NARSIMHA REDDY ENGINEERING COLLEGE
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Unit-II

TURNING MOMENT DIAGRAMS AND FLY WHEEL

- A flywheel is nothing but a rotating mass which is used as an energy reservoir in a machine which absorbs the energy when the speed in more and releases the energy when the speed is less, thus maintaining the fluctuation of speed within prescribed limits.

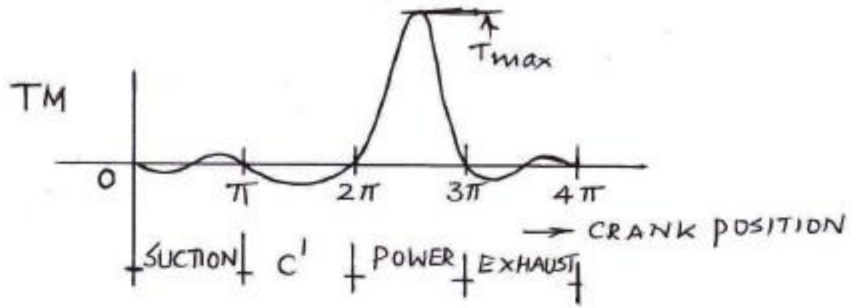
Difference between Governor and Flywheel:

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job. Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

Uses of turning moment Diagram

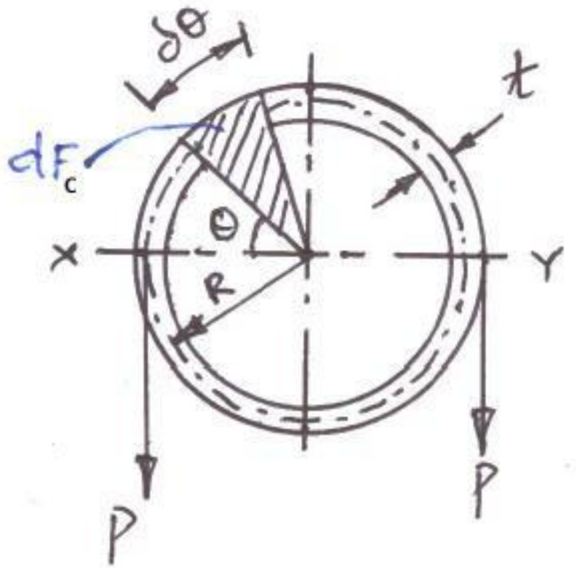
1) The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.
2) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us the find the fluctuation of energy.
3) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us the find diameter of the crank shaft.

TMD for a four stroke I.C. Engine



Size of fly wheel and hoop stress developed in a fly wheel.

Consider a rim of the fly wheel as shown in figure. Let D = mean diameter of rim, R = mean radius of rim, t = thickness of the fly wheel, A = cross sectional as area of rim in m^2 and ρ be the density of the rim material in Kg/m^3, N be the speed of the fly wheel in rpm, ω= angular velocity in rad/sec, V = linear velocity in m/σ, hoop stress in N/m^2 due to centrifugal force.



Consider small element of the rim. Let it subtend an angle $\delta\theta$ at the centre of flywheel.
Volume of the small element $= R\delta\theta.A$.
Mass of the small element $= dm = R\delta\theta.A\ \rho$

The centrifugal force on the small element

$$dF_C = dm\omega^2 R$$
$$= R\delta\theta.A\omega^2 R\,\rho$$
$$= R^2 A.\omega^2 \delta\theta\,\rho$$

Resolving the centrifugal force vertically

$$dF_C = dF_C Sin\theta$$

$$=\rho\ R^2 A\omega^2 Sin\theta.\ \delta\theta \quad ---(1)$$

Total Vertical upward force across diameter X & Y

$$= \int_0^{\pi} \rho R^2 A\omega^2 Sin\theta.\ \delta\theta$$
$$= \rho R^2 A\omega^2 \int_0^{\pi} Sin\theta.\ \delta\theta$$
$$2\rho\ = 2\rho A R^2 \omega^2$$

This vertical upward force will produce tensile stress on loop stress developed & it is resisted by 2P.

We know that, $\sigma = P/A$

$$P = \sigma A$$

$$\therefore\ 2P = 2\sigma A$$

$$PAR^2\omega^2 = 2\sigma A$$

$\boxed{\sigma = \rho R^2 \omega^2}$ % up to this deviation

Also,

Linear velocity V=Rx ω

$$\sigma = \delta V^2$$

$$\boxed{V = \sqrt{\sigma / \delta}}$$

Mass of the rim = volume x density

$$\boxed{m = \pi\, dA \times \rho}$$

Problem 1:

A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of the machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during ½ revolution and remains constant fore the following revolution. It then falls uniformly to 750 N-m during the next ½ revolution and remains constant for one revolution, the cycle being repeated thereafter.

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Solution.

