23MA301: MATHEMATICAL STATISTICAL FOUNDATION

Topic: RANDOM VARIABLES

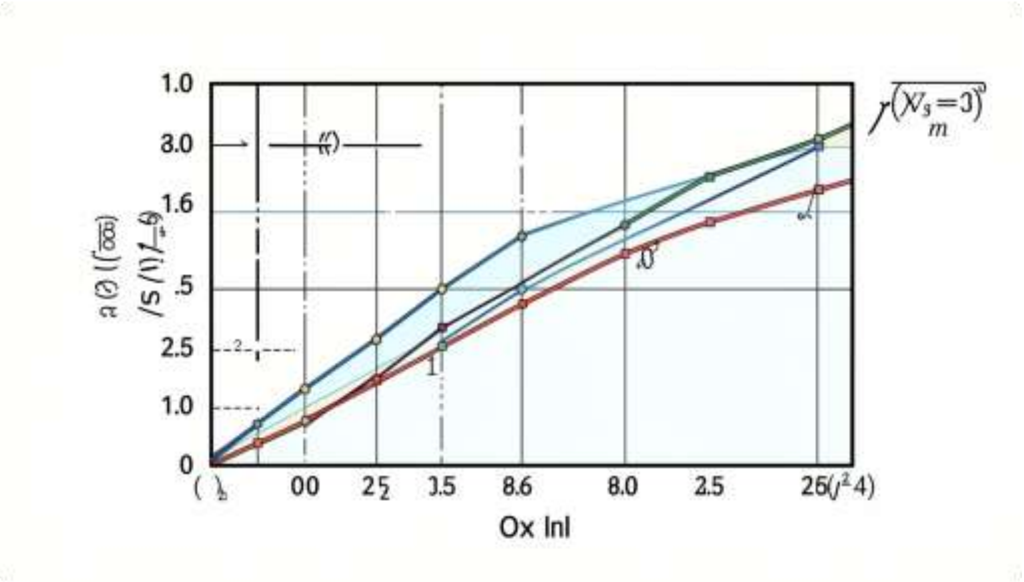
Mrs. K.Srividya

Assistant Professor, Freshman engineering
Narsimha Reddy Engineering College (Autonomous)
Secunderabad, Telangana, India- 500100.

Continuous Distributions & Sampling :

From the symmetry of the normal curve to the power of the Central Limit Theorem — this explores the continuous probability models and sampling frameworks that form the backbone of statistical inference.

The Uniform Distribution :



A continuous random variable X follows a **uniform distribution** on *[a, b]* when every subinterval of equal length has equal probability.

PDF

(f(x) = 1/b−a , a≤x≤b
0, otherwise.

f

Mean & Variance

E(X)=a+b/2

Var(X)= $(b-a)^2/12$

The Normal Distribution :

The **normal (Gaussian) distribution** is the most important continuous distribution in statistics, characterized by its symmetric bell-shaped curve.

