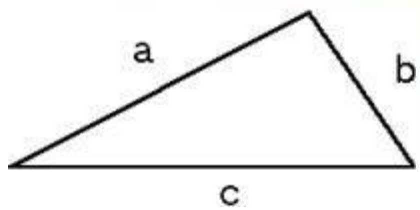
# UNIT-2

## COMPUTATION OF AREAS AND VOLUMES

Areas and Volumes are often required in the context of design, eg. We might need the surface area of a lake, the area of crops, of a car park or a roof, the volume of a dam embankment, or of a road cutting. Volumes are often calculated by integrating the area at regular intervals eg. along a road centre line, or by using regularly spaced contours. We simply use what you already know about numerical integration from numerical methods). Objectives

After completing this topic you should be able to calculate the areas of polygons and irregular figures and the volumes of irregular and curved solid



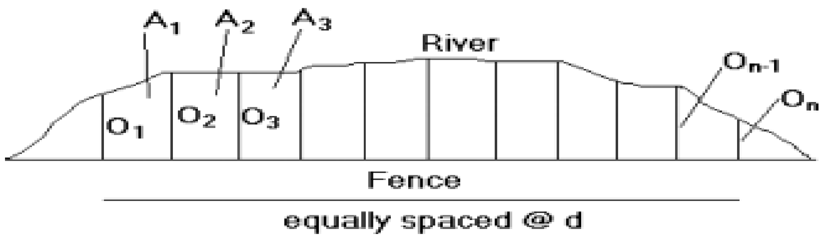

Triangles if s = (a + b + c) / 2 then area = S.(S-a)(S-b)(S-c)

Calculating area of a polygon from Coordinates: If the coordinate points are numbered clockwise: area = 1 2 ∑ i=1 n ( Ni . Ei+1 - Ei . Ni+1 ) This formula is not easy to remember, so let's look at a practical application

The computation of volumes of various quantities from the measurements done in the field is required in the design and planning on many engineering works. The volume of earth work is required for suitable alignment of road works, canal and sewer lines, soil and water conservation works, farm pond and percolation pond consent. The computation of volume of various materials such as coal, gravel and is required to check the stock files, volume computations are also required for estimation of capacities of bins tanks etc. For estimation of volume of earth work cross sections are taken at right angles to a fixed line, which runs continuously through the earth work. The spacing of the cross section

## Calculating areas with the Trapezoidal Rule

(as used in integrating functions)




$A_1 = d \,.\, (O_1 + O_2) / 2$
$A_2 = d \,.\, (O_2 + O_3) / 2$
$A_3 = d \,.\, (O_3 + O_4) / 2$

Hence, the total area is:

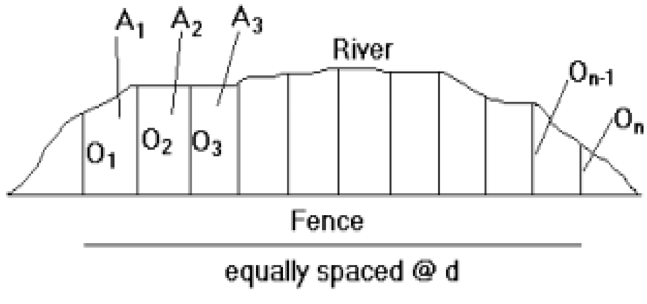
$$A = (d/2) \,.\, [O_1 + 2.O_2 + 2.O_3 + ... + 2.O_{n-1} + O_n]$$

The Trapezoidal Rule assumes **straight line segments** on the boundary.

## Doing better with Simpson's Rule

Simpson's Rule assumes a parabola fitted to 3 adjacent points, rather than the straight lines between adjacent points assumed by the Trapezoidal Rule.

This may be more accurate than the Trapezoidal Rule because boundaries are often curved.



Volumes can be calculated in a number of ways. It is common to calculate the area of each of several equally spaced slices (either vertical cross-sections, or horizontal contours), and integrate these using Simpson's Rule or similar. A second method is to use spot levels, and calculate the volume of a series of wedges or square cells. Cross-sections are well suited for calculating volumes of roads, pipelines, channels, dam embankments, etc. Formulae are given below for the most common cross-section cases.

