Surveying and Geomatics
Unit-2

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Introduction (Refer next slide)

- Leveling is the general term applied to any of the various processes by which elevations of points or differences in elevation are determined.

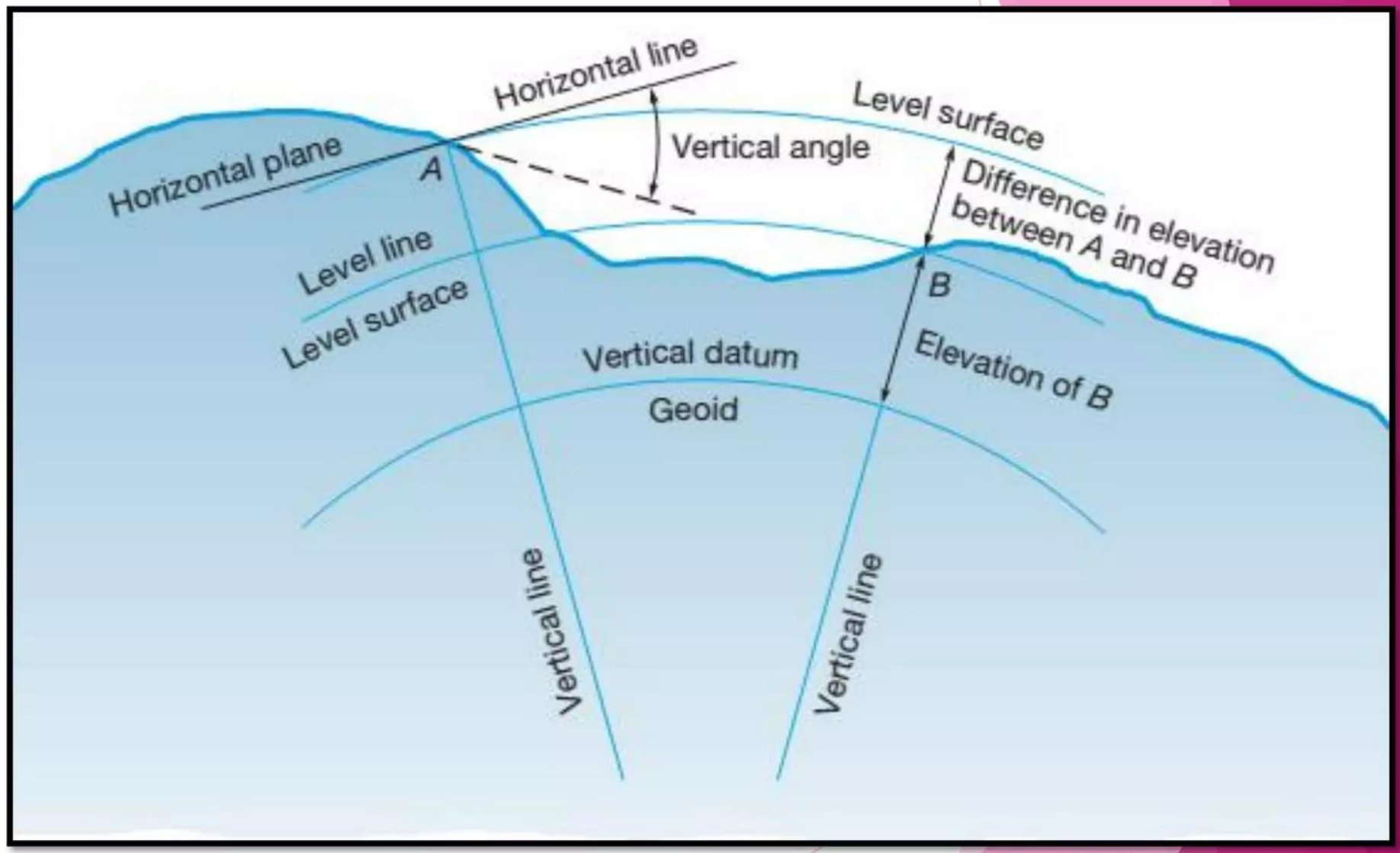
Basic terms:

- *Vertical line*. A line that follows the local direction of gravity as indicated by a plumb line.
- *Level surface*. A curved surface that, at every point is perpendicular to the local plumb line (the direction in which gravity acts).
- *Level line*. A line in a level surface
- *Horizontal plane*. A plane perpendicular to the local direction of gravity. In plane surveying, it is a plane perpendicular to the local vertical line.
- *Horizontal line*. A line in a horizontal plane. In plane surveying, it is a line perpendicular to the local vertical.
- *Vertical datum*. Any level surface to which elevations are referenced. This is the surface that is arbitrarily assigned an elevation of zero.
- Elevation. The distance measured along a vertical line from a vertical datum to a point or object.

Leveling terms.

- *Benchmark* (*BM*).

A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed.



Levelling terms

- Station: A point where the levelling staff is kept.
- Height of instrument: It is the elevation of the plane of sight with respect to assumed datum. It is also known as plane of collimation.
- Back sight(BS): It is the sight taken on the level staff, of a known elevation with the intention to obtain the elevation of plane of collimation. It is called PLUS sight because it is added to elevation of that point to get height of instrument or plane of collimation.
- Intermediate sights(IS): These are the sight taken after back sight and before sighting the final point. These are called MINUS sights. These are subtracted from plane of collimation to find the reduced level of different points.
- Fore sight(FS): The last reading taken from the instrument. This is also a MINUS sight.
- Change point(CP) or turning point(TP): The point at which both BS and FS are taken.
- Reduced level(RL): The elevations of the points with respect to assumed datum.

Types of levelling

- Simple levelling
- Differential levelling
- Fly levelling
- Profile levelling
- Cross sectional levelling
- Reciprocal levelling

There are two methods for obtaining the elevations at different points:

