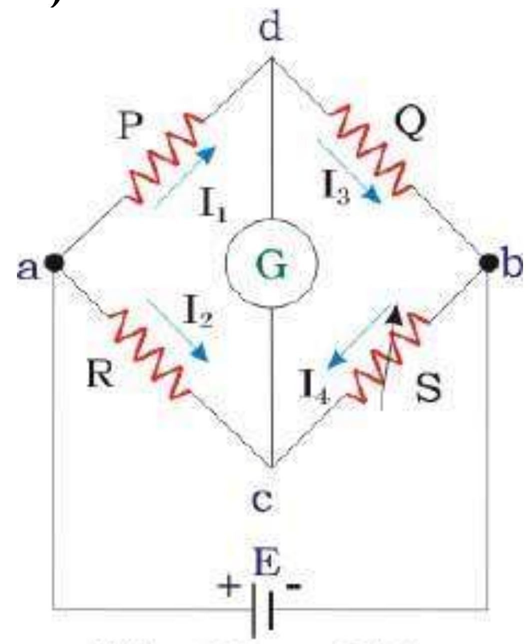
UNIT-IV

DC & AC Bridges

MEASUREMENT OF MEDIUM RESISTANCE (1Ω - 100KΩ)

$R = \frac{P}{Q} S$

A bridge circuit always works on the principle of null detection, i.e. we vary a parameter until the detector shows zero and then use a mathematical relation to determine the unknown in terms of varying parameter and other constants. Here also the standard resistance, S is varied in order to obtain null deflection in the galvanometer. This null deflection implies no current from point c to d, which implies that potential of point c and d is same. Hence



Wheatstone Bridge

$$I_1 P = I_2 R \, \ldots\ldots (4)$$

$$Also,\ I_1 = I_3 = \frac{E}{(P+Q)} \ and\ I_2 = I_4 = \frac{E}{(R+S)}$$

Combining the above two equations we get the famous equation –

$$R = \frac{P}{Q} S$$

Carey Foster Bridge

The Carey foster bridge principle is simple and similar to Wheatstone’s bridge working principle.

It works on the principle of null detection. That means the ratios of the resistances will be equal and the galvanometer records zero where there is no current flow.

Consider I1 be the distance from the left side where the bridge is balanced. Interchange the resistances R and S while the bridge gets balanced by sliding the contact with a distance of I2.

The switch is used to interchange the resistances R and S while testing. The galvanometer records zero when the bridge is balanced. The first bridge balance equation is,

$P/Q = (R+I1r) / [(S+(L+I1)\ r]$

Where r = resistance/unit length of the slide wire.

Now interchange the resistances R and S. Then the balanced equation for the bridge circuit is given as,

For the first balance equation, we get,

$P/Q + 1 = [(R+I1r+S+(L-I1)\ r] / [S + (L-I1)r]$ ……Eq (1)

$P/Q= (R+S+I1r) / (S+(L-I1)r)$

We get a second bridge balance equation as

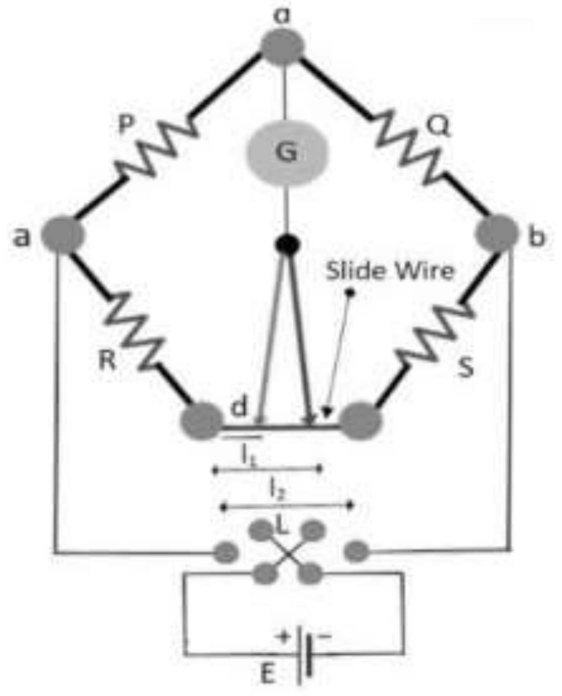
$P/Q + 1 = [(S+I2r+R+(L-I1)\ r] / [R + (L-I2)r]$ ….. Eq (2)

$P/Q +1 = (S+R+Ir) / (R+(L-I2)r)$

From the above equations (1) and (2)

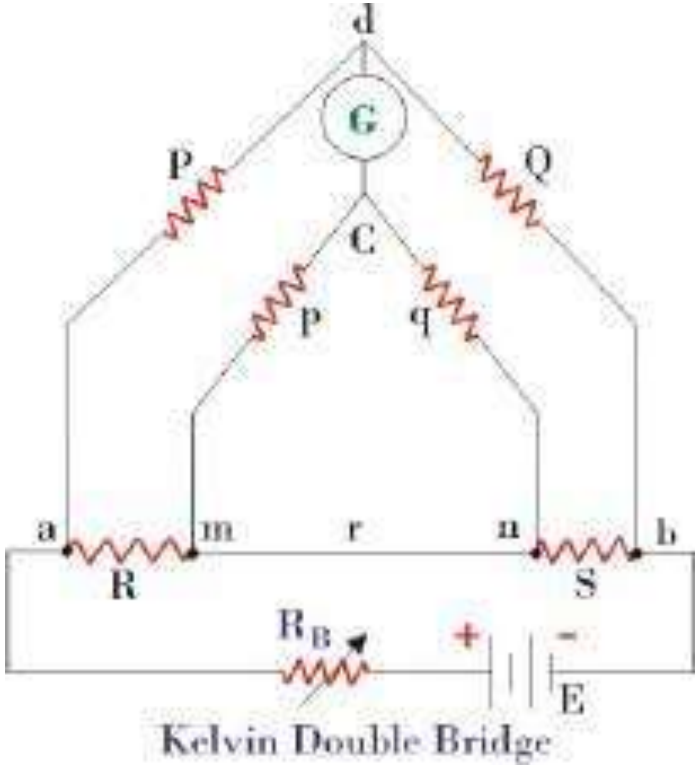
$S+(L-I1)\ r = R+(L-I1)\ r$

$S-R = (I1-I2)$



KELVIN'S DOUBLE BRIDGE

Kelvin's double bridge is a modification of simple Wheatstone bridge. Figure below shows the circuit diagram of Kelvin's double bridge.



