

UNIT – IV TRANSISTOR AMPLIFIERS

BJT HYBRID MODEL

Small signal low frequency transistor Models:

All the transistor amplifiers are two port networks having two voltages and two currents. The positive directions of voltages and currents are shown in **fig. 1**.

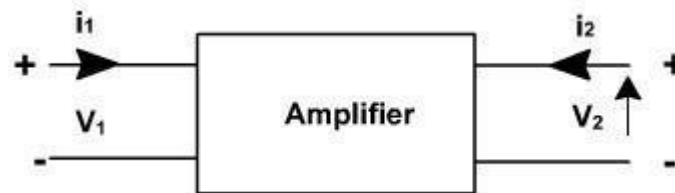


Fig. 1

A two-port network is represented by four external variables: voltage V_1 and current I_1 at the input port, and voltage V_2 and current I_2 at the output port, so that the two-port network can be treated as a black box modeled by the relationships between the four variables, V_1, V_2, I_1, I_2 . Out of four variables two can be selected as are independent variables and two are dependent variables. The dependent variables can be expressed in terms of independent variables. This leads to various two port parameters out of which the following three are important:

1. Impedance parameters (z-parameters)
2. Admittance parameters (y-parameters)
3. Hybrid parameters (h-parameters)

z-parameters

A two-port network can be described by z-parameters as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

In matrix form, the above equation can be rewritten as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

Input impedance with output port open circuited

$$z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

Reverse transfer impedance with input port open circuited

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

Forward transfer impedance with output port open circuited

$$z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

Output impedance with input port open circuited

Y-parameters

A two-port network can be described by Y-parameters as

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In matrix form, the above equation can be rewritten as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$

Input admittance with output port short circuited

$$y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

Reverse transfer admittance with input port short circuited

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

Forward transfer admittance with output port short circuited

$$y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

Output admittance with input port short circuited

Hybrid parameters (h-parameters)

If the input current I_1 and output voltage V_2 are taken as independent variables, the dependent variables V_1 and I_2 can be written as

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Where h_{11} , h_{12} , h_{21} , h_{22} are called as hybrid parameters.

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0}$$

Input impedance with o/p port short circuited

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}$$

Reverse voltage transfer ratio with i/p port open circuited

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$

Forward voltage transfer ratio with o/p port short circuited

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

output impedance with i/p port open circuited

THE HYBRID MODEL FOR TWO PORT NETWORK:

Based on the definition of hybrid parameters the mathematical model for two port networks known as h-parameter model can be developed. The hybrid equations can be written as:

$$V_1 = h_i I_1 + h_r V_2$$

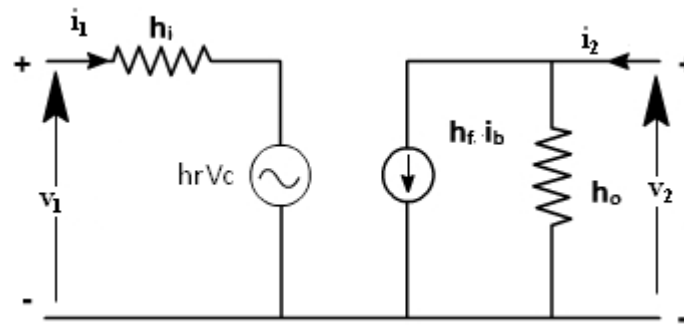
$$I_2 = h_f I_1 + h_o V_2$$

(The following convenient alternative subscript notation is recommended by the IEEE Standards:

$i=11$ = input $o = 22$ = output

$f=21$ = forward transfer $r = 12$ = reverse transfer)

We may now use the four h parameters to construct a mathematical model of the device of Fig.(1). The hybrid circuit for any device indicated in Fig.(2). We can verify that the model of Fig.(2) satisfies above equations by writing Kirchhoff's voltage and current laws for input and output ports.



If these parameters are specified for a particular configuration, then suffixes e, b or c are also included, e.g. h_{fe} , h_{ib} are h parameters of common emitter and common collector amplifiers

Using two equations the generalized model of the amplifier can be drawn as shown in fig. 2.

