UNIT- I

DC CIRCUITS

ELECTRICAL CIRCUIT ELEMENTS (R, L and C)

VOLTAGE AND CURRENT SOURCES

KVL and KCL

Analysis of Simple Circuits with DC Excitation

Superposition

Thevenin and

Norton Theorems

Time - domain analysis of first - Order

RL and RC Circuits

UNIT I

DC CIRCUITS

ELECTRICAL CIRCUIT ELEMENTS:

Basically there are three circuit elements in electrical circuit. They are Resistance, Inductance and Capacitance.

RESISTANCE:

The property of a material which opposes the flow of electrons (or) current in a conductor is called resistance.

The unit of resistance is OHM (Ω)

The unit of resistance is OHM. The resistance offered by a material when a current of 'I' ampere flows through it with 'V' volt potential difference across the material.

As $R = \frac{V}{I}$

Resistance

OHM'S LAW:

It is the fundamental law used in circuit analysis which provides a simple formula describing the voltage current relationship in a conducting material.

Statement: States that at constant temperature, potential difference (V) across the ends of a conductor is proportional to the current (I) flowing through conductor.

Mathematically,

Ohm's law can be stated as

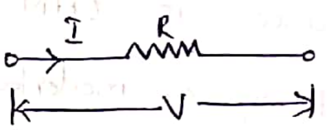
$$V \propto I$$

$$V = IR$$

(or) $I = V/R$

(or) $R = V/I$

where, $\frac{1}{R}$ is the proportionality constant and it is the resistance of the conductor between the two points.



- This law in a DC circuit first discovered by German scientist Georg Simon Ohm in 1827.
- According to this law the ratio V/I remains constant for the conductor if temperature is constant.

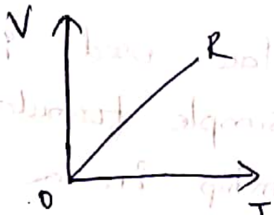


fig. (b)

$V \propto I$ will be a straight line passing through origin as shown in fig (b)

- Ohm's law can be applied for both AC & DC circuits provided the circuit must contain only linear elements.
- In case of AC circuits, resistance is replaced by Impedance.

Limitations of Ohm's law:

Ohm's law can't be applied for the following conditions

1. When temperature of the material varies.
2. For Non-linear elements like semiconductor devices.
3. Vacuum Tubes and Gas filled Valves.
4. For electrolytes when Enormous gases are produced on either Electrode.
5. For flourescent lamps.

Continuation... of Resistance:

When an electric current flows through any conductor heat is generated due to collision of free electrons with atoms. If 'I' amps to be the strength of current for a potential difference of 'V' volts across the conductor, the power absorbed by the resistor is given by

$$P = VI$$

$$P = (IR)I$$

$$P = I^2R \text{ Watts}$$

The Energy lost in the resistance in the form of heat is expressed as

$$W = \int_0^t P.\,dt$$

$$W = Pt$$

$$W = I^2Rt \qquad \left[\frac{V}{R}\cdot\frac{V}{\cancel{R}}\cdot \cancel{R}\cdot t\right]$$

$$W = \frac{V^2}{R}t \qquad \frac{V^2}{R}\cdot t$$

Prblm:

A $10\,\Omega$ resistor is connected across a 12V battery. How much current flows through the resistor,

Sol:

$V = 12V$

$R = 10\,\Omega$

$I = ?$

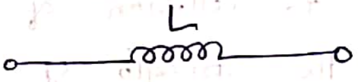
As, $V = I.R$

$12 = I \times 10$

$I = \frac{12}{10} = 1.2\ A$

INDUCTANCE:

It is the property of a coil which does not allow sudden changes of current and it is represented by 'L'. The symbolic representation is as shown



It stores the Energy in the form of magnetic field.

When current is made to pass through an inductor, an electromagnetic field is formed which induces the voltage across the coil by Faraday's law of Electromagnetic induction.

i.e., $V = L\frac{di}{dt}$

The unit of inductance is henry (H).

- By definition the inductance is one henry when current through coil changing at the rate of one ampere second induces one volt across the coil.

Above equation as,

So, $di = \frac{1}{L} V\,dt$

Also, power absorbed by the inductor is given by $P = V \times i$

$$P = L\frac{di}{dt} \times i$$

$$P = Li\frac{di}{dt} \text{ watts}$$

Energy absorbed by the inductor will thus be given by

$$W = \int_0^t P\, dt$$

$$W = \int_0^t Li\frac{di}{dt}\, dt \qquad \int x \cdot dx = \frac{x^2}{2} + C$$

$$W = Li\int_0^t di$$

$$W = \frac{1}{2}Li^2$$

Problem: The current in a 2H inductor varies at a rate of 2A/s. Find the voltage across the inductor and the energy stored in the magnetic field after 2s.

Sol: $L = 2H$ $\qquad di = 2\,A/s$

$\frac{di}{dt} = 2\,A/s$ $\qquad$ 1second $= 2A$

2 seconds $= 4A$

$V = L\frac{di}{dt}$ $\qquad V = L\frac{di}{dt}$

$V = 2 \times 4$ $\qquad V = 2 \times 4$

$V = 8V$ $\qquad V = 8V$

$W = \frac{1}{2}Li^2$

$W = \frac{1}{2}Li^2$ $\qquad W = \frac{1}{2} \times 2(4)^2$

$W = 16J$

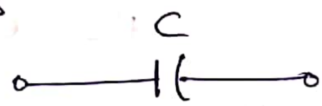
$= \frac{1}{2} \times 2 \times (4)^2$

$= \frac{1}{2} \times 2 \times 16$ $\qquad \boxed{W = 16J}$

CAPACITANCE:

It is the capability of an element to store electric charge within it.

- Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor.
- The conducting surfaces are called electrodes and the insulating medium is called Dielectric.
- A capacitor stores energy in the form of an Electric field that is established by lines of force between the positive and negative charges, and is concentrated within the dielectric.
- Capacitance does not allow sudden changes of voltage.
- By definition one farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates. Unit of capacitance is Farad
- The symbol for capacitance is as



As $C = \frac{Q}{V}$

- Q being the amount of charge that can be stored in a capacitor of capacitance 'C' against a potential difference of 'v' volts.

i.e., $i = C\frac{dv}{dt}$ $\left[i = \frac{dq}{dt} \right]$

where, v is the voltage across capacitor
i is the current through it

$$dv = \frac{1}{C} i \, dt$$

$$\int_0^t dv = \frac{1}{C} \int_0^t i \, dt$$

$$v(t) - v(0) = \frac{1}{C} \int_0^t i \, dt$$

$$v(t) = \frac{1}{C} \int_0^t i \, dt + v(0)$$

$v(t)$ – final voltage of capacitor
$v(0)$ – Initial voltage of capacitor

– Power absorbed by the capacitor is given by

$$P = Vi$$

$$P = V C \frac{dv}{dt}$$

– Energy stored by the capacitor is

$$W = \int_0^t P \, dt$$

$$W = \int_0^t V C \frac{dv}{dt} \, dt = C \left[\frac{V^2}{2} \right]_0^t$$

$$W = \frac{1}{2} C V^2$$

From the above discussion we can conclude

1. The current in a capacitor is zero if the voltage across it is constant that means the capacitor acts as an open circuit to DC.

2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly.

③ The capacitor can store a finite amount of energy even if the current through it is zero.

④ A pure capacitor never dissipates Energy, but only stores it that is why it is called Non-dissipative passive element.

Problems:

If $C = 2\mu F$ charging volts $1000V$. Calculate stored energy in Joules.

Sol:
$C = 2\mu F$

$V = 1000V$

$$W = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 2 \times 10^{-6} \times (1000)^2$$

$$W = \frac{1}{\cancel{2}} \times \cancel{2} \times 10^{-6} \times 1000 \times 1000$$

$$W = \frac{1000 \times 1000}{1000 \times 1000}$$

$$W = 1J$$

Table:

Exhibits the Voltage current relations of the three circuit elements R, L and C.

Voltage - Current Relationship of Circuit Elements

Circuit element	Voltage (volts)	Current (amps)	Power (watts)
R (Ω)	$V = Ri$	$i = \frac{V}{R}$	$P = i^2R$
L (H)	$V = L\frac{di}{dt}$	$i = \frac{1}{L}\int V\,dt + i_0$ i_0 being the initial current	$P = Li\frac{di}{dt}$
C (F)	$V = \frac{1}{C}\int i\,dt + V_0$ V_0 being the initial voltage	$i = C\frac{dV}{dt}$	$P = CV\frac{dV}{dt}$

Since, At steady state 'i' in the inductor and V in capacitor are zero hence Energy consumed (W) is zero for both inductor and capacitor in steady state.

* Resistance property – Opposes the flow of current.
* Inductance – stores energy in the form of magnetic field.
* Capacitance – stores Energy in the form of electric field.

ENERGY SOURCES :

The resistors, inductors and capacitors are incapable of generating voltage, current (or) power of their own.

So, we need other network components capable of generating voltage (or) current and transferring energy into the network. Such components are called, sources.

Based on this there are two principal types of sources, namely voltage source and current source.

Sources can be either independent (or) dependent upon some other quantities.

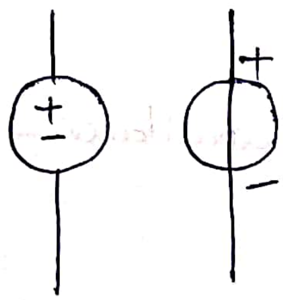
Independent Sources:

An independent voltage sources maintains a voltage (fixed . (or) Varying with time), which is not affected by any other quantity.

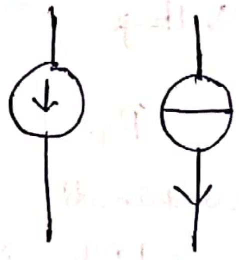
An independent current source maintains a current (fixed (or) time Varying) which is not affected by any other quantity.

Symbols used for independent sources are

Independent voltage source :



Independent Current Source



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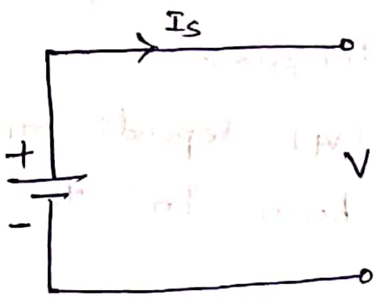
According to this, there are two independent Sources:

i, Ideal Voltage Source | practical voltage Source

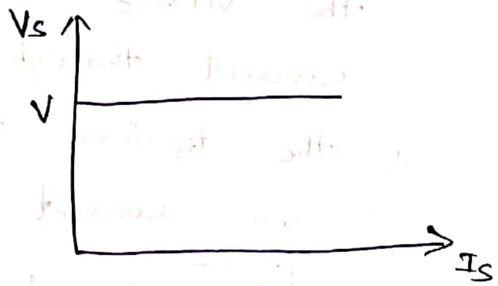
ii, Ideal Current source / practical Current source

(i) Ideal Voltage Source :

An ideal voltage Source is a two terminal element in which the voltage V_s is completely independent of the current I_s, through its terminals.



(a)



(b)

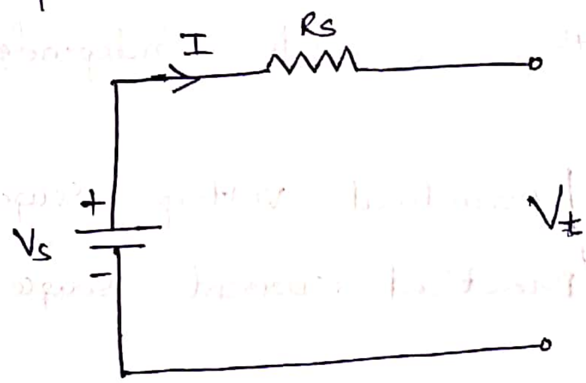
Fig (a) shows the representation of ideal voltage Source

Fig (b) Resembles the V-i characteristics of an ideal voltage.

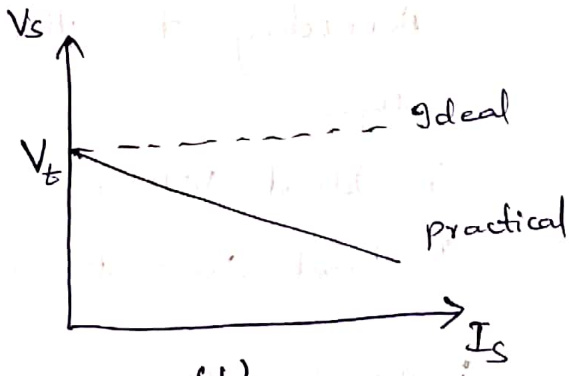
From the graph it is clear that at any time the value of the terminal voltage V_s is constant with respective current value I_s.

Practical Voltage Source:

Practical voltage Source considered as a series combination of ideal Voltage Source and a resistance which is termed as a Source Resistance (R_s).



(a)



(b).

Fig (a) represents the practical voltage source Fig (b) from the graph it is clear that the voltage across the terminals falls as the Current through it increases.

So, the terminal Voltage (V_t) depends on the Source Current as shown in the graph

$$\boxed{V_t = V_s - i_s R}$$

ii Ideal Current Source:

An ideal current source is a two terminal element in which the current I_s is completely independent of the voltage V_s across its terminals.

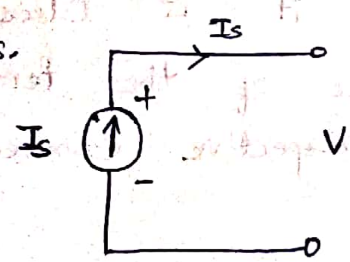
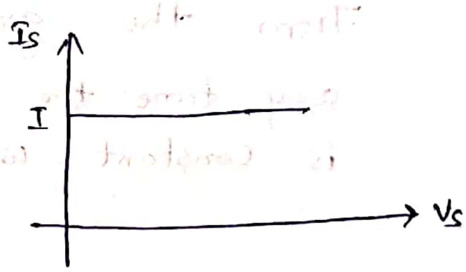


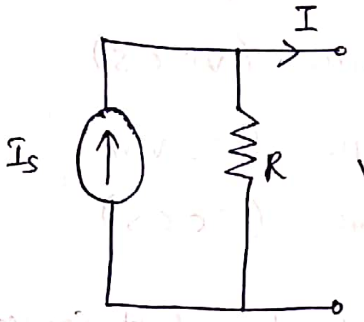
Fig (a)



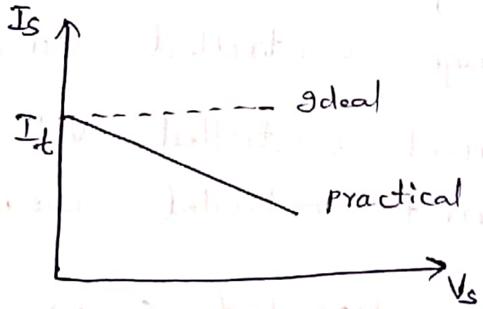
From the fig (b) it is clear that at any time the value of the current I_s is constant with respective to the voltage V_s across it.

Practical Vs Current Source:

Practical current source considered as a parallel combination of ideal current source and a resistance, which is termed as source Resistance.



(a)



(b)

Fig (a) represents the practical current source.

Fig (b) From the graph it is clear that there will be a decrease in current delivered by the source due to the current drawn by the parallel source resistance.

$$\boxed{i_t = i_s - V_s/R}$$

- If the value of the source resistance is infinite the ideal condition is achieved in the current source,
- Hence the source resistance (or) internal resistance of an ideal current source is infinity.

Dependent Sources:

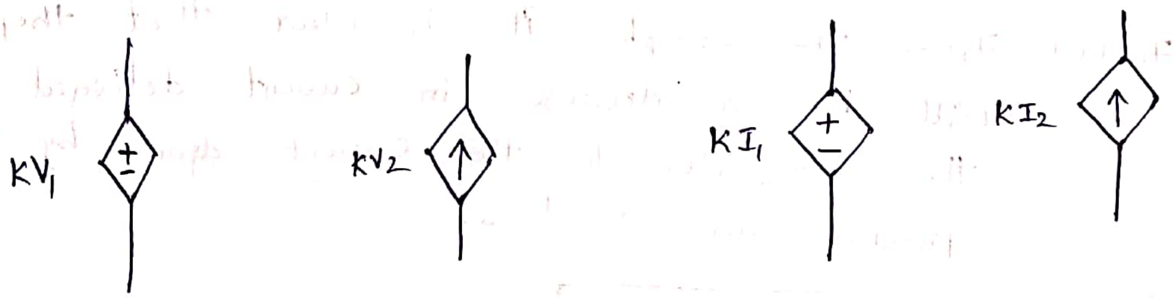
Dependent Sources are those whose values of Sources depends on voltage (or) current in the circuit. They are indicated (or) represented by symbol of diamond shaped. (◇)

Dependent sources are also called Controlled Sources and are of four (4) types:

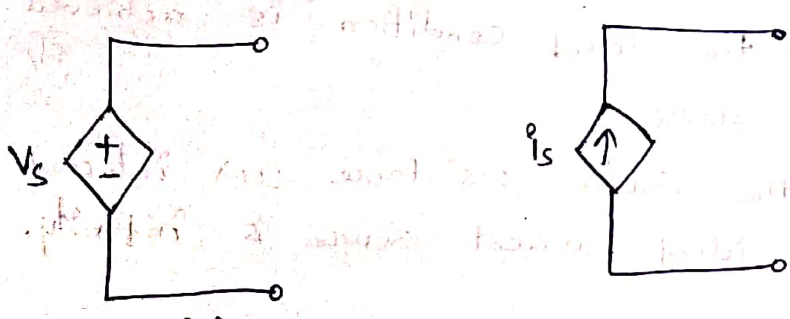
- Voltage Controlled Voltage Source (VCVS)
- Voltage Controlled Current Source (VCCS)
- Current controlled voltage Source (CCVS)
- Current Controlled Current Source (CCCS)

Voltage dependent Sources | Current dependent Sources

i, VCVS ii, VCCS i, CCVS ii, CCCS



→ All four dependent Sources are represented as shown.



