



Department of Electrical and Electronics Engineering

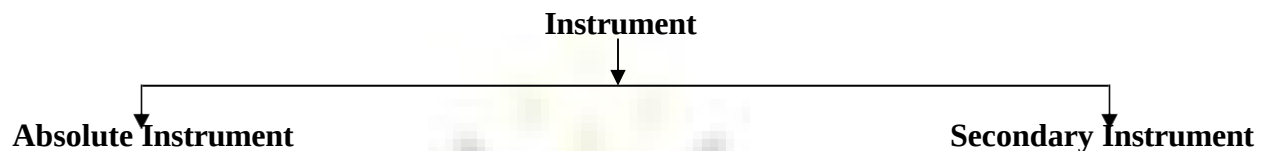
Electrical Measurements and Sensors (25EE305)

UNIT – I

INTRODUCTION TO MEASURING INSTRUMENTS:

1. Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified into two categories.



Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is rarely used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.

Secondary instruments

Indicating instruments

Recording

instruments

Integrating

Electromechanically Indicating

Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over aspecified period of time.

Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary. They are

- (a) Deflecting force
- (b) Controlling force
- (c) Damping force
- (d) **Deflecting force**

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

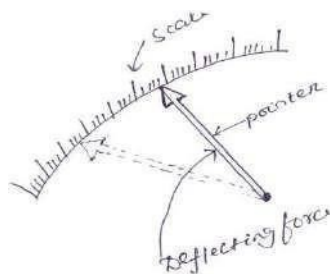


Fig. 1.1 Pointer scale

Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft- iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

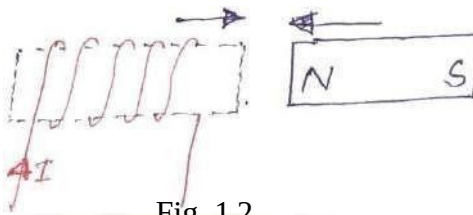


Fig. 1.2

If two soft iron pieces are placed near a current-carrying coil, there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

Force between a permanent magnet and a current-carrying coil

When a current-carrying coil is placed under the influence of a magnetic field produced by a permanent magnet, a force is produced between them. This principle is utilized in the moving coil type instrument.

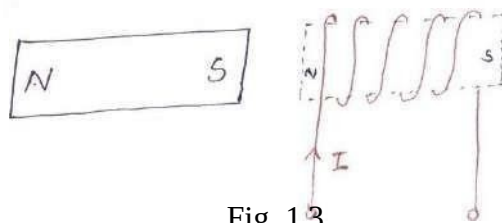


Fig. 1.3

Force between two current-carrying coils

When two current-carrying coils are placed closer to each other, there will be a force of repulsion between them. If one coil is movable and the other is fixed, the movable coil will move away from the fixed one. This principle is utilized in the electro-dynamometer type instrument.

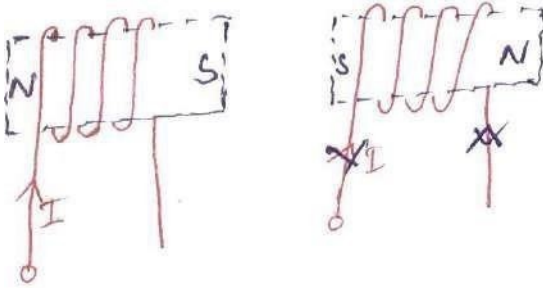


Fig. 1.4

Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \quad (1.1)$$

Spring control

Two springs are attached on either end of spindle (Fig. 1.5). The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ .

$$T_C \propto \theta \quad (1.2)$$

The deflecting torque produced T_d proportional to 'I'. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \quad (1.3)$$

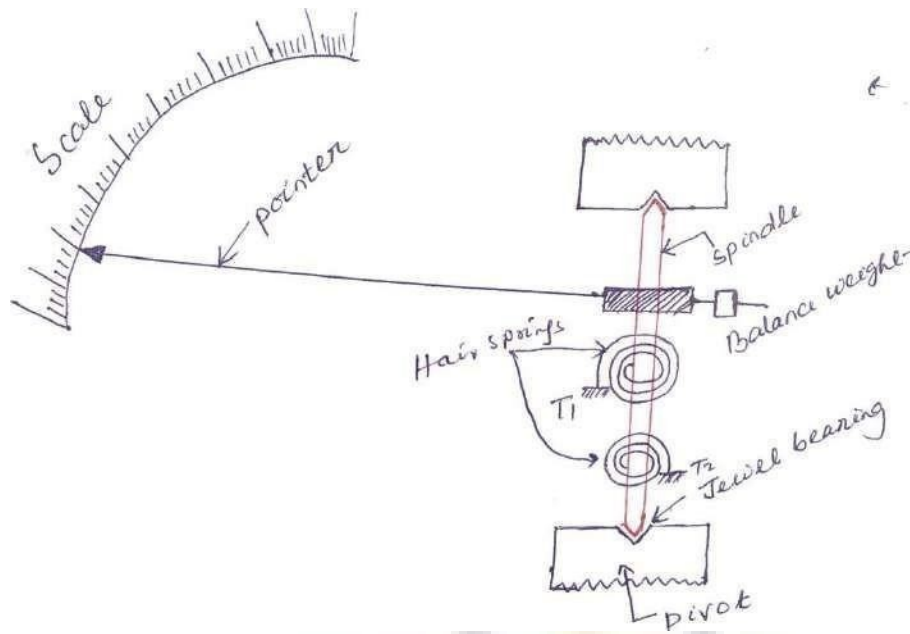


Fig. 1.5

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about its final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation quickly, a damping force is necessary. This force is produced by different systems.