- Height of instrument (or plane of collimation) method
- Rise and fall method

Height of Instrument method

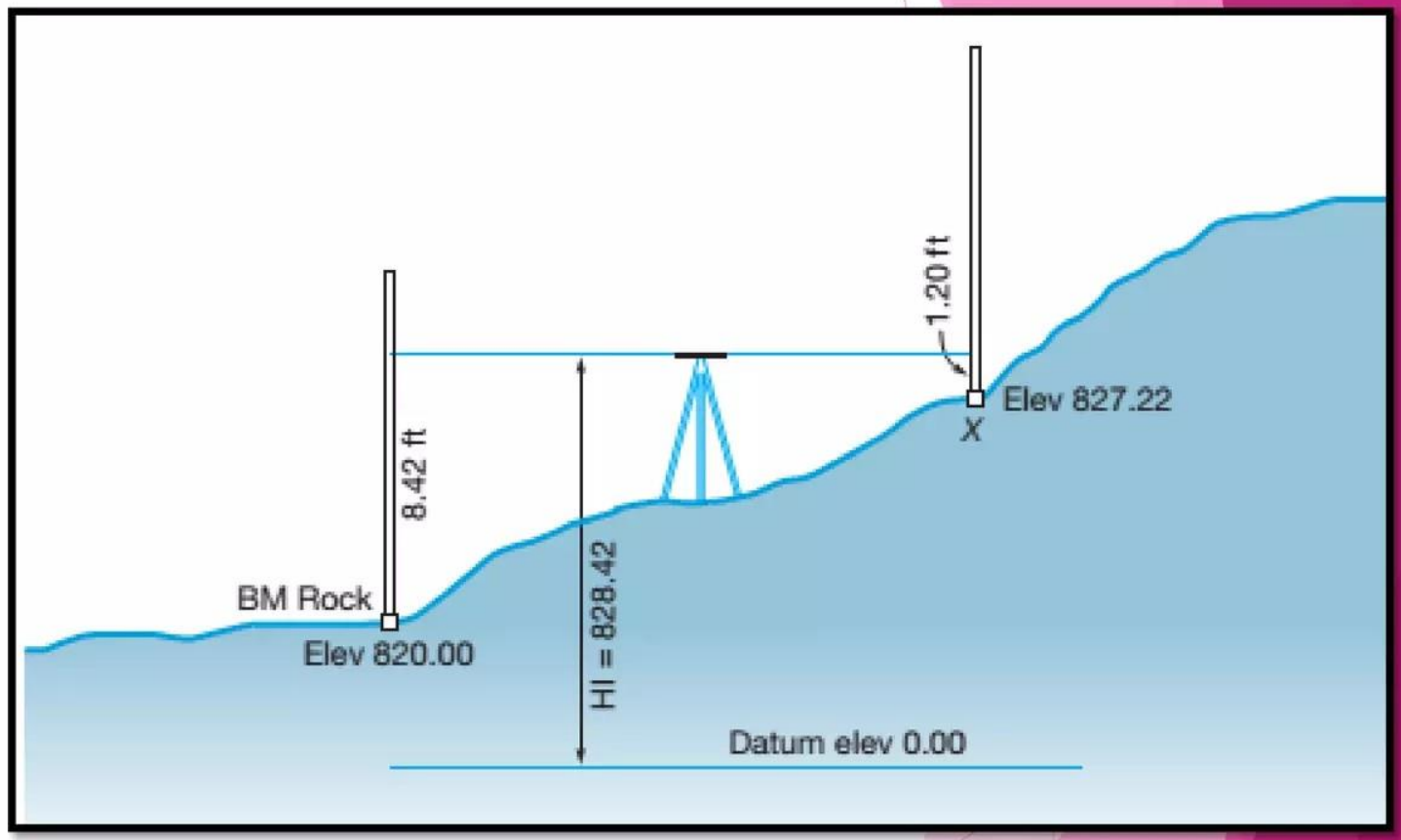
- The basic equations are
- Height of instrument for the first setting= RL of BM + BS(at BM)
- Subtract the IS and FS from HI to get RL of intermediate stations and change points.
- Checking: ∑BS - ∑FS = Last RL – First RL. **This is -ve for FALL and +ve for RISE.**

Rise and Fall method

- **In this method the difference of the present staff reading is subtracted from the previous staff reading.**
- **Previous reading - present staff reading = +ve, denotes RISE**
- **Previous reading - present staff reading = -ve, denotes FALL**
- **Checking:** ∑BS - ∑FS = Last RL – First RL= ∑Rise - ∑Fall

Simple leveling

- When the difference in the elevation of two nearby points is required then simple levelling is performed.
- Assume the elevation of BM Rock is known to be 820.00 ft.
- The BS at BM Rock is 8.42ft.
- So HI = (820 + 8.42)ft.
- Now the FS on “X” is 1.2ft.
- So the RL at “X” = HI – FS = 828.42ft
- Note that the RL of the instrument station will never comes in the calculation.



Differential levelling(table is in the next slide)

- Performed when the final point is very far from the final point.
- We have to find RL at B.
- It is given that RL at A is 100m and BS at A is 2.45m
- So, HI at L1=(100+2.45)=102.45m
- FS at CP1=2.41m
- RL at CP1=(102.45-2.14)=100.31m
- Now BS at CP1=1.43m
- HI at L2=(100.31+1.43)=101.74m
- FS at CP2=2.18m
- RL at CP2=(101.74-2.18)=99.56m
- BS at CP2=1.38m
- HI at L3=(99.56+1.38)m=100.94m
- FS at B=1.54m
- RL at B= (100.94-1.54)=**99.4**m (ans)

