COURSE CONTENT

SUBECT: DESIGN OF MACHINE ELEMENT

UNIT-V

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Shafts:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used. An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

Types of Shafts

The following two types of shafts are important from the subject point of view:

1. ***Transmission shafts*.** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts*.** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

a) Solid shaft:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

T

$$\overline{J} \quad r \; \overline{}$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre = d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J \; \frac{}{32} \; d^4$$

Then we get, $$T \quad \frac{d^3}{16}$$

From this equation, diameter of the solid shaft (d) may be obtained.

b) Hollow Shaft:

We also know that for hollow shaft, polar moment of inertia,

$$J \quad \frac{(d_0)^4 \;\; (d_i)^4}{32}$$

Where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_0 / 2$.

Substituting these values in equation ***(i)***, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o Now the equation ***(iii)*** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

From the equations, the outside and inside diameter of a hollow shaft may be determined. It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

2. The twisting moment (*T*) may be obtained by using the following relation: We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

Where *T* = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$T = (T_1 - T_2)R$

Where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and *R* = Radius of the pulley.

Shafts Subjected to Bending Moment Only

a) Solid Shaft:

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where *M* = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation, σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (*d*) may be obtained.

b) Hollow Shaft:

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \qquad \text{...(where } k = d_i / d_o)$$

And $y = d_0 / 2$

Again substituting these values in equation, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \, (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (*do*) may be obtained.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine’s theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

a) Solid Shaft:

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of σ_b and τ

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\text{or} \qquad \frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces **the same shear stress (τ) as the actual twisting moment. By limiting**

the maximum shear stress (τ_{max}) equal to the allowable shear stress (**τ**) for the material, the equation **(*i*)** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

$$\text{or} \qquad \frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

The expression $\frac{1}{2}[M + \sqrt{M^2 + T^2}]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (**σ_b**) as the actual bending moment. By limiting the m**aximum normal stress [σ_b(*max*)] equal to the allowable bending stress (σ*b*)**, then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

b) Hollow shaft:

In case of a hollow shaft, the equations ***(ii)*** and ***(v)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) - \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \, (1 - k^4)$$

It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

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Problem:

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another p ulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and **μ** = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in ten sion and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution. Given : $AB = 800$ mm ; $\alpha_C = 20°$; $D_C = 600$ mm or $R_C = 300$ mm ; $AC = 200$ mm ; $D_D = 700$ mm or $R_D = 350$ mm ; $DB = 250$ mm ; $\theta = 180° = \pi$ rad ; $W = 2000$ N ; $T_1 = 3000$ N ; $T_1/T_2 = 3$; $\tau = 40$ MPa $= 40$ N/mm^2

The space diagram of the shaft is shown in Fig (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right)R_D$$

$$= 3000\left(1 - \frac{1}{3}\right)350 = 700 \times 10^3 \text{ N-mm} \qquad ...(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. (*b*).

Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear *C*,

$$W_C = \frac{F_{tc}}{\cos\alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at *C*,

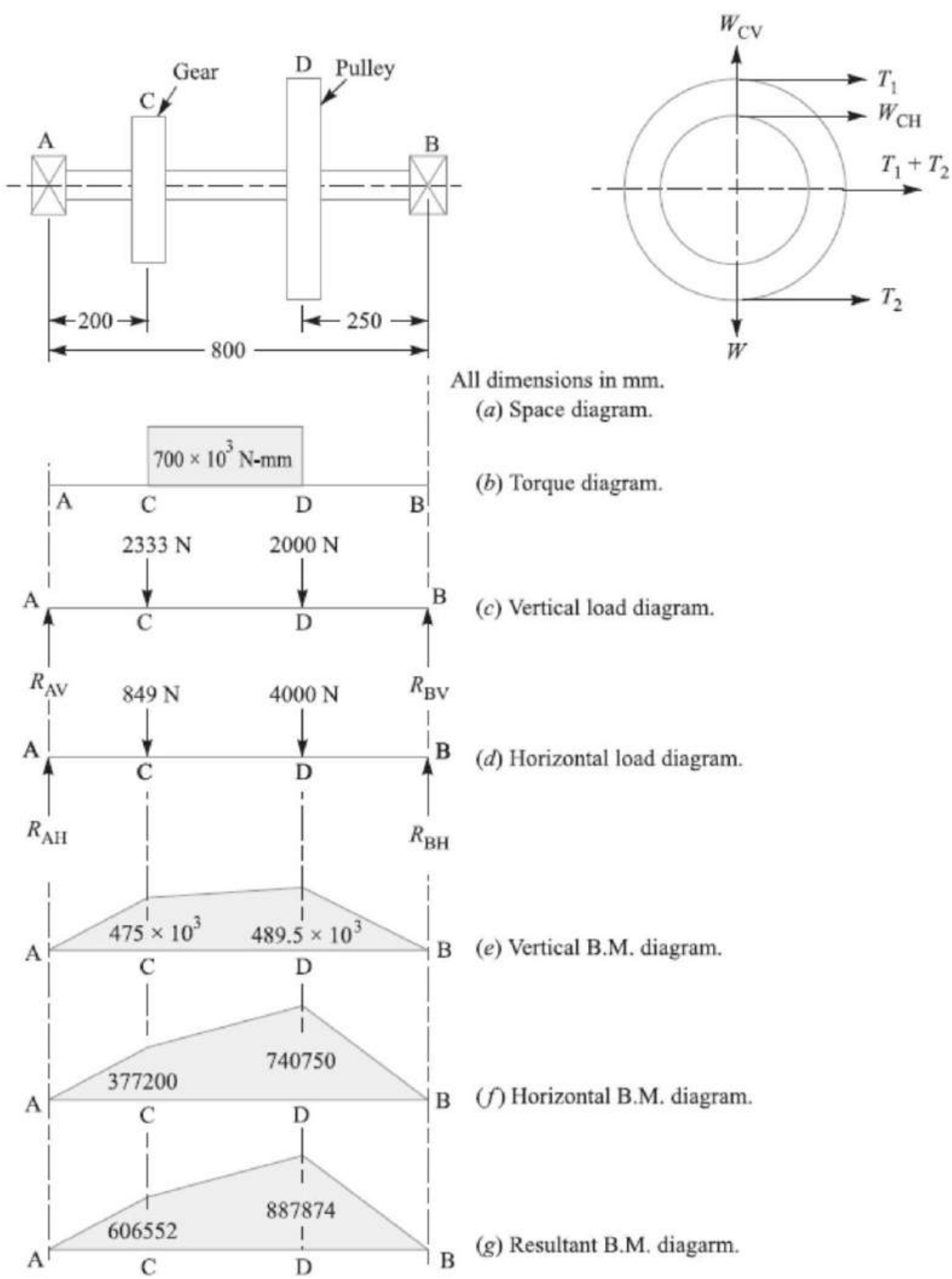
$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at *C*,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$



All dimensions in mm.

(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagarm.

∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram
respectively.

Now let us find the maximum bending mome

First of all considering the vertical loading at bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4$$

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\ (800 - 250)$$

$$= 1\ 566\ 600$$

$$\therefore \quad R_{BV} = 1\ 566\ 600 / 800 =$$

and $$R_{AV} = 4333 - 1958 = 2$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 200 = 237$$

$$= 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 250 = 195$$

The bending moment diagram for vertical loa

Now consider the horizontal loading at C and A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 48$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\ (800 - 250)$$

$$\therefore \quad R_{BH} = 2\ 369\ 800 / 800 =$$

and $$R_{AH} = 4849 - 2963 = 1$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 200 = 188$$

B.M. at D, $$M_{DH} = R_{BH} \times 250 = 296$$

The bending moment diagram for horizontal

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_C}$$

$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M}$$

$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig
maximum at D, therefore

Maximum B.M., $$M = M_D = 887\ 874 \text{ N-}$$

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\,874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Problem:

A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm m odule is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $AB = 750$ mm ; $T_D = 30$; $m_D = 5$ mm ; $BD = 100$ mm ; $T_C = 100$; $m_C = 5$ mm ; $AC = 150$ mm ; $\tau = 54$ MPa $= 54$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

$\therefore$ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716×10^3 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

