UNIT – IV
TRANSISTOR AMPLIFIERS

BJT HYBRID MODEL

**Small signal low frequency transistor Models:**

All the transistor amplifiers are two port networks having two voltages and two currents. The positive directions of voltages and currents are shown in **fig. 1**.


**Fig. 1**

A two-port network is represented by four external variables: voltage $V_1$ and current $I_1$ at the input port, and voltage $V_2$ and current $I_2$ at the output port, so that the two-port network can be treated as a black box modeled by the relationships between the four variables, $V_1$, $V_2$, $I_1$, $I_2$ . Out of four variables two can be selected as are independent variables and two are dependent variables.The dependent variables can be expressed interns of independent variables. This leads to various two port parameters out of which the following three are important:

1. Impedance parameters (z-parameters)
2. Admittance parameters (y-parameters)
3. Hybrid parameters (h-parameters)

**z-parameters**

A two-port network can be described by z-parameters as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In matrix form, the above equation can be rewritten as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

Input impedance with output port open circuited

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

Reverse transfer impedance with input port open circuited

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Forward transfer impedance with output port open circuited

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Output impedance with input port open circuited

**Y-parameters**

A two-port network can be described by Y-parameters as

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In matrix form, the above equation can be rewritten as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

Input admittance with output port short circuited

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Reverse transfer admittance with input port short circuited

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Forward transfer admittance with output port short circuited

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Output admittance with input port short circuited

**Hybrid parameters (h-parameters)**

If the input current I1 and output voltage V2 are taken as independent variables, the dependent variables V1 and I2 can be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Where $h_{11}$, $h_{12}$, $h_{21}$, $h_{22}$ are called as hybrid parameters.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

Input impedence with o/p port short circuited

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

Reverse voltage transfer ratio with i/p port open circuited

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

Forward voltage transfer ratio with o/p port short circuited

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

output impedence with i/p port open circuited

THE HYBRID MODEL FOR TWO PORT

NETWORK:

Based on the definition of hybrid parameters the mathematical model for two pert networks known as h-parameter model can be developed. The hybrid equations can be written as:

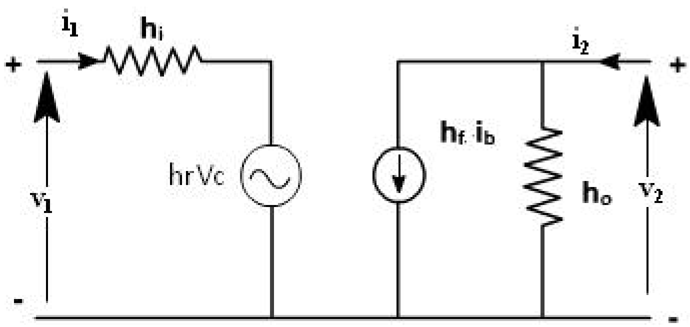
$$V_1 = h_i I_1 + h_r V_2$$

$$I_2 = h_f I_1 + h_o V_2$$

(The following convenient alternative subscript notation is recommended by the **IEEE Standards**:
***i*=11= input** ***o* = 22 = output**

***f* =21 = forward transfer *r* = 12 = reverse transfer)**

We may now use the four h parameters to construct a mathematical model of the device of Fig.(1). The hybrid circuit for any device indicated in Fig.(2). We can verify that the model of Fig.(2) satisfies above equations by writing Kirchhoff'svoltage and current laws for input and output ports.




If these parameters are specified for a particular configuration, then suffixes e,b or c are also included, e.g. $h_{fe}$ ,$h_{ib}$ are h parameters of common emitter and common collector amplifiers

Using two equations the generalized model of the amplifier can be drawn as shown in fig. 2.