Given: $N = 250\ r.p.m\ \ or\ \omega = 2\pi \times 250/60 = 26.2\, rad/s;\ m = 500\, kg;\ k = 600\ \ m = 0.6$

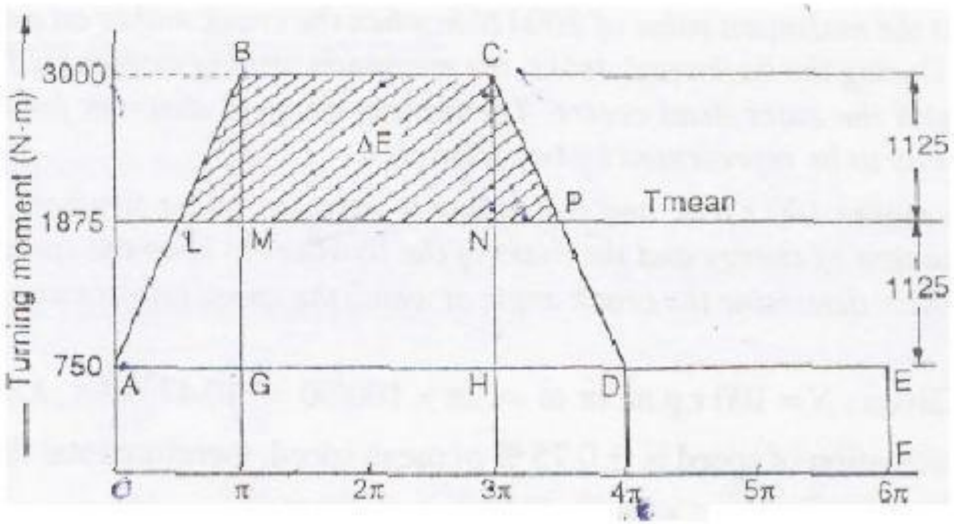
The turning moment diagram for the complete cycle is drawn.

The torque required for one complete cycle

$$
\begin{aligned}
&= Area\ of\ \ figure\ OABCDEF \\
&= Area\ \ OAEF + Area\ ABG + Area BCHG + Area\ CDH \\
&= OF \times OA + \frac{1}{2} \times AG \times\ BG + GH \times\ CH + \frac{1}{2} \times HD \times CH \\
&= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \frac{1}{2} \times \pi(3000 - 750) \\
&= 11250\ \pi\ N - m
\end{aligned}
$$

Torque required for one complete cycle $= T_{mean} \times \pi\, N - m$

$$\therefore T_{mean} = 11250\pi / 6\pi = 1875\ N - m$$



Power required to drive the machine, $P = T_{mean} \times \omega = 11875 \times 26.2 = 49125W = 49.125kW$.

To find Coefficient of fluctuation of speed, δ.

Find the values of *LM* and *NP*.

From similar triangles *ABG* and *BLM*,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad or \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad or \quad LM = 0.5\pi$$

From similar triangles *CHD* and *CNP*,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad or \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad or \quad NP = 0.5\pi$$

From the figure, we find that,

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

The area above the mean torque line represents the maximum fluctuation of energy. Therefore the maximum fluctuation of energy, ΔE

$$= \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\,\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\,\pi \times 1125 = 8837\ N-m$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.\delta = 500\,(0.6)^2\ (26.2)^2\ \delta = 123\,559\ \delta$$

$$\boxed{\delta = 0.071}$$

Problem 2

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

Solution

θ, deg.	$T, N-m$
0	500
90	1500
180	500
270	1500
360	500

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T \, d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta) \, d\theta$$

$$= 2000\pi \; N-m$$

$$T_{mean} = \frac{W.D / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \; N-m$$

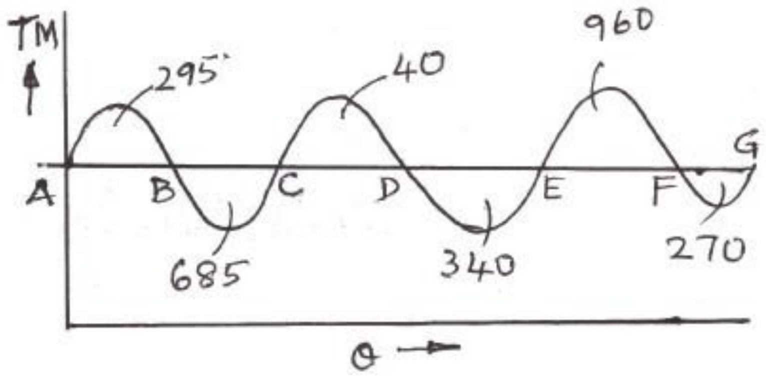
$$\text{Power developed } = T_{mean} \times \omega_{mean}$$

$$= 1000 \times \frac{2\pi \; N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

