23MA301: MATHEMATICAL STATISTICAL FOUNDATION

Topic: TEST OF HYPOTHESIS

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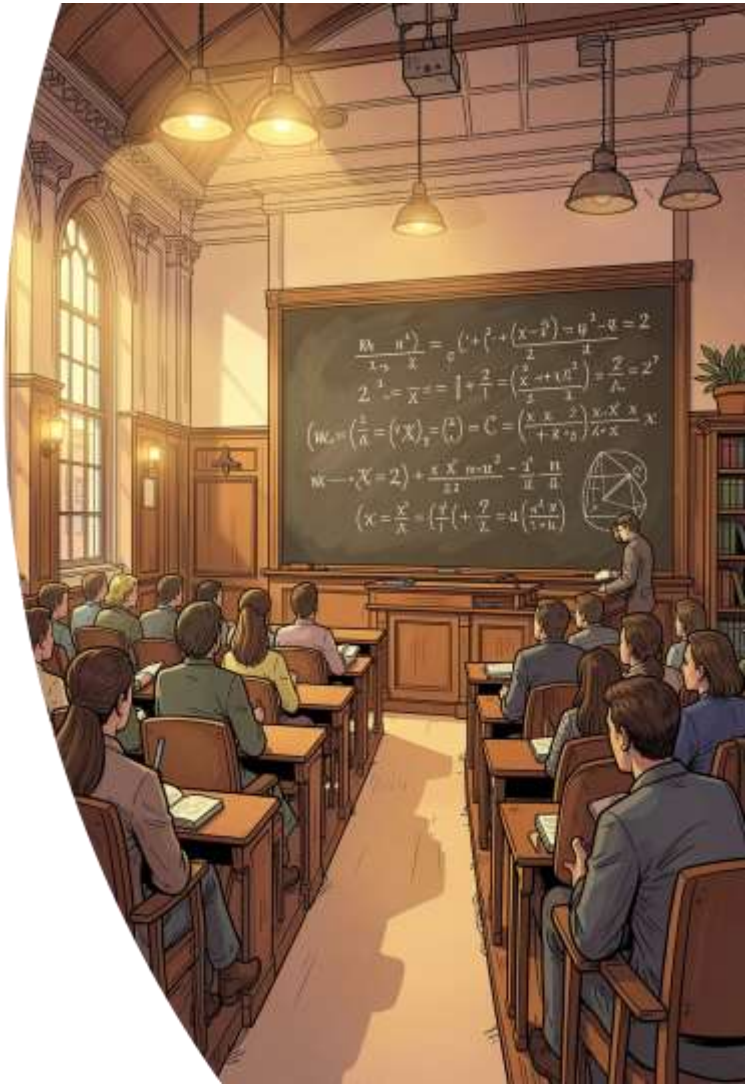
Tests of Hypotheses :

A **test of hypothesis** is a statistical procedure used to decide whether there is enough evidence from a sample to accept or reject a claim (hypothesis) about a population parameter.

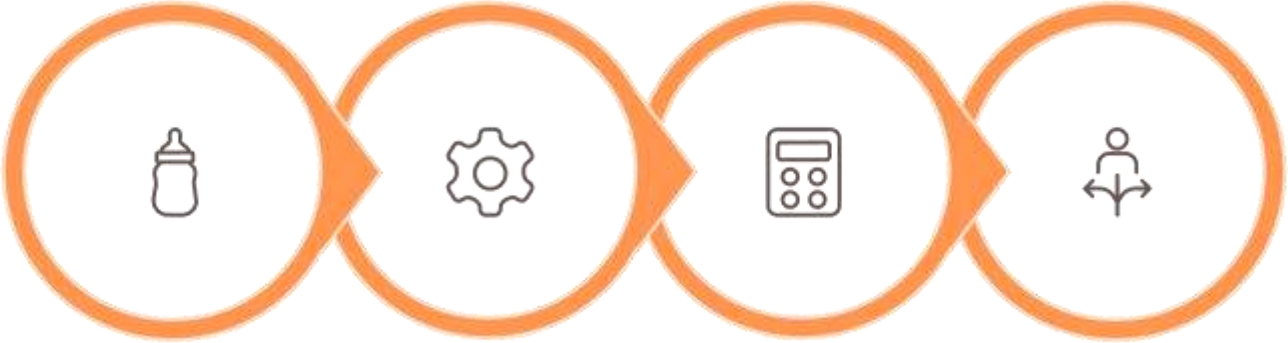
Key Terms

- **Null Hypothesis (H_0)**: The hypothesis assumed to be true unless evidence suggests otherwise.
- **Alternative Hypothesis (H_1 or H_a)**: The hypothesis accepted if the null hypothesis is rejected.
- **Level of Significance (α)**: The probability of rejecting a true null hypothesis (commonly 0.05 or 0.01).
- **Test Statistic**: A value calculated from the sample used to test the hypothesis.
- **Critical Region (Rejection Region)**: The set of values for which H_0 is rejected.
- **Acceptance Region**: The set of values for which H_0 is not rejected.
- **p-value**: The probability of obtaining the observed result if H_0 is true.