$$E_{ad} = E_{amc}$$

$$Or,\ \left\{\frac{P}{P+Q}\right\} E_{ab} = I\left[R + \frac{p}{p+q}\left\{\frac{r(p+q)}{p+q+r}\right\}\right] \ldots\ldots(1)$$

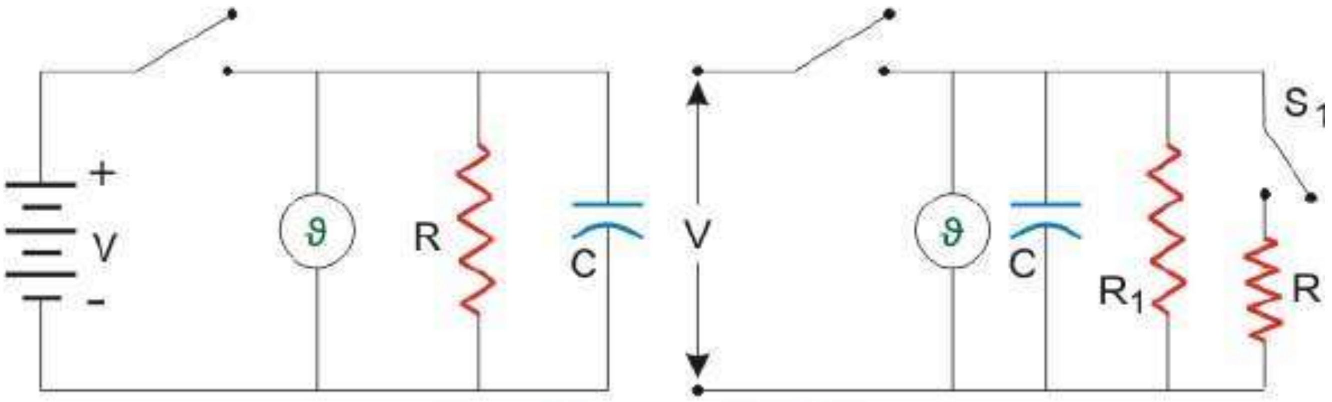
$$Where,\ E_{ab} = I\left[R + S + \frac{p}{p+q}\left\{\frac{r(p+q)}{p+q+r}\right\}\right] \ldots\ldots(2)$$

Putting eqn 2 in 1 and solving and using P/Q = p/q, we get-

$$R = \frac{P}{Q} S$$

Following are few methods used for measurement of high resistance values-

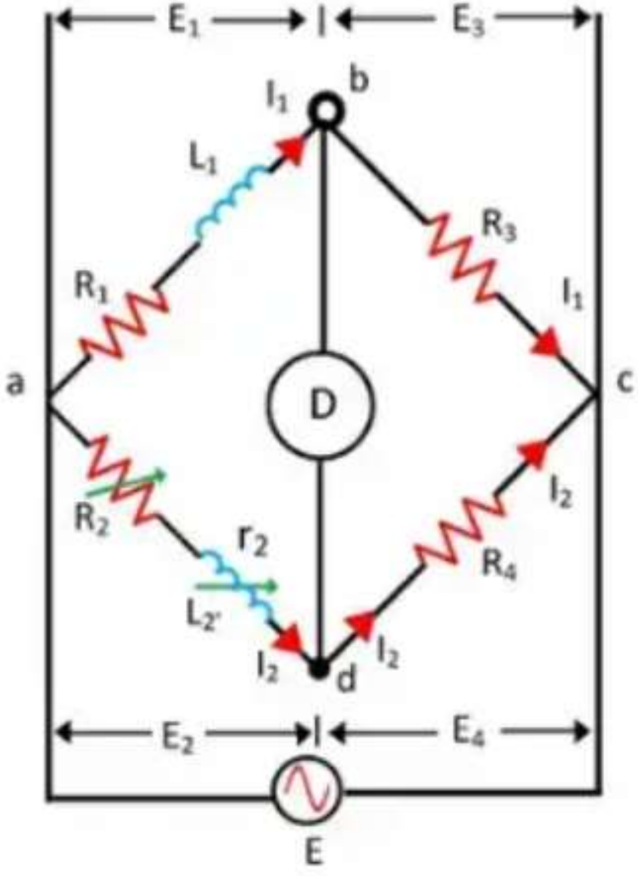
- Loss of Charge Method
- Megger
- Megohm bridge Method
- Direct Deflection MethodLOSS OF CHARGE METHOD

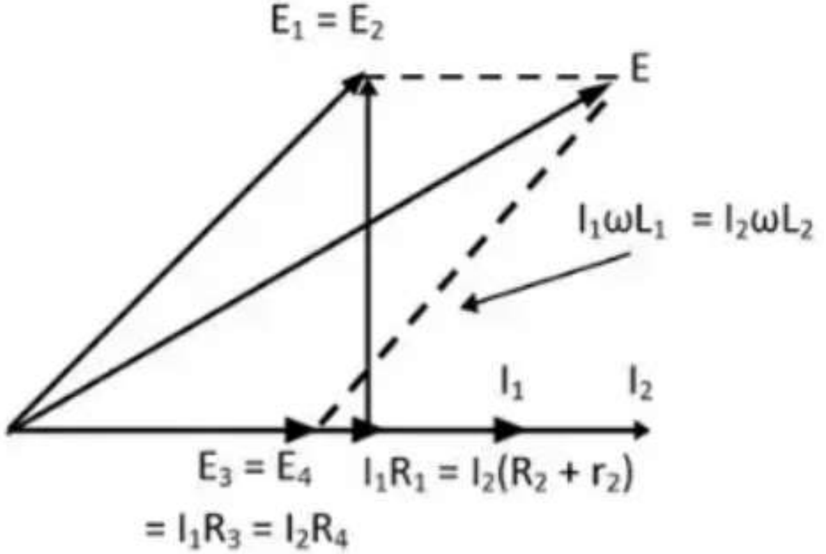


Loss of Charge Method

Maxwell's inductance bridge

The choke for which R_1 and L_1 have to measure connected between the points 'A' and 'B'.
In this method the unknown inductance is measured by comparing it with the standard inductance.