Computation of area using different methods

1. The following offsets were taken from a chain line to an irregular boundary line at an interval of 10 m. 0, 2.50, 3.50, 5.00, 4.60, 3.20, 0 m. Compute the area between the chain line, the irregular boundary line and the end offsets by:

(a) Trapezoidal Rule

(b) Simpson's Rule

**(a) Trapezoidal Rule**

Here d = 10

$$\text{Area} = \frac{10}{2}\{0+0+2(2.50+3.50+5.00+4.60+3.20)\} = 5 * 37.60 = \mathbf{188\ m^2}$$

**(b) Simpson's Rule**

D = 10

$$\text{Area} = \frac{10}{3}\{0+0+4(2.50+5.00+3.20)+2(3.50+4.60)\} = \frac{10}{3} * 59.00 = \mathbf{196.66\ m^2}$$

2. The following offsets were taken from a survey line to a curved boundary line:

| **Distance (m)** | 0 | 5 | 10 | 15 | 20 | 30 | 40 | 60 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| **Offset (m)** | 2.50 | 3.80 | 4.60 | 5.20 | 6.10 | 4.70 | 5.80 | 3.90 | 2.20 |

Find the area between the survey line, the curved boundary line and the first and last offsets by (a) Trapezoidal Rule and (b) Simpson's Rule.

Here, the intervals between the offsets are not regular throughout the length. Soothe section is divided into three compartments.

Let,

$\Delta_1$ = Area of the 1st section

$\Delta_2$ = Are of the 2nd section

$\Delta_3$ = Area of the 3rd section

Here,

$d_1 = 5$ m

$d_2 = 10$ m

$d_3 = 20$ m

**(a) Trapezoidal Rule:**

$$\Delta_1 = \frac{5}{2}\{2.50 + 6.10 + 2(3.80 + 4.60 + 5.20)\} = 89.50 \text{ m}^2$$

$$\Delta_2 = \frac{10}{2}\{6.10 + 5.80 + 2(4.70)\} = 106.50 \text{ m}^2$$

$$\Delta_3 = \frac{20}{2}\{5.80 + 2.20 + 2(3.90)\} = 158.00 \text{ m}^2$$

Total Area = 89.50 + 106.50 + 158.00 = **354.00 m²**

**(b) By Simpson's Rule**

$$\Delta_1 = \frac{5}{3}\{2.50 + 6.10 + 4(3.80 + 5.20) + 2(4.60)\} = 89.66 \text{ m}^2$$

$$\Delta_2 = \frac{10}{3}\{6.10 + 5.80 + 4.70\} = 102.33 \text{ m}^2$$

$$\Delta_3 = \frac{20}{3}\{5.80 + 2.20 + 4(3.90)\} = 157.33 \text{ m}^2$$

Total area = 89.66 + 102.33 + 157.33 = **349.32 m²**

# Simpson's rule

In this rule, the boundaries between the ends of ordinates are assumed to form an arc of a parabola. Hence Simpson's rule is sometimes called the parabolic rule.

Refer to Fig. 7.13.

Let

$O_1, O_2, O_3$ = three consecutive ordinates

$d$ = common distance between the ordinates

Area AFeDC = area of trapezium AFDC + area of segment FeDEF

Simpson's Rule

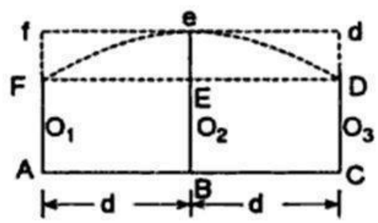



**Fig. 7.13**

Here,

$$\text{Area of trapezium} = \frac{O_1 + O_3}{2} \times 2d$$

$$\text{Area of segment} = \frac{2}{3} \times \text{area of parallelogram FfdD}$$

$$= \frac{2}{3} \times \text{Ee} \times 2d = \frac{2}{3} \times \left\{O_2 - \frac{O_1 + O_3}{2}\right\} \times 2d$$

