UNIT - IV

Boundary Layer theory

Navier Stokes Equation in Vector Form:

A general way of deriving the Navier-Stokes equations from the basic laws of physics.

- Consider a general flow field as represented in Fig. 4.1.

- Imagine a closed **control volume,** within the flow field. The control volume is **fixed in space** and the fluid is moving through it. The control volume occupies reasonably large finite region of the flow field.

- A **control surface , A_0** is defined as the surface which **bounds** the volume .

- According to **Reynolds transport theorem, "**The rate of change of momentum for a **system** equals the sum of the rate of change of momentum inside the **control volume** and the rate of **efflux** of momentum across the **control surface"**.

- **The rate of change of momentum for a system** (in our case, the control volume boundary and the system boundary are same) **is equal to the net external force acting on it.**

Now, we shall transform these statements into equation by accounting for each term,

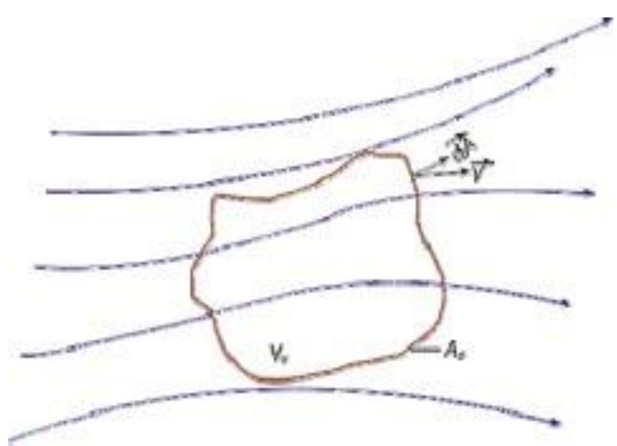


FIG 4.1 Finite control volume fixed in space with the fluid moving through it

- **Rate of change of momentum inside the control volume**

$$= \frac{\partial}{\partial t}\int \iint_{V_0} \rho \vec{V} dV$$

$$= \iint_{V_0} \int \frac{\partial}{\partial t}(\rho \vec{V}) dV$$

(since t is independent of space variable)

- **Rate of efflux of momentum through control surface**

$$\iint_{A_0} \rho \vec{V}(\vec{V}.d\vec{A}) = \iint_{A_0} \rho \vec{V}\vec{V}.\vec{n}dA$$

$$= \iint_{V_0} \int \left[\vec{V}(\nabla.\rho \vec{V}) + \rho \vec{V}.\nabla \vec{V}\right] dV$$

- **Surface force acting on the control volume**

$$= \iint_{A_0} d\vec{A}.\sigma$$

(σ is symmetric stress tensor)

$$= \iint_{V_0} \int (\nabla.\sigma) dV$$

- **Body force acting on the control volume**

$$\iint_{V_0} \int \rho \vec{f}_b dV$$

$\vec{f}_b$ in Eq. (25.4) is the body force per unit mass.

- **Finally, we get,**

$$\iint_{V_0} \int \left(\frac{\partial}{\partial t}(\rho \vec{V}) + \vec{V}(\nabla.\rho \vec{V}) + \rho \vec{V}.\nabla \vec{V}\right) dV$$

$$= \iint_{V_0} \int \left(\nabla.\sigma + \rho \vec{f}_b\right) dV$$

or

or, $$\rho \frac{\partial \vec{V}}{\partial t} + \vec{V}\frac{\partial \rho}{\partial t} + \rho \vec{V}.\nabla \vec{V} + \vec{V}(\nabla.\rho \vec{V}) = \nabla.\sigma + \rho \vec{f}_b$$

or $$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V}.\nabla \vec{V}\right) + \vec{V}\left(\frac{\partial \rho}{\partial t} + \nabla.\rho \vec{V}\right) = \nabla.\sigma + \rho \vec{f}_b$$

We know that $\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$ is the general form of **mass conservation equation** (popularly known as the **continuity equation**), valid for both **compressible** and **incompressible** flows.

Exact Solutions to Navier Stokes Equations:

Consider a class of flow termed as parallel flow in which **only one velocity term is nontrivial** and all the fluid particles move in one direction only.

- We choose x to be the direction along which all fluid particles travel , i.e. $u \neq 0, v = w = 0$. Invoking this in continuity equation, we get

$$\frac{\partial u}{\partial x} + \cancelto{0}{\frac{\partial v}{\partial y}} + \cancelto{0}{\frac{\partial w}{\partial z}} = 0$$

$$\frac{\partial u}{\partial x} = 0 \text{ which means } u = u(y, z, t)$$

- Now. Navier-Stokes equations for incompressible flow become

$$\frac{\partial u}{\partial t} + u\cancelto{0}{\frac{\partial u}{\partial x}} + \cancelto{0}{v\frac{\partial u}{\partial y}} + \cancelto{0}{w\frac{\partial u}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\cancelto{0}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

$$\cancelto{0}{\frac{\partial v}{\partial t}} + u\cancelto{0}{\frac{\partial v}{\partial x}} + v\cancelto{0}{\frac{\partial v}{\partial y}} + w\cancelto{0}{\frac{\partial v}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\cancelto{0}{\frac{\partial^2 v}{\partial x^2}} + \cancelto{0}{\frac{\partial^2 v}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 v}{\partial z^2}}\right]$$

$$\cancelto{0}{\frac{\partial v}{\partial t}}+u\cancelto{0}{\frac{\partial v}{\partial x}}+v\cancelto{0}{\frac{\partial v}{\partial y}}+w\cancelto{0}{\frac{\partial v}{\partial z}}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left[\cancelto{0}{\frac{\partial^2 v}{\partial x^2}}+\cancelto{0}{\frac{\partial^2 v}{\partial y^2}}+\cancelto{0}{\frac{\partial^2 v}{\partial z^2}}\right]$$

