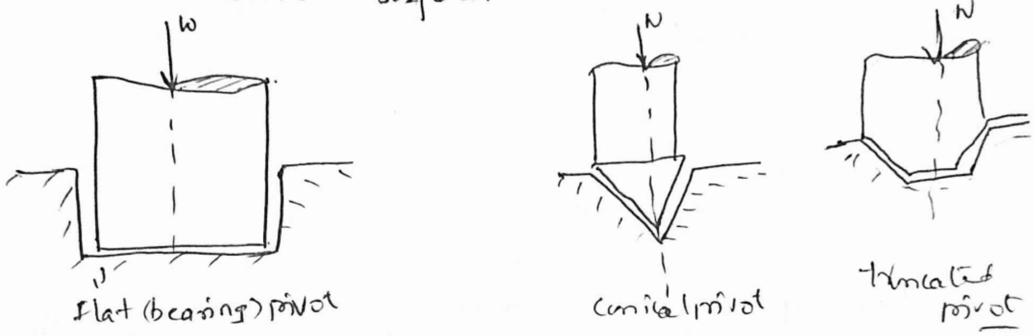
UNIT-III

Friction, Brakes & Dynamometers

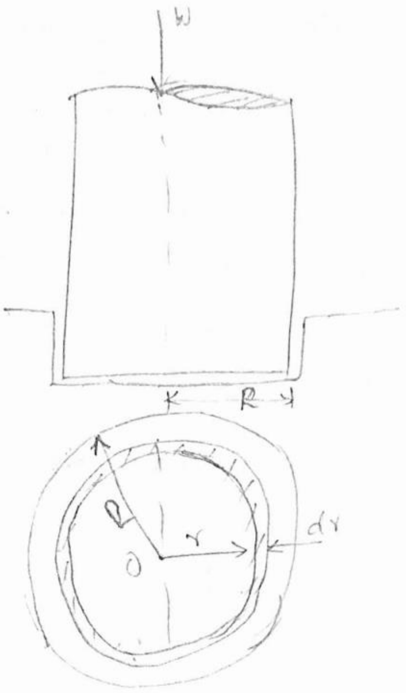
Pivot Bearing

①

The rotating shafts are frequently subjected to axial thrust. These shafts can be kept in correct axial position if bearing surfaces are provided. the bearing surfaces which are flat (or) conical carry the axial thrust. The bearing surfaces placed at the end of a shaft are known as pivots. The pivot may flat, conical (or) truncated conical surfaces.



* Flat Pivot :-



The bearing surface placed at the end of shaft is known as pivot. If the surface is flat as shown, then bearing surface is called flat-pivot (or)-foot-step.
There will be friction along the surface of contact between shaft & bearing. The power lost can be obtained by calculating torque.

let, W → Axial load, (or) load transmitted to the bearing surface

R → Radius of pivot.

μ → co-effecient of friction.

p → Intensity of Pr, N/mm².

T → Total frictnal torque.

r → radius of ring

dr → thickness of ring.

③

Consider a circular ring of ~~thickness~~ Radius 'r' & thickness dr as shown.

$\therefore$ Area of ring $= 2\pi r \cdot dr$.

We will consider 2 cases; namely;

(i) Uniform pressure over bearing surface &
(ii) Uniform wear over bearing surface

(i) <u>Case of Uniform Pr.</u>:

When the Pr is assumed to be uniform over the bearing surface, then intensity of pressure is given by,

$$p = \frac{\text{Axial load}}{\text{Area of c/s}} = \frac{W}{\pi R^2} \quad -(1)$$

Now, the load transmitted to the ring & frictional torque on the ring,

Load transmitted to the ring,

$$dW = \text{Pr on ring} \times \text{Area of ring}$$
$$= p \times 2\pi r dr$$

frictional force on ring,

$$dF = \mu \times dW$$
$$= \mu \times \text{load on ring}$$
$$= \mu \times p \times 2\pi r dr$$

But,

& frictional torque on the ring) Moment of frictional force about that axis,

$$dT = \text{frictional force} \times \text{Radius of ring}$$
$$= dF \times r.$$

$$\therefore \quad dT = \mu \times p \times 2\pi \cdot r \cdot dr \cdot r$$
$$= \mu \cdot p \times 2\pi r^2 \cdot dr \quad -ⓐ$$

Now, the total frictional torque will be obtained by integrating above eq. ⓐ.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi \mu p r^2 \cdot dr$

$$= 2\pi \mu p \int_0^R r^2 \cdot dr$$

$$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{2}{3} \mu \pi p R^3$$

$$= \frac{2}{3} \pi \times \mu \times R^{3} \times \frac{W}{\pi R^2} \qquad \left[\because p = \frac{W}{\pi R^2}\right]$$

$$\boxed{T = \frac{2}{3} \mu W R}$$

$\therefore$ Power lost in frictn $= T \times \omega$

$$= \frac{T \times 2\pi N}{60}$$

$$= \frac{2\pi N T}{60}$$

(ii) In case of Uniform Wear :- For uniform wear of bearing surface, The load transmitted to the various circular rings should be same.

But load transmitted to any circular ring is equal to the product of pressure & area of ring. Area of ring is directly proportional to the radius of ring.

Hence for uniform wear, the product of p r radius should be constant. i.e: $p \times r =$ constant.

For Uniform Wear, $p \times r =$ constant

i.e: $p \times r = C$.

$\therefore$ $p = \frac{C}{r}$ — ⓐ.

(5)

Load transmitted to the ring,

$$= p_r \times \text{Area of ring}$$

$$= p \times 2\pi r \cdot dr$$

$$= \frac{C}{r} \times 2\pi \cdot r \cdot dr$$

$$dW = 2\pi C \cdot dr \quad \text{— (6)}$$

Total load transmitted to the bearing, is obtained by integrating from '0 to R'

$\therefore$ Total load transmitting to the bearing,

$$W = \int_0^R dW$$

$$= \int_0^R 2\pi C\, dr = 2\pi C \int_0^R dr = 2\pi C [r]_0^R$$

$$W = 2\pi CR$$

$$\boxed{C = \frac{W}{2\pi R}}$$

Now frictional ~~torque~~ force on the ring,

$$dF = \mu \times \text{load on ring} = \mu \times dW$$

$$= \mu \times 2\pi C dr$$

Hence frictional torque on the ring,

$$dT = \text{Frictional force on ring} \times \text{radius}$$

$$= dF \times r$$

$$= \mu \times 2\pi C \cdot dr \cdot r$$

$$= \mu \cdot 2\pi C \cdot r \cdot dr$$

$\therefore$ Total frictional torque, $T = \int_0^R dT$

$$= \int_0^R \mu 2\pi C \cdot r \cdot dr$$

$$= 2\pi \cdot C \cdot \mu \cdot \int_0^R r\, dr$$

$$= 2\pi C \cdot \mu \left[\frac{r^2}{2}\right]_0^R = 2\pi C \cdot \mu \cdot \left[\frac{R^2}{2}\right]$$

$$= \cancel{2\pi} \times \frac{W}{\cancel{2\pi} R} \times \mu \left[\frac{R^2}{2}\right]$$

$$\boxed{T = \frac{1}{2} \mu W R}$$

$\therefore$ Power lost in friction, $P = \frac{2\pi N T}{60}$

Problem ①:- Find the power lost in friction assuming (i) uniform Pr. & (ii) Uniform wear. When a vertical shaft of 100mm dia. rotating at 150rpm rests on a flat end-foot step bearing. The coefficient of friction is equal to 0.05 & shaft carries a vertical load of 15 kN.

Sol:-

Given:-

Dia, $D = 100mm \Rightarrow 0.1m$ $\therefore R = \frac{0.1}{2} = 0.05m$

$N = 150$ rpm ; co-effecient of friction, $\mu = 0.05$

load, $W = 15KN = 15 \times 10^3 N$.

(i) Power lost in friction assuming uniform pressure,

For uniform Pr.,

$$T = \frac{2}{3}\mu W R$$

$$T = \frac{2}{3}(0.05)(15\times10^3)(0.05)$$

$$T = 25 \ N\text{-}m$$

Power lost,

$$P = \frac{2\pi N T}{60} \Rightarrow \frac{2\pi 150 \times 25}{60}$$

$$\boxed{P = 392.7 W}$$

(ii) For uniform wear,

$$T = \frac{1}{2}\mu W R.$$

$$= \frac{1}{2} \times 0.05 \times 15 \times 10^3 \times 0.05$$

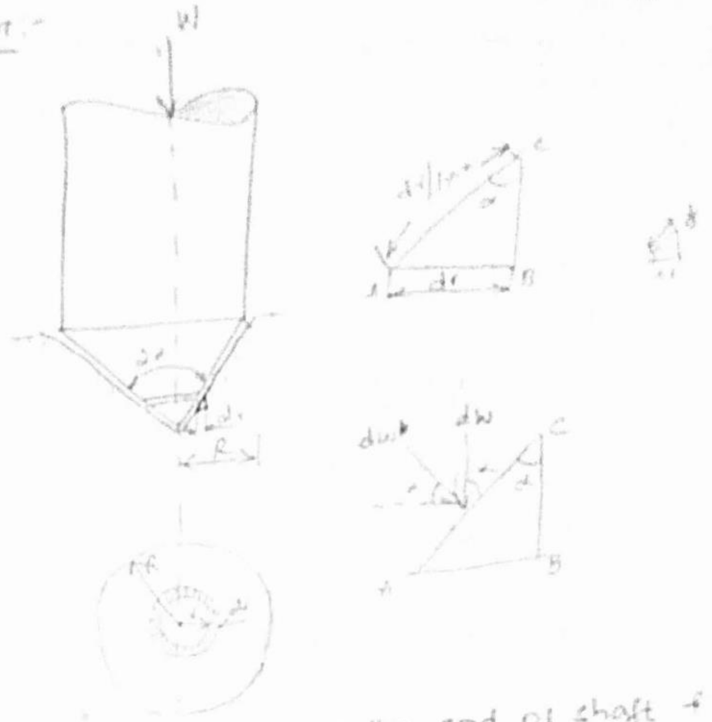
$$T = 18.75 Nm$$

$$P = \frac{2\pi N T}{60}$$

$$P = \frac{2\pi \times 150 \times 18.75}{60}$$

$$\boxed{P = 294.5 W}$$

CONICAL PIVOT :-



The bearing surface placed at the end of shaft & having a conical surface, is known as conical pivot bearing as shown.

let, $W \rightarrow$ Axial load

$\mu \rightarrow$ co-efficient of friction

$\alpha \rightarrow$ semi-angle of cone

$p \rightarrow$ Pr intensity to the cone surface

Consider a circular ring of radius 'r' & thickness dr. The actual thickness of the sloping ring will be $dr/\sin\alpha$ as shown in which $AB = dr$ (on enlarged scale)

angle $ACB = \alpha$ and sloping length of ring,

$$\sin\alpha = \frac{AB}{AC} \Rightarrow AC = \frac{AB}{\sin\alpha}$$

$$AC = dr/\sin\alpha$$

$\therefore$ Area of ring along conical surface

$= 2\pi r \times$ actual thickness of sloping ring

$= 2\pi r \times dr/\sin\alpha$

Now assuming 2 cases
(i) Uniform Pressure
(ii) Uniform Wear

(i) for Uniform Pressure :-

Load acting on the circular ring, normal to the conical surface.

$\therefore$ load on the ring normal to conical surface,

$$dW = p \times \text{Area of ring along conical surface}$$

$$dW^* = p \times 2\pi r \times \frac{dr}{\sin\alpha}$$

Vertical component of above load,

$$dW = \left[p \times 2\pi r \times \frac{dr}{\sin\alpha}\right] \cdot \sin\alpha$$

$$\therefore \quad dW = p \times 2\pi r \cdot dr.$$

$\therefore$ Total vertical load transmitted to bearing

$$W = \int_0^R p \times 2\pi r\, dr$$

$$= p \times 2\pi \int_0^R r\, dr$$

$$= p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \left[\frac{R^2}{2}\right] :) \quad p\pi R^2 = W \quad (a)$$

i.e. $W = p\pi R^2$

$$p = \frac{W}{\pi R^2} \quad (b)$$

[This eq (b) shows that pr intensity is independent on angle of conical surface].

Now the frictional force on the conical ring,

$$dF = \mu \times \text{load on ring normal to conical surface}$$

$$= \mu \times dW^*$$

$$dF = \mu \times p \times 2\pi r \times dr / \sin\alpha]$$

(9)

Total moment of this frictional force about the shaft [dT]

= Frictional torque on ring

= Frictional force × Radius ⇒ $dF \times r$

$$= \mu \times \left[p \times 2\pi r \cdot \frac{dr}{\sin\alpha} \right] \cdot r$$

$$= \mu p \cdot 2\pi \cdot r^2 \frac{dr}{\sin\alpha} \quad \text{—— (c)}$$

Total moment of the frictional force about shaft axis or total frictional torque on conical surface is obtained by integrating.

