\angle -54^\circ \text{ A}$$

$$I = I_R + I_L = 7.071\angle 36^\circ + 7.071\angle -54^\circ$$

$$= 5.72 + j4.156 + 4.156 - j5.72$$

$$= 9.876 - j1.564$$

$$I = 10\angle -9^\circ \text{ A}$$

Instantaneous Current (i):

$$i_R = 7.071 \times \sqrt{2}\ \sin(1000t + 36^\circ)$$

$$= 10\ \sin(1000t + 36^\circ)\text{ A}$$

$$i_L = 7.071 \times \sqrt{2}\ \sin(1000t - 54^\circ)$$

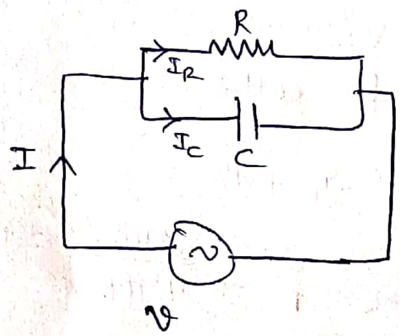
$$= 10\ \sin(1000t - 54^\circ)\text{ A}$$

$$i = 10 \times \sqrt{2}\ \sin(1000t - 9^\circ)$$

$$= 14.14\ \sin(1000t - 9^\circ)\text{ A}$$

SINUSOIDAL RESPONSE OF PARALLEL RC CIRCUIT:

Consider the RC parallel circuit excited by sinusoidal voltage source as shown in the figure.



By KCL:

$$I = I_R + I_C$$

$$= \frac{V}{R} + \frac{V}{-jX_C}$$

$$= \frac{V}{R} - \frac{V}{jX_C}$$

$$= V\left(\frac{1}{R} - \frac{1}{jX_C}\right)$$

$$= V\left(\frac{1}{R} + \frac{j}{X_C}\right)$$

$$\frac{1}{R} = G$$

$$\frac{1}{X_C} = B_C$$

As $Z = R + jX_C$

$|||^{ly}$ $Y = G + jB_C$

$$I = V(G + jB_C)$$

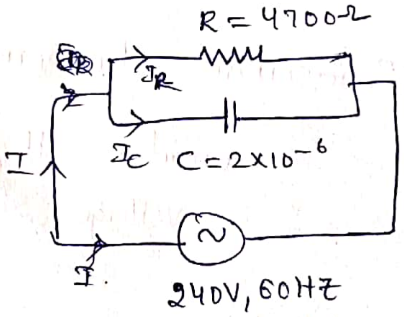
$$I = VY$$

$$|Y| = \sqrt{G^2 + B_C^2}$$

$$\theta = \tan^{-1}\left(\frac{B_C}{G}\right)$$

$\therefore$ In capacitor, the current leads the voltage by $90°$.

(P) A 4700Ω resistor & $2\mu F$ capacitor are connected in parallel across a 240V, 60 Hz Supply. Determine the circuit impedance and the line current



Sol:

$R = 4700\Omega$

$C = 2\mu F = 2\times10^{-6}$ F

$V = 240V$

$f = 60$ HZ

$$I = I_R + I_C$$

$$I_R = \frac{V}{R} = \frac{240}{4700} = 0.051A$$

$$I_C = \frac{V}{-jX_C} = Vj\omega C = j2\pi \times 60 \times 2\times10^{-6} \times 240$$

$$I_C = j0.1808$$

$$I = I_R + I_C = 0.051 + j0.1808 = 0.188 \angle 74.25° A$$

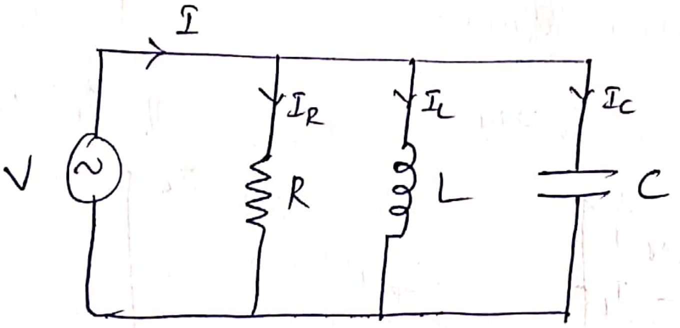
Impedance (Z) =

$$\frac{V}{I} = \frac{240}{0.188\angle 74.25°}$$

$$Z = 1276 \angle -74.25°$$

SINUSOIDAL RESPONSE OF PARALLEL RLC CIRCUIT:

Consider an RLC parallel circuit which is excited by a sinusoidal voltage source (V) as shown in the figure...,



By KCL:

$$I = I_R + I_L + I_C$$

$$= \frac{V}{R} + \frac{V}{jX_L} - \frac{V}{jX_C}$$

$$= V\left(\frac{1}{R} + \frac{1}{jX_L} - \frac{1}{jX_C}\right)$$

$$= V(G - jB_L + jB_C)$$

$$= V\left[G + j(B_C - B_L)\right]$$

$$Y = G + j(B_C - B_L)$$

$$Y = \sqrt{G^2 + (B_C - B_L)^2}$$

phase angle $\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$

Case (i) If $B_C - B_L > 0$ (Resistive - capacitive circuit)

$B_C > B_L$

then the phase angle $\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$

Case (ii) If $B_L - B_C > 0$ (Resistive - Inductive circuit)

$B_L > B_C$

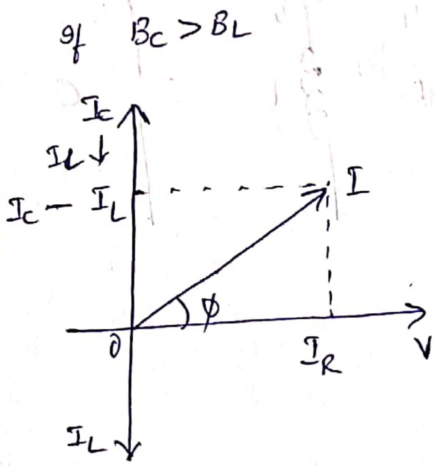
then the phase angle $\theta = \tan^{-1}\left(\frac{B_L - B_C}{G}\right)$

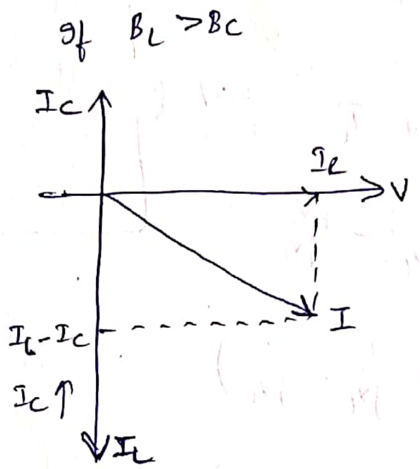
Case (iii) If $B_C = B_L$ (Resistive Circuit)

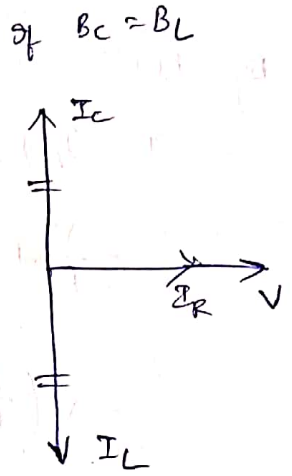
then the phase angle $\theta = 0^\circ$.

∵ This condition is called "RESONANCE CONDITION".