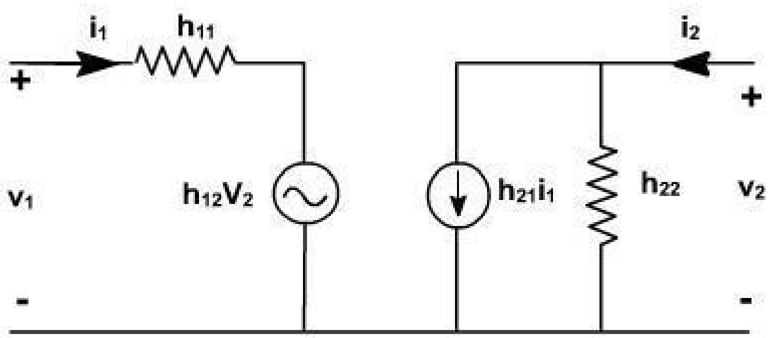



Fig. 2

**TRANSISTOR HYBRID MODEL:**

The hybrid model for a transistor amplifier can be derived as follow:

Let us consider CE configuration as show in fig. 3. The variables, $i_B$, $i_C$ ,$v_C$, and $v_B$ represent total instantaneous currents and voltages $i_B$ and $v_C$ can be taken as independent variables and $v_B$, $I_C$ as dependent variables.

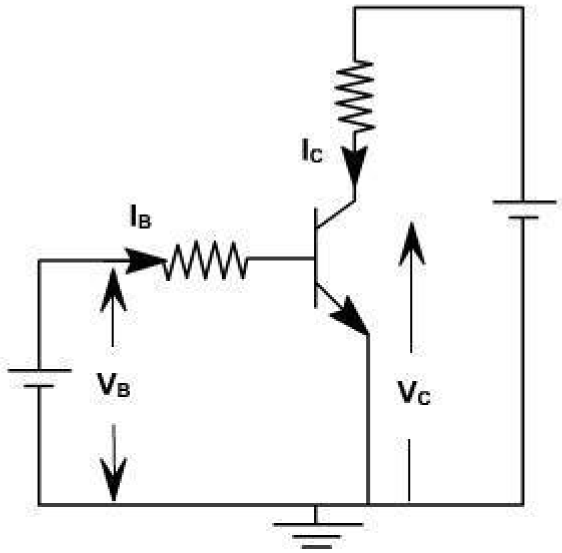



Fig. 3

$V_B = f1\ (i_B$

$, v_C)\ I_C = f2$

$(i_B\ , v_C)$.

Using Taylor 's series expression, and neglecting higher order terms we obtain.

$$\Delta v_B = \left.\frac{\partial f_1}{\partial i_B}\right|_{v_C} \Delta i_B + \left.\frac{\partial f_1}{\partial v_C}\right|_{i_B} \Delta v_C$$

$$\Delta i_C = \left.\frac{\partial f_2}{\partial i_B}\right|_{v_C} \Delta i_B + \left.\frac{\partial f_2}{\partial v_C}\right|_{i_B} \Delta v_C$$

The partial derivatives are taken keeping the collector voltage or base current constant. The $\Delta\ v_B$, $\Delta\ v_C$, $\Delta\ i_B$, $\Delta\ i_C$ represent the small signal (incremental) base and collector current and voltage and can be represented as $v_B$, $i_C$, $i_B$ ,$v_C$

$$\therefore v_b = h_{ie}\, i_B + h_{re}\, v_C$$
$$i_C = h_{fe}\, i_B + h_{oe}\, v_b$$

where

$$h_{ie} = \left.\frac{\partial f_1}{\partial i_B}\right|_{v_C} = \left.\frac{\partial v_B}{\partial i_B}\right|_{v_C} ; \qquad h_{re} = \left.\frac{\partial f_1}{\partial v_C}\right|_{i_B} = \left.\frac{\partial v_B}{\partial v_C}\right|_{i_B}$$

$$h_{fe} = \left.\frac{\partial f_2}{\partial i_B}\right|_{v_C} = \left.\frac{\partial i_C}{\partial i_B}\right|_{v_C} ; \qquad h_{oe} = \left.\frac{\partial f_2}{\partial v_C}\right|_{i_B} = \left.\frac{\partial v_B}{\partial v_C}\right|_{i_B}$$

The model for CE configuration is shown in fig. 4.

