UNIT II

FLUID KINEMATICS

Introduction

Kinematics is the geometry of Motion.

Kinematics of fluid describes the fluid motion and its consequences without consideration of the nature of forces causing the motion.

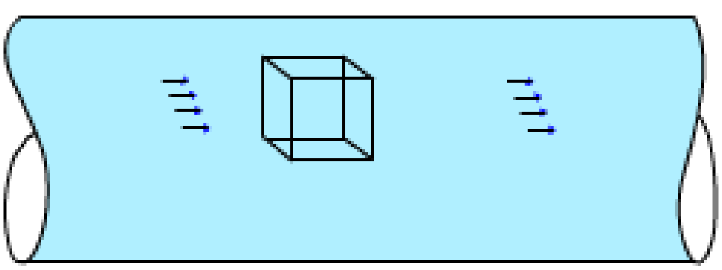
The fluid kinematics deals with description of the motion of the fluids without reference to the force causing the motion.

Thus it is emphasized to know how fluid flows and how to describe fluid motion. This concept helps us to simplify the complex nature of a real fluid flow.

When a fluid is in motion, individual particles in the fluid move at different velocities. Moreover at different instants fluid particles change their positions. In order to analyze the flow behavior, a function of space and time, we follow one of the following approaches

1. Lagarangian approach
2. Eularian approach

In the Lagarangian approach a fluid particle of fixed mass is selected. We follow the fluid particle during the course of motion with time



The fluid particles may change their shape, size and state as they move. As mass of fluid particles remains constant throughout the motion, the basic laws of mechanics can be applied to them at all times. The task of following large number of fluid particles is quite difficult. Therefore this approach is limited to some special applications for example re-entry of a spaceship into the earth's atmosphere and flow measurement system based on particle imagery.

In the Eularian method a finite region through which fluid flows in and out is used. Here we do not keep track position and velocity of fluid particles of definite mass. But, within the region, the field variables which are continuous functions of space dimensions (x , y , z) and time (t), are defined to describe the flow. These field variables may be scalar field variables, vector field variables and tensor quantities. For example, pressure is one of the scalar fields. Sometimes this finite region is referred as control volume or flow domain.

For example the pressure field 'P' is a scalar field variable and defined as

$$P = P(x, y, z, t)$$

Velocity field, a vector field, is defined as $\vec{V} = \vec{V}(x, y, z, t)$

Similarly shear stress τ is a tensor field variable and defined as

$$\tau = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

Note that we have defined the fluid flow as a three dimensional flow in a Cartesian co-ordinates system.

Advantages of Lagrangian Method:
1. Since motion and trajectory of each fluid particle is known, its history can be traced.
2. Since particles are identified at the start and traced throughout their motion, conservation of mass is inherent.

Disadvantages of Lagrangian Method:
1. The solution of the equations presents appreciable mathematical difficulties except

certain special cases and therefore, the method is rarely suitable for practical applications.

Types of Fluid Flow

Uniform and Non-uniform flow : If the velocity at given instant is the same in both magnitude and direction throughout the flow domain, the flow is described as uniform.

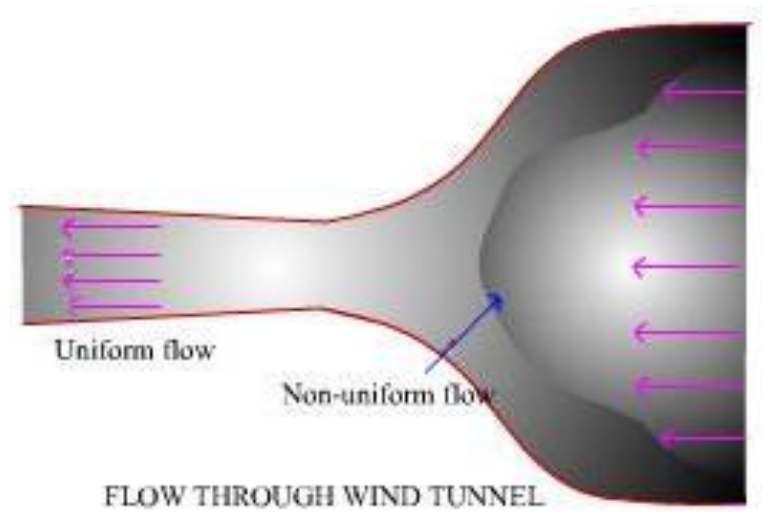


Fig. L-16.2 : Uniform and Non-uniform flow.