At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwL_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4}$$

Comparing the imaginary parts,

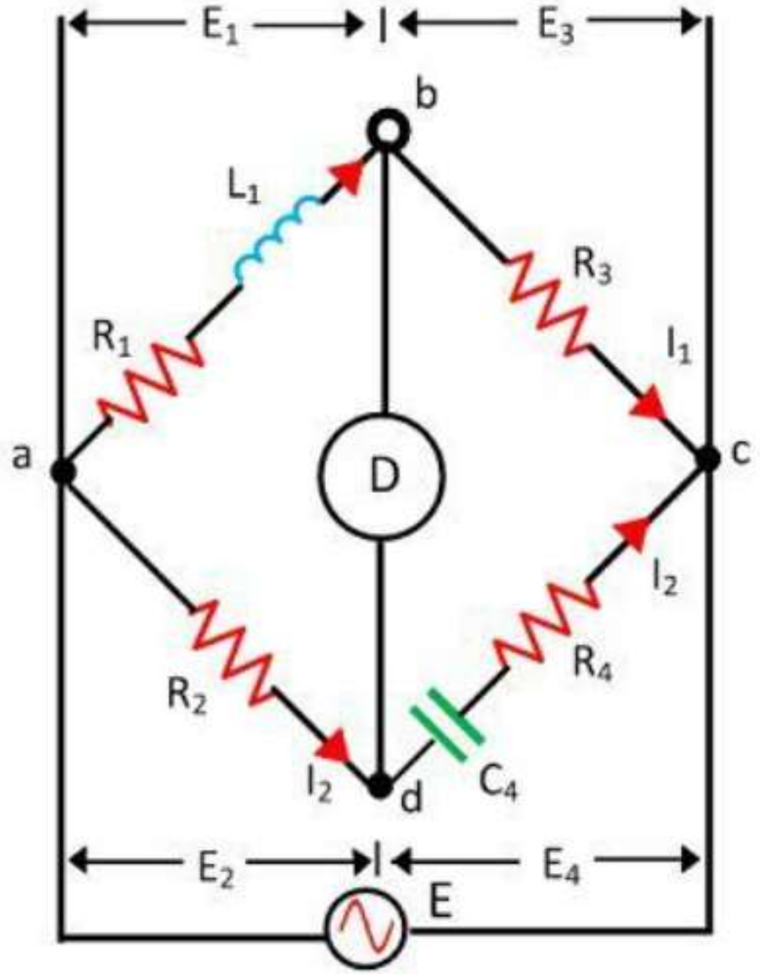
$$wL_1R_4 = wL_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4}$$

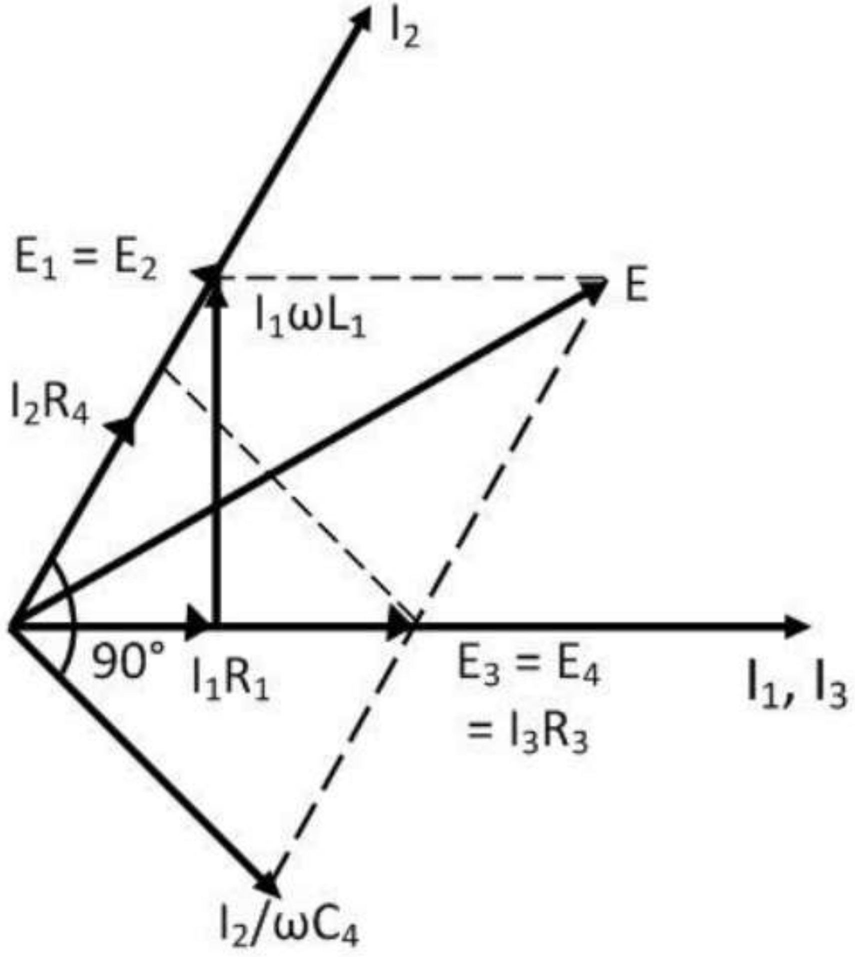
Q-factor of choke, $Q = \dfrac{WL_1}{R_1} = \dfrac{WL_2R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2}$$