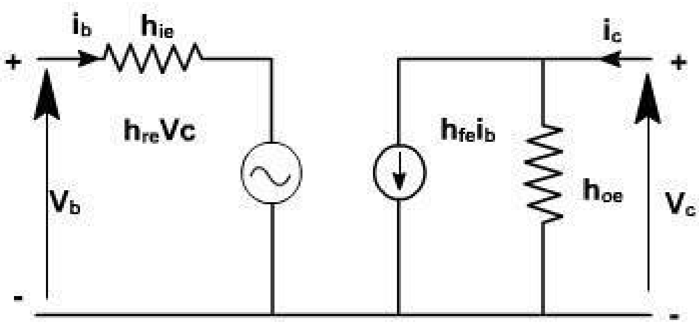



Fig. 4

To determine the four h-parameters of transistor amplifier, input and output characteristic are used. Input characteristic depicts the relationship between input voltage and input current with output voltage as parameter. The output characteristic depicts the relationship between output voltage and output current with input current as parameter. Fig. 5, shows the output characteristics of CE amplifier.

$$h_{fe} = \left.\frac{\partial i_C}{\partial i_B}\right|_{V_C} = \frac{i_{C2} - i_{C1}}{i_{b2} - i_{b1}}$$

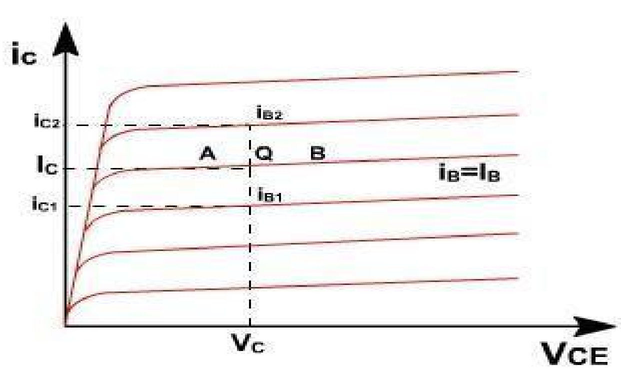



Fig. 5

The current increments are taken around the quiescent point Q which corresponds to $i_B = I_B$ and to the collector voltage $V_{CE} = V_C$

$$h_{oe} = \left.\frac{\partial i_C}{\partial V_C}\right|_{i_B}$$

The value of $h_{oe}$ at the quiescent operating point is given by the slope of the output characteristic at the operating point (i.e. slope of tangent AB).

$$h_{ie} = \frac{\partial V_B}{\partial i_B} \approx \left.\frac{\Delta V_B}{\Delta i_B}\right|_{V_C}$$

$h_{ie}$ is the slope of the appropriate input on fig. 6, at the operating point (slope of tangent EF at Q).

$$h_{re} = \frac{\partial V_B}{\partial V_C} = \left.\frac{\Delta V_B}{\Delta V_C}\right|_{I_B} = \frac{V_{B2} - V_{B1}}{V_{C2} - V_{C1}}$$

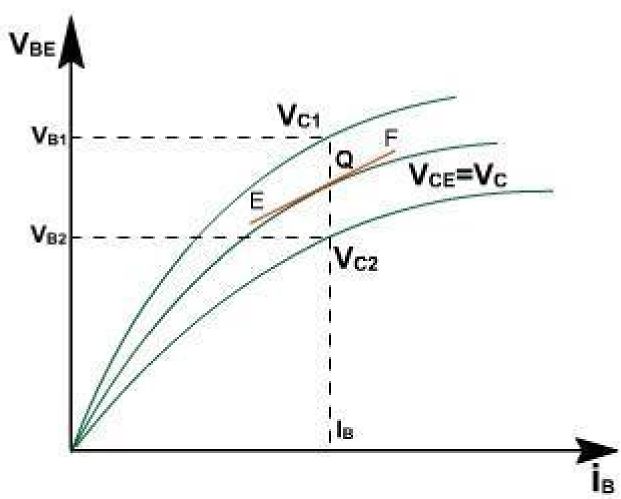



**Fig. 6**

A vertical line on the input characteristic represents constant base current. The parameter hre can be obtained from the ratio ($V_{B2}$– $V_{B1}$ ) and ($V_{C2}$– $V_{C1}$ ) for at Q.