(a)

Problems:

A 10Ω resistor is connected across a 12V battery. How much current flows through the resistor.

Sol:

$R = 10\Omega$

$V = 12V$

$I = ?$

$V = IR$

$I = \frac{V}{R} = \frac{12}{10} = 1.2A$

- A 100Ω resistance is directly switched ON across a 10V battery. What is the current through the resistor. How much is the power loss. Also find the energy consumed in 5 sec.

Sol: From given data:

$V = 10V$

$R = 100\Omega$

$I = ?$

$P = ?$

$E = ?$

$I = \frac{V}{R} = \frac{10}{100} = \frac{1}{10} = 0.1A$

Power loss : $I^2R = (0.1)^2 \times 100 = 1W$

[Also power loss : $\frac{V^2}{R} = \frac{10^2}{100} = \frac{100}{100} = 1W$]

Energy consumed : I^2Rt

$= 1^2 \times 5$

$= 1 \times 5 = 5$ joules

- The current in a 2H inductor varies at a rate of 2 A/s. Find the voltage across the inductor and the energy stored in the magnetic field after 2s.

Sol:

$L = 2H$

$di = 2 A/s$

$dt = 2s$

$V = ?$

$W = ?$

$V = L \frac{di}{dt}$

$= 2 \times 4$

$= 8V.$

$W = \frac{1}{2} L i^2 = \frac{1}{2} \times 2 \times (4)^2 = \frac{1}{2} \times 2 \times 16 = 16 J$

- Find the inductance of a coil through which flows a current of 0.2 A with an energy of 0.15 J.

Sol:

$W = 0.15 J$

$I = 0.2 A$

$L = ?$

$W = \frac{1}{2} L I^2$

$0.15 = \frac{1}{2} \times L \times (0.2)^2$

$L = \frac{2 \times 0.15}{(0.2)^2}$

$L = 7.5 H$

- The strength of current in 1 Henry inductor changes at a rate of 1 A/sec. Find the voltage across it and determine the magnitude of energy stored in the inductor after 2 secs.

Sol: $L = 1H$

$\frac{di}{dt} = 1 A/sec$

$V = L\frac{di}{dt} = 1 \times 1 = 1V$

Energy stored is given as $W = \frac{1}{2}LI^2$

$= \frac{1}{2} \times 1 \times 2^2$

$= \frac{1}{2} \times 4$

$\boxed{W = 2\ J}$

- A capacitor has a capacitance of 5μF. Calculate the stored Energy in it if a D.C voltage of 100V is applied across it.

Sol: When the capacitor will be charged full, the voltage across it would be 100V

Then $V = 100V$ $C = 5 \times 10^{-6} F$

$W = \frac{1}{2}CV^2$

$= \frac{1}{\cancel{2}} \times 5 \times 10^{-6} \times \cancel{100}^{50} \times 100$

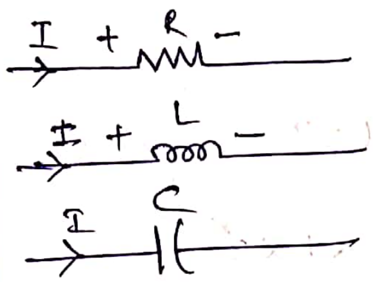
$= 250 \times 10^{-6} \times 100$

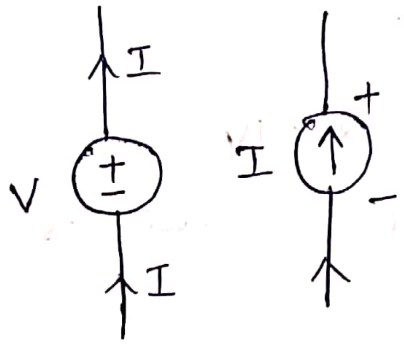
$\boxed{W = 2.5 \times 10^{-2} \text{ joules}}$

* PRESENCE OF SOURCES:

All the three passive Lumped Elements R, L, C will always absorb Energy.

Whenever they are absorbing Energy, current always Enters at the positive terminal of R, L, C Elements.





* ABSENCE OF SOURCES:

Energy storage Elements which are inductor & capacitor will deliver the Energy to the memory less element, which is "Resistor".

Then current enters at the negative (−ve) terminal of L & C where as in resistor always current Enters the positive terminal. Since it always dissipates Energy.

KIRCHOFF'S LAWS:

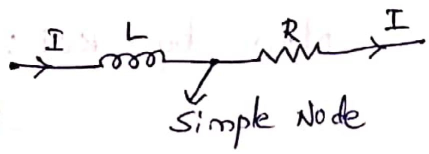
According to German Physicist in 1847 Kirchoff formulated two fundamental laws of Electricity.

For Network Simplification this laws are considered at simplification point of view.

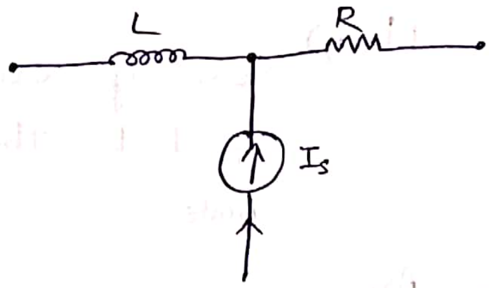
Kirchoff has formulated two Laws, they are Kirchoff's Current Law (KCL) and Kirchoff's voltage Law (KVL)

- **Kirchoff's Current Law:**
  - This law always defines at a Node.

Simple Node: Interconnection of two elements



Principle Node: Interconnection of atleast three Elements.



Statement:

It states that the algebraic sum of the currents meeting at a node (or) junction (or) point is Equal to zero.

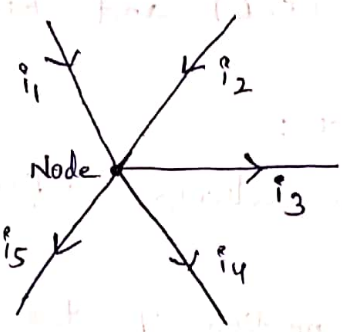
[OR]

It states that the sum of the currents Entering at a node is Equal to the sum of the currents leaving that node.

[OR]

In a Electric circuit for any of its nodes at a time 't', the algebraic sum of the currents at any node of a circuit is Equal to zero.

- This law also called as Kirchoff's point rule (or) Kirchoff's junction rule.



Entering -ve

Leaving +ve

Now by KCL:

$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$$i_1 + i_2 = i_3 + i_4 + i_5$$

(i.e.,) Sum of currents entering a node must be equal to the sum of the current leaving a node

As,

$$i = \frac{dq}{dt}$$

$$\Rightarrow \frac{dq_1}{dt} + \frac{dq_2}{dt} = \frac{dq_3}{dt} + \frac{dq_4}{dt} + \frac{dq_5}{dt}$$

At a given time 't' $\frac{d}{dt}$ is same for all branches

$$\Rightarrow q_1 + q_2 = q_3 + q_4 + q_5$$

(i.e.,) Sum of Entering Charges is Equal to sum of leaving charges.

Properties:

It is applicable for Linear, Non-linear, Active, passive, unilateral, Bilateral, Lumped, Time Variant & Time Invariant Elements.

(i.e.) KCL is independent of the nature of the Elements connected to the node.

KCL - Expresses "Conservation of Charge".

Kirchoff's Voltage Law:

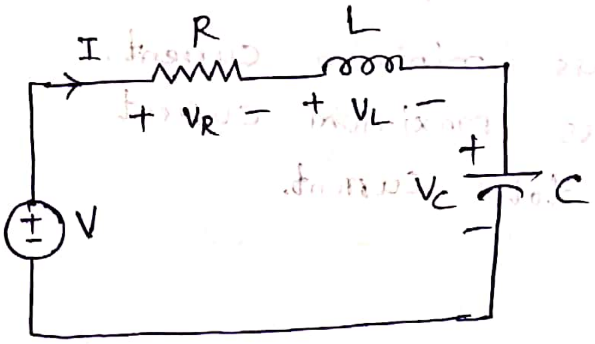
This law defines in a loop (or) mesh
ie, in a closed path.

Statement:

It states that "the algebraic sum of the potential differences around a circuit must be zero".

[OR]

In a electric circuit for any of its loops and at any time 't' the algebraic sum of branch voltages around the loop is zero.



By KVL:

$$-V + V_R + V_L + V_C = 0$$

$$\boxed{V = V_R + V_L + V_C}$$

since, $V = \frac{dw}{dq}$ and $i = \frac{dq}{dt}$

in series current is same & hence charge flow is same through all the elements

$$\frac{dw}{dq} = \frac{dW_R}{dq} + \frac{dW_L}{dq} + \frac{dW_C}{dq}$$

$$\boxed{W = W_R + W_L + W_C}$$

$\frac{W}{q} = \frac{\text{Energy}}{\text{charge}}$

$\frac{q}{t} = \frac{\text{charge}}{\text{time}}$

Properties:

i, KVL is applicable for Linear, Non-linear, Active passive, Unilateral, Bilateral, Lumped, time Variant, time invariant elements.

ii, KVL is independent of nature of element

iii, KVL expresses "Conservation of Energy". Neither created nor destroyed but convert into other form

Equivalent Circuit with respective to R, L, C:

→ Two elements are said to be in series only when currents through the elements are same and two elements are said to be in parallel only when Voltage across the elements are same.

i, Maximum resistance offers minimum current.

ii, Minimum resistance offers maximum current

iii, Zero resistance offers total current.

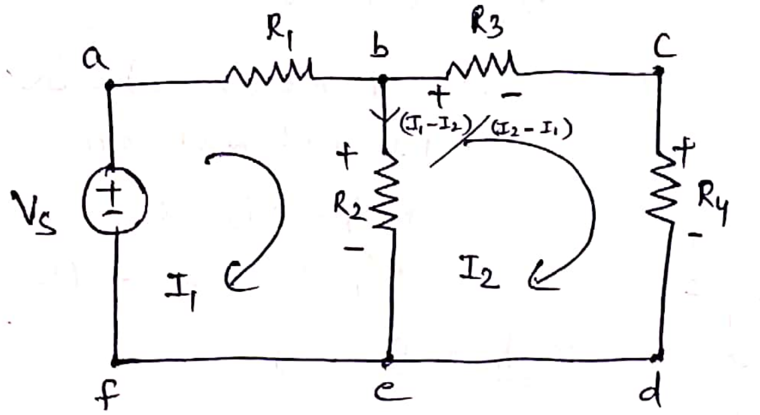
Problems: (KVL + OHM'S LAW:

Mesh Analysis:

A mesh is defined as a Loop which does not contain any other loops within it.

Steps:

1. Identify the Number of Meshes.
2. Assign loop / Mesh currents in the clockwise direction
3. By using KVL first and ohm's law next write the mesh Equations



Mesh | Loop ①: $[I_1 > I_2]$

$$-V_s + I_1 R_1 + (I_1 - I_2) R_2 = 0$$

$$I_1 R_1 + I_1 R_2 - I_2 R_2 = V_s$$

Mesh | Loop ②: $[I_2 > I_1]$

$$R_2 (I_2 - I_1) + I_2 R_3 + I_2 R_4 = 0$$

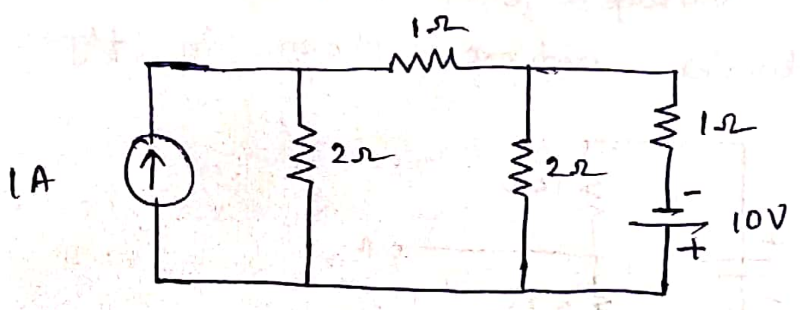
$$I_2 R_2 - I_1 R_2 + I_2 R_3 + I_2 R_4 = 0$$

By rearranging the above Equations, the mesh Equations are

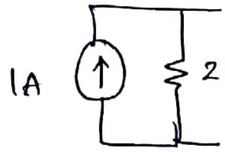
$$I_1 (R_1 + R_2) - I_2 R_2 = V_s$$

$$-I_1 R_2 + I_2 (R_2 + R_3 + R_4) = 0$$

Problem: Using Mesh analysis obtain the current through the 10V battery for the circuit shown in figure.



Sol: The current source 1A is converted into an equivalent voltage source

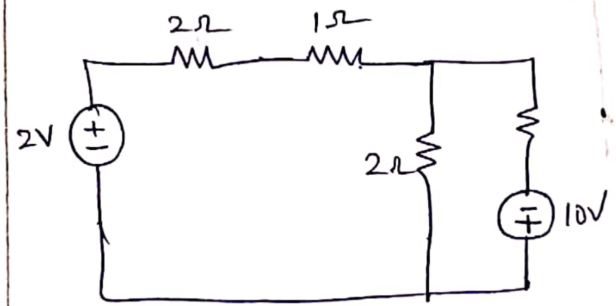


$V = IR$

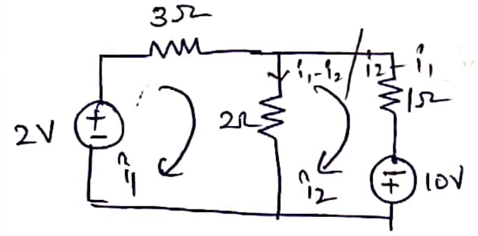
$V = 1 \times 2$

$V = 2V$

$\Downarrow$



$\Downarrow$



Mesh 1: $(i_1 > i_2)$

$$-2 + 3i_1 + 2(i_1 - i_2) = 0$$
$$-2 + 3i_1 + 2i_1 - 2i_2 = 0$$
$$-2 + 5i_1 - 2i_2 = 0$$
$$5i_1 - 2i_2 = 2 \quad \text{— (1)}$$

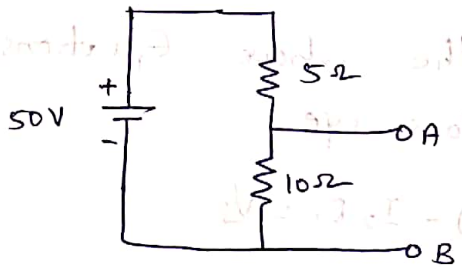
Mesh 2: $(i_2 > i_1)$

$$2(i_2 - i_1) + 1i_2 - 10 = 0$$
$$2i_2 - 2i_1 + i_2 - 10 = 0$$
$$3i_2 - 2i_1 - 10 = 0$$
$$-2i_1 + 3i_2 = 10 \quad \text{— (2)}$$

Solving (1) & (2)

$i_1 = 2.36A$ & $i_2 = 4.91A$

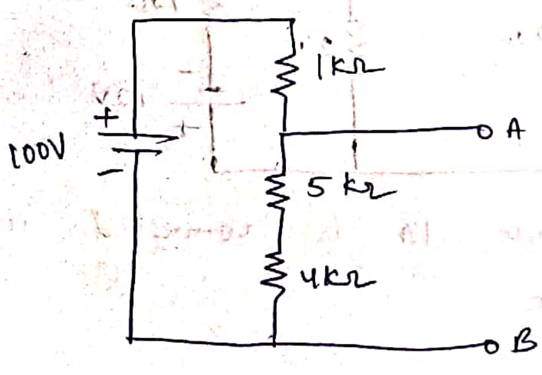
Problm: What is the voltage across 10Ω resistor



Voltage across 10Ω = V_{10}

ie, $50 \times \frac{10}{10+5} = \frac{500}{15} = 33.3V$

Problm: Find the voltage between A and B in a voltage divider network shown in figure



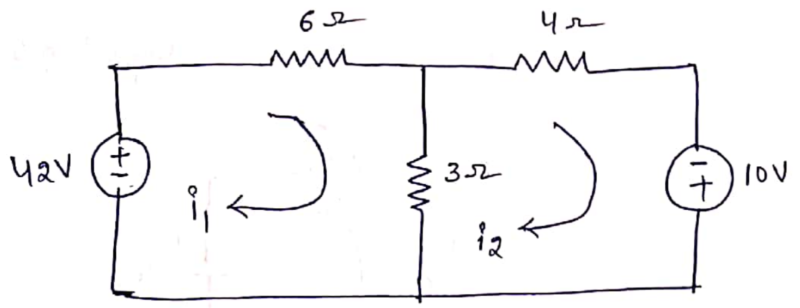
Voltage across 9 kΩ resistor = V_9

$$V_{AB} = 100 \times \frac{9}{10}$$

$$V_{AB} = 90V$$

Problem:

By using technique of mesh analysis determine i_1 and i_2 in the circuit given.



