23CE302 : STRENTH OF MATERIALS

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UNIT-IV Deflection of Beams & Conjugate Beam Method

- **Topics Covered**
- Elastic Curve
- Slope & Deflection
- Radius of Curvature
- Double Integration Method
- Macaulay's Method
- Moment Area Method
- Mohr's Theorems
- Conjugate Beam Method

Learning Outcomes

- After completing this unit, students will be able to:
- Define slope and deflection of beams.
- Derive differential equation of elastic curve.
- Determine slope and deflection using Double Integration Method.
- Apply Macaulay's Method.
- Use Moment Area Method.
- Explain Mohr's Theorems.
- Understand Conjugate Beam Method.
- Calculate deflections in determinate beams.

Introduction to Beam Deflection

- **Beam Deflection**
- Beam deflection is the displacement of a beam from its original position due to external loads.
- **Importance**
- Safety of structures
- Serviceability
- Prevents excessive deformation
- Ensures comfort and stability
- **Applications**
- Bridges
- Buildings
- Machine components
- Aircraft structures

Slope, Deflection and Radius of Curvature

- **Slope (θ)**
- Angle made by tangent to elastic curve.
- $\theta = \frac{dy}{dx}$
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- **Deflection (y)**
- Vertical displacement of beam axis.
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- **Radius of Curvature (R)**
- Radius of bent beam.
- $\frac{1}{R} = \frac{M}{EI}$
- Where
- M = Bending Moment
- E = Young's Modulus
- I = Moment of Inertia

Elastic Curve

- Elastic Curve
- The shape assumed by the beam after bending.
- **Characteristics**
- Depends on loading
- Depends on support conditions
- Depends on EI

Differential Equation of Elastic Curve

- Starting equation
- $\frac{1}{R} = \frac{M}{EI}$
- For small slopes
- $\frac{1}{R} = \frac{d^2y}{dx^2}$
- Hence,
- $EI\frac{d^2y}{dx^2} = M$
- This is called the **Differential Equation of Elastic Curve.**

Double Integration Method

- **Principle**
- Integrate bending moment equation twice.
- **Step 1**
- $EI\frac{d^2y}{dx^2} = M$
- **Step 2**
- Integrate once
- $EI\frac{dy}{dx} = \int M\,dx + C_1$
- **Step 3**
- Integrate again
- $EIy = \int\left(\int M\,dx\right)dx + C_1x + C_2$
- Boundary conditions are used to find constants.

Cantilever Beam using Double Integration

- Example
- Cantilever carrying point load at free end.
- Slope
- $\theta = \frac{WL^2}{2EI}$
- Maximum Deflection
- $y = \frac{WL^3}{3EI}$
- Occurs at free end.

Simply Supported Beam using Double Integration

- Simply Supported Beam with Central Load
- Maximum Deflection
- $y = \frac{WL^3}{48EI}$
- Maximum slope
- Occurs at supports.

Macaulay's Method

- Suitable for beams having multiple point loads.
- General Equation
- $EI\frac{d^2y}{dx^2} = M$
- Macaulay Bracket
- $(x-a)^n$
- where
- $(x-a) = 0, x < a$
- Advantages
- Single equation
- Multiple loads handled easily

Procedure of Macaulay's Method

- Steps
- Calculate reactions.
- Write bending moment equation.
- Introduce Macaulay brackets.
- Integrate twice.
- Apply boundary conditions.
- Find slope and deflection.

Deflection due to Different Loadings

- **Point Load**
- Maximum localized deflection
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- **Uniformly Distributed Load (UDL)**
- Smooth deflection curve
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- **Uniformly Varying Load (UVL)**
- Non-uniform curvature
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- **Couple (Moment)**
- Produces rotation without direct vertical load

Mohr's Theorems (Moment Area Method)

- Developed by Otto Mohr.
- Based on **M/EI diagram**.
- Useful for determining
- Slope
- Deflection
- without solving differential equations.

Mohr's First Theorem

- **Statement**
- Change in slope between two points equals
- Area of M/EI diagram.
- $\theta_B - \theta_A = \int \frac{M}{EI} dx$
- Applications
- Beam rotation
- Structural analysis

Mohr's Second Theorem

- **Statement**
- Tangential deviation between two points equals
- Moment of area of M/EI diagram.
- $\Delta = \int \frac{Mx}{EI} dx$
- Applications
- Beam deflection
- Frame analysis

Moment Area Method

- **Advantages**
- Faster calculations
- No integration constants
- Easy graphical interpretation
- **Limitations**
- Simple loading only
- Difficult for complex beams

Conjugate Beam Method

- **Introduction**
- Developed from Moment Area Method.
- Uses imaginary beam called
- **Conjugate Beam**
- Loading on conjugate beam
- $\frac{M}{EI}$
- Output
- Shear → Slope
- Moment → Deflection

Applications of Conjugate Beam Method

- Used for
- Simply supported beams
- Cantilever beams
- Continuous beams
- Structural design
- **Advantages**
- Simple procedure
- Less mathematics
- Quick solution

Summary

- **Key Points**
- Simple bending follows bending equation
- $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$
- Section modulus indicates bending strength.
- Flexural stress is maximum at extreme fibers.
- Shear stress is maximum at neutral axis.
- Shear stress formula:
- $\tau = \frac{VQ}{Ib}$
- **Applications**
- Building Beams
- Bridges
- Machine Components
- Steel Structures
- RCC Design