Typical CE h-parametersof transistor 2N1573 are given below:

$h_{ie}$ = 1000 ohm.
$h_{re}$ = 2.5 * 10 –4
$h_{fe}$ = 50
$h_{oe}$ = 25 µA/ V

**ANALYSIS OF A TRANSISTOR AMPLIFIER USING H-PARAMETERS:**

To form a transistor amplifier it is only necessary to connect an external load and signal source as indicated in **fig. 1** and to bias the transistor properly.

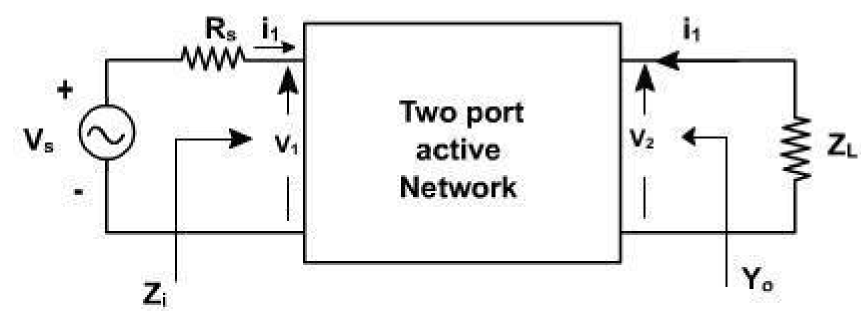
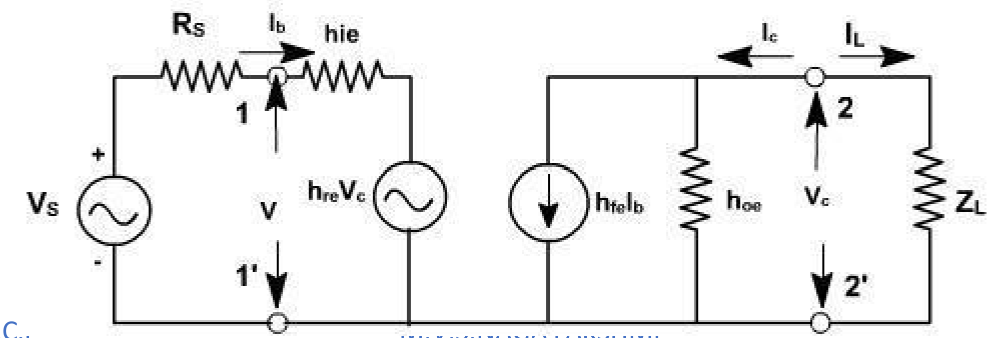



**Fig. 1**

Consider the two-port network of CE amplifier. $R_S$ is the source resistance and $Z_L$ is the load impedance h-parameters are assumed to be constant over the operating range. The ac equivalent circuit is shown in fig. 2. (Phasor notations are used assuming sinusoidal voltage input). The quantities of interest are the current gain, input impedance, voltage gain, and output impedance.



**Current gain:**

For the transistor amplifier stage, $A_i$ is defined as the ratio of output to input currents.

$$A_I = \frac{I_L}{I_1} = \frac{-I_2}{I_1}$$

**Input impedence:**

The impedence looking into the amplifier input terminals ( 1,1' ) is the input impedance $Z_i$

$$Z_i = \frac{V_b}{I_b}$$

$$V_b = h_{ie} I_b + h_{re} V_c$$

$$\frac{V_b}{I_b} = h_{ie} + h_{re} \frac{V_c}{I_b}$$

$$= h_{ie} - \frac{h_{re} I_c Z_L}{I_b}$$

$$\therefore Z_i = h_{ie} + h_{re} A_I Z_L$$

$$= h_{ie} - \frac{h_{re} h_{fe} Z_L}{1 + h_{oe} Z_L}$$

$$\therefore Z_i = h_{ie} - \frac{h_{re} h_{fe}}{Y_L + h_{oe}} \qquad (\text{since } Y_L = \frac{1}{Z_L})$$