- (a) Air friction damping
- (b) Fluid friction damping
- (c) Eddy current damping

Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle and moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

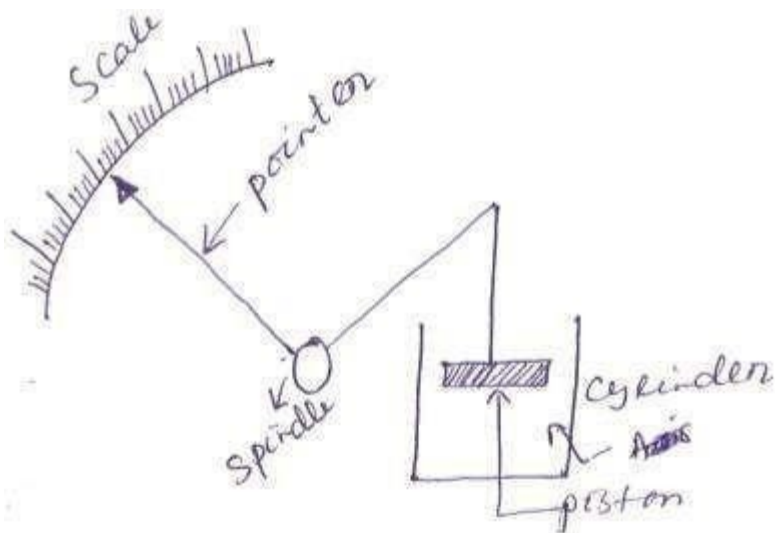


Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves downwards. The pointer tends to move in clockwise direction.

Eddy current damping

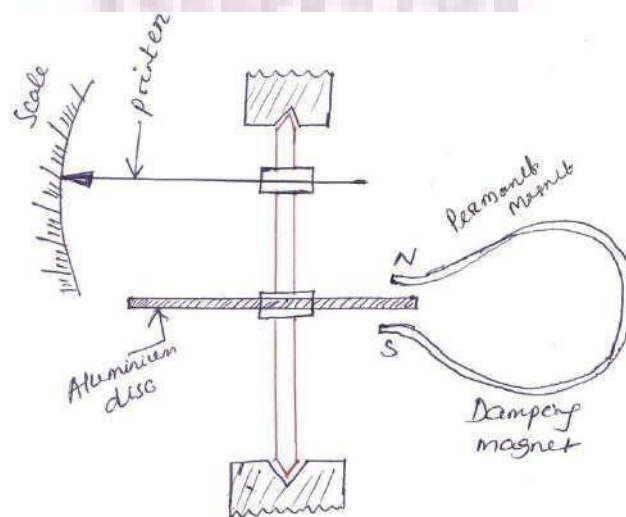


Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by Faraday's law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produces a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

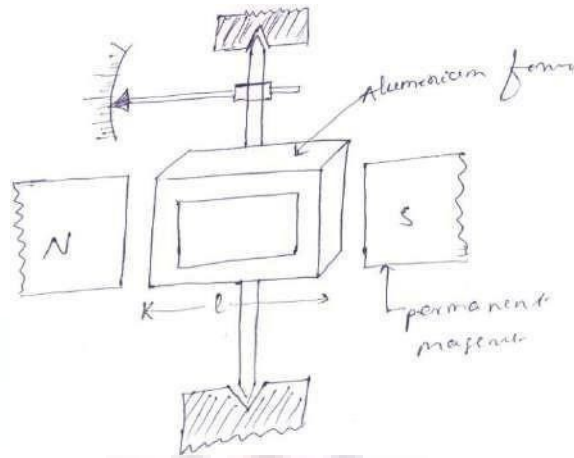


Fig. 1.6 Rectangular type

Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument. **Construction:** A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.

Control: Spring control is used.

