Ingoude University

THESIS DEFENSE

Design for Fatigue Strength

Presented by:

Sweshareefa Mahakul
Assistant Professor, ME Dept, NRCM

Introduction

What is Fatigue Strength?

Fatigue strength refers to the ability of a material to resist failure under repeated or cyclic loading conditions. Unlike static loading, fatigue failure occurs at stress levels well below the ultimate tensile strength of the material, making it a critical consideration in mechanical design.

Most engineering failures — over 80% — are attributed to fatigue. Components such as shafts, gears, springs, and connecting rods are subjected to fluctuating stresses during operation. Understanding fatigue behavior is essential to ensure safe, reliable, and durable mechanical systems.

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Assistant Professor, ME Dept, NRCM

Why Fatigue Analysis Matters

Fatigue failure is insidious — it initiates as a microscopic crack at a point of stress concentration and propagates gradually under cyclic loading until sudden fracture occurs. The failure gives little or no warning, making fatigue analysis a top priority in design.
Key topics covered in this course include stress concentration factors, notch sensitivity, endurance limits, and design criteria such as Gerber's curve, Goodman's line, and Soderberg's line.

Course Scope: Theoretical & Fatigue Stress Concentration Factors, Notch Sensitivity, Fluctuating Stresses, Endurance Strength Estimation, and Failure Theories for safe design under dynamic loading conditions.

Sweshareefa Mahakul | Asst. Prof., ME Dept, NRCM

Stress Concentration

Definition & Concept

Stress concentration refers to the localized increase in stress at geometric discontinuities such as holes, notches, fillets, and keyways. These irregularities disrupt the uniform stress flow, causing stress to "concentrate" at specific points far exceeding the nominal stress values in the component.

Causes of Stress Concentration

Common causes include abrupt changes in cross-section, surface irregularities, internal cracks or voids, sharp corners, keyways, oil holes, and press-fit assemblies. The severity depends on geometry sharpness — smaller radii produce higher stress concentrations and increase fatigue failure risk.

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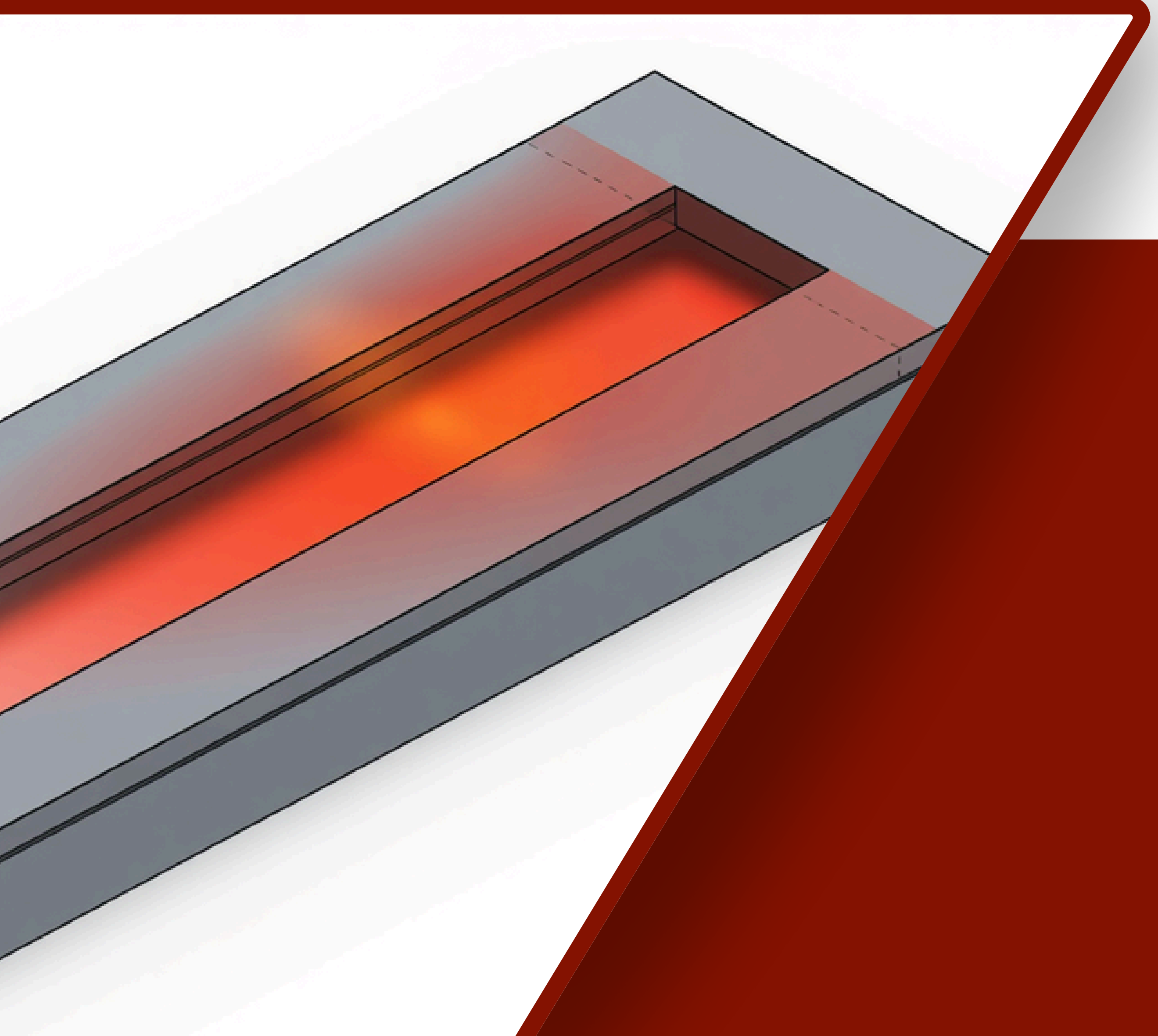
Theoretical Stress Concentration Factor

The Theoretical Stress Concentration Factor (Kt) quantifies the amplification of stress at geometric discontinuities in a component. It is defined as the ratio of the maximum stress at the notch to the nominal stress: $K_t = \sigma_{max} / \sigma_{nom}$. This factor depends solely on the geometry of the discontinuity and is independent of the material properties.