∴ Total frictional torque,

$$T = \int_0^R dT$$

$$= \int_0^R \mu p \cdot 2\pi r^2 \cdot dr / \sin\alpha$$

$$= \frac{\mu p \cdot 2\pi}{\sin\alpha} \int_0^R r^2 \cdot dr = \frac{2\pi \mu \cdot p}{\sin\alpha} \left[\frac{r^3}{3} \right]_0^R$$

$$T = \frac{2\pi \mu p}{\sin\alpha} \left(\frac{R^3}{3} \right)$$

$$= \frac{2\cancel{\pi} \mu}{\sin\alpha} \times \frac{W}{\cancel{\pi} R^{\cancel{2}}} \left[\frac{R^3}{3} \right]$$

$$\boxed{T = \frac{2}{3} \frac{\mu W R}{\sin\alpha}}$$

∴ Power lost in friction $= \frac{2\pi N T}{60}$

$$\boxed{P = \frac{2\pi N T}{60}}$$

(ii) Case of Uniform Wear: --(10)

For the uniform wear, the load transmitted to various circular rings to be constant:

$$p \times r = C.$$

$$p = \frac{C}{r}$$

The total vertical load transmitted to the bearing.

$$= \int_0^R p \times 2\pi r \times dr.$$

$$= \int_0^R \frac{C}{r} \times 2\pi r \times dr$$

$$= \int_0^R C\, 2\pi dr = 2\pi C \int_0^R dr = 2\pi C \,[r]_0^R$$

$$W = 2\pi C\,[R]$$

But ~~total~~ vertical load transmitted to bearing is also equal to W

$$\therefore W = 2\pi C R$$

$$C = W/2\pi R.$$

Now the frictional torque on ring is given by

$$dT = \text{Frictional force} \times \text{Radius}$$

$$= dF \times r.$$

$$= \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \times r$$

$$= \mu\, p \times 2\pi r^2 \times dr/\sin\alpha$$

$$= \mu \cdot \frac{C}{r} \times 2\pi r^2 \times dr/\sin\alpha$$

$$= \mu C\, 2\pi r\, dr/\sin\alpha \Rightarrow \mu \times \frac{W}{\cancel{2\pi} R} \times \cancel{2\pi} r \cdot \frac{dr}{\sin\alpha}$$

$$dT = \frac{\mu W}{R}\, r\, dr/\sin\alpha$$

Total frictional torque,

$$T = \int_0^R dT$$

$$= \int_0^R \frac{\mu W}{R} \cdot r\, dr/\sin\alpha$$

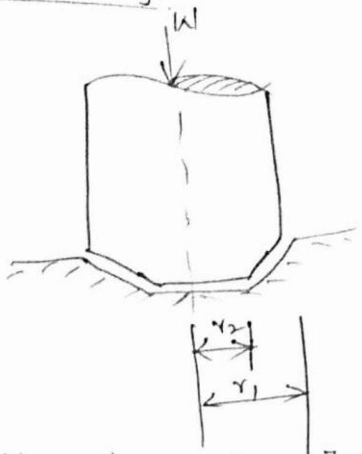
$$= \frac{\mu W}{R \sin\alpha} \int_0^R r \cdot dr \Rightarrow \frac{\mu W}{R\sin\alpha}\left[\frac{r^2}{2}\right]_0^R = \frac{\mu W}{R\sin\alpha}\left[\frac{R^2}{2}\right]$$

$$\boxed{T = \frac{1}{2}\frac{\mu W R}{\sin\alpha}}$$

Power lost in friction, $P = 2\pi N T/60$

(11).

Truncated Pivot Bearing:-



The above fig. shows truncated conical pivot of external & internal radii r_1 & r_2 respectively.

(i) Case of Uniform Pressure :-

vertical load transmitted to the bearing

$$dW = p \times 2\pi r \times dr \quad -ⓐ$$

For total vertical load, integrating with limits r_2 to r_1.

$$W = \int_{r_2}^{r_1} p \times 2\pi r \, dr$$

$$= p\, 2\pi \int_{r_2}^{r_1} r\,dr = p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$W = p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$p = \frac{W}{\pi\left[r_1^2 - r_2^2\right]} \quad -ⓑ$$

frictional force, $\mu \times dW$ normal to the surface

$= \mu \times p \times 2\pi r\, dr / \sin\alpha$

Frictional torque on the rings.

$$dT = \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \times r.$$

total frictional torque, $T = \int_{r_2}^{r_1} dT$

$$T = \int_{r_2}^{r_1} \mu \times p \times 2\pi r \times \frac{dr}{\sin\alpha} \cdot r$$

$$= \frac{2\pi \times \mu \cdot p}{\sin\alpha} \int_{r_2}^{r_1} r^2 \, dr$$

$$= \frac{2\pi \mu \cdot p}{\sin\alpha} \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = \frac{2\pi \times \mu \cdot p}{\sin\alpha} \left[\frac{(r_1^3 - r_2^3)}{3}\right]$$

$$= \frac{2\pi \times \mu}{3\sin\alpha} \cdot \frac{W}{\pi(r_1^2 - r_2^2)} \cdot (r_1^3 - r_2^3)$$

$$\therefore \boxed{T = \frac{2}{3} \frac{\mu W}{\sin\alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

Power lost in friction, $P = \frac{2\pi N T}{60}$.

(ii) Uniform Wear:

$$p \times r = C.$$
$$p = C/r.$$

Vertical load transmitted, $dW = p \times 2\pi r \, dr$

Total vertical load, $W = \int_{r_2}^{r_1} p \times 2\pi r \, dr$

$$= \int_{r_2}^{r_1} \frac{C}{r} \cdot 2\pi r \, dr \Rightarrow 2\pi C \int_{r_2}^{r_1} dr = 2\pi C \left[r\right]_{r_2}^{r_1}$$

$$W = 2\pi C [r_1 - r_2]$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

total frictional torque, $T = \int_{r_2}^{r_1} 2\pi \mu \times (C \times r \times dr) / \sin\alpha$

$$T = \frac{2\pi \mu \cdot C}{\sin\alpha} \int_{r_2}^{r_1} r \cdot dr \Rightarrow T = \frac{1}{\sin\alpha} \cdot 2\pi \cdot \mu \cdot C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

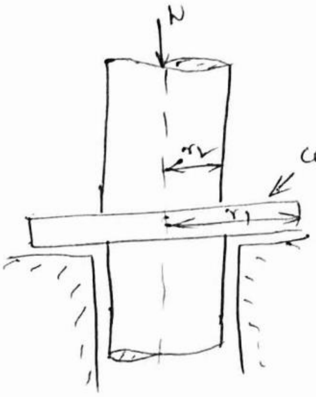
$$\Rightarrow T = \frac{1}{\sin\alpha} \cancel{2\pi} \cdot \mu \cdot \frac{W}{\cancel{2\pi}(r_1 - r_2)} \left[\frac{(r_1^2 - r_2^2)}{2}\right]$$

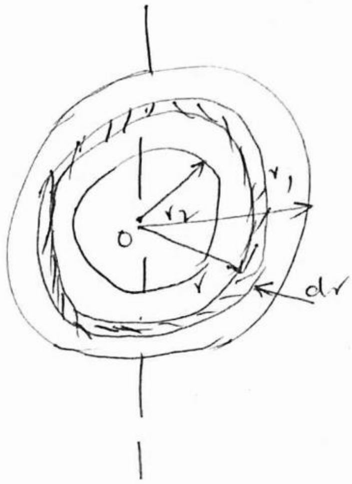
$$\boxed{T = \frac{1}{2} \frac{\mu W (r_1 + r_2)}{\sin\alpha}}$$

Power lost in friction, $\boxed{P = \frac{2\pi N T}{60}}$

(13).

Flat collar:- The bearing surface provided at any position on the shaft (but not at the end) to carry axial thrust is known as collar. Collar bearings are also known as thrust bearings





let,
$r_1 \rightarrow$ External radius of collar
$r_2 \rightarrow$ Internal radius of collar
$p \rightarrow$ intensity of Pr.
$W \rightarrow$ Axial load or total load transmitted to bearing surface
$\mu \rightarrow$ co-effecient of friction
$T \rightarrow$ Total frictional torque.

consider a circular ring of radius 'r' & thickness dr

$\therefore$ Area of ring, $= 2\pi r \cdot dr$.

load on ring, $= Pr \times$ Area of ring

$= p \times 2\pi \cdot r \cdot dr$.

Friction Force on ring, $= \mu \times$ load on ring

$= \mu \times p\, 2\pi r dr$.

Friction torque $=$ friction force $\times$ Radius

$= p \cdot \mu \times 2\pi r dr \times r$

$= 2\pi \mu p r^2 \cdot dr$.

$\therefore$ total frictional torque, $T = \int_{r_2}^{r_1} dT$

$$T = \int^{r_1} 2\pi \mu \cdot p r^2 \cdot dr.$$

(i) Uniform Pressure :-

$$p = \text{constant}.$$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} \text{load on ring } (dW)$$

$$= \int_{r_2}^{r_1} p \times 2\pi r \, dr$$

$$= p \times 2\pi \int_{r_2}^{r} r\,dr. \Rightarrow p \times 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$\Rightarrow p \times 2\pi \left[\frac{r_1^2 - r_2^2}{2}\right] \Rightarrow p \times \pi \left[r_1^2 - r_2^2\right] = W$$

$$p = \frac{W}{\pi \left[r_1^2 - r_2^2\right]}.$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu \cdot p r^2 \, dr$$

$$= 2\pi \mu \cdot p \int_{r_2}^{r_1} r^2 \cdot dr$$

$$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi \mu p \cdot \left(\frac{r_1^3 - r_2^3}{3}\right)$$

$$= \frac{2}{3} \pi \mu \cdot \frac{W}{\pi \left[r_1^2 - r_2^2\right]} \cdot \left[r_1^3 - r_2^3\right] \Rightarrow \boxed{T = \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

power lost in friction, $P = \frac{2\pi N T}{60}$

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ii). for Uniform Wear:-

$$p \times r = \text{constant}$$

$$p \times r = C$$

$$p = C/r$$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} \text{load on ring} = \int_{r_2}^{r_1} dW$$

$$W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$W = \int_{r_2}^{r_1} \frac{C}{r} \cdot \times 2\pi\, dr \cdot r$$

$$= 2\pi \cdot C \int_{r_2}^{r_1} dr$$

$$= 2\pi C \left[r\right]_{r_2}^{r_1} \Rightarrow 2\pi C\left[r_1 - r_2\right] = W$$

$$C = \frac{W}{2\pi\left[r_1 - r_2\right]}$$

Total frictional torque

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} dF \times r$$

$$= \int_{r_2}^{r_1} 2\pi \mu p \cdot r^2 \cdot dr$$

$$= 2\pi \mu \cdot \int_{r_2}^{r_1} \frac{C}{r} \cdot r^2 \cdot dr$$

$$= 2\pi \mu C \int_{r_2}^{r_1} r\, dr. = \pi \mu C\left[r_1^2 - r_2^2\right] = T.$$

$$= \pi \mu \cdot \frac{W}{2\pi(r_1 - r_2)} \left[r_1^2 - r_2^2\right]$$

$$\boxed{T = \frac{1}{2} \mu W (r_1 + r_2)}$$

Power lost in frictn, $P = \frac{2\pi N T}{60}$

Problems

1. A conical pivot with an angle of cone is 120°, supports a vertical shaft of dia 300 mm. It is subjected to load of 20 kN. The coeff. of friction is 0.05 & the speed of shaft is 210 rpm. Calculate the power lost in friction assuming (i) Uniform Pr., (ii) Uniform wear.

A load angle 350, ...

Sol: **Given**

$2\alpha = 120°$; $\alpha = 60°$.

$D = 300mm$; $R = 150mm$; $R = 0.15m$;

$W = 20 kN = 20\times10^3 N$; $\mu = 0.05$

$N = 210 rpm$

(i) Considering Uniform Pr.

$$T = \frac{2}{3}\frac{\mu W R}{\sin\alpha}$$

$$= \frac{2}{3}\times\frac{(0.05)(20\times10^3)\times(0.15)}{\sin(60°)}$$

$$\Rightarrow T = 115.53 \text{ N-m}$$

Power lost in friction, $P = \frac{2\pi N T}{60} \Rightarrow \frac{2\pi\times210\times115.53}{60}$

$$\boxed{P = 2540.6 W} \text{ Ans}$$

(ii) Considering Uniform Wear:

$$T = \frac{1}{2}\frac{\mu W R}{\sin\alpha}$$

$$= \frac{1}{2}\times\frac{(0.05)(20\times10^3)(0.15)}{\sin(60°)}$$

$$T = 86.60 \text{ N-m}$$

Power lost in friction,

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi\times210\times86.60}{60}$$

$$\boxed{P = 1904 W}$$

(17)

A load of 25 kN is supported by a conical pivot with angle of cone as 120°. The intensity of pr is not to exceed 350 kN/m². The external radius is 2 times the internal radius. The shaft is rotating at 180 rpm & $\mu = 0.05$. Find the power absorbed in friction assuming uniform pr.

Sol: Given:

$W = 25\text{ kN} = 25 \times 10^3$ N;

$2\alpha = 120°$; $\alpha = 60°$; $p = 350$ kN/m² $= 350 \times 10^3$ N/m²

$r_1 = 2r_2$; $N = 180$ rpm; $\mu = 0.05$.

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

$$350 \times 10^3 = \frac{25 \times 10^3}{\pi\left[(2r_2)^2 - r_2^2\right]} \Rightarrow \quad 350 \times \pi r_2^2 = \cancel{250}$$

$r_2 = 0.087$ m; $r_1 = 2r_2$

$r_1 = 2(0.087) \Rightarrow r_1 = 0.174$ m.

For uniform pr.:

$$T = \frac{2}{3}\frac{\mu W}{\sin\alpha}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3} \times \frac{(0.05)(25 \times 10^3)}{\sin 60}\left[\frac{(0.174)^3 - (0.087)^3}{(0.174)^2 - (0.087)^2}\right]$$

$T = 195.37$ N-m

power absorbed, $P = \frac{2\pi N T}{60} \Rightarrow$

$$= \frac{2\pi \times 180 \times 195.37}{60}$$

$P = 3682.6$ W $\Rightarrow \boxed{P = 3.68\text{ kW}}$

* NOTE:-

① If the axial load on the bearing is too great, then the bearing pr. on the collar will become more than limiting bearing pr. which is appr. equal to 400kN/m². Hence, to reduce the intensity of pr. on collar, two (or) more collars are used.

If $n \to$ no. of collars in multi-collar bearing, then

(i) $n = \dfrac{\text{Total load}}{\text{load permissible on one collar}}$

(ii) p = Intensity of Uniform Pr.