Sol: Mesh 1: $(i_1 > i_2)$

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

$$-42 + 6i_1 + 3i_1 - 3i_2 = 0$$

$$-42 + 9i_1 - 3i_2 = 0$$

$$9i_1 - 3i_2 = 42 \quad \text{—— (1)}$$

Mesh 2: $(i_2 > i_1)$

$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

$$3i_2 - 3i_1 + 4i_2 - 10 = 0$$

$$7i_2 - 3i_1 = 10$$

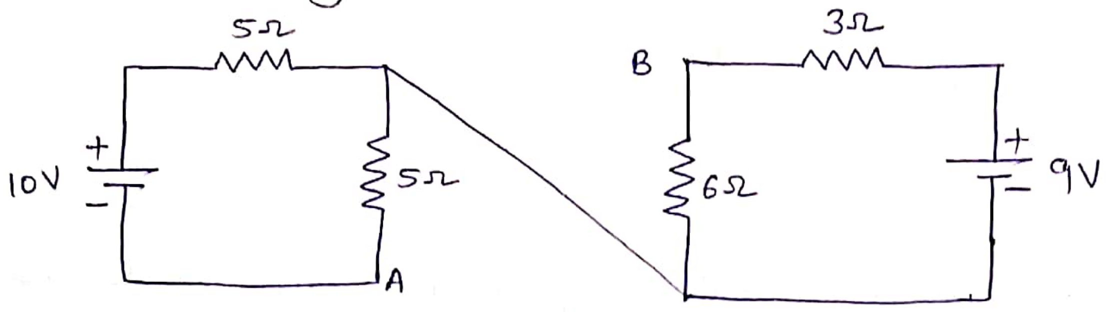
(or)

$$-3i_1 + 7i_2 = 10 \quad \text{—— (2)}$$

By solving Equ. (1) & Equ. (2) $\quad i_1 = 6A$
$i_2 = 4A$

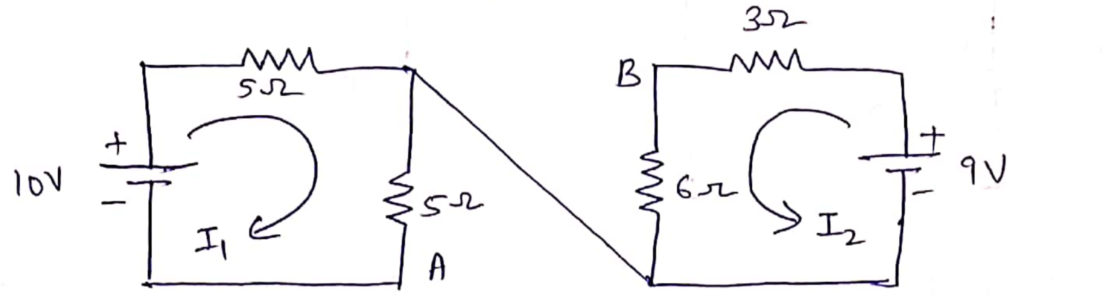
Prblm:

(1) Obtain the potential difference V_{AB} in the given circuit using Kirchoff's laws.



Sol: For the given circuit,

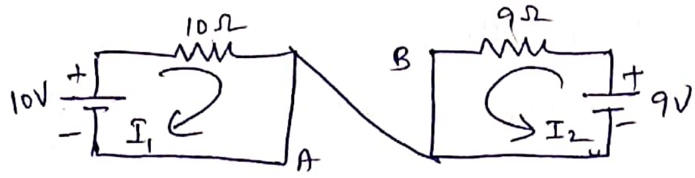
mark the loops (or) current directions



5Ω & 5Ω are in series

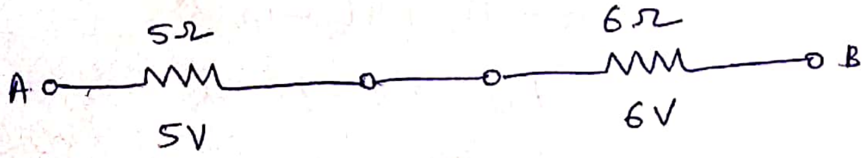
$5\Omega + 5\Omega = 10\Omega$

6Ω & 3Ω are in series



$I_1 = \frac{10}{10} = 1A$ $\qquad$ $I_2 = \frac{9}{9} = 1A$

As same current flowing then the resistors should be in series connection.



$V_{AB} = 5V + 6V$

$\boxed{V_{AB} = 11V}$

NODAL ANALYSIS (KCL + OHM'S LAW)

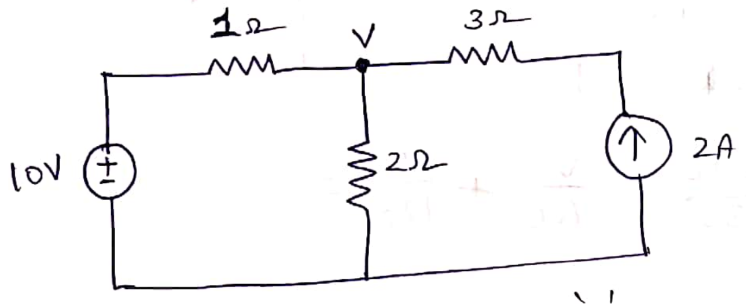
Steps:-

i) Identify number of Nodes

ii) Assign node voltages with respective to the ground node whose voltage is always equal to zero.

iii) By using KCL first and ohm's law next write the Nodal equations.

Prblm: Find the power in 2Ω resistor.



Sol: By applying KCL at 'V'

$$\frac{V-10}{1} + \frac{V}{2} = 2$$

$$2(V-10) + V = 2 \times 2$$

$$2V - 20 + V = 4$$

$$3V - 20 = 4$$

$$3V = 4 + 20$$

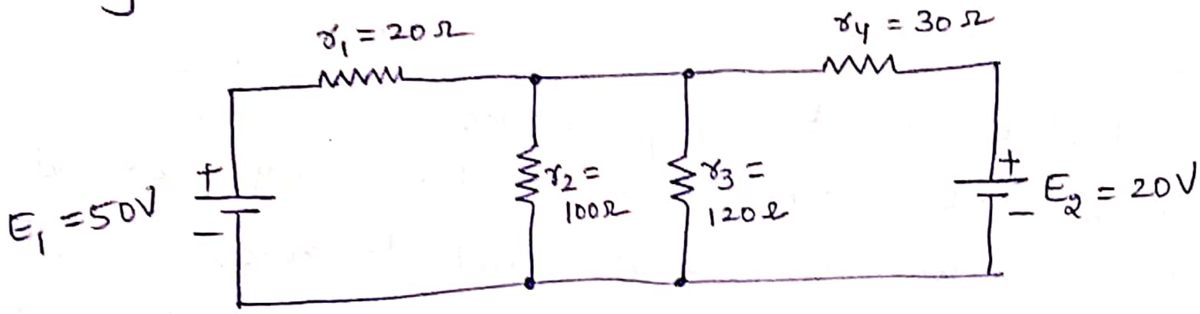
$$3V = 24$$

$$V = 24/3$$

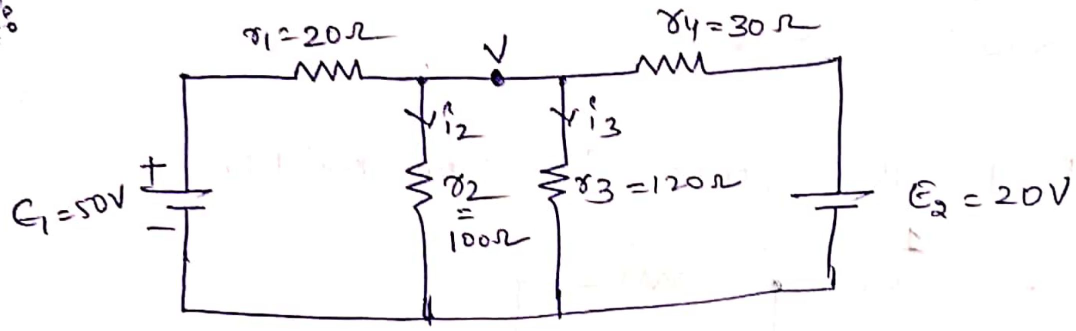
$$V = 8V$$

$$\therefore P_{2\Omega} = \frac{V^2}{R} = \frac{8^2}{2} = \frac{64}{2} = 32 \text{ Watts}$$

(P) Using Nodal method, find the current through R_2.



Sol:



$$\frac{V-50}{20} + \frac{V-20}{30} + \frac{V}{100} + \frac{V}{120} = 0$$

$$\frac{20(V-20) + (V-50)30 + 6(V) + 5(V)}{600} = 0$$

$$-400 + 20V + 30V - 1500 + 6V + 5V = 0$$

$$-1900 + 50V + 11V = 0$$

$$-1900 + 61V = 0$$

$$61V = 1900$$

$$V = 1900/61$$

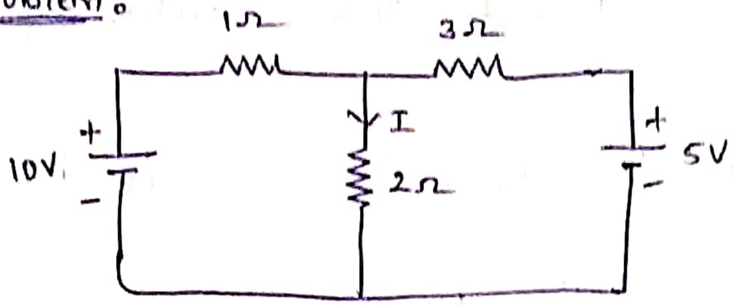
$$V = 31.1\,V$$

As current through r_2 resistor

is as $i_2 = \frac{V}{r_2} = \frac{31.1}{100}$

$$\boxed{i_2 = 0.3118A}$$

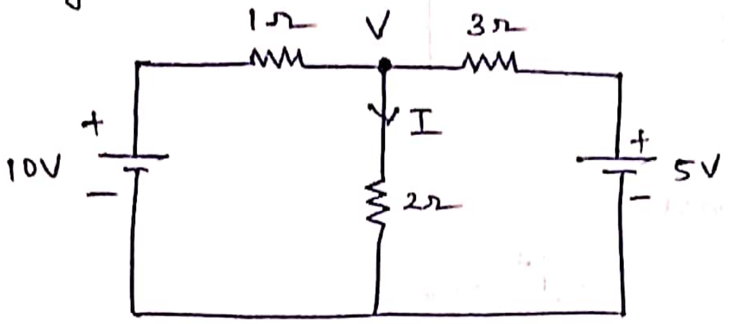
Problem:



Find the current 'I' through 2Ω resistor.

Sol:

By KCL:



$$\frac{V-10}{1} + \frac{V-0}{2} + \frac{V-5}{3} = 0$$

$$\frac{V-10}{1} + \frac{V}{2} + \frac{V-5}{3} = 0$$

$$(V-10)(2)(3) + V(1)(3) + (V-5)(1)(2) = 0$$

$$(V-10)(6) + 3V + (V-5)(2) = 0$$

$$6V - 60 + 3V + 2V - 10 = 0$$

$$11V - 60 - 10 = 0$$

$$11V - 70 = 0$$

$$11V = 70$$

$$V = 70/11$$

Current through 2Ω resistor

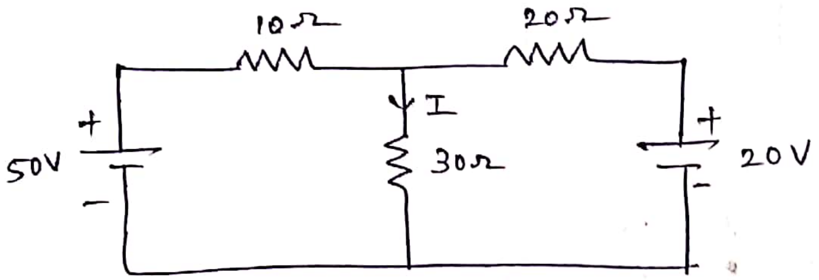
$$I_{(2\Omega)} = \frac{V}{2} = \frac{70/11}{2} = \frac{70}{11} \times \frac{1}{2} = \frac{70}{22} = 35/11$$

$$\boxed{I_{(2\Omega)} = 35/11}$$

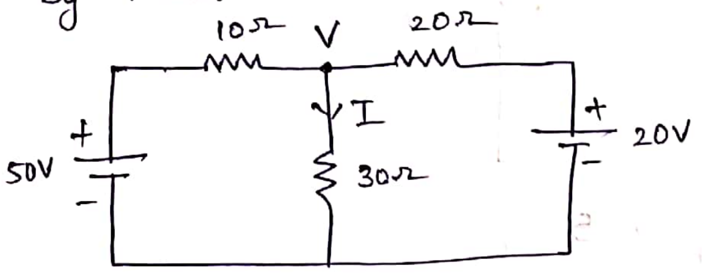
Problem:

~~Find the current "I" through 30Ω resistor. by using super positio~~

Find the current 'I' through 30Ω resistor.



Sol: By KCL:



$$\frac{V-50}{10} + \frac{V-0}{30} + \frac{V-20}{20} = 0$$

$$\frac{V-50}{10} + \frac{V}{30} + \frac{V-20}{20} = 0.$$

$$(V-50)(30)(20) + V(10)(20) + (V-20)(10)(30) = 0$$

$$(V-50)(600) + V(200) + (V-20)(300) = 0$$

$$600V - 30{,}000 + 200V + 300V - 6000 = 0$$

$$600V + 200V + 300V - 36{,}000 = 0$$

$$1100V = 36{,}000$$

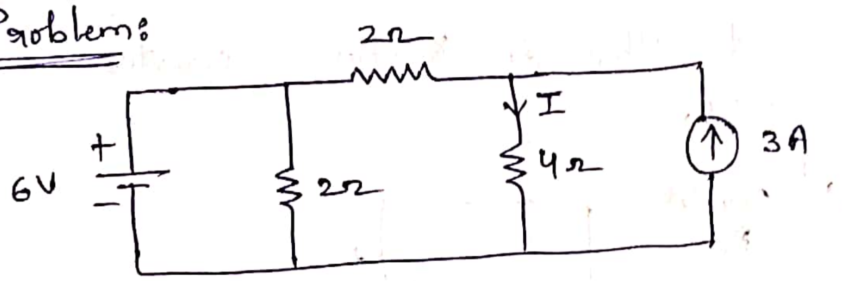
$$V = 36{,}000/1100$$

$$V = 32.7\ V$$

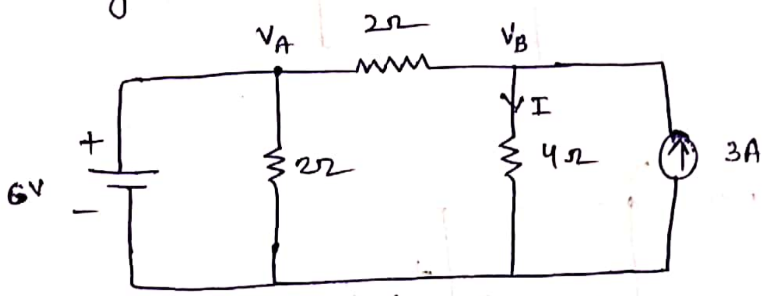
Current through 30Ω resistor $I_{(30\Omega)} = \frac{V}{R} = \frac{32.7}{30}$

$$\boxed{I_{(30\Omega)} = 1.09A}$$

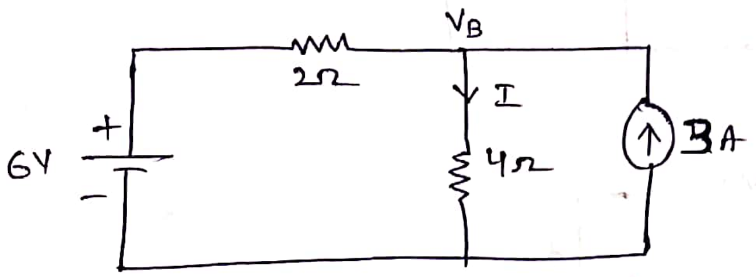
Problem:



Sol: By KCL:



⇓



$$\frac{V_B - 6}{2} + \frac{V_B - 0}{4} = 3$$

$$\frac{(V_B - 6)}{2} + \frac{V_B - 0}{4} = 3$$

$$\frac{(V_B - 6)}{2} + \frac{V_B}{4} = 3$$

$$\frac{(V_B - 6)(2) + V_B(1)}{4} = 3$$

$$2V_B - 12 + V_B = 12$$

$$2V_B + V_B = 12 + 12$$

$$3V_B = 24$$

$$V_B = \frac{24}{3}$$

$$\boxed{V_B = 8\,V}$$

Current through 4Ω resistor

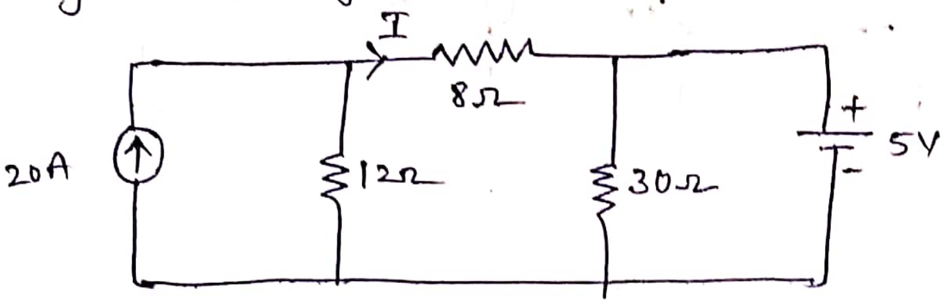
$$I_{(4\Omega)} = V/R$$

$$= \frac{8}{4}$$

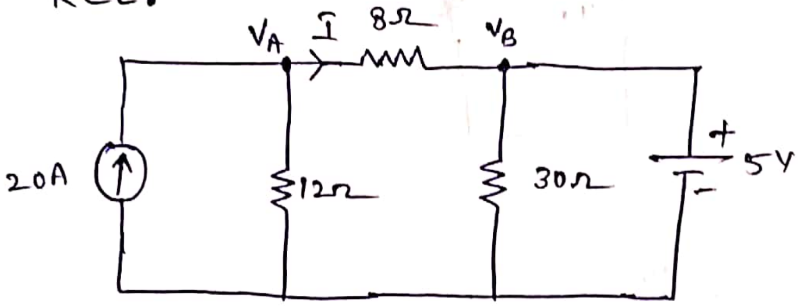
$$\boxed{I_{(4\Omega)} = 2\Omega}$$

Problem:

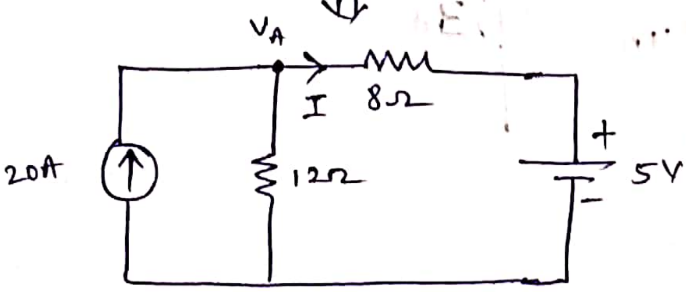
Find the current through 8 ohms resistor by concept of KCL.



sol: KCL:



⇓



$$20 = \frac{V_A - 0}{12} + \frac{V_A - 5}{8}$$

$$20 = \frac{V_A}{12} + \frac{V_A - 5}{8}$$

$$20 = \frac{2V_A + 3(V_A - 5)}{24}$$

$$20 \times 24 = 2V_A + 3V_A - 15$$

$$480 = 5V_A - 15$$

$$480 + 15 = 5V_A$$

$$495 = 5V_A$$

$$V_A = 495/5$$

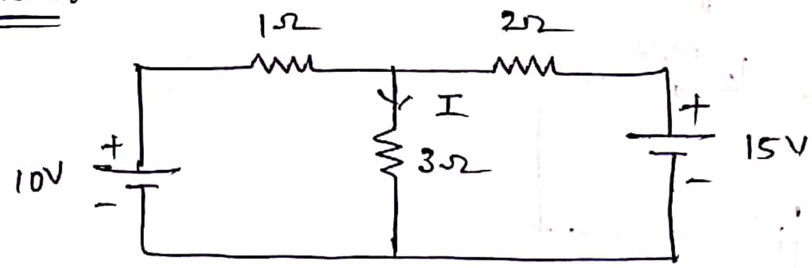
$$\boxed{V_A = 99\ V}$$

Current through 8Ω resistor,

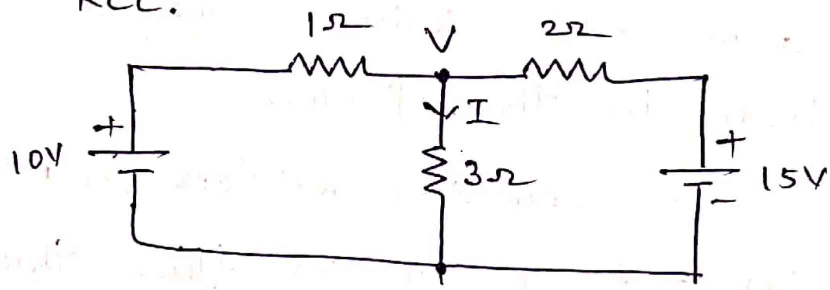
$$I_{(8\Omega)} = \frac{V}{R} = \frac{99}{8}$$

$$\boxed{(I_{8\Omega}) = 12.3\ A}$$

Problem:



Sol: KCL:



$$\frac{V-10}{1} + \frac{V-0}{3} + \frac{V-15}{2} = 0$$

$$\frac{V-10}{1} + \frac{V}{3} + \frac{V-15}{2} = 0$$

$$(V-10)(3)(2) + V(1)(2) + (V-15)(1)(3) = 0$$

$$(V-10)(6) + 2V + (V-15)(3) = 0$$

$$6V - 60 + 2V + 3V - 45 = 0$$

$$11V - 60 - 45 = 0$$

$$11V - 105 = 0$$

$$11V = 105$$

$$V = \frac{105}{11}$$

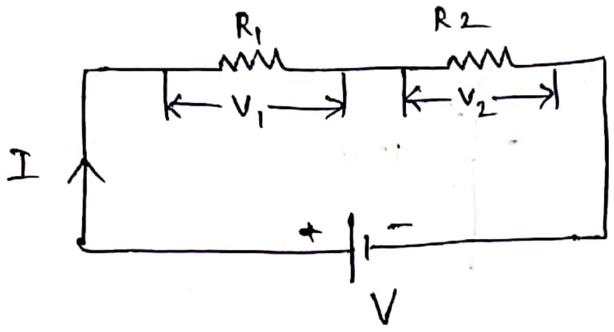
$$\boxed{V = 9.54\ V}$$

Current through 3Ω resistor,

$$I_{(3\Omega)} = \frac{V}{R} = \frac{9.54}{3} = 3.18\ A$$

$$\boxed{I_{(3\Omega)} = 3.18\ A}$$

VOLTAGE DIVISION RULE:



Voltage applied across the series connected elements as shown in the figure..,

- Circuit has two series connected resistors (R_1 & R_2)
- Voltage applied (V) and current (I) flows through the elements.
- As the elements are in series connected, the current flowing through the elements will be same, but the voltage gets divided among the elements.

Now,

By KVL

$$V = V_1 + V_2$$

AS $V_1 = I R_1$ (Voltage across resistor R_1).

$V_2 = I R_2$ (Voltage across resistor R_2)

$$V = I R_1 + I R_2$$

$$V = I (R_1 + R_2)$$

$$I = V / (R_1 + R_2)$$

$$V_1 = V \times \frac{R_1}{R_1 + R_2} \quad ; \quad V_2 = V \times \frac{R_2}{R_1 + R_2}$$

Voltage across any branch =

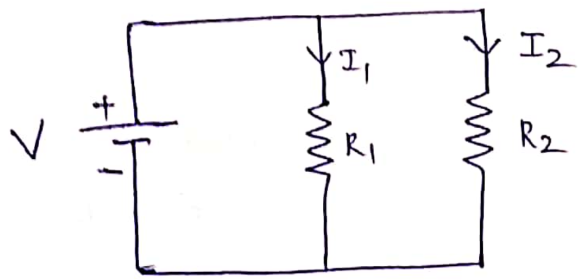
$$\boxed{\text{Total voltage applied} \times \frac{\text{Same Branch Resistance}}{\text{Sum of the resistances in series}}}$$

If 'n' number of resistors are connected in Series.

then,

$$V_n = V \frac{R_n}{R_1 + R_2 + R_3 + \cdots + R_n}$$

CURRENT DIVISION RULE:



Voltage applied across the parallel Connected elements as shown in figure.

- Circuit has two resistors (R_1 & R_2) Connected in parallel.
- Applied Voltage (V) and Current (I) flows through the Elements.
- As the Elements (Resistances) are in parallel, the voltage flowing across the elements will be Same and the current gets divided in Each branch Element.

By KCL,

$$I = I_1 + I_2$$

$$[\text{As } V = I R_{eq}]$$

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\{\because V = V_1 = V_2\}$$

$$V = I \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$\left\{\begin{array}{l} V_1 = I_1 R_1 \\ V_2 = I_2 R_2 \end{array}\right\}$$

$$I_1 \cancel{R_1} = I \cdot \frac{\cancel{R_1} R_2}{R_1 + R_2}$$

$$\boxed{I_1 = I \cdot \frac{R_2}{R_1 + R_2}}$$

Similarly,

$$V_2 = I_2 R_2$$

$$I_2 \cancel{R_2} = I \cdot \frac{R_1 \times \cancel{R_2}}{R_1 + R_2}$$

$$\boxed{I_2 = I \cdot \frac{R_1}{R_1 + R_2}}$$

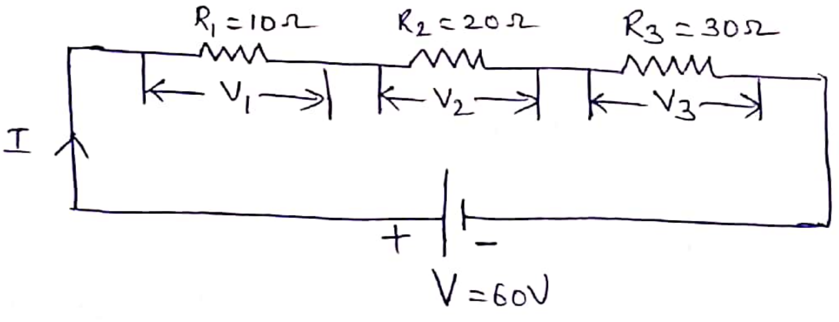
Current in any branch =

$$\boxed{\text{Total current} \times \frac{\text{Opposite branch resistance}}{\text{Sum of the resistances in parallel.}}}$$

CamScanner

Problems on Voltage and current division Rule:

→ Find the voltage across the three resistances given in the circuit.



Sol: In the circuit, as the resistances are connected in series, the current flowing through the branches will be same.

- The applied voltage gets divided across each branch element.

Now,
$$V = V_1 + V_2 + V_3$$

$V = 60V$; $R_1 = 10\Omega$; $R_2 = 20\Omega$; $R_3 = 30\Omega$

Voltage across the branch element (R_1)

$$V_1 = V \times \frac{R_1}{R_1 + R_2 + R_3}$$

$$V_1 = 60 \times \frac{10}{10+20+30}$$

$$V_1 = \cancel{60} \times \frac{10}{\cancel{60}}$$

$$\boxed{V_1 = 10V}$$

Voltage across the branch element (R_2):

$$V_2 = V \times \frac{R_2}{R_1+R_2+R_3}$$

$$V_2 = 60 \times \frac{20}{10+20+30}$$

$$V_2 = \cancel{60} \times \frac{20}{\cancel{60}}$$

$$\boxed{V_2 = 20\,V}$$

Voltage across the branch element (R_3):

$$V_3 = V \times \frac{R_3}{R_1+R_2+R_3}$$

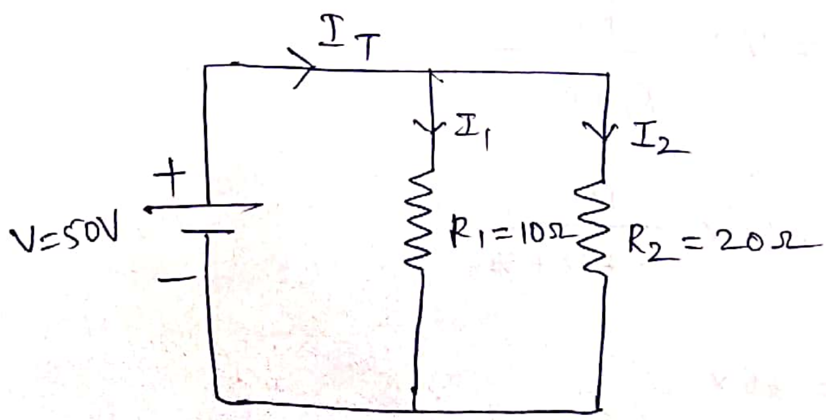
$$V_3 = 60 \times \frac{30}{10+20+30}$$

$$V_3 = \cancel{60} \times \frac{30}{\cancel{60}}$$

$$\boxed{V_3 = 30V}$$

→ Find the total current and branch currents through the resistors (R_1 & R_2), if $R_1 = 10\Omega$; $R_2 = 20\Omega$ & $V = 50V$

Sol:



Sol: In the circuit, as the resistances are connected in parallel, the current flowing through the branches will get divided.

- The applied voltage g will be same across the branch element.

Now, $I = I_1 + I_2$

$V = 50V$; $R_1 = 10\Omega$; $R_2 = 20\Omega$

Current through each branch element is as:

$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{10 \times 20}{10 + 20}$$

$$R_{eq} = 6.67\,\Omega$$

$$I_T = I = \frac{V}{R_{eq}} = \frac{50}{6.67} = 7.5A$$

$$\boxed{I_T = 7.5\,A}$$

Current through branch element (R_1):

$$I_1 = I_T \times \frac{R_2}{R_1 + R_2}$$

$$= 7.5 \times \frac{20}{10+20}$$

$$= 7.5 \times \frac{2\cancel{0}}{3\cancel{0}}$$

$$= \frac{7.5 \times 2}{3}$$

$$= \frac{\overset{5}{\cancel{75}} \times \cancel{2}}{\underset{15}{\cancel{30}}}$$

$$\boxed{I_1 = 5A}$$

Current through branch element (R_2):

$$I_2 = I_T \times \frac{R_1}{R_1+R_2}$$

$$= 7.5 \times \frac{10}{10+20}$$

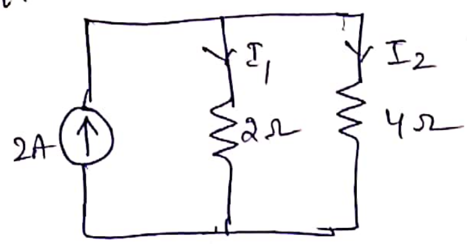
$$= 7.5 \times \frac{1\cancel{0}}{3\cancel{0}}$$

$$= \frac{7.5}{3}$$

$$= 75/30$$

$$\boxed{I_2 = 2.5\ A}$$

→ Find the ~~current~~ and branch currents from the given circuit.



Sol:

$R_1 = 2\Omega$

$R_2 = 4\Omega$

$I_T = 2A$

Current through branch element R_1:

$$I_1 = I_T \times \frac{R_2}{R_1+R_2}$$

$$= 2 \times \frac{4}{2+4}$$

$$= 2 \times \frac{\cancel{4}^{2}}{\cancel{6}_{3}}$$

$$= \frac{4}{3}$$

$$\boxed{I_1 = 4/3\ A}$$

Current through branch Element (R_2):

$$I_2 = I_T \times \frac{R_1}{R_1+R_2}$$

$$= 2 \times \frac{2}{2+4}$$

$$= 2 \times \frac{2}{6}$$

$$\boxed{I_2 = \frac{\cancel{4}2}{\cancel{6}3} A} = \frac{2}{3} A$$

Now,

$$I = I_1 + I_2$$

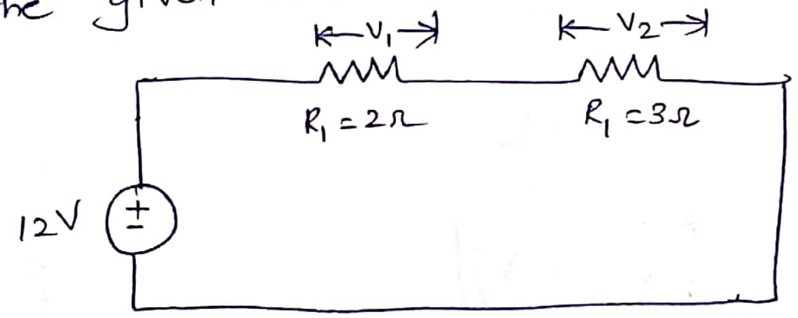
$$= \frac{4}{3} + \frac{2}{3}$$

$$= \frac{4+2}{3}$$

$$= \frac{\cancel{6}2}{\cancel{3}}$$

$$\boxed{I = 2A}$$

→ Find the branch voltages across each element from the given circuit.



Sol:

$V_T = 12V$

$R_1 = 2\Omega$

$R_2 = 3\Omega$

Voltage across branch element (R_1):

$$V_1 = V \times \frac{R_1}{R_1+R_2}$$

$$= 12 \times \frac{2}{2+3}$$

$$= 12 \times \frac{2}{5}$$

$$\boxed{V_1 = \frac{24}{5} V}$$

Voltage across branch Element (R_2):

$$V_2 = V \times \frac{R_2}{R_1 + R_2}$$

$$= 12 \times \frac{3}{2+3}$$

$$= 12 \times \frac{3}{5}$$

$$\boxed{V_2 = \frac{36}{5} V}$$

As $V_T = V_1 + V_2$

$$= \frac{24}{5} + \frac{36}{5}$$

$$= \frac{24 + 36}{5}$$

$$= \frac{60}{5}$$

$$\boxed{V = 12 V}$$

NOTE: When the resistors are connected in series, the current will be same across the Elements and voltage gets divided.

- When the resistors are connected in parallel, the current gets divided & voltage will be same.

