# UNIT-II

## Engine Force Analysis and Turning Moment Diagram

Forces acting on the engine are weight of reciprocating masses & CR, gas forces, Friction & inertia forces (due to acceleration & retardation of engine elements).

***i) Piston effort (effective driving force)***

-Net or effective force applied on the piston.

**In reciprocating engine:**

The reciprocating parts (masses) accelerate during the first half of the stroke and the inertia forces tend to resist the same. Thus, the net force on the piston is reduced. During the later half of the stroke, the reciprocating masses decelerate and the inertia forces oppose this deceleration or acts in the direction of applied gas pressure and thus effective force on piston is increased.

In vertical engine, the weights of the reciprocating masses assist the piston during out stroke (down) this increasing the piston effort by an amount equal to the weight of the piston. During the in stroke (up) piston effect is decreased by the same amount.

Force on the piston due to gas pressure; $FP = P_1A_1 - P_2A_2$

$P_1$=Pressure on the cover end,
$P_2$= Pressure on the rod
$A_1$=area of cover end,
$A_2$= area of rod end,
m=mass of the reciprocating parts.

Inertia force $(F_i) = m\,a$

$$= m.r\,\omega^2\left(Cos\theta + \frac{Cos2\theta}{n}\right) \quad \text{(Opposite to acceleration of piston)}$$

Force on the piston $F = F_P - F_i$

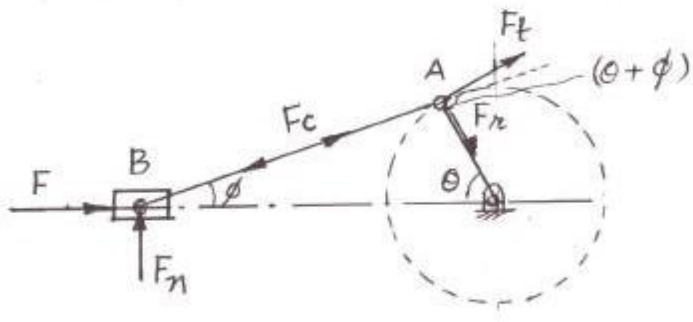
(if $F_f$ frictional resistance is also considered)

$$F = F_P - F_i - F_i$$

In case of vertical engine, weight of the piston or reciprocating parts also acts as force.

$$\therefore\ F = F_P + mg - F_i - F_i$$

***ii) Force (Thrust on the CR)***



$F_c$ = force on the CR

Equating the horizontal components;

$$F_c\,Cos\phi = F \;\; or \;\; F_c \frac{F}{Cos^2\phi}$$

***iii) Thrust on the sides of the cylinder***

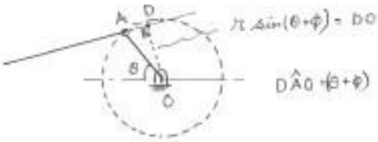
It is the normal reaction on the cylinder walls

$$F_n = F_c \sin\phi = F \tan\phi$$

***iv) Crank effort (T)***

It is the net force applied at the crank pin perpendicular to the crank which gives the required TM on the crank shaft.

$$F_t \times r = F_c\, r \sin(\theta+\phi)$$




$$F_t = F_c \sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)$$

***v) Thrust on bearings ($F_r$)***

The component of $F_C$ along the crank (radial) produces thrust on bearings

$$F_r = F_c\,Cos(\theta+\phi) = \frac{F}{Cos\,\phi}\,Cos(\theta+\phi)$$

***vi) Turning moment of Crank shaft***

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}\sin(\theta+\phi)\times r \;=\; \frac{F_r}{\cos\phi}\,(\sin\theta + \cos\phi + \cos\theta\sin\phi)$$

$$= F\times r\left(\sin\theta + \cos\theta\,\frac{\sin\phi}{\cos\phi}\right)$$

Proved earlier

$$= F\times r\left(\sin\theta + \cos\theta\,\frac{\sin\theta}{n}\;\frac{1}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}\right)$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$= F\times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

Also,

$$r\sin(\theta+\phi) = OD\,\cos\phi$$

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi}.\; r\,\sin(\theta+\phi)$$

$$= \frac{F}{\cos\phi}.\; OD\;\cos\phi$$

$$T = F\times OD.$$

# Turning Moment Diagram & Flywheels

**Introduction:** The torque of an engine crank shaft varies considerably throughout the working cycle, due to variations in crank position, the P in the cylinder & inertia force on piston & connecting rod. If the

**Turning Moment:** value of crank shaft torque, i.e., the turning moment T is plotted against crank angle θ, the diagram so obtained is turning moment dia.

Turning moment diagram is also known as crank-effort diagram, it is the graphical representation of the turning moment or crank-effort for various positions of the crank.

*** Turning moment Diagram for a Single cylinder double Acting Steam Engine:**

A turning moment diagram for a single cylinder double acting steam engine is:

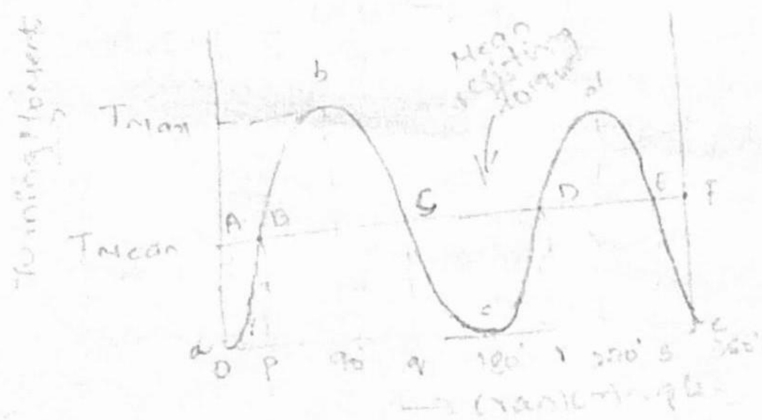



Fig ②: Turning moment diagram for a single cylinder double acting steam Engine.

The vertical ordinate represents Turning Moment, the horizontal ordinate represents crank angle.

Thus turning moment on crank shaft will be,

$$T = F_P \times r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

moment is zero, .

where, $F_p \rightarrow$ Piston effort

$r \rightarrow$ radius of crank.

$n \rightarrow$ ratio of connecting rod length & radius of crank and

$\theta \rightarrow$ Angle turned by crank from inner dead centre.

From the fig(a), we can say that 'T' → turning moment is zero, when the crank angle $\theta$ is zero. It is maximum when, the crank angle is $\left(\frac{180°}{2}\right) = 90°$. Again it is zero when the crank angle is 180° and so on.

This is shown by curve 'abc' in fig., and it represents turning moment of out stroke. The curve 'cde' represents turning moment of in stroke.

NOTE: 1. When the turning moment is '+ve', i.e., when the engine torque is more than mean resisting torque, as shown between point B & C in fig(a) the crank shaft accelerates and work is done by steam.

2. When the turning moment is '−ve', i.e., when the engine torque is less than the mean resisting torque, as shown between points C & D in fig(a). the crank shaft retards and the work is done on the steam.

3. If $T \rightarrow$ Torque on crank shaft at any instant &

$T_{mean} \rightarrow$ Mean resisting Torque

Then accelerating torque on rotating parts of engine,

$= T - T_{mean}$.

4. If $(T - T_{mean})$ is '+ve', the flywheel accelerate

if $(T - T_{mean})$ is '−ve', then the flywheel retards.

(2)

## ...ctuation of Energy:-

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in fig (a). We see that the mean resisting torque line AF cuts the turning moment diagram at B, C, D & E. When the crank moves from a to p, the work done by the engine equal to the area aBP, whereas the energy required is represented by the area ABP. In the other words, the engine has done less work, ~~the remaining~~ than the requirement. This amount, i.e. required amount of energy is taken from the flywheel and hence the speed of flywheel decreases. Now the crank moves from 'p to q', the work done by the engine, is equal to the area PBbcq, where as the requirement of energy is represented by the area PBCq. Therefore, the engine has done more work than the requirement. This excess energy stored in the flywheel, and the speed of flywheel increases while the crank moves from p to q.