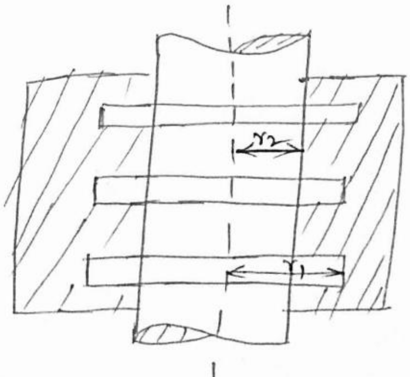
$= \dfrac{\text{Load}}{\text{No. of collars} \times \text{Area of one-collar}}$

$= \dfrac{W}{n \times \pi(r_1^2 - r_2^2)}$

(iii) Total torque transmitted remains constant i.e.,

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right].$$

② The frictional torque for uniform pr. is greater than that of uniform wear. Hence for safe design of bearing surfaces when power lost in frictn is to be determined & no assumption is mentioned, then assume uniform pr. But when power transmitted is to be determined & no assumptn is given, then assume uniform wear.



multi-collar Bearing

Q. (20)

In a thrust bearing, the external & internal radii of contact surfaces are 210mm & 160mm respectively. The total axial load is 60KN & coefficient of fricⁿ = 0.05. The shaft is rotating 380rpm. Intensity of pressure is not to exceed 350 kN/m². Calculate:

(i) power lost in overcoming the fricⁿ &

(ii) no. of collars reqd. for thrust bearing

Sol:- **Given:-**

External radius, $r_1 = 210mm = 0.21m$

Internal radius, $r_2 = 160mm = 0.16m$

$W = 60KN = 60\times10^3 N$; $\mu = 0.05$

$N = 380rpm$; $p = 350 kN/m^2$

$= 350\times10^3 N/m^2$.

Here, the power lost in overcoming the friction is to be determined. Also no assumptⁿ is given, hence it is safe to assume uniform pr.

(i) Power lost in overcoming fricⁿ.

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$T = 558.378\ N\text{-}m$$

$$\therefore P = \frac{2\pi N T}{60} = \frac{2\pi\times380\times558.3}{60} \Rightarrow \boxed{P = 22219.8W}$$

(ii) No. of collars reqd:

No. of collars, $n = \dfrac{\text{Total load}}{\text{load per collar}}$

load per collar, as we have, $p = \dfrac{W^*}{\pi(r_1^2 - r_2^2)}$

W^* is load per collar, $W^* = p\times[\pi(r_1^2 - r_2^2)]$

$W^* = 20341.8N$

$\therefore$ No. of collars $n = \dfrac{60\times10^3}{20341.8} = 2.95$

$\simeq$ 3 collars

Problems

1. In a collar thrust bearing the external & internal radii are 250mm & 150mm respectively. The total axial load is 50kN & shaft is rotating at 150 rpm. The coefficient of friction is equal to 0.05. Find the power lost in friction assuming uniform pressure.

Sol:- Given:-

External radius = 250mm = 0.25 m = r_1

Internal radius = 150mm = 0.15 m = r_2

$W = 50KN = 50\times10^3$ N ; N = 150 rpm

$\mu = 0.05$;

For uniform pr. total frictional torque,

$$T = \frac{2}{3}\mu W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3}\, 0.05 \times 50\times10^3 \left[\frac{(0.25)^3 - (0.15)^3}{(0.25)^2 - (0.15)^2}\right]$$

$T = 510.42$ N-m

∴ power lost in friction, $P = \frac{2\pi N T}{60}$

$$= \frac{2\pi(150)(510.42)}{60}$$

$P = 8017.6$ W

$P = 8.01$ kW

(21)

Single Plate clutch:

let, $r_1 \rightarrow$ External radius of friction lining on clutch plate

$r_2 \rightarrow$ Internal radius of friction lining

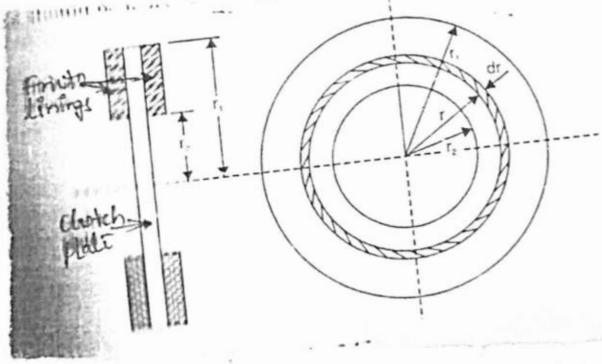
$p \rightarrow$ Intensity of Pr.

$W \rightarrow$ Total Axial load

$\mu \rightarrow$ Coeffecient of friction.

$T \rightarrow$ Torque transmitted.

The theory of single plate clutch is also based on same principle as that of collar bearing. In case of bearing, the power lost due to friction should be reduced & hence the value of coeffecient of friction should decrease. But in case of clutch the power transmitted by friction linings should be more & hence co-effecient of friction should be increased.



Also in case of a new clutch, the intensity of Pr is approximately uniform over the entire surface where as in an old clutch uniform wear theory is more approximate.

Consider a circular ring of radius 'r' & thickness dr as shown.

Area of ring, $dA = 2\pi r\,dr$

Axial load on ring, $dW = p \times$ Area of ring

$= p \times 2\pi r\,dr$

Frictional force on ring,

$dF = \mu \times$ load on ring

$= \mu \times p \times 2\pi r\,dr$

Frictional torque on ring, $dT = dF \times r$

$= \mu \times p \times 2\pi r\,dr \times r$

$= \mu \times p \times 2\pi r^2\,dr$

(i) <u>For Uniform Pressure:</u>

$p =$ constant

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

$\therefore$ Total friction torque.

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \times p \times 2\pi r^2 \cdot dr$$

$$= 2\pi \mu \cdot p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi \mu \cdot p \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$= \frac{2}{3} \mu \cdot \pi \cdot \frac{W}{\pi(r_1^2 - r_2^2)} \cdot (r_1^3 - r_2^3)$$

$$= \frac{2}{3} \mu W \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]. \quad -(a)$$

(23)

Total frictional torque acting on friction surface can also be expressed in terms of mean radius (R_M) of friction surface as,

$$T = \mu \cdot W \times R_M \quad \text{— (b)}$$

comparing eq's (a) & (b),

$$R_M = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

In a single clutch plate, there are 2 friction surfaces, one on each side of the frictional plate, hence, total torque on the clutch plate is given by,

$$T^* = 2T$$

$$= \cancel{2 \times \left[\frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\right]} \quad 2 \times \left[\frac{2}{3} \cdot \mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\right]$$

Where, $T^* \Rightarrow$ Total frictional torque on clutch plate.

(ii) Uniform Wear:-

$$p \times r = \text{constant}$$

$$p = c/r$$

W.K.T, axial load on ring

$$dW = p \times 2\pi r\, dr$$

∴ Total axial load is given by integrating above eq.

$$W = \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= \int_{r_2}^{r_1} c \cdot 2\pi\, dr$$

$$= 2\pi c\,[r]_{r_2}^{r_1} \Rightarrow 2\pi c\,[r_1 - r_2] \Rightarrow C = \frac{W}{2\pi(r_1 - r_2)}$$

The frictional torque on friction surface.

$$T = \int_{r_2}^{r_1} dT = \int_{r_2}^{r_1} \mu \cdot p \times 2\pi r^2 \cdot dr$$

$$= \int_{r_2}^{r_1} \mu \cdot \frac{C}{r} \times 2\pi r^2 \, dr = \mu \cdot C \cdot 2\pi \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$T = \frac{1}{2} \mu W (r_1 + r_2)$$

$$T = \mu W R_m$$

$$R_m = \frac{1}{2}(r_1 + r_2) : \text{mean Radius.}$$

∴ Total torque on a single clutch plate, is given by

$$T^* = 2T$$

$$= 2 \times \left[\frac{\mu W}{2}(r_1 + r_2)\right].$$

NOTE:- (i) For power transmission by friction through a clutch, uniform wear theory gives safer result. Hence, uniform wear should be assumed in case of friction clutch, unless it is specified otherwise.

Calculate the power transmitted by a single plate clutch at a speed of 2000rpm., if the outer & inner radii of frictn surfaces are 150mm & 100mm respectively. The max. intensity of pr. at any pt. of contact surface not to exceed 0.8×10^5 N/m², Take both sides of plate as effective & coeffecient of frictn = 0.3. Assume uniform wear.

Sol:- Given:-

Speed, N = 2000 rpm

$r_1 = 150mm = 0.15m$ | $r_2 = 100mm = 0.1m$ | $p_{max} = 0.8\times10^5$ N/m², $\mu = 0.3$; No. of effective sides = 2

For Uniform Wear, we have,

$$p\times r = \text{constant}$$

$$\text{(or)} \quad P_1\times r_1 = p_2\times r_2 = C.$$

As for Uniform wear, the product of pressure & radius is constant, hence pressure will be more where radius is less. Therefore, at inner radius, the pr. will be more.

$$\therefore \quad p_{max}\times r_2 = C. \qquad (\because r_2 \text{ inner radius})$$

$$0.8\times10^5\times0.1 = C \Rightarrow C = 0.8\times10^4$$

$$W = 2\pi C(r_1 - r_2)$$

$$W = 2\pi(0.8\times10^4)(0.15-0.1) = 2513.27\,N.$$

The torque due to both active surfaces

$$T^* = 2\left[\frac{\mu W}{2}(r_1+r_2)\right]$$

$$= 2\left[\frac{(0.3)(2513.27)}{2}(0.15+0.1)\right]$$

$$T^* = 188.49\,N\text{-}m$$

$$P = \frac{2\pi NT}{60} = \frac{2\pi\times2000\times(188.49)}{60} \Rightarrow P = 39477.25\,W$$

$$\boxed{P = 39.477\,kW} \text{ Ans}$$

(2) The external radius of a friction plate of single clutch having both sides as effective, is 150mm. The power transmitted is 20KW at a speed of 1000rpm. The maximum intensity of pr at any pt. of contact surface is 0.8×10^5 N/m². If the coefficient of friction is 0.30 Then find: (i) The internal radius of friction plate; (ii) Axial thrust with which the friction surfaces are held together.

Sol:- Given:-

External radius, $r_1 = 150mm = 0.15m$

Power transmitted, $P = 20KW = 20\times10^3$ W

$N = 1000rpm$

Max. Pr. $p_{max} = 0.8\times10^5$ N/m² ; $\mu = 0.3$

~~Find~~ (i) Internal radius, r_2

(ii) Axial thrust, W

Since, nothing is mentioned to what to assume, if power transmission in the problem it is clear that it is safer to assume uniform through a clutch and hence it is safer to assume uniform wear.

For uniform wear, $p\times r = c.$

hence, pr. will be max. where radius is minimum.

$$p_{max}\times r_2 = c.$$

$$(0.8\times10^5)\times r_2 = c.$$

$$P = \frac{2\pi NT}{60}$$

$$20\times10^3 = \frac{2\pi\times1000\times T}{60} \Rightarrow T = 190.986\ N\text{-}m \quad \text{--(i)}$$

Now using eq. of uniform wear.

$$W = 2\pi C(r_1 - r_2)$$

$$= 2\pi\,(0.8 \times 10^5) \times r_2 \times (0.15 - r_2)$$

$$= 502654.8\, r_2\,(0.15 - r_2)$$

The frictional torque due to both sides active surfaces

$$T = 2 \times \left[\frac{\mu W}{2}(r_1 + r_2)\right]$$

$$= 2 \times \left[\frac{0.3 \times 502654.8\, r_2 (0.15 - r_2)}{2}(0.15 + r_2)\right]$$

$$T = 150796.44\, r_2\,(0.15^2 - r_2^2) \quad \text{— (ii)}$$

Equating eq. (i) & (ii)

$$190.986 = 150796.44\, r_2\,(0.15^2 - r_2^2)$$

$$r_2^3 - 0.0225\, r_2 + 0.0012665 = 0$$

The above is the cubic eq. can be solved by trial & error method. i.e. LHS should be zero.

let, $r_2 = 0.095\,m$, then LHS = -0.0000137

$r_2 = 0.1\,m$, then LHS = +ve.

i.e. $0.1 > r_2 > 0.095$

let us look $r_2 = 0.097$

$r_2 = -0.0000137$

i.e. r_2 is slightly more than 0.097

let, $r_2 = 0.0974\,m = 97.4\,mm$

(ii) Axial thrust (W)

$$W = 502654.8\, r_2\,(0.15 - r_2)$$

$$= 502654.8\,(0.0974)\,(0.15 - 0.0974)$$

$$W = 2575.22\,N$$

(3) The external & internal radii of a friction clutch of disc type are 90mm & 50mm respectively. Both sides of friction clutch are effective & coefficient of friction is equal to 0.25. The friction clutch is used to rotate a machine from a shaft which is rotating at a constant speed of 2410 rpm. The moment of inertia of rotating parts of the machine is 5.5 kg.m². The intensity of pr is not to exceed 0.8×10^5 N/m². Assuming uniform wear, determine the time reqd. for the machine to attain the full speed when the clutch is suddenly applied. Also determine the energy lost in slipping of clutch.

Sol:

Given:

External radius, $r_1 = 90mm = 0.09m$

Internal radius, $r_2 = 50mm = 0.05m$

No. of effective sides = 2

coefficient of frictn, $\mu = 0.25$;

Constant speed of driving shaft, $N = 2410$ rpm

M.O.I of M/c parts = 5.5 kg-m².

Max. pr, $p = 0.8\times10^5$ N/m²

Theory assumed = uniform wear.

(i) Time Required for the machine to attain full speed of 2410 rpm:-

The driving shaft is rotating at a constant speed, whereas the machine is at rest. But when the clutch is engaged, the machine will attain its full speed not immediately but after some time. Let this time 't' sec.