Hay’sbridge



E1
E3
b
L1
R3
R1
I1
a
D
c
I2
R2
R4
I2
d
C4
E2
E4
E



I2
E1 = E2
E
I1ωL1
I2R4
90°
I1R1
E3 = E4
= I3R3
I1, I3
I2/ωC4

- $\dot{E}_1 = I_1 R_1 + jI_1 X_1$
- $\dot{E} = \dot{E}_1 + \dot{E}_3$
- $\dot{E}_4 = \dot{I}_4 R_4 + \frac{I_4}{jwC_4}$
- $\dot{E}_3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

$$L_1 = \frac{C_4 R_2 R_3}{1 + w^2 L_1 C_4{}^2 R_4{}^2}$$

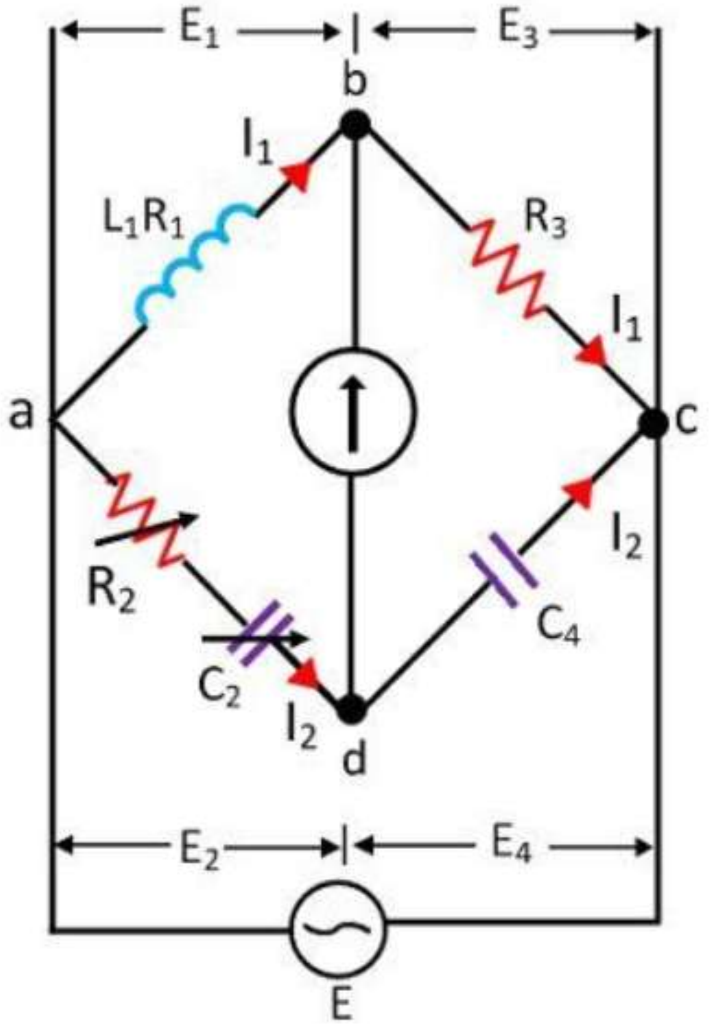
Substituting the value of L_1 in eqn. 2.14 , we have

$$R_1 = \frac{w^2 C_4{}^2 R_2 R_3 R_4}{1 + w^2 C_4{}^2 R_4{}^2}$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4 R_2 R_3}{1 + w^2 C_4{}^2 R_4{}^2} \times \frac{1 + w^2 C_4{}^2 R_4{}^2}{w^2 C_4{}^2 R_4 R_2 R_3}$$

$$Q = \frac{1}{wC_4R_4}$$

Owen's bridge



- $\dot{E} = \dot{E}_1 + \dot{E}_3$
- $\dot{E}_2 = I_2 R_2 + \dfrac{I_2}{jwC_2}$

Balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$Z_2 = R_2 + \frac{1}{jwC_2} = \frac{jwC_2R_2 + 1}{jwC_2}$$

$$\therefore (R_1 + jwL_1) \times \frac{1}{jwC_4} = \frac{(1 + jwR_2C_2) \times R_3}{jwC_2}$$