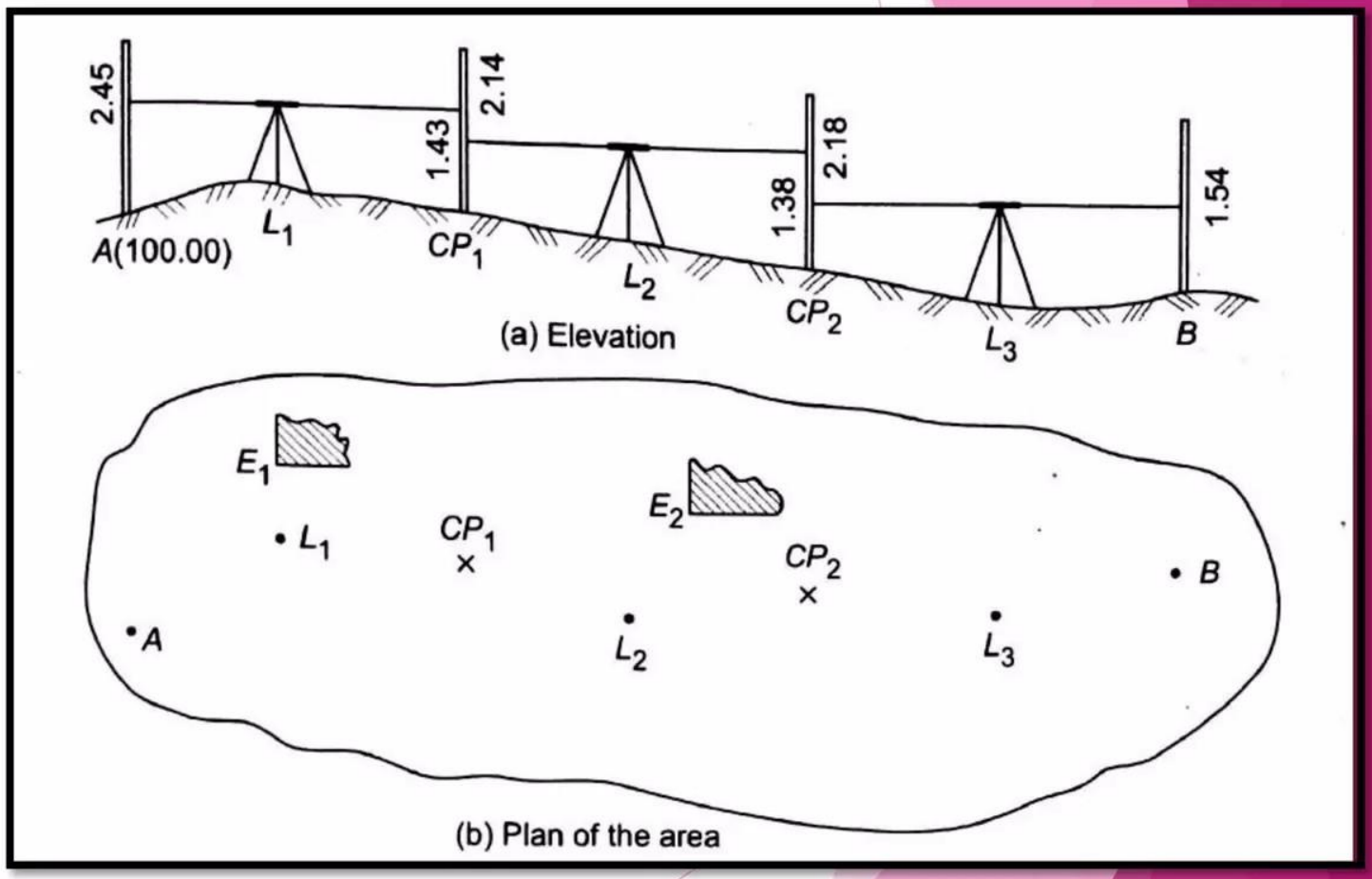


Table of previous example (height of instrument method)

Station	Reading			Height of Instrument	R.L	Remark
	BS	IS	FS			
A	2.45			102.45	100.00	Bench Mark
E_1		0.86			101.59	Plinth of Building
CP_1	1.43		2.14	101.74	100.31	Change Point
E_2		0.76			100.98	Plinth of Building
CP_2	1.38		2.18	100.94	99.56	Change Point
B			1.54		99.40	Station B, the last point shown
Check	Σ 5.26		Σ 5.86			
	Diff = −0.60 (Fall)				−0.60 (Fall)	

- Note that ∑BS- ∑FS=last RL – first RL

Table of previous example (Rise and Fall method)

Station	BS	IS	FS	Rise	Fall	R.L	Remarks
A	2.45					100.00	Bench Mark
		0.86		1.59		101.59	E_1
	1.43		2.14		1.28	100.31	CP_1
		0.76		0.67		100.98	E_2
	1.38		2.18		1.42	99.56	CP_2
			1.54		0.16	99.40	Last-Point
Σ	5.26		5.86	2.26	2.86		
	ΣBS – ΣIS = –0.6			ΣRise – ΣFall = –0.6		Last RL – First RL = –0.6	Checked

Note the rise and fall calculations:

From A to E_1, difference = 2.45 – 0.86 = 1.59, rise

From E_1 to CP_1, difference = 0.86 – 2.14 = –1.28 = 1.28 m, fall

From CP_1 to E_2, difference = 1.43 – 0.76 = 0.67, rise

From E_2 to CP_2, different = 0.76 – 2.18 = –1.42 = 1.42, fall

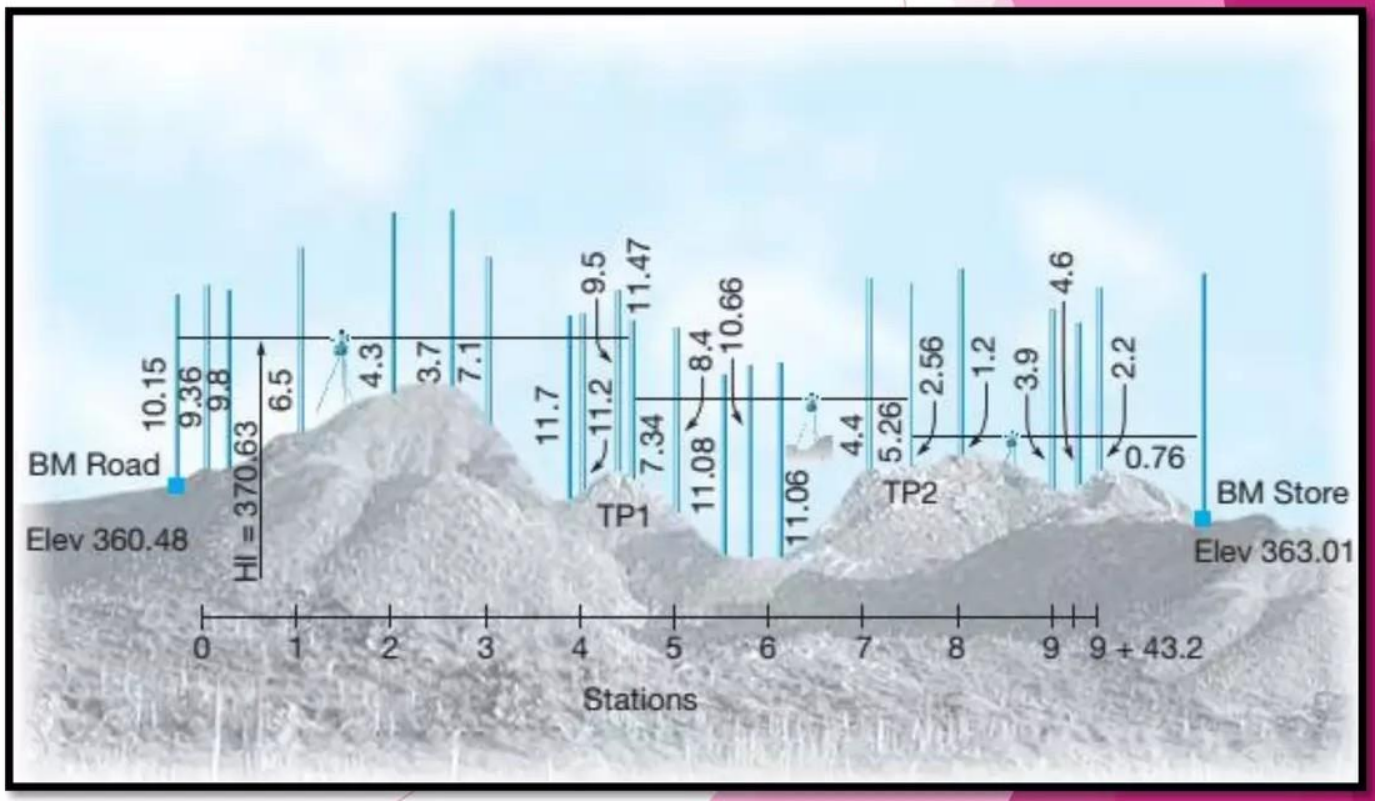
From CP_2 to Last Point, difference = 1.38 – 1.54 = –0.16 = 0.16, fall.