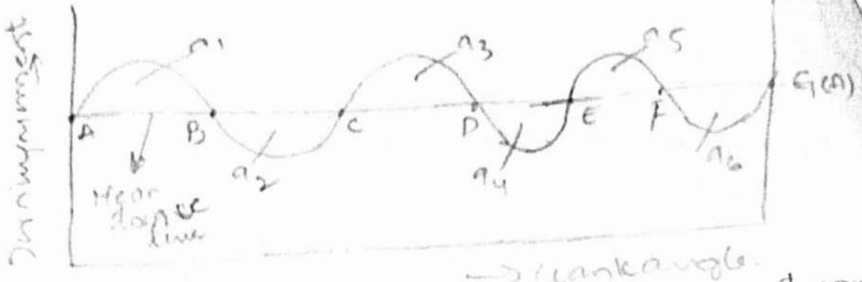
The variations of energy above and below the mean resisting torque line are called fluctuations of energy. The areas Bbc, CcD, DdE, etc.. represent fluctuations of energy.

The difference between the ~~maximum~~ maximum and minimum energies is known as maximum fluctuation of energy.

NOTE:- The area of the turning moment diagram is proportional to the work done per revolution as the work is the product of turning-moment & angle turned.

* Determination of the fluctuation of energy:

A turning diagram for a multicylinder engine is shown as:




The horizontal line AG represents Mean torque line.

Let, $a_1, a_3, a_5$ represents areas of above the mean torque line
$a_2, a_4, a_6$ " " below " "

Let the energy in the flywheel at A = E.

We have, Energy at $B = E + a_1$

" " $C = E + a_1 - a_2$

" " $D = E + a_1 - a_2 + a_3$

" " $E = E + a_1 - a_2 + a_3 - a_4$

" " $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

" " $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (i.e: cycle repeat after G).

Suppose, the greatest energy is at B and least at E.

∴ Max. energy in flywheel, $= E + a_1$

Min. " " " , $= E + a_1 - a_2 + a_3 - a_4$.

∴ Max. fluctuation of energy, $\Delta E$,

$\Delta E$ = Max. Energy − Min. energy.

$= (E + a_1) - [E + a_1 - a_2 + a_3 - a_4]$

$= \cancel{E + a_1 - E}$

$$\boxed{\Delta E = a_2 - a_3 + a_4}$$

## Co-effecient of Fluctuation of Energy [$C_E$]

- It may be defined as the, "ratio of maximum fluctuation of energy to the work done per cycle".

$$C_E = \frac{\text{max. fluctuatn of energy}}{\text{work done per cycle.}}$$

We may obtain work done per cycle in 2 methods:
work done per cycle, units → (N-m or J).

1. Work done per cycle = $T_{mean} \times \theta$.

where, $T_{mean}$ → Mean torque &
$\theta$ → Angle turned (in radians), in one revolution.
= $2\pi$, in case of steam engine & 2-stroke ~~Rate~~ I.C. engines.
= $4\pi$, in case of 4-stroke I.C. engines.

The mean torque can be obtained as,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where, P → Power Transmitted in watts,
N → Speed in rpm.
$\omega$ → Angular speed, $\frac{rad}{sec} = \frac{2\pi N}{60}$.

2. Work done per cycle,

$$= \frac{P \times 60}{n}.$$

where, n → no. of working strokes per minute,
$n = N$, in case of steam engines & 2 stroke I.C engines &
$n = \frac{N}{2}$, in case of 4-stroke I.C engines.

## * FLY WHEEL:-

A flywheel is used in machines used reservoirs, which stores the energy during the when the supply of energy is more than the require and releases it during the period when the requirement the energy is more than the supply.

In simple words, when the flywheel absorbs energy, its speed increases and when its releases the energy, its speed decreases.

We can say that, " A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation ".

NOTE: The function of a governor, in an engine is entirely different from that of flywheel. The governor regulates the mean speed of an engine when there are variations in the load. Where as The flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by varying load.

### Applications:-

Flywheels are provided in engines and fabricating machines such as presses, shearing machine, rivetting machines, punching machines, steel rollers, crushers etc..

(4).

## -effecient of Fluctuation of speed [$C_s$]:

The ratio of maximum fluctuation of speed to the mean speed is called co-effecient of fluctuation of speed.

"The difference b/w max. & min. speeds during a cycle is called max. fluctuatn of speed."

Let, $N_1$ & $N_2$ are max. & min. speeds during cycle.

$N \rightarrow$ mean position

$$= \frac{N_1 + N_2}{2}.$$

$$\therefore \quad C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{.. (In terms of Angular speed)}$$

$$= \frac{V_1 - V_2}{V} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{... (In terms of linear speed).}$$

**NOTE:** The reciprocal of co-effecient of friction of speed is known as "coeffecient of steadiness" and is denoted by 'm'.

$$m = \frac{1}{C_s} = \frac{N}{(N_1 - N_2)}$$

## * Energy Stored in a Flywheel:

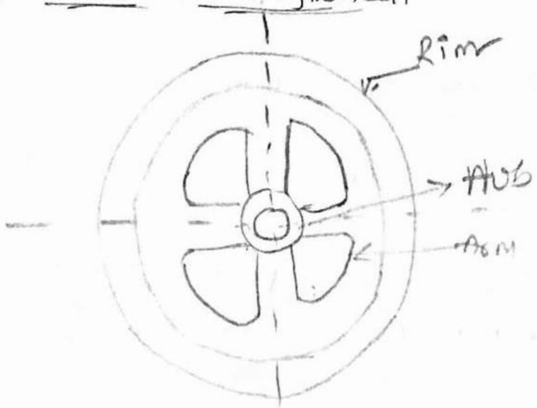



Fig: Flywheel.

We have already discussed that, when a flywheel absorbs energy, its speed increases and vice-versa.

Let, $m \rightarrow$ mass of flywheel.

$k \rightarrow$ radius of gyration.

$I \rightarrow$ mass moment of inertia, $= m \cdot k^2$.

$N_1 \,\&\, N_2 \rightarrow$ Max. & Min. speeds during the cycle, in rpm.

$\omega_1 \,\&\, \omega_2 \rightarrow$ Max. & Min. angular speeds during the cycle, $\frac{rad}{sec}$.

$N_{mean} \rightarrow$ Mean speed $= \frac{N_1+N_2}{2}$

$\omega \rightarrow$ Mean angular speed, $= \frac{\omega_1+\omega_2}{2}$.

$c_s \rightarrow$ co-efficient of fluctuation of speed $= \frac{N_1-N_2}{N}$ (or) $\frac{\omega_1-\omega_2}{\omega}$.

W.K.T, the mean kinetic energy of flywheel.

$$E = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} m k^2 \cdot \omega^2.$$

As the speed of flywheel changes from $\omega_1$ to $\omega_2$, max. fluc. of en.

$$E_f = \text{Max. K.E} - \text{Min. K.E.}$$

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2 = \frac{1}{2} \times I \left[\omega_1^2 - \omega_2^2\right].$$

$$= \frac{1}{2} \times I \left[(\omega_1+\omega_2)(\omega_1-\omega_2)\right] = I\, \omega\,(\omega_1-\omega_2) \qquad \because \frac{\omega_1+\omega_2}{2} = \omega$$

Multiply & divide by $\omega$.

$$E_f = I \omega^2 \left[\frac{\omega_1-\omega_2}{\omega}\right]$$

$$E_f = I\omega^2 (c_s) = m k^2 \omega^2 c_s.$$

$$= 2 E c_s. \qquad \left[E = \frac{1}{2} \times I \cdot \omega^2\right].$$

Now 'k' may be taken as 'R'. $k = R$

$$\boxed{\Delta E = m R^2 \omega^2 c_s}$$

$$= \boxed{m v^2 c_s = \Delta E} \qquad \because \boxed{v = \omega R} \text{ in m/s.}$$

$v \rightarrow$ linear velocity.

