

DESIGN OF SHAFTS AND KEYS

INTRODUCTION

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

MATERIAL USED FOR SHAFTS

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

DESIGN OF SHAFTS

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

- (a) Shafts subjected to twisting moment or torque only,*
- (b) Shafts subjected to bending moment only,*
- (c) Shafts subjected to combined twisting and bending moments,*
and
- (d) Shafts subjected to axial loads in addition to combined torsional and bending loads.*

SHAFTS SUBJECTED TO TWISTING MOMENT ONLY

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$T/J = \tau/r \dots (i)$$

where T = *Twisting moment (or torque) acting upon the shaft,*

J = *Polar moment of inertia of the shaft about the axis of rotation,*

τ = *Torsional shear stress, and*

r = *Distance from neutral axis to the outer most fibre*

= $d / 2$; *where d is the diameter of the shaft.*

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (i) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{d} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \quad \dots(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32} [(d_o)^4 - (d_i)^4]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (i), we have

$$\frac{T}{\frac{\pi}{32} [(d_o)^4 - (d_i)^4]} = \frac{\tau}{d_o} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] \quad \dots(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft
 $= d_i / d_o$

Now the equation (iii) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o} \left[1 - \left(\frac{d_i}{d_o} \right)^4 \right] = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \quad \dots(iv)$$

From the equations (iii) or (iv), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where

T = Twisting moment in N-m. and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2) R$$

where

T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 1. A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution. Given : $N = 200$ r.p.m. ; $P = 20$ kW = 20×10^3 W; $\tau = 42$ MPa = 42 N/mm²

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25 d^3$$

$\therefore d^3 = 955 \times 10^3 / 8.25 = 115\,733$ or $d = 48.7$ say 50 mm **Ans.**

Example 2. A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

Solution. Given : $P = 1 \text{ MW} = 1 \times 10^6 \text{ W}$; $N = 240 \text{ r.p.m.}$; $T_{max} = 1.2 T_{mean}$; $\tau = 60 \text{ MPa} = 60 \text{ N/mm}^2$

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\,784 \text{ N-m} = 39\,784 \times 10^3 \text{ N-mm}$$

\therefore Maximum torque transmitted,

$$T_{max} = 1.2 T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78 d^3$$

$$\therefore d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$$

or

$$d = 159.4 \text{ say } 160 \text{ mm Ans.}$$

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$M / I = \sigma_b / y \quad \dots(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64} [(d_o)^4 - (d_i)^4] = \frac{\pi}{64} (d_o)^4 (1 - k^4) \quad \dots(\text{where } k = d_i / d_o)$$

and

$$y = d_o / 2$$

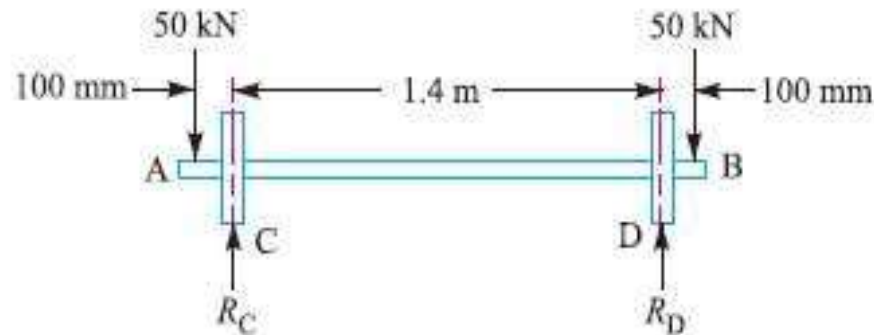
Again substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} (d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Example 1. A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution. Given : $W = 50 \text{ kN} = 50 \times 10^3 \text{ N}$; $L = 100 \text{ mm}$; $x = 1.4 \text{ m}$; $\sigma_b = 100 \text{ MPa} = 100 \text{ N/mm}^2$