So, we obtain

$$\frac{\partial p}{\partial y}=\frac{\partial p}{\partial z}=0 \text{ which means } p=p(x) \text{ alone}$$

and $$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

Couette Flow:

Couette flow is the flow between **two parallel plates**. Here, one plate is at **rest** and the other is **moving** with a velocity U . Let us assume the plates are infinitely large in z direction, so the z **dependence is not there**.

The **governing equation** is

$$\frac{dp}{dx}=\mu\frac{d^2u}{dy^2}$$

flow is independent of any variation in z-direction.

The **boundary conditions** are ---(i)**At y = 0, u = 0** (ii)**At y = h, u = U.**

Boundary Layer Concept:

Introduction

 The **boundary layer** of a flowing fluid is **the thin layer close to the wall**
 In a flow field, **viscous stresses are very prominent within this layer**.
 Although the layer is thin, it is very important to know the details of flow within it.
 The **main-flow velocity** within this layer **tends to zero** while approaching the wall **(noslip condition)**.
 Also the gradient of this velocity component in a direction normal to the surface is large as compared to the gradient in the streamwise direction.

Boundary Layer Properties:

Boundary Layer Equations

 In 1904, **Ludwig Prandtl,** the well known German scientist, introduced the concept of boundary layer and **derived the equations for boundary layer flow** by correct reduction of Navier-Stokes equations.

He hypothesized that **for fluids having relatively small viscosity**, **the effect of internal friction in the fluid is significant only in a narrow region surrounding solid boundaries or bodies over which the fluid flows.**
 Thus, close to the body is the boundary layer where **shear stresses exert an increasingly larger effect** on the fluid **as one moves from free stream towards the solid boundary.**

 However, **outside the boundary layer where the effect of the shear stresses on the flow is small compared to values inside the boundary layer (since the velocity gradient is negligible)**,---------

1. the fluid particles experience **no vorticity** and therefore**,**
2. the flow is similar to a **potential flow.**

 Hence, the ***surface*** **at the boundary layer interface** is a rather fictitious one, that **divides rotational and irrotational flow.** Fig 28.1 shows Prandtl's model regarding boundary layer flow.
 Hence with the exception of the immediate vicinity of the surface, the flow is frictionless (inviscid) and the velocity is U (the potential velocity).

In the region, very near to the surface (in the thin layer), there is friction in the flow which signifies that the fluid is retarded until it adheres to the surface (**no-slip condition).**
 The transition of the mainstream velocity from zero at the surface (with respect to the surface) to full magnitude takes place across the boundary layer.

About the boundary layer

 Boundary layer **thickness is** which is a **function of** the coordinate direction ***x*** .
 The thickness is considered to be **very small compared to the characteristic length *L*** of the domain.
 In the normal direction, **within this thin layer**, the gradient **is very large compared to** the gradient in the flow direction **.**

Now we take up the Navier-Stokes equations for : steady, two dimensional, laminar, incompressible flows.
Considering the Navier-Stokes equations together with the equation of continuity, the following dimensional form is obtained.

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

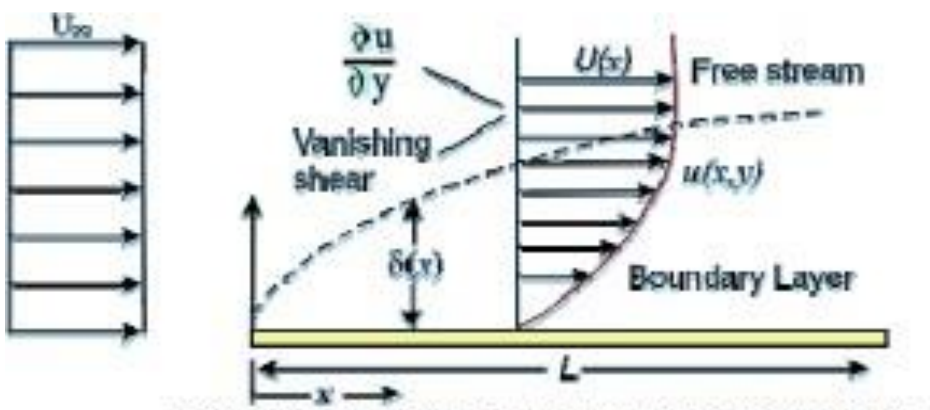


Fig 28.1 Boundary layer and Free Stream for Flow Over a flat plate

- u - velocity component along x direction.
- v - velocity component along y direction
- p - static pressure
- ρ - density.
- μ - dynamic viscosity of the fluid
- The equations are now non-dimensionalised.
- The l**ength and the velocity scales** are chosen as ***L*** **and respectively.**
- The non-dimensional variables are:

$$u^* = \frac{u}{U_\infty}, \quad v^* = \frac{v}{U_\infty}, \quad p^* = \frac{p}{\rho U_\infty^2}$$

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}$$

Derivation of Prandtl Bounday Layer Equation:

Order of Magnitude Analysis

- Let us examine what happens to the *u* velocity as we go across the boundary layer.