$$C_2(R_1 + jwL_1) = R_3C_4(1 + jwR_2C_2)$$

$$R_1C_2 + jwL_1C_2 = R_3C_4 + jwR_2C_2R_3C_4$$

Comparing real terms,

$$R_1C_2 = R_3C_4$$

$$R_1 = \frac{R_3 C_4}{C_2}$$

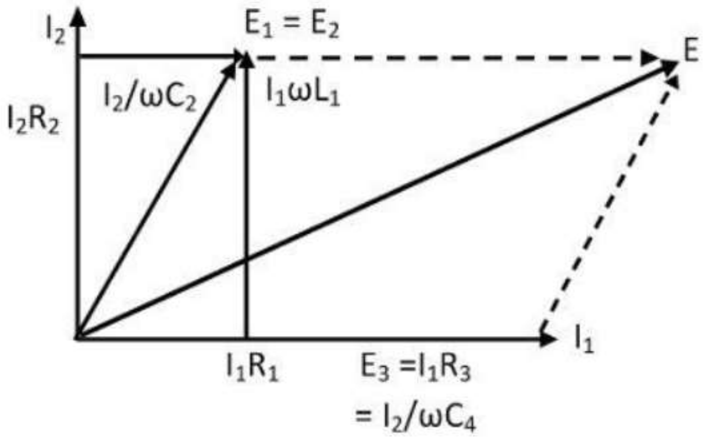
Comparing imaginary terms,

$$wL_1C_2 = wR_2C_2R_3C_4$$

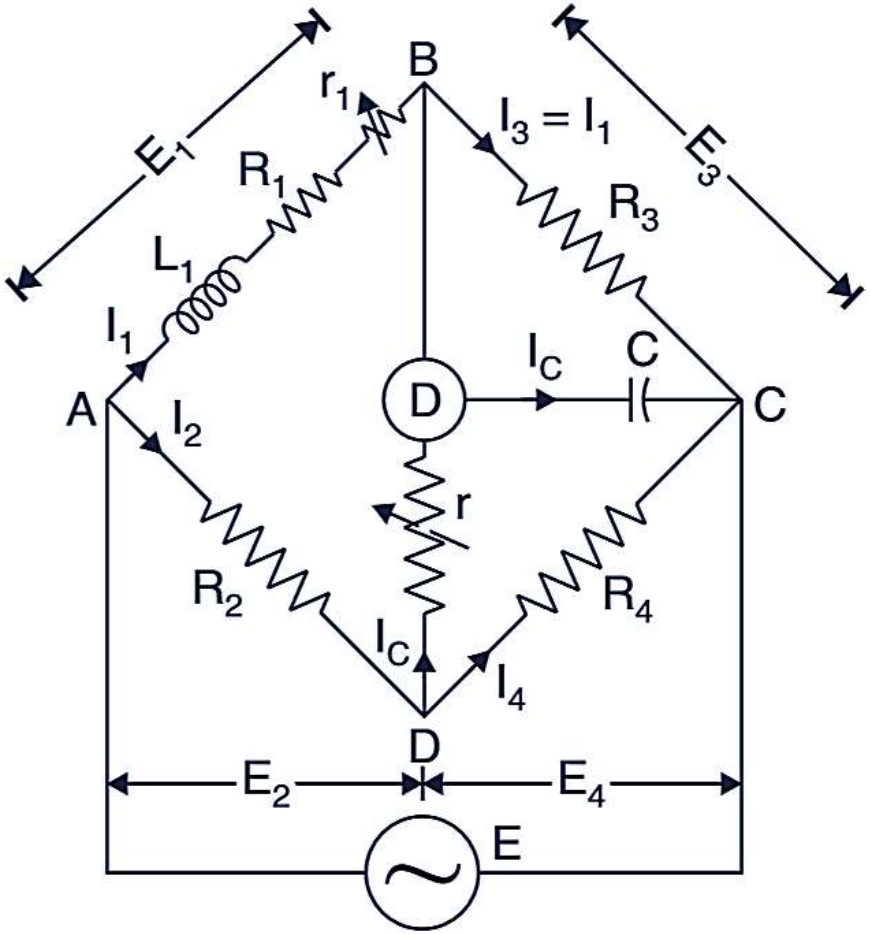
$$L_1 = R_2R_3C_4$$

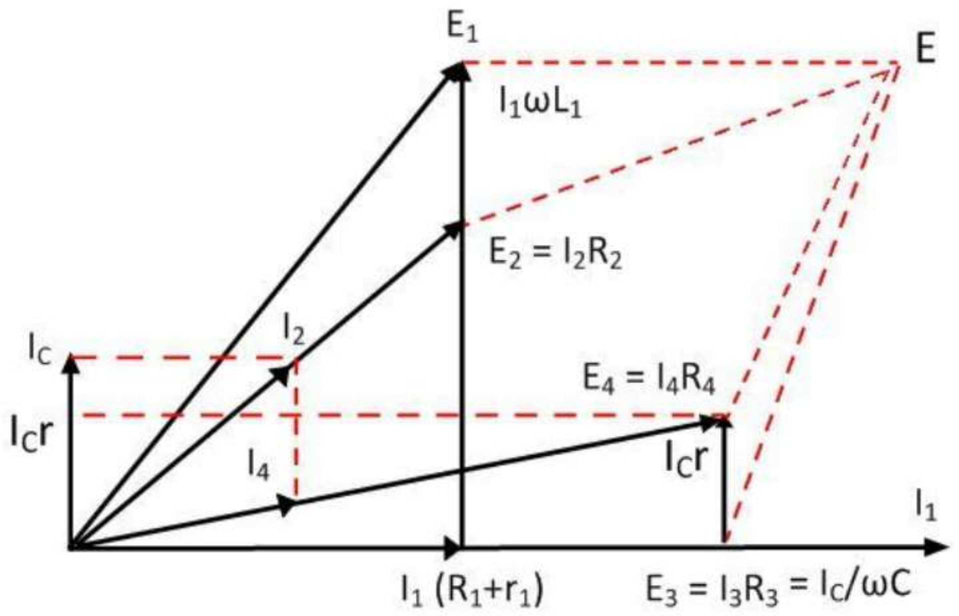
Q- factor = $\dfrac{WL_1}{R_1} = \dfrac{wR_2R_3C_4C_2}{R_3C_4}$

$$Q = wR_2C_2$$



Anderson’s bridge





Comparing real term,

$$R_1^1 R_4 = R_2 R_3$$

$$(R_1 + r_1) R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

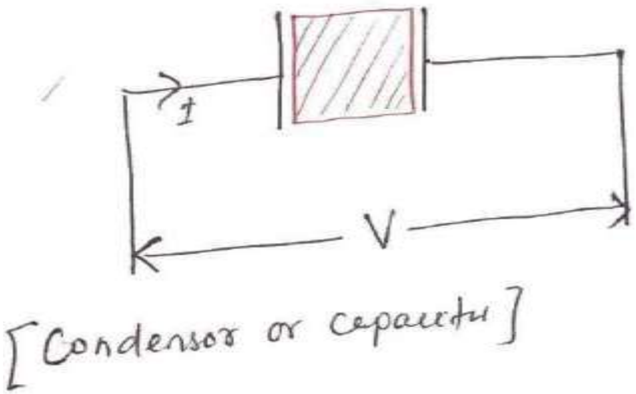
Comparing the imaginary term,

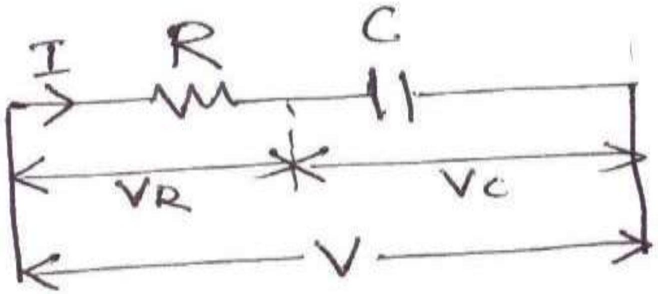
$$wL_1 R_4 = wCR_2 R_3 (r + R_4) + wcrR_3 R_4$$

$$L_1 = \frac{R_2 R_3 C}{R_4}(r + R_4) + R_3 rC$$

$$L_1 = R_3 C\left[\frac{R_2}{R_4}(r + R_4) + r\right]$$

Measurement of capacitance and loss angle





A dissipation factor is defined as 'tan δ'.

