UNIT-I

Gyroscopic Couple and Static & Dynamic Force Analysis

1.0 INTRODUCTION

'Gyre' is a Greek word, meaning 'circular motion. A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.

A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.1. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance to this motion is called gyroscopic effect.

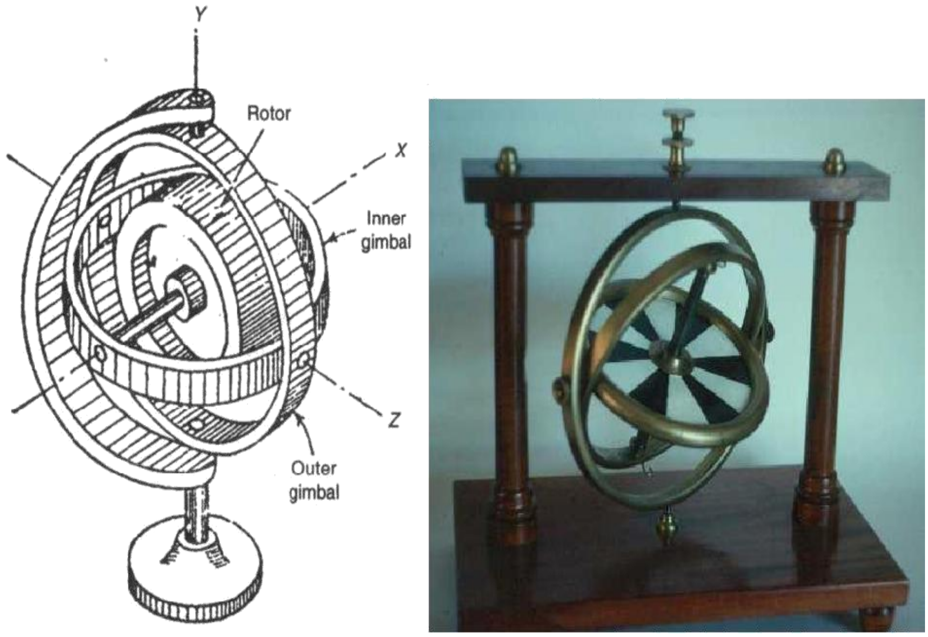


Fig.1: Gyroscope Mechanism

1.1 ANGULAR MOTION

A rigid body, (Fig.2) spinning at a constant angular velocity ω rad/s about a spin axis through the mass centre. The angular momentum 'H' of the spinning **vector** whose magnitude ω'. I represents the mass amount of inertia of the rotor about the axis of spin.

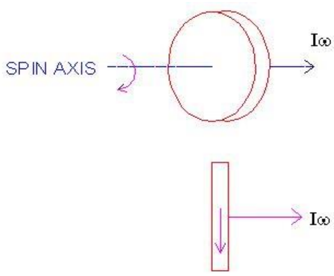
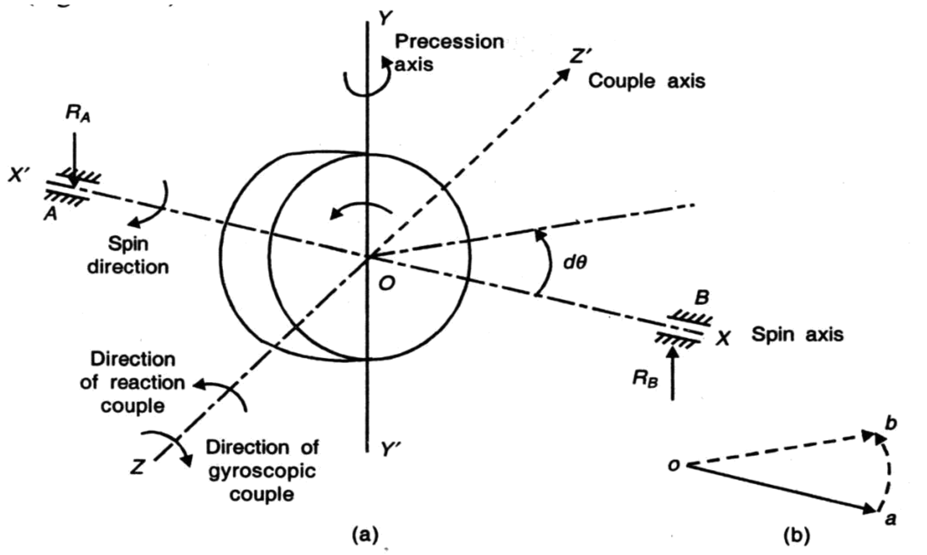


Fig.2: spinning body

The direction of the angular momentum can be found from the right hand screw rule or the right hand thumb rule. Accordingly, if the fingers of the right hand are bent in the direction of rotation of rotor, then the thumb indicates the direction of momentum.

1.2 GYROSCOPIC COUPLE

Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis (Fig.3).



The angular momentum of the rotating mass is given by,

$$H = mk^2\ \omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle δθabout Y-axis in the plane *XOZ,* then the angular momentum varies from *H* to *H* + δ*H*, where δ*H* is the change in the angular momentum, represented by vector ab [Figure 15.2(b)]. For the small value of angle of rotation 5^0, we can write

$$ab = oa \times \delta\theta$$

$$\delta H = H \times \delta\theta$$

$$= I\omega\delta\theta$$

However, the rate of change of angular momentum is:

$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\,\delta\theta}{\delta t}\right)$$

$$= I\omega\frac{d\theta}{dt}$$

$$\mathbf{C = I\omega\omega_p}$$

Where C = gyroscopic couple (N-m)

ω= angular velocity of rotary body (rad/s)

ω_p= angular velocity of precession (rad/s)

1.3 Direction of Spin vector, Precession vector and Couple/Torque vector with forced precession

To determine the direction of spin, precession and torque/couple vector, right hand screw rule or right hand rule is used. The fingers represent the rotation of the disc and the thumb shows the direction of the spin, precession and torque vector (Fig.4).

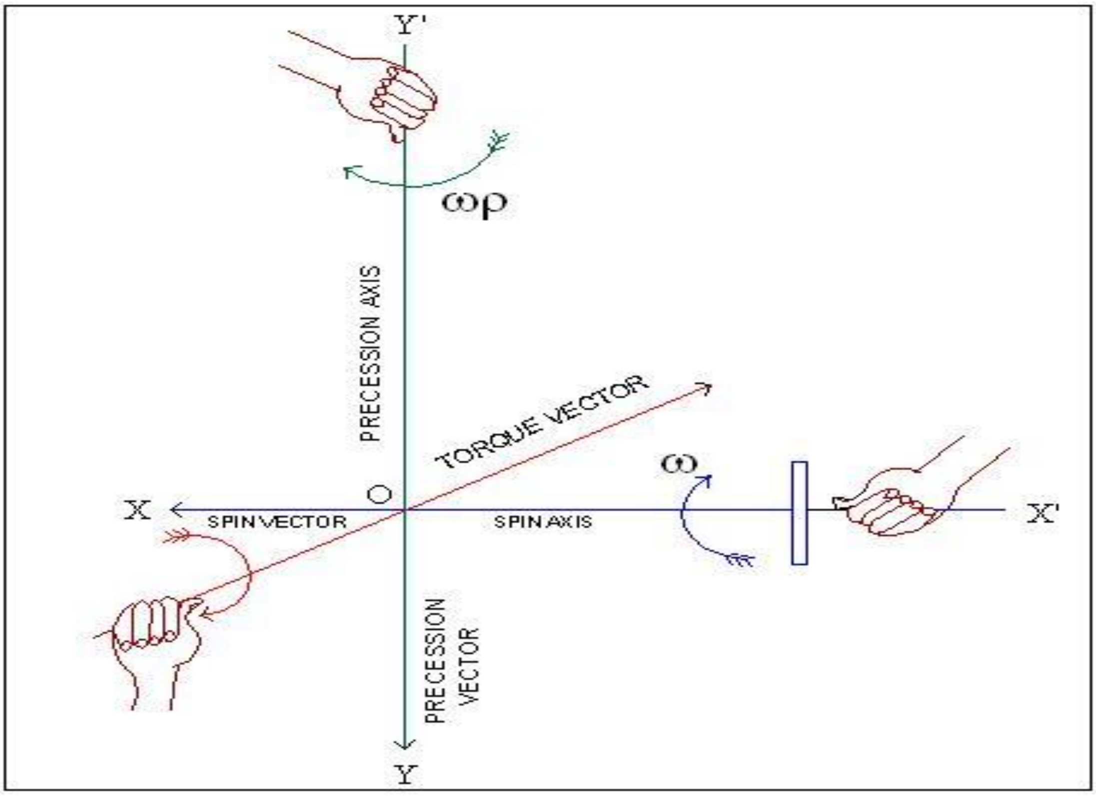


Fig.4. Direction of Spin vector, Precession vector and Couple/Torque vector

The method of determining the direction of couple/torque vector is as follows.

Case (i):

Consider a rotor rotating in anticlockwise direction when seen from the right (Fig.5 and Fig. 6), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector, the following procedure is used

- Turn the spin vector through 90^0 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction

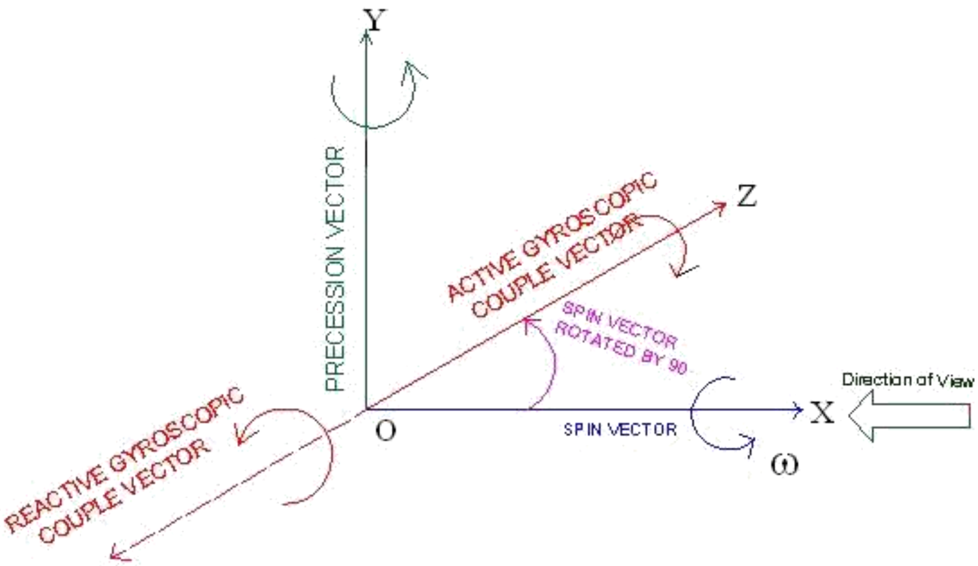


Fig. 5 Direction of active and reactive gyroscopic couple/torque vector

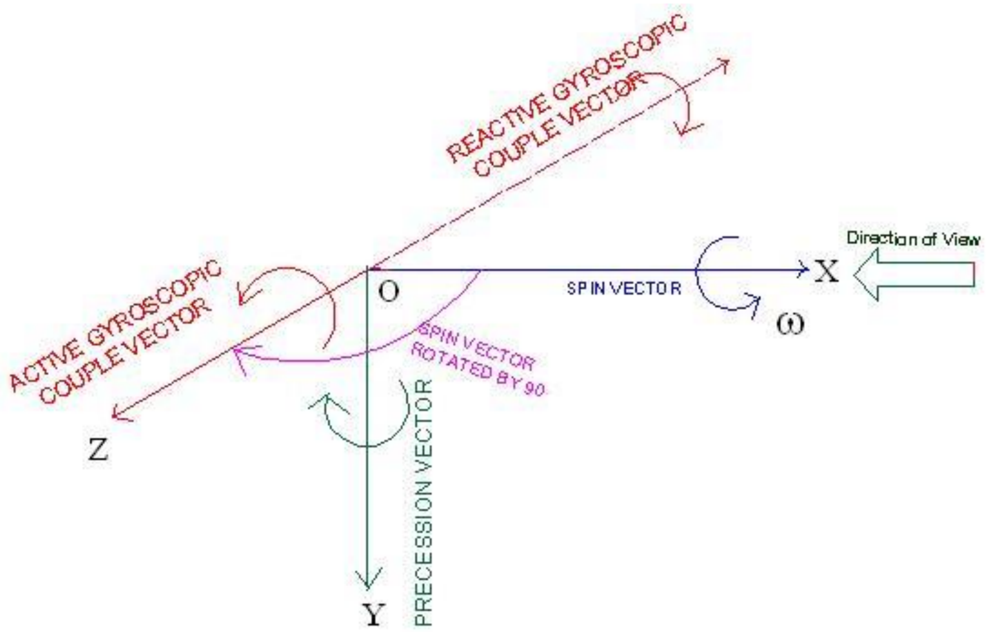


Fig. 6 Direction of active and reactive gyroscopic couple/torque vector

Case (ii):

Consider a rotor rotating in clockwise direction when seen from the right (Fig.7 and Fig. 8), and to precess the spin axis about precession axis in clockwise and anticlockwise direction when seen from top. Then, to determine the active/reactive gyroscopic couple vector,

- Turn the spin vector through 90^0 in the direction of precession on the XOZ plane
- The turned spin vector will then correspond to the direction of active gyroscopic couple/torque vector
- The reactive gyroscopic couple/torque vector is taken opposite to active gyro vector direction.

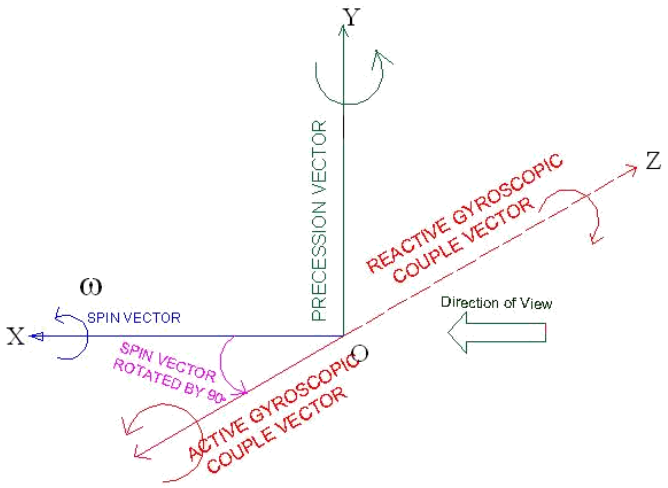


Fig. 7 Direction of active and reactive gyroscopic couple/torque vector

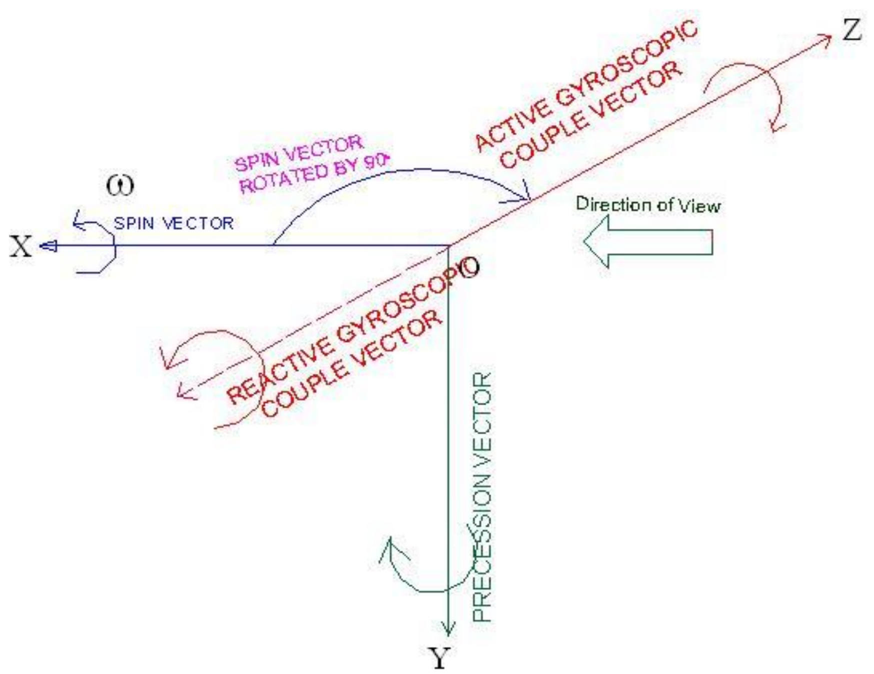


Fig. 8 Direction of active and reactive gyroscopic couple/torque vector

The resisting couple/ reactive couple will act in the direction opposite to that of the gyroscopic couple. This means that, whenever the axis of spin changes its direction, a **gyroscopic couple** is applied to it through the bearing which supports the spinning axis.

Please note that, for analyzing the gyroscopic effect of the body, always reactive gyroscopic couple is considered.

Problem 1

A disc of 5 kg mass with radius of gyration 70 mm is mounted at span on a horizontal shaft spins at 720 rpm in clockwise direction when viewed from the right hand bearing. If the shaft precesses about the vertical axis at 30 rpm in clockwise direction when viewed from the top, determine the reactions at each bearing due to mass of the disc and gyroscopic effect.

Solution Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 720}{60}$$

$$= 75.4 \text{ rad/s}$$

Angular velocity of precession: $\omega_p = \frac{2\pi N_p}{60}$

$$= \frac{2\pi \times 30}{60} = 3.14 \text{ rad/s}$$

Moment of inertia: $I = mk^2$

$$= 5 \times 0.07^2 = 0.0245 \text{ kg m}^2$$

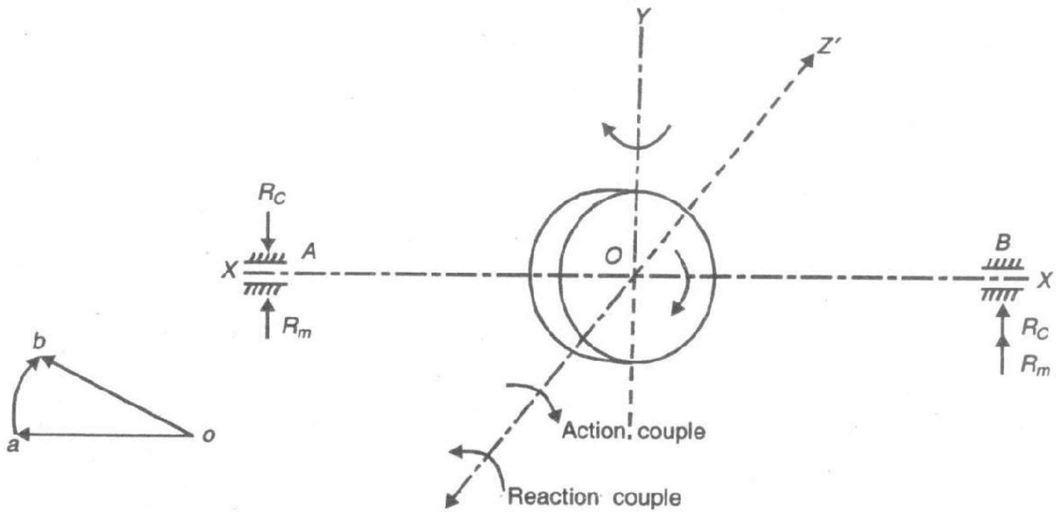


FIG. 9a

Gyroscopic couple: $C = I\,\omega\omega_p$

$$= 0.0245 \times 75.4 \times 3.14$$

$$= 5.8 \text{ Nm}$$

This couple induces reaction R_c at the bearing support.

$$R_c \times \frac{120}{1000} = 5.8$$

or $R_c = 48.3$ N

Reaction on the bearings due to weight of the disc, $R_m = mg/2 = 5 \times 9.81/2 = 24.53$ N

The angular momentum vector and induced reactive gyroscopic couple acting in anticlockwise direction as shown in fig.

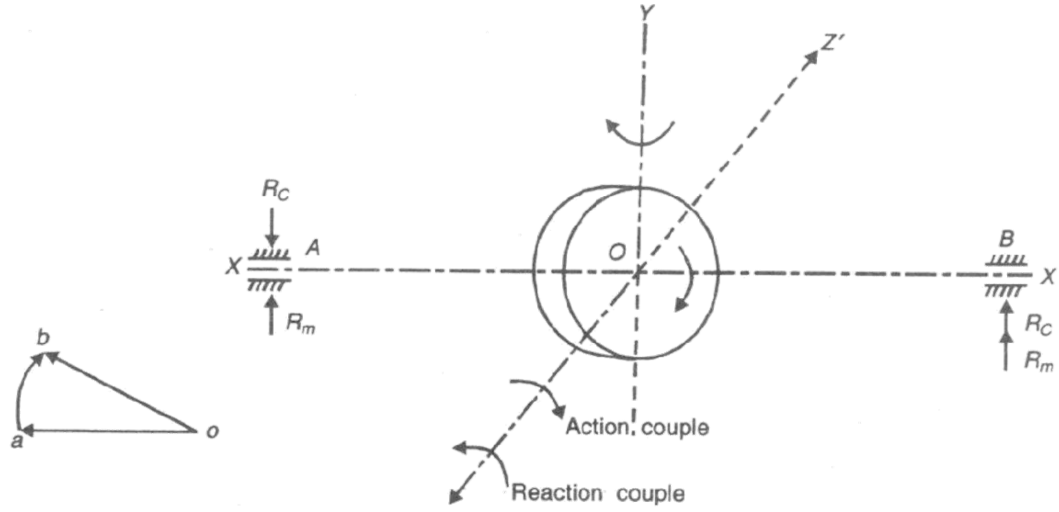


FIG.9b

Gyroscopic couple: $C = I\,\omega\omega_p$

$$= 0.0245 \times 75.4 \times 3.14$$

$$= 5.8 \text{ Nm}$$

This couple induces reaction R_c at the bearing support.

$$R_c \times \frac{120}{1000} = 5.8$$

or $$R_c = 48.3 \text{ N}$$

The reaction R_c acts in upward direction at right hand bearing and in downward d at left hand bearing.

The reaction due to weight of the disc acts in upward direction. Therefore,

Reaction at bearing A: $R_A = R_c - R_m$

$$= 48.43 - 24.53$$

$$= \mathbf{23.9\ N(\downarrow)}$$

Reaction at bearing B: $R_B = R_c + R_m$

$$= 48.43 + 24.53$$

$$= \mathbf{72.96\ N(\uparrow)}$$

1.4 GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion:

(i) Steering—The turning of ship in a curve while moving forward

(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis.

(iii)Rolling—Sideway motion of the ship about longitudinal axis

For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

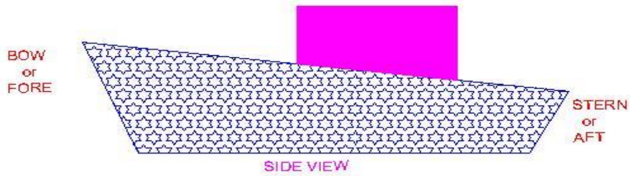
1.4.1 Ship Terminology

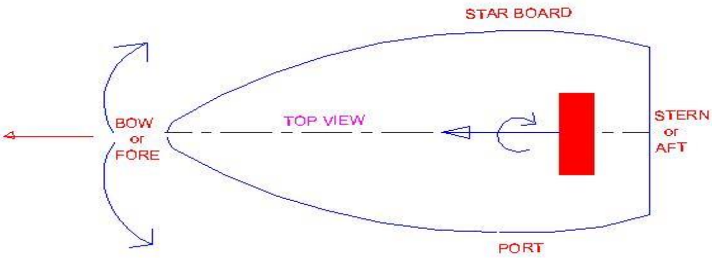
(i) Bow –It is the fore end of ship

(ii) Stern –It is the rear end of ship

(iii) Starboard –It is the right hand side of the ship looking in the direction of motion

(iv) Port –It is the left hand side of the ship looking in the direction of motion



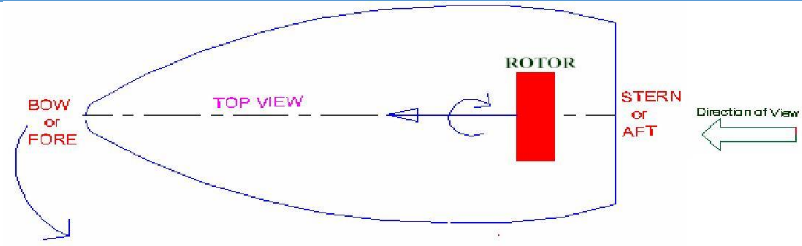


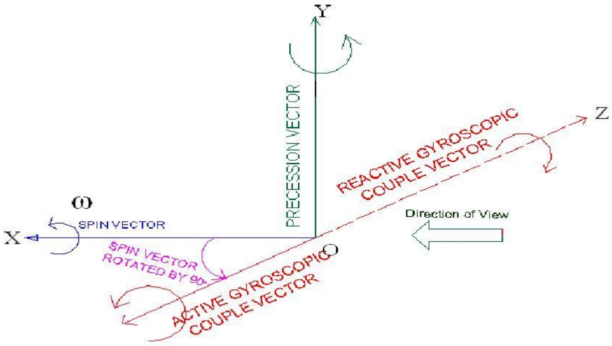
Consider a gyro-rotor mounted on the ship along longitudinal axis (X-axis) as shown in Fig. and rotate in clockwise direction when viewed from rear end of the ship. The angular speed of the rotor is rad/s. The direction of angular momentum vector *oa*, based on direction of rotation of rotor, is decided using right hand thumb rule as discussed earlier. The gyroscopic effect during the three types of motion of ship is discussed.

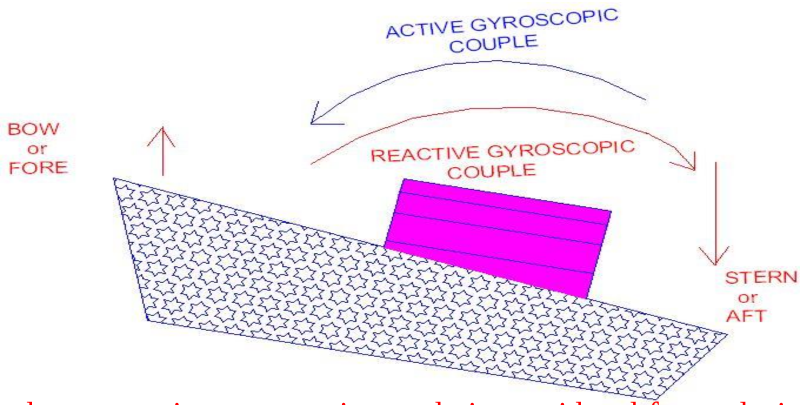
1.4.2 Gyroscopic effect on Steering of ship

(i) Left turn with clockwise rotor

When ship takes a left turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way.



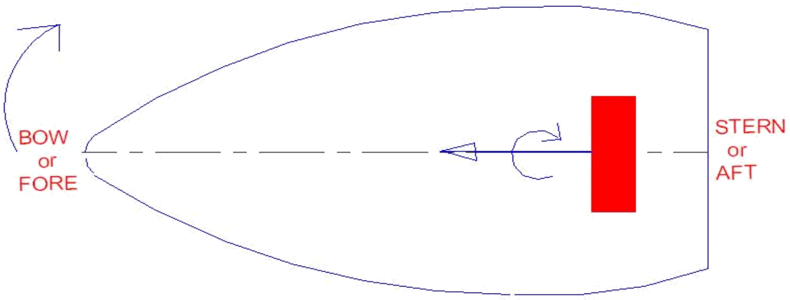


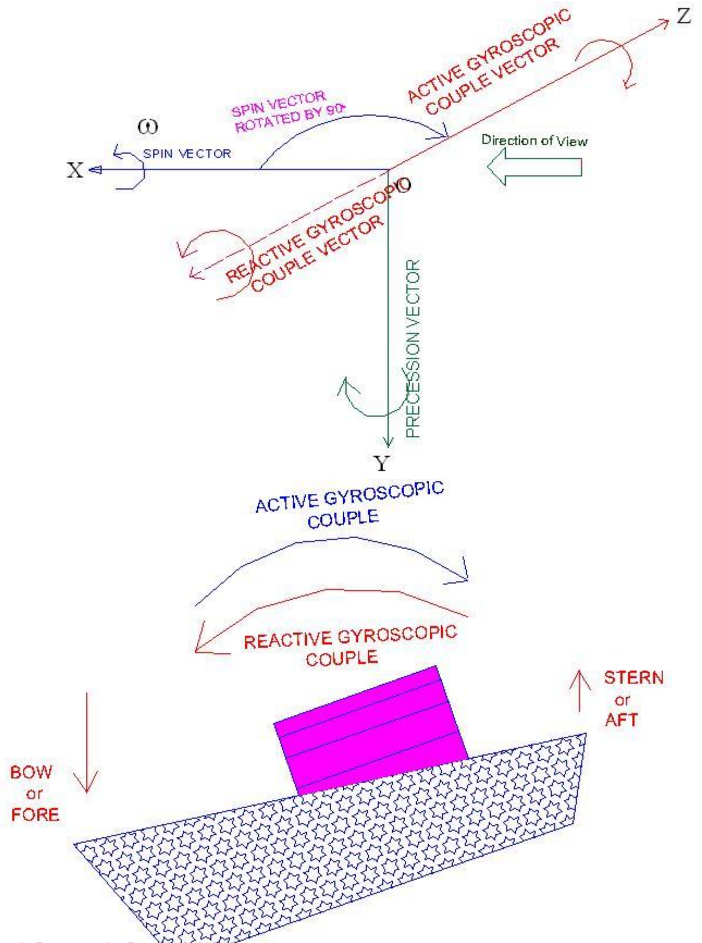


Note that, always reactive gyroscopic couple is considered for analysis. From the above analysis (Fig.12), the couple acts over the ship between stern and bow. This reaction couple tends to raise the front end (bow) and lower the rear end (stern) of the ship.

(ii) Right turn with clockwise rotor

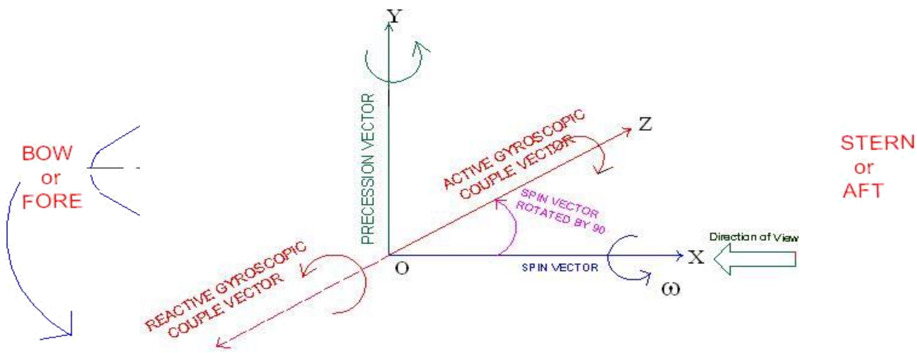
When ship takes a right turn and the **rotor rotates in clockwise direction** viewed from stern, the gyroscopic couple acts on the ship is analyzed (Fig). Again, the couple acts in vertical plane, means between stern and bow. Now the reaction couple tends to lower the bow of the ship and raise the stern.

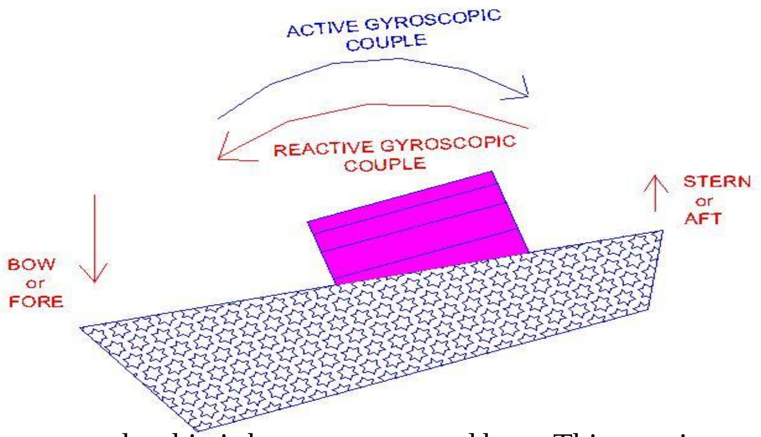




(iii) Left turn with anticlockwise rotor

When ship takes a left turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is analyzed in the following way (Fig.).





The couple acts over the ship is between stern and bow. This reaction couple tends to press or dip the front end (bow) and raise the rear end (stern) of the ship.

(iv) Right turn with anticlockwise rotor

When ship takes a right turn and the **rotor rotates in anticlockwise direction** viewed from stern, the gyroscopic couple act on the ship is according to Fig 20. Now, the reaction couple tends to raise the bow of the ship and dip the stern.