The mass of fly wheel of an engine is 6.5 tonns and radius of gyration is 1.8m. It is found from the turning moment diagram that the fluctuation of energy $(E_f)$ is 56 kN-m. If the mean speed of the engine is 120rpm. Find the max. & min. speeds.

Sol: Given Data:

$M = 6.5$ tonns $= 6500$ kgs.

$K = 1.8$ m; $E_f = 56$ kN-m $= 5600$ N-m. ; $N = 120$ rpm.

Let, $N_1$ = max speed ; $N_2$ = min speed.

We K.T, $N = \frac{N_1+N_2}{2}$

$120 = \frac{N_1+N_2}{2}$

$N_1+N_2 = 240$ —(1).

$E_f = \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2.$

$= \frac{1}{2} MK^2 [\omega_1^2 - \omega_2^2].$

$\frac{1}{2}(6500)(1.8)^2 \times \left[\left[\frac{2\pi N_1}{60}\right]^2 - \left[\frac{2\pi N_2}{60}\right]^2\right].$

$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right][N_1-N_2][N_1+N_2].$

$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left(\frac{4\pi^2}{3600}\right) \times 240 \times (N_1-N_2)$

$N_1 - N_2 = 0.2$

$N_1+N_2 = 240$
$N_1 - N_2 = 0.2$

$2N_1 = 240.2$

$\boxed{N_1 = 120.1 \text{ rpm}}$

$N_1 + N_2 = 240$

$120.1 + N_2 = 240$

$N_2 = 240 - 120.1$

$\boxed{N_2 = 119.9 \text{ rpm}}$

(2) The horizontal compound 2-slider cylinder engine develops at 90 rpm. The co-effecient of fluctuation of energy from the turning moment diagram is to be 0.1 and speed is to be at 0.5% of mean speed. The mass the flywheel required, if the radius of gyration is 2m

Sol: Given:

$P = 300\,kW$; $N = 90\,rpm$; $C_E = 0.1$; $C_s = \pm 0.5\%$ of N.

m → mass of flywheel

$$P = \frac{2\pi N T_{mean}}{60{,}000}$$

$$300 = \frac{2\pi \times 90 \times T_{mean}}{60{,}000}$$

$$\boxed{T_{mean} = 31830\ N\text{-}m}$$

$$C_E = \frac{E_f}{W.D/cycle}$$

$$0.1 = \frac{E_f}{T_{mean} \times 2\pi}$$

$$0.1 = \frac{E_f}{31830 \times 2\pi}$$

$$\boxed{E_f = 19999.9\ N\text{-}m}$$

$$C_s = \frac{N_1 - N_2}{N}$$

$N_1 = 90 - 0.5\%$ of N. ⇒ $90 - 0.5\% \times 90$

$\boxed{N_2 = 89.55}$ rpm.

$N_2 = 90 + 0.5\%$ of N.

$\boxed{N_2 = 90.45\ rpm}$

$$C_s = \frac{90.45 - 89.55}{90}$$

$\boxed{C_s = 0.01}$

$E_f = I\omega^2 C_s$.

$19999.9 = m k^2 \omega^2 C_s$.

$= m(2)^2 \left(\frac{2\pi \times 90}{60}\right)^2 \times 0.01$

$\boxed{m = 5628.91\,kg}$

## T–θ-diagram Problem

(6).

The turning moment for a petrol engine is drawn through following scales: for torque 1 mm = 5 N-m, crank angle 1 mm = $1^{\circ}$ = $1 \times \pi/180$. The turning moment diagram repeats itself at every revolution of engine and areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm².

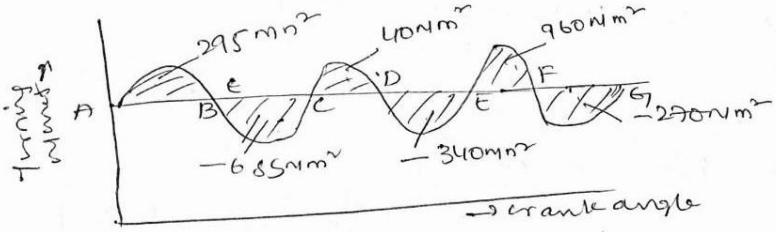
The rotating parts are equivalent to a mass of 36 kg at a radius of gyration 150 mm. Determine, coefficient of fluctuation of speed when the ranges at 1800 rpm.

Sol: **Given Data:**

for torque, 1 mm = 5 N-m

crank angle 1 mm = $1^{\circ}$ = $\frac{1\times\pi}{180}$ = 0.0174 rad.

The areas are, +295, −685, +40, −340, +960, −270 mm².




m = 36 kg.

K = 150 mm = 0.15 m; N = 1800 rpm. ∴ $C_s$ = ?

Let energy at pt. A = E m

" " pt. B = E + 295 ⇒ Max. K.E

pt. C = E + 295 − 685 ⇒ E − 390.

pt. D = E + 295 − 685 + 40 ⇒ E − 350

pt. E = E + 295 − 685 + 40 − 340 ⇒ E − 690 ⇒ Min K

pt. F = E + 295 − 685 + 40 − 340 + 960 ⇒ E + 270

pt. G = E + 295 − 685 + 40 − 340 + 960 − 270 = E.

$\therefore E_f = \text{Max. K.E} - \text{Min. K.E.}$

$= (E+295) - (E-690)$

$= E + 295 - E + 690$

$E_f = 985 \text{ mm}^2$

$= 985 \times mm \times mm$

$= 985 \times 5 \times \frac{\pi}{180}$

$\boxed{E_f = 85.96 \text{ N-m}}$

$E_f = I\omega^2 \times C_s$

$85.96 = mk^2 \cdot \omega^2 \times C_s$

$85.96 = (36)(0.15)^2 \times \left[\frac{2\pi \times 1800}{60}\right]^2 \times C_s$

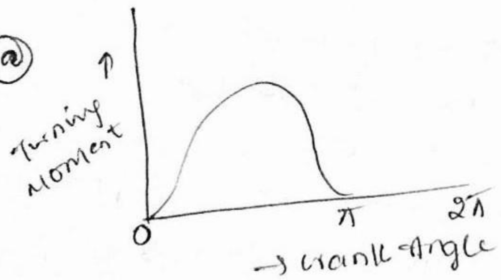
$C_s = 3 \times 10^{-3}$

$\boxed{C_s = 0.003}$

$C_s = 0.3\%$ (or) $\boxed{C_s = \pm 0.15\%}$

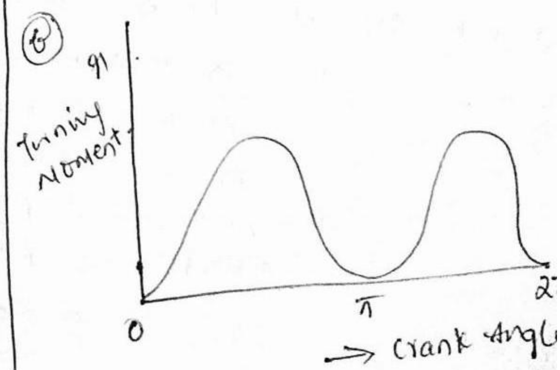
* Turning moment diagrams of common engines:-

(a)




(a) Single acting steam engine

(b)




(b) Double acting steam engine.

## T diagram for 4-stroke IC Engine: (7).

A Turning diagram for 4-stroke IC engine is shown in fig.

w.k.t, in a four stroke IC engine, There is one stroke after the crank has turned through 2-revolutions. i.e: 720° (or) 4π.

Since, the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke.