Mathematically the velocity field is defined as $\vec{V} = \vec{V}(t)$, independent to space dimensions (x , y , z).

When the velocity changes from point to point it is said to be non-uniform flow. Fig. shows uniform flow in test section of a well designed wind tunnel and describing non uniform velocity region at the entrance.

Steady and unsteady flows

The flow in which the field variables don't vary with time is said to be steady flow. For steady flow,

$$\frac{\delta V}{\delta t} = 0 \qquad \text{Or} \qquad \vec{V} = \vec{V}(x, y, z)$$

It means that the field variables are independent of time. This assumption simplifies the fluid problem to a great extent. Generally, many engineering flow devices and systems are designed to operate them during a peak steady flow condition.

If the field variables in a fluid region vary with time the flow is said to be unsteady flow.

$$\frac{\delta V}{\delta t} \neq 0 \qquad \vec{V} = \vec{V}(x, y, z, t)$$

Four possible combinations

Type	Example
1. Steady Uniform flow	Flow at constant rate through a duct of uniform cross-section (The region close to the walls of the duct is disregarded)
2. Steady non-uniform flow	Flow at constant rate through a duct of non-uniform cross-section (tapering pipe)
3. Unsteady Uniform flow	Flow at varying rates through a long straight pipe of uniform cross-section. (Again the region close to the walls is ignored.)
4. Unsteady non-uniform flow	Flow at varying rates through a duct of non-uniform cross-section.

One, two and three dimensional flows

Although fluid flow generally occurs in three dimensions in which the velocity field vary with three space co-ordinates and time. But, in some problem we may use one or two space components to describe the velocity field. For example consider a steady flow through a long straight pipe of constant cross-section. The velocity distributions shown in figure are independent of co-ordinate x and θ and a function of r only. Thus the flow field is one dimensional.

Fig. L-16.3

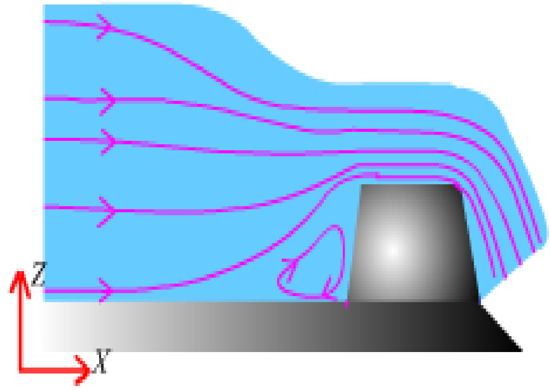


Fig. L-16.4

But in the case of flow over a weir of constant cross-section (), we can use two co-ordinate system x and z in defining the velocity field. So, this flow is a case of two dimensional flow. The reduction of independent space variable in a fluid flow problem makes it simpler to solve.

Laminar and Turbulent flow

In fluid flows, there are two distinct fluid behaviors experimentally observed. These behaviors were first observed by Sir Osborne Reynolds. He carried out a simple experiment in which water was discharged through a small glass tube from a large tank (the schematic of the experiment shown in Fig.). A colour dye was injected at the entrance of the tube and the rate of flow could be regulated by a valve at the out let.

When the water flowed at low velocity, it was found that the die moved in a straight line. This clearly showed that the particles of water moved in parallel lines. This type of flow is called laminar flow, in which the particles of fluid moves along smooth paths in layers. There is no exchange of momentum from fluid particles of one layer to the fluid particles of another layer.

This type of flow mainly occurs in high viscous fluid flows at low velocity, for example, oil flows at low velocity. Fig. shows the steady velocity profile for a typical laminar flow.

