UNIT V

Closed Conduit Flow

- Energy equation
- EGL and HGL
- Head loss
  - – major losses
  - – minor losses
- Non circular conduits

Conservation of Energy

- Kinetic, potential, and thermal energy

h_p = head supplied by a pump

h_t = head given to a turbine

h_L = head loss between sections 1 and 2

Cross section 2 is downstream from cross section 1!

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Energy Equation Assumptions

- Pressure is __hydrostatic__ in both cross sections
  - pressure changes are due to elevation only $p = \gamma h$
- section is drawn perpendicular to the streamlines (otherwise the __kinetic__ energy term is incorrect)
- Constant __density__ at the cross section
- __Steady__ flow

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Bernoulli Equation Assumption

- __Frictionless__ (viscosity can't be a significant parameter!)
- Along a __streamline__
- __Steady__ flow
- Constant __density__

$$z + \frac{V^2}{2g} + \frac{p}{\gamma} = const$$

287

Pipe Flow: Review

- We have the control volume energy equation for pipe flow.
- We need to be able to predict the head loss term.
- How do we predict head loss? Dimensional analysis.

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Pipe Flow Energy Losses

$$\frac{p_1}{\gamma} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + \cancel{z_1} + \cancel{h_p} = \frac{p_2}{\gamma} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + \cancel{z_2} + \cancel{h_t} + h_l$$

$$h_f = -\frac{\Delta p}{\gamma}$$ Horizontal pipe

$$f = \left(C_p \frac{D}{L}\right) = \text{function of } \left(\frac{e}{D}, \text{Re}\right)$$ Dimensional Analysis

$$C_p = \frac{-2\Delta p}{\rho V^2} \qquad C_p = \frac{2gh_f}{V^2}$$

$$f = \frac{2gh_f}{V^2}\frac{D}{L} \qquad h_f = f\frac{L}{D}\frac{V^2}{2g}$$ Darcy-Weisbach equation

289

Friction Factor: Major losses

- Laminar flow
  - Hagen-Poiseuille
- Turbulent (Smooth, Transition, Rough)
  - Colebrook Formula
  - Moody diagram
  - Swamee-Jain

Laminar Flow Friction Factor

$$V = \frac{\gamma D^2 h_l}{32\mu L}$$

Hagen-Poiseuille

$$h_f = \frac{32 m L V}{r g D^2} \qquad h_f = \frac{128 m L Q}{p r g D^4}$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

Darcy-Weisbach

$$\frac{32 m L V}{r g D^2} = f \frac{L}{D} \frac{V^2}{2g}$$

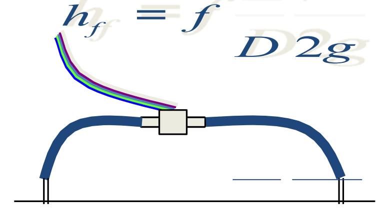
$$f = \frac{64 m}{r V D} = \frac{64}{\mathrm{Re}}$$

Slope of __-1__ on log-log plot

Turbulent Pipe Flow Head Loss

- Proportional to the length of the pipe
- Proportional to the square of the velocity (almost)
- Increases with surface roughness
- Is a function of density and viscosity
- Is independent of pressure

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$



Smooth, Transition, Rough Turbulent Flow

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

- Hydraulically smooth pipe law (von Karman, 1930)

$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{\mathrm{Re}\sqrt{f}}{2.51}\right)$$

- Rough pipe law (von Karman, 1930)