Initially let us find Axial load & frictional torque for uniform wear.

$$p \times r = C.$$

$$\therefore p_{max} \times r_2 = C.$$

$$0.8\times10^5 \times 0.05 = C$$

$$\Rightarrow C = 4000$$

3.V.K π,

$$W = 2\pi C(r_1 - r_2)$$

$$W = 2\pi\, 4000(0.09 - 0.05)$$

$$W = 1005.31\,\underline{N}$$

The frictional torque due to both active surface.

$$T^* = 2\left[\frac{\mu W}{2} \times (r_1 + r_2)\right]$$

$$= 2\left[\frac{(0.25)(1005.31)}{2} \times (0.09 + 0.05)\right]$$

$$T^* = 35.186\ N\text{-}m$$

Now, angular acceleration, when total torque is $35.186\,\underline{N\text{-}m}$

$$\text{Torque} = M.O.I \times \text{Angular acceleration}$$

$$= \underline{I} \times \alpha$$

$$35.186 = 5.5 \times \alpha$$

$$\alpha = 6.397\ \underline{rad/s^2}$$

The m/c starts from rest. After some time the final angular speed of m/c will be corresponding to speed of shaft

$\therefore$ final angular speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi 240}{60} = 8\pi$ rad/s.

Let $t \to$ time reqd. $\boxed{v = u + at}$

using, $\omega = \omega_0 + \alpha t$

Initial angular speed. $\omega_0 = 0$.

$$8\pi = 0 + 6.397\,t$$

$$t = \underline{\underline{3.928\ sec}}\ \text{Ans}$$

(ii) Energy lost in slipping of clutch:-

The driving shaft is rotating at a uniform speed of 2410 rpm i.e: uniform angular velocity 8π rad/s. Let us find the angles turned by driving shaft & the driven shaft.

Angle turned by driving shaft,

$$\theta_1 = \omega \times t$$

$$\theta_1 = 8\pi \times 3.928 = 98.72 \text{ rad}$$

The angle turned by driven shaft (or) m/c.

$$\theta_2 = \omega_0 t + \frac{1}{2} \alpha t^2 \qquad \left(\because s = ut + \tfrac{1}{2} at^2 \right)$$

$$= 0 \times 3.928 + \frac{1}{2} (6.397)(3.928)^2$$

$$\theta_2 = 49.35 \text{ rad}$$

Energy lost in friction due to clutch slip

= Frictional torque × Angle of slip

$$= T \times (\theta_1 - \theta_2)$$

$$= 35.186 \times (98.72 - 49.35)$$

$$= 1737.13 \text{ N-m}$$

(31)

Multi-Plate Clutch:-

Let, $r_1 \rightarrow$ External radius of friction lining on friction plate

$r_2 \rightarrow$ Internal radius " " " " "

$W \rightarrow$ Axial load

$p \rightarrow$ Intensity of pr.

$n_1 \rightarrow$ no. of friction plates on driving shaft

$n_2 \rightarrow$ " " disc on driven shaft

Then, no. of active surfaces (or) friction surfaces will be given as,

$$\boxed{n = n_1 + n_2 - 1}$$

Total torque transmitted is given by

$$T = n \times \mu \cdot W \times R_m$$

1) A multi-clutch has six plates (friction rings) on the driving shaft & six plates on driven shaft. The external radius of the friction surface is 115mm whereas the internal radius is 80mm. Assuming uniform wear & $\mu = 0.1$, find the power transmitted at 2000rpm. Axial intensity of pr is not to exceed $0.16\, N/mm^2$.

Sol:- Given:- no. of friction plates, $n_1 = 6$

no. of discs, $n_2 = 6$.

$\therefore$ no. of active surface, $n = n_1 + n_2 - 1$

$= 6 + 6 - 1 \Rightarrow 11$

External radius of friction surface, $r_1 = 115mm = 0.115m$

internal " " " , $r_2 = 80mm = 0.08m$

$\mu = 0.1$; $N = 2000rpm$

Max. intensity of pr, $p_{max} = 0.16\, N/mm^2$

$= 0.16 \times 10^6\, N/m^2$

Total torque transmitted

$T = n \times \mu \times W \times R_m$

Where, $R_m \rightarrow$ mean Radius.

$$R_m = \frac{r_1 + r_2}{2} = \frac{0.115 + 0.08}{2}$$

$$R_m = 0.0975m$$

For Uniform Wear,

$$p \times r = c$$

$$p_{max} \times r_2 = C$$

$$0.16 \times 10^6 \times 0.08 = C$$

$$\Rightarrow C = 128 \times 10^2$$

W.k.T, $W = 2\pi C (r_1 - r_2)$

$= 2\pi \cdot 128 \times 10^2 (0.115 - 0.08)$

$W = 2814.8670$

$\therefore T = \mu \cdot W \cdot R_m \times n$

$= 0.1 \times 2814.867 \times 0.0975 \times 11$

$T = 301.89\, N\,m$

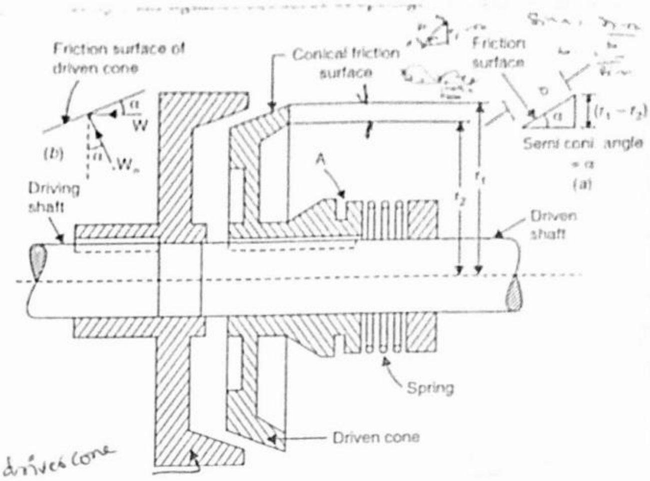
$\therefore$ Power transmitted, $P = \frac{2\pi N T}{60}$

$= \frac{2\pi \times 2000 \times 301.89}{60}$

$P = 63.22\, kW$

(b) 33

* Cone Clutch:-



Let $r_1 \rightarrow$ External radius of friction surface.

$r_2 \rightarrow$ Internal " " " " .

$\alpha \rightarrow$ Semi cone angle

$W \rightarrow$ Total axial load.

$R_m \rightarrow$ Mean Radius

$\mu \rightarrow$ coefficient of frictn.

$b \rightarrow$ width of contact surface (or) width of cone face.

$$= \frac{(r_1 - r_2)}{\sin\alpha}$$ Similar to that of conical surface

(i) In case of Uniform Pressure:

$$T = \frac{2}{3} \cdot \frac{\mu W}{\sin\alpha} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right].$$

(ii) uniform Wear:- $$T = \frac{1}{2} \frac{\mu W}{\sin\alpha} (r_1 + r_2)$$

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(iii) Driving torque based on Mean Radius:-

Let, $p_n \rightarrow$ intensity of pr at mean radius normal to frictn surface

$W_n \rightarrow$ Total load normal to frictn surface

$= p_n \times (2\pi R_m \times b)$

W = component of W_n in axial dirn.

$= W_n \times \sin\alpha$

$$T = \frac{1}{2} \cdot \frac{\mu W}{\sin\alpha} (r_1 + r_2)$$

$$= \mu \times \frac{W}{\sin\alpha} \left[\frac{r_1 + r_2}{2}\right]$$

$$\boxed{T = \mu \times W_n \times R_m}$$

$\therefore W_n = \frac{W}{\sin\alpha}$; $R_m = \frac{r_1 + r_2}{2}$.

The above eq. gives the torque in terms of W_n & R_m

Problem (35)

A cone clutch of cone angle 30° is used to transmit a power of 10 kW at 800 rpm. The intensity of Pr b/w the contact surfaces is not to exceed 85 kN/m². The width of conical friction surface is half of mean radius. If co-efficient of friction = 0.15, then find the dimensions of contact surfaces. Assume uniform wear. Also find the axial load or force reqd. to hold the clutch while transmitting the power. What is the width of friction surface?

Sol:- Given:-

Cone angle, $2\alpha = 30°$; $\alpha = 15°$.

Power $P = 10\ kW = 10\times10^3\ W$; $N = 800\ rpm$.

Max. Pr ; $P_{max} = 85\ kN/m^2 = 85\times10^3\ N/m^2$.

Width, $b = \frac{1}{2}\times$ mean Radius $= \frac{1}{2}\times R_m$

$= \frac{1}{2}\frac{(r_1+r_2)}{2}$; $\mu = 0.15$.

(i) Dimensions of contact surfaces i.e: r_1 & r_2:

W.K.T, $P = \frac{2\pi NT}{60}$ — (i)

$T = 119.366\ N\text{-}m$

width 'b' given as $= \frac{1}{2}R_m = \frac{1}{2}\frac{(r_1+r_2)}{2} = b$. — (a)

But $b = \frac{r_1-r_2}{\sin\alpha} = \frac{r_1-r_2}{\sin 15°} = \frac{r_1-r_2}{0.2588}$ — (b)

(36)

Equating (a) & (b)

$$\frac{r_1+r_2}{4} = \frac{r_1-r_2}{0.2588}$$

$$r_1 = 1.138 r_2$$

For uniform wear, $p \times r = C$

$$p_{max} \times r_2 = C.$$

$$85\times10^3 \times r_2 = C.$$

The value of 'W' uniform wear is given by.

$$W = 2\pi C (r_1-r_2)$$

$$= 2\pi \times 85\times10^3 \times r_2 (r_1-r_2)$$

$$= 534070 r_2 (r_1-r_2)$$

The frictional torque for uniform wear

$$T = \frac{1}{2} \cdot \frac{\mu W}{\sin\alpha} (r_1+r_2)$$

$$= \frac{1}{2} \times \frac{(0.15)(534070 r_2 (r_1-r_2))}{\sin\alpha} (r_1+r_2)$$

$$= 154762 r_2 (r_1+r_2)(r_1-r_2).$$

Sub. T from eq (i) in above eq.

$$119.366 = 154762 r_2 (r_1^2 - r_2^2)$$

$$\therefore r_1 = 1.138 r_2$$

$$119.366 = 154762 r_2 \left[(1.138 r_2)^2 - r_2^2\right]$$

$$r_2 = 0.138m = 138mm$$

$$r_1 = 0.157m = 157mm$$

Sub. values of r_1 & r_2 $W = 534070 r_2 (r_1-r_2)$

$$W = 1400.3N$$

width of friction surface $b = \frac{r_1-r_2}{\sin\alpha} = 73.4mm.$

(37)

(2). A cone clutch of semi-cone angle 15° is used to transmit a power of 30 kW at 800 rpm. The mean frictional surface radius is 150 mm. The normal intensity of pr at the mean radius is not to exceed 0.15 N/mm². The coefficient of friction is 0.2. Assuming uniform wear.

Determine: (i) Width of contact surface &

(ii) Axial load needed to engage the clutch.

Sol:- Given:-

$\alpha = 15°$; $P = 30\,kW = 30\times10^3\,W$;

$N = 800\,rpm$; $R_m = 150\,mm = 0.15\,m$; $p_n = 0.15\,N/mm^2 = 0.15\times10^6\,N/m^2$

$\mu = 0.2$;

$$P = \frac{2\pi NT}{60}$$

$$30\times10^3 = \frac{2\pi\times800\times T}{60}$$

$$T = 358.1\,N\cdot m$$

W.K.T,

$$T = \mu\cdot W_n \times R_m$$

$$358.1 = 0.2\times W_n\times 0.15$$

$$W_n = 11936.67\,N.$$

But, $W_n \rightarrow$ Total load normal to friction surface of cone.

$$W_n = p_n\times(2\pi R_m\times b)$$

Sub,... W_n to get 'b'

$$11936.67 = 0.15\times10^6\,(2\pi\,(0.2)\times b)$$

$$b = 0.084\,m \text{ (or) } 84\,mm$$

To get Axial load

$$W = W_n\times\sin\alpha$$

$$= 11936.67\times\sin 15°$$

$$W = 3089.4\,N$$

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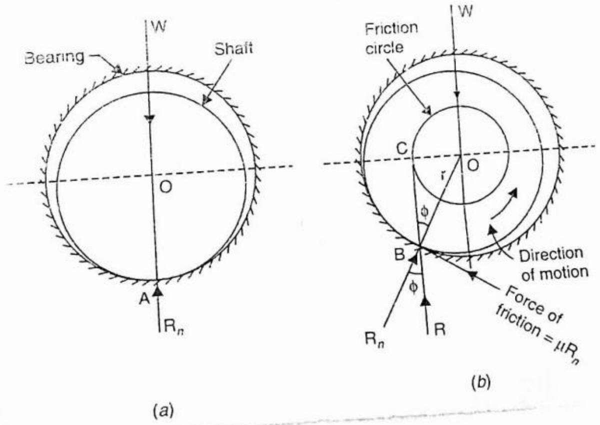
* Greasy friction of a Journal:

The following diagram shows a shaft inside a bearing. When the shaft is at rest in the bearing, the weight of shaft, W passes through centre of gravity at 'O'. A contact of shaft & bearing is maintained at pt. 'A' as shown in fig. ⓐ.



(a) (b)

The contact point 'A' is known as seat of pressure for bearing. The reaction of bearings acts at 'A' & is in line with W in the vertically upward dirn.

When the shaft is rotating because of clearance seat of pressure will roll or climb up the bearing in opposite dirn to that of rotation at pt. 'B' as shown. Metal to metal contact exists at pt. 'B' & greasy friction condition is applicable. as oil film is having very thin layer of lubricant.

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The climbing or rolling up will stop when following three forces are in equilibrium:

(i) Wt. of shaft W, acting vertically downwards.

(ii) Normal Reactn R_N at B, which is radial & passes through the pt. 'O' as shown.

