FLUID MECHANICS

Topic: UNIT II - 3D Continuity Equation in Cartesian Coordinates

Classification of Fluid Flows

- Steady vs Unsteady: Evaluates variations in flow field velocity vectors over discrete time cycles.
- Uniform vs Non-Uniform: Analyzes velocity variations relative to changing spatial coordinates.
- Laminar vs Turbulent: Identifies highly structured parallel fluid paths vs chaotic eddy mixtures.
- Rotational vs Irrotational: Tracks if fluid elements spin about localized internal geometric mass centers.

Flow Descriptions: Streamlines, Pathlines & Streaklines

- Streamline: Imaginary spatial curves tangent to velocity field directional vectors at any instant.
- Pathline: The absolute spatial path traced out by a single, distinct fluid particle over time.
- Streakline: Locus of all distinct fluid particles that passed sequentially through a fixed point.
- Stream Tube: Interconnected bundle of streamlines forming an impermeable fluid flow conduit boundary.

Velocity Potential & Stream Functions

- Velocity Potential (ϕ): Exists strictly within irrotational flow fields where velocity components $u = -\partial\phi/\partial x$.
- Stream Function (ψ): Defined strictly for 2D incompressible systems where velocity $u = -\partial\psi/\partial y$.
- Flow Nets: Formed via orthogonal crossings of equipotential lines and stream lines at 90° junctions.
- Laplace Equation: Satisfying $\nabla^2\phi = 0$ or $\nabla^2\psi = 0$ confirms structurally valid irrotational configurations.

Derivation of 3D Continuity: Mass Conservation

- Built upon the physical axiom of Conservation of Mass within a differential control volume.
- Net mass flow rate entering the spatial control grid must equal internal mass accumulation rates.
- Mass flux tracking is conducted systematically across parallel coordinate planes (dx, dy, dz).
- Establishes spatial balance between coordinate space velocities and fluid densities over time.

The Mathematical 3D Continuity Form

- General Equation: $\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$.
- Tracks density changes alongside coordinate components (u, v, w) across X, Y, and Z axes.
- Vector Operator Notation: $\partial\rho/\partial t + \nabla \cdot (\rho V) = 0$, where ∇ represents the spatial gradient vector.
- Governs all multi-dimensional aerodynamic, hydraulic, and process flow transport analyses.

Simplifications: Steady & Incompressible Flow

- Steady Flow Simplification: Eliminates transient time-dependent variations ($\partial\rho/\partial t = 0$).
- Incompressible Fluid Flow: Mass density remains spatially and temporally invariant (ρ = Constant).
- Resulting Classical Continuity Form: $\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$.
- Direct Vector Representation: $Div(V) = \nabla \cdot V = 0$, representing zero localized net volume divergence.

Fluid Dynamics: Surface and Body Forces

- Body Forces: Act externally across the complete structural mass volume (e.g., gravity, magnetic pull).
- Surface Forces: Act directly on boundary cross-sections via direct contact (e.g., pressure, shear stress).
- Forms the fundamental force balance framework required to build Navier-Stokes and Euler formulations.
- Governs momentum fluxes within highly dynamic open channels and closed internal pipeline structures.

Euler's Equation of Motion

- Formulated by applying Newton's Second Law of Motion directly along a streamlined fluid path.
- Core Assumptions: Fluid is non-viscous (inviscid), completely continuous, and homogenous.
- Governing Equation: $(dp / \rho) + g \cdot dz + V \cdot dV = 0$, reflecting static balance forces.
- Links pressure acceleration differentials directly with geometric elevation shifts and kinetic changes.

Bernoulli's Equation Integration

- Derived by performing mathematical integration on Euler's differential form along a streamline.
- Core Constraints: Fluid must be ideal (inviscid), steady, incompressible, and completely irrotational.
- Classic Integration Form: $(p / \rho g) + (V^2 / 2g) + z = \text{Constant}$.
- Relates Pressure Head, Velocity Head, and Datum Head across discrete flow sections.

Kinetic Energy Correction Factor (α)

- Accounts for non-uniform, real velocity profiles across internal pipeline cross-sections.
- Mathematical Form: $\alpha = (1/A) \int (v/V)^3 \, dA$, where V is mean velocity and v is local stream velocity.
- Laminar Flow through pipes: $\alpha = 2.0$ (highly non-uniform parabolic profile configuration).
- Turbulent Flow through pipes: α ranges from 1.02 to 1.15 (flatter, highly uniform velocity core).

Vortex Flow Mechanics: Free vs Forced

- Free Vortex: Fluid rotates without external power; velocity varies inversely with radius ($V \propto 1/r$).
- Examples include whirlpools, basin drains; total energy head remains completely constant across loops.
- Forced Vortex: Fluid rotates via constant external mechanical torque input; velocity $V = \omega \cdot r$.
- Fluid surface deforms into a paraboloid; total energy increases radially from the center rotation line.

UNIT II Summary & High-Yield Viva-Voce Key

- Comprehensive tracking of Unit II core syllabus parameters and theoretical frameworks.
- High-yield focus on governing mathematical derivations and mechanical configurations.

□ VIVA-VOCE ASSESSMENT CORE QUESTIONS:

1. Prove that stream lines and velocity potential lines intersect orthogonally.

2. Under what exact fluid criteria does the simplified continuity equation div(V) = 0 hold true?

3. Why does the kinetic energy correction factor (α) equal 2.0 for laminar pipe profiles?

Thank You..