Definition & Formula

Kt is determined from stress concentration charts (Peterson's charts) based on geometry parameters such as notch radius (r), width (d), and overall dimension (D). For a circular hole in an infinite plate under uniaxial tension, Kt = 3. Higher Kt values indicate sharper notches and greater stress amplification at the discontinuity location.

Significance in Design

A high Kt value significantly reduces fatigue life by initiating cracks at stress raisers. Designers must minimize Kt by using larger fillet radii, gradual transitions, and smooth surface finishes. Kt serves as the foundation for calculating the Fatigue Stress Concentration Factor (Kf) used in actual fatigue analysis.

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Fatigue Stress Concentration Factor

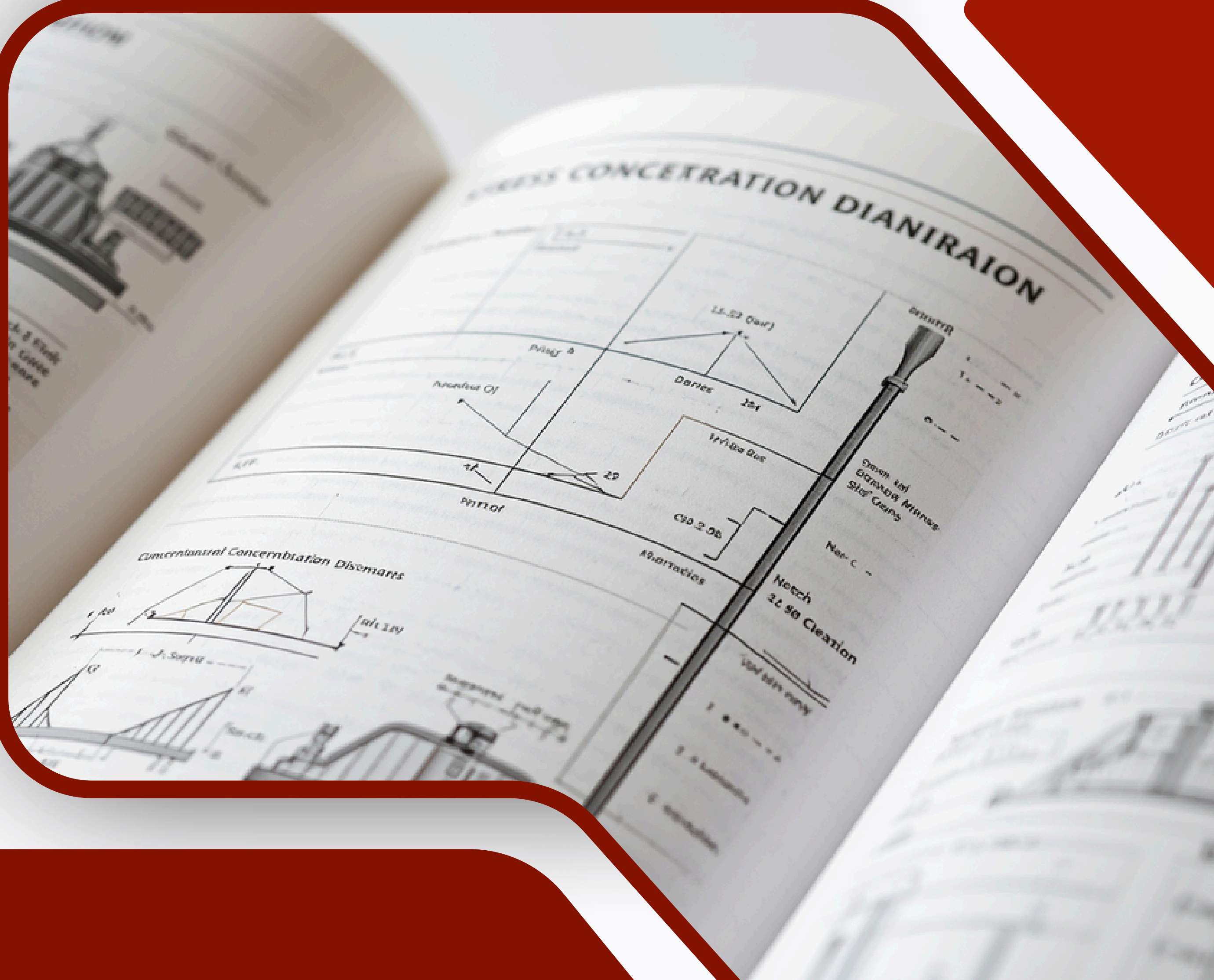
$K_f = 1 + q(K_t - 1)$, where q is the Notch Sensitivity Index ($0 \leq q \leq 1$). When $q = 0$, the material is fully insensitive to notches ($K_f = 1$). When $q = 1$, the material is fully notch-sensitive ($K_f = K_t$). Kf is used to modify the endurance limit: $Se' = Se / K_f$.

Key Relationship

Kf is critical in fatigue design as it directly reduces the effective endurance limit. Using Kt instead of Kf in design would be overly conservative. Accurate Kf estimation ensures safe yet efficient component design under fluctuating stresses.

