UNIT-III

MECHANICAL PROPERTIES STIFNESS AND STRENGTH

Mechanical Properties Stiffness and Strength

Volume Fractions

Consider a composite consisting of fiber and matrix. Take the following symbol notations:

$v_{c,f,m}$ = volume of composite, fiber, and matrix, respectively

$\rho_{c,f,m}$ = density of composite, fiber, and matrix, respectively.

Now define the fiber volume fraction

Vf and the matrix volume fraction Vm as

$$V_f = \frac{v_f}{v_c}, \qquad V_m = \frac{v_m}{v_c}.$$

Note that the sum of volume fractions is

$$V_f + V_m = 1, \quad v_f + v_m = v_c.$$

Mass Fractions

Consider a composite consisting of fiber and matrix and take the following

symbol notation:

wc,f,m= mass of composite, fiber, and matrix, respectively. The mass fraction (weight fraction) of the fibers (Wf) and the matrix (Wm) are defined as

$$W_f = \frac{w_f}{w_c}, \text{ and}$$

$$W_m = \frac{w_m}{w_c}.$$

Note that the sum of mass fractions is

$$W_f + W_m = 1, \quad w_f + w_m = w_c.$$

From the definition of the density of a single material,

$$w_c = r_c v_c,$$

$$w_f = r_f v_f, \text{ and}$$

$$w_m = r_m v_m.$$

Substituting above equations, the mass fractions and volume fractions are related as

$$W_f = \frac{\rho_f}{\rho_c} V_f \text{, and}$$

$$W_m = \frac{\rho_m}{\rho_c} V_m,$$

in terms of the fiber and matrix volume fractions. In terms of individual constituent properties, the mass fractions and volume fractions are related by

$$W_f = \frac{\frac{\rho_f}{\rho_m}}{\frac{\rho_f}{\rho_m} V_f + V_m} V_f,$$

$$W_m = \frac{1}{\frac{\rho_f}{\rho_m}(1 - V_m) + V_m} V_m .$$

One should always state the basis of calculating the fiber content of a composite. It is given in terms of mass or volume. Based on above Equation, it is evident that volume and mass fractions are not equal and that the mismatch between the mass and volume fractions increases as the ratio between the density of fiber and matrix differs from one.

Density

The derivation of the density of the composite in terms of volume fractions is found as follows. The mass of composite *wc* is the sum of the mass of the fibers *wf* and the mass of the matrix *wm* as

$$w_c = w_f + w_m.$$

$$\rho_c v_c = \rho_f v_f + \rho_m v_m, \quad \rho_c = \rho_f \frac{v_f}{v_c} + \rho_m \frac{v_m}{v_c} .$$

Using the definitions of fiber and matrix volume fractions from Equation

$$\rho_c = \rho_f V_f + \rho_m V_m.$$

Now, consider that the volume of a composite *vc* is the sum of the volumes of the fiber *vf* and matrix (*vm*):

$$v_c = v_f + v_m .$$

The density of the composite in terms of mass fractions can be found as

$$\frac{1}{\rho_c} = \frac{W_f}{\rho_f} + \frac{W_m}{\rho_m}.$$

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	μm/m/°C	–1.3	5	–5.0
Transverse coefficient of thermal expansion	μm/m/°C	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	μm/m/°C	63	23	90
Coefficient of moisture expansion	m/m/kg/kg	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

Idealization of Microstructure of Fibrous Composite:

As mentioned earlier, the micromechanics is a study at fibre and matrix level. Thus, the geometry of arrangement of the fibres and matrix in a composite is an essential requirement to develop a model for the study. Some of the methods do not use the geometry of arrangement. Most of the methods developed for micromechanical analysis assume that:

1. The fibers and matrix are perfectly bonded and there is no slip between them.
2. The fibres are continuous and parallel.
3. The fibres are assumed to be circular in cross section with a uniform diameter along its length.
4. The space between the fibres is uniform throughout the composite.
5. The elastic, thermal and hygral properties of fibre and matrix are known and uniform.
6. The fibres and matrix obey Hooke's law.
7. The fibres and the matrix are only two phases in the composite.
8. There are no voids in the composite.

There are many ways to idealize the cross section of a lamina. In Figure 1 are shown two popular idealizations. The most commonly preferred arrangements are square packed and hexagonal packed arrays of fibres in matrix. The square and hexagonal packed arrays can be as shown in Figure 1(a), and (b), respectively.

In these idealizations it is seen that due to symmetry and periodicity of these arrays one can consider only one array to analyze the lamina at micro scale. Further, if this one array represents the general arrangement of fibres with respect to matrix and the interactions of fibre and matrix phases, then such array is called ***Representative Volume Element*** **(RVE)**. Further, this RVE as a volume of material statistically represents a homogeneous material. In the analysis of an RVE the boundary conditions are chosen such that they reflect the periodicity. Thus, the arrays shown in Figure 1 are various RVEs. One should be able to see that the RVE also reflects the volume fractions. The term RVE was first coined by Hill in 1963.
For example, the square RVE represents a lower fibre volume fraction than a hexagonal RVE. Note that RVE is also called as ***Unit Cell.***

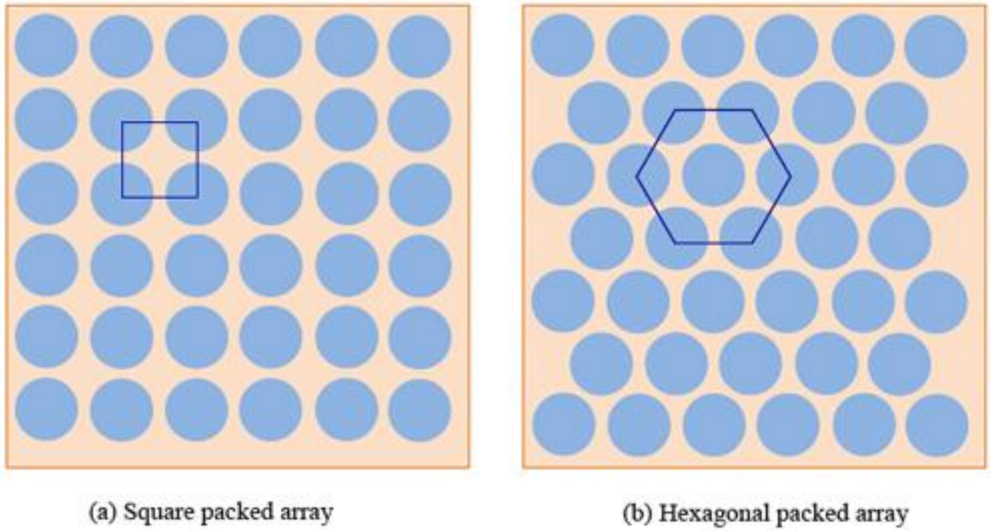

Figure 1.: Idealization of cross section of lamina

StrengthofMaterialApproximations:

In general, the laminates made are thin. Hence, for such laminates the analysis done using Kirchhoff and plane stress assumptions is reasonably good. For such analysis, one needs the engineering constants that occur in defining planar constitutive equations. These engineering constants are:

1. E_1^* - **the axial modulus**
2. $E_2^* = E_3^*$ - **transverse modulus**
3. $\nu_{12}^* = \nu_{13}^*$ - **axial Poisson's ratio (for loading in x_1- direction)**
4. $G_{12}^* = G_{13}^*$- **axial shear modulus (shear stress parallel to the fibers)**

Further, it is seen that for transversely isotropic composite, four out of five (the fifth one is G_{23}^*) properties can be developed from this approach. For the planar hygro-thermal analysis of such laminates, one can also obtain the in-plane coefficients of thermal expansions α_1^* and α_2^* and hygroscopic expansion β_1^* and β_2^* as well.

It is important to note that this approach involves assumptions which do not necessarily satisfy the requirements of an exact elasticity solution. In this approach the effective properties will be expressed in terms of the elastic properties and volume fractions of the fiber and matrix. The lamina is considered to be an alternate arrangement of fibres and matrix. The RVE chosen in these derivations is shown in Figure 2. The RVE here does not take into account the cross sectional arrangement of fibres and matrix, rather it represents volume of the material through the cross sectional area of fibre and matrix.