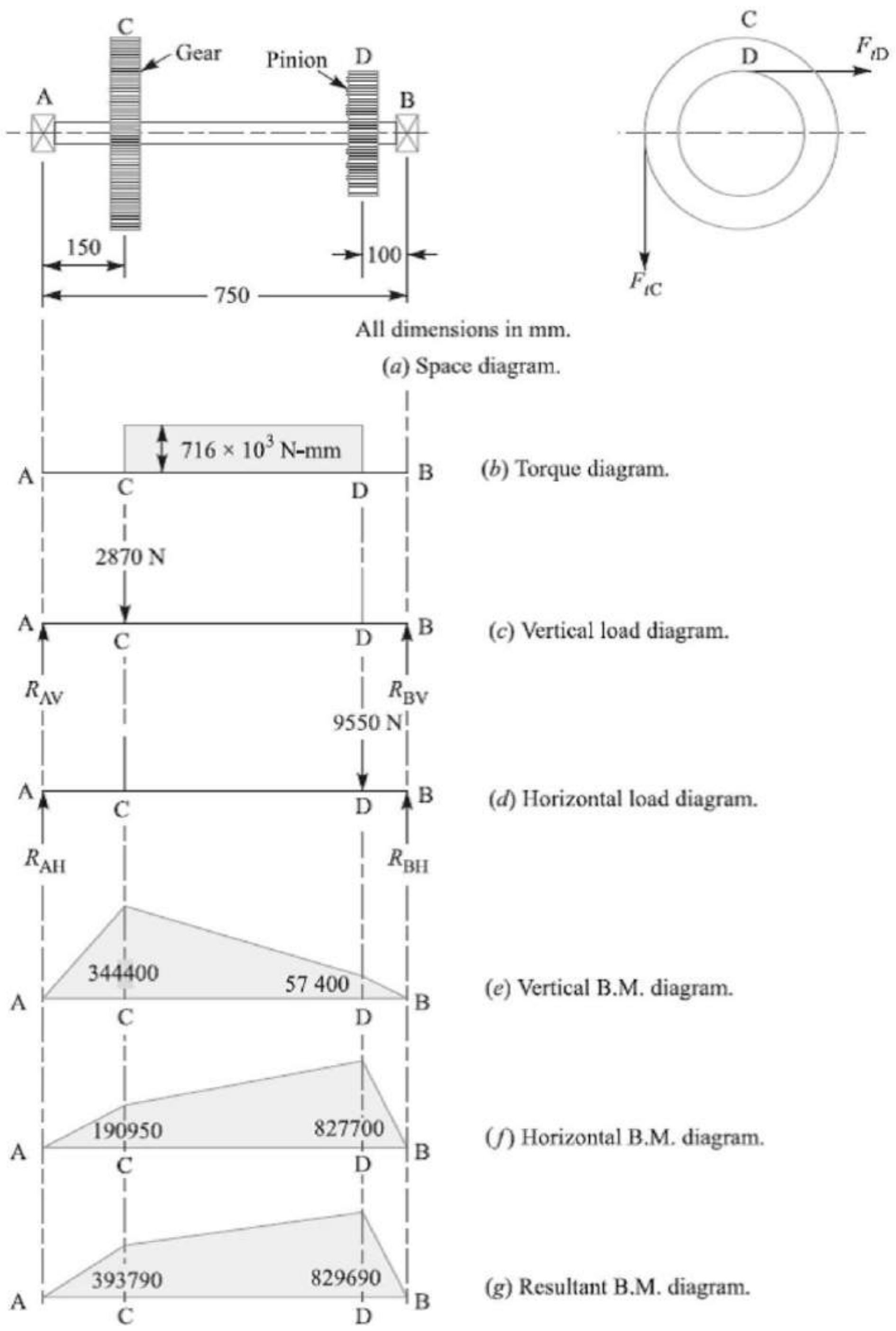
Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 750 = 2870 \times 150$$



(*a*) Space diagram.

(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

$\therefore \quad R_{BV} = 2870 \times 150 / 750 = 574$ N

and $R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $\quad M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\,400$ N-mm

B.M. at D, $\quad M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\,400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 750 = 9550\,(750 - 100) = 9550 \times 650$$

$$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277 \text{ N}$$

and $\quad R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $\quad M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\,950$ N-mm

B.M. at D, $\quad M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\,700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\,400)^2 + (190\,950)^2}$$
$$= 393\,790 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\,400)^2 + (827\,700)^2}$$
$$= 829\,690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 829\,690 \text{ N-mm}$$

Let $\quad d$ = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\,690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$$\therefore \quad d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$$

or $\quad d = 47$ say 50 mm **Ans.**

Shafts Subjected to Axial Load in addition to Combined Torsion and Bennding Loads:

When the shaft is subjected to a n axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and sha fts for driving worm gears, then the stress due to axial load must be added to the bending stress (B). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

And stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi (d_o)^2 (1 - k^2)} \qquad \text{... } (\because k = d_i/d_o)$$

Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32 M_1}{\pi d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi (d_o)^3 (1 - k^4)} + \frac{4F}{\pi (d_o)^2 (1 - k^2)}$$

$$= \frac{32}{\pi (d_o)^3 (1 - k^4)}\left[M + \frac{F d_o (1 + k^2)}{8}\right] = \frac{32 M_1}{\pi (d_o)^3 (1 - k^4)}$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***COLUMN FACTOR*** **(■) must be introd uced to take the column effect into account.** Therefore, Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2}$$

or

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \, (1 - k^2)}$$

The value of column factor (▒) for compressive loads* may be obtained from the following relation :

Column factor,

$$\alpha = \frac{1}{1 - 0.0044\,(L/K)}$$

This expression is used when the slenderness ratio (*L* / *K*) is less than 115. When the slenderness ratio (*L* / *K*) is more than 115, then the value of column factor may be obtained from the following relation:

Column factor, α)mustbeintroducedtotakethecolumneffectintoaccount.

$$\alpha = \frac{\sigma_y \, (L/K)^2}{C \pi^2 E}$$

Where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_E) and equivalent bending moment (M_E) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha\, F\, d_o\, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o\, (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o\, (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b\, (d_o)^3\, (1 - k^4)$$

It may be noted that for a solid shaft, $K = 0$ and $D_0 = D$. When the shaft carries no axial load, then $F = 0$ and **when the shaft carries axial tensile load, then α)mustbeintroducedtotakethecolumneffectintoaccount. = 1.**

Problem:

A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maxi mum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diam eter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.

SOLUTION. Given: T = 1.5 kN-m = 1.5×10^3 N-m ; M = 3 kN-m = 3×10^3 N-m ; F = 10 kN = 10×10^3 N ; k = d_i / d_o = 0.5 ; d_o = 80 mm = 0.08 m

Let **τ = Shear stress induced in th e shaft.**

Since the load is applied gradually, therefore from DDB, we find that K_m =1.5 ; and K_t = 1.0 We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o \, (1+k^2)^2}{8}\right] + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 \, (1+0.5^2)^2}{8}\right] + (1 \times 1.5 \times 10^3)^2}$$

$$= \sqrt{(4500+125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1-k^4) = \frac{\pi}{16} \times \tau \, (80)^3 \, (1-0.5^4) = 94\,260 \, \tau$$

$$\therefore \quad \tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6 \text{ MPa} \; \textbf{Ans.}$$

Problem:

A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart an d it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN.

Determine:

1. The maximum shear stress de veloped in the shaft, and

2. The angular twist between the bearings.

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\,460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\,500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L/K = 6/0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \quad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also

$$k = d_i/d_o = 0.3/0.5 = 0.6$$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 \quad (K_t \times T)^2}$$

$$= \sqrt{\left[\ldots \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{\ldots}\right]^2 \ldots}$$ [illegible]

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. **Torsional rigidity.** The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where θ = Torsional deflection or angle of twist in radians, T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation, G = Modulus of rigidity for the shaft material, and L = Length of the shaft.

2. **Lateral rigidity.** It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *I.E.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

BIS codes of Shafts

The standard sizes of transmission shafts are:

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps. The standard length of the shafts are 5 m, 6 m and 7 m.

Problem:

A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa = 84×10^9 N/m^2 = 84×10^3 N/mm^2

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

$\therefore$ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\ (35)^3 = 8420\ \tau$$

$\therefore$ $\tau = 47\ 740 / 8420 = 5.67$ N/mm^2 = 5.67 MPa **Ans.**

Problems:

Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diiameter of the hollow shaft being half the exterrnal diameter. Both the shafts have the same material and length.

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1/2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad \text{...(i)}$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad \text{...(ii)}$$

Since both the shafts have the same material and length, therefore by dividing equation (*i*) by equation (*ii*), we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad \text{...}(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \quad \text{...(iii)}$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad \text{...(iv)}$$

Dividing equation (*iii*) by equation (*iv*), we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \quad \text{...}(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

$\therefore$ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(v)}$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \quad \text{...}(vi)$$

Dividing equation (v) by equation (vi), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \quad \text{...}(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Shaft Coupling

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(a) Sleeve or muff coupling.

(b) Clamp or split-muff or compression coupling,

and **(*c*)** Flange coupling.

2. Flexible coupling. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(a) Bushed pin type coupling,

(b) Universal coupling,

and **(*c*)** Oldham coupling.