At the **wall** the *u* velocity is **zero** [with respect to the wall and absolute zero for a stationary wall (which is normally implied if not stated otherwise)].

The value of *u* on the **inviscid side**, that is on the free stream side beyond the boundary layer is ***U.***

For the case of external flow over a flat plate, this ***U*** is equal to .

- Based on the above, we can identify the following scales for the boundary layer variables:

Variable	Dimensional scale	Non-dimensional scale
u	U_∞	1
x	L	1
y	δ	$\varepsilon = \delta / L$

The **symbol describes a value much smaller than 1.**

Now we analyse equations 28.4 - 28.6, and look at the order of magnitude of each individual term

8.4 Blasius Solution:

Blasius Flow Over A Flat Plate

- The classical problem considered by H. Blasius was
    1. Two-dimensional, steady, incompressible flow over a flat plate at zero angle of incidence with respect to the uniform stream of velocity U_∞.
    2. The fluid extends to infinity in all directions from the plate.

The physical problem is already illustrated in Fig. 28.1

- Blasius wanted to determine
  (a) the velocity field solely within the boundary layer,
  (b) the boundary layer thickness (δ),
  (c) the shear stress distribution on the plate, and
  (d) the drag force on the plate.
- The Prandtl boundary layer equations in the case under consideration are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (28.15)$$

$$\nu=\mu/\rho$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

The boundary conditions are

$$at\ y=0,\ u=v=0 \qquad (28.16)$$

$$at\quad y=\infty,\quad u=U_\infty$$

- Note that the substitution of the term $-\frac{1}{\rho}\frac{dp}{dx}$ in the original boundary layer momentum equation in terms of the free stream velocity produces $U_\infty\frac{dU_\infty}{dx}$ which is equal to zero.
- Hence the governing Eq. (28.15) does not contain any pressure-gradient term.

- However, the characteristic parameters of this problem are U_∞, ν, x, y that is, $u = u(U_\infty, \nu, x, y)$
- This relation has five variables U_∞, ν, x, y.
- It involves two dimensions, length and time.
- Thus it can be reduced to a dimensionless relation in terms of (5-2) =3 quantities (**Buckingham Pi Theorem**)
- Thus a similarity variables can be used to find the solution
- Such flow fields are called self-similar flow field .

Law of Similarity for Boundary Layer Flows

- It states that the *u* component of velocity with two velocity profiles of *u(x,y)* at different *x* locations differ only by scale factors in *u* and *y* .
- Therefore, the velocity profiles *u(x,y)* at all values of *x* can be made congruent if they are plotted in coordinates which have been made dimensionless with reference to the scale factors.
- The local free stream velocity *U(x)* at section *x* is an obvious scale factor for *u*, because the dimensionless *u(x)* varies between zero and unity with *y* at all sections.
- The scale factor for *y* , denoted by *g(x)* , is proportional to the local boundary layer thickness so that *y* itself varies between zero and unity.
- Velocity at two arbitrary *x* locations, namely x_1 and x_2 should satisfy the equation

$$\frac{u[x_1, (y/g(x_1))]}{U(x_1)} = \frac{u[x_2, (y/g(x_2))]}{U(x_2)} \qquad (28.17)$$

- Now, for Blasius flow, it is possible to identify g(x) with the boundary layers thickness δ we know

$$g = \frac{\delta}{L} \sim \frac{1}{\sqrt{\mathrm{Re}_L}}$$

Thus in terms of x we get

$$\frac{\delta}{x} \sim \frac{1}{\sqrt{\frac{U_\infty x}{\nu}}}$$

$$\delta \sim \sqrt{\frac{\nu x}{U_\infty}}$$

Turbulent Boundary Layer over Flat Plate:

Derivation of Governing Equations for Turbulent Flow

- For incompressible flows, the Navier-Stokes equations can be rearranged in the form

$$\rho\left[\frac{\partial u}{\partial t}+\frac{\partial (u^2)}{\partial x}+\frac{\partial (uv)}{\partial y}+\frac{\partial (uw)}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\nabla^2 u \qquad (33.1a)$$

$$\rho\left[\frac{\partial v}{\partial t}+\frac{\partial (uv)}{\partial x}+\frac{\partial (v^2)}{\partial y}+\frac{\partial (vw)}{\partial z}\right]=-\frac{\partial p}{\partial y}+\mu\nabla^2 v \qquad (33.1b)$$

$$\rho\left[\frac{\partial w}{\partial t}+\frac{\partial (uw)}{\partial x}+\frac{\partial (vw)}{\partial y}+\frac{\partial (w^2)}{\partial z}\right]=-\frac{\partial p}{\partial z}+\mu\nabla^2 w \qquad (33.1c)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (33.2)$$