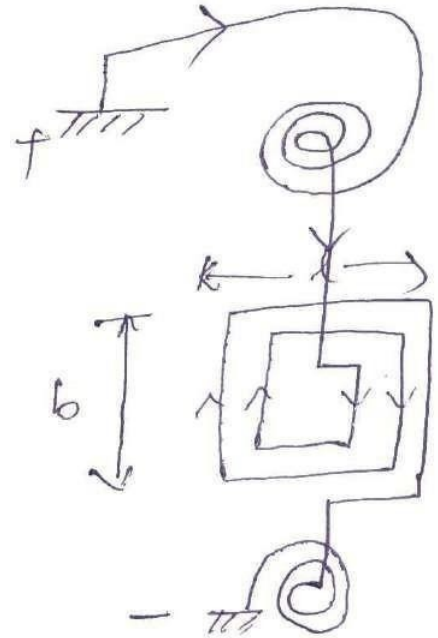
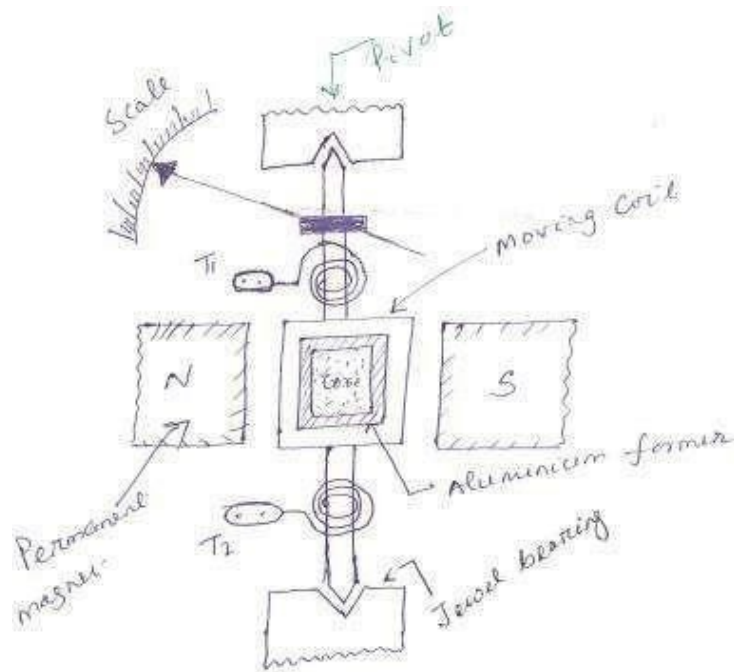


Fig. 1.7

Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument. If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC

Let T_d = deflecting torque

T_C = controlling torque

θ = angle of deflection

K = spring constant b = width
of the coil

l =height of the coil or length of coil N =No. of turns

I =current B =Flux density A =area of the coil

The force produced in the coil is given by

$$F = BIL \sin\theta$$

When $\theta = 90^\circ$ (1.4)

For N turns, $F = NBIL$ (1.5)

Torque produced $T_d = F \times \perp r$ distance (1.6)

$$T_d = NBIL \times b = BINA$$
 (1.7)

$$T_d = BAN I$$
 (1.8)

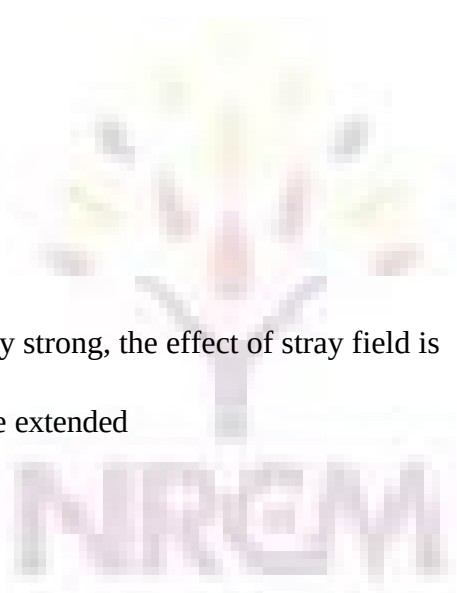
$$T_d \propto I$$
 (1.9)

Advantages

- ✓ Torque/weight is high
- ✓ Power consumption is less
- ✓ Scale is uniform
- ✓ Damping is very effective
- ✓ Since operating field is very strong, the effect of stray field is negligible
- ✓ Range of instrument can be extended

Disadvantages

- ✓ Use only for D.C.
- ✓ Cost is high
- ✓ Error is produced due to ageing effect of PMMC
- ✓ Friction and temperature error are present



| 1 + $R \rightarrow$ Multiplication facto

Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let ' θ ' be the deflection corresponding to a current of ' i ' amp

Let the current increases by di , the corresponding deflection is ' $\theta + d\theta$ '

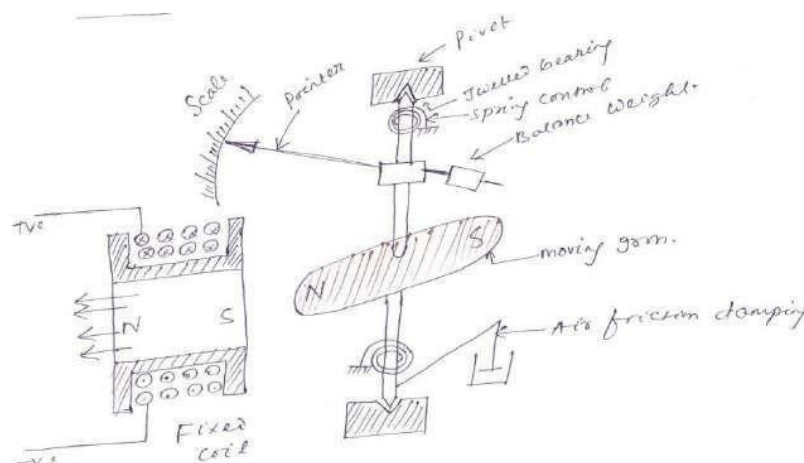


Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ' $L+dL$ '. The current change by ' di ' is dt seconds.

Let the emf induced in the coil be ' e ' volt.

$$e = \frac{d}{dt}(Li) = L \frac{di}{dt} + i \frac{dL}{dt} \quad (1.22)$$

Multiplying by ' idt ' in equation (1.22)

$$e \times idt = L \frac{di}{dt} \times idt + i \frac{dL}{dt} \times idt \quad (1.23)$$

$$e \times idt = Lidi + i^2 dL \quad (1.24)$$

Eqⁿ (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.



Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$\begin{aligned}
 &= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2}Li^2 \\
 &= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\} \\
 &= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\} \\
 &= \frac{1}{2}\{Li^2 + 2Lidi + i^2dL + 2ididL - Li^2\} \\
 &= \frac{1}{2}\{2Lidi + i^2dL\} \\
 &= Lidi + \frac{1}{2}i^2dL
 \end{aligned} \tag{1.25}$$

Mechanical work to move the pointer by $d\theta$

$$= T_d d\theta \tag{1.26}$$

By law of conservation of energy,

Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi + i^2dL = Lidi + \frac{1}{2}i^2dL + T_d d\theta \tag{1.27}$$

$$\frac{1}{2}i^2dL = T_d d\theta \tag{1.28}$$

$$T_d = \frac{1}{2}i^2 \frac{dL}{d\theta} \tag{1.29}$$

At steady state condition $T_d = T_C$

$$\frac{1}{2}i^2 \frac{dL}{d\theta} = K\theta \tag{1.30}$$

$$\theta = \frac{1}{2K}i^2 \frac{dL}{d\theta} \tag{1.31}$$

$$\theta \propto i^2 \tag{1.32}$$

When the instruments measure AC, $\theta \propto i_{rms}^2$

Advantages

- ✓ MI can be used in AC and DC
- ✓ It is cheap
- ✓ Supply is given to a fixed coil, not in moving coil.
- ✓ Simple construction
- ✓ Less friction error.

Disadvantages

- ✓ It suffers from eddy current and hysteresis error
- ✓ Scale is not uniform
- ✓ It consumed more power
- ✓ Calibration is different for AC and DC operation

Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation. Control: Spring control is used.

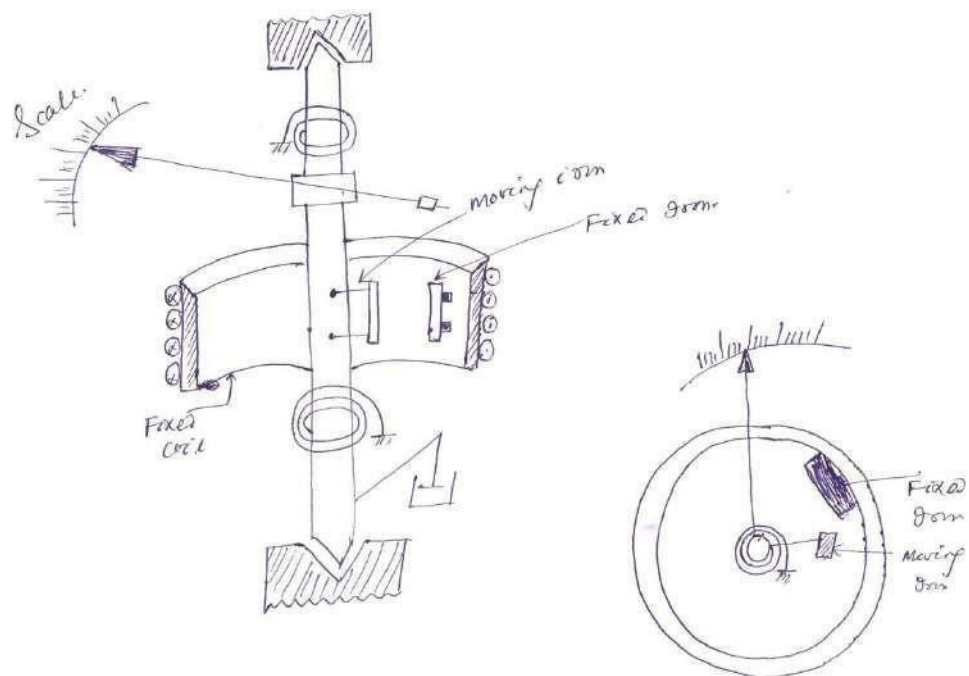


Fig. 1.12

Dynamometer (or) Electromagnetic moving coil instrument(EMMC)

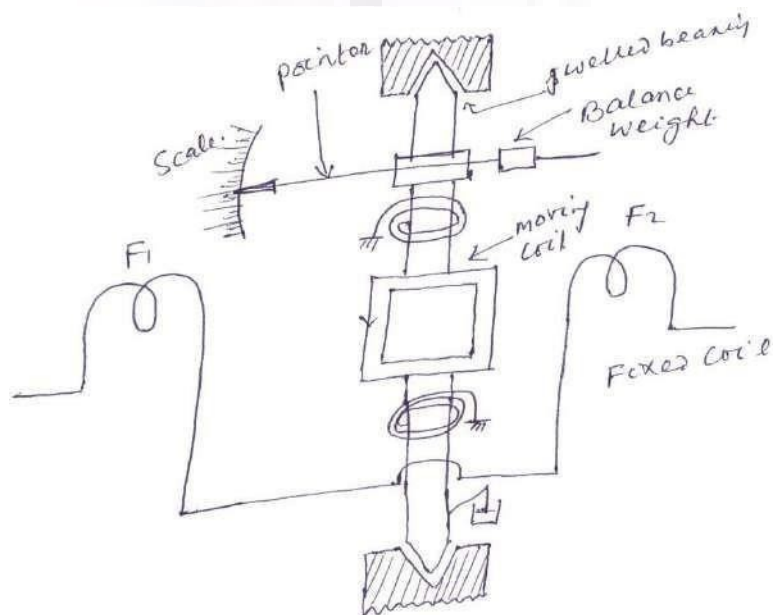


Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction: A fixed coil is divided into two equal halves. The moving coil is placed between the two halves of the fixed coil. Both the fixed and moving coils are air-cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non-metallic former the moving coil is wound.