Sleeve or Muff-coupling

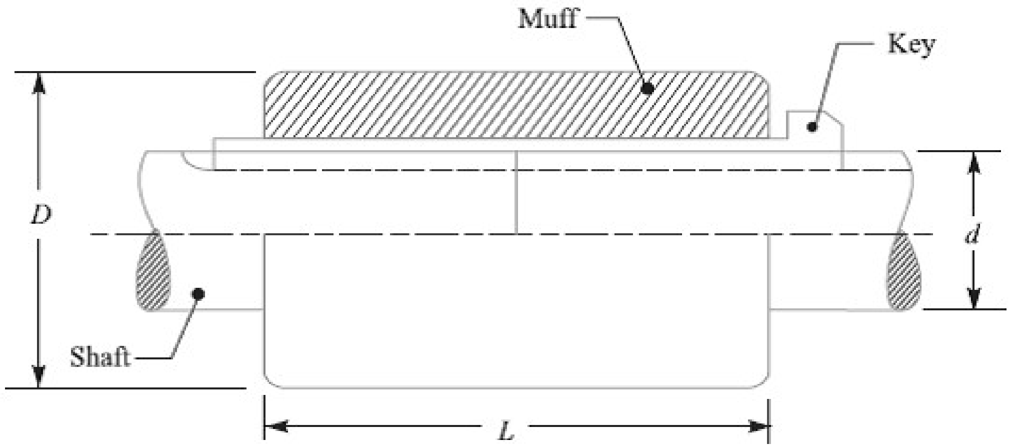
It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, D = 2d + 13 mm

And length of the slee ve, L = 3.5 d

Where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.



1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast ir on.

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \, (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. Design for key

The key for the coupling may be designed in the similar way as discussed i n Unit-5. The width and thickness of the coupling key is obtained from the proportions. The length of the coupling key is at least equal to the length of the sleeve (i.e. 3.5 d). The coupling key is usually made into two parts so th at the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\, d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torqu e transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \ldots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots \text{(Considering crushing of the key)}$$

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

Problem: Design and make a ne at dimensioned sketch of a muff coupling which is used to connect two steel shafts transmit ting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution.

Given: P = 40 kW = 40 × 10^3 W ; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm2; $\sigma_{c\,s}$ = 80 MPa = 80 N/mm^2; σ_c = 15 MPa = 15 N/mm^2.

$$T - \frac{P \times 60}{2\,\pi N} - \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} - 1100 \text{ N-m}$$
$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$
$$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm Ans.}$$

2. Design for sleeve

We know that outer diameter of the muff,

D = 2d + 13 mm = 2 × 55 + 13 = 123 say 125 mm **Ans.**

and length of the muff,

L = 3.5 d = 3.5 × 55 = 192.5 say 195 mm **Ans.**

Let us now check the induced shear stress in the muff. Let τ_c be the induced shea r stress in the muff which is made of cast ir on. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$
$$= 370 \times 103\ \tau_c$$
$$\therefore \quad \tau_c = 1100 \times 10^3/370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the desi gn of muff is safe.

3. Design for key

From Design data Book, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

Then, Thickness of key, t = w = 18 mm **Ans.**

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3 \, \tau_s$$

$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\sigma_{cs} - 1100 \times 10^3 / 24.1 \times 10^3 - 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

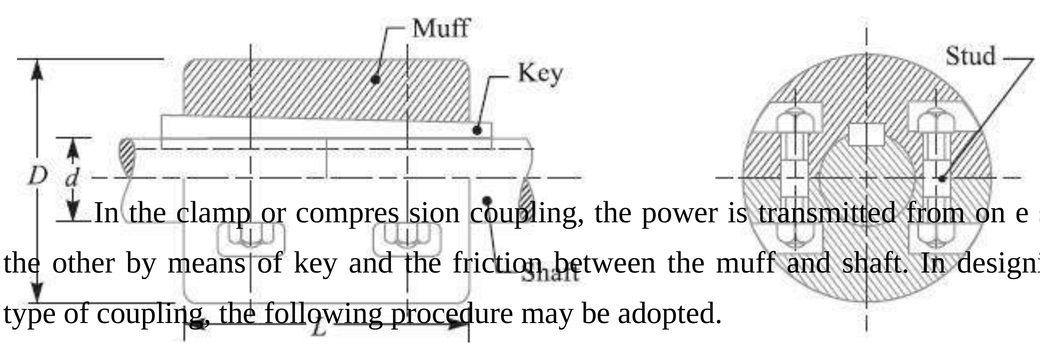
<u>Clamp or Compression Coupling or split muff coupling</u>

It is also known as **split muff coupling.** In this case, the muff or sleeve is m ade into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of th e muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are:

Diameter of the muff or s leeve, D = 2d + 13 mm

Length of the muff or sleeve, L = 3.5 d

Where d = Diameter of the shaft.



In the clamp or compres sion coupling, the power is transmitted from on e shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.

1. Design of muff and key

The muff and key are designed i n the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmitted by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of frictio n between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4}(d_b)^2 \sigma_t$$

Then, Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

Then, Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2}\pi d \times L$$

$$= \mu \times \frac{\pi}{4}(d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n$$

And the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8}(d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu\,(d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flanges has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adapted to heavy loads and hence it is used on large shafting. The flange couplings are of the following three types:

1. **Unprotected type flange co upling**. In an unprotected type flange coupling, as shown in Fig.1, each shaft is keyed to the boss of a flange with a counter sunk key and t he flanges are coupled together by means of bo lts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order t o divide the weakening effect caused by key ways.

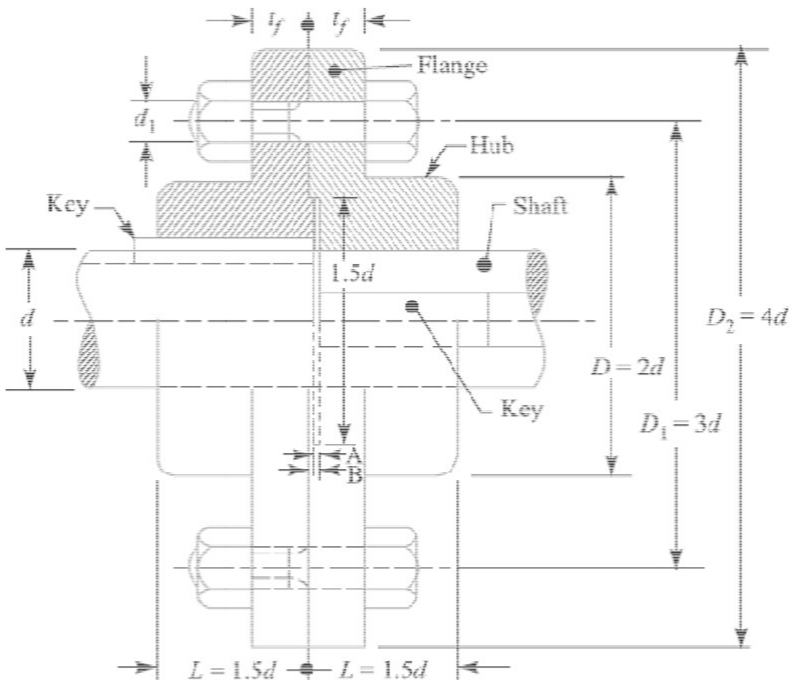


Fig.1 Unprotected Type Flange Coupling.

The usual proportions for an unprotected type cast iron flange couplings, as sho wn in Fig.1, are as follows:

If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$D = 2\,d$

Length of hub, $L = 1.5\,d$

Pitch circle diameter of bolts, $D_1 = 3d$

Outside diameter of flange,

$D_2 = D_1 + (D_1 – D) = 2\,D_1 – D = 4\,d$

Thickness of flange, t_f = 0.5 d

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

2. **Protected type flange coupling.** In a protected type flange coupling, as shown in Fig.2, the protruding bolts and nuts are protected by flanges on the two halves of th e coupling, in order to avoid danger to the workman. The thickness of the protective circumfe rential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for un protected type flange coupling.

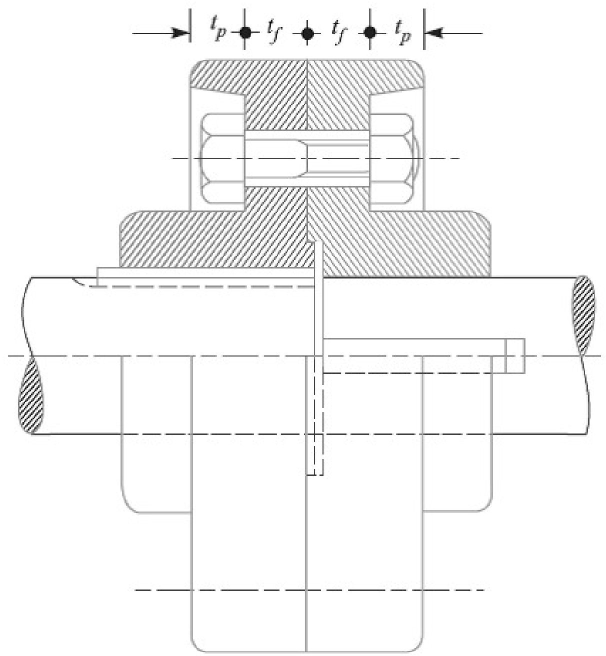
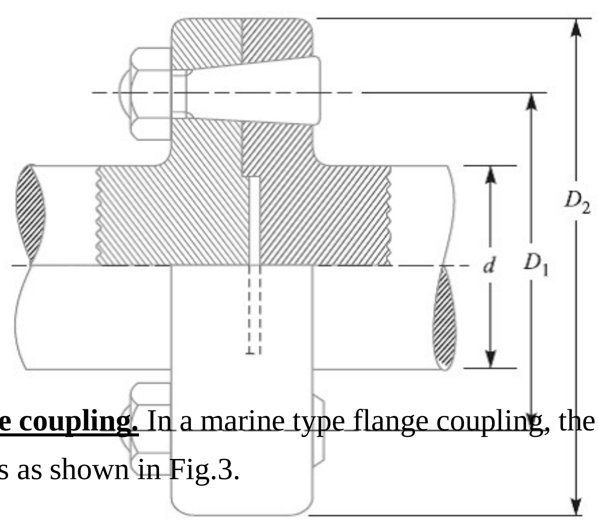


Fig.2 . Protected Type Flange Coupling.