- Express the velocity components and pressure in terms of time-mean values and corresponding fluctuations. In **continuity equation**, this substitution and subsequent time averaging will lead to

$$\overline{\frac{\partial(\bar{u}+u')}{\partial x}}+\overline{\frac{\partial(\bar{v}+v')}{\partial y}}+\overline{\frac{\partial(\bar{w}+w')}{\partial z}}=0$$

or,

$$\left(\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}\right)+\left(\frac{\partial \overline{u'}}{\partial x}+\frac{\partial \overline{v'}}{\partial y}+\frac{\partial \overline{w'}}{\partial z}\right)=0$$

Since, $$\frac{\partial \overline{u'}}{\partial x} + \frac{\partial \overline{v'}}{\partial y} + \frac{\partial \overline{w'}}{\partial z} = 0$$

We can write $$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \qquad (33.3a)$$

From Eqs (33.3a) and (33.2), we obtain

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \qquad (33.3b)$$

- It is evident that **the time-averaged velocity components** and **the fluctuating velocity components**, each satisfy the continuity equation for incompressible flow.
- Imagine a two-dimensional flow in which the turbulent components are independent of the z -direction. Eventually, Eq.(33.3b) tends to

$$\frac{\partial u'}{\partial x} = -\frac{\partial v'}{\partial y} \qquad (33.4)$$

On the basis of condition (33.4), it is postulated that if at an instant there is an increase in u' in the x -direction, it will be followed by an increase in v' in the negative y -direction. In other words, $\overline{u'v'}$ **is non-zero and negative**. (see Figure 33.2)

Fig 33.2 Each dot represents uv pair at an instant

c), we obtain expressions in terms of mean and fluctuating components. Now, forming time averages and considering the rules of averaging we discern the following. The terms which are linear, such as $\frac{\partial u'}{\partial t}$ and $\frac{\partial^2 u'}{\partial x^2}$ vanish when they are averaged [from (32.6)]. The same is true for the mixed terms like $\bar{u}\cdot u'$, or $\bar{u}\cdot v'$, but the quadratic terms in the fluctuating components remain in the equations. After averaging, they form $\overline{u'^2}$, $\overline{u'v'}$ etc.

- If we perform the aforesaid exercise on the x-momentum equation, we obtain

$$\rho\left[\frac{\partial \bar{u}}{\partial t}+\cancel{\frac{\partial \overline{u'}}{\partial t}}^{0}+\frac{\partial\left(\bar{u}^2+\overline{u'^2}\right)}{\partial x}+\frac{\partial\left(\overline{\bar{u}\cdot\bar{v}}+\overline{u'v'}\right)}{\partial y}+\frac{\partial\left(\bar{u}\cdot\bar{w}+\overline{u'w'}\right)}{\partial z}\right]$$

$$=-\frac{\partial \bar{p}}{\partial x}-\cancel{\frac{\partial \overline{p'}}{\partial x}}^{0}+\mu\left[\frac{\partial^2 \bar{u}}{\partial x^2}+\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{\partial^2 \bar{u}}{\partial z^2}+\left(\cancel{\frac{\partial^2 \overline{u'}}{\partial x^2}}^{0}+\cancel{\frac{\partial^2 \overline{u'}}{\partial y^2}}^{0}+\cancel{\frac{\partial^2 \overline{u'}}{\partial z^2}}^{0}\right)\right]$$

using rules of time averages,

$$\frac{\partial \overline{u'}}{\partial t}=0,\ \frac{\partial \overline{p'}}{\partial x}=0,\ \frac{\partial^2 \overline{u'}}{\partial x^2}=\frac{\partial^2 \overline{u'}}{\partial y^2}=\frac{\partial^2 \overline{u'}}{\partial z^2}=0$$

We obtain

$$\rho\left\{\frac{\partial \bar{u}}{\partial t}+\frac{\partial\left(\bar{u}^2\right)}{\partial x}+\frac{\partial\left(\bar{u}\cdot\bar{v}\right)}{\partial y}+\frac{\partial\left(\bar{u}\cdot\bar{w}\right)}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial x}+\mu\nabla^2\bar{u}-\rho\left[\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}\right]$$

- Introducing simplifications arising out of continuity Eq. (33.3a), we shall obtain.

$$\rho\left\{\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial x}+\mu\nabla^2\bar{u}-\rho\left[\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}\right]$$

- Performing a similar treatment on y and z momentum equations, finally we obtain the momentum equations in the form.