## Trapezoidal rule

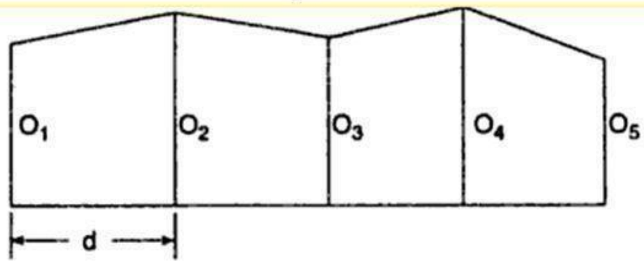



**Fig. 7.12**

Let $O_1, O_2, \ldots, O_n$ = ordinates at equal intervals

$d$ = common distance

$$\text{1st area} = \frac{O_1 + O_2}{2} \times d$$

$$\text{2nd area} = \frac{O_2 + O_3}{2} \times d$$

$$\text{3rd area} = \frac{O_3 + O_4}{2} \times d$$

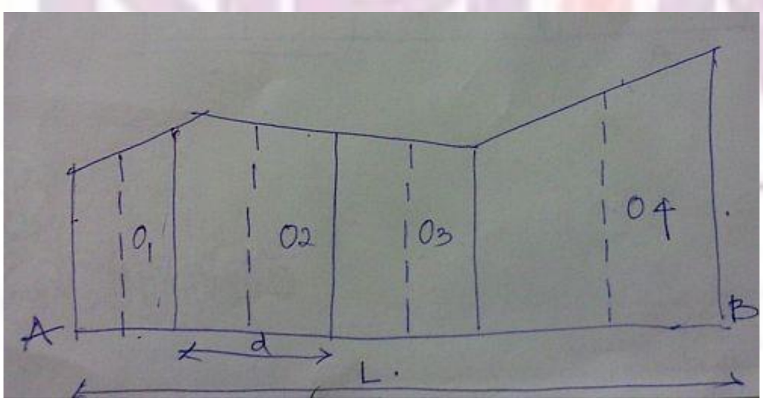
$$\text{Last area} = \frac{O_{n-1} + O_n}{2} \times d$$

$$\text{Total area} = \frac{d}{2}\{O_1 + 2O_1 + 2O_2 + \ldots + 2O_{n-1} + O_n\}$$

$$= \frac{\text{common distance}}{2}\{(\text{1st ordinate} + \text{last ordinate} + 2\ (\text{sum of other ordinate})\}$$

**Midpoint-ordinate rule**

The rule states that if the sum of all the ordinates taken at midpoints of each division multiplied by the length of the base line having the ordinates (9 divided by number of equal parts).



The following perpendicular offsets were taken at 10m interval from a survey line to an irregular boundary line. The ordinates are measured at midpoint of the division are 10, 13, 17, 16, 19, 21, 20 and 18m. Calculate the are enclosed by the midpoint ordinate rule.