3. **Marine type flange coupling.** In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig.3.

Fig.3. Solid Flange Coupling or Marine Type flange coupling.

The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft. The other proportions for the marine type flange coupling are taken as follows:

Thickness of flange = d / 3

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, $D_1 = 1.6$ d

Outside diameter of flange, $D_2 = 2.2$ d

Design of Flange Coupling

Consider a flange coupling as shown in Fig.1 and Fig.2.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

D_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively τ_c = Allowable shear stress for the flange material i.e. cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below:

1. Design for hub

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. Design for key

The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. Design for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the torque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. Design for bolts

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

Load on each bolt

$$= \frac{\pi}{4} (d_1)^2 \tau_b$$

Then, Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \tau_b \times n$$

And torque transmitted,

$$T = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts = $n \times d_1 \times$

t_f And crushing strength of all the bolts = $(n \times d_1 \times t_f)\ \sigma_{cb}$

Torque,

$$T = (n \times d_1 \times t_f \times \sigma_{cb}) \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

Problem: Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a co mpressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :

Shear stress for shaft, bolt and k ey material = 40

MPa Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupli ng.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm ^{2}. The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diam eter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress fo r the hub material (i.e. cast iron) is less than th e permissible value of 8 MPa, therefore the de sign of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 - l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} - 11\ 025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, t_f = 0.5 d = 0.5 × 35 = 17.5 mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

D_1 = 3d = 3 × 35 = 105 mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$(d_1)^2$ = 215 × 103/4950 = 43.43 or d_1 = 6.6

mm Assuming coarse threads, the nearest standard size of bolt is M

8. Ans. Other proportions of the flange are taken as follows:

Outer diameter of the flange,

D_2 = 4 d = 4 × 35 = 140 mm Ans.

Thickness of the protective circumferential flange,

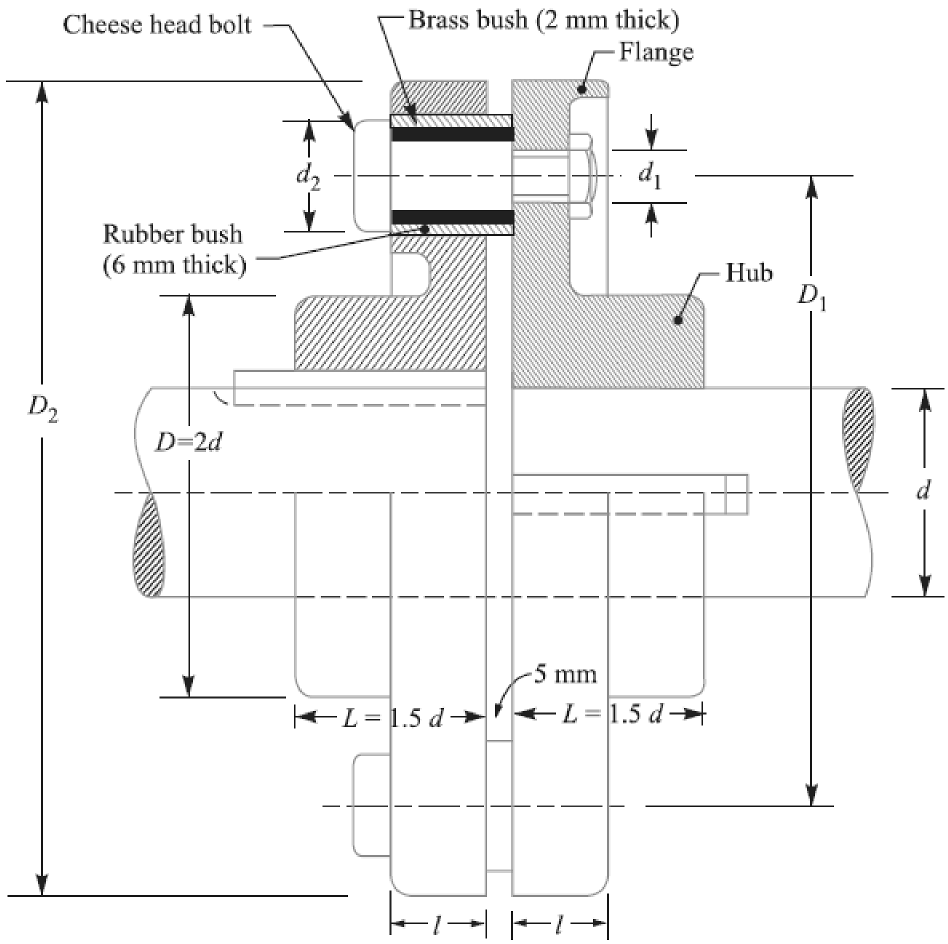
t_p = 0.25 d = 0.25 × 35 = 8.75 say 10 mm Ans.

Flexible Coupling:

We have already discussed that a flexible coupling is used to join the abutting e nds of shafts. when they are not in exact alig nment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a compa ratively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalig nemnt of the shaft without undue absorption o f the power which the shaft are transmitting.

Bushed-pin Flexible Coupling

A bushed-pin flexible co upling, as shown in Fig., is a modification of th e rigid type of flange coupling. The coupling bolts are known as pins.



The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid c onnection between them and the drive takes plac e through the medium of the compressible rub ber or leather bushes.

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2. In order to keep the low bearing pressure, the pitch circle diamete r and the pin size is increased. Let l = Length of bush in the flange,

D_2 = Diameter of bush,

P_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing lo ad acting on each pin,

$W = p_b \times d_2 \times l$

Then, Total bearing load on the bush or pins

$= W \times n = p_b \times d_2 \times l \times n$

And the torque transmitted by the coupling,

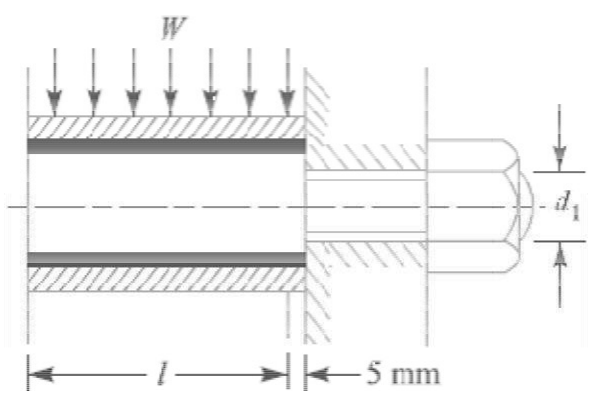
$$T = W \times n \left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n \left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit i n the coupling hole to avoid bending stresses.

The threaded length of the pin should be as small as possible so that the direc t shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$



Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enla rged portion will exert a bending action on the pin as shown in Fig. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distributi on of the load W along the bush, the maxi mum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5\text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations: Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

Problem:

Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows:

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm2.

(d) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Solution. Given: P = 32 kW = 32×10^3 W; N = 960 r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau_s = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cs} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 15 MPa = 15 N/mm^2 ; p_b = 0.8 N/mm^2.

1. Design for pins and rubber bush

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$$d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

$$d_1 = \frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. Ans.

The length of the pin of least diameter i.e. d_1 = 20 mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

So, Overall diameter of rubber bush,

$d_2 = 24 + 2 \times 2 + 2 \times 6 = 40$ mm Ans.

and diameter of the pitch circle of the pins,

$D_1 = 2\,d + d_2 + 2 \times 6 = 2 \times 40 + 40 + 12 = 132$ mm Ans.

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\,l$ N

And the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} - 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

l = 382 × 103/12 672 = 30.1 say 32 mm

And W = 32 l = 32 × 32 = 1024 N

So, Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\ 504 \text{ N-mm}$$

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

$$\sigma = \frac{M}{Z} = \frac{21\ 504}{785.5} = 27.4 \text{ N/mm}^2$$

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\ (3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

And maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\ (3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

$$D = 2\ d = 2 \times 40 = 80 \text{ mm}$$

And length of hub, $L = 1.5\ d = 1.5 \times 40 = 60$ mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3\ \tau_c$$

$\tau_c = 382 \times 103 / 94.26 \times 103 = 4.05 \text{ N/mm}^2 = 4.05$ MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\ \tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, w = 14 mm Ans.

and thickness of key,t = w = 14 mm Ans.

