Unit-IV

Governors & Balancing of Masses

UNIT- VI (1)

GOVERNORS

The function of the governor is to regulate the mean speed of an engine, when there are variations in the load.
i.e: ~~when there are variations~~
when the load increases, its speed decreases, then, the supply of working fluid is increased.
On the other hand, when the load decreases, its speed increases, ~~theof~~ then less working fluid is required to supply.

Governor automatically controls the supply of working fluid to the engine with varying load conditions & keeps the mean speed within certain limits.

The governors may be classified into:
(1) Centrifugal governor.
(2) Inertia governor.

(1) Centrifugal governor:- These governors further may be classified as

Centrifugal Governors
- Pendulum type
  - Watt Governor
- Loaded Type
  - Dead weight governors
    - Porter Governor
    - Proell Governor
  - Spring controlled gover
    - Hartnell governor
    - Hartung governor
    - Wilson-Hartnell governor
    - Pickering governor

are worked on the balancing of the . (2)
gal force on rotating balls by an equal and
site radial force, known as "controlling force".
consists of 2 balls which are known as
governor balls.

→ These balls revolve with in the spindle. The upper ends of the arms are pivoted to the spindle. So that the balls may rise up or fall down as they revolve about vertical axis.

→ The arms are connected by the links to the sleeve which is keyed to the spindle.

→ This sleeve revolves with the spindle, but can slide up and down. The balls rises up when the spindle speed increases and falls when the spindle speed decreases.

→ In order to limit the travel of sleeve in upward & downward direction, two stops are provided on the spindle.

→ The supply of working fluid increases, when the sleeve falls and it decreases when the sleeve raises.

→ If the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls moves inwards and sleeves comes downwards.

→ If the load on the engine decreases, the engine and the governor speed increases. This results in the increase in centrifugal force of balls. Thus the balls moves upwards and sleeve raises upwards.

*__Note:__ The controlling force is provided by the action of gravity as in __Watt Governor__ "__OR__" by a spring in case of "__Hartnell Governor__"

(3)

Used In Governors:-

...ght of the Governor: It is the vertical (or) radial distance from the centre of the ball to the (height) point where the axes of arms intersect on the spindle axes. It is denoted by 'h'.

Equilibrium Speed:- It is the speed at which the governor balls, arms etc.. are in complete equilibrium & the sleeve doesnot move downwards (or) upwards.

3. **Sleeve Lift:** It is the vertical distance which the sleeve travels due to change in equilibrium speed.

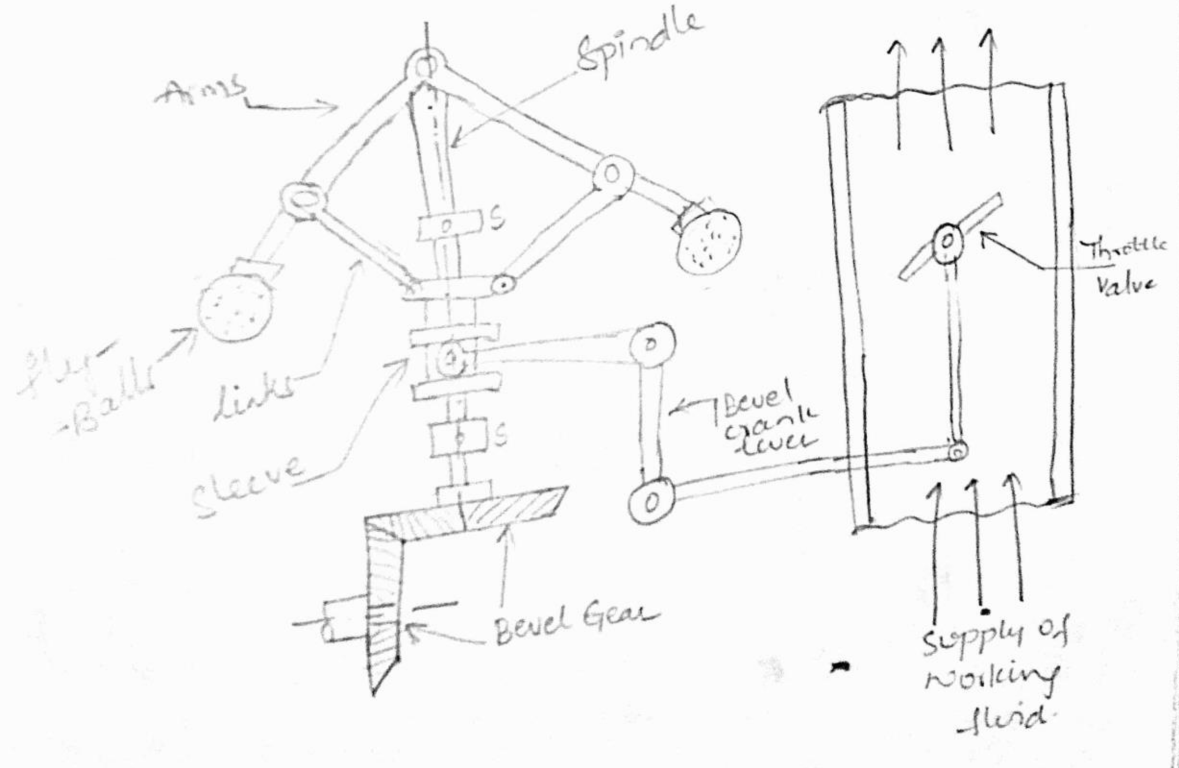


fig: Centrifugal Governor.

(4)

...vernor:

The simplest form of a centrifugal governor is a ... governor.

It is basically a conical pendulum with links attached to sleeve of negligible mass. The arms of governor may be connected to the spindle in 3 ways;

(i) The pivot P, may be on the spindle axis as shown in fig ⓐ.

(ii) The pivot P, may be offset from the axis and the arms intersects at O, when they are produced as shown in fig ⓑ.

(iii) The pivot P, may be offset, but the arms cross the axis at O, as shown in fig ⓒ.

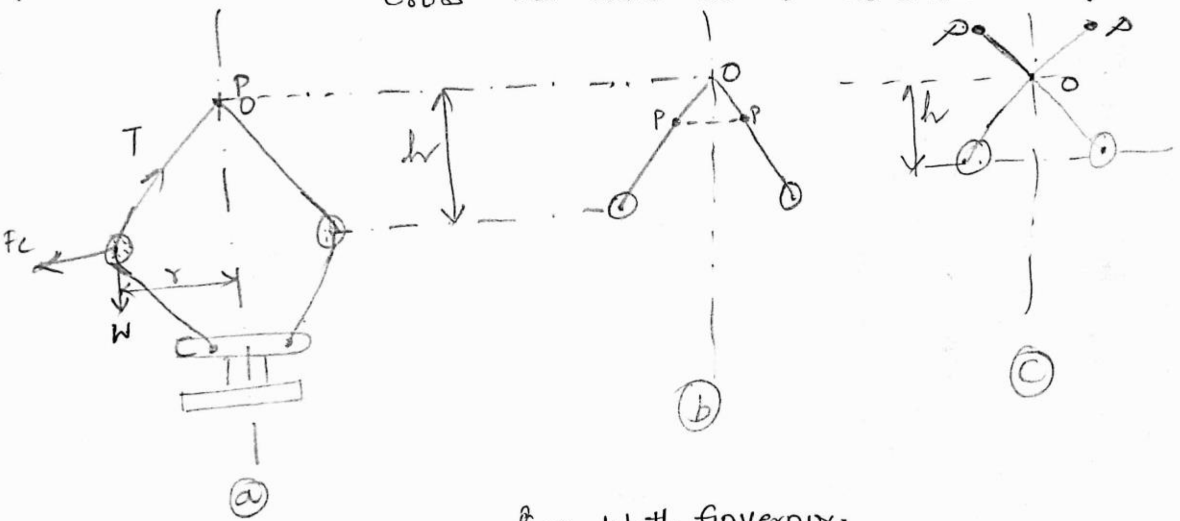


fig: Watt Governor.

Here, m → mass of the ball in kg.

W → weight of the ball in newtons = m.g.

T → Tension in arm in newtons.

ω → Angular velocity of the arm and the ball about spindle axis, in rad/sec.

r → radius of the path of rotation of the ball i.e., horizontal distance from the centre of ball to the spindle axis, in m.

Fc → centrifugal force acting on ball in N = $m r \omega^2$.

h → height of the governors in m.

c is assumed that the weight of arms, links the sleeve are negligible when compared to weight the balls. Now, the ball is in equilibrium condition under the action of (5).

(i) The centrifugal force (F_c) acting on ball.

(ii) The tension (T) in the arm. and.

(iii) The weight of the ball.

Taking the moments about point O, we have,

$$F_c \times h = W \times r$$

$\because F_c = m r \omega^2$ & $W = mg$.

$$\cancel{m}\cancel{r}\omega^2 \times h = \cancel{m} \cdot g \cdot \cancel{r}$$

$$h = \frac{g}{\omega^2} \quad -(1).$$

W.K.T, $\omega = \frac{2\pi N}{60}$.

$$h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2} \Rightarrow h = \frac{9.81}{\left(\frac{2\pi N}{60}\right)^2}$$

$$\therefore \boxed{h = \frac{895}{N^2} \text{ mts}} \quad -(2).$$

NOTE:- from the above expression we can notice that, h is inversely proportional to the N^2. i.e; the height of governor decreases at (larger) high speeds. The governor may only work relatively at low speeds i.e; ~~from~~ 60 to 80 rpm.

Problem (6).

Calculate the vertical height of a watt governor when it rotates at 60 rpm. Also find the change in vertical height when its speed increases to 61 rpm.

Sol: Given data:

$N_1 = 60$ rpm & $N_2 = 61$ rpm.

Initial height:

W.K.T, $h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2}$

$= 0.248$ m.

final height, $h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2}$

$= 0.24$ m.

∴ change in vertical height,

$= h_1 - h_2$

$= 0.248 - 0.24$

$= 0.008$ m

$= 8$ mm.

Problems

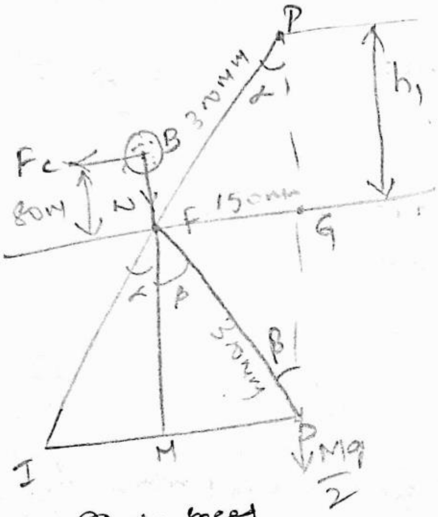
(16).

…oell governor has equal arms of length 300mm. The upper & lower ends of arms are pivoted on the axis of governor. The extension arms of the lower links are each 80mm long & parallel to the axis when the radii of rotation of the balls are 150mm & 200mm. The mass of each ball is 10kg. and the mass of central load is 100kg. Determine the range of speed of governor.