A **statistical hypothesis** is a claim about a population parameter — such as a mean μ or proportion p — that can be evaluated using sample data.

The Hypothesis Testing Framework



Every hypothesis test follows this logical sequence. The **significance level** α (commonly 0.05) controls the probability of a Type I error — falsely rejecting a true null hypothesis.

One-Tailed and Two-Tailed Tests

(Hypothesis Testing)

1. Left-Tailed Test (Left One-Tailed Test)

Used when the alternative hypothesis is

$$H_1 : \mu < \mu_0$$

The rejection region lies in the **left tail** of the normal distribution.

```
                Normal
Distribution

                    /\
                  /    \
                /        \
______________/
\______________
```

[redacted]

```
Reject H0                Accept
H0

Critical Value = -Zα
```

Right-Tailed Test (Right One-Tailed Test)

Used when the alternative hypothesis is

$$H_1 : \mu > \mu_0$$

The rejection region lies in the **right tail**.

```
                Normal Distribution

                    /\
                  /    \
                /        \
______________/
\______________
```

[redacted]

```
                   Accept H0
Reject H0

                 Critical Value =
+Zα
```

Two-Tailed Test

Used when the alternative hypothesis is

$$H_1 : \mu \neq \mu_0$$

The rejection regions lie in **both tails** of the distribution.

```
                Normal Distribution


                    /\
                  /    \
                /        \
______________/            \______________
█████                      █████
Reject H0   Accept H0     Reject H0

     -Zα/2                 +Zα/2
```

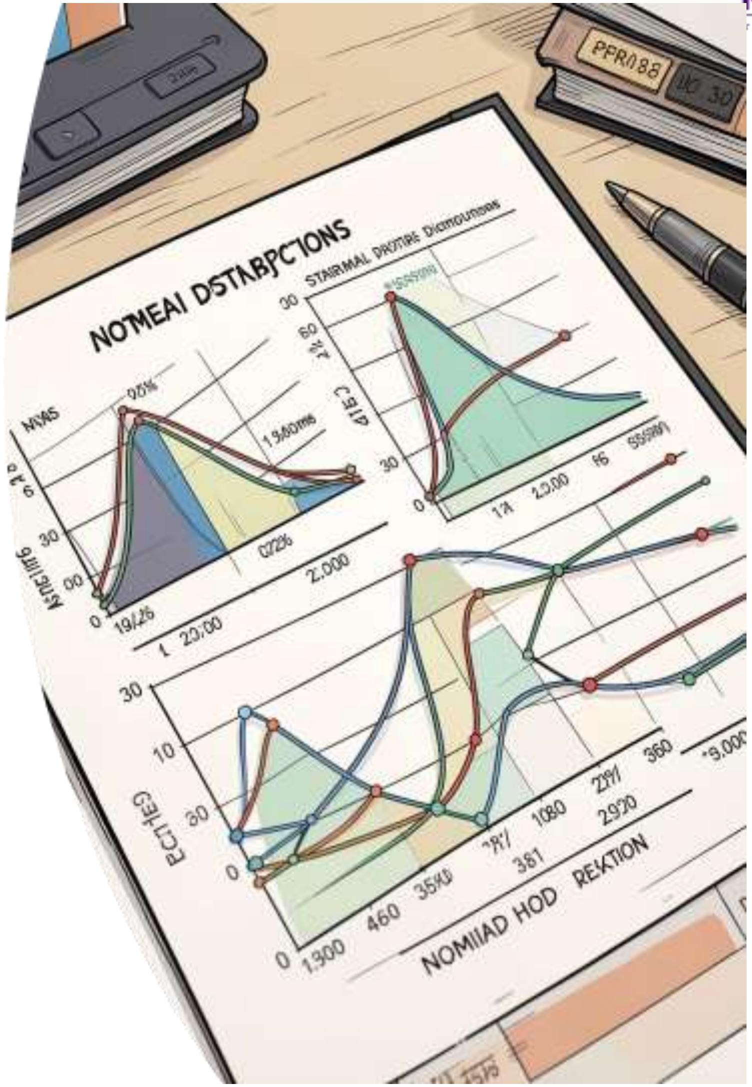
Types of Errors

- **Type I Error**: Rejecting a true H_0.
  - Probability = α
- **Type II Error**: Failing to reject a false H_0.
  - Probability = β