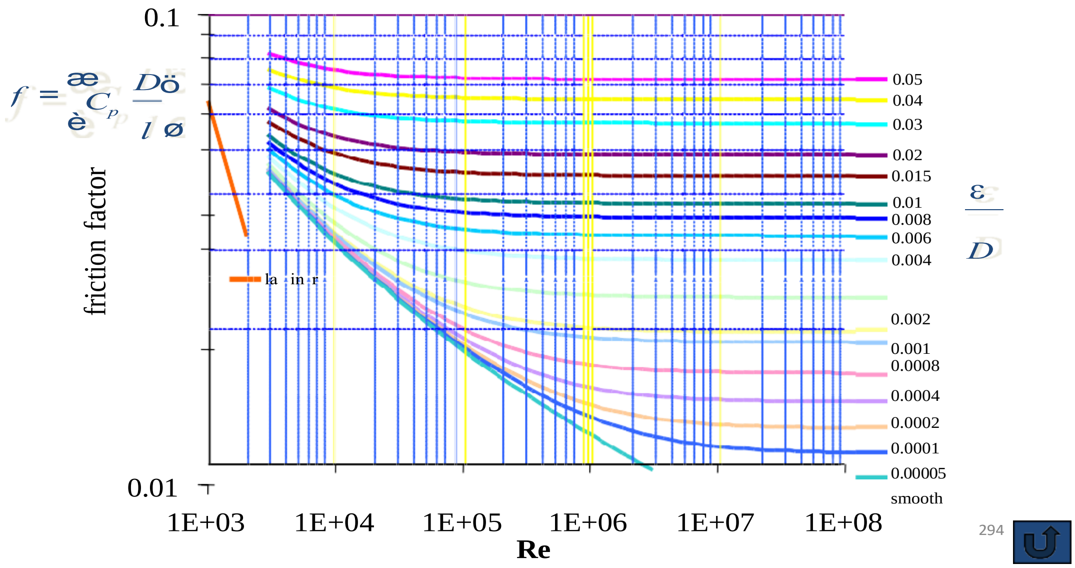
$$\frac{1}{\sqrt{f}} = 2\log\left(\frac{3.7D}{e}\right)$$

- Transition function for both smooth and rough pipe laws (Colebrook)

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

(used to draw the Moody diagram)

Moody Diagram



Pipe roughness

pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18-0.6
rivet steel	0.9-9.0
corrugated metal	45
PVC	0.12

Exponential Friction Formulas

- Commonly used in commercial and industrial settings

$$h_f = \frac{RLQ^n}{D^m}$$

- Only applicable over __range of data__collected
- Hazen-Williams exponential friction formula

$$R = \begin{cases} \dfrac{4.727}{C^n} & \text{USC units} \\ \dfrac{10.675}{C^n} & \text{SI units} \end{cases}$$

$$h_f = \frac{10.675L}{D^{4.8704}}\left(\frac{Q}{C}\right)^{1.852} \quad \text{SI units}$$

C = Hazen-Williams coefficient 296

Head loss:
Hazen-Williams Coefficient

C	Condition
150	PVC
140	Extremely smooth, straight pipes; asbestos cement
130	Very smooth pipes; concrete; new cast iron
120	Wood stave; new welded steel
110	Vitrified clay; new riveted steel
100	Cast iron after years of use
95	Riveted steel after years of use
60-80	Old pipes in bad condition

297

Hazen-Williams

$$h_f = \frac{10.675L}{D^{4.8704}}\left(\frac{Q}{C}\right)^{1.852} \quad \text{SI units}$$

vs

Darcy-Weisbach

$$h_f = f\frac{8}{p^2 g}\frac{LQ^2}{D^5}$$

- Both equations are empirical
- Darcy-Weisbach is rationally based, dimensionally correct, and preferred .
- Hazen-Williams can be considered valid only over the range of gathered data.
- Hazen-Williams can't be extended to other fluids without further experimentation.