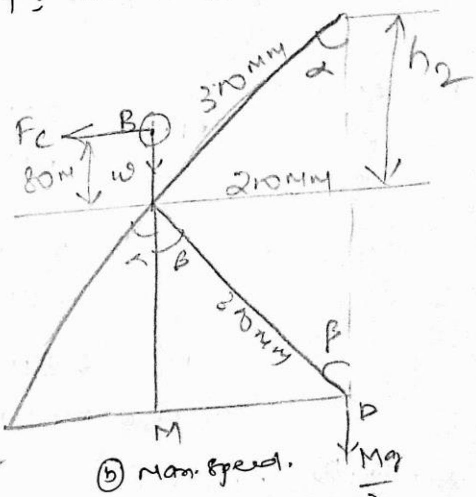
Sol: Given Data:

$PF = DF = 300mm$;

$BF = 80mm$; $r_1 = 150mm$; $r_2 = 200mm$; $m = 10kg$; $M = 100kg$.



(a) Min speed



(b) Max. speed.

Here, we need to find out the Min. & Max. speeds of the governor

i.e; $N_1 \rightarrow$ Min. speed of governor, when $r_1 = 150mm$

$N_2 \rightarrow$ Max. speed of governor, when $r_2 = 200mm$

Before that we have to find out the height's of the governor.

$h_1 = PG.$

By applying pythogorus theorem.

$PG^2 = PF^2 - FG^2$

$= (300)^2 - (150)^2$

$PG = 259.8 \simeq 260\,MM$

$\therefore \boxed{PG = 0.26M = h_1}$

$\& \, FM = PG = h_1$

$= 0.26m.$

$\therefore BM = BF + FM \Rightarrow 80 + 260$

$= 340\,MM \Rightarrow \boxed{BM = 0.34m}$

We know that,

$$N_1^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\frac{895}{h_1} \qquad [\because \alpha = \beta \text{ or } r = 1].$$

$$= \frac{0.26}{0.34}\left[\frac{10+100}{10}\right]\frac{895}{0.26}$$

$$N_1^2 = 28929.15$$

$\therefore N_1 = 170\,rpm.$

Similarly, $h_2 = PG$

$h_2 \Rightarrow PG = \sqrt{PF^2 - FG^2} = \sqrt{300^2 - 210^2} = 224\,mm. = h_2.$

$FM = h_2 = 0.224m = 224MM.$

$BM = BF + FM = 80 + 224 = 304MM = 0.304m.$

Now, $N_2^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right]\frac{895}{h_2}$

$= \frac{0.224}{0.304}\left[\frac{100+10}{10}\right]\frac{895}{0.224}$

$N_2 = 180\,rpm.$

Range of speed $\Rightarrow N_2 - N_1 = 180 - 170$

$= 10\,rpm.$

Following particulars refer to a proell governor with open arms: (11).

→ Length of all arms = 200 mm;

→ Distance of pivot of arms from the axis of rotation = 40 mm

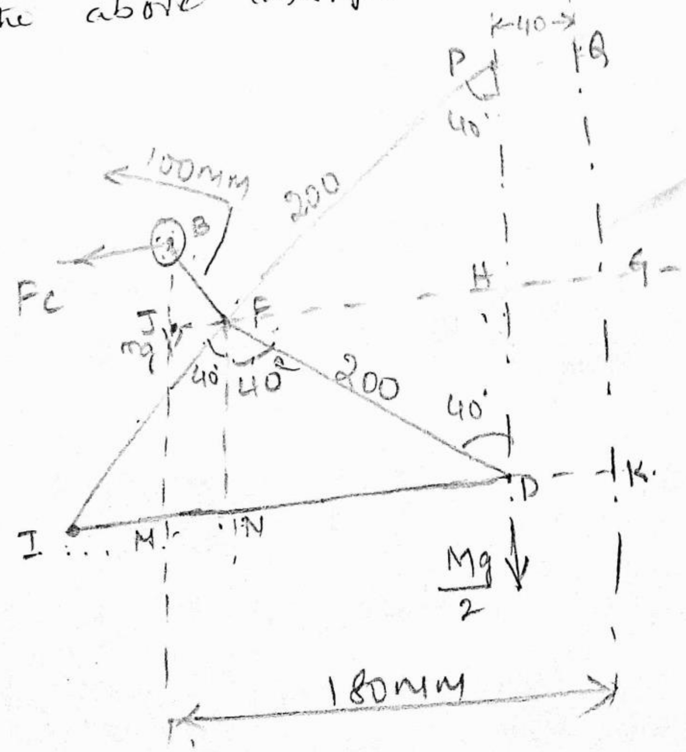
→ length of extension of lower arms to which each ball is attached = 100 mm;

→ mass of each ball = 6 kg.

→ mass of central load = 150 kg.

If the radius of rotation of balls is 180 mm when the arms are inclined at an angle of 40° to the axis of rotation, find the equilibrium speed for the above configuration.

Sol:



(12).

Given Data:

$DF = 200mm$;

$\emptyset = DK = 40\ mm = HG$.

$BF = 100mm$;

$m = 6kg$; $M = 150kg$;

$r = JG = 180mm$; $\alpha = \beta = 40°$.

Let, $N \rightarrow$ Equilibrium Speed.

Taking the moments about I,

$$F_c \times BM = m \cdot g * IM + \frac{M \cdot g}{2} \cdot ID. \quad -(1).$$

calculating BM, IM & ID values.

from the equilibrium position of the governor,

consider Δ^{le} PFH,

$$\cos\alpha = \frac{PH}{PF} \Rightarrow PH = PF \times \cos 40°.$$

$$= 200 \times \cos 40°.$$

$$\boxed{PH = 153.2mm}$$

Similarly, $\sin\alpha = \frac{FH}{PF} \Rightarrow FH = PF \times \sin 40°$

$$= 200 \times \sin 40°$$

$$\boxed{FH = 128.5\ mm}$$

Now, $JF = JG - HG - FH$

$$= 180 - 40 - 128.5$$

$$JF = 11.5\ mm$$

Now from Δ^{le} BJF

applying pythogorus theorem,

$$BJ^2 = BF^2 - JF^2$$

$$= 100^2 - 11.5^2$$

$$BJ = 99.3\ mm.$$

(13).

$BM = BJ + JM$ $\quad [JM = HD = PH]$

$= BJ + PH$

$BM = 99.3 + 153.2$

$\boxed{BM = 252.5 \text{ mm}}$

Now, $IM = IN - MN$

$= FH - JF$

$IM = 128.6 - 11.5$

$\boxed{\therefore IM = 117.1 \text{ mm}}$

$ID = IN + ND$

$= 2 \times IN$ $\quad [\because IN = FH]$

$= 2 \times FH$

$ID = 2 \times 128.6 \Rightarrow 257.2 \text{ mm}$

Sub... BM, ID, IM in eq. (1).

$F_c \times BM = m \cdot g \times IM + \frac{M \cdot g}{2} \times I \cdot D$

$mrw^2 \times 252.5 = (6)(9.81)(117.1) + \frac{150 \times 9.81}{2} \times (257.2)$.

$\left\{(6)(180)\left[\frac{2\pi N}{60}\right]^2\right\} \times (252.5) = 196127.4$

$\left(\frac{2\pi N}{60}\right)^2 = \cancel{\ldots}$ $\quad (6 \times 0.18) \left[\frac{2\pi N}{60}\right]^2 = 776.74$

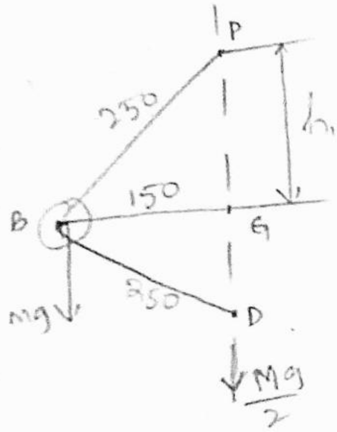
$N = 256 \text{ rpm}$

Problems

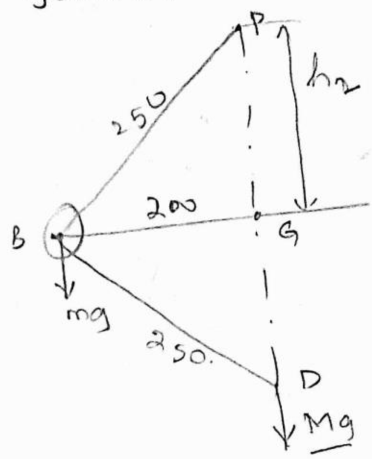
(16)

1. A porter governor has equal arms each 250mm long & pivoted on axis of rotation. Each ball has a mass of 5Kg and the mass of central load on sleeve is 15kg. The radius of rotation of ball is 150mm when the governor begins to lift & 200mm when the governor is at maximum speed. Find the min. & Max. speeds and range of speed of governor.

Sol:



(a) min. pos.



(b) Max. pos.

Given Data:

$BP = BD = 250mm$; $m = 5Kg$; $M = 15Kg$;

$r_2 = 200mm = 0.2m$

$r_1 = 150mm = 0.15m$;

case(i) Minimum speed, when, $r_1 = BG = 0.15m$

Let, N_1 = min. speed.

First we have to ht. of governor (h_1).

$h_1 = PG^2 = PB^2 - BG^2 \Rightarrow \sqrt{(250)^2-(150)^2}$

$\Rightarrow h_1 = 0.2m$.

W.K.T; $N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1}$

$= \frac{5+15}{5} \times \frac{895}{0.2}$

$N_1 = 133.8$ rpm.

case(ii) Max. speed, $BG = 200m$, $r_2 = 0.2m$.

$h_2 = PG^2 = \sqrt{(250)^2-(200)^2}$

$h_2 = 0.15m$.

$N_2^2 = \frac{m+M}{m} \times 895/h_2$

$= \frac{5+15}{5} \times \frac{895}{0.15}$

$N_2 = 154.5$ rpm.

$\therefore$ Range of speed,

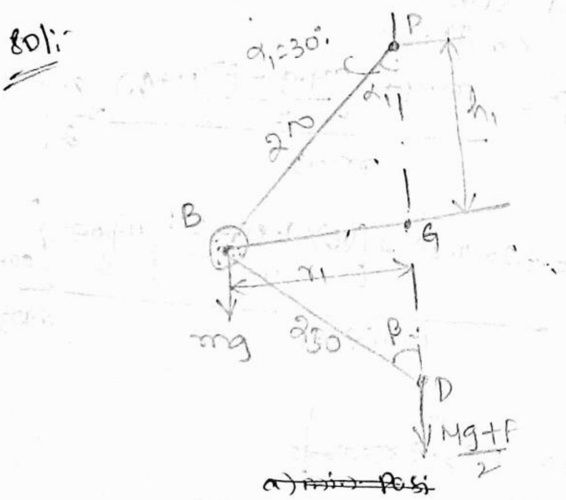
$N_2 - N_1 = 154.5 - 133.8$

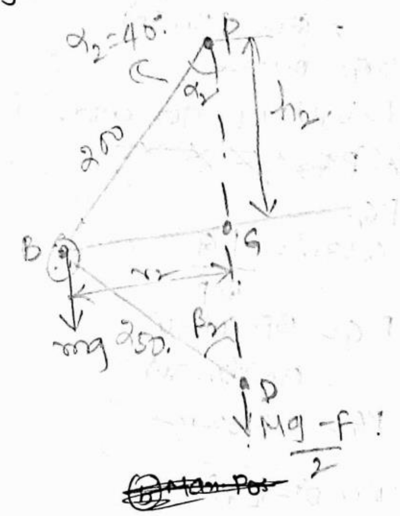
$= 20.7$ rpm.

(17)

...gine governor of the Porter type, the upper & lower arms ...200mm ~~long~~ & 250mm respectively & pivoted on the axis of ...ation. The mass of central load is 15kg and the mass of each ball is 2kg & friction of sleeve together with resistance of operating gear is equal to a load of 24N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° & 40°, find, taking friction into account, range of speed of governor.