∴ Negative loop is formed, as shown in fig,

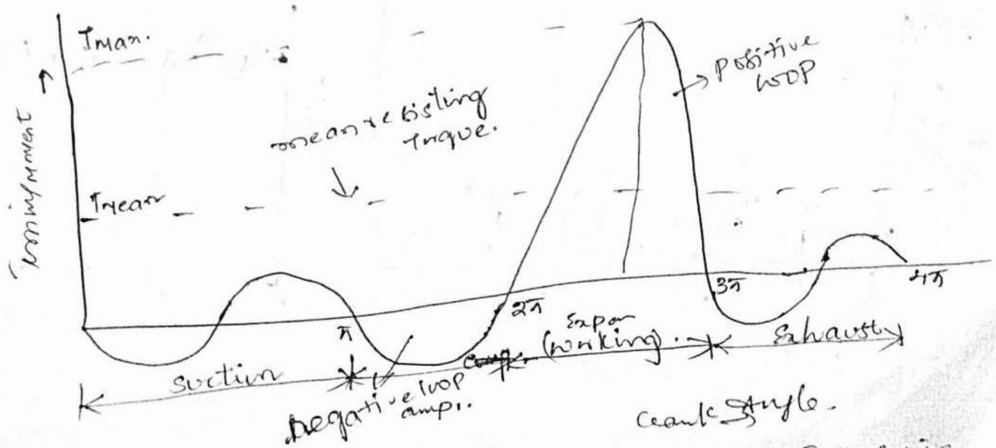
During the compression, the work is done ~~other~~ on the gases: therefore, higher negative loop is formed.

During the expansion stroke the fuel burns & the gases expands,

∴ a large +ve loop is obtained. In this stroke, the work is done by the gases.

During exhaust stroke, the work is done on the gases.

∴ There is a '-ve' loop during the exhaust stroke.




T-θ diagram for 4-stroke IC engine

Problem (2).

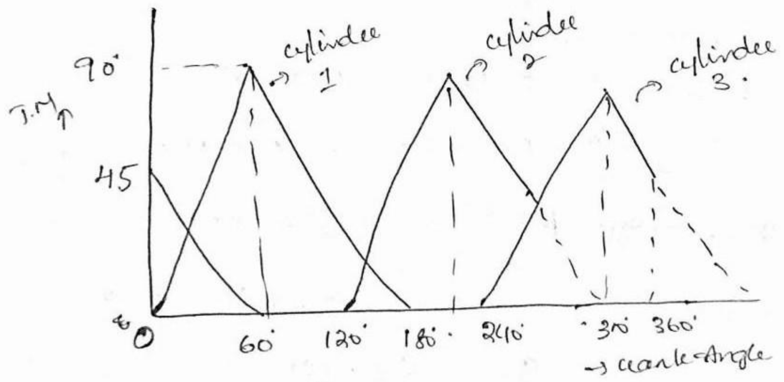
~~Single~~ 3 cylinder single acting engine has its crank set equally at 120° and its runs at 600 rpm. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque 90 N-m at 60° from dead centre of corresponding crank. The torque on return stroke is sensibly zero. Determine:

1. Power developed; 2. coefficient of fluctuation of speed, if the mass of flywheel is 12 kg & has a radius of gyration of 80 mm, 3. coefficient of fluctuation of energy, &
4. Max. angular acceleration of flywheel.

Sol: Given $N = 600$ rpm $\Rightarrow \omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$

$\omega = 62.84 \frac{rad}{sec}$;

$T_{max} = 90$ N-m ; $m = 12$ kg ; $k = 80$ mm $= 0.08$ m




(a)

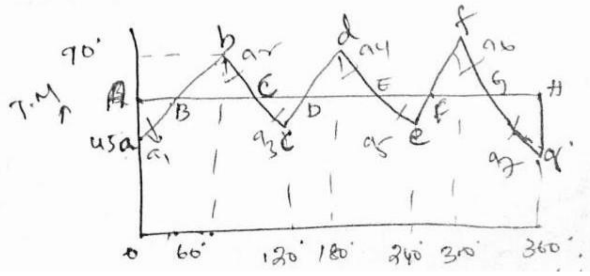


Fig ⓐ represents T-θ diagram for 3 cylinders in set of & Fig ⓑ represents resultant T-θ diagram for 3 cylinder

1. Power developed.

$$P = T_{mean} \times \omega$$

WKT, $\frac{W.D}{cycle}$ = Area of $3\Delta^{les}$

$$= 3 \times \frac{1}{2} \times \pi \times 90^{\circ}.$$

$$= 424 N\text{-}m$$

$$T_{mean} = \frac{W.D/cycle}{crank\ Angle/cycle} = \frac{424}{2\pi} = 67.5 N\text{-}m$$

$\therefore P = T_{mean} \times \omega = 67.5 \times 62.84 = 4240 W = 4.24 kW.$

2. Coeff of fluctuation of speed:

Let, $C_s \rightarrow$ coeff. of fluctn of speed. $E_f = I\omega^2 C_s$.

So, that, initially we have to find Max. fluctuation of energy.

from fig ⓑ we have to find,

$a_1$ = Area of triangle $aAB = \frac{1}{2} AB \times Aa$. $(\because AB = 30^{\circ} = \frac{\pi}{3})$

$$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45^{\circ}) = 5.89 N\text{-}m = a_7.$$

$a_2$ = Area of triangle $Bbc = \frac{1}{2} \times Bc \times bb'$. $(\because Bc = 60^{\circ} = \pi/6)$

$$= \frac{1}{2} \times \frac{\pi}{3} \times (90 - 67.5)$$

$$a_2 = 11.78 N\text{-}m.$$

$$a_2 = a_3 = a_4 = a_5 = a_6.$$

(9)

Let the total energy at $A = E$

then the energy at $B = E - 5.89$.

" " $C = E - 5.89 + 11.78 = E + 5.89$.

" " $D = E + 5.89 - 11.78 = E - 5.89$

" " $E = E - 5.89 + 11.78 = E + 5.89$

" " $F = E + 5.89 - 11.78 = E - 5.89$

" " $G = E - 5.89 + 11.78 = E + 5.89$

" " $H = E + 5.89 - 5.89 = E$. = Energy at A.

∴ The max. Energy = $E + 5.89$

" Min. Energy = $E - 5.89$.

∴ $E_f = \Delta E$ = max. fluctuation of Energy

$= (E + 5.89) - [E - 5.89]$

$\Delta E = 11.78$ N-m.

W.K.T, max. fluctuation of energy

$E_f = \Delta E = I \omega^2 C_s$.

$11.78 = mk^2 . \omega^2 C_s$.

$11.78 = (12)(0.08)^2 \times (62.84)^2 C_s$.

$C_s = 0.04$ (or) $4\%$.

3. Coeff. of fluctuation of energy.

W.K.T, coeff. of fluctuation of energy,

$$C_E = \frac{\text{max. fluctuation of energy}}{\text{W.D/cycle}} = \frac{11.78}{424}$$

$= 0.0278$

$C_E = 2.78\%$

4. Max. angular acceleration of flywheel,

$\alpha \rightarrow$ angular acceleration.

$T_{max} - T_{mean} = I . \alpha . = mk^2 . \alpha$.

$90 - 67.5 = (12)(0.08)^2 . \alpha$

$$\alpha = \frac{90 - 67.5}{0.077} \Rightarrow \alpha = 292 \ \frac{\text{rad}}{\text{sec}^2}.$$

② A single cylinder, single acting four stroke engine develops 20KW at 300 rpm. The work done by the gases during the expansion stroke is three times the work done the gases during compression stroke, the work done during suction stroke & exhaust stroke being negligible. If the total fluctuation of speed is not to ±2% of mean speed & Turning moment diagram during compression & expansion is assumed to be triangular in shape. find the moment of inertia of flywheel.

Sol: Given: $P = 20KW = 20\times10^3 W$; $N = 300$ rpm.

Total fluctuation of speed $(\omega_1-\omega_2)$ is not to exceed ±2% of mean speed.