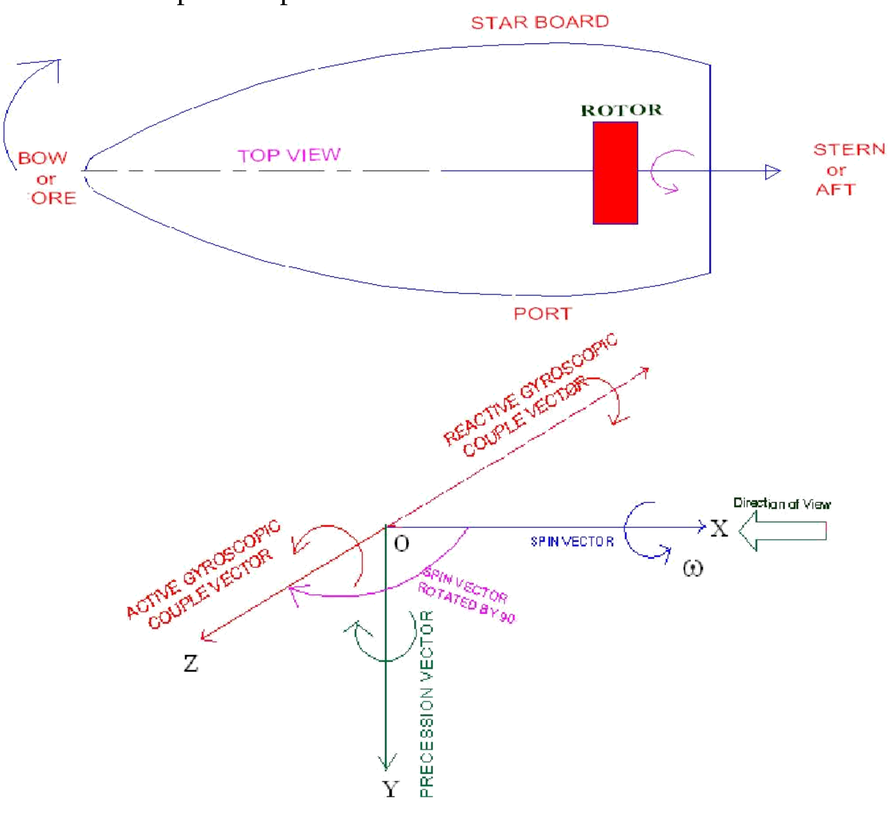


Fig. 21

1.4.3 Gyroscopic effect on Pitching of ship

The pitching motion of a ship generally occurs due to waves which can be approximated as sine wave. During pitching, the ship moves up and down from the horizontal position in vertical plane (Fig.22. & Fig. 23)

Fig.22 Pitching action of ship

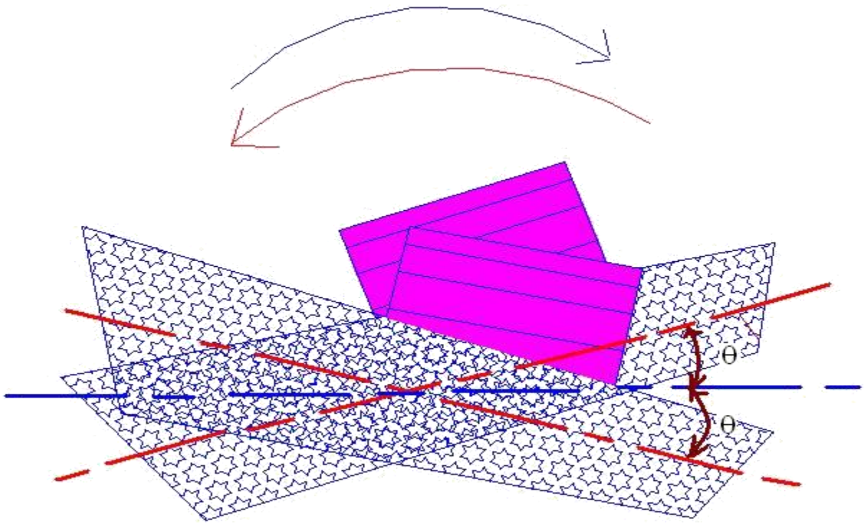


Fig.23 Pitching action of ship

Let θ = angular displacement of spin axis from its mean equilibrium position

A = amplitude of swing

(= angle in degree $\times \frac{2\pi}{360°}$)

and ω_0 = angular velocity of simple hormonic motion $\left(= \frac{2\pi}{\text{time period}}\right)$

The angular motion of the rotor is given as

$$\theta = A \sin \omega_0 t$$

Angular velocity of precess: $$\omega_p = \frac{d\theta}{dt}$$

$$= \frac{d}{dt}(A \sin \omega_0 t)$$

or $$\omega_p = A\omega_0 \cos \omega_0 t$$

The angular velocity of precess will be maximum when $\cos \omega_0 t = 1$

or $$\omega_{p\max} = A\omega_0$$

$$= A \times \frac{2\pi}{t}$$

Thus the gyroscopic couple: $$C = I\,\omega\,\omega_p$$

Consider a rotor mounted along the longitudinal axis and rotates in clockwise direction when seen from the rear end of the ship. The direction of momentum for this condition is shown by vector *ox* (Fig.24). When the ship moves up the horizontal position in vertical plane by an angle $\delta\theta$from the axis of spin, the rotor axis (X-axis) processes about Z-axis in XY-plane and for this case Z-axis becomes precession axis. The gyroscopic couple acts in anticlockwise direction about Y-axis and the reaction couple acts in opposite direction, i.e. in clockwise direction, which tends to move towards **right side** (Fig.25). However, when the ship pitches down the axis of spin, the direction of reaction couple is reversed and the ship turns towards **left side** (Fig. 26).

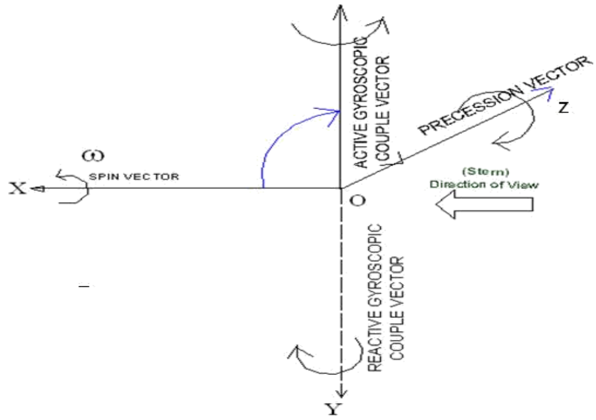


Fig. 24

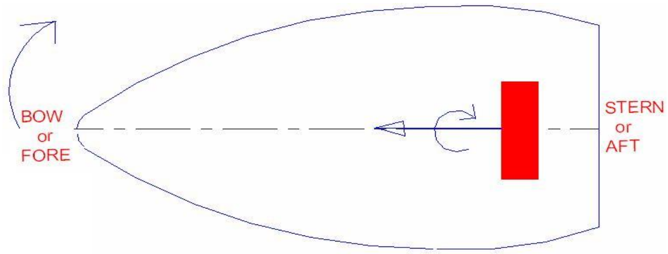


Fig. 25

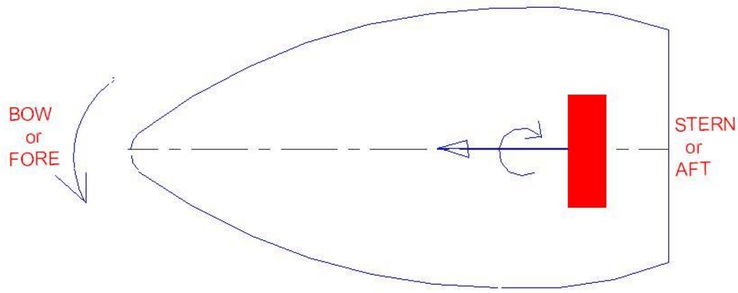


Fig. 26

Similarly, for the anticlockwise direction of the rotor viewed from the rear end (Stern) of the ship, the analysis may be done.

1.4.4 Gyroscopic effect on Rolling of ship.

The axis of the rotor of a ship is mounted along the longitudinal axis of ship and therefore, there is **no** precession of this axis. Thus, no effect of gyroscopic couple on the ship frame is formed when the ship rolls.

Fig.27

Problem 2

A turbine rotor of a ship has a mass of 3500 kg and rotates at a speed of 2000 rpm. The rotor has a radius of gyration of 0.5 m and rotates in clockwise direction when viewed from the stern (rear) end. Determine the magnitude of gyroscopic couple and its direction for the following conditions

(i) When the ship runs at a speed of 12 knots and steers to the left in a curve of 70 m radius

(ii) When the ship pitches 6° above and 6° below the horizontal position and the bow (Front) end is lowered. The pitching motion is simple harmonic with periodic time 30 sec.

(iii) When the ship rolls and at a certain instant, it has an angular velocity of 0.05 rad/s clockwise when viewed from the stern

Also find the maximum angular acceleration during pitching.

Solution Given, 1 knot = 1.86 kmph, the linear velocity of the ship:

$$V = 1.86 \times 12 = 22.32 \text{ kmph}$$

$$= \frac{22.32 \times 1000}{3600} = 6.2 \text{ m/s}$$

Angular velocity of the rotor:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60}$$

$$= 209.44 \text{ rad/s}$$

Precession velocity: $\omega_p = \frac{V}{R} = \frac{6.2}{70} = 0.08857$ rad/s

Moment of inertia: $I = mk^2 = 3500 \times 0.5^2 = 875$ kg m^2

Gyroscopic couple: $C = I\omega\omega_p$

$$= 875 \times 209.44 \times 0.08857$$

$$= \mathbf{16231.34 \text{ Nm}}$$

When ship steers to the left, the reaction gyroscopic couple action is in anticlockwise direction and the bow of the ship is raised and stern is lowered, as shown in Fig.28.

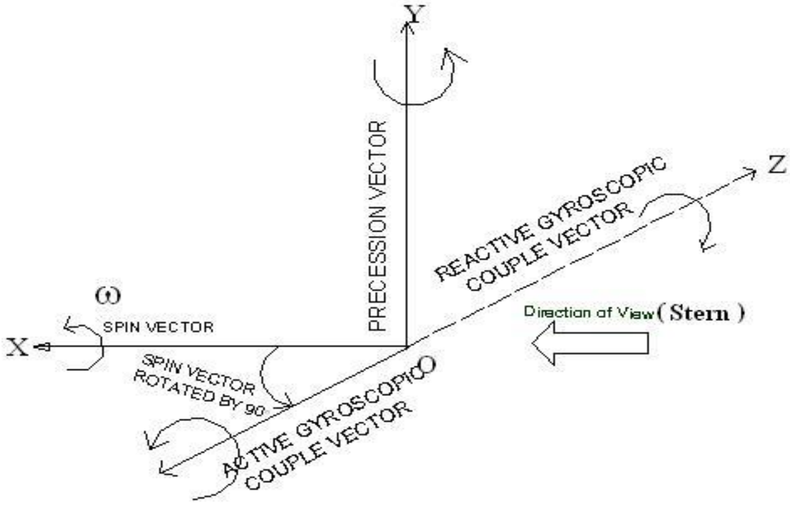


Fig.28

(ii) Amplitude of swing: $A = \frac{6° \times 2\pi}{360°} = 0.1047$ rad

Angular displacement: $\theta = A \sin \omega_0 t$

Angular velocity of precession: $\omega_p = \frac{d\theta}{dt} = A\omega_0 \cos \omega_0 t$

Maximum angular velocity of precession:

$$\omega_{pmax} = \omega_0 A$$

where $\omega_0 = \frac{2\pi}{\text{time period of oscillation}} = \frac{2\pi}{30}$

$= 0.2094$ rad/s

$$\omega_{pmax} = 0.2094 \times 0.1047 = 0.022 \text{ rad/s}$$

Maximum couple for pitching:

$$C_{max} = I\omega\omega_{pmax}$$
$$= 875 \times 209.44 \times 0.022$$
$$= 4031.72 \text{ Nm}$$

The effect of gyroscopic couple due to pitching is shown in Fig.29. the reactive gyroscopic couple will act in anticlockwise direction seen from top and it will turn ship towards the left side.

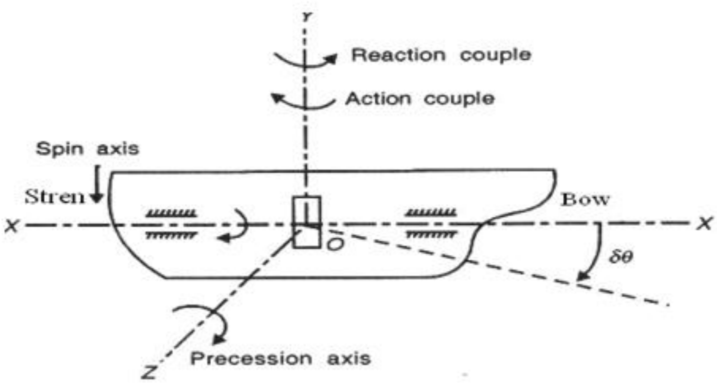


Fig.29

iii) Angular velocity of precession while the ship rolls is:

$$\omega p = 0.05 \text{ rad/s}$$

and gyroscopic couple : C $= I\omega\omega p$
$= 875 \times 209.44 \times 0.05$
$= 9163$ Nm

Since the ship rolls in the same plane as the plane of spin, there **is no gyroscopic effect**.

Angular velocity of precess during pitching is:

$$\omega_p = \frac{d\theta}{dt} = A\omega_0 \cos \omega_0 t$$

Therefore, angular acceleration:

$$\alpha = \frac{d^2\theta}{dt^2} = -A\omega_0^2 \sin \omega_0 t$$

Maximum angular acceleration:

$\alpha_{max} = -A\omega_0^2$
$= 0.1047 \times 0.2094^2$
$= 0.00459 \text{ rad/s}^2$

Problem 3

A ship is propelled by a rotor of mass of 2000 kg rotates at a speed of 2400 rpm. The radius of gyration of rotor is 0.4 m and spins clockwise direction when viewed from bow (front) end. Find the gyroscopic couple and its effect when;

(i) the ship takes left turn at a radius of 350 m with a speed of 35 kmph
(ii) the ship pitches with the bow rising at an angular velocity of 1 rad/s
(iii) the ship rolls at an angular velocity of 0.15 rad/s

Angular velocity:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2400}{60} = 251.33 \text{ rad/s}$$

Linear velocity: $V = 35 \text{ kmph} = \frac{35 \times 1000}{3600} = 9.72 \text{ m/s}$

Moment of inertia: $I = mk^2 = 2000 \times 0.4^2 = 320 \text{ kg m}^2$

Steering towards left

Angular velocity of precession: $\omega_p = \frac{V}{R} = \frac{9.72}{350} = 0.0278 \text{ rad/s}$

Gyroscopic couple: $C = I\omega\omega_p$

$= 320 \times 251.33 \times 0.0278$

$= 2235.8 \text{ Nm}$

The reaction gyroscopic couple will act in anticlockwise and will tend to lower the bow as shown in Figure 30.

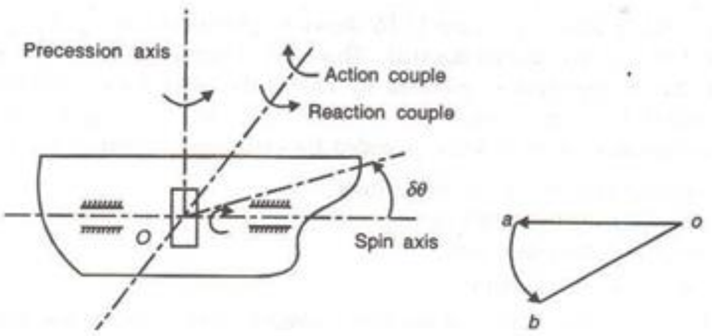


Fig.30

Pitching. Angular velocity of precession during pitching ω_p = 1.0 rad/s

Gyroscopic couple: C = 320 x 251.33 x 1.0

= 80425.6 **Nm Ans.**

The reaction gyroscopic couple acting in anticlockwise direction will tend to turn the bow towards the Right side as shown in Figure 31.

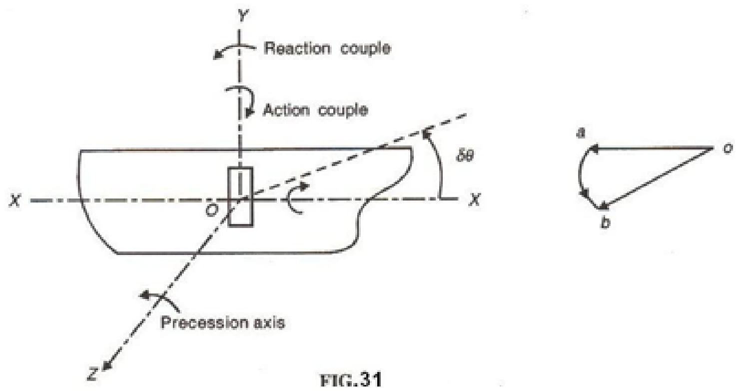


FIG.31

Rolling, Gyroscopic couple: C= $I\omega\omega_p$

= 320 x 251.33 x 0.15 = 12063.84 Nm

During rolling, the ship rolls in the same plane as the plane of spin and there will be no gyroscopic effect.

1.5 Gyroscopic Effect on Aeroplane

Aero planes are subjected to gyroscopic effect when it taking off, landing and negotiating left or right turn in the air.

Let

ω = Angular velocity of the engine rotating parts in rad/s,
m = Mass of the engine and propeller in kg,
r_W = Radius of gyration in m,
I = Mass moment of inertia of engine and propeller in kg m^2,
V = Linear velocity of the aeroplane in m/s,
R = Radius of curvature in m,
ω_p =Angular velocity of precession = $\frac{V}{R}$ rad/s

∴ Gyroscopic couple acting on the aero plane = $C = I\,\omega\omega_p$

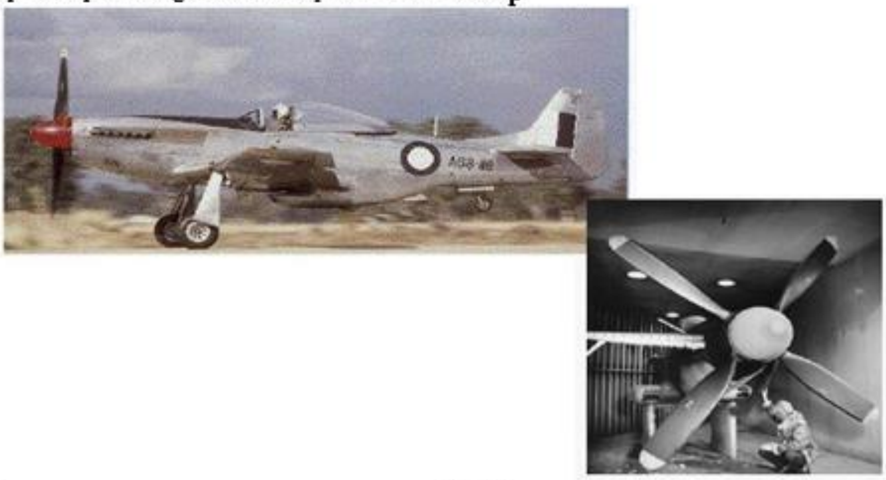


Fig.32

Let us analyze the effect of gyroscopic couple acting on the body of the aero plane for various conditions.

Case (i): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT

Fig.33

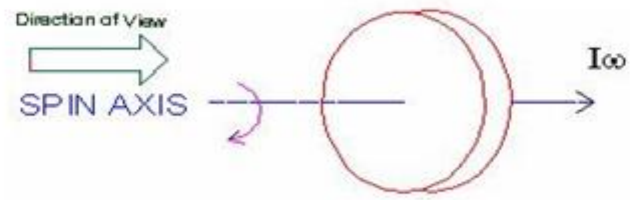


Fig.34

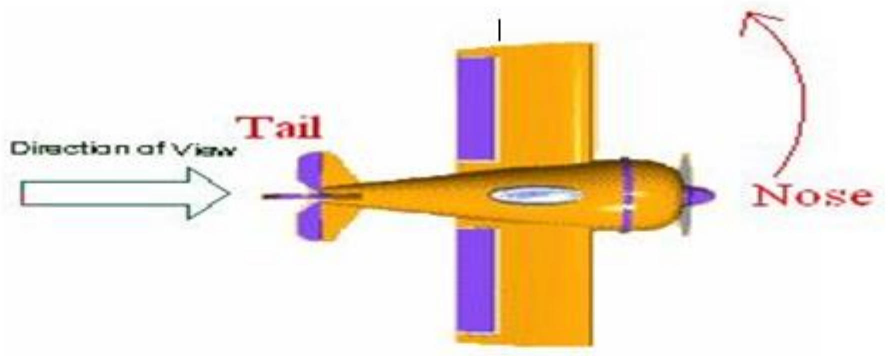


Fig.35

Fig.36

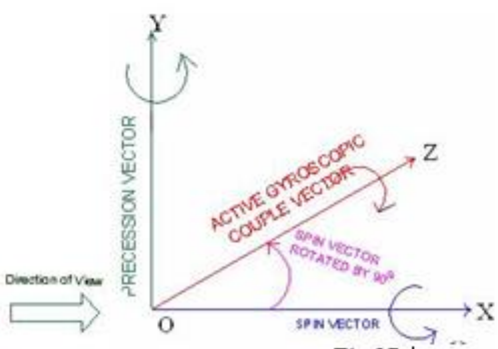


Fig.37

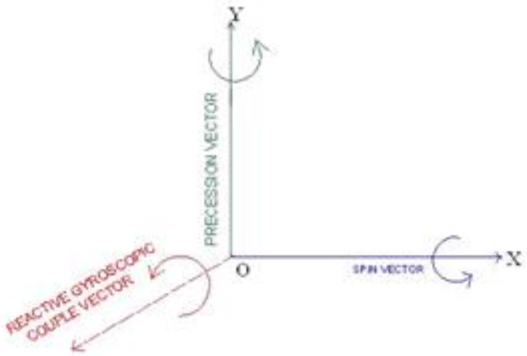


Fig.38

According to the analysis, the reactive gyroscopic couple tends to dip the tail and raise the nose of aeroplane.

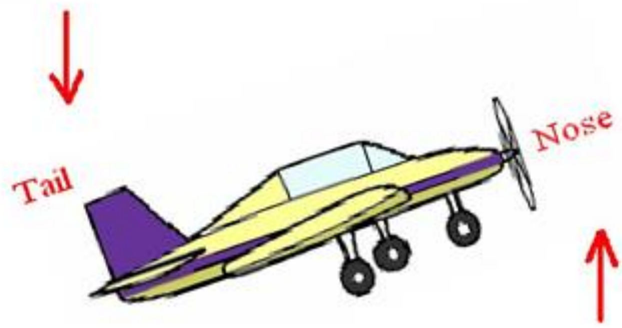
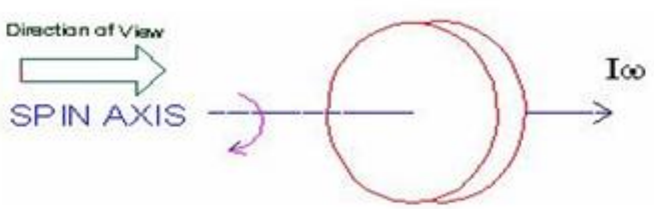


Fig.39

Case (ii): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane turns towards RIGHT

Fig.40



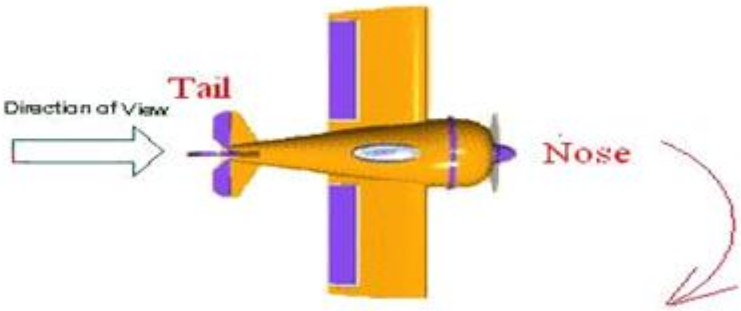


Fig.42

Fig.43

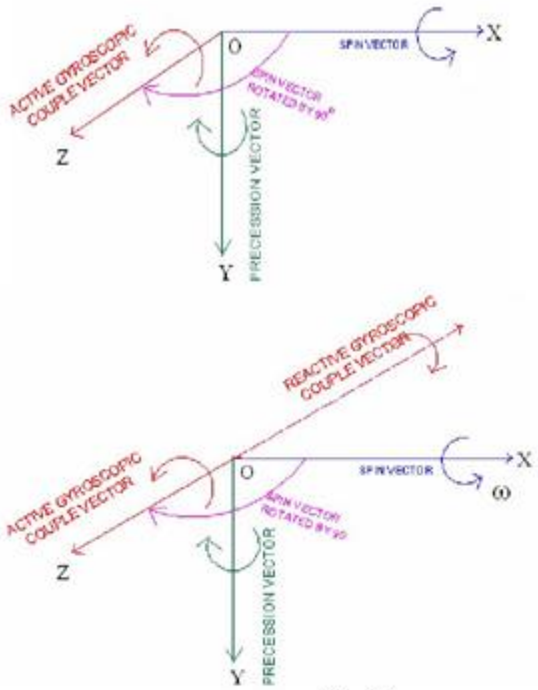


Fig. 44

According to the analysis, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane.

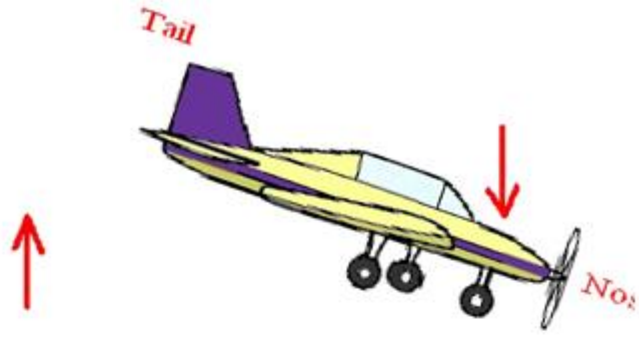


Fig.45

Case (iii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane turns towards LEFT

Fig.46

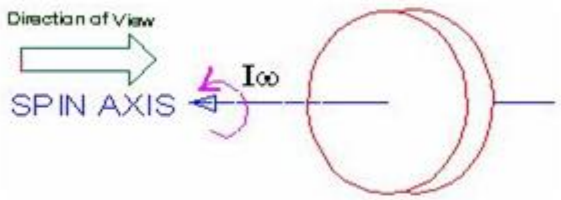


Fig.47

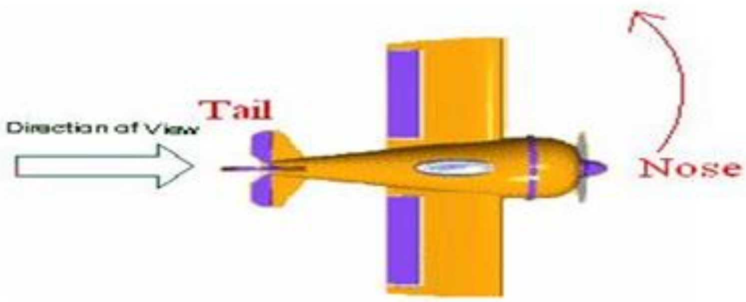


Fig.48

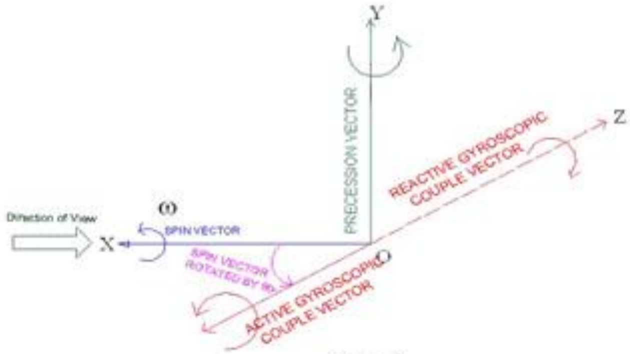
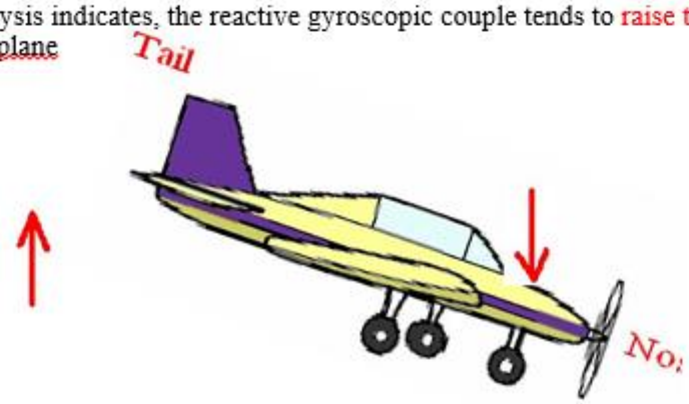


Fig.49

The analysis indicates, the reactive gyroscopic couple tends to raise the tail and dip the nose of aeroplane



Case (IV): **PROPELLER direction when seen from rear end and Aero plane turns towards RIGHT**

Fig.51

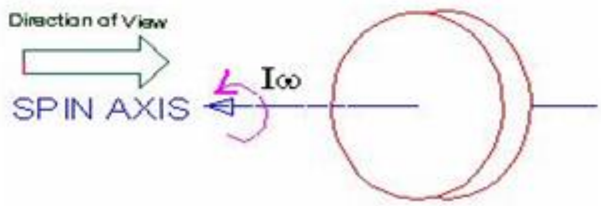


Fig.52

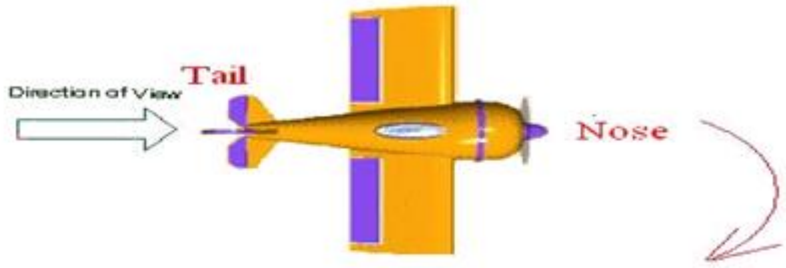


Fig.53

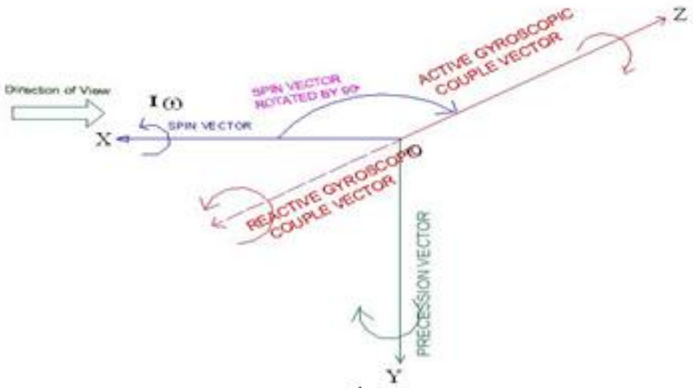


Fig.54

The analysis and dip the nose of aeroplane.