Control: Spring control is used.

Damping: Air friction damping

is used. **Principle of operation:**

When the current flows through the fixed coil, it produces a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC

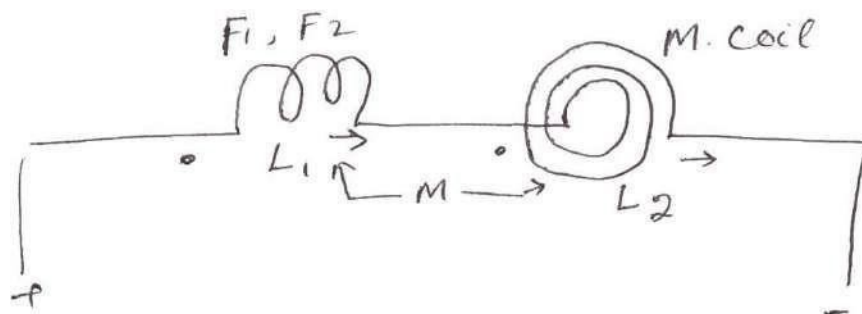


Fig. 1.14

Let

L_1 =Self inductance of fixed coil

L_2 =Self inductance of moving coil

M =mutual inductance between fixed coil and moving coil i_1 =current through fixed coil

i_2 =current through moving coil Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \quad (1.33)$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \quad (1.34)$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta} (L_1 + L_2 + 2M) \quad (1.35)$$

The value of L_1 and L_2 are independent of ' θ ' but ' M ' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \quad (1.36)$$

$$T_d = i^2 \frac{dM}{d\theta} \quad (1.37)$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \quad (1.38)$$

$$T_C = T_d \quad (1.39)$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} \quad (1.40)$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

Extension of PMMC instrument Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are redesigned such that the resistance of each branch is same.

Therefore

$$I_1 = I_2 = I$$

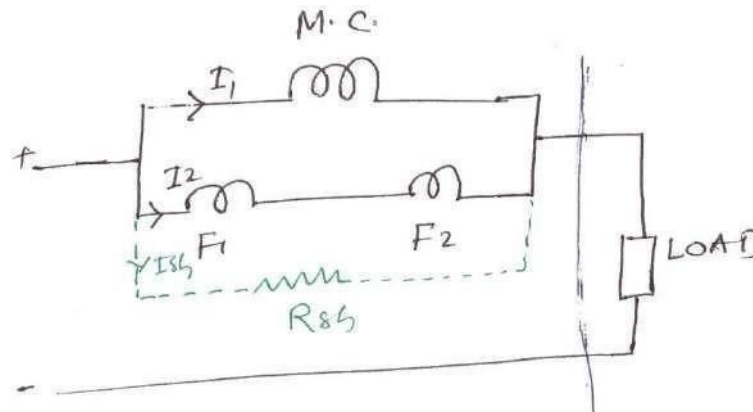


Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value R_{sh} is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \quad (1.41)$$

$$\text{Or } \therefore T_d = I^2 \frac{dM}{d\theta} \quad (1.42)$$

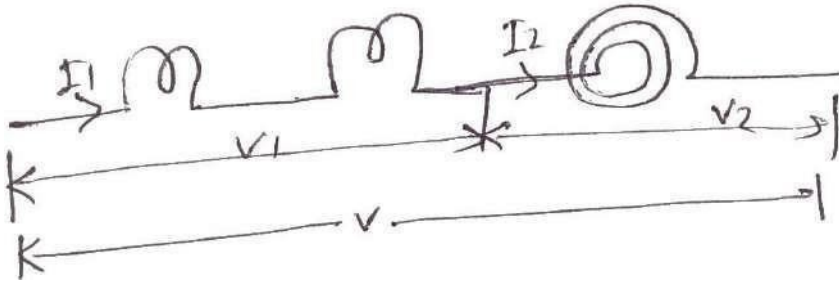
$$T_C = K\theta \quad (1.43)$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \quad (1.44)$$

$$\therefore \theta \propto I^2 \text{ (Scale is not uniform)} \quad (1.45)$$

Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extend the range of voltmeter.