In x direction,

$$\rho\left\{\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial x}+\mu\nabla^2\bar{u}-\rho\left[\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}\right] \quad (33.5a)$$

In y direction,

$$\rho\left\{\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial y}+\mu\nabla^2\bar{v}-\rho\left[\frac{\partial}{\partial x}\overline{u'v'}+\frac{\partial}{\partial y}\overline{v'^2}+\frac{\partial}{\partial z}\overline{v'w'}\right] \quad (33.5b)$$

In z direction,

$$\rho\left\{\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}-\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial z}+\mu\nabla^2\bar{w}-\rho\left[\frac{\partial}{\partial x}\overline{u'w'}+\frac{\partial}{\partial y}\overline{v'w'}+\frac{\partial}{\partial z}\overline{w'^2}\right] \quad (33.5c)$$

- Comments on the governing equation :
  1. The left hand side of Eqs (33.5a)-(33.5c) are essentially similar to the steady-state Navier-Stokes equations if the velocity components u, v and w are replaced by $\bar{u}$, $\bar{v}$ and $\bar{w}$.
  2. The same argument holds good for the first two terms on the right hand side of Eqs (33.5a)-(33.5c).
  3. However, the equations contain some additional terms which depend on turbulent fluctuations of the stream. **These additional terms can be interpreted as components of a stress tensor.**
- Now, the resultant surface force per unit area due to these terms may be considered as

In x direction,

$$\rho\left\{\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial x}+\mu\nabla^2\bar{u}+\left[\frac{\partial}{\partial x}\sigma'_{xx}+\frac{\partial}{\partial y}\tau'_{yx}+\frac{\partial}{\partial z}\tau'_{zx}\right] \quad (33.6a)$$

In y direction,

$$\rho\left\{\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial x}+\bar{v}\frac{\partial \bar{v}}{\partial y}+\bar{w}\frac{\partial \bar{v}}{\partial z}\right\}=-\frac{\partial \bar{p}}{\partial y}+\mu\nabla^2\bar{v}+\left[\frac{\partial}{\partial x}\tau'_{xy}+\frac{\partial}{\partial y}\sigma'_{yy}+\frac{\partial}{\partial z}\tau'_{zy}\right] \quad (33.6b)$$

In z direction,

$$\rho\left(\frac{\partial \bar{w}}{\partial t}+\bar{u}\frac{\partial \bar{w}}{\partial x}+\bar{v}\frac{\partial \bar{w}}{\partial y}+\bar{w}\frac{\partial \bar{w}}{\partial z}\right)=-\frac{\partial \bar{p}}{\partial z}+\mu\nabla^2\bar{w}+\left[\frac{\partial}{\partial x}\tau'_{xz}+\frac{\partial}{\partial y}\tau'_{yz}+\frac{\partial}{\partial z}\sigma'_{zz}\right] \quad (33.6c)$$

- Comparing Eqs (33.5) and (33.6), we can write

$$\begin{bmatrix}\sigma'_{xx} & \tau'_{xy} & \tau'_{xz}\\ \tau'_{xy} & \sigma'_{yy} & \tau'_{yz}\\ \tau'_{xz} & \tau'_{yz} & \sigma'_{zz}\end{bmatrix}=-\rho\begin{bmatrix}\overline{u'^2} & \overline{u'v'} & \overline{u'w'}\\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'}\\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2}\end{bmatrix} \quad (33.7)$$

- It can be said that the mean velocity components of turbulent flow satisfy the same Navier-Stokes equations of laminar flow. However, for the turbulent flow, the laminar stresses must be increased by additional stresses which are given by the stress tensor (33.7). These additional stresses are known as **apparent stresses of turbulent flow** or **Reynolds** stresses . Since turbulence is considered as eddying motion and the aforesaid additional stresses are added to the viscous stresses due to mean motion in order to explain the complete stress field, it is often said that the apparent stresses are caused by eddy viscosity . The total stresses are now