Fig.55

Case (v): PROPELLER rotates in CLOCKWISE direction when seen from rear upwards

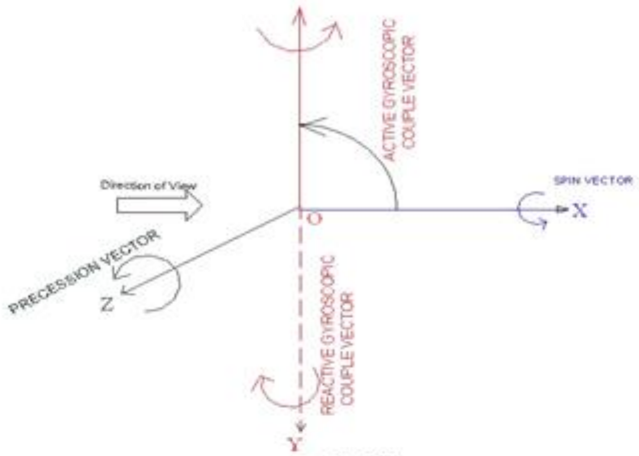
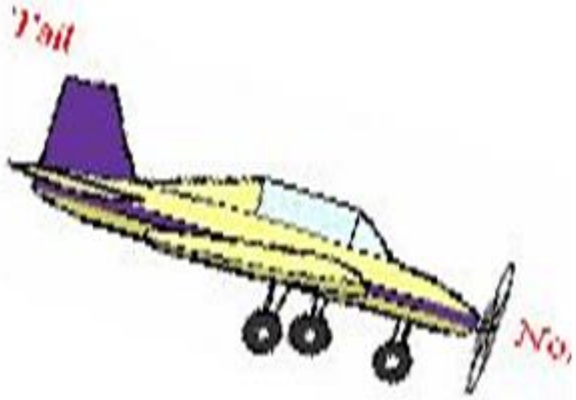


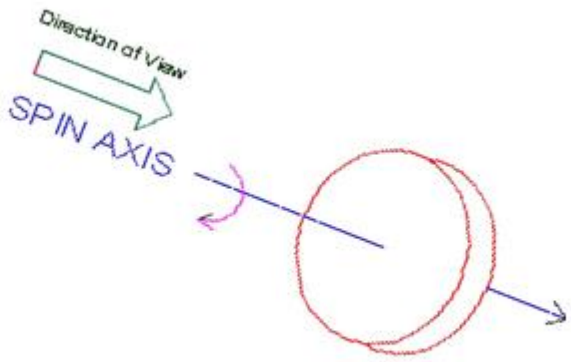
Fig.57

The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right

Fig.58

Case (vi): PROPELLER rotates in CLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards





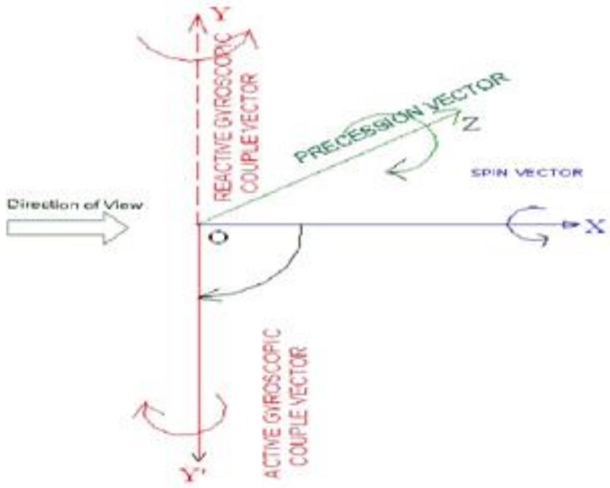


Fig. 61

The reactive gyroscopic couple tends to turn the nose of aeroplane toward left

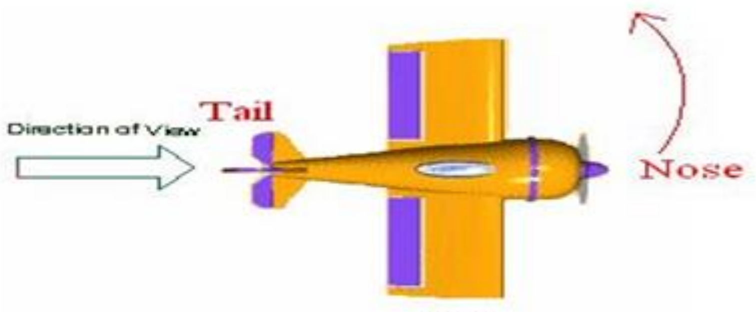


Fig.62

Case (vii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane takes off or nose move upwards

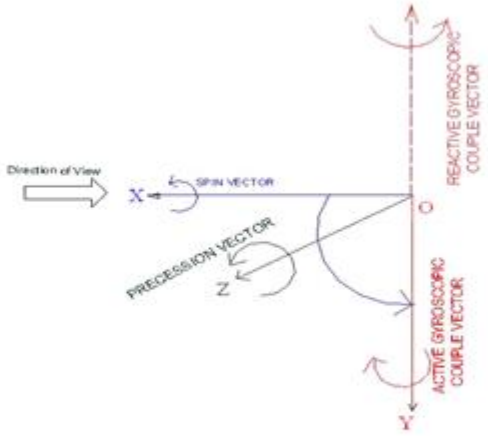


Fig.63

The reactive gyroscopic couple tends to turn the nose of aeroplane toward left

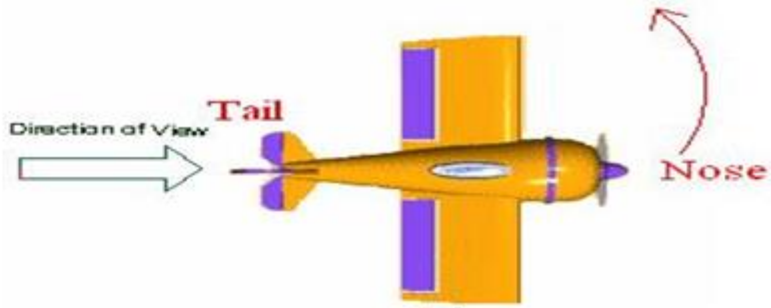


Fig.64

Case (viii): PROPELLER rotates in ANTICLOCKWISE direction when seen from rear end and Aeroplane is landing or nose move downwards

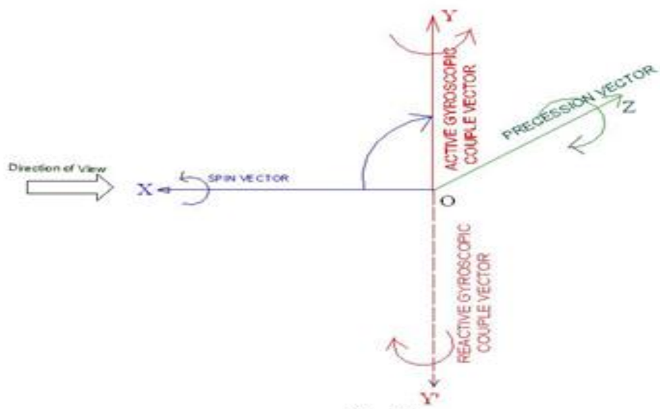


Fig.65

The analysis show, the reactive gyroscopic couple tends to turn the nose of aeroplane toward right

Problem 4

An aeroplane flying at a speed of 300 kmph takes right turn with a radius of 50 m. The mass of engine and propeller is 500 kg and radius of gyration is 400 mm. If the engine runs at 1800 rpm in clockwise direction when viewed from tail end, determine the gyroscopic couple and state its effect on the aeroplane. What will be the effect if the aeroplane turns to its left instead of right?

Solution Angular velocity of aeroplane engine:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.49 \text{ rad/s}$$

Angular velocity of precession: $\omega_p = \frac{V}{R}$

or $$\omega_p = \frac{300 \times 1000}{3600} \times \frac{1}{50}$$

$$= 1.67 \text{ rad/s}$$

Moment of inertia: $I = mk^2 = 500 \times 0.4^2$

$$= 80 \text{ kg m}^2$$

Gyroscopic couple: $c = I\omega\omega_p$

$$= 80 \times 188.49 \times 1.67$$

$$= \mathbf{25182.26 \text{ Nm}}$$ **Ans.**

1.6 Stability of Automotive Vehicle

A vehicle running on the road is said to be stable when no wheel is supposed to leave the road surface. In other words, the resultant reactions by the road surface on wheels should act in upward direction. For a moving vehicle, one of the reaction is due to gyroscopic couple produced by the rotating wheels and rotating parts of the engine. Let us discuss stability of two and four wheeled vehicles when negotiating a curve/turn.

1.6.1 Stability of Two Wheeler negotiating a turn

Fig.71

Fig. 71 shows a two wheeler vehicle taking **left turn** over a curved path. The vehicle is inclined to the vertical for equilibrium by an angle known as angle of heel.

Let

m = *Mass of the vehicle and its* rider in kg,

W = *Weight of the vehicle and* its rider in newtons = $m.g$,

h = *Height of the Centre of* gravity of the vehicle and rider,

r_W = *Radius of the wheels,*

R = *Radius of track or curvature,*

I_W = *Mass moment of inertia of each wheel,*

I_E = *Mass moment of inertia of the rotating parts of the engine,*

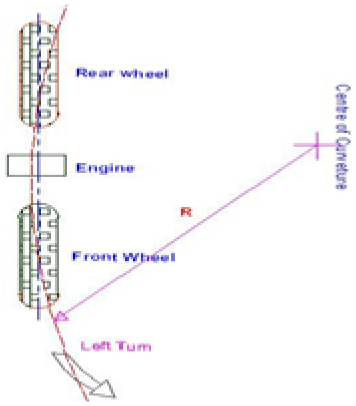
ω_W = Angular velocity of the wheels,

ω_E = Angular velocity of the engine rotating parts,

G = *Gear ratio* = ω_E / ω_W,

v = *Linear velocity*$_W \times r_W$, *of the vehicle* = ω

θ = Angle of heel. It is inclination of the



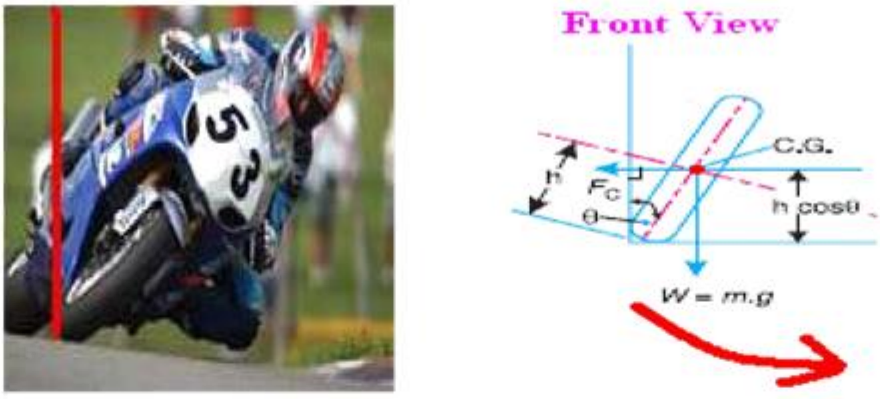
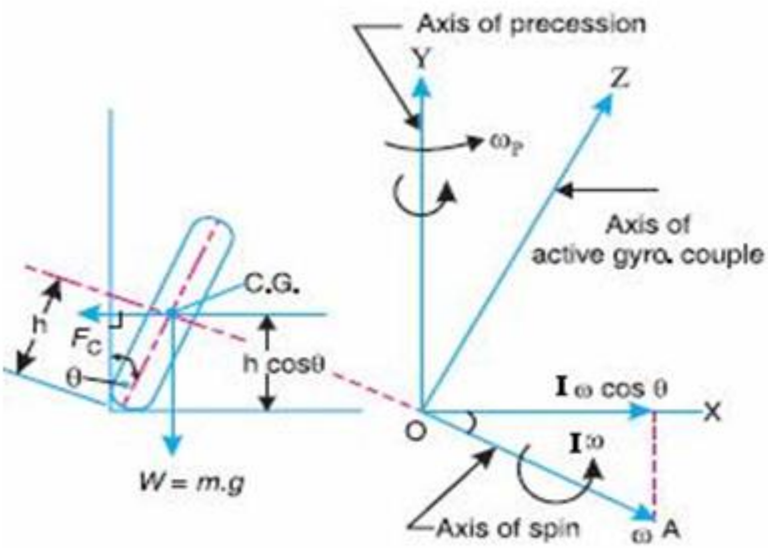


Fig.73



Let us consider the effect of the gyroscopic couple and centrifugal couple on the wheels.

1. Effect of Gyroscopic Couple

We know that, $V = \omega_W \times r_W$

$\omega_E = G.\omega_W$ or $\omega_E = G.v/r_W$

Angular momentum due to wheels = $2 I_W \omega_W$

Angular momentum due to engine and transmission = $I_E \omega_E$

Total angular momentum ($I x\omega$) = $2 I_W \omega_W \pm I_E \omega_E$

$$= 2I_W \frac{v}{r_W} \pm I_E G \frac{v}{r_W}$$

$$= \frac{v}{r_w}(2I_w \pm GI_E)$$

Also, Velocity of precession = $\omega p = \frac{V}{R}$

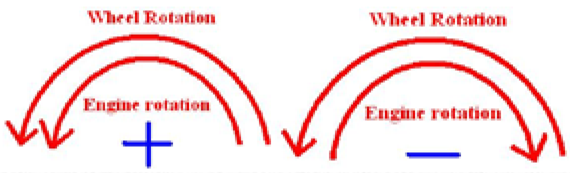
It is observed that, when the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown of heel'. In other inclined words, to the horizontal axis of spin at a in Fig.73 Thus, the angular momentum vector I ω due to spin is represented by OA inclined to OX at an angle θ. But, the precession axis is in vertical. Therefore, the spin vector is resolved along OX.

Gyroscopic Couple,

$$C_g = (I\omega)\cos\theta \times \omega_p$$

$$C_g = \frac{v^2}{Rr_w}(2I_w \pm GI_E)\cos\theta$$

Note: When the engine is rotating in the same direction as that of wheels, then the positive sign is used in the above equation. However, if the engine rotates in opposite direction to wheels, then negative sign is used.



The gyroscopic couple will act over the vehicle outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown

Analysis:

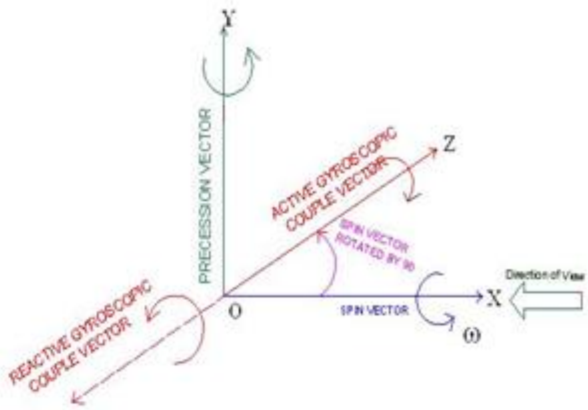


Fig.75

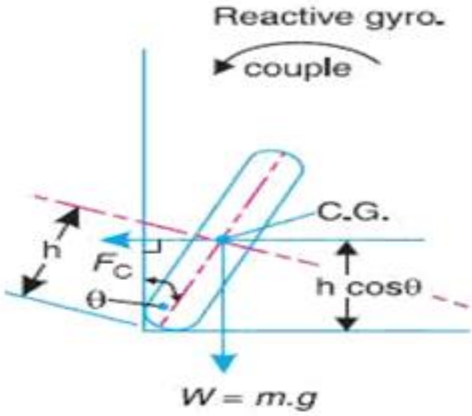


Fig.76

2. Effect of Centrifugal Couple

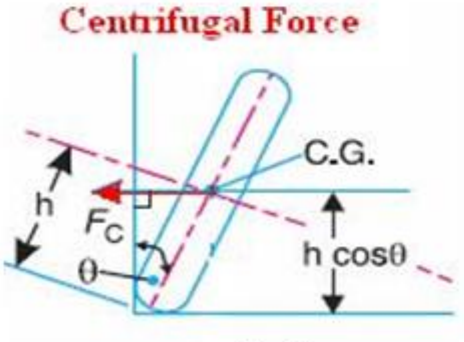


Fig. 77

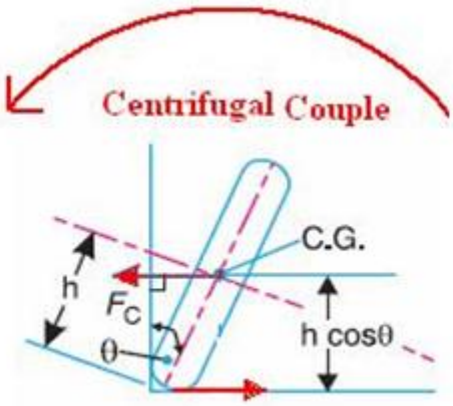
We have,

Centrifugal force, $F_c = \frac{mv^2}{R}$

Or

Centrifugal Couple, $C_c = F_c \times h\cos\theta$

$$= \frac{mv^2}{R} h\cos\theta$$



The Centrifugal couple will act over the two wheeler outwards i.e., in the anticlockwise direction when seen from the front of the two wheeler. This couple tends to overturn/topple the vehicle in the outward direction as shown in Fig.78

Therefore, the total Over turning couple: $C = C_g + C_c$

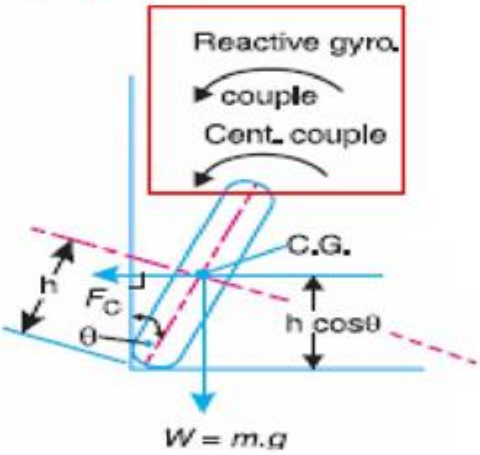


Fig.79

$$C = \frac{v^2}{Rr}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta$$

For the vehicle to be in equilibrium, overturning couple should be equal to balancing couple acting in clockwise direction due to the weight of the vehicle and rider.

∴ $C = mgh\sin\theta$

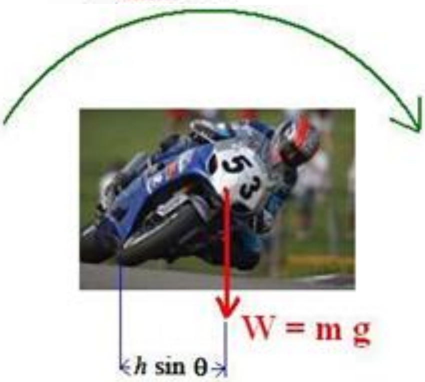


Fig.80

For the stability, overturning couple must be equal to balancing couple,

$$\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta = mgh\sin\theta$$

Therefore, from the above equation, the value of angle of heel (θ) may be determined, so that the vehicle does not skid. Also, for the given value of the maximum vehicle speed in the turn without skid may be determined.

Problem 5

A motorcycle and its rider together weighs 2000 N and their combined centre of gravity is 550 mm above the road when motorcycle is upright. Each wheel is of 580 mm diameter and has a moment of inertia of 1.0 kgm^2. The moment of inertia of rotating parts of engine is 0.15 kg m^2. The engine rotates at 5 times the speed of the vehicle and the same sense. Determine the angle of heel necessary when motorcycle is taking a turn over a track of 35 m radius at a speed of 60 kmph.

Solution:

Velocity of vehicle: $v = \frac{60 \times 1000}{3600} = 16.67$ m/s

Angular velocity of wheel: $\omega = \frac{2v}{d} = \frac{2 \times 16.67}{0.58} = 57.48$ rad/s

Angular velocity of precession: $\omega_p = \frac{v}{R} = \frac{16.67}{35} = 0.476$ rad/s

(i) Gyroscopic couple due to two wheels:

$C_W = 2I_W \omega \omega_p \cos\theta$
$= 2 \times 1.0 \times 57.48 \times 0.476 \times \cos\theta$
$= 54.72 \cos\theta$ Nm

(ii) Gyroscopic couple due to rotating parts of engine:

$C_E = I_E G \omega \omega_p \cos\theta$
$= 0.15 \times 5 \times 57.48 \times 0.476 \times \cos\theta$
$= 20.52 \cos\theta$ Nm

(iii) Centrifugal force due to angular velocity of die wheel:

$$F_c = \frac{mv^2}{R} = \frac{2000 \times 16.67^2}{9.81 \times 35} = 1618.7 \text{ N}$$

Centrifugal couple: $C_c = 1618.7 \times 0.55 \cos\theta$
$= 890.28 \cos\theta$ Nm

Total overturning couple: $C = C_W + C_e + C_c$
$= (54.72 + 20.52 + 890.28) \cos\theta$
$= 965.52 \cos\theta$ Nm

Balancing couple $= mgh \sin\theta$

$$= \frac{2000}{9.81} \times 9.81 \times 0.55 \sin\theta$$

$= 1100 \sin\theta$ Nm

For the stability of motorcycle, overturning couple should be equal to resisting couple.

$\therefore$ $1100 \sin\theta = 965.52 \cos\theta$

Or

$$\tan\theta = \frac{965.52}{1100} = 0.877$$

heel angle: $\theta = 41.27°$

Problem 6

A motor cycle with its rider has a mass of 300 kg. The centre of gravity of the machine and rider combined being 0.6 m above the ground with machine in vertical position. Moment of inertia of each wheel is 0.525 kg m^2 and the rolling diameter of 0.6 m. The engine rotates 6 times the speed of the road wheels and in the same sense. The engine rotating parts have a mass moment of inertia of 0.1686 kg m^2. Find (i) the angle of heel necessary if the vehicle is running at 60 km/hr round a curve of 30 m (ii) If the road and tyre friction allow for the angle of heel not to exceed 50^o, what is the maximum road velocity of the motor cycle.

Solution:

m = 300 kg, h = 0.6 m, I_W = 0.525 kg m^2, dw=0.6 m; r_W = 0.3 m, G = 6, I_E = 0.1686 m,
V= 60km/hr = 16.66 m/s, R = 30 m (i) θ=? (ii) θ= 50^o V=?

(i) Angle of heel,

We have,

$$\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta = mgh\sin\theta$$

$$\therefore \frac{16.66^2}{30}\left[\frac{2x0.525 + 6x0.1685}{0.3} + 300\ x\ 0.6\right]\cos\theta = 300x9.81x0.6x\sin\theta$$

$$\theta = 45°$$

(ii) Given, $\theta = 50°$, V=?,

$$\frac{v^2}{Rr_w}(2I_w + GI_e)\cos\theta + \frac{mv^2}{R} h\cos\theta = mgh\sin\theta$$

$$\therefore \frac{V^2}{30}\left[\frac{2\ x\ 0.525 + 6\ x\ 0.1685}{0.3} + 300\ x\ 0.6\right]\cos 50 = 300\ x\ 9.81\ x\ 0.6\ x\ \sin 50$$

$$\therefore V = 66\ Kmph$$

1.6.2 Stability of Four Wheeled Vehicle negotiating a turn.

Stable condition

Unstable Condition

Fig.81

Consider a four wheels automotive vehicle as shown in Figure 82. The engine is mounted at the rear with its crank shaft parallel to the rear axle. The centre of gravity of the vehicle lies vertically above the ground where total weight of the vehicle is assumed to be acted upon.

Let

m = *Mass of the vehicle* (kg)

W = *Weight of the vehicle* (N) = $m.g$,

h = *Height of the centre of* gravity of the vehicle (m)

r_W = *Radius of the wheels (m)*

R = *Radius of track or* curvature (m)

I_W = *Mass moment of inertia of each wheel (kg-m²)*

I_E = *Mass moment of inertia of the rotating parts of the engine (kg-m²)*

ω_W = Angular velocity of the wheels (rad/s)

ω_E = Angular velocity of the engine (rad/s)

G = *Gear ratio* = ω_E / ω_W,

v = *Linear velocity* $_W \times r_W$, *of the vehicle (m/s)* = ω x = Wheel track (m)

b = Wheel base (m)

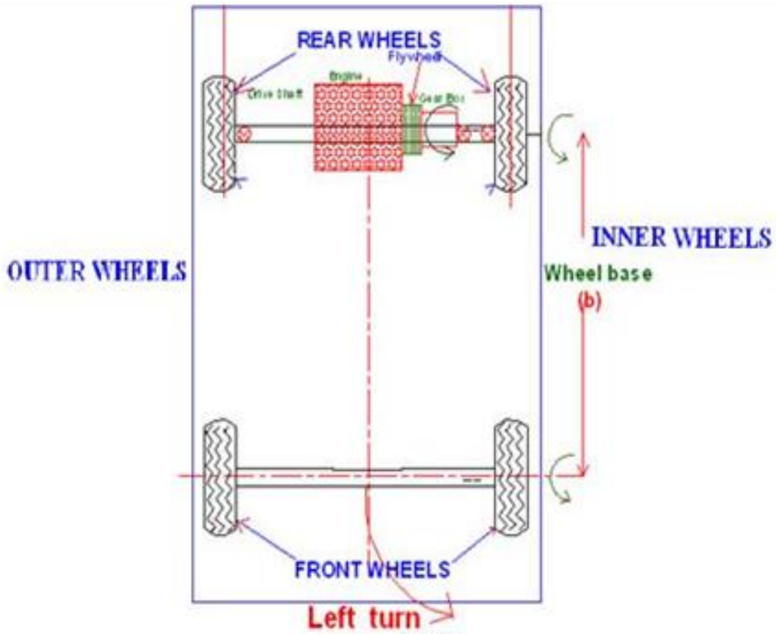


Fig.82

(i) Reaction due to weight of Vehicle

Weight of the vehicle. Assuming that weight of the vehicle *(mg)* is equally distributed over four wheels. Therefore, the force on each wheel acting downward is $mg/4$ and the reaction by the road surface on the wheel acts in upward direction.

$$R_w = \frac{mg}{4} \qquad 38$$

(ii) Effect of Gyroscopic couple due to Wheel

Gyroscopic couple due to four wheels is,

$$C_W = 4\, I_W \omega \omega_p$$

(iii) Effect of Gyroscopic Couple due to Engine

Gyroscopic couple due to rotating parts of the engine

$$C_E = I_E\, \omega \omega_p = I_E\, G\, \omega \omega_p$$

Therefore, total gyroscopic couple:

$$C_g = C_W + C_E = \omega \omega_p\, (4I_W \pm I_E G)$$

When the wheels and rotating parts of the engine rotate in the same direction, then positive sign is used in the above equation. Otherwise negative sign should be considered.

Assuming that the vehicle takes a left turn, the reaction gyroscopic couple on the vehicle acts between outer and inner wheels.

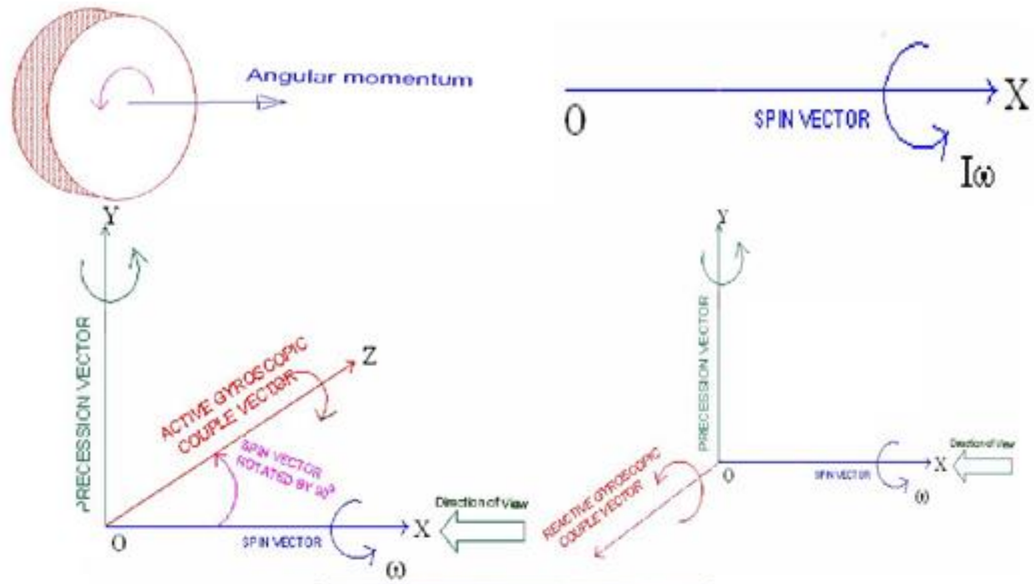


Fig.83

This gyroscopic couple tends to press the outer wheels and lift the inner wheels.

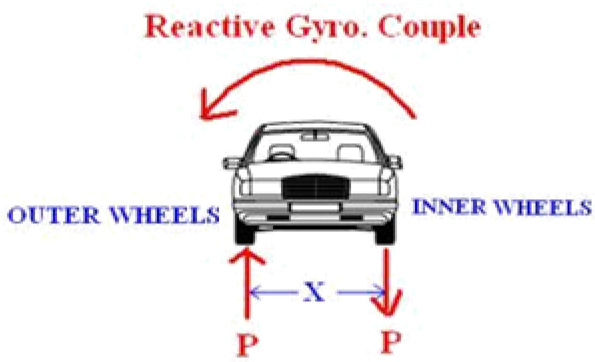


Fig.84

Due to the reactive gyroscopic couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwords on the outer wheels and vertically downwords on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be P Newtons, then,

$$P \times X = C_g$$

$$P = \frac{C_g}{x}$$

Road reaction on each outer/Inner wheel,

$$\frac{P}{2} = \frac{C_g}{2X}$$

(iii) **Effect of Centrifugal Couple**

When a vehicle moves on a curved path, a centrifugal force acts on the vehicle in outward direction through the centre of gravity of the vehicle (Fig)

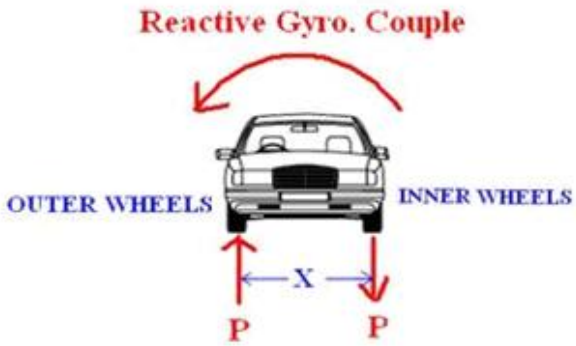


Fig.85

Centrifugal force,

$$F_c = m\omega_p^2 R = \frac{mv^2}{R}$$

This force forms a Centrifugal couple.

$$C_c = \frac{mv^2 h}{R}$$

This centrifugal couple tends to press the outer and lift the inner

Fig.86

Due to the centrifugal couple, vertical reactions on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer and inner wheels be F Newtons, then,

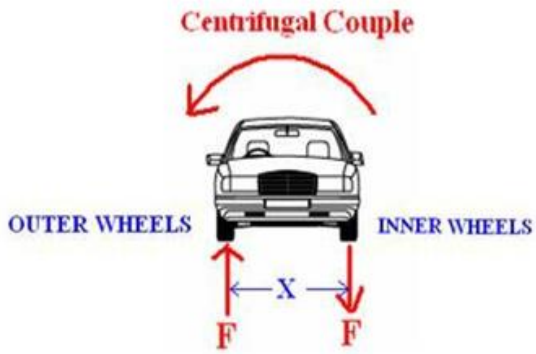


Fig.87

Road reaction on each outer/Inner wheel,

$$\frac{F}{2} = \frac{C_c}{2X}$$

The reactions on the outer/inner wheels are as follows,

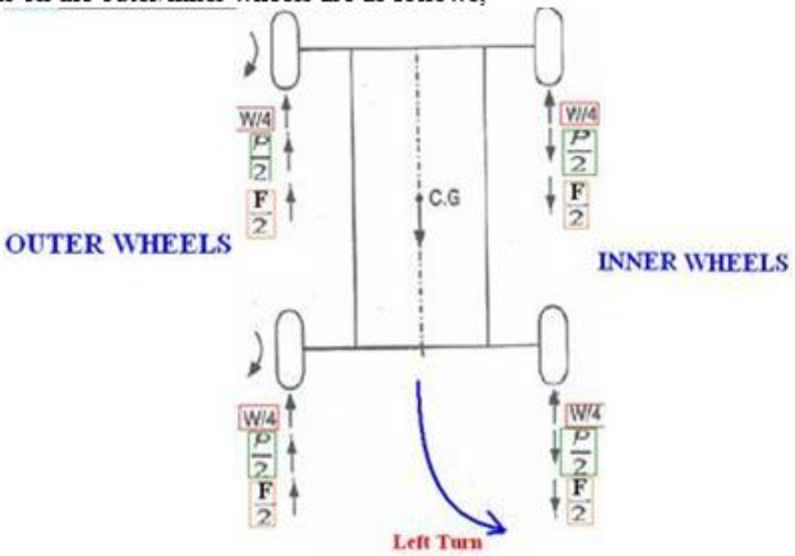


Fig.88

Total vertical reaction at each outer wheels

$$P_o = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

Total vertical reaction at each inner wheels

$$P_i = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

Problem 7

An automobile car is travelling along a track of 100 m mean radius. The moment of inertia of 500 mm diameter wheel is 1.8 kg m^2. The engine axis is parallel to the rear axle and crank shaft rotates in the same sense as the wheel. The moment of inertia of rotating parts of the engine is 1 kg m^2. The gear ratio is 4 and **the mass of** the vehicle is 1500 kg. If the centre of gravity of the vehicle is 450 mm above the road level and width of the track of the vehicle is 1.4 m, determine the limiting speed of the vehicle for condition that all four wheels maintain contact with the road surface.

Solution Let = limiting velocity of the vehicle.