$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \quad (1.46)$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \quad (1.47)$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \quad (1.48)$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \quad (1.49)$$

$$T_d \propto V^2 \quad (1.50)$$

$$\therefore \theta \propto V^2 \text{ (Scale is not uniform)} \quad (1.51)$$



Case-III As wattmeter

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

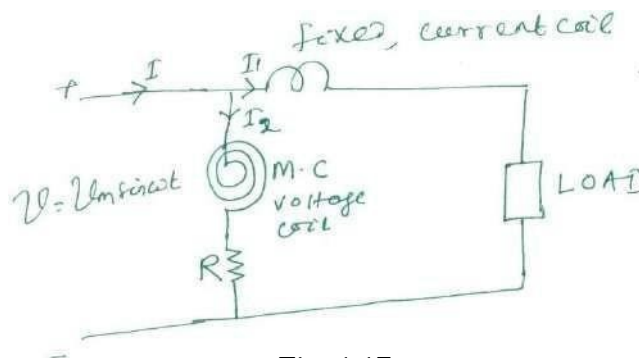


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R'. The current,

$$I_2 = \frac{V_m \sin \omega t}{R}$$

Let the phase difference between the currents I_1 and I_2 is ϕ

$$I_1 = I_m \sin(\omega t - \phi)$$

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

$$T_d = I_m \sin(\omega t - \phi) \times \frac{V_m \sin \omega t}{R} \frac{dM}{d\theta}$$

$$T_d = \frac{1}{R} (I_m V_m \sin \omega t \sin(\omega t - \phi)) \frac{dM}{d\theta}$$

$$T_d = \frac{1}{R} I_m V_m \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta}$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} T_d \times d(\omega t) \quad (1.58)$$

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{R} \times I_m V_m \sin \omega t \sin(\omega t - \phi) \frac{dM}{d\theta} \times d(\omega t) \quad (1.59)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\pi} \times \frac{1}{R} \times \frac{dM}{d\theta} \left[\int_0^{2\pi} (\cos \phi - \cos(2\omega t - \phi)) d\omega t \right] \quad (1.60)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} \left[\int_0^{2\pi} \cos \phi d\omega t - \int_0^{2\pi} \cos(2\omega t - \phi) d\omega t \right] \quad (1.61)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} [\cos \phi]_0^{2\pi} \quad (1.62)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} [\cos \phi (2\pi - 0)] \quad (1.63)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos \phi \quad (1.64)$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos \phi \times \frac{1}{R} \times \frac{dM}{d\theta} \quad (1.65)$$

$$(T_d)_{avg} \propto KVI \cos \phi$$

$$T_C \propto \theta$$

$$\theta \propto KVI \cos \phi$$

$$\theta \propto VI \cos \phi$$

Advantages

- ✓ It can be used for voltmeter, ammeter and wattmeter
- ✓ Hysteresis error is nill
- ✓ Eddy current error is nill
- ✓ Damping is effective
- ✓ It can be measure correctively and accurately the rms value of the voltage

Disadvantages

- ✓ Scale is not uniform
- ✓ Power consumption is high(because of high resistance)
- ✓ Cost is more
- ✓ Error is produced due to frequency, temperature and stray field.
- ✓ Torque/weight is low.(Because field strength is very low)

Errors in PMMC

- ✓ The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- ✓ The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- ✓ When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

Difference between attraction and repulsion type instrument

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

Characteristics of meter

Full scale deflection current(I_{FSD})

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between 2 μ A to 30mA.

Resistance of the coil(R_m)

This is ohmic resistance of the moving coil. It is due to ρ , L and A. For an ammeter this should be as small as possible.

Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}} (\Omega / \text{volt}), \uparrow S = \frac{Z}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in Ω/volt . When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading effect is negligible. It is also defined as one over full scale deflection current.

Error in M.I instrument

Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_s)^2 + X^2} \quad (1.70)$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_s)^2 + X^2}} \quad (1.71)$$

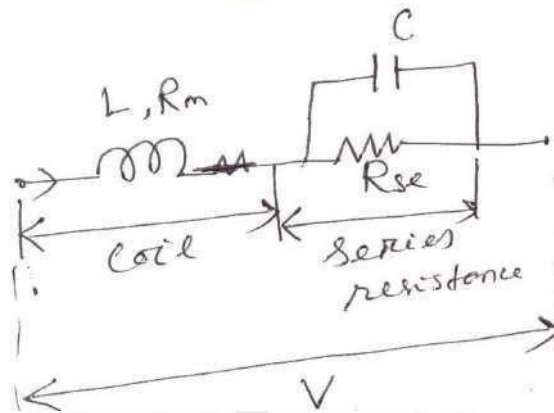


Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, $(X_L - X_C)$ is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_s)^2} \quad (1.72)$$

Electrostatic instrument

In multi cellular construction several vanes and quadrants are provided. The voltage to be measured is applied between the vanes and quadrant. The force of attraction between the vanes