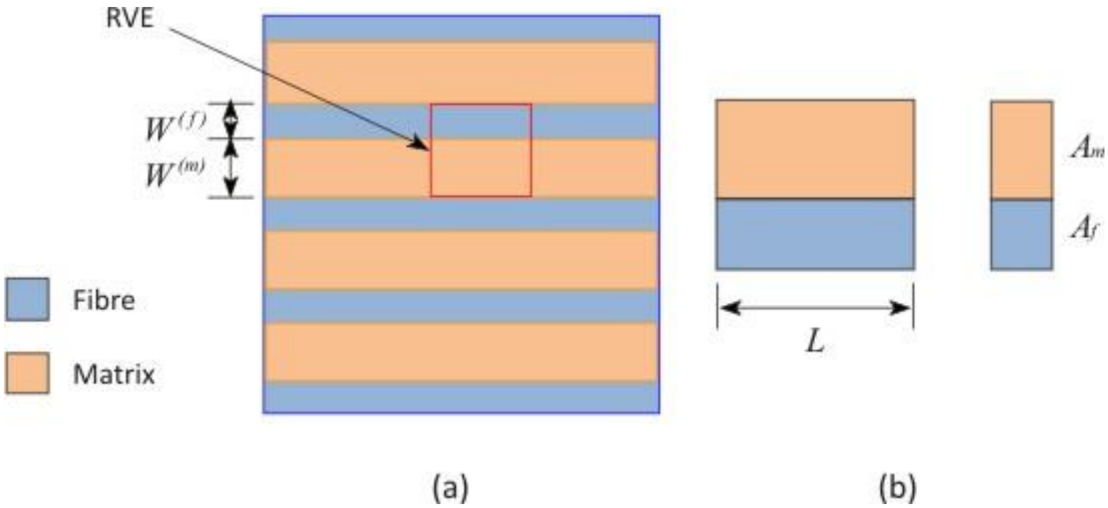

Figure 2: (a) Unidirectional lamina, (b) RVE for unidirectional composite for prediction of elastic properties

Let, A_f and A_m represent fibre area and matrix area, respectively. $W^{(f)}$ and $W^{(m)}$ represent fibre and matrix widths, respectively. L be the length of the RVE.

Effective Axial Modulus E_1^***:**

The unit cell as shown in Figure 2 is used to compute the effective axial modulus E_1^*. It should be noted that the thickness of the unit cell is not important in this computation. Further, the cross sectional shapes are not considered in this calculation. However, the cross sectional areas are important in this calculation. The thicknesses of the fibre and matrix constituents are same in the unit cell. Hence, the areas of the constituents represent the volume fractions of the constituents.

In the calculation of effective axial modulus, it is assumed that the axial strain in the composite is uniform such that the axial strains in the fibers and matrix are identical. This assumption is justified by the fact that the fibre and the matrix in the unit cell are perfectly bonded. Hence, the elongation in the axial direction of the fibre and matrix will also be identical. Thus, the strains in the fibre and matrix can be given as

$$\bar{\varepsilon}_1 = \varepsilon_1^{(f)} = \varepsilon_1^{(m)} = \frac{\Delta L}{L} \qquad (1)$$

where, $\bar{\varepsilon}_1$ is the axial strain in the composite and $\varepsilon_1^{(f)}$ and $\varepsilon_1^{(m)}$ are the axial strains in fibre and matrix, respectively. Now, let $E_1^{(f)}$ and $E^{(m)}$ be the axial Young's moduli of the fibre and matrix, respectively. We can give the axial stress in the fibre, $\sigma_1^{(f)}$ and matrix, $\sigma_1^{(m)}$ as

$$\sigma_1^{(f)} = E_1^{(f)}\varepsilon_1^{(f)} \quad \text{and} \quad \sigma_1^{(m)} = E^{(m)}\varepsilon_1^{(m)}$$

Using the above equation and the cross section areas of the respective constituent in the unit cell, we can calculate the forces in them as

$$F_1^{(f)} = \sigma_1^{(f)}A_f \quad \text{and} \quad F_1^{(m)} = \sigma_1^{(m)}A_m$$

The total axial force in the composite is sum of the axial forces in fibre and matrix. Thus, the total axial force in the composite substituting the expressions for axial strains in fibre and matrix from Equation (1) in above equation, can be given as

$$F_1 = F_1^{(f)} + F_1^{(m)} = \sigma_1^{(f)}A_f + \sigma_1^{(m)}A_m = \left(E_1^{(f)}A_f + E^{(m)}A_m\right)\frac{\Delta L}{L} \quad (2)$$

Now $\bar{\sigma}_1$ be the average axial stress in composite. The total cross sectional area of the composite is $A = A_f + A_m$. Thus, using the average axial stress and cross sectional area of the composite, the axial force is

$$F_1 = \bar{\sigma}_1 A \quad (3)$$

Thus, combining Equation (2) and Equation (3) and rearranging, we get

$$\bar{\sigma}_1 = \left(E_1^{(f)}\frac{A_f}{A} + E^{(m)}\frac{A_m}{A}\right)\frac{\Delta L}{L} \quad (4)$$

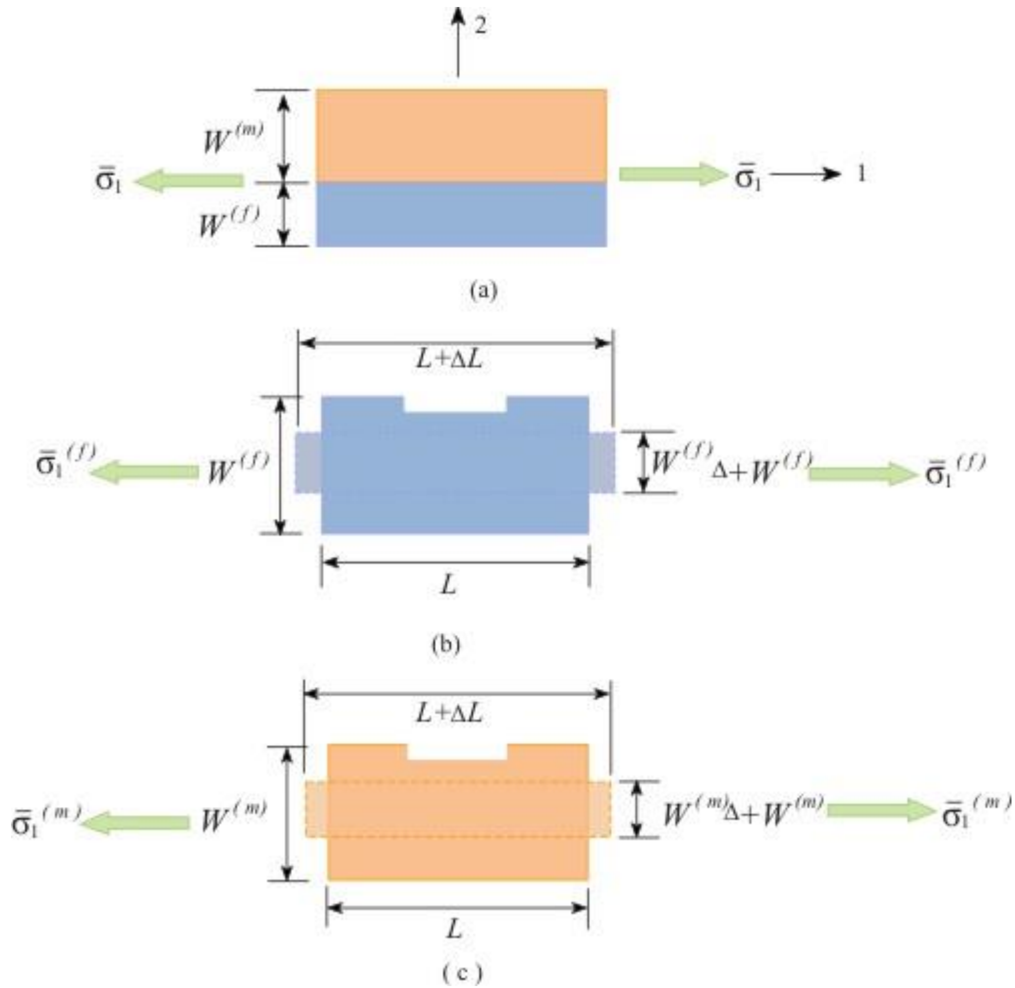

Figure 3: (a) Undeformed unit cell under $\bar{\sigma}_1$ (b) and (c) deformed individual constituents of the unit cell

Let us define

$$\bar{\sigma}_1 = E_1^* \bar{\varepsilon}_1 = E_1^* \frac{\Delta L}{L} \qquad (5)$$

Further, noting that the ratios $\frac{A_f}{A}$ and $\frac{A_m}{A}$ for same length of fibre and matrix represent the fibre and matrix volume fractions, respectively. Thus, combining Equations (4) and (5), we get

$$E_1^* = E_1^{(f)} V_f + E^{(m)} V_m = E_1^{(f)} V_f + E^{(m)} \left(1 - V_f\right) \quad (6)$$

The above equation relates the axial modulus of the composite to the axial moduli of the fibre and matrix through their volume fractions. Thus, the effective axial modulus is a linear function of the fiber volume fraction. This equation is known as rule of mixtures equation. It should be noted that the effective properties are functions of the fiber volume fractions; hence it should always be quoted in reporting the effective properties of a composite.