Sol:





Given Data:

BP = 200mm; BD = 250mm;

BP = 0.2m; BD = 0.25m;

M = 15kg; m = 2kg; F = 24N; $\alpha_1 = 30°$; $\alpha_2 = 40°$.

Let, $N_1 \rightarrow$ min. speed and

$N_2 \rightarrow$ max. speed.

case(i):

We know that, when the sleeve ~~[illegible]~~ move downwards, the frictional force acts upwards, then the min. speed of governor may be,

$$\boxed{N_1^2 = \frac{m \cdot g + \left[\frac{M \cdot g - F}{2}\right](1+q_1)}{m \cdot g} \times \frac{895}{h_1}} \quad \text{--- (1)}.$$

Here, height, & q are to be calculated and also the value of β.

→ Minimum radius of rotation,

$r_1 = BG$

$\sin\alpha_1 = \frac{BG}{BP}$

$BG = BP \cdot \sin\alpha_1$

$= (200)(\sin 30)$

$r_1 = BG = 0.1m$

→ Now height of governor,

$h_1 = PG$

$\cos 30^\circ = \frac{PG}{BP}$

$PG = BP \cdot \cos 30$

$= (200)(\cos 30)$

$h_1 = PG = 0.1732m$

$DG = \sqrt{(BD)^2 - (BG)^2}$

$= \sqrt{(0.25)^2 - (0.1)^2}$

$DG = 0.23m$

$\tan\beta_1 = \frac{BG}{PG} = \frac{0.1}{0.23}$

$\tan\beta_1 = 0.4348$

$\tan\alpha_1 = \tan 30 = 0.5774$

$\therefore q = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4348}{0.5774}$

$q_1 = 0.753$

Now substitute, $h_1, q_1, F, \ldots$ in eq. ①.

$$N_1^2 = \frac{m \cdot g + \left[\frac{M \cdot g - F}{2}\right](1+q_1)}{m \cdot g} \times \frac{895}{h_1}$$

$$= \frac{(2 \times 9.81) + \left[\frac{(15)(9.81) - 24}{2}\right](1+0.753)}{(2)(9.81)} \times \frac{895}{0.1232}$$

$$\boxed{N_1 = 183.3 rpm}$$

(18)

When the sleeve moves upwards, the frictional force acts downwards, then the max speed of governor will be,

$$\boxed{N_2^2 = \frac{m \cdot g + \left[\frac{M \cdot g + F}{2}\right](1+q_2)}{m \cdot g} \times \frac{895}{h_2}} \quad (2).$$

Now calculating the values of h_2, β_2 & q_2,

Now, $\alpha_2 = 40°$.

Similarly,

→ radius of rotation,

$r_2 = BG$

$\sin\alpha_2 = \frac{BG}{BP}$

$BG = BP \sin\alpha_2$

$= (0.2)(\sin 40°)$

$r_2 = BG = 0.1268 m$

→ Height of governor,

$h_2 = PG$

$\cos\alpha_2 = \frac{PG}{BP}$

$PG = BP\cos\alpha_2$

$= (0.2)\cos 40°$

$PG = 0.1532 m$

Now,

$DG = \sqrt{(BD)^2 - (BG)^2}$

$= \sqrt{(0.25)^2 - (0.1268)^2}$

$\boxed{DG = 0.2154 m}$

$\tan\beta_2 = \frac{BG}{DG} = \frac{0.1268}{0.2154}$

$\tan\beta_2 = 0.59$.

$\tan\alpha_2 = \tan 40° = 0.839$

$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.59}{0.839}$

$q_2 = 0.703$.

Now sub. q_2, h_2 .. in (2).

$$N_2^2 = \frac{(2 \times 9.81) + \left[\frac{(15 \times 9.81) + 24}{2}\right](1+0.703)}{2 \times 9.81} \times \frac{895}{0.1532}$$

$\boxed{N_2 = 222 \text{ rpm}}$

∴ Range of speed,

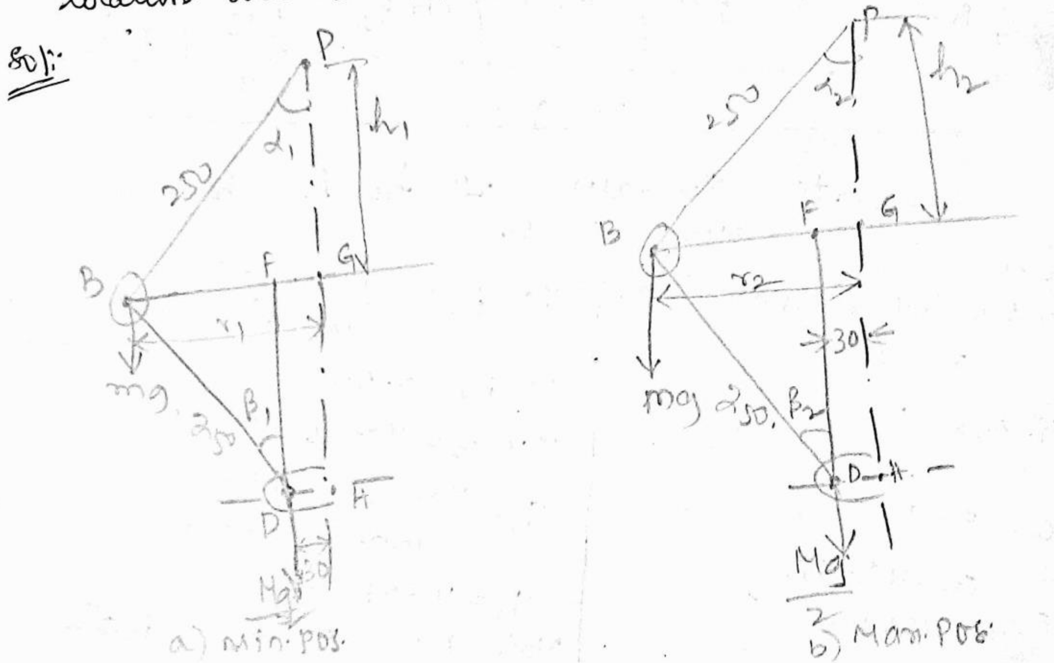
$N_2 - N_1$

$= 222 - 183.3$

$= 38.7$ rpm

③ A porter governor has all four arms 250 mm l... upper arms are attached to axis of rotation & ... arms are attached to the sleeve at a distance 30mm from the axis. The mass of each ball is 5kg the sleeve has a mass of 50kg. The extreme radii of rotation are 150mm & 200mm. Determine the range of speed...

Sol:



a) Min. pos. b) Max. pos.

Given Data:

BD = BP = 250 mm = 0.25 m ; DH = 30 mm ; m = 5 kg ; M = 50 kg ; r_1 = 150 mm & r_2 = 200 mm

Let, N_1 → Min. speed. N_2 → max. speed.

W.K.T,

$$\boxed{N_1^2 = \frac{m + \frac{M}{2}(1+q_1)}{m} \times \frac{895}{h_1}} \quad (1)$$

here, we have to calculate h_1 & q_1.

(19)

of the governor,

$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(250)^2 - (150)^2}$

$h_1 = PG = 200 mm$

$h_1 = 0.2 m$

$BF = BG - \cancel{BG}$

$= 150 - 30$

$\boxed{BF = 120 mm}$

$\tan\alpha_1 = \frac{BG}{PG} = \frac{150}{200} = 0.75.$

$DF = \sqrt{(250)^2 - (120)^2}$

$DF = 219 mm.$

$\tan\beta_1 = \frac{BF}{DF}$

$= \frac{120}{219}$

$\tan\beta_1 = 0.548$

$q_1 = \frac{\tan\beta_1}{\tan\alpha_1}$

$q_1 = \frac{0.548}{0.75}$

$q_1 = 0.731$

$N_1^2 = \frac{m + \frac{M}{2} \cdot g(1+q_1)}{m} \times \frac{895}{h_1}$

$= \frac{5 + \frac{50}{2}(1+0.731)}{5} \times \frac{895}{0.2}$

$N_1 = 208 rpm.$

Case(ii):-

$$\boxed{N_2^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2}}$$

→ Height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2}$$

$$= \sqrt{(250)^2 - (200)^2}$$

$$h_2 = 150\ mm = 0.15\ m$$

$$BF = BG - FG$$

$$= 200 - 30 = 170\ mm = 0.17\ m.$$

$$DF = \sqrt{(DB)^2 - (BF)^2}$$

$$= \sqrt{(250)^2 - (170)^2}$$

$$DF = 183\ mm$$

$$\tan\beta_2 = \frac{BF}{DF} = \frac{170}{183} \Rightarrow \tan\beta_2 = 0.93$$

$$\tan\alpha_2 = \frac{BG}{PG} = \frac{200}{150} \Rightarrow \tan\alpha_2 = 1.33$$

$$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.93}{1.33} \Rightarrow q_2 = 0.7.$$

Now,

$$N_2^2 = \frac{5 + \frac{50}{2}(1+0.7)}{5} \times \frac{895}{0.15}$$

$$N_2 = 238\ rpm.$$

∴ The range of speed, $= N_2 - N_1$

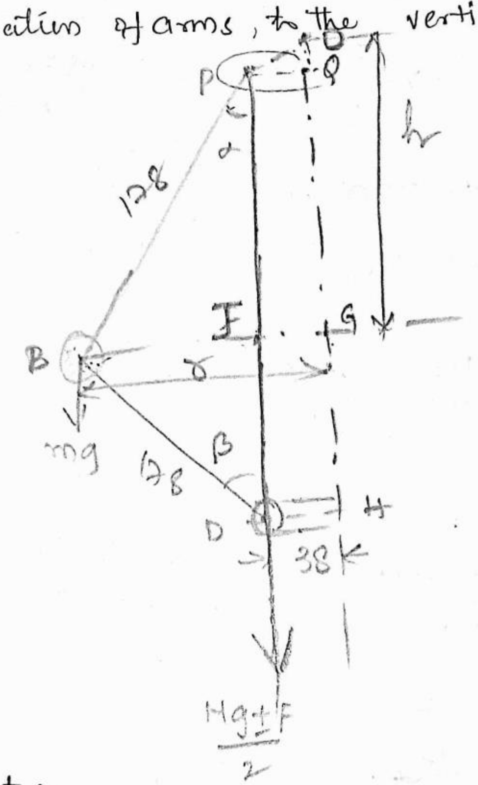
$$= 238 - 208$$

$$= 30\ rpm.$$

(5) (20).

...rms of a porter governor are 178 mm long and are ...ged at a distance of 38 mm from the axis of rotation. ...e mass of each ball is 1.15 kg and mass of sleeve is 20 kg. The governor sleeve begins to rise at 280 rpm. when the links are at an angle of 30° to the vertical. Assuming the frictional force to be constant, determine the minimum & maximum speed of rotation when the inclination of arms, to the vertical is 45°.

Sol:-



Given Data:-

BP = BD = 178 mm; DH = JG = 38 mm; N = 280 rpm;

m = 1.15 kg; M = 20 kg; α = β = 30°.

case(i):-

First we find the friction when inclination will be 30°.