Large Sample Tests (Large n) : $n \geq 30$

Common Tests

- Z-test for mean
- Z-test for proportion
- Difference between two means
- Difference between two proportions



Formula (One Sample Mean)

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

If population standard deviation is unknown:

$$Z = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

For two means

$$\boxed{Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

Small Sample Tests : $n < 30$

One Sample Mean (Student's t-test)

Formula:

$$\boxed{t = \frac{\bar{X} - \mu}{s/\sqrt{n}}}$$

Degrees of freedom:

$$df = n - 1$$

Difference Between Two Means (Independent Samples t-test)

When population variances are assumed equal:

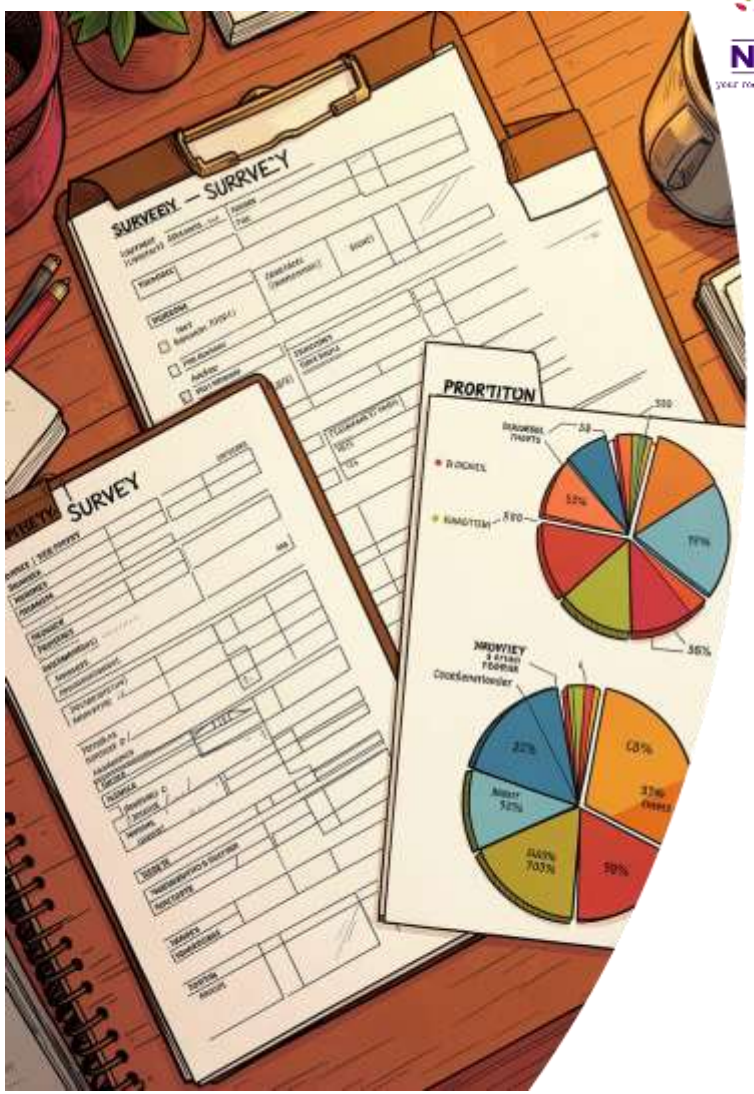
$$\boxed{t = \frac{\bar{X}_1 - \bar{X}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}}$$

where the pooled variance is

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Degrees of freedom:

$$df = n_1 + n_2 - 2$$



Large Sample Test for One Proportion:

Formula: $Z = \frac{p-P}{\sqrt{\frac{PQ}{n}}}$

where
- p= sample proportion
- P= population proportion
- $Q = 1 - P$

Large Sample Test for Difference Between Two Proportions

Formula: $\boxed{Z = \frac{p_1-p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}}$

where

$$P = \frac{x_1 + x_2}{n_1 + n_2}$$
$$Q = 1 - P$$

and
- $p_1 = \frac{x_1}{n_1}$
- $p_2 = \frac{x_2}{n_2}$

F-Test :

The **F-test** is used to compare the variances of two independent normal populations and to determine whether they are significantly different.