and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

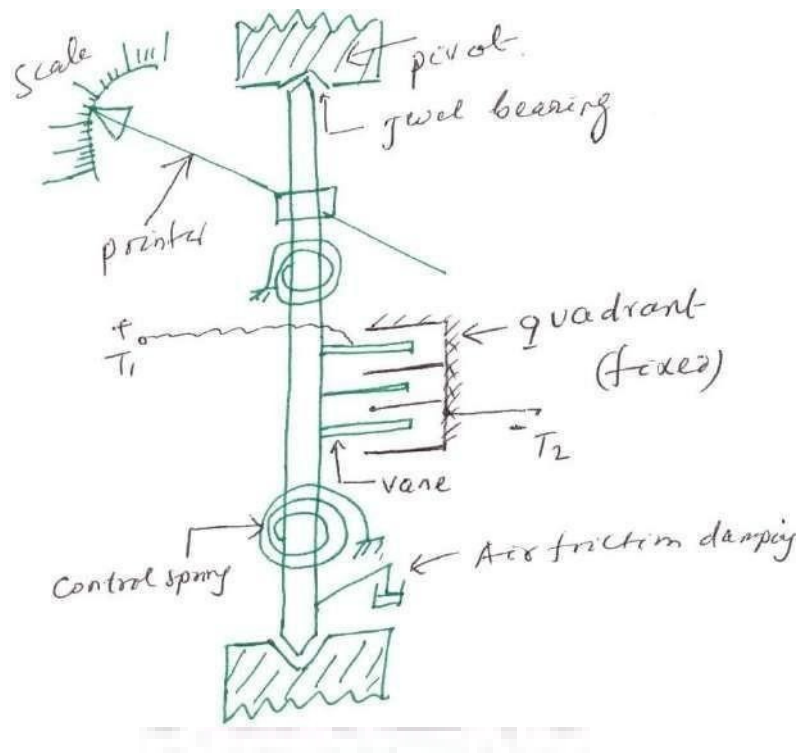


Fig. 1.19

torque develop by electrostatic instrument

V = Voltage applied between vane and quadrant

C = capacitance between vane and quadrant

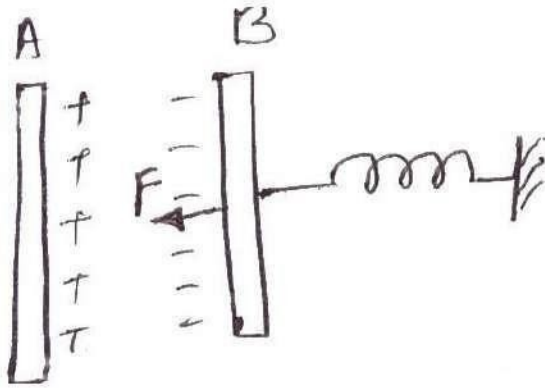


Fig. 1.20

$$Vidt = \frac{dC}{dt} V^2 dt + CV \frac{dV}{dt} dt \quad (1.75)$$

$$Vidt = V^2 dC + CVdV \quad (1.76)$$

$$\text{Change in stored energy} = \frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2 \quad (1.77)$$

$$\begin{aligned} &= \frac{1}{2}[(C + dC)V^2 + dV^2 + 2VdV] - \frac{1}{2}CV^2 \\ &= \frac{1}{2}[CV^2 + CdV^2 + 2CVdV + V^2dC + dCdV^2 + 2VdVdC] - \frac{1}{2}CV^2 \\ &= \frac{1}{2}V^2dC + CVdV \\ V^2dC + CVdV &= \frac{1}{2}V^2dC + CVdV + F \times rd\theta \end{aligned} \quad (1.78)$$

$$T_d \times d\theta = \frac{1}{2}V^2dC \quad (1.79)$$

$$T_d = \frac{1}{2}V^2 \left(\frac{dC}{d\theta} \right) \quad (1.80)$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2 \left(\frac{dC}{d\theta} \right) \quad (1.81)$$

$$\theta = \frac{1}{2K}V^2 \left(\frac{dC}{d\theta} \right) \quad (1.82)$$

Advantages

- ✓ It is used in both AC and DC.
- ✓ There is no frequency error.
- ✓ There is no hysteresis error.
- ✓ There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- ✓ It is used for high voltage

- ✓ Power consumption is negligible.

Disadvantages

- ✓ Scale is not uniform
- ✓ Large in size
- ✓ Cost is more

Multi range Ammeter

When the switch is connected to position (1), the supplied current I_1

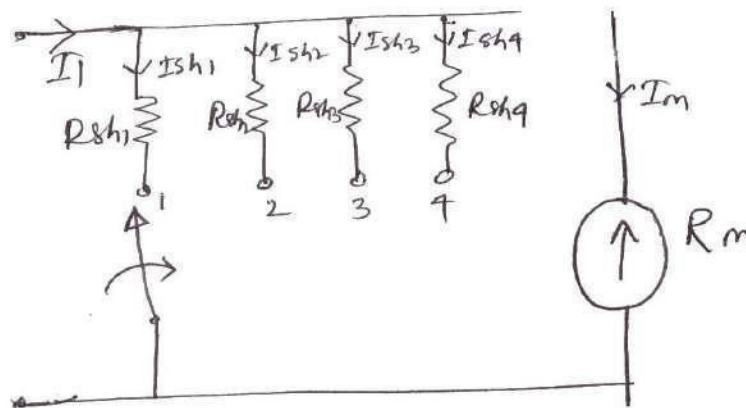


Fig. 1.21

$$I_{sh1}R_{sh1} = I_m R_m \quad (1.83)$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \quad (1.84)$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, R_{sh1} = \frac{R_m}{m_1 - 1}, m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m} \quad (1.85)$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m} \quad (1.86)$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m} \quad (1.87)$$