Comparison: Kt vs Kf

The Theoretical Stress Concentration Factor (Kt) is based purely on geometry and is determined from elastic theory. The Fatigue Stress Concentration Factor (Kf) accounts for actual material behavior under cyclic loading and is always less than or equal to Kt.



Notch Sensitivity

Notch Sensitivity (q) is a material property that measures how sensitive a material is to the presence of stress raisers (notches, holes, keyways, etc.). It defines the degree to which the theoretical stress concentration factor (Kt) is actually realized in terms of fatigue damage. The value of q ranges from 0 (fully insensitive) to 1 (fully sensitive), and depends on both the material and the notch radius.

Notch Sensitivity Factor

The notch sensitivity factor q is defined as:

$q = (K_f - 1) / (K_t - 1)$

where Kf is the Fatigue Stress Concentration Factor and Kt is the Theoretical Stress Concentration Factor.

- When q = 0: Kf = 1 (material is insensitive to notch)
- When q = 1: Kf = Kt (material is fully sensitive)
- $K_f = 1 + q(K_t - 1)$

Higher notch sensitivity means greater reduction in endurance limit.

Material Dependence & Design

Notch sensitivity depends on:

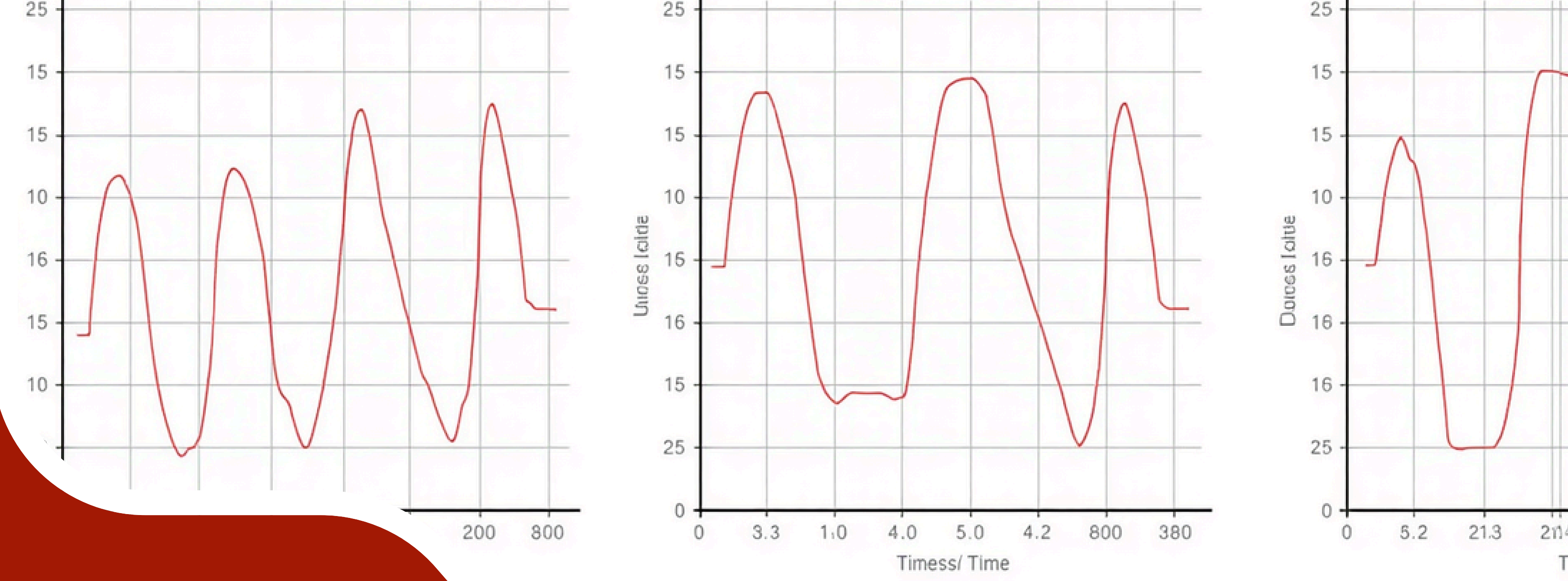
- Material type: Ductile materials (mild steel) have lower q; brittle materials (hardened steel) have higher q.
- Notch radius: Larger radius → lower q; sharper notch → higher q.
- Ultimate tensile strength (Sut): Higher Sut → higher q value.

Design Implication: Use Neuber's constant 'a' to determine q from charts. Modified endurance limit:

$S_e' = S_e / K_f$

Always use Kf instead of Kt in fatigue calculations for accuracy.

Fluctuating Stresses



Types of Cyclic Loading

Fluctuating stresses vary in magnitude and direction over time. Three primary types are identified in fatigue analysis:

- Repeated Stress: Varies between zero and a maximum value ($\sigma_{min} = 0$). The stress cycles from no load to peak load repeatedly.
- Reversed Stress: Alternates between equal tensile and compressive values ($\sigma_{mean} = 0$). A rotating beam under bending is a classic example.
- Fluctuating Stress: Varies between two non-zero values with both a mean stress component and an alternating stress component present simultaneously.

Key Parameters in Fluctuating Stress Analysis:

Mean Stress (σ_m): $\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$

Alternating Stress (σ_a): $\sigma_a = (\sigma_{max} - \sigma_{min}) / 2$

Stress Ratio (R): $R = \sigma_{min} / \sigma_{max}$

Amplitude Ratio (A): $A = \sigma_a / \sigma_m$

For reversed loading: $R = -1$, $\sigma_m = 0$

For repeated loading: $R = 0$, $\sigma_m = \sigma_{max}/2$

Proper identification of loading type is essential for accurate fatigue life prediction and safe component design.