Angular velocity: $\omega = \frac{v}{r} = \frac{v}{0.25}$ rad/s

Precession velocity: $\omega_p = \frac{v}{R} = \frac{v}{100}$ rad/s

(i) Reaction due to gyroscopic couple:

(a) Gyroscopic couple due to four wheels:

$$C_w = 4I_w \omega \omega_p$$

$$= 4 \times 2 \times \frac{v}{0.25} \times \frac{v}{100} = 0.32\ v^2 \text{ Nm}$$

(b) Gyroscopic couple due to engine parts:

$$C_e = I_e G \omega \omega_p$$

$$= 1 \times 4 \times \frac{v}{0.25} \times \frac{v}{100} = 0.16\ v^2 \text{ Nm}$$

Reaction due to total gyroscopic couple on each outer wheel:

$$R_g = \frac{C_g}{2b} = \frac{0.48v^2}{2 \times 1.5} = 0.16v^2 \text{N} (\uparrow)$$

Reaction due to total gyroscopic couple on each inner wheel:

$$C_g = 0.16\ v^2 N\ (\downarrow)$$

(ii) Reaction due to centrifugal couple:

Centrifugal force: $$F_c = \frac{mv^2}{R} = \frac{1500 \times v^2}{100} = 15v^2\ \text{N}$$

Overturning couple due to centrifugal force:

$$C_c = F_c \times h$$

$$= 15\ v^2 \times 0.45 = 6.75\ v^2\ \text{Nm}$$

Vertical downward reaction on each inner wheel is:

$$R_c = \frac{C_c}{2b} = \frac{6.75v^2}{2 \times 1.5} = 2.25v^2\ \text{N}\ (\downarrow)$$

(iii) Reaction due to weight of the vehicle:

$$R_w = \frac{mg}{4} = \frac{1500 \times 9.81}{4} = 3678.75\ \text{N}\ (\uparrow)$$

The limiting condition to avoid lifting of inner wheels from the road surface is:

$$R_i = R_w - R_c - R_g > 0$$

Or

$$R_w > R_c + R_g$$

$$3678.75 \geq 2.25v^2 + 0.16\ v^2$$

$$v = 39.07\ \text{m/s}\quad \text{or}\quad \mathbf{140.65\ kmph}$$

Or

Force Analysis

Static Force Analysis

Introduction

A machine is a device that performs work and, as such, transmits energy by means mechanical force from a power source to a driven load. It is necessary in the design machine mechanisms to know the manner in which forces are transmitted from input to the output, so that the components of the machine can be properly size withstand the stresses that are developed. If the members are not designed to strong enough, then failure will occur during machine operation; if, on the other hand, the machine is over designed to have much more strength than required, then the machine may not be competitive with others in terms of cost, weight, size, power requirements, or other criteria. The bucket load and static weight loads may far exceed any dynamic loads due to accelerating masses, and a static-force analysis would be justified. An analysis that includes inertia effects is called a dynamic-force analysis and will be discussed in the next chapter. An example of an application where a dynamic-force analysis would be required is in the design of an automatic sewing machine, where, due to high operating speeds, the inertia forces may be greater than the external loads on the machine.

Another assumption deals with the rigidity of the machine components. No material is truly rigid, and all materials will experience significant deformation if the forces, either external or inertial in nature, are great enough. It will be assumed in this chapter and the next that deformations are so small as to be negligible and, therefore, the members will be treated as though they are rigid. The subject of mechanical vibrations, which is beyond the scope of this book, considers the flexibility of machine components and the resulting effects on machine behaviour. A third major assumption that is often made is that friction effects are negligible. Friction is inherent in all devices, and its degree is dependent upon many factors, including types of bearings, lubrication, loads, environmental conditions, and so on. Friction will be neglected in the first few sections of this chapter, with an introduction to the subject presented. In addition to assumptions of the types discussed above, other assumptions may be necessary, and some of these will be addressed at various points throughout the chapter.

The first part of this chapter is a review of general force analysis principles and will also establish some of the convention and terminology to be used in succeeding sections. The remainder of the chapter will then present both graphical and analytical methods for static-force analysis of machines.

Free-Body Diagrams:

Engineering experience has demonstrated the importance and usefulness of free-body diagrams in force analysis. A free-body diagram is a sketch or drawing of part or all of a system, isolated in order to determine the nature of forces acting on that body. Sometimes a free-body diagram may take the form of a mental picture; however, actual sketches are strongly recommended, especially for complex mechanical systems.

Generally, the first, and one of the most important, steps in a successful force analysis is the identification of the free bodies to be used. Figures 5.1B through 5.1E show examples of various free bodies that might be considered in the analysis of the four-bar linkage shown in Figure 5.1A. In Figure 5.1B, the free body consists of the three moving members isolated from the frame; here, the forces acting on the free body include a driving force or torque, external loads, and the forces transmitted:

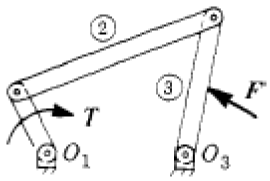


Figure 5.1(A) A four-bar linkage.

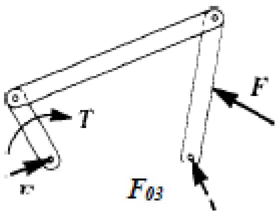


Figure 5.1(B) Free-body diagram of the three moving links

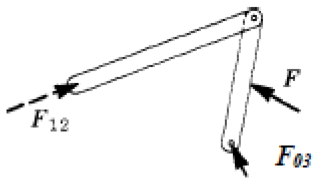


Figure 5.1(C) Free-body diagram of two connected links

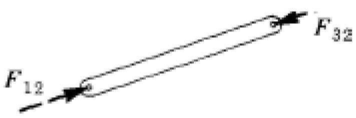


Figure 5.1(D) Free-body diagram of a single link

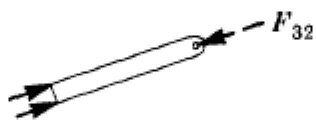


Figure 5.1(E) Free body diagram of part of a link.

► *5.1.2 Static Equilibrium:*

For a free body in static equilibrium, the vector sum of all forces acting on the body must be zero and the vector sum of all moments about any arbitrary point must also be zero. These conditions can be expressed mathematically as follows:

$$\sum F = 0 \qquad (5.1A)$$

$$\sum T = 0 \qquad (5.1B)$$

Since each of these vector equations represents three scalar equations, there are a total of six independent scalar conditions that must be satisfied for the general case of equilibrium under three-dimensional loading.

There are many situations where the loading is essentially planar; in which case, forces can be described by two-dimensional vectors. If the xy plane designates the plane of loading, then the applicable form of Eqs. 5.1A and 5.1B is:-

$$\sum F_x = 0 \qquad (5.2A)$$

$$\sum F_y = 0 \qquad (5.2B)$$

$$\sum T_z = 0 \qquad (5.2C)$$

Eqs. 5.2A to 5.2C are three scalar equations that state that, for the case of two-dimensional xy loading, the summations of forces in the x and y directions must individually equal zero and the summation of moments about any arbitrary point in the plane must also equal zero. The remainder of this chapter deals with two-dimensional force analysis. A common example of three-dimensional forces is gear forces.

5.1.3 Superposition:

The principle of superposition of forces is an extremely useful concept, particularly in graphical force analysis. Basically, the principle states that, for linear systems, the net effect of multiple loads on a system is equal to the superposition (i.e., vector summation) of the effects of the individual loads considered one at a time. Physically, linearity refers to a direct proportionality between input force and output force. Its mathematical characteristics will be discussed in the section on analytical force analysis. Generally, in the absence of Coulomb or dry friction, most mechanisms are linear for force analysis purposes, despite the fact that many of these mechanisms exhibit very nonlinear motions. Examples and further discussion in later sections will demonstrate the application of this principle.

5.1.4 Graphical Force Analysis:

Graphical force analysis employs scaled free-body diagrams and vector graphics in the determination of unknown machine forces. The graphical approach is best suited for planar force systems. Since forces are normally not constant during machine motion. Analyses may be required for a number of mechanism positions; however, in many cases, critical maximum-force positions can be identified and graphical analyses performed for these positions only. An important advantage of the graphical approach is that it provides useful insight as to the nature of the forces in the physical system.

This approach suffers from disadvantages related to accuracy and time. As is true of any graphical procedure, the results are susceptible to drawing and measurement errors. Further, a great amount of graphics time and effort can be expended in the iterative design of a machine mechanism for which fairly thorough knowledge of force-time relationships is required. In recent years, the physical insight of the graphics approach and the speed and accuracy inherent in the computer-based analytical approach have been brought together through computer graphics systems, which have proven to be very effective engineering design tools. There are a few special types of member loadings that are repeatedly encountered in the force analysis of mechanisms, These include a member subjected to two forces, a member subjected to three forces, and a member subjected to two forces and a couple. These special cases will be considered in the following paragraphs, before proceeding to the graphical analysis of complete mechanisms.

► *5.2.1 Analysis of a Two-Force Member:*

A member subjected to two forces is in equilibrium if and only if the two forces (1) have the same magnitude, (2) act along the same line, and (3) are opposite in sense. Figure 5.2A shows a free-body diagram of a member acted upon by forces F_1 and F_2 where the points of application of these forces are points A and B. For equilibrium the directions of F_1 and F_2 must be along line AB and F_1 must equal $-F_2$ graphical vector addition of forces F_1 and F_2 is shown in Figure 5.2B, and, obviously, the resultant net force on the member is zero when $F_1 = -F_2$. The resultant moment about any point will also be zero.

Thus, if the load application points for a two-force member are known, the line of action of the forces is defined, and it the magnitude and sense of one of the forces are known, then the other force can immediately be determined. Such a member will either be in tension or compression.

Figure 5.2(A) A two-force member. The resultant force and the resultant moment both equal Zero.

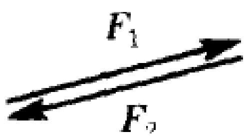


Figure 5.2(B) Force summation for a two-force member

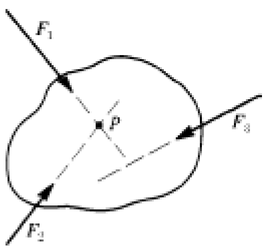
► *5.2.2 Analysis of a Three-Force Member:*

A member subjected to three forces is in equilibrium if and only if (1) the resultant of the three forces is zero, and (2) the lines of action of the forces all intersect at the same point. The first condition guarantees equilibrium of forces, while the second condition guarantees equilibrium of moments. The second condition can be under-stood by considering the case when it is not satisfied. See Figure 5.3A. If moments are summed about point *P*, the intersection of forces F_1 and F_2, then the moments of these forces will be zero, but F_3 will produce a nonzero moment, resulting in a nonzero net moment on the member. On the other hand, if the line of action of force F_3 also passes through point *P* (Figure 5.3B), the net moment will be zero. This common point of intersection of the three forces is called the point of concurrency.

A typical situation encountered is that when one of the forces, F_1, is known completely, magnitude and direction, a second force, F_2, has known direction but unknown magnitude, and force F_3 has unknown magnitude and direction. The graphical solution of this case is depicted in Figures 5.4A through 5.4C. First, the free-body diagram is drawn to a convenient scale and the points of application of the three forces are identified. These are points *A, B,* and *C*. Next, the known force F_1 is drawn on the diagram with the proper direction and a suitable magnitude scale. The direction of force F_2 is then drawn, and the intersection of this line with an extension of the line of action of force F_1 is the concurrency point *P*. For equilibrium, the line of action of force F_3 must pass through points *C* and *P* and is therefore as shown in Figure 5.4A.

The force equilibrium condition states that

$$F_1 + F_2 + F_3 = 0$$



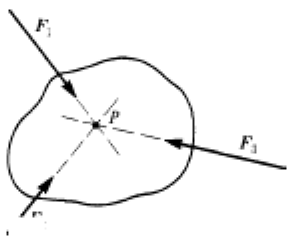
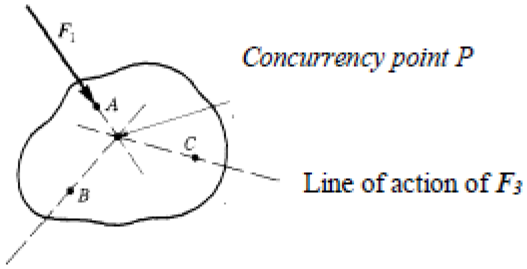


Figure 5.3(B) The three forces intersect at the same point *P*, called the *concurrency point,* and the net moment is zero.

Figure 5.3(A) The three forces on the member do not intersect at a common point and there is a nonzero resultant moment.

Since the directions of all three forces are now known and the magnitude of F_1 were given, this equation can be solved for the remaining two magnitudes. A graphical Solution follows from the fact that the three forces must form a closed vector loop, called a force polygon. The procedure is shown in Figure 5.4B. Vector $_1F$ is redrawn. From the head of this vector, a line is drawn in the direction of force F_2, and from the tail, a line is drawn parallel to F_3. The intersection of these lines closes the vector loop and determines the magnitudes of forces $_2F$ and F_3. Note that the same solution is obtained if, instead, a line parallel to $_3F$ is drawn from the head of F_1, and a line parallel to F_2 is drawn from the tail of F_1. See Figure 5.4C.

Figure 5.4(A) Graphical force analysis of a three- force member.



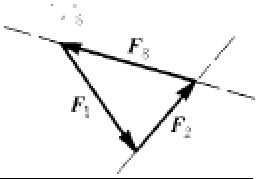


Figure 5.4(B) Force polygon for the three forces member.

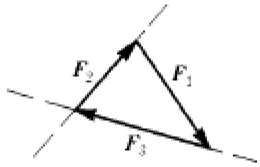


Figure 5.4(C) An equivalent force polygon for the three force member

This is so because vector addition is commutative, and, therefore, both force polygons are equivalent to the vector equation above. It is important to remember that, by the definition of vector addition, the force polygon corresponding to the general force equation

$$\sum F = 0$$

Will have adjacent vectors connected head to tail. This principle is used in identifying the sense of forces F_2 and F_3 in Figures 5.4B and 5.4C. Also, if the lines of action of F_1 and F_2 are parallel," then the point of concurrency is at infinity, and the third force F_3 must be parallel to the other two. In this case, the force polygon collapses to a straight line.

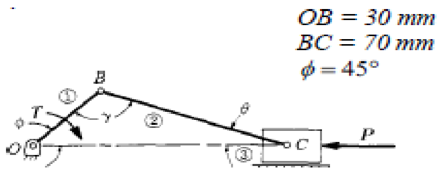
► 5.3.1 Graphical Force Analysis of the Slider Crank Mechanism:

The slider crank mechanism finds extensive application in reciprocating compressors, piston engines, presses, toggle devices, and other machines where force characteristics are important. The force analysis of this mechanism employs most of the principles described in previous sections, as demonstrated by the following example.

▼ EXAMPLE 5.1

Static-force analysis of a slider crank mechanism is discussed. Consider the slider crank linkage shown in Figure 5.5A, representing a compressor, which is operating at so low a speed that inertia effects are negligible. It is also assumed that gravity forces are small compared with other forces and that all forces lie in the same plane. The dimensions are OB = 30 mm and BC = 70 mm, we wish to find the required crankshaft torque T and the bearing forces for a total gas pressure force P = 40N at the instant when the crank angle $\phi = 45°$.

Figure 5.5(A) Graphical force analysis of a slider crank mechanism, which is acted on by piston force *P* and crank torque *T*



SOLUTION

The graphical analysis is shown in Figure 5.5B. First, consider connecting rod 2. In the absence of gravity and inertia forces, this link is acted on by two forces only, at pins B and C. These pins are assumed to be frictionless and, therefore, transmit no torque. Thus, link 2 is a two-force member loaded at each end as shown. The forces F_{12} and F_{32} lie along the link, producing zero net moment, and must be equal and opposite for equilibrium of the link. At this point, the magnitude and sense of these forces are unknown.

Next, examine piston 3, which is a three-force member. The pressure force *P* is completely known and is assumed to act through the center of the piston (i.e., the pressure distribution on the piston face is assumed to be symmetric). From Newton's third law, which states that for every action there is an equal and opposite reaction, it follows that $F_{23} = -F_{32}$, and the direction of F_{23} is therefore known. In the absence of friction, the force of the cylinder on the piston, F_{03}, is perpendicular to the cylinder wall, and it also must pass through the concurrency point, which is the piston pin *C*. Now, knowing the force directions, we can construct the force polygon for member 3 (Figure 5.5B). Scaling from this diagram, the contact force between the cylinder and piston is $F_{03} = 12.70N$, acting upward, and the magnitude of the bearing force at *C* is $F_{23} = F_{32} = 42.0N$. This is also the bearing force at crankpin *B*, because $F_{12} = -F_{32}$. Further, the force directions for the connecting rod shown in the figure are correct, and the link is in compression.

Finally, crank 1 is subjected to two forces and a couple *T* (the shaft torque *T* is assumed to be a couple). The force at *B* is $F_{12} = -F_{21}$ and is now known. For force equilibrium, $F_{01} = -F_{21}$ as shown on the free-body diagram of link 1. However these forces are not collinear, and for equilibrium, the moment of this couple must be balanced by torque *T*. Thus, the required torque is clockwise and has magnitude

$$T = F_{21}h = (42.0N)(26.6mm) = 1120N.mm = 1.120N.m$$

It should be emphasized that this is the torque required for static equilibrium in the position shown in Figure 9.10A. If torque information is needed for a complete compression cycle, then the analysis must be repeated at other crank positions throughout the cycle. In general, the torque will vary with position.

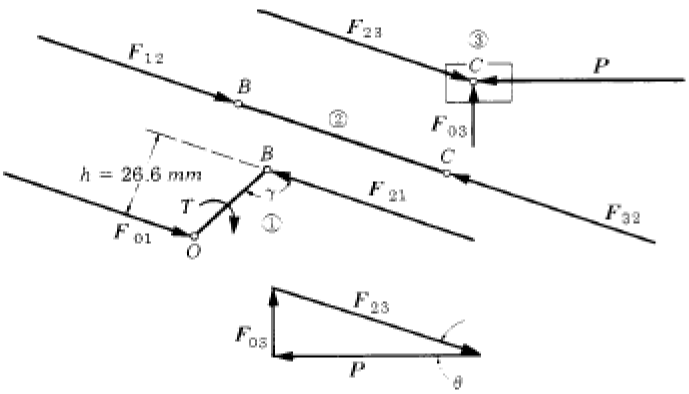


Figure 5.5(B) Static force balances for the three moving links, each considered as a free body

► *5.3.1 Graphical Force Analysis of the Four-Bar Linkage:*

The force analysis of the four-bar linkage proceeds in much the same manner as that of the slider crank mechanism. However, in the following example, we will consider the case of external forces on both the coupler and follower links and will utilize the principle of superposition.

▼ EXAMPLE 5.2

Static-force analysis of a four-bar linkage is considered. The link lengths for the four-bar linkage of Figure 5.6 A are given in the figure. In the position shown, coupler link 2 is subjected to force F_2 of magnitude *47 N*, and follower link 3 is subjected to force F_3, of magnitude *30 N*. Determine the shaft torque *Ti* on input link1 and the bearing loads for static equilibrium.

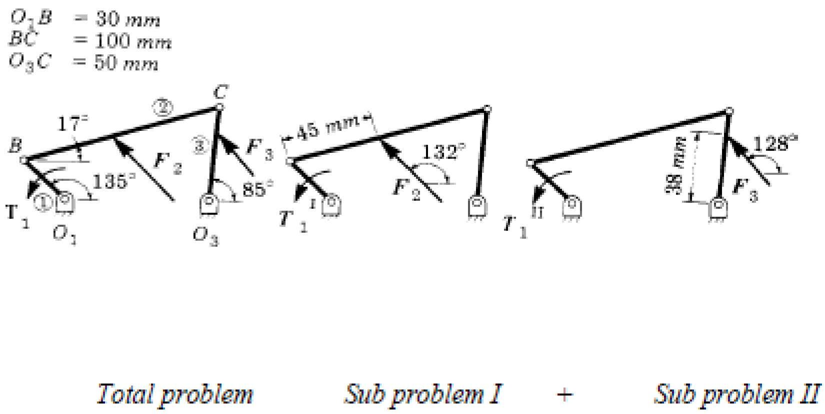


Figure 5.6(A) Graphical force analysis of a four-bar linkage, utilizing the principle of the superposition

SOLUTION

As shown in Figure 5.6A, the solution of the stated problem can be obtained by superposition of the solutions of sub problems *I* and *II*. In sub problem *I*, force F_3 is neglected, and in sub problem *II*, force F_2 is neglected. This process facilitates the solution by dividing a more difficult problem into two simpler ones.

The analysis of sub problem *I* is shown in Figure 5.6B, with quantities designated by superscript *I*. Here, member 3 is a two-force member because force F_3 is neglected. The direction of forces F_{23}^{1} and F_{03}^{1} are as shown, and the forces are equal and opposite (note that the magnitude and sense of these forces are as yet unknown), This information allows the analysis of member 2, which is a three-force member with completely known force F_2, known direction for F_{32}^{1}, and, using the concurrency point, known direction for F_{12}^{1}. Scaling from the force polygon, the following force magnitudes are determined (the force directions are shown in Figure (5.6B):

$$F_{32}^{1} = F_{23}^{1} = F_{03}^{1} = 21.0N \qquad F_{12}^{1} = F_{21}^{1} = 36N$$

Link 1 is subjected to two forces and couple $T_1^{\,1}$, and for equilibrium,

$$F_{03}^{11} = 29.0N \qquad F_{23}^{11} = F_{21}^{11} = F_{01}^{11}$$

And; $T_1^{\,1} = F_{21}^{1} h^{1} = (36N)(11mm) = 396N.mm \quad CW$

The analysis of sub problem *II* is very similar and is shown in Figure 5.6C, where superscript II is used. In this case, link 2 is a two-force member and link 3 is a three-force member, and the following results are obtained:

$$F_{03}^{11} = 29N \qquad F_{23}^{11} = F_{21}^{11} = F_{01}^{11} = 17N$$

And; $T_1^{\,11} = F_{21}^{11} h^{11} = (17N)(26mm) = 442N.mm \quad CW$

The superposition of the results of Figures 5.6B and 5.6C is shown in Figure 5.6D. The results must be added vectorially, as shown. By scaling from the free-body diagrams, the overall bearing force magnitudes are

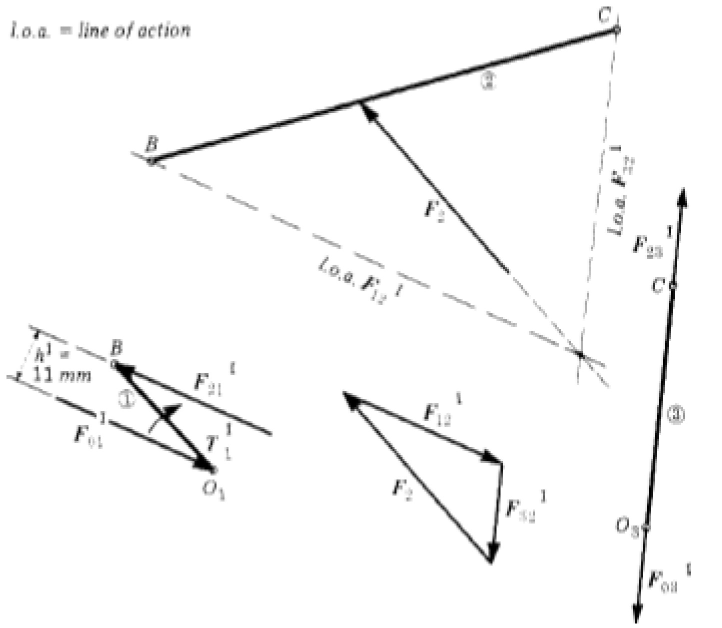


Figure 5.6B The solution of *sub problem I*

$F_{01} = 50N \qquad F_{23} = 31N$

$F_{12} = 50N \qquad F_{03} = 49N$

And the net crankshaft torque is

$$T_1 = T_1^{I} + T_1^{II} = 396N\ mm + 442N\ mm = 838N\ mm \quad CW$$

The directions of the bearing forces are as shown in the figure. These resultant quantities represent the actual forces experienced by the mechanism. It can be seen from the analysis that the effect of the superposition principle, in this example, was to create sub problems containing two-force members, from which the separate analyses could begin. In an attempt of a graphical analysis of the original problem without superposition, there is not enough intuitive force information to analyze three-force members 2 and 3, because none of the bearing force directions can be determined by inspection.

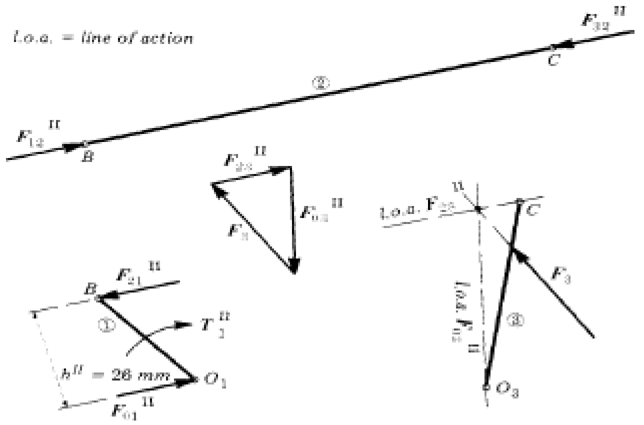


Figure 5.6C
The solution of
sub problem II

Dynamic Force Analysis

► 5.4.1 D'Alembert's Principle and Inertia Forces:

An important principle, known as d'Alembert's principle, can be derived from Newton's second law. In words, d'Alembert's principle states that *the reverse-effective forces and torques and the external forces and torques on a body together give statical equilibrium*.

$$F + (-ma_G) = 0 \qquad (5.3A)$$

$$T_{eG} + (-I_G \alpha) = 0 \qquad (5.3B)$$

The terms in parentheses in Eqs. 5.3A and 5.3B are called the reverse-effective force and the reverse-effective torque, respectively. These quantities are also referred to as inertia force and inertia torque. Thus, we define the inertia force F_i as

$$F_i = -ma_G \qquad (5.4A)$$

This reflects the fact that a body resists any change in its velocity by an inertia force proportional to the mass of the body and its acceleration. The inertia force acts through the center of mass G of the body. The inertia torque or inertia couple C_i is given by:

$$C_i = -I_G \alpha \qquad (5.4B)$$

As indicated, the inertia torque is a pure torque or couple. From Eqs. 5.4A and 5.4B, their directions are opposite to that of the accelerations. Substitution of Eqs. 5.4A and 5.4B into Eqs, 5.3A and 5.3B leads to equations that are similar to those used for static-force analysis:

$$\sum F = \sum F_e + F_i = 0 \qquad (5.5A)$$

$$\sum T_G = \sum T_{eG} + C_i = 0 \qquad (5.5B)$$

Where $\sum F$ refers here to the summation of external forces and, therefore, is the resultant external force, and $\sum T_{eG}$ is the summation of external moments, or resultant external moment, about the center of mass G. Thus, the dynamic analysis problem is reduced in form to a static force and moment balance where inertia effects are treated in the same manner as external forces and torques. In particular for the case of assumed mechanism motion, the inertia forces and couples can be determined completely and thereafter treated as known mechanism loads.

Furthermore, d'Alembert's principle facilitates moment summation about any arbitrary point P in the body, if we remember that the moment due to inertia force F_i must be included in the summation. Hence,

$$\sum T_P = \sum T_{eP} + C_i + R_{PG} \times F_i = 0 \qquad (5.5C)$$

Where; $\sum T_P$ is the summation of moments, including inertia moments, about point P. $\sum T_{eP}$ is the summation of external moments about P, C_i is the inertia couple defined by Eq. 5.4B, F_i is the inertia force defined by Eq. 5.4A, and R_{PG} is a vector from point P to point C. It is clear that Eq. 5.5B is the special case of Eq.5.5C, where point P is taken as the center of mass G (i.e., $R_{PG} = 0$).

For a body in plane motion in the xy plane with all external forces in that plane. Eqs. 5.5A and 5.5B become:

$$\sum F_x = \sum F_{ex} + F_{ix} = \sum F_{ex} + (-ma_{Gx}) = 0 \quad (5.6A)$$

$$\sum F_y = \sum F_{ey} + F_{iy} = \sum F_{ey} + (-ma_{Gy}) = 0 \quad (5.6B)$$

$$\sum T_G = \sum T_{eG} + C_i = \sum T_{eG} + (-I_G \alpha) = 0 \quad (5.6C)$$

Where a_{Gx} and a_{Gy} are the x and y components of a_G. These are three scalar equations, where the sign convention for torques and angular accelerations is based on a right-hand xyz coordinate system; that is. Counterclockwise is positive and clockwise is negative. The general moment summation about arbitrary point P, Eq. 5.5C, becomes:

$$\sum T_P = \sum T_{eP} + C_i + R_{PGx} F_{iy} - R_{PGy} F_{ix}$$
$$= \sum T_{eP} + (-I_G \alpha) + R_{PGx}(-ma_{Gy}) - R_{PGy}(-ma_{Gx}) = 0 \quad (5.6D)$$

Where R_{PGx} and R_{PGy} are the x and y components of position vector R_{PG}. This expression for dynamic moment equilibrium will be useful in the analyses to be presented in the following sections of this chapter.

► *5.4.2 Equivalent Offset Inertia Force:*

For purposes of graphical plane force analysis, it is convenient to define what is known as the equivalent offset inertia force. This is a single force that accounts for both translational inertia and rotational inertia corresponding to the plane motion of a rigid body. Its derivation will follow, with reference to Figures 5.7A through 5.7D.

Figure 5.7A shows a rigid body with planar motion represented by center of mass acceleration a_C and angular acceleration α. The inertia force and inertia torque associated with this motion are also shown. The inertia torque $-I_G\alpha$ can be expressed as a couple consisting of forces Q and (- Q) separated by perpendicular

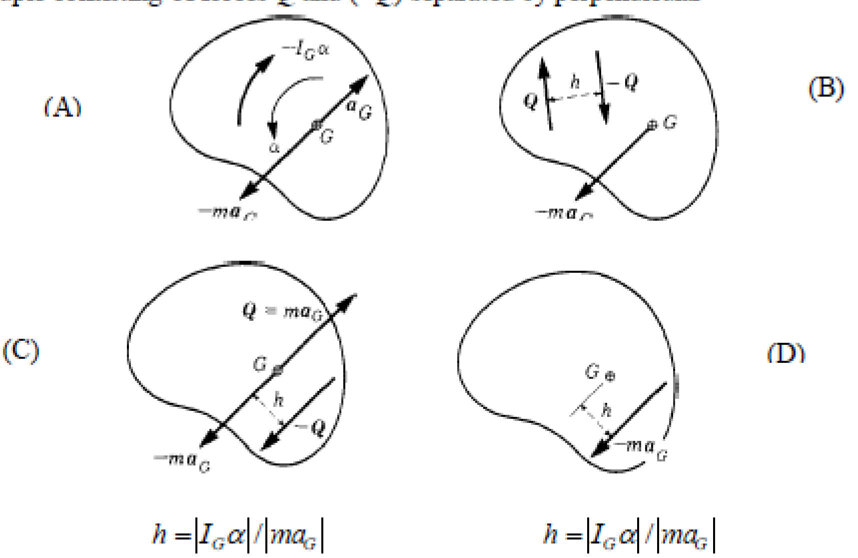


Figure 5.7 (A) Derivation of the equivalent offset inertia force associated with planer motion of a rigid body. (B) Replacement of the inertia torque by a couple. (C) The strategic choice of a couple. (D) The single force is equivalent to the combination of a force and a torque in figure 5.7(A)

Distance h, as shown in Figure 5.7B. The necessary conditions for the couple to be equivalent to the inertia torque are that the sense and magnitude be the same. Therefore, in this case, the sense of the couple must be clockwise and the magnitudes of Q and h must satisfy the relationship

$$|Q.h| = |I_G.\alpha|$$

Otherwise, the couple is arbitrary and there are an infinite number of possibilities that will work. Furthermore, the couple can be placed anywhere in the plane.

Figure 5.7C shows a special case of the couple, where force vector Q is equal to ma_G and acts through the center of mass. Force $(-Q)$ must then be placed as shown to produce a clockwise sense and at a distance;

$$h = \frac{|I_G \alpha|}{|Q|} = \frac{|I_G \alpha|}{|ma_G|} \qquad (5.7)$$

Force Q will cancel with the inertia force $F_i = -ma_G$, leaving the single equivalent offset force shown in Figure 5.7D, which has the following characteristics:

1. The magnitude of the force is $|ma_G|$.
2. The direction of the force is opposite to that of acceleration α.
3. The perpendicular offset distance from the center of mass to the line of action of the force is given by Eq. 5.7.
4. The force is offset from the center of mass so as to produce a moment about the center of mass that is opposite in sense to acceleration a.

The usefulness of this approach for graphical force analysis will be demonstrated in the following section. It should be emphasized, however, that this approach is usually unnecessary in analytical solutions, where Eqs. 5.6A to 5.6D. Including the original inertia force and inertia torque, can be applied directly.

