Department of Civil Engineering

Study Material

UNIT–II: Shear Force and Bending Moment

Introduction

A **beam** is a structural member that carries loads acting perpendicular to its longitudinal axis. When loads are applied, the beam develops **shear force (SF)** and **bending moment (BM)** at every section.

The study of SF and BM helps in designing safe beams for buildings, bridges, machines, and other structures.

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1. Types of Beams

A beam is classified based on its support conditions.

(a) Simply Supported Beam

- Supported by a hinge at one end and a roller at the other.
- Free to rotate at supports.
- No fixing moment.

Example: Bridge girders.

(b) Cantilever Beam

- Fixed at one end.
- Free at the other end.

Example:

- Balcony
- Traffic signal pole

(c) Overhanging Beam

- One or both ends extend beyond the supports.

Example:

- Roof projections

(d) Fixed Beam

- Both ends are fixed.

(e) Continuous Beam

- Supported on more than two supports.

2. Load Types Acting on Beams

(a) Point Load (Concentrated Load)

Acts at one point.

Unit

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kN

(b) Uniformly Distributed Load (UDL)

Load is uniformly distributed over the beam.

Unit

kN/m

Example

5 kN/m over 4 m.

Equivalent Load

W=wLW=wLW=wL

Acts at the center of the loaded length.

(c) Uniformly Varying Load (UVL)

Load varies uniformly from zero to maximum.

Equivalent Load

W=wL2W=\frac{wL}{2}W=2wL

Acts at

L3\frac{L}{3}3L

from the heavier end.

(d) Couple (Moment)

Produces only bending.

Unit

kN·m

3. Shear Force (SF)

Definition

Shear force at any section is the algebraic sum of all vertical forces acting on one side of the section.

Formula

SF=∑VSF=\sum VSF=∑V

Sign Convention

Positive SF

- Left side moves upward
- Right side moves downward

Negative SF

- Left side moves downward
- Right side moves upward

4. Bending Moment (BM)

Definition

Bending moment at any section is the algebraic sum of moments of all forces about that section on one side.

Formula

BM=∑MBM=\sum MBM=∑M

Unit

kN·m

Sign Convention

Positive Moment

Sagging (Smile shape)

Negative Moment

Hogging (Upside-down smile ☹)

5. Shear Force Diagram (SFD)

Definition

Graph showing variation of shear force along the beam length.

Characteristics

- Sudden jump at point load.
- Sloping line under UDL.
- Curved line under UVL.

6. Bending Moment Diagram (BMD)

Definition

Graph showing variation of bending moment along the beam.

Characteristics

- Straight line when SF is constant.
- Parabola under UDL.
- Cubic curve under UVL.

7. SFD and BMD for Standard Cases

(A) Cantilever with Point Load at Free End

Load = P

Reaction at fixed end

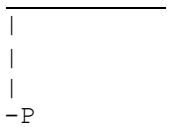
R=PR=PR=P

Shear Force

Constant throughout

SF=−PSF=−PSF=−P

SFD



Bending Moment

At free end

BM=0BM=0BM=0

Maximum at fixed end

BM=−PLBM=−PLBM=−PL

BMD

Linear variation.

(B) Cantilever with UDL

Load = w kN/m

Total load

W=wLW=wLW=wL

Maximum SF

SF=−wLSF=−wLSF=−wL

Maximum BM

BM=−wL22BM=−\frac{wL^2}{2}BM=−2wL2

SFD

Straight inclined line

BMD

Parabola

(C) Cantilever with UVL

Maximum SF

SF=−wL2SF=-\frac{wL}{2}SF=−2wL

Maximum BM

BM=−wL26BM=-\frac{wL^2}{6}BM=−6wL2

SFD

Parabolic

BMD

Cubic curve

(D) Simply Supported Beam with Central Point Load

Load = P

Support reactions

RA=RB=P2R_A=R_B=\frac{P}{2}RA =RB =2P

Maximum SF

+P2+\frac{P}{2}+2P

and

−P2-\frac{P}{2}−2P

Maximum BM

At center

BM=PL4BM=\frac{PL}{4}BM=4PL

SFD

Rectangle

BMD

Triangle

(E) Simply Supported Beam with UDL

Load = w

Reactions

RA=RB=wL2R_A=R_B=\frac{wL}{2}RA =RB =2wL

Maximum BM

BM=wL28BM=\frac{wL^2}{8}BM=8wL2

Occurs at center.