Stress Components & Significance

In fluctuating stress conditions, both the mean stress and the alternating stress contribute to fatigue failure. The mean stress affects the endurance limit of the material, while the alternating stress drives crack initiation and propagation.

Wöhler's S-N curve forms the basis for understanding fatigue behavior under cyclic loading. The endurance limit (Se) represents the stress amplitude below which a material can theoretically withstand infinite load cycles without fatigue failure.

Design Significance: Components subjected to fluctuating loads must be designed considering both static and dynamic stress components. Failure theories such as Goodman, Gerber, and Soderberg lines are used to evaluate safety under combined mean and alternating stresses — covered in subsequent sections.

Presented by: Sweshareefa Mahakul, Asst. Professor, ME Dept, NRCM

SWESHAREEFA MAHAKUL | Asst. Prof., ME Dept, NRCM

Stress Components in Fluctuating Loads

Fluctuating loads produce stresses that vary cyclically between maximum (σ_{max}) and minimum (σ_{min}) values. Unlike static loading, fatigue failure depends on both the mean stress level and the amplitude of stress variation. Decomposing total stress into mean and alternating components is the foundational step in all fatigue strength calculations and endurance limit assessments.

Mean & Alternating Stress

The mean stress (σ_m) is the average of maximum and minimum stresses: $\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$. The alternating stress (σ_a) is half the stress range: $\sigma_a = (\sigma_{max} - \sigma_{min}) / 2$. These two components together define the nature of any fluctuating stress cycle in fatigue analysis.

Fluctuating Stress Analysis

In fluctuating stresses, the total stress at any instant = $\sigma_m + \sigma_a \cdot \sin(\omega t)$. For design purposes, both mean and alternating components must be considered simultaneously. The stress range $\Delta\sigma = \sigma_{max} - \sigma_{min}$, and the peak-to-mean ratio determines severity. Proper identification of these components is essential before applying Gerber, Goodman, or Soderberg criteria for safe fatigue design.

Stress Ratio & Amplitude Ratio

Stress Ratio $R = \sigma_{min} / \sigma_{max}$ defines the load cycle type. $R = -1$ indicates fully reversed loading; $R = 0$ indicates repeated loading; $R = 1$ indicates static loading. Amplitude Ratio $A = \sigma_a / \sigma_m = (1 - R) / (1 + R)$ relates alternating to mean stress and is critical for Goodman and Soderberg fatigue diagrams.

SWESHAREEFA MAHAKUL | Asst. Professor, ME Dept, NRCM

Endurance Limit

Definition & Significance

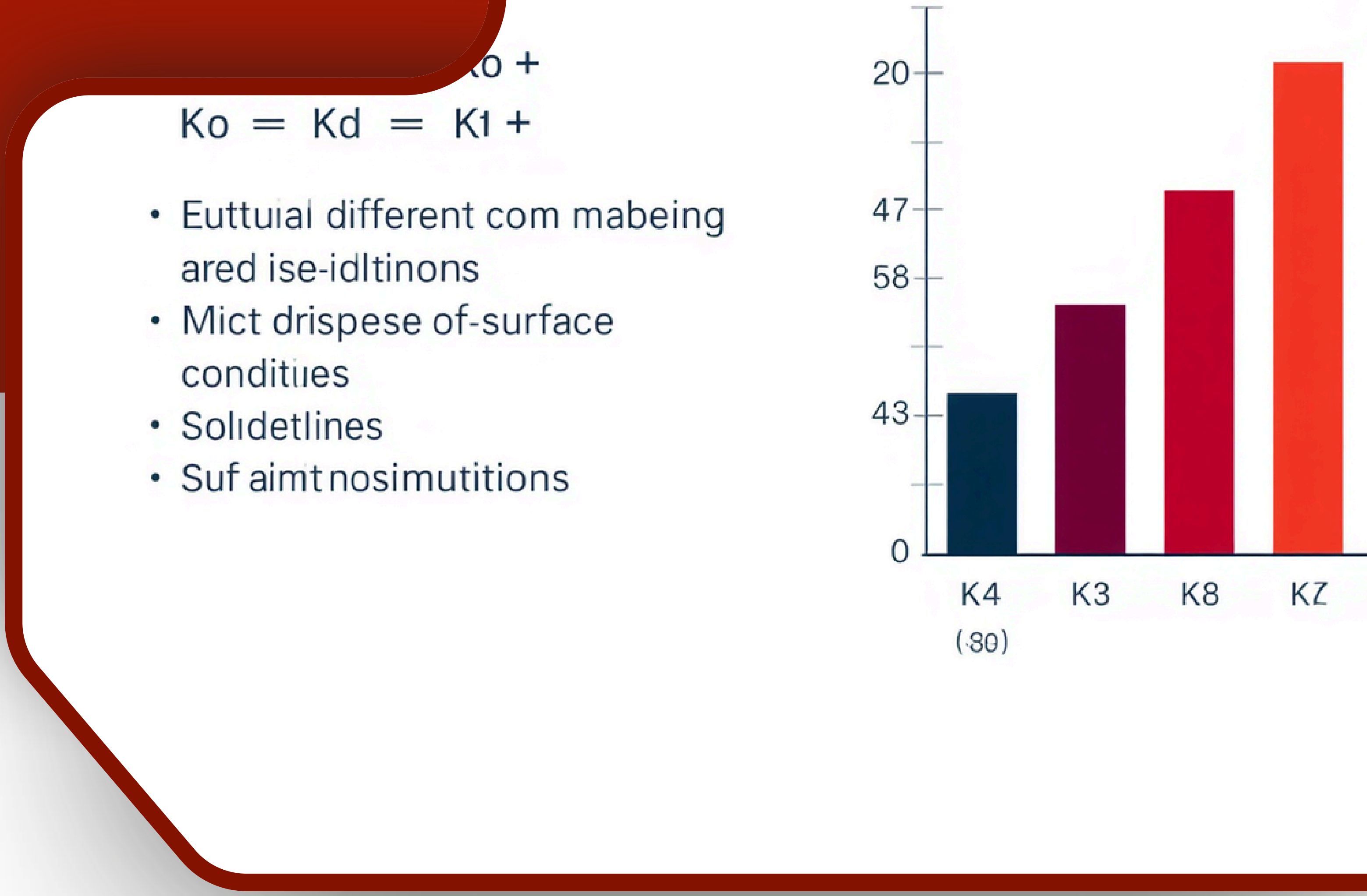
Endurance limit (Se) is the maximum stress amplitude a material can withstand for an infinite number of loading cycles without fatigue failure. It is the stress below which fatigue life is considered infinite. Critical in designing components subjected to fully reversed cyclic loading for long service life.