**Voltage gain:**

The ratio of output voltage to input voltage gives the gain of the transistors.

$$A_v = \frac{V_C}{V_b} = -\frac{I_C Z_L}{V_b}$$

$$\therefore A_v = \frac{I_B A_i Z_L}{V_b} = \frac{A_i Z_L}{Z_i}$$

**Output Admittance:**

$$Y_0 = \left.\frac{I_c}{V_c}\right|_{V_s} = 0$$

$$I_c = h_{fe} I_b + h_{oe} V_c$$

$$\frac{I_c}{V_c} = h_{fe} \frac{I_b}{V_c} + h_{oe}$$

when $V_s = 0$, $R_s . I_b + h_{ie} . I_b + h_{re} V_c = 0$.

$$\frac{I_b}{V_c} = - \frac{h_{re}}{R_s + h_{ie}}$$

$$\therefore Y_0 = h_{oe} - \frac{h_{re}\, h_{fe}}{R_s + h_{ie}}$$

Voltage amplification taking into account source impedance ($R_S$) is given by

$$A_{VS} = \frac{V_c}{V_s} = \frac{V_c}{V_b} * \frac{V_b}{V_S} \qquad \left(V_b = \frac{V_s}{R_s + Z_i} * Z_i\right)$$

$$= A_V . \frac{Z_i}{Z_i + R_s}$$

It is defined as

$$= \frac{A_i\, Z_L}{Z_i + R_s}$$

$A_v$ is the voltage gain for an ideal voltage source ($R_v = 0$).

Consider input source to be a current source $I_S$ in parallel with a resistance $R_S$ as shown in fig. 3.

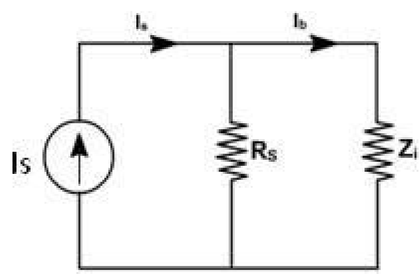



**Fig. 3**

In this case, overall current gain $A_{IS}$ is defined as

$$A_{I_s} = \frac{I_L}{I_s}$$
$$= -\frac{I_c}{I_s}$$
$$= -\frac{I_c}{I_b} * \frac{I_b}{I_s} \qquad \left( I_b = \frac{I_s * R_s}{R_s + Z_i} \right)$$
$$= A_I \cdot \frac{R_s}{R_s + Z_i}$$
$$\text{If } R_s \to \infty, \quad A_{I_s} \to A_I$$

h-parameters

To analyze multistage amplifier the h-parameters of the transistor used are obtained from manufacture data sheet. The manufacture data sheet usually provides h-parameter in CE configuration. These parameters may be converted into CC and CB values. For example fig. 4 hrc in terms of CE parameter can be obtained as follows.