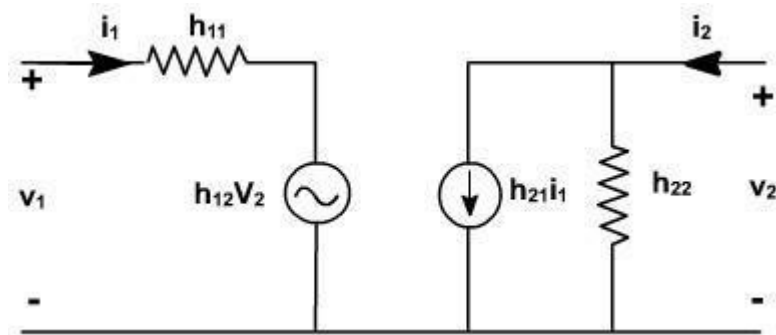


Fig. 2

TRANSISTOR HYBRID MODEL:

The hybrid model for a transistor amplifier can be derived as follow:

Let us consider CE configuration as show in fig. 3. The variables, i_B , i_C , v_C , and v_B represent total instantaneous currents and voltages i_B and v_C can be taken as independent variables and v_B , I_C as dependent variables.

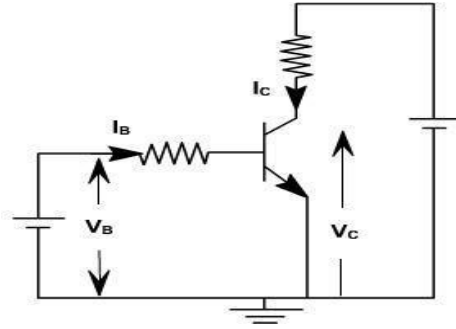


Fig. 3

$$V_B = f_1(i_B, v_C)$$

$$I_C = f_2(i_B, v_C).$$

Using Taylor 's series expression, and neglecting higher order terms we obtain.

$$\Delta v_B = \left. \frac{\partial f_1}{\partial i_B} \right|_{v_C} \Delta i_B + \left. \frac{\partial f_1}{\partial v_C} \right|_{i_B} \Delta v_C$$

$$\Delta i_C = \left. \frac{\partial f_2}{\partial i_B} \right|_{v_C} \Delta i_B + \left. \frac{\partial f_2}{\partial v_C} \right|_{i_B} \Delta v_C$$

The partial derivatives are taken keeping the collector voltage or base current constant. The Δv_B , Δv_C , Δi_B , Δi_C represent the small signal (incremental) base and collector current and voltage and can be represented as v_b , i_c , i_b , v_c

$$\therefore v_b = h_{ie} i_b + h_{re} v_c$$

$$i_c = h_{fe} i_b + h_{oe} v_b$$

where

$$h_{ie} = \left. \frac{\partial f_1}{\partial i_B} \right|_{v_C} = \left. \frac{\partial v_B}{\partial i_B} \right|_{v_C}; \quad h_{re} = \left. \frac{\partial f_1}{\partial v_C} \right|_{i_B} = \left. \frac{\partial v_B}{\partial v_C} \right|_{i_B}$$

$$h_{fe} = \left. \frac{\partial f_2}{\partial i_B} \right|_{v_C} = \left. \frac{\partial i_C}{\partial i_B} \right|_{v_C}; \quad h_{oe} = \left. \frac{\partial f_2}{\partial v_C} \right|_{i_B} = \left. \frac{\partial i_C}{\partial v_C} \right|_{i_B}$$

The model for CE configuration is shown in fig. 4.

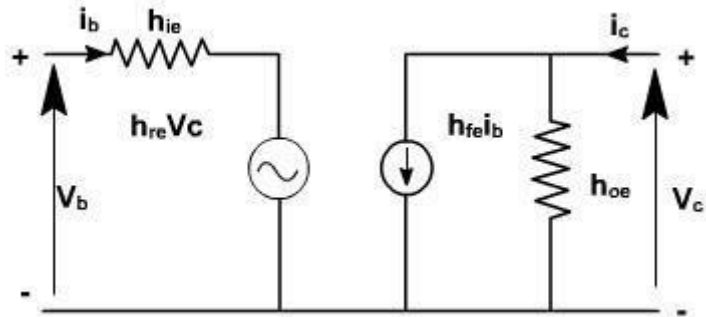


Fig. 4

To determine the four h-parameters of transistor amplifier, input and output characteristic are used. Input characteristic depicts the relationship between input voltage and input current with output voltage as parameter. The output characteristic depicts the relationship between output voltage and output current with input current as parameter. Fig. 5, shows the output characteristics of CE amplifier.