Head Loss: Minor Losses

- Head loss due to outlet, inlet, bends, elbows, valves, pipe size changes
- Losses due to expansions are greater than losses due to contractions
- Losses can be minimized by gradual transitions

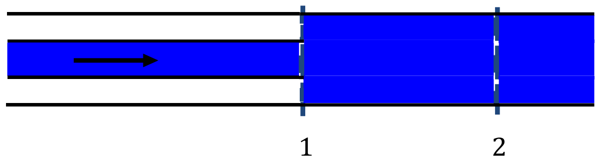
Minor Losses

- Most minor losses can not be obtained analytically, so they must be measured
- Minor losses are often expressed as a loss coefficient, K, times the velocity head.

$$C_p = f(\text{geometry}, \text{Re})$$

$$C_p = \frac{-2\Delta p}{\rho V^2} \qquad C_p = \frac{2gh_l}{V^2} \qquad h_l = C_p \frac{V^2}{2g} \qquad h = K\frac{V^2}{2g}$$

Head Loss due to Sudden Expansion: Conservation of Energy



$$\frac{p_1}{\gamma_1}+z_1+\alpha_1\frac{V_1^2}{2g}+H_p=\frac{p_2}{\gamma_2}+z_2+\alpha_2\frac{V_2^2}{2g}+H_t+h_l$$

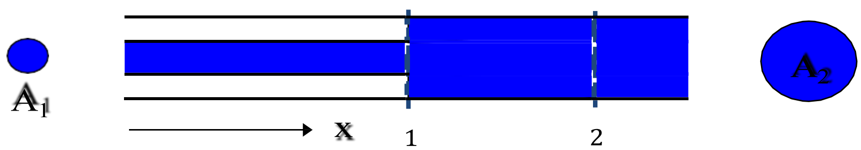
$$\frac{p_1-p_2}{\gamma}=\frac{V_2^2-V_1^2}{2g}+h_l$$

$z_1 = z_2$

$$h_l=\frac{p_1-p_2}{\gamma}+\frac{V_1^2-V_2^2}{2g}$$

What is $p_1 - p_2$?

Head Loss due to Sudden Expansion: Conservation of Momentum



$$\mathbf{M}_1+\mathbf{M}_2=\mathbf{W}+\mathbf{F}_{p_1}+\mathbf{F}_{p_2}+\mathbf{F}_{ss}$$

Apply in direction of flow

$$M_{1x}+M_{2x}=F_{p_{1x}}+F_{p_{2x}}$$

Neglect surface shear

$$M_{1x}=-\rho V_1^2A_1 \qquad M_{2x}=\rho V_2^2A_2$$

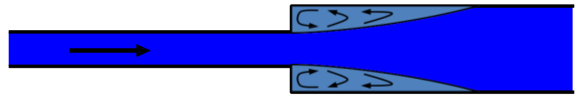
$$-\rho V_1^2A_1+\rho V_2^2A_2=p_1A_2-p_2A_2$$

Pressure is applied over all of section 1.
Momentum is transferred over area corresponding to upstream pipe diameter.
V_1 is velocity upstream.

