Department of Civil Engineering

Study Material

UNIT–III: Flexural Stresses and Shear Stresses

FLEXURAL STRESSES

1. Introduction

When a beam is subjected to transverse loads, it bends. Due to bending:

- The upper fibers are compressed.
- The lower fibers are stretched.
- Between them lies a **Neutral Axis (NA)** where stress is zero.

The stresses developed due to bending are called **Flexural (Bending) Stresses**.

2. Theory of Simple Bending

Definition

The theory of simple bending explains the relationship between bending moment and the stresses produced in a beam.

It is also known as the **Euler–Bernoulli Beam Theory**.

3. Assumptions of Simple Bending Theory

1. The beam is initially straight.
2. The material is homogeneous and isotropic.
3. Hooke's law is valid (within elastic limit).
4. Young's modulus is the same in tension and compression.
5. Plane sections remain plane after bending.
6. Radius of curvature is large compared to beam depth.
7. The beam is subjected to pure bending only.
8. Longitudinal fibers are free to expand or contract.

4. Neutral Axis (NA)

Definition

The line in the cross-section where bending stress is zero is called the **Neutral Axis**.

Properties:

- Passes through the centroid.
- Separates tension and compression zones.

5. Neutral Surface

The surface formed by joining all neutral axes along the beam length is called the **Neutral Surface**.

6. Derivation of Bending Equation

The fundamental bending equation is

MI= σ y=ER\boxed{\frac{M}{I}=\frac{\sigma}{y}=\frac{E}{R}}IM =y σ =RE

Where

- M = Bending Moment
- I = Moment of Inertia
- σ = Bending Stress
- y = Distance from Neutral Axis
- E = Young's Modulus
- R = Radius of Curvature

Meaning of Each Term

(1) Bending Moment

External moment acting on the beam.

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(2) Moment of Inertia

Resistance offered by a section against bending.

Unit

mm^4

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(3) Radius of Curvature

Radius of the curved beam after bending.

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(4) Flexural Stress

Stress produced due to bending.

7. Bending Stress Formula

From bending equation,

σ =MyI\boxed{\sigma=\frac{My}{I}} σ =IMy

Maximum bending stress

σ max=MymaxI\boxed{\sigma_{max}=\frac{My_{max}}{I}} σ max =IMymax

8. Section Modulus

Definition

Section modulus indicates the bending strength of a beam section.

Formula

Z=Iymax\boxed{Z=\frac{I}{y_{max}}}Z=ymax I

Hence,

σ =MZ\boxed{\sigma=\frac{M}{Z}} σ =ZM

Unit

mm^3

Larger section modulus means stronger beam.

9. Section Modulus of Standard Sections

(A) Rectangular Section

Width = b

Depth = d

Moment of inertia

I=bd312I=\frac{bd^3}{12}I=12bd3

Section modulus

Z=bd26\boxed{Z=\frac{bd^2}{6}}Z=6bd2

(B) Solid Circular Section

Diameter = D

Moment of inertia

$$I=\frac{\pi D^4}{64}$$

Section modulus

$$\boxed{Z=\frac{\pi D^3}{32}}$$

(C) Hollow Circular Section

Outer diameter = D

Inner diameter = d

Moment of inertia

$$I=\frac{\pi(D^4-d^4)}{64}$$

Section modulus

$$\boxed{Z=\frac{\pi(D^4-d^4)}{32D}}$$

(D) I Section

Procedure

1. Find centroid.
2. Find moment of inertia using parallel-axis theorem.
3. Divide by maximum distance from NA.

$$Z=\frac{I}{y_{max}}$$

(E) T Section

Same procedure.

(F) Angle Section

1. Locate centroid.
2. Compute moment of inertia.
3. Calculate section modulus.

(G) Channel Section

Same procedure as above.

10. Design of Simple Beam Sections

The design equation is

Z=Mσallow\boxed{Z=\frac{M}{\sigma_{allow}}}Z=σallow M

Where

- M = Maximum bending moment
- σallow = Allowable bending stress

Procedure:

1. Calculate maximum bending moment.
2. Choose allowable stress.
3. Find required section modulus.
4. Select a suitable beam section.

PART–B: SHEAR STRESSES

11. Introduction

When a beam carries transverse loads, **shear stresses** develop in addition to bending stresses.

These stresses are maximum near the neutral axis.

12. Shear Stress Formula

General formula

τ=VQIb\boxed{\tau=\frac{VQ}{Ib}}τ=IbVQ

Where

- τ = Shear stress
- V = Shear force
- Q = First moment of area about NA
- I = Moment of inertia
- b = Width at the section

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13. Derivation of Shear Stress Formula

The derivation is based on:

1. Bending equation
2. Equilibrium of beam element
3. Longitudinal force balance

Final result:

τ =VQIb\boxed{\tau=\frac{VQ}{Ib}} τ =IbVQ

This is called the **Beam Shear Formula**.