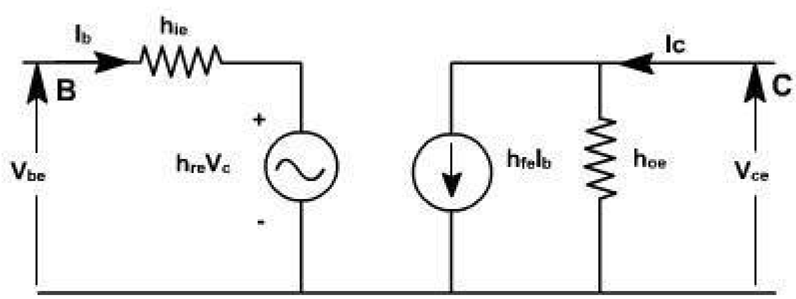



Fig. 4

For CE transistor configuaration

Vbe = hie Ib + hre Vce

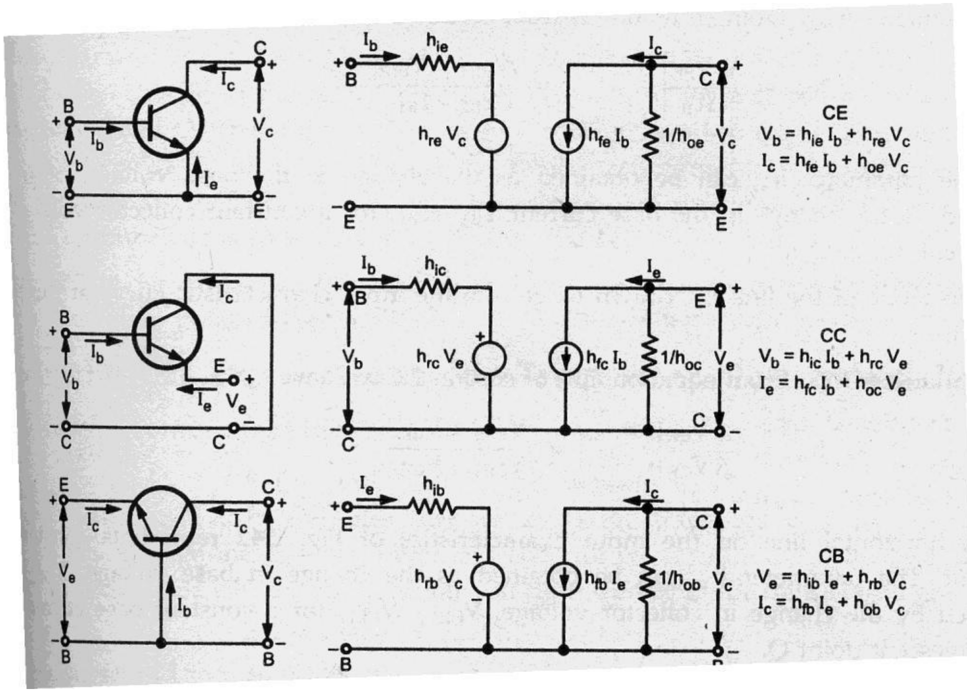
Ic = h fe Ib + hoe Vce

The circuit can be redrawn like CC transistor configuration as shown in fig. 5.

Vbc = hie Ib + hrc Vec

Ic = hfe Ib + hoe Vec

hybrid model for transistor in three different configurations




Typical h-parameter values for a transistor

| Parameter | CE | CC | CB |
|---|---|---|---|
| $h_i$ | 1100 Ω | 1100 Ω | 22 Ω |
| $h_r$ | $2.5 \times 10^{-4}$ | 1 | $3 \times 10^{-4}$ |
| $h_f$ | 50 | -51 | -0.98 |
| $h_o$ | 25 µA/V | 25 µA/V | 0.49 µA/V |

**Analysis of a Transistor amplifier circuit using h-parameters**

A transistor amplifier can be constructed by connecting an external load and signal source and biasing the transistor properly.

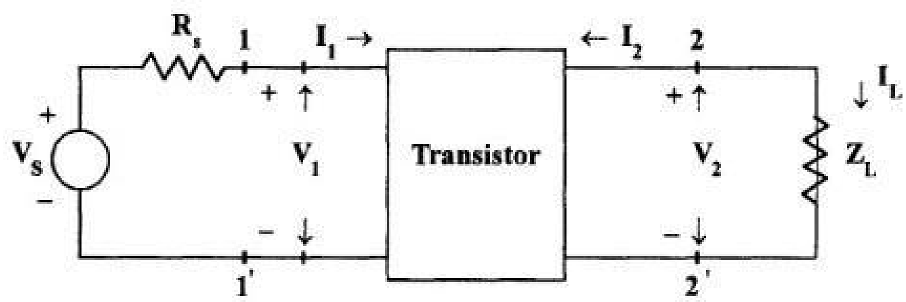



Fig.1.4 Basic Amplifier Circuit

The two port network of Fig. 1.4 represents a transistor in any one of its configuration. It is assumed that h-parameters remain constant over the operating range.The input is sinusoidal and $I_1$,V- $_1$,$I_2$ and $V_2$ are phase quantities