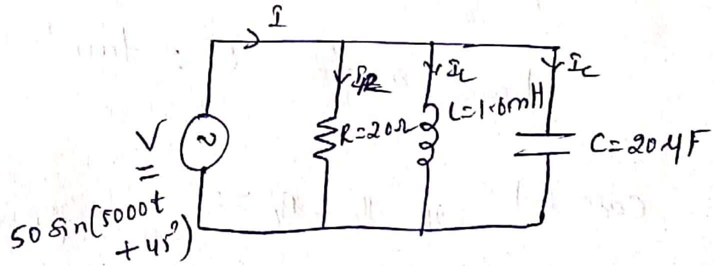
phasor diagrams:







(P) The applied voltage in a parallel RLC circuit is given by $V = 50 \sin(5000t + 45^\circ)$ volts. If the values of R, L and C are given as 20Ω, $1.6mH$ & $20\mu F$ resp... then find the total current supplied by the source and power factor.



Sol: Given Data:

$R = 20\Omega$

$L = 1.6mH$

$= 1.6 \times 10^{-3} H$

$C = 20\mu F = 20 \times 10^{-6} F$

$V = 50 \sin(5000t + 45^\circ)$

$V = V_m \sin(\omega t + \theta)$

$\omega = 5000 = 2\pi f$

$V_m = 50$

$I = I_R + I_L + I_C$

$I_R = \frac{V}{R}$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 35.35 \angle 45^\circ$

$I_L = \frac{V}{jX_L} \Rightarrow X_L = 2\pi f L = \omega L = 5000 \times 1.6 \times 10^{-3}$

$\boxed{X_L = 8\Omega}$

$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} = \frac{1}{5000 \times 20 \times 10^{-6}} = 10\Omega$

$\boxed{X_C = 10\Omega}$

$I_R = \frac{V}{R} = \frac{35.35 \angle 45^\circ}{20} = 1.77 \angle 45^\circ = 1.25 + j1.25 \text{ A}$

$I_L = \frac{V}{jX_L} = \frac{35.35 \angle 45^\circ}{8 \angle 90^\circ} = 4.42 \angle -45^\circ = 3.125 - j3.125 \text{ A}$

$I_C = \frac{V}{-jX_C} = \frac{35.35 \angle 45^\circ}{10 \angle -90^\circ} = 3.535 \angle 135^\circ = (-2.5 + j2.5) \text{ A}$

i) Total current supplied by the source, $I = I_R + I_L + I_C$

$= 1.25 + j1.25 + 3.125 - j3.125 + (-2.5 + j2.5)$

$= 1.25 + j1.25 + 3.125 - j3.125 - 2.5 + j2.5$

$= 1.875 + j0.625$ $\quad \sqrt{(1.875)^2 + (0.625)^2} = 1.976$

$= 1.976 \angle 18.44^\circ$ $\quad \tan^{-1}\left(\frac{0.625}{1.875}\right) = 18.43^\circ$

ii) Power factor $\cos\phi = \cos(18.44^\circ)$

$= 0.95$ lagging.

POWER IN AC CIRCUITS:

Power: It is the rate at which work is done.

- In DC circuits, the voltage & current are constant. So, the power is constant.
- This constant power is called Average power (or) power.
- Denoted by 'P'. $\boxed{P = VI}$

- In AC circuits, the voltage and current are sinusoidal which vary with time.
- When V and I are time varying quantities, then the power P is also a time varying quantity, which is is called as 'instantaneous power'.
- Denoted by 'p'.

Instantaneous power $\boxed{p = v\,i}$

- So, According to this the complex power is given by

Apparent power $S = VI^{*} = VI\angle\phi$

ϕ is the phase difference between V and I

$$S = VI\cos\phi + jVI\sin\phi$$

$$S = P + jQ$$

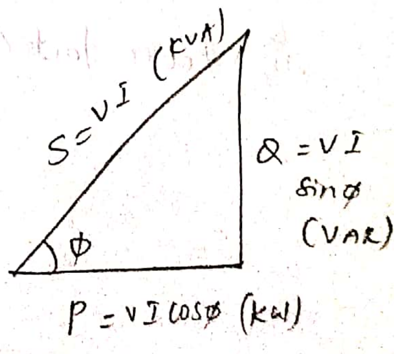
$$S = \sqrt{P^2 + Q^2} \qquad \left\{ \begin{array}{l} P = VI\cos\phi \\ Q = VI\sin\phi \end{array} \right\}$$

where,

S – apparent power in VA (or) KVA

P – Active Power in KW $= VI\cos\phi$

Q – Reactive Power in KVAR $= VI\sin\phi$



ACTIVE POWER (P):

It is defined as, the real part of of S.

(or)

defined as the product of voltage (V) and current (I) and cosine of the phase angle (ϕ).

$$\therefore P = VI\cos\phi \text{ (Watts)}$$

REACTIVE POWER (Q):

defined as the product of voltage and current and sine of the phase angle (ϕ)

$$\therefore Q = VI\sin\phi \text{ (VAR)}$$

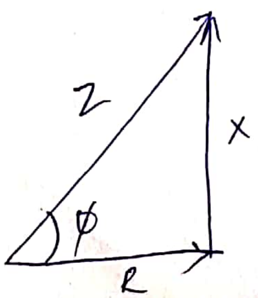
- If Q is leading ~~then~~ the reactive power is taken as positive and it is capacitive.
- If Q is lagging then reactive power is taken as negative and it is Inductive

POWER FACTOR:

The cosine of angle between voltage and current.

$$\cos\phi = \frac{\text{Active (or) Real power}}{\text{Apparent power}} = \frac{P}{S}$$

phase angle (ϕ): The phase different between V and I



$$\cos\phi = \frac{R}{Z} = \left(\frac{\text{Resistance}}{\text{Impedance}}\right)$$

(P) → Find the power delivered from a sinusoidal voltage Source with $V=220V$ to an impedance of $Z=(6+j8)\Omega$ and also calculate the power factor.

Sol: Given Data:

$$V=220V$$

$$Z=(6+j8)\Omega$$

$$=10\angle 53.13^\circ$$

$$I=\frac{V}{Z}=\frac{220}{10\angle 53.13}=22\angle -53.13^\circ$$

Power delivered $(P)=VI\cos\phi = 220\times 22\times\cos(53.13^\circ)$

$$=2904\,W$$

power factor $(\cos\phi)=\cos 53.13^\circ = 0.6$

(P) The voltage and current across a load are given by $V=110V$ and $I=20\angle 50^\circ A$. Find Active power, Reactive Power and Apparent Power.

Sol:

$$V=110V$$

$$I=20\angle 50^\circ A$$

Active power $(P)=VI\cos\phi = 110\times 20\times\cos 50^\circ = 1414W = 1.414\,KW$

Reactive power $(Q)=VI\sin\phi = 110\times 20\times\sin 50^\circ = 1685\,VAR$

Apparent power $(S)=VI=110\times 20=2200\,VA$

(P) A certain passive network has Equivalent impedance of $Z=(3+j4)\Omega$ and an applied voltage $V(t)=75\cos(1000t+30^\circ)$. Give the complete power information.

Sol: Given Data:

$$Z=(3+j4\Omega)$$

$$V(t)=75\cos(1000t+30^\circ)V$$

$$V=V_m\cos(\omega t+\theta)$$

$$V_m=75$$

$$V_{rms}=\frac{V_m}{\sqrt{2}}=\frac{75}{\sqrt{2}}=53\angle 30^\circ$$

$Z = 3 + j4 = 5 \angle 53.13^\circ$

$I = \frac{V}{Z} = \frac{53 \angle 30^\circ}{5 \angle 53.13^\circ} = 10.6 \angle -23.13^\circ A$

$S = VI = 53 \angle 30^\circ \times 10.6 \angle -23.13^\circ$

$S = 562 \angle \cancel{53.13^\circ} 6.87$

$=$

RESONANCE:

Resonance occurs in every kind of system as electrical, mechanical, optical etc...,

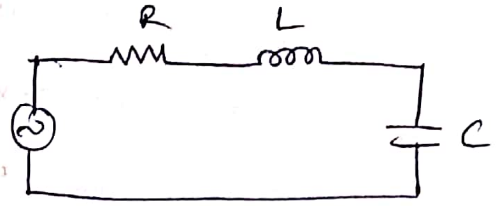
- Usually Resonance occurs in any of these systems when energy storage elements interchange exactly equal amount of energy.
- Resonance cannot takes place when there is only one type of energy storing element.
- There must be a two types of energy storing elements capable of interchanging energy between one another.

