23MA301: MATHEMATICAL STATISTICAL FOUNDATION

Topic: RANDOM VARIABLES

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Concept of a Random Variable

A **random variable** assigns a numerical value to each outcome of a random experiment. Represented by it maps uncertain outcomes to numbers — depending entirely on chance.

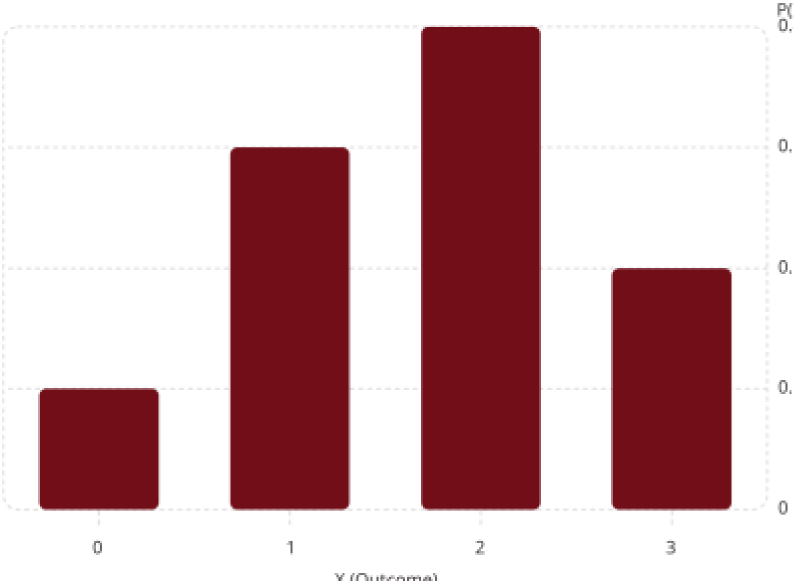
- **Discrete:** Countable outcomes — e.g., rolling a die, coin toss
- **Continuous:** Measurable values — e.g., height, temperature

Discrete Probability Distribution

A **discrete distribution** deals with **countable outcomes**. The Probability Mass Function (PMF) must satisfy:

$$0 \le P(X = x) \le 1$$

$$\sum P(X = x) = 1$$



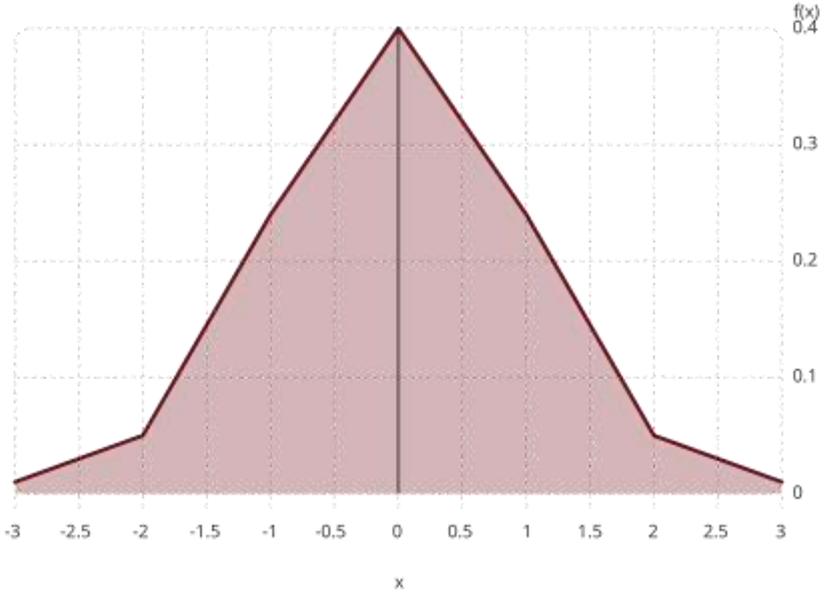
Coin Toss

Number of heads in 3 flips

Defective Bulbs

Defects in a batch of 10

Continuous Probability Distribution



A **continuous distribution** handles measurable values over a range. Probability is the **area under the curve** between two points.

$$P(a < X < b) = \int_a^b f(x)\,dx$$

- **f(x) ≥ 0** for all x
- **Total area = 1**

Examples: student height, temperature, body weight

Mean / Expected Value

Discrete Formula

$$\mu = \sum x \quad P(x)$$

Continuous Formula

$$\mu = \int xf(x)\,dx$$

Solved Example

Find the expected value when X takes values 1, 2, 3 with probabilities 0.2, 0.5, 0.3:

$$\mu = 1(0.2) + 2(0.5) + 3(0.3) = 2.1$$

The mean μ = 2.1 is the long-run average — the balance point of the distribution.