The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = WL = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let

d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82 d^3$$

\therefore

$$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm Ans.}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and
 σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2} \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \left[\sqrt{M^2 + T^2}\right]$$

or
$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \quad \dots(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as *equivalent twisting moment* and is denoted by T_e . The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (i) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \quad \dots(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\begin{aligned} \sigma_{b(max)} &= \frac{1}{2} \sigma_b + \frac{1}{2} \sqrt{(\sigma_b)^2 + 4\tau^2} \quad \dots(iii) \\ &= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2} \sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} \\ &= \frac{32}{\pi d^3} \left[\frac{1}{2} (M + \sqrt{M^2 + T^2})\right] \end{aligned}$$

or
$$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2} [M + \sqrt{M^2 + T^2}] \quad \dots(iv)$$

The expression $\frac{1}{2} \left[(M + \sqrt{M^2 + T^2}) \right]$ is known as *equivalent bending moment* and is denoted by M_e . The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (iv) may be written as

$$M_e = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right] = \frac{\pi}{32} \times \sigma_b \times d^3 \quad \dots(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (ii) and (v) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

and

$$M_e = \frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right) = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 1. A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000 \text{ N-m} = 3 \times 10^6 \text{ N-mm}$; $T = 10\ 000 \text{ N-m} = 10 \times 10^6 \text{ N-mm}$;
 $\sigma_u = 700 \text{ MPa} = 700 \text{ N/mm}^2$; $\tau_u = 500 \text{ MPa} = 500 \text{ N/mm}^2$

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_u}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let $d =$ Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36 d^3$$

$$\therefore d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$\begin{aligned}M_e &= \frac{1}{2} \left(M + \sqrt{M^2 + T^2} \right) = \frac{1}{2} (M + T_e) \\ &= \frac{1}{2} (3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}\end{aligned}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46 d^3$$

$$\therefore d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm Ans.}$$

KEYS

Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys,
- and 5. Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

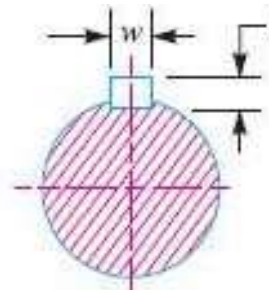
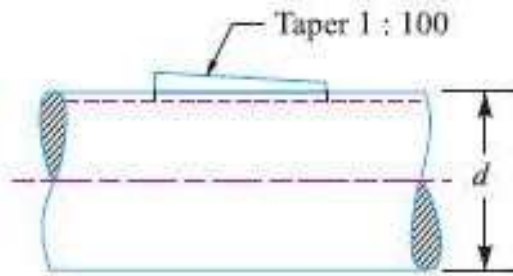
1. Rectangular sunk key. A rectangular sunk key is shown in Fig. The usual proportions of this key are :

$$\text{Width of key, } w = d / 4 ;$$
$$\text{and thickness of key, } t = 2w / 3 = d / 6$$

where

d = Diameter of the shaft or diameter of the hole in the hub.

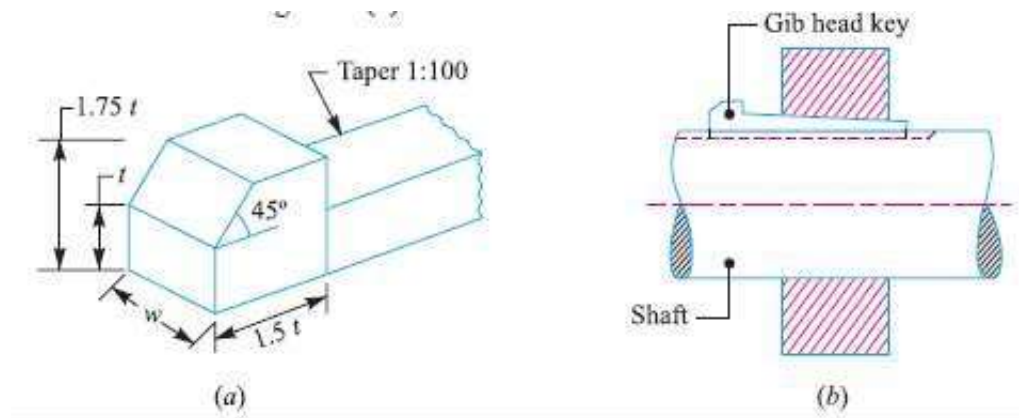
The key has taper 1 in 100 on the top side only.



2. Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.* $w = t = d / 4$

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. Gib-head key. It is a rectangular sunk key with a head at one end known as *gib head*. It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. (a) and its use in shown in Fig. (b).

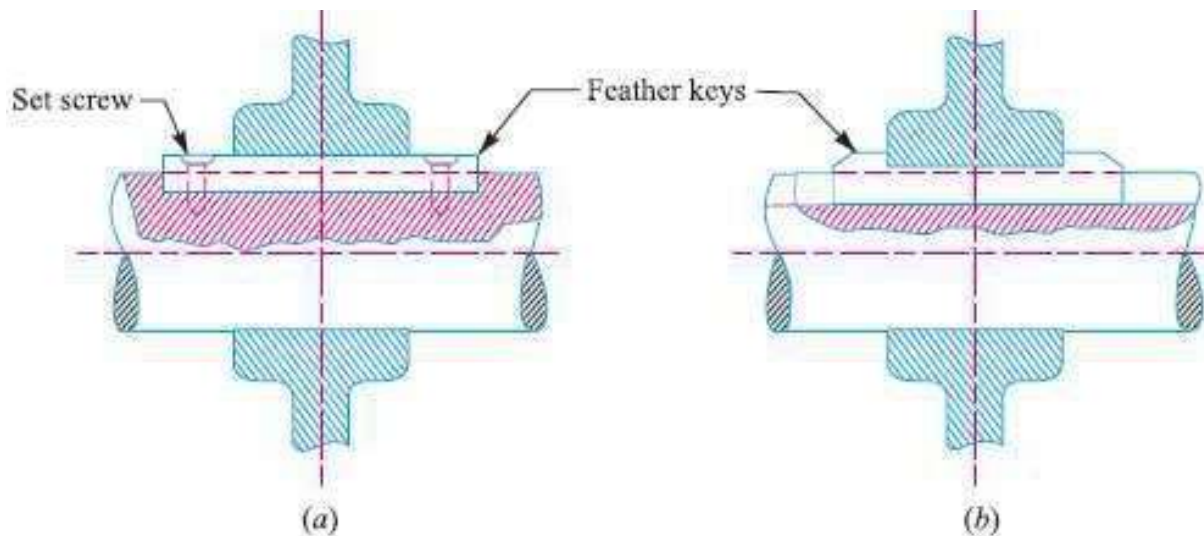


The usual proportions of the gib head key are :

Width, $w = d / 4$;and thickness at large end, $t = 2w / 3 = d / 6$

5. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

The feather key may be screwed to the shaft as shown in Fig. (a) or it may have double gib heads as shown in Fig. (b). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.



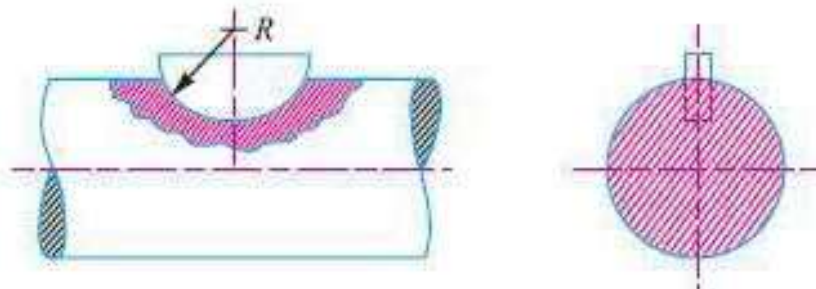
6. Woodruff key. *The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.*