(iii) Frictional force, tangential to shaft at B & acting in opp. dirn of motn of shaft.

$$F = \mu \cdot R_N$$

The frictional force & normal reactn can be combined in to single resultant force R which is inclined at ϕ. Hence the shaft is in equilibrium now under following forces:

1. Weight of shaft W, acting vertically downwards.
2. Single resultant reactn R.

For equilibrium, R must be equal to W. & must act vertically ↑. R & W are equal & parallel. & they form a couple. This couple is called as frictn couple.

Moment of friction couple

$$= W \times \perp \text{ distance b/w } R \& W$$

$$= W \times OC.$$

$$= W \times r \sin\phi.$$

The angle ϕ is very small, $\sin\phi = \tan\phi$.

$$\therefore W \times r \tan\phi \qquad \therefore \boxed{\tan\phi = \mu}$$

$$= W \times r \times \mu.$$

This frictn couple acts in a dirn. opposite to dirn of rotatn as is clear. This frictn couple opposes the driving ~~shaft~~ torque on shaft. And it will be equal to driving torque for equilibrium.

(41)

* Friction Circle :-

The circle of radius equal to $OC = r \tan\phi$
$= r \times \mu$

is known as "friction circle". This radius of friction circle, which is equal to $r \times \mu$, will be constant as the values of 'r' & μ are constant. Hence the radius of friction circle is independent of load or weight of shaft.

Power loss in friction:

frictional torque, T = Moment of friction couple

$= W \times OC$

$= W \times r \times \mu$

Power lost in friction,

$= T \times \omega$

$= (W \times r \times \mu) \times \omega$

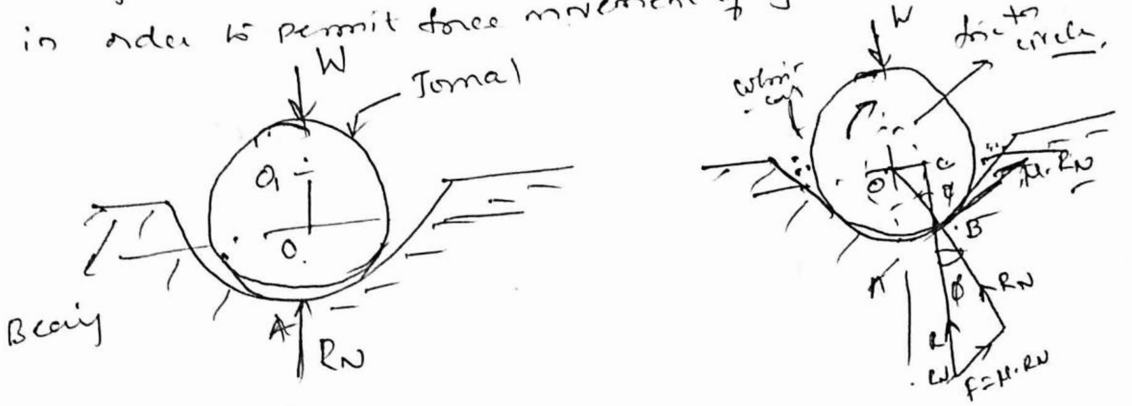
$= (W \times r \times \mu) \cdot \left(\frac{v}{r}\right)$

$= W \times \mu \cdot v$ watts

$= \frac{\mu W v}{1000}$ kW

* Friction Circle:-

A journal bearing forms a turning pair. The fixed outer element of turning pair is called a bearing. The inner element which fits the bearing is called journal. The journal is slightly less in dia than that of bearing, in order to permit free movement of journal in bearing.



When bearing is not lubricated then there is line contact b/w 2 elements. The load W & normal Reaction R_N of bearing acts through centre. Reactn R_N acts vertically upwards at pt. A. This pt. called pt of contact @ seat.

Now consider a shaft rotating inside a bearing in c.w dirn. The reactn, R doesnot act vertically upwards, but acts at another pt. of contact 'B'. This is due to fact that the shaf rotates., $F = \mu . R_N$ acts at circumference of shaft which has a tendency to rotate the shaft in opp. dirn of motion & this shifts pt. A to pt B.

(Resultant of $F + R_N$ = R)

$\phi \to$ Angle b/w R & R_N

$\mu \to$.

$T \to$ friction torque

$r \to$ rad. of shaft

for uniform motn, resultant force acting on shaft must be zero. & resultant T.m should be zero

$$R = W \quad \& \quad T = W \times OC = W \times OB \sin\phi$$
$$= W r \sin\phi.$$

∴ since ϕ is very small sob.. $\sin\phi = \tan\phi$.

$$T = W \cdot r \tan\phi = \mu \cdot W r.$$

If shaft rotatin with angle velocity ω ∴ then power lost

$$P = T \cdot \omega$$

NOTE:- ① . if a circle is drawn with centre O & radius as OC = then the circle is called a friction circle.

② force R exerted by one element of a turning pair acts along a tangent to friction circle

Brakes & Dynamometers.

Brake:

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of machine.

The capacity of a brake depends upon the following factors:

1. The unit pressure between the braking surfaces.
2. The co-effecient of friction between braking surfaces.
3. The peripheral velocity of brake drum
4. The projected area of friction surfaces, &
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The materials used for the brake lining should have the following characteristics:

1. The co-effecient of fricth should remain constant, with change in temperature.
2. It should have low wear rate.
3. It should have high heat resistance.
4. It should have high heat dissipation capacity.
5. It should have adequate mechanical strength.
6. It should not affected by moisture (or) oil.

(2)

Types of Brakes:

The brakes, according to the means used for transforming the energy by braking elements are classified as:

1. Hydraulic Brakes e.g., pumps (or) hydrodynamic brake.
2. Electric Brake. e.g., Generators.
3. Mechanical Brake.

Hydraulic & electric brakes cannot bring the member to rest and are largely used where large amount of energy is to be transformed.

These brakes are also used for retarding (or) controlling the speed of vehicle for down-hill travel.

Mechanical brakes, according to the direction of acting force, may be divided in following 2 groups:

a) Radial brakes &
b) Axial brakes.

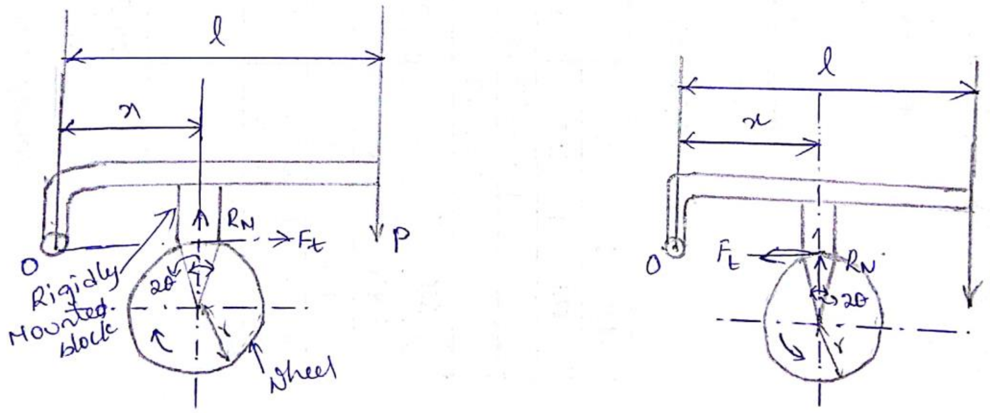
a) Radial Brakes:- In these brakes, the force acting on the brake drum is in radial direction. These may subdivided in to external brakes & internal brakes. According to the shape of the friction elements, these brakes may block (or) shoe brakes & band brakes.

In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes & cone brakes.

* Single Block (or) Shoe Brake:-

It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made up of a softer material than the rim of wheel. This type of brake is commonly used in trains, and tram cars.

The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retards the rotation of wheel. The block is pressed against wheel by a force applied to one end of lever is pivoted on a fixed fulcrum O.



(a) clockwise Rotation of brake wheel

(b) Anti clockwise direction rotation of brake wheel.

Fig:) Line of action of F_t passes through the fulcrum

4

Let,
$P \rightarrow$ force applied at the end of lever,
$R_N \rightarrow$ Normal force pressing the brake block on wheel.
$r \rightarrow$ radius of wheel.
$2\theta \rightarrow$ Angle of contact surface of block.
$\mu \rightarrow$ co-effecient of friction.
$F_t \rightarrow$ Tangential braking force or frictional force acting at the surface of the block & wheel.

If the angle of contact is less than 60°, then it may be assumed that normal pressure between the block & wheel is uniform. In such cases, tangential breaking force on wheel, $F_t = \mu \cdot R_N$ — ①

& Braking Torque, $T_B = F_t \cdot r = \mu \cdot R_N \cdot r$ — ②.

Let us consider the following cases:

case 1: When the line of action of tangential braking force passes through the fulcrum O of the lever, and the brake wheel rotates clockwise as shown in fig 1(a), then for equilibrium, taking moments about fulcrum (O),

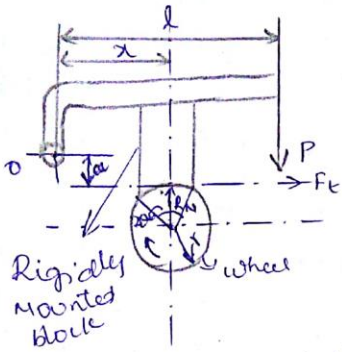
then,
$$R_N \times x = P \times l.$$
$$R_N = \frac{P \times l}{x}.$$

Then,
$$T_B = \mu \cdot R_N \cdot r.$$
$$= \mu \cdot \frac{P \times l}{x} \cdot r$$
$$\boxed{T_B = \frac{\mu \times P \times l \times r}{x}}$$

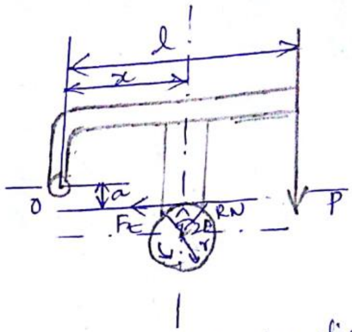
The braking torque in this case will also be same for anticlockwise direction.

(5).

Case 2: When the line of action of tangential braking force passes through a distance 'a' below the fulcrum 'O', and the brake wheel rotates clockwise as shown in fig 2(a),



(a) clockwise rotation of Brake Wheel.



(b) Anti-clockwise direction of Brake wheel.

Now, when the brake wheel rotates in clockwise direction, then for equilibrium, taking moments about the fulcrum O,

$$R_N \times x + F_t \times a = P \times l$$

$$R_N \times x + \mu \cdot R_N \times a = P \cdot l.$$

$$R_N [x + \mu \cdot a] = P \cdot l$$

$$\boxed{R_N = \frac{P \cdot l}{x + \mu \cdot a}}$$

& Braking torque, $T_B = \mu \cdot R_N \cdot r.$

$$= \mu \cdot \frac{P \cdot l}{(x + \mu \cdot a)} \cdot r.$$

$$\boxed{T_B = \frac{\mu \cdot P \, l \cdot r}{(x + \mu a)}}$$

When the brake wheel rotating in A.C.W then for equilibrium, taking moments about fulcrum then,

$$R_N \times x = F_t \times a + P \times l.$$

$$R_N \times x = \mu R_N \times a + P \times l.$$

$$R_N \times x - \mu R_N \times a = P \times l.$$

$$R_N [x - \mu a] = P \times l$$

$$R_N = \frac{P \times l}{x - \mu a}.$$

then Braking torque,

$$T_B = \mu \cdot R_N \cdot r.$$

$$= \mu \left[\frac{P \times l}{x - \mu a}\right] \cdot r$$

$$\boxed{T_B = \frac{\mu \times P \times l \cdot r}{(x - \mu a)}}$$

Case 3: When the line of action of the tangential braking force (F_t) passes through a distance 'a' above the fulcrum O,

Now, the brake wheels rotates in the clockwise direction then ~~these~~ for equilibrium moments about fulcrum O,

$$R_N \cdot x = P \cdot l + F_t \cdot a$$

$$R_N \cdot x = P \cdot l + \mu \cdot R_N \cdot a$$

$$R_N \cdot x - \mu \cdot R_N \cdot a = P \cdot l$$

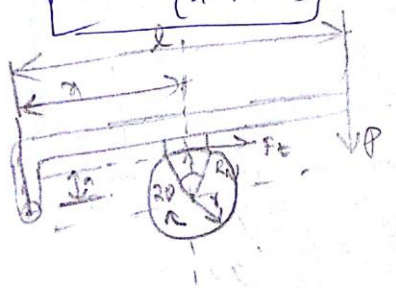
$$R_N [x - \mu \cdot a] = P \cdot l$$

$$R_N = \frac{P \cdot l}{[x - \mu a]}$$

Then $T_B = \mu \cdot R_N \cdot r$

$$= \mu \frac{P \cdot l}{x - \mu a} \cdot r$$

$$\boxed{T_B = \left(\frac{\mu \cdot P \cdot l \cdot r}{x - \mu a}\right)}$$



a) clockwise rotation of a wheel

When the brake wheels rotates in counter clockwise direction, then for equilibrium taking moments along fulcrum O,

$$R_N \cdot x + F_t \cdot a = P \times l.$$

$$R_N \cdot x + \mu \cdot R_N \times a = P \times l.$$

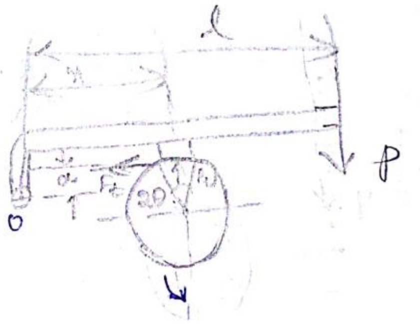
$$R_N \{x + \mu \cdot a\} = P \times l$$

$$R_N = \frac{P \times l}{x + \mu a}$$

Then, $T_B = \cancel{F_t} \mu \cdot R_N \cdot r$

$$= \mu \cdot \frac{P \times l}{x + \mu a} \cdot r$$

$$\boxed{T_B = \left(\frac{\mu \cdot P \times l \cdot r}{x + \mu a}\right)}$$



(b) Anticlockwise rotation of a brake wheel.