→ radius of rotation will be,

$r = BG = BJ + JG$

$BJ = BP \sin\alpha$

$BJ = 178 \times \sin 30°$

~~BJ = ?~~

$BG = 178 \times \sin 30 + 38$

$\therefore r = BG = 127\,mm$

$r = BG$

$= BJ + JG$

$= BP \sin\alpha + JG$

$= 178 \sin 45° + 38$

$\boxed{r = 164\,mm}$

and height of the governor,

$h = \frac{BG}{\tan\alpha}$

$= \frac{164}{\tan 45°}$

$h = 164\,mm = 0.164\,m.$

let, $N_1 \rightarrow$ min. speed of rotatn of
$N_2 \rightarrow$ max. speed of rotatn.

w.k.t,
$$N_1^2 = \frac{m.g + [M.g - F]}{m.g} \times \frac{895}{h}$$

$$= \frac{(1.15)(9.81) + \{(20 \times 9.81) - 10\}}{(1.15)(9.81)} \times \frac{895}{0.164}$$

$N_1 = 309$ rpm.

and height of Governor,

$$h = \frac{BG}{\tan\alpha}$$

$$h = 127/\tan 30^\circ.$$

$$h = 220\,mm = 0.22\,m.$$

W.K.T,

$$N^2 = \frac{m.g + [M.g \pm F]}{m.g} \times \frac{895}{h} \qquad [\because \alpha = \beta = 30^\circ; \; q = 1].$$

$$(280)^2 = \frac{(1.15 \times 9.81) + [(20)(9.81) \pm F]}{(1.15 \times 9.81)} \times \frac{895}{0.22}$$

$$\pm F = \frac{(280)^2 \times (1.15) \times (9.81) \times 0.22}{895} - (1.15 \times 9.81) - (20 \times 9.81)$$

$$\boxed{F = 10\,N}$$

When the inclination of the arms to the vertical is 45°, i.e; $\alpha = \beta = 45^\circ$. with considering frictional force, $F = 10\,N$.

$$N_1^2 = \frac{m.g + [M.g - F]}{m.g} \times \frac{895}{h} \quad \because [\alpha = \beta = 45^\circ. \; q = 1].$$

$$N_2^2 = \frac{m.g + [M.g + F]}{m.g} \times \frac{895}{h}$$

and $N_2^2 = \dfrac{m \cdot g + [M \cdot g + F]}{m \cdot g} \times \dfrac{895}{h}$

$$= \frac{(1.15)(9.81) + [(20)(9.81) + 10]}{(1.15)(9.81)} \times \frac{895}{0.164}$$

$N_2 = 324$ rpm

and range of speed, $= N_2 - N_1$

$= 324 - 309$

$= 15$ rpm.

(22)

Governor:-

A Hartnell Governor is a spring loaded governor. consists of two bell crank levers pivoted at points O, O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB & a roller at the end of horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on sleeve.

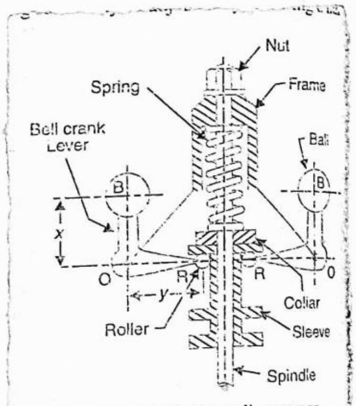


Fig. 18.18. Hartnell governor.

Let, $m \rightarrow$ mass of each ball in kg,

$M \rightarrow$ mass of sleeve, in kg

$r_1 \rightarrow$ minimum radius of rotation, mts.

$r_2 \rightarrow$ maximum radius of rotation, mts.

$\omega_1 \rightarrow$ Angular speed of governor at min. radius, rad/s.

$\omega_2 \rightarrow$ Angular speed of governor at max. radius, rad/s.

$S_1 \rightarrow$ Spring force exerted on the sleeve at ω_1, in N.

$S_2 \rightarrow$ Spring force exerted on the sleeve at ω_2, in N

$F_{c1} \rightarrow$ centrifugal force at ω_1, N $F_{c1} = m r \omega_1^2$

$F_{c2} \rightarrow$ centrifugal force at ω_2, N, $F_{c2} = m r \omega_2^2$.

$s \rightarrow$ Stiffness of the spring

$x \rightarrow$ length of the vertical or ball arm of lever, mts.

$y \rightarrow$ length of horizontal or sleeve arm of lever, mts.

$r \rightarrow$ Distance of fulcrum 'O' from the governor axis (or) radius of rotation when the governor is in mid position.

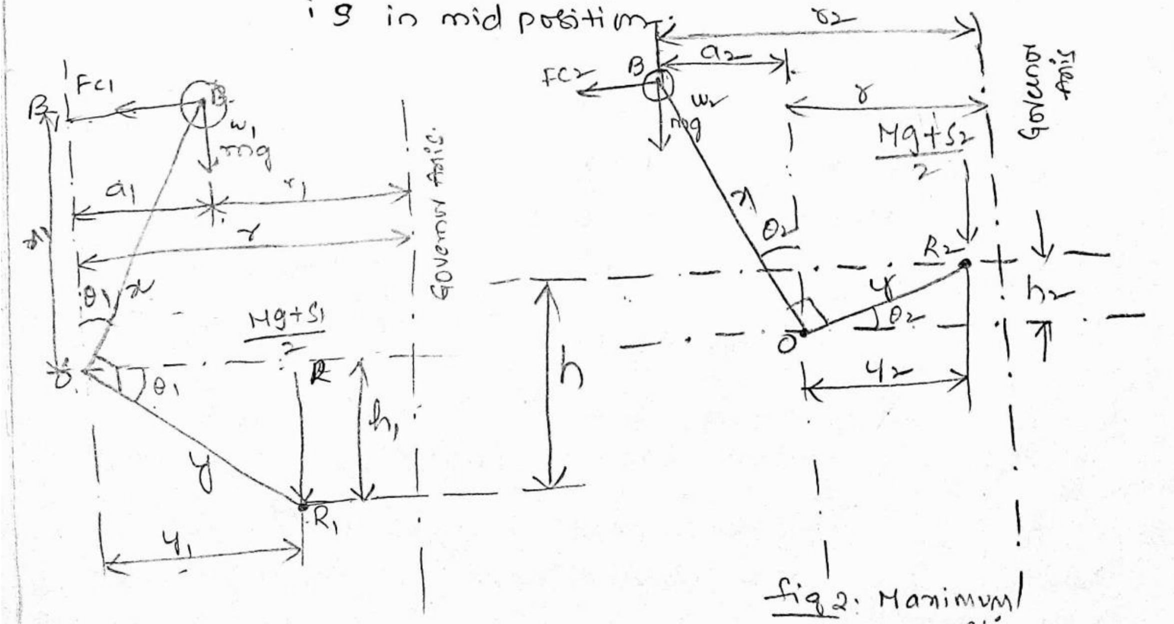


fig 1: Minimum Position

fig 2: Maximum position

(53).

...nsider the forces acting at one ball crank lever. The minimum & maximum position are shown in fig① & fig.②.

Let, $h \rightarrow$ the compression of the spring when radius of rotation changes from r_1 to r_2.

* For minimum position :
i.e; when the radius of rotation changes from ~~...~~ r to r_1, as shown in fig①.
the compression of the spring, or lift of sleeve h_1 is given by,

$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{BB_1}{OB}$

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad — ①.$$

for maximum position, i.e; when the radius of rotation changes from r to r_2, as shown in fig②, the compression of spring, or lift of sleeve h_2,

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad — ②$$

Adding Eq. ① + ②.

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x}$$

Sleeve lift, h (or) $\boxed{h = (r_2 - r_1) \times \frac{y}{x}}$ — ③.

Now for minimum position, taking moments about 'O',

$$\frac{M\cdot g + S_1}{2} \times y_1 = F_{C_1} \times r_1 - m\cdot g \times a_1 \quad -(4)$$

$$\left(M\cdot g + S_1 = \frac{2}{y_1}\left[F_{C_1} \times r_1 - m\cdot g \times a_1\right] \quad -(4) \right)$$

Now taking moments again 'O' for maximum position,

$$\frac{M\cdot g + S_2}{2} \times y_2 = F_{C_2} \times r_2 + m\cdot g \times a_2 \quad -(5)$$

$$\left(Mg + S_2 = \frac{2}{y_2}\left[F_{C_2} \times r_2 + m\cdot g \times a_2\right] \quad -(5) \right)$$

Substracting eq. (4) from eq. (5).

$$S_2 - S_1 = \frac{2}{y_2}\left[F_{C_2} \times r_2 + m\cdot g \times a_2\right] - \frac{2}{y_1}\left[F_{C_1} \times r_1 - m\cdot g \times a_1\right]$$

Let, $S_2 - S_1 = h\cdot S$ & w.k.t,

$$h = (r_2 - r_1)\cdot\frac{y}{x}.$$

$$\boxed{S = \frac{S_2 - S_1}{h} = \frac{S_2 - S_1}{(r_2 - r_1)} \times \frac{x}{y}}$$

Neglecting obliquity effect of the arms (i.e; $x_1 = x_2 = x$ & $y_1 = y_2 = y$) and the moment due to weight of ball, (m.g.),

(5) (24).

minimum position,

$$\frac{M.g+S_1}{2} \times y = F_{C1} \times r \quad \text{or}$$

$$M.g+S_1 = \frac{2}{y}[F_{C1}\times r]. \quad -(6)$$

for maximum position,

$$\frac{Mg+S_2}{2} \times y = F_{C2} \times r \quad \text{or.}$$

$$Mg+S_2 = \frac{2}{y}[F_{C2}\times r]. \quad -(7).$$

$Mg+S_2 - Mg+S_1$

substituting (6) from (7),

$$S_2 - S_1 = \frac{2}{y}[F_{C2}-F_{C1}]\times r$$

$$\boxed{S_2 - S_1 = 2[F_{C2}-F_{C1}]\times \frac{r}{y}} \rightarrow (8)$$

let, $S_2 - S_1 = h.s$; $h = (r_2 - r_1)\left(\frac{y}{r}\right)$.

~~h.s~~, sub. ... in eq (8).

$$h.s = 2[F_{C2}-F_{C1}]\times \frac{r}{y}$$

$$(r_2-r_1)\cdot\frac{y}{r}\cdot s = 2[F_{C2}-F_{C1}]\times\frac{r}{y}$$

$$\boxed{s = 2\left[\frac{F_{C2}-F_{C1}}{r_2-r_1}\right]\times\left[\frac{r}{y}\right]^2} \quad -(9).$$

s → stiff

NOTE:-

1. When the friction is taken into account, the weight of the sleeve M.g may be replaced by M.g±F.

2. The centrifugal force for any intermediate position i.e; between minimum & Maximum position, at a radius of rotation "r" may be obtained as;

Since, the stiffness for a given spring is constant for all positions, therefore for minimum & intermediate position,

$$S = 2\left[\frac{F_C - F_{C1}}{r - r_1}\right]\left[\frac{x}{y}\right]^2 \quad - \quad (10)$$

& for intermediate & Maximum position,

$$S = 2\left[\frac{F_{C2} - F_C}{r_2 - r}\right]\left[\frac{x}{y}\right]^2$$

∴ from eq's (8), (9) & (10).