The main advantages of a woodruff key are as follows :

1. It accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.



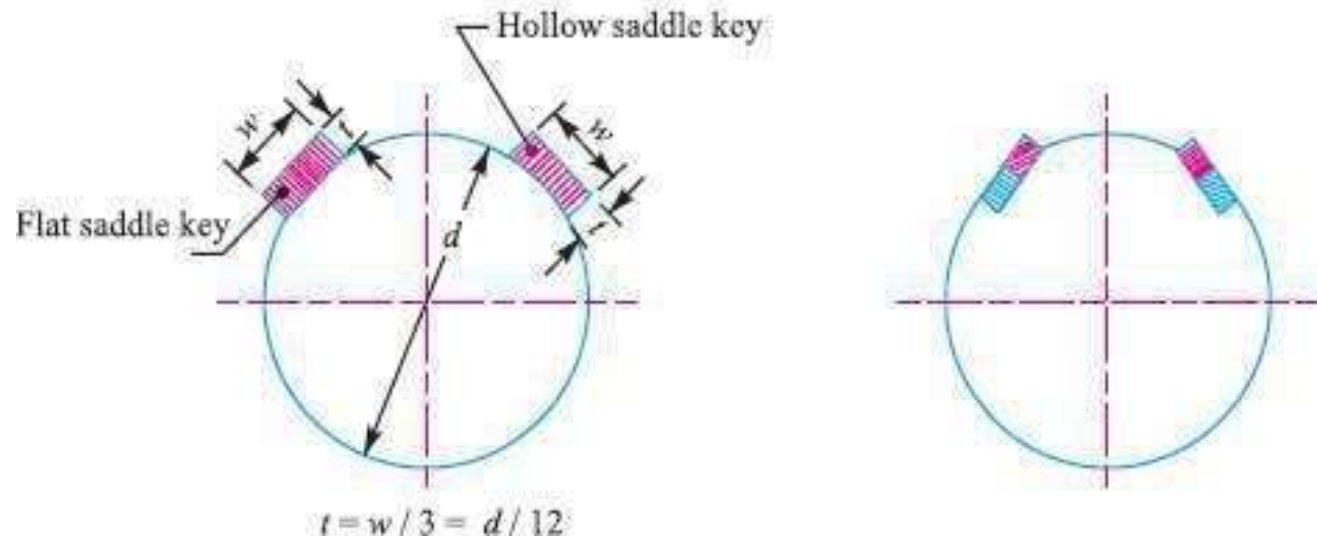
Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and
2. Hollow saddle key.

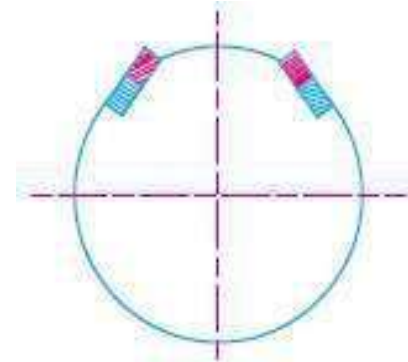
A *flat saddle key* is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

A *hollow saddle key* is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.



Tangent Keys

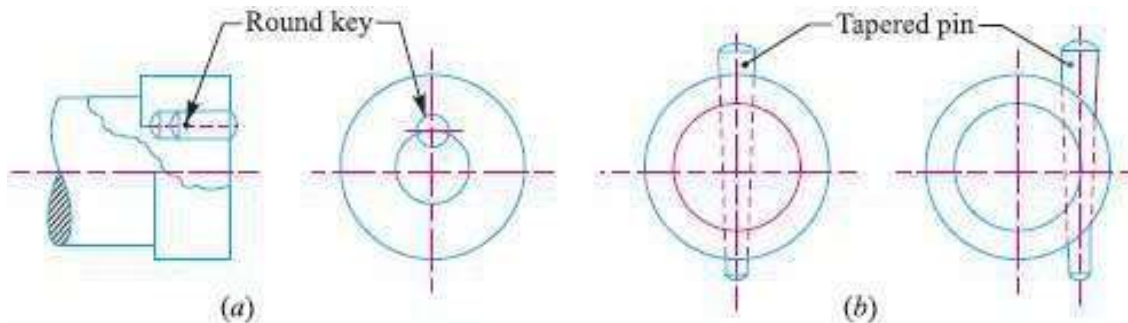
The tangent keys are fitted in pair at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