Fig: Line of Action of F_t passes below fulcrum.

7

NOTE :

1. from the above we see that when the brake wheels rotates anticlockwise in case 2 and when it rotates in clockwise in case 3, the equations are same.

(a) Here, the frictional force helps to apply the ~~brake~~.
Such type of brakes are said to self-energising brake.

(b) When the frictional force is ~~not enough~~ i.e. greater enough to apply the brake with no external force, then the brake is said to be self-locking brake.
No external force is needed to apply the ~~brake~~ &
hence the brake is self locking.
∴ the condition will be,

$$\boxed{x \leq \mu . a}$$

2. The brake should be self energising & not self locking.

3. In order to avoid self locking & to prevent the brake from grabbing, x is kept greater than $(\mu . a)$.

4. If A_b is the projected bearing area of shoe brake, then the bearing pressure on the shoe,

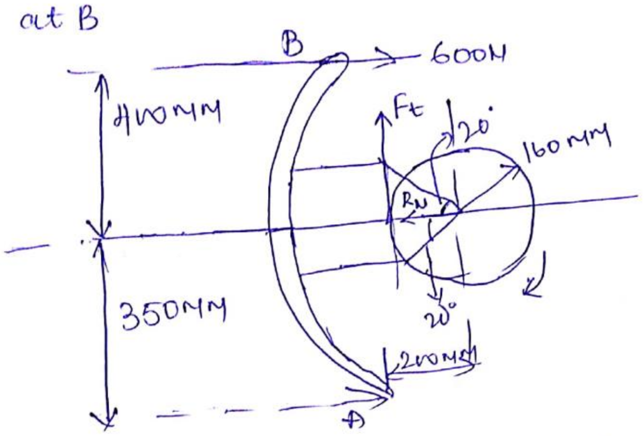
$$P_b = \frac{R_N}{A_b}$$

where, $P_b \rightarrow$ bearing pressure.
$A_b \rightarrow$ width of shoe x projected length of shoe
$= w(2r\sin\theta)$

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) & produces bending of shaft.
To overcome this drawback, double shoe brake is used.

Problem (8).

(1) Following fig shows a brake applied to a drum by a lever AB which is pivoted at a fixed point A & rigidly fixed to the shoe. The radius of drum is 160MM. The coeff. of friction at brake lining is 0.3. If the drum rotates in C.W, find the braking torque due to horizontal force of 600N at B



Sol: Given, $r = 160MM = 0.16M$; $\mu = 0.3$; $P = 600N$.

Since the angle subtended by the shoe at the drum is $40°$.

let, $R_N \rightarrow$ Normal force pressing the block of brake drum

$F_t \rightarrow$ Tangential force $= \mu \cdot R_N \Rightarrow F_t = \mu \cdot R_N$

$R_N = \frac{F_t}{\mu}$

Taking moments about point A,

$$R_N \times 350 + F_t(200-160) = 600(410+350)$$

$$\frac{F_t}{\mu} \times 350 + F_t(40) = 600(750)$$

$$\frac{F_t}{0.3} \times 350 + 40F_t = 450 \times 10^3.$$

$$F_t\left[\frac{350}{0.3} + 40\right] = 450 \times 10^3.$$

$$\therefore \boxed{F_t = 372.8N}$$

We know that,

$$T_B = F_t \cdot r$$

$$= 372.8 \times 0.16$$

$$\boxed{T_B = 59.6 N\text{-}M}$$

(9).

(2). A bicycle and rider of mass 100kg are travelling at the rate of 16km/hr on a level road. A brake is applied to the rear wheel which is 0.9m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100N & μ=0.05

Sol: Given Data:

$m = 100kg$; $v = 16km/hr$

$v = 4.44m/sec$

$D = 0.9m$; $R_N = 100N$; $\mu = 0.05$.

(i) Distance travelled by a bicycle before it comes to rest

Let, $x \rightarrow$ distance travelled by the bicycle before it comes to rest.

W.K.T, Tangential breaking force acting at the point of contact of brake & wheel.

$F_t = \mu \cdot R_N$

$= 0.05 \times 100 = 5N$.

and Work done, $\Rightarrow F_t \times x = 5x$ N-m.

In order to bring the bicycle to rest, Work done against friction must be equal to be kinetic energy.

Kinetic Energy, $K.E = \frac{mv^2}{2} = \frac{(100)(4.44)^2}{2}$ $KJ = N\frac{m}{sec}$ N-m

$= 986$ N-m

$\therefore$ Work done $= KE$

$5x = 986$

$x = \frac{986}{5}$

$\boxed{x = 197.2m}$

(ii) no. of revolutions made by bicycle

distance travelled $= \pi DN$

$x = \pi DN$

$197.2 = \pi(0.9)N$

$\boxed{N = 70}$ revolutions

$\therefore$ 70 revolutions made by bicycle before it comes to rest.

(18)

* Pivoted Block (or) Shoe Brake:-

In the last case we have discussed that, if the angle of contact is less than 60°, then the normal pressure between block & wheel is uniform.

If the angle of contact is greater than 60° between block & wheel, then the unit normal pressure to the surface of contact is less at the ends than at centre. In such cases, block is pivoted to lever. as shown, instead being rigidly attached to lever. This gives uniform wear of a brake lining in the direction of applied force.

The braking torque for a pivoted block when 2θ > 60° will be,

$T_B = F_t \times r$

$= \mu' R_N \cdot r$

where, $\mu' \rightarrow$ Equivalent frictional force, $= \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

$\mu \rightarrow$ actual friction

These brakes have more life time & may provide a higher braking torque.

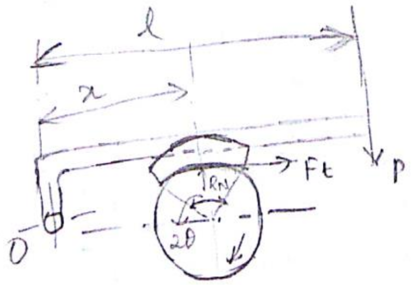
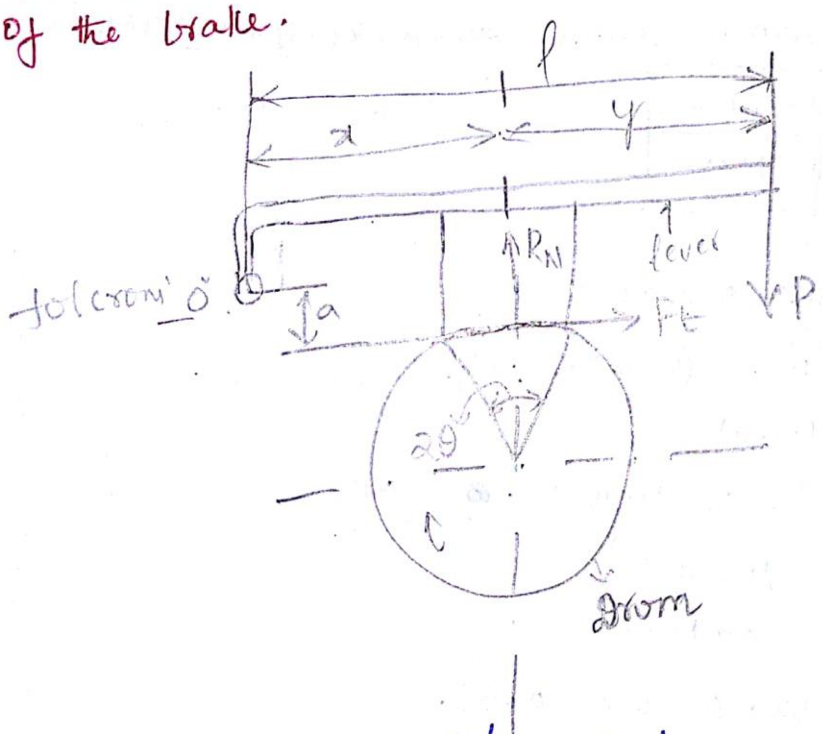


fig: Pivoted block

The diameter of the brake drum of a single block shown in fig. is 1m. It sustains 240N-m of torque at 400rpm. The coeffecient of friction is 0.32. Determine the required force to be applied when the rotation of drum is (a) clockwise; (b) counter-cw. and the angle of contact is (i) 35° ; & (ii) 100°: Given that, $l = 800mm$; $x = 150\,mm$; $a = 25mm$. Also find the new value of 'a' for self-locking of the brake.



C.W. rotation of drum.

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(c) New value of 'a' for self-locking brake:-

For self-locking, the external force must be zero: ie: 'p' must be zero and thus the condition is,

$$x \leq \mu \cdot a$$

or

$$x = \mu \cdot a$$

$$\therefore a = \frac{x}{\mu}$$

$$a = \frac{0.15}{0.32} = 0.4688\,m \Rightarrow \boxed{a = 468.8\,mm} \text{ Ans}$$

(ii) When $2\theta = 100°$:

Since, the angle of contact is more than 60°: then the coeffecient of friction [μ] is replaced by, μ'.

$$\therefore \mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.32 \times \sin 50}{100 \times \frac{\pi}{180} + \sin 100}$$

$$\therefore \mu' = 0.359.$$

(a) When the rotation of drum is clockwise :-

$$\tau_B = \frac{\mu' . P . l . r}{x + \mu' a}$$

$$240 = \frac{0.359 \times P . (0.8)(0.5)}{[0.15 + (0.359)(0.025)]}$$

$\therefore$ $P = 265.7 N.$

(b) When the rotation of drum is A.C.W.

$$\tau_B = \frac{\mu' . p . l . r}{x - \mu' a.}$$

$$240 = \frac{(0.359) . P . (0.8)(0.5)}{[0.15 - (0.359)(0.025)]}$$

$P = 235.7 N.$

(c) New value of 'a' for selflocking of brake:-

Again the condition is,

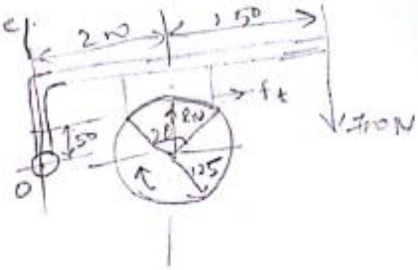
$$x = \mu' . a.$$

$$a = \frac{x}{\mu'} = \frac{0.15}{0.359} = 0.417 m$$

$$\therefore \boxed{a = 417 mm}$$

Ans

(1) A Single block brake as shown in fig. (11)
The diameter of the drum is 250mm and the angle of contact is 90°. If the operating force of 700N is applied at the end of lever & coefficient of friction between drum & lining is 0.35. Determine the torque that may be transmitted by the block brake.



Sol: Given Data:

$d = 250mm$; $r = 125mm$

$2\theta = 90°$; $P = 700N$; $\mu = 0.35$.

$T_B = F_t \times r$

$\therefore F_t = \mu' \cdot R_N$.

$\mu' \rightarrow$ Equivalent frictional force, $\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

$\mu' = \frac{4 \times 0.35 (\sin 45°)}{90 + \sin 90} = 0.385$.

Taking moments about fulcrum O, we have,

$R_N \times 200 = 700(200+250) + F_t \times 50$

$\frac{F_t}{\mu'} \times 200 = 315000 + F_t \times 50$.

$\frac{F_t}{0.385} \times 200 = 315000 + F_t \times 50$

$520 F_t = 315000 + F_t \times 50$

$520 F_t - 50 F_t = 315000$

$\boxed{F_t = 670N}$

Now Torque transmitted by a brake may be,

$T_B = F_t \times r$

$= 670 \times 125$

$= 83750 \text{ N-mm}$

$\boxed{T_B = 83.75 \text{ N-m}}$

(5)

* Simple Band Brake:

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of circumference of drum. A simple band brake in which one end of the band is fixed to fixed point (or) pin (or) fulcrum of lever and the other end is attached to lever at a distance b from fulcrum.

When a force P is applied to the lever at C, the lever turns about fulcrum pin 'O' & tightens the band on the drum and hence the brakes are applied. The friction between the band & the drum provides brake force. The force 'P' on the lever at C may be determined as: &

$T_1 \rightarrow$ Tension in the tight side of band,

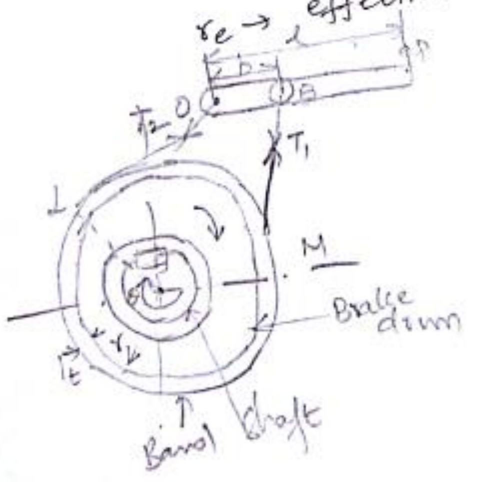
$T_2 \rightarrow$ Tension in the slack side of band.

$\theta \rightarrow$ Angle of lap of band on drum.

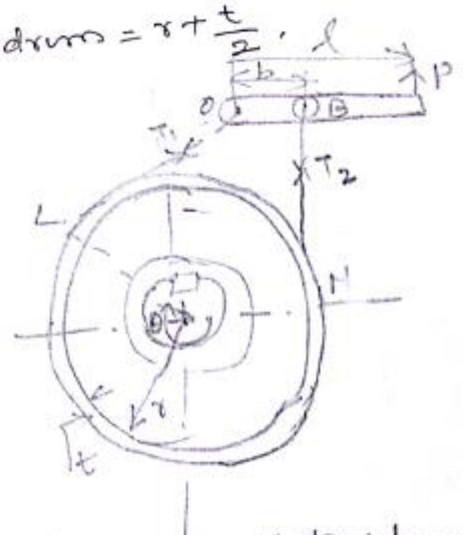
$r \rightarrow$ radius of drum,

$t \rightarrow$ thickness of band &

$r_e \rightarrow$ effective radius of drum $= r + \frac{t}{2}$.