Typical Values for Materials

Ferrous metals (steel): Se ≈ 0.5 × Sut (for Sut ≤ 1400 MPa). Non-ferrous metals (aluminum, copper): No true endurance limit; fatigue strength specified at 10^8 cycles. Cast iron: Se ≈ 0.4 × Sut. These values are used as baseline before applying correction factors.

Sweshareefa Mahakul | Asst. Professor, ME Dept, NRCM

Endurance Strength



Estimation of Endurance Strength

The endurance limit of a machine component differs from the standard specimen value due to several modifying factors. The actual endurance limit is: $S_e = K_a \times K_b \times K_c \times K_d \times K_e \times S_e'$
Where Se' is the endurance limit of the test specimen and Ka through Ke are correction factors for real-world conditions.

Surface & Size Factors

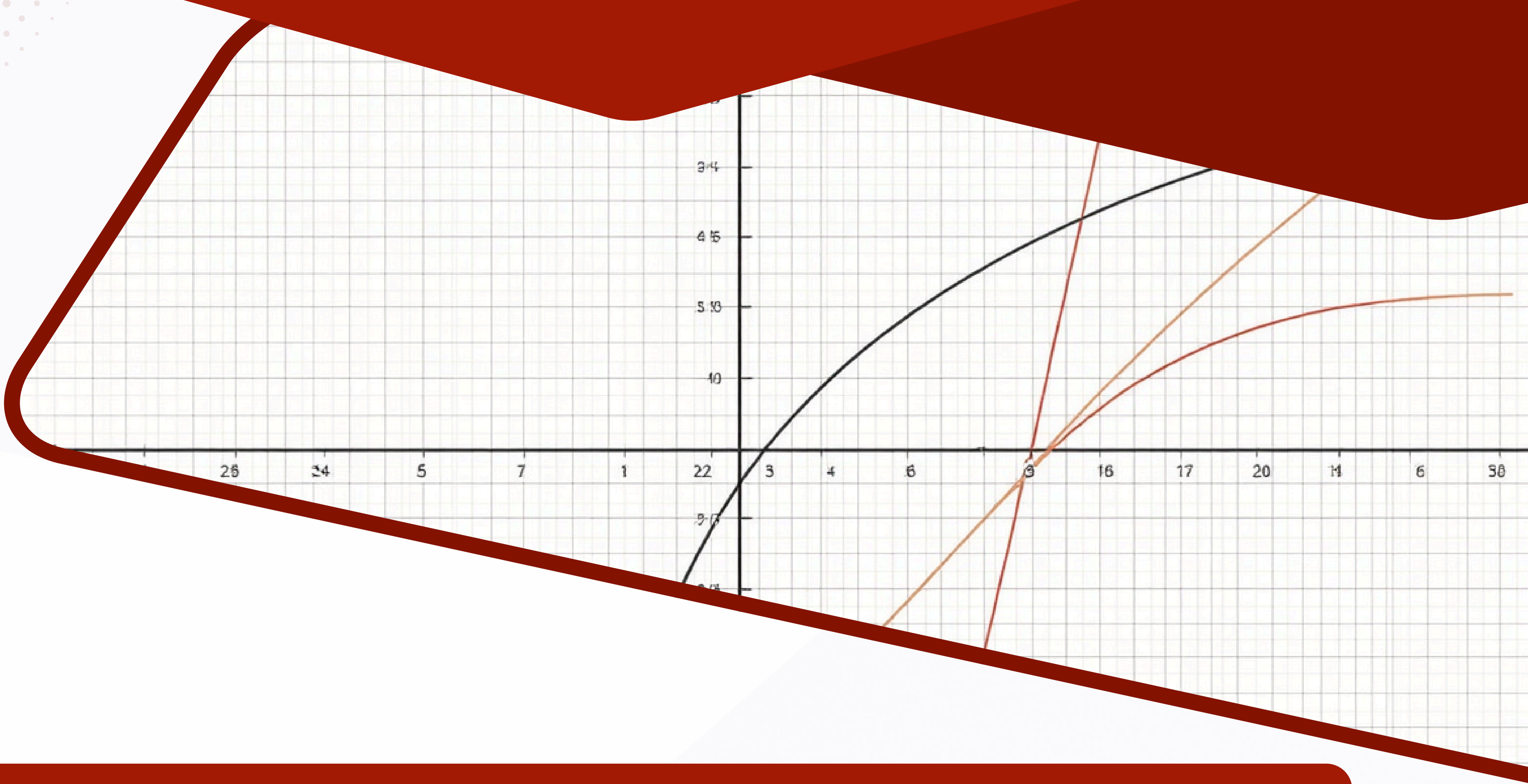
Surface Finish Factor (Ka): Accounts for the effect of surface roughness. Polished surfaces have $K_a \approx 1.0$; machined surfaces have lower values. Rough surfaces introduce stress concentrations reducing fatigue life.
Size Factor (Kb): Larger components have lower endurance limits due to higher probability of defects. For $d \leq 7.6$mm, $K_b = 1.0$; for larger diameters, Kb decreases accordingly.

