UNIT-1
INTRODUCTION TO ELECTRO-STATICS

The Electrostatic Field

1. Introduction

2. Coulomb's Law

3. The Electric Field

4. Continuous Charge Distribution

1.1 Introduction

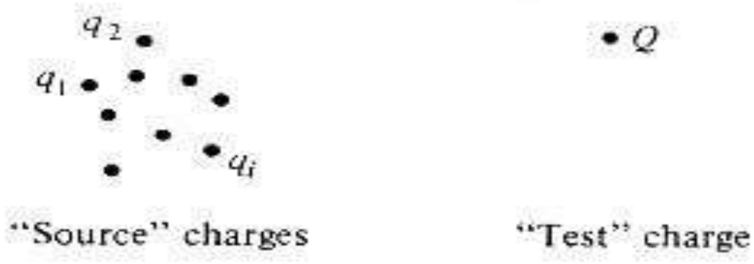


Figure 2.1

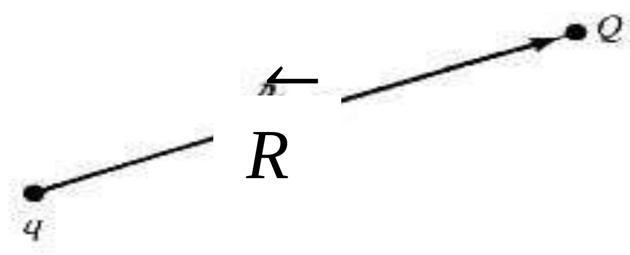


Figure 2.2

The fundamental problem for electromagnetic to solve is to calculate the interaction of charges in a given configuration.

That is, what force do they exert on another charge Q ?
The simplest case is that the source charges are stationary.

Principle of Superposition:
The interaction between any two charges is completely unaffected by the presence of other charges.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$\vec{F}_i$ is the force on Q due to q_i

1.2 Coulomb's Law

The force on a charge Q due to a single point charge q is given by Coulomb`s law.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R^2}\hat{R} \qquad \vec{R} = \vec{r}_Q - \vec{r}_q = R\,\hat{R}$$

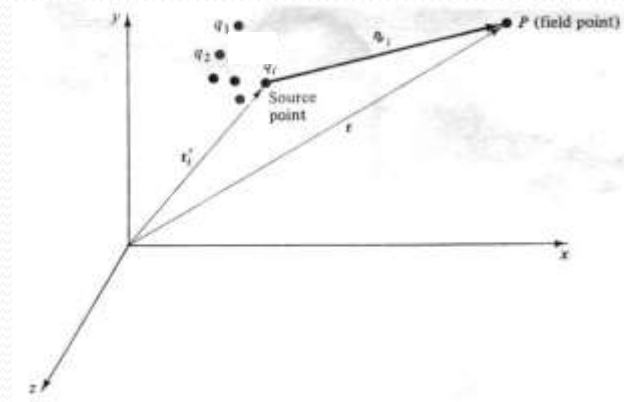
$$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{c^2}{N \cdot m^2}$$

1.3 The Electric Field

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{R_1^2}\hat{R}_1 + \frac{q_2 Q}{R_2^2}\hat{R}_2 + \cdots\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1^2}\hat{R}_1 + \frac{q_2}{R_2^2}\hat{R}_2 + \frac{q_3}{R_3^2}\hat{R}_3 + \cdots\right)$$