(a) C.W rotation of drum



(b) A.C.W rotation of drum

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I. Operating force when drum rotates in Anticlockwise:-

Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates in Anticlockwise, then, the end of the band attached to O will be tight side with tension T_1 and the end of the band attached to other end B, will be slack side with tension T_2.

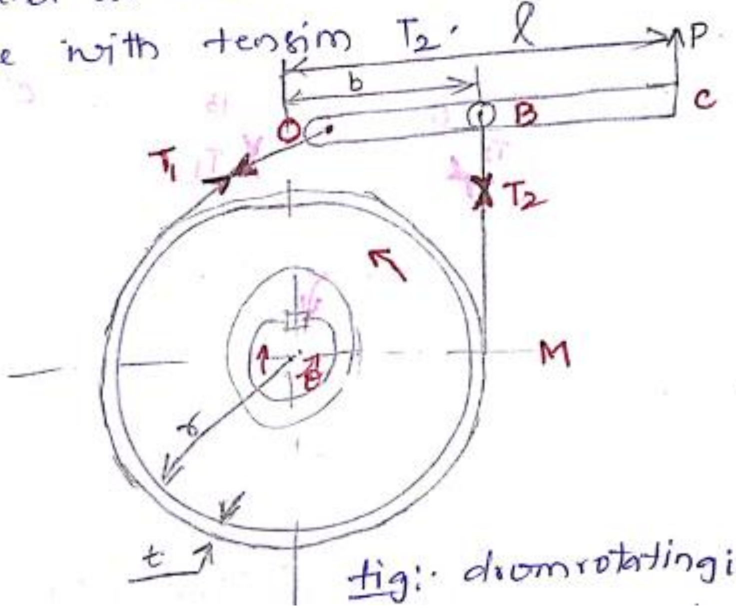


fig:- drum rotating in A.C.W

Now taking moments about fulcrum O, we have.

$$P \times l = T_2 \cdot b$$

$$P \cdot 0.5 = 444 \times 0.1$$

$$\therefore \boxed{P = 88.8\,N}$$ Ans

② Operating force, when the drum rotates in C.W:-

As we know that, the operating force acts in upward dir^n. and the drum is rotating in clockwise dir^n, then the end of the band attached to the fulcrum 'O' will be slack side, with tension T_2 and the other end of the band attached to 'B' will be tight side with Tight side T_1.

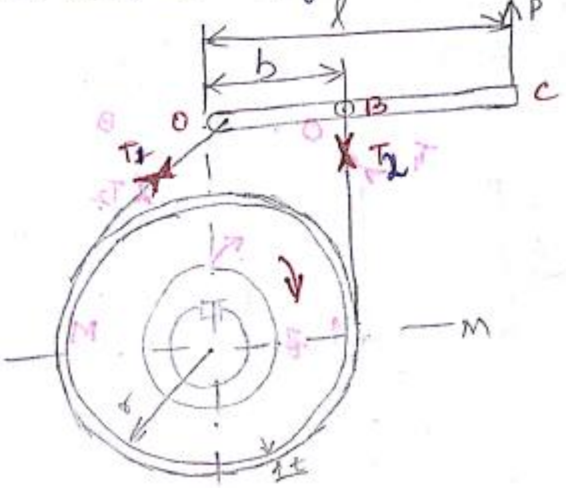


fig: Drum rotating in C.W

Now taking moments about 'O',

$$P.l = T_1.b.$$

$$P.0.5 = 1443.8 \times 0.1$$

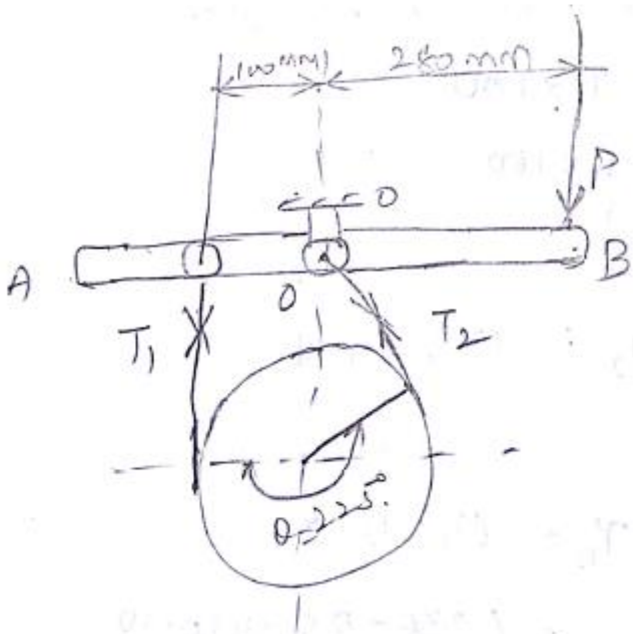
$$\boxed{P = 288.76N}$$ Ans

A simple band brake shown in fig. applied to a shaft carrying a flywheel of mass of 250kg and of radius of gyration 300mm. The shaft speed is 200 rpm. The drum diameter is 200 mm & coefficient of friction is 0.25. The angle of lap of band on drum is 225°.

Determine: (i) The brake torque when a force of 120N is applied at lever end.

(ii) The no. of turns of flywheel before it comes to rest, and.

(iii) The time taken by the flywheel ~~before~~ to come to rest:



Sol: Given Data:-

$D = 200mm$
$R = 100mm$
$r = 0.10m$

$m = 250kg$; $k = 300mm = 0.3m$; $N = 200rpm$;
$\mu = 0.25$; $\theta = 225^\circ = 225^\circ \times \pi/180 = 3.92\ rad$; $P = 120N$.

(i) Brake Torque applied at lever end;

Tension ratio, $2.3 \log\left[\frac{T_1}{T_2}\right] = \mu \cdot \theta$.

$$2.3 \log\left[\frac{T_1}{T_2}\right] = (0.25)(3.92)$$

$$\log\left[\frac{T_1}{T_2}\right] = 0.426$$

$$\frac{T_1}{T_2} = 2.67. \Rightarrow T_1 = 2.67 T_2 \quad -(a).$$

Taking moments about fulcrum O, we get,

$$P \times 280 = T_1 \times 100$$

$$120 \times 280 = T_1 \times 100$$

$$\boxed{T_1 = 336\ N}$$

$T_1 = 2.67\ T_2$
$336 = 2.67\ T_2$

$T_2 = 125.84\ N.$

$\therefore$ Braking torque, $\tau_B = (T_1 - T_2) r$.

$$= (336 - 125.84) 0.10$$

$$\boxed{\tau_B = 21.01\ N\text{-}m}$$

(ii) No. of turns of flywheel before it comes to rest,

$$K.E = \cancel{\;}\ \frac{1}{2} I \omega^2 . \qquad \because \omega = \frac{2\pi N}{60}$$

$$\omega = \frac{2\pi \times 210}{60}$$

$$= \frac{1}{2} m k^2 \cdot \omega^2$$

$$= \frac{1}{2} \times 250 \times (0.3)^2 \times \left[\frac{2\pi \times 200}{60}\right]^2 ,$$

$$K.E = 4934.80 \text{ N-m.}$$

This kinetic energy is used to overcome the work done due to braking torque.

$$\therefore \text{ Kinetic energy of flywheel} = T_B \times 2\pi n .$$

$$4934.80 = 21.01 \times 2\pi (n)$$

$$\therefore n = 37.38 \text{ revolutions.}$$

$$\boxed{n \simeq 37} .$$

(iii) Time taken by the flywheel to come to rest:-

$$\text{Time taken} = \frac{n}{N} = \frac{37}{200} = 0.1868 \text{ min}$$

$$= 11.208 \text{ sec's}$$

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Let,

$\alpha \Rightarrow$ Angle of Inclination of plane to horizontal;

$m \rightarrow$ mass of vehicle;

$W \rightarrow$ Weight of vehicle; $W = m \cdot g$,

$h \Rightarrow$ height of C·G of vehicle above road surface.

$x \rightarrow$ Perpendicular distance of C·G from rear axle.

$L \Rightarrow$ Wheel base of vehicle.

$R_A \Rightarrow$ total normal reaction between the ground and front wheels.

$R_B \Rightarrow$ total normal reaction between the ground and rear wheels.

$\mu \Rightarrow$ coeffecient of friction b/w the types of road surfaces.

$a \rightarrow$ Retardation of vehicle.

$F_A \rightarrow \mu \cdot R_A \Rightarrow$ Total braking force acting at the front wheels due to application of brakes and.

$F_B \Rightarrow \mu \cdot R_B \rightarrow$ Total braking force acting at rear wheels due to application of brakes.

Substituting F_A and R_A in (iii)

$$\mu \cdot R_A \cdot h + (mg\cos\alpha - R_A)x = R_A(L-x)$$

$$\mu \cdot R_A \cdot h + mg\cos\alpha \cdot x = R_A \cdot L$$

$$\therefore \quad R_A = \frac{mg\cos\alpha \cdot x}{(L - \mu \cdot h)} \quad \text{— (A)}$$

and $R_B = mg\cos\alpha - R_A$

$$= mg\cos\alpha - \frac{mg\cos\alpha \cdot x}{L - \mu \cdot h}$$

$$= mg\cos\alpha\left[1 - \frac{x}{L-\mu \cdot h}\right]$$

$$\therefore \quad R_B = mg\cos\alpha\left[\frac{L - \mu h - x}{L - \mu \cdot h}\right] \rightarrow \text{(B)}$$

From equation (i) the retardation of vehicle is given by,

$$a = \frac{F_A + mg \cdot \sin\alpha}{m} = \frac{\mu \cdot R_A + mg\sin\alpha}{m}$$

Substituting the value of R_A,

$$\cancel{a = \frac{\mu \cdot mg\cos\alpha \cdot x + mg\sin\alpha}{(L - \mu \cdot h)\, m}}$$

$$\Rightarrow a = \frac{\mu \cdot R_A}{m} + \frac{mg \sin\alpha}{m}$$

$$\Rightarrow a = \frac{\mu \cdot mg\cos\alpha \cdot x}{(L - \mu h) \cdot m} + \frac{mg\sin\alpha}{m}$$

$$\boxed{a = \frac{\mu g \cos\alpha \cdot x}{L - \mu \cdot h} + g\sin\alpha.} \quad -(C).$$

NOTE:- 1. When the vehicles moves on a level track, then, $\alpha = 0$.

$$\therefore \boxed{a = \frac{\mu g x}{L - \mu \cdot h}}$$

2. When the vehicle moves down the plane, then equation (i) becomes.

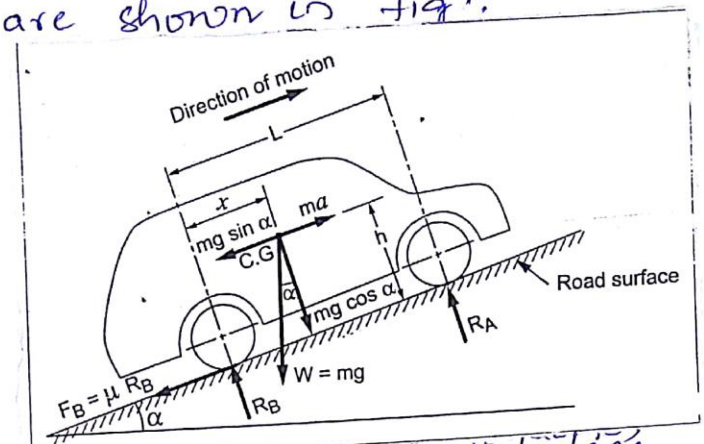
$$F_A - mg\sin\alpha = m \cdot a$$

$$\therefore a = \frac{F_a}{m} - g\sin\alpha = \frac{\mu \cdot R_A}{m} - g\sin\alpha$$

$$\therefore \boxed{a = \frac{\mu g \cos\alpha \cdot x}{L - \mu \cdot h} - g\sin\alpha.}$$

Case (ii): When the brakes applied to the rear wheels only:

Consider a car moving up an inclined plane. For equilibrium of vehicle, the various forces acting on vehicle are shown in fig.