$\boxed{W_E = 3W_C}$

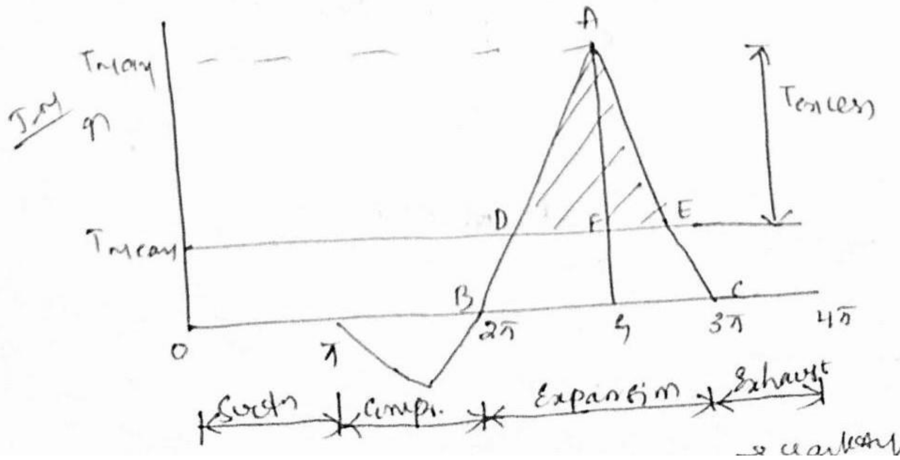
$\therefore \omega_1-\omega_2 = 4\%\omega$.

& co-efficient of fluctuation of speed,

$$C_S = \frac{\omega_1-\omega_2}{\omega} = 4\%$$

$$(i.e.) \frac{\omega_1-\omega_2}{\omega} = 0.04$$

The following will be the T-θ diagram for the four stroke engine neglecting suction and exhaust strokes.



(10).

$T_1$ - for four stroke no. of working strokes / cycle.

$$n = \frac{N}{2} = \frac{300}{2} = 150.$$

$$\therefore \frac{W.D}{cycle} = P \times \frac{60}{n} = 20 \times 10^3 \times \frac{60}{150} \Rightarrow$$

$$\frac{W.D}{cycle} = 8000 \text{ N-m}. \quad (1).$$

Since workdone during suction & exhaust are negligible,

& net w.D/cycle ⇒

$\because W_E = 3W_C.$

$$\boxed{W_C = \frac{W_E}{3}}$$

$$\Rightarrow W_E - W_C$$

$$\Rightarrow \cancel{3W_C - W_C}$$

$$= W_E - \frac{W_E}{3}$$

$$W.D/cycle = \frac{2W_E}{3} \quad -(2).$$

Equating (1) & (2).

$$8000 = \frac{2}{3} W_E$$

$$W_E = 12000 \text{ N-m}$$

W.K.T, work done during expansion stroke $[W_E]$.

In order to get $T_{max}$.

12000 = Area of $\triangle^{le}$ ABC

$$= \frac{1}{2} BC \times AG$$

$$12000 = \frac{1}{2} \pi \times AG.$$

$$\therefore T_{max} = AG = \frac{12000 \times 2}{\pi} = 7638 \text{ N-m}.$$

$$\text{Now } T_{mean} = FG = \frac{w.D/cycle}{\text{Crank Angle/cycle}} = \frac{8000}{4\pi} = 637 \text{ N-m}.$$

$\therefore$ Tencess = $AF = AG - FG$

$= 7638 - 637$

Tencess = $7001$ N-m.

Now, from similar $\triangle$les ADE & ABC.

$$\frac{DE}{BC} = \frac{AF}{FG}$$

(or) $DE = BC \times \frac{AF}{FG}$

$= \pi \times \frac{7001}{637}$

$\therefore \boxed{DE = 2.88\text{ mm}}$

$\therefore$ The area above the mean represents max. fluctuation energy, $\therefore$ Max. fluctuation of Energy,

$\Delta E = E_f$ = Area of $\triangle^{le}$ ADE

$= \frac{1}{2} DE \times AF$

$= \frac{1}{2} \times 2.88 \times 7638$

$E_f = 10081$ N-m.

$\therefore$ Moment of inertia is to be calculated,

$E_f = I\omega^2 C_s$.

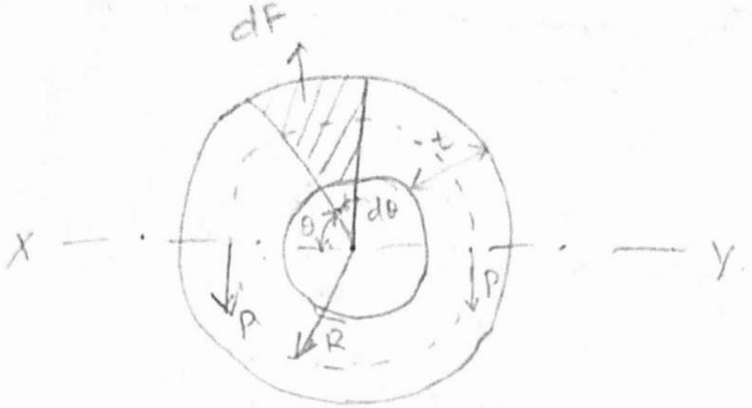
$10081 = I \cdot \left(\frac{2\pi 300}{60}\right)^2 \times 0.04$

$10081 = 39.5 I$

$\boxed{I = 255.2\ \text{kg-m}^2}$

(11).

## Dimensions of the Flywheel Rim:-




Rim of a flywheel.

Consider a rim of the flywheel as shown.

Let, $D \rightarrow$ Mean Diameter of rim, mts.

$R \rightarrow$ Mean Radius of rim, mts.

$A \rightarrow$ cross-sectional area of rim, $m^2$.

$\rho \rightarrow$ Density of rim material, $kg/m^3$.

$N \rightarrow$ Speed of the flywheel, rpm

$\omega \rightarrow$ Angular ~~speed~~ velocity of flywheel, rad/sec

$v \rightarrow$ linear velocity at mean radius in m/s

$= \omega \cdot R = \frac{\pi D N}{60}$, m/s.

$\sigma \rightarrow$ Tensile stress or hoop stress, $N/m^2$ due to centrifugal force.

Consider a small element of the rim as shown shaded. Let it subtends at an angle of $\delta\theta$ at the centre of flywheel.

Volume of small element.

$$dV = A \times R \cdot d\theta.$$

$\therefore$ mass of the small element,

$$dm = \text{Density} \times \text{Volume}$$

$$dm = \rho \times A \times R \cdot d\theta.$$

Centrifugal force on the element, acting radially outwards,

$$dF = dm \times R \cdot \omega^2$$

$$= \rho \cdot A \cdot R \times d\theta \cdot R \cdot \omega^2$$

$$dF = \rho \times A \cdot R^2 \omega^2 \cdot d\theta$$

Vertical component of force of the element dF

$$= dF \cdot \sin\theta = \rho \cdot A \cdot R^2 \cdot \omega^2 \, d\theta \, \sin\theta$$

$\therefore$ Total vertical upward force tending to burst rim across the dia. X Y.

$$= \rho A \cdot R^2 \omega^2 \int_0^{\pi} \sin\theta \cdot d\theta. \qquad \cos\pi = -1 \,;\, \cos 0 = 1$$

$$= \rho A R^2 \omega^2 \left[ -\cos\theta \right]_0^{\pi} = 2 \rho A R^2 \omega^2. \quad -(1)$$

$$\left[ -\cos\pi - (-\cos 0) \right] \quad 1 + 1 = 2$$

The vertical upward force will produce hoop stress or circumferential force, & it is resisted by 2P.

$$2P = 2 \cdot \sigma \cdot A. \quad -(2).$$

$\boxed{A = b \times t}$

Equating (1) & (2).

$$2 \rho A R^2 \omega^2 = 2 \cdot \sigma \cdot A$$

$$\sigma = \rho R^2 \omega^2 = \rho v^2 \qquad \because v = R\omega, \; \omega = \frac{v}{R}$$

$$\boxed{v = \sqrt{\frac{\sigma}{\rho}}}$$

We know mass of rim, $m = \text{Volume} \times \text{density} = \pi \cdot D A \cdot \rho$. $\therefore \boxed{A = \frac{m}{\pi D \rho}}$

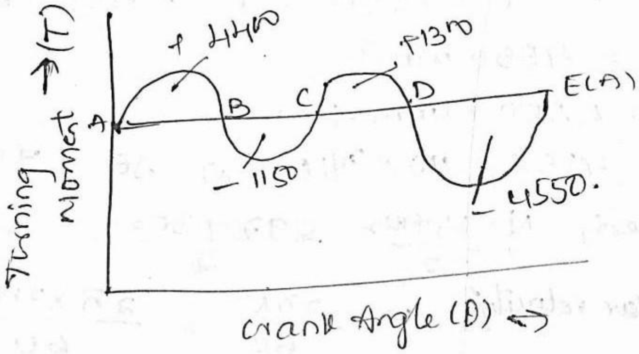
Date................................ Page No.......6(a).........