$$h_{fe} = \left. \frac{\partial i_c}{\partial i_b} \right|_{V_c} = \frac{i_{c2} - i_{c1}}{i_{b2} - i_{b1}}$$

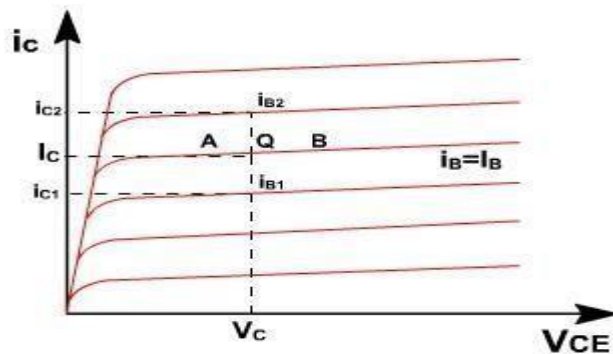


Fig. 5

The current increments are taken around the quiescent point Q which corresponds to $i_B = I_B$ and to the collector voltage $V_{CE} = V_C$

$$h_{oe} = \left. \frac{\partial i_c}{\partial V_c} \right|_{i_B}$$

The value of h_{oe} at the quiescent operating point is given by the slope of the output characteristic at the operating point (i.e. slope of tangent AB).

$$h_{ie} = \frac{\partial V_B}{\partial i_B} \approx \left. \frac{\Delta V_B}{\Delta i_B} \right|_{V_C}$$

h_{ie} is the slope of the appropriate input on [fig. 6](#), at the operating point (slope of tangent EF at Q).

$$h_{re} = \frac{\partial V_B}{\partial V_C} = \left. \frac{\Delta V_B}{\Delta V_C} \right|_{i_B} = \frac{V_{B2} - V_{B1}}{V_{C2} - V_{C1}}$$

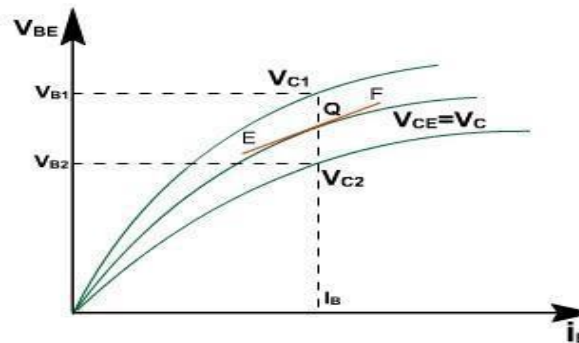


Fig. 6

A vertical line on the input characteristic represents constant base current. The parameter h_{re} can be obtained from the ratio $(V_{B2} - V_{B1})$ and $(V_{C2} - V_{C1})$ for at Q.

Typical CE h-parameters of transistor 2N1573 are given below:

$$h_{ie} = 1000 \text{ ohm.}$$

$$h_{re} = 2.5 \times 10^{-4}$$

$$h_{fe} = 50$$

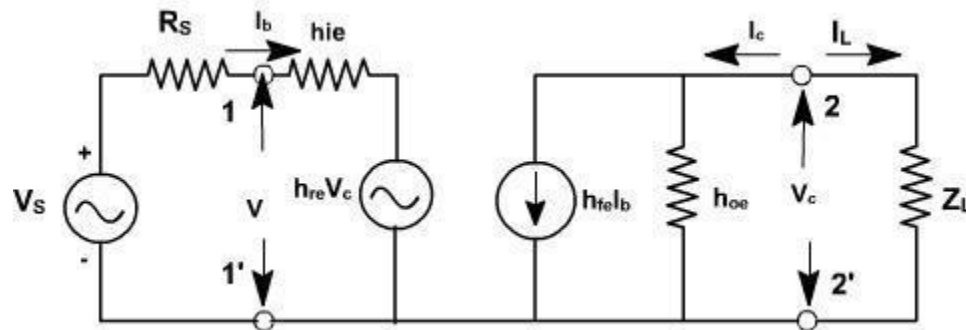
ANALYSIS OF A TRANSISTOR AMPLIFIER USING H-PARAMETERS:

To form a transistor amplifier it is only necessary to connect an external load and signal source as indicated in [fig. 1](#) and to bias the transistor properly.