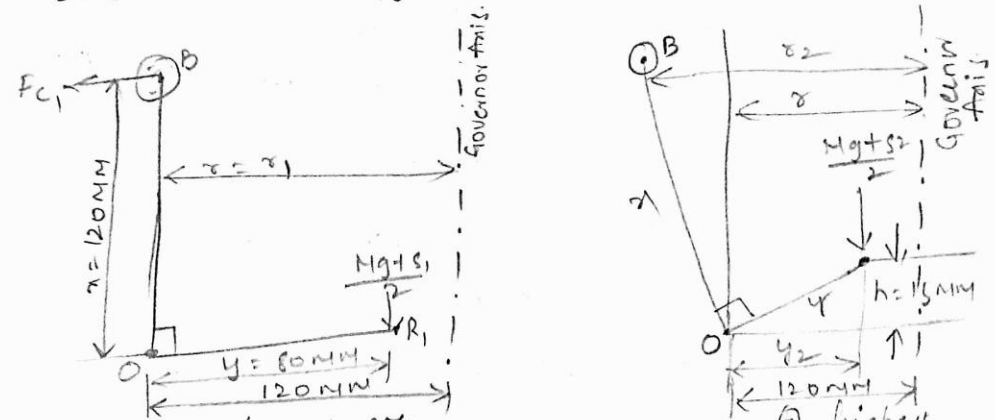
$$\frac{F_{C2} - F_{C1}}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1} = \frac{F_{C2} - F_C}{r_2 - r}$$

Problem

(25)

...nell governor having a central sleeve spring & 2 right angled bell crank levers moves between 290 r.p.m & 310 rpm for a sleeve lift of 15mm. The sleeve arms and ball arms are 80mm & 120mm resp. The levers are pivoted at 120mm from governor axis and mass of each ball is 2.5kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine: 1. loads on the spring at the lowest and highest equilibrium speeds & 2. stiffness of spring.

Sol:



a) lowest pos. (b) highest pos

Given Data:-

$N_1 = 290$ rpm

$\omega_1 = \frac{2\pi \times 290}{60}$

$\omega_1 = 30.4 \frac{rad}{sec}$

$N_2 = 310$ rpm

$\omega_2 = \frac{2\pi \times 310}{60}$

$\omega_2 = 32.5 \frac{rad}{sec}$

$h = 15MM$

$h = 0.015M$; $y = 80MM$

$y = 0.08m$

$x = 120MM = 0.12m$; $r = 120MM$

$r = 0.12m$;

$m = 2.5kg$.

1. Loads on the spring at lowest & highest equilibrium speeds.

Let, S_1 = Spring load at lowest equilibrium speed.

S_2 = " " " highest " " .

Since, the ball arms are parallel to governor axis at lowest equilibrium speed (i.e; $N_1 = 290$ rpm)

$\therefore r = r_1 = 120MM = 0.12m$.

W.k.t, F_c at min. speed,

$F_{c_1} = m r_1 \omega_1^2 = (2.5) \times (0.12)(30.4)^2 = 277\,N$

Before finding F_c for max. speed, i.e; $F_{c_2} = m r_2 \omega_2^2$.
we have to know r_2.

Firstly let us calculate 'r_2' i.e. radius of rotation at highest speed.

W.k.t, $h = (r_2 - r_1) \cdot \frac{y}{x}$

$$r_2 = (h + r_1) \times \frac{x}{y}$$

$$= (0.015 + 0.12) \times \frac{0.12}{0.08}$$

$$r_2 = 0.1425\,m.$$

Now, $F_{c_2} = m r_2 \omega_2^2 = (2.5) \times (0.1425)(32.5)^2$

$$F_{c_2} = 376\,N.$$

Neglecting obliquity effect of ~~arms~~ & moment due to weights,

for min. posi. $(M \cdot g) + S_1 = 2 F_{c_1} \times \frac{x}{y}$

$$S_1 = 2 \times 277 \times \frac{0.12}{0.08} = 831\,N.$$

for max. pos. $M \cdot g + S_2 = 2 F_{c_2} \times \frac{x}{y}$

$$= 2 \times 376 \times \frac{0.12}{0.08}$$

$$S_2 = 1128\,N$$

2. Stiffness of spring:

$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8\ N/mm$$

26

Governor:

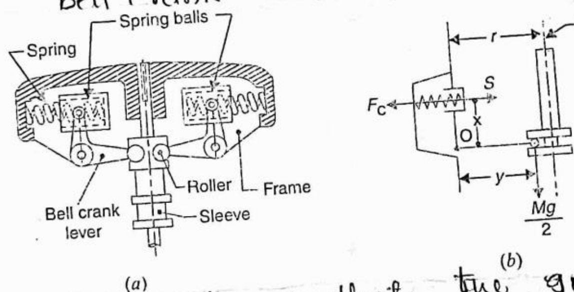
A Spring controlled governor of the Hartung type is shown in fig. In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of governor when the rollers at the horizontal arm press against the sleeve.

Let, S → Spring force.

F_c → centrifugal force.

M → mass on the sleeve &

x & y → lengths of vertical & horizontal arm of bell crank lever respectively.



The fig. (a) & (b) shows that the governor in mid-position. Neglecting the effect of obliquity of arms, taking moments about fulcrum 'O',

$$\boxed{F_c \times x = S \times x + \frac{Mg}{2} \times y.}$$

Problem

(1) In a spring-controlled governor of the Hartung type, the length of the ball and sleeve arms are 80mm and 120mm respectively. The total travel of the sleeve is 25mm. In the mid position, each spring is compressed by 50mm and the radius of rotation of mass centres is 140mm. Each ball has a mass of 4kg and the spring has a stiffness of 10kN/m of compression. The equivalent mass of the governor gear at the sleeve is 16kg. Neglecting the moment due to revolving masses when the arms are inclined, determine the ratio of the range of speed to the mean speed of the governor, find, also, the speed in the mid-position.

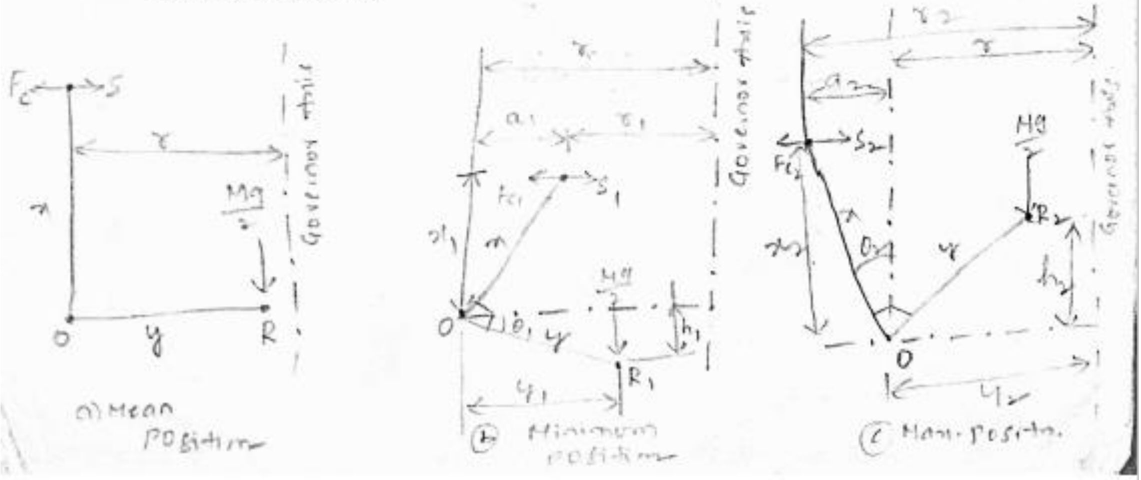
Sol: Given, $x = 80mm = 0.08m$; $y = 120mm = 0.12m$; $h = 25mm$, $h = 0.025m$

$r = 140mm = 0.14m$; $m = 4kg$; $M = 16kg$;

$s = 10 kN/m = 10 \times 10^3 N/m$; initial compression $= 50mm = 0.05m$.

* Mean Speed of the Governor:-

Let us find out the mean speed of the governor, i.e., the speed when the governor is in mid-position.



$\omega \rightarrow$ Mean Angular speed in rad/s (27).

$N \rightarrow$ Mean Speed, rpm.

centrifugal force acting on ball spring,

$$F_c = mr\omega^2 = 4 \times \omega^2 \times 0.14$$

$$F_c = 0.56\omega^2 \text{ N.}$$

Spring force, S = Stiffness X Initial compression.

$$S = 10 \times 10^3 \times 0.05$$

$$S = 500 \text{ N.}$$

Now taking moments about pt. O, neglecting ~~amount~~ moment due to revolving masses, we have,

$$F_c \times x = S \times x + \frac{Mg}{2} \times y.$$

$$0.56\omega^2 \times 0.08 = 500 \times 0.08 + \frac{16 \times 9.81}{2} \times 0.12$$

$$0.56\omega^2 = \frac{49.42}{0.08}$$

$$0.56\omega^2 = 617.7$$

$$\omega^2 = 1103.0$$

$$\omega = 33.2 \text{ rad/s.}$$

$$\frac{2\pi N}{60} = 33.2$$

$$N = 317 \text{ rpm.}$$

* Ratio of Range of Speed to Mean Speed.

Let, $\omega_1 \rightarrow$ min. angular speed $\frac{rad}{s}$, at min. radius of rotation r_1.

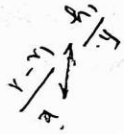
$\omega_2 \rightarrow$ Max. angular speed, at max. radius of rotation. r_2.

N_1 & $N_2 \rightarrow$ corresponding min & Max. speeds, rpm.

first let us find the minimum speed, N_1.

from fig (b).

$$\frac{r-r_1}{h_1} = \frac{x}{y} \qquad \therefore h_1 = h/2.$$



$$\frac{0.14 - r_1}{\frac{0.025}{2}} = \frac{0.08}{0.12}$$

$$\boxed{r_1 = 0.132\,m}.$$

w.k.t, centrifugal force at minimum position.

$$F_C = m r_1 \omega_1^2$$

$$= (4)(0.132)(\omega_1^2)$$

$$F_C = 0.528\,\omega_1^2\ N$$

Spring force at the min. pos.

$$S_1 = \left[\text{initial compr.} - (r-r_1)\right] \times \text{Stiffness}$$

$$= \left[0.05 - (0.14 - 0.132)\right] \times 10 \times 10^3.$$

$$S_1 = 420\,N.$$

(28)

Taking moments about 'O', neglecting obliquity of arms.

$F_{c_1} \times x = S_1 \times x + \frac{M \cdot g}{2} \cdot y$

$0.528\omega_1^2 \times 0.08 = (420) \times (0.08) + \frac{16 \times 9.81}{2} \times 0.12$

$\omega_1^2 = 1019$

$\omega_1 = 31.92$.

$\frac{2\pi N_1}{60} = 31.92$

$N_1 = 304.83$ rpm.

Similarly for max. position,

max. speed N_2, $\frac{r_2 - r}{h_2} = \frac{x}{y}$

$h_2 = h/2$

$\underline{r_2 = 0.148\,m}$

$F_{c_2} = m r_2 \omega_2^2$: $F_c = 4 \times (0.148)\omega_2^2 \Rightarrow F_c = 0.592\,\omega_2^2$.

+ Spring force, $S_2 =$ [Initial compr. + $(r_2 - r)$] × Sti

$= [0.05 + (0.148 - 0.14)] \times 10 \times 10^3$

$\underline{S_2 = 580 N}$.

Taking moments about 'O',

$F_c \times x = S_2 \times x + \frac{Mg}{2} \times y$.

$0.592\omega_2^2 \times 0.08 = 580 \times 0.08 + \frac{16 \times 9.81}{2} \times 0.12$

$\omega_2 = 34.32$

$N_2 = 327.7$ rpm

Range of speed, $N_2 - N_1 = 327.7 - 304.83$

$= 22.8$ rpm.