Based on this there are two types of Resonances:

- Series Resonance
- Parallel Resonance

SERIES RESONANCE:

Consider an RLC series circuit with a 1-ϕ variable frequency (F) supply as shown in figure.



- In series RLC circuit the impedance is given by

$$Z = R + j(X_L - X_C)$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

- The circuit is said to be under Resonance when the applied voltage 'V' and the resulting current 'I' are in phase.
- Thus, a Series RLC circuit under resonance behaves like a pure resistive network and the Net reactance of the circuit should be zero.

So, at Resonance Condition

$$X_L - X_C = 0$$
$$X_L = X_C$$
$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega = \sqrt{\frac{1}{LC}}$$
$$\omega = 1/\sqrt{LC}$$
$$\omega = 2\pi f$$
$$2\pi f = \frac{1}{\sqrt{LC}}$$
$$f = \frac{1}{2\pi\sqrt{LC}}$$
$$f = f_r = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

f_0 – f_r – Resonant frequency of the circuit

* If $f = f_0$, the total impedance of the circuit is $Z = R$ So, the Net reactance is zero, impedance is minimum, then the power factor is unity $[P.f \cos\phi = \frac{R}{Z} = R/R = 1]$

* If $f > f_0$, the net reactance is inductive, power factor is Lagging.

* If $f < f_0$, the net reactance is capacitive, power factor is leading.

CONDITIONS AT RESONANCE:

i, At resonance condition the circuit is like a pure resistive,

ii, X_L and X_C are exactly Equal.

iii, Impedance is minimum & Equal to $Z = R$

iv, Current is maximum (i.e, $I = \frac{V}{R} = \frac{V}{Z}$)

v, Applied voltage & current are in phase

vi, power factor of the circuit is unity.

vii, $V_L = V_C$ (must be Equal).

viii, Series Resonance is also called as Acceptor circuit (or) voltage magnification circuit.

ix, Resistance is independent of frequency,

x, Inductor is dependent of frequency

Inductive reactance $(X_L) = 2\pi f L$

$X_L \propto f$

X_L is directly proportional to frequency

xi, Capacitive reactance is dependent of frequency

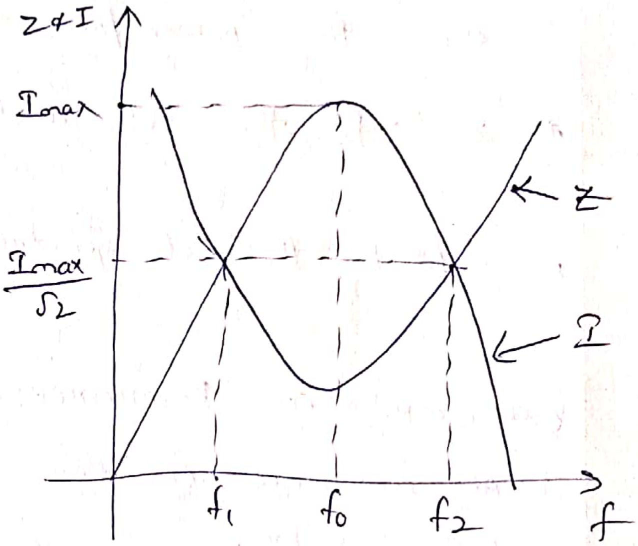
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$X_C \propto \frac{1}{f}$ (Inversely proportional to freq)

From the graph, it is observed that the impedance varies with frequency, the current in the circuit changes.

- As the frequency is varied from 0 to ∞ the current increases from 0 to maximum [i.e, I_{max} at $f = f_0$ & then decreases to zero]

Response of series RLC circuit



(P) A coil with resistance $R = 80\,\Omega$ and inductance $L = 100$ mH is connected in series with a 500 μF capacitor and a generator of 200V source. Determine the i, Resonant frequency ii, Circuit impedance at resonant frequency

Sol: Given that;

$R = 80\,\Omega$

$L = 100\,mH = 100 \times 10^{-3} H$

$V = 200V$

$C = 500\,\mu F = 500 \times 10^{-6} F$

i, $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 500 \times 10^{-6}}} = 22.5\,Hz$

ii, Circuit impedance at Resonant frequency: $Z = R = 80\,\Omega$

QUALITY FACTOR: defined as the ratio of voltage magnification in series Resonant circuit.

- Measure of selectivity of series Resonant circuit

– denoted by 'Q'.

$$\text{Quality factor } (Q) = \frac{\text{Voltage across inductor (or) Capacitor}}{\text{Applied voltage}}$$

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

$$Q = \frac{V_L}{V} = \frac{I X_L}{I R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \frac{1}{\cancel{\sqrt{L}}\sqrt{C}} \cdot \frac{\cancel{\sqrt{L}} \cdot \sqrt{L}}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$Q = \frac{V_C}{V} = \frac{I X_C}{I R} = \frac{X_C}{R} = \frac{1}{\omega C} \cdot \frac{1}{R} = \frac{1}{\frac{1}{\sqrt{LC}} \cdot C} \cdot \frac{1}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

– To improve the quality factor of a coil, the Resistance (R) should be designed as low as possible

– Results in Band width.
Reduction of.

(P) A 40 μF capacitor is connected in series with the coil whose inductance is 8 mH. Determine the

i, Resonant frequency

ii, Resistance of the coil if a 100V source operating at the resonant frequency causes a circuit current of 4.6A

iii, quality factor of the coil

Sol: $C = 40\mu F$; $L = \cancel{8 mH}$ 8 mH ; $V = 100V$; $I = 4.8A$

i, $f_r = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{8\times10^{-3} \times 40\times10^{-6}}}$

$f_r = 281.5 HZ$

ii. $V = 100V$; $I = 4.6A$

Resistance of the coil $(R) = \frac{V}{I_m} = \frac{100}{4.6} = 17.86\Omega$

iii, Quality factor of the coil

$$Q = \frac{\omega_r L}{R} = \frac{2\pi f_r \times 8\times10^{-3}}{R} = \frac{2\times\pi\times281.4\times8\times10^{-3}}{17.86}$$

$$\boxed{Q = 0.8}$$

(P) A circuit having resistance of 5Ω & inductance of 0.4H and a variable capacitance in series is connected across a 110V, 50HZ supply. calculate the:

i, Value of capacitance to given resonance

ii, Current flowing through the circuit

iii, Voltage across the Inductor

iv, Voltage across the Capacitor

v, Quality factor of the circuit

Sol: $R = 5\Omega$; $L = 0.4H$; $V = 110V$; $f_r = 50HZ$

i, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$C = \frac{1}{4\pi^2 f_r^2 L} = \frac{1}{4\pi^2\times50^2\times0.4} = 25.34F$$

ii, $I = \frac{V}{R} = \frac{110}{5} = 22A$

iii, $V_L = I\cdot X_L = I\cdot 2\pi f L = 22\times2\pi\times50\times0.4$

$V_L = \cancel{2763.4}\ 2763.4V$

iv, $V_C = I\cdot X_C = I\cdot\frac{1}{2\pi f C} = 22\times\frac{1}{2\pi\times50\times25.3\times10^{-6}}$

$V_C = 2763.4V$

v, $Q = \frac{\omega_r L}{R} = \frac{2\pi\times f_r\times0.4}{5} = \frac{2\pi\times50\times0.4}{5}$