The length of key (L) is taken equal to the length of hub, i.e.

$L = 1.5\ d = 1.5 \times 40 = 60$ mm

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\ 800\ \tau_k$$

$$\tau_k = 382 \times 10^3 / 16\ 800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\ \sigma_{ck}$$

σ_{ck} = 382 × 103/8400 = 45.48 N/mm2 = 45.48 MPa

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

$t_f = 0.5\ d = 0.5 \times 40 = 20$ mm

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3\ \tau_c$$

τ_c =382 × 103 / 201 × 103 = 1.9 N/mm2 = 1.9 MPa

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

Problem:

Design a cast iron protective ty pe flange coupling to transmit 15 kW at 900 r .p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. T he following permissible stresses may be used:

Shear stress for shaft, bolt and key material = 40

MPa Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupli ng.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/m m^2.

The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diam eter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\ 147\ \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress fo r the hub material (i.e. cast iron) is less than th e permissible value of 8 MPa, therefore the de sign of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, $t_f = 0.5\ d = 0.5 \times 35 = 17.5$ mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$(d_1)^2$ = 215 × 103/4950 = 43.43 or d_1 = 6.6 mm

Assuming coarse threads, the nearest standard size of bolt is M 8. Ans.

Other proportions of the flange are taken as follows:

Outer diameter of the flange,

$D_2 = 4\ d = 4 \times 35 = 140$ mm Ans.

Thickness of the protective circumferential flange,

$t_p = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm Ans.

Problem:

Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 125 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 r.p.m. For the safe stresses mentioned below, calculate 1. Diameter of bolts; 2. Thickness of flanges; 3. Key dimensions ; 4. Hub length; and 5. Power transmitted. Safe shear stress for shaft material = 63 MPa Safe stress for bolt material = 56 MPa Safe stress for cast iron coupling = 10 MPa Safe stress for key material = 46 MPa

Solution. Given: d = 35 mm; n = 6; D1 = 125 mm; T = 800 N-m = 800 × 103 N-mm; N = 350 r.p.m.; τs = 63 MPa = 63 N/mm^2; τ_b = 56 MPa = 56 N/mm^2 ; τ_c = 10 MPa = 10 N/mm^2 ; τ_k = 46 MPa = 46 N/mm^2.

1. Diameter of bolts

Let d_1 = Nominal or outside diameter of bolt. We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \, 56 \times 6 \times \frac{125}{2} = 16\,495\,(d_1)^2$$

$(d_1)^2 = 800 \times 10^3 / 16\,495 = 48.5$ or $d_1 = 6.96$ say 8 mm **Ans.**

2. Thickness of flanges

Let t_f = Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (2 \times 35)^2}{2} \times 10 \times t_f = 76\,980\, t_f \qquad \ldots (\because D = 2d)$$

$t_f = 800 \times 103 / 76\,980 = 10.4$ say 12 mm **Ans.**

3. Key dimensions

From Table 13.1, we find that the proportions of key for a 35 mm diameter shaft are:

Width of key, w = 12 mm **Ans.**

And thickness of key, t = 8 mm**Ans.**

The length of key (l) is taken equal to the length of hub (L).

$l = L = 1.5\, d = 1.5 \times 35 = 52.5$ mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} \times = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\, \tau_k$$

$\tau_k = 800 \times 103 / 11\,025 = 72.5$ N/mm2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length o f key by substituting the value of τk = 46 MPa in the above equation, i.e.

$$800 \times 10^3 = l \times 12 \times 46 \times \frac{35}{2} = 9660\, l$$

$l = 800 \times 103 / 9660 = 82.8$ say 85 mm **Ans.**

4. Hub length

Since the length of key is taken e qual to the length of hub, therefore we shall take hub length,

L = l = 85 mm **Ans.**

5. Power transmitted

We know that the power transmitted,

$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60} = 29\,325 \text{ W} = 29.325 \text{ kW} \textbf{ Ans.}$$

Problem:

The shaft and the flange of a ma rine engine are to be designed for flange coup ling, in which the flange is forged on the end of the shaft. The following particulars are to considered in the design:

Power of the engine = 3 MW

Speed of the engine = 100 r.p.m.

Permissible shear stress in bolts and shaft = 60

MPa Number of bolts used = 8

Pitch circle diameter of bolts = 1.6 × Diameter of shaft

Find: 1. diameter of shaft; 2. diameter of bolts; 3. thickness of flange; and 4 . diameter of flange.

Solution. Given: P = 3 MW = 3 × 106 W; N = 100 r.p.m.; τb = τs = 60 MPa = 60 N/mm^2; n = 8; D1 = 1.6 d

1. Diameter of shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$d_3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$$

or $d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. Diameter of bolts

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\, 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$= 90\,490\,(d_1)^2$ ... (Since $D_1 = 1.6$ d)

So, $(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160$ or $d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

3. Thickness of flange

The thickness of flange (t_f) is taken as d / 3.

So, tf = d / 3 = 300/3 = 100 mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6 \tau_s$$

$$\tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2 \text{ N/mm}^2 = 20.2 \text{ MPa}$$

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (tf = 100 mm) is safe.

4. Diameter of flange

The diameter of flange (D_2) is taken as 2.2 d.

So, D_2 = 2.2 d = 2.2 × 300 = 660 mm **Ans.**

References:

1. Machine Design - V.Bandari
2. Machine Design – R.S. Khurmi
3. Design Daa hand Book - S MD Jalaludin.

Introduction

A spring is defined as an elast ic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

1. To cushion, absorb or control energy due to either shock or vibration as i n car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

2. To apply forces, as in brakes, clutches and spring loaded valves.

3. To control motion by maintain ing contact between two elements as in cams a nd followers.

4. To measure forces, as in spring balances and engine indicators.

5. To store energy, as in watches, toys, etc.

Types of springs:

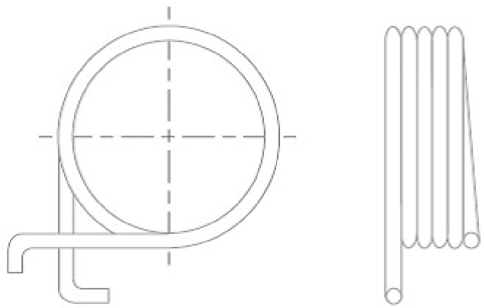
1. **Helical springs.** The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.

(*a*) Compression helical spring. (*b*) Tension helical spring.

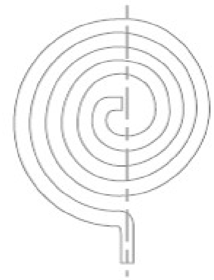
2. **Conical and volute springs.** The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired

(*a*) Conical spring. (*b*) Volute spring.

3. **Torsion springs.** These spri ngs may be of **helical** or **spiral** type as show n in Fig. The **helical type** may be used only i n applications where the load tends to wind up the spring and are used in various electrical me chanisms.

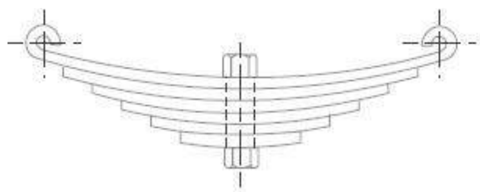


(*a*) Helical torsion spring.

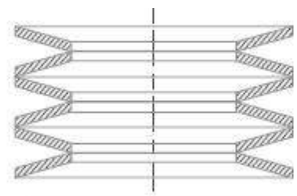


(*b*) Spiral torsion spring.

4. **Laminated or leaf springs.** The laminated or leaf spring (also known as **flat spring** or **carriage spring**) consists of a n umber of flat plates (known as leaves) of varyin g lengths held together by means of clamps and bolts.



Laminated or leaf springs.



Disc or bellevile springs.

5. **Disc or bellevile springs.** Th ese springs consist of a number of conical discs held together against slipping by a central bolt or tube.

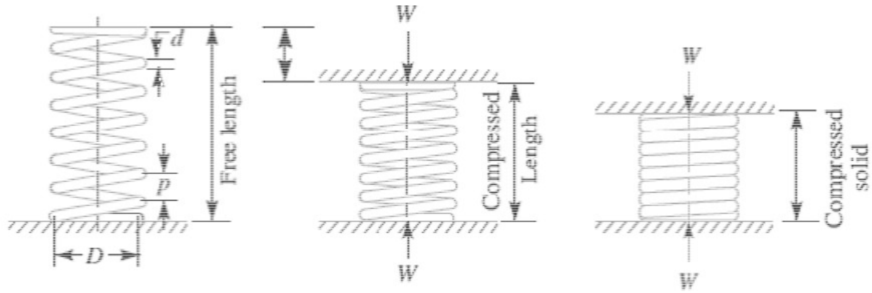
6. **Special purpose springs.** These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or li quid) can behave as a compression spring. These springs are used for special types of application only.