∴ Ratio Range of speed to mean speed,

$\frac{22.8}{317} = 0.07 = \underline{\underline{7\%}}$

* Sensitiveness of Governor:-

Consider 2 governors A & B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that is sufficient to change the energy supplied to the engine by required amount. For this reason, the sensitiveness is defined, ~~for the~~ as the "ratio of difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed."

Let, $N_1 \rightarrow$ minimum equilibrium speed.

$N_2 \rightarrow$ maximum equilibrium speed.

$N \rightarrow$ Mean equilibrium speed $= \dfrac{N_1+N_2}{2}$.

Sensitiveness of governor, $= \dfrac{N_2-N_1}{N} = \dfrac{2(N_2-N_1)}{N_1+N_2}$.

$$= \frac{2(\omega_2-\omega_1)}{(\omega_1+\omega_2)} \rightarrow \text{(In terms of angular speed)}.$$

...ning ... cases of slee...

~~Power~~ Effort of Governor:

The effort of a governor is the mean force exerted at sleeve for given % change of speed. It may be noted that when G is running steadily, ~~there~~ there is no force at sleeve. But when the speed changes, there is a resistance at the sleeve which opposes the motion. It is assumed that this resistance which is equal to the effort, varies uniformly from a max. value to zero while governor moves into its new positn of equilibrium

~~Power:~~

~~P = Effort x height~~

Power of the ~~G~~

(29)

Stability of Governors:

A governor is said to be stable, when for every speed within the working range there is only one radius of rotatn of governor balls at which the governor is in equilibrium. For a stable governor if the ~~governor~~ equilibrium speed ↑, the radius of rotatn of ↑.

NOTE: for an unstable governor, radius of rotatn decreases for ↑ in speed.

Isochronous

Governor is said to be isochronous when equilibrium speed is constant (range of speed '0') for all radii of rotatn of balls within working range, neglecting frictn. The isochronism is the stage of infinite sensitivity. $N_2 - N_1 = 0$.

Hartnell G: $\frac{Mg + S_1}{Mg + S_2} = \frac{r_1}{r_2}$

Porter governor (isochronous) $h_1 = h_2$ & $(N_1 = N_2)$

No practical use for isochronism, because the sleeve moves to its extreme positn immediately the speed deviates from its isochronous speed.

*** Hunting:**

A governor is said to be hunt, if the speed of engine fluctuates continously above & below the mean speed. This is caused by too sensitive Governor which changes fuel supply by a large amount when a small change in speed of rotation takes place.

*** Power of a Governor:-** (30)

Power of the governor is the work done at the sleeve for a given % change of speed. It is the product of the mean value of the effort & the distance through which the sleeve moves.

Mathematically, Power = Mean Effort × lift of sleeve.

*** Effort & Power of a Porter Governor**

The effort & power of a porter governor may be discussed below:

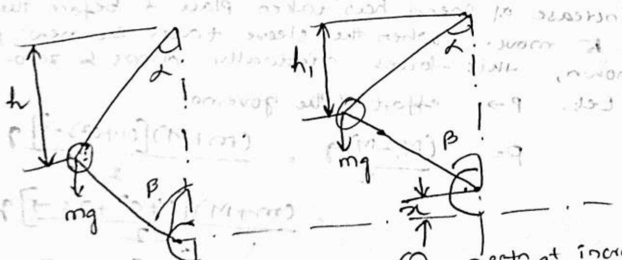
Let, $N \rightarrow$ Equilibrium speed corresponding to the configuration as shown

$c \Rightarrow$ % increase in speed.

∴ increased in speed $= c \cdot N$

∴ increased speed $= N + c \cdot N = N(1+c)$

The equilibrium position of the governor at the increased speed



(a) Position at Equilibrium speed (b) Position at increased speed.

Assuming $\alpha = \beta$ ∴ $q = 1$

then $$h = \frac{m+M}{m} \times \frac{895}{N^2} \quad -(1)$$

When the increase of speed takes place, a downward force P will have to exert on sleeve in order to prevent the sleeve from rising. If the speed increases to $N(1+c)$ & ht. of the governor remains same, the load on the sleeve increases to $M_1 g$

$$\therefore \quad h = \frac{m+M_1}{m} \times \frac{895}{(1+c)^2 N^2} \qquad -(2)$$

Equating eq's (1)&(2)

$$m+M = \frac{m+M_1}{1+c^2}$$

$$M_1 = (m+M)(1+c^2) - m$$

Substituting M on b.s.

$$M_1 - M = (m+M)(1+c^2) - m - M$$

$$M_1 - M = (m+M)\left[(1+c^2) - 1\right] \qquad -(3)$$

A little consideratn will show that, $(M_1 - M)g$ is the downward force which must be applied in order to prevent the sleeve from rising as speed increases. It is the same force which acts on the governor sleeve immediately after the increase of speed has taken place & before the sleeve begins to move. When the sleeve takes the new positn, this forces gradually drops to zero.

as shown,

Let, $P \rightarrow$ effort of the governor

$$P = \frac{(M_1 - M)g}{2} = \frac{(m+M)\left[(1+c^2) - 1\right]g}{2}$$

$$= \frac{(m+M)\left[\cancel{1}+c^2+2c-\cancel{1}\right]g}{2}$$

$$= \cancel{\frac{(m+M)\, c\,[c+2]\, g}{2}}$$

$$\cancel{P = c(m+M)\,2\,g.}$$

neglecting c^2 being very small

$$= \frac{(m+M)(2c)\cdot g}{2}$$

$$\boxed{P = c\cdot(m+M)\,g} \qquad -(4)$$

31

If the frictional force F acts at the sleeve, then

$$P = c\,[m\cdot g + Mg \pm F]$$

W.K.T, Power is the product of governor effort & sleeve lift

Let $x \rightarrow$ sleeve lift.

$\therefore$ Governor power $= P \times x$ — (5)

If the height of the governor at speed N is h & at an increased speed (1+c)N is h_1, then,

$$x = 2(h - h_1) \quad \text{— (6)}$$

$$\therefore \quad h = \frac{m+M}{m} \times \frac{895}{N^2} \quad \& \quad h_1 = \frac{m+M}{m} \cdot \frac{895}{(1+c^2)N^2}$$

$$\frac{h_1}{h} = \frac{\left(\frac{m+M}{m}\right)\frac{895}{N^2(1+c^2)}}{\left(\frac{m+M}{m}\right)\frac{895}{N^2}} = \frac{1}{1+c^2}$$

$$\frac{h_1}{h} = \frac{1}{1+c^2} \Rightarrow h_1 = \frac{h}{1+c^2}$$

$\therefore$ sub. h_1 in (6) eq.

$$x = 2\left[h - \frac{h}{1+c^2}\right] = 2h\left[1 - \frac{1}{1+c^2}\right]$$

$$= 2h\left[\frac{\cancel{1}+c^2+2c-\cancel{1}}{1+c^2+2c}\right]$$

neglecting c^2

$$x = 2h\left[\frac{2c}{1+2c}\right] \quad \text{— (7)}$$

sub. x in eq (5).

$$\therefore \text{Governor Power} = c(m+M)\cdot g \times 2h\left[\frac{2c}{1+2c}\right]$$

$$= \left(\frac{4c^2}{1+2c}\right)(m+M)\cdot g h$$

Prob (1) A porter governor has equal arms each 250mm long & pivoted on the axis of rotatn. Each ball has a mass of 5kg. & central mass 25kg. The radius of rotatn of the ball is 150mm when the governor begins to lift & 200mm at max. speed. Find the range of speed, sleeve lift, governor effort & power of governor when:

(i) when frictn neglected, 1. frictn is considered as 10N
2. " is neglected.

$N_2 - N_1 = 25$ rpm

Sol:
sleeve lift,

$$x = 2(h_1 - h_2) = 2(200 - 150) = 0.1m$$

~~Power~~ effort of governor,

$$P = C(m+M)g.$$

$$\boxed{C N_1 = N_2 - N_1} \Rightarrow CN_1 = 25$$

$$C = \frac{25}{164} = 0.152.$$

$$P = 0.152(5+25)9.81) = 44.7N$$

Power of Governor,

$$= P.x = 44.7 \times 0.1 = 4.47 \text{ N-m}.$$

* when frictn is considered

$$N_2 - N_1 = 31.4 \text{rpm}$$

$$C \cdot N_1 = N_2 - N_1$$

$$C = \frac{31.4}{161} \Rightarrow C = 0.195$$

Governor effort, $P = C\left[mg + Mg + F\right]$

$$= 0.195\left[(5 \times 9.81) + (25 \times 9.81 + 10)\right]$$

$$= 57.4N$$

Power $= P \times x$

$$= 57.4 \times 0.1$$

$$\boxed{\text{Power } 5.74 \text{ N-m}}$$

Balancing of Masses

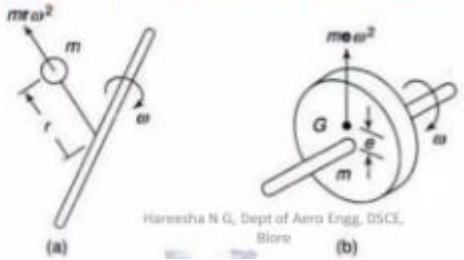
Unit 4: Balancing of Rotating Masses

- **Static and dynamic balancing**
- **Balancing of** single rotating mass by balancing masses in same plane and in different planes.
- Balancing of several rotating masses by balancing masses in same plane and in different planes.

What is Balancing ?

- Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses.
- Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

- A particle or mass moving in a circular path experiences a centripetal acceleration and a force is required to produce it.
- An equal and opposite force acting radially outwards acts on the axis of rotation and is known as centrifugal force .
- This is a disturbing force on the axis of rotation, the magnitude of which is constant but the direction changes with the rotation of the mass.
- In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of the rotor lies on the axis of the shaft.
- When the centre of mass does not lie on the axis or there is an eccentricity, an unbalanced force is produced



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Why Balancing is necessary?

- The high speed of engines and other machines is a common phenomenon now-a-days.
- It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible.
- If these parts are not properly balanced, the dynamic forces are set up.
- These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Balancing of Rotating Masses

- Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it.
- In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass.
- This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite.
- The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called ***balancing of rotating masses.***

Balancing of Rotating Masses

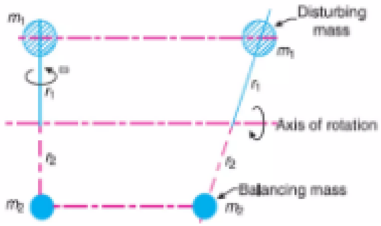
- The following cases are important from the subject point of view:
  1. Balancing of a single rotating mass by a single mass rotating in the same plane.
  2. Balancing of a single rotating mass by two masses rotating in different planes.
  3. Balancing of different masses rotating in the same plane.
  4. Balancing of different masses rotating in different planes.

Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

- Consider a disturbing mass *m*1 attached to a shaft rotating at ω rad/s as shown in Fig.
- Let *r*1 be the radius of rotation of the mass *m*1 (*i.e. distance between the axis of rotation of the shaft* and the centre of gravity of the mass *m*1).
- We know that the centrifugal force exerted by the mass *m*1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

- This centrifugal force acts radially outwards and thus produces bending moment on the shaft.
- In order to counteract the effect of this force, a balancing mass (*m*2) may be attached in the same plane of rotation as that of disturbing mass (*m*1) such that the centrifugal forces due to the two masses are equal and opposite.



Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1.\omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Notes : **1.** The product $m_2.r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.