Parameters

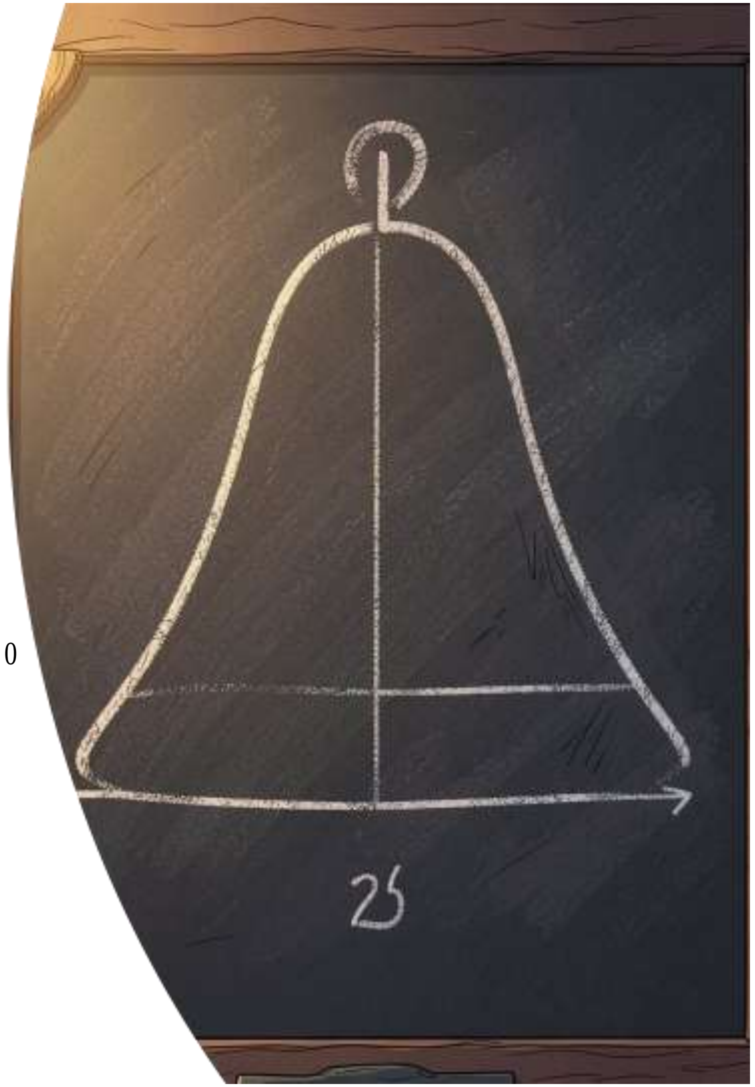
Mean μ and variance σ ^2

PDF Formula

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \; ; \; -\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

Notation

F(x, μ,σ)

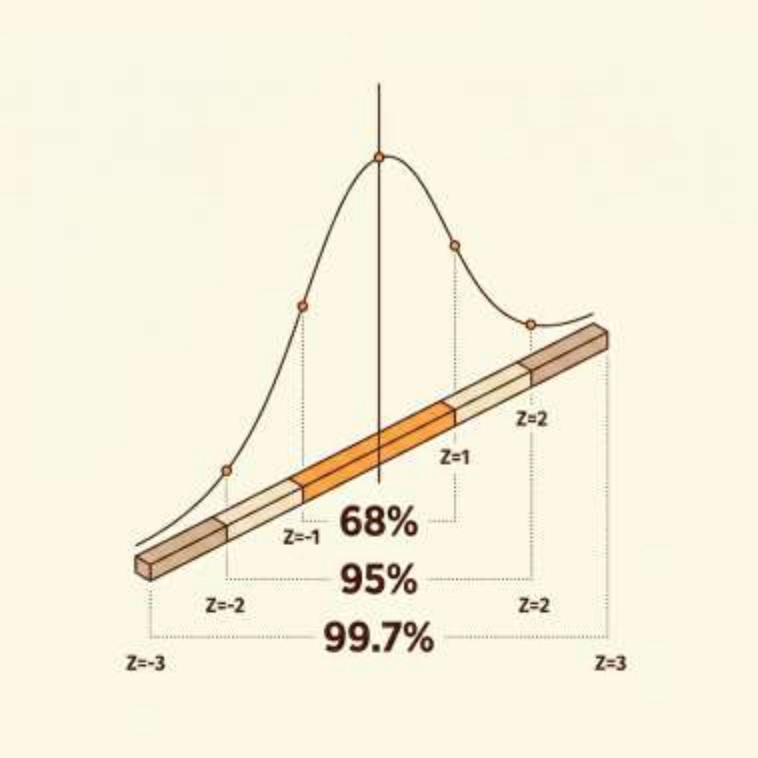


Areas Under the Normal Curve

Probabilities correspond to **areas under the curve**. We standardize using the **Z-score**:

$$Z = \frac{X - \mu}{\sigma}$$

Empirical Rule: ~68% within 1\ σ, ~95% within 2\ σ, ~99.7% within 3\ σ.