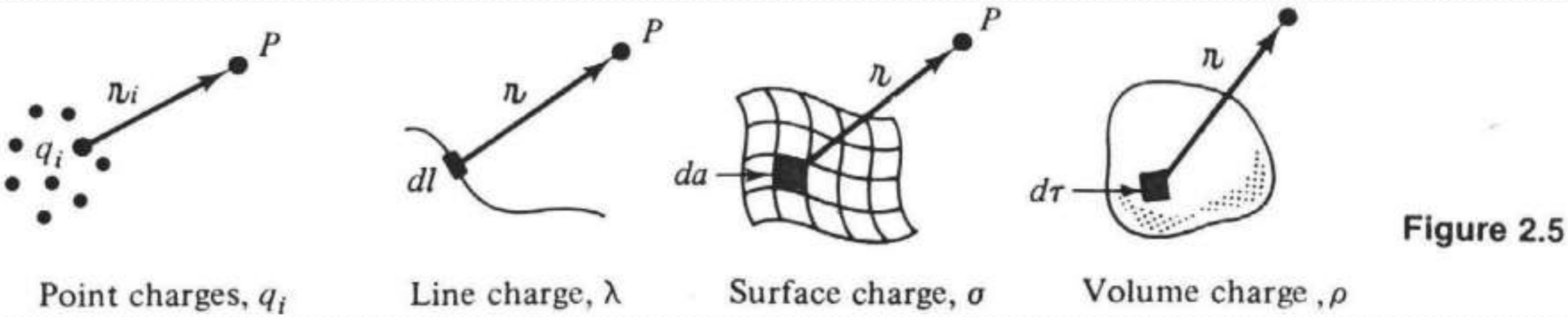


$$\vec{F} = Q\vec{E}$$

$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{R_1^2}\hat{R}_i$$

the electric field of the source charge

1.4 Continuous Charge Distributions



$$\sum_{i=1}^{n} (\quad) q_i \sim \int_{line} (\quad) \lambda \, dl \sim \int_{surface} (\quad) \sigma \, da \sim \int_{volume} (\quad) \rho \, d\tau$$

$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{R_i^2} \hat{R}_i \qquad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \int_{line} \frac{\hat{R}}{R^2} \lambda \, dl$$

$$\vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \int_{surface} \frac{\hat{R}}{R^2} \sigma \, da \qquad \vec{E}(P) = \frac{1}{4\pi\varepsilon_0} \int_{volume} \frac{\hat{R}}{R^2} \rho \, d\tau$$

Example:

the test point is at $\vec{p} = (x, y, z)$ the source point is at (x', y', z')

$$\vec{R} = (x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}$$

$$\vec{R} = \left[(x - x')^2 + (y - y')^2 + (z - z')^2\right]^{1/2} \qquad \hat{R} = \frac{\vec{R}}{|R|}$$

$$\vec{E}(x, y, z) = \frac{1}{4\pi\varepsilon_0}\int_{volume} \frac{(x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}} \rho(x', y', z')\, dx'\, dy'\, dz'$$

$$\vec{E}(x, y, z) = \frac{1}{4\pi\varepsilon_0}\int_{volume} \frac{\hat{R}}{R^2}\rho(x', y', z')\, dx'dy'dz'$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{volume} \frac{\hat{R}}{R^2} \cdot \frac{R}{R}\rho(x', y', z')\, dx'dy'dz'$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{volume} \frac{\vec{R}}{R^3}\rho(x', y', z')\, dx'dy'dz'$$

Example 2.1 Find the electric field a distance z above the midpoint of a straight of length 2L, which carries a uniform line chargλe

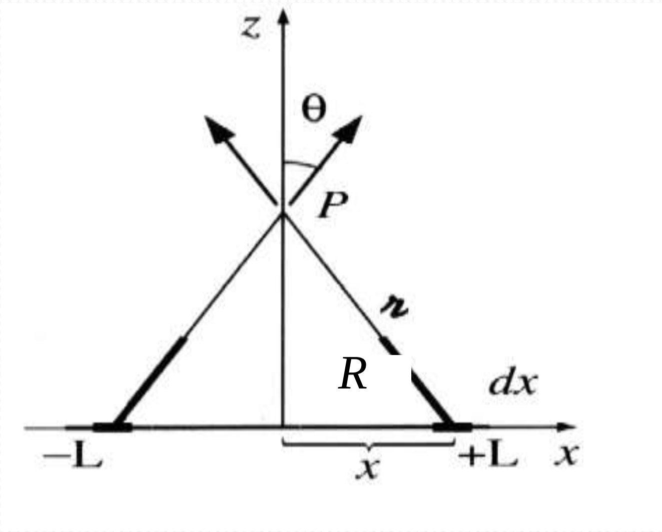
Solution:

$$d\vec{E} = 2\frac{1}{4\pi\varepsilon_0}\left(\frac{\lambda\, dx}{R^2}\right)\cos\theta\ \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{2\lambda z}{(z^2+x^2)^{3/2}}dx$$

$$= \frac{2\lambda z}{4\pi\varepsilon_0}\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]\Bigg|_0^L$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}$$



(1) z>>*L* $\quad \vec{E} \cong \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z^2}$

(2) $\quad L \to \infty \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}$