**Given:**

Ordinates

O1 = 10

O2 = 13

O3 = 17

O4 = 16

O5 = 19

O6 = 21

O7 = 20

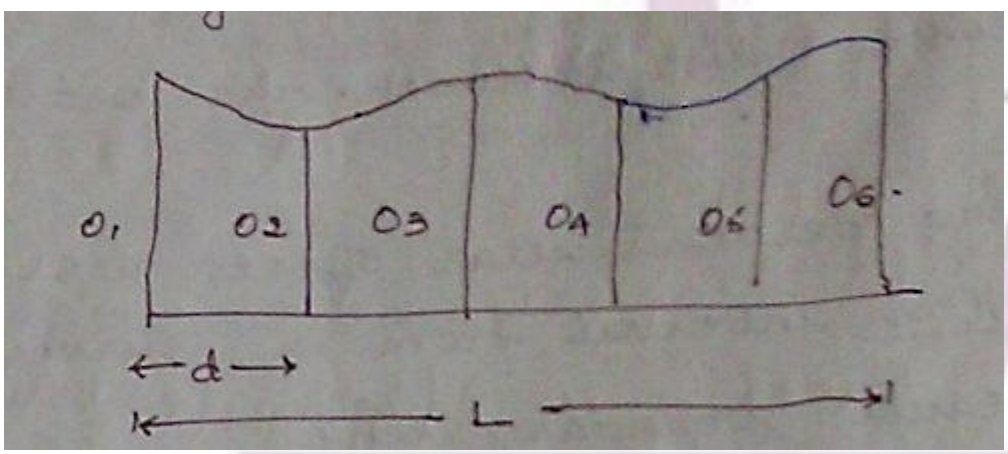
O8 = 18

Common distance, d =10m

Number of equal parts of the baseline, n = 8

Length of baseline, L = n *d = 8*10 = 80m

Area = [(10+13+17+16+19+21+20+18)*80]/8




**Average Ordinate Rule**

The rule states that (to the average of all the ordinates taken at each of the division of equal length multiplies by baseline length divided by number of ordinates)

Problems

The following perpendicular offsets were taken at 10m interval from a survey line to an irregular boundary line.

9, 12, 17, 15, 19, 21, 24, 22, 18

Calculate area enclosed between the survey line and irregular boundary line.

Area = [(O1+ O2+ O3+ …. + O9)*L]/(n+1)

= [(9+12+17+15+19+21+24+22+18)*8*10]/(8+1)

= 139538sqm

**Simpson's Rule Statement**

It states that, sum of first and a last ordinate has to be done. Add twice the sum of remaining odd ordinates and four times the sum of remaining even ordinates. Multiply to this total sum by $1/3^{rd}$ of the common distance between the ordinates which gives the required area.

Problem

| Chainage | 0 | 25 | 50 | 75 | 100 | 125 | 150 |
|---|---|---|---|---|---|---|---|
| Offset „m" | 3.6 | 5.0 | 6.5 | 5.5 | 7.3 | 6.0 | 4.0 |

The following offsets are taken from a chain line to an irregular boundary towards right side of the chain line.

Common distance, d = 25m

Area = $d/3[(O_1+O_7) + 2 (O_3+O_5)+4(O_2+O4+O_6)]$

= 25/3[(3.6+4)+2(6.5+7.3)+4(5+5.5+6)]

Area = 843.33sqm

COMPUTATION OF VOLUMES

INTRODUCTION

The computation of volumes of various quantities from the measurements done in the field is required in the design and planning on many engineering works. The volume of earth work is required for suitable alignment of road works, canal and sewer lines, soil and water conservation works, farm pond and percolation pond consent. The computation of volume of various materials such as coal, gravel and is required to check the stock files, volume computations are also required for estimation of capacities of bins tanks etc.

For estimation of volume of earth work cross sections are taken at right angles to a fixed line, which runs continuously through the earth work. The spacing of the cross sections will depend upon the accuracy required. The volume of earth work is computed once the various cross-sections are known, adopting Prismoidal rule and trapezoidal rule

**Measurement of Volume of Earth work from Cross-Sections**

The length of the project along the centre line is divided into a series of solids known as Prismoidal by the planes of cross-sections. The spacing of the sections should depend upon the character of ground and the accuracy required in measurement. They are generally run at 20m or 30m intervals, but sections should also be taken at points of change from cutting to filling, if these are known, and at places where a marked change of slop occurs either longitudinally or transversely. The areas of the cross-sections which have been taken are first calculated and the volumes of the Prismoidal between successive cross- sections are then obtained by using the Trapezoidal formula or the Prismoidal formula. The former is used in the preliminary estimates and for ordinary results, while the latter is employed in the final estimates and for precise results. The Prismoidal formula can be used directly or indirectly. In the indirect method, the volume is firstly calculated by trapezoidal formula and the Prismoidal correction is then applied to this volume so that the corrected volume is equal to that as if it has been calculated by applying the Prismoidal formula directly. The indirect method being simpler is more commonly used.

When the centre line of the project is curved in plan, the effect of curvature is also taken into account specially in final estimates of earthwork where much accuracy is needed. It is the common practice to calculate the volumes as straight as mentioned above and then to apply the correction for curvature to them.