(4). In a turning moment dia, the areas above & below the mean torque line taken in order are 4400, 1150, 1300 & 4550 mm² resp. The scales of the turning moment diagram are;

Turning moment, 1mm = 100 N-m; Crank Angle, 1mm = 1°.

Find the mass of the fly wheel reqd. to keep the speed b/w 297 & 303 rpm. If the radius of gyration is 0.525 m.

Sol:




Sol: Given Data:

$N_1 = 297$ & $N_2 = 303$ rpm., $k = 0.525$ m

Turning moment, 1mm = 100 N-m

Crank Angle, 1mm = 1° = $(1 \times \pi/180)$.

Let the total energy, at, $A = E$,
the energies at diff. pt's.

at, $A = E$.

$B = E + 4410$.

$C = E + 4410 - 1150 = E + 3250$.

$D = E + 4410 - 1150 + 1300 = E + 4550$ (max.

$E = E + 4410 - 1150 + 1300 - 4550 = E$. (min eng)

W.K.T, max. fluctuation of energy,

$\Delta E =$ max. energy $-$ min. energy

$= E + 4550 - E = 4550\ mm^2$.

$\Delta E = 4550\ mm^2$

$= 4550 \times 1mm \times 1mm$

$= 4550 \times 110 \times \pi/180 \Rightarrow \Delta E = 7939.75\ N\text{-}m$.

mean speed, $N = \frac{N_1 + N_2}{2} = \frac{297 + 303}{2} = 300$ rpm.

mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} \Rightarrow \omega = 31.416\ rad/sec$

coeff. of fluctuation of speed, $C_s = \frac{N_1 - N_2}{N} = \frac{303 - 297}{300} = 0.02$.

W.K.T max. fluctuation of energy,
$\Delta E = I \omega^2 C_s$.

$\Delta E = mk^2 \omega^2 C_s$.

$7939.73 = m(0.525)^2 \times (31.416)^2 \times (0.02)$

$\boxed{m = 1459.3\ Kg}$

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(3). In a machine, the intermittent operatio demand the torque to be applied as follows:

- During the first half revolution, the torque increases from 1200 N-m to 3600 N-m.
- During the next one revolution, the torque remains constant.
- During the next one revolution, the torque decreases uniformly from 3600 N-m to 1200 N-m.
- During last 1½ revolution, the torque remains constant.

Thus a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 200 rpm. A flywheel of mass 2000kg and radius of gyration 600mm is fitted to shaft.

Determine: (i) The power of motor, P.
(ii) The total coeffecient of fluctuation of speed of m/c shaft.

Sol:- Given:

$N_{mean} = 200$ rpm; $m = 2000$ kg;

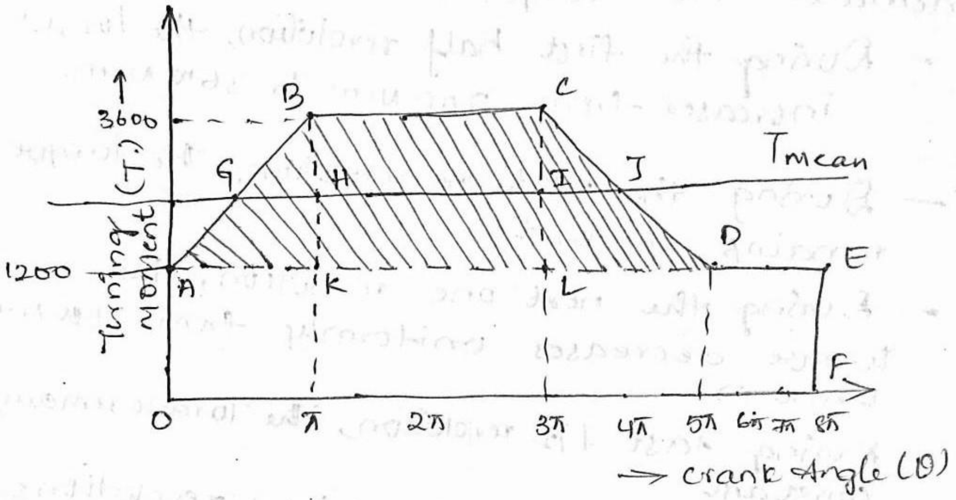
$K = 600$ mm $= 0.6$ m

Mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60}$

$\omega = 20.94$ rad/sec.

Moment of inertia, $I = m \cdot k^2$
$= (2000)(0.6)^2 = 720$ kg-m$^2$.

The turning moment diagram for complete cycle is as below,




We know that the torque reqd. for one complete cycle

= Area of OABCDEFO.

= Area OAEF + Area ABK + Area BCLK + Area CDL

= (OF × OA) + [½ × AK × BK] + [KL × CL] + [½ × DL × CL]

= [8π × 1200] + [½ × π × (3600−1200)] + [2π × (3600−1200)] + [½ × 2π × (3600−1200)].

= 30159.28 + 3769.91 + 15079.6 + 7539.8

T = 56548.61 N-m. — (a).

Date.................................  Page No. 6 (1).

If $T_{mean}$ is the mean torque, then torque required for 1 complete cycle

$$\tau = T_{mean} \times 8\pi \quad — (b)$$

Equating eq's (a) & (b),

$$56548.61 = T_{mean} \times 8\pi$$

$$T_{mean} = 2250 \text{ N-m}$$

(i) Power of the motor :

W.K.T, Power, $P = \frac{2\pi N T}{60}$

$$P = \frac{2\pi \times 200 \times 2250}{60}$$

$$P = 47123.8 \text{ W}$$

$$= 47.124 \text{ kW}$$

(ii) Total coeffecient of fluctuation of speed $[C_s]$ :-

$$\Delta E = I \omega^2 C_s$$

we need to find out the fluctuation of energy $\Delta E$.

for that first need to find out the values of GH & IJ.

from similar triangles ABK & GBH, we get,

$$\frac{GH}{AK} = \frac{BH}{BK} \quad \text{or} \quad \frac{GH}{\pi} = \frac{(3600-2250)}{(3600-1200)}$$

or $GH = 1.767$ rad.

from similar triangles CIJ and CLD.

$$\frac{IJ}{LD} = \frac{CI}{CL}$$

$$\frac{IJ}{2\pi} = \frac{(3600-2250)}{(3600-1200)}$$

$IJ = 3.534$ rad.

W.K.T the area ~~abo~~ above mean torque line represents max. fluctuatn of energy [$\Delta E$]

$\therefore \Delta E =$ Area of GBCJ = Area GBH + Area BCIH + Area CJI.

$$= \left[\tfrac{1}{2} \times GH \times BH\right] + \left[HI \times CI\right] + \left[\tfrac{1}{2} \times IJ \times CI\right].$$

$$= \left[\tfrac{1}{2} \times 1.767 \times (3600-2250)\right] + \left[2\pi \times (3600-2250)\right] + \left[\tfrac{1}{2} \times 3.534 \times (3600-2250)\right]$$

$\Delta E = 12060.475$ N-m.