Fig. 1

Consider the two-port network of CE amplifier. R_s is the source resistance and Z_L is the load impedance. h-parameters are assumed to be constant over the operating range. The ac equivalent circuit is shown in fig. 2. (Phasor notations are used assuming sinusoidal voltage input). The quantities of interest are the current gain, input impedance, voltage gain, and output impedance.



Current gain:

For the transistor amplifier stage, A_i is defined as the ratio of output to input currents.

$$A_i = \frac{I_L}{I_1} = \frac{-I_2}{I_1}$$

Input impedance:

The impedance looking into the amplifier input terminals (1, 1') is the input impedance Z_i

$$Z_i = \frac{V_b}{I_b}$$

$$V_b = h_{ie} I_b + h_{re} V_c$$

$$\frac{V_b}{I_b} = h_{ie} + h_{re} \frac{V_c}{I_b}$$

$$= h_{ie} - \frac{h_{re} I_c Z_L}{I_b}$$

$$\therefore Z_i = h_{ie} + h_{re} A_v Z_L$$

$$= h_{ie} - \frac{h_{re} h_{fe} Z_L}{1 + h_{oe} Z_L}$$

$$\therefore Z_i = h_{ie} - \frac{h_{re} h_{fe}}{Y_L + h_{oe}} \quad \left(\text{since } Y_L = \frac{1}{Z_L} \right)$$

Voltage gain:

The ratio of output voltage to input voltage gives the gain of the transistors.

$$A_v = \frac{V_c}{V_b} = - \frac{I_c Z_L}{V_b}$$

$$\therefore A_v = \frac{I_b A_i Z_L}{V_b} = \frac{A_i Z_L}{Z_i}$$

Output Admittance:

$$Y_o = \left. \frac{I_c}{V_c} \right|_{V_s=0} = 0$$

$$I_c = h_{fe} I_b + h_{oe} V_c$$

$$\frac{I_c}{V_c} = h_{fe} \frac{I_b}{V_c} + h_{oe}$$

when $V_s = 0$, $R_s \cdot I_b + h_{ie} \cdot I_b + h_{re} V_c = 0$.

$$\frac{I_b}{V_c} = - \frac{h_{re}}{R_s + h_{ie}}$$

$$\therefore Y_o = h_{oe} - \frac{h_{re} h_{fe}}{R_s + h_{ie}}$$

Voltage amplification taking into account source impedance (R_s) is given by

$$A_{vS} = \frac{V_c}{V_s} = \frac{V_c}{V_b} * \frac{V_b}{V_s} \quad \left(V_b = \frac{V_s}{R_s + Z_i} * Z_i \right)$$

$$= A_v * \frac{Z_i}{Z_i + R_s}$$

It is defined as

$$= \frac{A_i Z_L}{Z_i + R_s}$$

A_v is the voltage gain for an ideal voltage source ($R_v = 0$).

Consider input source to be a current source I_s in parallel with a resistance R_s as shown in fig. 3.

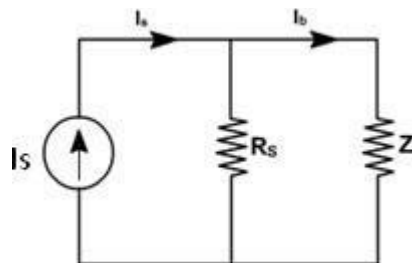


Fig. 3

In this case, overall current gain A_{IS} is defined as

$$\begin{aligned}
 A_{I_s} &= \frac{I_L}{I_s} \\
 &= -\frac{I_c}{I_s} \\
 &= -\frac{I_c}{I_b} \cdot \frac{I_b}{I_s} \quad \left(I_b = \frac{I_s \cdot R_s}{R_s + Z_i} \right) \\
 &= A_I \cdot \frac{R_s}{R_s + Z_i}
 \end{aligned}$$

If $R_s \rightarrow \infty$, $A_{I_s} \rightarrow A_I$

h-parameters

To analyze multistage amplifier the h-parameters of the transistor used are obtained from manufacture data sheet. The manufacture data sheet usually provides h-parameter in CE configuration. These parameters may be converted into CC and CB values. For example [fig. 4](#) hrc in terms of CE parameter can be obtained as follows.