► *5.4.3 Dynamic Analysis of the Four-Bar Linkage:*

The analysis of a four-bar linkage will effectively illustrate most of the ideas that have been presented; furthermore, the extension to other mechanism types should become clear from the analysis of this mechanism.

▼ EXAMPLE 5.3

The four-bar linkage shown in Figure 5.8A has the dimensions shown in the figure where G refers to center of mass, and the mechanism has the following mass properties:

$$m_1 = 0.10kg \qquad I_{G1} = 20kg.mm^2$$
$$m_2 = 0.20kg \qquad I_{G2} = 400kg.mm^2$$
$$m_3 = 0.30kg \qquad I_{G3} = 20kg.mm^2$$

Determine the instantaneous value of drive torque T required to produce an assumed motion given by input angular velocity $\omega = 95 rad/s$ counterclockwise and input angular acceleration $a_1 = 0$ for the position shown in the figure. Neglect gravity and friction effects.

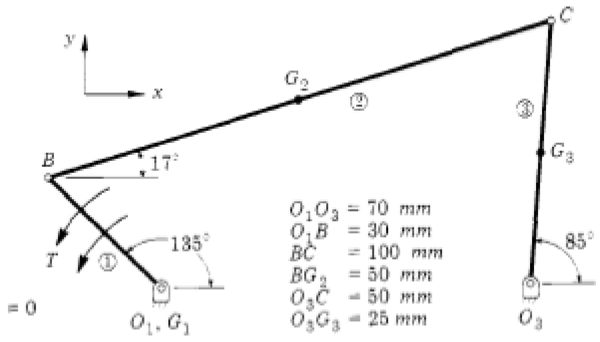


Figure 5.8(A) The four-bar linkage of Example 5.3

SOLUTION

This problem falls in the first analysis category that is given the mechanism motion, determine the resulting bearing forces and the necessary input torque. Therefore, the first step in the solution process is to determine the inertia forces and inertia torques. Thereafter, the problem can be treated as though it were a static-force analysis problem.

Kinematics analysis of the mechanism can be accomplished by using any of the methods presented in earlier chapters. Figure 5.8B shows a graphical analysis employing velocity and acceleration polygons. From the analysis, the following accelerations are determined:

$$a_{C1} = 0(Stationary\ Center\ of\ mass) \qquad \alpha_1 = 0(given)$$
$$a_{C2} = 235{,}000\angle 312°mm/Sec^2 \qquad \alpha_2 = 520 rad/s^2 \quad ccw$$
$$a_{C3} = 235{,}000\angle 308°mm/Sec^2 \qquad \alpha_3 = 2740 rad/s^2 \quad cw$$

Where the angles of the acceleration vectors are measured counterclockwise from the positive x direction shown in Figure 5.8A. From Eqs. 5.4A and 5.4B, the inertia forces and inertia torques are;

$$F_{i1} = 0$$
$$F_{i2} = -m_2 a_{G2} = 47{,}000\angle 132° kg.mm/s^2 = 47\angle 132° N$$
$$F_{i3} = -m_3 a_{G3} = 30{,}000\angle 128° kg.mm/s^2 = 30\angle 132° N$$
$$C_{i1} = 0$$
$$C_{i2} = -I_{G2}\alpha_2 = 208{,}000 kg.mm^2/s^2 cw = 208 N.mm \quad cw$$
$$C_{i3} = -I_{G3}\alpha_3 = 274{,}000 kg.mm^2/s^2 ccw = 274 N.mm \quad ccw$$

The inertia forces have lines of action through the respective centers of mass, and the inertia torqueses are pure couples.

The inertia forces have lines of action through the respective centres of mass, and the inertia torques are pure couples.

Velocity polygon

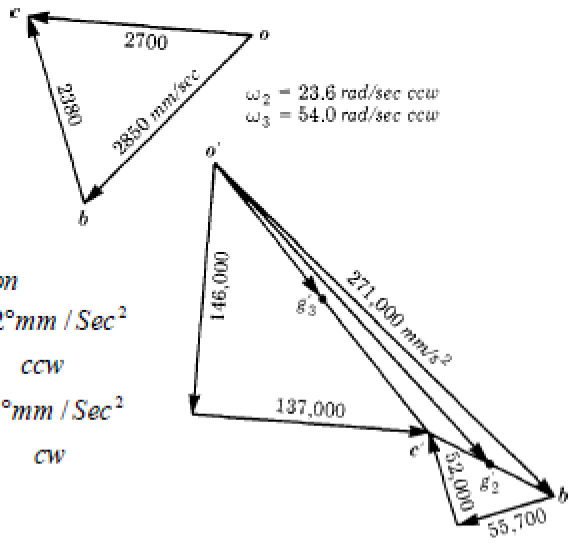


Figure 5.8(B) the velocity and acceleration analysis necessary for determination of inertia forces and inertia

Acceleration polygon

$a_{G2} = 235{,}000\angle 312° mm/Sec^2$

$\alpha_2 = 520 rad/Sec \quad ccw$

$a_{G3} = 100{,}000\angle 308° mm/Sec^2$

$\alpha = 2740 rad/Sec \quad cw$

GRAPHICAL SOLUTION

In order to simplify the graphical force analysis, we will account for the inertia torques by introducing equivalent offset inertia forces. These forces are shown in Figure 2.8C, and their placement is determined according to the previous section. For link 2, the offset force F_2 is equal and parallel to inertia force F_{12}. Therefore,

$$F_2 = 47\angle 132° N$$

It is offset from the center of mass G_2 by a perpendicular amount equal to

$$h_2 = \frac{|I_{G2}\alpha_2|}{|m_2 a_{G2}|} = \frac{208}{47} = 4.43mm$$

And this offset is measured to the left as shown to produce the required clockwise direction for the inertia moment about point G_2. In a similar manner, the equivalent offset inertia force for link 3 is

$$F_3 = 30\angle 128° N \quad \text{at an offset distance} \quad h_3 = \frac{|I_{G3}\alpha_3|}{|m_3 a_{G3}|} = \frac{274}{30} = 9.13mm$$

Where this offset is measured to the right from G_3 to produce the necessary counterclockwise inertia moment about G_3. From the values of h_2 and h_3 and the angular relationships, the force positions r_2 and r_3 in Figure 5.8C are computed to be

$$r_2 = BG_2 - \frac{h_2}{\cos(132° - 17° - 90°)} = 45.10mm$$

$$r_3 = O_3G_3 + \frac{h_3}{\cos(90° + 85° - 128°)} = 38.40mm$$

Now, we wish to perform a graphical force analysis for known forces F_2 and F_3. This has been done in Example Problem 9.2, and the reader is referred to that

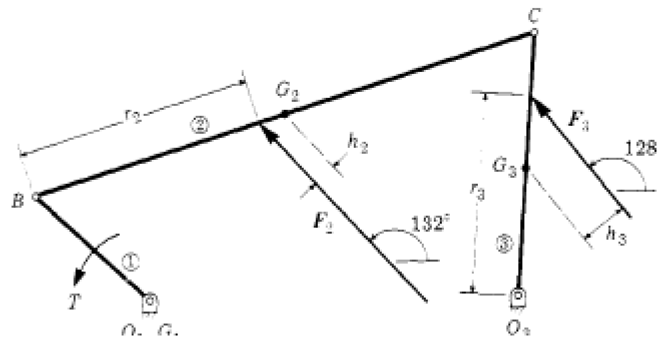


Figure 5.8(C) Equivalent offset inertia forces for members 2 and 3

Analysis. The required input torque was found to be $T = 383N.mm$ *cw*

ANALYTICAL SOLUTION

Having determined the equivalent offset inertia forces F_2 and F_3 the analytical solution could proceed according to Example Problem 9, 6, which examined the same problem. However, it is not necessary to convert to the offset force, and here we will carry out the analytical solution in terms of the original inertia forces and inertia couples.

Figure 5.8D shows the linkage with the inertia torques and the inertia forces in *xy* coordinate form. Consistent with Figure 9.15A, we define the following quantities:

$\ell_1 = 30mm \qquad \ell_2 = 100mm \qquad \ell_3 = 50mm$

$\phi_1 = 135° \qquad \phi_2 = 17° \qquad \phi_3 = 85°$

$r_1 = 0 \qquad r_2 = 50mm \qquad r_3 = 25mm$

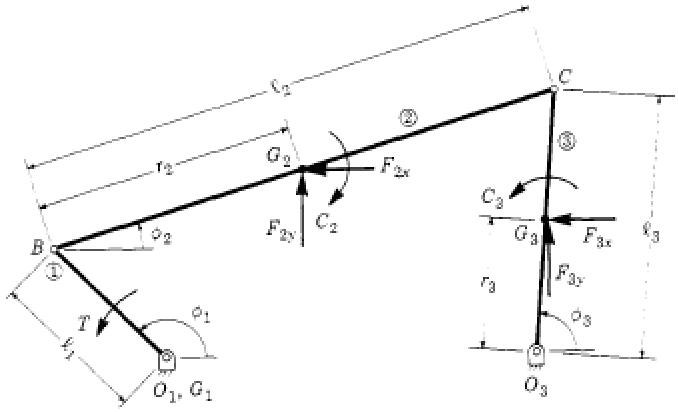
$F_{2x} = 47\cos(132°) = -31.40N \qquad F_{2y} = 47\sin(132°) = 34.90N$

$F_{3x} = 30\cos(128°) = -18.50N \qquad F_{3y} = 30\sin(128°) = 23.60N$

$C_2 = -208N.mm \qquad C_3 = 274N.mm$

$F_{1x} = F_{1y} = C_1 = 0$

Figure 5.8(D) Combinations of inertia forces and inertia torques for members 2 and 3



Where the differences are due to round off:

$a_{11} = -49.8 \quad a_{21} = 29.2 \quad b_1 = -786$

$a_{12} = 4.36 \quad a_{22} - 95.6 \quad b_2 = -1920$

Then, $F_{23} = 31.30N \quad F_{12} = 50.30N$

$F_{03} = 49.20N \quad F_{01} = 50.30N$

And $T = -851N.mm$

Thus, it can be seen that the general analytical solution of the four-bar linkage presented in this Chapter for static-force analysis is equally well suited for dynamic-force analysis. Before leaving this example, a couple of general comments should be made.

First, the torque determined is the instantaneous value required for the prescribed motion, and the value will vary with position. Furthermore, for the position considered, the torque is opposite in direction to the angular velocity of the crank. This can be explained by the fact that the inertia of the mechanism in this position is tending to accelerate the crank in the counterclockwise direction, and, therefore, the required torque must be clockwise to maintain a constant angular speed. If a constant speed is to be maintained throughout the mechanism cycle, then there will be other positions of the mechanism for which the required torque will be counterclockwise. The second comment is that it may be impossible to find a mechanism actuator, such as an electric motor, that will supply the required torque versus position behavior. This problem can be alleviated, however, in the case of a "constant" rotational speed mechanism through the use of a device called a flywheel, which is mounted on the input shaft and produces a relatively large mass moment of inertia for crank 1. The flywheel can absorb mechanism torque and energy- variations with minima] speed fluctuation and. thus, maintains an essentially constant input speed. In such a case. The assumed-motion approach to dynamic-force analysis is appropriate.

► *5.4.3 Dynamic Analysis of the Slider-Crank Mechanism:*

Dynamic forces are a very important consideration in the design of slider crank mechanisms for use in machines such as internal combustion engines and reciprocating compressors. Dynamic-force analysis of this mechanism can be carried out in exactly the same manner as for the four-bar linkage in the previous section. Following such a process a kinematics analysis is first performed from which expressions are developed for the inertia force and inertia torque for each of the moving members, These quantities may then be converted to equivalent offset inertia forces for graphical analysis or they may be retained in the form of forces and torques for analytical solution, utilizing, in either case, the methods presented in this chapter. In fact, the analysis of the slider crank mechanism is somewhat easier than that of the four-bar linkage because there is no rotational motion and, in turn, no inertia torque for the piston or slider, which has translating motion only. The following paragraphs will describe an analytical approach in detail.

Figure 5.9A is a schematic diagram of a slider crank mechanism, showing the crank 1, the connecting rod 2, and the piston 3, all of which are assumed to be rigid. The center of mass locations are designated by letter **G**, and the members have masses m, and moments of inertia I_{Gi}, i = 1, 2, 3. The following analysis will consider the relationships of the inertia forces and torques to the bearing reactions and the drive torque on the crank, at an arbitrary mechanism position given by crank angle ϕ Friction will be neglected.

Figure 5.9B shows free-body diagrams of the three moving members of the linkage. Applying the dynamic equilibrium conditions. Eqs. 5.6A to 5.6D, to each member yields the following set of equations. For the piston (moment equation not included):

$$F_{23x} + (-m_3 a_{G3}) = 0 \qquad (5.8A)$$

$$F_{03y} + F_{23y} = 0 \qquad (5.8B)$$

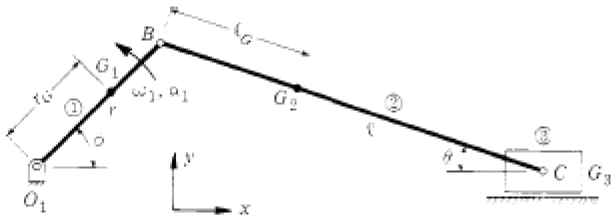


Figure 5.9(A) Dynamic-force analysis of a slider crank mechanism

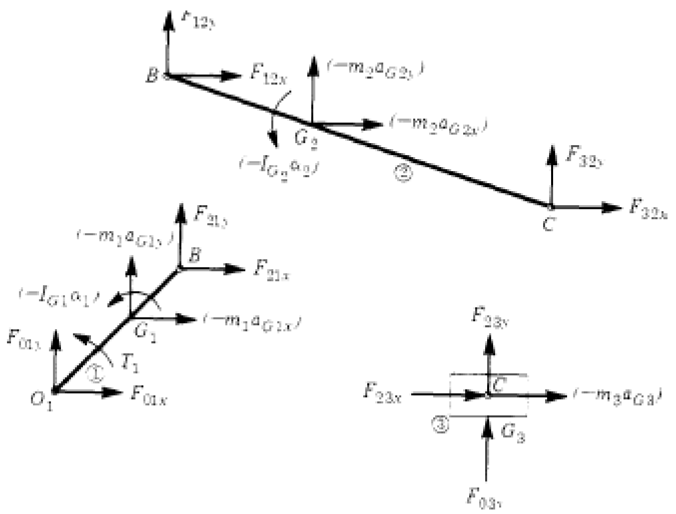


Figure 5.9(B) Free-body diagrams of the moving members

For the connecting rod (moments about point *B*):

$$F_{12x} + F_{32x} + (-m_2 a_{G2x}) = 0 \quad (5.8C)$$

$$F_{12y} + F_{32y} + (-m_2 a_{G2y}) = 0 \quad (5.8D)$$

$$F_{32x} \ell \sin\theta + F_{32y} \ell \cos\theta + (-m_2 a_{G2x}) \ell_G \sin\theta + (-m_2 a_{G2y}) \ell_G \cos\theta + (-I_{G2}\alpha_2) = 0 \quad (5.8E)$$

For the crank (moments about point O_1):

$$F_{01x} + F_{21x} + (-m_1 a_{G1x}) = 0 \quad (5.8F)$$

$$F_{01y} + F_{21y} + (-m_1 a_{G1y}) = 0 \quad (5.8G)$$

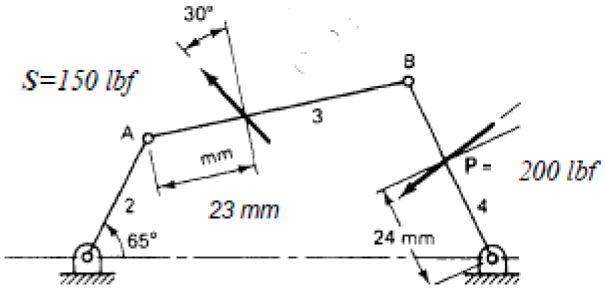
$$T_1 - F_{21x} r \sin\phi + F_{21y} r \cos\phi + (-m_1 a_{G1x}) r_G \sin\phi + (-m_1 a_{G1y}) r_G \cos\phi + (-I_{G1}\alpha_1) = 0 \quad (5.8H)$$

Where *T* is the input torque on the crank. This set of equations embodies both of the dynamic-force analysis approaches described in Newton's Laws. However, its form is best suited for the case of known mechanism motion, as illustrated by the following example.

Question 1:

The four-bar mechanism of Figure has one external force ***P = 200 Ibf*** and one inertia force ***S = 150 Ibf*** acting on it. The system is in dynamic equilibrium as a result of torque T_2 applied to link 2. *Find T_2 and the pin forces*.

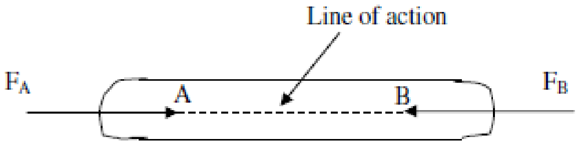
(a) Use the graphical method based on free-body diagrams.



O_2A = 30 mm
AB = 60 mm
O_4B = 45 mm
O_2O_4 = 90 mm

Very useful & important principles.

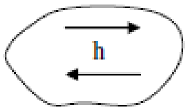
(i) Equilibrium of a body under the action of two forces only (no torque)



For body to the in Equilibrium under the action of 2 forces (only), the two forces must the equal opposite and collinear. The forces must be acting along the line joining A&B.

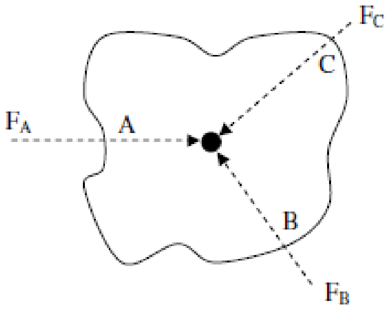
That is,

$F_A = - F_B$ (for equilibrium)



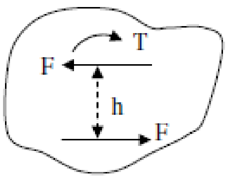
If this body is to be under equilibrium 'h' should tend to zero

(ii) Equilibrium of a body under the action of three forces only (no torque / couple)



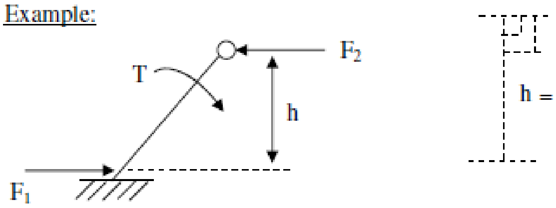
For equilibrium, the 3 forces must be concurrent and the force polygon will be a triangle.

(iii) Equilibrium of a body acted upon by 2 forces and a torque.



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

Example:



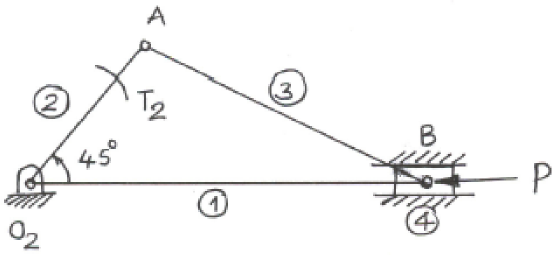
h = Perpendicular distance between F_1 & F_2

Free body diagram

The mass is separated from the system and all the forces acting on the mass are represented.

Problem No.1: Slider crank mechanism

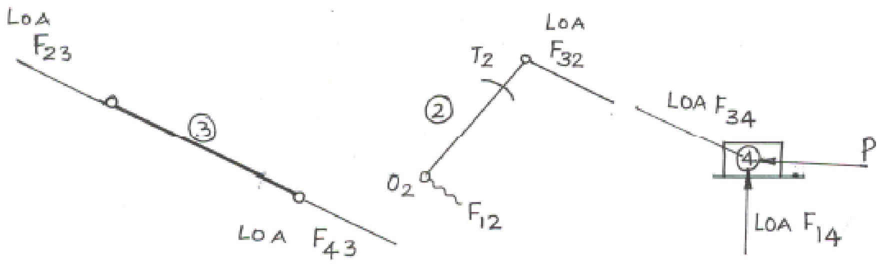
Figure shows a slider crank mechanism in which the resultant gas pressure 8 x 10^4 Nm^{-2} acts on the piston of cross sectional area 0.1 m^2. The system is kept in equilibrium as a result of the couple applied to the crank 2, through the shaft at O_2. Determine forces acting on all the links (including the pins) and the couple on 2.



$$P = (8 \times 10^4) \times (0.1)$$

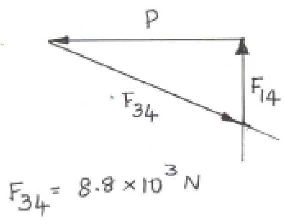
$$= 8 \times 10^3\ N$$

Free body diagram

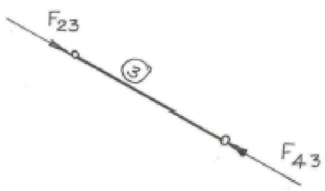


Force triangle for the forces acting on (4) is drawn to some suitable scale.

Magnitude and direction of P known and lines of action of F_{34} & F_{14} known.



Measure the lengths of vectors and multiply by the scale factor to get the magnitudes of F_{14} & F_{34}. Directions are also fixed.

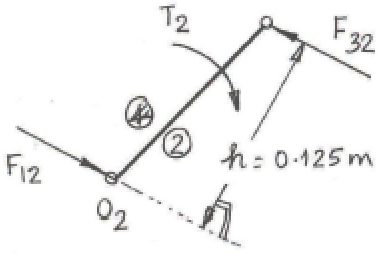


i.e, $F_{23} = -F_{32}$

Since link 3 is acted upon by only two forces, F_{43} and F_{23} are collinear, equal in magnitude and opposite in direction

i.e., $F_{43} = -F_{23} = 8.8 \times 10^3\ N$

Also, $F_{23} = -F_{32}$ (equal in magnitude and opposite in direction).



Link 2 is acted upon by 2 forces and a torque (stated in the problem), for equilibrium the two forces must be equal, parallel and opposite and their sense must oppose T_2.

There fore,

$$F_{32} = -F_{12} = 8.8\times10^3 N$$

F_{32} & F_{12} form a counter clock wise couple of magnitude,

$$(F_{23} \times h) = (F_{12} \times h) = (8.8\times10^3)\times0.125 = 1100 Nm.$$

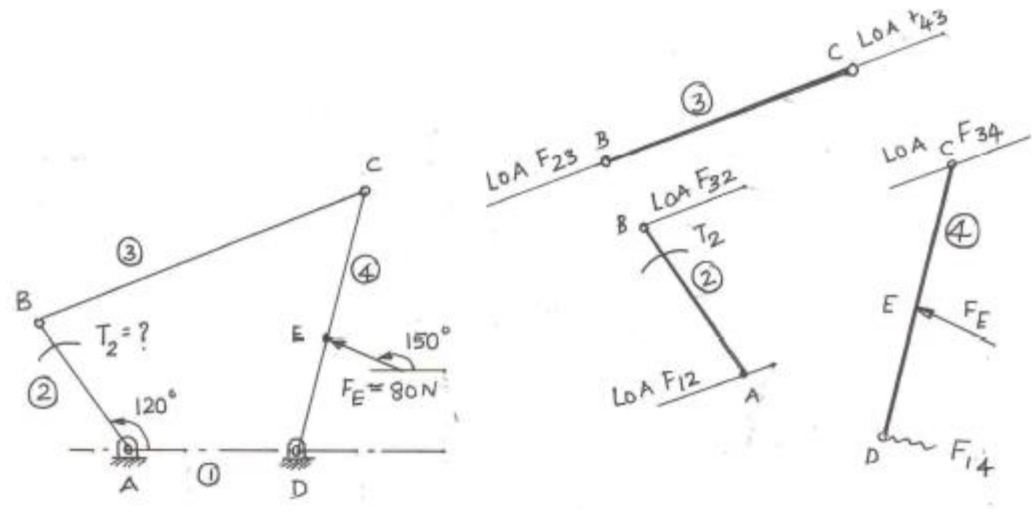
To keep 2 in equilibrium, T_2 should act clockwise and magnitude is 1100 Nm.
Important to note;

i) h is measured perpendicular to F_{32} & F_{12};
ii) always multiply back by scale factors.

Problem No 2. Four link mechanism.

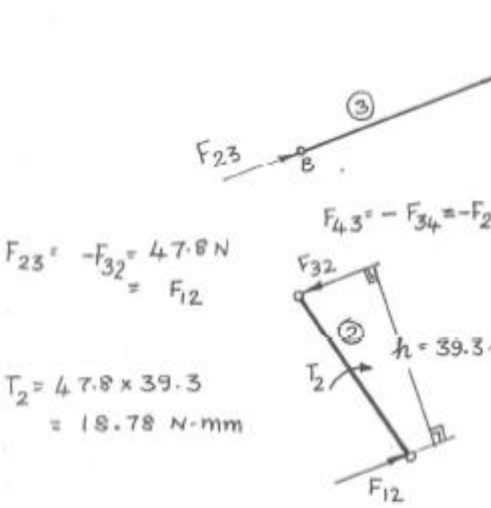
A four link mechanism is acted upon by forces as shown in the figure. Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium.

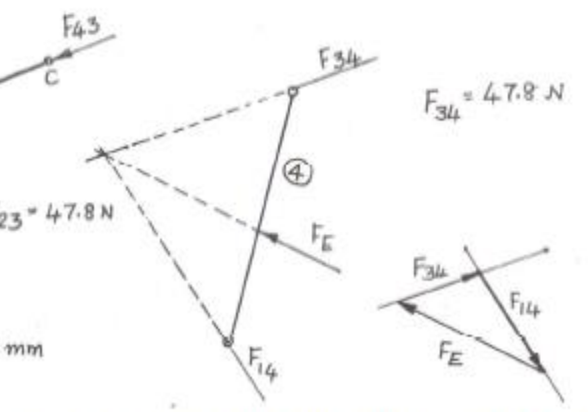
AD=50mm, AB=40mm, BC=100mm, Dc=75mm, DE= 35mm,



Link 3 is acted upon by only two forces F_{23} & F_{43} and they must be collinear & along BC.

Link 4 is acted upon by three forces F_{14}, F_{34} & F_4 and they must be concurrent. LOA F_{34} is known and F_E completely given.



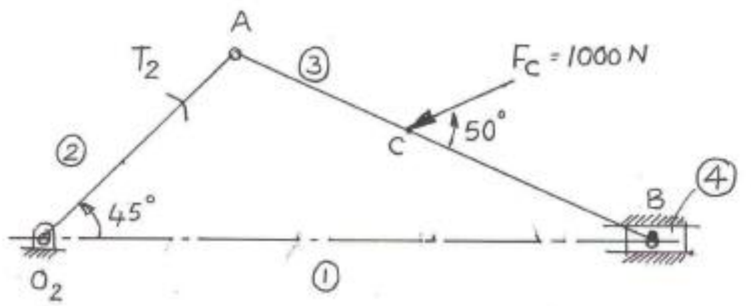


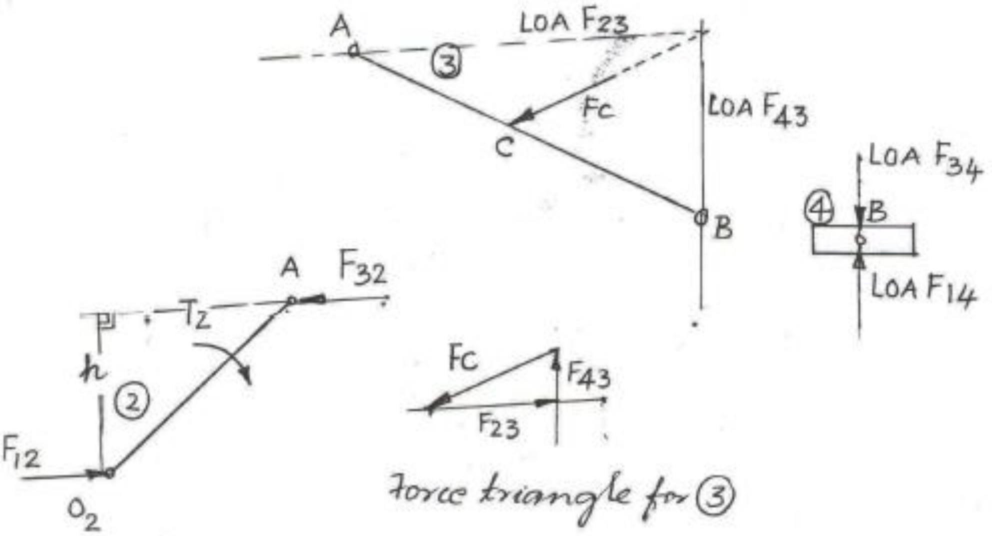
F_{32} & F_{12} from a CCW couple which is equaled by a clockwise couple T_2

Problem No 3.

Determine T_2 to keep the mechanism in equilibrium

AC=70mm,
AB=150mm,
O_2A= 40mm





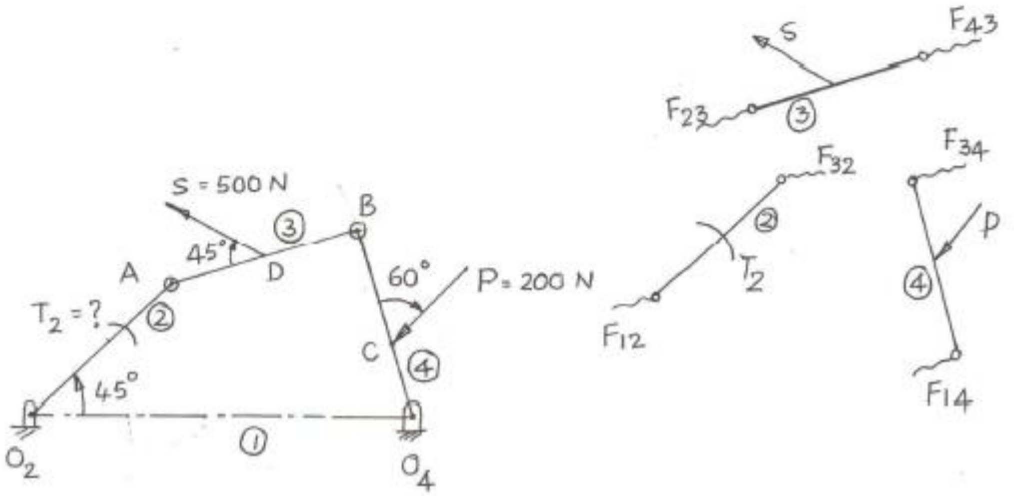
$$T_2 = F_{32} \times h = F_{12} \times h$$

F_{32} and F_{12} form a CCW couple and hence T_2 acts clock wise.

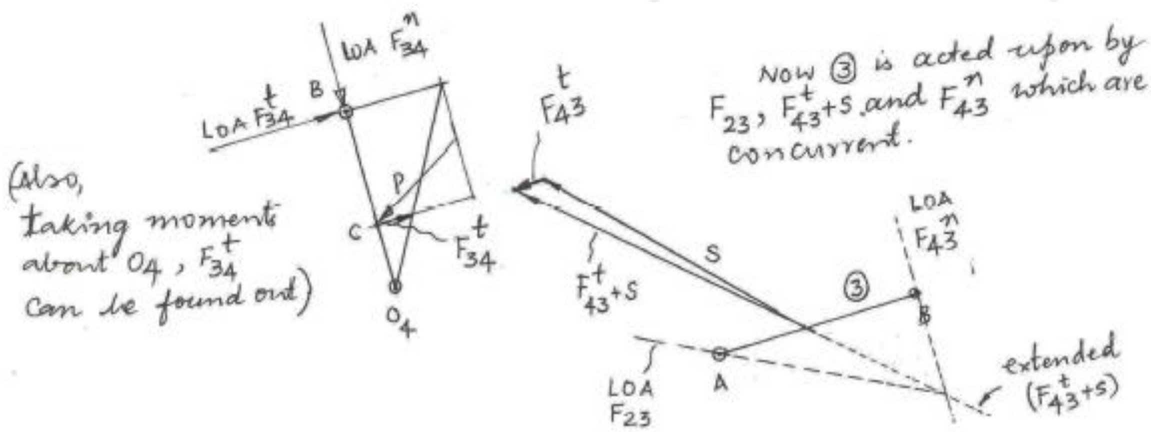
Problem No 4.