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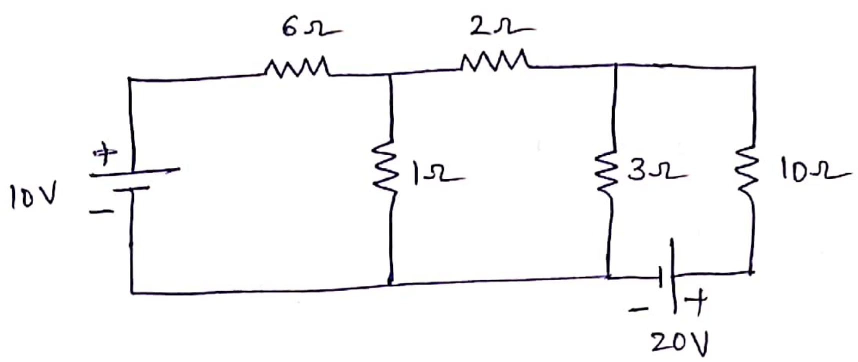
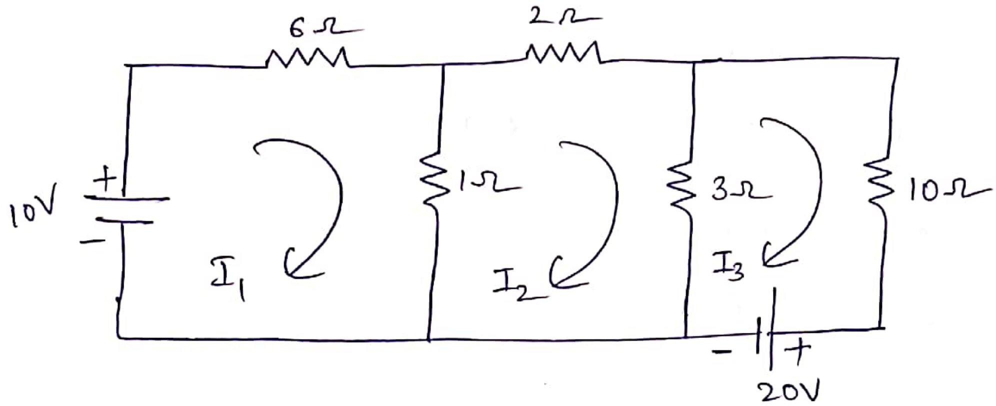
→ Find the mesh currents from the given circuit.



Sol: Assume Loop currents to the given circuit



loop ①: $(I_1 > I_2)$

$$-10 + 6I_1 + 1(I_1 - I_2) = 0$$

$$-10 + 6I_1 + I_1 - I_2 = 0$$

$$-10 + 7I_1 - I_2 = 0$$

$$7I_1 - I_2 = 10 \quad \text{—— ①}$$

loop ②: $(I_2 > I_3)$

$$1(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$$

$$I_2 - I_1 + 2I_2 + 3I_2 - 3I_3 = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \quad \text{—— ②}$$

loop 3:

$$3(I_3 - I_2) + 10 I_3 + 20 = 0$$

$$3I_3 - 3I_2 + 10I_3 + 20 = 0$$

$$-3I_2 + 13I_3 = -20 \quad \text{--- } (3)$$

Solving Equations 1 & 2

We get,

$$7I_1 - I_2 + 0 = 10 \quad \times 6$$

$$-I_1 + 6I_2 - 3I_3 = 0$$

$$42I_1 - \cancel{6I_2} + 0 = 60$$

$$-I_1 + \cancel{6I_2} - 3I_3 = 0$$

$$41I_1 - 3I_3 = 60 \quad \text{--- } (4)$$

Solving 2 & 3

$$-I_1 + 6I_2 - 3I_3 = 0$$

$$0 - 3I_2 + 13I_3 = -20 \quad \times 2$$

$$-I_1 + \cancel{6I_2} - 3I_3 = 0$$

$$0 - \cancel{6I_2} + 26I_3 = -40$$

$$-I_1 + 23I_3 = -40 \quad \text{--- } (5)$$

Now, solving equ. 4 & 5

$$41 I_1 - 3 I_3 = 60$$
$$-I_1 + 23 I_3 = -40 \quad \times 41$$

$$\cancel{41 I_1} - 3 I_3 = 60$$
$$\cancel{-41 I_1} + 943 I_3 = -1640$$

$$940 I_3 = -1580$$
$$I_3 = -\frac{1580}{940}$$
$$\boxed{I_3 = -1.68 A}$$

Substitute the value of i_3 in Equ. (4)

$$41 I_1 - 3(-1.68) = 60$$
$$41 I_1 + 5.04 = 60$$
$$41 I_1 = 60 - 5.04$$
$$41 I_1 = 54.96$$
$$I_1 = 54.96/41$$
$$\boxed{I_1 = 1.34 A}$$

To find the value of I_2; substitute the values of I_1 & I_3 in Equ. (2)

$$-I_1 + 6 I_2 - 3 I_3 = 0$$
$$-1.34 + 6 I_2 - 3(-1.68) = 0$$
$$-1.34 + 6 I_2 + 5.04 = 0$$
$$6 I_2 = 1.34 - 5.04$$
$$I_2 = -3.7/6 \Rightarrow \boxed{I_2 = -0.62 A}$$

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CIRCUIT THEOREMS:

Necessity of Theorems:

If a circuit (or) network is complicated consisting of several nodes, loops (or) meshes and if the response in a single element is desired, then theorems are used (or) applicable.

Based on this necessity we have some types of Theorems:

- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem
- Reciprocity Theorem
- Tellegen's Theorem
- Milliman's Theorem
- Compensation Theorem
- Substitution Theorem

⇒ SUPER POSITION THEOREM:

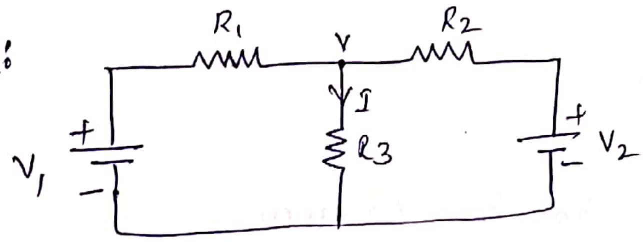
"If a linear network containing more than one independent source and dependent source, the response in a particular branch is equal to the algebraic sum of the responses when individual source acting alone".

- The independent voltage sources are represented by their internal resistances if given (or) simply with zero resistances (i.e., short circuits if internal resistances are not mentioned.

- The independent current sources are represented by infinite resistances (i.e., open circuit)

- This theorem is applicable only for linear elements R, L, C.

Ex:



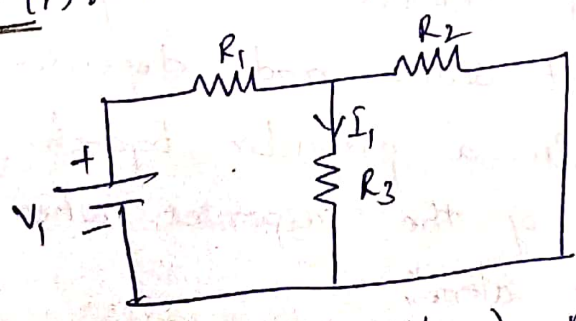
By using Nodal Analysis

$$\frac{V-V_1}{R_1} + \frac{V}{R_3} + \frac{V-V_2}{R_2} = 0$$

$V = \_\_\_\_\_ V$

$I = \frac{V}{R_3} = \_\_\_\_\_ A$

Case (i): When V_1 source acting alone



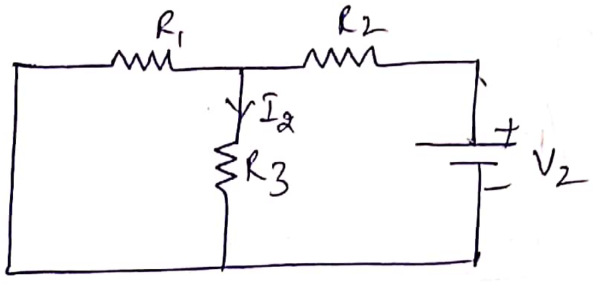
$$R_{eq} = (R_2 \| R_3) + R_1$$

(43)

$$I = \frac{V_1}{R_{eq}}$$

$$I_1 = I \times \frac{R_2}{R_2 + R_3}$$

Case (ii): When V_2 source acting alone



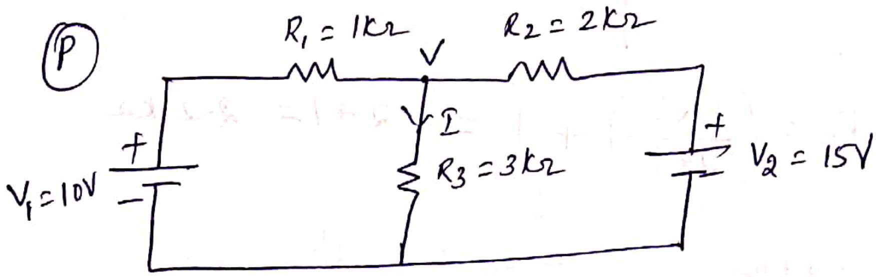
$$R_{eq} = (R_1 // R_3) + R_2$$

$$I = \frac{V_2}{R_{eq}}$$

$$I_2 = I \times \frac{R_1}{R_1 + R_3}$$

$$\boxed{I = I_1 + I_2}$$

(P)



By using Nodal Analysis

$$\frac{V-10}{1} + \frac{V}{3} + \frac{V-15}{2} = 0$$

$$(V-10)(3)(2) + V(1)(2) + (V-15)(1)(3) = 0$$

$$(V-10)(6) + V(2) + (V-15)(3) = 0$$

$$6V - 60 + 2V + 3V - 45 = 0$$

$$6V - 60 + 5V - 45 = 0$$

$$11V - 60 - 45 = 0$$

$$11V - 105 = 0$$

$$11V = 105$$

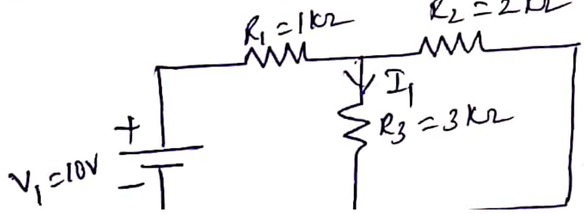
$$V = 105/11$$

$$\boxed{V = 9.54\ V.}$$

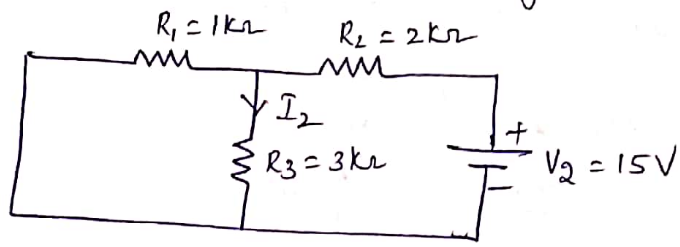
$$I_{(3\Omega)} = \frac{9.54}{3} = 3.18$$

$$\boxed{I_{(3\Omega)} = 3.18\ mA}$$

Case (i) when V_1 source acting alone



Case (ii) when V_2 source acting alone



$$R_{eq} = (R_1 // R_3) + R_2$$

$$= \left(\frac{1 \times 3}{1+3}\right) + 2$$

$$= \frac{3}{4} + 2$$

$$R_{eq} = 2.75 \ k\Omega$$

$$I = \frac{V_2}{R_{eq}} = \frac{15}{2.75} = 5.45 \ A$$

$$I_2 = I \times \frac{R_1}{R_1 + R_3} = 5.45 \times \frac{1}{1+3}$$

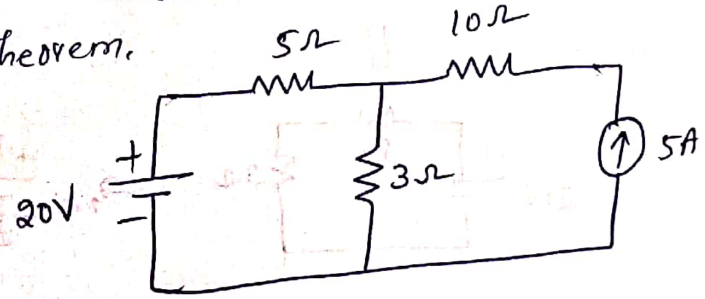
$$I_2 = 5.45/4$$

$$\boxed{I_2 = 1.36 \ mA}$$

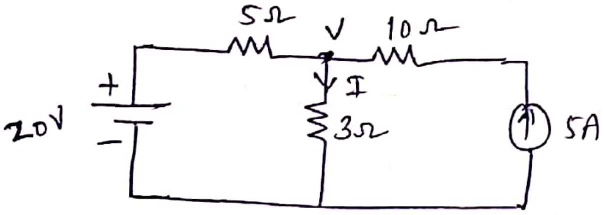
$$I_1 + I_2 = I_T$$

$$\boxed{1.816 + 1.36 = 3.176 \approx 3.18 \ mA}$$

(P) Find the current through 3Ω resistor by using Super position Theorem.



Sol: By applying Nodal Analysis!



$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$3(V-20)+5(V)=15\times 5$$

$$3V-60+5V=75$$

$$8V-60=75$$

$$8V=75+60$$

$$8V=135$$

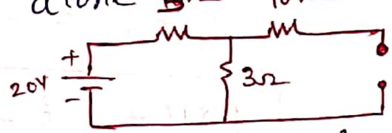
$$V=135/8$$

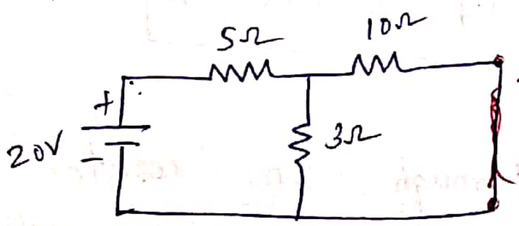
$$\boxed{V=16.875\ V}$$

$$I=\frac{V}{R}=\frac{16.875}{3}=5.625\ A$$

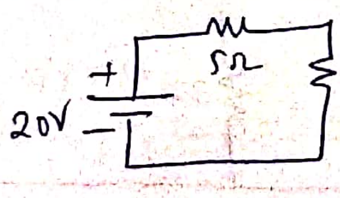
$$\boxed{I=5.625\ A}$$

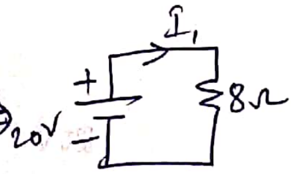
Step i, 20V source acting alone





As 10Ω is in series with 5A current source, (Negligible 10Ω)

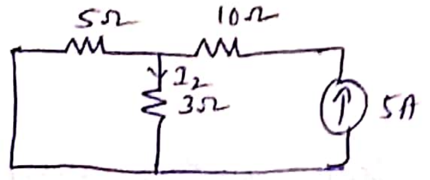




$$\Rightarrow I_1=\frac{20}{8}$$

$$\boxed{I_1=2.5\ A}$$

Step (iii): When 5A source acting alone



$I = 5A$

$$I_2 = I \times \frac{\cancel{10}\,5}{5+3} = 5 \times \frac{5}{8} = \frac{25}{8} = 3.125A$$

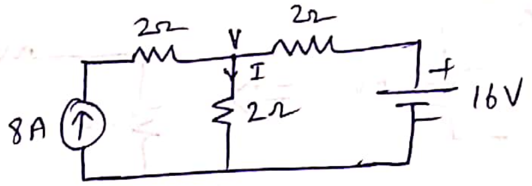
$$\boxed{I_2 = 3.125A}$$

$$I = I_1 + I_2$$

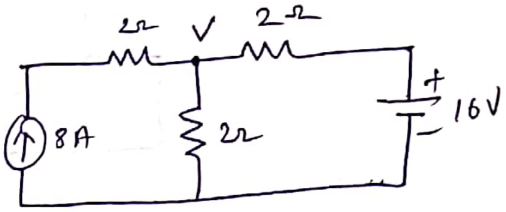
$$= 2.5 + 3.125$$

$$\boxed{I = 5.625\ A}$$

(P) Consider the following circuit, Determine (I).



Sol: By using Nodal



$$8 = \frac{V}{2} + \frac{V-16}{2}$$

$$8 = \frac{V+V-16}{2}$$

$$8 \times 2 = 2V - 16$$

$$16 = 2V - 16$$

$$16 + 16 = 2V$$

$$32 = 2V$$

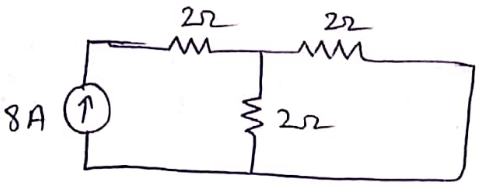
$$\boxed{V = 16V}$$

As 2Ω is in series with current source 8A, Neglecting the resistor (2Ω)

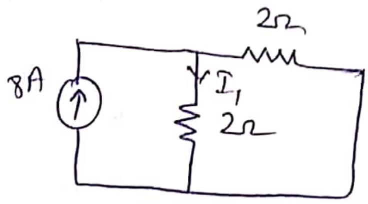
$$I = \frac{V}{2} = \frac{16}{2} = 8\,A$$

$$\boxed{I = 8\,A}$$

Step (i): When 8A source acting alone



$\Rightarrow$



$$I = 8\,A$$

$$I_1 = I \times \frac{2}{2+2}$$

$$I_1 = \cancel{8}^{4} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}}$$

$$\boxed{I_1 = 4\,A}$$

Step (ii): when 16V source acting alone

THEVENIN'S THEOREM:

To find particular current in a network as the resistance of that branch is varied while all other resistances and sources remains constant.