Effective Axial (Major) Poison's Ratio ν_{12}^{*}**:**

To determine the effective axial Poisson's ratio we consider the loading as in the case applied for determining the effective axial modulus. Here, for this loading we have $\bar{\sigma}_1 \neq 0$ and other stresses are zero. We define the effective axial Poisson's ratio as

$$\nu_{12}^{*} = -\frac{\bar{\varepsilon}_2}{\bar{\varepsilon}_1}$$

The effective strain in direction 2 from Figure 3(b) and (c) can be given as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W^{(f)} + W^{(m)}}$$

Now, the changes in $W^{(f)}$ and $W^{(m)}$ can be obtained using the Poisson's ratio of individual constituents. The axial Poisson's ratios for fibre and matrix are given as

$$\nu_{12}^{(f)} = -\frac{\varepsilon_2^{(f)}}{\varepsilon_1^{(f)}} = -\frac{\Delta W^{(f)} / W^{(f)}}{\Delta L / L} \quad \text{and} \quad \nu^{(m)} = -\frac{\varepsilon_2^{(m)}}{\varepsilon_1^{(m)}} = -\frac{\Delta W^{(m)} / W^{(m)}}{\Delta L / L} \quad (7.24)$$

Thus, the changes in $W^{(f)}$ and $W^{(m)}$ are given as

$$\Delta W^{(f)} = -\nu_{12}^{(f)} W^{(f)} \frac{\Delta L}{L} \quad \text{and} \quad \Delta W^{(m)} = -\nu^{(m)} W^{(m)} \frac{\Delta L}{L} \quad (7.25)$$

The total change in W is given as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} \qquad (7)$$

The strain in direction 2 for the composite can be given using Equation (6) and Equation (7) as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \frac{\Delta W^{(f)} + \Delta W^{(m)}}{W} = -\left(\nu_{12}^{(f)} \frac{W^{(f)}}{W} + \nu^{(m)} \frac{W^{(m)}}{W}\right) \frac{\Delta L}{L} \quad (8)$$

Here, $\frac{W^{(f)}}{W}$ and $\frac{W^{(m)}}{W}$ denote the fibre and matrix volume fractions for same length of fibre and matrix. Note that $\frac{\Delta L}{L}$ denotes the effective axial strain $\bar{\varepsilon}_1$. Thus, from Eq. (8) the effective axial Poisson's ratio is written as

$$\nu_{12}^{(*)} = \nu_{12}^{(f)} V_f + \nu^{(m)} V_m$$

The above equation is the rule of mixtures expression for composite axial Poisson's ratio.

Effective Transverse Modulus E_2^*:

Here, we are going to derive the effective transverse modulus by loading the RVE in direction 2 as shown in Figure 4(a). There are two considerations while deriving this effective modulus. The first approach considers that the deformation of the each constituent is independent of each other as shown in Figure 4(b) and (c) and the deformation in direction 1 is not considered. The second approach considers that deformations of the fibre and matrix in direction 1 are identical as they are perfectly bonded.

To calculate the effective modulus in direction 2, a stress $\bar{\sigma}_2$ is applied to the RVE as shown in Figure 4(a).

***First Approach*:**

As mentioned, the fibre and matrix deform independently of each other. The resulting deformation in direction 1 is not considered here. This assumption is simplistic and was used by early researchers.

The fibre and matrix are subjected to same state of stress. The state of stress is unidirectional, that is, $\sigma_2^{(f)} = \sigma_2^{(m)} = \bar{\sigma}_2$. Now, using the individual moduli and deformations in direction 2, these stresses can be given as

$$\sigma_2^{(f)} = E_2^{(f)} \varepsilon_2^{(f)} = E_2^{(f)} \frac{\Delta W^{(f)}}{W^{(f)}}$$

$$\sigma_2^{(m)} = E^{(m)} \varepsilon_2^{(m)} = E^{(m)} \frac{\Delta W^{(m)}}{W^{(m)}}$$

From this equation we can write the individual deformations, which give the total deformation in direction 2 as

$$\Delta W = \Delta W^{(f)} + \Delta W^{(m)} = \left(\frac{W^{(f)}}{E_2^{(f)}} + \frac{W^{(m)}}{E^{(m)}} \right) \bar{\sigma}_2$$

Now, the composite strain in direction 2 can be calculated from the definition as

$$\bar{\varepsilon}_2 = \frac{\Delta W}{W} = \left(\frac{W^{(f)}}{W} \frac{1}{E_2^{(f)}} + \frac{W^{(m)}}{W} \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

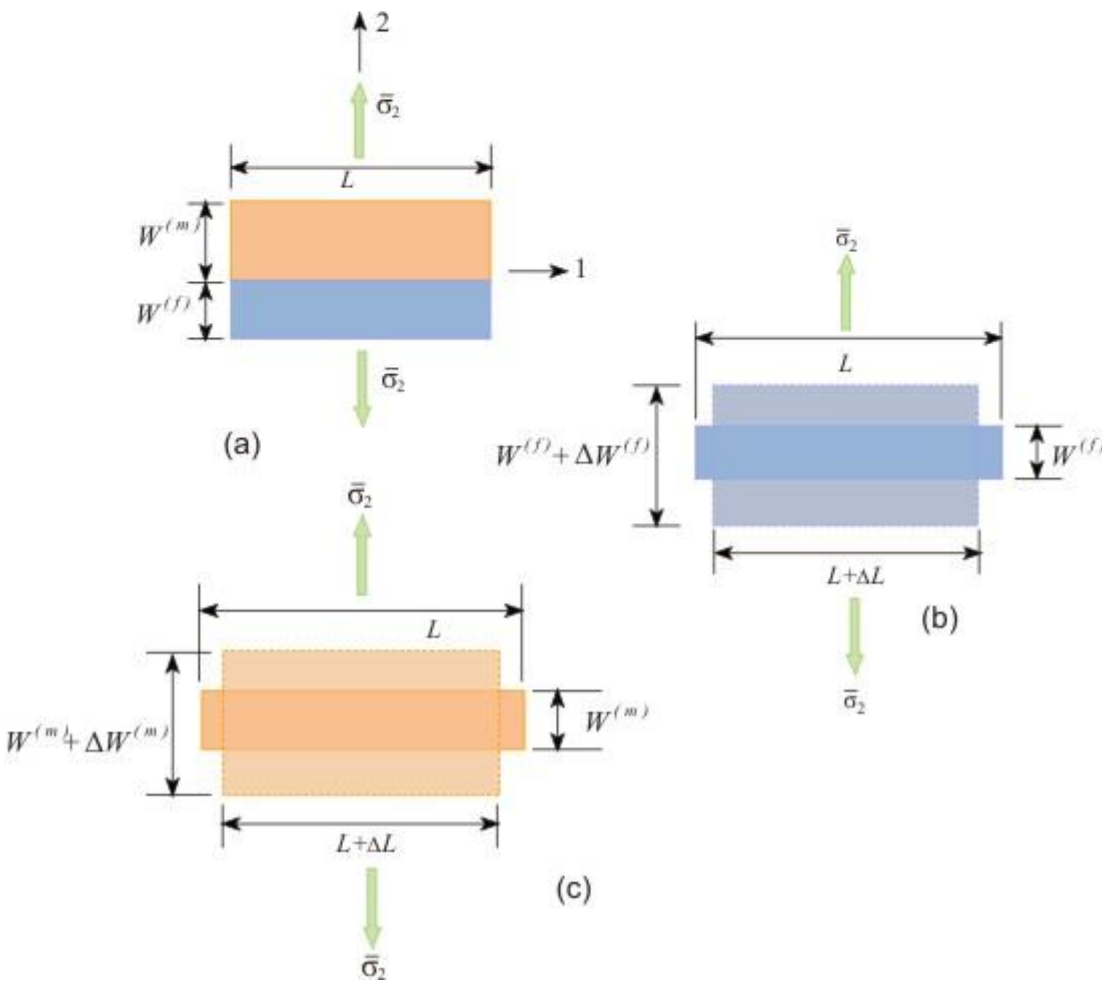

Figure 4: (a) Undeformed unit cell under uniform $\bar{\sigma}_2$ stress (b) and (c) deformed individual constituents of the unit cell

Introducing the volume fractions in the above equation,

$$\bar{\varepsilon}_2 = \left(V_f \frac{1}{E_2^{(f)}} + V_m \frac{1}{E^{(m)}} \right) \bar{\sigma}_2$$

Noting that $\frac{\bar{\sigma}_2}{\bar{\varepsilon}_2} = E_2^*$, from the above equation, we get

$$\frac{1}{E_2^*} = \frac{V_f}{E_2^{(f)}} + \frac{V_m}{E^{(m)}} = \frac{V_f}{E_2^{(f)}} + \frac{(1-V_f)}{E^{(m)}}$$

This equation is the rule of mixtures equation for effective modulus E_2^*.

