23CE302 : STRENTH OF MATERIALS

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UNIT-II Shear Force and Bending Moment

- **Shear Force and Bending Moment**
- **Topics Covered**
- Types of Beams
- Shear Force (SF)
- Bending Moment (BM)
- SFD and BMD
- Cantilever Beams
- Simply Supported Beams
- Overhanging Beams
- Point of Contraflexure
- Relationship between Load, SF, and BM

Learning Outcomes

- After studying this unit, students will be able to:
- Classify different types of beams.
- Define shear force and bending moment.
- Draw Shear Force Diagrams (SFD).
- Draw Bending Moment Diagrams (BMD).
- Analyze different loading conditions.
- Locate the point of contraflexure.
- Explain the relationship between load, shear force, and bending moment.

Introduction to Beams

- **Definition**
- A **beam** is a structural member designed to resist transverse loads by developing shear forces and bending moments.
- **Applications**
- Buildings
- Bridges
- Roof Structures
- Machine Frames
- Railway Sleepers

Types of Beams

- **Classification Based on Supports**
- Cantilever Beam
- Simply Supported Beam
- Overhanging Beam
- Fixed Beam
- Continuous Beam
- Propped Cantilever Beam
- **Importance**
- Different support conditions produce different reactions and bending moments.

Types of Loads on Beams

- **1. Point (Concentrated) Load**
- Acts at a single point.
- **2. Uniformly Distributed Load (UDL)**
- Load is constant per unit length.
- **3. Uniformly Varying Load (UVL)**
- Load varies linearly along the beam.
- **4. Couple (Moment)**
- Produces rotation without translation.

Shear Force (SF)

- **Definition**
- The **shear force** at any section of a beam is the algebraic sum of all vertical forces acting on one side of the section.
- **Formula**
- $SF = \sum F_y$
- **Units**
- Newton (N)
- kN
- **Sign Convention**
- Upward force on the left → Positive
- Downward force on the left → Negative

Bending Moment (BM)

- **Definition**
- The **bending moment** at a section is the algebraic sum of the moments of all external forces about that section.
- **Formula**
- $BM = \sum (Force \times Distance)$
- **Units**
- N·m
- kN·m
- **Sign Convention**
- Sagging → Positive
- Hogging → Negative

Shear Force Diagram (SFD)

- **Definition**
- A graphical representation showing the variation of shear force along the length of the beam.
- **Characteristics**
- Constant between point loads
- Linear under UDL
- Parabolic under UVL
- **Uses**
- Beam Design
- Structural Analysis
- Identifying Maximum Shear

Bending Moment Diagram (BMD)

- **Definition**
- A graphical representation showing the variation of bending moment along the beam.
- **Characteristics**
- Straight line under point load
- Parabolic under UDL
- Cubic curve under UVL
- **Uses**
- Determining Maximum BM
- Reinforcement Design
- Beam Safety

Cantilever Beam – Point Load

- **Conditions**
- One end fixed
- One end free
- Point load at free end
- **Shear Force**
- Constant throughout the beam.
- **Maximum Bending Moment**
- $M = PL$
- Occurs at the fixed end.

Cantilever Beam – UDL

- **Loading**
- Uniformly Distributed Load over the entire span.
- **Maximum Shear Force**
- $V = wL$
- **Maximum Bending Moment**
- $M = \frac{wL^2}{2}$
- Occurs at the fixed support.

Simply Supported Beam – Point Load

- **Load at Centre**
- **Support Reactions**
- $R_A = R_B = \frac{P}{2}$
- **Maximum Bending Moment**
- $M = \frac{PL}{4}$
- Occurs at the centre.

Simply Supported Beam – UDL

- **Load**
- Uniformly Distributed Load over the entire span.
- **Support Reactions**
- $R_A = R_B = \frac{wL}{2}$
- **Maximum BM**
- $M = \frac{wL^2}{8}$
- Occurs at the centre.

Simply Supported Beam – UVL

- **Characteristics**
- Shear Force Diagram → Parabolic
- Bending Moment Diagram → Cubic
- **Applications**
- Retaining Walls
- Earth Pressure
- Water Pressure

Overhanging Beam

- **Definition**
- A beam extending beyond one or both supports.
- **Loading**
- Point Load
- UDL
- UVL
- Combination of Loads
- **Importance**
- Used in balconies, bridges, and roof projections.

Beams Subjected to Couple and Combined Loads:

- **Couple (Moment)**
- Produces a sudden jump in the Bending Moment Diagram without changing the Shear Force Diagram.
- **Combined Loading**
- May include:
- Point Load
- UDL
- UVL
- Applied Couple
- **Analysis Steps**
- Calculate support reactions.
- Draw SFD.
- Draw BMD.

Point of Contraflexure

- **Definition**
- The point where the bending moment changes sign from positive to negative or vice versa.
- **Characteristics**
- BM = 0 at the point.
- Common in overhanging and continuous beams.
- **Importance**
- Indicates change in curvature of the beam.

Relationship Between Load, SF, and BM

- **Differential Relationships**
- $\frac{dV}{dx} = -w$
- $\frac{dM}{dx} = V$
- Where:
- w= Rate of loading
- V= Shear force
- M= Bending moment
- **Key Observations**
- Constant load → Linear SF.
- Linear SF → Parabolic BM.
- Maximum BM occurs where SF = 0.

Numerical Problems:

- **Example**
- A simply supported beam of span **6 m** carries a **20 kN** point load at its center.
- **Find**
- Support reactions
- Maximum Shear Force
- Maximum Bending Moment
- **Solution**
- $R_A = R_B = 10\ kN$
- Maximum SF = **10 kN**
- Maximum BM:
- $M = \frac{PL}{4} = \frac{20\times6}{4} = 30\ kN\cdotpm$

Summary

- **Key Takeaways**
- A beam resists transverse loads through **shear force** and **bending moment**.
- **SFD** represents variation of shear force along the beam.
- **BMD** represents variation of bending moment.
- Different loading conditions produce characteristic SFD and BMD shapes:
  - Point Load → Constant SF, Linear BM
  - UDL → Linear SF, Parabolic BM
  - UVL → Parabolic SF, Cubic BM
- The **Point of Contraflexure** is where the bending moment changes sign.
- Fundamental relationships:
- $\frac{dV}{dx} = -w, \frac{dM}{dx} = V$