Variance and Standard Deviation

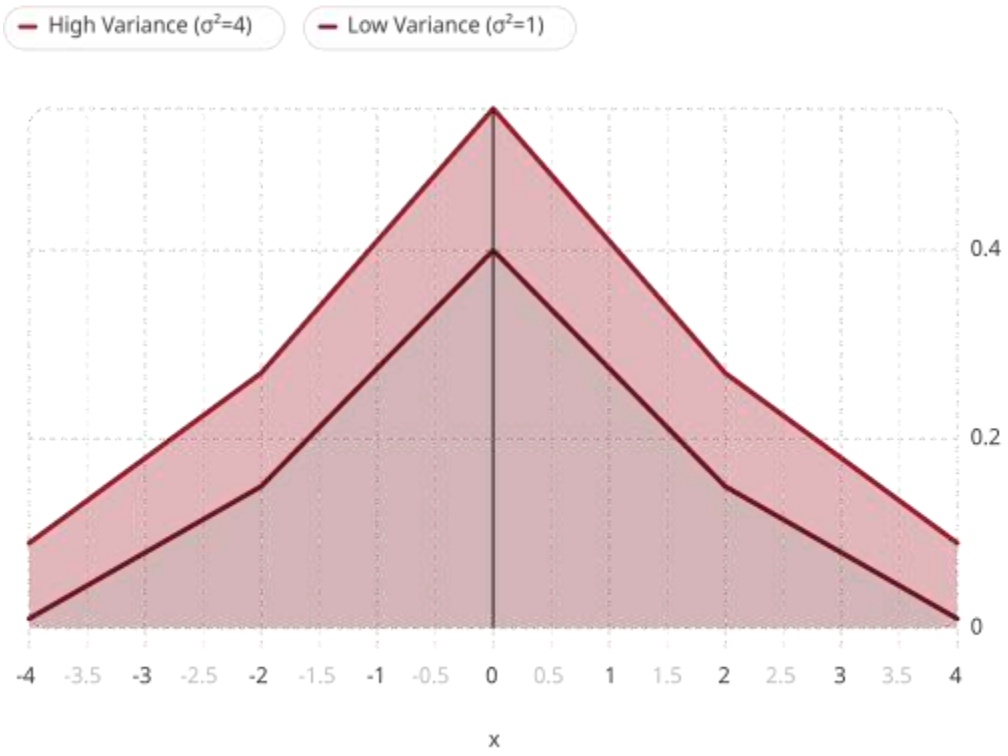
Formulas

$$\sigma^2 = \sum (x - \mu)^2 P(x)$$

$$\sigma = \sqrt{\sigma^2}$$

Small variance → values clustered near the mean.

Large variance → data spread widely.



Binomial Distribution

Models the number of successes in n fixed, independent trials, each with constant probability p.

Formula

$$P(X = x) = \binom{n}{x} p^{x} (1 - p)^{n-x}$$

Mean & Variance

$$\mu = np$$

$$\sigma^2 = np(1 - p)$$

Example: P(exactly 3 heads in 5 tosses) = C(5,3) · $(0.5)^3$ · $(0.5)^2$ ≈ **0.3125**

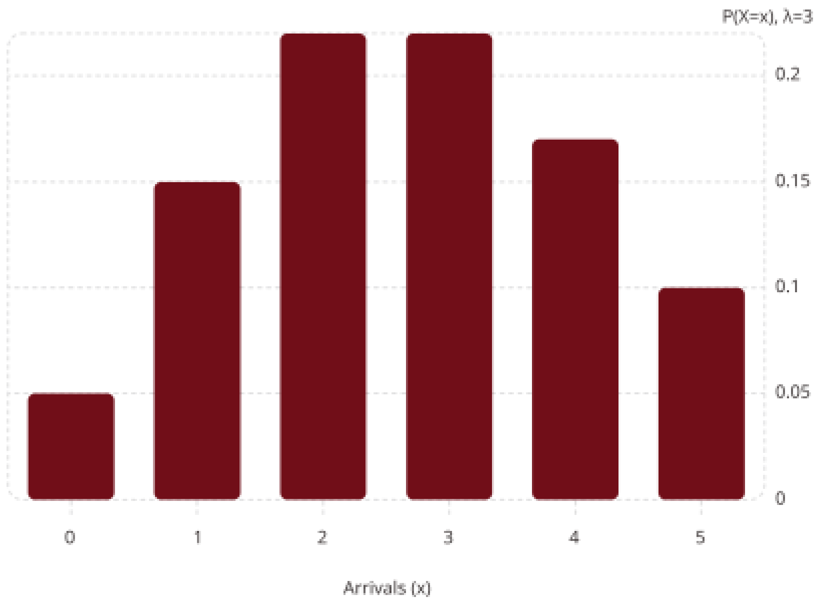
Poisson Distribution

Models the number of **events** occurring in a fixed interval at a constant average rate λ.

$$P(X = x) = \frac{e^{-\lambda}\ \lambda^{x}}{x!}$$

- **Mean:** $\mu = \lambda$
- **Variance:** $\sigma^2 = \lambda$

Example: $\lambda = 3$ customers/hour.
$P(X = 2) = e^{-3} \cdot 3^2 / 2! \approx$ **0.224**



Binomial vs. Poisson: Comparison

Both model discrete outcomes but apply to different scenarios.

Feature	Binomial	Poisson
Trial Type	Fixed number of trials (n)	Fixed time or space interval
Outcomes	Success / Failure	Count of events
Parameters	n (trials), p (probability)	λ (average rate)
Mean	$\mu = np$	$\mu = \lambda$
Variance	$\sigma^2 = np(1 - p)$	$\sigma^2 = \lambda$
Example	3 heads in 5 coin tosses	2 customers arriving per hour

Remember

Random Variables

Can be **discrete** (countable) or **continuous** (measurable)

Mean (μ)

Measures the **center** or long-run average of a distribution

Variance (σ^2)

Measures the **spread** of values around the mean

Binomial

Used for **fixed trials** with two outcomes

Poisson

Used for **events over time or space** at a constant rate

Thank You..