Background to Mechanical Testing of Composites:

Objectives of Mechanical Testing:

The development of the mechanical testing of the materials depends upon other scientific factors. These factors help in better understanding and facilitate the progress in evaluating the various processes. These processes include:

1. quality control of a process
2. quality assurance for the material developed and structure fabricated from thereof
3. better material selection
4. comparisons between available materials
5. can be used as indicators in materials development programmes
6. design analysis
7. predictions of performance under conditions other than test conditions
8. starting points in the formulation of new theories

It should be noted that these processes are dependent upon each other. However, if they are considered individually then the data required can be different for the evaluation. For example, some tests are carried out as multipurpose tests using various processes. A conventional tensile test carried out under fixed conditions may serve quality control function whereas one carried out varying factors like temperature, strain rate, humidity etc. may provide information on load bearing capacity of the material.

The properties evaluated for materials like composite is very sensitive to various internal structure factors. However, these factors depend mainly upon the fabrication process or other factors. The internal structure factors that affect the properties are, in general, at atomic or molecular level. These factors mostly affect the matrix and fibre-matrix interface structure.

The mechanical properties of the fibrous composite depend on several factors of the composition. These factors are listed below again for the sake of completeness.

1. properties of the fibre
2. surface character of the fibre
3. properties of the matrix material
4. properties of any other phase
5. volume fraction of the second phase (and of any other phase)
6. spatial distribution and alignment of the second phase (including fabric weave)
7. nature of the interfaces

Another important factor is processing of the composites. There are many parameters that control the processing of composites that access the quality of adhesion between fibre and matrix, physical integrity and the overall quality of the final structure.

In case of composite the spatial distribution and alignment of fibres are the most dominating factor which causes the variation of properties. The spatial distribution and alignment of the fibres can change during the same fabrication process. Thus, for a given fabrication process the property evaluated from the composite material may show a large variation.

Tensile Testing

The well known purpose of the tensile testing is to measure the ultimate tensile strength and modulus of the composite. However, one can measure the axial Poison's ratio with additional instrumentations. The standard specimen used for tensile testing of continuous fiber composites is a flat, straight-sided coupon. A flat coupons in ASTM standard D 3039/D 3039M-93 for 0° and 90° have been shown.

The specimen, as mentioned above is flat rectangular coupon. The tabs are recommended for gripping the specimen. It protects the specimen from load being directly applied to the specimen causing the damage. Thus, the load is applied to the specimen through the grips. Further, it protects the outer fibres of the materials. The tabs can be fabricated from a variety of materials, including fiberglass, copper, aluminum or the material and laminate being tested. When the tabs of composite material are used then according to ASTM specifications the inner plies of the tabs should match with the outer plies of the composite. This avoids the unwanted shear stresses at the interface of the specimen and tabs. However, the recent versions of the ASTM standards allow the use of tabs with reinforcement at $\pm 45^\circ$. Further, end-tabs can also facilitate accurate alignment of the specimen in the test machine, provided that they are symmetrical and properly positioned on the specimen. The tabs are pasted to the specimen firmly with adhesive.

This specimen can provide data on:

1. The axial modulus E_x,
2. In-plane and through thickness Poisson's ratio γ_{xy}, γ_{xz}
3. Tensile ultimate stress σ_x^{ult},
4. Tensile ultimate strain ε_x^{ult} ,
5. Any nonlinear, inelastic response

In general, the tensile tests are done on coupons with 0° laminae/laminate for corresponding axial properties and coupons with 90° laminae/laminate for corresponding transverse properties. The off axis laminae specimen also provides data on coefficient of mutual influence and the in-plane shear response.

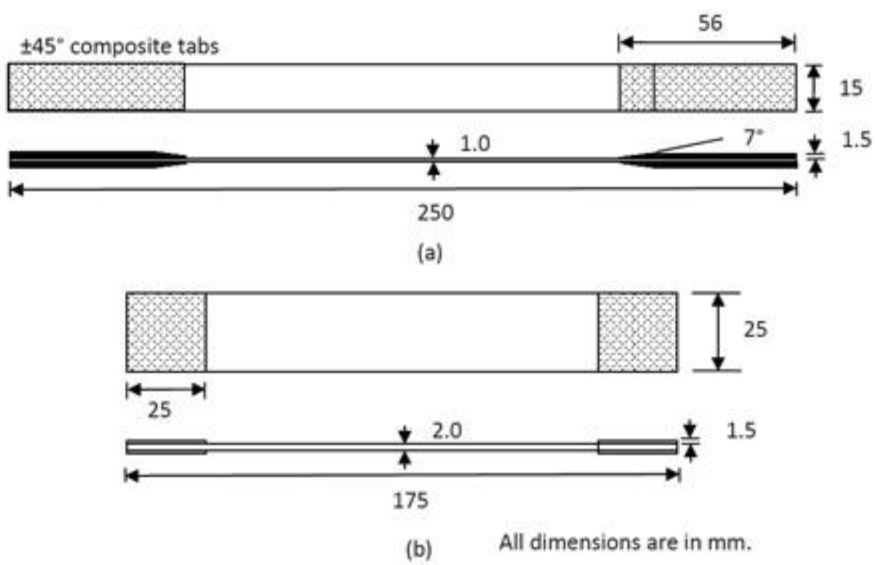

Figure 4:Composite tensile test specimens
(a) ASTM D 3039 for 0° and
(b) ASTM D 3039 for 90°.

***Orthotropic Laminae and Laminate*:**

For orthotropic, symmetric laminates with 0° and 90° laminae, the effective axial modulus and Poisson's ratio is given as

$$E_x = \frac{1}{a_{11}^*}, v_{xv} = \frac{-a_{12}^*}{a_{11}^*}$$

where, the quantities with asterisk are for laminate as mentioned in Chapter on Laminate Theory. These properties can be measured directly from a tensile test on a specimen of thickness t under axial force per unit length N_x as follows:

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{N_{xx}}{t\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}$$

The tensile strength is defined as the average stress at failure. Thus, the tensile strength can be given using the maximum applied force per unit length N_x and thickness t as

$$\bar{\sigma}_x^{ult} = \frac{N_x^{\max}}{t}$$

It should be noted that the failure of laminates is often influenced by inter laminar stresses along the free edge effects of the coupon. These factors will be explained in brief in one of the lecture.

The measurement of tensile strength by experiments can also provide information on the comparison of laminate theory with experiments.

Off-Axis Laminae

One can measure the tensile properties by conducting experiments on off-axis laminae. However, there are certain issues associated with this kind of experiments. For example, the presence of axial-shear coupling is associated with the nonzero a_{16}^* . Alternately, one can say that this term is associated with coefficient of mutual influence $\eta_{xy,x}$. Hence, these tests are not straight forward as in case of symmetric laminates with 0° and 90° laminae. Therefore, sometimes these tests are called as specialized tests.

When the experiments are conducted to measure the properties like E_x, v_{xy} and v_{xz} one can get the other properties along with these tests. For example, the coefficient of mutual influence $\eta_{xy,x}$, the nonlinear response and strength of an off-axis lamina for given fibre orientation can also be obtained.

There is an important issue associated with these tests is that what boundary conditions one should impose on the specimen? If a pure, uniform state of axial stress $\sigma_{xx} \neq 0, \sigma_{yy} = \tau_{xy} = 0$ can be applied to the ends and sides of a specimen and the specimen is free to assume any desired deformation pattern, the state of stress will be uniform and constant through-out the specimen. The deformation pattern is shown in Figure 8.4(a).

For uniform, far-field axial stress loading, that is $\sigma_{xx} \neq 0$, the stresses in principal material directions can be given as

$$\sigma_{11} = m^2\sigma_{xx}, \sigma_{22} = n^2\sigma_{xx}, \tau_{12} = -mn\sigma_{xx}$$

Further, the global elastic constants associated with axial stress loading are measured as

$$E_x = \frac{\sigma_{xx}}{\varepsilon_{xx}}, v_{xy} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}, \eta_{xy,x} = \frac{\gamma_{xy}}{\varepsilon_{xx}}$$

Combining above two equations, we get

$$\begin{aligned}
\varepsilon_{11} &= \frac{\sigma_{11}}{E_1} - \frac{v_{21}\sigma_{22}}{E_2} = (m^2 - n^2 v_{12})\frac{\sigma_{xx}}{E_1} \\
\varepsilon_{22} &= -\frac{v_{12}\ \sigma_{11}}{E_1} + \frac{\sigma_{22}}{E_2} = \left(-\frac{v_{12}m^2}{E_1} + \frac{n^2}{E_2}\right)\sigma_{xx} \\
\gamma_{12} &= \frac{\tau_{12}}{G_{12}} = \frac{-mn\ \sigma_{xx}}{G_{12}}
\end{aligned}$$

From the above equation all three strain components can be obtained for non zero value of axial stress. Thus, from the third of the above equation we can find the shear modulus.