2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

- In the previous arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings.
- Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for *static balancing*.
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1) and (2) together give *dynamic balancing.***

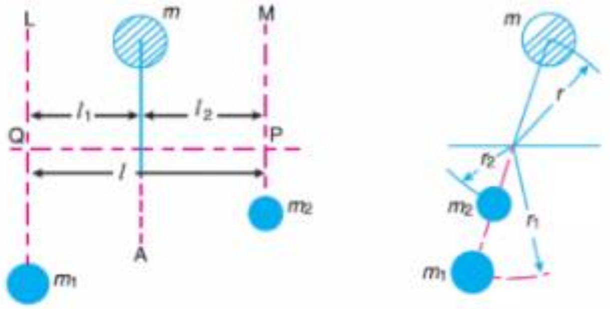
Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

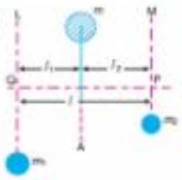
- The following two possibilities may arise while attaching the two balancing masses :
  1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
  2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

1. *When the plane of the disturbing mass lies in between the planes of the two balancing masses*

- Consider a disturbing mass *m lying in a plane A to be balanced by two rotating masses m*1 and *m*2 lying in two different planes *L and M as shown in Fig.*
- *Let r, r*1 and *r*2 be the radii of rotation of the masses in planes *A, L and M respectively.*

Let l_1 = Distance between the planes A and L,
l_2 = Distance between the planes A and M, and
l = Distance between the planes L and M.





We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

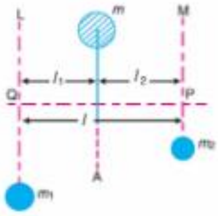
$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (ii)$$

Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

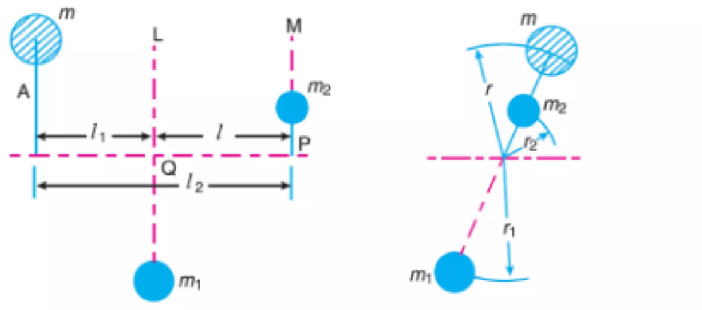
$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (iii)$$

- It may be noted that equation (*i) represents the condition for static balance, but in order to* achieve dynamic balance, equations (*ii) or (iii) must also be satisfied.*

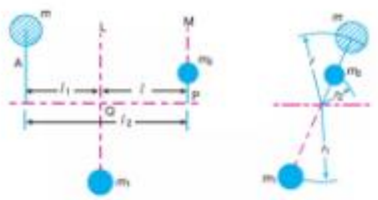


When the plane of the disturbing mass lies on one end of the planes of the balancing masses

- In this case, the mass *m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig.*



As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*



$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \qquad \ldots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (v)$$

. . . [Same as equation (ii)]

Similarly, to find the balancing force in the plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad \ldots (vi)$$

. . . [Same as equation (iii)]

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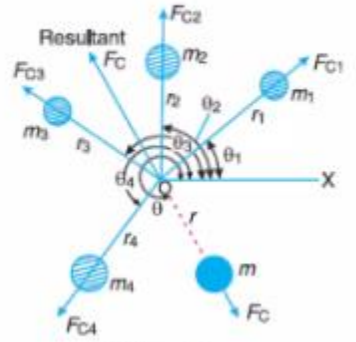
Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line *OX*, as shown in Fig. 21.4 (*a*). Let these masses rotate about an axis through *O* and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

- The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



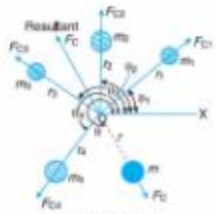
(*a*) Space diagram.

1. *Analytical method*

- The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :
  1. First of all, find out the centrifugal force exerted by each mass on the rotating shaft.
  2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV. We know that

     Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$



(*a*) Space diagram.

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

  3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

  4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

  5. The balancing force is then equal to the resultant force, but in ***opposite direction.***
  6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

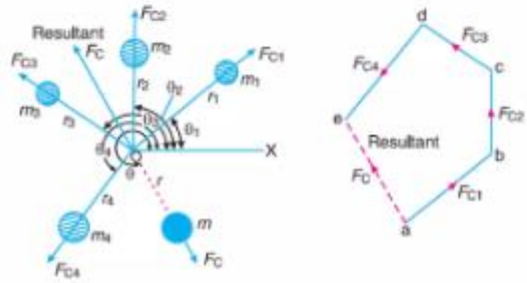
where m = Balancing mass, and

r = Its radius of rotation.

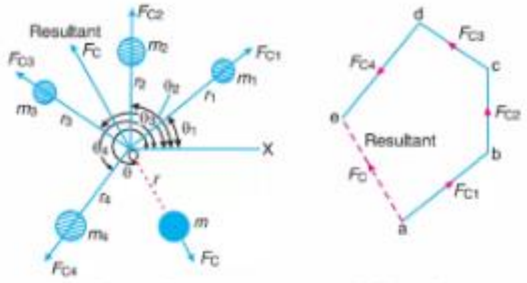
2. Graphical method

- The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig. (*a*).
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).



2. Graphical method



(*a*) Space diagram. (*b*) Vector diagram.

4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 21.4 (*b*).
5. The balancing force is, then, equal to the resultant force, but in ***opposite direction.***
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

Example 21.1. *Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.*

Solution. Given : m_1 = 200 kg ; m_2 = 300 kg ; m_3 = 240 kg ; m_4 = 260 kg ; r_1 = 0.2 m ; r_2 = 0.15 m ; r_3 = 0.25 m ; r_4 = 0.3 m ; θ_1 = 0° ; θ_2 = 45° ; θ_3 = 45° + 75° = 120° ; θ_4 = 45° + 75° + 135° = 255° ; r = 0.2 m

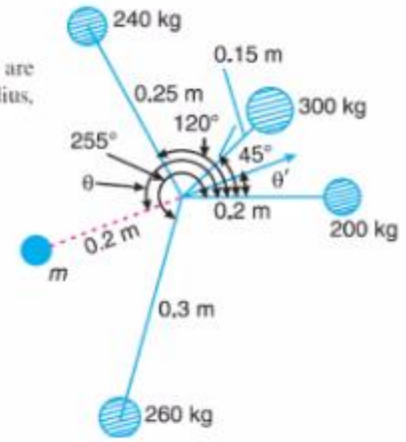
Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore

$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

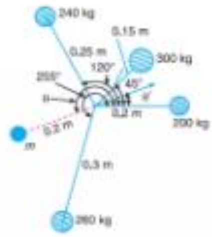
$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$



1. *Analytical method*

The space diagram is shown in Fig.

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,



$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + m_3 \cdot r_3 \cos\theta_3 + m_4 \cdot r_4 \cos\theta_4$$
$$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$$
$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m}$$

Now resolving vertically,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + m_3 \cdot r_3 \sin\theta_3 + m_4 \cdot r_4 \sin\theta_4$$
$$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$$
$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m}$$

$\therefore$ Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2$ kg-m

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2/0.2 = 116 \text{ kg} \quad \textbf{Ans.}$$

and $\tan\theta' = \Sigma V / \Sigma H = 8.5/21.6 = 0.3935$ or $\theta' = 21.48°$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180° + 21.48° = 201.48° \quad \textbf{Ans.}$$

2. *Graphical method*

The magnitude and the position of the balancing mass may also be found graphically as discussed below :

1. First of all, draw the space diagram showing the positions of all the given masses

2. Since the centrifugal force of each mass is proportional to the product of the mass and radius, therefore

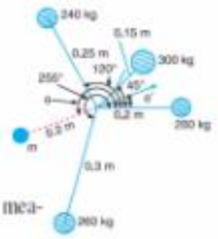
$m_1.r_1 = 200 \times 0.2 = 40$ kg-m

$m_2.r_2 = 300 \times 0.15 = 45$ kg-m

$m_3.r_3 = 240 \times 0.25 = 60$ kg-m

$m_4.r_4 = 260 \times 0.3 = 78$ kg-m

Take: 10kg-m=1cm



3. Now draw the vector diagram with the above values, to some suitable scale.

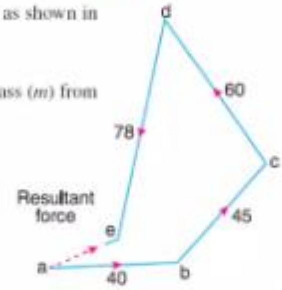
The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.

4. The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in Fig. Since the balancing force is proportional to *m.r*, therefore

$m \times 0.2$ = vector *ea* = 23 kg-m or $m = 23/0.2 =$ **115 kg Ans.**

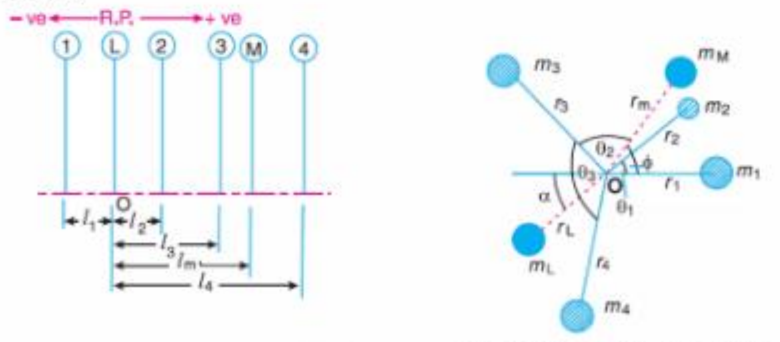
By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal mass of 200 kg.

$\theta = 201°$ **Ans.**



Balancing of Several Masses Rotating in Different Planes

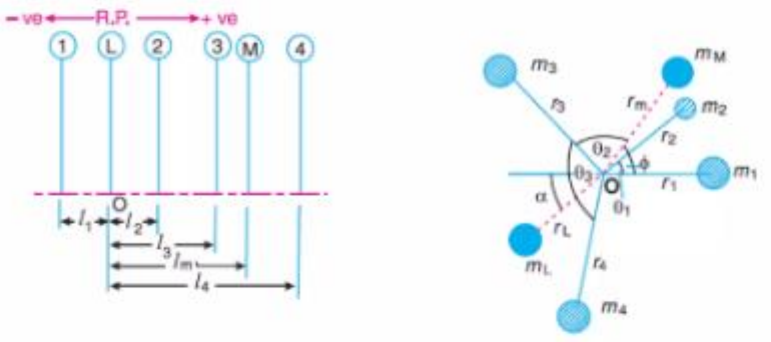
- When several masses revolve in different planes, they may be transferred to a *reference plane (briefly written as R.P.), which may be defined as the plane passing through a* point on the axis of rotation and perpendicular to it.
- The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane.



(b) Angular position of the masses.