& also $\Delta E = I\omega^2 C_s$.

$12060.475 = 720(20.94)^2 \times C_s$.

$\therefore$ coefficient of fluctuatn of speed,

$C_s = 0.0382$ or $3.8\%$

Date..............................  Page No. 40

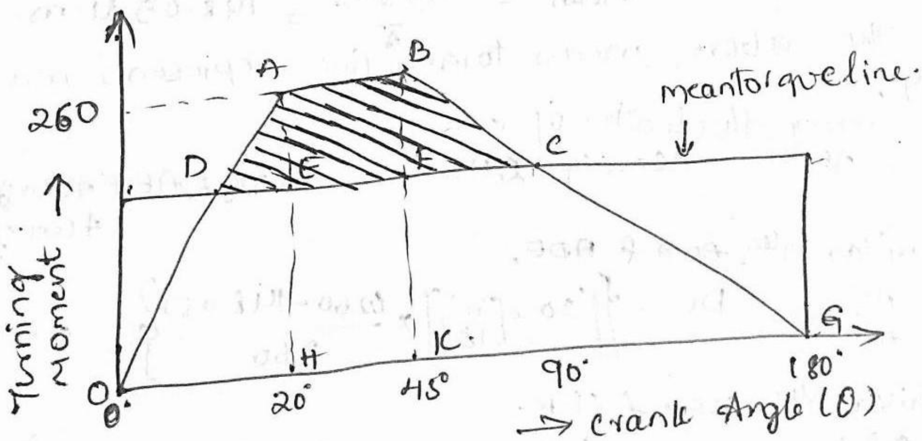
(4) The variation of crankshaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° & 180° and as 260 N-m for crank angles 20° & 45°, the intermediate portions of torque graph being straight lines. The cycle is being repeated in every half revolutn. The average speed is 600 rpm. Supposing that engine drives a machine at constant torque, determine the mass of flywheel of radius of gyration 250 mm, which must be provided so that total variation of speed shall be one percent.

Sol: Given Data:-

$N = 600$ rpm; $K = 250$ mm $= 0.25$ m

$C_s = 1\% = 0.01$

The turning moment diagram,



Work done for half revolution

= Area of turning moment diagram

= Area of OABG.

= [Area of OAH] + [Area of HABK] + [Area of KBG]

$$= \left[\frac{1}{2}\left(20^\circ \times \frac{\pi}{180}\right) \times 260\right] + \left[(45^\circ - 20^\circ)\left[\frac{\pi}{180}\right] \times 260\right] + \left[\frac{1}{2} \times (180 - 45^\circ)\left[\frac{\pi}{180}\right] \times 260\right].$$

= 465.13 N-m.

If $T_{mean}$ is the mean torque, then work done corresponding of mean torque for half revolution, is given by,

$$T_{mean} \times \pi = \text{work done} = 465.13.$$

$$T_{mean} = \frac{465.13}{\pi} = 148.05 \text{ N-m.}$$

Since the above mean torque line represents max. fluctuation of energy,

∴ max. fluctuation of energy,

$\Delta E$ = Area of $\Delta ABC$ = Area of $\Delta AE$ + Area of EABF + Area of CFB.

from similar $\Delta^{les}$ AOH & ADE,

$$\frac{DE}{OH} = \frac{AE}{AH} \Rightarrow DE = \left\{\left[20^\circ \times \left[\frac{\pi}{180}\right]\right] \times \frac{(260 - 148.05)}{260}\right\} = 0.15 \text{ rad}.$$

from similar $\Delta^{les}$, BGK & CFB.

$$\frac{FC}{KG} = \frac{BFB}{BK} \Rightarrow FC = \left\{\left[(180 - 45^\circ)\left[\pi/180\right]\right] \times \frac{(260 - 148.05)}{260}\right\} = 1.014 \text{ rad}.$$

$$\therefore \Delta E = \left[\frac{1}{2} \times 0.15 \times 111.95\right] + [0.4363 \times 111.95] \times \left[\frac{1}{2} \times 111.95 \times 1.014\right]$$

$$\Delta E = 114 \text{ N-m}.$$

W.K.T, $\Delta E = m \cdot k^2 \omega^2 C_s$.

$$114 = m \cdot (0.25)^2 \cdot (62.83)^2 \cdot (0.01)$$

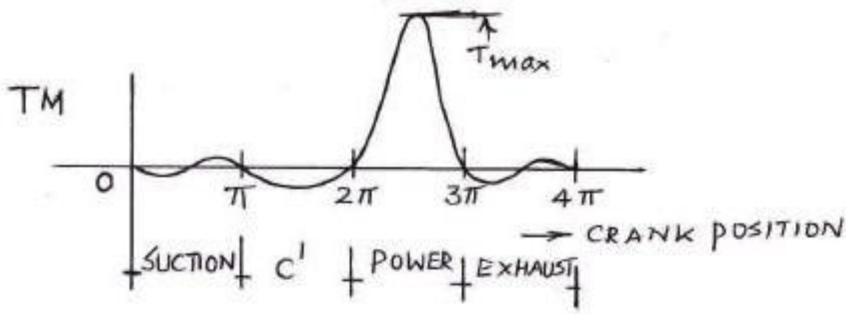
$$m = 46.2 \text{ kg}$$

**Difference between Governor and Flywheel:**

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job.
Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

**Crank effort diagrams or Turing Moment diagrams:**
It is the graphical representation of turning moment or crank effort for the various positions of the crank. The TM is plotted on 'y' axis and crank angle on 'x' axis.

**Use of turning moment**
**Diagram:** The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.
1) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us to find the fluctuation of energy.
2) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us to find diameter of the crank shaft.TMD for a four strokeI.C. Engine




We know that four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions (4p or 720°). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke therefore a negative loop is formed as shown in figure. During compression stroke the work done on engine the gases therefore a higher negative loop is obtained. During expansion or working stroke the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases; therefore a negative loop is firmed.

**Problem 2**

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

***Solution***

| $\theta$, deg. | $T, N-m$ |
|---|---|
| 0 | 500 |
| 90 | 1500 |
| 180 | 500 |
| 270 | 1500 |
| 360 | 500 |

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T \, d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta) \, d\theta$$

$$= 2000\pi \ N-m$$

$$T_{mean} = \frac{W.D / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \ N-m$$

$$\text{Power developed} = T_{mean} \times \omega_{mean}$$

$$= 1000 \times \frac{2\pi \ N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

**Problem: 3**

The turning moment curve for an engine is represented by the equation,

$T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:

1. Power developed by the engine;
2. Moment of inertia of flywheel in kg-$m^2$, if the total fluctuation of speed is not the exceed 1% of mean speed which is 180 r.p.m. and
3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.

**Solution:**

Given, $T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m ;

N = 180 r.p.m. or $\omega = 2\pi \times 180/60 = 18.85$ rad/s

Since the total fluctuation of speed ($\omega_1$-$\omega_2$) is 1% of mean speed (ω), coefficient of fluctuation of speed,

$$\delta = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. Power developed by the engine.