The TMD for a petrol engine is drawn to the following scale, turning moment, 1mm = 5Nm, crank 1mm = 1°. The TMD repeats itself at every half revolution of the engine & areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150mm. Calculate the maximum fluctuation of energy & co-efficient of fluctuation of speed when engine runs at 1800rpm



Energy at $A = E$
Energy at $B = E + a_1$
$= E + 295$
Energy at $C = E + 295 - 685 = E - 390$
Energy at $D = E + 295 - 685 + 40 = E - 350$
Energy at $E = E - 350 - 340 = E - 690$
Energy at $F = E - 690 + 960 = E + 270$
Energy at $G = E + 270 - 270 = E$
$\therefore A = G$

Max Energy $= E + 295$
Min Energy $= E - 690$

$m = 36kg, k = 150mm, \quad N = 1800rpm$

Maximum Fluctuation of Energy $\Delta E = E + 295 - (E - 690)$

$= 985mm^2$

Scale: $1mm = 5Nm$ & $1mm = 1°$

$$Torque \times \theta = \frac{5}{180}\pi \times 1 = \frac{\pi}{36} Nm$$

$$\Delta E = 985 \times \frac{\pi}{36} = 85.95Nm$$

$$\Delta E = mk^2\omega^2\delta$$

$$86 = 36 \times 0.15^2 \times \left(\frac{2\Pi(1800)}{60}\right)^2 \delta$$

$$\delta = 0.003 \quad or \quad 0.3\%$$

The TMD for a multi cylinder engine has been drawn to a scale 1mm to 500Nm torque & 1mm to 6° of crank displacement. The intercepted area in order from one end is mm^2 are -30, 410, -280, 320, -330, 250, -360, +280, -260 mm^2 when engine is running at 800rpm. The engine has a stroke of 300mm & fluctuation of speed is not to exceed ±2% of the mean speed, determine

1. a suitable diameter & cross section of the fly wheel rim for a limiting value of the safe centrifugal stress of 7MPa. The material density may be assumed as 7200 kg/m^3. The width of the rim is to be 5times the thickness.

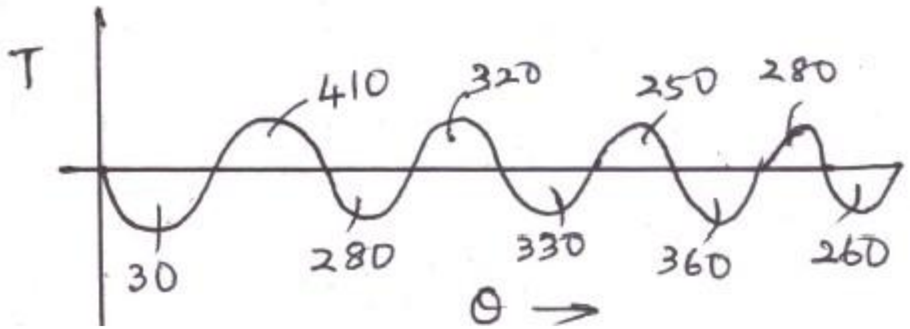
Solution:

$N = 800 \ \ rpm$

$\pm \ 2\ \% means, \ \delta = 4\% = 0.04$

$\sigma = 7\,Mpa = 7 \ N / m2$

$\rho = 7\,200 \ kg / m^3$



Energy at $A = E$
Energy at $B = E - 30$
Energy at $C = E - 30 + 410 = E + 380$
Energy at $D = E + 380 - 280 = E + 100$
Energy at $E = E + 100 + 320 = E + 420$
Energy at $F = E + 420 - 330 = E + 90$
Energy at $G = E + 90 + 250 = E + 340$
Energy at $H = E + 340 - 360 = E - 20$
Energy at $I = E - 20 + 280 = E + 260$
Energy at $J = E + 260 - 260 = E$

$$\Delta E = E + 420 - (E - 30)$$
$$= 450mm^2$$

$1mm = 500Nm$, $1mm = 6°$ (0.1047 *radians*), $1mm^2 = 52.35Nm$

$$\Delta E = 450 \times 52.35 = 23557.5 \text{ Nm}$$

$$\sigma = \rho V^2$$

$$7 \times 10^6 = 7200\ V^2$$

$$V = 31.18\ m/s$$

$$\Delta E = mr^2\ \omega^2 \delta$$
$$= mV^2 \delta$$
$$V = r\omega$$

$$V = \frac{\pi DN}{60},\ D = 0.745\,m$$

Cross sectional area $A = bt$

$A = (5t)t = 5t^2$

Fluctuation of energy $\Delta E = mV^2 \delta$

$$23.56 \times 10^3 = m(31.18)^2\ (0.04)$$

$$m = 605\ kg$$

$m = Volume \times Density$

$\pi DA \times \rho$

$605 = \pi(0.745)(5t^2)7200$

$t = 0.084\,m$

Area $= 5t^2 = 0.035m^2$