14. Shear Stress Distribution

(A) Rectangular Section

Maximum shear stress

τ max=3V2A\boxed{\tau_{max}=\frac{3V}{2A}} τ max =2A3V

Distribution

- Zero at top.
- Zero at bottom.
- Maximum at neutral axis.
- Parabolic shape.

(B) Circular Section

Maximum shear stress

τ max=4V3A\boxed{\tau_{max}=\frac{4V}{3A}} τ max =3A4V

Distribution

- Zero at surface.
- Maximum at center.
- Parabolic.

(C) Triangular Section

Distribution

- Zero at top.
- Zero at bottom.
- Maximum above the neutral axis.
- Parabolic.

(D) I Section

Characteristics

- Very small in flanges.
- Maximum in web.

- Web carries most of the shear.

(E) T Section

Maximum in web.

Lower in flange.

(F) Angle Section

Irregular distribution.

Maximum near the neutral axis.

(G) Channel Section

Similar to I section.

Web carries most of the shear.

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15. Comparison Between Bending Stress and Shear Stress

Bending Stress	Shear Stress
Due to bending moment	Due to shear force
Maximum at outer fibers	Maximum at neutral axis
Zero at neutral axis	Zero at extreme fibers
Varies linearly	Varies parabolically (for rectangular sections)
Formula: σ =MyI\sigma=\frac{My}{I} σ =IMy	Formula: τ =VQIb\tau=\frac{VQ}{Ib} τ =IbVQ

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16. Important Formula Sheet

Quantity	Formula
Bending Equation	MI=σ y=ER\frac{M}{I}=\frac{\sigma}{y}=\frac{E}{R} IM =yσ =RE
Flexural Stress	σ =MyI\sigma=\frac{My}{I} σ =IMy
Section Modulus	Z=IyZ=\frac{I}{y}Z=yI
Design Equation	Z=Mσ allowZ=\frac{M}{\sigma_{allow}}Z=σ allow M
Shear Stress	τ =VQIb\tau=\frac{VQ}{Ib} τ =IbVQ
Rectangular Section Modulus	bd26\frac{bd^2}{6}6bd2
Circular Section Modulus	πD332\frac{\pi D^3}{32}32πD3
Hollow Circular Section Modulus	π(D4−d4)32D\frac{\pi(D^4-d^4)}{32D}32Dπ(D4−d4)
Rectangular Shear Stress	3V2A\frac{3V}{2A}2A3V
Circular Shear Stress	4V3A\frac{4V}{3A}3A4V

17. Typical Shear Stress Distributions

Section	Shear Stress Distribution	Maximum Location
Rectangular	Parabolic	Neutral axis
Circular	Parabolic	Center
Triangular	Parabolic	Above neutral axis
I Section	Mostly in web	Web at neutral axis
T Section	Mostly in web	Web
Angle Section	Irregular	Near neutral axis
Channel Section	Mostly in web	Web

18. Frequently Asked University Exam Questions (2 Marks)

1. Define flexural stress.
2. State the assumptions of simple bending theory.
3. What is the neutral axis?
4. Write the bending equation.
5. Define section modulus.
6. Define shear stress in beams.
7. Write the shear stress formula.
8. Where is bending stress maximum in a beam?
9. Where is shear stress maximum in a rectangular beam?
10. What is the significance of the section modulus?

19. Frequently Asked University Exam Questions (5–10 Marks)

1. Derive the simple bending equation MI=σy=ER\frac{M}{I}=\frac{\sigma}{y}=\frac{E}{R}IM =yσ =RE .
2. Explain the assumptions of the simple bending theory.
3. Derive the shear stress formula τ=VQIb\tau=\frac{VQ}{Ib}τ=IbVQ .
4. Explain the shear stress distribution in rectangular, circular, and I sections with neat sketches.
5. Determine the section modulus of rectangular, circular, and hollow circular sections.
6. Calculate bending stresses in rectangular, T, I, angle, and channel sections.
7. Design a simple beam section for a given bending moment and allowable stress.
8. Compare bending stress and shear stress with suitable diagrams.

20. Quick Revision Points

- **Bending equation:** MI=σy=ER\frac{M}{I}=\frac{\sigma}{y}=\frac{E}{R}IM =yσ =RE
- **Flexural stress:** σ=MyI\sigma=\frac{My}{I}σ=IMy
- **Section modulus:** Z=IyZ=\frac{I}{y}Z=yI
- **Design equation:** Z=MσallowZ=\frac{M}{\sigma_{allow}}Z=σallow M
- **Shear stress:** τ=VQIb\tau=\frac{VQ}{Ib}τ=IbVQ
- **Maximum bending stress:** At the extreme (outer) fibers.
- **Maximum shear stress (rectangular beam):** At the neutral axis.
- **I-sections:** The web carries most of the shear force, while the flanges primarily resist bending.