Always, it is not necessary to analyse the complete circuit, it requires that the voltage, current (or) power in only one resistance of a circuit be found,

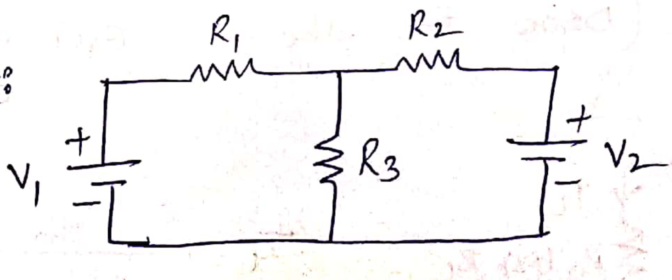
So, by using Thevenin's Theorem we can reduce the circuit to a single source and resistance which is the substitution for original Network.

Statement:

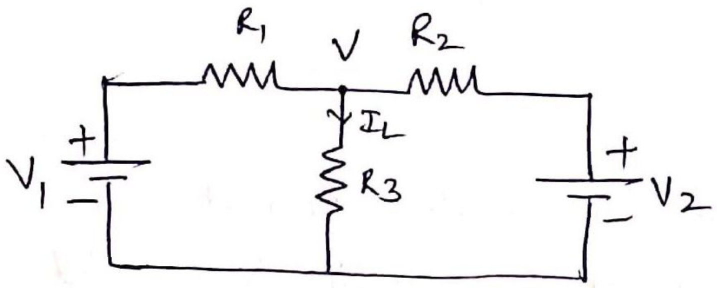
"In a linear network consisting of number of voltage sources, current sources and resistances can be replaced by a simple equivalent circuit which consists of a single voltage source in series with a internal resistance".

- Voltage source is equal to the open circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals.

Ex:



Apply Nodal (KCL)

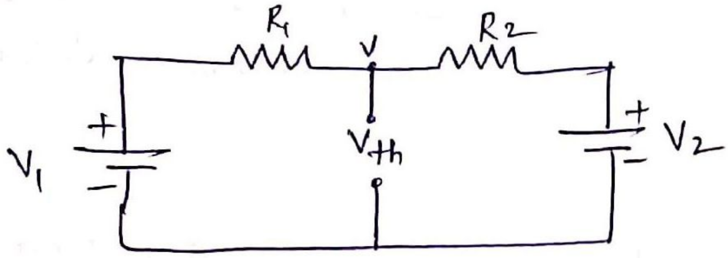


$$\frac{V-V_1}{R_1} + \frac{V}{R_3} + \frac{V-V_2}{R_2}$$

$$V = \_\_\_\_\_ \ V$$

$$I_L = \frac{V}{R_3 \text{ (or) } R_L} = \_\_\_\_\_\_$$

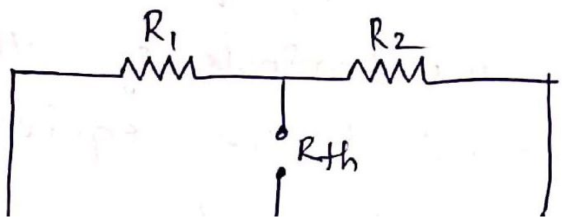
step ①: To find V_{th} (Remove R_L - Load resistance)



$$\frac{V-V_1}{R_1} + \frac{V-V_2}{R_2} = 0$$

$$V = V_{th} = V_{oc} = \_\_\_\_\_ \ V$$

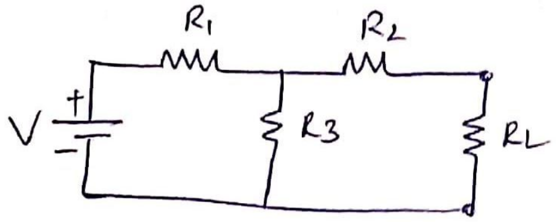
step ②: To find R_{th} (Thevenin's Resistance)



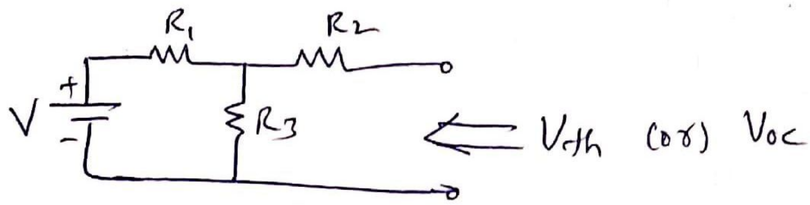
$$R_{th} = R_1 || R_2$$

$$R_1 \times R_2$$

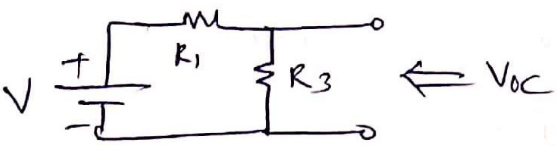
Ex: with Single Source



① To find Vth (Remove R_L)

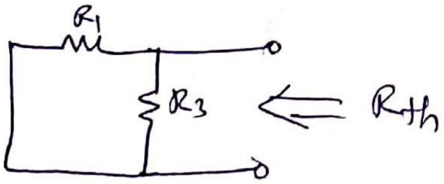


⇓



$$V_{oc} = V_{th} = V \times \frac{R_3}{R_1 + R_3} \ V$$

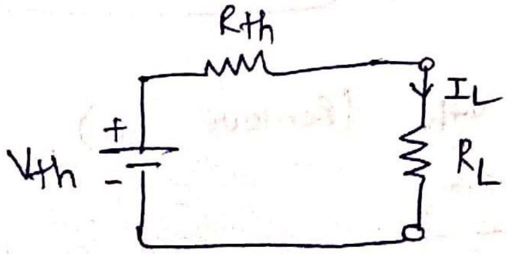
② To find Rth



$$R_{th} = R_1 \parallel R_3$$

$$R_{th} = \frac{R_1 \times R_3}{R_1 + R_3} \ \Omega$$

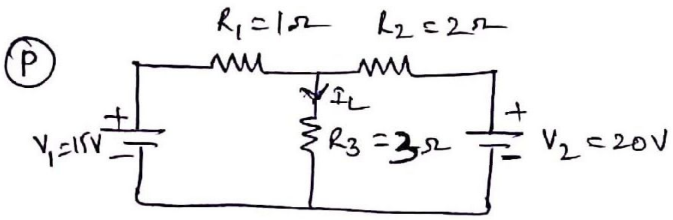
③ Thevenin's Equivalent Circuit (finding I_L)



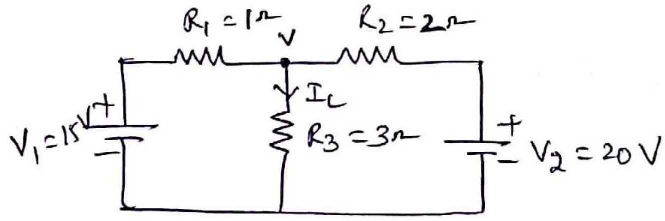
$$I_L = \frac{V_{th}}{R_{th} + R_L} \ A$$

$$\boxed{P = I_L^2 R_L \ W}$$

(P)



Apply Nodal (KCL)



$$\frac{V-15}{1} + \frac{V}{3} + \frac{V-20}{2} = 0$$

$$(V-15)(3)(2) + V(1)(2) + (V-20)(1)(3) = 0$$

$$(V-15)(6) + 2V + (V-20)(3) = 0$$

$$6V - 90 + 2V + 3V - 60 = 0$$

$$3V + 8V - 90 - 60 = 0$$

$$11V - 150 = 0$$

$$11V = 150$$

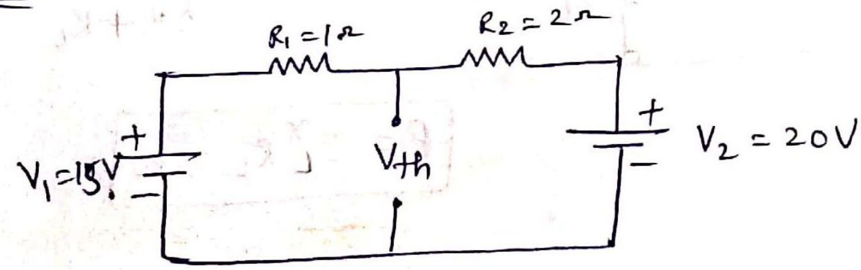
$$V = 150/11$$

$$V = \cancel{13.63}\ V \quad 13.64V$$

$$I_L = \frac{V}{R_L} = \frac{13.64}{3} = 4.54\ A$$

$$\boxed{I_L = 4.54\ A}$$

<u>step ①</u>: Calculation of V_{th} (Remove R_L)



$$\frac{V-15}{1} + \frac{V-20}{2} = 0$$

$$2(V-15)+1(V-20)=0$$

$$2V-30+V-20=0$$

$$3V-30-20=0$$

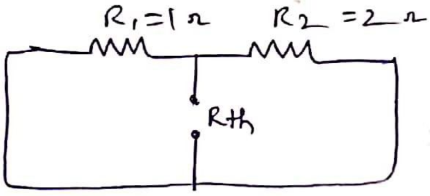
$$3V-50=0$$

$$3V=50$$

$$V=50/3$$

$$\boxed{V=16.67\,V = V_{th} \text{ (or) } V_{oc}}$$

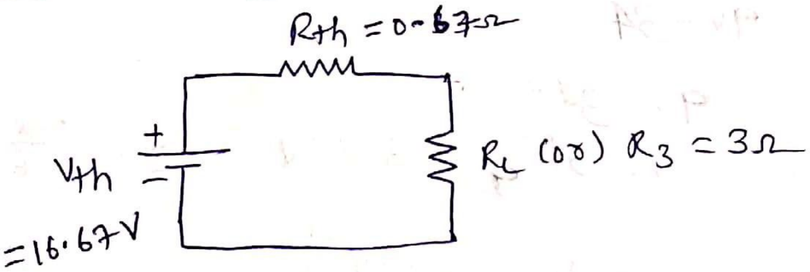
Step 2: Calculation of R_{th}



$$R_{th} = 1 \| 2 = \frac{1\times 2}{1+2} = \frac{2}{3}$$

$$\boxed{R_{th} = 0.67\,\Omega}$$

Step 3: Finding (I_L) & Thevenin's Equivalent circuit



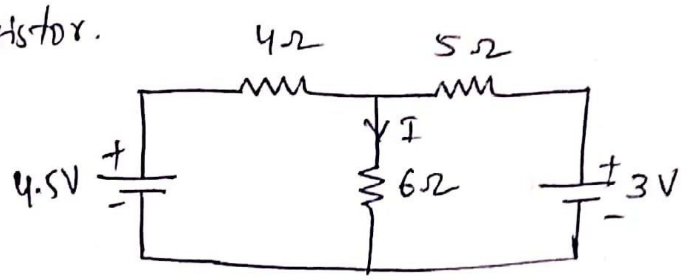
$$I_L = \frac{V_{th}}{R_{th}+R_L}$$

$$= \frac{16.67}{0.67+3}$$

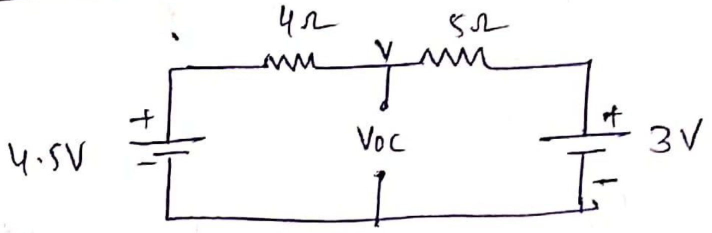
$$= \frac{16.67}{3.67}$$

$$\boxed{I_L = 4.54\,A}$$

(P) Using thevenin's theorem find the current in 6Ω resistor.



Sol: Step ①: calculation of V_{th} (or) V_{oc}:



$$\frac{V-4.5}{4}+\frac{V-3}{5}=0$$

$$(V-4.5)5+4(V-3)=0$$

$$5V-22.5+4V-12=0$$

$$9V-34.5=0$$

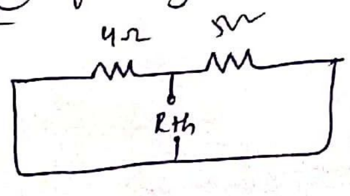
$$9V=34.5$$

$$V=34.5/9=3.83V$$

$I=\frac{3.83}{6}=$

$$\boxed{V_{th}\ (\text{or})\ V_{oc}=3.83V}$$

step ②: finding R_{th}

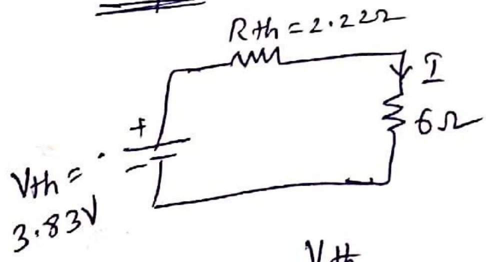


$$R_{th}=\frac{4\times5}{4+5}$$

$$R_{th}=\frac{20}{9}$$

$$\boxed{R_{th}=2.22\Omega}$$

step ③ Finding $I_{6\Omega}$



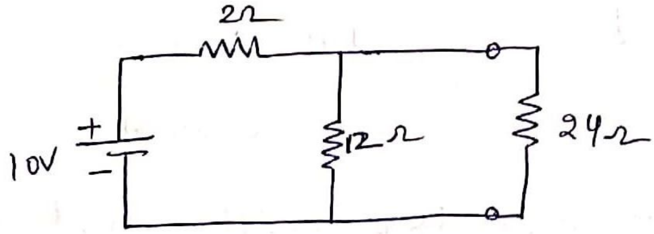
$$I=\frac{V_{th}}{R_{th}+R}$$

$$=\frac{3.83}{2.22+6}$$

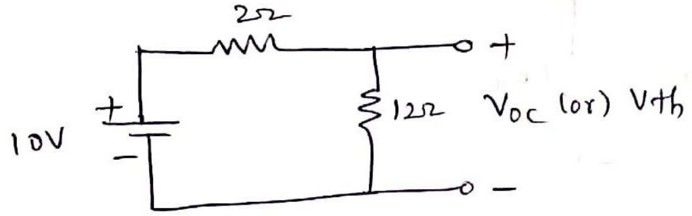
$$=\frac{3.83}{8.22}$$

$$\boxed{I=0.465A}$$

(P) Find the current through 24Ω resistor by using Thevenin's Theorem.



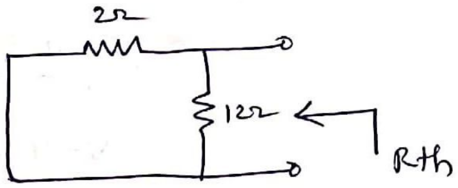
Sol: step ①: Calculation of V_{th} (or) V_{oc}



$$V_{oc} \text{ (or) } V_{th} = 10 \times \frac{12}{2+12} = 10 \times \frac{12}{14} = 8.57V$$

$$\boxed{V_{oc} \text{ (or) } V_{th} = 8.57V}$$

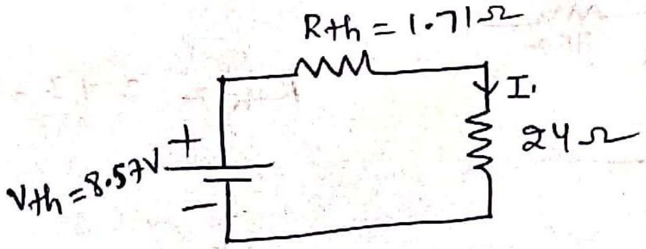
step ②: calculation of R_{th}:



$$R_{th} = 2 \| 12 = \frac{2 \times 12}{2+12} = \frac{24}{14}$$

$$\boxed{R_{th} = 1.71\Omega}$$

step ③: calculation of current

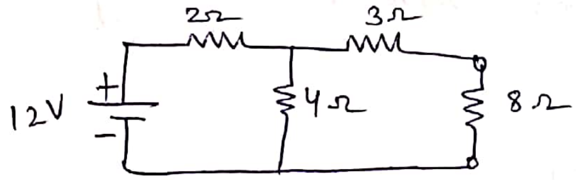


$$I = \frac{V_{th}}{R_{th}+R}$$

$$I = \frac{8.57}{1.71+24}$$

$$I = \frac{8.57}{25.71} = 0.33\text{ A}$$

Ⓟ Find the current through 8Ω resistor.