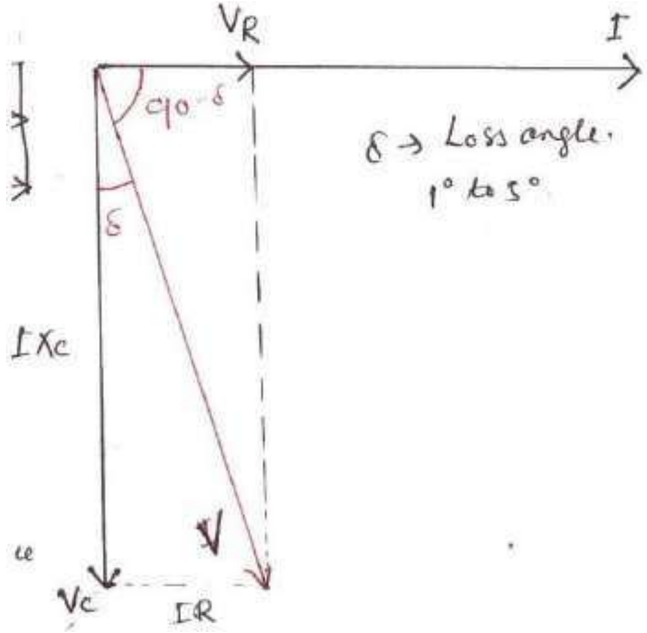
$$\therefore \tan\delta = \frac{IR}{IX_C} = \frac{R}{X_C} = wCR$$

$$D = wCR$$

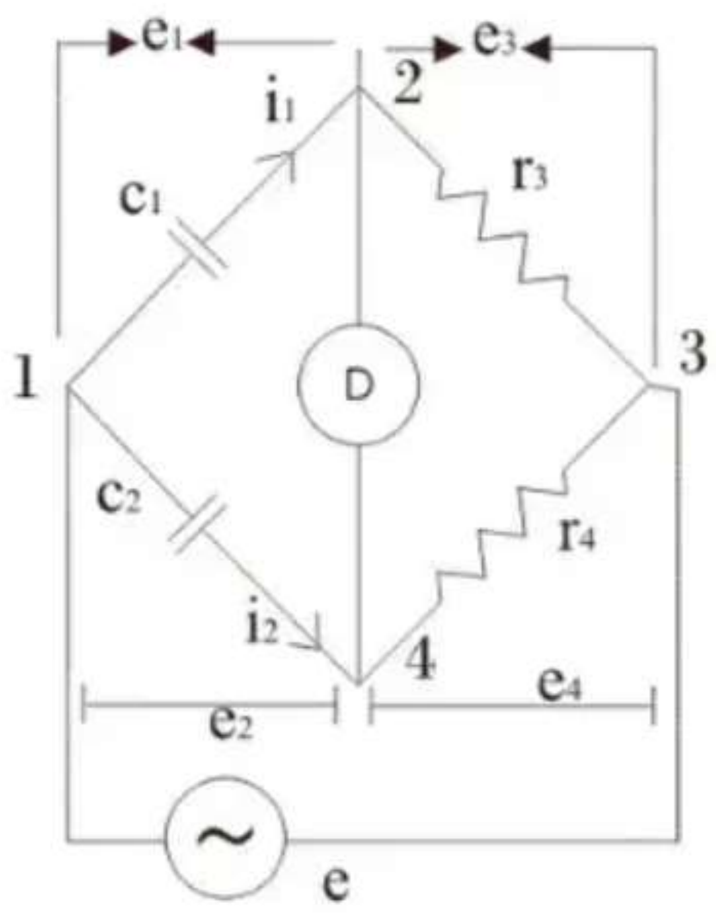
$$D = \frac{1}{Q}$$

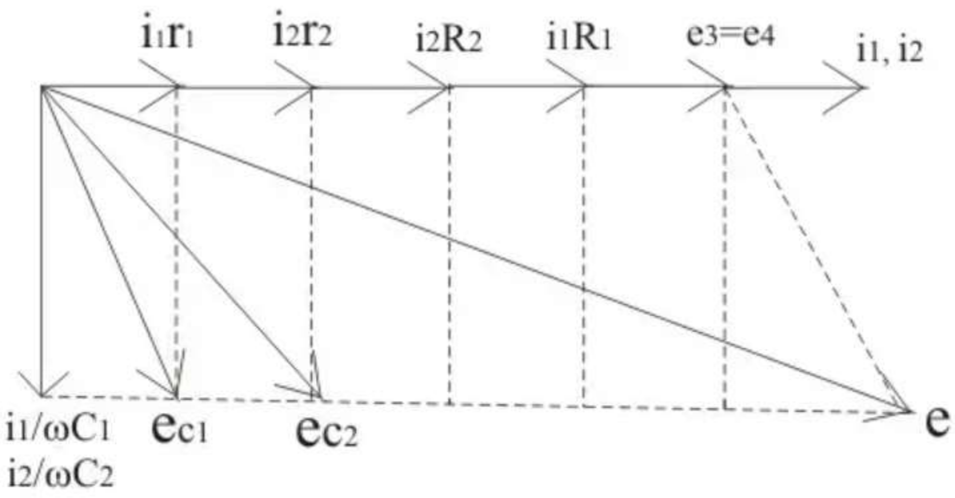
$$D = \tan\delta = \frac{\sin\delta}{\cos\delta} \cong \frac{\delta}{1}$$ For small value of 'δ' in radians

$D \cong \delta \cong$ Loss Angle ('δ' must be in radian)