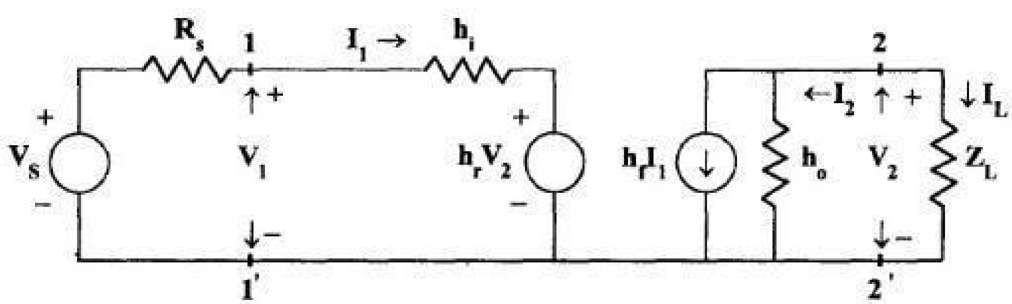



Fig. 1.5 Transistor replaced by its Hybrid Model

**Current Gain or Current Amplification ($A_i$)**

For transistor amplifier the current gain $A_i$ is defined as the ratio of output current to input current,i.e,

$$A_i = I_L / I_1 = -I_2 / I_1$$

From the circuit of Fig

$$I_2 = h_f I_1 + h_o V_2$$

Substituting $V_2 = I_L Z_L = -$

$I_2 Z_L I_2 = h_f \quad I_1 - I_2 Z_L h_o$

$I_2 + I_2 Z_L h_o = h_f I_1$

$I_2( 1+ Z_L h_o) = h_f$

$I_1$

$A_i = -I_2 / I_1 = - h_f / ( 1+ Z_L h_o)$

Therefore,

$A_i = - h_f / ( 1+ Z_L h_o)$

**Input Impedence ($Z_i$)**

In the circuit of Fig , $R_S$ is the signal source resistance .The impedence seen when looking into the amplifier terminals (1,1') is the amplifier input impedence $Z_i$,

$Z_i = V_1 / I_1$

From the input circuit of Fig $V_1 = h_i I_1 +$

$h_r V_2$ $Z_i = ( h_i I_1 + h_r V_2) / I_1$

$= h_i + h_r V_2 / I_1$

Substituting

$V_2 = -I_2 Z_L = A_1 I_1 Z_L$

$Z_i = h_i + h_r A_1 I_1 Z_L / I_1$

$= h_i + h_r A_1 Z_L$

Substituting for $A_i$

$Z_i = h_i - h_f h_r Z_L / (1+ h_o Z_L)$

$= h_i - h_f h_r Z_L / Z_L(1/Z_L + h_o)$

Taking the Load admittance as $Y_L = 1/$

$Z_L$ $Z_i = h_i - h_f h_r / (Y_L + h_o)$

**Voltage Gain or Voltage Gain Amplification Factor($A_v$)**

The ratio of output voltage $V_2$ to input voltage $V_1$ give the voltage gain of the transistor i.e,

$$A_v = V_2 / V_1$$

Substituting

$$V_2 = -I_2 Z_L = A_1 I_1 Z_L$$

$$A_v = A_1 I_1 Z_L / V_1 = A_i Z_L / Z_i$$

**Output Admittance ($Y_o$)**

$Y_o$ is obtained by setting $V_S$ to zero, $Z_L$ to infinity and by driving the output terminals from a generator $V_2$. If the current $V_2$ is $I_2$ then $Y_o = I_2/V_2$ with $V_S=0$ and $R_L = \infty$.