Work done per revolution

$$= \int_0^{2\pi} T d\theta = \int_0^{2\pi} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\, d\theta$$

$$= \left[ 20000\,\theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20000 \times 2\pi = 40\ 000\ \pi\ N-m$$

Mean resisting torque of the engine,

$$T_{mean} = \frac{Work\ done\ per\ revolution}{2\pi} = \frac{40\ 000\ \pi}{2\pi} = 20000\ N-m$$

Power developed by the engine

$$= T_{mean}.\omega = 20\ 000 \times 18.85 = 377\ 000 W = 377 kW.$$

2. Moment of inertia of the flywheel

The turning moment diagram for one stroke (i. e. half revolution of the crankshaft) is shown in the Fig. Since at points *B* and *D*, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

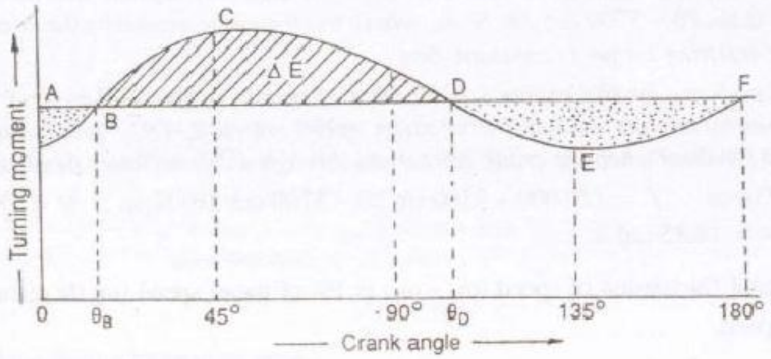
$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000$$

or $$9500 \sin 2\theta = 5700 \cos 2\theta$$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$\therefore$ $$2\theta = 31^\circ \; or \theta = 15.5^\circ$$

$\therefore$ $$\text{i.e.}, \theta_B = 15.5^\circ \; and \; \theta_D = 90^\circ + 15.5^\circ = 105.5^\circ$$




Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta$$

$$= \int_{15.5^\circ}^{105.5^\circ} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000) d\theta$$

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta = \left[ -\frac{9500 \sin 2\theta}{2} - \frac{5700 \cos 2\theta}{2} \right]_{15.5^\circ}^{105.5^\circ} = 11078 \; N - m$$

Maximum fluctuation of energy ( $\Delta E$ ),

$$11\,078 = I.\omega \;\; \delta = I(18.85)^2 \, 0.01 = 3.55 \, I$$

$$I = 11078/3.55 = 3121 \text{ kg-m}^2.$$

3. Angular acceleration of the flywheel

Let $\alpha$ = Angular acceleration of the flywheel, and

$\theta$ = Angle turned by the crank from inner dead centre = 45°... (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. Excess torque at any instant,

$$T_{excess} = T - T_{mean}$$

$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\,000$$

$$9500 \sin 2\theta - 5700 \cos 2\theta$$

$\therefore$ Excess torque at 45°= 9500 sin 90° - 5700 cos 90° =9500Nm

We also know that excess torque= $I.\alpha = 3121 \times \alpha$

From equations (i) and (ii),

$\alpha = 9500 / 3121 = 3.044$ rad/s$^2$.

**Problem 5:** The equation of the turning moment diagram of a three crank engine is 21000+7000 sin3θ Nm. Where θ in radians is the crank angle. The moment of inertia of the flywheel is $4.5 \times 10^3$ $Nm^2$ and the mean engine speed is 300 rpm. Calculate the power of the engine and the total percentage fluctuation of speed of the flywheel (i) if the resisting torque is constant (ii) if the resisting torque is 21000 + 3000 sinθ Nm.

a) $T_m = 21000\, Nm.$

$$\text{Power} = \frac{2\pi \times 21000 \times 300}{60} = 660\, kW.$$

b) (i) $\Delta E = \int_0^{\frac{\pi}{3}} 7000 \sin 3\theta d\theta = 4666.7\ Nm$.

$$\therefore \text{Total percent fluctuation of speed} = \frac{100\ \Delta E}{I \omega^2_{mean}}$$

$$= \frac{100 \times 4666.7 \times 9.8}{45 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.04\%$$

(ii) Engine torque = load torque, at crack angles given by

7000 sin3θ = 3000 sinθ

i.e., 2.33 (3sinθ - 4 sin$^3$θ) = sinθ

One solution is sinθ =0, i.e., θ = 0 and 180º, and the other is sinθ = ±0.803, i.e., θ = 53°24' or 126°36' between 0° and 180º. The intersections are shown in figure and the areas between the curves represent increase or decrease of total energy. The numerically longest is between θ = 53° 24' and 126º36'.

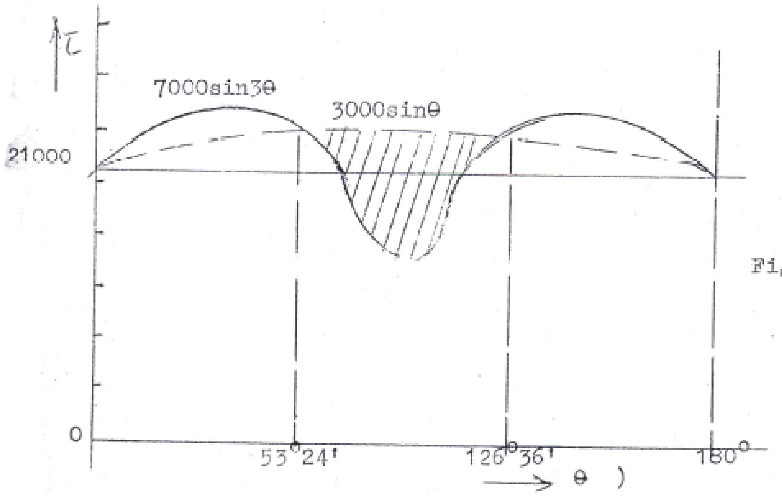



Fig.66.

$$i.e.,\ \Delta E = \int_{53^\circ 24'}^{126^\circ 36'} (7000 \sin 3\theta - 3000 \sin \theta)\, d\theta$$

$$= 7960 \text{ Nm.}$$

Therefore, the total (percentage) fluctuation of speed $\dfrac{100\ \Delta E}{I\omega^2_{mean}}$

$$= \frac{100 \times 7960 \times 9.8}{4.5 \times 10^3 \times \left(\dfrac{300\pi}{30}\right)^2}$$

$$= 1.65\%$$

A machine punching 3.8 cm dia hole in a 3.2 cm thick plate does 600 J of work / sq. cm of sheared area. The punch has a stroke of 10.2 cm and punches 6 holed / min. The maximum speed of the flywheel at its radius of gyration is 27.5 m/s. Find the mass of the flywheel so that its speed at the same radius does not fall below 24.5 m/s. Also determine the power of the motor, driving this machine.

d = 3.8cm, t = 3.2 cm, A= 38.2 cm$^2$

Energy required / punch = 600 x 38.2 = 22.920 J

$$\text{Assuming, } \frac{(\theta_2 - \theta_1)}{(2\pi)} = \frac{t}{2S} = \frac{3.2}{20.4}$$

$$\therefore (\Delta K_E)_{\max} = E\left[1 - \frac{t}{2S}\right] = \frac{1}{2} I\left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$= 22.920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} m k^2 \left(\omega_{\max}^2 - \omega_{\min}^2\right)$$

$$V_{\max} = k\ \omega_{\max} = 27.5 m/s$$

$$V_{\min} = k\ \omega_{\min} = 24.5 m/s$$

We get,

$$22920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} m\left(27.5^2 - 24.5^2\right) = \frac{1}{2} m\ 158$$

$$\therefore\ m = 244 kg.$$

The energy required / minute is $6 \times 22920 J$

$$\therefore\ Motor\ \ power\ = \frac{6 \times 22920}{1000 \times 60} k\omega = 2.292 kW$$

A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute.

Solution.

Given: $P = 3\ kW$; $m = 150\ kg$; $k = 0.6$m; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$rad/s

*Speed of the flywheel immediately after riveting*

Let $\omega_2$ = Angular speed of the flywheel immediately after riveting.

We know that, energy supplied by the motor,

$$E_2 = 3\ kW = 3000\ W = 3000\ N-m/s \qquad (\because 1W = 1\ N-m/s)$$

But, energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\ 000\ N-m$$

$\therefore$ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\ 000 - 3000 = 7000 \text{ N-m}$$

We know that maximum fluctuation of energy ($\Delta E$),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150\ (0.6)^2 \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

$$\therefore \qquad (\omega_2)^2 = 987.2 - 7000/27 = 728 \ or\ \omega_2 = 26.98\ rad/s$$

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60 / 2\pi = 257.6\ r.p.m.$$

*Number of rivets that can be closed per minute.*

Since, the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\ 000} \times 60 = 18\ rivets$$