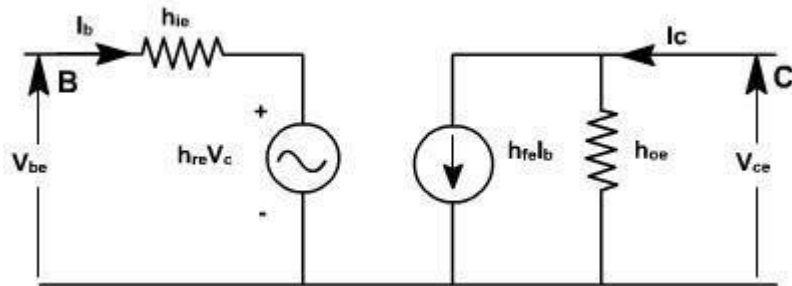


Fig. 4

For CE transistor configuration

$$V_{be} = h_{ie} I_b + h_{re} V_{ce}$$

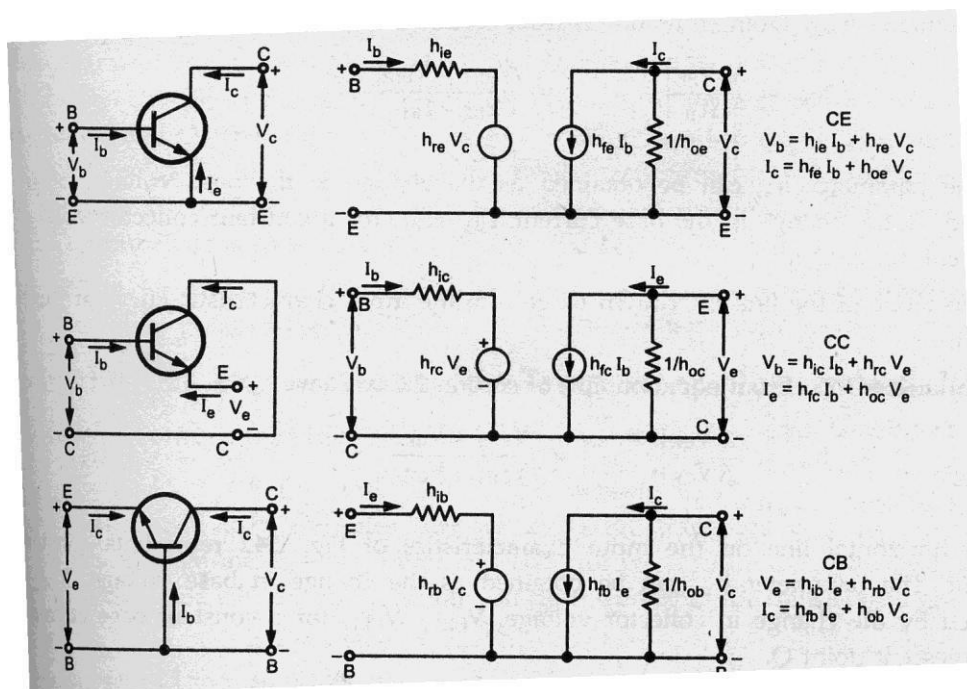
$$I_c = h_{fe} I_b + h_{oe} V_{ce}$$

The circuit can be redrawn like CC transistor configuration as shown in [fig. 5](#).

$$V_{bc} = h_{ie} I_b + h_{rc} V_{ec}$$

$$I_c = h_{fe} I_b + h_{oe} V_{ec}$$

hybrid model for transistor in three different configurations



Typical h-parameter values for a transistor

Parameter	CE	CC	CB
h_i	1100 Ω	1100 Ω	22 Ω
h_r	2.5×10^{-4}	1	3×10^{-4}
h_f	50	-51	-0.98
h_o	25 $\mu\text{A/V}$	25 $\mu\text{A/V}$	0.49 $\mu\text{A/V}$

Analysis of a Transistor amplifier circuit using h-parameters

A transistor amplifier can be constructed by connecting an external load and signal source and biasing the transistor properly.