<u>Terms used in Compression Sp rings</u>

1. Solid length. When the comp ression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.**

Solid length of the spring, L_s = n'.d where n' = Total number of coils, and d = Diameter of the wire.

2. Free length. The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition.



Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

3. Spring index. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Spring index, C = D / d where D = Mean diameter of the coil, and d = Diameter of the wire.

4. Spring rate. The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically, Spring rate, k = W / δ where W = Load, and δ = Deflection of the spring.

5. Pitch. The pitch of the coil is defined as the axial distance between adj acent coils in uncompressed state. Mathematically, Pitch of the coil,

$$p = \frac{\text{Free Length}}{n' - 1}$$

Stresses in Helical Springs of C ircular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load *W*, as shown in Fig.(*a*).

Let *D* = Mean diameter of th e spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity f or the spring material,

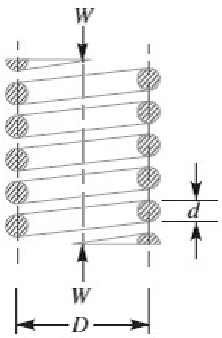
W = Axial load on the sp ring,

τ = Maximum shear stress induced in the wire,

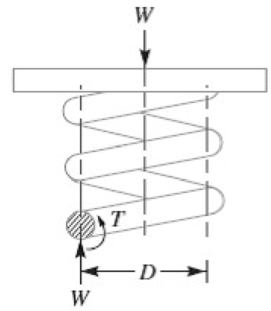
C = Spring index = *D*/*d*,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load *W*.



a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. (*b*). The load *W* tends to rotate the wire due to the twistin g moment (*T*) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig.(*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8W.D}{\pi\, d^3} \qquad \ldots(i)$$

The torsional shear stress diagra m is shown in Fig. (*a*).

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire:

1. Direct shear stress due to the load *W*, and

2. Stress due to curvature of wire .

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

Maximum shear stress induced i n the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

$\therefore$ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad ...(iv)$$

where $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

<u>Deflection of Helical Springs o f Circular Wire</u>

Total active length of the wire,

l = Length of one coil × No. of active coils = $\pi\,D \times n$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\,D.n}{\frac{\pi}{32} \times d^4\,G} = \frac{16W.D^2.n}{G.d^4} \qquad ...(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \qquad ... (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

<u>Buckling of Compression Spri ngs</u>

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch dia meter (D), then the spring behaves like a column and may fail by buckling at a comparatively l ow load.

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

Surge in springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

Problem: A helical spring is ma e from a wire of 6 mm diameter and has outsid e diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm^2, find the axial load which the spring can carry and the deflection per active turn.

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm^2 ; $G = 84$ kN/mm^2 $= 84 \times 10^3$ N/mm^2

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $C = \frac{D}{d} = \frac{69}{6} = 11.5$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$\therefore$ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta - \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$\therefore$ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm Ans.}$$

Problem: Design a spring for a balance to measure 0 to 1000 N over a scale of le ngth 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approxiimate number of turns is 30. The modulus of rigidity is 85 kN/mm^2. Also calculate the maximu m shear stress induced.

Solution:

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm } \textbf{Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

∴ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa } \textbf{Ans.}$$

Problem: Design a helical compr ession spring for a maximum load of 1000 N fo r a deflection of 25 mm using the value of sp ring index as 5. The maximum permissible sh ear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm^2.

Take Wahl's factor, $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\,d^2} = \frac{16\,677}{d^2}$$

$\therefore$ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3\,n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \text{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm Ans.}$$

Problem: Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial de flection of the spring for the load range is 6 m m. Assume a spring index of 5. The permissib le shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm^2. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm^2 ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of W_2 = 2750 N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\,d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\,d^3$$

$$\therefore \quad d^2 = 6875 / 82.48 = 83.35 \quad \text{or} \quad d = 9.13 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for W = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$$\therefore \quad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$
$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

Where V = Volume of the spring wire

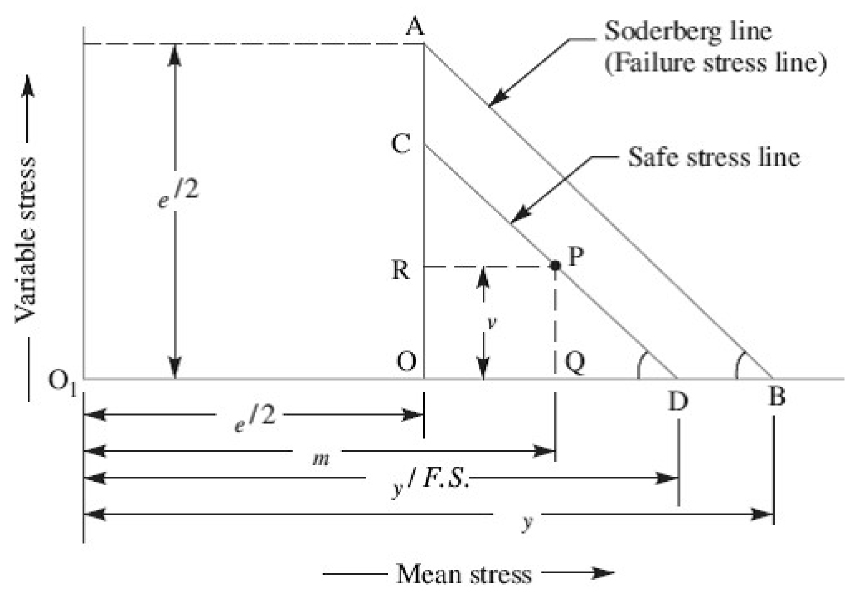
= Length of spring wire × Cross-sectional area of spring wire

Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to τe / 2 and the variable shear stress is also equal to τe / 2. A line drawn from A to B (the yield point in shear, τy) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength (τy), a safe stress line CD may be drawn
parallel to the line AB, as show n in Fig. Consider a design point P on the line CD.

Now the value of factor of safety may be obtained as discussed below:



From similar triangles PQD and AOB, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\frac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e / 2}{\tau_y - \frac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v \cdot \tau_y - \tau_v \cdot \tau_e = \frac{\tau_e \cdot \tau_y}{F.S.} - \tau_m \cdot \tau_e$$

$$\therefore \quad \frac{\tau_e \cdot \tau_y}{F.S.} = 2\,\tau_v \cdot \tau_y - \tau_v \cdot \tau_e + \tau_m \cdot \tau_e$$

Dividing both sides by $\tau_e \cdot \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

Springs in Series

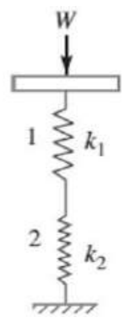
Total deflection of the springs,

$$\delta - \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

k = Combined stiffness of the springs.



Springs in Parallel

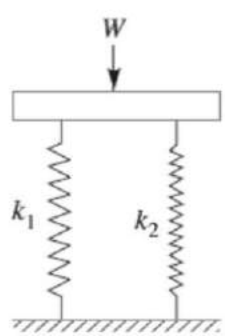
$$W = W_1 + W_2$$

$$\delta.k = \delta.k_1 + \delta.k_2$$

$$k = k_1 + k_2$$

k = Combined stiffness of the springs, and

δ = Deflection produced.



Surge in Springs or finding na tural frequency of a helical spring:

When one end of a heli cal spring is resting on a rigid support and the other end is loaded suddenly, then all the co ils of the spring will not suddenly deflect equ ally, because some time is required for the propagation of stress along the spring w ire. A little consideration will show that in t he beginning, the end coils of the spring in contact with the applied load takes up whole o f the deflection and then it transmits a larg e part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge*.**

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W . \delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\, W . D}{\pi\, d^3} \text{ or } W = \frac{\pi\, d^3 . \tau}{8\, K . D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\, d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

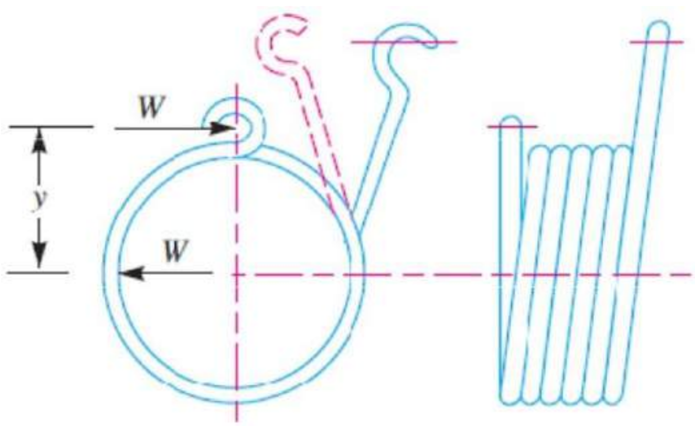
$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

Where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

Helical Torsion Springs

The helical torsion springs as shown in Fig., may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is



$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

Where K = Wahl's stress factor = $\frac{4C^2\ C\ 1}{4C^{2}\ 4C}$

C = Spring index,

M = Bending moment = W × y,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

And

Total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\, d^4 / 64} = \frac{64\ M.D.n}{E.d^4}$$

Where l = Length of the wire = π.D.n,

D = Diameter of the spring, and

n = Number of turns.