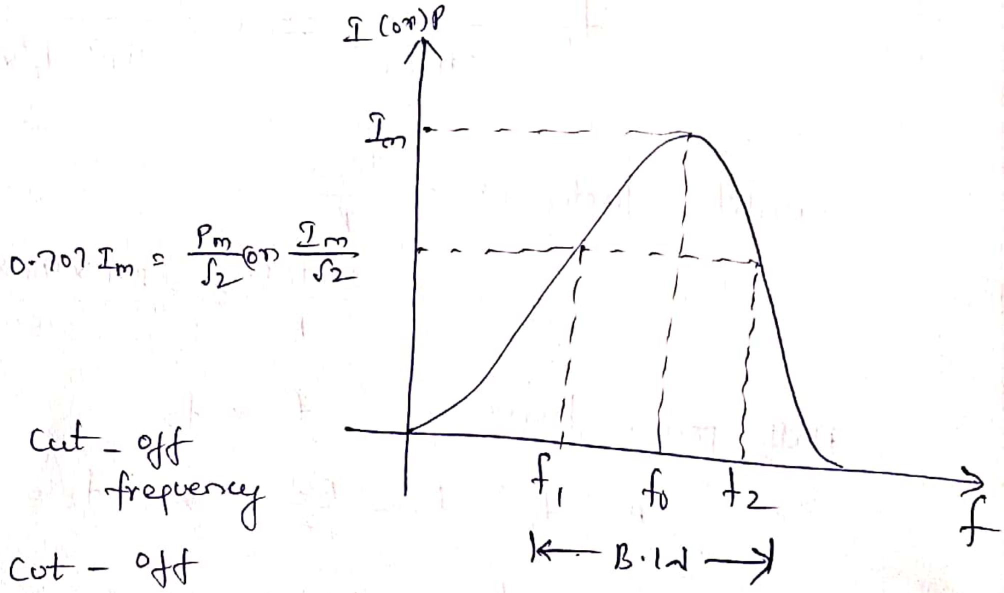
$Q = 25.12$ //

BAND WIDTH:

defined as the band of frequencies which lies between two specified points where power falls to half of its maximum value and current falls to $1/\sqrt{2}$ times of its maximum value.

[or]

The band width of a series circuit is defined as the range of frequency over which circuit current is greater than 70.7% of maximum current (i.e, current at Resonance).



f_1 – Lower cut-off frequency

f_2 – Upper cut-off frequency

$$\boxed{B.W = f_2 - f_1} \text{ Hz}$$

$$f_2 = f_r + \frac{B.W}{2} \quad ; \quad f_1 = f_r - \frac{B.W}{2}$$

→ Band width $= \frac{R}{2\pi L}$ and $Q = \frac{\omega_r L}{R}$

→ $Q = \frac{W_r L}{R} = \frac{2\pi f_r L}{R} = \frac{f_r}{B.W}$ $\left\{\frac{2\pi L}{R} = \frac{1}{B.W}\right\}$

$$\boxed{Q = \frac{f_r}{B.W}}$$

→ $B.W \propto \frac{1}{Q}$

(P) A 220V, 100 HZ AC source supplies a series RLC circuit with a capacitor and a coil. If the coil has 50 mΩ resistance and 5mH inductance, find at a resonance frequency of 100 HZ. What is the value of capacitor. Also calculate the Q-factor & half power frequencies of the circuit.

Sol: Given that: $V = 220V$; $f = 100 HZ$; $R = 50 m\Omega = 50\times10^{-3}\Omega$

$L = 5 mH = 5\times10^{-3} H$

When $f_r = 100 HZ$; $C = ?$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 \times f_r^2 \times L} \Rightarrow \frac{1}{4\pi^2\times100^2\times5\times10^{-3}}$$

$$\boxed{C = 507\mu F}$$

Quality factor $(Q) = \frac{\omega_r L}{R}$

$$= \frac{2\pi f_r L}{R} = \frac{2\pi\times100\times5\times10^{-3}}{50\times10^{-3}} = 62.83$$

Half power frequencies: f_1 & f_2

As $f_1 = f_r - \frac{BW}{2}$ & $f_2 = f_r + \frac{BW}{2}$

$$B.W = \frac{f_r}{Q} = \frac{100}{62.83} = 1.59 HZ$$

$f_1 = f_r - \frac{BW}{2}$ $\qquad$ $f_2 = f_r + \frac{BW}{2}$

$f_1 = 100 - \frac{1.59}{2}$ $\qquad$ $f_2 = 100 + \frac{1.59}{2}$

$f_1 = 99.205 HZ$ $\qquad$ $f_2 = 100.795 HZ$

THREE - PHASE BALANCED CIRCUITS:

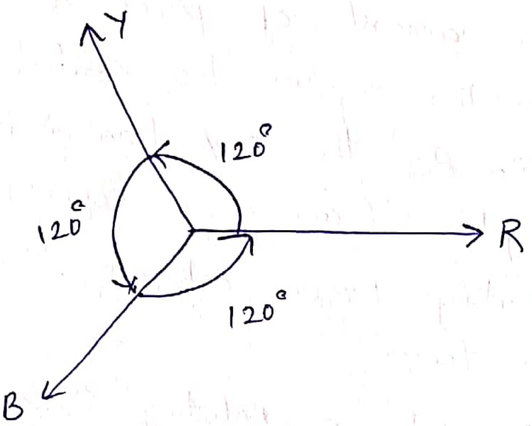
Advantages of 3-ϕ System

* The power delivered in 1-ϕ circuit is pulsating where as 3-ϕ system is constant at every instant of time.
* In a given size, a three phase machine either motor (or) generator produces more output than 1-ϕ machine.
* 3-ϕ I.M is self starting and more efficient whereas 1-ϕ I.M are not self starting.
* For transmitting the same amount of power at same voltage, a 3-ϕ transmission line requires less conductor material than a single phase line. As the 3-ϕ transmission system is cheaper than single phase (saving of copper)
* 1-ϕ motors has a pulsating torque whereas three phase motors have a uniform torque.
* Polyphase system (3-ϕ) can set up rotating magnetic field in stationary windings.
* The operating characteristics of a three-phase machine are superior than those of single phase machines.
* Compare to 1-ϕ generators, the parallel operation of 3-ϕ generators is much simple.
* Using 3-ϕ transmission i.e., the power generated by machine is increased by 50% from 1-ϕ to 3-ϕ.
* Voltage Regulation of 3-ϕ transmission line is better than that of 1-ϕ line.

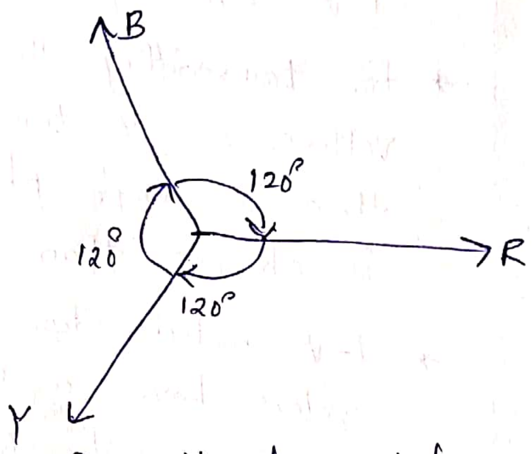
3-ϕ System:

* **Phase Sequence:** The order in which the voltages in three phases (or) three coils of an alternator reach their maximum possible values is called phase sequence (or) phase.

- The phase sequence is determined by the direction of rotation of alternator.
- In 3-ϕ each windings (or) coils are displaced by 120°.