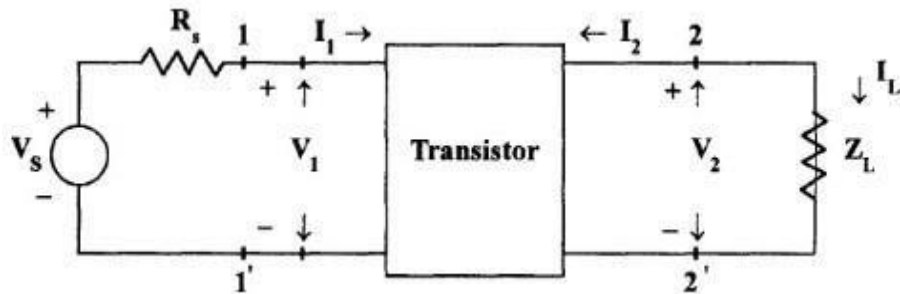


Fig.1.4 Basic Amplifier Circuit

The two port network of Fig. 1.4 represents a transistor in any one of its configuration. It is assumed that h-parameters remain constant over the operating range. The input is sinusoidal and I_1, V_1, I_2 and V_2 are phase quantities

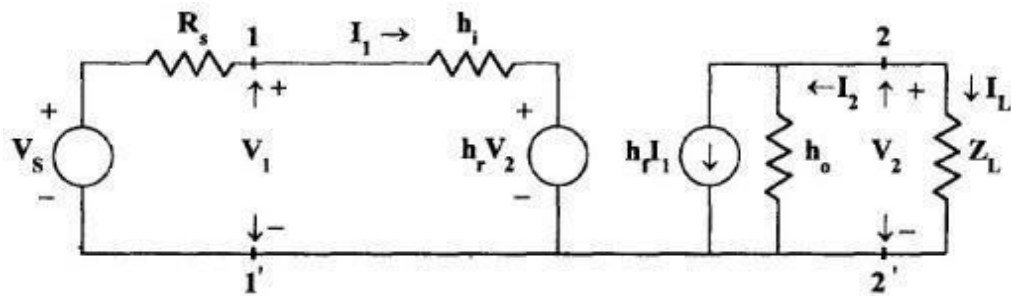


Fig. 1.5 Transistor replaced by its Hybrid Model

Current Gain or Current Amplification (A_i)

For transistor amplifier the current gain A_i is defined as the ratio of output current to input current, i.e.,

$$A_i = I_L / I_1 = -I_2 / I_1$$

From the circuit of Fig

$$I_2 = h_f I_1 + h_o V_2 \quad \text{Substituting } V_2 = I_L Z_L = -I_2 Z_L \quad I_2 = h_f I_1 - I_2 Z_L h_o$$

$$I_2 + I_2 Z_L h_o = h_f I_1$$

$$I_2 (1 + Z_L h_o) = h_f I_1$$

$$A_i = -I_2 / I_1 = -h_f / (1 + Z_L h_o)$$

Therefore,

$$A_i = -h_f / (1 + Z_L h_o)$$

Input Impedance (Z_i)

In the circuit of Fig , R_S is the signal source resistance .The impedance seen when looking into the amplifier terminals (1,1') is the amplifier input impedance Z_i ,

$$Z_i = V_1 / I_1$$

From the input circuit of Fig $V_1 = h_i I_1 +$

$$h_r V_2 \quad Z_i = (h_i I_1 + h_r V_2) / I_1$$

$$= h_i + h_r V_2 / I_1$$

Substituting

$$V_2 = -I_2 Z_L = A_i I_1 Z_L$$

$$Z_i = h_i + h_r A_i I_1 Z_L / I_1$$

$$= h_i + h_r A_i Z_L$$

Substituting for A_i

$$Z_i = h_i - h_f h_r Z_L / (1 + h_o Z_L)$$

$$= h_i - h_f h_r Z_L / Z_L (1/Z_L + h_o)$$

Taking the Load admittance as $Y_L = 1/Z_L$

$$Z_L \quad Z_i = h_i - h_f h_r / (Y_L + h_o)$$

Voltage Gain or Voltage Gain Amplification Factor(A_v)

The ratio of output voltage V_2 to input voltage V_1 give the voltage gain of the transistor i.e,

$$A_v = V_2 / V_1$$

Substituting

$$V_2 = -I_2 Z_L = A_i I_1 Z_L$$

$$A_v = A_i I_1 Z_L / V_1 = A_i Z_L / Z_i$$

Output Admittance (Y_o)

Y_o is obtained by setting V_S to zero, Z_L to infinity and by driving the output terminals from a generator V_2 . If the current V_2 is I_2 then $Y_o = I_2/V_2$ with $V_S=0$ and $R_L = \infty$.