Desauty'sBridge





C_1= Unknown capacitance

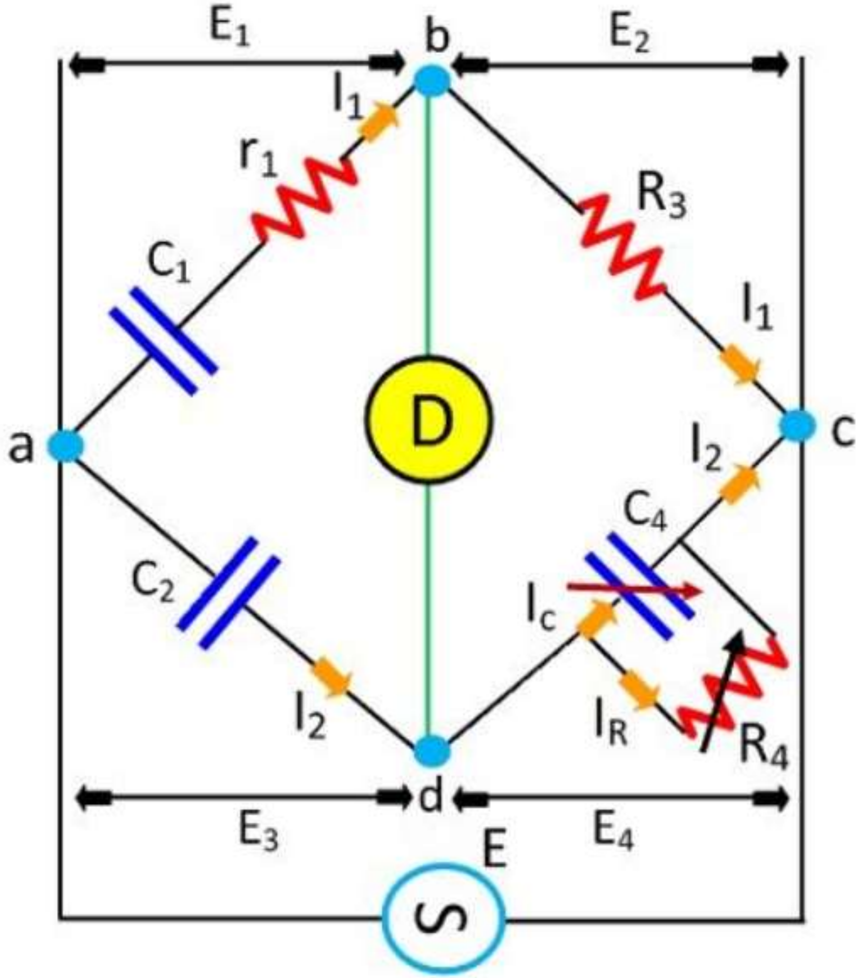
At balance condition,

$$\frac{1}{jwC_1} \times R_4 = \frac{1}{jwC_2} \times R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

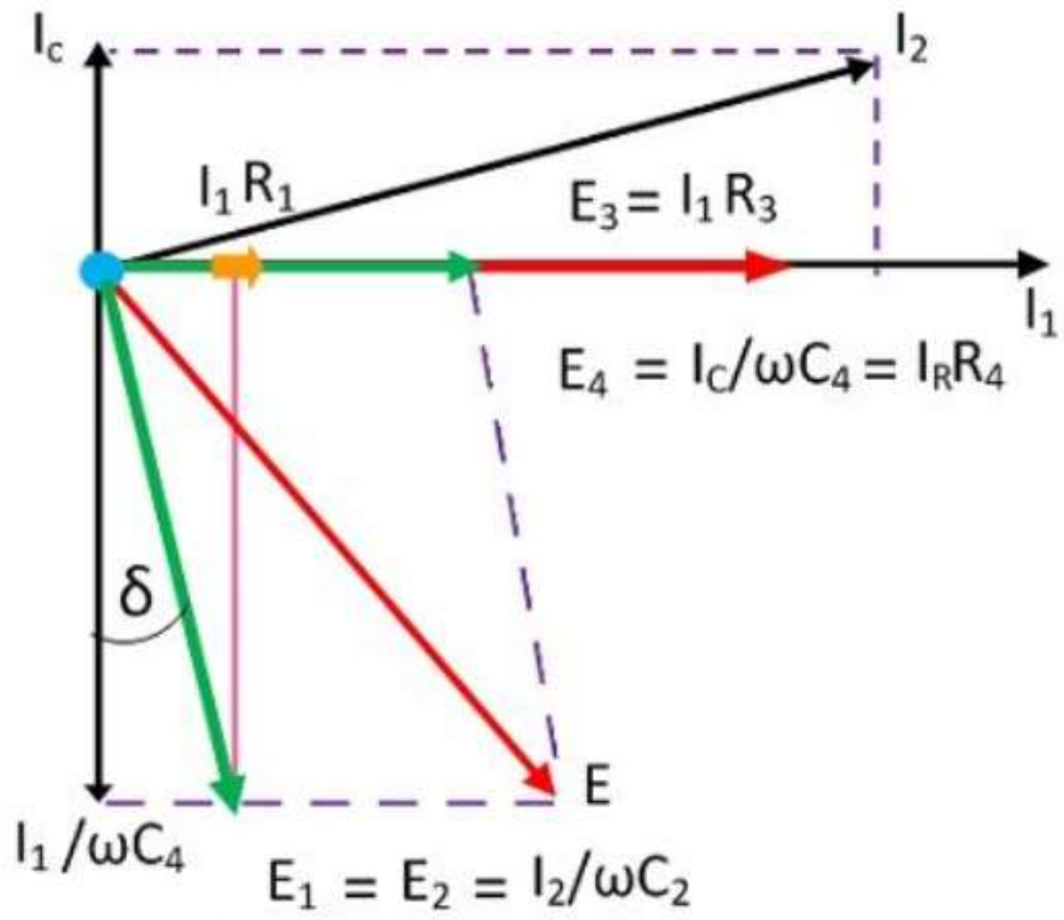
$$\Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

Schering bridge



$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}} = \frac{R_4}{1 + jwC_4 R_4}$$



I_c
I_2
$I_1 R_1$
$E_3 = I_1 R_3$
I_1
$E_4 = I_C/\omega C_4 = I_R R_4$
δ
E
$I_1/\omega C_4$
$E_1 = E_2 = I_2/\omega C_2$