Reliability, Temperature & Load Factors

Reliability Factor (Kc): Accounts for statistical scatter. At 50% reliability $K_c = 1.0$; at 99% reliability $K_c = 0.814$.
Temperature Factor (Kd): Endurance limit decreases at elevated temperatures. For $T \leq 350°C$, $K_d \approx 1.0$.
Load Factor (Ke): Bending = 1.0, Axial = 0.85, Torsion = 0.59. These factors adjust Se for the type of loading applied to the component.

SWESHAREEFA MAHAKUL | Asst. Professor, ME Dept, NRCM

Fatigue Failure Theories



Gerber's Curve

A parabolic curve relating mean stress and alternating stress. It is the least conservative of the three criteria. Defined by: $(\sigma a/Se) + (\sigma m/Sut)^2 = 1$. Accounts for the non-linear effect of mean stress. Suitable for ductile materials where experimental data fits a parabolic trend. Provides the widest safe design region.

Goodman's Line

A linear relationship between alternating stress and mean stress. Defined by: $(\sigma a/Se) + (\sigma m/Sut) = 1$. More conservative than Gerber's curve. Widely used in engineering practice for its simplicity and safety. Recommended for brittle materials and components under dynamic loading with stress concentrations.

Soderberg's Line

The most conservative linear failure criterion. Defined by: $(\sigma a/Se) + (\sigma m/Syt) = 1$. Uses yield strength (Syt) instead of ultimate tensile strength. Ensures no yielding occurs under fluctuating loads. Preferred for ductile materials in safety-critical applications. Provides the smallest safe design envelope.

Gerber's Curve

Gerber's curve is a parabolic relationship used in fatigue design under fluctuating stresses. The equation is:

$(\sigma m / Sut)^2 + (\sigma a / Se) = 1$

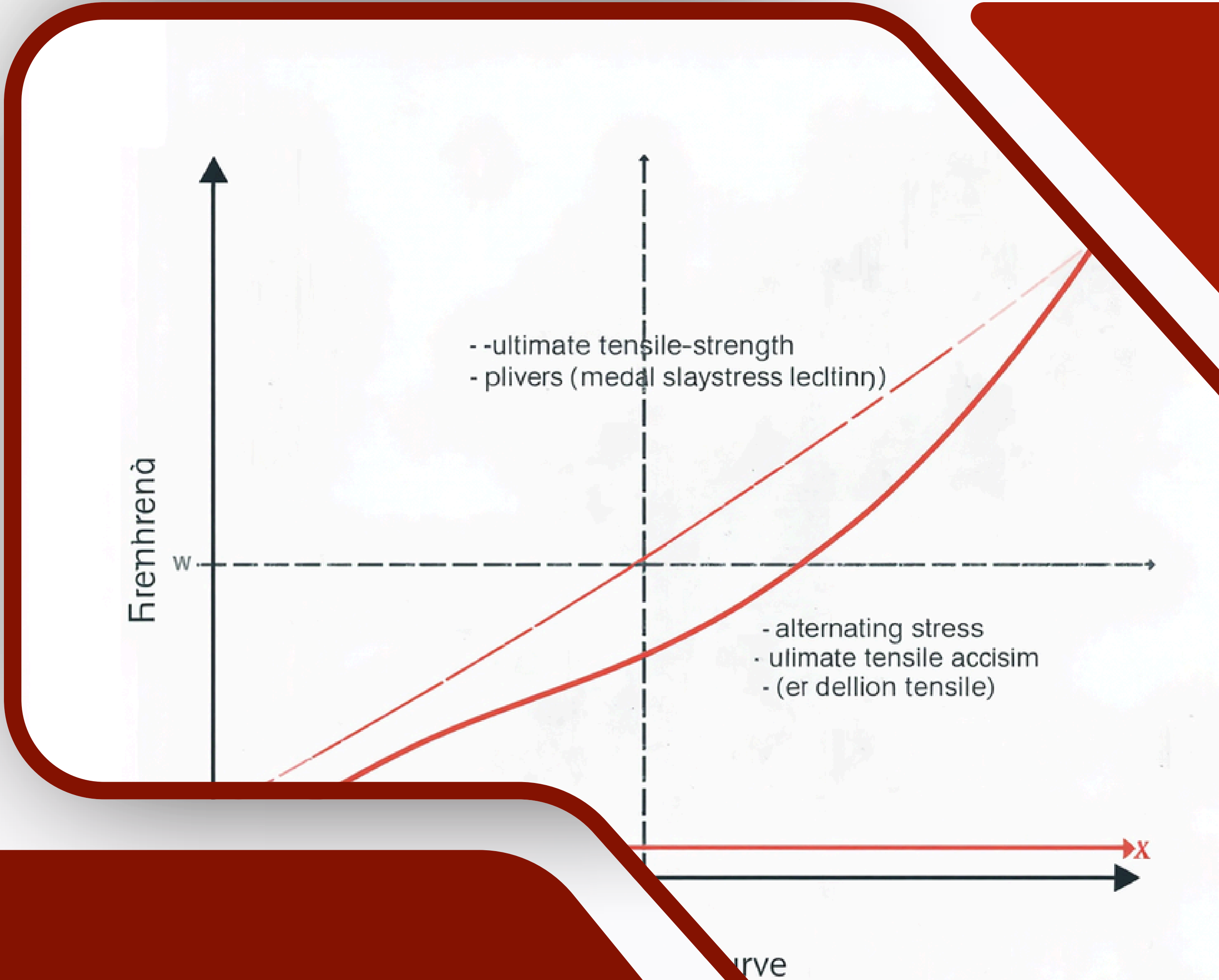
where σm is mean stress, σa is alternating stress, Sut is ultimate tensile strength, and Se is endurance limit. The parabola passes through Sut on the mean stress axis and Se on the alternating stress axis, enclosing a larger safe zone than linear criteria.