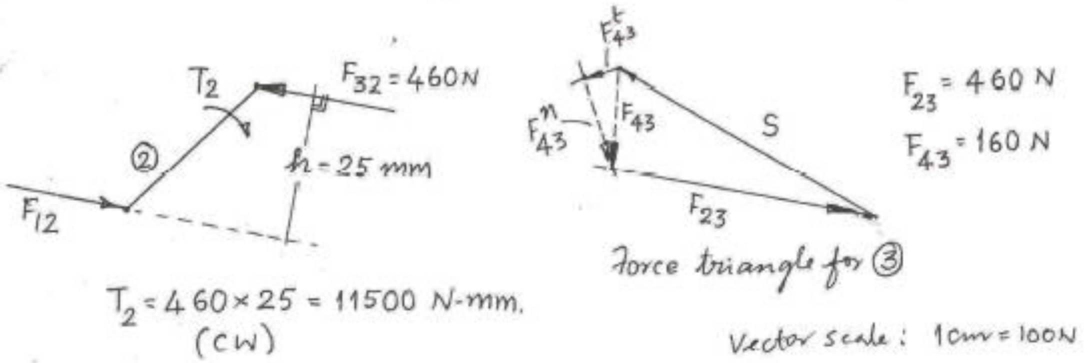
Determine the torque T_2 required to keep the given mechanism in equilibrium.

O_2A= 30mm, = AB =O_4B, O_2O_4 =60mm, $A\hat{O_2}O_4 = 60^o$, BC = 19mm, AD=15mm.



None of the links are acted upon by only 2 forces. Therefore links can't be analyzed individually.

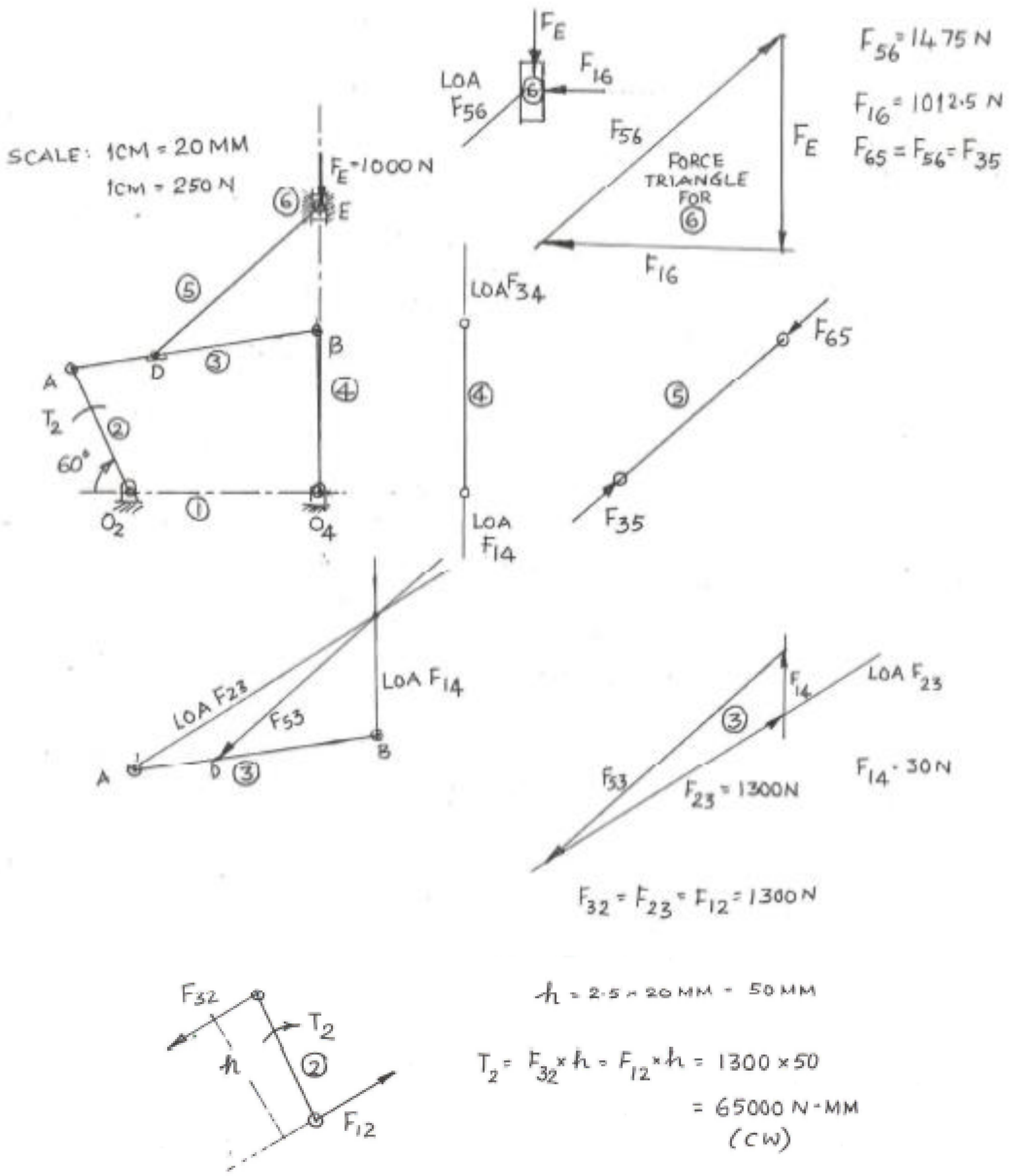




Problem No 5.

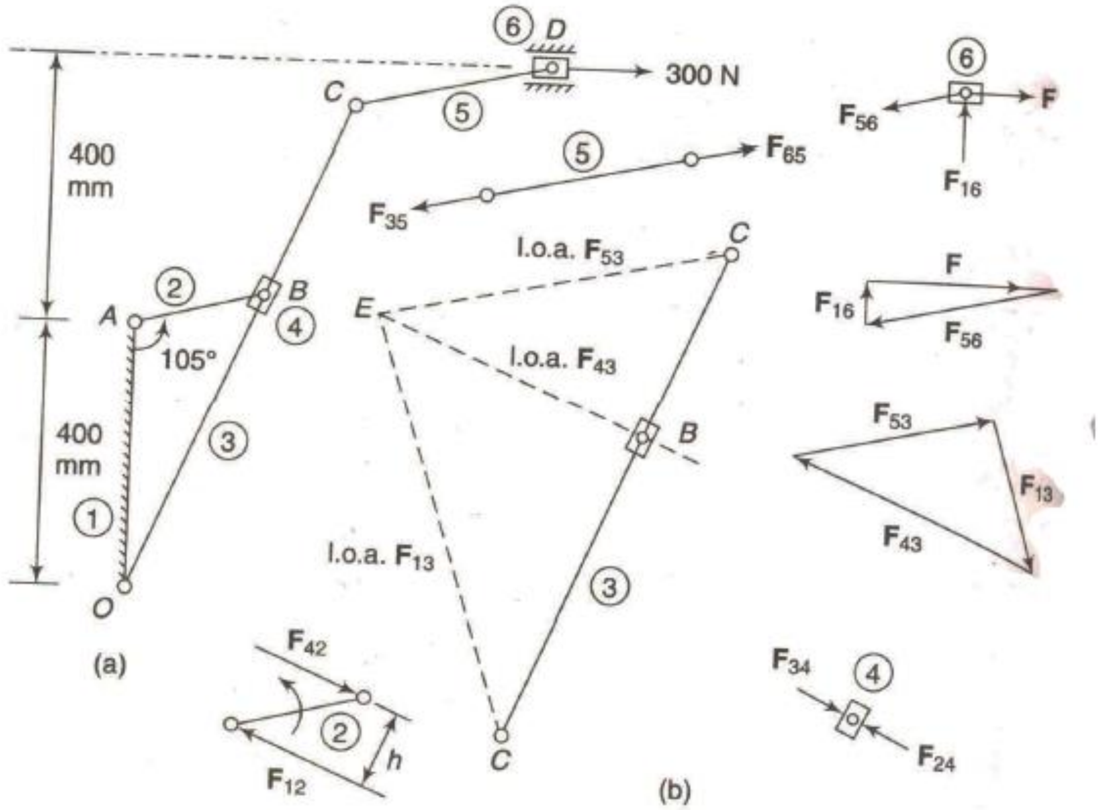
Determine the torque T_2 required to overcome the force F_E along the link 6.

AD=30mm, AB=90mm, O_4 B=60mm, DE=80mm, O_2 A=50mm, O_2 O_4 =70mm



Problem No 6

For the static equilibrium of the quick return mechanism shown in fig. 12.11 (a), determine the input torque T_2 to be applied on link AB for a force of 300N on the slider D. The dimensions of the various links are OA=400mm, AB=200mm, OC=800mm, CD=300mm



Than, torque on link 2,

$T_2 = F_{42} x\ h = 403x120 = 48\ 360$ N counter - clockwise

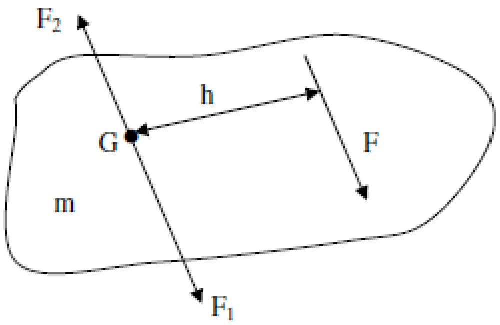
DYNAMIC FORCE ANALYSIS:

It is defined as the study of the force at the pin and guiding surfaces and the forces causing stresses in machine parts, such forces being the result of forces due to the motion of each part in the machine. The forces include both external and inertia forces. Inertia forces in high speed machines become very large and cannot be neglected, Ex: Inertia force of the piston of an automobile travelling at high speed might be thousand times the weight of the piston. The dynamic forces are associated with accelerating masses.

If each link, with its inertia force and force applied to the link can be considered to be in equilibrium, the entire system can also be considered to be in equilibrium.

Determination of force & couple of a link

(Resultant effect of a system of forces acting on a rigid body)



G = c .g point
F_1& F_2: equal and opposite forces acting through G (Parallel to F)

F: Resultant of all the forces acting on the rigid body.
h: perpendicular distance between F & G.
m = mass of the rigid body

Note: F_1=F_2 & opposite in direction; they can be cancelled with out affecting the equilibrium of the link. Thus, a single force 'F' whose line of action is not through G, is capable of producing both linear & angular acceleration of CG of link.

F and F_2 form a couple.

$T = F \times h = I\,\alpha = mk^2\,\alpha$ (Causes angular acceleration) . . . . . . (1)

Also, F_1 produces linear acceleration, f.

$F_1 = mf$

Using 1 & 2, the values of 'f' and 'α' can be found out if F_1, m, k & h are known.

D'Alembert's principle:

Final design takes into consideration the combined effect of both static and dynamic force systems. D'Alembert's principle provides a method of converting dynamics problem into a static problem.

Statement: The vector sum of all external forces and inertia forces acting upon a rigid body is zero. The vector sum of all external moments and the inertia torque, acting upon the rigid body is also separately zero. In short, sum of forces in any direction and sum of their moments about any point must be zero.

Inertia force and couple: Inertia: Tendency to resist change either from state of rest or of uniform motion Let 'R' be the resultant of all the external forces acting on the body, then this 'R' will be equal to the product of mass of the body and the linear acceleration of c.g of body. The force opposing this 'R' is the inertia force (equal in magnitude and opposite in direction).

(Inertia force is an Imaginary force equal and opposite force causing acceleration).

If the body opposes angular acceleration (α) in addition to inertia force R, at its cg, there exists an inertia couple Ig x α, Where Ig= M I about cg. The sense of this couple opposes α. i.e., inertia force and inertia couple are equal in magnitude to accelerating force and couple respectively but, they act in opposite direction.

Inertia force (F_i) = M x f,
(mass of the rigid body x linear acceleration of cg of body)

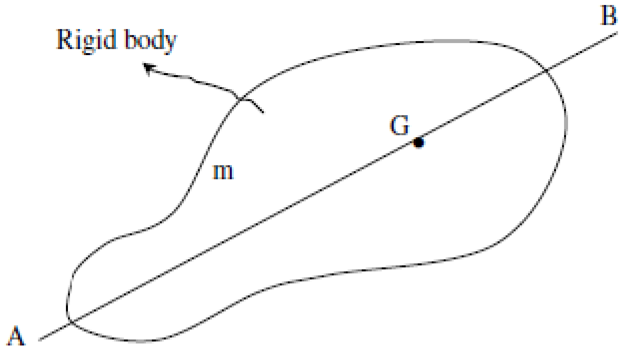
Inertia couple (C_i)=I x α, [MMI of the rigid body about an axis perpendicular to the plane of motion] [Angular acceleration]

Dynamic Equivalence:

The line of action of the accelerating force can also be determined by replacing the given link by a dynamically equivalent system. Two systems are said to be dynamically equivalent to one another, if by application of equal forces, equal linear and angular accelerations are produced in the two systems.

i.e., the following conditions must be satisfied;

i) The masses of the two systems must be same.
ii) The cg's of the two systems must coinside.
iii) The moments of inertia of the two systems about same point must be equal, Ex: about an axis through cg.

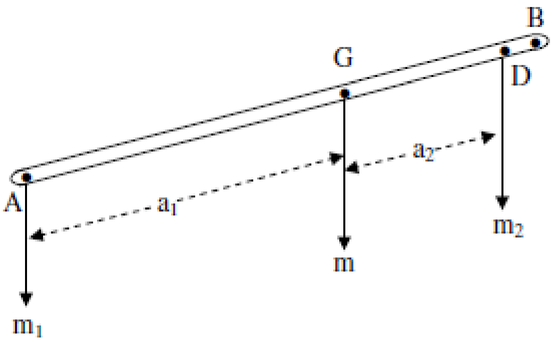


G = c.g.

m = mass of the rigid body

k_g = radius of gyration about an axis through G and perpendicular to the plane

Now, it is to be replaced by dynamically equivalent system.



m_1, m_2 – masses of dynamically equivalent system at a_1 & a_2 from G (respectively)

As per the conditions of dynamic equivalence,

$$m = m_1 + m_2 \quad \text{.. (a)}$$
$$m_1 a_1 = m_2 a_2 \quad \text{.. (b)}$$
$$mk_g^2 = m_1 a_1^2 + m_2 a_2^2 \quad \text{.. (c)}$$

Substituting (b) in (c),

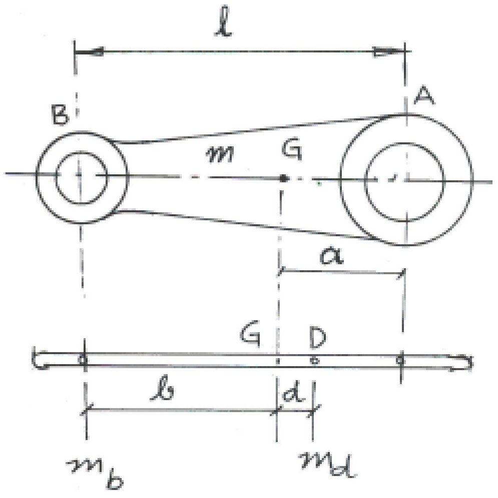
$$mk_g^2 = (m_2 a_2) a_1 + (m_1 a_1) a_2$$

$$= a_1 a_2 (m_2 + m_1) = a_1 a_2 (m)$$

i.e., $k_g^2 = a_1 a_2$ $\quad [I_g = mk_g^2 \text{ or } k_g^2 = \frac{I_g}{m}]$

or $\frac{I_g}{m} = a_1 a_2$

Inertia of the connecting rod:



Connecting rod to be replaced by a massless link with two point masses m_b & m_d .

m = Total mass of the CR m_b& m_d point masses at B& D.

$$m_b + m_d = m \qquad -- \quad (i)$$

$$m_b \times b = m_d \times d \qquad -- \quad (ii)$$

Substituting (ii) in (i);

$$m_b + \left(m_b \times \frac{b}{d} \right) = m$$

$$m_b \left(1 + \frac{b}{d} \right) = m \qquad \text{or} \quad m_b \left(\frac{b+d}{d} \right) = m$$

$$\text{or } m_b = m\left(\frac{d}{b+d} \right) \qquad -- \quad (1)$$

Similarly; $$m_d = m\left(\frac{b}{b+d} \right) \qquad -- \quad (2)$$

Also; $I = m_b b^2 + m_d d^2$

$$= m\left(\frac{d}{b+d} \right) b^2 + m\left(\frac{b}{b+d} \right) d^2 \qquad [\textit{from } (1) \,\&\, (2)]$$

$$I = mbd \left(\frac{b+d}{b+d} \right) = mbd$$

Then, $mk_g^2 = mbd$, (since $I = mk_g^2$)

$k_g^2 = bd$

The result will be more useful if the 2 masses are located at the centers of bearings A & B.

Let m_a = mass at A and dist. AG = a

Then,

$$m_a + m_b = m$$

$$m_a = m\left(\frac{b}{a+b}\right) = m\,\frac{b}{l}\ ; \quad Since\ (a+b=l)$$

Similarly, $m_b = m\left(\frac{a}{a+b}\right) = m\,\frac{a}{l}\ ; \quad (Since,\ a+b=l)$

$$I^1 = m_a a^2 + m_b b^2 = .\ .\ .\ . = mbd$$

(Proceeding on similar lines it can be proved)

Assuming; $a > d,\ I^1 > I$

i.e., by considering the 2 masses A & B instead of D and B, the inertia couple (torque) is increased from the actual value. i.e., there exists an error, which is corrected by applying a correction couple (opposite to the direction of applied inertia torque).

The correction couple,

$$\Delta T = \alpha_c (mab - mbd)$$

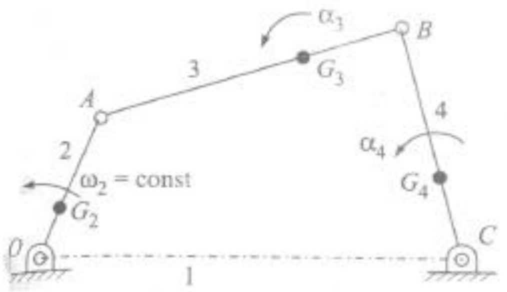
$$= mb\ \alpha_c\ (a-d)$$

$$= mb\ \alpha_c \left[(a+b) - (b+d)\right]$$

$$= mb\ \alpha_c\ (l-L) \qquad because\,(b+d=L)$$

As the direction of applied inertia torque is always opposite to the direction of angular acceleration, the direction of the correction couple will be same as that of angular acceleration i.e., in the direction of decreasing angle β.

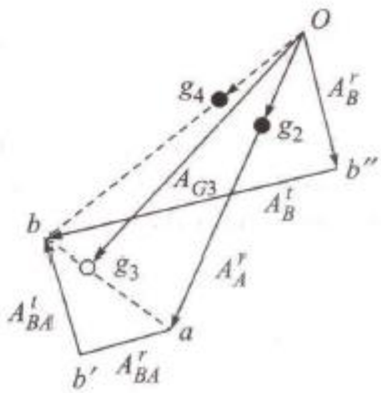
Dynamic force Analysis of a 4 – link mechanism.



OABC is a 4–bar mechanism. Link 2 rotates with constant ω_2. G_2, G_3 & G_4 are the cgs and M_1, M_2 & M_3 the masses of links 1, 2 & 3 respectively.

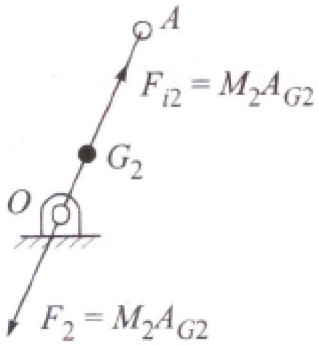
What is the torque required, which, the shaft at o must exert on link 2 to give the desired motion?

1. Draw the velocity & acceleration polygons for determing the linear acceleration of G_2, G_3 & G_4.
2. Magnitude and sense of α_3 & α_4 (angular acceleration) are determined using the results of step 1.



To determine inertia forces and couples

Link 2

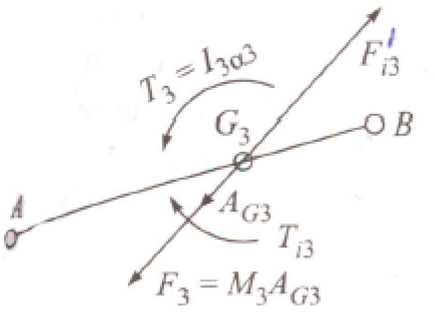


F_2 = accelerating force (towards O)

F_{i2} = inertia force (away from O)

Since ω_2 is constant, $\alpha_2 = 0$ and no inertia torque involved.

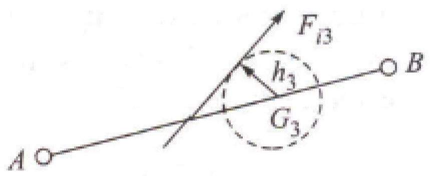
Link 3



Linear acceleration of G_3 (i.e., AG_3) is in the direction of Og_3 of acceleration polygon.

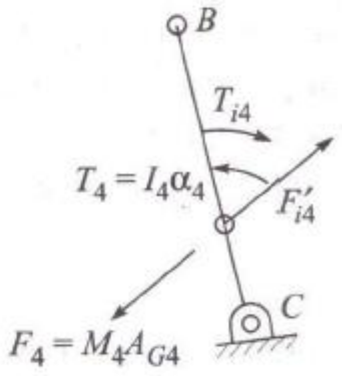
F_3 = accelerating force

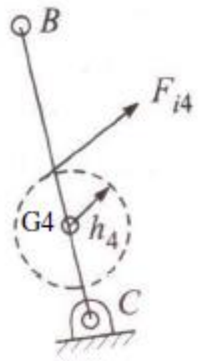
Inertia force $F_{i3}^{'}$ acts in opposite direction. Due to α_3, there must be a resultant torque $T_3 = I_3\,\alpha_3$ acting in the sense of α_3 (I_3 is MMI of the link about an axis through G_3, perpendicular to the plane of paper). The inertia torque T_{i3} is equal and opposite to T_3.



F_{i3} can replace the inertia force $F_{i3}^{'}$ and inertia torque T_{i3}. F_{i3} is tangent to circle of radius h_3 from G_3, on the top side of it so as to oppose the angular acceleration α_3. $h3 = \frac{I_3\alpha_3}{M_3 AG_3}$

Link 4





$h4 = \frac{I_4\alpha_4}{M_4 AG_4}$

Problem 1 :

It is required to carryout dynamic force analysis of the four bar mechanism shown in the figure.

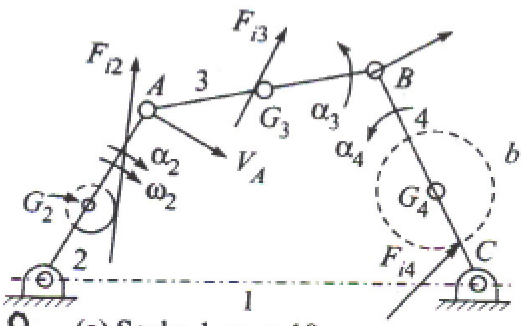
ω_2 = 20rad /s (cw), α_2 = 160 rad/s^2 (cw)

OA= 250mm, OG$_2$= 110mm, AB=300mm, AG$_3$=150mm, BC=300mm, CG$_4$=140mm, OC=550mm, $\angle AOC = 60°$

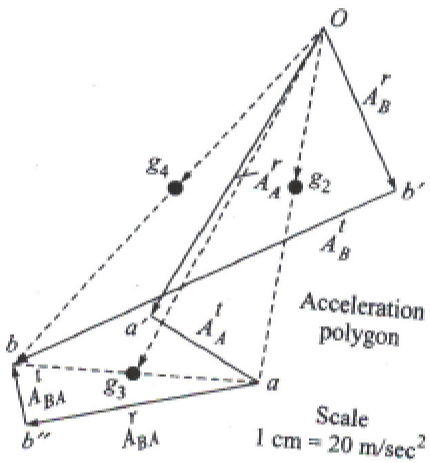
The masses & MMI of the various members are

Link	Mass, m	MMI (I_G, Kgm^2)
2	20.7kg	0.01872
3	9.66kg	0.01105
4	23.47kg	0.0277

Determine i) the inertia forces of the moving members
ii) Torque which must be applied to (2)



(a) Scale: 1 cm = 10 cms



A) Inertia forces:

(i) (from velocity & acceleration analysis)

$V_A = 250\times 20;\ 5m/s,\quad V_B = 4\ m/s,\ V_{BA} = 4.75\ m/s$

$a_A^r = 250\times 20^2;\ 100m/s^2,\quad a_A^t = 250\times 160;\ 40m/s^2$

Therefore;

$$A_B^r = \frac{V_B^2}{CB} = \frac{(4)^2}{0.3} = 53.33\ m/s^2$$

$$A_{BA}^r = \frac{V_{BA}^2}{B_A} = \frac{(4.75)^2}{0.3} = 75.21\ m/s^2$$

$$Og_2 = A_{G2} = 48\ m/s^2;\ Og_3 = AG_3 = 120m/s^2$$

$$Og_4 = A_{G4} = 65.4\ m/s^2$$

$$\alpha_3 = \frac{A_{BA}^t}{AB} = \frac{19}{0.3} = 63.3\ rad/s^2$$

$$\alpha_4 = \frac{A_B^t}{CB} = \frac{129}{0.3} = 430\ rad/s^2$$

Inertia forces (accelerating forces)

$$F_{G2} = m_2\ A_{G2} = \frac{20.7}{9.81} \times 48 = 993.6\,N\ (in\ the\ direction\ of\ Og_2)$$

$$F_{G3} = m_3\ A_{G3} = 9.66 \times 120 = 1159.2\,N\ (in\ the\ direction\ of\ Og_3)$$

$$= F_{G4} = m_4\ A_{G4} = 23.47 \times 65.4 = 1534.94\,N\ (in\ the\ direction\ of\ Og_4)$$

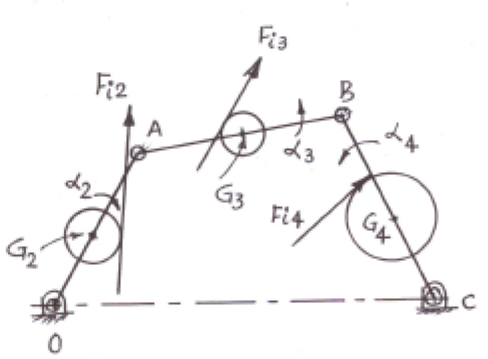
$$h_2 = \frac{I_{G2}(\alpha_2)}{F_2} = \frac{(0.01872 \times 160)}{993.6} = 3.01 \times 10^{-3}\,m$$

$$h_3 = \frac{I_{G3}(\alpha_3)}{F_3} = \frac{(0.01105 \times 63.3)}{1159.2} = 6.03 \times 10^{-4}\,m$$

$$h_4 = \frac{I_{G4}(\alpha_4)}{F_4} = \frac{(0.0277 \times 430)}{1534.94} = 7.76 \times 10^{-3}\,m$$

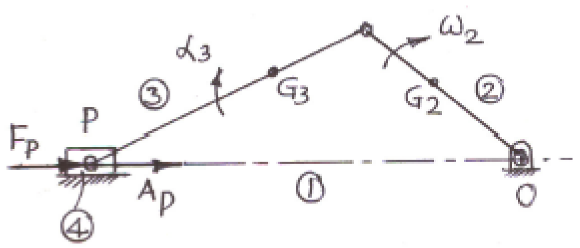
The inertia force F_{i2}, F_{i3} & F_{i4} have magnitudes equal and direction opposite to the respective accelerating forces and will be tangents to the circles of radius h_2, h_3 & h_4 from G_2, G_3 & G_4 so as to oppose α_2, α_3 & α_4.

$F_{i2} = 993.6\,N$, $F_{i3} = 1159.2\,N$ $F_{i4} = 1534.94\,N$



Further, each of the links is analysed for static equilibrium under the action of all external force on that link plus the inertia force.

Dynamic force analysis of a slider crank mechanism.



F_p = load on the piston

Link	mass	MMI
2	m_2	I_2
3	m_3	I_3
4	m_4	-

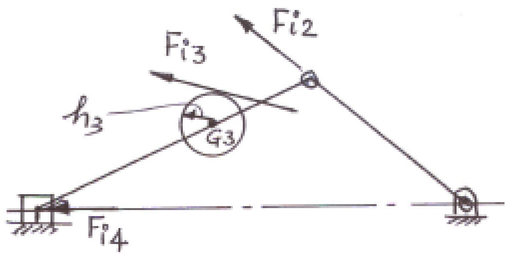
ω_2 assumed to be constant

Steps involved:

1. Draw velocity & acceleration diagrams
2. Consider links 3 & 4 together and single FBD written (elimination F_{34} & F_{43})
3. Since, weights of links are smaller compared to inertia forces, they are neglected unless specified.
4. Accelerating forces F_2 , F_3 & F_4 act in the directions of respective acceleration vectors Og_2, Og_3 & Og_p

Magnitudes: $F_2 = m_2\,AG_2$ $F_3 = m_3\,AG_3$ $F_4 = m_4\,A_p$

$F_{i2} = F_2$, $F_{i3} = F_3$, $F_{i4} = F_4$ (Opposite in direction)



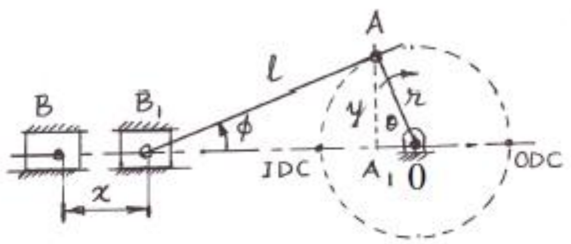
$$h_3 = \frac{I_3 \alpha_3}{M_3 \alpha_{g3}}$$

F_{i3} is tangent to the circle with h_3 radius on the RHS to oppose α_3

Solve for T_2 by solving the configuration for both static & inertia forces.

Dynamic Analysis of slider crank mechanism (Analytical approach)

Displacement of piston



x = displacement from IDC

$$x = BB_1 = BO - B_1O$$

$$= BO - (B_1A_1 + A_1O)$$

$$= (l + r) - (l\cos\phi + r\cos\theta) \qquad \left(\text{since}, \frac{l}{r} = n\right)$$

$$= (nr + r) - (rn\cos\phi + r\cos\theta)$$

$$= r[(n+1) - (n\cos\phi + \cos\theta)] \qquad \cos\phi = \sqrt{1 - \sin^2\phi}$$

$$= r\left[(n+1) - \left(\sqrt{n^2 - \sin^2\theta} + \cos\theta\right)\right]$$

$$= r\left[(1-\cos\theta) + \left(n - \sqrt{n^2 - \sin^2\theta}\right)\right]$$

$$(similary\ l >> r, \frac{l}{r} = n >> 1 \ \&\ \max\ value\ of\ \sin\theta = 1)$$

$$\therefore \sqrt{n^2 - \sin^2\theta} \to \sqrt{n^2}\ or\ n),$$

$$\boxed{x = r\ (1 - \cos\theta)}$$

$$= \sqrt{1 - \frac{y^2}{l^2}}$$

$$= \sqrt{1 - \frac{(r\sin\theta)^2}{l^2}}$$

$$= \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

This represents SHM and therefore Piston executes SHM.

Velocity of Piston:

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\ \frac{d\theta}{dt}$$

$$\frac{d}{d\theta}\left[r(1-\cos\theta) + n - (n^2 - \sin 2\theta)^{-\frac{1}{2}}\right]\frac{d\theta}{dt}$$

$$= r\left[0 + \sin\theta + 0 - \frac{1}{2}(n^2 - \sin 2\theta)^{-1/2}(-2\sin\theta\cos\theta)\right]\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right]$$

Since, $n^2 >> \sin^2\theta$,

$$\therefore\ v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

Since n is quite large, $\frac{\sin 2\theta}{2n}$ can be neglected.

$$\boxed{\therefore\ v = r\omega\sin\theta}$$

Acceleration of piston:

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\,(\sin\theta + \frac{\sin 2\theta}{2n})\right]\omega$$

$$= r\omega\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right]$$

$$= r\omega\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

If n is very large;

$$\boxed{a = r\omega^2 \cos\theta} \quad \textbf{(as in SHM)}$$

When θ = 0, at IDC,

$$a = r\omega^2\left(1 + \frac{1}{n}\right)$$

When θ = 180, at 0DC,

$$a = r\omega^2\left(-1 + \frac{1}{n}\right)$$

At θ = 180, when the direction is reversed,

$$a = r\omega^2\left(1 - \frac{1}{n}\right)$$

Angular velocity & angular acceleration of CR (α_c)

$$y = l\sin\phi = r\,\sin\theta$$

$$\sin\,\phi = \frac{\sin\,\theta}{n}$$

Differentiating w.r.t time,

$$\cos\,\phi\,\frac{d\phi}{dt} = \frac{1}{n}\cos\,\theta\,\frac{d\theta}{dt} \qquad \frac{d\phi}{dt} = \omega_c$$

$$\omega_c = \omega\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \qquad \frac{d\theta}{dt} = \omega$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$\omega_c = \omega \frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta}\left[\cos\theta\,(n^2 - \sin^2\theta)^{-\frac{1}{2}}\right]\omega$$

$$= \omega^2 \left[\cos\theta \frac{1}{2}(n^2 - \sin^2\theta)^{-\frac{3}{2}}(-2\sin\theta\cos\theta) + (n^2 - \sin^2\theta)^{-\frac{1}{2}}(-\sin\theta)\right]$$

$$= \omega^2 \sin^2\theta \left[\frac{\cos^2\theta - (n^2 - \sin^2\theta)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

$$= -\omega^2 \sin\theta \left[\frac{(n^2 - 1)}{(n^2 - \sin^2\theta)^{\frac{3}{2}}}\right]$$

Negative sign indicates that, ϕ reduces (in the case, the angular acceleration of CR is CW)

UNIT-II

Engine Force Analysis and Turning Moment Diagram

Forces acting on the engine are weight of reciprocating masses & CR, gas forces, Friction & inertia forces (due to acceleration & retardation of engine elements).

i) Piston effort (effective driving force)

- Net or effective force applied on the piston.