(a) Alternator rotates in anti-clockwise direction [BRY]



(b) Alternator rotates in clockwise direction [RYB]

- R (Red) - phase
  Y (Yellow) - phase
  B (Blue) phase

(or)

A - phase
B - phase
C - phase

RYB (or) ABC

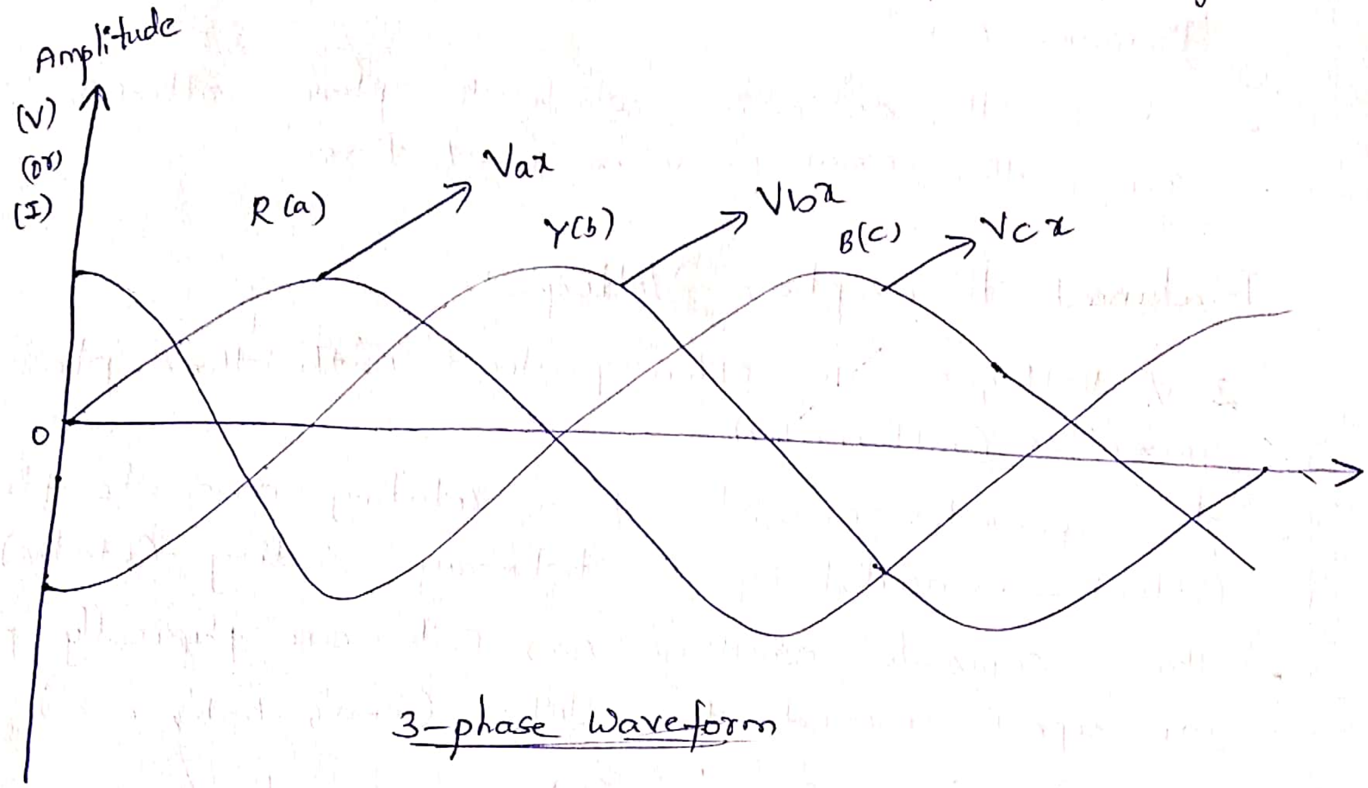
- The phase sequence can be changed by inter-changing any of the two phase.

  if RYB is +ve sequence
  then

  $\left.\begin{matrix} RBY \\ YRB \end{matrix}\right\}$ -ve sequence

- Three phase system can be connected either in Star (Y) and Delta (Δ) manner.

- Three phase system can be i, 3-ϕ 3-wire system
  ii, 3-ϕ 4wire system



3-phase Waveform

- V_{ax}, V_{bx}, V_{cx} are the phase voltages i.e., Voltage between phase a, b, c and neutral line respectively.
- The voltage between any two phases is said to be line voltage V_{ab}, V_{bc}, V_{ca} are line voltages.
- The phase voltages can be expressed as

$$V_{ax} = V_m \sin \omega t = \frac{V_m}{\sqrt{2}} \angle 0^\circ = V \angle 0^\circ$$

$$V_{bx} = V_m \sin(\omega t - 120^\circ) = \frac{V_m}{\sqrt{2}} \angle -120^\circ = V \angle -120^\circ$$

$$V_{cx} = V_m \sin(\omega t - 240^\circ) = \frac{V_m}{\sqrt{2}} \angle -240^\circ = \frac{V_m}{\sqrt{2}} \angle +120^\circ$$

$$= V \angle -240 \quad (or) \quad V \angle +120$$

$$V_{ax} + V_{bx} + V_{cx} = V \angle 0^\circ + V \angle -120^\circ + V \angle 120^\circ$$

$$\boxed{V_{ax} + V_{bx} + V_{cx} = 0}$$

- Phase Sequence is determined by the order in which the phasors pass through a fixed point in the phasor diagram (or).
- It is the order in which the phase voltages reach their maximum value w.r.t time.

Balanced three-phase Voltages:

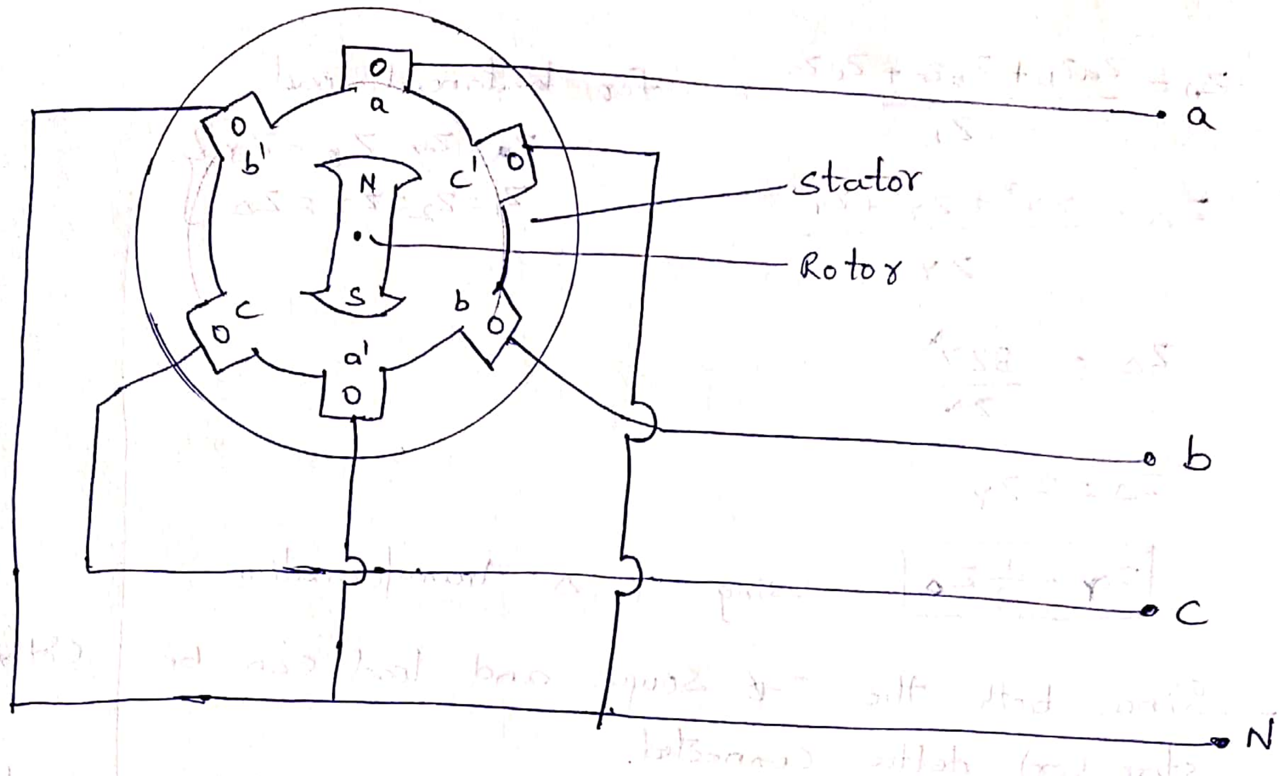
- 3-ϕ Voltages are often produced with three phase AC generator (alternator).
- The generator consists of a rotating magnetic field (rotor) surrounded by a stationary winding (stator)
- Three separate windings (or) Coils are physically placed 120° apart around the stator (a-a', b-b', c-c')

  a – Incoming & a' – Outgoing currents
  b – b' –
  c – c' –

- As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coil.
- Since coils are placed 120° apart, the voltage induced will be equal in magnitude but 120° out of phase with each other.