Gerber's Parabolic Equation

Key Assumptions: Material is ductile; mean stress ranges from 0 to Sut; alternating stress ranges from 0 to Se. The parabolic curve provides the closest approximation to actual experimental results for most engineering metals under combined mean and alternating stresses.

Theoretical Framework

Gerber's curve is considered the most accurate fit to experimental fatigue data. It is more optimistic than Goodman's and Soderberg's lines, predicting higher allowable stress combinations. It is preferred when ductile materials are used and when a best-fit to test data is required rather than a conservative design estimate.



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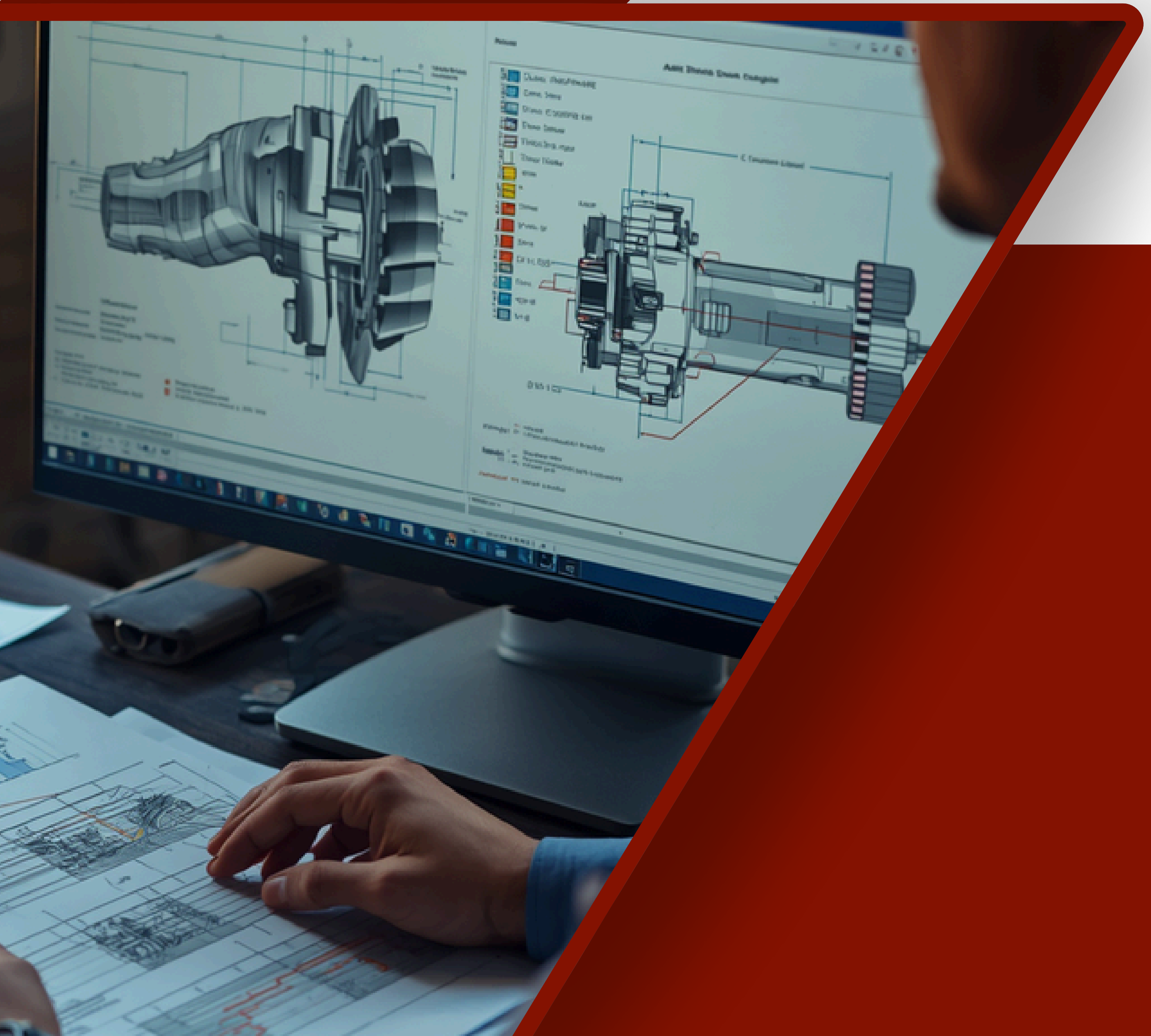
Goodman's Line

Goodman's Line establishes a linear relationship between mean stress and alternating stress to define the boundary of safe fatigue design. The Modified Goodman Diagram plots alternating stress (σ_a) on the Y-axis and mean stress (σ_m) on the X-axis, connecting the endurance limit (Se) to the ultimate tensile strength (Sut). Any stress state falling below this line is considered safe from fatigue failure.