Round Keys

The round keys, as shown in Fig.(a), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

Sometimes the tapered pin, as shown in Fig.(b), is held in place by the friction between the pin and the reamed tapered holes.

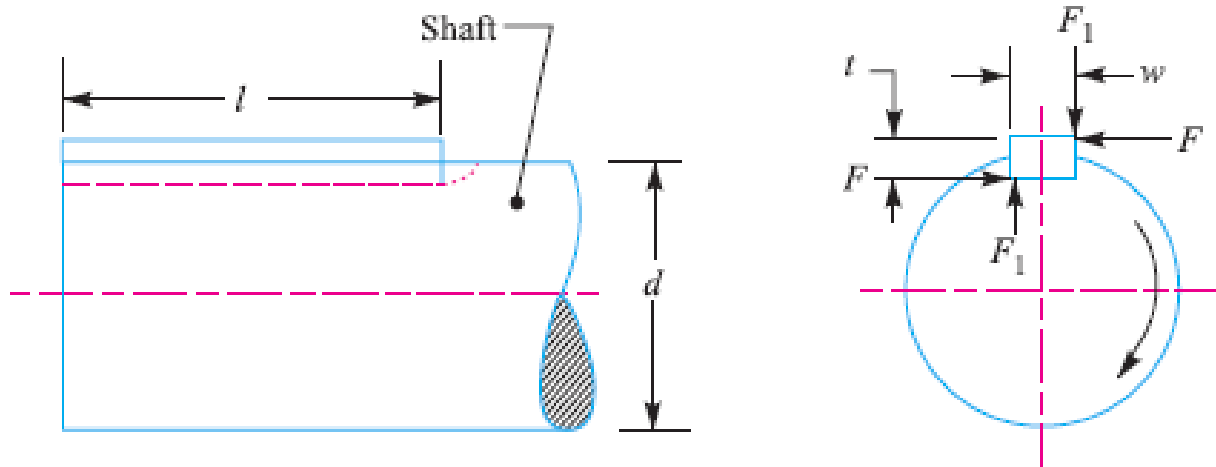


Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. **Forces (F_1) due to fit of the key in its keyway**, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. **Forces (F) due to the torque transmitted by the shaft**. These forces produce shearing and compressive (or crushing) stresses in the key.



Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \dots(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots[\text{Equating equations (i) and (ii)}]$$

or
$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad \dots(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation (iii), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \dots(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (iv) and (v), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571 d \times \frac{\tau_1}{\tau} \quad \dots \text{(Taking } w = d/4 \text{)} \quad \dots(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore l = 1.571 d \quad \dots [\text{From equation (vi)}]$$

Example 1. Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

Solution. Given : $d = 50 \text{ mm}$; $\tau = 42 \text{ MPa} = 42 \text{ N/mm}^2$; $\sigma_c = 70 \text{ MPa} = 70 \text{ N/mm}^2$

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16 \text{ mm}$ **Ans.**

and thickness of key, $t = 10 \text{ mm}$ **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let $l =$ Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800 \text{ l N-mm} \quad \dots(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times (50)^3 = 1.03 \times 10^6 \text{ N-mm} \quad \dots(ii)$$

From equations (i) and (ii), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750 \text{ l N-mm} \quad \dots(iii)$$

From equations (ii) and (iii), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm} \text{ **Ans.**}$$

Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following :

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. It should have no projecting parts.