In reciprocating engine:

The reciprocating parts (masses) accelerate during the first half of the stroke and the inertia forces tend to resist the same. Thus, the net force on the piston is reduced. During the later half of the stroke, the reciprocating masses decelerate and the inertia forces oppose this deceleration or acts in the direction of applied gas pressure and thus effective force on piston is increased.

In vertical engine, the weights of the reciprocating masses assist the piston during out stroke (down) this increasing the piston effort by an amount equal to the weight of the piston. During the in stroke (up) piston effect is decreased by the same amount.

Force on the piston due to gas pressure; $FP = P_1A_1 – P_2A_2$

P_1 = Pressure on the cover end,
P_2 = Pressure on the rod
A_1 = area of cover end,
A_2 = area of rod end,
m = mass of the reciprocating parts.

Inertia force (F_i) = m a

$$= m.r\omega^2\left(Cos\theta + \frac{Cos2\theta}{n}\right) \quad \text{(Opposite to acceleration of piston)}$$

Force on the piston $F = F_P - F_i$

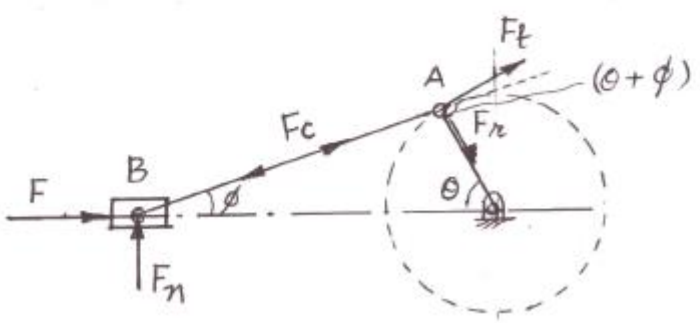
(if F_f frictional resistance is also considered)

$$F = F_P – F_i – F_i$$

In case of vertical engine, weight of the piston or reciprocating parts also acts as force.

$$\therefore F = F_P + mg – F_i – F_i$$

ii) Force (Thrust on the CR)



F_c = force on the CR

Equating the horizontal components;

$$F_c \, Cos\phi = F \quad or \quad F_c \frac{F}{Cos^2\phi}$$

iii) Thrust on the sides of the cylinder

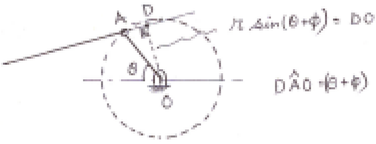
It is the normal reaction on the cylinder walls

$$F_s = F_c \sin\phi = F \tan\phi$$

iv) Crank effort (T)

It is the net force applied at the crank pin perpendicular to the crank which gives the required TM on the crank shaft.

$$F_t \times r = F_c \, r \sin(\theta + \phi)$$



$$F_t = F_c \sin(\theta + \phi)$$

$$= \frac{F}{\cos\phi} \sin(\theta + \phi)$$

v) Thrust on bearings (F_r)

The component of F_C along the crank (radial) produces thrust on bearings

$$F_r = F_c \, Cos(\theta + \phi) = \frac{F}{Cos\,\phi} \, Cos(\theta + \phi)$$

vi) Turning moment of Crank shaft

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi} \sin(\theta + \phi) \times r \; = \frac{F_r}{\cos\phi} (\sin\theta + \cos\phi + \cos\theta \sin\phi)$$

$$= F \times r \left(\sin\theta + \cos\theta \frac{\sin\phi}{\cos\phi} \right)$$

$$= F \times r \left(\sin\theta + \cos\theta \, \frac{\sin\theta}{n} \, \frac{1}{\frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \right)$$

Proved earlier

$$\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$= F \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

Also,

$$r \sin(\theta + \phi) = OD \, \cos\phi$$

$$T = F_t \times r$$

$$= \frac{F}{\cos\phi} . \, r \sin(\theta + \phi)$$

$$= \frac{F}{\cos\phi} . \, OD \, \cos\phi$$

$$T = F \times OD.$$

Turning Moment Diagram & Flywheels

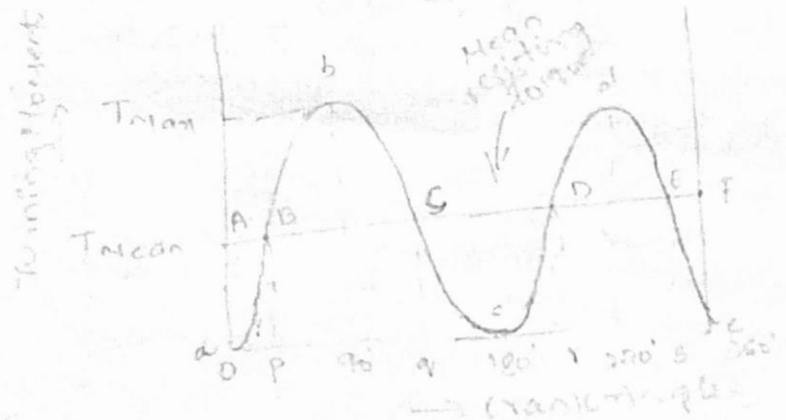
Introduction: The torque of an engine crank shaft varies considerably throughout the working cycle, due to variations in crank position, the P in the cylinder & inertia force on piston & connecting rod. If the

Turning Moment: Value of crank shaft torque, i.e., the turning moment T is plotted against crank angle θ, the diagram so obtained is turning moment dia.

Turning moment diagram is also known as crank-effort diagram, it is the graphical representation of the turning moment or crank-effort for various positions of the crank.

* Turning moment Diagram for a Single cylinder double Acting Steam Engine:

A turning moment diagram for a single cylinder double acting steam engine is:



Fig②: Turning moment diagram for a single cylinder double acting steam Engine.

The vertical ordinate represents Turning Moment, the horizontal ordinate represents crank angle.

Thus turning moment on crank shaft will be,

$$T = F_p \times r \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right]$$

moment is zero, .

where, $F_p \rightarrow$ Piston effort

$r \rightarrow$ radius of crank.

$n \rightarrow$ ratio of connecting rod length & radius of crank and

$\theta \rightarrow$ Angle turned by crank from inner dead centre.

From the fig ⓐ, we can say that 'T' → turning moment is zero, when the crank angle θ is zero. It is maximum when, the crank angle is $\left(\frac{180°}{2}\right) = 90°$. Again it is zero when the crank angle is 180°. and som

This is shown by curve 'abc' in fig., and it represents turning moment of out stroke. The curve 'cde' represents turning moment in stroke.

NOTE: 1. when the turning moment is '+ve' i.e; when the engine torque is more than mean resisting torque, as shown between points B & C in fig ⓐ the crank shaft accelerates and work is done by steam.

2. When the turning moment is '−ve', i.e; when the engine torque is less than the mean resisting torque, as shown between points C & D in fig ⓐ. the crank shaft retards and the work is done on the steam.

3. If $T \rightarrow$ Torque on crank shaft at any instant &

$T_{mean} \rightarrow$ Mean resisting Torque

Then accelerating torque on rotating parts of engine,

$= T - T_{mean}$.

4. If $(T - T_{mean})$ is '+ve', the flywheel accelerate.

if $(T - T_{mean})$ is '−ve', then the flywheel retards.

(2)

...ctuation of Energy:-

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in fig (a). We see that the mean resisting torque line AF cuts the turning moment diagram at B, C, D & E. When the crank moves from a to p, the work done by the engine equal to the area aBP, whereas the energy required is represented by the a ABP. In the other words, the engine has done less work, ~~the remaining~~ than the requirement. This amount, i.e. required amount of energy is taken from the flywheel and hence the speed of flywheel decreases. Now the crank moves from 'p to q', the work done by the engine, is equal to the area PBbcq, where as the requirement of energy is represented by the area PBCq. Therefore, the engine has done more work than the requirement. This excess energy stored in the flywheel, and the speed of flywheel increases while the crank moves from p to q.

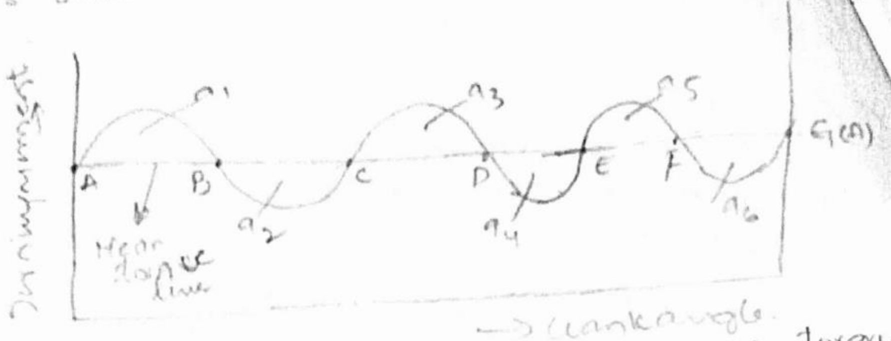
The variations of energy above and below the mean resisting torque line are called fluctuations of energy. The areas Bbc, CcD, DdE, etc. represent fluctuations of energy.

The difference between the ~~minimum~~ maximum and minimum energies is known as Maximum fluctuation of energy.

NOTE:- The area of the turning moment diagram is proportional to the work done per revolution as the work is the product of turning-moment & angle turned.

* Determination of the fluctuation of energy:

A turning diagram for a multicylinder engine is shown as:



The horizontal line AG represents mean torque line

Let, a_1, a_3, a_5 represents areas of above the mean torque line

a_2, a_4, a_6 " " below " "

Let the energy in the flywheel at A = E.

we have, Energy at $B = E + a_1$

" " $C = E + a_1 - a_2$

" " $D = E + a_1 - a_2 + a_3$

" " $E = E + a_1 - a_2 + a_3 - a_4$

" " $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

" " $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

= Energy at A (i.e. cycle repeat after G).

Suppose, the greatest energy is at B and least at E.

∴ Max. energy in flywheel, $= E + a_1$

min. " " " , $= E + a_1 - a_2 + a_3 - a_4$.

∴ Max. fluctuation of energy, ΔE,

ΔE = Max. Energy − Min. energy.

$= (E + a_1) - [E + a_1 - a_2 + a_3 - a_4]$

$$\boxed{\Delta E = a_2 - a_3 + a_4}$$

Co-efficient of Fluctuation of Energy [C_E]

- It may be defined as the, "ratio of maximum fluctuation of energy to the work done per cycle".

$$C_E = \frac{\text{max. fluctuatn of energy}}{\text{work done per cycle}}$$

We may obtain work done per cycle in 2 methods:
Work done per cycle, units → (N-m or J).

1. Work done per cycle = $T_{mean} \times \theta$.

where, T_{mean} → Mean torque &
θ → Angle turned (in radians), in one revolution.
= 2π, in case of steam engine & 2-stroke I.C. engines.
= 4π, in case of 4-stroke I.C. engines.

The mean torque can be obtained as,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where, P → Power Transmitted in watts,
N → Speed in rpm.
ω → Angular speed, $\frac{rad}{sec} = \frac{2\pi N}{60}$.

2. Work done per cycle,

$$= \frac{P \times 60}{n}.$$

where, n → no. of working strokes per minute,
$n = N$, in case of steam engines & 2 stroke I.C engines &
$n = \frac{N}{2}$, in case of 4-stroke I.C engines.

* FLY WHEEL:-

A flywheel is used in machines used reservoirs, which stores the energy during the when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than the supply.

In simple words, when the flywheel absorbs energy, its speed increases and when its releases the energy, its speed decreases.

We can say that, "A flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation".

NOTE: The function of a governor, in an engine is entirely different from that of flywheel. The governor regulates the mean speed of an engine when there are variations in the load. Where as the flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. It does not control the speed variations caused by varying load.

Applications:-

Flywheels are provided in engines and fabricating machines such as presses, shearing machines, rivetting machines, punching machines, steel rollers, crushers etc..

(A).

Co-effecient of Fluctuation of speed [Cs]:

The ratio of maximum fluctuation of speed to the mean speed is called co-effecient of fluctuation of speed.

"The difference b/w max. & min. speeds during a cycle is called max. fluctuation of speed."

Let, N_1 & N_2 are max. & min. speeds during cycle.

$N \rightarrow$ mean position

$$= \frac{N_1 + N_2}{2}.$$

$$\therefore \; C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \ldots \text{(In terms of Angular speed)}$$

$$= \frac{V_1 - V_2}{V} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \ldots \text{(In terms of linear speed)}.$$

NOTE: The reciprocal of co-effecient of friction of speed is known as "coefficient of steadiness" and is denoted by 'm'.

$$m = \frac{1}{C_S} = \frac{N}{(N_1 - N_2)}$$

* Energy Stored in a Flywheel:

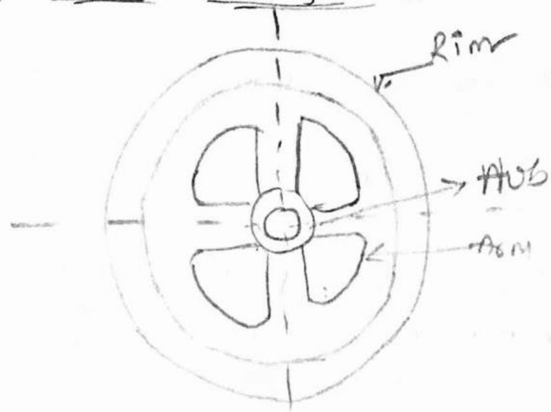


Fig: Flywheel.

We have already discussed that, when a flywheel absorbs energy, its speed increases and vice-versa.

let, $m \rightarrow$ mass of flywheel.

$k \rightarrow$ radius of gyration.

$I \rightarrow$ mass moment of inertia, $= m \cdot k^2$.

N_1 & $N_2 \rightarrow$ Max. & Min. Speeds during the cycle, in rpm.

ω_1 & $\omega_2 \rightarrow$ Max. & Min. angular speeds during the cycle, $\frac{rad}{sec}$.

$N_{mean} \Rightarrow$ Mean speed $= \frac{N_1+N_2}{2}$

$\omega \rightarrow$ Mean angular speed, $= \frac{\omega_1+\omega_2}{2}$.

$c_s \rightarrow$ co-efficient of fluctuation of speed $= \frac{N_1-N_2}{N}$ (or) $\frac{\omega_1-\omega_2}{\omega}$.

W.K.T, the mean kinetic energy of flywheel.

$$E = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2} m k^2 \cdot \omega^2.$$

As the speed of flywheel changes from ω_1 to ω_2, max. fluc. Acc.

$$E_f = \text{Max. K.E} - \text{Min. K.E.}$$

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2 = \frac{1}{2} \times I \left[\omega_1^2 - \omega_2^2\right].$$

$$= \frac{1}{2} \times I \left[(\omega_1+\omega_2)(\omega_1-\omega_2)\right] = I\,\omega(\omega_1-\omega_2) \qquad \because \frac{\omega_1+\omega_2}{2} = \omega$$

multiply & divide by ω.

$$E_f = I\omega^2\left[\frac{\omega_1-\omega_2}{\omega}\right]$$

$$E_f = I\omega^2 (c_s) = m k^2 \omega^2 c_s.$$

$$= 2 E c_s. \qquad \left[E = \frac{1}{2} \times I \cdot \omega^2\right].$$

'k' may be taken as 'R'. $k = R$

$$\Delta E = m R^2 \omega^2 c_s$$

$$= m V^2 c_s = \Delta E \qquad \therefore V = \omega R \text{ in m/s}.$$

$V \rightarrow$ linear velocity.

The mass of fly wheel of an engine is 6.5 tonns. and radius of gyration is 1.8m. It is found from the turning moment diagram that the fluctuation of energy (E_f) is 56 kN-m. If the mean speed of the engine is 120 rpm. find the max. & min. speeds.

Sol: Given Data:

$M = 6.5 \text{ tonns} = 6500 \text{ kgs}$.

$K = 1.8 \text{ m}$; $E_f = 56 \text{ kN-m} = 5600 \text{ N-m}$; $N = 120 \text{ rpm}$.

Let, N_1 = max speed ; N_2 = min speed.

We K.T, $N = \frac{N_1 + N_2}{2}$

$120 = \frac{N_1 + N_2}{2}$

$N_1 + N_2 = 240$ —(1).

$E_f = \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$.

$= \frac{1}{2} MK^2 [\omega_1^2 - \omega_2^2]$.

$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\left(\frac{2\pi N_1}{60}\right)^2 - \left(\frac{2\pi N_2}{60}\right)^2\right]$.

$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right][N_1 - N_2][N_1 + N_2]$.

$5600 = \frac{1}{2}(6500)(1.8)^2 \times \left[\frac{4\pi^2}{3600}\right] \times 240 \times (N_1 - N_2)$

$N_1 - N_2 = 0.2$

$N_1 + N_2 = 240$
$N_1 - N_2 = 0.2$

$2N_1 = 240.2$

$\boxed{N_1 = 120.1 \text{ rpm}}$

$N_1 + N_2 = 240$

$120.1 + N_2 = 240$

$N_2 = 240 - 120.1$

$\boxed{N_2 = 119.9 \text{ rpm}}$

(2) The horizontal compound 2-stroke cylinder engine develops at 90 rpm. The co-effecient of fluctuation of energy from the turning moment diagram is to be 0.1 and speed is to be at 0.5% of mean speed. The mass ... the flywheel required, if the radius of gyration is 2 ... joining ... following ... crank angle ... diagram ...

Sol:

Given:

$P = 300\,kW$; $N = 90\,rpm$; $C_E = 0.1$; $C_s = \pm 0.5\%$ of N.

m = mass of flywheel

$$P = \frac{2\pi N T_{mean}}{60{,}000}$$

$$300 = \frac{2\pi \times 90 \times T_{mean}}{60{,}000}$$

$$\boxed{T_{mean} = 31830\ N\text{-}m}$$

$$C_E = \frac{E_f}{W.D/cycle}$$

$$0.1 = \frac{E_f}{T_{mean} \times 2\pi}$$

$$0.1 = \frac{E_f}{31830 \times 2\pi}$$

$$\boxed{E_f = 19999.9\ N\text{-}m}$$

$$C_s = \frac{N_1 - N_2}{N}$$

$N_1 = 90 - 0.5\%$ of N. $\Rightarrow 90 - 0.5\% \times 90$

$\boxed{N_2 = 89.55}$ rpm.

$N_2 = 90 + 0.5\%$ of N.

$\boxed{N_1 = 90.45\ rpm}$

$$C_s = \frac{90.45 - 89.55}{90}$$

$$\boxed{C_s = 0.01}$$

$E_f = I\omega^2 C_s$.

$19999.9 = m K^2 \omega^2 C_s$.

$= m (2)^2 \left(\frac{2\pi \times 90}{60}\right)^2 \times 0.01$

$\boxed{m = 5628.9\ kg}$

T-θ-diagram Problems

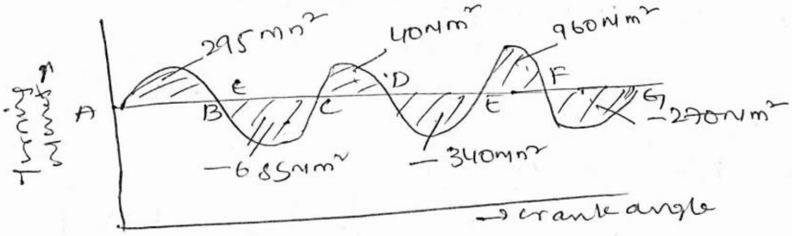
(6). Turning moment for a petrol engine is drawn through following scales: for torque 1mm = 5 N-m, crank angle 1mm = 1° = 1 × π/180. The turning moment diagram repeats itself at every revolution of engine and areas above & below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm². The rotating parts are equivalent to a mass of 36 kg at a radius of gyration 150 mm. Determine, coefficient of fluctuation of speed when the ranges at 1800 rpm.

Sol: Given Data:

for torque, 1mm = 5 N-m

crank angle 1mm = 1° = $\frac{1 \times \pi}{180}$ = 0.0174 rad.

The areas are, +295, −685, +40, −340, +960, −270 Nm².



m = 36 kg.

K = 150 mm = 0.15 m; N = 1800 rpm. ∴ C_s = ?

Let energy at pt. A = E m

" " pt. B = E + 295 ⇒ Max. K.E

pt. C = E + 295 − 685 ⇒ E − 390.

pt. D = E + 295 − 685 + 40 ⇒ E − 350

pt. E = E + 295 − 685 + 40 − 340 ⇒ E − 690 ⇒ Min k

pt. F = E + 295 − 685 + 40 − 340 + 960 ⇒ E + 270

pt. G = E + 295 − 685 + 40 − 340 + 960 − 270 = E.

$\therefore E_f = \text{Max. K.E} - \text{Min. K.E.}$

$= (E+295) - (E-690)$

$= E+295-E+690$

$E_f = 985 \text{ mm}^2$

$= 985 \times mm \times mm$

$= 985 \times 5 \times \frac{\pi}{180}$

$E_f = 85.96 \text{ N-m}$

$E_f = I\omega^2 \times C_s$

$85.96 = mk^2 \cdot \omega^2 \times C_s$

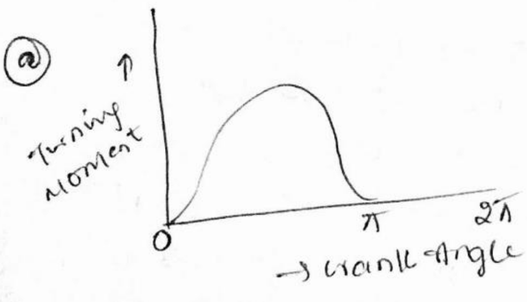
$85.96 = (36)(0.15)^2 \times \left[\frac{2\pi \times 1800}{60}\right]^2 \times C_s$

$C_s = 3 \times 10^{-3}$

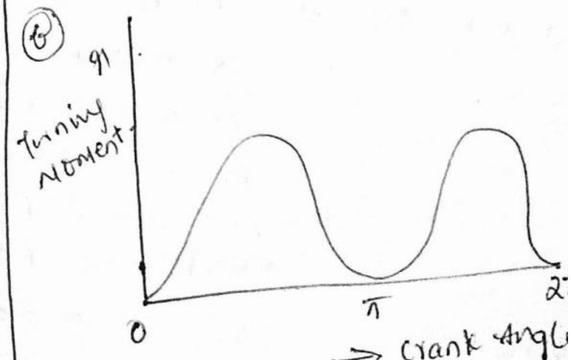
$C_s = 0.003$

$C_s = 0.3\%$ (or) $C_s = \pm 0.15\%$

* Turning moment diagrams of common engines:-



(a) Single acting steam engine



(b) Double acting steam engine

T diagram for 4-stroke IC Engine: (7).

A Turning diagram for 4-stroke IC engine is shown in fig. W.K.T, in a four stroke IC engine, there is one stroke after the crank has turned through 2-revolutions. i.e. 720° (or) 4π.

Since, the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke.

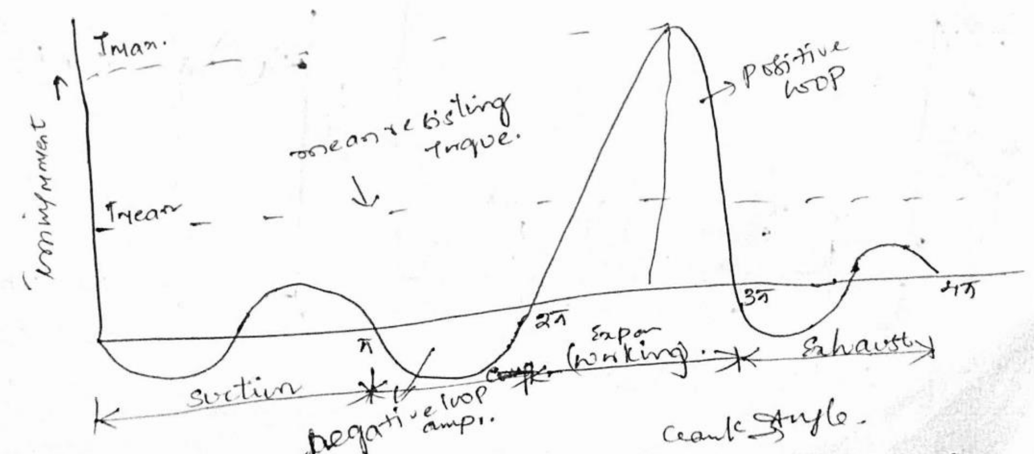
∴ Negative loop is formed, as shown in fig. During the compression, the work is done on the gases; therefore, higher negative loop is formed.

During the expansion stroke the fuel burns & the gases expands,

∴ a large +ve loop is obtained. In this stroke, the work is done by the gases.

During exhaust stroke, the work is done on the gases.

∴ There is a '-ve' loop during the exhaust stroke.



T-θ diagram for 4-stroke IC engine

Problem (8).

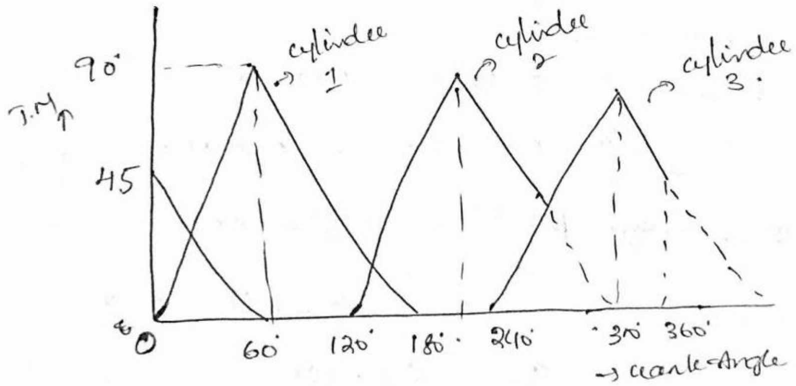
~~Single~~ 3 cylinder single acting engine has its crank set equally at 120° and its runs at 600 rpm. The torque-crank angle diagram for each cycle is a triangle for the power stroke with a maximum torque 90N-m at 60° from dead centre of corresponding crank. The torque on return stroke is sensibly zero. Determine:

1. Power developed; 2. coeffecient of fluctuation of speed, if the mass of flywheel is 12kg & has a radius of gyration of 80mm, 3. coeffecient of fluctuation of energy, & 4. Max. angular acceleration of flywheel.

Sol: Given $N = 600 \text{ rpm}$; $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$

$\omega = 62.84 \text{ rad/sec}$;

$T_{max} = 90 \text{ N-m}$; $m = 12 \text{ kg}$; $k = 80 \text{ mm} = 0.08 \text{ m}$.



(a)

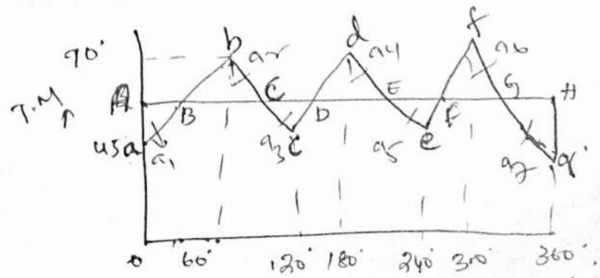


Fig ⓐ represents T-θ diagram for 3 cylinders in ... & Fig ⓑ represents resultant T-θ diagram for 3 cylinder

1. Power developed.

$P = T_{mean} \times \omega$

W.K.T, $\frac{W.D}{cycle}$ = Area of 3 D's

$= 3 \times \frac{1}{2} \times \pi \times 90$

$= 424$ N-m

$T_{mean} = \frac{W.D/cycle}{crank\ angle/cycle} = \frac{424}{2\pi} = 67.5$ N-m

$\therefore P = T_{mean} \times \omega = 67.5 \times 62.84 = 4240 W = 4.24 kW$.

2. Coeff of fluctuation of speed:

Let, $C_s \to$ coeff. of fluctn of speed. $E_f = I\omega^2 C_s$.

So, that, initially we have to find Max. fluctuation of energy.

from fig ⓑ we have to find,

a_1 = Area of triangle aAB $= \frac{1}{2} AB \times Aa$. ($\because AB = 30° = \frac{\pi}{3}$)

$= \frac{1}{2} \times \frac{\pi}{6} \times (67.5 - 45°) = 5.89$ N-m $= a_7$.

a_2 = Area of triangle Bbc $= \frac{1}{2} \times Bc \times bb'$. ($\because Bc = 60 = \pi/6$)

$= \frac{1}{2} \times \frac{\pi}{3} \times (90 - 67.5)$

$a_2 = 11.78$ N-m.

$a_2 = a_3 = a_4 = a_5 = a_6$.

(9)

Let the total energy at A = E

then the energy at B = $E - 5.89$.

" " $C = E - 5.89 + 11.78 = E + 5.89$.

" " $D = E + 5.89 - 11.78 = E - 5.89$

" " $E = E - 5.89 + 11.78 = E + 5.89$

" " $F = E + 5.89 - 11.78 = E - 5.89$

" " $G = E - 5.89 + 11.78 = E + 5.89$

" " $H = E + 5.89 - 5.89 = E$. = Energy at A.

∴ The max. Energy = $E + 5.89$

" Min. Energy = $E - 5.89$.

∴ $E_f = \Delta E$ = max. fluctuation of Energy

$= (E + 5.89) - [E - 5.89]$

$\Delta E = 11.78$ N-m.

W.K.T, max. fluctuation of energy

$E_f = \Delta E = I\omega^2 C_s$.

$11.78 = mk^2 . \omega^2 C_s$.

$11.78 = (12)(0.08)^2 \times (62.84)^2 C_s$.

$C_s = 0.04$ (or) 4%.

3. Coeff. of fluctuation of energy.

W.K.T, coeff. of fluctuation of energy,

$C_E = \frac{\text{max. fluctuation of energy}}{\text{W.D/cycle}} = \frac{11.78}{424}$

$= 0.0278$

$C_E = 2.78\%$

4. Max. angular acceleration of flywheel,

$\alpha \rightarrow$ angular acceleration.

$T_{max} - T_{mean} = I . \alpha . = mk^2 . \alpha$.

$90 - 67.5 = (12)(0.08)^2 . \alpha$

$\alpha = \frac{90 - 67.5}{0.077} \Rightarrow \alpha = 292 \ \frac{rad}{sec^2}$.

② A single cylinder, single acting four stroke engine develops 20KW at 300 rpm. The work done by the gases during the expansion stroke is three times the work done on the gases during compression stroke, the work done during suction stroke & exhaust stroke being negligible. If the total fluctuation of speed is not to ± 2% of mean speed & Turning moment diagram during compression & expansion is assumed to be triangular in shape. Find the moment of inertia of flywheel.

Sol: Given: $P = 20KW = 20\times10^3 W$; $N = 300$ rpm.

Total fluctuation of speed $(\omega_1-\omega_2)$ is not to exceed ±2% of mean speed.

$W_E = 3W_C$

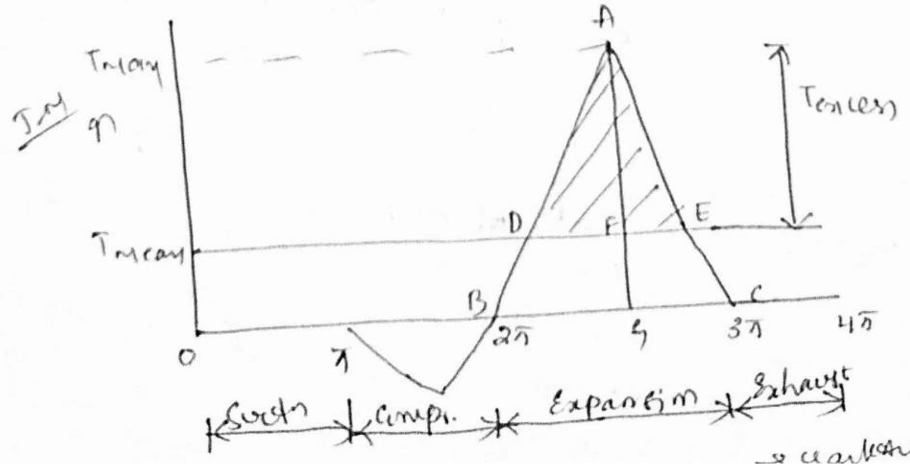
$\therefore \omega_1-\omega_2 = 4\omega$..