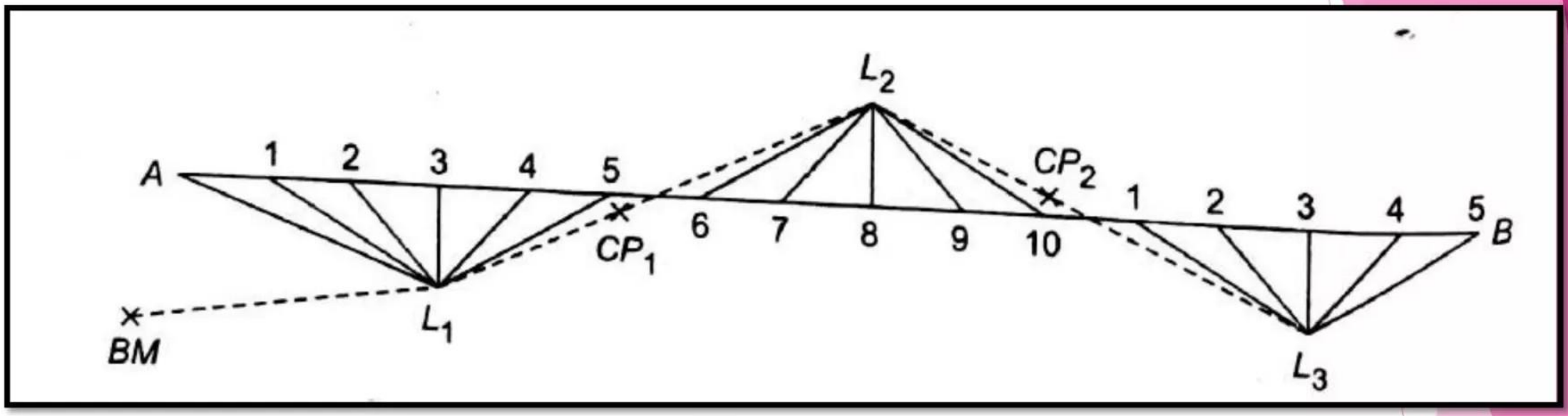
Fly levelling

- Performed when the work site is very far away from the bench mark.
- The surveyor starts by taking BS at BM and proceed towards worksite till he finds a suitable place for temporary BM. All works are done with respect to temporary BM.
- At the end of the day the surveyor comes back to original BM.
- This is called fly levelling.

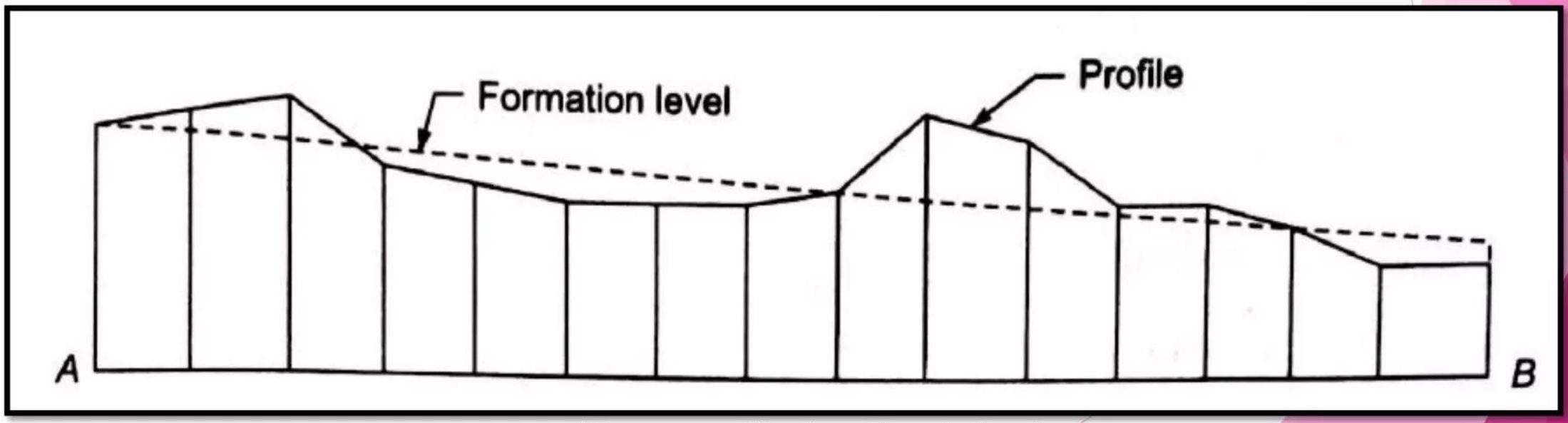
Profile levelling (figures are in next slide)

- Profile leveling, which yields elevations at definite points along a reference line, provides the needed data for designing facilities such as highways, railroads, transmission lines.
- Reduced levels at various points at regular interval along the line is calculated.
- After getting the RL of various points the profile is drawn. Normally vertical scale is much larger than horizontal scale for the clear view of the profile.



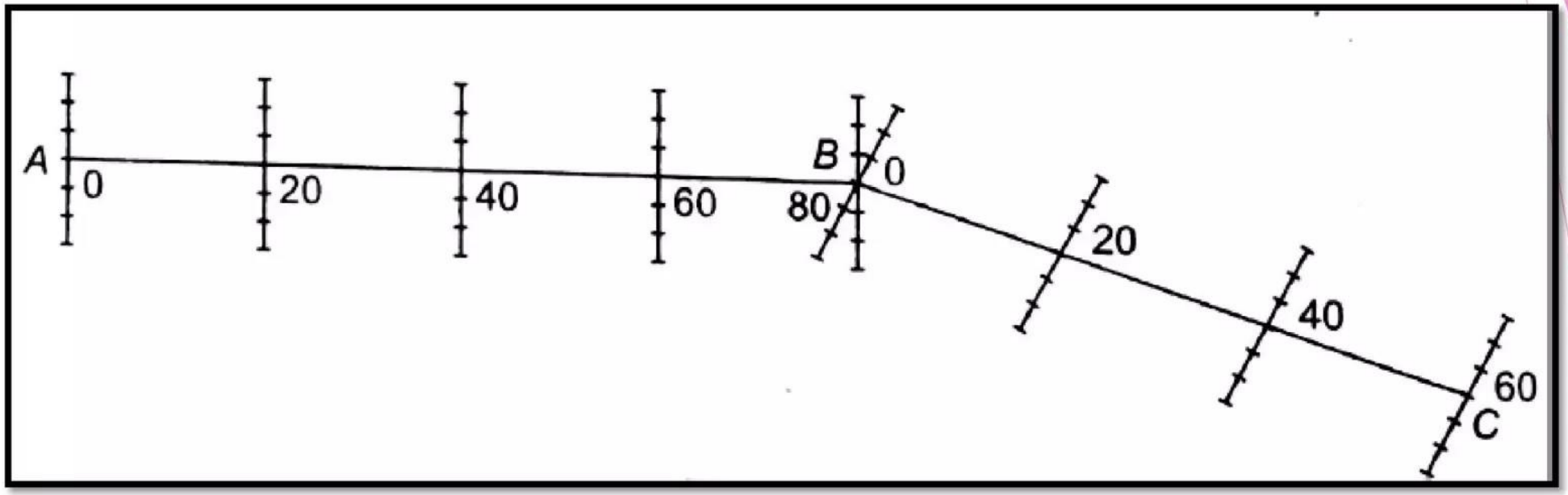


Plan view



Drawn profile (sectional view)