And deflection,

$$\delta = \theta \times y = \frac{64\ M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\ M}{t.b^2} = K \times \frac{6\ W \times y}{t.b^2}$$

Where

$$K = \frac{3C^2 \quad C \quad 0.8}{3C^2 - 3C}$$

Angular deflection,

$$\theta = \frac{12\ \pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12\ \pi M.D.n}{E.t.b^3} \times y$$

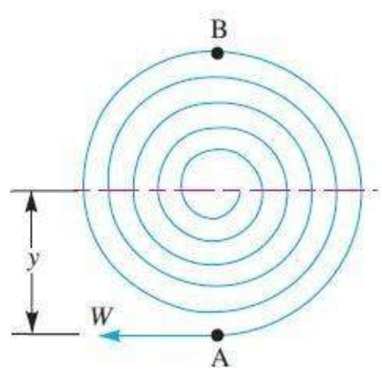
In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have

$$\sigma_b = K \times \frac{6\ M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\ \pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\ \pi M.D.n}{E.b^4} \times y$$

<u>Flat Spiral Spring</u>

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. These springs are frequently used in watches and gramophones etc. When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be



pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending. Let W = Force applied at the outer end A of the spring,

y = Distance of centre of gravity of the spring from

A, l = Length of strip forming the spring,

b = Width of strip,

t = Thickness of strip,

I = Moment of inertia of the spring section = $b.t^3/12$,

and Z = Section modulus of the spring section = $b.t^2/6$

When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at B which is at a maximum distance from the application of W.

Bending moment at B,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2 / 6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3}$$

And the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t}$$

The strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\,W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\,W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\,W^2y^2}{Eb^2t^4} \times \frac{btl}{24}$$

... (Multiplying the numerator and denominator by 24 bt)

$$= \frac{(\sigma_b)^2}{24\,E} \times btl = \frac{(\sigma_b)^2}{24\,E} \times \text{Volume of the spring}$$

Problem: A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stres s induced and the angular deflection of the spring in degrees. The spring index is 10 an d modulus of elasticity for the spring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$; $E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2 ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa } \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \textbf{ Ans.}$$

Problem: A spiral spring is mad e of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assumin g the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns t o wind up the spring and the strain energy stor ed in the spring. Take E = 200 kN/mm^2.

Bending moment in the spring

Let M = Bending moment in the spring.

We know that the maximum bending stress in the spring material (σ_b),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{8\,(0.25)^2} = 32\,M$$

$\therefore$ $M = 800 / 32 = 25$ N-mm **Ans.**

Number of turns to wind up the spring

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to 2π radians, therefore number of turns to wind up the spring

$$= 40 / 2\pi = 6.36 \text{ turns } \textbf{Ans.}$$

Strain energy stored in the spring

We know that strain energy stored in the spring

$$= \frac{1}{2}\,M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm } \textbf{Ans.}$$

Concentric or Composite Spri ngs or coaxial springs or nested springs

A concentric or composite sprin g is used for one of the following purposes:

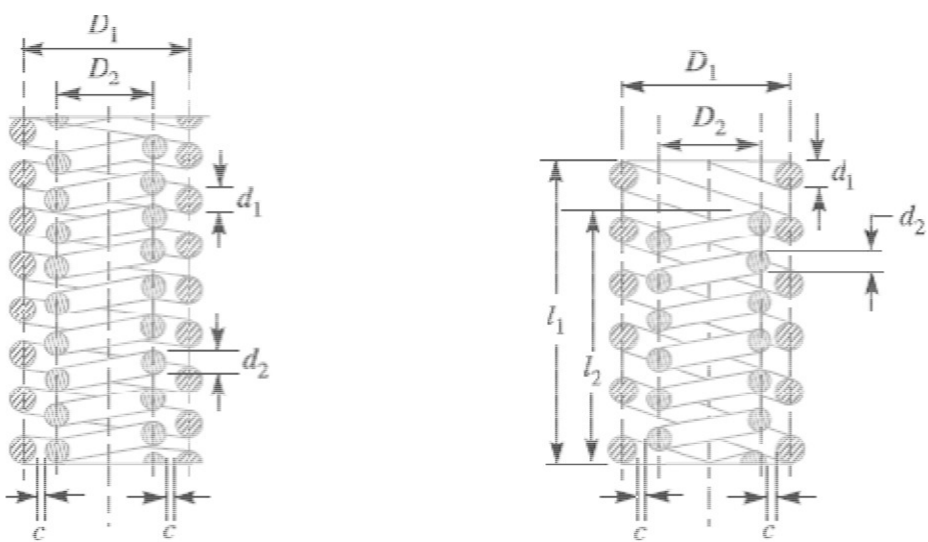
1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more spri ngs and have the same free lengths as shown i n Fig. (*a*) And are compressed equally.

Such springs are used in autom obile clutches; valve springs in aircraft, heavy duty diesel engines and rail-road car susp ension systems. Sometimes concentric spring s are used to obtain a spring force which d oes not increase in a direct relation to the d eflection but increases faster. Such springs are made of different lengths as shown in Fig. (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variab le centrifugal force. The adjacent coils of t he concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same s tress. In order to get the same stress factor (*K*), it is desirable to have the same spring index (*C*).



(*a*) (*b*)

Consider a concentric spring as s hown in Fig. (*a*).

Let W = Axial load,

W_1 = Load shared by outer sprin g,

W_2 = Load shared by inner sprin g,

d_1= Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of in ner spring.

Assuming that both the springs are made of same material, then the maximu m shear stress induced in both the springs is ap proximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3}$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

$$\frac{8 W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8 W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad ...(ii)$$

When both the springs are comp ressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$n_1.d_1 = n_2.d_2$

The equation ***(ii)*** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation ***(iii)*** by e quation ***(i)***, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same. From equations ***(i)*** and ***(iv)***, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find tha the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance bet ween the two springs is taken as

$$\frac{d_1 - d_2}{2}$$

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or $$\frac{D_1 - D_2}{2} = d_1 \qquad \text{.........(vi)}$$

From equation **(*iv*)**, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation **(*vi*)**, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2}$$

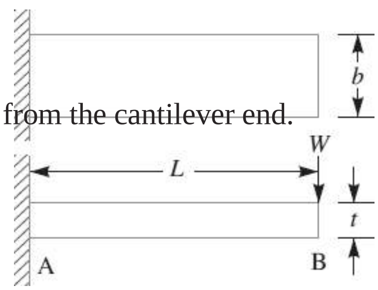
<u>Leaf Springs</u>

Leaf springs (also known as **fla t springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a d efinite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in additi on to shocks. Consider a single plate fixed at one end and loaded at the other end as show n in Fig. This plate may be used as a flat spring .

Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or d istance of the load W from the cantilever end.



We know that the maximum bending moment at the cantilever end A,

$$M = W.L$$

And section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\ W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load a t the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\ W.L^3}{E.b.t^3}$$

$$= \frac{2\ \sigma.L^2}{3\ E.t}$$

If the spring is not of cantilever type but it is like a simply supported beam, w ith length 2*L* and load 2*W* in the centre, as sh own in Fig. then Maximum bending moment in the centre,

$M = W.L$

Section modulus, $Z = b.t2 / 6$

Bending stress,

$$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$$

$$= \frac{6\ W.L}{b.t^2}$$

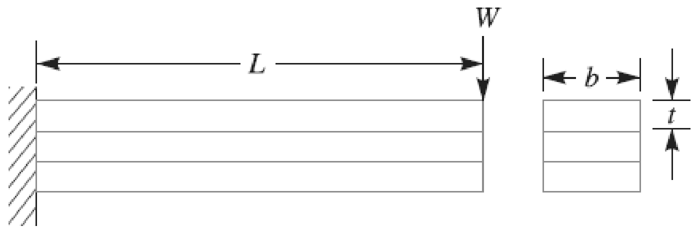
We know that maximum deflection of a simply supported beam loaded in the centre is given by

$$\delta = \frac{W_1\ (L_1)^3}{48\ E.I} = \frac{(2W)\ (2L)^3}{48\ E.I} = \frac{W.L^3}{3\ E.I}$$

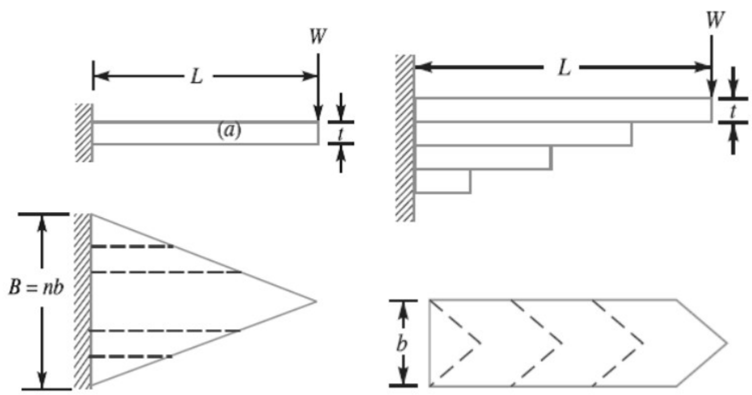
From above we see that a spring such as automobile spring (semi-elliptical spring) with length 2*L* and loaded in the centre by a load 2*W*, may be treated as a double can tilever. If the plate of cantilever is cut into a series of *n* strips of width *b* and these are place d as shown in Fig., then equations ***(i)*** and ***(ii)*** m ay be written as

$$\sigma = \frac{6\ W.L}{n.b.t^2} \qquad \ldots(iii)$$

And $$\delta = \frac{4\ W.L^3}{n.E.b.t^3} = \frac{2\ \sigma.L^2}{3\ E.t} \qquad \ldots(iv)$$



The above relations give the stress and deflection of a leaf spring of uniform cross section. The stress at such a spring is ma ximum at the support.