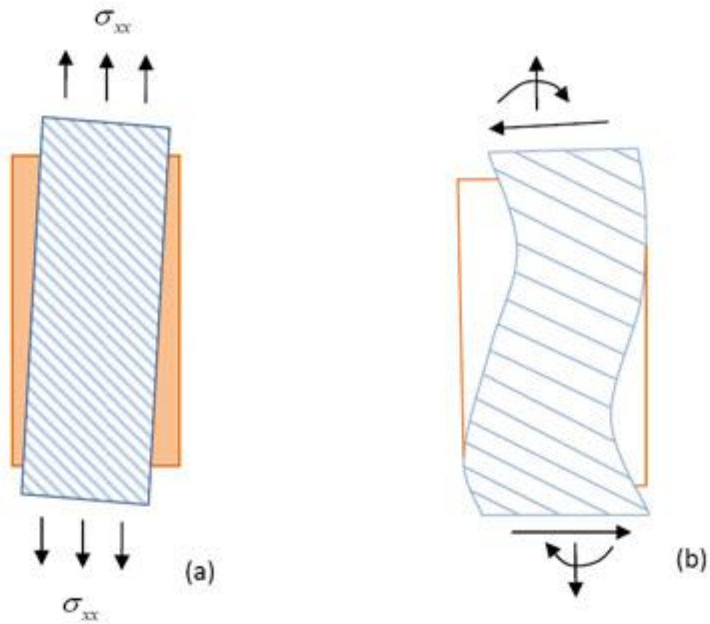

Figure 5: Axial load on off-axis laminae with effect of end constraint
(a) unconstrained displacement and
(b) constrained displacement

It is not easy to apply pure, uniform tensile stress to an off-axis coupon. The specimens are gripped in such a manner that the ends of the specimen are constrained and boundary condition is actually a specification of the axial end displacement. Further, there are more issues with these tests like the constrained displacement induces a doubly curved displacement field in the specimen. The deformed shape of the coupon with restrictions on the ends is depicted in Figure 5(b). We will not deal the complete analysis for the measurements of the properties with tests on off-axis laminae.

The bone shaped specimens for chopper-fiber, metal matrix composite tensile tests. More details can be seen in ASTM D3552-77(1989). Further, for the tensile testing for transverse properties of hoop-wound polymer matrix composite cylinders are used. The details of this testing can be seen in ASTM D5450/D5450M-93.

Measurement of modulus

It should be noted that due to progressive damage the stiffness of the lamina or laminae/laminate changes causing the stress strain curve to be non-linear. The measurement of modulus in a tensile testing from a non-linear loading curve can be done by three methods.

In the first method the modulus is taken as a tangent to the initial part of the curve. In the second method a tangent is constructed at a specified strain level. For example, in the Figure 5 the modulus is measured at 0.25% strain or 0.0025 strain (Point B). In the third method, a secant is constructed between two points. For example in Figure 8.5 a secant is constructed between points A and B.

Typically, the strain values at these points are 0.0005 and 0.0025. In ASTM standards the secant is called as *chord*. The modulus measured by these methods is known as **'*initial tangent modulus*', '*B% modulus*' and '*A%-B% secant (chord) modulus*',** respectively.

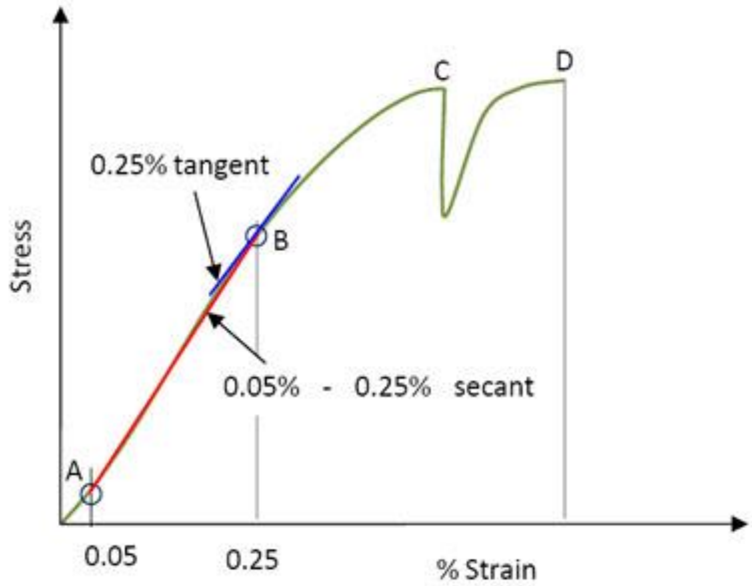

Figure 6: Typical tensile stress-strain curve with details

Compression Testing

Most of the structural members include the compression members. Such members can be loaded directly in compression or under a combination of flexural and compression loading. The axial stiffness of such members depends upon the cross-sectional area. Thus, it is proportional to the weight of the structure. One can alter the stiffness by changing the geometry of the cross section within limits. However, some of the composites have low compressive strength and this fact limits the full potential application of these composites.

The compression testing of the composites is very challenging due to various reasons. The application of compressive load on the cross section can be done in three ways: directly apply the compressive load on the ends of a specimen, loading the edges in shear and mixed shear and direct loading. These three ways of imposing the loads for compression testing are shown .

During compression loading the buckling of the specimen should be avoided. This demands a special requirement on the holding of the specimen for loading purpose. Further, it demands for special geometry of the specimen. These specimens are smaller in size as compared to the tensile testing specimens. A compression test specimen according to ASTM D695 (modified) standard is shown.

The compression testing of composites is a vast topic. Additional reading on this topic from other literature is suggested to readers.

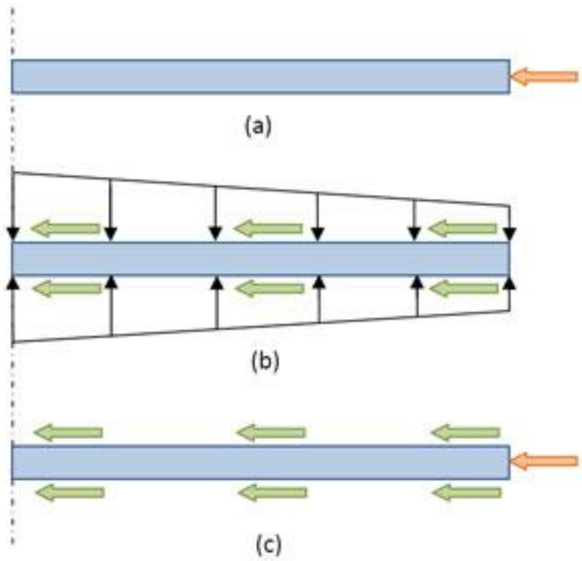

Figure 7: Load imposition methods for compression testing. (a) Direct end loading (b) Shear loading and (c) Mixed shear and direct loading

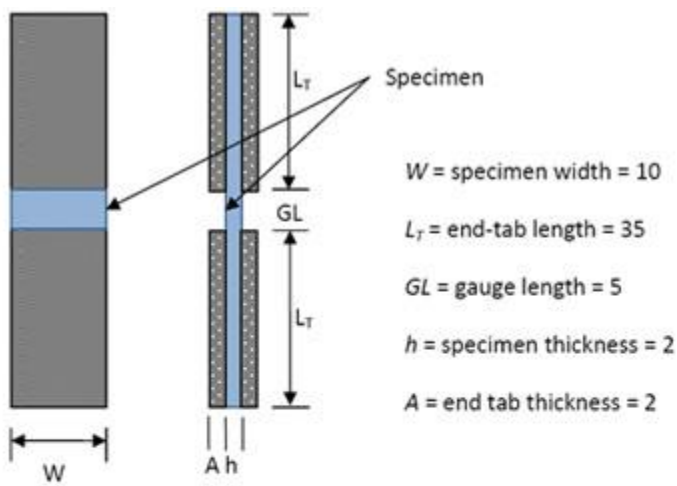

Figure 8: Composite compression test specimen according to ASTM D695 (modified) standard.