Resolving the forces parallel to the plane,

$$F_B + mg\cdot\sin\alpha = m\cdot a \rightarrow (i)$$

Resolving the forces per to the plane,

$$R_A + R_B = mg\cos\alpha \rightarrow (ii)$$

Now, taking moments about C.G,

$$F_B\cdot h + R_B\cdot x = R_A\,[L-x] \rightarrow (iii)$$

W.K.T,

$$F_B = \mu\cdot R_B \text{ and}$$

$$R_A = mg\cos\alpha - R_B \;\&$$

$$R_B = mg\cos\alpha - R_A.$$

(23)

Substituting the values of R_A & R_B in (iii).

$$\mu \cdot R_B \cdot h + R_B \cdot x = (mg\cos\alpha - R_B)(L - x)$$

$$R_B \cdot \mu \cdot h + R_B \cdot x = mg\cos\alpha[L - x] - R_B[L - x]$$

$$R_B \cdot \mu \cdot h + \cancel{R_B \cdot x} + R_B \cdot L - \cancel{R_B \cdot x} = mg\cos\alpha[L - x]$$

$$R_B[\mu \cdot h + L] = mg\cos\alpha[L - x]$$

$$\therefore R_B = \frac{mg\cos\alpha[L - x]}{[L + \mu \cdot h]}.$$

and, $R_A = mg\cos\alpha - R_B$

$$= mg\cos\alpha - \frac{mg\cos\alpha[L - x]}{L + \mu \cdot h}$$

$$= mg\cos\alpha\left[1 - \frac{(L - x)}{L + \mu \cdot h}\right]$$

$$= mg\cos\alpha\left[\frac{\cancel{L} + \mu \cdot h - \cancel{L} + x}{L + \mu \cdot h}\right]$$

$$R_A = \frac{mg\cos\alpha\,(x + \mu \cdot h)}{L + \mu \cdot h}.$$

From eq.(i) retardation of vehicle is,

$$a = \frac{F_B + m \cdot g \sin\alpha}{m}$$

$$= \frac{F_B}{m} + \frac{mg \sin\alpha}{m}$$

$$= \frac{\mu \cdot R_B}{m} + g \sin\alpha$$

$$= \frac{\mu \cdot mg \cos\alpha (L-x)}{(L+\mu h) \cdot m} + g \sin\alpha$$

$$\boxed{a = \frac{\mu g \cos\alpha [L-x]}{(L+\mu \cdot h)} + g \sin\alpha.}$$

NOTE:- 1. When the vehicle moves on a level track, then $\alpha = 0$.

$$R_B = \frac{m \cdot g [L-x]}{L+\mu \cdot h}; \quad R_A = \frac{mg (x+\mu h)}{L+\mu \cdot h}.$$

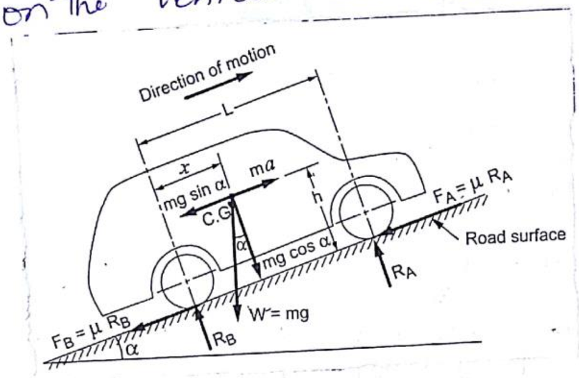
$$\therefore \quad a = \frac{\mu g [L-x]}{L+\mu \cdot h}$$

2. when vehicle moves in a downward to the plane, then.

$$a = \frac{\mu \cdot g \cos\alpha [L-x]}{L+\mu \cdot h} - g \sin\alpha$$

Case iii :- When the brakes are applied to all the four wheels:-

consider a car moving up an inclined plane. For the equilibrium of the vehicle, the various forces acting upon the vehicle as shown.



Resolving the forces parallel to the plane, we get,

$$F_A + F_B + mg\sin\alpha = m \cdot a \quad - (i).$$

Resolving the forces ⊥ar to the plane,

$$R_A + R_B = mg\cos\alpha \quad - (ii)$$

Taking moments about C.G, we have,

$$(F_A + F_B) \cdot h + R_B \cdot x = R_A [L - x]. \quad -(iii).$$

W.K.T;

$$F_A = \mu \cdot R_A.$$

$$F_B = \mu \cdot R_B$$

$$R_B = mg\cos\alpha - R_A$$

$$R_A = mg\cos\alpha - R_B$$

Substituting the values in eq (iii) we get,

$$\mu (R_A+R_B).h + (mg\cos\alpha - R_A)x = R_A[L-x].$$

$$\Rightarrow \mu[R_A + mg\cos\alpha - R_A].h + [mg\cos\alpha - R_A].x = R_A[L-x].$$

$$\Rightarrow h\cancel{\mu}.R_A + \mu mg\cos\alpha.h - \cancel{R_A.h\mu} + mg\cos\alpha.x - \cancel{R_A.x} = R_A L - \cancel{R_A.x}$$

$$\Rightarrow \mu mg\cos\alpha.h + mg\cos\alpha.x = R_A.L.$$

$$\therefore R_A = \frac{m.g\cos\alpha(\mu.h+x)}{L}.$$

$$\therefore R_B = mg\cos\alpha - R_A$$

$$= mg\cos\alpha - \left[\frac{mg\cos\alpha[\mu.h+x]}{L}\right]$$

$$= mg\cos\alpha\left[1 - \frac{(\mu.h+x)}{L}\right]$$

$$R_B = m.g\cos\alpha\left[\frac{L-\{\mu h+x\}}{L}\right].$$

From eq. (i) Retardation 'a' of the Vehicle will be,

$$F_A + F_B + m \cdot g \sin\alpha = m \cdot a.$$

$$\mu \cdot R_A + \mu \cdot R_B + mg \sin\alpha = m \cdot a$$

$$\mu (R_A + R_B) + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu \left[\frac{mg \cdot \cos\alpha (\mu \cdot h + x)}{L} + \frac{mg \cdot \cos\alpha (L - \mu \cdot h - x)}{L} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu \, mg \cos\alpha \left[\frac{\mu \cdot h + x}{L} + \frac{L - \mu \cdot h - x}{L} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \mu \cdot m \cdot g \cos\alpha \left[\frac{\cancel{\mu h} + \cancel{x} + \cancel{L} - \cancel{\mu h} - \cancel{x}}{\cancel{L}} \right] + mg \sin\alpha = m \cdot a.$$

$$\Rightarrow \quad \mu \cdot mg \cos\alpha + mg \sin\alpha = m \cdot a$$

$$\Rightarrow \quad \therefore a = \mu g \cos\alpha + g \sin\alpha$$

$$\boxed{a = g [\mu \cos\alpha + \sin\alpha]}$$

NOTE :- 1. When the vehicle moves on a level track, $\alpha = 0$.

$\therefore \boxed{a = \mu \cdot g}$

2. When the vehicle moves ~~to the~~ downward to the plane, $\boxed{a = g [\mu \cos\alpha - \sin\alpha]}$

Problems

1. A truck has 3.15m wheel base and the centre of gravity is 1.28m in the front of the rear axle and 0.9m above the ground level. The coeffecient of adhesion between types & roads is 0.6 and the brakes are applied to rear wheels only. What is the minimum distance in which the truck can be stopped on a level road when travelling at 48km/hr?. If the weight of truck is 8tons., find the P.r on each wheel during braking.

Sol: Given: $L = 3.15m$; $x = 1.28m$; $h = 0.9m$; $\mu = 0.6$;
$u = 48 km/hr \Rightarrow u = 13.3 m/s$; $m = 8 tons = 8000kg$.

$\Rightarrow$ minimum distance travelled by truck before it comes to rest.

$u^2 = 2as$

$s = \frac{u^2}{2a}$

Let, $S \Rightarrow$ Distance travelled by truck.

W.K.T, $\alpha = 0$ [$\because$ at level road] and brakes applied to rear wheels only.

$$\therefore \quad a = \frac{\mu \cdot g[L - x]}{L + \mu \cdot h}$$

$$a = \frac{0.6 \times 9.81[3.15 - 1.28]}{3.15 + (0.6 \times 0.9)}$$

$$a = 2.98 m/s^2$$

$\therefore$ Distance travelled. $S = \frac{u^2}{2a}$

$$S = \frac{(13.3)^2}{2 \times 2.98} = 29.65 m.$$

Pressure on each wheel during braking:-

Let, $R_A \rightarrow$ Normal reaction between ground & front wheels

$R_B \rightarrow$ Normal reaction between ground & rear wheels.

W.K.T when brakes are applied to rear wheels only, then,

$$R_A = \frac{m \cdot g\,(x + \mu \cdot h)}{L + \mu \cdot h}$$

$$= \frac{(8000)(9.81)(1.28 + (0.6)(0.9))}{3.15 + (0.6)(0.9)}$$

$$R_A = 38708.3\ N$$

Pressure on each front wheel, $\frac{R_A}{2} = 19354.14\ N$. Ans

For Rear wheels,

$$R_B = \frac{m \cdot g\,(L - x)}{L + \mu \cdot h} = \frac{8000\,(9.81)\,[3.15 - 1.28]}{3.15 + (0.6)(0.9)}$$

$$\therefore R_B = 39771.70\ N$$

Pressure on each Rear wheel, $\frac{R_B}{2} = 19885.86\ N$ Ans

Summary

Table 5.1 summarises the expressions used for determining the retardation of the vehicle for different cases.

Table 5.1.

Sl. No.	Case	Vehicle moves up an inclined plane	Vehicle moves on a level track	Vehicle moves down the inclined plane
1.	Brakes are applied to front wheels only	$a = \frac{\mu\, g \cos\alpha \times x}{(L - \mu\, h)} + g \sin\alpha$	$a = \frac{\mu\, g\, x}{L - \mu\, h}$	$a = \frac{\mu\, g \cos\alpha \times x}{L - \mu\, h} - g \sin\alpha$
2.	Brakes are applied to rear wheels only	$a = \frac{\mu\, g \cos\alpha\,(L - x)}{L + \mu\, h} + g \sin\alpha$	$a = \frac{\mu\, g\,(L - x)}{L + \mu\, h}$	$a = \frac{\mu\, g \cos\alpha\,(L - x)}{L + \mu\, h} - g \sin\alpha$
3.	Brakes are applied to all four wheels	$a = g\,(\mu \cos\alpha + \sin\alpha)$	$a = g \cdot \mu$	$a = g\,(\mu \cos\alpha - \sin\alpha)$

* DYNAMOMETERS:

A dynamometer is a brake incorporating a device to measure the frictional resistance applied. This is used to for measuring the driving force or torque transmitted and also the power developed by machine.

Types of Dynamometer:

Dynamometer

- I. Absorbtion dynamometer
  - 1. Prony brake Dynamometer
  - 2. Rope brake dynamometer
- II. Transmission Dynamometer
  - 1. Belt Transmission dynamometer
  - 2. Epicyclic dynamometer
  - 3. Torsion Dynamo-meter.

* Absorption Dynamometer:

→ In absorption type dynamometer, the entire power developed by the prime mover is absorbed by frictional resistance of the brake and is transformed to heat during the process of measurement.

→ These dynamometers are suitable for measuring output of machines of moderate powers.

→ Some examples: (a) Prony brake dynamometer &
(b) Rope brake dynamometer.

5.8.1. Prony Brake Dynamometer

The prony brake dynamometer is the simplest form of absorption dynamometer. A typical form of prony brake dynamometer is shown in Fig.5.23. It is suitable for engine tests in laboratory.

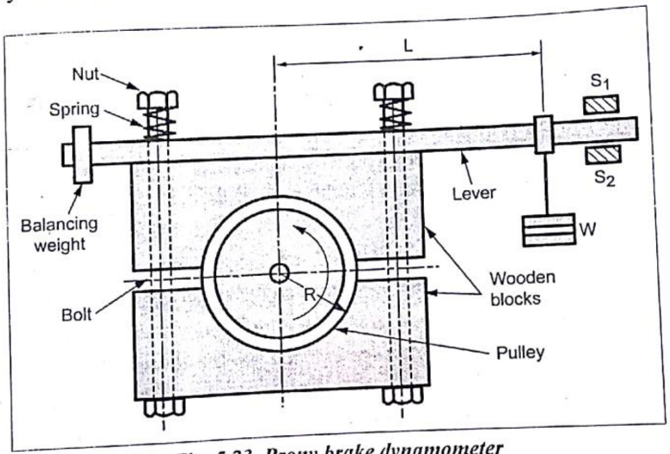


Fig. 5.23. Prony brake dynamometer

Arrangement

It consists of two wooden blocks placed around a pulley fixed to the shaft of the prime mover, whose power is to be measured. The blocks are clamped by means of bolts and nuts. The pressure of the blocks over the pulley is adjusted with the help of nut-helical spring-bolt arrangement. The upper block is attached with a long lever which carries a weight W at its one end. A counter/balancing weight is placed at the other end of the lever to balance the brake when unloaded. Two stoppers S_1 and S_2 are provided to limit the motion of lever.

Working

The friction between the blocks and the pulley tends to rotate the blocks in the direction of shaft rotation. Power is absorbed due to friction. However, the motion is prevented by the suspended weight W provided at the end of lever. The lever remains in horizontal position for the required speed of the engine.

Therefore for measuring the power of the engine, (*i*) attach a known weight W at the end of the lever, and (*ii*) tighten the nuts until the shaft runs at a constant speed and the lever is in horizontal position. At this instant, the moment due to weight W will balance the moment of the frictional resistance between the blocks and the pulley.

Power of the Prime mover:

Let, $W \rightarrow$ weight at the end of lever.

$R \rightarrow$ Radius of pulley.

$L \rightarrow$ Horizontal distance of weight from centre of pulley.

$N \rightarrow$ Speed of shaft, rpm

$F \rightarrow$ frictional resistance b/w blocks & pulley.

W.K.T, braking torque on shaft i.e., the moment of frictional resistance,

$$T = W.L = F.R.$$

$\therefore$ Brake power of engine, = Braking Torque x Angular Speed

$$= T.\omega.$$

$$= \frac{T \times 2\pi N}{60}$$

$$\boxed{P = \frac{W.L \times 2\pi N}{60} \text{ Watt}}$$

Braking power of prime mover is independent on (i) radius of pulley.

(ii) coeffecient of frictn.

(iii) Pr excited by tightening of the nuts.

(18)

Prob:

1. In a prony brake dynamometer, The spring balance reading is 200N, Radius of brake drum is 300mm & distance between The drum axis & hinge of the blocks is 600mm. Determine the pr exerted on drum by tightening the screw, tangential force acting on brake drum & the o/p power of prime mover if record speed is 300 rpm. Take $\mu = 0.25$.

Sol:

$W = 200N$; $R = 300mm = 0.3m$; $L = 600mm = 0.6m$

$N = 300 rpm$; $\mu = 0.25$.

Tangential force,

$$F = \mu \cdot R_N = \mu \cdot W = 0.25 \times (200) = 50N$$

Power of prime mover,

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times N \times W \cdot L}{60}$$

$$= \frac{2\pi \times 300 \times 200 \times 0.6}{60}$$

$$P = 3.77kW$$