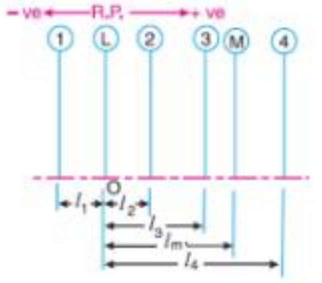
Balancing of Several Masses Rotating in Different Planes

- In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :
  1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
  2. The couples about the reference plane must balance, *i.e. the resultant couple must be zero.*

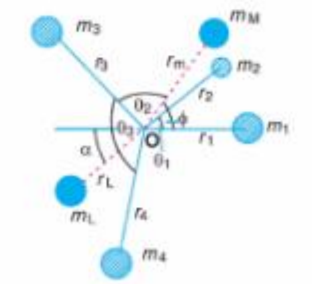


Balancing of Several Masses Rotating in Different Planes

- Let us now consider four masses $m1$, $m2$, $m3$ and $m4$ revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. (*a*).
- *The relative angular positions of these masses are shown in the end view [Fig. (b)].*



(*a*) Position of planes of the masses.

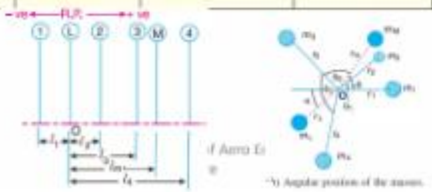


(*b*) Angular position of the masses.

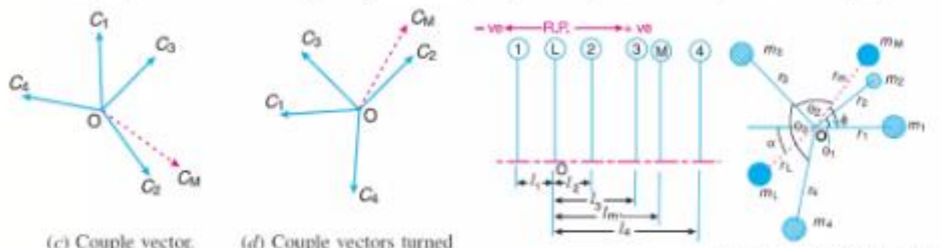
The magnitude of the balancing masses m_L and m_M in planes *L and M may be obtained as* discussed below :

1. Take one of the planes, say L as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive***.
2. Tabulate the data as shown in Table   The planes are tabulated in the same order in which they occur, reading from left to right.

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is proportional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. 21.7 (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.



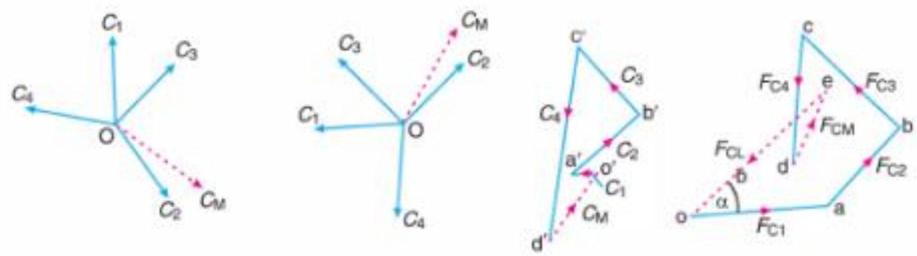
(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*b*) Angular position of the masses.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. 21.7 (*e*). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane M may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (*b*).

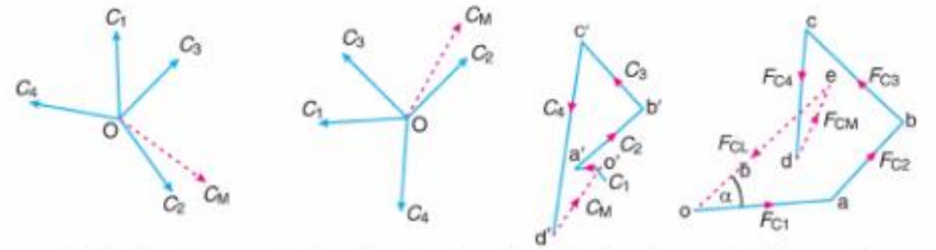


(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

6. Now draw the force polygon as shown in Fig. 21.7 (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

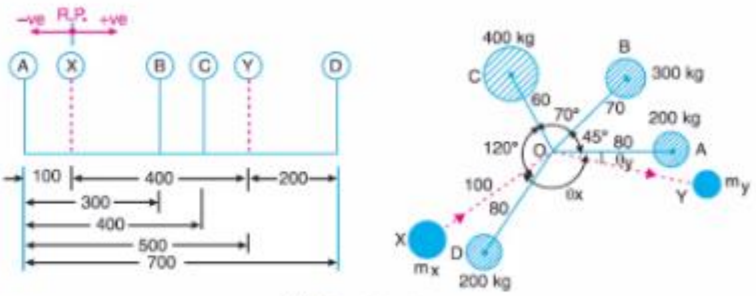
From this expression, the value of the balancing mass m_L in the plane *L* may be obtained



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

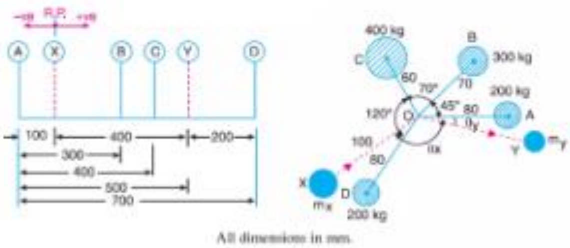
Example 21.2. *A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.*

Solution. Given : m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; r_X = r_Y = 100 mm = 0.1 m

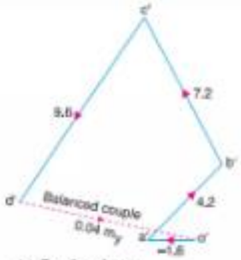


All dimensions in mm.

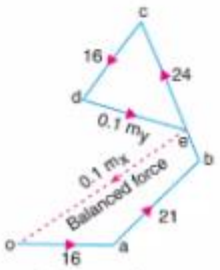
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from Plane x(l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m² (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6



All dimensions in mm.
(a) Position of planes. (b) Angular position of masses.



(c) Couple polygon.



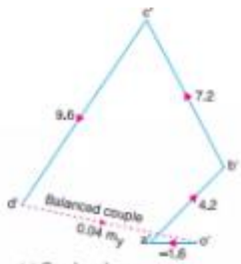
(d) Force polygon.

$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$ or $m_Y = 182.5$ kg **Ans.**

$0.1\, m_X = \text{vector } eo = 35.5$ kg-m or $m_X = 355$ **kg Ans.**

$0.04\, m_Y = \text{vector } d'o' = 7.3 \text{ kg-m}^2$ or $m_Y = 182.5$ kg **Ans.**

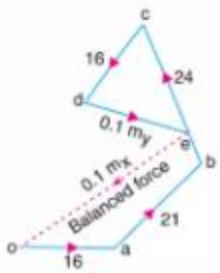
$0.1\, m_X = \text{vector } eo = 35.5$ kg-m or $m_X = 355$ **kg Ans.**



(c) Couple polygon.

$\theta_Y = 12°$ in the clockwise direction from mass m_A

$\theta_X = 145°$ in the clockwise direction from mass m_A



(d) Force polygon.

Example 21.3. *Four masses A, B, C and D as shown below are to be completely balanced.*

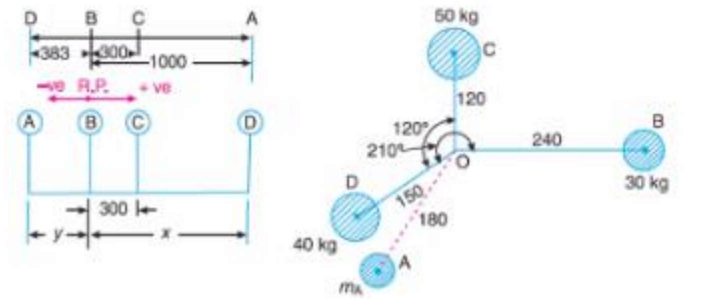
	A	*B*	*C*	*D*
Mass (kg)	—	*30*	*50*	*40*
Radius (mm)	*180*	*240*	*120*	*150*

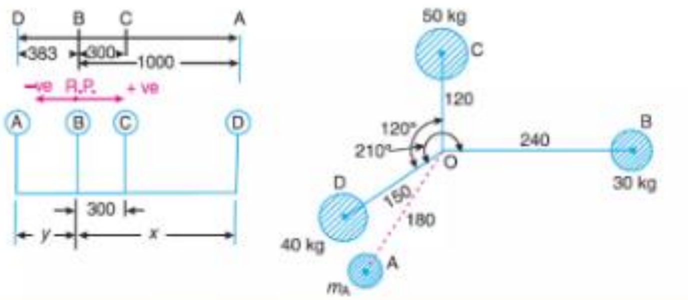
The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

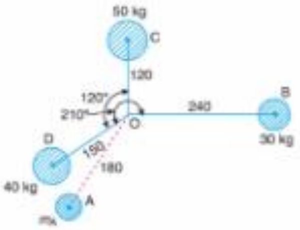
Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$





Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 m_A y
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

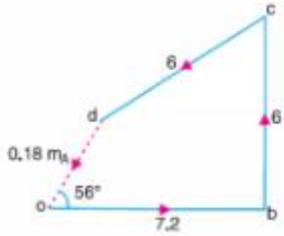
Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 m_A y
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

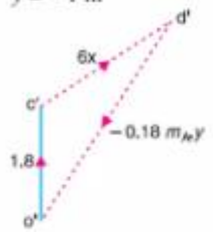


$6x$ = vector $c'd'$ = 2.3 kg-m^2 or x = 0.383 m

– 0.18 $m_A.y$ = vector $o'd'$ = 3.6 kg-m^2

– 0.18 × 20 y = 3.6 or y = – 1 m



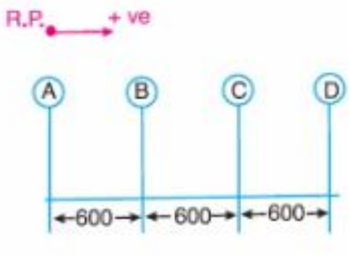


(d) Couple polygon.

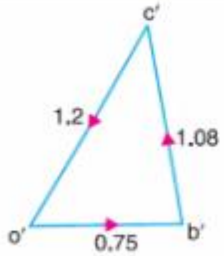
Example 21.4. *A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.*

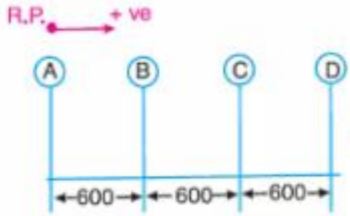
Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

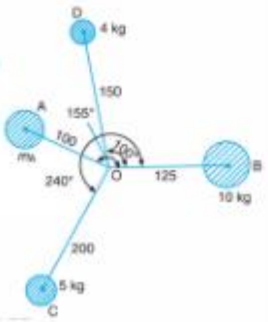
Solution. Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ; r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg



Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent. Force* ÷ ω^2 (m.r)kg-m (4)	*Distance from plane A (l)m* (5)	*Couple* ÷ ω^2 (m.r.l) kg-m² (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08







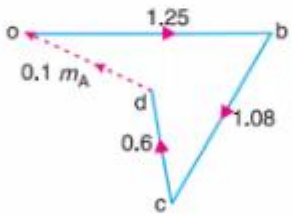
(*c*) Couple polygon.

$\angle BOA = 155°$ **Ans.**

$\angle BOC = 240°$ **Ans.**

$\angle BOD = 100°$ **Ans.**

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent. Force* ÷ ω^2 (m.r)kg-m (4)	*Distance from plane A (l)m* (5)	*Couple* ÷ ω^2 (m.r.l) kg-m² (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08



(*d*) Force polygon.

$0.1\, m_A = 0.7$ kg-m² or $m_A = 7$ kg **Ans.**