When the water flowed at high velocity, it was found that the dye colour was diffused over the whole cross section. This could be interpreted that the particles of fluid moved in very irregular paths, causing an exchange of momentum from one fluid particle to another. This type of flow is known as turbulent flow. The time variation of velocity at a point for the turbulent flow is shown in Fig

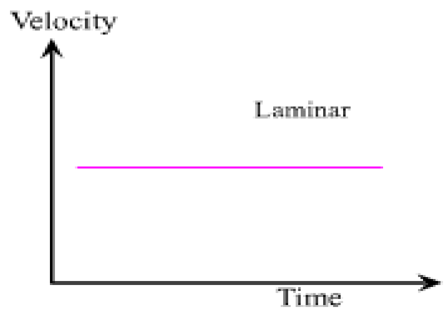


Fig-L16.5 : Velocity profile for laminar flow

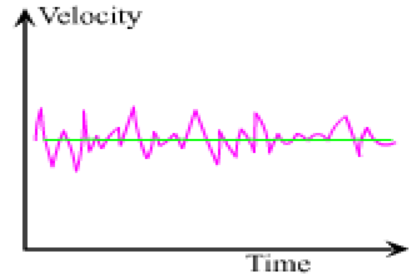


Fig. L-16.6 : Velocity profile for turbulent flow

It means that the flow is characterized by continuous random fluctuations in the magnitude and the direction of velocity of the fluid particles.

Velocity Field

Consider a uniform stream flow passing through a solid cylinder (Fig.). The typical velocities at different locations within the fluid domain vary from position to position at a particular time t . At different time instants this velocity distribution may change. Keeping this observation in mind, the velocity within a flow domain can be represented as function of position (x , y , z) and time t .

In the Cartesian co-ordinates the variation of velocity can be represented as a vector $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ where u , v , w are the velocity scalar components in x , y and z directions respectively.

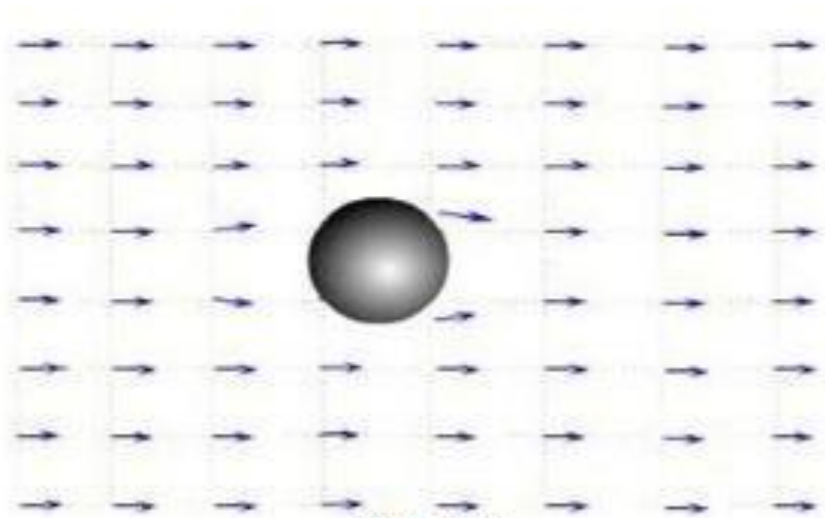


Fig. L-17.1

The scalar components u , v and w are dependent functions of position and time. Mathematically we can express them as

$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t)$$
$$w = w(x, y, z, t)$$

This type of continuous function distribution with position and time for velocity is known as velocity field. It is based on the Eularian description of the flow. We also can represent the Lagrangian description of velocity field.