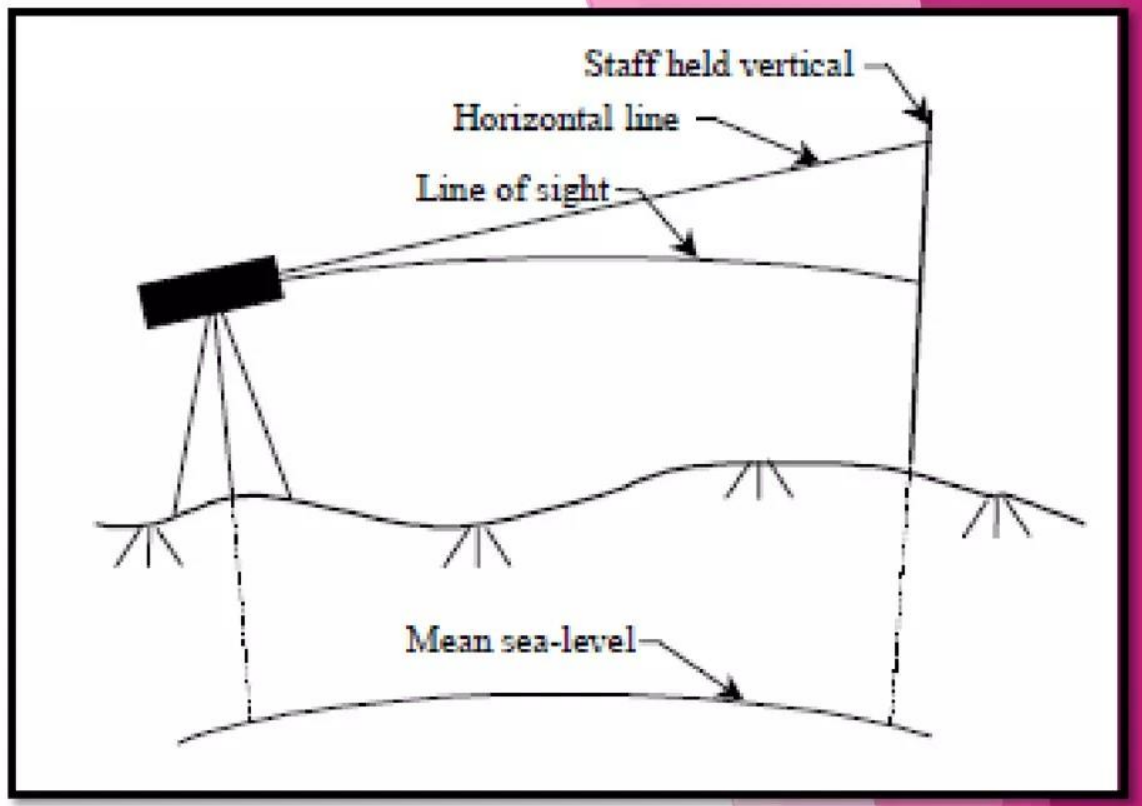
Cross sectioning



- In many projects not only profile along the line is required but also profile of cross – section at a regular interval is required. (here 20m)
- Levels are taken at every 3m distance (till 9m i.e. 3 readings on one side) to left and right of the line AB and BC for cross section profile.

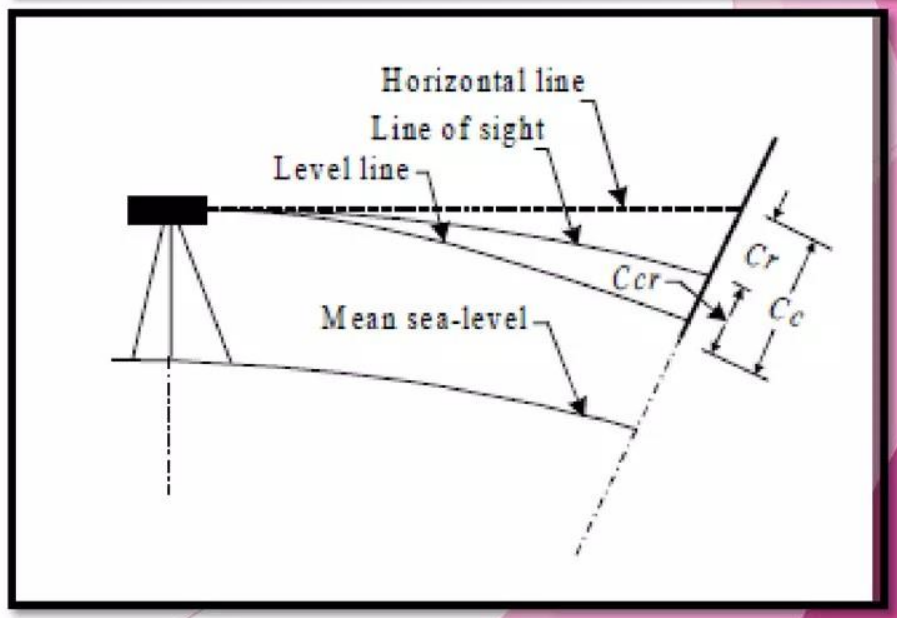
curvature correction(Cc)

- Cc is always negative.
- A properly adjusted instrument take the readings with respect to horizontal line.
- So something has to be subtracted from the staff reading to get actual reading.
- $Cc = d^2/2R$ and -ve
- D is distance of the staff from the point of tangency and R is the radius of the Earth = 6370km
- $Cc = -7.849 \times 10^{-8} \times d^2$ meters, if d is in meters.



Effect of refraction (Cr)

- Air is denser near the earth and becomes thinner as we go far from the surface of the earth. This causes refraction.
- $Cr = (1/7) \times (d^2/2R)$ and +ve



Combined effect

- $Cc + Cr = - (6/7) \times (d^2/2R) = - 6.728 \times 10^{-8} \times d^2$ meters, if d is in meters.