Shear Testing

Here we will see measurement of in-plane shear modulus G_{12} only. The methods are listed below:

1. Tension of a $[\pm 45]_s$ laminate

2. Tension of an off-axis lamina
3. Torsion of a unidirectional tube
4. Iosipescu shear of unidirectional laminae and cross ply laminates
5. Rail shear of unidirectional laminae
6. Picture frame test

1. $[\pm 45]_s$ ***Tensile Test*:**

A tension test on $[\pm 45]_s$ laminate is popularly used test for the measurement of in-plane shear modulus G_{12}. The more details of this test are available in ATSM standard D3518/D3518/M–91. According to ASTM standard the method uses a 250 mm long rectangular specimen with width 25 mm and thickness 2 mm. Further, it is recommended that for materials constructed with layers thicker than 0.125mm, the laminate should consist of 16 layers, that is, $[\pm 45]_{4s}$.The specimen is shown . The dimensions in this figure are in mm.

When a $[\pm 45]_{4s}$ is subjected to axial tensile stress $\bar{\sigma}_{xx}$ then stresses in principal material coordinates developed in each of the $+45^\circ$ and -45° lamina are given as

$$
\begin{aligned}
\sigma_{11} &= B\bar{\sigma}_{xx} \\
\sigma_{22} &= (1-B)\bar{\sigma}_{xx} \\
\tau_{12} &= \frac{-1}{2mn}[B(1-2m^2)+m^2]\bar{\sigma}_{xx}
\end{aligned}
$$

where,

$$
B = \left[\frac{m^2(2m^2-1)+4m^2n^2\frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}v_{12}+1\right)}{4m^2n^2\ \frac{G_{12}}{E_2}\left(\frac{E_2}{E_1}+2\frac{E_2}{E_1}\ v_{12}+1\right)+(2m^2-1)(m^2-n^2)}\right]
$$

and other quantities as defined in earlier chapters. For a special case with $\theta = 45^\circ$ we get the shear stress as

$$
\tau_{12} = (\pm\theta) = \mp\frac{\bar{\sigma}_{xx}}{2}
$$

Thus, from this equation one can see that the shear stress in principal material directions is statically determinate, that is, it is independent of material properties of the specimen and only depends upon the magnitude of the applied stress. The magnitude of this stress is half of the applied stress.

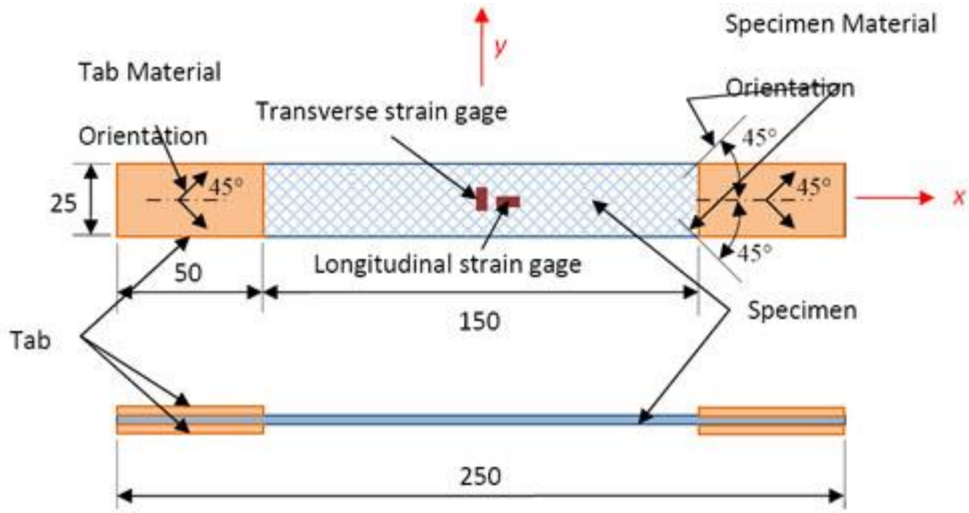

Figure 9: Specimen geometry and strain gage positioning for $[\pm 45]_{4s}$ tensile testing

From the knowledge of linear elastic behaviour of the orthotropic materials it is clear that the shear response is uncoupled from the normal response. Hence, in-plane shear modulus G_{12} can be determined directly from a tensile test on a $[\pm 45]_{4s}$ laminate.

Now the shear strain γ_{12} in principal material coordinates can be found by transformation of the measured axial and transverse strains ε_{xx} and ε_{yy}. It should be noted that the shear strain γ_{xy} is zero for orthotropic laminates under tension and γ_{12} is independent of γ_{xy} for $\theta = \pm 45^{\circ}$ (see the strain transformation relations). Thus, from the strain transformation relations, we can get the shear strain in principal material directions as

$$\gamma_{12} = -(\varepsilon_{xx} - \varepsilon_{yy})$$

Thus, from the definition of the shear modulus we get

$$G_{12} = \frac{\bar{\sigma}_{xx}}{2(\varepsilon_{xx} - \varepsilon_{yy})}$$

The above equation can be rearranged in the following manner to express the shear modulus in terms of effective properties of $[\pm 45]_{4s}$ laminate.

$$G_{12} = \frac{\frac{\bar{\sigma}_{xx}}{\varepsilon_{xx}}}{2\left(\frac{\varepsilon_{xx}}{\varepsilon_{xx}} - \frac{\varepsilon_{yy}}{\varepsilon_{xx}}\right)} = \frac{E_x}{2(1+v_{xy})}$$

Here, E_x is the effective modulus of the $[\pm 45]_{4s}$ laminate.

The measurement of in-plane shear modulus from shear stress-strain curve is done as follows. The shear stress-strain curve for $\pm 45^{\circ}$ specimen is obtained first. A typical shear stress-strain curve for such a specimen is shown. The shear modulus is obtained from the initial slope of the this curve in the range of 0.1-0.5% strain as

$$G_{12} = \frac{\tau_{12}' - \tau_{12}'}{\gamma_{12}' - \gamma_{12}'}$$

The tensile test on $\pm 45^{\circ}$ specimen provides an acceptable method for the measurement of in-plane shear modulus. However, one should be careful while interpreting the ultimate shear strength and strain. It should be noted that the laminae are subjected to a biaxial state of stress and not a pure shear. The normal stresses act along the shear planes causing the onset of mixed mode fracture. Other kind of failure like multiple ply cracking, fibre rotation and edge or internal delaminations occur prior to final failure. Therefore, the true failure is very difficult to determine. The shear strength is specified by different standards corresponding either to the ultimate load generated during the test or to a specified strain level. It is recommended in ISO standard that the test be terminated at $\gamma_{12} = 5\%$. The shear strength is taken as the peak load at or before 5% strain.

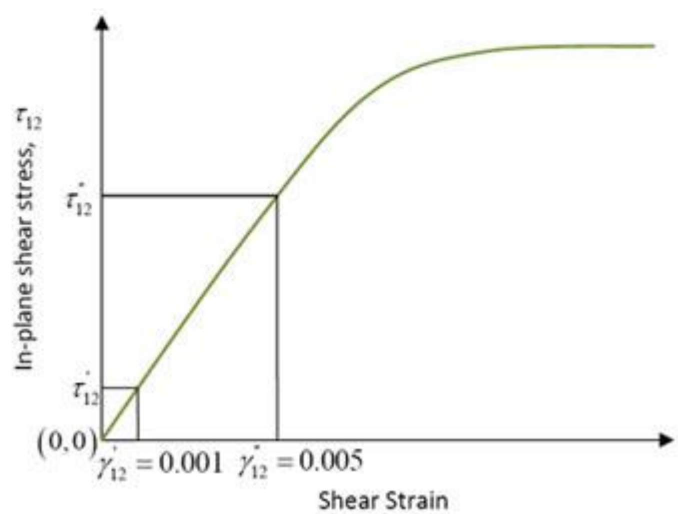

Figure 10: Typical shear stress-strain curve for specimen

***2. Shear Of an Off-Axis Lamina*:**

In similar way to the tensile testing of a $[\pm 45]_{4s}$ laminate one can use a unidirectional off-axis tensile coupon to determine the shear response of a composite in the principal material coordinates. A tensile test on 10° off-axis lamina is a commonly used. Specimen has same geometry . The state of stress in principal material coordinate directions can be obtained from transformation relations. Since, the shear response in the principal material coordinates is uncoupled from the normal response we can write the shear modulus as

$$G_{12} = \frac{\tau_{12}}{\gamma_{12}}$$

The shear stress in the principal material directions due to axial tensile stress can be given using transformation relations as

$$\bar{\tau}_{12} = -mn\bar{\sigma}_{xx}$$

The shear strain is measured from the strains $\varepsilon_{xx}, \varepsilon_{yy}$ and γ_{xy} with the help of strain transformation relations. Then the apparent shear modulus $\bar{G}_{12}$ can be given as

$$\bar{G}_{12} = \frac{-mn\bar{\sigma}_{xx}}{\gamma_{12}}$$

***1. Rail Shear Test*:**

This is a very popular method used to measure in-plane shear properties. This method is extensively used in aerospace industry. The shear loads are imposed on the edges of the laminate using specialized fixtures. There are two types of such fixtures: Two rail and three rail fixture. The ASTM D4255 standard covers the specification for two and three rail specimens for both continuous and discontinuous (0° and 90° fibre alignment), symmetric laminates and randomly oriented fibrous laminates.

a. Two Rail Shear Test

The two rail shear test fixture along with a laminate to be tested is shown . The Figure shows the specimen geometry according to ASTM D4255 standard. The two rail shear test fixture has two rigid parallel steel rails for loading purpose. The rails are aligned to the loading direction as shown . Thus, it induces the shear load in the specimen which is bolted to these rails. A strain gage is bonded at 45° to the longitudinal axis of the specimen.