Another method of finding curved volumes is to apply the correction for curvature to the areas of cross-sections, and then to compute the required volumes from the corrected areas from Prismoidal formula

The following are the various cross-sections usually met with whose areas are to be computed:

1. Level section.
2. Two-level section.
3. Side-hill two-level section.
4. Three-level section.
5. Multi-level section

**1. Level-Section (Fig. 12.2):**

In this case the ground is level transversely.

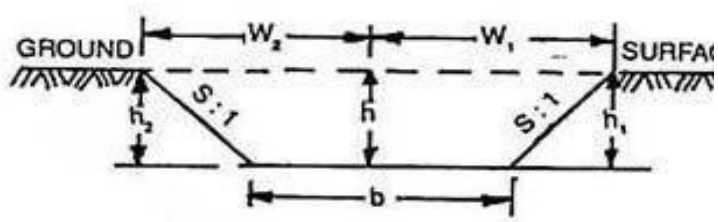



*Fig. 12.2*

$$h_1 = h_2 = h$$
$$w_1 = w_2$$
$$= \frac{b}{2} + sh$$
$$A = \frac{1}{2}[b + (b + 2sh)]h$$
$$= (b + sh)\,h \qquad \dots \quad \dots \quad \dots$$

### 2. Two-Level Section (Fig. 12.1):

In this case, the ground is sloping transversely, but the slope of the ground does not intersect the formation level.

$$w_1 = \frac{b}{2} + \frac{rs}{r-s}\left(h + \frac{b}{2r}\right)$$

$$w_2 = \frac{b}{2} + \frac{rs}{(r+s)}\left(h - \frac{b}{2r}\right)$$

$$h_1 = h + \frac{w_1}{r}$$

$$h_2 = h - \frac{w_2}{r}$$

$$A = \frac{1}{2}\left[(w_1 + w_2)\left(h + \frac{b}{2s}\right) - \frac{b^2}{2s}\right]$$

$$= \left[\frac{s(\frac{b}{2})^2 + r^2bh + r^2sh^2}{(r^2 - s^2)}\right] \qquad \ldots \qquad \ldots \qquad \text{(Eqn. 12.2)}$$

## 4. Three-Level Section (Fig. 12.4):

In this case, the transverse slope of the ground is not uniform.

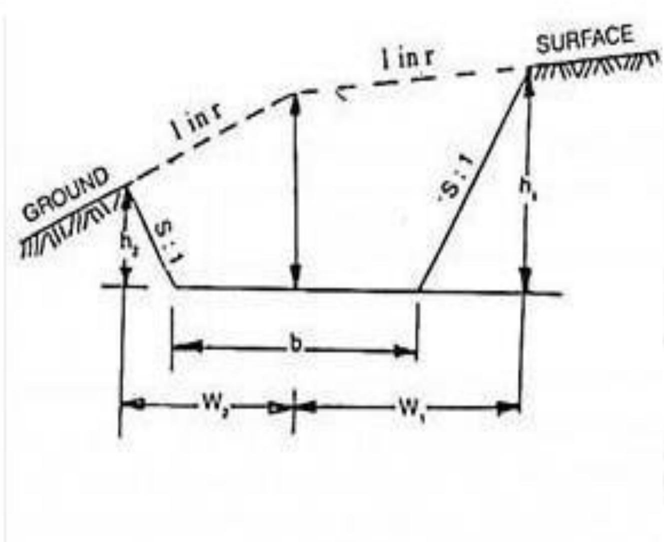



Fig. 12.4

$$w_1 = \frac{r_1 s}{(r_1 - s)}\left(h + \frac{b}{rs}\right)$$

$$w_2 = \frac{r_2 s}{(r_2 + s)}\left(h + \frac{b}{2s}\right)$$

The formulae for $w_1$ and $w_2$ may apply to both side widths according as the ground rises or falls from the centre to both sides.