& co-effecient of fluctuation of speed,

$$C_S = \frac{\omega_1-\omega_2}{\omega} = 4\%$$

$$(or) \Rightarrow \frac{\omega_1-\omega_2}{\omega} = 0.04$$

The following will be the T-θ diagram for the four stroke engine neglecting suction and exhaust strokes.



(10).

W.K.T, for four stroke no. of working strokes / cycle.

$$n = \frac{N}{2} = \frac{300}{2} = 150.$$

$$\therefore \frac{W.D}{cycle} = P \times \frac{60}{n} = 20 \times 10^3 \times \frac{60}{150} \Rightarrow$$

$$\frac{W.D}{cycle} = 8000\, N\text{-}m. \quad (1).$$

Since workdone during suction & exhaust are negligible,

& net w.D/cycle ⇒

$$\Rightarrow W_E - W_C \qquad \because W_E = 3W_C.$$

$$\boxed{W_C = \frac{W_E}{3}}$$

$$= W_E - \frac{W_E}{3}$$

$$W.D/cycle = \frac{2W_E}{3} \quad -(2).$$

Equating (1) & (2).

$$8000 = \frac{2}{3} W_E$$

$$W_E = 12000\, N\text{-}m$$

W.K.T, work done during expansion stroke [W_E].

In order to get T_{max}.

12000 Area of Δ^{le} ABC

$$= \frac{1}{2} BC \times AG$$

$$12000 = \pi \times AG.$$

$$\therefore T_{max} = AG = \frac{12000 \times 2}{\pi} = 7638\, N\text{-}m.$$

$$\text{Now } T_{mean} = FG = \frac{W.D/cycle}{\text{Crank Angle/cycle}} = \frac{8000}{4\pi} = 637\, N\text{-}m.$$

$\therefore$ Tences = $AF = AG - FG$

$= 7638 - 637$

Tences = 7001 N-m.

Now, from similar Δles ADE & ABC.

$$\frac{DE}{BC} = \frac{AF}{FG}$$

(or) $DE = BC \times \frac{AF}{FG}$.

$= \pi \times \frac{7001}{637}$

$\therefore$ $DE = 2.88$ rad.

$\therefore$ The area above the mean represents max. fluctuation energy, $\therefore$ Max. fluctuation of Energy,

$\Delta E = E_f =$ Area of Δle ADE

$= \frac{1}{2} DE \times AF$

$= \frac{1}{2} \times 2.88 \times 7638$

$E_f = 10081$ N-m.

$\therefore$ Moment of inertia is to be calculated,

$E_f = I \omega^2 C_s$.

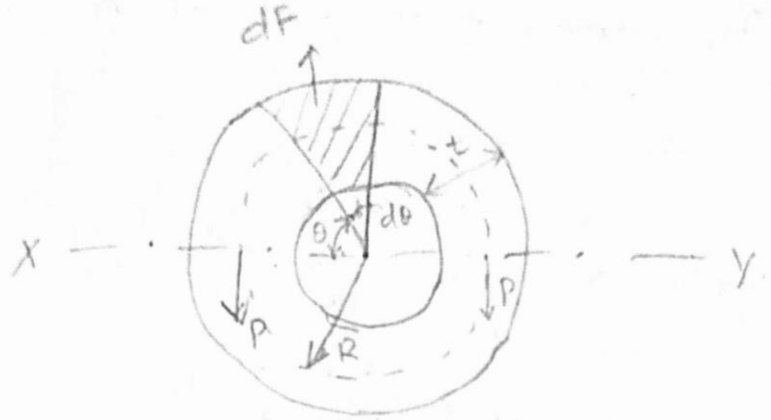
$10081 = I \cdot \left(\frac{2\pi 300}{60}\right)^2 \times 0.04$

$10081 = 39.5 I$

$I = 255.2$ kg-m²

(11)

Dimensions of the Flywheel Rim:-



Rim of a flywheel.

Consider a rim of the flywheel as shown,

Let, $D \rightarrow$ Mean Diameter of rim, mts.

$R \rightarrow$ Mean Radius of rim, mts.

$A \rightarrow$ cross-sectional area of rim, m^2.

$\rho \rightarrow$ Density of rim material, kg/m^3.

$N \rightarrow$ speed of the flywheel, rpm

$\omega \rightarrow$ Angular ~~speed~~ velocity of flywheel. rad/sec

$v \rightarrow$ linear velocity at mean radius in m/s

$= \omega \cdot R = \frac{\pi D N}{60}$, m/s.

$\sigma \rightarrow$ Tensile stress or hoop stress, N/m^2. due to centrifugal force.

Consider a small element of the rim as shown shaded.

- let it subtends at an angle of $\delta\theta$ at the centre flywheel.

Volume of small element.

$$dV = A \times R \cdot d\theta.$$

∴ mass of the small element,

$$dm = \text{Density} \times \text{Volume}$$

$$dm = \rho \times A \times R \cdot d\theta.$$

Centrifugal force on the element, acting radially outwards,

$$dF = dm \times R \cdot \omega^2$$

$$= \rho \cdot A \cdot R \times d\theta \cdot R \cdot \omega^2$$

$$dF = \rho \times A \cdot R^2 \omega^2 \cdot d\theta$$

Vertical component of force of the element dF

$$= dF \cdot \sin\theta = \rho A R^2 \omega^2 d\theta \sin\theta$$

∴ Total vertical upward force tending to burst rim across the dia. X & Y.

$$= \rho A \cdot R^2 \omega^2 \int_0^{\pi} \sin\theta \cdot d\theta.$$

$\cos\pi = -1$; $\cos 0 = 1$

$$\left[\cos\pi - (-\cos 0)\right]$$

$1 + 1 = 2$

$$= \rho A R^2 \omega^2 \left[-\cos\theta\right]_0^{\pi} = 2\rho A R^2 \omega^2. \quad -(1)$$

The vertical upward force will produce hoop stress (or) circumferential force, & it is resisted by 2P.

$$2P = 2 \cdot \sigma \cdot A \quad -(2).$$

$A = b \times t$

Equating (1) & (2).

$$2\rho A R^2 \omega^2 = 2 \cdot \sigma \cdot A$$

$$\sigma = \rho R^2 \omega^2 = \rho v^2$$

$\because v = R\omega$; $\omega = \frac{v}{R}$

$$v = \sqrt{\frac{\sigma}{\rho}}$$

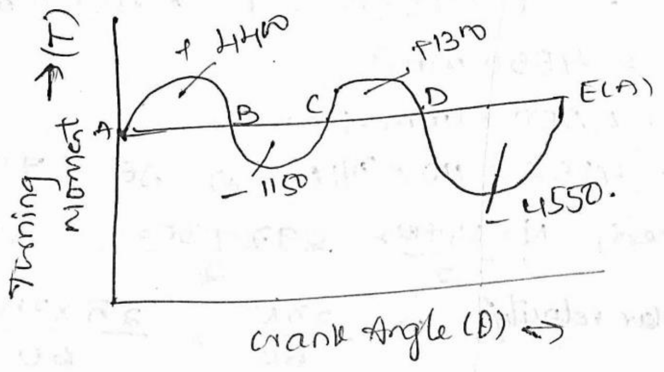
We know mass of rim, m = Volume × density = $\pi \cdot D A \cdot \rho$. ∴ $A = \frac{m}{\pi D \rho}$

Date................................ Page No. 6(a)

(A). In a turning moment dia, the areas above & below the mean torque line taken in order are 4400, 1150, 1300 & 4550 mm^2 resp. The scales of the turning moment diagram are;

Turning moment, 1mm = 100 N-m; Crank Angle, 1mm = 1°. Find the mass of the fly wheel reqd. to keep the speed b/w 297 & 303 rpm. If the radius of gyration is 0.525 m.

Sol:



Sol: Given Data:

$N_1 = 297$ & $N_2 = 303$ rpm., $k = 0.525$ m

Turning moment, 1mm = 100 N-m

Crank Angle, 1mm = 1° = $(1 \times \pi/180)$.

Let the total energy at, A = E,
the energies at diff. pts.

at, A = E.

B = E + 4410.

C = E + 4410 − 1150 = E + 3250.

D = E + 4410 − 1150 + 1310 = E + 4550 (max. energy)

E = E + 4410 − 1150 + 1310 − 4550 = E. (min energy)

W.K.T, max. fluctuation of energy,

ΔE = max. energy − min. energy

= E + 4550 − E = 4550 mm².

$\Delta E = 4550\ mm^2$

$= 4550 \times 1mm \times 1mm$

$= 4550 \times 110 \times \pi/180 \Rightarrow \Delta E = 7939.75\ N\text{-}m.$

mean speed, $N = \frac{N_1 + N_2}{2} = \frac{297 + 303}{2} = 300$ rpm.

mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} \Rightarrow \omega = 31.416\ rad/sec$

coeff. of fluctuation of speed, $C_s = \frac{N_1 - N_2}{N} = \frac{303 - 297}{300} = 0.02.$

W.K.T max. fluctuation of energy,

$\Delta E = I\omega^2 C_s.$

$\Delta E = mk^2 \omega^2 C_s.$

$7939.75 = m(0.525)^2 \times (31.416)^2 \times (0.02)$

m = 1459.3 Kg

Date..............................

Page No. 6(c)

(3). In a machine, the intermittent operation demand the torque to be applied as follows:

- During the first half revolution, the torque increases from 1200 N-m to 3600 N-m.
- During the next one revolution, the torque remains constant.
- During the next one revolution, the torque decreases uniformly from 3600 N-m to 1200 N-m.
- During last 1½ revolution, the torque remains constant.

Thus a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 240 rpm. A flywheel of mass 2000 kg and radius of gyration 600 mm is fitted to shaft.

Determine: (i) The power of motor, P.
(ii) The total coefficient of fluctuation of speed of m/c shaft.

Sol: Given:

$N_{mean} = 240$ rpm; $m = 2000$ kg;

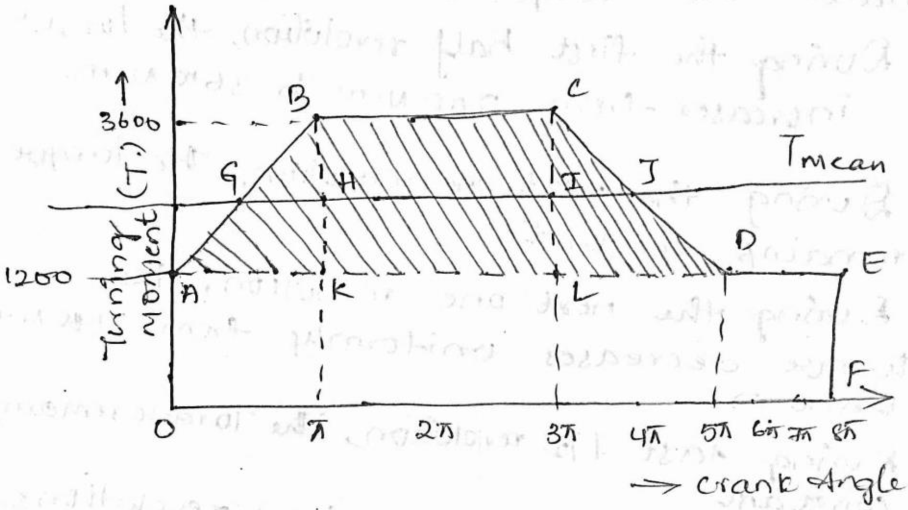
$K = 600$ mm $= 0.6$ m

Mean Angular velocity, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$

$\omega = 25.13$ rad/sec.

Moment of Inertia, $I = m \cdot k^2$
$= (2000)(0.6)^2 = 720$ kg-m².

The turning moment diagram for complete cycle is as below,



We know that the torque reqd. for one complete cycle

= Area of OABCDEFO.

= Area OAEF + Area ABK + Area BCLK + Area CDL

$= (OF \times OA) + \left[\frac{1}{2} \times AK \times BK\right] + \left[KL \times CL\right] + \left[\frac{1}{2} \times DL \times CL\right]$

$= \left[8\pi \times 1200\right] + \left[\frac{1}{2} \times \pi \times (3600-1200)\right] + \left[2\pi \times (3600-1200)\right] + \left[\frac{1}{2} \times 2\pi \times (3600-1200)\right].$

$= 30159.28 + 3769.91 + 15079.6 + 7539.8$

T = 56548.61 N-m. — (a)

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If T_{mean} is the mean torque, then torque required for 1 complete cycle

$$\tau = T_{mean} \times 8\pi \quad \text{— (b)}$$

Equating eq^n (a) & (b),

$$56548.61 = T_{mean} \times 8\pi$$

$$T_{mean} = 2250 \text{ N-m}$$

(i) Power of the motor :

W.K.T, Power, $P = \frac{2\pi N T}{60}$

$$P = \frac{2\pi \times 200 \times 2250}{60}$$

$$P = 47123.8 \text{ W}$$

$$= 47.124 \text{ kW}$$

(ii) Total coeffecient of fluctuation of Speed $[C_s]$:-

$$\Delta E = I\omega^2 C_s$$

we need to find out the fluctuation of energy ΔE.

for that first need to find out the values of GH & IJ.

from similar triangles ABK & GBH, we get,

$$\frac{GH}{AK} = \frac{BH}{BK} \quad (\text{or}) \quad \frac{GH}{\pi} = \frac{(3600-2250)}{(3600-1200)}$$

or $GH = 1.767$ rad.

from similar triangles CIJ and CLD.

$$\frac{IJ}{LD} = \frac{CI}{CL}$$

$$\frac{IJ}{2\pi} = \frac{(3600-2250)}{(3600-1200)}$$

$IJ = 3.534$ rad.

W.K.T the area above mean torque line represents max. fluctuation of energy [ΔE]

∴ ΔE = Area of GBCJ = Area GBH + Area BCIH + Area CJI

$= \left[\frac{1}{2} \times GH \times BH\right] + \left[HI \times CI\right] + \left[\frac{1}{2} \times IJ \times CI\right].$

$= \left[\frac{1}{2} \times 1.767 \times (3600-2250)\right] + \left[2\pi \times (3600-2250)\right] + \left[\frac{1}{2} \times 3.534 \times (3600-2250)\right]$

$\Delta E = 12060.475$ N-m.

& also $\Delta E = I\omega^2 C_s$.

$12060.475 = 720(20.94)^2 \times C_s$.

∴ coefficient of fluctuation of speed,

$C_s = 0.0382$ (or) 3.8%

Date................................ Page No..........

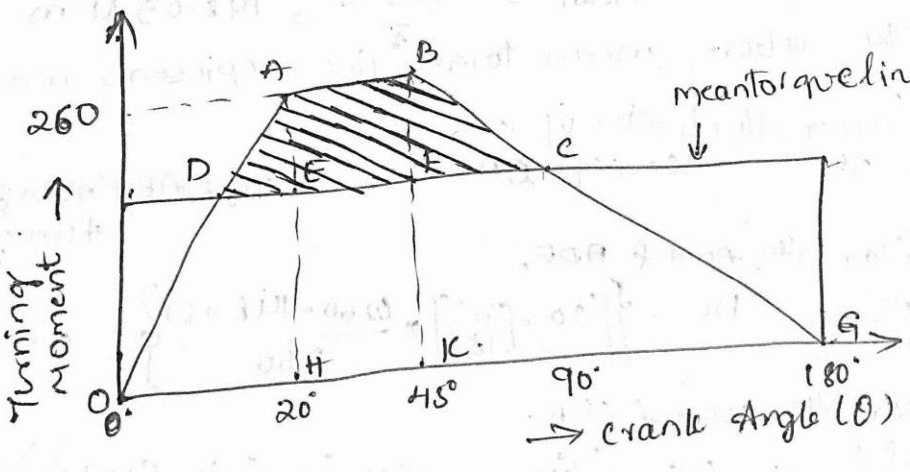
(4) The variation of crank shaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° & 180° and as 260 N-m for crank angles 20° & 45°, the intermediate portions of torque graph being straight lines. The cycle is being repeated in every half revolution. The average speed is 600 rpm. Supposing that engine drives a machine at constant torque, determine the mass of flywheel of radius of gyration 250 mm, which must be provided so that total variation of speed shall be one percent.

Sol: Given Data:

$N = 600$ rpm; $K = 250$ mm $= 0.25$ m

$C_s = 1\% = 0.01$

The turning moment diagram,



Work done for half revolution

= Area of turning moment diagram

= Area of OABG.

= [Area of OAH] + [Area of HABK] + [Area of KBG]

$$= \left[\frac{1}{2}\left(20^\circ \times \frac{\pi}{180}\right) \times 260\right] + \left[(45^\circ - 20^\circ)\left[\frac{\pi}{180}\right] \times 260\right] + \left[\frac{1}{2} \times (180 - 45^\circ)\left[\frac{\pi}{180}\right] \times 260\right].$$

$= 465.13$ N-m.

If T_{mean} is the mean torque, then work done corresponding of mean torque for half revolution, is given by,

$$T_{mean} \times \pi = \text{work done} = 465.13.$$

$$T_{mean} = \frac{465.13}{\pi} = 148.05 \text{ N-m}.$$

Since the above mean torque line represents max. fluctuation of energy,

∴ max. fluctuation of energy,

ΔE = Area of ΔABC = Area of ΔAE + Area of EABF + Area of CFB.

From similar Δ^{les} AOH & ADE,

$$\frac{DE}{OH} = \frac{AE}{AH} \Rightarrow DE = \left\{\left[20^\circ \times \left[\frac{\pi}{180}\right]\right] \times \frac{(260 - 148.05)}{260}\right\} = 0.15 \text{ rad}.$$

From similar Δ^{les}, BGK & CFB.

$$\frac{FC}{KG} = \frac{BFB}{BK} \Rightarrow FC = \left\{\left[(180 - 45^\circ)\left[\pi/180\right]\right] \times \frac{(260 - 148.05)}{260}\right\} = 1.014 \text{ rad}.$$

$$\therefore \Delta E = \left[\frac{1}{2} \times 0.15 \times 119.95\right] + [0.4363 \times 111.95] \times \left[\frac{1}{2} \times 111.95 \times 1.014\right]$$

$\Delta E = 114$ N-m.

W.K.T, $\Delta E = m \cdot k^2 \omega^2 C_s$.

$114 = m \cdot (0.25)^2 \cdot (62.83)^2 (0.01)$.

$m = 46.2$ kg

Difference between Governor and Flywheel:

A governor controls the speed of the output shaft within close limits, but its action depends upon controlling the amount of working fluid to the engine as required by the load on the engine. The flywheel, on the other hand, serves only to smooth out the energy transfer in each energy cycle. For example, if an engine is operating at quarter load, with the governor in a particular position controlling the amount of working media to the engine; the flywheel would take care of redistributing the energy throughout a cycle. If the load was increased to full load the governor action would permit more working fluid to the engine maintaining the speed of the engine, but when balance of working fluid to the engine and load on the engine was reached, the flywheel would continue its action of redistributing the energy throughout a cycle. Changes of seed in an engine will cause the governor to respond and attempt to do the flywheels job.

Usually, the effect of the governor is disregarded in the design of the flywheel. The flywheel analysis is limited to engines receiving power at a variable rate and delivering it to a shaft at an approximately constant rate.

Crank effort diagrams or Turing Moment diagrams:

It is the graphical representation of turning moment or crank effort for the various positions of the crank. The TM is plotted on 'y' axis and crank angle on 'x' axis.

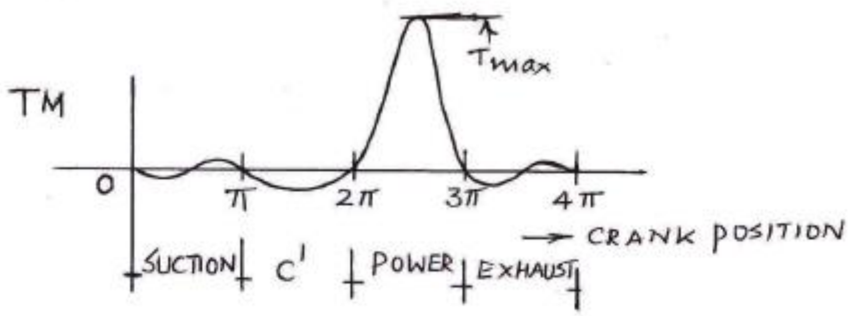
Uses of turning moment Diagram:

1) The area under the turning moment diagram represents work done per cycle. The area multiplied by number of cycles per second gives the power developed by the engine.

2) By dividing the area of the turning moment diagram with the length of the base we get the mean turning moment. This enables us the find the fluctuation of energy.

3) The max. ordinate of the turning moment diagram gives the maximum torque to which the crank shaft is subjected. This enables us the find diameter of the crank shaft.

TMD for a four stroke I.C. Engine



We know that four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions (4p or 720°). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke therefore a negative loop is formed as shown in figure. During compression stroke the work done on engine the gases therefore a higher negative loop is obtained. During expansion or working stroke the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases; therefore a negative loop is firmed.

Problem 2

The torque delivered by two stroke engine is represented by T = 1000+300 sin 2θ-500 cos 2θ where θ is angle turned by the crack from inner dead under the engine speed. Determine work done per cycle and the power developed.

Solution

θ, deg.	$T, N-m$
0	500
90	1500
180	500
270	1500
360	500

Work done / cycle = Area under the turning moment diagram.

$$= \int_0^{2\pi} T \, d\theta$$

$$= \int_0^{2\pi} (1000 + 300\sin 2\theta - 500\cos 2\theta)\, d\theta$$

$$= 2000\pi \; N-m$$

$$T_{mean} = \frac{W.D / cycle}{2\pi}$$

$$= \frac{2000\pi}{2\pi} = 1000 \; N-m$$

$$\text{Power developed } = T_{mean} \times \omega_{mean}$$

$$= 1000 \times \frac{2\pi \, N}{60}$$

$$= 1000 \times \frac{2\pi \times 200}{60}$$

$$= 26179 W$$

Problem: 3

The turning moment curve for an engine is represented by the equation,

$T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find:

1. Power developed by the engine;
2. Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not the exceed 1% of mean speed which is 180 r.p.m. and
3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre.

Solution:

Given, $T = (20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m ;

N = 180 r.p.m. or $\omega = 2\pi \times 180/60 = 18.85$ rad/s

Since the total fluctuation of speed (ω_1-ω_2) is 1% of mean speed (ω), coefficient of fluctuation of speed,

$$\delta = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. Power developed by the engine.

Work done per revolution

$$= \int_0^{2\pi} T\,d\theta = \int_0^{2\pi} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\,d\theta$$

$$= \left[20000\,\theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2}\right]_0^{2\pi}$$

$$= 20000 \times 2\pi = 40\ 000\ \pi\ N-m$$

Mean resisting torque of the engine,

$$T_{mean} = \frac{Work\ done\ per\ revolution}{2\pi} = \frac{40\ 000\ \pi}{2\pi} = 20000\ N-m$$

Power developed by the engine

$$= T_{mean}.\omega = 20\ 000 \times 18.85 = 377\ 000 W = 377 kW.$$

2. Moment of inertia of the flywheel

The turning moment diagram for one stroke (i. e. half revolution of the crankshaft) is shown in the Fig. Since at points *B* and *D*, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

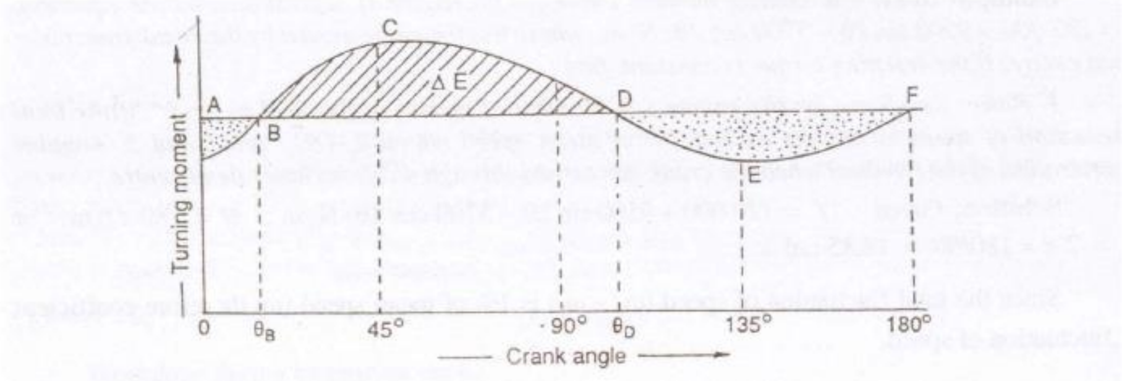
$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000$$

or $$9500 \sin 2\theta = 5700 \cos 2\theta$$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$\therefore$ $$2\theta = 31^\circ \; or \theta = 15.5^\circ$$

$\therefore$ $$\text{i.e.,} \theta_B = 15.5^\circ \; and \; \theta_D = 90^\circ + 15.5^\circ = 105.5^\circ$$



Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta$$

$$= \int_{15.5^\circ}^{105.5^\circ} (20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\ 000) d\theta$$

$$\Delta E = \int_{\theta_B}^{\theta_D} (T - T_{mean}) d\theta = \left[-\frac{9500 \sin 2\theta}{2} - \frac{5700 \cos 2\theta}{2} \right]_{15.5^\circ}^{105.5^\circ} = 11078 \; N - m$$

Maximum fluctuation of energy (ΔE),

$$11\,078 = I.\omega \;\; \delta = I(18.85)^2 \, 0.01 = 3.55 \, I$$

$$I = 11078/3.55 = 3121 \text{ kg-m}^2.$$

3. Angular acceleration of the flywheel

Let α = Angular acceleration of the flywheel, and

θ = Angle turned by the crank from inner dead centre = 45°... (Given)

The angular acceleration in the flywheel is produced by the excess torque over the mean torque. Excess torque at any instant,

$$T_{excess} = T - T_{mean}$$

$$20\ 000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\ 000$$

$$9500 \sin 2\theta - 5700 \cos 2\theta$$

$\therefore$ Excess torque at 45°= 9500 sin 90° - 5700 cos 90° =9500Nm

We also know that excess torque= $I.\alpha$ = 3121 xα

From equations (i) and (ii),

α = 9500 / 3121 = 3.044 rad/s^2.

Problem 5: The equation of the turning moment diagram of a three crank engine is 21000+7000 sin3θ Nm. Where θ in radians is the crank angle. The moment of inertia of the flywheel is $4.5 \times 10^3\ Nm^2$ and the mean engine speed is 300 rpm. Calculate the power of the engine and the total percentage fluctuation of speed of the flywheel (i) if the resisting torque is constant (ii) if the resisting torque is 21000 + 3000 sinθ Nm.

a) $T_m = 21000\ Nm.$

$$\text{Power} = \frac{2\pi \times 21000 \times 300}{60} = 660\,kW.$$

b) (i) $\Delta E = \int_0^{\frac{\pi}{3}} 7000 \sin 3\theta d\theta = 4666.7\ Nm.$

$$\therefore \text{Total percent fluctuation of speed} = \frac{100\ \Delta E}{I\omega^2_{mean}}$$

$$= \frac{100 \times 4666.7 \times 9.8}{45 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.04\%$$

(ii) Engine torque = load torque, at crack angles given by

7000 sin3θ = 3000 sinθ

i.e., 2.33 (3sinθ - 4 sin^{3}θ) = sinθ

One solution is sinθ =0, i.e., θ = 0 and 180º, and the other is sinθ = ±0.803, i.e., θ = 53°24' or 126°36' between 0° and 180º. The intersections are shown in figure and the areas between the curves represent increase or decrease of total energy. The numerically longest is between θ = 53° 24' and 126º36'.

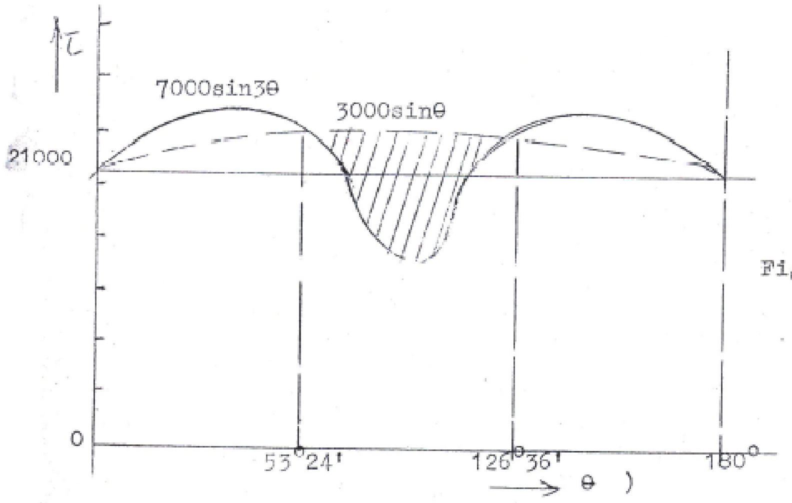


Fig.66.

$$i.e.,\ \Delta E = \int_{53°24'}^{126°36'} (7000 \sin 3\theta - 3000 \sin\theta)\, d\theta$$

$$= 7960 \text{ Nm.}$$

Therefore, the total (percentage) fluctuation of speed $\dfrac{100\ \Delta E}{I\omega^2_{mean}}$

$$= \frac{100 \times 7960 \times 9.8}{4.5 \times 10^3 \times \left(\frac{300\pi}{30}\right)^2}$$

$$= 1.65\%$$

A machine punching 3.8 cm dia hole in a 3.2 cm thick plate does 600 J of work / sq. cm of sheared area. The punch has a stroke of 10.2 cm and punches 6 holed / min. The maximum speed of the flywheel at its radius of gyration is 27.5 m/s. Find the mass of the flywheel so that its speed at the same radius does not fall below 24.5 m/s. Also determine the power of the motor, driving this machine.

d = 3.8cm, t = 3.2 cm, A= 38.2 cm^2

Energy required / punch = 600 x 38.2 = 22.920 J

$$\text{Assuming, } \frac{(\theta_2 - \theta_1)}{(2\pi)} = \frac{t}{2S} = \frac{3.2}{20.4}$$

$$\therefore (\Delta K_E)_{max} = E\left[1 - \frac{t}{2S}\right] = \frac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right)$$

$$= 22.920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} mk^2\left(\omega_{max}^2 - \omega_{min}^2\right)$$

$$V_{max} = k\ \omega_{max} = 27.5 m/s$$

$$V_{min} = k\ \omega_{min} = 24.5 m/s$$

We get,

$$22920\left[1 - \frac{3.2}{20.4}\right] = \frac{1}{2} m\left(27.5^2 - 24.5^2\right) = \frac{1}{2} m\ 158$$

$$\therefore\ m = 244 kg.$$

The energy required / minute is $6 \times 22920 J$

$$\therefore\ Motor\ \ power\ = \frac{6 \times 22920}{1000 \times 60} k\omega = 2.292 kW$$

A riveting machine is driven by a constant torque 3 kW motor. The moving parts including the flywheel are equivalent to 150 kg at 0.6 m radius. One riveting operation takes 1 second and absorbs 10 000 N-m of energy. The speed of the flywheel is 300 r.p.m. before riveting. Find the speed immediately after riveting. How many rivets can be closed per minute.

Solution.

Given: $P = 3\ kW$; $m = 150\ kg$; $k = 0.6$m; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$rad/s

Speed of the flywheel immediately after riveting

Let ω_2 = Angular speed of the flywheel immediately after riveting.

We know that, energy supplied by the motor,

$$E_2 = 3\ kW = 3000\ W = 3000\ N-m/s \qquad (\because 1W = 1\ N-m/s)$$

But, energy absorbed during one riveting operation which takes 1 second,

$$E_1 = 10\ 000\ N-m$$

∴ Energy to be supplied by the flywheel for each riveting operation per second or the maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 10\ 000 - 3000 = 7000 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$7000 = \frac{1}{2} \times m.k^2 \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times 150\ (0.6)^2\ \left[(31.42)^2 - (\omega_2)^2\right]$$

$$= 27\left[987.2 - (\omega_2)^2\right]$$

$$\therefore \qquad (\omega_2)^2 = 987.2 - 7000/27 = 728 \ or\ \omega_2 = 26.98\ rad/s$$

Corresponding speed in r.p.m.,

$$N_2 = 26.98 \times 60 / 2\pi = 257.6\ r.p.m.$$

Number of rivets that can be closed per minute.

Since, the energy absorbed by each riveting operation which takes 1 second is 10 000 N-m, therefore number of rivets that can be closed per minute,

$$= \frac{E_2}{E_1} \times 60 = \frac{3000}{10\ 000} \times 60 = 18\ rivets$$