Let a fluid particle exactly positioned at point A moving to another point A' during time interval Δt. The velocity of the fluid particle is the same as the local velocity at that point as obtained from the Eulerian description

At time t , $\vec{V}$ particle at x , y , z $(t) = \vec{V}(x, y, z, t)$

At time $t + \Delta t$, $\vec{V}$ particle at x', y', z' $(t + \Delta t) = \vec{V}(x', y', z', t + \Delta t)$

This means that instead of describing the motion of the fluid flow using the Lagrangian description, the use of Eularian description makes the fluid flow problems quite easier to solve. Besides this difficult, the complete description of a fluid flow using the Lagrangian description requires to keep track over a large number of fluid particles and their movements with time. Thus, more computation is required in the Lagrangian description.

The Acceleration field

At given position A, the acceleration of a fluid particle is the time derivative of the particle's velocity.

Acceleration of a fluid particle: $\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$

Since the particle velocity is a function of four independent variables (x , y , z and t), we can express the particle velocity in terms of the position of the particle as given below

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}\left(x_{particle}, y_{particle}, z_{particle}\right)}{dt}$$

Applying chain rule, we get

$$\vec{a}_{particle} = \frac{\delta\vec{V}}{\delta t}\cdot\frac{dt}{dt} + \frac{\delta\vec{V}}{\delta x_{particle}}\frac{dx_{particle}}{dt} + \frac{\delta\vec{V}}{\delta y}\frac{dy_{particle}}{dt} + \frac{\delta\vec{V}}{\delta z}\frac{dz_{particle}}{dt}$$

Where δ and d are the partial derivative operator and total derivative operator respectively.

The time rate of change of the particle in the x -direction equals to the x -component of velocity vector, u . Therefore

$$\frac{dx_{particle}}{dt} = u$$

Similarly, $\frac{dy_{particle}}{dt} = v$

$$\frac{dz_{particle}}{dt} = w$$

As discussed earlier the position vector of the fluid particle (x particle , y particle , z particle) in the Lagranian description is the same as the position vector (x , y , z) in the Eulerian frame at time t and the acceleration of the fluid particle, which occupied the position (x , y , z) is equal to $a(x,y,z,t)$ in the Eularian description.

Therefore, the acceleration is defined by

$$\vec{a}_{(x,y,z,t)} = \frac{\delta\vec{V}}{\delta t} + u\frac{\delta\vec{V}}{\delta x} + v\frac{\delta\vec{V}}{\delta y} + w\frac{\delta\vec{V}}{\delta z}$$

in vector form

$$\vec{a}_{(x,y,z,t)} = \underbrace{\frac{\delta\vec{V}}{\delta t}}_{\text{(local accelaration)}} + \underbrace{\left(\vec{V}.\nabla\right)\vec{V}}_{\text{(convective accelartion)}}$$

where ∇ is the gradient operator.

The first term of the right hand side of equation represents the time rate of change of velocity field at the position of the fluid particle at time t . This acceleration component is also independent to the change of the particle position and is referred as the local acceleration. However the term $\left(\vec{V}.\nabla\right)\vec{V}$ accounts for the affect of the change of the velocity at various positions in this field. This rate of change of velocity because of changing position in the field is called the convective acceleration.

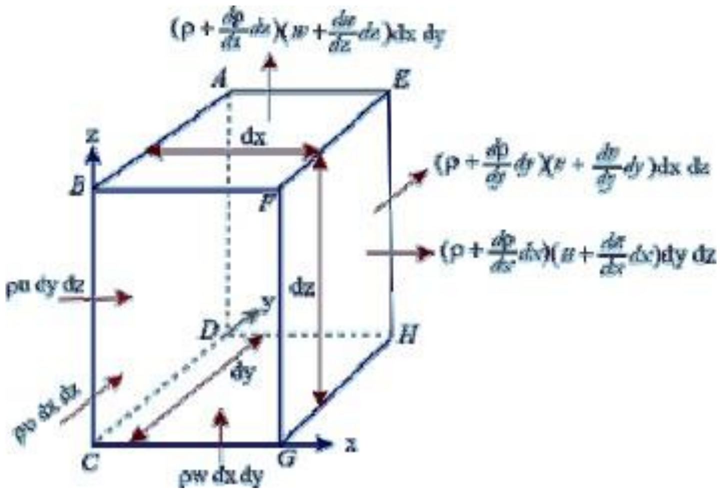
Type of Flow	Material Acceleration	
	Temporal	Convective
1. Steady Uniform flow	0	0
2. Steady non-uniform flow	0	exists
3. Unsteady Uniform flow	exists	0
4. Unsteady non-uniform flow	exists	exists