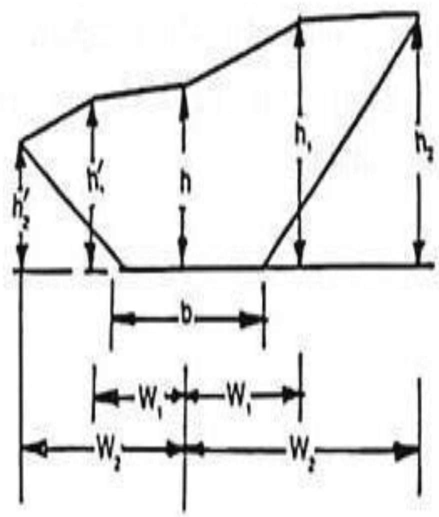
$$h_1 = \left(h + \frac{w_1}{r_1}\right)$$

$$h_2 = \left(h - \frac{w_2}{r_2}\right)$$

$$A = \left[\frac{1}{2}h(w_1 + w_2) + \frac{b}{4}(h_1 + h_2)\right] \quad \ldots \quad \ldots \qquad \text{(Eqn. 12.5}$$

### 5. Multi-Level Section (Fig. 12.5):

In this case, the transverse slope of the ground is not uniform but-has multiple cross-slopes as is clear from the figure.




**The notes regarding the cross-section are recorded as follows:**

| *Left* | *Centre* | *Right* |
|---|---|---|
| $\frac{\pm h'_2}{w'_2}\ \frac{\pm h'_1}{w'_1}$ | $\frac{\pm h}{0}$ | $\frac{\pm h_1}{w_1}\ \frac{\pm h_2}{w_2}$ |

The numerator denotes cutting (+ve) or filling (-ve) at the various points, and the denominator their horizontal distances from the centre line of the-section. The area of the section is calculated from these notes by coordinate method. The co-ordinates may be written in the determinant form irrespective of the signs.

$$\frac{0}{b/2} \times \frac{h'_2}{w'_2} \times \frac{h'_1}{w'_1} \times \frac{h}{0} \times \frac{h_1}{w_1} \times \frac{h_2}{w_2} \times \frac{0}{b/2}$$

Formula

Let Σ F= sum of the product of the co-ordinates joined by full lines.

Σ D= sum of the products of the co-ordinates joined by dotted lines.

Then, A= 1/2 (ΣF- ΣD)

Volume of a reservoir

**Formulae for volume:**

- To calculate die volumes of the solids between sections, it must be assumed that they have some geometrical from. They must nearly take the form of Prismoidal and therefore, in calculation work, they are considered to be Prismoidal.
- Let $A_1$, $A_2$, $A_3$…………….. $A_n$ = the areas at the 1st, 2nd, 3rd…....................last cross-section.
- D = the common distance between the cross-section.

  V = the volume of cutting or filling

  **Measurement of Volumes from contours:**
- **Mass Diagram:**
- The mass diagram is a graph plotted between distances along centre line, taken as base and algebraic sum of the mass of the earth work, taken as ordinates. The volume of cutting is considered as positive where as that of filling as negative.

  For determining in advance, the proper distribution of excavated material and the amount of waste and borrow, a mass diagram is commonly used. From the mass diagram, it is possible to determine by trial, the earthwork distribution plan that will result in the minimum cost of overhaul and the economical expenditure for overhaul and borrow

  **Lift and Lead:**
- **Lift:**
- Vertical distance through which the excavated earth is lifted beyond a certain depth is called lift. Excavation up to 1.5m depth below ground level and excavated material deposited on the ground shall be included in the item of work as specified. The lift shall be measured from the C.G. of the excavated earth to that of the deposited earth. Extra lift shall be measured in unit of 1.5m or as per pre-accepted condition.
- **Lead:**

- The horizontal distance from borrow pit to the site of work is called lead. It shall be measured from the centre of the area of excavation to the centre of the placed earth. Normally a lead up to 30m or as per pre- accepted condition is not paid-extra.

  Beyond a lead of 30m and lift of 1.5m rates will be different for every unit of 30m lead and 1.5m lift or fraction thereof

  **Converting Lift into lead:**

- **The lift is converted into lead by the following rules:**
- 1. The lift up to 3 .6m is multiplied by 10
- 2. Lift more than 3.6m and less than 6m is squared and multiplied by 3.3.

  Lift more than 6m is multiplied by 20