1.15 Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \quad (1.88)$$

$$R_2 = R_{sh2} - R_{sh3} \quad (1.89)$$

$$R_3 = R_{sh3} - R_{sh4} \quad (1.90)$$

$$R_4 = R_{sh4} \quad (1.91)$$

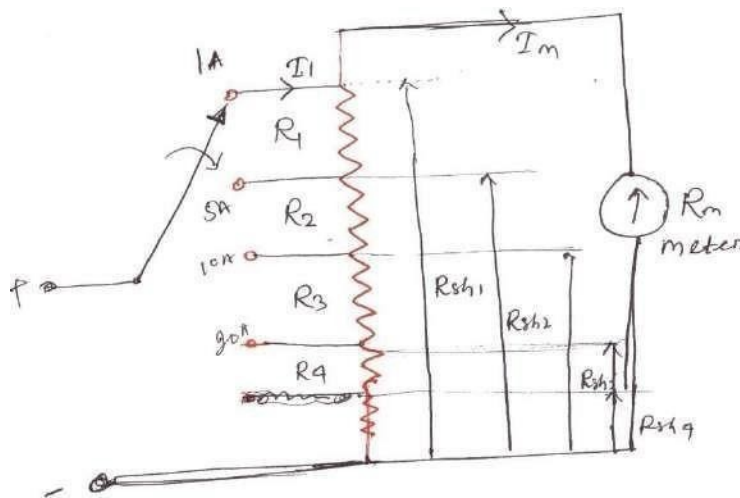
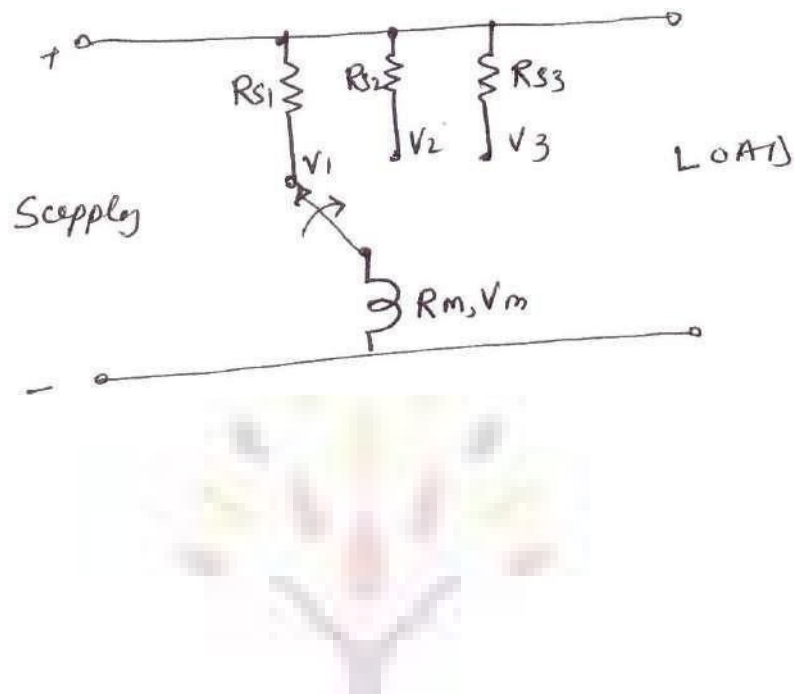


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Ayrton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

Multi range D.C.voltmeter



$$R_{s1} = R_m(m_1 - 1)$$

$$R_{s2} = R_m(m_2 - 1)$$

$$R_{s3} = R_m(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m}, m_2 = \frac{V_2}{V_m}, m_3 = \frac{V_3}{V_m}$$

Consider for voltage V_1 , $(R_1 + R_m)I_m = V_1$

$$\therefore R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_m}{R_m}\right)} - R_m = \left(\frac{V_1}{V_m}\right)R_m - R_m$$

$$R_1 = (m_1 - 1)R_m$$

For V_2 , $(R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m$

$$R_2 = \frac{V_2}{\left(\frac{V_m}{R_m}\right)} - (m_1 - 1)R_m - R_m$$

$$\begin{aligned} R_2 &= m_2 R_m - R_m - (m_1 - 1)R_m \\ &= R_m(m_2 - 1 - m_1 + 1) \end{aligned}$$

$$R_2 = (m_2 - m_1)R_m$$

For V_3 $(R_3 + R_2 + R_1 + R_m)I_m = V_3$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$\begin{aligned} &= \frac{V_3}{V_m} R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ &= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ R_3 &= (m_3 - m_2)R_m \end{aligned}$$

For V_4 $(R_4 + R_3 + R_2 + R_1 + R_m)I_m = V_4$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$\begin{aligned} &= \left(\frac{V_4}{V_m}\right)R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ R_4 &= R_m[m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1] \\ R_4 &= (m_4 - m_3)R_m \end{aligned}$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required.

Potential divider arrangement

The resistance R_1, R_2, R_3 and R_4 is connected in series to obtained the ranges V_1, V_2, V_3 and V_4

Example: 1.1

A PMMC ammeter has the following specification

Coil dimension are $1\text{cm} \times 1\text{cm}$. Spring constant is $0.15 \times 10^{-6} \text{ N-m / rad}$, Flux density is $1.5 \times 10^{-3} \text{ wb / m}^2$. Determine the no. of turns required to produce a deflection of 90° when a current 2mA flows through the coil.

Solution:

Solution:

At steady state condition $T_d = T_C$

$$BANl = K\theta$$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

$$A = 1 \times 10^{-4} \text{ m}^2$$

$$K = 0.15 \times 10^{-6} \frac{\text{N-m}}{\text{rad}}$$

$$B = 1.5 \times 10^{-3} \text{ wb / m}^2$$

$$I = 2 \times 10^{-3} \text{ A}$$

$$\theta = 90^\circ = \frac{\pi}{2} \text{ rad}$$

$$N = 785 \text{ ans.}$$