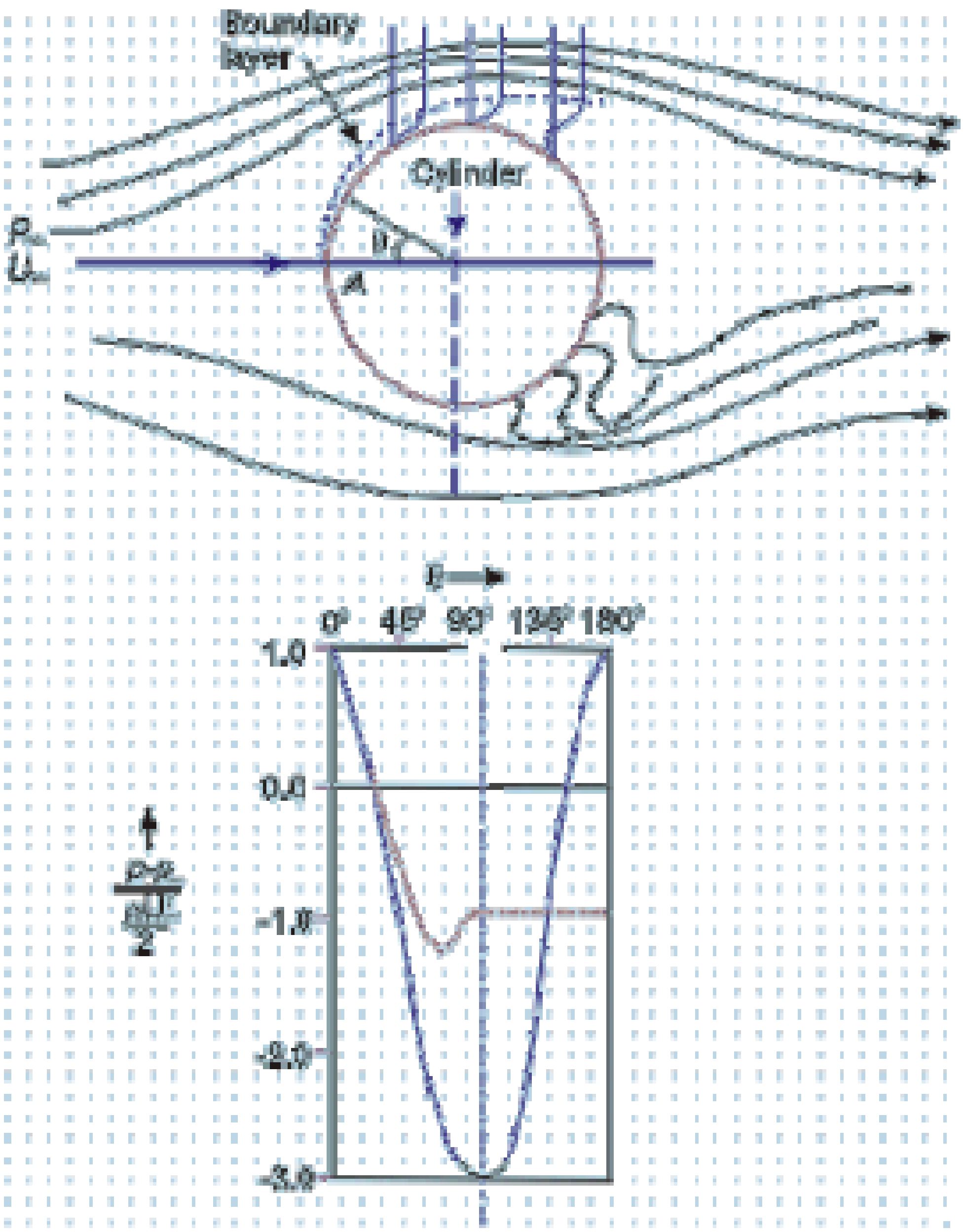
$$\left.\begin{aligned}\sigma_{xx}&=-\bar{p}+2\mu\frac{\partial \bar{u}}{\partial x}-\rho\overline{u'^2}\\ \tau_{xy}&=\mu\left(\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{v}}{\partial x}\right)-\rho\overline{u'v'}\end{aligned}\right] \quad (33.8)$$

and so on. The apparent stresses are much larger than the viscous components, and the viscous stresses can even be dropped in many actual calculations.

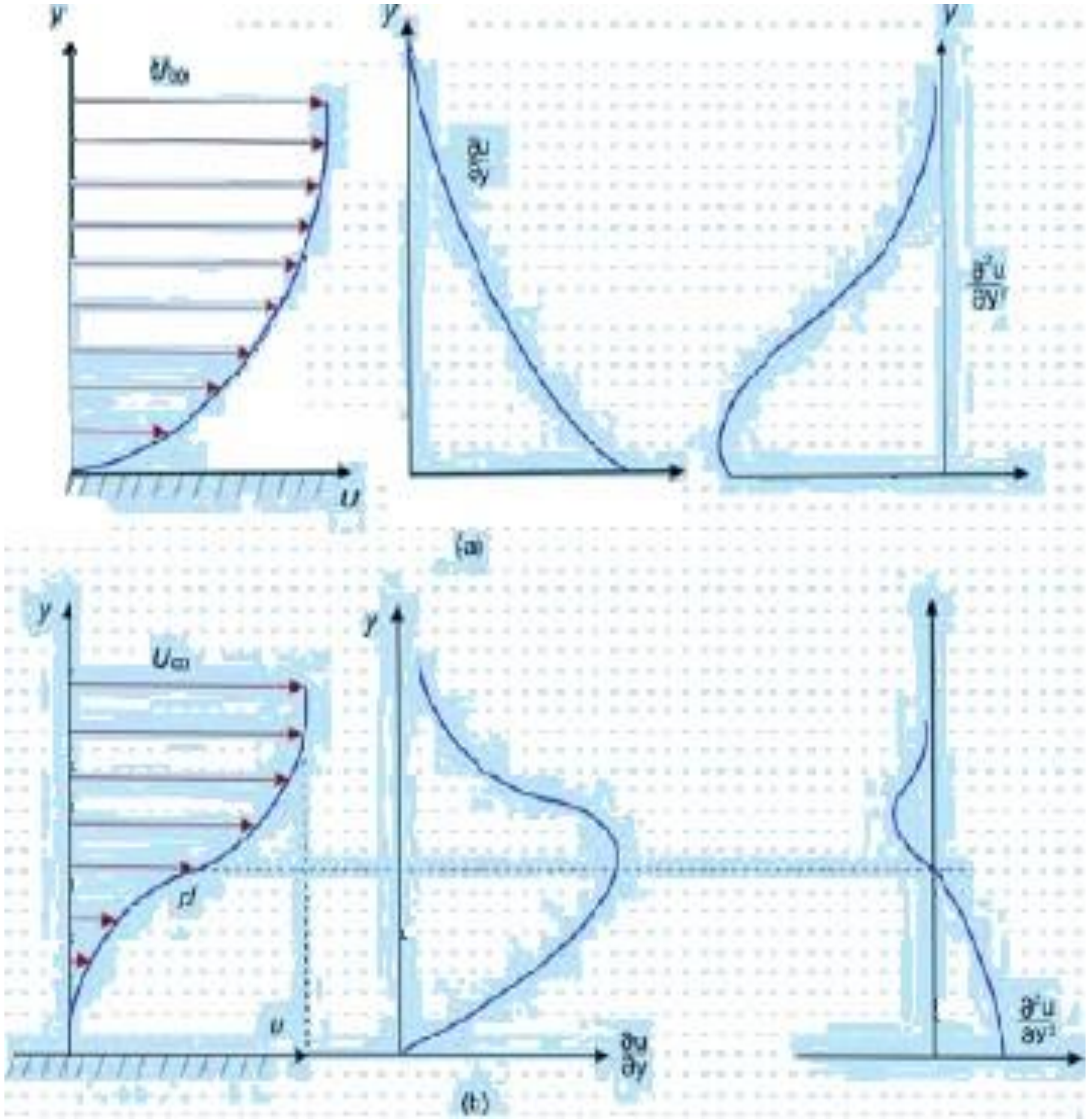
Boundary Layer Control:

Seperation of Boundary Layer

- It has been observed that the **flow is reversed at the vicinity of the wall** under certain conditions.
- The phenomenon is termed as **separation of boundary layer**.
- Separation takes place **due to excessive momentum loss near the wall in a boundary layer trying to move downstream against increasing pressure, i.e.,** $\frac{dp}{dx} > 0$ **, which is called** ***adverse pressure gradient.***
- Figure 29.2 shows the flow past a circular cylinder, in an infinite medium.
    1. Up to $\theta = 90^\circ$, the flow area is like a constricted passage and the flow behaviour is like that of a nozzle.
    2. Beyond $\theta = 90^\circ$ the flow area is diverged, therefore, the flow behaviour is much similar to a diffuser



Flow separation and formation of wake behind a circular cylinder



Velocity distribution within a boundary layer

(a) Favourable pressure gradient, $\frac{dp}{dx} < 0$

(b) adverse pressure gradient $\frac{dp}{dx} > 0$

Let us reconsider the flow past a circular cylinder and continue our **discussion on the wake behind a cylinder.** The pressure distribution which was shown by the firm line in Fig. 21.5 is obtained from the potential flow theory. However. somewhere near **(in experiments it has been observed to be at) . the boundary layer detaches itself from the wall.**

2. Meanwhile, **pressure in the wake remains close to separation-point-pressure** since the eddies (formed as a consequence of the retarded layers being carried together with the upper layer through the action of shear) cannot convert rotational kinetic energy into pressure head. The actual pressure distribution is shown by the dotted line in Fig. 29.3.

3. Since the **wake zone pressure is less than that of the forward stagnation point** (pressure at point A in Fig. 29.3), the cylinder experiences a drag force which is basically attributed to the pressure difference.

The drag force, brought about by the pressure difference is known as *form drag* whereas the shear stress at the wall gives rise to *skin friction drag*.
Generally, these two drag forces together are responsible for resultant drag on a body

Control Of Boundary Layer Separation –

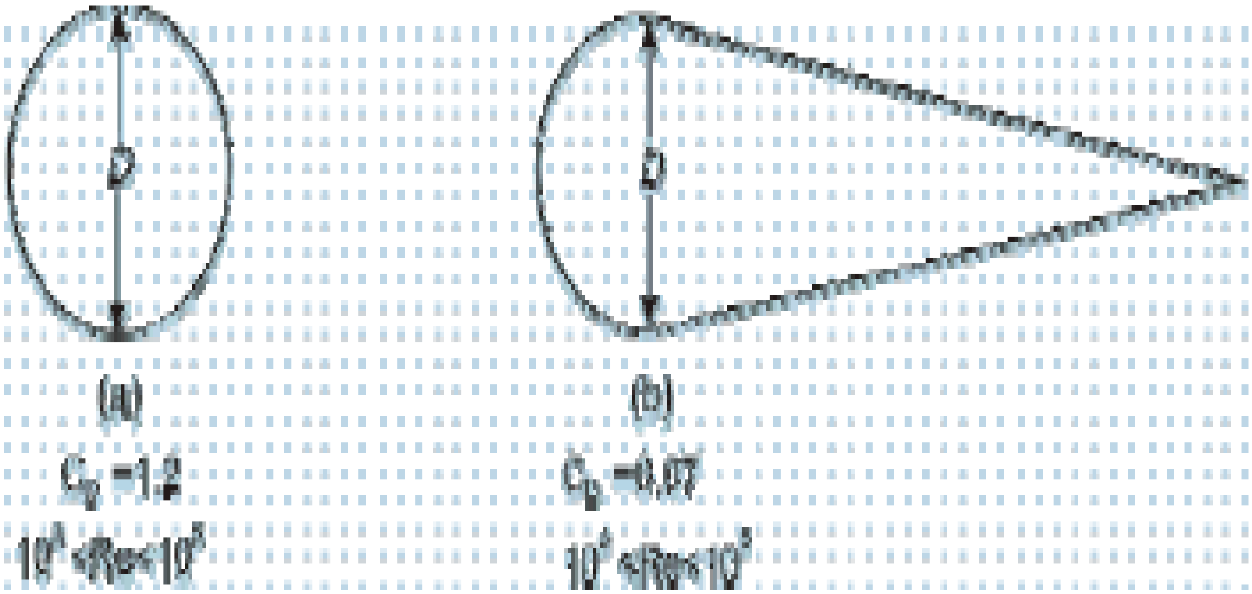
 The total drag on a body is attributed to form drag and skin friction drag. In some flow configurations, the contribution of form drag becomes significant.