- Therefore, each coil can be regarded as a 1-ϕ generator by itself, the 3-ϕ generator can supply power to both single-phase and 3-ϕ loads.

- The circuit of 3-ϕ generator is as shown in figure below:

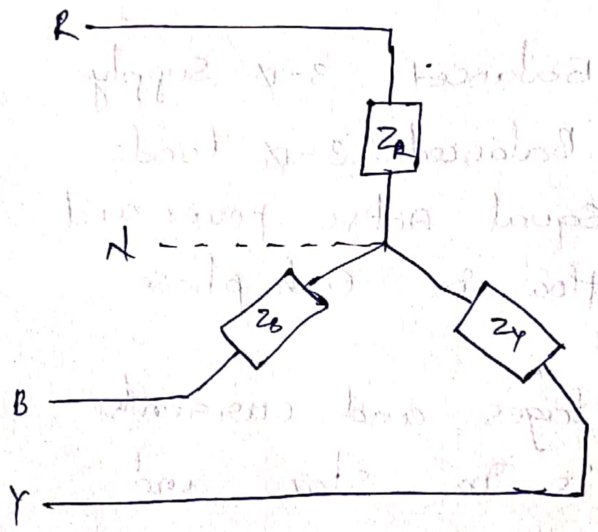


A 3-ϕ Generator (AC)

Balanced Load:

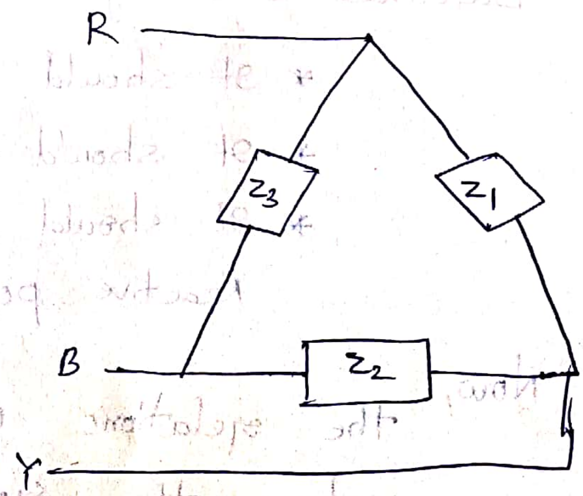
A balanced load is one in which the phase impedances are equal in magnitude and in phase.

– A 3-ϕ load can be star (Y) or Delta (Δ):



(Y – load)

$\boxed{Z_R = Z_B = Z_y = Z_Y}$



(Δ – load)

$\boxed{Z_1 = Z_2 = Z_3 = Z_\Delta}$

$$Z_A = \frac{Z_aZ_b + Z_bZ_c + Z_cZ_a}{Z_b}$$

for balanced load

$$\left.\begin{array}{l} Z_A = Z_y = Z_B = Z_Y \\ Z_1 = Z_2 = Z_3 = Z_\Delta \end{array}\right\}$$

$$Z_\Delta = \frac{Z_Y^2 + Z_Y^2 + Z_Y^2}{Z_Y}$$

$$Z_\Delta = \frac{3Z_Y^{\cancel{2}}}{\cancel{Z_Y}}$$

$$Z_\Delta = 3Z_Y$$

$\boxed{Z_Y = \frac{1}{3}Z_\Delta}$ using Y-Δ transformation

- Since both the 3-φ source and load can be Either star (or) delta Connected.
- Based on this there are four possible connections

i, Star - star (Y-Y) → star source & star load
ii, star - delta (Y-Δ) → star source & delta load
iii, Delta - Delta (Δ-Δ) → Delta source & Delta load
iv, Delta - star (Δ-Y) → Delta source & star load

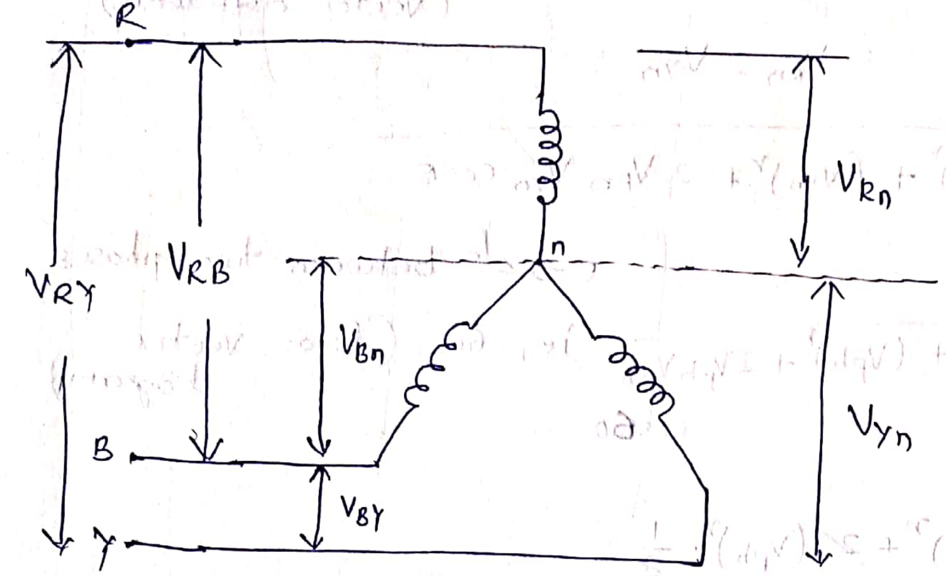
Balanced 3-φ system: (Adv)

* It should have Balanced 3-φ Supply
* It should have Balanced 3-φ Loads
* It should have Equal Active power and Reactive power flow in Each phase.

Now, the relations of Voltages and currents when the system is in star and delta connections

⇒ Voltage and Current relations in Y (star) Connection:

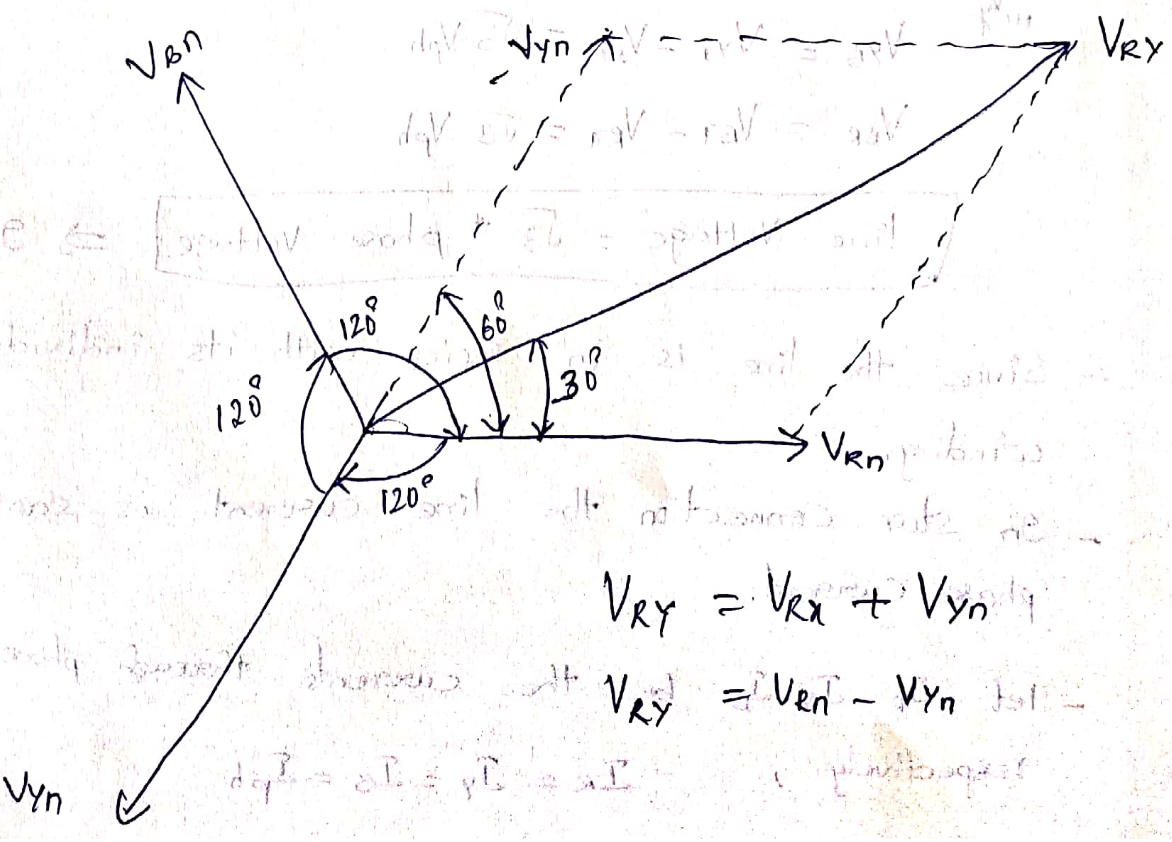
- let V_{Rn} V_{Yn} V_{Bn} represents the phase voltages.



- V_{RB}, V_{YB}, V_{RY} represents the line voltages of a star connection.

$$|V_{Rn}| = |V_{Yn}| = |V_{Bn}| = V_{ph}$$

- The vector representation of the phase voltages is as shown in figure



$$V_{RY} = V_{Rn} + V_{Yn}$$

$$V_{RY} = V_{Rn} - V_{Yn}$$

- The line voltage is the vector sum of phase voltages from the phasor diagram.

$$\bar{V}_{RY} = \bar{V}_{Rn} + (-\bar{V}_{Yn})$$

(vector difference)

$$= V_{Rn} - V_{Yn}$$

$$V_{RY} = \sqrt{(V_{Rn})^2 + (V_{Yn})^2 + 2 V_{Rn} V_{Yn} \cos\theta}$$

$\theta \rightarrow \angle^{le}$ between two phases

i.e., $60°$ (from vector diagram)

$$V_{RY} = \sqrt{(V_{ph})^2 + (V_{ph})^2 + 2 V_{ph} V_{ph} \cos 60}$$

$$V_{RY} = \sqrt{2(V_{ph})^2 + \cancel{2}(V_{ph})^2 \cdot \frac{1}{\cancel{2}}}$$

$$V_{RY} = \sqrt{3 (V_{ph})^2}$$

$$\boxed{V_{RY} = \sqrt{3}\ V_{ph}} \text{ Volts}$$

$(\because V_{RY} = V_{YB} = V_{BR} = V_L$ line voltages$)$

(or)

$$\boxed{V_L = \sqrt{3}\ V_{ph}} \text{ Volts}$$

$|||^{ly}$

$$V_{YB} = V_{Yn} - V_{Bn} = \sqrt{3}\ V_{ph}$$

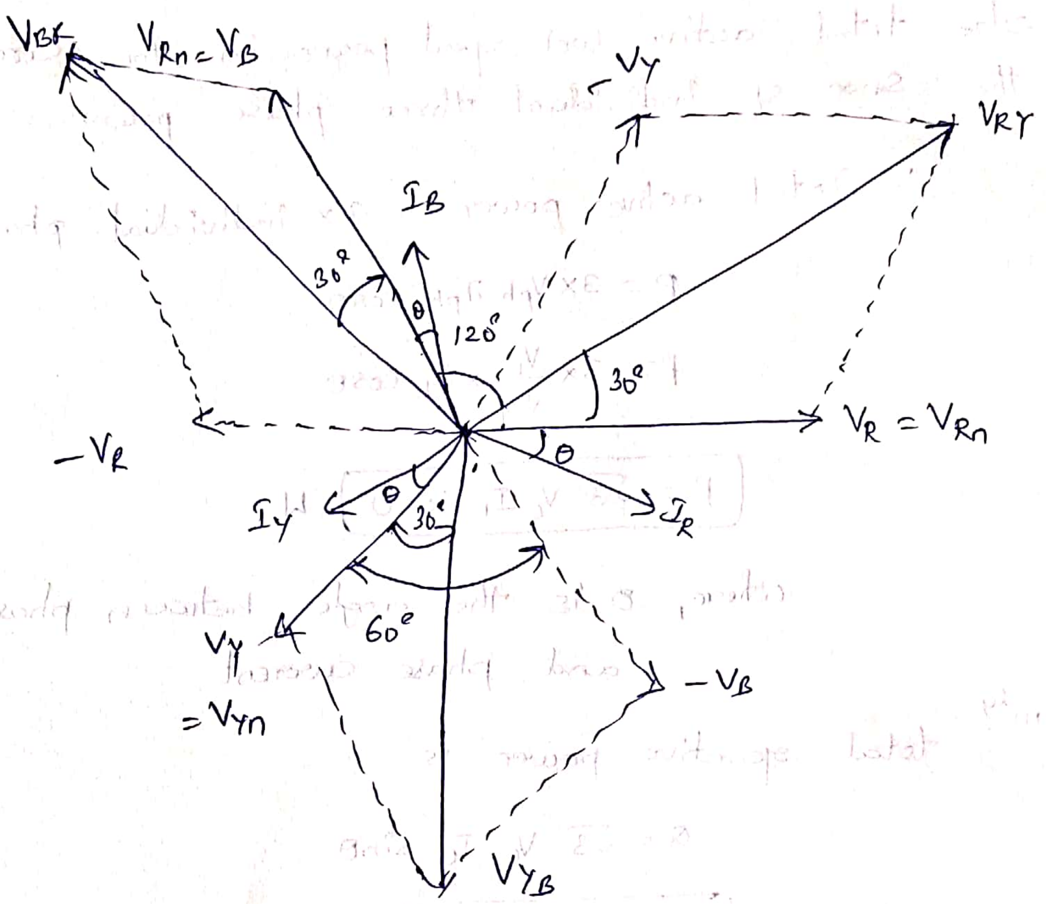
$$V_{BR} = V_{Bn} - V_{Rn} = \sqrt{3}\ V_{ph}$$

$$\boxed{\text{line Voltage} = \sqrt{3} * \text{phase Voltage}} \Rightarrow \text{In Y- Connect}$$

- Since the line is in series with its individual phase winding.
- In star connection the line current is same as phase current.
- let I_R I_Y I_B be the currents through phase R, Y, B respectively.., $I_R = I_Y = I_B = I_{ph}$

- In star Connection

$\boxed{I_L = I_{ph}}$ i.e., line Current = phase currents



Note:

- The angle between two line Voltages is 120°.
- The angle between two adjustment phase voltages is 120°.
- The line voltage is 30° ahead of their respective phase Voltage.
- The current lags phase voltage by an angle (θ) and it lags the line voltage by an angle (30° + θ), if it is an inductive load.
- line voltage is $\sqrt{3}$ times the phase Voltage
- line current is equal to phase current.
- for a balanced star Connection;
  $I_R + I_Y + I_B = 0$

- If their is a Neutral line then

$$I_R + I_Y + I_B = I_n$$

Power in star - Connection:

- The total active (or) real power in the circuit is the sum of individual three phase powers.