If a triangular plate is used as s hown in Fig., the stress will be uniform throu ghout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. to form a graduated or la minated leaf spring, then

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

where *n* = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becom es compact so that the space occupied by the spring is considerably reduced.

When bending stress alo ne is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one o r more leaves of uniform cross-section extendi ng clear to the end. We see from equations ***(iv)*** and ***(vi)*** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full le ngth leaves) is 50% greater than in the grad uated leaves, assuming that each spring eleme nt deflects according to its own elastic curve. I f the suffixes

F and G are used to indicate the full length (or uniform cross section) and gra duated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\,W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2}\times\frac{W_G}{n_G}$$

$$\frac{W_F}{W_G} = \frac{3\,n_F}{2\,n_G} \qquad ...(vii)$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\,n_F}{2\,n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\,n_F + 2\,n_G}{2\,n_G}$$

$$W_G = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)(W_F + W_G) = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)W \qquad ...(viii)$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation ***(vii)***, we may write

$$\frac{W_G}{W_F} = \frac{2\,n_G}{3\,n_F}$$

or

$$\frac{W_G}{W_F} + 1 = \frac{2\,n_G}{3\,n_F} + 1$$

$$\frac{W_G + W_F}{W_F} = \frac{2\,n_G + 3\,n_F}{3\,n_F}$$

$$\therefore \qquad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W \qquad ...(ix)$$

Bending stress for full length leaves,

$$\sigma_F - \frac{6\,W_F.L}{n_F.b\,t^2} - \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W - \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

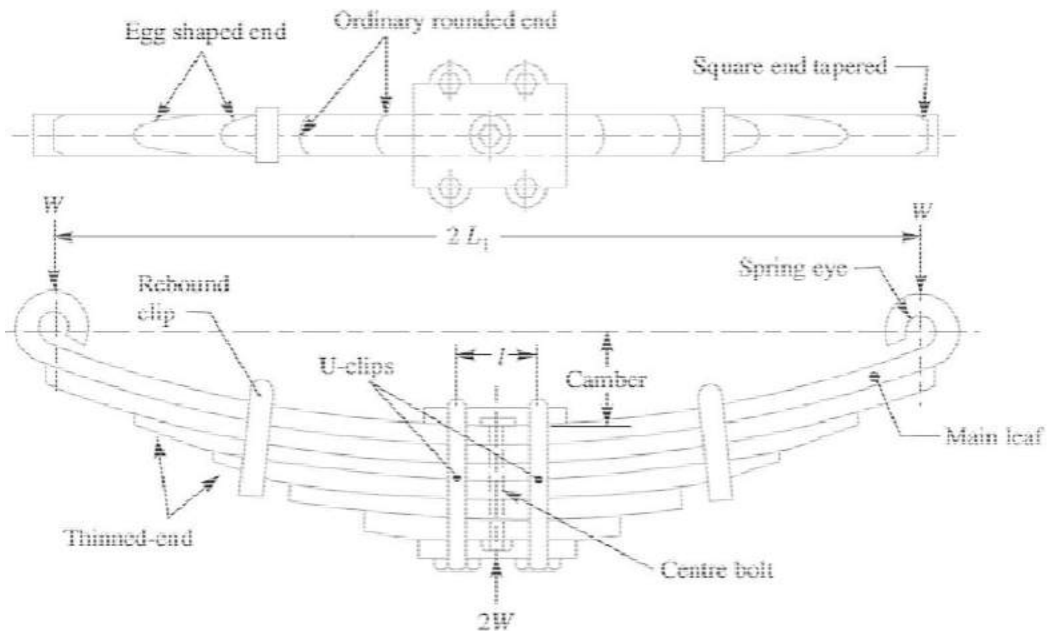
Since

$$\sigma_F - \frac{3}{2}\sigma_G, \text{ therefore}$$

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3}\times\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$



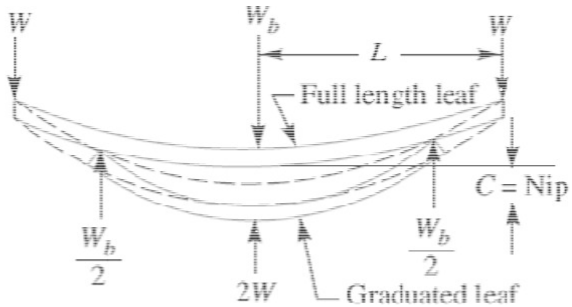
<u>Equalised Stress in Spring Leaves (Nipping)</u>

We have already discussed that the stress in the full length leaves is 50% gr eater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed .

This condition may be obtained i n the following two ways:

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small dist ance from the neutral axis to the edge of the lea f.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C* in Fig, is called ***nip.***



Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount qual to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$C = \delta_G - \delta_F = \frac{6\, W_G \,.\, L^3}{n_G\, E.b.t^3} - \frac{4\, W_F.L^3}{n_F.E.b.t^3} \qquad \ldots(i)$$

Since the stresses are equal, ther efore

$$\sigma_G = \sigma_F$$

$$\frac{6\, W_G.L}{n_G\, b.t^2} = \frac{6\, W_F\, L}{n_F\, b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} - \frac{W_F}{n_F}$$

$$\therefore \qquad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of *W*G an d *W*F in equation ***(i)***, we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad \ldots(ii)$$

The load on the clip bolts (W_b) r equired to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\, L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

Or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\, W_b}{2\, n_G} = \frac{2\, n_G.W_b + 3\, n_F.W_b}{2\, n_F.n_G} = \frac{W_b\, (2\, n_G + 3\, n_F)}{2\, n_F.n_G}$$

$$W_b = \frac{2\, n_F.n_G.W}{n\, (2\, n_G + 3\, n_F)} \qquad \ldots(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

Final stress,

$$\sigma = \frac{6\, W_F . L}{n_F . b . t^2} - \frac{6\, L}{n_F . b . t^2} \times \frac{W_b}{2} = \frac{6\, L}{n_F . b . t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\, L}{n_F . b . t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F . n_G . W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6 W . L}{b . t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\, W . L}{b . t^2}\left[\frac{3n - n_G}{n(2n_G + 3n_F)}\right]$$

$$= \frac{6\, W . L}{b . t^2}\left[\frac{3(n_F + n_G) - n_G}{n(2n_G + 3n_F)}\right] = \frac{6\, W . L}{n . b . t^2} \qquad \text{...}(iv)$$

Length of Leaf Spring Leaves

The length of the leaf spring leav es may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or dist ance between centres of *U*-bolts. It is the in effective length of the spring,

n_F = Number of full leng th leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discuss ed that the effective length of the spring,

$2L = 2L_1 - l$ ... (When band is used)

Problem: Design a leaf spring for the following specifications:

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.

Take Young's modulus, E = 200 kN/mm2 and allowable stress in sprin g material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \qquad \text{...}(i)$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \qquad \text{...}(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm } \textbf{Ans.}$$

Problem:

A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness an d width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.

Solution. Given : $n = 12$; $n_F = 2$; $2L_1 = 1.05$ m = 1050 mm ; $l = 85$ mm ; $2W = 5.4$ kN = 5400 N or $W = 2700$ N ; σ_F 280 MPa = 280 N/mm^2

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\,W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$$\therefore \quad t^3 = 225\,476 / 280 = 805.3 \quad \text{or} \quad t = 9.3 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $$b = 4\,t = 4 \times 10 = 40 \text{ mm } \textbf{Ans.}$$

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\,(2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\,(2 \times 10 + 3 \times 2)} \text{ mm}$$

$$= 16.7 \text{ mm } \textbf{Ans.}$$ ... (Taking $E = 210 \times 10^3$ N/mm^2)

References:

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2. Machine Design – R.S. Khurmi
3. Design Data hand Book - S MD Jalaludin..