 In order **to reduce the form drag, the boundary layer separation** should be **prevented or delayed** so that **better pressure recovery takes place** and the form drag is reduced considerably. There are some popular methods for this purpose which are stated as follows.

i. By giving the profile of the body a streamlined shape

1. This has an elongated shape in the rear part to reduce the magnitude of the pressure gradient.

2. The optimum contour for a streamlined body is the one for which the wake zone is very narrow and the form drag is minimum.



Reduction of drag coefficient (CD) by giving the profile a streamlined shape

The injection of fluid through porous wall can also control the boundary

layer separation. This is generally accomplished by blowing high energy fluid particles tangentially from the location where separation would have taken place otherwise.

1. The **injection of fluid promotes turbulence**

2. This **increases skin friction.** But the **form drag is reduced** considerably due to suppression of flow separation

3. The reduction in form drag is quite significant and **increase in skin friction drag can be ignored.**

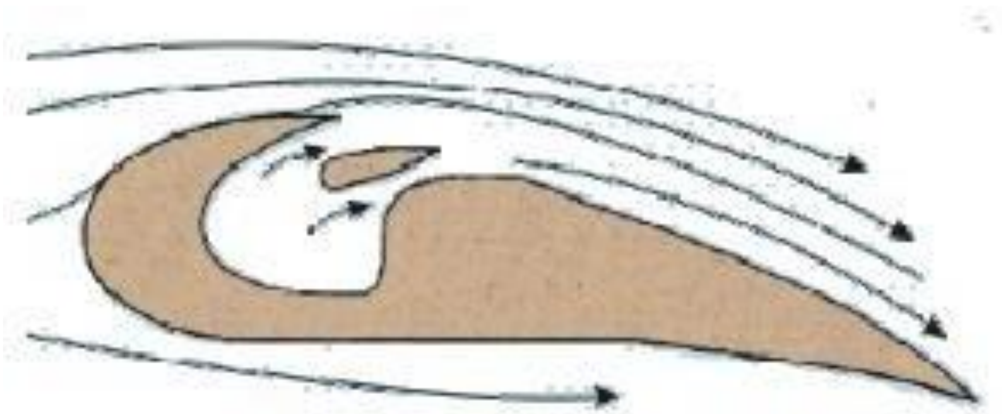
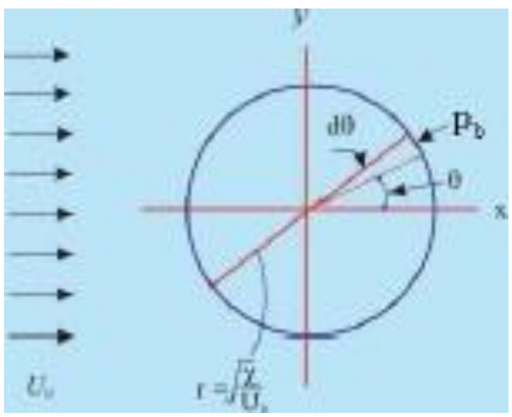


Fig. 31.3 Boundary layer control by blowing

Lift and Drag

Lift :force acting on the cylinder (per unit length) in the direction normal to uniform flow.

Drag: force acting on the cylinder (per unit length) in the direction parallel to uniform flow.



Calculation of Drag in a Cylinder

The drag is calculated by integrating the force components arising out of pressure, in the x direction on the boundary. Referring to Fig.22.4, the drag force can be written as

$$D = -\int_0^{2\pi} p_s \cos\theta r d\theta \qquad r d\theta = ds \rightarrow$$ infinitesimal length on the circumference

Since, $$r = \left(\frac{K}{\pi U_0}\right)^{1/2}$$

$$D = -\int_0^{2\pi} p_s \cos\theta \left(\frac{K}{\pi U_0}\right)^{1/2} d\theta$$

or, $$D = -\int_0^{2\pi} \rho g \left(\frac{K}{U_0}\right)^{1/2} \left[\frac{U_0^2}{2g} + \frac{p_0}{\rho g} - \frac{(2U_0 \sin\theta)^2}{2g}\right] \cos\theta d\theta$$

$$D = -\int_0^{2\pi} \left[p_0 + \frac{\rho U_0^2}{2}\left(1 - 4U_0 \sin^2\theta\right)\right]\left(\frac{K}{U_0}\right)^{1/2} \cos\theta d\theta$$

Similarly, the lift force may be calculated as

$$L = -\int_0^{2\pi} p_s \sin\theta \left(\frac{K}{U_0}\right)^{1/2} d\theta$$

However, in reality, the cylinder will always experience some drag force. This contradiction between the inviscid flow result and the experiment is usually known as D 'Almbert paradox.