SFD

Straight line

BMD

Parabola

(F) Simply Supported Beam with UVL

Equivalent Load

W=wL2W=\frac{wL}{2}W=2wL

Reaction forces are obtained by equilibrium equations.

SFD

Parabolic

BMD

Cubic

(G) Overhanging Beam

Procedure

1. Calculate reactions.
2. Draw SF from left to right.
3. Draw BM.
4. BM at free end = 0.

(H) Beam with Couple

A couple changes only the bending moment.

Shear force remains unchanged.

BMD shows sudden jump equal to the applied couple.

8. Procedure to Draw SFD and BMD

Step 1

Draw beam.

Step 2

Calculate support reactions.

Use

$\sum V=0$\sum V=0

and

$\sum M=0$\sum M=0

Step 3

Move from left to right.

Calculate SF after every load.

Step 4

Use SF values to calculate BM.

Step 5

Draw diagrams carefully.

9. Point of Contraflexure

Definition

The point where bending moment changes sign from positive to negative or vice versa.

At this point,

BM=0BM=0BM=0

but the beam is still loaded.

Occurs mainly in

- Overhanging beams
- Continuous beams
- Fixed beams

10. Relationship Between Load, Shear Force, and Bending Moment

These are the most important formulas in the chapter.

Relation 1

Rate of change of Shear Force

dVdx=w\boxed{\frac{dV}{dx}=w}dxdV =w

where

w = Load intensity

Relation 2

Rate of change of Bending Moment

dMdx=V\boxed{\frac{dM}{dx}=V}dxdM =V

where

V = Shear Force

Relation 3

Second derivative

d2Mdx2=w\boxed{\frac{d^2M}{dx^2}=w}dx2d2M =w

Important Conclusions

If Load = 0

Shear force remains constant.

Bending moment varies linearly.

If SF = 0

Bending moment is maximum or minimum.

If BM = 0

Beam may have

- Support
- Free end
- Point of contraflexure

If Load is Constant (UDL)

Shear force varies linearly.

Bending moment becomes parabolic.

If Load is UVL

Shear force becomes parabolic.

Bending moment becomes cubic.

Formula Sheet

Quantity	Formula
Equivalent load of UDL	W=wLW=wLW=wL
Equivalent load of UVL	W=wL2W=\frac{wL}{2}W=2wL
Cantilever BM (Point Load)	PLPLPL
Cantilever BM (UDL)	wL22\frac{wL^2}{2}2wL2
Cantilever BM (UVL)	wL26\frac{wL^2}{6}6wL2
Simply Supported BM (Point Load at Center)	PL4\frac{PL}{4}4PL
Simply Supported BM (UDL)	wL28\frac{wL^2}{8}8wL2
Rate of change of BM	dMdx=V\frac{dM}{dx}=VdxdM =V
Rate of change of SF	dVdx=w\frac{dV}{dx}=wdxdV =w

Comparison of Shear Force Diagram and Bending Moment Diagram

Shear Force Diagram	Bending Moment Diagram
Shows variation of shear force	Shows variation of bending moment
Unit: kN	Unit: kN • m

Shear Force Diagram	Bending Moment Diagram
Changes suddenly at point loads	Changes suddenly only at applied couples
Linear under UDL	Parabolic under UDL
Parabolic under UVL	Cubic under UVL

Frequently Asked University Exam Questions (2 Marks)

1. Define shear force.
2. Define bending moment.
3. What is a simply supported beam?
4. What is an overhanging beam?
5. Define point of contraflexure.
6. State the relation between load and shear force.
7. State the relation between shear force and bending moment.
8. What is a UDL?
9. What is a UVL?
10. What is a couple?

Frequently Asked University Exam Questions (5–10 Marks)

1. Draw the SFD and BMD for a cantilever beam carrying a point load at the free end.
2. Draw the SFD and BMD for a cantilever beam carrying a uniformly distributed load.
3. Draw the SFD and BMD for a simply supported beam carrying a central point load.
4. Draw the SFD and BMD for a simply supported beam carrying a UDL over the entire span.
5. Draw the SFD and BMD for an overhanging beam subjected to different loading conditions.
6. Explain the concept of shear force and bending moment with sign conventions.
7. Derive the relationships between load intensity, shear force, and bending moment.
8. Explain the significance of the point of contraflexure