Modified Goodman Criterion

The Goodman equation is expressed as: $\sigma_a/S_e + \sigma_m/S_{ut} = 1$. The safety factor (n) is calculated as: $1/n = \sigma_a/S_e + \sigma_m/S_{ut}$. This linear criterion is widely preferred in industrial applications due to its conservative and straightforward nature. It is especially suitable for brittle materials and components under high mean stress conditions.

Industrial Applications

Goodman's Line is extensively used in the design of shafts, axles, springs, and pressure vessels subjected to fluctuating loads. Engineers use the Modified Goodman Diagram to evaluate design safety margins, compare multiple load cases, and optimize component geometry. It provides a reliable and practical approach for fatigue life prediction in mechanical design.

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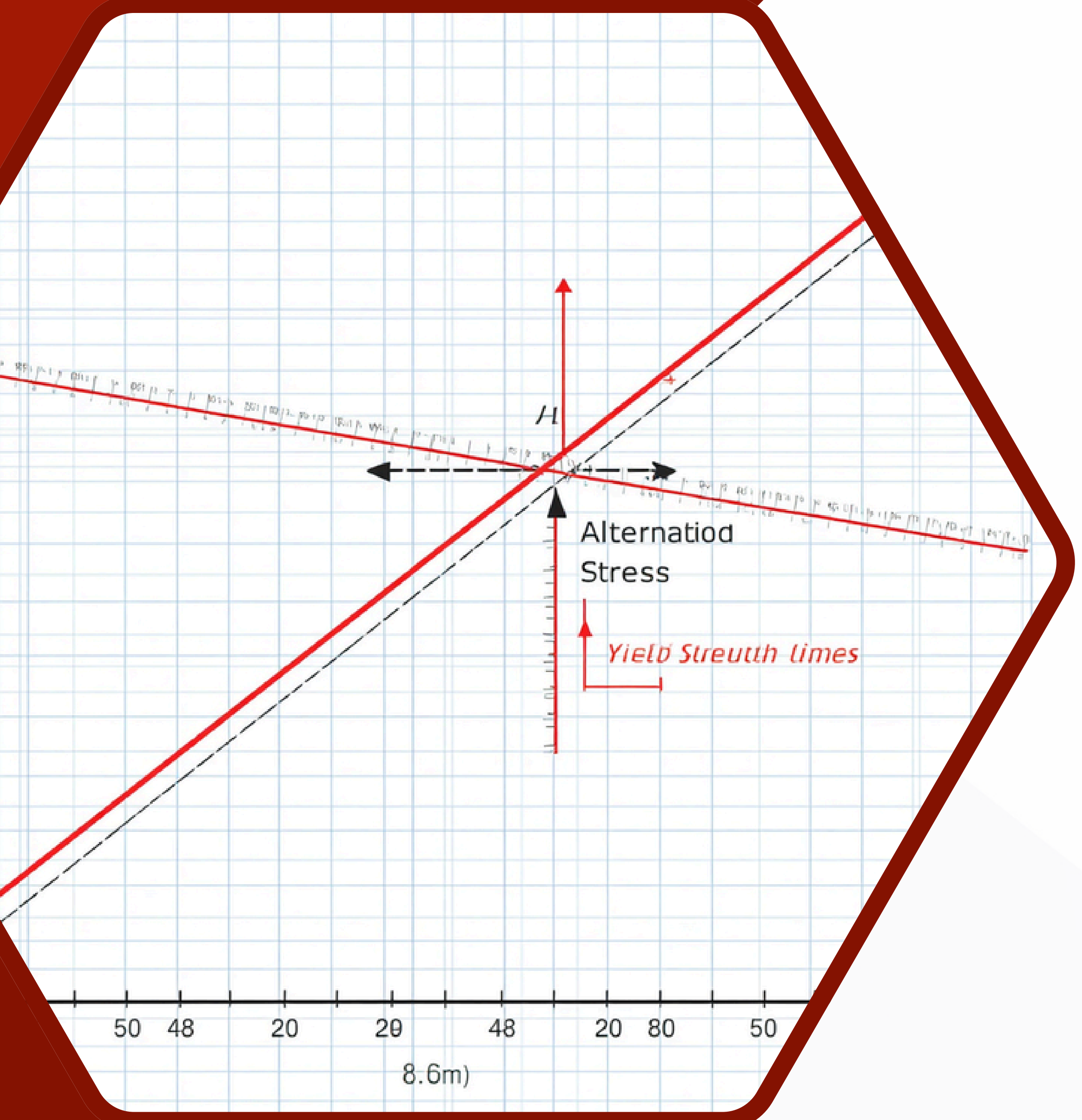
Soderberg's Line

Soderberg's line is the most conservative fatigue failure criterion. It connects the endurance limit (Se) on the stress amplitude axis to the yield strength (Syt) on the mean stress axis. The design equation is: σa/Se + σm/Syt = 1/FOS. It ensures the component remains safe against both fatigue and yielding under fluctuating stresses.



Soderberg Equation & Design

The Soderberg line equation: σa/Se + σm/Syt = 1/FOS, where σa = alternating stress, σm = mean stress, Se = endurance limit, Syt = yield strength, FOS = factor of safety. It lies below both Goodman's line and Gerber's parabola, making it the safest but most conservative design approach for machine elements under cyclic loading.

Comparison with Other Criteria

Unlike Goodman's line (uses ultimate strength Sut) and Gerber's parabola (curved, least conservative), Soderberg's line uses yield strength (Syt), giving the most conservative estimate. It is preferred for ductile materials and critical components. Though it may lead to over-design, it guarantees no yielding or fatigue failure under fluctuating stress conditions.

THANK YOU!

Questions & Discussion

Fatigue Strength & Design

ME Department, NRCM

Slide 15 of 15

Presented by:

Sweshareefa Mahakul
Assistant Professor, ME Dept, NRCM