From the circuit of fig

$$I_2 = h_f I_1 + h_o V_2$$

Dividing by $V_2$,

$$I_2 / V_2 = h_f I_1/V_2 + h_o$$

With $V_2 = 0$, by KVL in input circuit,

$$R_S I_1 + h_i I_1 + h_r V_2 =$$

$$0 \; (R_S + h_i) I_1 +$$

$$h_r V_2 = 0$$

Hence, $\quad I_2 / V_2 = -h_r / (R_S + h_i)$

$$= h_f(-h_r/(R_S + h_i) + h_o$$

$$Y_o = h_o - h_f h_r/(R_S + h_i)$$

The output admittance is a function of source resistance. If the source impedence is resistive then $Y_o$ is real.Voltage Amplification Factor($A_{vs}$) taking into account the resistance ($R_s$) of the source

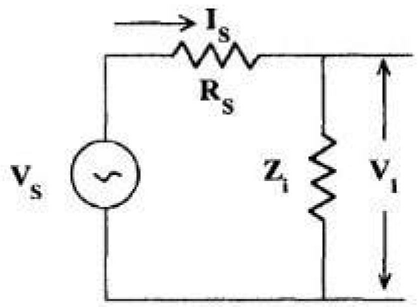



Fig. 5.6 Thevenin's Equivalent Input Circuit

This overall voltage gain $A_{vs}$ is given by

$$A_{vs} = V_2 / V_S = V_2 V_1 / V_1 V_S = A_v V_1 / V_S$$

From the equivalent input circuit using Thevenin's equivalent for the source shown in Fig. 5.6

$V_1 = V_S Z_i / (Z_i + R_S)$

$V_1 / V_S = Z_i / (Z_i + R_S)$

Then, $A_{vs} = A_v Z_i / (Z_i +$

$R_S)$ Substituting $A_v = A_i Z_L / Z_i$

$$A_{vs} = A_i Z_L / (Z_i + R_S)$$

$$A_{vs} = A_i Z_L R_S / (Z_i + R_S) R_S$$

$$A_{VS} = A_{IS} Z_L / R_S$$

**Current Amplification ($A_{is}$) taking into account the sourse Resistance($R_S$)**

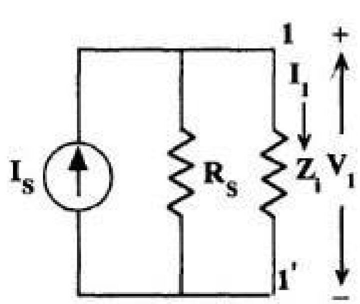



Fig. 1.7 Norton's Equivalent Input Circuit

The modified input circuit using Norton's equivalent circuit for the calculation of $A_{is}$ is shown in Fig. 1.7 Overall Current Gain, $A_{is} = -I_2 / I_S = - I_2 I_1 / I_1 I_S = A_i I_1/I_S$

From Fig. 1.7

$I_1 = I_S R_S / (R_S + Z_i)$

$I_1 / I_S = R_S / (R_S + Z_i)$

and hence, $A_{is} = A_i R_S / (R_S + Z_i)$

**Operating Power Gain ($A_P$)**

The operating power gain $A_P$ of the transistor

is defined as $A_P = P_2 / P_1 = -V_2 I_2 / V_1$

$I_1 = A_v A_i = A_i A_i Z_L / Z_i$

$A_P = A_i^2 (Z_L / Z_i)$

**Small Signal analysis of a transistor amplifier**

| | |
|---|---|
| $A_i = - h_f / (1 + Z_L h_o)$ | $A_v = A_i Z_L / Z_i$ |
| $Z_i = h_i + h_r A_1 Z_L = h_i - h_f h_r / (Y_L + h_o)$ | $A_{vs} = A_v Z_i / (Z_i + R_S) = A_i Z_L / (Z_i + R_S)$<br>$= A_{is} Z_L / R_S$ |
| $Y_o = h_o - h_f h_r / (R_S + h_i) = 1/ Z_o$ | $A_{is} = A_i R_S / (R_S + Z_i) = A_{vs} = A_{is} R_S / Z_L$ |