Requirements of a Good Shaft Coupling

A good shaft coupling should have the following requirements :

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

1. Rigid coupling. *It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view :*

- (a) Sleeve or muff coupling.*
- (b) Clamp or split-muff or compression coupling, and*
- (c) Flange coupling.*

2. Flexible coupling. *It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view :*

- (a) Bushed pin type coupling,*
- (b) Universal coupling, and*
- (c) Oldham coupling.*

Sleeve or Muff-coupling

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows :

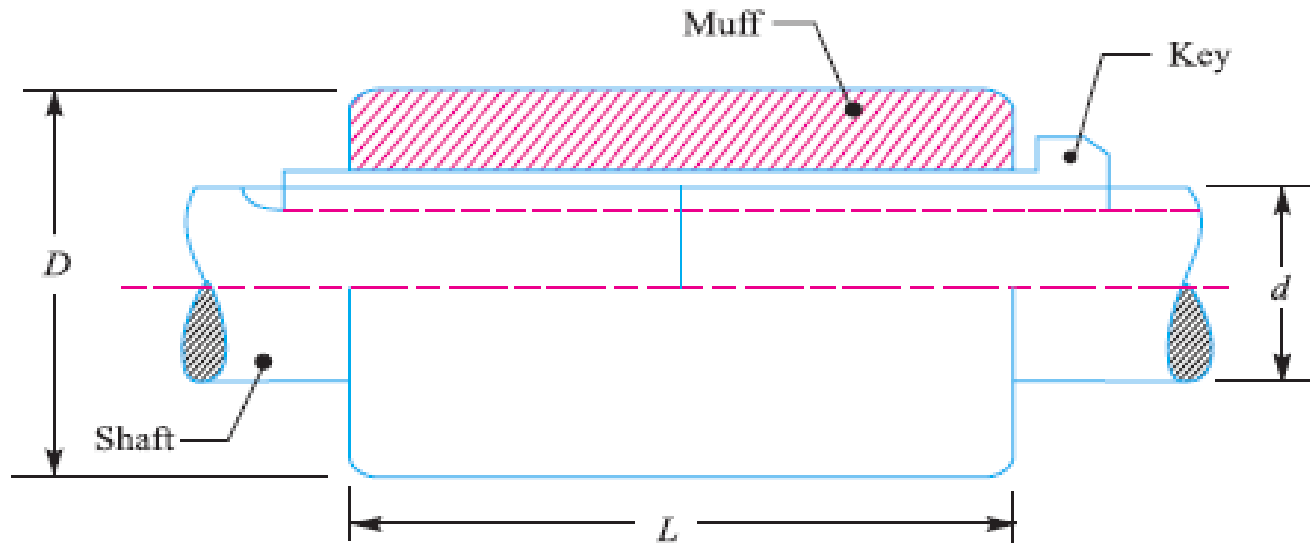
Outer diameter of the sleeve, $D = 2d + 13 \text{ mm}$
and length of the sleeve, $L = 3.5 d$

where d is the *diameter of the shaft*.

In designing a sleeve or muff-coupling, the following procedure may be adopted.

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft.



Let $T =$ Torque to be transmitted by the coupling, and
 $\tau_c =$ Permissible shear stress for the material of the sleeve which is cast iron.

The safe value of shear stress for cast iron may be taken as 14 MPa.
We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \quad \dots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. Design for key

The key for the coupling may be designed in the similar way as discussed in Art. 13.9. The width and thickness of the coupling key is obtained from the proportions.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.* $3.5 d$). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5 d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \dots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \dots \text{(Considering crushing of the key)}$$

Example 1. Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 Mpa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution. Given : $P = 40 \text{ kW} = 40 \times 10^3 \text{ W}$; $N = 350 \text{ r.p.m.}$; $\tau_s = 40 \text{ MPa} = 40 \text{ N/mm}^2$; $\sigma_{cs} = 80 \text{ MPa} = 80 \text{ N/mm}^2$; $\tau_c = 15 \text{ MPa} = 15 \text{ N/mm}^2$

1. Design for shaft

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2 \pi N} = \frac{40 \times 10^3 \times 60}{2 \pi \times 350} = 1100 \text{ N-m}$$
$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86 d^3$$

$$\therefore d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm Ans.}$$

2. Design for sleeve

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm Ans.}$$

and length of the muff,

$$L = 3.5 d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$\begin{aligned} 1100 \times 10^3 &= \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right] \\ &= 370 \times 10^3 \tau_c \end{aligned}$$

$$\therefore \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2 , therefore the design of muff is safe.