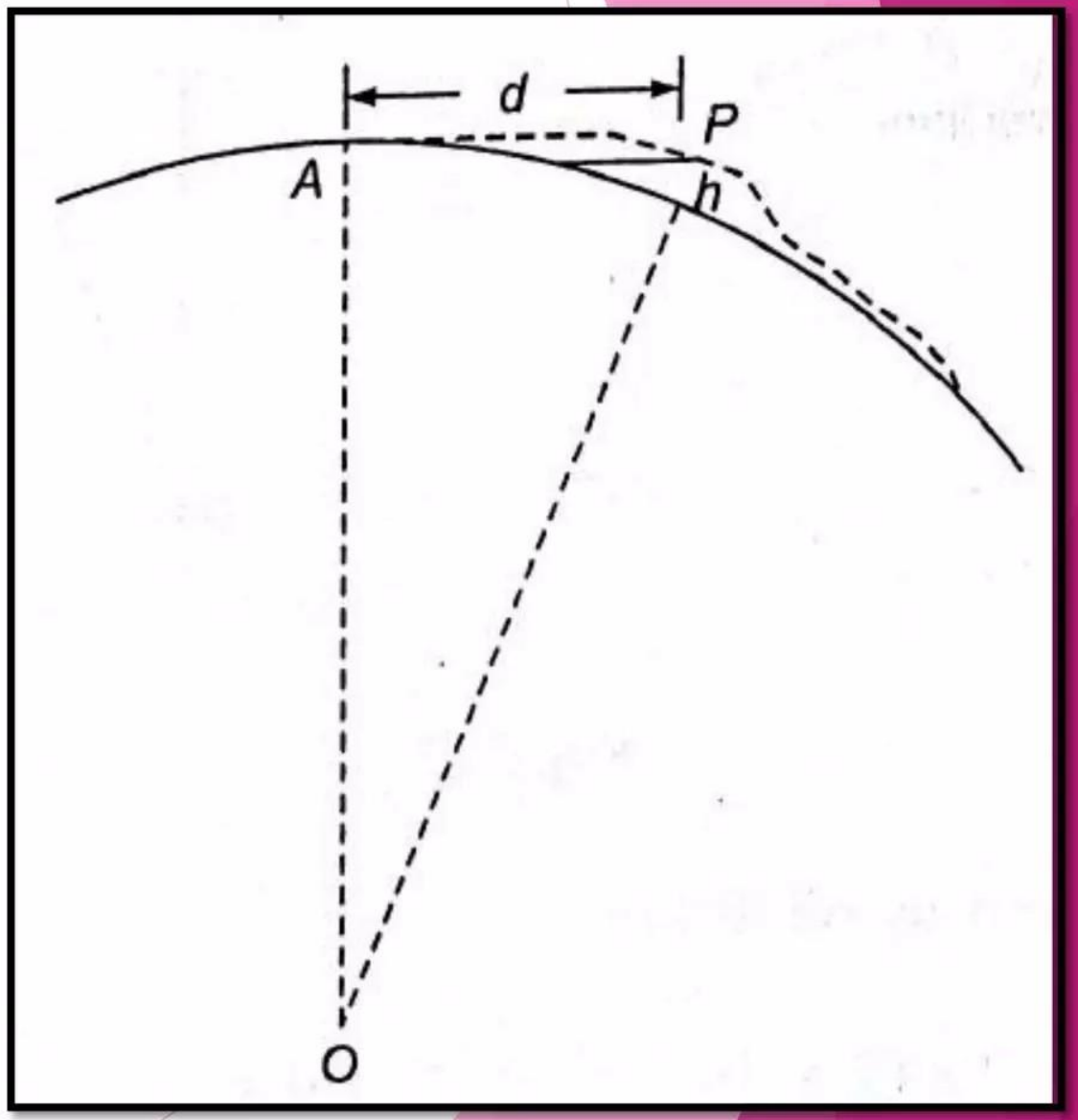
Distance of visible horizon

- Let P be the point of observation which is “h” meters above mean seal level.
- “d” is called distance of visible horizon.
- Taking refraction and curvature into account.