∴ Total active power = 3 × individual phase power

$$P = 3 \times V_{ph} I_{ph} \cos\theta$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\theta$$

$$\boxed{P = \sqrt{3}\, V_L I_L \cos\theta} \ W$$

where, θ is the angle between phase voltage and phase current.

III^{ly} total reactive power is

$$Q = \sqrt{3}\, V_L I_L \sin\theta$$

$$\boxed{Q = \sqrt{3}\, V_{ph} I_{ph} \sin\theta} \ VAR$$

∴ Total Apparent Power $S = \sqrt{P^2 + Q^2}$

$$\boxed{S = \sqrt{3}\, V_L I_L} \ VA$$

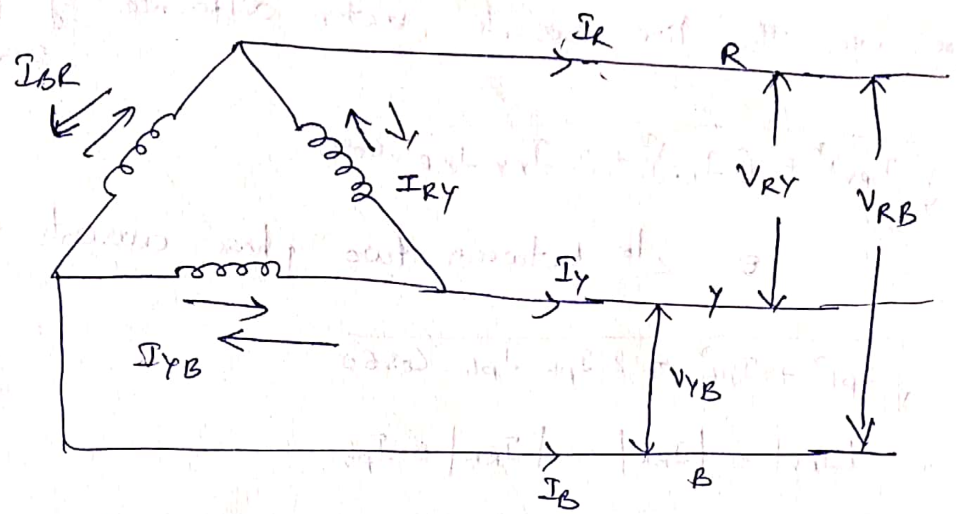
* Voltage and current relations in Delta (Δ) Connect

- let the phase currents be I_{RY} I_{YB} I_{BR} and the line currents be I_R I_Y & I_B.

- The line current is the vector difference of two phase currents.

- The phasor representation of line currents & phase currents as shown in fig (b)

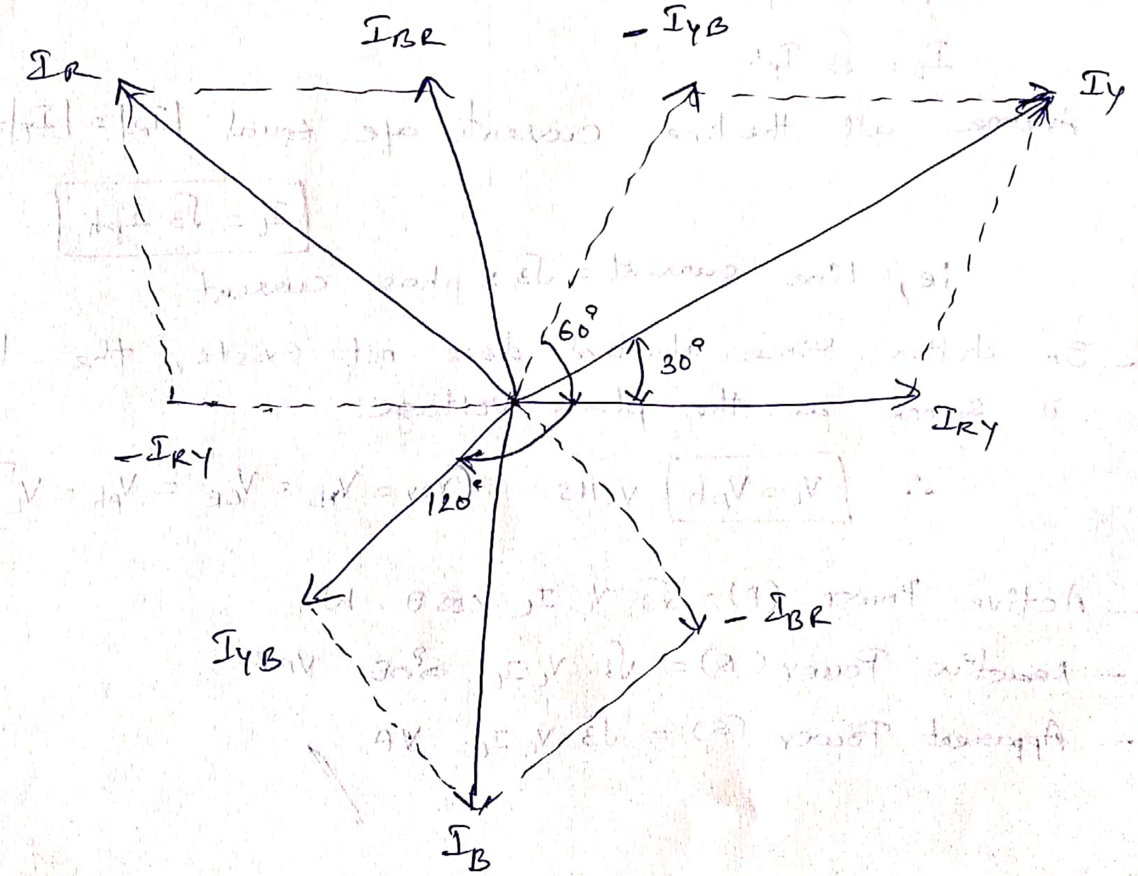
Fig. (a) Δ - Connected Source



$$I_R + I_{RY} = I_{BR} = I_R$$

$$I_Y + I_{YB} = I_{BY} = I_Y$$

Fig (b) phase diagram (relation between phase & line currents)



From phasor diagram

the line current $\bar{I_Y} = \bar{I_{RY}} - \bar{I_{YB}}$

$\bar{I_B} = \bar{I_{YB}} - \bar{I_{BR}}$

$$\bar{I}_R = \bar{I}_{BR} - \bar{I}_{RY}$$

These are the line currents vector difference of phase currents

$$I_R = \sqrt{(I_{RY})^2 + (I_{BR})^2 + 2 I_{RY} I_{BR} \cos\theta}$$

θ – $\angle$le between two phase currents $= 60°$

$$I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} I_{ph} \cos 60}$$

Assume $|I_{RY}| = |I_{BR}| = |I_{RB}| = I_{ph}$

$$I_R = \sqrt{2 I_{ph}^2 + 2 \cdot I_{ph}^2 \tfrac{1}{2}}$$

$$= \sqrt{3 I_{ph}^2}$$

$$I_R = \sqrt{3}\, I_{ph}$$

$$I_Y = \sqrt{3}\, I_{ph}$$

$$I_B = \sqrt{3}\, I_{ph}$$

Assume all the line current are equal $|I_R| = |I_Y| = |I_B| =$

$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

ie, Line current $= \sqrt{3} \times$ phase current

- In delta since Neutral does not Exists, the line volt is same as the phase voltage.

$\therefore$ $\boxed{V_L = V_{ph}}$ Volts $[V_{RY} = V_{YB} = V_{BR} = V_{ph} = V_L]$

- Active Power $(P) = \sqrt{3}\, V_L I_L \cos\theta$ W
- Reactive Power $(Q) = \sqrt{3} \cdot V_L I_L \sin\theta$ VAR
- Apparent Power $(S) = \sqrt{3}\, V_L I_L$ VA