From the circuit of fig

$$I_2 = h_f I_1 + h_o V_2$$

Dividing by V_2 ,

$$I_2 / V_2 = h_f I_1 / V_2 + h_o$$

With $V_2 = 0$, by KVL in input circuit,

$$R_S I_1 + h_i I_1 + h_r V_2 =$$

$$0 \quad (R_S + h_i) I_1 +$$

$$h_r V_2 = 0$$

$$\text{Hence, } I_2 / V_2 = -h_r / (R_S + h_i)$$

$$= h_f (-h_r / (R_S + h_i)) + h_o$$

$$Y_o = h_o - h_f h_r / (R_S + h_i)$$

The output admittance is a function of source resistance. If the source impedance is resistive then Y_o is real. Voltage Amplification Factor(A_{vs}) taking into account the resistance (R_S) of the source

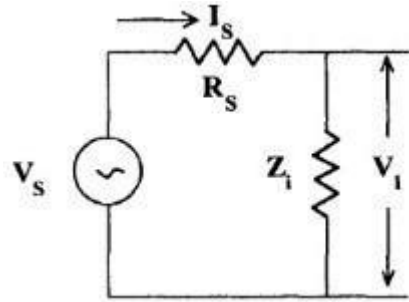


Fig. 5.6 Thevenin's Equivalent Input Circuit This overall voltage gain A_{vs} is given by

$$A_{vs} = V_2 / V_s = V_2 V_1 / V_1 V_s = A_v V_1 / V_s$$

From the equivalent input circuit using Thevenin's equivalent for the source shown in Fig. 5.6

$$V_1 = V_s Z_i / (Z_i + R_s)$$

$$V_1 / V_s = Z_i / (Z_i + R_s)$$

Then, $A_{vs} = A_v Z_i / (Z_i +$

$R_s)$ Substituting $A_v = A_i Z_L / Z_i$

$$A_{vs} = A_i Z_L / (Z_i + R_s)$$

$$A_{vs} = A_i Z_L R_s / (Z_i + R_s) R_s$$

$$A_{vs} = A_{is} Z_L / R_s$$

Current Amplification (A_{is}) taking into account the source Resistance(R_s)

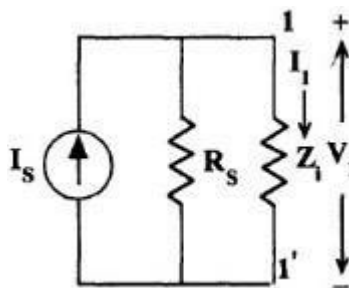


Fig. 1.7 Norton's Equivalent Input Circuit

The modified input circuit using Norton's equivalent circuit for the calculation of A_{is} is shown in Fig. 1.7 Overall Current Gain, $A_{is} = -I_2 / I_S = -I_2 I_1 / I_1 I_S = A_i I_1 / I_S$

From Fig. 1.7 $I_1 = I_S R_S / (R_S + Z_i) I_1 / I_S$

$$= R_S / (R_S + Z_i)$$

and hence, $A_{is} = A_i R_S / (R_S + Z_i)$

Operating Power Gain (A_p)

The operating power gain A_p of the transistor is

defined as $A_p = P_2 / P_1 = -V_2 I_2 / V_1 I_1 =$

$$A_v A_i = A_i A_i Z_L / Z_i$$

$$A_p = A_i^2 (Z_L / Z_i)$$

Small Signal analysis of a transistor amplifier

$A_i = -h_f / (1 + Z_L h_o)$	$A_v = A_i Z_L / Z_i$
$Z_i = h_i + h_r A_i Z_L = h_i - h_f h_r / (Y_L + h_o)$	$A_{vs} = A_v Z_i / (Z_i + R_S) = A_i Z_L / (Z_i + R_S)$ $= A_{is} Z_L / R_S$
$Y_o = h_o - h_f h_r / (R_S + h_i) = 1 / Z_o$	$A_{is} = A_i R_S / (R_S + Z_i) = A_{vs} = A_{is} R_S / Z_L$