3. Design for key

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

∴ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm } \mathbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \tau_s$$

$$\therefore \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Clamp or Compression Coupling

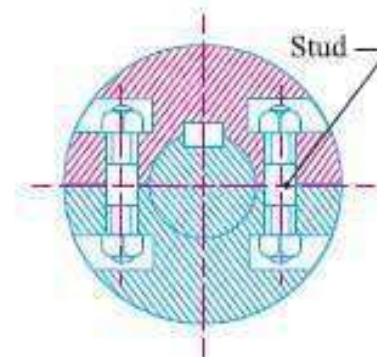
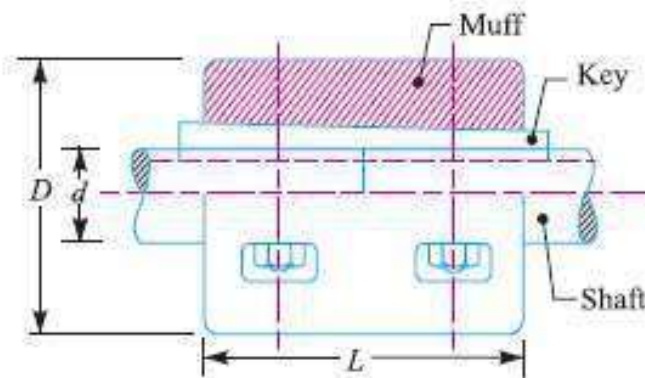
It is also known as *split muff coupling*. In this case, the muff or sleeve is made into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of the muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are :

Diameter of the muff or sleeve, $D = 2d + 13 \text{ mm}$

Length of the muff or sleeve, $L = 3.5 d$

where

d = Diameter of the shaft.



In the clamp or compression coupling, the power is transmitted from one shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. Design of muff and key

The muff and key are designed in the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmitted by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft,

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4} (d_b)^2 \sigma_t$$

∴ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

∴ Frictional force between each shaft and muff

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n$$

and the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Note: The value of μ may be taken as 0.3.

Example 1. Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Solution. Given : $P = 30 \text{ kW} = 30 \times 10^3 \text{ W}$; $N = 100 \text{ r.p.m.}$; $\tau = 40 \text{ MPa} = 40 \text{ N/mm}^2$;
 $n = 6$; $\sigma_t = 70 \text{ MPa} = 70 \text{ N/mm}^2$; $\mu = 0.3$

1. Design for shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{30 \times 10^3 \times 60}{2 \pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86 d^3$$

$\therefore d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. Design for muff

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm } \mathbf{Ans.}$$

and total length of the muff,

$$L = 3.5 d = 3.5 \times 75 = 262.5 \text{ mm } \mathbf{Ans.}$$

3. Design for key

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, $w = 22 \text{ mm}$ **Ans.**

Thickness of key, $t = 14 \text{ mm}$ **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. Design for bolts

Let $d_b =$ Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3 (d_b)^2 \cdot 70 \times 6 \times 75 = 5830 (d_b)^2$$

$$\therefore (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

<i>Shaft diameter (mm) upto and including</i>	<i>Key cross-section</i>		<i>Shaft diameter (mm) upto and including</i>	<i>Key cross-section</i>	
	<i>Width (mm)</i>	<i>Thickness (mm)</i>		<i>Width (mm)</i>	<i>Thickness (mm)</i>
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50