Continuity Equation - Differential Form

Derivation

1. The point at which the continuity equation has to be derived, is enclosed by an elementary control volume.

2. The influx, efflux and the rate of accumulation of mass is calculated across each surface within the control volume.



A Control Volume Appropriate to a Rectangular Cartesian coordinate system

Consider a rectangular parallelopiped in the above figure as the control volume in a rectangular cartesian frame of coordinate axes.

- Net efflux of mass along x -axis must be the excess outflow over inflow across faces normal to x -axis.

- Let the fluid enter across one of such faces ABCD with a velocity u and a density ρ.The velocity and density with which the fluid will leave the face EFGH will be $u+\frac{\partial u}{\partial x}dx$ and respectively (neglecting the higher order terms in δx).

- Therefore, the rate of mass entering the control volume through face ABCD = $\rho u\ dy\ dz$.

- The rate of mass leaving the control volume through face EFGH will be

$$=\left(\rho+\frac{\partial \rho}{\partial x}dx\right)\left(u+\frac{\partial u}{\partial x}dx\right)dydz$$

$$=\left(\rho u+\frac{\partial}{\partial x}(\rho u)dx\right)dydz$$

(neglecting the higher order terms in dx)

Similarly influx and efflux take place in all y and z directions also.

Rate of accumulation for a point in a flow field

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t}\rho(d\forall) = \frac{\partial \rho}{\partial t}d\forall$$

Using, Rate of influx = Rate of Accumulation + Rate of Efflux

$$\rho u\,dy\,dz + \rho v\,dx\,dz + \rho w\,dx\,dy = \frac{\partial \rho}{\partial t}d\forall + \left(\rho + \frac{\partial \rho}{\partial x}dx\right)\left(u + \frac{\partial u}{\partial x}dx\right)dy\,dz$$
$$+ \left(\rho + \frac{\partial \rho}{\partial y}dy\right)\left(v + \frac{\partial v}{\partial y}dy\right)dx\,dz + \left(\rho + \frac{\partial \rho}{\partial z}dz\right)\left(w + \frac{\partial w}{\partial z}dz\right)dx\,dy$$

Transferring everything to right side

$$0 = \left[\left(\rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x}\right) + \left(\rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y}\right) + \left(\rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z}\right)\right]dx\,dy\,dz + \left(\frac{\partial \rho}{\partial t}\right)d\forall$$

$$\Rightarrow \left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]d\forall = 0$$

This is the Equation of Continuity for a compressible fluid in a rectangular Cartesian coordinate system.

Continuity Equation - Vector Form

The continuity equation can be written in a vector form as

$$\frac{\partial \rho}{\partial t} + \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\left[\rho u\hat{i} - \rho v\hat{j} + \rho w\hat{k}\right] = 0$$

or,

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{V}) = 0$$

Streamlines, Pathlines and Streakline

Streamlines

Definition: Streamlines are the Geometrical representation of the of the flow velocity.

Description:

- In the **Eulerian** method, the velocity vector is defined as a function of time and space coordinates.
- If for a fixed instant of time, a **space curve** is drawn so that it is **tangent** everywhere to the **velocity** vector, then this curve is called a **Streamline**.

Therefore, the Eulerian method gives a series of instantaneous streamlines of the state of motion.

Streamlines

Alternative Definition: A streamline at any instant can be defined as an imaginary curve or line in the flow field so that the tangent to the curve at any point represents the direction of the **instantaneous velocity** at that point.
In an **unsteady flow** where the velocity vector changes with time, the pattern of streamlines also **changes from instant to instant.**
In a **steady flow**, the orientation or the pattern of streamlines will be **fixed**.

From the above definition of streamline, it can be written as $\vec{V} \times d\vec{S} = 0$

1. $d\vec{S}$ is the length of an infinitesimal line segment along a streamline at a point .
2. $\vec{V}$ is the instantaneous velocity vector.