$$h = 6.728 \times 10^{-8} d^2$$

$$d = \sqrt{\frac{h}{6.728 \times 10^{-8}}}$$

$$= 10^4 \sqrt{\frac{h}{6.728}} = 3855.3\sqrt{h} \text{ m}$$

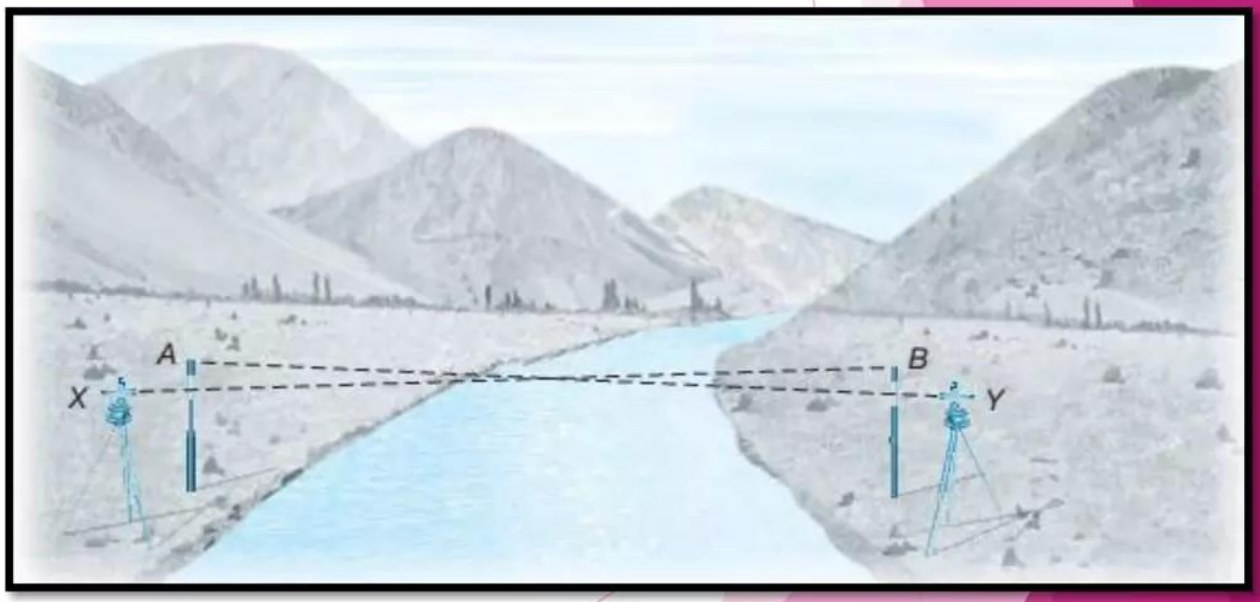


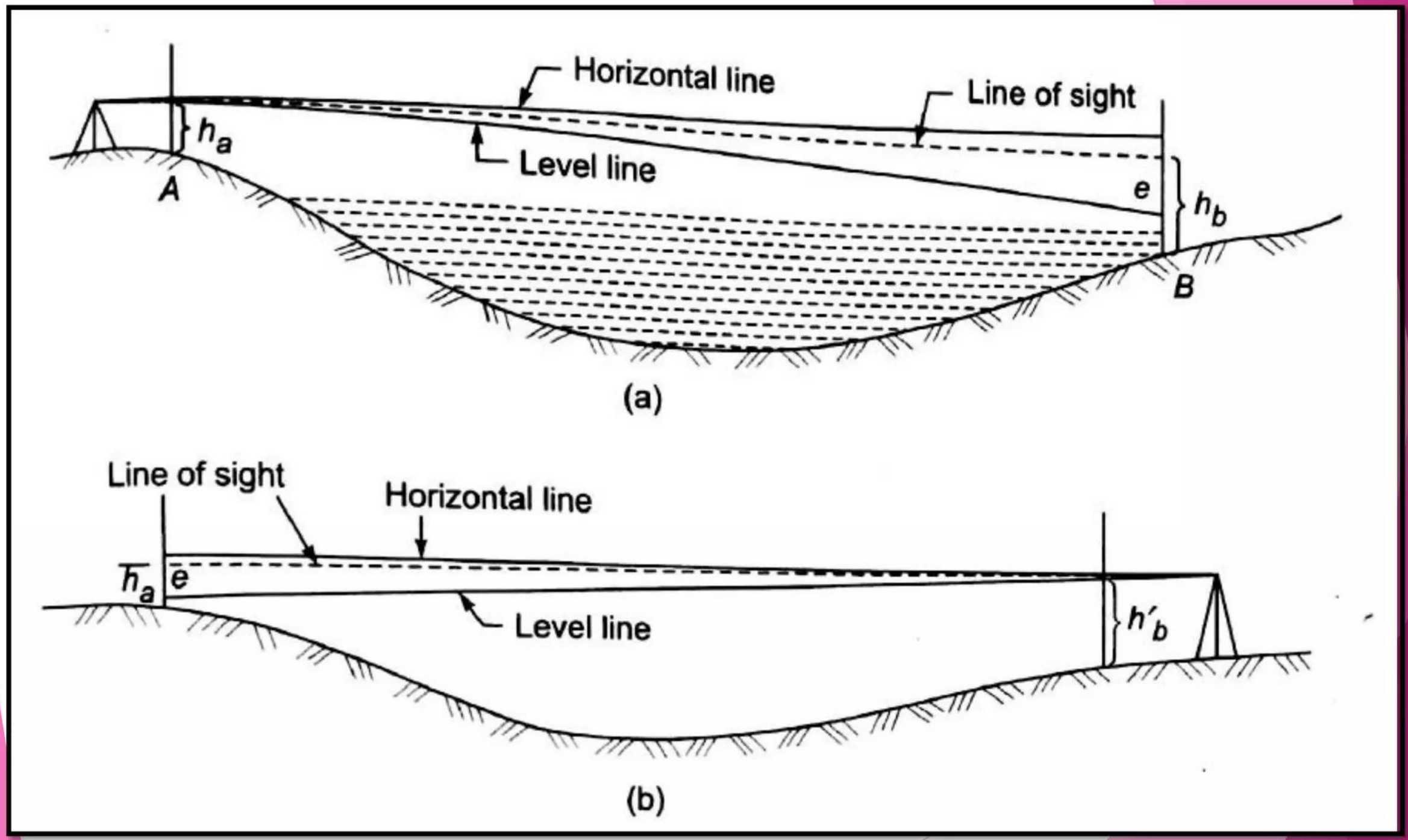
Note

- If the instrument is kept, such that the distance of BS point and the FS point are same (from the instrument) then the difference in elevation of the two points will be free from curvature and refraction error.

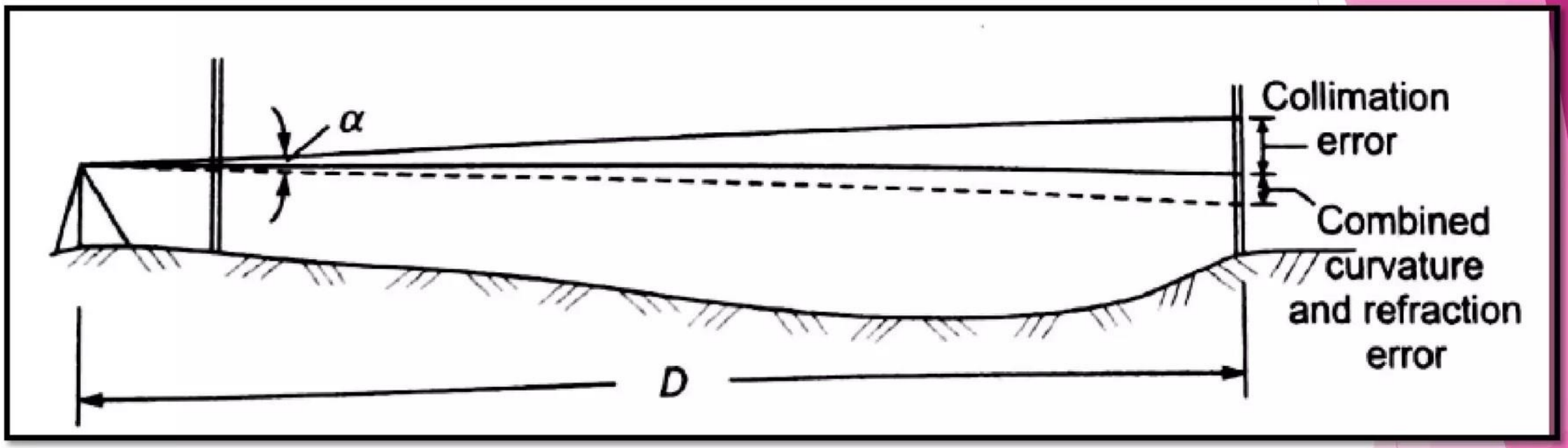
Reciprocal levelling (refer next slide for pictures)

- When levelling across river is required then this method is applied to get rid of various errors.
- The correct elevation is given as:
- The instrument is set very close to A. so ha will be correct. Let the error in h_b be "e". Therefore, correct reading at B is $(h_b - e)$
- Difference in the elevation= $h_a - (h_b - e)$
- The instrument is set very close to B. so h'_b will be correct. Let the error in h'_a be "e". Therefore, correct reading at B is $(h'_a - e)$
- Difference in the elevation= $(h_a - e) - h_b$
- $2H = [h_a - (h_b - e) + (h'_a - e) - h'_b]$
- **$H = [(h_a - h_b) + (h'_a - h'_b)]/2$**
- **$e = [(h'_a - h'_b) - (h_a - h_b)]/2$**
- Thus the true elevation is given by mean of two apparent differences in elevations.




