NARSIMHA REDDY ENGINEERING COLLEGE

UGC AUTONOMOUS INSTITUTION

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FLUID MECHANICS

Topic: UNIT V - Boundary Layer Thickness & Growth Profiles

Laminar Pipe Flow: Hagen-Poiseuille Law

- Models fully developed laminar flow through circular pipes at low Reynolds numbers.
- Velocity profile assumes a parabolic curve shape: $u(r) = U_{max} \cdot [1 - (r/R)^2]$.
- Maximum velocity at the center is exactly twice the average velocity ($U_{max} = 2 \cdot V_{avg}$).
- Pressure drop expression: $\Delta p = (32\ \mu\ U\ L) / d^2$, with shear stress peaking linearly at the walls.

Boundary Layer Concept Foundations

- Prandtl's Hypothesis: Viscous effects are confined to a thin layer near solid walls.
- Outside this boundary layer, fluid behavior can be modeled as ideal and inviscid.
- No-Slip Condition: Fluid velocity drops to zero directly at the solid wall surface.
- The boundary layer grows thicker along the plate in the downstream direction.

Nominal Boundary Layer Thickness (δ)

- Quantifies the physical height of the fluid layer affected by wall friction.
- Defined as the vertical distance from the wall where local velocity reaches 99% of free-stream velocity.
- Mathematical criterion: δ is located where local velocity $u = 0.99 \cdot U_\infty$.
- Growth rates match local flow regimes, expanding as $x^{1/2}$ under laminar conditions.

Displacement Thickness Mechanics (δ*)

- Measures the reduction in mass flow rate caused by boundary layer friction.
- Represents the distance the solid wall must be shifted to restore mass flow capacity.
- Governing Equation: $\delta^* = \int_0^n (1 - u / U\_\infty)\, dy$, integrated vertically.
- Key parameter used in aerodynamic design to correct airfoil profile geometries.

Momentum Thickness Formulations (θ)

- Measures the reduction in momentum flux caused by skin friction shear forces.
- Represents the fluid layer thickness required to carry the missing momentum.
- Governing Equation: $\theta = \int_0^n (u / U\_\infty) \cdot (1 - u / U\_\infty) \, dy$.
- Directly linked to total drag forces via von Kármán momentum integral equations.

Energy Thickness Principles (δ_E)

- Measures the loss of kinetic energy within the boundary layer.
- Represents the fluid layer thickness required to carry the missing kinetic energy.
- Governing Equation: $\delta_E = \int_0^n (u / U_\infty) \cdot (1 - u^2 / U_\infty^2)\, dy$.
- Quantifies viscous dissipation rates in high-velocity boundary layer fields.

Shape Factor & Profile Transition

- Shape Factor (H): Dimensionless ratio of displacement to momentum thickness ($H = \delta^* / \theta$).
- Blasius laminar solution yields H = 2.59; turbulent profiles drop to $H \approx 1.3 - 1.4$.
- Boundary layer transition occurs as the plate Reynolds number (Re_x) passes $\approx 5 \times 10^5$.
- Turbulent layers exhibit full velocity profiles with higher wall shear stresses.

Von Kármán Momentum Integral Form

- Provides an engineering analysis framework applicable to both laminar and turbulent boundary layers.
- Integral Form: $\tau_0 / (\rho U_\infty^2) = d\theta / dx$, linking wall shear to momentum growth.
- Used to estimate drag forces without requiring exact solutions to Navier-Stokes equations.
- Forms the foundation for semi-empirical modeling of rough engineering boundaries.

Adverse Pressure Gradients & Separation

- Favorable Gradient ($dp/dx < 0$): Pressure drops downstream, stabilizing boundary layer growth.
- Adverse Gradient ($dp/dx > 0$): Pressure rises downstream, slowing down fluid particles.
- Flow Separation: Occurs when fluid particles near the wall run out of kinetic energy.
- Separation Criterion: Wall velocity gradient drops to zero: $(\partial u/\partial y)$ at $y=0$ is less than or equal to 0.

Separation Control & Aerodynamic Drag

- Separation triggers a wake zone downstream, increasing form drag forces.
- Control Methods: Boundary layer suction, micro vortex generators, or geometric streamlining.
- Drag Components: Total Drag = Skin Friction Drag + Pressure (Form) Drag.
- Streamlining shifts separation points downstream, reducing overall pressure drag.

Aerodynamic Lift & The Magnus Effect

- Lift Force: Component of fluid force acting perpendicular to the relative approach velocity.
- Magnus Effect: Generation of lift force on a spinning cylinder or sphere in a fluid stream.
- Rotation alters local velocity fields, creating a pressure difference between opposite sides.
- Provides the operating principle for spinning sports balls and rotor ship propulsion systems.

UNIT V Summary & High-Yield Viva-Voce Key

- Comprehensive tracking of Unit V core syllabus parameters and theoretical frameworks.
- High-yield focus on governing mathematical derivations and mechanical configurations.

 VIVA-VOCE ASSESSMENT CORE QUESTIONS:

1. Define the nominal boundary layer thickness (δ) using the velocity profile criterion.
2. Explain the physical distinction between displacement thickness (δ^*) and momentum thickness (θ).
3. State the exact mathematical boundary criterion that marks the onset of flow separation.

Thank You..