The above expression therefore represents the **differential equation of a streamline**. In a cartesian coordinate-system, representing

$$d\vec{S} = \hat{i}dx + \hat{j}dy + \hat{k}dz \qquad \vec{V} = \hat{i}u + \hat{j}v + \hat{k}w$$

the above equation may be simplified as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Stream tube:
A bundle of neighboring streamlines may be imagined to form a passage through which the fluid flows. This passage is known as a **stream-tube**.

Properties of Stream tube:

1. The stream-tube is bounded on all sides by streamlines.

2. Fluid velocity does not exist across a streamline, no fluid may enter or leave a stream-tube except through its ends.

3. The entire flow in a flow field may be imagined to be composed of flows through streamtubes arranged in some arbitrary positions

Path Lines

Definition: A path line is the trajectory of a fluid particle of **fixed identity**

Path lines

A family of path lines represents the **trajectories of different particles**, say, P1, P 2, P3, etc.

Differences between Path Line and Stream Line

Path Line	Stream Line
This refers to a path followed by a fluid particle over a period of time.	This is an imaginary curve in a flow field for a fixed instant of time, tangent to which gives the instantaneous velocity at that point .
Two path lines can intersect each other as or a single path line can form a loop as different particles or even same particle can arrive at the same point at different instants of time.	Two stream lines can never intersect each other, as the instantaneous velocity vector at any given point is unique.

In a steady flow **path lines** are **identical** to **streamlines** as the **Eulerian and Lagrangian** versions become the **same**.

Vorticity

Definition: The vorticity Ω in its simplest form is defined as a vector which is equal to **two times the rotation vector**

$$\vec{\Omega} = 2\vec{\omega} = \nabla \times \vec{V}$$

For an **irrotational** flow, vorticity components are zero.

Vortex line:

If tangent to an imaginary line at a point lying on it is in the direction of the Vorticity vector at that point , the line is a **vortex line**.

The **general equation** of the **vortex line** can be written as,

$$\vec{\Omega} \times d\vec{s} = 0$$

In a rectangular cartesian cartesian coordinate system, it becomes

$$\frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$$

where,

$$\Omega_x = 2\omega_x$$

$$\Omega_y = 2\omega_y$$

$$\Omega_z = 2\omega_z$$

Vorticity components as vectors:

The vorticity is actually an **anti symmetric tensor** and its three distinct elements transform like the components of a vector in cartesian coordinates.

This is the reason for which the vorticity components can be treated as vectors.

Existence of Flow

- A fluid must obey the law of conservation of mass in course of its flow as it is a material body.
- For a Velocity field to exist in a fluid continuum, the velocity components must obey the **mass conservation principle**.
- Velocity components which follow the mass conservation principle are said to constitute a possible fluid flow
- Velocity components violating this principle, are said to describe an impossible flow.
- The existence of a physically possible flow field is verified from the principle of conservation of mass.

The detailed discussion on this is deferred to the next chapter along with the discussion on principles of**conservation of momentum and energy**.

Definition of rotation at a point:

The rotation at a point is defined as the **arithmetic mean** of the **angular velocities** of two perpendicular linear segments meeting at that point.

Example: The angular velocities of AB and AD about A are

$\frac{d\alpha}{dt}$ and $\frac{d\beta}{dt}$ respectively.

Considering the **anticlockwise direction as positive**, the rotation at A can be written as,

$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} - \frac{d\beta}{dt}\right)$$

or

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

The suffix z in ω represents the rotation about z-axis.

When $u = u\ (x, y)$ and $v = v\ (x, y)$ the **rotation and angular deformation of a fluid element exist simultaneously**.

Special case : Situation of pure Rotation

$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$, $\dot{\gamma}_{xy} = 0$ and $\omega_z = \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

Observation:

- The linear segments AB and AD move with the same angular velocity (both in magnitude and direction).
- The included angle between them remains the same and no angular deformation takes place. This situation is known as **pure rotation.**