The Shear strength is obtained as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{Lh}$$

where, $P_{\max}$ is ultimate failure load, L is the specimen length along the rails and h is the specimen thickness.

The shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{2Lh\Delta\varepsilon_{45}}$$

where, ΔP is the change in applied load and $\Delta\varepsilon_{45}$ is the change in strain for $+45^{\circ}$ or -45° strain gage in the initial linear stress-strain regime. It is suggested that the change in the strain is taken as the average of the change in strains on the both sides of the specimen.

Various modes of failure are seen. The modes are highly dependent upon the microstructure of the material.

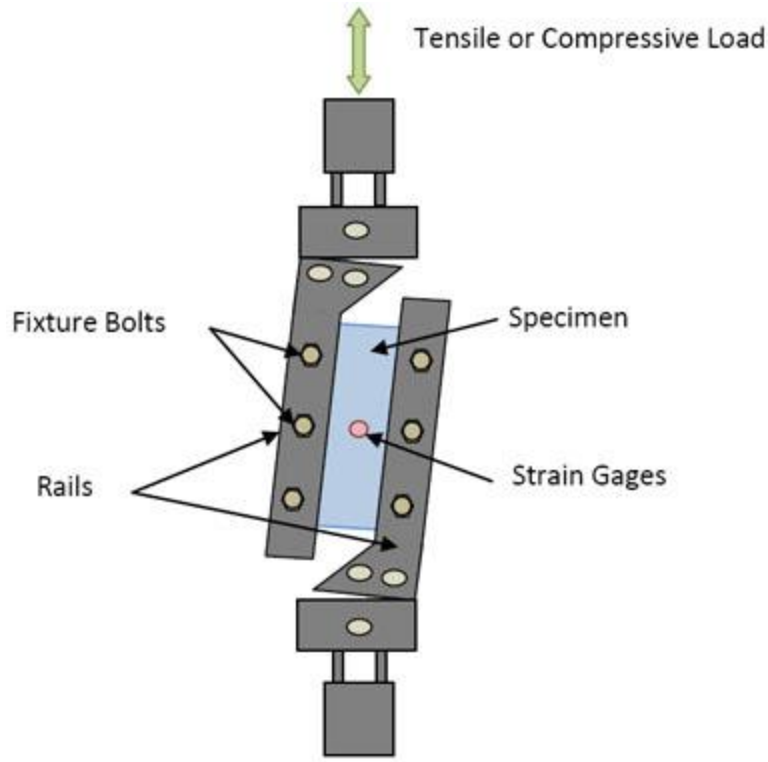

Figure 11: Two rail shear fixture for shear testing

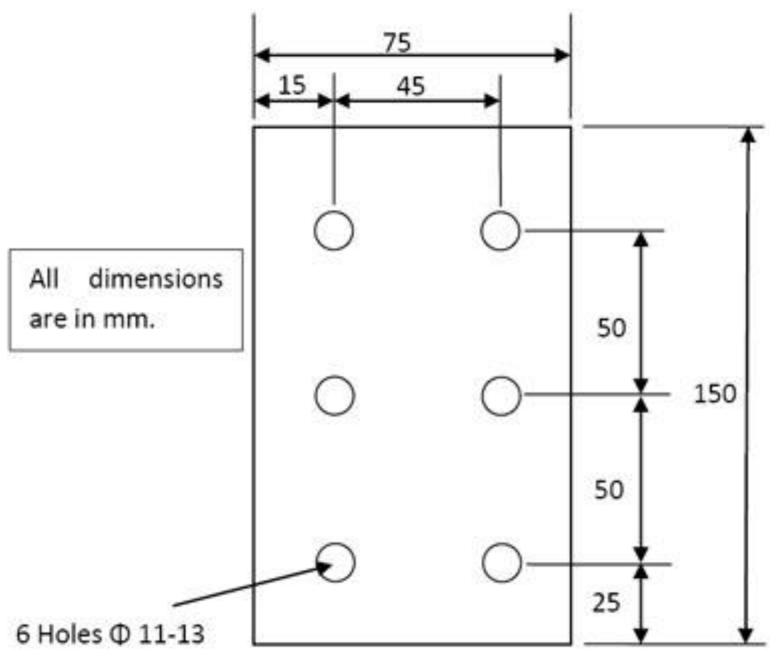

Figure 12: Two rail shear test specimen

***b. Three Rail (Symmetric) Shear Test*:**

The three rail shear test is an improved version of the rail shear test. Using one more rail in two rail shear test fixture it can produce a closer approximation to pure shear. The fixture consists of 3 pairs of rails clamped to the test specimen as shown . The outside pairs are attached to a base plate which rests on the test machine. Another pair (third middle) pair of rails is guided through a slot in the top of the base fixture. The middle pair loaded in compression. The shear force in laminate is generated via friction between rail and specimen. The strain gages bonded to the specimen at 45° to the specimen's longitudinal axis. The specimen geometry is shown.
The shear strength is given as

$$\tau_{xy}^{ult} = \frac{P_{\max}}{2Lh}$$

And the shear modulus is given as

$$G_{xy} = \frac{\Delta\tau_{xy}}{\Delta\gamma_{xy}} = \frac{\Delta P}{4Lh\Delta\varepsilon_{45}}$$

where, all variables in these two equations are given previously.

It should be noted that the holes in the specimen are slightly oversized than the bolts used for clamping. Further, the bolts are tightened in such a manner to ensure that there is no bearing contact between the bolt and specimen in the loading direction. It is recommended that each bolt is tightened with a 100 Nm torque.

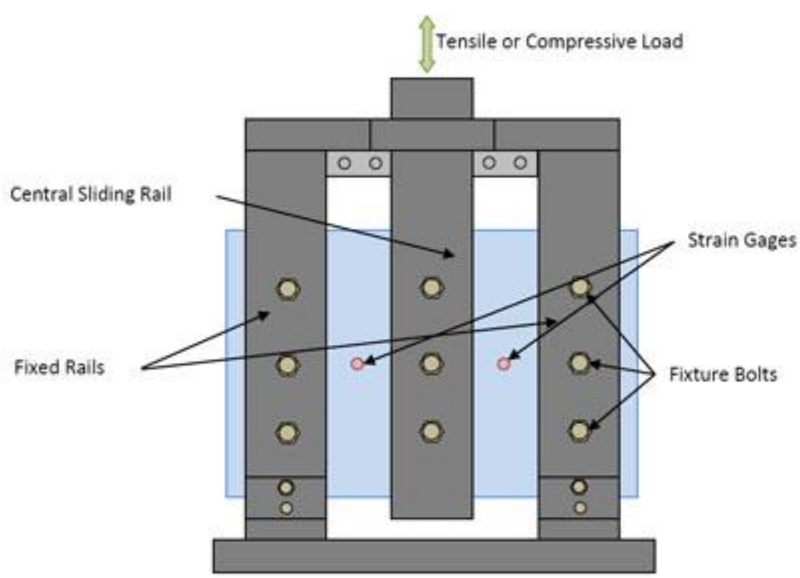

Figure 12: Three rail shear fixture for shear testing

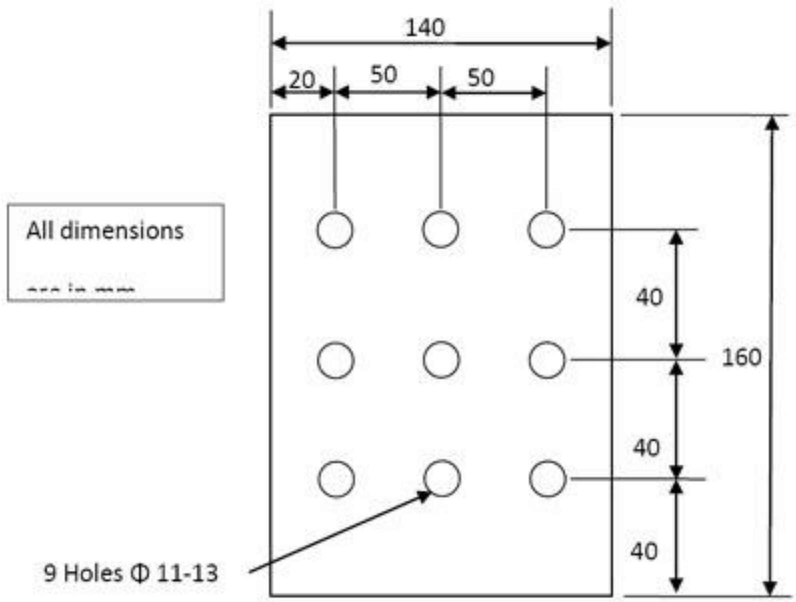

Figure 13: Specimen dimensions for three-rail shear test

Flexural Tests:

The flexural tests are conducted to determine the mechanical properties of resin and laminated fiber composite materials. Further, these tests are used to determine the interlaminar shear strength of a laminate, shear modulus, shear strength, tensile and compression moduli along with flexural and shear stiffness. These tests are not only used for composites but also for sandwich beams.