The **standard normal** *Z \ sim N(0, 1)* is used with Z-tables to find cumulative probabilities.

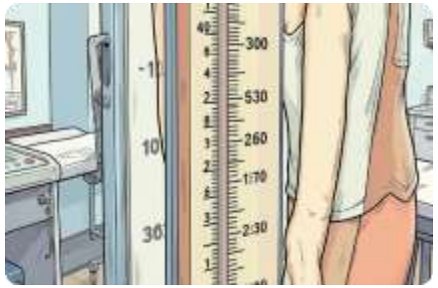
Applications of the Normal Distribution :

Quality Control

Monitor product dimensions and detect deviations from target specifications using control limits.

Test Scores

Standardized exams (SAT, GRE) are often modeled as normal to set percentiles and cutoffs.



Biological Data

Height, weight, and blood pressure in large populations typically follow a normal distribution.

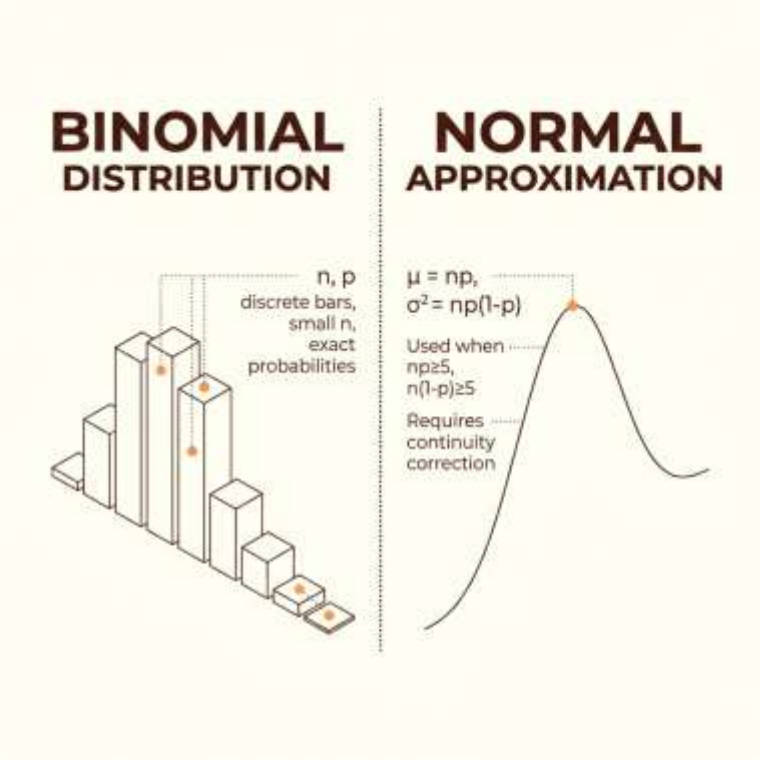
Normal Approximation to the Binomial :

When *n* is large, the binomial *B(n, p)* is well-approximated by a normal distribution:

$$X \approx N(np, np(1-p))$$

⚠ Validity condition: *np ₩geq 5* and *n(1-p) ₩geq 5*.

Continuity Correction: Add or subtract 0.5 when approximating a discrete binomial probability with a continuous normal curve.



Fundamental Sampling Distributions :

We now shift from populations to **samples** — understanding how sample statistics behave and vary is the foundation of statistical inference.

Random Sampling :

Random sampling is the process of selecting a sample from a population so that every member has an equal chance of being selected.

Types of Random Sampling

1. **Simple Random Sampling (SRS)** – Every member has an equal chance of selection.
2. **Systematic Sampling** – Select every k-th item after a random start.
3. **Stratified Sampling** – Divide the population into strata and randomly sample from each.
4. **Cluster Sampling** – Divide the population into clusters and randomly select whole clusters.