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{1+jwC_1r_1}{jwC_1} \times \frac{R_4}{1+jwC_4R_4} = \frac{R_3}{jwC_2}$$

$$(1+jwC_1r_1)R_4C_2 = R_3C_1(1+jwC_4r_4)$$

$$R_2C_2 + jwC_1r_1R_4C_2 = R_3C_1 + jwC_4R_4R_3C_1$$

Comparing the real part,

$$\therefore C_1 = \frac{R_4C_2}{R_3}$$

Comparing the imaginary part,

$$wC_1r_1R_4C_2 = wC_4R_3R_4C_1$$

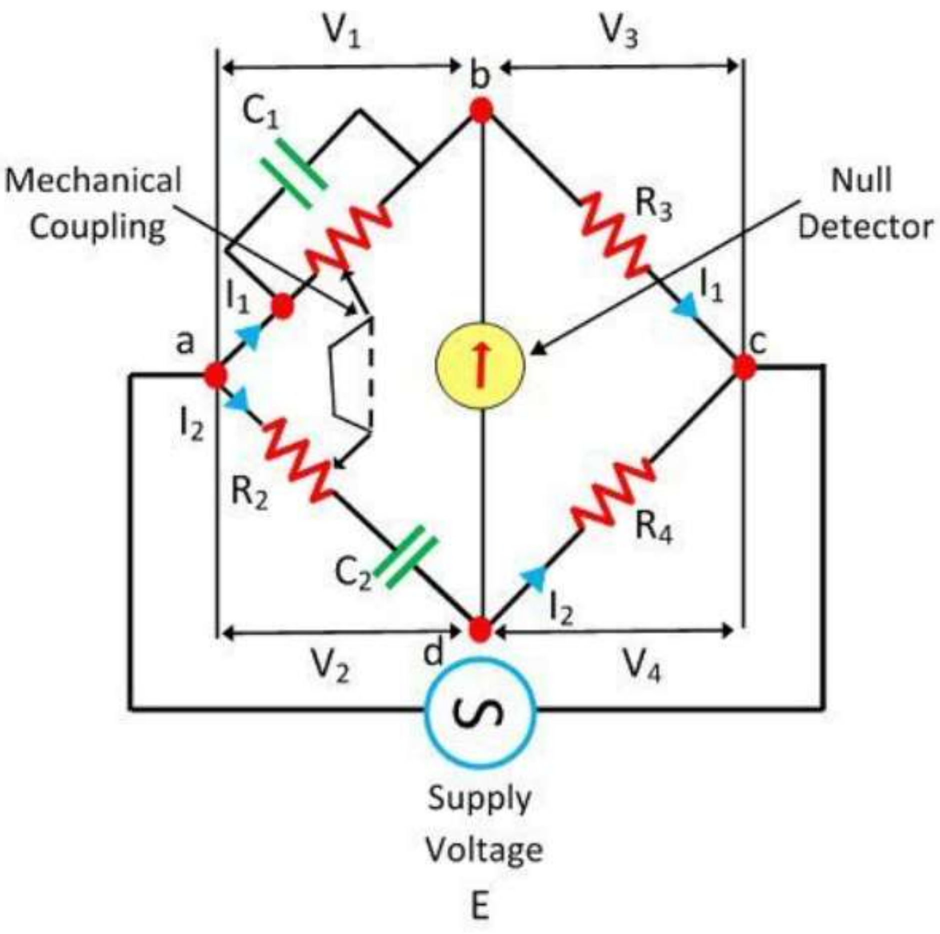
$$r_1 = \frac{C_4R_3}{C_2}$$

Dissipation factor of capacitor,

$$D = wC_1r_1 = w \times \frac{R_4C_2}{R_3} \times \frac{C_4R_3}{C_2}$$

$$\therefore D = wC_4R_4$$

Wein's bridge



$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

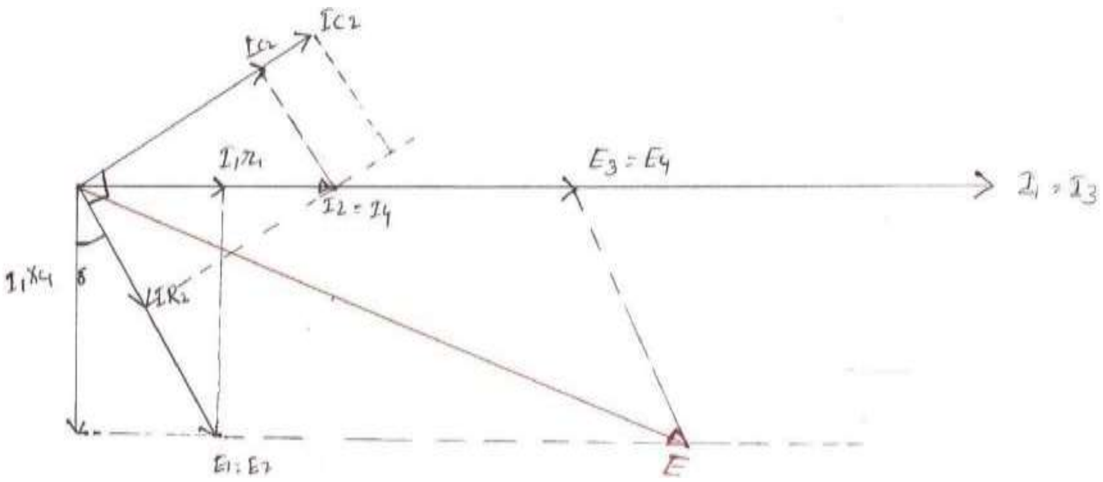
$$Z_2 = \frac{R_2}{1 + jwC_2 R_2}$$

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{jwC_1 r_1 + 1}{jwC_1} \times R_4 = \frac{R_2}{1 + jwC_2 R_2} \times R_3$$

$$(1 + jwC_1 r_1)(1 + jwC_2 R_2) R_4 = R_2 R_3 \times jwC_1$$

$$\left[1 + jwC_2 R_2 + jwC_1 r_1 - w^2 C_1 C_2 r_1 R_2\right] = jwC_1 \frac{R_2 R_3}{R_4}$$



Comparing real

term,

$$1 - w^2 C_1 C_2 r_1 R_2$$

$$= 0$$

$$w^2 C_1 C_2 r_1 R_2 = 1$$

$$w^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$w = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}} \quad , f = \frac{1}{2\Pi \sqrt{C_1 C_2 r_1 R_2}}$$