Horizontal line
Line of sight
h_a
Level line
A
e
h_b
B
(a)
Line of sight
Horizontal line
h_a
e
Level line
h'_b
(b)

Note



- e = collimation error + combined effect of curvature and refraction error
- **e = D x tan(α) + (6/7) x (D^2 / 2R)**
- **This "e" has to be subtracted from the staff reading to get original reading.**

$e = e_l + e_c - e_r$

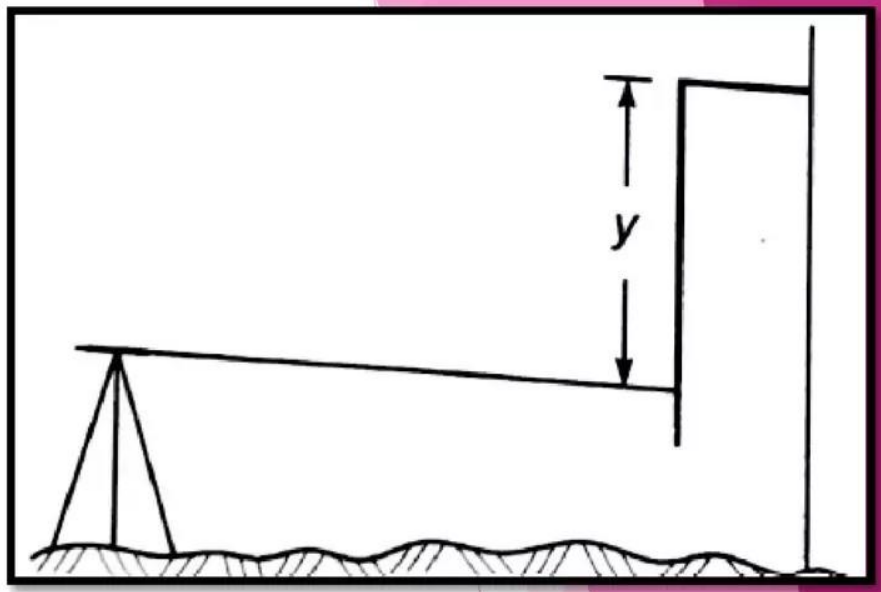
e_l = the collimation error assumed positive for the line of sight inclined upward

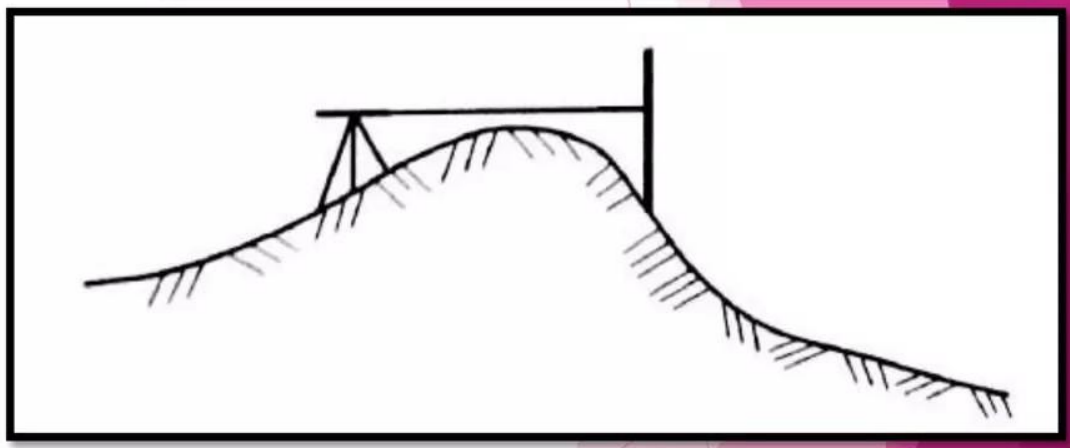
e_c = the error due to the earth's curvature, and

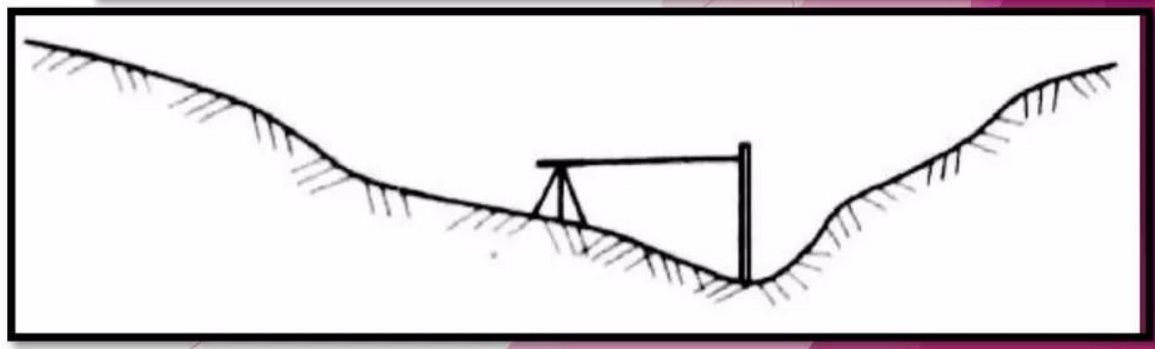
e_r = the error due to the atmospheric refraction.

Difficulties in the levelling

- Taking level of an overhead point
  - If we need RL of points like on *chejja* of the window, then the staff is inverted. This reading is entered as a negative reading.
  - To get RL
    - RL = HI – (- y), where “y” is the inverted reading.
    - RL = HI + y
- Levelling across the summit
  - The instrument set on one side of the summit, such that line of sight just passes over the summit.
- Levelling across a hollow
  - Instrument should set such that the reading at lowest point of the hollow is maximum staff reading.







Example 3.8. Reciprocal levelling was conducted across a wide river to determine the difference in level of points A and B, A situated on one bank of the river and B situated on the other. The following results on the staff held vertically at A and B from level stations 1 and 2, respectively, were obtained. The level station 1 was near to A and station 2 was near to B.

Instrument at	Staff reading on	
	A	B
1	1.485	1.725
2	1.190	1.415

(a) If the reduced level of B is 55.18 m above the datum, what is the reduced level of A?

(b) Assuming that the atmospheric conditions remain unchanged during the two sets of the observations, calculate (*i*) the combined curvature and refraction correction if the distance AB is 315 m, and (*ii*) the collimation error.

Solution:

To eliminate the errors due to collimation, curvature of the earth and atmospheric refraction over long sights, the reciprocal levelling is performed.

From the given data, we have

$$a_1 = 1.485 \text{ m}, \quad a_2 = 1.725 \text{ m}$$
$$b_1 = 1.190 \text{ m}, \quad b_2 = 1.415 \text{ m}$$

The difference in level between A and B is given by

$$\Delta h = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$
$$= \frac{(1.485 - 1.190) + (1.725 - 1.415)}{2} = 0.303 \text{ m}$$

R.L. of B = R.L. of A + Δh

R.L. of A = R.L. of B − Δh

= 55.18 − 0.303 = **54.88 m.**

The total error $e = e_l + e_c - e_r$

where

$$e = \frac{(b_1 - a_1) - (b_2 - a_2)}{2}$$
$$= \frac{(1.190 - 1.485) - (1.415 - 1.725)}{2} = 0.008 \text{ m}$$

and $e_c - e_r = 0.067\ d^2$

$= 0.067 \times 0.315^2 = 0.007$ m.

Therefore collimation error $e_l = e - (e_c - e_r)$

$= 0.008 - 0.007 =$ **0.001 m.**

Thank you