These tests are simple one. Further, they need simple instrumentation and equipment required. These tests conducted on beams of uniform cross section. These beam specimens do not require the end tabs.

There are two methods to carry out these tests. The beam is a flat rectangular specimen and is simply supported close to its ends. In the first method the beam is centrally loaded. Thus gives three point bending. Since there are three important points (two end supports and one central loading point) along the span of the beam this method is called as *three-point bending* test. In the second method the beam is loaded by two loads placed symmetrically between the supports. In this method there are four important points (two end supports and two loading points) along the span of the beam. Thus, it gives four-point bending. Hence, this method is called *four point bending*. These methods are shown schematically. Also shown in this figure are the shear force diagram (SFD) and bending moment diagrams (BMD) related to the particular loading regimes.

From the shear force and bending moment diagrams it is clear that there is a stress concentration at the point of loading. However, for four point bending there is uniform bending moment and both shear force and interlaminar shear stress are zero between the loading points. Thus, it leads to the pure bending loading. Such a state of stress is desirable in testing.

The properties are assumed to be uniform through the thickness as composite as it is a unidirectional composite or isotropic material. For such a material the normal stress varies linearly across the thickness. The maximum in compression is on one side and an equal maximum in tension on other side of the thickness and passes through zero at the mid-plane. The maximum normal stress is given as

$$|\sigma_C| = |\sigma_T| = \frac{6M}{bh^2}$$

where, M is the bending moment, b is width and h is the thickness of the specimen. Further, σ_C and σ_T denote compressive and tensile normal stresses, respectively.

The shear stress varies parabolic through the thickness with maximum at mid plane and zero at the outer surface. The maximum shear stress at the mid plane is given as

$$\tau = \frac{3Fs}{2bh}$$

where Fs is the shear force on the specimen cross section. The normal stress and shear force variation through the thickness is shown.The flexural response of the beam in three or four point bending test is obtained by recording the load applied and the resulting strain. The resulting strains are measured using the strain gages bonded on the beam in the gage length. It is clear from the distribution of the shear force and bending moment that the state of stress in specimens subjected to three and four-point bending tests are somewhat different. Thus, it may lead to differences in the results.

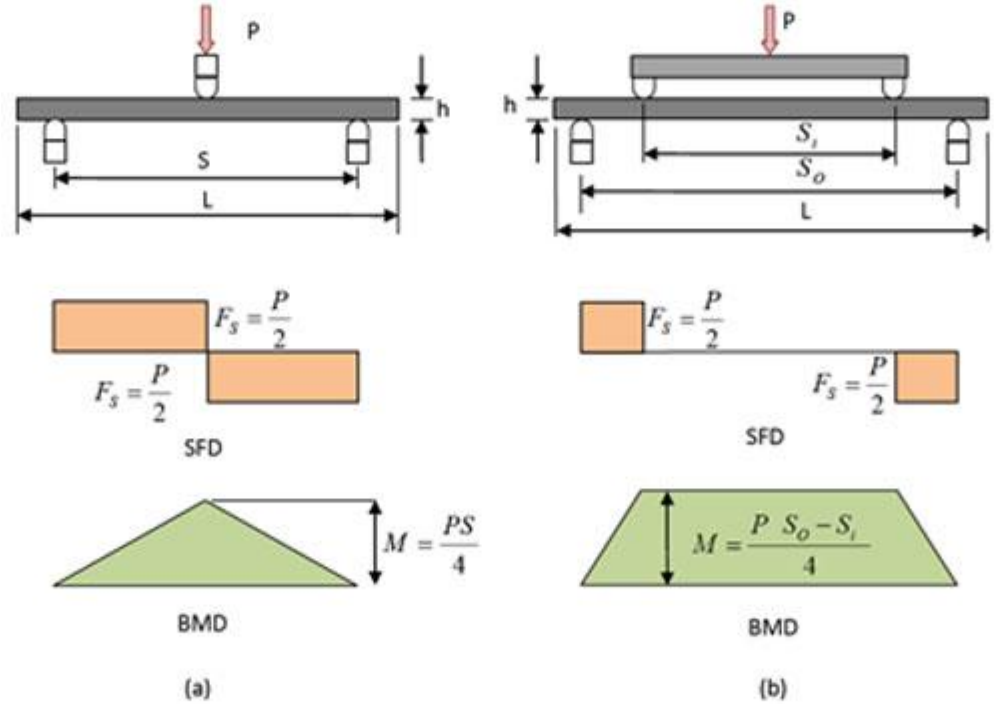

Figure 13: Shear force and bending moment diagrams for (a) three point and (b) four point bending test

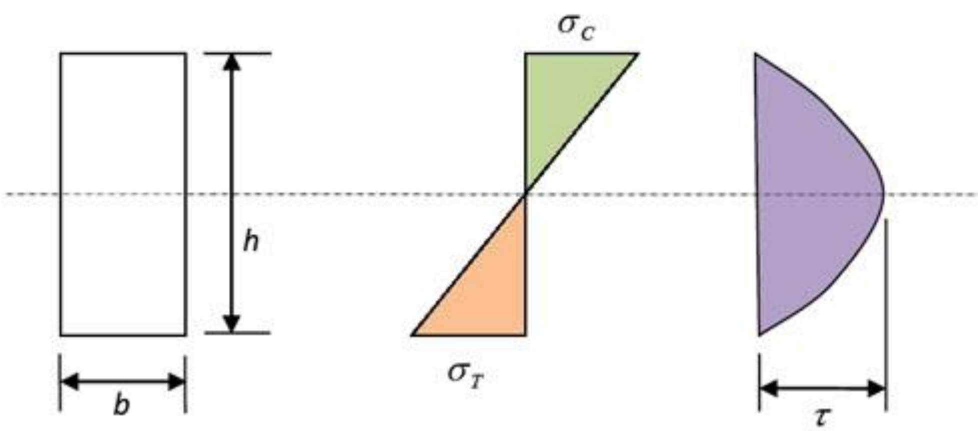

Figure 14: Bending and shearing stresses in the thickness direction

In the following we will see the measurement of flexural modulus and maximum stress on the outer surface of the beam.

Flexural strength**:** This is the stress on the surface of the specimen at failure, which should be accompanied by the breaking of fibers, rather than inter laminar shear.

In the three point bending method the flexural modulus E_f is given as

$$E_f = \frac{S^3 m}{4b\, h^3}$$

where, E_f is flexural modulus, S is the support span, m is the slope of the load-deflection curve, b and h are the width and thickness of the specimen, respectively.

In case of four point bending there are two options according to ASTM D790 standard. In the first option the loading span is one third of the support span. For this case the flexural modulus is given as

$$F_f = 0.21\frac{S^3 m}{b h^3}$$

In the second option the loading span is half of the support span. The flexural modulus for this case is given as

$$F_f = 0.17\frac{S^3 m}{b h^3}$$

where, the parameters in these two equations are as defined earlier.

The maximum stress on outer surface of the beam is given below for all the cases.

$$\sigma = \frac{3PS}{2b\, h^2} \quad \text{3 point bending}$$
$$\sigma = \frac{PS}{b\, h^2} \quad \text{4 point bending with loading span equal to one third support span}$$
$$\sigma = \frac{3PS}{4b\, h^2} \quad \text{4 point bending with loading span equal to one third support span}$$

It is important to note that the measurement of width and thickness of the beam is important for accurate measurement of flexural modulus and maximum stresses.

For more details on these tests one can refer to ASTM D790-92 and ASTM D790M-93.