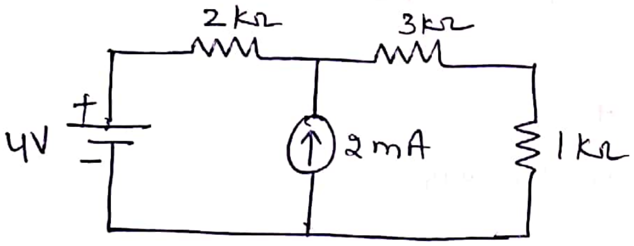


$V_{th} = 8V$

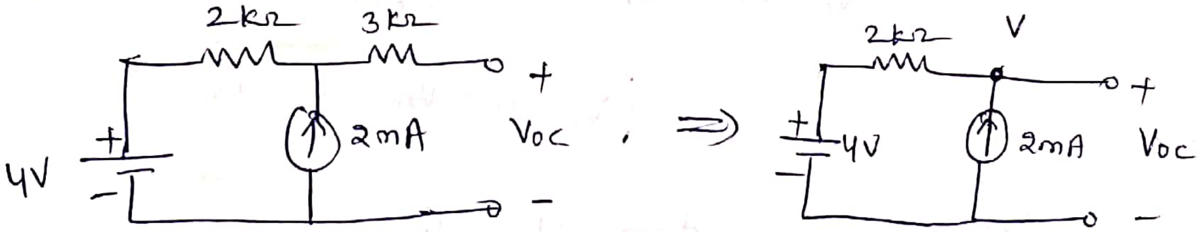
$R_{th} = \cancel{4.33\Omega}$ 1.33Ω

$I = 0.648A$ $0.857A$

Ⓟ Find the current through 1kΩ resistor and draw the thevenin's equivalent circuit.



Sol: Step ①: Calculation of V_{th} (or) V_{oc}



$$\frac{V-4}{2} - \frac{2}{1} = 0$$

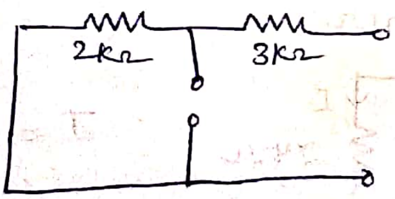
$$\frac{V-4-2}{2} = 0$$

$$V - 4 - 4 = 0$$

$$V - 8 = 0$$

$$V = 8V = V_{th} \text{ (or) } V_{oc}$$

Step ②: Calculation of R_{th}:



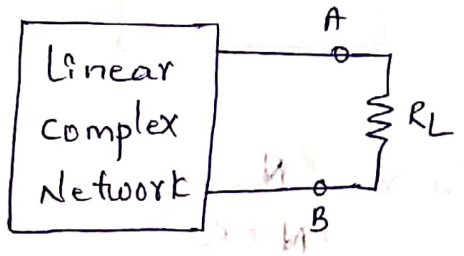
$R_{th} = 2 + 3 = 5k\Omega$

NORTON'S THEOREM:

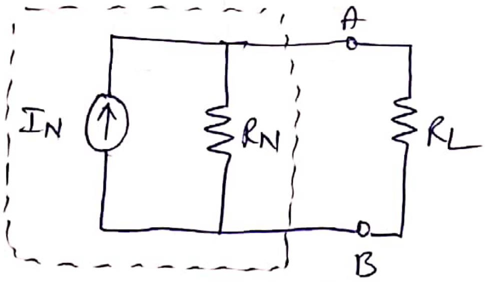
Another method of analysing the circuit by this Theorem.

Statement: Any two terminal linear network with current sources, voltage sources and resistances can be replaced by an simple equivalent circuit which consisting of current source in parallel with a equivalent resistance.

The Value of the current source is the short circuit current between the two terminals of the network, and the resistance is the equivalent resistance measured between the terminals of the network with all Energy sources replaced by their internal resistance.

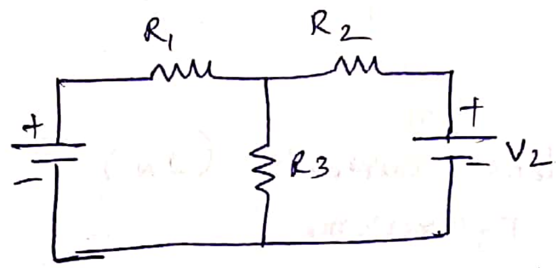


$\cong$



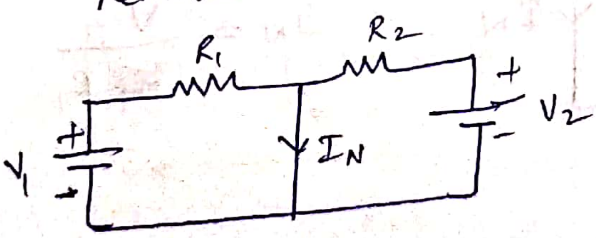
Norton's Equivalent circuit

Ex:



Step ① I_N (Norton's current) (OR) I_{sc} (short circuit current)

Remove the resistor (R_3) and short circuit it.



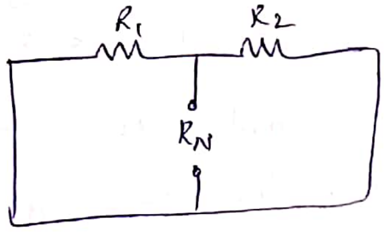
$I_N = I_1 + I_2$

$I_1 = \frac{V_1}{R_1}$

$I_2 = \frac{V_2}{R_2}$

$I_N = I_1 + I_2$

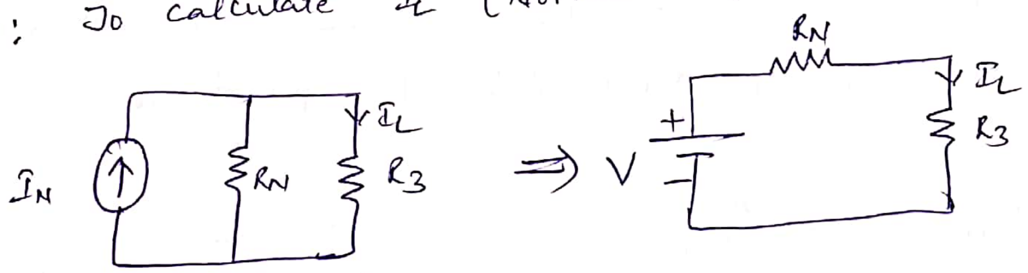
Step ②: To find Norton's Resistance (R_N)



$R_N = R_1 \| R_2$

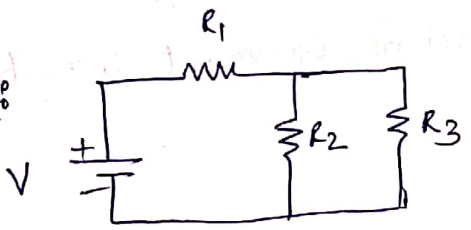
$= \frac{R_1 \times R_2}{R_1 + R_2}$

step ③: To calculate I_L (Norton's Equivalent circuit)

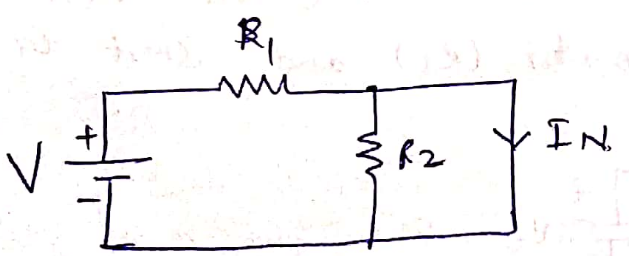


$I_L = I_N \times \frac{R_N}{R_N + R_L}$

Ex:



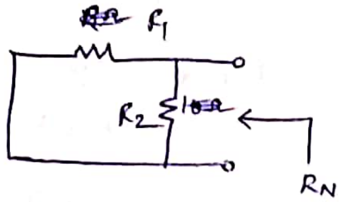
Step ①: calculation of Norton's current (I_N) short circuit the R_3 resistor.



$\cancel{R_{eq}} = \cancel{R_1 \| R_2}$

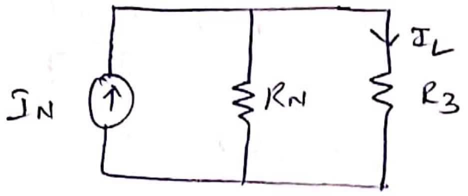
$I_N = \frac{V}{R_1}$

step ②: Calculation of R_N



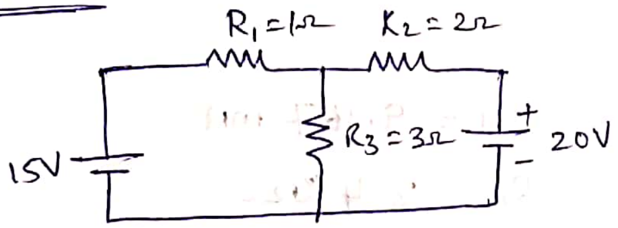
$$R_N = R_1 \| R_2 = \frac{R_1 \times R_2}{R_1 + R_2}$$

step ③: Calculation of I_L

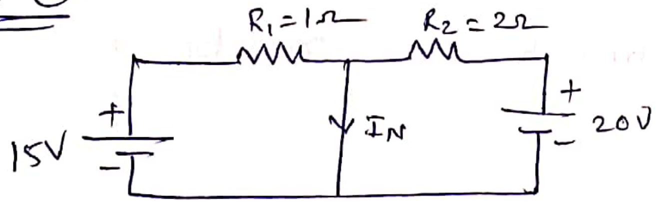


$$I_L = I_N \times \frac{R_N}{R_N + R_L}$$

Problems:



step ①: find Norton's current (I_N)



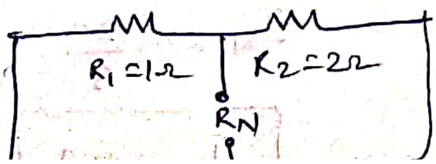
$$I_N = I_1 + I_2$$

$$I_1 = \frac{V_1}{R_1} = \frac{15}{1} = 15A$$

$$I_2 = \frac{V_2}{R_2} = \frac{20}{2} = 10A$$

$$\boxed{I_N = 15 + 10 = 25\,A}$$

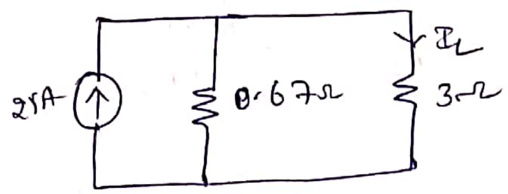
step ②: Finding (R_N) Norton's resistance



$$R_N = R_1 \| R_2$$

$$= \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1 \times 2}{1+2} = \frac{2}{3}$$

Step ③: Finding (I_L) Norton's Equivalent circuit



$$I_L = I_N \times \frac{R_N}{R_N + R_3}$$

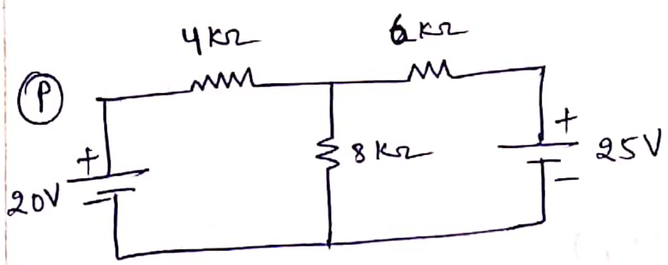
$$= 25 \times \frac{0.67}{0.67 + 3}$$

$$= 25 \times \frac{0.67}{3.67}$$

$$= 25 \times 0.18$$

$$\boxed{I_L = 4.5\,A}$$

Ⓟ

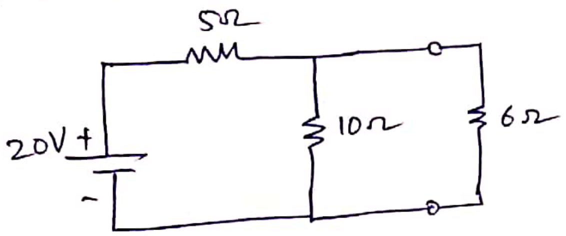


$I_N = 9.167$ mA

$R_N = 2.4$ kΩ

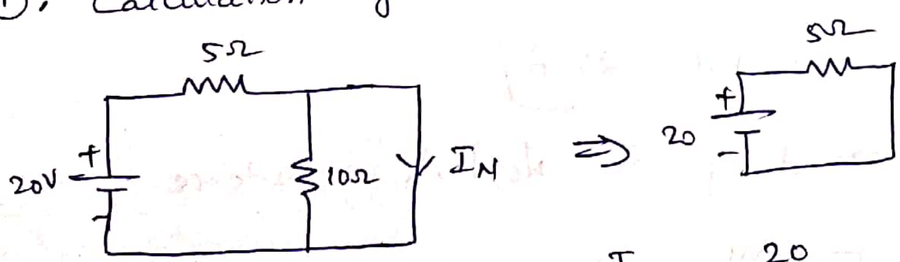
$I_L = 2.1$ mA

Ⓟ Find the current through 6Ω resistor.



Sol:

Step ①: Calculation of I_N

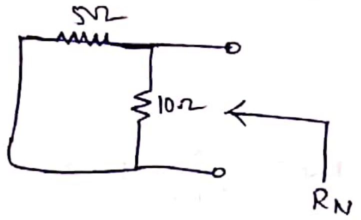


$$I_N = \frac{20}{5}$$

$$\boxed{I_N = 4A}$$

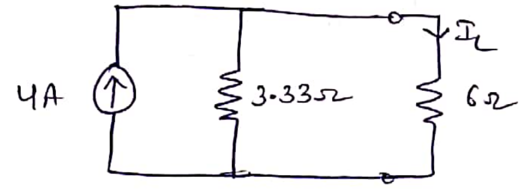
CamScanner

step ②: Calculation of R_N



$$R_N = \frac{5 \times 10}{5+10} = 3.33\,\Omega$$

step ③: Calculation of I_L

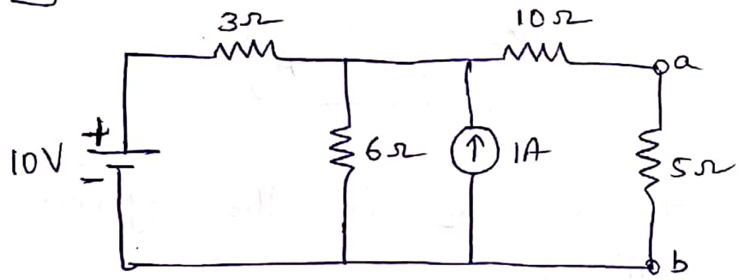


$$I_L = 4 \times \frac{3.33}{3.33+6} = 4 \times \frac{3.33}{9.33}$$

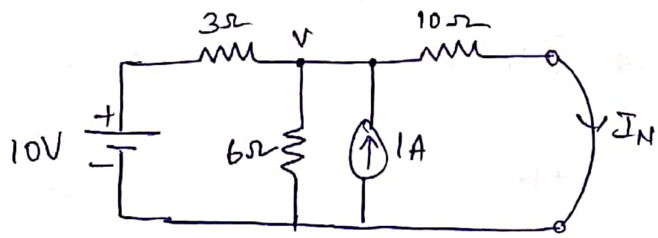
$$= 4 \times 0.35$$

$$\boxed{I_L = 1.4\,A}$$

(P) Find the current in the 5Ω resistor for the given circuit.



Sol: step ①: Calculation of I_N



By Nodal

$$\frac{V-10}{3} + \frac{V}{6} - 1 + \frac{V}{10} = 0$$

$$10V - 100 + 5V - 30 + 3V = 0$$

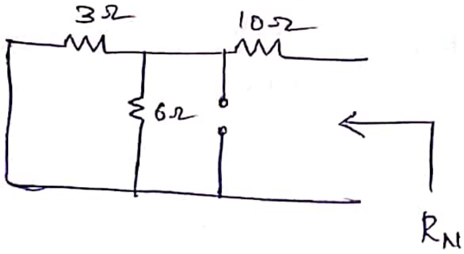
$$18V = 130$$

$$V = \frac{130}{18}$$

$$\boxed{V = 7.22\,V}$$

$\therefore I_N = \frac{V}{10} = \frac{7.22}{10} = 0.722\,A$

step ②: calculation of R_N

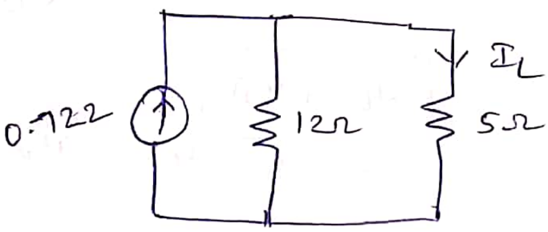


$$R_N = \frac{3 \times 6}{3+6} + 10$$

$$= 2 + 10$$

$$\boxed{R_N = 12\,\Omega}$$

step ③: calculation of I_L



$$I_L = 0.722 \times \frac{12}{12+5}$$

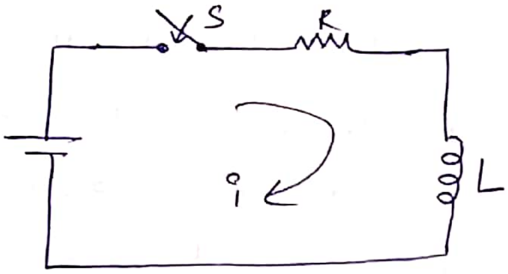
$$= 0.722 \times \frac{12}{17}$$

$$\boxed{I_L = 0.5\,A}$$

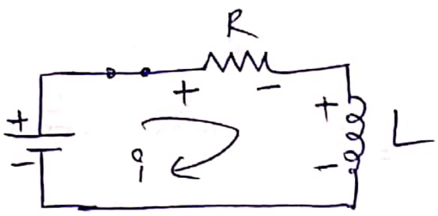
TRANSIENT RESPONSE OF FIRST ORDER RL CIRCUIT USING DC EXCITATION

Consider a circuit consisting of a resistance and inductance as shown in figure.

The inductor in the circuit is initially uncharged & it is in series with resistor.



Now, when the switch 's' is closed.