$$\frac{p_1-p_2}{\gamma}=\frac{V_2^2-V_1^2\frac{A_2}{A_1}}{g}$$

Divide by $(A_2\gamma)$

Head Loss due to Sudden Expansion



Energy $$h_l = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g}$$

Mass $$\frac{A_1}{A_2} = \frac{V_2}{V_1}$$

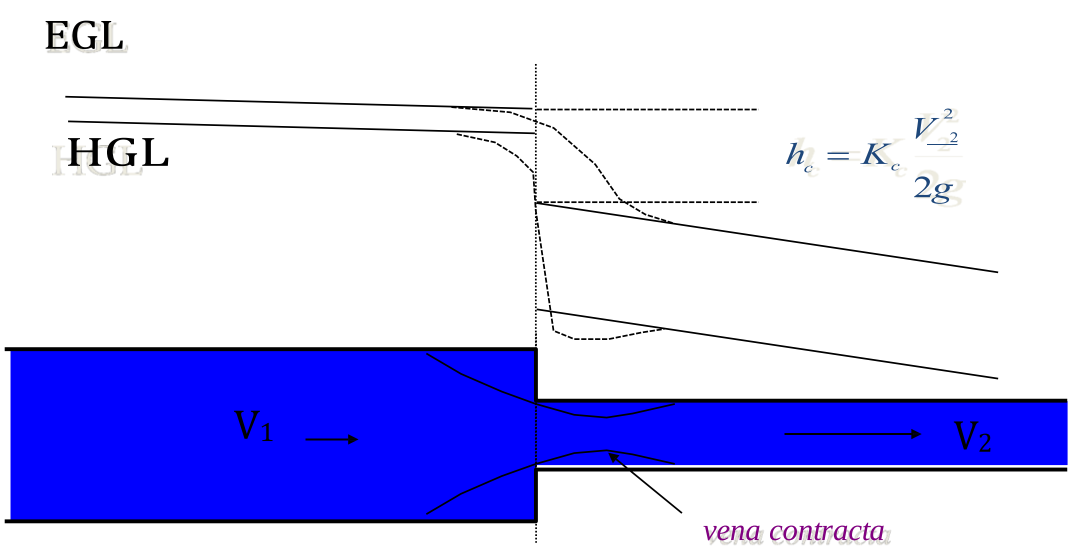
Momentum $$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2 \frac{A_1}{A_2}}{g}$$

$$h_l = \frac{V_2^2 - V_1^2 \frac{V_2}{V_1}}{g} + \frac{V_1^2 - V_2^2}{2g} \qquad h_l = \frac{V_2^2 - 2V_1V_2 + V_1^2}{2g}$$

$$h_l = \frac{(V_1 - V_2)^2}{2g} \qquad h_l = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 \qquad K = \left(1 - \frac{A_1}{A_2}\right)^2$$

303

Contraction

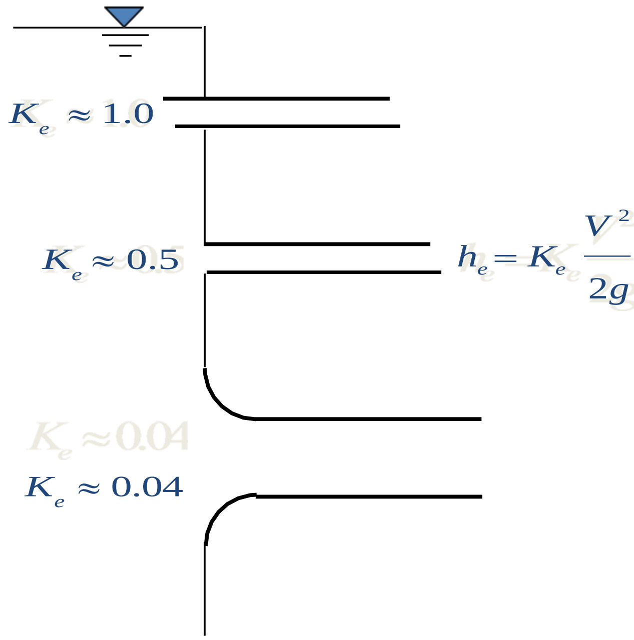


$$h_c = K_c \frac{V_2^2}{2g}$$

losses are reduced with a gradual contraction

Entrance Losses

- Losses can be reduced by accelerating the flow gradually and eliminating the *vena contracta*



Head Loss in Valves

- Function of valve type and valve position
- The complex flow path through valves often results in high head loss
- What is the maximum value that K_v can have?

$$h_v = K_v \frac{V^2}{2g}$$

Non-Circular Conduits: Hydraulic Radius Concept

- A is cross sectional area
- P is wetted perimeter
- R_h is the "Hydraulic Radius" (Area/Perimeter)
- Don't confuse with radius!

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

$$R_h = \frac{A}{P} = \frac{\frac{\pi}{4}D^2}{\pi D} = \frac{D}{4}$$

For a pipe

$$D = 4R_h$$

$$h_f = f\frac{L}{4R_h}\frac{V^2}{2g}$$

We can use Moody diagram or Swamee Jain with D = 4R!

311