Formula

Assuming $S_1^2 > S_2^2$

$$F = \frac{S_1^2}{S_2^2}$$

where:

•S_1^2= larger sample variance

•S_2^2= smaller sample variance

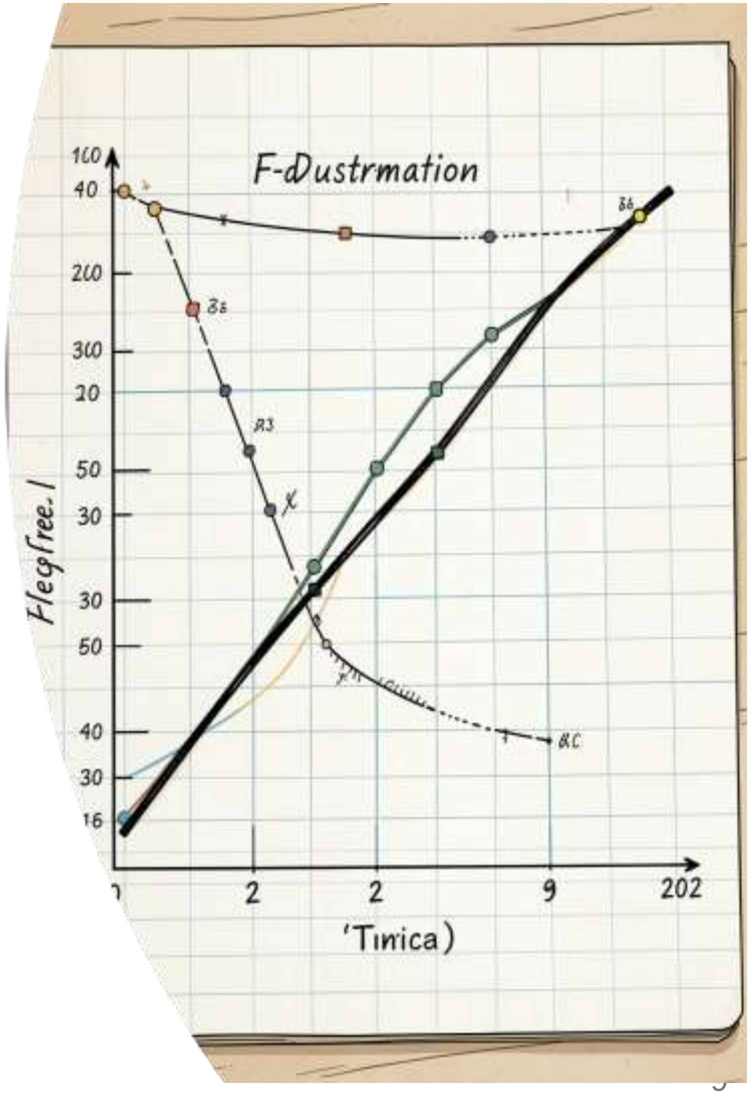
Degrees of freedom:

•Numerator:

$$df_1 = n_1 - 1$$

•Denominator:

$$df_2 = n_2 - 1$$



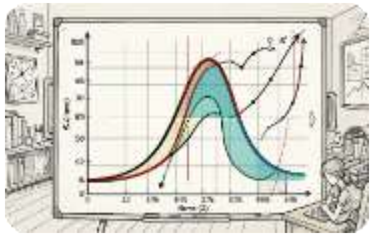
Conditions for F-Test

•Samples are independent.

•Populations are normally distributed.

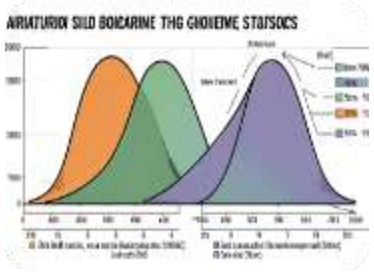
•Data are randomly selected.

•Variances are compared.



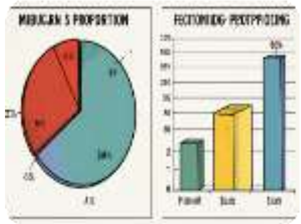
Single Mean

Use Z-test (known \ *sigma*) or t-test (unknown \ *sigma*). Check sample size and normality conditions.



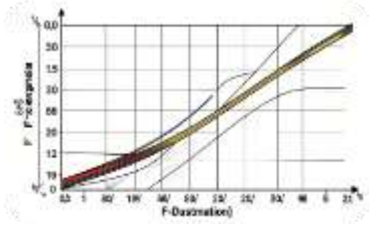
Two Means

Pooled t-test when variances are equal; Welch's t-test otherwise. Degrees of freedom differ in each case.



Proportions

Single and two-sample proportion tests use the Z-statistic. Verify large-sample conditions before applying.



Variances & F-Test

The F-distribution governs two-sample variance tests. Always place the larger variance in the numerator.

Thank You..