Apply KVL

$$-V + iR + L\frac{di}{dt} = 0$$

$$iR + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad \text{——— (1)}$$

In this equation 'i' is the solution to be found.

V is the applied constant voltage.

This equation is a linear differential equation of first order, Comparing it with a Non-Homogenous differential Equation,

$$\frac{dx}{dt} + Pk = k \quad \text{——— (2)}$$

whose solution is

$$x = e^{-Pt}\int k e^{Pt}\,dt + Ce^{-Pt} \quad \text{— (3)}$$

In similar way, we can write the current equation as

$$i = e^{-(R/L)t} \int \frac{V}{L} e^{(R/L)t} \, dt + C e^{-(R/L)t}$$

$$i = e^{-(R/L)t} \, \frac{V}{L} \cdot \frac{e^{(R/L)t}}{R/L} + C e^{-(R/L)t}$$

$$i = \frac{V}{R} + C e^{(-R/L)t} \quad \text{———} \quad (4)$$

To determine the value of 'C' in Equ. (4) we need to use the initial condition

At $t = 0$, $i = 0$ $\left[\because L \rightarrow \text{at } t = 0^{+} \longrightarrow \text{o.c}\right]$

Now, substituting the above conditions in Equ. (4)

$$i = \frac{V}{R} + C e^{(-R/L)t}$$

$$0 = \frac{V}{R} + C e^{(-R/L)(0)}$$

$$0 = \frac{V}{R} + C e^{0} \qquad \{e^{0} = 1\}$$

$$0 = \frac{V}{R} + C$$

$$\boxed{C = \frac{-V}{R}}$$

Substitute the value of 'C' in Equ. (4)

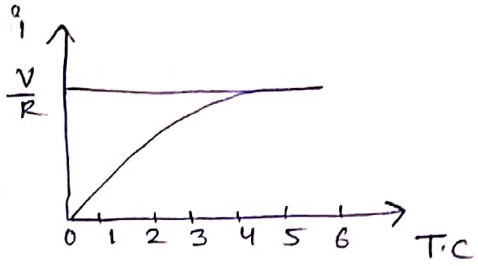
$$i = \frac{V}{R} + C e^{(-R/L)t}$$

$$i = \frac{V}{R} - \frac{V}{R} e^{(-R/L)t}$$

$$i = \frac{V}{R}\left[1 - e^{(-R/L)t}\right] \quad \text{———} \quad (5)$$

Equ. (5) consists of two parts, the steady state part $\frac{V}{R}$ and transient part $\frac{V}{...} \, (-R/L)t$

When Switch 'S' is closed, the response reaches a steady state value after a time interval as shown in figure.



Time Constant $(\tau) = \frac{L}{R}$

As $i = \frac{V}{R} - \frac{V}{R} e^{-(R/L)t} = \frac{V}{R} - \frac{V}{R} e^{-t/\tau}$

$t = 1\tau = i(\tau) = \frac{V}{R} - \frac{V}{R} e^{-t/\tau} = \frac{V}{R} - \frac{V}{R} e^{-1} = -0.632 \frac{V}{R}$

$t = 2\tau = i(2\tau) = \frac{V}{R} - \frac{V}{R} e^{-2} = \frac{V}{R}(1 - 0.135)$

$t = 3\tau = i(3\tau) = \frac{V}{R}[1 - e^{-3}] = \frac{V}{R}(1 - 0.0498)$

$t = 4\tau = i(4\tau) = \frac{V}{R}[1 - e^{-4}] = \frac{V}{R}$

$t = 5\tau = i(5\tau) = \frac{V}{R}[1 - e^{-5}] = \frac{V}{R}$

i) Voltage across the resistor

$$V_R = Ri = \cancel{R} \times \frac{V}{\cancel{R}}\left[1 - e^{-(R/L)t}\right]$$

$$V_R = V\left[1 - e^{(-R/L)t}\right]$$

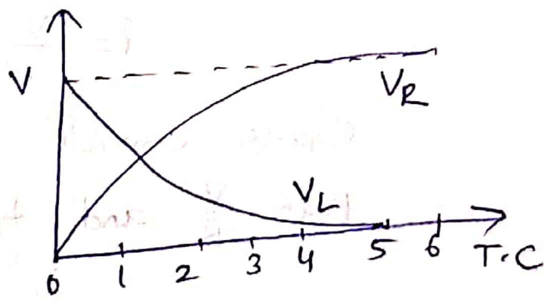
ii) Voltage across the Inductor

$$V_L = L\frac{di}{dt} = L\frac{d}{dt}\left[\frac{V}{R}(1 - e^{(-R/L)t})\right]$$

$$V_L = \cancel{L} \times \frac{V}{\cancel{R}} \times \frac{\cancel{R}}{\cancel{L}} e^{(-R/L)t}$$

$$V_L = V \cdot e^{(-R/L)t}$$

Now, the responses of V_R and V_L are as shown in figure...



Power in the resistor:

$$P_R = V_R i = V\left[1-e^{(-R/L)t}\right] \times \frac{V}{R}\left(1-e^{-(R/L)t}\right)$$

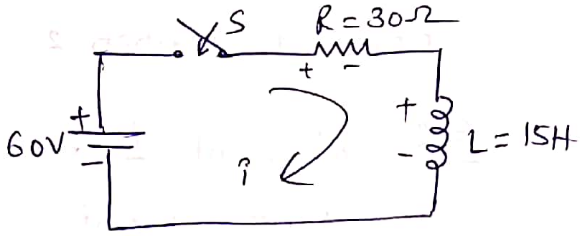
$$= \frac{V^2}{R}\left[1-2e^{(R/L)t} + e^{-\left(-\frac{2R}{L}+t\right)}\right]$$

Power in the inductor:

$$P_L = V_L i = V\cdot e^{-(R/L)t} \times \frac{V}{R}\left[1-e^{-(R/L)t}\right]$$

$$= \frac{V^2}{R}\left[e^{-(R/L)t} - e^{-2(R/L)t}\right]$$

Problem: A series RL circuit with $R = 30\Omega$ and $L = 15H$ has a constant voltage $V = 60V$ applied at $t=0$ as shown in figure. Determine the current i, the Voltage across resistor V_R & Voltage across inductor V_L.

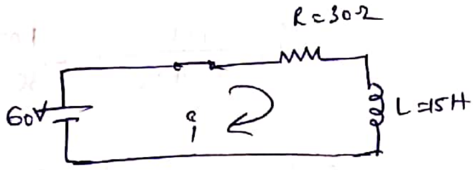


Sol: By applying KVL

$$-60 + 30i + 15\frac{di}{dt} = 0$$

$$15\frac{di}{dt} + 30i = 60$$

$$\frac{di}{dt} + 2i = 4$$



fig(2)

General solution for linear differential equation

$$i = ce^{-Pt} + e^{-Pt}\int k e^{Pt}\,dt$$

where, $P = 2$ $k = 4$

$$i = ce^{-2t} + e^{-2t}\int 4e^{2t}\,dt$$

$$i = ce^{-2t} + 2$$

At $t=0$, the switch 'S' is closed (fig. 2) $t=0^+$, $i=0$

$$0 = c + 2$$

$$\boxed{c = -2}$$

Substitute the value of 'c'

$$i = -2e^{-2t} + 2$$

$$i = 2(1 - e^{-2t})\ A$$

Voltage across resistor $V_R = iR$

$$V_R = 2(1 - e^{-2t})\ 30$$

$$V_R = 60(1 - e^{-2t})\ V$$

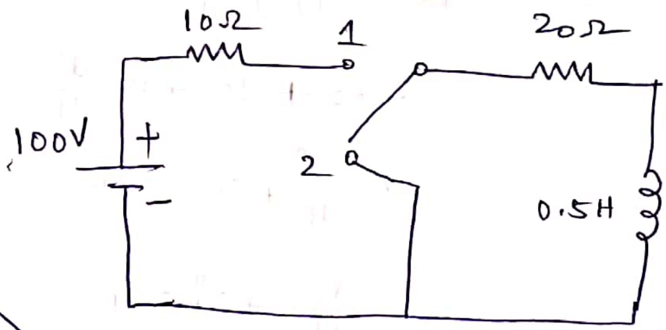
Voltage across inductor: $V_L = L\dfrac{di}{dt}$

$$V_L = 15\frac{d}{dt}\left[2(1 - e^{-2t})\right]$$

$$V_L = 30 \times 2e^{-2t}$$

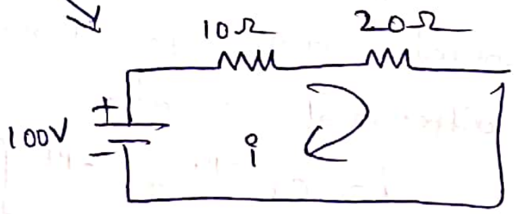
$$V_L = 60e^{-2t}\ V$$

(P) In fig. the switch 'S' is kept first at position 1 and steady state condition is reached. At $t=0$ the switch is moved to position 2. Find current in both.

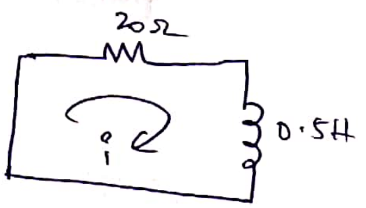


Sol: Switch at '1' the steady state current

$$i = \frac{100}{10+20} = \frac{100}{30} = 3.33A$$



Switch is at '2'



Apply to circuit 'KVL'

$$20i + 0.5\frac{di}{dt} = 0$$

$$0.5\frac{di}{dt} + 20i = 0$$

$$\frac{di}{dt} + 40i = 0 \quad \text{—— (1)}$$

Solution of Equ. (1)

$$i = ce^{-40t}$$

To determine 'c' use initial condition (At $t=0$; $i = 3.33$)

$$3.33 = c\,e^{0}$$

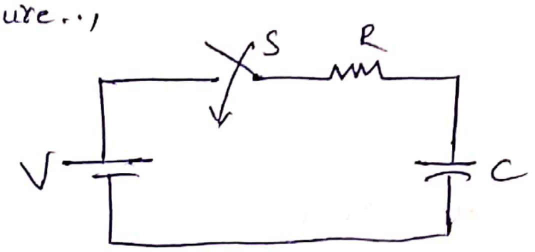
$$c = 3.33$$

$$\therefore\ i = 3.33e^{-40t}$$

TRANSIENT RESPONSE OF FIRST ORDER RC CIRCUIT USING DC EXCITATION

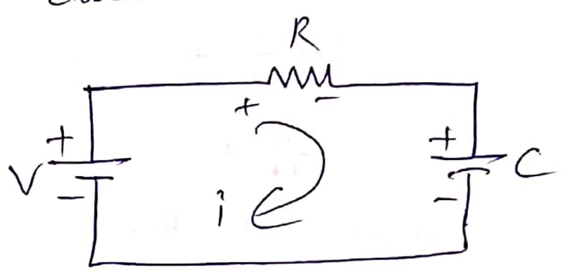
ns

Consider a circuit consisting of resistance and capacitance as shown in figure..,



The capacitor in the circuit is initially uncharged and it is in series with a resistor.

Now, when switch 's' is closed



Apply KVL to the circuit

$$-V + Ri + \frac{1}{C}\int i\,dt = 0$$

$$Ri + \frac{1}{C}\int i\,dt = V$$

By differentiating the above equation we get,

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

$$\frac{di}{dt} + \frac{i}{RC} = 0$$

$$\frac{di}{dt} + \frac{1}{RC}i = 0 \quad \text{—— (1)}$$

Equ (1) is a like a linear differential equation with only complementary ~~solution~~ function,

The particular function for the above equation is zero,

So, the solution for such type of differential equation is as:

$$i = Ce^{-t/RC} \quad \text{--- (2)}$$

To find the Value of 'c' use initial conditions

At $t=0^+$ capacitor is short circuited

Then $t=0$ & $i = \frac{V}{R}$

substitute this Values in Equ. (2)

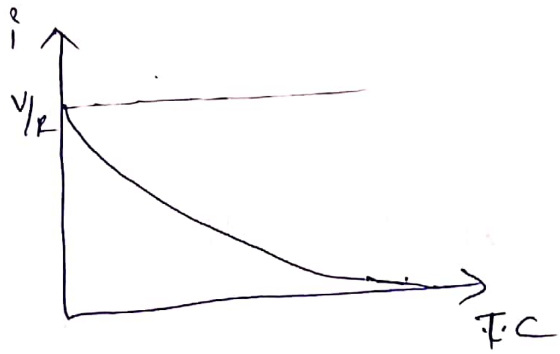
$$\frac{V}{R} = Ce^{-0/RC}$$

$$\frac{V}{R} = Ce^{-0}$$

$$\boxed{\frac{V}{R} = C}$$

Now, Equ. (2) becomes $\boxed{i = \frac{V}{R} e^{-t/RC}}$ → Complete solution.

Current Response of RC circuit



for the RC series circuit the Time constant $(\tau) = RC$

Now, $i = \frac{V}{R} e^{-t/\tau}$

At $t = 1\tau \Rightarrow i = \frac{V}{R} e^{-1\tau/\tau} = \frac{V}{R} e^{-1} = 0.368 \frac{V}{R}$

$t = 2\tau \Rightarrow i = \frac{V}{R} e^{-2\tau/\tau} = \frac{V}{R} e^{-2} = 0.135 \frac{V}{R}$

$t = 3\tau \Rightarrow i = \frac{V}{R} e^{-3\tau/\tau} = \frac{V}{R} e^{-3} = 0.0498 \frac{V}{R}$

$t = 4\tau \Rightarrow i = \frac{V}{R} e^{-4\tau/\tau} = \frac{V}{R} e^{-4} = 0.0183 \frac{V}{R}$

At $t = 5\tau \Rightarrow i = \frac{V}{R} e^{-t/\tau} = \frac{V}{R} e^{-5\tau/\tau} = \frac{V}{R} e^{-5} = 0.0067 \frac{V}{R}$

After 5τ time constants the current has reduced almost to zero.

Hence Transient period is completed.

Voltage across resistor : $V_R = R i$

$$= \cancel{R} \times \frac{V}{\cancel{R}} e^{-t/RC}$$

$$\boxed{V_R = V e^{-t/RC}}\ V$$

Voltage across ~~Inductor~~ Capacitor : $V_C = \frac{1}{C} \int i\,dt$

$$= \frac{1}{C} \int \frac{V}{R} e^{-t/RC}\,dt$$

$$= \frac{V}{RC} \int e^{-t/RC}\,dt$$

$$= \frac{-V}{\cancel{RC}} e^{-t/RC} \times \cancel{RC} + K$$

$$V_C = -V e^{-t/RC} + K$$

At $t=0$, voltage across capacitor is zero ($V_C = 0$)

$$0 = -V e^{-0/RC} + K$$

$$0 = -V e^{-0} + K$$

$$0 = -V + K$$

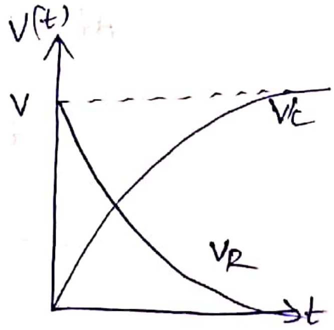
$$\boxed{K = V}$$

Now,

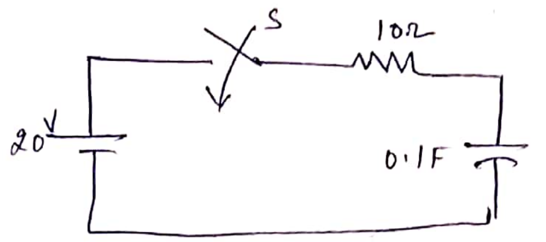
$$V_C = V - V e^{-t/RC}$$

$$\boxed{V_C = V\left[1 - e^{-t/RC}\right]}$$

Voltage Response of V_R & V_C

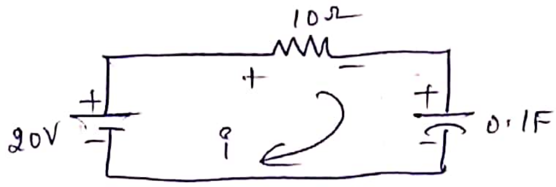


(P) A series RC circuit consists of resistor 10Ω and capacitor of $0.1F$ as shown in figure. A constant voltage of 20V is applied to the circuit at $t=0$. Obtain the current equation, Determine the voltages across resistor and the capacitor.



Sol: When switch 'S' is closed (ie., $t=0$)

Apply KVL to the circuit



$$-20 + 10i + \frac{1}{0.1}\int i\,dt = 0$$

$$10i + \frac{1}{0.1}\int i\,dt = 20$$

On differentiating we get

$$10\frac{di}{dt} + \frac{1}{0.1}i = 0$$

$$\frac{di}{dt} + i = 0 \quad \text{--- (1)}$$

Solution is $i = ce^{-t}$

At $t=0$ $i = V/R = \frac{20}{10} = 2A$

$i = 2A$ $t=0$

$\Rightarrow i = ce^{-t}$

$2 = ce^{-0}$

$$\boxed{c=2}$$

$\Rightarrow i = 2e^{-t}$

$V_R = 10 \times 2e^{-t} = 20e^{-t}\,V$

$V_c = 20(1-e^{-t})V.$

$V_c = V(1-e^{-t/RC})$

$= V(1-e^{-t/10\times0.1})$

$10\times\frac{1}{10}$