Population Mean

$$\mu = \frac{\sum X}{N}$$

Sample Mean

$$\bar{x} = \frac{\sum x_i}{n}$$

Sampling Distribution of the Mean

The **sampling distribution of {$\bar{x}$}** describes how the sample mean varies across all possible samples of size *n* from a population.

- **Population Variance**

$$\sigma^2 = \frac{\sum(x_i - \mu)^2}{N}$$

- **Sample Variance**

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

Shrinking Error

Standard error σ/√n decreases as n grows.

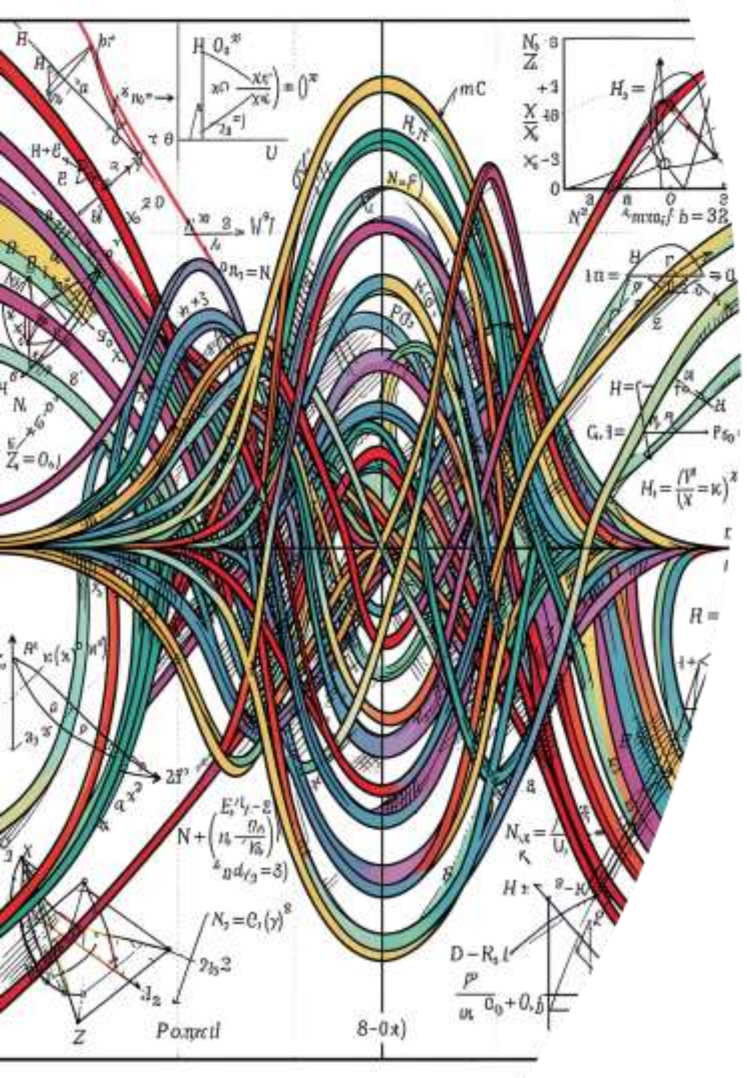
Unbiased Mean

Mean equals population mean μ.

Normal Shape

Distribution approaches normal with larger n.

As *n* grows, the distribution tightens around *\mu* and becomes increasingly bell-shaped — the foundation of the Central Limit Theorem.



The Central Limit Theorem

For *any* population with mean μ and variance σ^2, the sampling distribution of $\bar{X}$ approaches $N\left(\mu, \frac{\sigma^2}{n}\right)$ as $n \to \infty$.

Population Shape

Does not matter — skewed, uniform, or irregular

Rule of Thumb

$n \geq 30$ is typically sufficient for a good approximation

Why It Matters

Justifies normal-based inference even when the population is non-normal

Thank You..